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"*Knowledge of what is possible is the beginning of happiness*"
George Santayana

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Abstract

In today's wireless networks, there is an increasing demand for high service quality, data rates, and network coverage. However, when addressing these demands, noise, interference, fading, power constraints, and bandwidth limitation are some of the fundamental challenges. *Spatial diversity* is one way to deal with these challenges and is achieved by sending and receiving a signal using multiple transmit and/or multiple receive antennas. The use of multiple transmit and receive antennas in spatial diversity results in a technique called Multiple-Input Multiple-Output (MIMO). In practice, however, one shortcoming of MIMO is that installing multiple antennas per wireless node may not be feasible because of limitations in power, cost, and/or device size.

When nodes are limited in the number of antennas, distributed nodes in the network can be engaged to emulate MIMO. This technique of gaining spatial diversity is called *cooperative transmission*. Information-theoretic studies have shown substantial capacity improvements as compared to traditional point-to-point wireless networks. In recent years, network-coded cooperation was proposed as one protocol to realize cooperation in wireless networks. Most of the previous work done in this area considers error-free inter-user channels; however, this is usually not the case in wireless networks.

This thesis investigates the performance of two types of network-coded cooperation protocols under a more practical scenario of erroneous wireless channels, transmissions using orthogonal channels, and energy constraints. Specifically, we provide the analytical tools to compute the error rate bounds of these two network-coded cooperation protocols, study their outage behavior, and show that these protocols can achieve full diversity. We then investigate the coverage area by using network-coded cooperation and study the effect of network topology on outage performance. In large networks where a source has potential partners in its surrounding to choose from, metrics that provide insight on how to select a partner are required. One option would be to select a partner that minimizes the total energy spent in the network. With energy minimization in mind, we finally analyze the energy consumption of network-coded cooperation considering transmission, reception, and processing energy at all cooperating nodes.

Zusammenfassung

In heutigen drahtlosen Netzen wächst die Anforderung an guter Servicequalität, hohen Datenraten und umfassender Netzabdeckung ständig. Gleichzeitig gibt es jedoch Bandbreiten- und Sendeleistungsbeschränkungen sowie Interferenz und Fading auf den drahtlosen Kanälen, die das Erreichen dieser Anforderungen erschweren.

Kooperative Übertragung ist ein neues Paradigma in der drahtlosen Kommunikation um Kanal-Fading zu handhaben. Bei der kooperativen Übertragung werden verteilte Knoten in einem Netzwerk gruppiert und emulieren so Antennendiversität. Informationstheoretische Studien haben gezeigt, dass sich die Kapazität im Vergleich zu herkömmlicher drahtloser Punkt-zu-Punkt-Übertragung verbessern lässt. In den vergangenen Jahren wurde "network-coded" Kooperation vorgeschlagen und untersucht; ein Verfahren, bei dem Network Coding in der kooperativen Übertragung eingesetzt wird. In der Vergangenheit werden in der Literatur größtenteils fehlerfreie Kanäle zwischen den Nutzern angenommen. Dies ist jedoch üblicherweise in drahtlosen Netzen nicht der Fall.

Diese Doktorarbeit untersucht die Performanceleistung zweier Typen von network-coded Kooperationsverfahren in einem realitätsnahen Szenario mit Energiebeschränkung bei fehlerbehafteter drahtloser Übertragung über orthogonale Kanäle. Konkret entwickeln wir ein Framework, um die Outage Wahrscheinlichkeit der zwei network-coded Kooperationsprotokolle zu berechnen, ihr Outage Verhalten zu untersuchen und um zu zeigen, dass diese Protokolle Diversität ausnutzen können. Wir untersuchen, wie sich aufgrund von network-coded Kooperation die abgedeckte Fläche erweitert und betrachten den Effekt der Netzwerktopologie auf die Outage Performance. Abschließend analysieren wir den Energieverbrauch eines der network-coded Kooperationsverfahren unter Berücksichtigung des individuellen Energieverbrauchs der Sende-, Empfang- und Verarbeitungsoperationen an allen kooperierenden Knoten.

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List of Abbreviations

LNA Low Noise Amplifier

ACK Acknowledgment

NACK Negative Acknowledgment

BER Bit Error Rate

BPSK Binary Phase-Shift Keying

CRC Cyclic Redundancy Check

LLR Log-likelihood Ratio

CSI Channel-State Information

FEC Forward Error Correction

LNC Linear Network Coding

MAC Medium-Access Control Layer

OSI Open Systems Interconnection

PER Packet Error Rate

PHY Physical Layer

SNR Signal-to-Noise Ratio

TDMA Time-Division Multiple Access

WSN Wireless Sensor Network

BS Base Station

MS Mobile Station

PPT Point-to-Point Transmission

ASK Amplitude Shift Keying

PSK Phase Shift Keying

8-PSK 8 Phase Shift Keying

FSK Frequency Shift Keying

ARQ Automatic Repeat Request

ADC Analog-to-Digital Converter

AWGN Additive White Gaussian Noise

CDMA Code-Division-Multiple-Access

DAC Digital-to-Analog Converter

DSP Digital Signal Processor

EGC Equal-Gain Combining

CC Code Combining

FDMA Frequency Division Multiple Access

ISI Inter-symbol Interference

MIMO Multiple-Input Multiple-Output

SISO Single-Input Single-Output

MRC Maximum-Ratio Combining

SC Selection Combining

QoS Quality of Service

QPSK Quadrature Phase Shift Keying

TCP Transmission Control Protocol

1. Introduction

In today's wireless networks, there is an increasing demand for service quality, high data rates, network coverage, and lesser processing time. The scarcity of two fundamental resources for communications, namely, energy and bandwidth, is a serious challenge to fulfill these demands [1]. Moreover, wireless channels feature fading, shadowing, interference, and other impairments that make the channel unpredictable.

Signal *fading* is the most severe among these impairments. In a wireless channel, random scattering from reflectors with different attenuation coefficients results in multiple copies of a transmitted signal arriving (and interfering) at a receiver with different gains, phase shifts, and delays. These multiple signal replicas can add together in a constructive or destructive way, amplifying or attenuating the received signal's amplitude. Destructive interference results in fading which causes temporary failure of communication as the amplitude of the received signal may be low to the extent that the receiver may not be able to distinguish it from thermal noise [2, 3].

Diversity is one technique to combat fading in a wireless channel. Diversity can be defined as the technique by which multiple copies of a signal are delivered to the receiver via independently fading channels. If one or more copies are highly degraded due to severe fading, then the receiver can still receive the signal from the other received copies. *Diversity gain* is used to quantify the gain from diversity. It is related to the number of independent channels over which the signal is being received. Independent channels can be generated in three physical domains: time, frequency, and space. Time diversity is achieved by transmitting the same signal via different time slots; these slots should be well separated to ensure that the channels at these slots are uncorrelated. Drawbacks of time diversity include loss in the system signal rate and an increase in transmission delay. Frequency diversity is achieved by transmitting the same data on different frequency bands. This diversity is inefficient in bandwidth utilization. Spatial diversity, on the other hand, is achieved by sending and receiving a signal using multiple transmit and/or multiple receive antennas that are physically separated from one another. The received copies of the signal are likely to be uncorrelated as they propagate along spatially separated paths. The use of spatial diversity has gained lots of interest in recent years as it does not increase the bandwidth or the transmission delay [1].

The use of multiple transmit and receive antennas in spatial diversity results in a Multiple-Input Multiple-Output (MIMO) system. In MIMO, the transmitter has the capability to transmit different signals from each antenna and the receiver can observe different signals from each antenna as an input. The signal present at each receive antenna is the combination of signals from the transmit antennas, after each has traveled through possibly different fading channels. Statistically independent channels between any pair of transmit and receive antennas are obtained by placing the antenna pairs a few wavelengths apart from each other.

Depending on whether multiple antennas are used for transmission or reception, the diversity from MIMO can further be classified as *transmit antenna diversity* or *receive antenna diversity*, or both. In transmit antenna diversity, multiple transmit antennas are deployed at the transmitter to send multiple copies of a signal. If we take a mobile communication system as an example, transmit antenna diversity is feasible in a downlink transmission (i.e., from Base Station (BS) to Mobile Station (MS)) because multiple antennas can easily be deployed at the BS. On the other hand, in receive antenna diversity, multiple receive antennas are deployed at the receiver to receive multiple copies of the transmitted signal. This diversity can be used to improve the uplink (i.e., MS to BS) transmission of the mobile communication system.

The gains of MIMO in terms of increasing channel capacity, higher throughput, improved error performance, and better energy efficiency are well established by now. In practice, however, one limitation of MIMO is that installing multiple antennas on wireless nodes (e.g., MS in a mobile communication system) may not be feasible because of limitations in power, cost, and/or size. To achieve full diversity gain in MIMO, there must be sufficient separation between the antenna elements at the transmitter and receiver sides. If not sufficiently separated, the fades of the channels between different antenna pairs will be correlated, thereby reducing the diversity gain. In the mobile communication system example above, transmit diversity in the uplink transmission may not be feasible, i.e., the uplink has some bottleneck in current mobile communication systems.

To overcome the above mentioned drawback of MIMO, distributed wireless nodes (active terminals or fixed relays) can be engaged in a cooperative fashion to emulate antenna diversity. This mode of gaining transmit diversity is called *cooperative transmission* (also called *virtual antenna arrays* [4], *cooperative diversity* [5], or *user cooperation* [6, 7]). In cooperative transmission, nodes can share their time, frequency, and/or other resources to form a *distributed or virtual MIMO*.

Figure 1.1 shows cooperation in a mobile communication deployment. Nodes in one shaded region are partnered to cooperatively transmit each other's message to the BS, i.e., the MSs form (virtual) transmit antenna diversity for the uplink transmission. The lower part of the

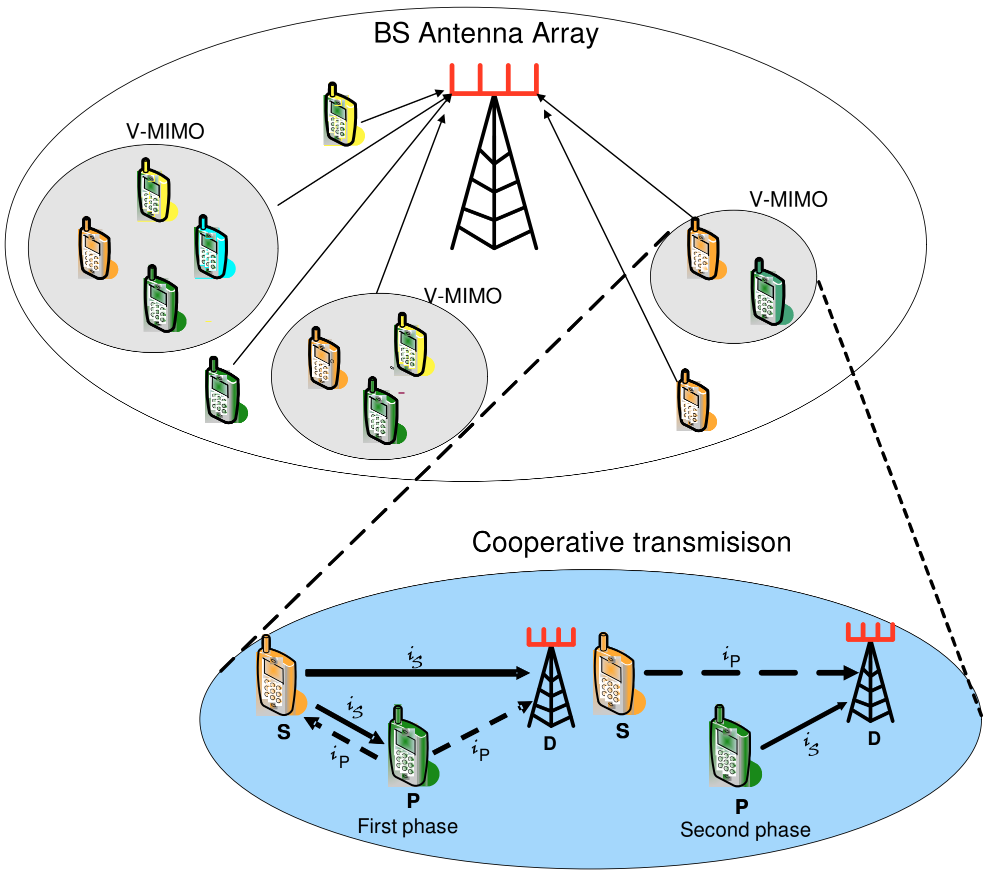

Figure 1.1.: Cooperative transmission in a mobile communication deployment. The lower part of the figure shows two mobile stations, called source, S, and partner, P, cooperatively sending their messages to a base station, D. Solid and dashed lines show the transmission of the source's message, i_s, and partner's message, i_p, respectively, and V-MIMO stands for virtual MIMO.

figure shows a *three-node cooperation* scenario where two mobile stations, called source, S, and partner, P, cooperate while communicating with the base station, D. Each node forwards the same message received from its partner (this is called repetition coding). The antenna elements in the cooperative schemes are widely separated and connected through wireless links, unlike the physical cabling in MIMO [8]; this necessitates cooperation to benefit from the broadcast nature of the wireless channel.

Because of the spatial diversity gain, transmission failures are reduced in cooperative transmissions which leads to an increase of aggregate throughput. On the other hand, when users share their resources, e.g., time, the effective transmission rate of an individual user will reduce; hence, it may not be intuitive at a glance whether the loss in data rate is compensated for the spatial diversity gain. The performance of cooperative transmission is influenced by a number of factors. To mention but two: type of cooperative protocol implemented and quality of channels among cooperating nodes (also called *inter-user* channels). If the inter-user

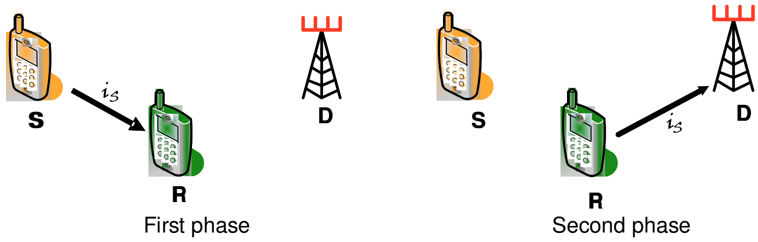

Figure 1.2.: Relay-channel system in the uplink of the mobile communication system, where S, R, and D stands for the source, relay, and destination, respectively, and i_s is the message of the source.

channels are error free, then cooperative transmission behaves like conventional MIMO. In contrast, the performance of cooperative transmission is worse if the quality of the inter-user channels is bad. Performance analysis, conditioned on the quality of inter-user channels, is usually followed in the study of cooperative transmission protocols (e.g., [5, 9, 10]) and is also adopted in this thesis. In the next section, the working principle of the three-node cooperation and various protocols to implement cooperation are revised.

1.1. Review of Cooperative Transmission Protocols

In this section, various cooperative protocols for the three-node cooperation are discussed. One way to view this cooperation is as an extension of the classical *relay-channel system*, which consists of a source, a relay, and a destination [11]. Figure 1.2 shows a typical relay-channel system in the uplink of the mobile communication system. The source broadcasts its message[1] to the relay and destination in a *first phase*; the relay forwards the message it has received to the destination in a *second phase*. The destination recovers the source's information bits based on only the message received from the relay. Usually, the relay is located in the path between the source and destination to split a longer path into shorter segments so that the effect of overall path loss is reduced. Even more so, if energy and propagation environment constraints preclude Point-to-Point Transmission (PPT), relaying emerges as the only option to provide connectivity [9]. However, spatial diversity is ignored in the relay-channel system.

In the case of cooperative transmission, the relay node, after receiving the source's message, resends a processed version of this message to the destination. Consider the solid lines in the first phase of Figure 1.3 that indicate transmission of the source's message. Because of

[1] In the course of this thesis, we use 'message', 'codeword', and sometimes 'packet' interchangeably to refer to a group of channel-coded bits.

the *broadcast nature* of the wireless channel, the same message sent by the source is likely to be received by the destination as well. The destination *combines* the two copies received from the source and relay. This way, *spatial diversity* is exploited as the two messages are received from potentially uncorrelated channels (contrary to the relay-channel system). Moreover, depending on the location of the relay, cooperative transmission also benefits from the path-loss reduction as in the relay-channel system [9].

The cooperative transmission explained above assumes the presence of a "dedicated" relay node. In a case that both the source and relay have messages of their own, the two nodes can be partnered such that they cyclically interchange their roles, i.e., in a next cycle, the source becomes the relay and vice versa. This partnering of nodes is for mutual benefit and hereafter we call the relay node *partner* node. As an example, consider the network in Figure 1.3 where the source and partner send their messages i_s and i_p, respectively, to the destination, D. In the first phase, the source and partner, using orthogonal channels, transmit their messages to the destination as well as exchange each other's message (because of the broadcast nature of the channel). The orthogonality could be in time as in Time-Division Multiple Access (TDMA) or frequency as in Frequency Division Multiple Access (FDMA). In the following discussions, we assume TDMA-based channel sharing by the source and partner as it is frequently used; however, all discussions hold for FDMA-based counterparts.

In the second phase, the source and partner forward "processed" versions of each others' messages to the destination. In repetition-coding-based cooperation shown in Figure 1.3, the partner forwards the very message received from the source. The destination combines identical messages received from the source and partner (e.g., by using Maximum-Ratio Combining (MRC), Equal-Gain Combining (EGC), or Selection Combining (SC)) and decides if the message is correctly received. On the other hand, if the source and partner forward different versions of the received messages (e.g., in coded cooperation where nodes forward incrementally redundant symbols [10]), then the destination uses Code Combining (CC) to form stronger messages (in coding sense) from messages/symbols received in the two phases, decodes, and makes decision on the correct reception of messages.

In the following, three salient contributions in the study of cooperative transmission are revised. Sendonaris et al. were the first to introduce cooperative transmission and used the name *user cooperation* [6, 7]. In their two-part paper, a three-node cooperative transmission was developed for a Code-Division-Multiple-Access (CDMA) system (orthogonal codes to avoid multiple-access interference) in which the source and partner operate in half-duplex mode (i.e., cannot transmit and receive at the same time). Assuming knowledge of channel phases at the source and partner, increased data rates for the cooperating users have been demonstrated.

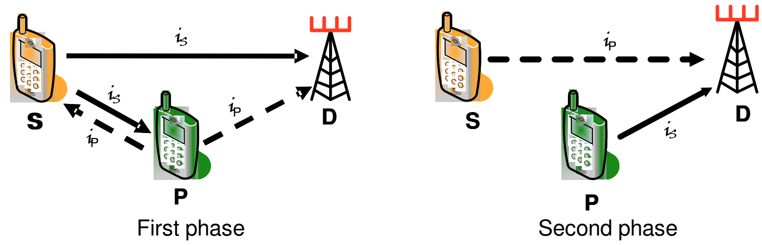

Figure 1.3.: Cooperative transmission in the uplink transmission of the mobile communication system; it is based on repetition coding. Solid and dashed lines show the source's message, i_s, and partner's message, i_p.

Laneman et al. introduced *cooperative diversity* for the same three-node cooperation [5]. Several cooperative protocols were proposed and their outage behavior was analyzed. In this work, more practical considerations such as half-duplex and orthogonality constraint based on TDMA (i.e., a node cannot transmit its own and partner's data simultaneously at the same frequency) were considered.

Hunter et al. proposed *coded cooperation* and analyzed its outage behavior [10]. In coded cooperation, error control coding was incorporated into cooperation such that the source and partner cooperate by transmitting incrementally redundant symbols to each other. In this protocol, the source encodes k of its source bits into a codeword of N symbols, and the N symbols are further partitioned into a weaker codeword of N_1 symbols and $N_2 = N - N_1$ incremental symbols. The source transmits the weaker codeword in the first phase, and if the partner successfully decodes the k bits from this codeword, then it generates and transmits the source's N_2 remaining symbols. If decoding fails, then the partner will transmit additional N_2 symbols of its own message. At the destination, the symbols received from the source and partner are code combined to form a codeword of N symbols, and this codeword is further decoded to generate information bits of the source [10, 12].

In general, the cooperative protocols mentioned in the above prominent (and plenty of follow-up) literature can be broadly categorized based on various parameters, to mention but few: relaying/forwarding strategy, level of adaptiveness to decoding error, and type of coding used in the second phase. Considering the relaying strategy at the relaying node (in the following, we consider the transmission of the source's message only such that the partner will be the relaying node; the same argument holds for the partner's message), some of the common relaying strategies are:

- *Amplify-and-forward:* In this relaying strategy, the partner simply forwards an amplified version of the received message (works in the analog domain) [5]. Because of the noise added at the partner and as there is no error-checking mechanism, the forwarded message is a noisy version of the original message from the source. In spite of the noise propagation, it was shown that amplify-and-forward can achieve full diversity gain, which is equal to two for one relay.

- *Decode-and-forward:* In this relaying protocol, the partner decodes the received message, re-encodes it (using the same codebook as in repetition coding [13] or a different codebook as in coded cooperation [10]) and then forwards it to the destination [5, 9]. Decode-and-forward requires correct decoding of the message at the partner for the forwarded message to be usable at the destination; otherwise the forwarded message leads to error propagation and further decoding error at the destination. The performance of this protocol is limited by the worst link of either source-partner or source-destination, as the protocol will not benefit from either the relayed transmission or the direct transmission [14, 15]. Comparing the above two forwarding strategies, decode-and-forward protocol performs better when the two inter-user channels are of high quality (where chance of correctly decoding is high); on the other hand, amplify-and-forward performs better when the inter-user channels are of poor quality.

- *Compress-and-forward:* In this protocol, the partner first samples, quantizes, and compresses (in order to reduce redundancy) the received message. Second, it encodes the compressed message into a new message (as if they were information bits) and forwards it to the destination [16, 17]. The destination jointly processes the observations from the source and partner. The compression at the partner is realized using Wyner-Ziv source coding [18, 19]. As this protocol works in semi-analog and semi-digital domain, it incorporates benefits of both the amplify-and-forward and decode-and-forward protocols.

In the decode-and-forward-based protocol, when the inter-user channels are bad, the partner is more likely to forward an erroneous message. Moreover, when the source-destination channel is not very bad (or even better than the partner-destination channel), a high percentage of messages transmitted by the source are likely to be received correctly by the destination; in this case transmissions from the partner are a waste. To overcome these drawbacks, relaying protocols can be designed to adapt to decoding results at the partner and/or destination. This leads to further classification of protocols as static and adaptive.

- *Static protocols:* In static (or fixed) protocols, the partner always forwards the source's message without checking errors [5, 9]. This protocol is easy to implement, but is prone to error propagation.

- *Adaptive protocol:* Here, the partner decides whether to forward or not, depending on its success of decoding the source's message. If successful, then it may forward the same message or a modified version of it. If decoding fails, then the partner has the options to switch to amplify-and-forward, transmit its own message, or even remain silent [9].

In adaptive protocols, the decision at the partner to either forward the received message or switch to the other options can be made at different levels of granularity. To mention two:

- *Cyclic Redundancy Check (CRC)*: The received message/packet is completely decoded and then a CRC is performed. This approach is useful in a packet-based transmission where the CRC is done on a *group* of bits, and it insures that the partner forwards only correctly received packets [15].

- *Threshold-based decoding*: Using this approach, the partner forwards if the Signal-to-Noise Ratio (SNR) of the received signal exceeds a certain threshold [9]. Alternatively, soft information, which is obtained from the Log-likelihood Ratio (LLR) value of the received signals, can be used instead of the SNR [20, 21]. In threshold-based decoding, the choice of an appropriate threshold is not easy, decision is done on signal/bit level rather than on packet level, and is less reliable than the CRC-based approach because a message may be received in error even if its SNR (or soft information) is above a threshold.

Depending on the level of Channel-State Information (CSI) available at the source and partner, adaptive protocols are further categorized as *selective* or *incremental* relaying [5].

- *Selective relaying:* When the source knows the fading state of the inter-user channels, then it can reasonably estimate if the partner correctly receives and forwards its message to the destination. In the case of decoding failure at the partner, the source can transmit a copy of its message to the destination instead. One drawback of this relaying is the high overhead of acquiring CSI, which may overwhelm the gain from cooperation. Alternatively, *reciprocity* in reception can be assumed, i.e., if the source decodes the partner's message, then the source can assume that its message is also likely to be decoded by the partner.

- *Incremental relaying:* Here, the destination feedbacks an acknowledgment to the partner if it was able to receive the source's message correctly from the first-phase transmission (assuming a feedback channel from the destination to the partner). Accordingly, the partner stops forwarding the source's message. In this relaying scheme, nodes cooperate only when the direct link to the destination fails. It was shown that the protocol has better spectral efficiency as the partner does not need to always transmit (especially when the source-destination channel is good and a high percentage of the messages transmitted by the source can be received correctly by the destination) [5].

In the decode-and-forward-based schemes, once the partner receives the source's message correctly it can employ the following coding strategies before forwarding the message to the destination.

- Repetition coding: The partner uses the same codebook used at the source. At the destination, either MRC, EGC, or SC is used to combine the two messages received from the source and partner [12]. The forwarded message helps to accumulate SNR (i.e., increase the strength of the signal); however, the partner forwards no additional information that would help decoding.

- Incremental redundancy coding: The partner uses a different codebook to re-encode the received message and forwards the resulting message to the destination. At the destination, this message is used as incrementally redundant information and code combining is used to combine it with the message received from the source in the first phase [11, 10]. One drawback of repetition coding and incremental redundancy coding is that all the resources in the second phase are dedicated to either the source or partner only. This leads to unfair cooperation especially when the source-partner and partner-source channels have different quality, where one user may forward for the other but not vice versa. To overcome this drawback, the following coding approaches are proposed.

- The partner combines the source's message with its own message (e.g., network coding [22, 13], space-time coding [23, 24, 25], superposition modulation [26], or differential modulation [27, 28]) and forwards to the destination. Using one of these coding strategies, resource in the second phase is shared between the two users instead of being dedicated to one user. At the destination, this message is *"combined"* with the messages received in the first phase and then decoded to recover the source's message.

In addition to the above classification of protocols, other considerations in the study of cooperation are: purpose of cooperation, resource allocation, and type of diversity gain. Purpose of cooperation is related to whether cooperation is beneficial for a single user (called *user-level* cooperation and occurs, for example, when the partner is a dedicated relay with no data of its own) or both users (called *system-level* cooperation and is for mutual benefit). Resource allocation refers to how users allocate their resources to cooperation; this includes total energy (how the total energy is allocated in the two phases of cooperation) and avoiding multiple access interference (orthogonality constraints and half-duplex transmission). Type of diversity gain refers to whether the gain is from spatial diversity, temporal diversity, or a combination of these two gains. Temporal diversity is achieved when the fading statistics of a channel changes from phase one to phase two, and in this case diversity gain is achieved by repeating an own message. If the channel state remains constant over the two phases, the channel is called *block fading*. Block fading channel is the most common assumption in almost all work in the area of cooperation.

The various protocols discussed above are summarized in Figure 1.4. Shaded areas in the diagram indicate protocols/parameters considered in this thesis. We see that by combining the various degrees of freedom (e.g., relaying strategy, level of adaptiveness, coding strategy) one can design a variety of cooperative transmission protocols, trading-off between complexity of a protocol and its performance [9]. Section 1.2 briefly discusses the working principle of network-coded cooperation protocols, which are the focus of this thesis, and reviews some of the related work.

1.2. Network-Coded Cooperation

In the previous section, we have presented various cooperative protocols and mentioned that network coding is one coding strategy for decode-and-forward-based protocols. This thesis focuses on protocols that use network coding; hereafter, we call the protocols "*network-coded cooperation*", or a variant of it. In this section, we briefly talk about two possible implementations of network-coded cooperation and present a literature survey on existing approaches and their limitations.

1.2.1. Review of network-coded cooperation

In network-coded cooperation, the partner, instead of simply forwarding a received message, transmits a network-coded version of its own and the source's messages (assuming it has correctly received the source's message). The network-coded message is a weighted sum (or linear combination) of the received and locally generated (own) messages, where the weighing coefficients are generated either randomly or deterministically. This coded message contains information bits of both users, unlike other cooperative schemes where a message forwarded in the second phase contains information originating from one user only. At the destination, network-coded message(s) received from the source and/or partner are used to recover both the source's and partner's information bits, i.e., both users benefit from any one or both of the two coded messages sent by the source and partner. Conceptually, the network-coded cooperation resembles the distributed space-time cooperation protocols in [23, 10], where, for example, space-time block codes based upon orthogonal designs are proposed to implement the coding.

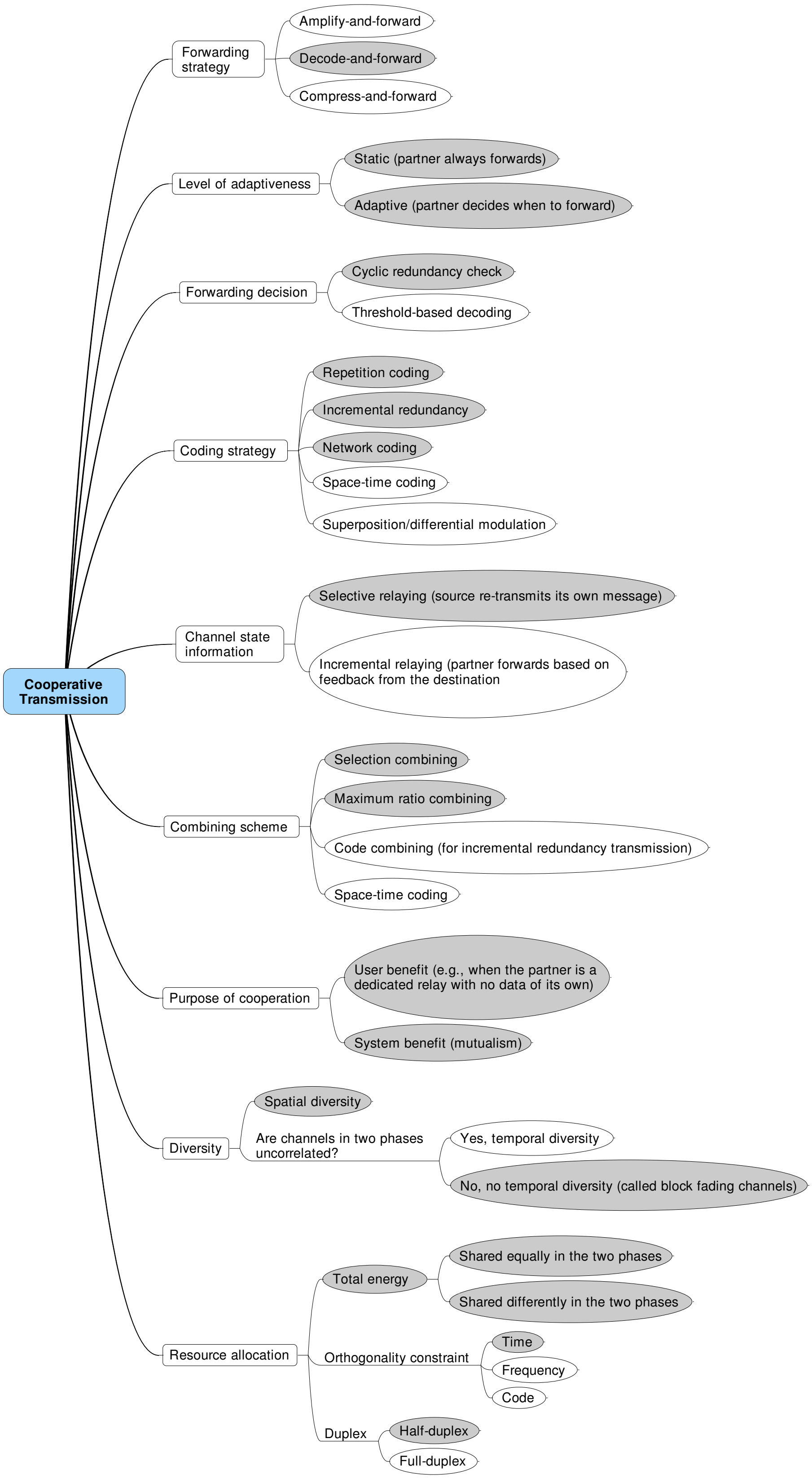

Figure 1.4.: Classification of cooperative transmission schemes. Shaded areas indicate the focus in this thesis.

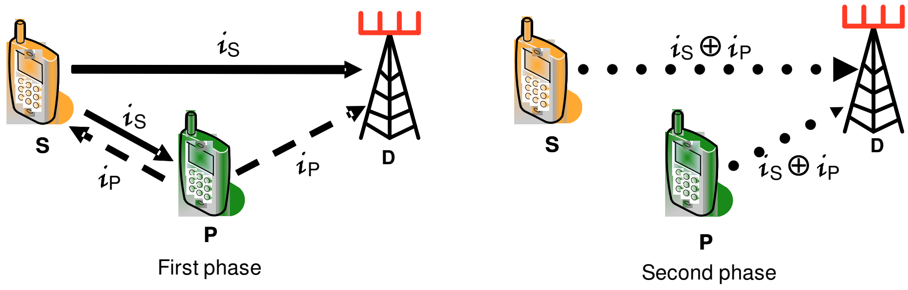

Figure 1.5.: System diagram of network-coded cooperation in the uplink transmission. Solid, dashed, and dotted lines show the transmission of the source's message, partner's message, and network-coded message, respectively.

One simple implementation of network coding is by *modulo-2* summation of received messages. Figure 1.5 shows that the network-coded cooperation and transmissions in the second phase are the modulo-2 summed messages $i_s \oplus i_p$. Once the destination receives the four messages, it first combines the two network-coded messages using either SC or MRC. Then the source's message, for example, is recovered using one of the two options: either from the source's first-phase transmission, provided no error occurs, or through additional network coding of the partner's and network-coded messages, i.e., $i_s = i_p \oplus (i_s \oplus i_p)$, provided both messages are correctly received. Note that, at the destination, the network-coded message is considered as an independent message and will be usable if it is received error-free. We call this cooperation *conventional network-coded cooperation* to differentiate it from other network-coding-based cooperation to be discussed in the next paragraph. Intuitively, as information bits are recovered using either of the two options, a maximum *diversity order* of two can be obtained. The actual value of the diversity order depends on other system parameters such as received SNR and transmission rate. A detailed discussion on the working principle of this protocol and investigation of its outage behavior are presented in Chapter 4.

Alternatively, when both network and channel codings are used, network coding can be done on parity symbols of the two users. We coin this cooperation *incremental redundancy network-coded cooperation*. In this cooperation protocol, the partner, for example, after receiving the source's message in the first phase, generates two groups of parity symbols: one for its own and the other for the source. Then it network codes these groups of parity symbols to form *network-coded parity symbols* and forwards the symbols to the destination in the second phase. At the destination, the network-coded parity symbols are *code combined* with message(s) received in the first phase to form a stronger codeword. This is unlike conventional network-coded cooperation where the network-coded message is considered as an independent message. We see that this approach embeds network coding into channel coding in a way that the redundancy in the network code is used to support the channel code for better error protection [29, 30]. A detailed discussion of the working principle, various coding approaches, and the outage behavior of incremental network-coded cooperation are discussed in Chapter 6.

1.2.2. Literature survey on network-coded cooperation

So far, we briefly explained the use of network coding in cooperative transmission; next, we review some of the work in network-coding-based cooperation that are related to our study. In general, different terminologies are used for various ways how network coding is implemented. To mention some: joint vs. separate network-channel coding [31], symbol vs. packet level [32], analog (or continuous) vs. digital [33], physical layer vs. network layer [33].

The idea of using network coding in a *two-way relay channel* system is investigated in [34, 30] and references therein. The two-way relay channel consists of two sources that want to exchange information with the help of a third dedicated relay. The relay is usually located in the path between the two sources, and this relay is referred to as a "routing node" [34]. If no network coding is used, a total of *four time slots* is required to complete the exchange of information: two slots for each source to transmit to the relay and another two slots to forward/route from the relay to each source. If network coding is used instead, the relay modulo-2 sums the messages received from the two sources and sends the message in the third time slot, i.e., three slots are sufficient to exchange the information and this improves system throughput. This network coding is sometimes called *digital network coding* or *hard-decision network coding*, since the coding is done on digital/hard bits [34]. The joint network-channel coding can also be designed to exploit the broadcast nature of the wireless channel [29]. As a message sent by one source may reach both the relay and the other source, the relay forwards incrementally redundant information that provides additional error protection at the two sources. This approach is called joint network-channel coding [29]. Almost all work in the area of two-way relaying assumes that the relay always decodes messages from the two users correctly.

The same joint network-channel coding approach for a *multiple-access relay channel* system is discussed in [30, 31, 35]. The multiple-access relay channel consists of two sources, one dedicated relay and one destination. In [30, 36], the authors consider the design of a joint network-channel code based on turbo codes and allocation of transmission time among the two sources and relay. Three transmissions from the two sources and the relay are required (instead of four if no network coding is used). It is reported in [29, 31, 30] that joint network-channel coding exploits the redundancy from the relay more efficiently than separate network-channel coding (where the relay forwards the incremental messages of the two sources separately using two time slots). The results presented in [29, 31, 30] were based on simulation, and the channels between the source(s) and relay were assumed to be ideal, i.e., the relay always decodes messages from the two sources. Moreover, the relay has no messages of its own and serves the sources only. For non-ideal source-relay channels, we have performed the outage analysis of this scheme in [37].

Recall that in conventional cooperation systems, the available time slots are divided into two subslots: one for the first-phase transmission and one for the second-phase transmission. The use of such orthogonal and interference-free channels simplifies receiver implementation but results in loss of spectral efficiency (as messages of high information rate should be transmitted in each phase). To form low-rate codes that overcome the loss in spectral efficiency, the idea of algebraic superposition of either *channel codes* [22], *modulated bits* [26], or superposition in analog domain [38] (also called analog network coding) are introduced. The basic idea is the source, for example, assuming knowledge of the partner's previous message, always sends the superposition of its own current and partner's previous message; where the superposition could be after channel coding, on modulated bits, or in the analog domain (e.g., using dirty paper coding as in [38]). The partner extracts the source's message from the received superimposed message, does further superposition of its current message with the source's message, and further transmits to the source and destination. We see that, as the source transmits for its own and simultaneously forwards for its partner, there is no need for two-phased transmissions. The destination implements some kind of iterative decoding on hard or soft information or use dirty paper decoding. Complexity of the receiver is one drawback of this approach.

For the three-node cooperation, the work in [39] proposed a cooperation protocol in which the partner concatenates its own and the source's information bits to form longer information bits (as opposed to modulo-2 summation of the two information bits). The longer information bits form systematic bits of a systematic code. The partner channel codes the longer information bits and transmits the codeword. In this way, in a single transmission a user transmits its own message, forwards for its partner, and transmits parity bits that protect messages of the two users. The notion of network coding is embodied on the parity bits as it is the result of the information bits from the two users. It was shown, by analysis and simulations, that this network coding scheme is more tolerant to poor inter-user channels than the repetition-coding-based cooperation.

Katti et al. proposed, for the two-way relay channel system, to include wireless interference through analog network coding in [34]. This scheme allows the two sources to transmit simultaneously so that they interfere at the relay node; the relay forwards the interfered signals.

Each source, given knowledge of its own message, can extract the other source's message from the message received from the relay. The scheme is called *analog network coding* because the relay mixes analog signals, not bits. We see that only two time slots are required to complete the information exchange between nodes and this gives an improvement in spectral efficiency as compared to the digital network coding discussed above, which requires three slots. Note the analogy between analog network coding and its digital counterpart. In digital network coding, the sources transmit using orthogonal channels; the relay superimposes (mixes) the content of the messages/packets and broadcasts the mixed version. In analog network coding, the sources transmit simultaneously and the wireless channel mixes these signals. Instead of forwarding mixed packets, the relay amplifies and forwards mixed signals. Drawbacks of this approach are: the two sources should synchronize their transmissions such that they interfere at the relay with no delay; this is difficult, if not impossible to achieve. Moreover, like any analog scheme, there is no error checking at the relay which makes error propagation inevitable.

Zhang et al. also proposed a similar approach to analog network coding and used the name *physical layer network coding* [40]. The authors explained, through a proper design of modulation and demodulation techniques, how mapping/de-mapping of modulated/demodulated symbols at the physical layer of relaying nodes can emulate the modulo-2 summation of digital bit streams as in digital network coding. Historically, network coding is implemented at the *network layer* of Open Systems Interconnection (OSI) architecture [41, 42]; however, the use of analog, physical-layer, and joint network-channel network codings pushes the implementation down to the *physical layer*.

The use of *soft-bit information* in network-coding-based cooperation is another recent development [21, 43, 44]. Soft-bit information refers to the confidence interval of a received bit and is computed from the LLR value of that bit. For the multiple-access relay channel system, the soft-bit-based protocols are proposed mainly to overcome one drawback of the hard-decision network coding protocols in [30, 31, 35], where the relay must correctly receive transmissions from the two sources. In this regard, for Additive White Gaussian Noise (AWGN) channels, Koetter et al. investigate the network coding gain even when transmissions to the relay cannot be recovered correctly [44]. Instead of decoding the messages received from the two sources, the relay computes the LLR values of the two messages and performs an operation on the LLR values that emulates the modulo-2 summation in network coding. As the resulting LLR takes on continuous values, it is modulated using one of the known analog modulation techniques and the signal is forwarded to the destination. At the destination, an iterative decoding algorithm based on the three observations (two from the two sources and one from the relay) is run. Simulation is used to measure performance and Bit Error Rate (BER) results for various sources-relay, relay-destination, and source-

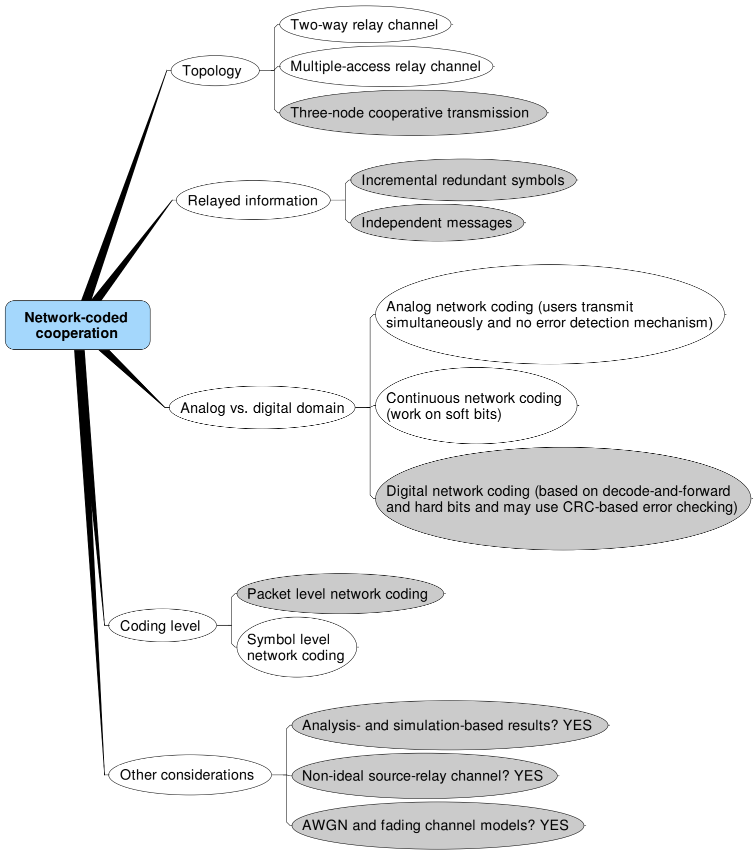

Figure 1.6.: Classification of network-coded cooperation schemes. Shaded areas indicate the focus in this thesis.

destinations channels SNR values are presented (unfortunately, no comparison with other protocols is presented). Pu et al. have also investigated the multiple-access relay channel system with soft-bit information and used the name *continuous network coding* [43]. In their approach, the soft-bit-based information forwarded by the relay is used as incremental redundancy information at the destination. The authors reported that relaying a network-coded version of the soft information gives significant gain over traditional network coding (i.e., based on hard-decision at the relay).

Finally, a network of one source communicating to one destination using multiple relays is introduced in [32]. In this scheme, the relays forward selected groups of bits of a packet instead of the entire bits of a packet, and it can be extended to a network of multiple sources and multiple destinations. Katti et al. called the scheme MIXIT [32]. The basic motivation is to minimize the packet discarded by relaying nodes in the event of incorrect reception, as some of the bits in a packet are likely to be correct. MIXIT works as follows: at a source node, a given packet is divided into sub-packets (which are groups of bits also called *symbols*), network code the symbols, and send the coded symbols to the relays and destination.

A relaying node estimates the correct reception of each coded symbol using soft-bit information, discards those that are more likely to be incorrect, and forwards the remaining coded symbols. The destination also estimates and collects symbols that are more like to be correct, and re-computes the original message using these symbols. The network coding approach is also called *symbol-level network coding*. By forwarding symbols that are more likely to be correct, it is reported that MIXIT achieves higher throughput than the traditional approach, were only correctly received packets are forwarded. Figure 1.6 summarizes the various protocols revised above and the shaded areas indicate the focus of this work.

1.3. Thesis Motivation and Contributions

In the previous section, we have revised some of the network-coded cooperation protocols related to this work. The advantages and limitations of each protocol were mentioned. In the following, we point out some of the common assumptions in most of the network-coded cooperation papers and limitations of the assumptions.

1.3.1. Thesis motivation

Network coding in general, and its application in cooperative transmission in particular, is a recent field of study. So far, research is mainly focused on introducing the concept of network coding in various application scenarios and designing new protocols. In the design of a new protocol, various parameters and assumptions come into play. At an initial stage of research, keeping the overhead of the assumptions as small as possible, at the same time elaborating on the concept is a procedure usually followed; and this is also the case in network-coded cooperation. While reviewing existing work during the inception of this thesis, we came across few assumptions in already published papers that, to our believe, are unrealistic. Investigating network-coded cooperation by making more realistic assumptions was the initial motivation of this thesis. In the following, we point out some of the limiting assumptions common to most previous work.

- The inter-user channels are assumed reliable such that relaying nodes[2] reliably decode received messages and always forward network-coded message only [29, 35]. This assumption simplifies the protocol design as no decision and adaptation strategies are required in the case of reception failure. Moreover, the assumption simplifies analysis (e.g., outage analysis). One notes that the decoding requirement is most likely fulfilled

[2]Partner in the case of three-node cooperation and dedicated relay in the case of multiple-access relay channel and two-way relay channel systems.

when the source and relaying node are close to each other; restricting the two nodes to be closer to each other is a bottleneck as it inhibits further study in terms of node deployment, partner selection, and coverage area extension (which requires the relaying node to be located anywhere in the network where the chance of decoding all the time may not be possible).

- Recent results show that network-coded cooperation performs better than repetition-coding-based cooperation when the inter-user channels are of poor quality [13, 39]; this contradicts the ideal inter-user channel assumption.

- A thorough information theoretic analysis of outage probability, which leads to a further study of outage behavior and diversity-multiplexing tradeoff, is missing. Simulation is usually used to investigate protocols.

Based on the above three observations, our initial motivation, in a nut shell, was to theoretically analyze outage probability of the network-coded cooperation when the inter-user channels are non-ideal. Using the developed outage probability expression, we have investigated the performance of network-coded cooperation. A summary of the main contributions of this thesis are presented next.

1.3.2. Thesis contributions

One problem when considering non-ideal inter-user channels was that the protocols have to be redesigned to take decision in the case of decoding failure (i.e., should adapt to decoding errors). This adaptiveness has to be reflected in the analysis, which makes the analysis at first challenging and difficult to approach. For non-ideal inter-user channels, here are the main achievements in this work:

1. The outage behavior of network-coded cooperation protocol is examined by deriving its outage probability; this is reported in Chapter 4 and is already published in [45, 13]. Outage probability helps to study the protocol independent of any particular coding scheme and is also shown to be a lower bound on block error rate for sufficiently large block lengths [10]. To make the outage probability analysis more tractable and convenient for exposition, quasi-static (or block) Rayleigh fading channels, orthogonal transmission, and half-duplex constraints are assumed. Approximating the outage result at high SNR values, we shown that this protocol achieves full diversity (order two for two users) asymptotically in user transmit power [45, 46].

2. We investigate the outage behavior for various inter-user and uplink (between a transmitting node and destination) channel qualities; we compare various cooperative protocols based on the inter-user channels. Based on the outage results, network-coded cooperation protocols are found to be suitable when the inter-user channels are lower quality; when the inter-user channels are good, protocols without network coding perform better [13].

3. The outage results are further extended to study the diversity-multiplexing tradeoff and the coverage area extension and results are also published in [46].

Once outage results are available, the next step is to address energy efficiency of network-coded cooperation in wireless sensor networks. The motivation is that cooperation helps wireless nodes to achieve spatial diversity, which allows to reach equal error rate at lower transmit power. However, relaying redundant messages consumes considerable energy at both transmitting and receiving nodes. Striking a balance between the diversity gain and additional consumed energy is the intention of this step. By defining appropriate energy consumption metrics, a study is conducted if and when cooperation is energy efficient than ; detailed discussion is given in Chapter 5.

The outage behavior of *incremental redundancy network-coded cooperation* is investigated next; results are reported in Chapter 6 and also published in [47]. As explained above, in incremental redundancy network-coding, the network coding is embedded into the channel coding such that the redundancy in the network code is used to support the channel code for better error protection [29, 30]. With this in mind, the steps followed and results obtained are as follows.

1. Two decoding approaches, namely *joint network-channel* decoding and *individual network-channel* decoding, are proposed first. In the former, one 'big' codeword is formed from all symbols received in the two phases, and the information bits of both source and partner are obtained from a single decoding of this big codeword. In the latter approach, the source's and partner's information bits are recovered by two independent decodings.

2. The outage behavior, for quasi-static Rayleigh fading channels, orthogonal transmission, and half-duplex constraints, is studied next. The outage results show that this scheme also achieves full diversity order of two.

3. Then, using the outage result, 'optimal' *rate* and *energy* allocations that minimize the outage probability are studied. The results show that outage performance is more sensitive to the energy allocation than to the rate allocation.

All in all, the outage probability analysis, based on realistic non-ideal channel assumption, of both types of network-coded cooperations help to investigate if and under what condition(s) cooperation benefits from network coding. Also, the outage probability approximations at high SNR values paved the way for further study in diversity-multiplexing tradeoff, energy efficiency computation, and rate and power allocation. Finally, our analysis approach has been followed in recent publications, to mention but few: [48, 49, 50, 51].

1.4. Organization of the Thesis

The rest of the thesis is organized as follows. In the following chapter, we present an overview of the salient work in network coding, encoding and decoding operations in a linear network coding, and symbol- and packet-level network coding. Chapter 3 overviews the system and channel models used in this work. It also discusses the layering issue in wireless networks and briefly explains some of the performance measures used in the study. In Chapter 4, we compute the outage probability for network-coded cooperation and show the performance benefits, in comparison with the point-to-point transmission. The diversity-multiplexing trade-off and coverage area are also discussed. By defining appropriate energy efficiency metrics for sensor networks, the energy efficiency of network-coded cooperation is studied in Chapter 5. The model takes into account all the transmission and processing energy costs and shows when network-coded cooperation is better than point-to-point transmission. The outage behavior of the incremental redundancy network-coded cooperation is investigated in Chapter 6; moreover, optimal energy and rate allocations were investigated. Chapter 7 summarizes the findings of this thesis and gives the main conclusions. In addition, some recommendations for future research are also outlined.

2. Introduction to Network Coding

Network coding is a relatively new field of study which, initially, was proposed to increase network throughput. It is concerned with coding at a node in a network with error-free point-to-point links. This error-free-links requirement distinguishes network coding from channel coding, which is designed for error protection in noisy links [52]. In this chapter, we will give a brief overview of network coding. Section 2.1 introduces network coding using a butterfly network and reviews some of the fundamental research in the area. A type of network coding, called *linear network coding*, is discussed in Section 2.2 and the encoding and decoding operations are explained. Section 2.3 highlights the interaction of network coding and channel coding in error-prone networks.

2.1. Network Coding in Error-Free Networks

Consider a multicast network, where information is usually transmitted from a source node to each sink node through a chain of intermediate (also called *routing* or *forwarding*) nodes by a method known as *store-and-forward*. In this method, data received from an input link of a routing node is stored and a copy is forwarded to the next node via an output link [53]. In this method, routing nodes only replicate incoming data without processing them.

In order to improve the throughput of a network, Ahlswede et al. recently introduced *network coding* as an alternative approach to simple forwarding of data at routing nodes and demonstrated its advantage over the store-and-forward approach [41]. In a network with error-free links, one general definition of network coding is an arbitrary mapping of data at the inputs of a node to output data [52]. The error-free-links assumption distinguishes the function of network coding from that of channel coding, which is designed for error protection in noisy links. In network coding, a node in the network is allowed to *combine* (usually linearly) several messages it has received or created into one or several outgoing messages and forwards to nodes to which it is connected [41, 54, 42]. The sink node, after receiving sufficient number of encoded messages, extracts the messages that were originally intended for it.

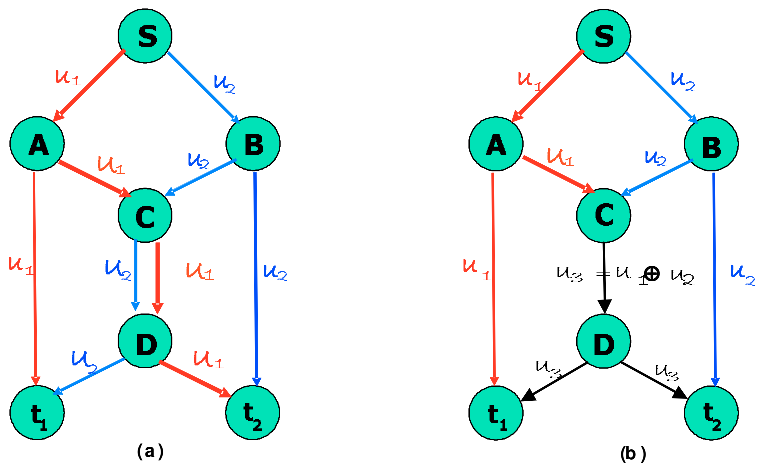

Figure 2.1.: Butterfly topology where node S is a source with two messages, u_1 and u_2. Both sink nodes, denoted as t_1 and t_2, need to receive both messages and all links are assumed to have a capacity of one packet per unit of time. (a) Transmission using the store-and-forward approach. (b) Using network coding, one can send $u_1 \oplus u_2$ down the middle link from node **C** to node **D** and deliver the two messages.

The principle of network coding is best explained with the butterfly example depicted in Figure 2.1 [41]. The butterfly network illustrates that, by sending fewer messages than the store-and-forward approach, network coding can increase throughput in multicast networks. Assume that all links are error free and have a capacity of one packet per unit of time. The source, **S**, wants to deliver messages u_1 and u_2 to two sink nodes t_1 and t_2. Figure 2.1 (left) depicts the conventional store-and-forward approach. Router nodes **C** and **D** can deliver either u_1 to t_2 or u_2 to t_1. The link between nodes **C** and **D** acts as a bottleneck as it has to be used twice and only one packet can be sent at a time. The other links are used once.

Figure 2.1 (right) depicts the solution with network coding. Node **C** performs a modulo-2 addition of the two incoming messages to get $u_3 = u_1 \oplus u_2$ and forwards u_3 to both sinks via node **D**. Sink t_1 gets u_1 from node **A** and can recover u_2 with another modulo-2 addition as $u_2 = u_1 \oplus (u_1 \oplus u_2)$. Similarly, t_2 recovers u_1 as $u_1 = u_2 \oplus (u_1 \oplus u_2)$. Contrary to the store-and-forward solution, all links have to be used once and one channel use is saved through network coding. Network coding can also be used to improve robustness, complexity, and security of a network and a detailed discussion is available in [52].

Once the concept of network coding is clear, we briefly discuss the history of network coding by revising some of the pioneer work that has greatly contributed to the emergence of network coding.[1] The concept of network coding originates from the seminal paper of Ahlswede et al. [41]. Their work focused on improving network multicast capacity for applications in

[1] An exhaustive list of literature on network coding is available at [55].

computer networks (e.g., Internet backbone). In an error-free point-to-point network (assuming that the effect of the channel noise is removed by using powerful channel codes or retransmission in the link layer), the authors in [41] proved that a source can multicast k messages to a set of sinks, provided the min-cut between the source and each sink has capacity k per unit time.

The work by Li et al. has laid the theoretical foundation for constructing network coding by showing that the min-cut capacity for the multicast problem can always be achieved using a *linear network code* [54]. They presented how to explicitly construct the linear network codes. This focused attention on linear codes in particular raised the question of whether they can be used to solve a wider array of network coding problems [56]. Their proof of the existence of a linear solution can be viewed as the first deterministic algorithm for network coding. However, its run time is exponential in the size of the network.

Koetter et al. established a simple and effective algebraic framework for network coding [42]. This reduced the problem of finding a linear solution for a general network coding problem to finding a non-zero point for a multivariate polynomial. Ho et al. proposed a *random linear network coding* algorithm [57]. Each node in the network independently and randomly encodes the received messages over some finite field. They gave a lower bound on the probability that an independent, random linear code design at every node achieves the multicast capacity. The probability approaches one as the size of the finite field approaches infinity. The main benefits of random network codes are distributed implementation without coordination between nodes in the network and robustness with respect to network change or channel failure. In the next section, we will discuss the encoding and decoding operations in linear network codes.

2.2. Linear Network Coding

Much of the work in network coding has concentrated around *Linear Network Coding (LNC)* [52]. LNC requires messages, being communicated through the network, to be accompanied by some degree of extra information, in this case, a vector of encoding coefficients. In packet networks, data is divided into packets, network coding is applied to the content of the packets (hereafter, we call this content the information vector), and the extra information can be placed in the packet header [52].

In LNC, each node in the network forwards a new packet on its outgoing link. This packet is formed by linearly combining earlier received information vectors (or locally generated information vectors if it is a source node) and then appending the encoding vector [58].

Assume that each information vector consists of l bits. When the information vectors to be combined do not have the same size, the shorter ones are padded with trailing 0s. For coding, the s consecutive bits of an information vector are treated as a *symbol* over the Galois field $\mathbb{F}_q$ with $q = 2^s$; hence, an information vector consists of $m = \frac{l}{s}$ symbols. The coding coefficients are from the finite field and addition and multiplication are performed over the field $\mathbb{F}_q$.

2.2.1. Encoding

Assume that n original information vectors, $x_1, \dots, x_n$, are generated by a source node k; we call these vectors *source vectors*. In LNC, each encoded vector from the source, denoted as $y_k \in \mathbb{F}_q^m$, is associated with a sequence of coefficients $g_k = (g_{k,1}, \dots, g_{k,n}) \in \mathbb{F}_q^n$, also called *global encoding vector*.[2] The encoding vectors are used to linearly combine the source vectors x_i in order to produce the encoded vector y_k as

$$y_k = \sum_{i=1}^{n} g_{k,i} \cdot x_i. \tag{2.1}$$

Note that encoded vectors have the same size as the source vectors and contain only a fraction of the information contained in the source vectors. To recover the n source vectors at a sink node, the source should perform the encoding at least n times, each time using a new set of encoding vectors to insure that the encoded packets are linearly independent. Moreover, knowledge of the global encoding vector is required at the sink node. For this reason, the source forwards a packet containing both the global encoding vector, g_k, and the encoded vector, y_k. Hereafter, we use a 2-tuple (g_k, y_k) to represent a transmitted packet.

The encoding in Equation (2.1) is on a packet level as the summation is done on each packet. In *symbol-level* network coding, the summation has to occur for every symbol of the source vectors, and each symbol of the encoded vector is given by

$$y_{k,m} = \sum_{i=1}^{n} g_{k,i} \cdot x_{i,m} \tag{2.2}$$

where $x_{i,m}$ and $y_{k,m}$ are the m^{th} symbols of the vectors x_i and y_k, respectively.

At forwarding nodes, it is not necessary to decode received packets (i.e., recover the source vectors) in order to create new encoded packets. Instead, the same encoding operation as in the source node can be applied recursively to already encoded (and received) packets. Consider that a routing node r is connected to the source k and has received and stored a set

[2] If not stated otherwise, vectors are row vectors.

of h encoded packets, namely $(g_k^1, y_k^1), \ldots (g_k^h, y_k^h)$, directly from the source. The superscripts in (g_k^j, y_k^j) are added to differentiate the various packets generated at the source k. The forwarding node may generate a new packet (g_r, y_r) by picking a *local encoding vector* $l_r = (l_{r,1}, \ldots, l_{r,h}) \in \mathbb{F}_q^h$ and performing the linear combination

$$y_r = \sum_{j=1}^{h} l_{r,j} \cdot y_k^j. \tag{2.3}$$

One can substitute for each y_k^j from Equation (2.1) and show that the encoded vector y_r is also a linear sum of the source vectors, x_i, and the corresponding encoding vectors are formed from the global encoding vector g_k (at the source) and the local encoding vector l_r (at the routing node). The new global encoding vector, g_r, to be transmitted together with y_r is not simply equal to l_r. Instead, it is shown in [58] that it is also a linear combination of the received global encoding vector and is given as

$$g_r = \sum_{j=1}^{h} l_{r,j} \cdot g_k^j. \tag{2.4}$$

To summarize, at an intermediated node the linear combination is always applied to both the encoded vectors and global encoding vectors; this operation is repeated at several routing nodes in the network. By doing so, we ensure that, even after several packet combinations at different routing nodes, the global encoding vector always gives the linear combination of the source vectors.

Example 2.2.1.1. Let us consider the butterfly example of Figure 2.1, where the field is $\mathbb{F}_2 = \{0, 1\}$ and a symbol is a bit. In this field, addition corresponds to bitwise modulo-2 summation and multiplication corresponds to bitwise `and`. The linear combination sent by node **C**, after receiving $x_1 = u_1$ and $x_2 = u_2$, is $u_1 \oplus u_2$. Part (a) of Figure 2.2 shows the global encoding vectors used at the source and routing nodes.

Example 2.2.1.2. Consider again LNC over $\mathbb{F}_2$, where a source node has three source vectors, namely $x_1 = (1111)$, $x_2 = (1100)$ and $x_3 = (1010)$. Figure 2.3 shows a routing node in the network that has received three network-coded packets (g_1, y_1), (g_2, y_2) and (g_2, y_2). As g_1 has only a single one at position one, the payload (i.e., encoded vector) y_1 is equal to x_1. From the global encoding vector g_2, we learn that the payload y_2 is equal to $x_2 \oplus x_3$. Likewise, g_3 indicates that y_3 is computed from $x_1 \oplus x_2$. The node generates its outgoing packet (g_k, y_k) by choosing a *local encoding vector*; in this case $l_k = (110)$. From Equation 2.1, the payload y_k is then given by $y_k = 1111 \oplus 0110 = 1001$. The same linear combination is used to produce the new *global encoding vector*, g_k, which is, $g_k = g_1 \oplus g_2 = 100 \oplus 011 = 111$.

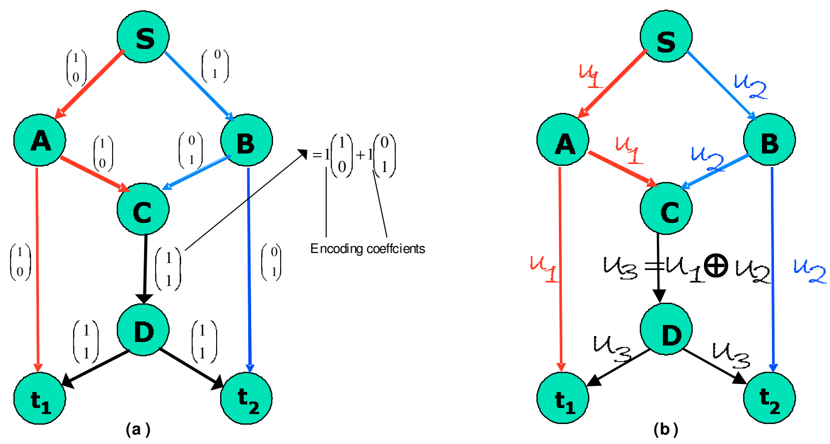

Figure 2.2.: Linear network coding in the butterfly network example of Figure 2.1. The global encoding vectors are shown next to the links.

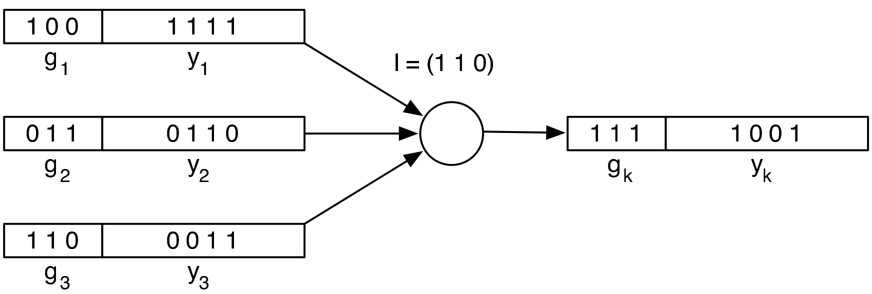

Figure 2.3.: Local encoding over the Galois field $\mathbb{F}_2$.

2.2.2. Decoding

To recover the source vectors, the sink node should receive as many encoded packets as possible. Consider that it has received m packets, denoted as (g_k, y_k), where $1 \leqslant k \leqslant m$. As discussed before, each encoded vector within a packet is a linear combination of the source vectors. With the *global encoding vectors* included in the network-coded packet, the source vectors x_i can be decoded by solving the system of linear equations $\{y_k = \sum_{i=1}^{n} g_{k,i} x_i\}$ for received packets (g_k, y_k), $1 \leqslant k \leqslant m$. Writing this system in matrix form leads to

$$\underbrace{\begin{pmatrix} y_1 \\ \vdots \\ y_m \end{pmatrix}}_{Y} = \underbrace{\begin{pmatrix} g_1 \\ \vdots \\ g_m \end{pmatrix}}_{G} \cdot \underbrace{\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}}_{X} \tag{2.5}$$

where Y, G, and X are matrices of dimension $m \times 1$, $m \times n$, and $n \times 1$, respectively. With Y and G extracted from the received packets, the unknowns are the components of the source vector, X. This is a linear system with m equations and n unknowns. As long as $m \geqslant n$ and at least n of the vectors g_i are linearly independent (also called *innovative*), the decoding matrix G formed by these vectors has rank n. Taking n of these linearly independent equations (accordingly the dimension of G will be $n \times n$), the source vectors x_i can be recovered by inverting G, i.e., $X = G^{-1} \cdot Y$. Hence, one task of the network is to ensure that it delivers at least n linearly independent packets to the sink. One approach is to have each node in the network select the local encoding vector over the field F_{2^s}, uniformly at random in a completely independent and decentralized manner. This is called *random network coding* and a detailed discussion is available in [59, 60]. A well understood way to solve a system of linear equations is via Gaussian elimination. For the **Example** 2.2.1.2, the decoding can be done intuitively as follows.

Example 2.2.2.1. Suppose the node in Figure 2.3 is interested in obtaining the source vectors x_1, x_2 and x_3. As all three *global encoding vectors* g_i are linearly independent, it is guaranteed that decoding is possible. We see from g_1 that $y_1 = x_1 = (1111)$ and have decoded the first packet. Packet g_3 states that $y_3 = x_1 \oplus x_2$. We can then obtain x_2 by computing $x_2 = y_3 \oplus x_1 = (0011) \oplus (1111) = (1100)$. With the same argument we can then decode the last packet by evaluating $x_3 = y_2 \oplus x_2 = (0110) \oplus (1100) = (1010)$.

2.3. Network Coding and Channel Coding

In the previous section, we have described network coding for multicast in error-free networks. However, in wireless networks links are not error-free and packet loss arises in the network for various reasons. To mention but a few: link outage, buffer overflow, and collision [52]. There are a number of ways to deal with such losses. The most straightforward is to set up a system of acknowledgments, where packets received by the sink are acknowledged by a message sent back to the source and, if the source does not receive the acknowledgment for a particular packet, it retransmits the packet. Alternatively, *channel coding* is a method that is sometimes used. Through careful design, the network coding can also be used to support the purpose of the channel coding, i.e., error protection. The interaction of network coding and channel coding will be the focus of this section. In the following, we briefly describe the interaction of these two coding schemes. A significant part of this section is taken from [30].

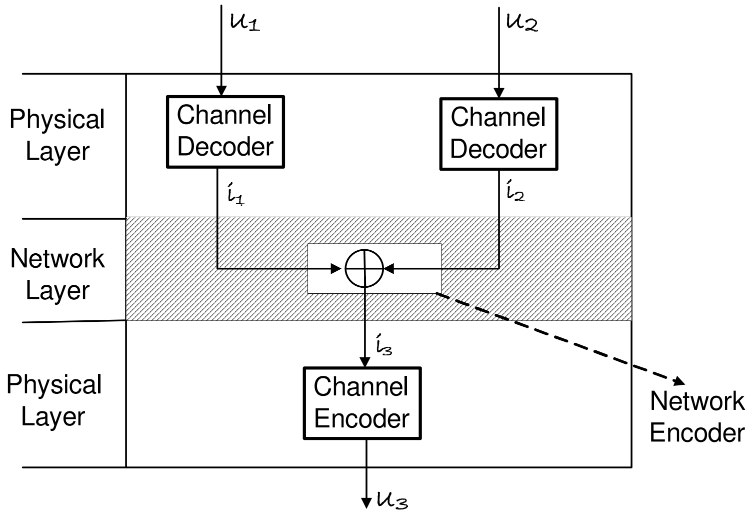

Figure 2.4.: Network encoding at a node for a system with separate network-channel coding.

2.3.1. Separate network-channel coding

According to the OSI model, channel coding is part of the *physical layer* whereas network coding is located in the *network layer* [30]. The purpose of network coding is different from channel coding. Channel coding is performed to protect the communication over point-to-point links against transmission errors. Network coding is performed with the aim to transfer information efficiently through the (error-free) network. The layered architecture model separates the two encodings and simplifies the complexity of the system because the system design is split into the following two tasks: channel coding for a point-to-point communication and network coding for the error-free point-to-point network. We call this approach *separate network-channel coding*.

In order to illustrate separate network-channel coding, consider a node with two incoming links which performs network encoding (example node **C** in Figure 2.1). Figure 2.4 depicts the channel coding and network coding at this node. First, the two incoming packets are channel decoded in the physical layer to obtain the information packets i_1 and i_2. The physical layer delivers i_1 and i_2 to the network layer. Then, the network encoder in the network layer outputs $i_3 = i_1 \oplus i_2$ and then i_3 is delivered to the physical layer. Finally, the channel encoder generates the packet u_3. Assuming a channel encoder with generator matrix $\mathbf{G}$, the output of the network encoder can be expressed as

$$u_3 = (i_1 \oplus i_2) \cdot \mathbf{G}. \tag{2.6}$$

A sink node, such as node t_1 in Figure 2.1 which performs network decoding, works according to the same principle. First, the incoming packets are channel decoded in the physical layer to obtain i_1 and i_3. Then, $i_2 = i_1 \oplus i_3$ is obtained by the network decoder in the network layer. Chapter 4 investigates the implementation and performance of separate network-channel coding in cooperative transmission.

2.3.2. Joint network-channel coding

Joint network-channel coding is a more general approach than separate network-channel coding. This approach tries to merge the network and channel codings in such a way that the network coding also provides support for error protection. Instead of guaranteeing the error-free transmission for each point-to-point link, the interest here is to guarantee error-free decoding at the sink nodes. With this approach, a node has to decode the data using the input from all incoming links, i.e., error-free decoding at the sink can be possible even if error-free decoding of a single link is not possible. In joint network-channel coding, the network-coded packets are designed to carry redundant information.

Analogous to joint source-channel coding where the remaining redundancy after the source encoding helps the channel code to combat noise, joint network-channel coding allows to exploit the redundancy in the network code to support the channel code for a better error protection. Accordingly, the implementation of the network coding is pushed to a layer below the network layer. Let us re-consider the channel coding given in Equation (2.6). Assuming a linear channel encoder, the encoding can be written as

$$u_3 = (i_1 \oplus i_2) \cdot \mathbf{G} = i_1 \cdot \mathbf{G} \oplus i_2 \cdot \mathbf{G}. \tag{2.7}$$

We learn from Equation (2.7) that the network coding can be done on channel-coded bits, i.e., the network coding is performed after channel coding. This is a (joint) realization of the two encodings on the same layer. As a second example, consider next how joint network-channel coding can be used to provide additional redundancy. The encoding in Equation (2.6) is written once again in the form

$$u_3 = [i_1, i_2] \begin{bmatrix} \mathbf{G}_1 & \mathbf{G}_2 \\ 0 & \mathbf{G}_2 \end{bmatrix} \quad = \quad [i_1 \cdot \mathbf{G}_1, (i_2 \oplus i_2) \cdot \mathbf{G}_2] \,. \tag{2.8}$$

where G_1 and G_2 are linear encoding matrices. We see that the first part of the encoded packet, namely $i_1 G_1$, forms the systematic bits in a systematic code and the remaining network-coded bits, namely $(i_2 \oplus i_2) G_2$, forms the parity bits. Chapter 6 presents a detailed study on the implementation and performance of joint network-channel coding in cooperative transmission.

2.4. Summary

Network coding generalizes routing and increases throughput and robustness in communication networks. We have discussed this with the help of the butterfly network. Moreover, discussion on one type of network coding, namely linear network coding, and its encoding and decoding operations are presented. With wireless networks in mind, the interaction of the network and channel coding is also briefly discussed. We also pointed out that, through joint realization of the network and channel codes, network coding can be used to generate redundant information that can support the channel codes. In the next chapters, we will show how separate and joint network-channel codings can be used in cooperative transmission networks for the purpose of diversity gain; we will also study the outage behavior of the two realizations.

3. Wireless Channels and Networks

A transmission in a wireless channel experiences significant attenuation, called *path loss*, as well as self-interference, resulting in *fading*, induced by multipath propagation of a transmitted signal. Moreover, nodes in a wireless network share a common transmission medium, which leads to interference from users operating in the same spectrum. The presence of attenuation, interference, and fading makes the design of wireless networks particularly complex and challenging. To reliably communicate over longer distance, these channel impairments require increasing transmission power, bandwidth, and/or receiver complexity.

The objective of this chapter is to give some background on wireless channels and networks. Considering the point-to-point transmission, we explain a general system model, channel coding, and modulation operations in Section 3.1. Section 3.2 discusses interference, attenuation, and fading in the wireless channel and presents how these impairments are modeled. The information-theoretic capacity of a fading channel, assuming various channel state information, is presented in Section 3.3. The conventional and incremental-redundancy network-coded cooperations, which were introduced in Chapter 1, are revisited in Section 3.4. Finally, a brief discussion of existing network architectures and cross-layer considerations in cooperative wireless networks is given in Section 3.5.

3.1. System Model of the Point-to-Point Transmission

In today's wireless network implementations, channel coding and modulation are two of the widely used techniques. Channel coding is used to detect and possibly correct transmission errors whereas modulation is used to transmit messages into the wireless channel. For a general point-to-point communication system shown in Figure 3.1, we next explain channel coding and also introduce notations relevant for upcoming discussions.

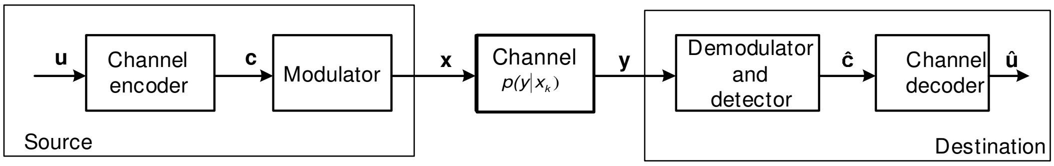

Figure 3.1.: System model for the point-to-point transmission.

3.1.1. Forward error correction with channel coding

Many wireless communication systems require a reliable communication, in the sense that a destination node obtains messages with minimal error. The use of Forward Error Correction (FEC) with channel coding is one way to protect messages against channel impairments. Consider the system diagram in Figure 3.1 and suppose the source generates a message **u** that consists of k independent and uniformly distributed bits. The channel encoder encodes the message

$$\mathbf{u} = (\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_k)$$

and outputs a message

$$\mathbf{c} = (\mathbf{c}_1, \mathbf{c}_2, \dots, \mathbf{c}_n)$$

of $n \geqslant k$ coded bits (or symbols). The block **c** is referred to as *codeword*, whereas the ensemble from which **c** is chosen is called *codebook*. The channel encoder includes $n - k$ redundant bits in order to allow the channel decoder at the destination to detect and correct (within some bound) erroneous bits without the need to ask the source for retransmission. Hence, FEC is usually applied in situations where retransmissions are relatively costly, too late, or impossible. The code rate of the channel code is defined as $R_c = k/n$. Channel coding should provide reliable communication with as little redundancy as possible. A small amount of redundancy means to use a high code rate and vice versa.

A practically important special cases are so called *linear codes*. In linear channel codes, the channel encoder is defined by the generator matrix, **G**, and the output of this encoder is given by

$$\mathbf{c} = \mathbf{u} \cdot \mathbf{G}. \tag{3.1}$$

At the destination, the task of the channel decoder is to generate the estimate $\hat{\mathbf{u}}$ from the erroneous reception $\hat{\mathbf{c}}$, which is the output of the demodulator and detector. For the linear block code given in Equation (3.1), the decoding matrix H, also called *parity check* matrix, can be designed to fulfill $\mathbf{G} \cdot \mathbf{H}^{\mathbf{T}} = 0$, where the **T** in the superscript indicates the transpose of the matrix. Accordingly, codeword **c** generated by Equation (3.1) should fulfill $\mathbf{c} \cdot \mathbf{H}^{\mathbf{T}} =$

$\mathbf{u} \cdot \mathbf{G} \cdot \mathbf{H}^{\mathbf{T}} = 0$. The destination multiplies a received codeword with the transpose of the parity check matrix. If the product is zero, then the codeword is a valid codeword (i.e., either received correctly or undetectable error(s) occurred). If the product is different from zero, then the codeword is received in error.

3.1.2. Modulation and demodulation

Modulation is used in order to transmit coded bits over the wireless channel, as it is not possible to directly send bits into this channel. A modulator takes L coded bits from the output of a channel encoder and maps them, depending on the L bits, into one of 2^L waveforms that will be transmitted over the channel. The waveforms are selected such that their characteristics, e.g., bandwidth, matches that of the channel. Due to practical constraints, the 2^L waveforms usually have a similar shape but differ in amplitude (e.g., in Amplitude Shift Keying (ASK)), phase (e.g., in Phase Shift Keying (PSK)), and/or frequency (e.g., in Frequency Shift Keying (FSK)). Although the modulated signals are often *continuous-time* and *passband* (i.e., centered at carrier frequencies ranging from kHz to GHz), it is often conceptually convenient to model them as *discrete-time* and *baseband* (i.e., centered at 0 Hz) signals. Baseband-equivalent models are convenient because they suppress the issues of frequency up- and down-conversion and discrete-time models are appealing because architectures designed for them can be efficiently implemented in digital signal processing hardware.

Assuming discrete-time and basedband-equivalent representation of a signal, the modulator can be described as an alphabet $\mathcal{X}$ of 2^L complex numbers and it is not necessary to consider the shape of the waveform. In reference to Figure 3.1, the modulator maps L coded bits of the channel encoder into modulated symbol $\mathbf{x}_i$, where $\mathbf{x}_i$ is from alphabet $\mathcal{X}$. Each $\mathbf{x}_i$ is used to scale the amplitude of the waveform in ASK, shift the phase in PSK, or shift the frequency in FSK. To mention two examples: Binary Phase-Shift Keying (BPSK) with alphabets $\mathcal{X}_2 = \{-1, +1\}$, where each symbol carries $L = 1$ bit, and Quadrature Phase Shift Keying (QPSK) with $\mathcal{X}_4 = \{-j, -1, j, 1\}$, where each symbol carries $L = 2$ bit. In a block-based transmission of Figure 3.1, the modulator maps the codeword $\mathbf{c}$ of n coded symbols to a block

$$\mathbf{x} = (\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_m)$$

of $m = n/L$ symbols where each $\mathbf{x}_i$ is from the $\mathcal{X}$.

The rate of both the channel encoder and modulator is defined as $R = k/m = (k/n) \cdot (n/m) = R_c \cdot L$. We note that by increasing L, the data rate can be increased as a single modulated symbol contains more number of coded bits. However, when the energy of the waveform is fixed, the Euclidean distance between the constellation points decreases and the probability of wrong detection increases as well. The Euclidean distance can be increased by increasing the energy of the waveform; however, this may not be desired for energy efficiency reasons or to decrease interference to other nodes. *This energy vs. rate relationship becomes even more interesting in the context of cooperative transmission and will be addressed in Chapter 5.*

The modulator sends the block $\mathbf{x}$ through the channel. The channel outputs a block

$$\mathbf{y} = (\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_m),$$

which is a distorted version of the block $\mathbf{x}$. Based on the block $\mathbf{y}$, the detector and demodulator generates the estimate $\hat{\mathbf{c}}$, which is further processed by the channel decoder. The aim of channel coding and modulation is to minimize the bit errors between $\mathbf{u}$ and $\hat{\mathbf{u}}$ given the allowed rate R and other constraints, for example the transmission power or energy.

3.2. Wireless Channel Models

A wireless channel generally suffers from large-scale fading and small-scale fading. Also, interference from nodes transmitting in the same spectrum and noise generated at a destination node cause significant signal distortions. Large-scale fading is attributed to *path loss*, which is the loss because of the separation of the source and destination, and *shadowing*, which is observed when moving over several tens of wavelengths. Small-scale fading is a random effect observed in the temporal and spatial dimensions, which can be categorized as *slow fading* vs. *fast fading* and *flat fading* vs. *frequency selective fading*. In the following, we will describe the significant channel distortions affecting wireless transmissions and provide their fairly general mathematical description.

3.2.1. Noise and interference

Thermal noise is the main type of noise at the destination generated by thermal agitation of electrons in the receiver circuit [30]. The received signal at the destination is passed through a bandpass filter with a bandwidth large enough not to distort the transmitted waveform. On the other hand, interference results when nodes in the network use the same radio frequency band. In cellular mobile radio communication systems, for example, frequency is reused so that users in geographically separated cells can use the same frequency. This introduces co-channel interference coming from the cells using the same carrier frequency. There is also adjacent channel interference due to partial spectral overlap between neighboring radio channels.

Noise is usually modeled as *additive* (superimposed on the signal), *white* (has a flat power spectral density within the bandwidth), and *Gaussian* distributed (due to the central limit theorem and the fact that noise is the cumulative result of contributions from a large number of independent sources). Noise according to this model is called Additive White Gaussian Noise (AWGN). The AWGN channel model, in baseband-equivalent form, is given as

$$\mathbf{y}[m] = \mathbf{x}[m] + \mathbf{z}[m] \tag{3.2}$$

where $\mathbf{x}[m]$ is the transmitted signal, $\mathbf{y}[m]$ is the received signal at the destination, and $-\infty < m < \infty$ is the time index. The term $\mathbf{z}[m]$ is a zero-mean AWGN process with variance N and captures the effects of thermal noise and interferences.

3.2.2. Fading channels

In free space propagation, the signal power decay is proportional to the square of the propagation distance. In more general settings, a signal can travel from the source to destination over multiple paths. This phenomenon is referred to as *multipath propagation* and arises because a propagating signal reflects off, refracts through, and diffuses around objects in the channel environment. The multiple copies of the transmitted signal might add constructively, thereby increasing the SNR, or destructively, thereby decreasing the SNR and this phenomena is generally called *fading*. The fading in wireless channels can be categorized as large-scale fading and small-scale fading [2, 61].

Large-scale fading channels

Large-scale fading represents the average signal power attenuation due to separation of the source and destination, called propagation *path loss*, or scattering from prominent terrain contours (e.g. hills, forests, billboards, clumps of buildings, etc.) between the transmitter and receiver, called *shadowing* [62]. Shadowing is observed when moving over large areas and the receiver is often represented as being “shadowed” by such prominences. Large-scale fading is considered constant in time for a specific non-changing environment, distance, and frequency.[1]

[1] Strictly speaking, path loss is a deterministic effect whereas shadowing is a random effect observed in the spatial dimension. However, for low-mobility communication scenario assumption, the effect of shadowing is observed when moved over large geographic area and it is usually incorporated into the path loss.

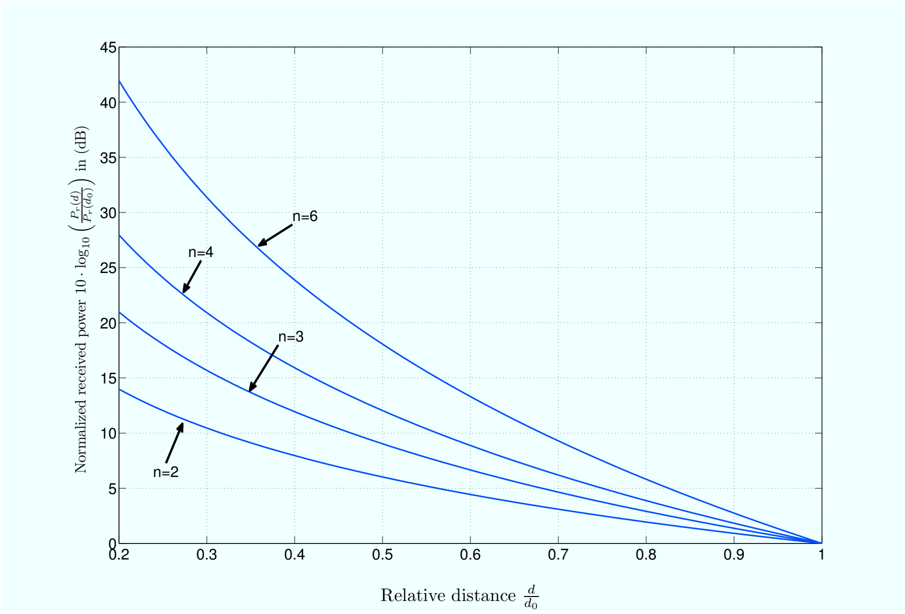

Figure 3.2.: Normalized received power, in dB, vs. distance.

In practical indoor and outdoor radio channels, the average signal power decays with distance, with some path-loss exponent, n, greater than two. The free-space transmission formula of Friis allows to calculate the received power, $P_r(d)$, at the destination when the source transmits with power, P_t, as [30, 62]

$$P_r(d) \;=\; P_t \cdot \left(\frac{\lambda}{4\pi d}\right)^n \cdot G_t \cdot G_r = \alpha_{PL} P_t \tag{3.3}$$

where G_t and G_r are the gains of the transmitter and receiver antennas, λ is the wavelength of the signal, and d is the separation distance of the source and destination. The path-loss exponent, typically $2 \leqslant n \leqslant 6$, depends on frequency, antenna height, and propagation environment and is equal to 2 for free-pace propagation. The term $\alpha_{\mathrm{PL}} = G_t \cdot G_r \cdot \left(\frac{\lambda}{4\pi d}\right)^n$ is called the *path-loss coefficient* and represents the effect of the loss because of the separation, frequency, and the antenna gains. Equation (3.3) is valid only in the far-field of the transmission antenna, where the distance d is larger than the Fraunhofer distance, $d_F = \frac{2D^2}{\lambda}$, where D is the size of the transmission antenna. At distance d, the received power is related to a power received at a reference distance, d_0, by

$$P_r(d) \;=\; P_r(d_0) \cdot \left(\frac{d_0}{d}\right)^n \tag{3.4}$$

where $P_r(d_0)$ is the received power at the reference distance. Sometimes, it is convenient to assume the reference distance $d_0 = 1$ unit and work on the normalized power $\left(\frac{P_r(d)}{P_r(d_0)}\right) = \left(\frac{1}{d}\right)^n$. This representation will be used in later chapters when we address node deployment and coverage area extension issues. The normalized power, in dB, is plotted in Figure 3.2 for various values of the path-loss coefficient, n.

The effect of large-scale fading on the transmitted signal is modeled to be multiplicative and the channel model in Equation (3.2) can be modified as

$$\mathbf{y}[m] = \sqrt{\frac{P_r(d)}{P_t}} \cdot \mathbf{x}[m] + \mathbf{z}[m] = \sqrt{\alpha_{\mathrm{PL}}} \cdot \mathbf{x}[m] + \mathbf{z}[m]. \tag{3.5}$$

One can define the instantaneous SNR of the received signal, $\mathbf{x}[m]$, as the ratio of the received signal power and the noise power and is given by

$$\gamma \;\; = \;\; \frac{P_r(d)}{N} = \alpha_{\mathrm{PL}} \cdot \frac{P_t}{N}. \tag{3.6}$$

where N is the noise variance. The term $\frac{P_t}{N}$ is sometimes called *transmit SNR*.

Example 3.2.2.1. Consider the uplink transmission in the first phase of the cooperative network shown in Figure 3.3, where the source and partner are located at a distance of 450 meters and 100 meters from the destination, respectively. Assume that each node is transmitting at a carrier frequency $f_c = 5.3$ GHz, has access to $W = 100$ MHz bandwidth, both nodes transmit at a power $P_t = 50$ mW (17 dBm), and the noise level is -94 dBm.

Using the free space path-loss model given in Equation (3.3), the received signal, in dB, at the destination is computed as

$$P_r(d) = P_t + \alpha_{\mathrm{PL}}, \tag{3.7}$$

where the path-loss term $\alpha_{\mathrm{PL}} = 20\log_{10}\left(\frac{\lambda}{4\pi d}\right) + G_a$, where $G_a = G_t + G_r = 7$ dB represents the combined antennas gain, all in dB, and $\lambda = \frac{c}{f_c}$, where c is the speed of the light. One can compute that the partner and source, respectively located at distance of 100 and 450 meters from the destination, experience path loss $\alpha_{\mathrm{PL}} = 87$ and $\alpha_{\mathrm{PL}} = 100$ dB, respectively. The received power level of the partner is -77 dBm while that of the source is -90 dBm. Note that the received power at the partner, because of its location, is greater than at the source. We will use this example in the next sections to motivate relaying in cooperative networks.

Small-scale fading channels

For most practical channels, the above propagation model is inadequate to describe the channel and predict system performance. Small-scale fading occurs due to multipath components in a channel, and refers to changes in signal amplitude and phase that can be experienced

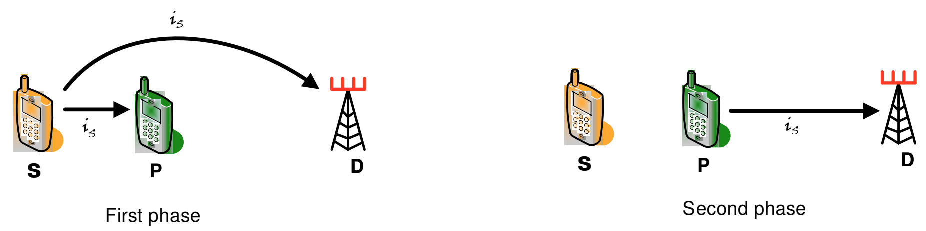

Figure 3.3.: Cooperative transmission in the uplink transmission of the mobile communication system based on repetition coding; i_s is the source's message.

as a result of small changes in the spatial separation between the source and destination. In addition to the multipath propagation, other factors that affect small-scale fading are speed of the destination, speed of surrounding objects, and signal bandwidth. Small-scale fading manifests itself in two mechanisms, namely:

- Time spreading of the signal and
- Time-variant behavior of the channel.

Based on either the time-spreading or time-variant nature of the channel, fading is further classified as follows.

Flat fading vs. frequency-selective fading Assume that a narrow pulse (e.g., impulse signal) is transmitted from the source. Because of the multipath nature of the wireless channel, the destination receives a sequence of delayed pulses of various magnitude and delay, i.e., spreading of the impulse in time. For a single transmitted impulse, the *excess delay* T_m between the first and last received component represents the maximum delay during which the multipath signal power falls to some threshold level below that of the strongest component.

Viewed in the frequency domain, for an impulse signal of infinite bandwidth, the bandwidth of the received series of impulses is finite. This means that the channel has filtered out some frequency content of the signal. The *coherence bandwidth*, f_0, is a statistical measure of the range of frequencies over which the channel passes all spectral components with approximately equal gain and linear phase. Excess delay and coherence bandwidth are approximately related by

$$f_0 \approx \frac{1}{T_m}. \tag{3.8}$$

In a multipath environment, the time spreading causes a signal to undergo either *flat fading* or *frequency-selective fading*. In flat fading, the coherence bandwidth of the channel is greater than the bandwidth of the transmitted signal, f, such that all frequency components of the signal will experience the same magnitude of fading. This means $f_0 > f \approx \frac{1}{T_s}$, where $\frac{1}{T_s}$ is the symbol rate and is nominally taken to be equal to f. Flat fading could also be viewed, in the time domain, to be the result of a multipath propagation whose excess delay, T_m, is so small compared to the symbol duration, T_s, that they add up to one undistorted signal (or the received signal is not distorted by Inter-symbol Interference (ISI)).

In frequency-selective fading, the coherence bandwidth of the channel is smaller than the bandwidth of the signal, i.e., $f_0 < \frac{1}{T_s}$, such that different frequency components of the signal will be affected differently by the channel (or experience decorrelated fading). In the time domain, the multipath components of the signal will have significant time dispersion compared to the symbol period and this results in ISI.

Fast fading vs. slow fading In wireless channels, the time-varying nature of the channel is caused by changes in the propagation path (because of a relative motion between the source and destination and/or by movement of objects within the channel). Thus, for a transmitted signal the destination sees variations in the signal's amplitude and phase. The time-variant mechanism will be characterized in the time domain by the channel *coherence time* T_c, which is a measure of the expected time duration over which the channel is essentially invariant.

The coherence time determines whether the channel can be described as *slow fading* or *fast fading*. In slow fading channel, the coherence time of the channel is greater than the symbol duration of the transmitted signal, i.e., $T_c >> T_s$. In this regime, the amplitude and phase change imposed by the channel can be considered roughly constant over the period of channel use. In contrast, fast fading occurs when the coherence time of the channel is small relative to the symbol duration of the transmitted signal, i.e., $T_c < T_s$. In this regime, the amplitude and phase change imposed by the channel varies considerably over the period of channel use.

In the fast-fading channel, the source, using *time diversity*, may take advantage of channel variations. Although a deep fade may temporarily erase some of a transmitted codeword, use of channel coding coupled with successfully transmitted bits during other time instances can allow the erased bits to be recovered. In a slow-fading channel, it is not possible to use time diversity because the transmitter sees only a single realization of the channel within its delay constraints (i.e., coherence time). A deep fade therefore lasts the entire duration of transmission and cannot be mitigated using coding.

The coherence time of the channel is related to a quantity known as the *Doppler spread* of the channel. When a user (or reflectors in its environment) is moving, the user's velocity causes a shift in the frequency of the signal transmitted along each signal path. This phenomenon is known as the *Doppler shift*. Signals traveling along different paths can have different Doppler shifts, such that when they add-up at the destination, the resulting signal will have a broader (and possibly shifted) bandwidth than the transmitted signal. This is known as the *Doppler spread*, represented as f_d, and measures this spectral broadening of the signal. In general, coherence time is inversely related to Doppler spread and typically expressed as

$$f_d \approx \frac{V}{\lambda} \approx \frac{k}{T_c}, \tag{3.9}$$

where V is the relative velocity, λ is the signal wavelength, and k is a constant taking values in the range of 0.25 to 0.5.

Model of small-scale fading Small-scale fading is sometimes called *Rayleigh fading* because if the multiple reflective paths are large in number and there is no line-of-sight signal component, the envelope of the received signal is statistically described by a *Rayleigh* probability density function given as:

$$p(x) = \begin{cases} \frac{x}{\sigma^2} \cdot \exp\left(\frac{-x^2}{2\sigma^2}\right) & x \geqslant 0; \\ 0 & \text{otherwise} \end{cases} \tag{3.10}$$

where x is the amplitude of the received signal, and $2\sigma^2$ is the mean power of the multipath signal envelope. When there is a dominant non-fading signal component, such as a line-of-sight propagation path, the small-scale fading envelope is described by a *Rician* distribution.

Flat and slow fading (which is one case of small-scale fading where there is no ISI and the channel remains the same during the period of channel use), like the large-scale fading, results in multiplicative distortion of the transmitted signal, $\mathbf{x}[m]$ [62]. For this channel, let α represents the fading attenuation term. The channel model given in Equation (3.5) can be modified to include the flat fading as

$$\mathbf{y}[m] = \sqrt{\frac{P_r(d)}{P_t}} \cdot \alpha \cdot \mathbf{x}[m] + \mathbf{z}[m] = h \cdot \mathbf{x}[m] + \mathbf{z}[m] \tag{3.11}$$

where h is a new fading coefficient that combines both the large-scale fading and flat fading. Figure 3.4 depicts this channel model. Note that Equation (3.11) assumes an ideal coherent detection (i.e., the channel fading is sufficiently slow that the phase shift can be estimated from the received signal without error). In a case that the separation distance of the source and receiver is of interest (e.g. in a coverage area study), the channel model in Equation

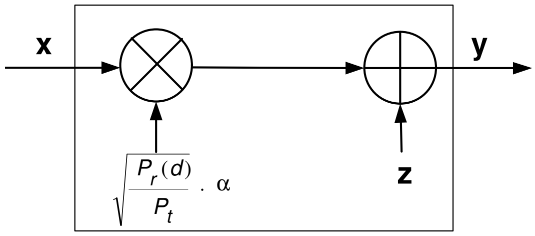

Figure 3.4.: Channel model with small-scale fading coefficient α and noise samples $\mathbf{z}$.

(3.11) can be represented as

$$\mathbf{y}[m] = \frac{h}{d^{n/2}} \cdot \mathbf{x}[m] + \mathbf{z}[m]. \tag{3.12}$$

The instantaneous SNR of the received signal, $\mathbf{x}[m]$, which was defined in Equation (3.6) can now be written as

$$\gamma \;=\; |\alpha|^2 \cdot \alpha_{PL} \cdot \frac{P_t}{N} = \frac{|h^2|}{d^n} \cdot \frac{P_t}{N} \tag{3.13}$$

and the average SNR, Γ, is given by

$$\Gamma = E\left[\gamma\right] = \frac{E\left[|h^2|\right]}{d^n} \cdot \frac{P_t}{N} \tag{3.14}$$

where $E\left[\cdot\right]$ is the expectation operation. For $|\alpha|$ Rayleigh-distributed random variable representing the magnitude of the small-scale fading term, the random variable $|\alpha|^2$, corresponding to the signal's power, is *exponentially distributed*.

3.3. Information Theory – Fading Channel Capacity

The aim of a communication system is to provide a reliable communication with as little overhead as possible. Reliable communication means that an arbitrarily small probability of decoding error can be achieved. Information theory defines capacity of a channel and provides a limit for it. Re-considering the system model in Figure 3.1, let $\mathbf{x}$ and $\mathbf{y}$ be random variables representing the channel input and output with alphabets $\mathcal{X}$ and $\mathcal{Y}$, respectively. The alphabets $\mathcal{X}$ and $\mathcal{Y}$ can assume either discrete values or continuous values.

3.3.1. Capacity of Additive White Gaussian Noise Channel

Consider a discrete-input, continuous-output channel. A block $\mathbf{x} = (\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_m)$ is transmitted by the modulator, where each input symbol, $\mathbf{x}_i$, for $1 \leqslant i \leqslant m$, is drawn from the alphabet $\mathcal{X}$. The AWGN channel is modeled by

$$\mathbf{y}_i = \mathbf{x}_i + \mathbf{z}_i,$$

where $\mathbf{z}_i$ is the zero-mean Gaussian random variable with variance N. This channel is a typical example of a discrete-input, continuous-output channel. The output $\mathbf{y}_i$ of the channel is continuous (or unquantized) and can assume any value on the real line, i.e., $\mathcal{Y} = (-\infty, \infty)$. Assume that the power constraint on the input block is given as

$$\sum_{i=1}^{m} E\left[\mathbf{x}_i^2\right]/m \leqslant P_t. \tag{3.15}$$

This channel is described by the conditional probability density function[2] $p(\mathbf{y}_i|\mathbf{x}_i)$. The capacity of this channel is the maximum rate $R = k/m$ bits per channel use[3] for which, for sufficiently large m, there exists a **u**-to-**x** mapping (encoder and modulator) and a **y**-to-$\hat{\mathbf{u}}$ mapping (detector, demodulator, and decoder) so that the error probability $Pr[\hat{\mathbf{u}} \neq \mathbf{u}]$ can be made as close to 0 as desired (but not necessarily exactly 0). The capacity, in bits per channel use, is the maximum average mutual information between the discrete input $\mathbf{x}_i \in \mathcal{X}$ and output $\mathbf{y}_i \in (-\infty, \infty)$ and is given as

$$\begin{aligned} C &= \max_{P(\mathbf{x}_i)} I(\mathbf{x}_i; \mathbf{y}_i) \\ &= \max_{P(\mathbf{x}_i)} \sum_{\mathbf{x}_i \in \mathcal{X}} \int_{-\infty}^{\infty} p(\mathbf{y}_i|\mathbf{x}_i) P(\mathbf{x}_i) \log_2 \frac{p(\mathbf{y}_i|\mathbf{x}_i)}{p(\mathbf{y}_i)} d\mathbf{y}_i \quad \text{[bits/channel use]}, \end{aligned} \tag{3.16}$$

where $P(\mathbf{x}_i)$ is the probability that $\mathbf{x}_i$ is sent, and $I(\mathbf{x}_i; \mathbf{y}_i)$ is referred to as *mutual information* and physically represents the amount of information that can be deduced about $\mathbf{x}_i$, based on observing $\mathbf{y}_i$.

Capacity is essentially the maximum, over the input distribution of $\mathbf{x}_i$, of the amount of information about $\mathbf{x}_i$ that can be inferred from $\mathbf{y}_i$. Note that if $\mathcal{Y}$ is from a discrete alphabet, then $p(\mathbf{y}_i|\mathbf{x}_i)$ and $p(\mathbf{y}_i)$ in Equation (3.16) are replaced by the probability functions $P(\mathbf{y}_i|\mathbf{x}_i)$ and $P(\mathbf{y}_i)$, respectively, and also the integral is replaced by a sum. From Equation (3.16), an

[2] Also called transition probability density function.

[3] Bits per channel use means bits per input symbol into the channel. If a symbol enters the channel every T_s seconds (for every symbol period a symbol is transmitted), the channel capacity, in bits per second, is $\frac{C}{T_s}$, where C is the capacity in bits per channel use.

arbitrarily reliable communication can be achieved for all rates $R < C$. For rates $R > C$, the probability of error is strictly bounded above zero. It is shown that the best $\mathbf{x}_i$ that maximizes the capacity in Equation (3.16) is Gaussian with zero mean and variance P_t (power constraint in Equation (3.15)) and the capacity is given as [63]

$$C = \frac{1}{2}\log_2(1 + \frac{P_t}{N}) = \frac{1}{2}\log_2(1+\gamma) \quad \text{[bits/channel use]} \tag{3.17}$$

where $\gamma = \frac{P_t}{N}$ is the instantaneous SNR. The capacity in Equation (3.19) is sometimes called called *Shannon capacity* or the *instantaneous capacity*. In a case that both $\mathbf{x}_i$ and $\mathbf{z}_i = \mathbf{z}_{i,R} + j\mathbf{z}_{i,I}$ are complex with $\mathbf{z}_{i,R}$ and $\mathbf{z}_{i,I}$ are independent, real, Gaussian random variables with variance $N/2$ and $j = \sqrt{-1}$, the Shannon capacity becomes

$$C = \log_2(1+\gamma) \quad \text{[bits/channel use].} \tag{3.18}$$

The Nyquist-Shannon sampling theorem states that a complex signal, $\mathbf{x}_i$, of bandwidth W can be represented by approximately W complex samples per second, and the channel capacity in Equation (3.18), in [bits/second], is then

$$C \;=\; W \cdot \log_2(1+\gamma) \quad \text{[bits/second].} \tag{3.19}$$

3.3.2. Capacity of flat and slow fading channels

Consider the case that the channel is flat fading, slow fading, and AWGN, whose channel model is given by Equation (3.12) as

$$\mathbf{y}_i = \frac{h}{d^{n/2}}\mathbf{x}_i + \mathbf{z}_i$$

where $1 \leqslant i \leqslant m$, the same power constraint as given by Equation (3.15). The only difference between a fading channel and the AWGN channel is the random channel gain, h, and the Shannon capacity formula for the AWGN channel works for fading channel as well, i.e.,

$$C = \log_2\left(1 + \frac{|h|^2}{d^n} \cdot \frac{P_t}{N}\right) = \log_2(1+\gamma) \quad \text{[bits/channel use].} \tag{3.20}$$

The capacity in Equation (3.20) is plotted in Figure 3.5 as the curve labeled “Gaussian”. Equation (3.20) is based on a general complex Gaussian input $\mathbf{x}_i$ with zero-mean and variance P_t assumption. In a digitally modulated system, however, $\mathbf{x}_i$ assumes only a limited

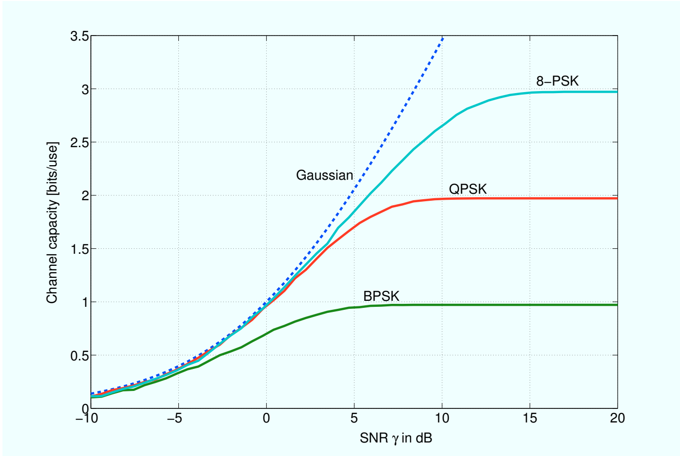

Figure 3.5.: Channel capacity in [bits/channel use] for input symbols $\mathbf{x}_i$ from a Gaussian distribution, BPSK, QPSK, or 8-PSK modulation.

number of values. For example, in M-*ary phase-shift keying* (M-PSK) $\mathbf{x}_i$ is uniform over the alphabet

$$\mathcal{X} = \left\{ \sqrt{E_s}, \sqrt{E_s}\exp^{j2\pi/M}, \ldots, \sqrt{E_s}\exp^{j2\pi(M-1)/M} \right\},$$

where $E_s = P_t$ is the (average) per-symbol-energy. The capacity of the complex AWGN channel with M-PSK is derived in [63, 64]. Figure 3.5 depicts the capacity of BPSK, QPSK, and 8 Phase Shift Keying (8-PSK). From this plot, we note the following points

1. Sending at full rate through the channel is possible only at high SNR values (called *high*-SNR region). This is the region of interest in the study of cooperative protocols and will be used in the next chapters, when we compute outage probability of various protocols.

2. The capacity of the Gaussian input is the upper bound. Moreover, at high SNRs, one must use large symbol sets, while at low SNRs it seems that BPSK suffices [63].

3.3.3. Channel state information and channel capacity

Channel State Information (CSI) is an important issue affecting the design and analysis of communication systems. CSI refers to knowledge of channel realization (or the channel coefficient, h) at a node in a network. Depending on which node in the network has knowledge of the channel, here are the possible cases to consider.

CSI available at the destination This is a standard assumption in many wireless systems and is reasonable when the channel is slow fading. In slow fading channels, the destination may use, for example, training signals such as pilot tones to "learn" the channel. The capacity of the fading channel with destination side information is given by Equation (3.20).

CSI available at the source This assumption is reasonable when the channel is fading slowly and when the destination can share the CSI with the source by means of a separate feedback channel. In this case, the source can adjust its power level and rate according to the channel state. One adaptation rule would be to conserve power by not transmitting when the CSI falls below a certain threshold, and transmit if the CSI lies above the threshold. Any adaptation must be performed subject to an appropriate power constraint. In this case, the Shannon capacity expression can be written in the form

$$C = \max_{P_t(\cdot)} \log_2\left(1 + \frac{|h|^2}{d^n} \cdot \frac{P_t(h)}{N}\right) \tag{3.21}$$

where $P_t(\cdot)$ represents the power allocation function subject to an average power constraint $E[P_t(h)] \leqslant P_t$.

No CSI at the source and destination In reality, the CSI is never exactly known to the source or destination. Here, the capacity depends not only on the marginal density of the channel (Rayleigh, Rician, etc.), but also on the temporal correlation of the channel and computing the channel capacity is an open problem.

3.3.4. Ergodic fading channels

In stationarity channel, the statistical properties of h (such as its mean and variance) will not change over time. When h is also ergodic, then the time average of the channel coefficient h is equal to its ensemble average and these prosperities can be deduced from a single and sufficiently long sample of the process h. In other words, the randomness of h can be averaged out (removed) over time so that long-term constant bit rates can be supported (like AWGN channels). In general, an *ergodic channel* is defined as a channel whose h varies (or fades fast enough) over a (possibly finite but very long) codeword but all its moments are the same from codeword to codeword. In that case, the capacity is given as

$$C = E\left[\log_2\left(1 + \frac{|h|^2}{d^n} \cdot \frac{P}{N}\right)\right] \tag{3.22}$$

where $E[\cdot]$ denotes the expectation with respect to h and this can be computed using the distribution of h. Equation (3.22) means that as the mean of the fading statistics can be observed with high reliability, such a channel can support any rate not exceeding the capacity C. In terms of diversity, these temporal variations allows the coding strategy to fully exploit temporal diversity.

3.3.5. Non-ergodic fading channels and outage probability

Consider a slow fading channel where the channel state is random but held constant for the duration of a codeword. This channel is called a *non-ergodic* channel or, sometimes *block-fading* channel. In non-ergodic channel, the randomness of the channel gain can not be averaged out (removed) over time so that long-term constant bit rates can not be supported. Shannon capacity is not a useful performance measure because, for example, if a channel is in a deep fade, then the channel state, h, as well as the capacity will be zero. However, since the channel realization is random and kept constant over the codeword transmission, there is a non-zero probability that a given transmission rate can be supported by the channel, and this probability is called *outage probability*. The largest rate of reliable communication at a certain outage probability is called the *outage capacity*.

Outage probability helps to examine the tradeoff between a fixed rate and the probability that this rate is achievable over the channel. For example, for a fixed desired rate, R, those channel states with

$$C \;=\; \log_2\left(1+\frac{|h|^2}{d^n}\cdot\frac{P_t}{N}\right) < R \tag{3.23}$$

will not support the rate, i.e., there is an outage. The event $\left\{\log_2\left(1+\frac{|h|^2}{d^n}\cdot\frac{P_t}{N}\right) < R\right\}$ is called an *outage event*, and the outage probability is the probability that this event occurs. In block-fading channels, lack of temporal variation in the channel state over a codeword duration prevents a coding strategy from exploiting temporal diversity. In the fast fading channel, in contrast, outage can be avoided due to the ability to average over the time variation of the channel, and one can define a capacity at which arbitrarily reliable communication is possible.

Example 3.3.5.1. In this example, we will compute the outage probability in a block-fading channel. We observe that the capacity is a random variable whose distribution depends on the distribution of the fading coefficient. From the outage event $\left\{\log_2\left(1+\frac{|h|^2}{d^n}\cdot\frac{P_t}{N}\right) < R\right\}$, we deduce that $|h|^2 < \frac{2^R-1}{\Gamma_T}$, where $\Gamma_\mathrm{T} = \frac{P_t}{d^n N}$. The outage probability, denoted as $P_{out,ppt}$, is then computed as

$$P_{out,ppt} = \Pr\left[|h|^2 < \frac{2^R - 1}{\Gamma_{\mathrm{T}}}\right]. \tag{3.24}$$

If the channel state $|h|$ is Rayleigh distributed, $|h|^2$ will have an exponential distribution with parameter $\frac{1}{\Gamma_h}$, where $\Gamma_h = E\left[|h|^2\right]$. The outage probability in Equation (3.24) evaluates to

$$P_{out,ppt} = \int_0^{\frac{2^R-1}{\Gamma_T}} \frac{1}{\Gamma_h} \cdot \exp\left(-\frac{h}{\Gamma_h}\right) dh = 1 - \exp\left(-\frac{2^R - 1}{\Gamma_h \Gamma_{\mathrm{T}}}\right) \approx \frac{2^R - 1}{\Gamma_h \Gamma_T}. \tag{3.25}$$

The approximation in Equation (3.25) holds for high $\Gamma_h \Gamma_{\mathrm{T}}$ values (or in the high SNR region). In block-fading channels, outage probability is used to compute *diversity*, an important figure of merit that measures reliability of communication. Diversity measures the rate at which the outage probability decays with respect to SNR. In Equation (3.25), this decay is proportional to $(\Gamma_h \Gamma_{\mathrm{T}})^{-1}$, and hence the diversity gain is 1. In cooperative transmission, higher diversity gain is obtained by the use of relaying nodes in the networks.

Example 3.3.5.2. Re-consider the link budget calculation Example 3.2.2.1 whose network was shown in Figure 3.3. Let us additionally assume that $E\left[|h|^2\right] = 1$ and a capacity C of 400 Mb/s is desired. Because of $E\left[|h|^2\right] = 1$ assumption, the average received power will remain the same as calculated, i.e., -77 dBm for the partner and -90 dBm for the source.

From the Shannon formula $C = W \log_2(1 + \gamma)$, we compute that a SNR of 12 dB is required to support the desired rate. For a noise level of -94 dBm, either the source or partner will not be in outage if the received signal power level is at least -82 (= -94 + 12) dBm. As the received power level of the partner is -77 dBm while that of the source is -90 dBm, the partner is not in outage; however, the source is in outage since the received power is 8 dB below the required power level. However, one way for the source to avoid outage is to use the partner as a relaying node such that the overall path loss is reduced.

3.3.6. Capacity vs. combining schemes

Consider the case of *repetition coding* where the destination node receives multiple copies of the same codeword (either through retransmission from the source or other nodes in the network, which correctly receive from the source and forward to the destination). Maximum-Ratio Combining is used to combine the multiply received codewords. As a result of this combining, the SNR of the codeword accumulates. If the destination receives m copies of the codeword and if all channels are block fading, the Shannon capacity is given as

$$C = \frac{1}{m} \cdot \log_2\left(1 + \sum_{i=1}^{m} \frac{|h_i|^2}{d_i^n} \cdot \frac{P_t}{N}\right) \tag{3.26}$$

where h_i and d_i^n are the channel coefficient and separation distance from node i to the destination, respectively. The $\frac{1}{m}$ is included if each transmission takes $\frac{1}{m}$ of the available time.

If the source and relaying nodes in the network re-transmit incrementally redundant information, the destination can employ *code combining* to combine parts of the received codeword. By using redundant transmission, more information bits are delivered to the destination and this is reflected as addition in the channel capacity. The amount of incremental information need not necessarily be the same as the information sent at the first time. If each transmission is finished in α_i of the available time and $\sum_{i=1}^{m} \alpha_i = 1$, then the capacity of this channel for m re-transmissions is given as

$$C = \sum_{i=1}^{m} \left[\alpha_i \cdot \log_2 \left(1 + \frac{|h_i|^2}{d_i^n} \cdot \frac{P_t}{N} \right) \right]. \tag{3.27}$$

3.4. Network-Coded Cooperation

As explained in the above sections, the presence of time-varying fading channels makes the design of wireless networks complex. One way to combat the effect of fading is by using spatial diversity. As explained in Section 1.1, network-coded cooperation is a cooperative transmission approach that realizes spatial diversity. In this section, we will formulate a generalized channel model and define general assumptions that will be used in the next chapters.

3.4.1. System model for network-coded cooperation

Consider the system model of the network-coded cooperation shown in Figure 3.6. In network-coded cooperation, in the *first phase* of cooperation, both the source and partner send a codeword of $N_1 = \alpha N$ symbols, where α is the cooperation level. In the conventional network-coded cooperation of Chapter 4, $\alpha = 1/2$ and in the incremental network-coded cooperation of Chapter 6, $0 \leqslant \alpha \leqslant 1$. N is the number of symbols per codeword if the point-to-point transmission were used. Then both nodes, using orthogonal channels, transmit their respective codewords to the destination and simultaneously try to decode each other's codeword.

If, for example, the source decodes the partner's message correctly, then it forms a network-coded message by *linearly combining* its own and the received message (details of the content of this message will be explained in the upcoming chapters). This network-coded mes-

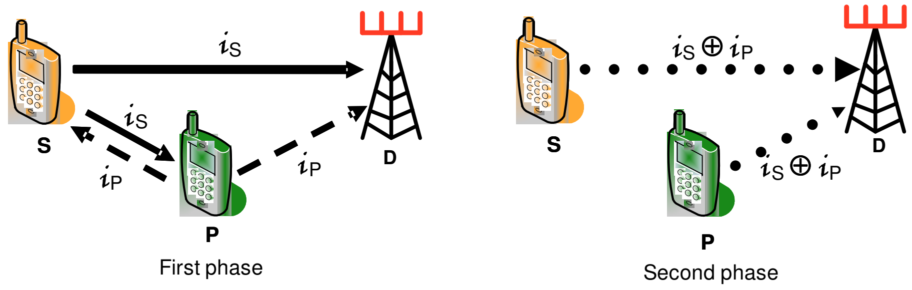

Figure 3.6.: System diagram of network-coded cooperation in the uplink transmission. Solid, dashed, and dotted lines show the transmission of the source's message, partner's message, and network-coded message, respectively.

sage of $N_2 = (1 - \alpha)N$ symbols is transmitted to the destination in the *second phase*. The partner will also perform the same operation in the next cycle.

3.4.2. Equivalent channel model

In our model of the wireless channels, narrowband transmissions suffer from the effects of frequency non-selective slow fading and additive noise. The coherence time of the fading is long enough such that the fading does not change for the transmission of a large number of (block of) transmitted symbols, i.e., block fading channel. The medium-access control in our model imposes the practical system constraints of orthogonal transmission and half-duplex relaying. We also assume that CSI is available at the destination, i.e., channel coefficients are perfectly estimated at the destination, and perfect synchronization exists between nodes and each receiving node is capable of coherent detection.

Consider a baseband-equivalent, discrete-time channel model for the continuous-time channel, and also consider $2N$ consecutive uses of the channel, where N is a large integer is utilized. For point-to-point transmission, our baseline for comparison, the channel model for the transmission from the source to destination, is

$$y_{s,d}[m] = h_{s,d} i_s[m] + z_d[m] \tag{3.28}$$

for, say, $m = 1, \ldots, N$, where $i_s[m]$ is the source-transmitted signal, and $y_{s,d}[m]$ is the destination-received signal. The partner transmits for $m = N + 1, \ldots, 2N$. The fading $h_{s,d}$ captures the effects of path loss, shadowing, and frequency non-selective fading, and $z_d[m]$ captures the effects of receiver noise and other forms of interference in the system.

For network-coded cooperation, the source uses the channel for $m = 1, \ldots, \alpha N$ in the first phase. The source-destination and source-partner channels are modeled as

$$\begin{aligned} y_{s,d}[m] &= h_{s,d}x_s[m] + z_d[m] \\ y_{s,p}[m] &= h_{s,p}x_s[m] + z_p[m]. \end{aligned} \tag{3.29}$$

Likewise, for $\alpha N + 1, \ldots, 2\alpha N$ the partner sends its message to the destination and the source. The partner-destination and partner-source channel models are given by

$$\begin{aligned} y_{p,d}[m] &= h_{p,d}x_p[m - \alpha N] + z_d[m] \\ y_{p,s}[m] &= h_{p,s}x_p[m - \alpha N] + z_s[m]. \end{aligned} \tag{3.30}$$

Similarly, in the second phase of transmission, the destination receives from the source for channel use $2\alpha N + 1, \ldots, (1 + \alpha)N$ and from the partner for channel use $(1 + \alpha)N + 1, \ldots, 2N$. In the case of conventional network-coded cooperation, the model of the source-destination and partner-destination channels are given by

$$\begin{aligned} y_{s,d}[m] &= h_{s,d}\left(x_s[m - 2\alpha N] \oplus x_p[m - 2\alpha N]\right) + z_d[m] \\ y_{p,d}[m] &= h_{p,d}\left(x_s[m - (1 + \alpha)N] \oplus x_p[m - (1 + \alpha)N]\right) + z_d[m]. \end{aligned} \tag{3.31}$$

In the case of incremental redundancy network-coded cooperation, the channel model in Equation (3.31) should be modified a little as only a part of the message of the source and partner are forwarded.

3.5. Cross-Layer Design in Cooperative Wireless Network

In wireless networks, to reliably transmit messages among nodes, the network should mitigate channel impairments, e.g. fading and interference, and efficiently allocate and utilize network resources, e.g. power and bandwidth. One approach to realize reliable communication is to partition the network architecture into a set of protocol layers. Figure 3.7 illustrates layers in an existing wireless network architecture and indicates the functions they usually serve. As examples, the Medium-Access Control Layer (MAC) manages interference in the network and the Physical Layer (PHY) combats fading with channel coding, spread-spectrum, or multiple antennas. In the next sections, we will discuss the existing network architecture as well as considerations when cooperative transmission is included in the existing networks.

3.5.1. Existing wireless network architecture

In the point-to-point transmission, consider the source runs an application layer process that wishes to transmit messages to an application layer process at the destination node. Messages are encoded as data packets with appropriate headers that identify the application process, the source node, and the destination node.[4] Roughly, the tasks allocated to each layer are as follows. The *application layer* generates user messages, and conveys them through an interface to the *transport layer*. These packets are buffered and sequenced by the transport layer, typically Transmission Control Protocol (TCP), that implements both reliable end-to-end connection as well as end-to-end congestion and flow control [63]. The release of packets to the network layer is controlled by a reverse stream of TCP Acknowledgment (ACK)s from the destination TCP process.

Finding routes (via a sequence of point-to-point links) to the destination, maintaining these routes, and forwarding packets along these routes is the task of the *network layer*. The *data link layer* ensures reliable packet transmission on a single point-to-point link. This layer may include a MAC sublayer that regulates channel access [63]. At the data link layer of the source, it is a common practice to append error-detecting codes, such as parity-bit or CRC, to each packet. The CRC allows the data link layer at the destination to detect packet reception errors. Sequence numbers may also be added to facilitate Automatic Repeat Request (ARQ) retransmission protocols at the link layer. The retransmission is triggered by the result of the error detecting code. If the destination determines that the packet is in error, it sends a Negative Acknowledgment (NACK) to the source, otherwise it sends an ACK. In the former case, the packet is retransmitted.

The *physical layer* PHY, which incorporates a majority of the analog circuitry and signal processing, transmits signals at the source and receives and processes signals at the destination. Channel coding and modulation are two of the basic functionalities of the physical layer. Other schemes that are used in the physical layer are bit-rate adaptation, channel selection, and recently techniques such as MIMO smart antennas, cooperative transmission, interference cancellation, and ultra-wide band transmission (UWB).

The throughput of ARQ protocols can be improved by combining them with channel coding in the form of Hybrid ARQ. Hybrid ARQ lets erroneously received packets be collected and combined in various ways before decoding. Packet combining can be based on hard decisions or soft channel outputs. In soft channel outputs, noisy versions of the same packet

[4] In the context of network layering, a message is information/data that the application layer of the source wishes to communicate and a packet generated and transmitted by a source and addressed to a particular destination. Along the way, several intermediate nodes may contribute to the communication of the message to the destination, but each node may transmit its own unique data packets [63].

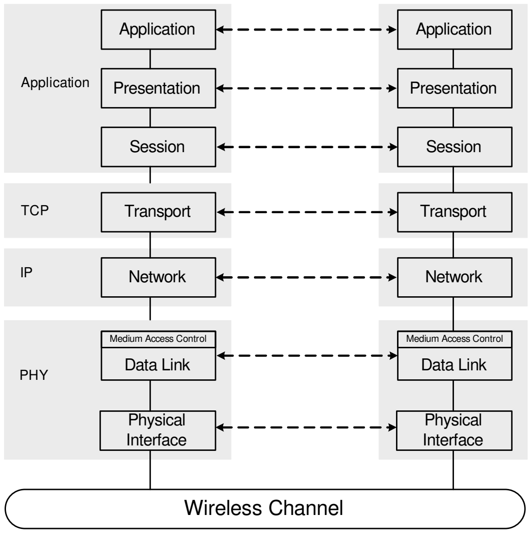

Figure 3.7.: Existing wireless network protocol stack.

are combined by MRC, EGC, or SC techniques. The transmitted packets can thus be viewed as symbols of a repetition code. Incremental redundancy ARQ can also be realized, with both hard decisions and soft channel outputs, e.g. using rate-compatible punctured convolutional codes (RCPC), by first sending the highest rate code from the RCPC code family and then sending additional bits as needed.

When Hybrid ARQ is employed, the line between the physical layer, link layer, and the MAC sub layer is blurred (see the gray boxes of Figure 3.7) [63]. These three layers are lumped together as a PHY layer and this combined PHY layer is just an interface queue that accepts IP packets. This becomes even more complicated in the case of network-coded cooperation as it involves interactions among the physical, link and network layers.

3.5.2. Cooperative wireless network architecture

Like the Hybrid ARQ which involves interaction of the physical and link layers, cooperative networks need a more complex set of interactions between the physical, link, and network layers. In cooperative networks, transmissions are not point-to-point and routing is not store-and-forward. In a cooperative network, a packet received in error at the PHY layer is not necessarily discarded; instead, such a packet may be saved by the link layer and subsequently combined with other received packets or perhaps pushed up to the network layer as a packet with errors (e.g. at a relaying node of the static decode-and-forward cooperation) [65]. Thus in cooperative transmission, we distinguish between receiving a packet reliably or unreliably

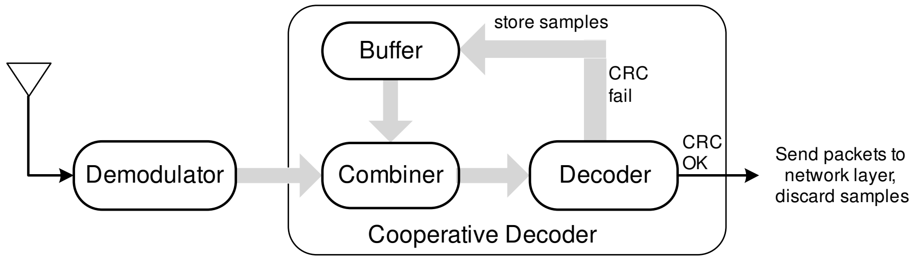

Figure 3.8.: Cooperative transmission physical layer.

as a synonym for error-free or erroneous reception, respectively [63]. The reliability/unreliability decision could be based on a CRC check. The next section discusses the physical layer at the destination and partner nodes of cooperative networks.

3.5.3. Destination node receiver

At the destination node of cooperative networks, possibly unreliable reception of multiple transmitted packets are first combined and then decoded. As mentioned above, packet combining can be based on either hard decision or soft decision channel outputs [63]. A cooperative transmission PHY layer receiver architecture is shown in Figure 3.8. As in a conventional receiver, demodulation and sampling occurs in the demodulator module, yielding a soft symbol output stream. The soft symbols of previously received packets are stored in a buffer and a combiner merges the stored packets with the newly received packet. The precise action of the combiner and the buffer depends on the type of cooperative protocol implemented.

When diversity is achieved through repetition coding, the new packet is simply a copy of a previously received packet and the combiner performs MRC on the soft symbols of the packet copies. In this case, the buffer can store the soft symbols corresponding to a linear combination of past received packets, regardless of how many packet copies are received for a particular message. This applies to the conventional network-coded cooperation as well. In this cooperation, in the case that the inter-user channel is good, two copies of a network-coded packet are received at the destination from the source and partner. When the inter-user channel has bad quality, the source may repeat its packet in the second phase. In either case, the combiner of Figure 3.8 is used to combine twice received packets. The network layer performs network decoding on correctly received packets sent from the decoder.

In incremental-redundancy-based relaying strategies (e.g. coded cooperation or incremental redundancy network-coded cooperation), the new packet contains new coded symbols. These approaches require the destination to store soft samples for all received packets for decoding.

The storage requirements of the buffer increase linearly with the number of received packets for that message. Furthermore, the decoder component becomes considerably more complex because decoding is based on multiple transmissions from multiple transmitters employing multiple codebooks.

3.5.4. Partner node receiver

In decode-and-forward cooperation, the partner passes (reliably or unreliably) received packets to the network layer. The network layer reads the packet header, possibly modifies this header, and sends the packet back to the PHY layer, from where it will be forwarded to the destination. In network-coded cooperation, the network layer combines locally generated packets (i.e., from application layer) and received packets from the PHY layer. A key observation is that all data packet transmissions are generated by the network layer. The network-coded packet can be sent to the PHY and forwarded to the destination. In incrementally redundant network-coded cooperation, the PHY can puncture the network-coded packet received from the network layer and forwards incremental information to the destination.

3.6. Summary

In this chapter, considering the point-to-point transmission, we have in general explained channel coding and modulation operations, discussed common causes of distortion in a wireless channel and how they are modeled, and gave background knowledge on the information-theoretic capacity of fading channels. Section 3.1, Section 3.2, and Section 3.3 were written to explain these points. The conventional and incremental-redundancy network-coded cooperations were revisited in Section 3.4, and the appropriate channel model was presented. The last section of the chapter was dedicated to briefly discuss existing network architecture and cross-layer considerations in cooperative wireless networks. In this section, we have seen that cooperative networks require a more complex set of interactions between the physical, link, and network layers.

4. Outage Behavior of Network-Coded Cooperation

In Chapter 3, we have introduced *network-coded cooperation* as a cooperative transmission protocol with additional network coding, as well as its extension which we call *incremental network-coded cooperation*. In this chapter, we first examine the outage behavior of the network-coded cooperation protocol by deriving its outage probability. Outage probability helps to study the protocol independent of any particular coding scheme and is also shown to be a lower bound on block error rate for sufficiently large block lengths [10]. To make the outage probability analysis more tractable and convenient for exposition, quasi-static (or block) Rayleigh fading channels, orthogonal transmission, and half-duplex constraints are assumed. Approximating the outage result at high SNR values, we first show that this protocol achieves full diversity (order two for two users) asymptotically in transmit power. Second, we investigate the outage behavior for various inter-user (i.e., between transmitting nodes) and uplink (between a transmitting node and destination) channel qualities; and we compare various cooperative protocols based on the inter-user channels. Based on the outage results, network-coded cooperation protocols are found to be suitable when the inter-user channels are lower quality; when the inter-user channels are good, protocols without network coding perform better. Third, the outage results are further extended to study the diversity-multiplexing tradeoff and the coverage area extension.

Section 4.1 describes the system and channel models under investigation. The outage probability of the point-to-point transmission and network-coded cooperation (considering both SC and MRC at the destination) are derived in Section 4.2. In Section 4.3, numerical results are presented, results are discussed, and conclusions on the general outage behavior and comparison results are drawn. Finally, the diversity-multiplexing tradeoff of the network-coded cooperation and its coverage area analysis are investigated in Section 4.4 and Section 4.5, respectively.

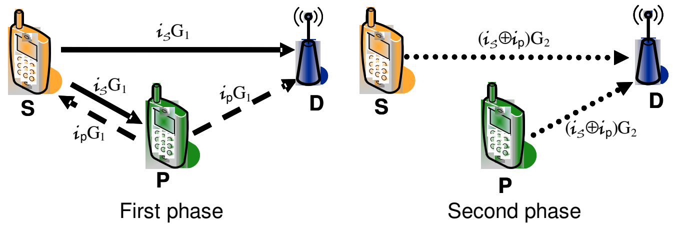

Figure 4.1.: System diagram of network-coded cooperation. Solid, dashed, and dotted lines show the transmission of S, P, and the network-coded codewords, respectively.

4.1. System model

We focus on the case of two cooperating terminals, called source (S) and partner (P), communicating to the same destination terminal (D) as depicted in Figure 4.1. In our model of the wireless channel, transmissions suffer from the effect of quasi-static Rayleigh fading channels, in which the fading remains constant over the two phases (also called *block-fading* Rayleigh channels), and also additive noise. This model is appropriate for many types of ad hoc and sensor networks in which the nodes move slowly, or are fixed but with the exact geometry unknown at the time of design [10]. In addition, practical system constraints such as orthogonal transmission and half-duplex constraints are considered (refer Figure 4.2). The orthogonal transmission constraint allows for the system to be readily integrated into existing networks and makes the analysis of outage probability more tractable and convenient for exposition. The orthogonality constraint is fulfilled by dividing the available bandwidth into orthogonal channels and allocate these channels to the transmitting terminals. The medium-access control (MAC) sublayer typically performs this function.

The system model considered in this work is shown Figure 4.1 whose timing diagram shown in Figure 4.2. In network-coded cooperation, in the *first phase* of cooperation, the source and partner first encode k of their information bits into a codeword of $N_1 = \alpha N$ symbols, where α is the cooperation level[1] and N is the number of symbols per codeword if the point-to-point transmission was used. Then both nodes, using orthogonal channels, transmit their respective codewords to the destination and simultaneously try decoding each other's codeword. If, for example, the source node decodes the partner's k bits correctly, then it forms *network-coded bits* by *linearly combining* its own and the decoded bits. These network-coded bits are further channel encoded (using the same or a different codebook) to form a *network-coded codeword* of length $N_2 = (1 - \alpha)N$, and the resulting codeword is transmitted to the destination in the *second phase*. The partner will also perform the same operation. If decoding fails, then the source (respectively the partner) transmits additional N_2 symbols for its own or remains

[1] In this chapter, we only consider the case $\alpha = 1/2$.

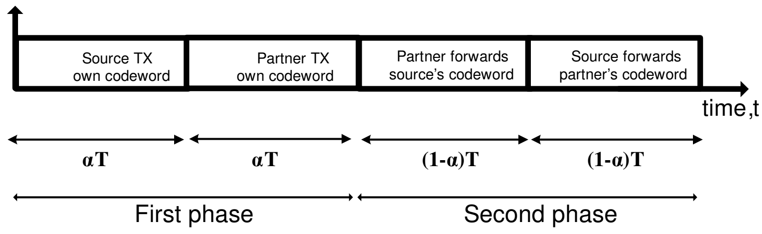

Figure 4.2.: Timing diagram of network-coded cooperation. T is the total time allocated to transmit N symbols if the point-to-point transmission was used, and α is the fraction of the time T in which users allocate to phase one. In network-coded cooperation protocol $\alpha = 1/2$ and in incremental network-coded cooperation $0 < \alpha \leqslant 1$.

silent. Cyclic redundancy check (CRC) is assumed to detect any decoding error. Moreover, incorporating one additional bit in the second-phase transmission would help the destination to know the success of decoding at the source and partner. As the N_1 and N_2 symbols contain the same n information bits as the N symbols of point-to-point transmission, for $\alpha = \frac{1}{2}$ the information rates in both phases are $n/N_1 = n/N_2 = 2n/N$, which are double the rate of point-to-point transmission n/N.

Once the destination receives the four codewords, it first combines the two network-coded codewords using either SC, MRC, or EGC. Then the source's, partner's, and network-coded codewords are decoded to get the respective information bits. Note that the network coding at transmitting nodes is performed before channel coding and the network decoding at the destination is performed after channel decoding. The network-coded cooperation can be seen as a *separate network-channel coding* scheme in that the two codings are done separately. If the information bits of the source and partner pass the CRC check, then the network-coded bits are discarded. However, if for example the information bits from the source fail the CRC check, then they can still be received by *network decoding* of the partner and the network-coded bits, provided both are correctly recovered. A simple modulo-2 summation can implement the network encoding and decoding at the transmitting nodes and destination node, respectively. The same operation is performed to recover the partner's information bits. Intuitively, information bits are recovered using either of the two options, which means that a maximum *diversity order* of two can be obtained. The actual value of the diversity order depends on received SNR and transmission rate.

The last point concerns energy consumption as in [9], we confine the total energy[2] spent to transmit N symbols is the same in both network-coded cooperation and point-to-point

[2]This energy, also called *radiated energy*, refers to the energy available at the transmitter antenna. In Chapter 5, we also include the processing energy at both transmitting and receiving nodes as well as the energy spent at the power amplifier.

transmission. This is referred to *total energy constraint*, and this total energy is shared in the two phases of cooperation. Let β be the fraction of the total energy allocated to the first phase. If E_s is the radiated energy per symbol in point-to-point transmission, then $\frac{\beta N E_s}{\alpha N} = 2\beta E_s$ is the energy per symbol in phase one for $\alpha = \frac{1}{2}$. Similarly, the energy per symbol in phase two is $2(1-\beta)E_s$. We designate $2\beta_k E_s$ to represent the energy in phase $k \in \{1, 2\}$, and the energy allocation term, β_k, takes on the value

$$\beta_k = \begin{cases} \beta & \text{if } k = 1; \\ (1-\beta) & \text{if } k = 2. \end{cases} \tag{4.1}$$

The SNR relationship in the two phases of the network-coded cooperation and point-to-point transmission follow the same relations as the energy per symbol discussed above, and are used to compute outage probability in the coming sections. Note that the SNR, which takes the channel impairments and additive noise into account, is measured at the destination; whereas the energy per symbol refers to the radiated energy at the transmitting nodes.

4.2. Outage Probability Computation

This section presents the outage probability of network-coded cooperation. First, as a baseline for comparison, we consider point-to-point transmission between source and destination. An outage probability computation requires knowledge of the *outage event*, which occurs when the channel capacity between the source and destination falls below a target information rate, R. Since the channel capacity is a function of the fading coefficient of the channel, it too is a random variable [5]. The outage event is converted into an equivalent event defined in terms of the fading coefficients of the channel and the probability that this outage event occurs is referred to as the *outage probability*. In terms of the Shannon formula, $C_s^{(s,d)} = \log(1 + \gamma_{s,d})$ b/s/Hz is the capacity of the source-destination channel (represented by the superscript (s, d)) when the information of the source is transmitted (represented by the subscript s) and $\gamma_{s,d}$ is the instantaneous SNR of the channel. The corresponding *outage event* is $\left\{C_s^{(s,d)} < R\right\}$, where R is the information rate at which the source transmits, or equivalently $\left\{\gamma_{s,d} < 2^R - 1\right\}$. The *outage event probability* or simply the *outage probability* is thus defined as

$$P\left(\gamma_{s,d} < 2^R - 1\right) = \int_0^{2^R-1} p(\gamma_{s,d}) d\gamma_{s,d} \tag{4.2}$$

where $p(x)$ denotes the probability density function (pdf) of random variable x. For the case of Rayleigh fading, γ has an exponential pdf with parameter $\frac{1}{\Gamma_{s,d}}$, where $\Gamma_{s,d}$ denotes the mean value of SNR over the fading and accounts for the combination of transmit power and large-scale path loss and shadowing effects [10]. The outage probability for Rayleigh fading can thus be evaluated as

$$P\left(\gamma_{s,d} < 2^R - 1\right) \;=\; \int_0^{2^R-1} \frac{1}{\Gamma_{s,d}} \exp\left(-\frac{\gamma_{s,d}}{\Gamma_{s,d}}\right) d\gamma = 1 - \exp\left(-\frac{2^R-1}{\Gamma_{s,d}}\right). \quad (4.3)$$

Equation (4.3) implies that increasing $\Gamma_{s,d}$ by 10 dB reduces the outage probability by only a factor of 10. As will be discussed in the next section, the network-coded cooperation protocol decrease the outage probability by roughly a factor of 100 when $\Gamma_{s,d}$ is increased by 10 dB, for SNR large, i.e., a diversity order of two is achievable.

4.2.1. Network-coded cooperation

In the outage analysis of network-coded cooperation, we assume symmetrical inter-user channels (i.e., source-partner and partner-source channels have different instantaneous SNR but may have the same average SNR). If both the source and partner act independently in the second phase with no knowledge of whether each other's codeword was correctly decoded, there are four possible cases of cooperation. In **Case 1**, both the source and partner successfully decode each other, so that each user transmits the network-coded codeword in the second phase, resulting in the *full cooperation* scenario. In ***Case 2***, neither user successfully decodes its partner's first-phase transmission correctly and the system reverts to the *non-cooperative* scenario for that pair of codewords. In ***Case 3***, the partner successfully decodes the source codeword, but not vice versa. Consequently, the partner transmits the network-coded codeword which helps both nodes, but the source repeats its own codeword. The two independent copies of the source are combined at the destination prior to decoding. ***Case 4*** is identical to ***Case 3*** with the roles of the source and partner reversed. Clearly the destination must know which of these four cases has occurred and this is achieved by incorporating one additional bit in the second-phase transmission and assuming that this bit is received correctly.

Selection combining and *maximum-ratio combing* are the two types of combing considered at the destination. In the former, from two received codewords the one with stronger SNR is selected and decoded, and in the latter a codeword whose SNR is the weighted sum of the received codewords is formed and decoded. We focus on the outage probability computation of the source only; the same analysis holds for the partner.

Selection Combing at the Destination

The general approach to compute the outage probability is as follow: we first compute the outage event and probability conditioned on the occurrence of each case, and then compute the total probability by taking the sum of the probabilities in each case, assuming that each case occurs independently.

For compactness of representation, let $X_l^{(i,j)}$ be a codeword of node l (could be its own codeword or the network-coded codeword) transmitted from node i and received by node j. The subscript l takes on either s, p, or $s \oplus p$ to represent the source, partner, and network-coded codewords, respectively. Instead of introducing additional notation, let us abuse $X_l^{(i,j)}$ such that it also represents the event that the transmission is working correctly; hence $\overline{X}_i^{(j,k)}$ denotes the outage event. The outage event of a codeword in terms of Shannon channel capacity as

$$\overline{X}_i^{(j,k)} \cong \left\{ C_l^{(i,j)}(\gamma_{i,j}) = \log_2(1 + 2\beta_k \gamma_{i,j}) < 2R \right\} \tag{4.4}$$

where $i \in \{s, p\}$, $j \in \{s, p, d\}$, $i \neq j$, $k \in \{1, 2\}$ represents the two phases, and $\gamma_{i,j}$ is the instantaneous SNR. Note that actually the capacity is also a function of R and β, were R is the rate used in point-to-point transmission. Here $2R$ has to be used since only half the time is available per phase. Rearranging Equation (4.4), the outage event can also be written in terms of the instantaneous, $\gamma_{i,j}$, as

$$\overline{X}_i^{(j,k)} \cong \left\{ \gamma_{i,j} < \frac{2^{2R} - 1}{2\beta_k} \right\}. \tag{4.5}$$

For the Rayleigh fading channel assumption with $\gamma_{i,j}$ and $\Gamma_{i,j}$ as the instantaneous SNR and the average SNR between nodes i and j, respectively, the outage probability of the event in Equation (4.5) is computed using Equation (4.2) and is written as follow

$$P\left(\overline{X}_i^{(j,k)}\right) = P\left(\gamma_{i,j} < \frac{2^{2R} - 1}{2\beta_k}\right) = 1 - \exp\left(-\frac{g(R)}{\beta_k \Gamma_{i,j}}\right). \tag{4.6}$$

where $g(R) = \frac{2^{2R}-1}{2}$. Equation (4.6) will be used to determine the outage of the inter-user and first-phase source/partner-destination transmissions. Now let us look the four cases one-by-one.

- **Case 1:** Both the source and partner succeed in correctly decoding each other's codeword. In this case X_s and X_p in the first phase and $X_s \oplus X_p \cong X_{s \oplus p}$ in the second phase are transmitted by the source and partner (see Figure 4.3). The *success event*

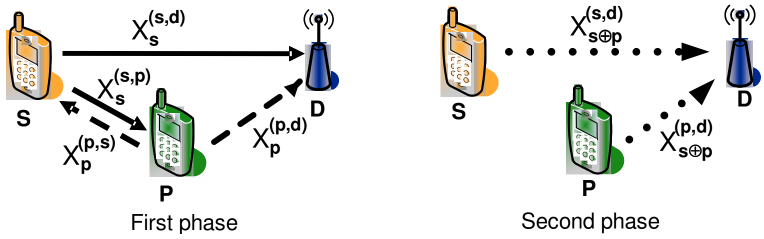

Figure 4.3.: Both users decode each others' codeword correctly. In the first phase, the source and partner transmit their own codewords X_s and X_p, respectively; in the second phase both the source and partner transmit the network-coded codeword $X_{s\oplus p}$.

probability of the inter-user channel transmission is obtained by subtracting Equation (4.6) from 1.

Under this condition, a successful recovery of the source's information bits at the destination depends on the success of the transmissions of the source and partner information bits in the two phases. The destination recovers the source's information bits either from the direct transmission in the first phase or by network decoding[3] of the partner's information bits, received in the first phase, and the network-coded bits, provided both are correctly recovered.

Using the above notation, the outage event of the source occurs if X_s and at least one of X_p and $X_{s\oplus p}$ (after selection combining) are incorrectly decoded, i.e.,

$$\overline{X}_{(s,1)} \cong X_s^{(s,p)} \wedge X_p^{(p,s)} \wedge \overline{X}_s^{(s,d)} \wedge \left(\overline{X}_p^{(p,d)} \vee \overline{X}_{s\oplus p}^{SC}\right). \tag{4.7}$$

where the subscript '1' refers to the case one, '$\wedge$'and '$\vee$' are the logical 'and' and 'or' operators. $\overline{X}_{s\oplus p}^{SC}$ is the outage event of the network-coded codeword after selection combining.

The events $X_s^{(s,p)}$ and $X_p^{(p,s)}$ in Equation (4.7) show the success of reception of the source and partner codewords at the destination, respectively, and the corresponding probabilities are computed using Equation (4.6). We see that even if X_s from the first-phase transmission fails, the destination is still able to recover X_s by combining X_p and $X_{s\oplus p}$, provided both are correctly recovered.[4] Expressed equivalently, for the source to be in outage $X_s^{(s,d)}$ and at least one of $X_p^{(p,d)}$ and $X_{s\oplus p}^{SC}$ must be in outage.

The selection combining is implemented by picking one of the two copies of the network-coded bits with higher instantaneous SNR value. The outage event of the

[3]The network decoding at the destination is identical to the network coding at transmitting nodes.
[4]Note that $(x \oplus y) \oplus x = y$ and $(x \oplus y) \oplus y = x$.

selection combining, $\overline{X}^{SC}_{s\oplus p}$, for the given values of R and β, is then written as

$$\overline{X}^{SC}_{s\oplus p} \cong \left\{\max\left(\gamma_{s,d}\,,\,\gamma_{p,d}\right) < \frac{g(R)}{\beta_2}\right\}. \tag{4.8}$$

Equation (4.8) states that, given the instantaneous SNR of the two received codewords as $\gamma_{s,d}$ and $\gamma_{p,d}$, the larger of the two SNRs is selected first, and then compared to $g(R)$ to determine the outage of the source after the selection combining. From Equation (4.8), the outage condition "*if the maximum of the two received SNR is less then the threshold* $g(R)$" is equivalent to "*if both SNRs are less than the threshold*". Accordingly, Equation (4.8) can be re-written in the form

$$\overline{X}^{SC}_{s\oplus p} \cong \left\{\gamma_{s,d} < \frac{g(R)}{\beta_2}\right\} \wedge \left\{\gamma_{p,d} < \frac{g(R)}{\beta_2}\right\}. \tag{4.9}$$

Assuming that the source-destination and partner-destination channels are uncorrelated, the probability that the event in (4.9) occurs can be written in terms of the individual probabilities[5] of the two transmissions as

$$P\left(\overline{X}^{SC}_{s\oplus p}\right) = \left(1-\exp\left(-\frac{g(R)}{\beta_2\Gamma_{s,d}}\right)\right)\left(1-\exp\left(-\frac{g(R)}{\beta_2\Gamma_{p,d}}\right)\right). \tag{4.10}$$

With the selection combining outage probability quantified in Equation (4.10), the probability that the event in Equation (4.7) occurs is then[6]

$$P\left(\overline{X}_{(s,1)}\right) = P(X^{(s,p)}_s)P(X^{(p,s)}_p)P(\overline{X}^{(s,d)}_s)\left[P(\overline{X}^{(p,d)}_p) + P(\overline{X}^{SC}_{s\oplus p})(1-P(\overline{X}^{(p,d)}_p))\right]. \tag{4.11}$$

The probabilities $P(\overline{X}^{(i,j)}_l)$ and $P(\overline{X}^{SC}_{s\oplus p})$ in Equation (4.11) can be computed from Equations (4.6) and (4.10), respectively. The outage probability for the partner node can be computed similarly.

Asymptotic Analysis and Diversity Order

To examine the behavior of the outage probability at high-SNR values, where the promised full diversity order of two is achieved for two-users cooperative schemes, we use the approximation $1-\exp^{-\frac{1}{x}} \approx \frac{1}{x}$ for large values of x. Consequently, the

[5]For two independent events A and B, the probability $P(A\cap B) = P(A/B)P(B) = P(A)P(B)$.

[6]In Equation (4.11), for two events A and B, the probability property $P(A\cup B) = P(A)+P(B)-P(A\cap B)$ is used for the term inside [..].

outage probability in Equation (4.6) is approximated at high $\Gamma_{i,j}$ values as

$$P(\overline{X}_i^{(j,k)}) = 1 - \exp\left(-\frac{g(R)}{\beta_j \Gamma_{j,k}}\right) \approx \frac{g(R)}{\beta_k \Gamma_{i,j}}. \tag{4.12}$$

Similarly, for the network-coded codeword given in Equation (4.10)

$$P\left(\overline{X}_{s\oplus p}^{SC}\right) \approx \left(\frac{g(R)}{\beta_2}\right)^2 \frac{1}{\Gamma_{s,d}\Gamma_{p,d}}. \tag{4.13}$$

Substituting Equations (4.12) and (4.13) into Equation (4.11), we get

$$\begin{aligned} P(\overline{X}_{(s,1)}) &\approx \underbrace{\left(1-\frac{g(R)}{\beta_1\Gamma_{s,p}}\right)\left(1-\frac{g(R)}{\beta_1\Gamma_{p,s}}\right)}_{\text{inter-user txs.}}\underbrace{\left(\frac{g(R)}{\beta_1\Gamma_{s,d}}\right)}_{\text{direct tx.}} \\ &\quad \underbrace{\left[\left(\frac{g(R)}{\beta_1\Gamma_{p,d}}\right)+\frac{g^2(R)}{\beta_2^2\Gamma_{s,d}\Gamma_{p,d}}\left(1-\frac{g(R)}{\beta_1\Gamma_{p,d}}\right)\right]}_{\text{network decoding}} \\ &= \left(1-\frac{g(R)}{\beta_1\Gamma_{s,p}}\right)\left(1-\frac{g(R)}{\beta_1\Gamma_{p,s}}\right)\left(\frac{g(R)}{\beta_1\Gamma_{s,d}}\right)\left(\frac{g(R)}{\beta_1\Gamma_{p,d}}\right) \\ &\quad \left[1+\frac{\beta_1}{\beta_2^2}\frac{g(R)}{\Gamma_{s,d}}\left(1-\frac{g(R)}{\beta_1\Gamma_{p,d}}\right)\right]. \end{aligned} \tag{4.14}$$

The $\left[1-\frac{g(R)}{\beta_k\Gamma_{i,j}}\right]$ terms in (4.14) approach 1 for high $\Gamma_{i,j}$ values. For symmetrical uplink channels, i.e., $\Gamma_{s,d} = \Gamma_{p,d}$, the outage probability reduces to

$$P(\overline{X}_{(s,1)}) \approx \left(\frac{g(R)}{\beta_1\Gamma_{s,d}}\right)^2 \left[1+\frac{\beta_1}{\beta_2^2}\frac{g(R)}{\Gamma_{s,d}}\right] \approx \left(\frac{g(R)}{\beta_1\Gamma_{s,d}}\right)^2. \tag{4.15}$$

From the outage probability approximation given in Equation (4.15), we observe the following points:

1. When the inter-user channels are reliable, the outage-probability decay is proportional to the square of $\Gamma_{s,d}$; hence a diversity order of two is achievable using network-coded cooperation.

2. The outage probability is less sensitive to the inter-user channels variation.

3. At high-SNR values, the contribution of the direct transmission from the source is dominant compared to the contribution of the network coding. As will be discussed in the later sections when comparing the network-coding-based cooperation approach with other cooperative approaches, the former performs better when the inter-user channels are reciprocal and the uplink channels are of poor

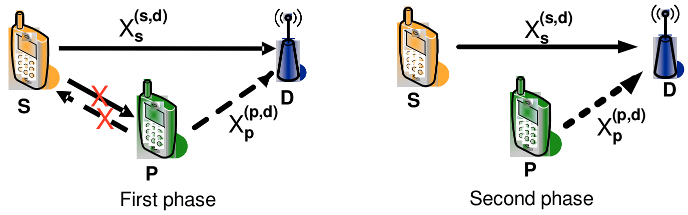

Figure 4.4.: Neither user decods its partner's codeword correctly. The source transmits and partner transmit codewords X_s and X_p, respectively, to the destination in phase one and repest the same codeword in the second phase.

quality, i.e., at lower SNR values.

4. Controlling the energy allocation term β is one way to optimize the performance of the network-coded cooperation.

- **Case 2:** Neither user decodes its partner's codeword correctly and hence the source and partner repeat their own codewords in the second phase (Figure 4.4). For a block-fading channel assumption where the fading is the same over the two phases, the strength of the received SNR is dictated by the energy allocation term β. Under this condition, selection combining is performed on the source's codewords received over the two phases (unlike on the network-coded codeword in the above case) and an outage occurs to the source if X_s is incorrectly decoded from either of the two transmissions. In terms of the instantaneous SNR, the outage event occurs if both $2\beta_1\gamma_{s,d}$ and $2\beta_2\gamma_{s,d}$ are less than the threshold $g(R)$, and for a given value of β, this event is written as

$$\overline{X}_s^{SC} \cong \left\{\gamma_{s,d} < \textbf{min}\left(\frac{g(R)}{\beta_1}, \frac{g(R)}{\beta_2}\right)\right\}. \tag{4.16}$$

Note that, unlike the event in Equation (4.10), in Equation (4.16) a single random variable, $\gamma_{s,d}$, is compared to the minimum of two fixed threshold values, namely $\frac{g(R)}{\beta_1}$ and $\frac{g(R)}{\beta_2}$. The probability that the event in Equation (4.16) occurs is then

$$\begin{aligned} P\left(\overline{X}_s^{SC}\right) &= 1 - \exp\left(-\frac{g(R)}{\Gamma_{s,d}}\textbf{min}\left(\frac{1}{\beta_1}, \frac{1}{\beta_2}\right)\right) \\ &\approx \frac{g(R)}{\Gamma_{s,d}}\textbf{min}\left(\frac{1}{\beta_1}, \frac{1}{\beta_2}\right). \end{aligned} \tag{4.17}$$

The second line in Equation (4.17) is the result of a high-SNR approximation. The overall outage event is then

$$\overline{X}_{(s,2)} \cong \overline{X}_s^{(s,p)} \cdot \overline{X}_p^{(p,s)} \cdot \overline{X}_s^{SC}. \tag{4.18}$$

The outage probability $P(\overline{X}_{(s,2)})$ that the event in (4.18) occurs is then given as

$$\begin{aligned} P(\overline{X}_{(s,2)}) &= \underbrace{\exp\left(-\frac{g(R)}{\beta_1\Gamma_{s,p}}\right)\exp\left(-\frac{g(R)}{\beta_1\Gamma_{p,s}}\right)}_{\text{inter-user txs.}}\underbrace{\left[1-\exp\left(-\frac{g(R)}{\Gamma_{s,d}}\mathbf{min}\left(\frac{1}{\beta_1},\frac{1}{\beta_2}\right)\right)\right]}_{\text{direct txs.}} \\ &\approx \left(\frac{g(R)}{\beta_1\Gamma_{s,p}}\right)\left(\frac{g(R)}{\beta_1\Gamma_{p,s}}\right)\left[\frac{g(R)}{\Gamma_{s,d}}\,\mathbf{min}\left(\frac{1}{\beta_1},\frac{1}{\beta_2}\right)\right]. \end{aligned} \tag{4.19}$$

The second line in Equation (4.19) is the result of a high SNR approximation. From the outage probability approximation given in Equation (4.19), we observe the following points:

1. When the inter-user channels are unreliable, the outage probability decays proportionally to $\Gamma_{s,d}$ (not to the square of $\Gamma_{s,d}$ as in case one above); hence, even at high SNR values, no diversity gain is obtained in such a case. The result would have been different if the channels were time varying per phase, i.e., the scheme would have benefited from *time diversity* as there is no diversity gain by repeatedly sending on a correlated channel (i.e., block-fading channels).

2. As a consequence of the above observation, if we allow nodes to *remain silent* when they fail to decode their partner's codeword, the outage probability in Equation (4.19) reduces to

$$P(\overline{X}_{(s,2)}) \approx \left(\frac{g(R)}{\beta_1\Gamma_{s,p}}\right)\left(\frac{g(R)}{\beta_1\Gamma_{p,s}}\right)\left(\frac{g(R)}{\beta_1\Gamma_{s,d}}\right). \tag{4.20}$$

   where the outage probability is more or less the same. In correlated channels, it is better to remain silent instead of repeating own message, and the former is more advantageous if energy saving is a concern as in sensor networks.[7]

3. Minimizing the outage probability given in Equation (4.19), among other factors, requires minimizing the minimum of $\frac{1}{\beta_1}=\frac{1}{\beta}$ and $\frac{1}{\beta_2}=\frac{1}{(1-\beta)}$. As the former and the latter are decreasing and increasing functions of β, respectively, allocating the same power in the two phases, i.e., $\beta=\frac{1}{2}$ is an optimal choice when the inter-user channels are poor quality. Of course, optimal allocation of energy should consider all four cases.

- **Case 3:** The partner correctly decodes the source's codeword, but the source cannot correctly decode the partner's codeword (see Figure 4.5). In the second phase, the source transmits its own codeword, X_s, and the partner transmits $X_{s\oplus p}$. Selection combining done on the source's codeword and its outage event and probability are

[7]The energy efficiency in sensor network discussion in Chapter 5 is based on the idea of remaining silent.

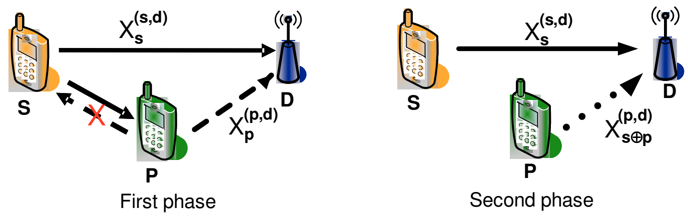

Figure 4.5.: The partner correctly decodes the source's codeword, but the source cannot correctly decode the partner's codeword. In the second phase, the source transmits its own codeword X_s whereas the partner transmits the network-coded codeword $X_{s\oplus p}$.

given as in **Case 2** of Equations (4.16) and (4.17), respectively. The overall outage event of this case is given as

$$\overline{X}_{(s,3)} \cong X_s^{(s,p)} \wedge \overline{X}_p^{(p,s)} \wedge \overline{X}_s^{SC} \wedge \left[\overline{X}_p^{(p,d)} \vee \overline{X}_{s\oplus p}^{(p,d)}\right]. \tag{4.21}$$

As in Equations (4.8) and(4.16), one can write the event $\left[\overline{X}_p^{(p,d)} \vee \overline{X}_{s\oplus p}^{(p,d)}\right]$ in Equation (4.21) in the form

$$\overline{X}_p^{(p,d)} \vee \overline{X}_{s\oplus p}^{(p,d)} \cong \overline{X}_{p,s\oplus p}^{(p,d)} \cong \left\{\gamma_{p,d} < \ \mathbf{max}\left(\frac{g(R)}{\beta_1}, \frac{g(R)}{\beta_2}\right)\right\}. \tag{4.22}$$

and the probability that this event occurs is written similar to the probability in Equation (4.17). Therefore, the overall outage probability in this case is then

$$P\left(\overline{X}_{(s,3)}\right) \;=\; P\left(X_s^{(s,p)}\right) P\left(\overline{X}_p^{(p,s)}\right) P\left(\overline{X}_s^{SC}\right) P(\overline{X}_{p,s\oplus p}^{(p,d)}). \tag{4.23}$$

Substituting the individual outage probability results at high-SNR regime, Equation (4.23) becomes

$$\begin{aligned} P\left(\overline{X}_{(s,3)}\right) \;\approx\; & \underbrace{\left(1-\frac{g(R)}{\beta_1\Gamma_{s,p}}\right)\left(\frac{g(R)}{\beta_1\Gamma_{p,s}}\right)}_{\text{inter-user txs.}}\underbrace{\left(\frac{g(R)}{\Gamma_{s,d}}\right)\mathbf{min}\left(\frac{1}{\beta_1},\frac{1}{\beta_2}\right)}_{\text{direct tx.}} \\ & \underbrace{\left(\frac{g(R)}{\Gamma_{p,d}}\right)\mathbf{max}\left(\frac{1}{\beta_1},\frac{1}{\beta_2}\right)}_{\text{network coding}}. \end{aligned} \tag{4.24}$$

Note that in Equation (4.24), $\mathbf{max}\left(\frac{1}{\beta_1},\frac{1}{\beta_2}\right)\mathbf{min}\left(\frac{1}{\beta_1},\frac{1}{\beta_2}\right) = \left(\frac{1}{\beta_1\beta_2}\right)$. For the case that the source-destination and partner-destination uplink channels have the same average SNR, i.e., $\Gamma_{s,d} = \Gamma_{p,d}$, and the inter-user channels are symmetrical, Equation (4.24)

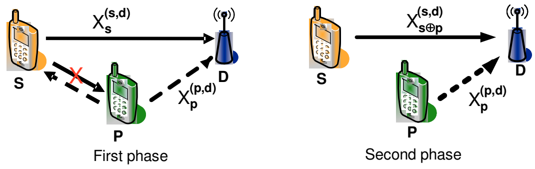

Figure 4.6.: The source correctly decodes the partner's codeword, but not vice versa. In the second phase, the partner transmits its own codeword X_p whereas the source transmits the network-coded codeword $X_{s\oplus p}$.

can be further approximated as

$$P\left(\overline{X}_{(s,3)}\right) \approx \left(\frac{g(R)}{\beta_1 \Gamma_{p,s}}\right)\left(\frac{\beta_1}{\beta_2}\right)\left(\frac{g(R)}{\beta_1 \Gamma_{s,d}}\right)^2. \tag{4.25}$$

From the outage probability result in Equation (4.25), we conclude the following points:

1. Like **Case 1**, diversity order of two is achievable as the outage probability decay is proportional to the square of $\Gamma_{s,d}$.

2. For *asymmetric cooperation* (i.e., one node cooperates but not the other), the outage probability depends on the inter-user channels quality, and this is unlike **Case 1** above, where both the source and partner send the network-coded codeword in the second phase. This means that network coding is more beneficial when asymmetric cooperation exists; this is because the network-coded codeword sent by either the source or partner benefits both the source and partner.

3. Like the above two cases, the energy allocation term plays an important role in controlling the outage probability.

- **Case 4:** The source correctly decodes the partner's codeword, but the partner cannot decode the source's codeword correctly (see Figure 4.6). Here, X_p is received twice and its outage event, considering selection combining, is then

$$\overline{X}_p^{SC} \cong \left\{\gamma_{p,d} < \textbf{min}\left(\frac{g(R)}{\beta_1}, \frac{g(R)}{\beta_2}\right)\right\} \tag{4.26}$$

and the probability that this event occurs is then

$$P\{\overline{X}_p^{SC}\} \approx \frac{g(R)}{\Gamma_{p,d}}\, \textbf{min}\left(\frac{1}{\beta_1}, \frac{1}{\beta_2}\right). \tag{4.27}$$

The overall outage event under this case is given as

$$\begin{aligned}\overline{X}_{(s,4)} &\cong \overline{X}_s^{(s,p)} \wedge X_p^{(p,s)} \wedge \overline{X}_s^{(s,d)} \wedge \left[\overline{X}_p^{SC} \vee \overline{X}_{s\oplus p}^{(s,d)}\right] \\ &\cong \overline{X}_s^{(s,p)} \wedge X_p^{(p,s)} \wedge \left[\left(\overline{X}_s^{(s,d)} \wedge \overline{X}_{s\oplus p}^{(s,d)}\right) \vee \left(\overline{X}_s^{(s,d)} \wedge \overline{X}_p^{SC}\right)\right]. \end{aligned} \tag{4.28}$$

As in **Case 3** above, the even $\left(\overline{X}_s^{(s,d)} \wedge \overline{X}_{s\oplus p}^{(s,d)}\right)$ in Equation (4.28) is written in the form

$$\overline{X}_s^{(s,d)} \vee \overline{X}_{s\oplus p}^{(s,d)} \cong \overline{X}_{s,s\oplus p}^{(s,d)} \cong \left\{\gamma_{s,d} < \ \mathbf{max}\left(\frac{g(R)}{\beta_1}, \frac{g(R)}{\beta_2}\right)\right\}. \tag{4.29}$$

The overall outage probability to user S is computed from Equation (4.28) as

$$P\left(\overline{X}_{(s,4)}\right) \approx \left(\frac{g(R)}{\beta_1 \Gamma_{s,p}}\right)\left[\frac{g(R)}{\Gamma_{s,d}}\mathbf{max}\left(\frac{1}{\beta_1}, \frac{1}{\beta_2}\right) + \left(\frac{g(R)}{\beta_1 \Gamma_{s,d}}\right)^2 \mathbf{min}\left(1, \frac{\beta_1}{\beta_2}\right)\right]. \tag{4.30}$$

Performing the same manipulations as above, the outage probability in Equation (4.30) reduces to the form

$$P\left(\overline{X}_{(s,4)}\right) \approx \left(\frac{g(R)}{\beta_1 \Gamma_{s,p}}\right)\mathbf{min}\left(1, \frac{\beta_1}{\beta_2}\right)\left(\frac{g(R)}{\beta_1 \Gamma_{s,d}}\right) \tag{4.31}$$

From the outage probability result in Equation (4.31), we conclude the following points:

1. In this case, only diversity order of one is achievable as the outage probability decay is proportional to $\Gamma_{s,d}$.

2. As in **Case 3** above, for asymmetric cooperation the outage probability depends on the inter-user channels' quality, which indicates that network coding is more beneficial in asymmetric cooperation cases. In this case the network-coded codeword sent by the source benefits both the source and partner.

3. The energy allocation term plays an important role in controlling the outage probability.

Total Outage Expression

In the four cases seen so far, we computed the outage probability conditioned on the occurrence of each case. Assuming that the occurrence events of the four cases are mutually exclusive (for independent inter-user channels), the total outage probability, $P\left(\overline{X}_s\right)$, is the sum of the outage probabilities in the four cases, and its high-SNR approximation is given as

$$\begin{aligned} P\left(\overline{X}_s\right) &= P\left(\overline{X}_{(s,1)}\right) + P\left(\overline{X}_{(s,2)}\right) + P\left(\overline{X}_{(s,3)}\right) + P\left(\overline{X}_{(s,4)}\right) \\ &\approx \underbrace{\left(\frac{g(R)}{\beta_1\Gamma_{s,d}}\right)^2}_{\text{Case 1}} + \underbrace{\left(\frac{g(R)}{\beta_1\Gamma_{s,p}}\right)\left(\frac{g(R)}{\beta_1\Gamma_{p,s}}\right)\left[\frac{g(R)}{\Gamma_{s,d}}\,\mathbf{min}\left(\frac{1}{\beta_1},\frac{1}{\beta_2}\right)\right]}_{\text{Case 2}} + \\ &\quad \underbrace{\left(\frac{g(R)}{\beta_1\Gamma_{p,s}}\right)\left(\frac{1}{\beta_1\beta_2}\right)\left(\frac{g(R)}{\beta_1\Gamma_{s,d}}\right)^2}_{\text{Case 3}} + \underbrace{\left(\frac{g(R)}{\beta_1\Gamma_{s,p}}\right)\mathbf{min}\left(1,\frac{\beta_1}{\beta_2}\right)\left(\frac{g(R)}{\beta_1\Gamma_{s,d}}\right)}_{\text{Case 4}}. \end{aligned} \tag{4.32}$$

Plotted in Figure 4.7 are the exact and approximation (given in Equation (4.32)) outage probability results when the average SNR of all the uplink and inter-user channels are the same, i.e., $\Gamma_{s,p} = \Gamma_{p,s} = \Gamma_{s,d} = \Gamma_{p,d}$. One can see that at higher $\Gamma_{s,d}$ values, the approximation captures the exact result, and the approximation error is on the order of 1 dB.

Asymptotic Analysis and Diversity Order

To determine the diversity order achieved by network-coded cooperation, the mean SNR $\Gamma_{i,j}$ is re-parameterize by decoupling the user transmit power from the physical impairments of the channel itself as in [66]:

$$\Gamma_{i,j} \;\Rightarrow\; \Gamma_T\Gamma_{i,j} \tag{4.33}$$

where Γ_T is the ratio of transmit power to the received noise, and $\Gamma_{i,j}$ is a finite constant accounting for large-scale, path-loss, and shadowing effects. Further, we assume that Γ_T is the same for both the source and partner, and the relative differences in quality between the various channels are captured by the $\Gamma_{i,j}$ values. Thus, by expressing outage probability as a function of $1/\Gamma_T$, and then letting $\Gamma_T \to \infty$ (e.g., the high-SNR regime), the diversity order is given by the smallest exponent of $1/\Gamma_T$.

To obtain the outage probability as a function of $\frac{1}{\Gamma_T}$, we re-write Equation (4.32) by collecting like-order terms of $\frac{1}{\Gamma_T}$, and this results in the following expression:

$$P\left(\overline{X}_s\right) \;\approx\; \frac{1}{\Gamma_T^2}\cdot\left[\frac{g(R)}{\beta_1\Gamma_{s,d}}\right]^2 + O\left(\frac{1}{\Gamma_T^3}\right) \tag{4.34}$$

where $O\left(\frac{1}{\Gamma_T^3}\right)$ denotes the higher-order terms. It is interesting to note that, in the high-SNR regime, the dependence of outage probability of the source on the inter-user channels, i.e., $\Gamma_{s,p}$ and $\Gamma_{p,s}$, appears only in the third-order term. So, we once again see from Equation

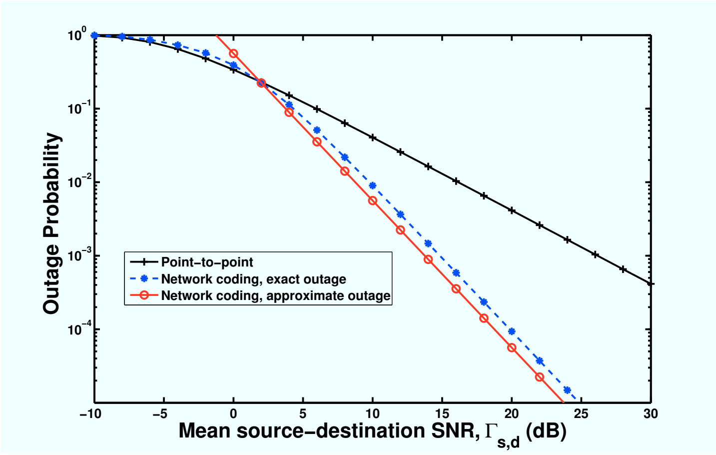

Figure 4.7.: Outage probability curves of the point-to-point transmission and the network-coded cooperation (both approximate and exact).

(4.34) that the outage probability decays proportional to the square of the source-destination SNR, hence the diversity order of two can be achieved.

Reciprocal inter-user channels

In reciprocal inter-user channels, the instantaneous SNRs $\gamma_{s,p} = \gamma_{p,s}$, such that **Case 3** and **Case 4** vanish. The overall outage probability becomes

$$P\left(\overline{X}_s\right) \approx \left(\frac{g(R)}{\beta_1 \Gamma_{s,d}}\right)^2 + \left(\frac{g(R)}{\beta_1 \Gamma_{s,p}}\right)\left(\frac{g(R)}{\beta_1 \Gamma_{p,s}}\right)\left[\frac{g(R)}{\Gamma_{s,d}}\,\mathbf{min}\left(\frac{1}{\beta_1}, \frac{1}{\beta_2}\right)\right]. \quad (4.35)$$

We see from Equation (4.35) that outage probability is dependent on the partner-destination channel only in **Case 1**. Hence, in the case of reciprocal inter-user channels, the cooperative protocol does not benefit much from network coding.

Maximum-Ratio Combing at the Destination

So far, the outage behavior of the network-coded cooperation was investigated assuming selection combining at the destination. Next, we study the outage behavior when MRC is used to combine identical codewords at the destination. In its simplest form, MRC is realized by adding the instantaneous SNR of each codeword received from the two channels.[8] This accumulation of the instantaneous SNR increases the rate at which the destination can

[8] A more complex implementation of MRC requires estimation of the channel coefficients and then taking the weighted sum of the received codewords.

reliably decode the codeword (in other words, increases the channel capacity). Following similar four cases as in selection combining, the outage results are presented next.

- **Case 1:** Both the source and partner decode each others' codeword correctly. As the network-coded codeword $X_{s\oplus p}$ is received twice from the source and partner, its instantaneous SNR accumulates at the destination. We can define an outage event as

$$\overline{X}^{MRC}_{s\oplus p} \quad \cong \quad \left\{ (\gamma_{s,d} + \gamma_{p,d}) < \frac{g(R)}{\beta_2} \right\}. \tag{4.36}$$

This event depends on two random variables $\gamma_{s,d}$ and $\gamma_{b,d}$, which are exponentially distributed under a Rayleigh fading channel assumption. Let **u** and **v** be two exponentially distributed random variables with parameters λ_u and λ_v, and **w = u + v**. Referring to [5, appendix I], the probability $P\{\mathbf{w} \leqslant w\}$ is given as

$$P(\mathbf{w} \leqslant w) \quad = \quad \begin{cases} 1 - \left[\left(\frac{\lambda_u}{\lambda_u - \lambda_v} \right) \exp(-\lambda_v w) + \left(\frac{\lambda_v}{\lambda_v - \lambda_u} \right) \exp(-\lambda_u w) \right] \\ 1 - (1 + \lambda w) \exp(-\lambda w) \end{cases} \tag{4.37}$$

where the first equation is when $\lambda_u \neq \lambda_v$ and the second for $\lambda_u = \lambda_w = \lambda$. In our case, setting $w = \frac{g(R)}{\beta_2}$, $\lambda_u = \frac{1}{\Gamma_{s,d}}$, $\lambda_v = \frac{1}{\Gamma_{p,d}}$, and $\mathbf{w} = \gamma_{s,d} + \gamma_{p,d}$, the probability $P\left(\overline{X}^{MRC}_{s\oplus p}\right)$ is readily computed. For identical source/partner-destination channels, i.e., $\Gamma_{s,d} = \Gamma_{p,d}$, the outage probability at high SNR is approximated as

$$\begin{aligned} P\left(\overline{X}^{MRC}_{s\oplus p}\right) &= P\left((\gamma_{s,d} + \gamma_{p,d}) < \frac{g(R)}{\beta_2} \right) \\ &= 1 - \left(1 + \frac{1}{\Gamma_{s,d}} \frac{g(R)}{\beta_2} \right) \exp\left(-\frac{1}{\Gamma_{s,d}} \frac{g(R)}{\beta_2} \right) \approx \frac{1}{2} \left(\frac{g(R)}{\beta_2 \Gamma_{s,d}} \right)^2 \end{aligned} \tag{4.38}$$

We see the high-SNR approximation in Equation (4.38) (which is for MRC at the destination) differs from the approximation in Equation (4.13) (which is for selection combing at the destination) by a factor of $\frac{1}{2}$. The outage event probability of the source is the same as the probability given in Equation (4.11) except that the term $P\left(\overline{X}^{SC}_{s\oplus p}\right)$ is replaced by $P\left(\overline{X}^{MRC}_{s\oplus p}\right)$. For symmetrical inter-user channels, the outage probability in this case is approximated as follows:

$$P\left(\overline{X}_{(s,1)}\right) \quad \approx \quad \left(\frac{g(R)}{\beta_1 \Gamma_{s,d}} \right)^2 \left[1 + \frac{1}{2} \frac{\beta_1}{\beta_2^2} \frac{g(R)}{\Gamma_{s,d}} \right] \approx \left(\frac{g(R)}{\beta_1 \Gamma_{s,d}} \right)^2 \quad . \tag{4.39}$$

This is the same as for selection combining at the destination, and it is not surprising because at high-SNR regime, the direct transmission from the the source dictates the outage behavior. The conclusions for the **Case 1** of the selection combining holds true for MRC as well.

- **Case 2:** Neither the source nor the partner decodes each other's codeword correctly. Each node repeats its own codeword and MRC is performed at the destination. The resulting instantaneous SNR of the source's codeword at the destination is $\beta_1\gamma_{s,d} + \beta_2\gamma_{s,d} = \gamma_{s,d}$, and the outage probability is computed as

$$P\left(\overline{X}_s^{MRC}\right) = P\left(\gamma_{s,d} < g(R)\right) \approx \frac{g(R)}{\Gamma_{s,d}} \ . \tag{4.40}$$

Note that the difference between the above equation and Equation (4.17) (which is for selection combining) is that the term $\mathbf{min}\left(\frac{1}{\beta_1}, \frac{1}{\beta_2}\right)$, which accounts for the energy allocation, is missing in the former. The overall outage probability, for symmetrical inter-user channels, is given as

$$P\left(\overline{X}_{(s,2)}\right) \approx \left(\frac{g(R)}{\beta_1\Gamma_{s,p}}\right)^2 \left(\frac{g(R)}{\Gamma_{s,d}}\right) \ . \tag{4.41}$$

From the outage probabilities given in Equations (4.40) and (4.41), we note the following points:

1. As a result of the MRC operation at the destination, the outage probability of the uplink transmission is independent of the energy allocation.

2. No diversity gain is obtained in the case of unreliable inter-user channels, as the outage is a function of the source-destination instead of the source/partner-destination channels.

- **Case 3:** The partner correctly decodes the source's codeword, but the source cannot decode the partner's codeword correctly. In this case, X_s is received twice from the source and its outage probability is given by Equation (4.40), and the codeword $X_{s\oplus p}$ is received from the partner only. The overall outage probability is approximated as

$$P\left(\overline{X}_{(s,3)}\right) \approx \underbrace{\left(\frac{g(R)}{\beta_1\Gamma_{p,s}}\right)}_{\text{inter-user txs.}} \underbrace{\mathbf{max}\left(\frac{1}{\beta_1}, \frac{1}{\beta_2}\right)}_{\text{power allo.}} \underbrace{\left(\frac{g(R)}{\Gamma_{s,d}}\right)^2}_{\text{uplink tx.}} . \tag{4.42}$$

- **Case 4:** The source correctly decodes the partner's codeword, but the partner cannot decode the source's codeword correctly. In this case, X_p is received twice from the partner and its outage probability is given by Equation (4.40) with $\Gamma_{s,d}$ replaced by $\Gamma_{p,d}$. The outage probability is similar to the probability of **Case 4** given in Equation (4.31) above, and at high SNR, symmetrical inter-user, and identical uplink channels,

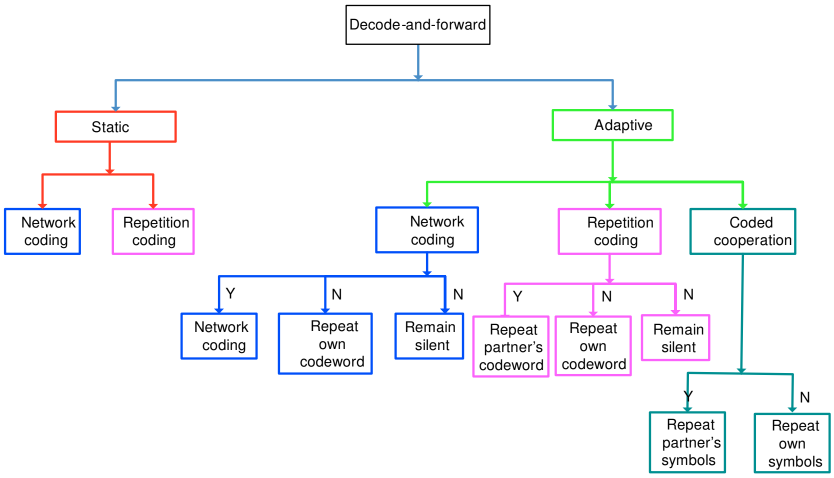

Figure 4.8.: A block diagram showing the various protocols considered for comparison, 'Y' and 'N' stands for 'YES' and 'NO' to mean that decoding of partner's codeword is successful or not successful.

it is approximated as

$$P\left(\overline{X}_{(s,4)}\right) \approx \underbrace{\left(\frac{g(R)}{\beta_1 \Gamma_{s,p}}\right)}_{\text{inter-user txs.}} \underbrace{\mathbf{max}\left(\frac{1}{\beta_1}, \frac{1}{\beta_2}\right)}_{\text{energy allo.}} \underbrace{\left(\frac{g(R)}{\Gamma_{s,d}}\right)}_{\text{uplink tx.}} . \tag{4.43}$$

The total outage probability is the sum of the outage probabilities under **Cases 1-4**. The asymptotic behavior of the total outage probability can be written as in (4.34) and after summing the probabilities in each case and simplifying, we get

$$P\left(\overline{X}_s\right) \approx \frac{1}{\Gamma_T^2} \cdot \left[\frac{g(R)}{\beta_1 \Gamma_{s,d}}\right]^2 + O\left(\frac{1}{\Gamma_T^3}\right) . \tag{4.44}$$

Comparing on the asymptotic outage probabilities of (4.34) and (4.44), we conclude that at high-SNR regime, both selection combining and maximum-ratio combining behave the same and also that both achieve diversity order of two.

4.3. Numerical Results and Discussion

4.3.1. Basic assumptions and parameters

In this section, the outage probability results for the network-coded cooperation and point-to-point transmission are presented. For ease of exposition, we set $\Gamma_{s,p} = \Gamma_{p,s}$, i.e., symmetrical

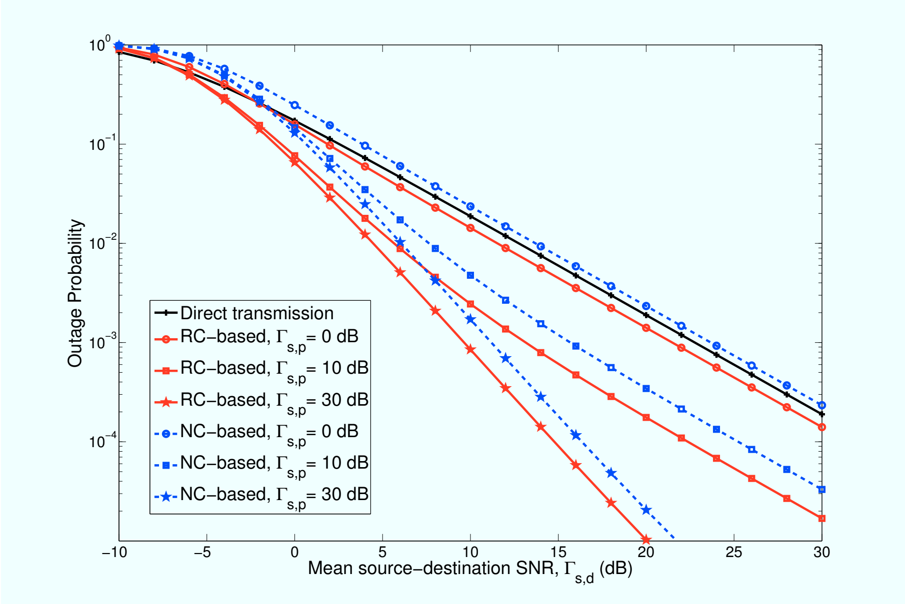

Figure 4.9.: Outage probability vs. $\Gamma_{s,d}$ results of static protocols without (solid line) and with (dashed lines) network coding. Results are based on $\Gamma_{s,p}$ = 0, 10, and 30 dB; NC stands for network coding.

inter-user channels (this is a true assumption for reciprocal inter-user channels, and reasonable for independent inter-user channels since path loss is a reciprocal phenomenon, and large-scale shadowing, i.e., from buildings or other large obstructions, is also in many cases [10]). As a result of this, we note that the outage probabilities for both the source and partner are equal if the uplink channels have equal mean SNR, i.e., $\Gamma_{s,d} = \Gamma_{p,d}$. Unless stated otherwise, we assume an information rate of $R = 1/4$, energy allocations of $\beta = 1/2$, $\Gamma_{s,d} = \Gamma_{p,d}$ channels, and all plots are results of the exact outage probability (i.e., not the high SNR approximations).

4.3.2. List of investigated protocols

The list of protocols considered for comparison are shown in the block diagram of Figure 4.8, and a detailed description of each protocol can be referred from [13]. Here is a brief description of how the protocols are categorized. Based on their level of adaptiveness when decoding of each other's codeword fails, decode-and-forward-based protocols can be categorized as *static* or *adaptive*. In static protocols, the partner always forwards the source's codeword using either repetition or network coding. In the group of adaptive protocols, the partner decides whether to forward or not, depending on its success of decoding the source's codeword. If successful, then it may forward using network coding, repetition coding, or

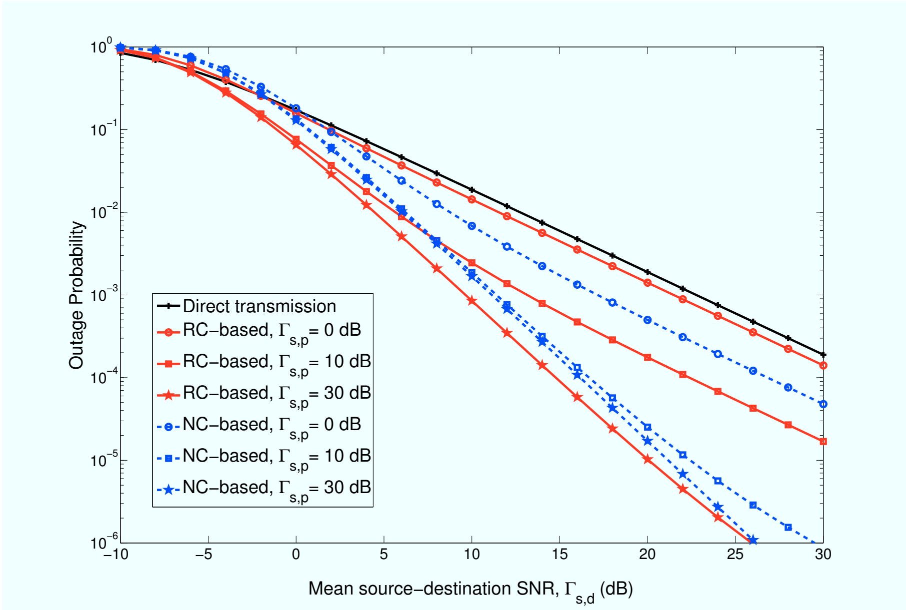

Figure 4.10.: Outage probability vs. $\Gamma_{s,d}$ results of adaptive protocols based on repetition coding (solid lines) and network coding (dashed lines), both with the option to remain silent if decoding fails. $\Gamma_{s,p} = 0$, 10, and 30 dB are used; RC stands for repetition coding.

coded cooperation; if decoding fails, then the partner has the options to transmit its own codeword (symbols for coded cooperation) or remain silent [13].

4.3.3. Numerical results

Below, the numerical results for the protocols listed in Figure 4.8 are presented. Figure 4.9 depicts the outage probability vs. $\Gamma_{s,d}$ (dB) of static protocols based on repetition and network coding, and for various inter-user channel qualities. We see that at lower $\Gamma_{s,p}$ values (example 0 dB), the performance of these protocols is closer to the point-to-point transmission; and at higher $\Gamma_{s,p}$ values (example 30 dB), the performance improves substantially and a diversity order of two, taking point-to-point transmission as reference, can be achieved.

Static protocols are relatively simple to implement as the destination does not need to know decoding results at the source and partner, and the partner (respectively the source) also does not need to worry about errors contained in the codeword it has received from the source (respectively partner). This simplicity of implementation, if supported by the presence of reliable inter-user channels, would make static protocols attractive. Within static protocols, the performance of the protocol with network coding is poor as compared to the

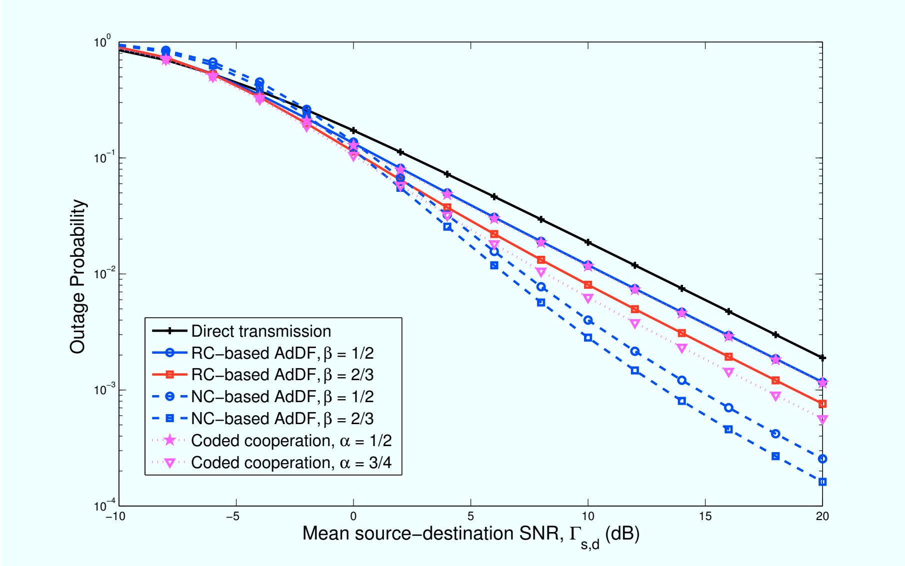

Figure 4.11.: Outage probability vs. $\Gamma_{s,d}$ results of adaptive protocols based on repetition coding (solid lines) and network coding (dashed lines), both with the option to repeat their own codewords if decoding fails, and coded cooperation (dotted lines). $\Gamma_{s,p}$ = 0 dB; $\beta = 1/2$ and 2/3 for repetition and network-coding-based protocols; $\beta = 1/2$, and $\alpha = 1/2$ and 3/4 for coded cooperation. AdDF stands for adaptive DF.

repetition-coding-based protocol; hence there is no incentive to use network coding in static protocols.

Next, let us investigate the option of remaining silent during decoding failure in adaptive protocols, and for that we consider the two groups of protocols: repetition and network coding. The outage probability results of the two protocols are shown in Figure 4.10. At lower $\Gamma_{s,p}$ values of 0 and 10 dB, the protocol with network coding performs better than the repetition-coding-based protocol; this improvement is appreciable when $\Gamma_{s,d}$ is greater than 10 dB. On the other hand, the repetition-coding-based protocol performs better than its network-coded counterpart when the inter-user channels are of high quality (example 30 dB); and at this $\Gamma_{s,p}$ value both protocols achieve a diversity order of two.

Now, let us include coded cooperation protocols on top of the repetition-coding and network-coding-based adaptive protocols, both with the option of repeating own codeword, whose comparison is shown in Fig. 4.10. For clarity of presentation, three separate plots are made for three inter-user channel qualities. Figure 4.11 is when $\Gamma_{s,p}$ = 0 dB, Figure 4.12 is when $\Gamma_{s,p}$ = 10 dB, and Figure 4.13 is when $\Gamma_{s,p}$ = 30 dB. From Figure 4.11 and Figure 4.12, we see that at lower $\Gamma_{s,p}$ values, namely 0 and 10 dB, the network-coding-based protocol outperforms the other two protocols. Also, for the first two protocols, we see the advantage of allocating more energy in the first phase (i.e., higher β value). This is logical in that at

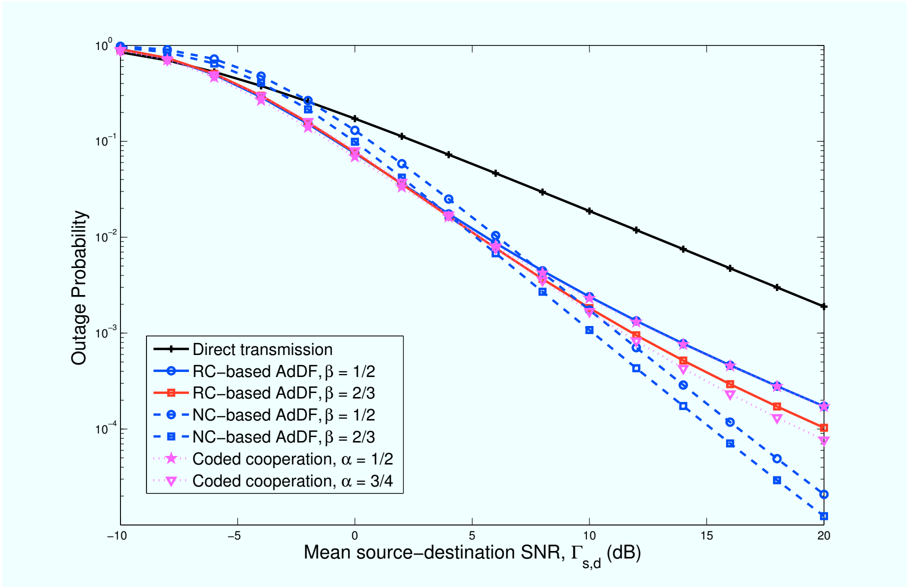

Figure 4.12.: Outage probability vs. $\Gamma_{s,d}$ results for the same three adaptive protocols considered in Fig. 4.11; the only exception is here $\Gamma_{s,p}$ = 10 dB used. In both figures, the curves of coded cooperation with $\alpha = 1/2$ and repetition coding with $\beta = 1/2$ appear overlapping.

lower inter-user SNR values, the source and partner should allocate more power in the first phase than in the second phase. In the coded cooperation protocol, a larger α shows a better performance. Here, the increased redundancy in the first phase (i.e., large α) compensates for the low $\Gamma_{s,p}$. With $\alpha = 3/4$ it achieves higher performance than the repetition-coding-based protocol.

In Figure 4.13, the comparison is done for $\Gamma_{s,p}$ = 30 dB (a high-quality link). The network-coded cooperation with $\beta = 1/2$ performs worse; and coded cooperation with $\alpha = 1/2$ performs relatively better than the other protocols. It is interesting that compared to the cases with lower $\Gamma_{s,p}$ (Figures 4.11 and 4.12), here the effect of α has reversed. Due to the good inter-user links, here it is beneficial to allocate more redundancy to the second phase, i.e., choosing a smaller α than with low $\Gamma_{s,p}$.

Finally, let us investigate if there is an advantage by remaining silent instead of repeating one's own codeword in repetition and network-coding-based adaptive protocols. The results in Figure 4.14 show that, in repetition-coding-based protocol, at lower $\Gamma_{s,p}$ values (example 0 dB) there is a slight advantage by repeating one's own codeword during decoding failure; and at $\Gamma_{s,p}$ values higher than 0 dB, there is only negligible advantage by repeating one's own codeword, and the curves for $\Gamma_{s,p}$ 10 dB and 30 dB appear overlapping. In the network-coding-based protocols, as illustrated in Figure 4.15, there is a clear advantage of repeating one's own codeword at lower $\Gamma_{s,p}$ values (example 0 dB); and at a moderate value of 10

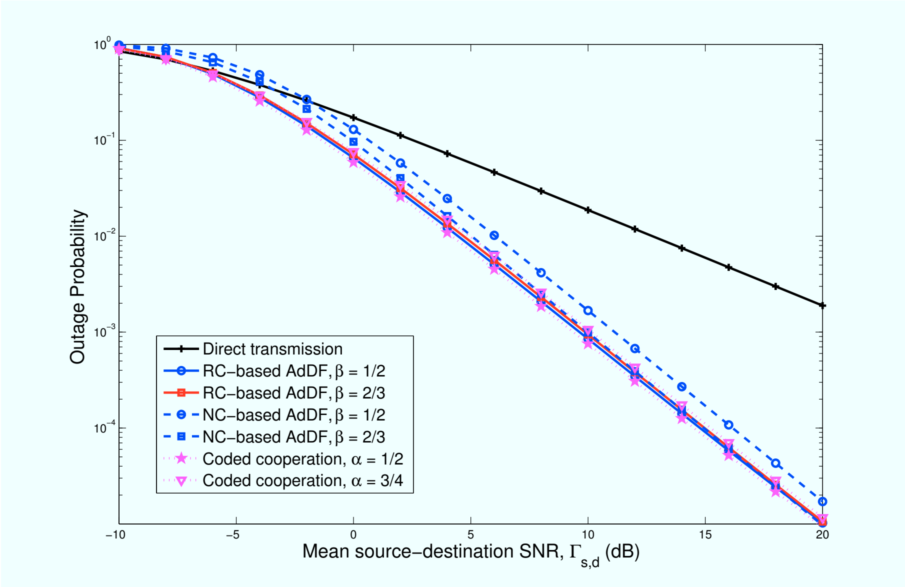

Figure 4.13.: Outage probability vs. $\Gamma_{s,d}$ results for the same three adaptive protocols considered in Fig. 4.11 and Fig. 4.12 above. The only difference is, here $\Gamma_{s,p} = 30$ dB is used.

dB, the performance improvement is evident at higher values of $\Gamma_{s,d}$. In the presence of a reliable source-relay link (example $\Gamma_{s,p} = 30$ dB), like the results in Figure 4.14, there is negligible performance gain by repeating one's own codeword during decoding failure; this is not surprising as decoding failure at the relay, at high $\Gamma_{s,p}$ values, rarely happens.

4.3.4. Conclusion and remarks

In this section, we have presented the outage probability results of the static and adaptive cooperative protocols without and with network coding, and inter-user-channels-based performance comparisons are done. Based on the results, we draw the following conclusions.

- Static protocols are found to achieve full diversity, provided the destination decodes the source's first-phase transmission before attempting to decode from the combined codeword.
- Static protocols do not benefit from network coding.
- In adaptive protocols, contrary to static protocols, incorporating network coding delivers performance improvement, and this improvement is noticeable when the inter-source channels are poor quality.

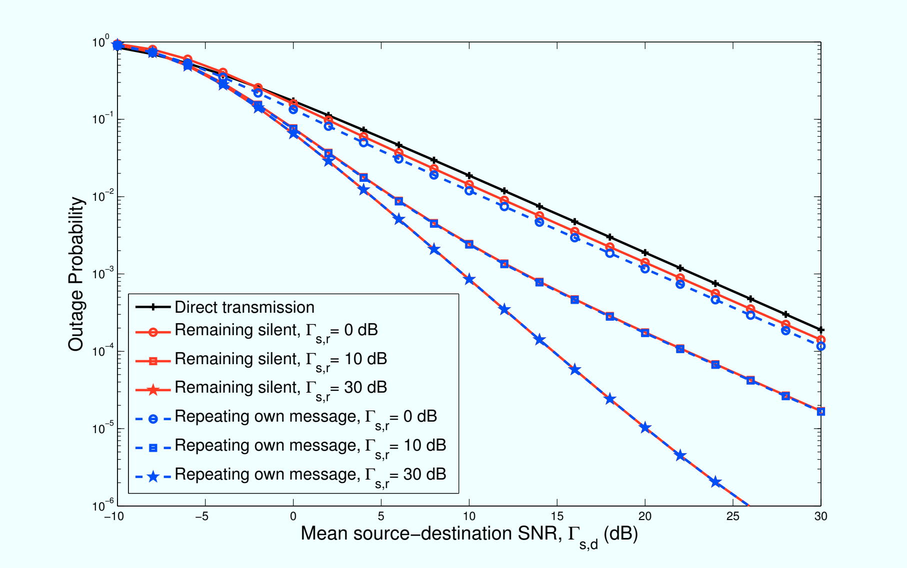

Figure 4.14.: Outage probability vs. $\Gamma_{s,d}$ results of repetition-coding-based adaptive protocols with the options of either remaining silent (solid lines) or repeating one's own codeword (dashed lines) when decoding failure occurs. The curves with identical $\Gamma_{s,p}$ values of 10 dB and 30 dB appear overlapping with each other.

- With reliable inter-user channels, the coded cooperation protocol, with low cooperation level α, performs relatively better. In unreliable channels, higher values of α give better performance.
- In repetition-coding-based protocols, when the inter-user channels are unreliable, repeating one's own codeword during decoding failure offers slight advantage over remaining silent, but this advantage vanishes even at moderate inter-user channels (example 10 dB).
- But in network-coded cooperation protocols, there is a clear advantage of repeating one's own codeword; and this advantage is significant at poor inter-user link qualities.
- We have demonstrated the effect of the energy allocation on the performance of adaptive protocols. Based on the results, allocating more energy in the first phase gives better performance.

4.4. Diversity-Multiplexing Tradeoff

Generally, MIMO and cooperative transmission schemes provide both *diversity gain* and *multiplexing gain*. But there is a fundamental tradeoff between how much of each type of gain a cooperative scheme can get, and *diversity-multiplexing tradeoff* is one performance

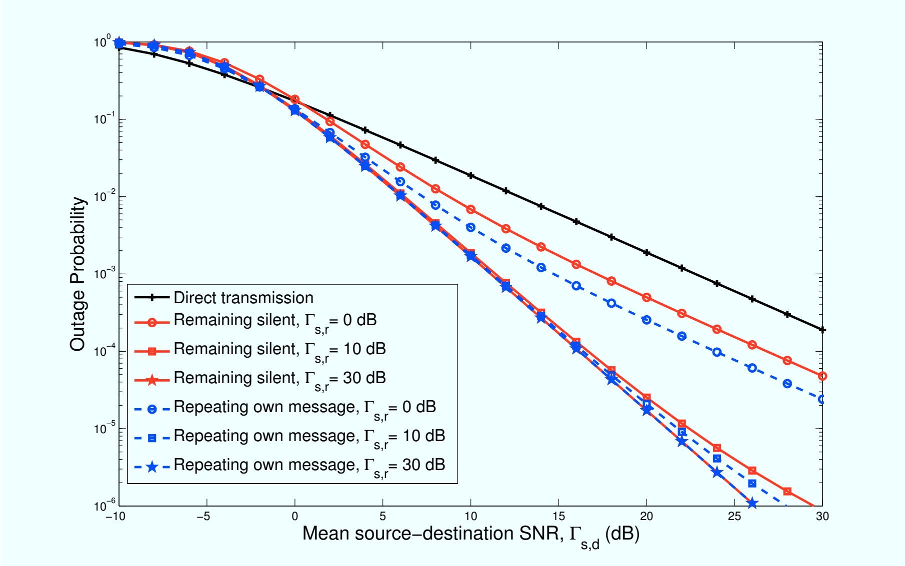

Figure 4.15.: Outage probability versus $\Gamma_{s,d}$ results of network-coding-based adaptive protocols with the option of either remaining silent (solid lines) or repeating one's own codeword (dashed lines) during decoding failure.

measure of this tradeoff. This tradeoff illustrates the relationship between reliability of data transmission in terms of *diversity gain*, and spectral efficiency in terms of *multiplexing gain* [5, 67].

In point-to-point transmission with the source's information rate R (bits/second/Hz), multiplexing gain helps to examine the high spectral-efficiency regime as SNR becomes large. This is done by allowing R to grow with increasing SNR [5]. For slower growth, the outage results essentially behave like fixed R with sufficiently large SNR, while for faster growth, the outage probability tend to 1. In other words, the multiplexing gain m is defined as [67]

$$m := \lim_{\Gamma_{s,d}\to\infty} \frac{R(\Gamma_{s,d})}{\log_2(\Gamma_{s,d})}. \tag{4.45}$$

The diversity gain d is defined as

$$d := -\lim_{\Gamma_{s,d}\to\infty} \frac{\log_2 P_{\overline{X}_s}(\Gamma_{s,d}, R)}{\log_2(\Gamma_{s,d})} \tag{4.46}$$

where $P_{\overline{X}_s}$ is the outage probability of the source. Larger d implies more robustness to fading (faster decay in the outage probability with increasing SNR). The relationship between diversity gain and multiplexing gain can be characterized by mapping d as a function of m. At large $\Gamma_{s,d}$ values, if we approximate the outage probability of the point-to-point transmission by $P_{\overline{X}_s} \approx \frac{2^R}{\Gamma_{s,d}}$, then we get

$$d := -\lim_{\Gamma_{s,d}\to\infty} \frac{\log_2 P_{\overline{X}_s}(\Gamma_{s,d}, R)}{\log_2(\Gamma_{s,d})} \approx 1 - m \ . \tag{4.47}$$

In network-coded cooperation with selection combing at the destination, substituting (4.32) into (4.46) and using (4.45), the diversity-multiplexing tradeoff of this scheme is given as

$$\begin{aligned} d: \ &= \ -\lim_{\Gamma_{s,d}\to\infty} \frac{\log_2 P_{\overline{X}_s}(\Gamma_{s,d}, R)}{\log_2(\Gamma_{s,d})} = -\lim_{\Gamma_{s,d}\to\infty} 2\left[\frac{\log_2(g(R)) - \log_2(\beta) - \log_2(\Gamma_{s,d})}{\log_2(\Gamma_{s,d})}\right] \\ &\approx \ 2(1-2m). \end{aligned} \tag{4.48}$$

Figure 4.16 shows the tradeoff curves based on Equations (4.47) and (4.48). We see that the maximum multiplexing gain of the network-coded cooperation is $\frac{1}{2}$, and this loss in rate is expected as total available time per codeword in the point-to-point transmission is split into two for the two phases of cooperation. As a result, to send the same information bits in each phase of cooperation and point-to-point transmission, the codewords in the former should have double the information rate of the latter. To conclude, we see that comparing the network-coded cooperation and point-to-point transmission, the latter is good for multiplexing gain and bad in terms of reliability of transmission, and vice versa.

4.5. Coverage Area Analysis

So far, we have seen the diversity gain of using network coding in cooperative transmission. In large networks, e.g., ad hoc networks, cooperative transmission can be implemented to expand coverage area (i.e., increase the range of signal radiation); however, this increase in coverage also increases the interference range. To implement cooperative transmission in a network of multiple nodes, partner node selection mechanisms are required and a study on coverage area gives some clues. Moreover, relative location of nodes and coverage area extension are other performance measures that can be used to compare various protocols [17, 68, 69, 70].

As pointed out in the previous sections, the level of cooperation between the source and partner node depends on the quality of inter-user channels, which in turn depend, among other factors, on relative location of the two nodes. Network-coded cooperation performs better than the repetition coding counterpart when the quality of the inter-user channels is poor (i.e., when, for example, the partner is closer to the destination than the source). In [17], it was found that the latter scheme works better when the partner is closer to the source than the destination. In the following, we will show that the former scheme preforms better when

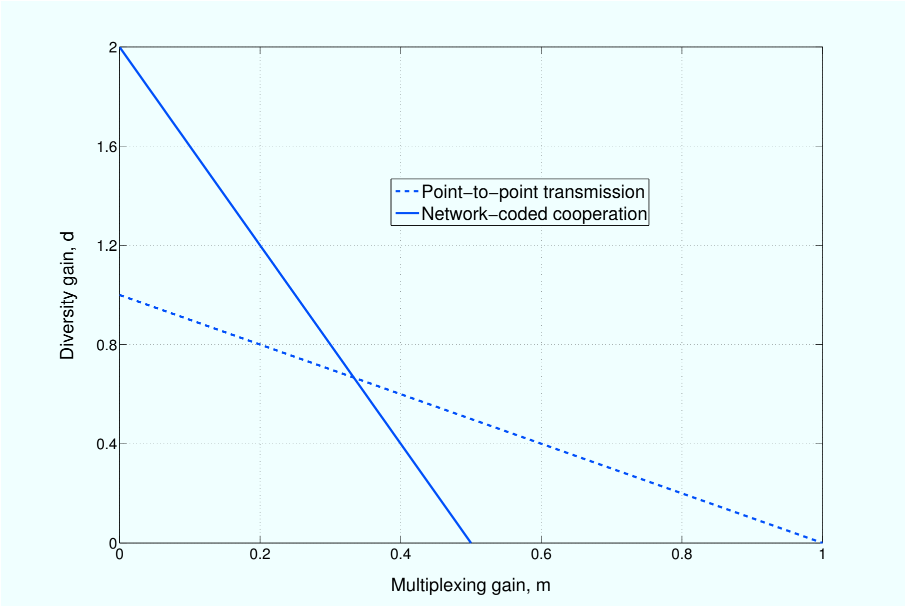

Figure 4.16.: Diversity-multiplexing tradeoff plots of the network-coding-based cooperative protocol (solid line) and point-to-point transmission scheme (dashed line).

the partner is located closer to the destination than the source. Specifically, the coverage area and location of the partner node, where outage is minimized, is studied in this section; the exact outage probability results are used (see [13, 46] for further details). In the course of discussion, the following questions will be addressed:

- For the given network topology (i.e., location of the source, partner, and destination nodes), which cooperative protocol to use?
- In which geographic region does the network-coded cooperation perform better than the repetition-coding-based protocol, and vice versa?
- Within the network-coding-based (respectively the repetition coding) protocol, how do the static and adaptive protocols perform?

To address these points, the following approach is followed: the channel coefficient is split into path-loss and fading coefficients. Then in all channels, the fading coefficient is assumed to have unity mean power, but the pathloss varies as it depends on node locations. The source-destination separation and the transmit power at both the source and partner are fixed. The partner's location is varied, such that the inter-user and partner-destination link quality vary because of the pathloss. To aid the discussion, the following three terms are defined as follows.

Definition *Coverage area* is defined as the region or area in which the partner can be placed

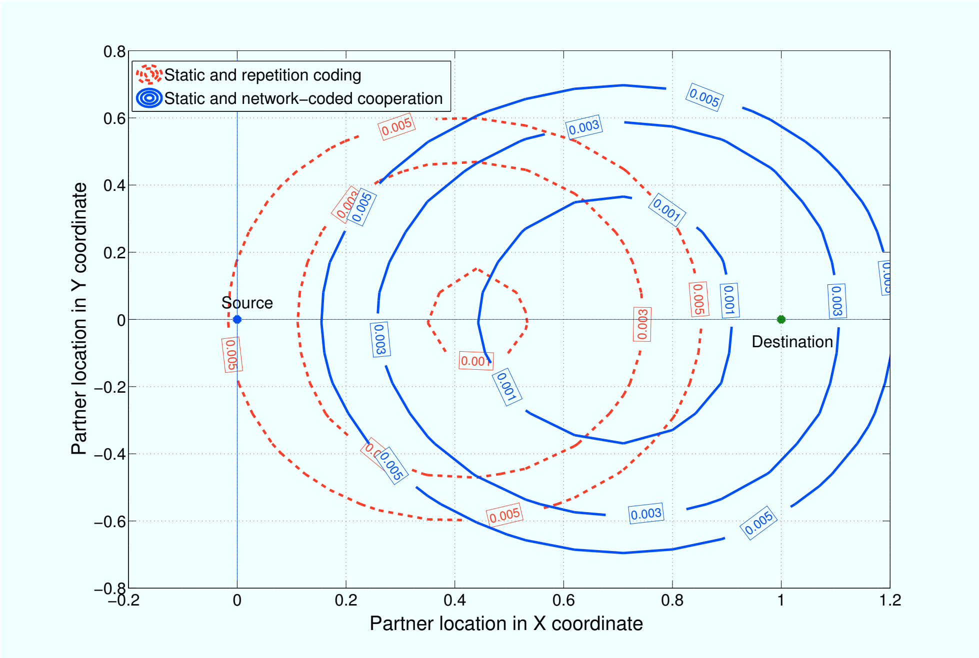

Figure 4.17.: Outage probability contours of static protocols: repetition coding (dashed lines) and with network coding (solid lines). The source and the destination are placed at the coordinates $(0, 0)$ and $(0, 1)$, respectively.

such that for a given resource allocation (i.e transmission power and bandwidth) and end-to-end spectral efficiency, the outage probability (or ratio of outage probabilities) is less than or equal to some threshold value.

Definition *Intra-cooperation gain* is the ratio of outage probabilities of two protocols; a gain of unity demarcates the region into two parts, where one protocol performs better than the other.

Definition *Transmitter cluster* is formed when the partner is deployed closer to the source than the destination. *Receiver cluster* is formed when the partner is deployed closer to the destination than the source.

To plot the coverage area, the transmit SNR at both the source and partner is set to 20 dB and the source-destination distance is taken as a reference, i.e., $d_{s,d} = 1$ such that the other channels' normalized pathloss become $q_{i,j}^2 = \left(\frac{1}{d_{i,j}}\right)^{\alpha}$, where i, $j \in \{s, p, d\}$ and α is the pathloss coefficient.

Figure 4.17 depicts the outage probability contours of the repetition coding (dashed lines) and network-coding-based (solid lines) static protocols. For a given outage probability value,

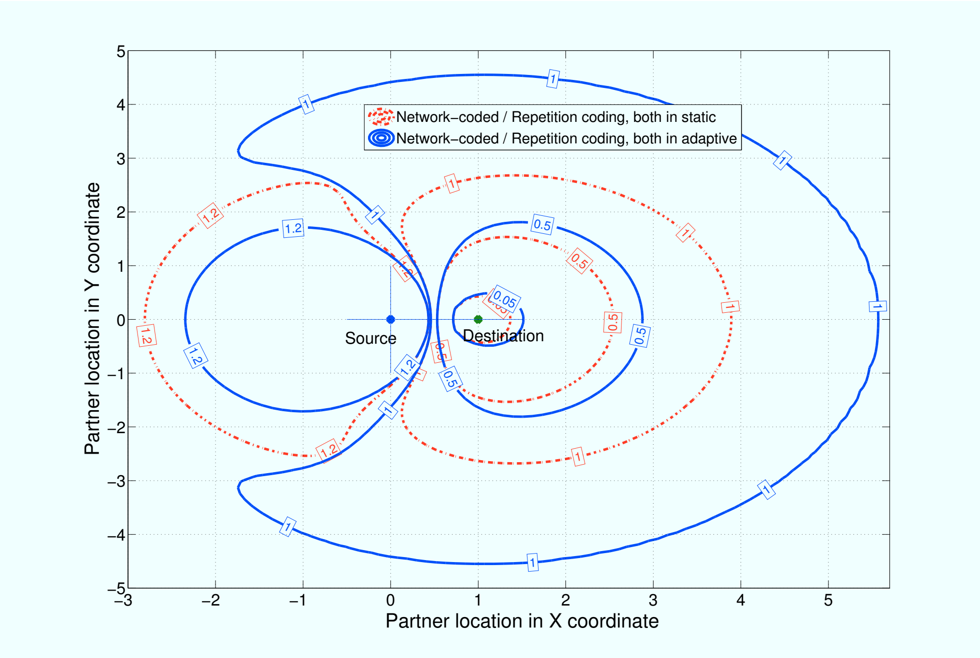

Figure 4.18.: Intra-cooperation gain contours, where the gain is computed by dividing the outage probability results of the network-coding-based static protocol by the repetition coding static protocol (dashed lines) and network-coding-based adaptive protocol by the repetition coding adaptive protocol (solid lines).

e.g. 0.001, the area span by the network coding protocol is larger than the repetition-coding-based protocol. As long as the partner is confined to these areas, we are guaranteed that the outage probability does not exceed 0.001. The probability contours of the network-coding-based and repetition-coding-based static protocol are approximately concentric to the coordinates (0, 0.7) and (0, 0.45), respectively. The coordinates (0, 0.7) and (0, 0.45) can also be seen as the outage contours where the outage probability approaches 0. Hence, the network-coding-based static protocol is more appropriate when a node closer to the destination is selected as the partner; at such locations the quality of the inter-user channels is poor and the uplink channels are more asymmetrical. In the repetition coding protocol, a node closer to the source (or in the center) should be selected as the partner. This gives more choice of selecting partner.

Shown in Figure 4.18 is the intra-cooperation gain contours of the static protocols (solid lines) and adaptive protocols (dashed lines). The gain is computed by dividing the outage probability of the network-coding-based and repetition coding protocols when both work either in static or adaptive manner. These contours help to answer the question, given the location of the relay, is it better to use the repetition coding or network-coding-based protocol. When the gain is greater than 1 (outside the unity-gain contour), the repetition coding protocol performs better; when it is less than 1 (inside the unity-gain contour), the protocol

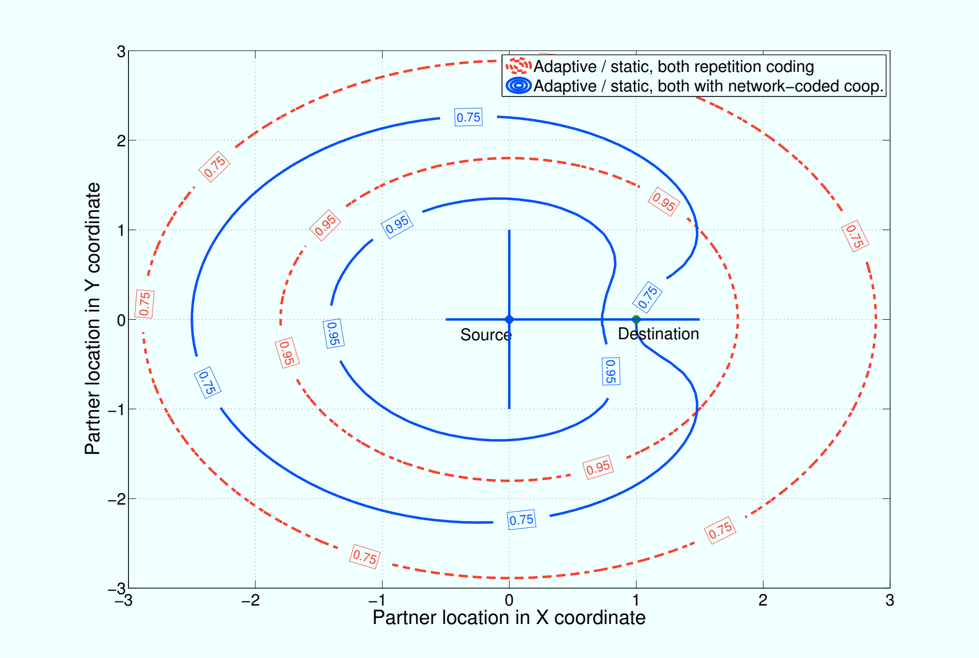

Figure 4.19.: Intra-cooperative gain contours, where the gain is computed by dividing the outage probability of the repetition coding adaptive protocol by the repetition coding static protocol (solid lines) and the network-coding-based adaptive protocol by the network-coding-based static protocol (dashed lines).

with network coding performs better. From the figure, we note that the region in which the gain is greater than 1 (or the repetition coding protocols perform better) is located closer to the source, and the region in which the gain is less than 1 is located closer to the destination. Moreover, the gain is less than one for sufficiently large geographic area.

Finally shown in Figure 4.19 is the intra-cooperative gain contours, where the comparison is performed within the repetition-coding-based and network-coding-based protocols, i.e., by dividing the outage probability of the repetition coding (respectively with network coding) adaptive protocol by the the repetition coding (respectively with network coding) static protocol. These contours help to illustrate the advantage, within either the repetition coding or network-coding-based protocol, when we switch from static to adaptive. We see that as we go from the bigger to the smaller contours, the intra-cooperation gain approaches unity. This means that over such large area, adaptive protocols are more suited than the static protocols. We conclude that, in general, adaptive protocols outperform their static counterparts over wider geographic area.

4.6. Summary

In this chapter, we have derived the outage probability of the network-coded cooperation protocol and examined its outage behavior. All channels are assumed to be block-fading Rayleigh distributed; moreover, orthogonal transmission and half-duplex constraints are considered. The system model was given in Section 4.1. The outage derivation was presented in Section 4.2; approximating this outage result at high SNR values, we have shown that the protocol achieves full diversity order 2. The outage behavior of various cooperative protocols is compared in Section 4.3 and we showed that network-coded cooperation is suitable when the inter-user channels are lower quality. When the inter-user channels are good, protocols without network coding perform better. Based on the outage probability result, the diversity-multiplexing tradeoff and the coverage area extension of the protocol were studied in Section 4.4 and Section 4.5. Based the coverage area results, network-coded cooperation static protocol is more appropriate when a node closer to the destination is selected as the partner; and at such locations the quality of the inter-user channels is poor and the uplink channels are more asymmetrical. In the repetition coding protocol, a node closer to the source (or in the center) should be selected as the partner.

5. Energy Efficiency in Wireless Sensor Networks with Network-Coded Cooperation: Diversity-Energy Saving tradeoff

In energy-constrained WSNs, designing energy efficient transmission protocols is a key requirement. As discussed in Chapter 4, network-coded cooperation allows wireless nodes to exploit spatial diversity, which reduces transmission energy or increases communication reliability. However, relaying redundant messages consumes considerable energy at both transmitting and receiving nodes. Hence, one has to strike a balance between the diversity gain and additional energy expenditure when designing cooperative protocols in WSN. This chapter investigates the energy consumption of network-coded cooperation in WSN. First, for a given error rate requirement at the destination node, an *energy consumption* model is formulated. The model describes the average energy per information bit and takes into account transmission, reception, and processing energy spent at all cooperating nodes. Second, the impact of various parameters of the model, for example node separation distance, on the energy consumption will be evaluated.

Section 5.1 presents a brief introduction to energy considerations in a WSN that uses network-coded cooperation. A typical WSN transceiver circuit is presented in Section 5.2. The energy consumption modeling of point-to-point transmission is described in Sections 5.3. For the network-coded cooperation, general assumptions are presented in Section 5.4 and the average energy consumption model is derived in Section 5.5. In this section, we formulate the energy consumption as an optimization problem and present numerical results. Finally, an alternative formulation of energy consumption, called *energy efficiency* model is given in Section 5.6 and results for various payload sizes and source-partner separation distance are presented.

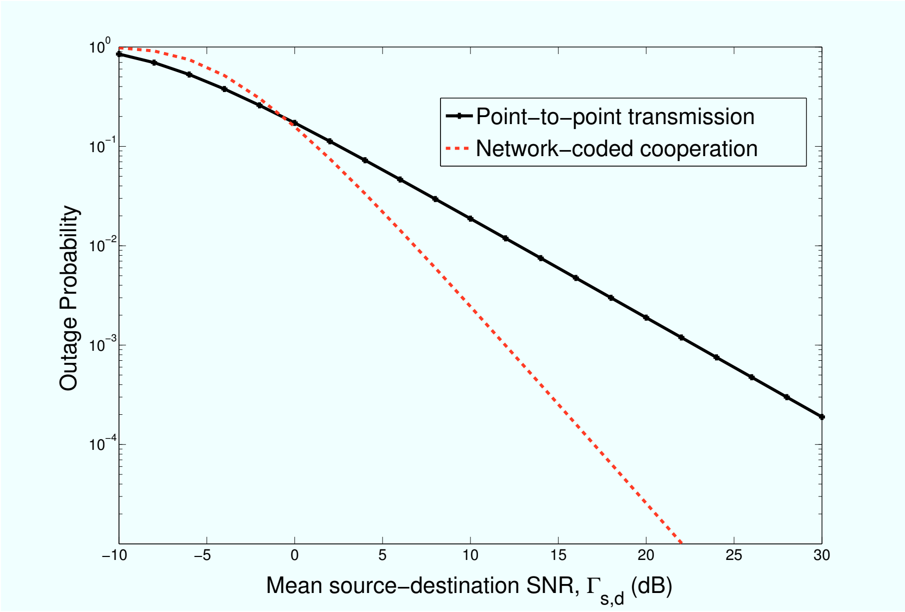

Figure 5.1.: Outage probability vs. average source-destination SNR $\Gamma_{s,d}$ of network-coded cooperation.

5.1. Introduction

This section presents the motivation to study energy consumption in network-coded cooperation as well as a brief literature survey on related studies.

5.1.1. Motivation

A WSN is generally composed of a collection of sensor nodes that are capable of data collection, signal processing, and wireless communication; this network is expected to be widely applicable in various fields [71]. The sensor nodes in WSNs are small and low cost devices that typically operate with small batteries for which replacement, if not impossible, is very difficult and expensive [72]. Consequently, minimizing energy consumption is one of the primary objectives in WSN design [73]. Energy-efficient communication techniques typically focus on minimizing transmission energy only, which is reasonable in long-range applications where this energy is dominant in total energy consumption. However, in short-range applications such as sensor networks, energy consumed in transmitting, receiving, and processing circuitry can constitute a significant portion of the total consumed energy. The circuit energy consumption includes Analog-to-Digital Converter (ADC), Digital-to-Analog Converter (DAC), frequency synthesizer, mixer, Low Noise Amplifier (LNA), power amplifier, and baseband Digital Signal Processor (DSP) (See Figure 5.3). Moreover, startup

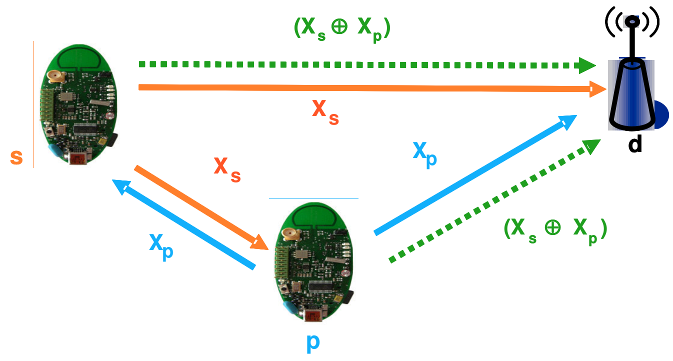

Figure 5.2.: Network-coded cooperation in WSN.

and idle mode (i.e. when a sensor is not transmitting or receiving) energy consumptions are sometimes significant. Hence, a comprehensive study of energy efficiency in WSNs requires a consideration of all these sources of energy consumption.

Network-coded cooperation can be used in WSNs to combat channel fading by diversity gain, which can be tradedoff to provide savings in transmit energy or an increase in communication range. Figure 5.1 shows the diversity gain of network-coded cooperation in comparison to the point-to-point transmission. However, when nodes transmit cooperatively, we need to consider the extra transmission and processing energy consumed in all cooperating nodes as a result of the redundant transmissions at relaying nodes.

The extra energy consumption in network-coded cooperation is explained using the network shown in Figure 5.2, where all transmissions from the source and partner in the two phases are shown. Assume the following: Inter-user channels are reciprocal, transmitting nodes use forward error correcting code, a relaying node remains silent if it fails to decode its partner's message, and all nodes are identical. Moreover, nodes once started up will be awake during the two phases. Idle mode occurs when either a node remains silent while its partner is transmitting, to avoid collision due to the orthogonality assumption, or when a node is not able to decode its partner's message and will remain silent in the second phase. Table 5.1 compares the network-coded cooperation and point-to-point transmission in terms of total number of transmissions, receptions, decodings, startup and idle times (including the source, partner, and destination).

We see that the total number of transmissions, processing, and receptions (accordingly the energy spent for processing these transmissions) are different in the two systems. Within the network-coded cooperation, the numbers differ based on the inter-user channel's quality. In fading wireless channels, quality of a channel is a random process that follows the behavior of the channel. Hence, energy consumption in cooperative transmission is also a random process that follows the quality of the inter-user channels. A stochastic approach is required

Table 5.1.: Network-coded cooperation vs. point-to-point transmissions.

Number of . . .	Cooperative transmission		Point-to-point transmission
	Good S-P link	Bad S-P link	
Transmissions	4	2	2
Receptions	6	4	2
Encodings	4	2	2
Decodings	5	4	2
Idle times	2	6	2
Start-ups	3	3	3

when modeling energy consumption in these networks. From the curves in Figure 5.1 and the number of transmissions in Table 5.1, there is obviously a tradeoff between the diversity gain and the loss due to additional transmitting, receiving, and processing energy. This needs to be taken into consideration in a network design [74]. We will address such tradeoff and characterize the gain of cooperation under such extra overhead in energy consumption.

In this chapter, we first formulate an *energy consumption model* for a given error rate (i.e, Packet Error Rate or Bit Error Rate) at the destination. Based on the model, we compute the optimal radiated power at the source and partner that fulfills the error rate requirement and minimizes the total energy consumption. Second, we define an *energy efficiency* metric as the product of reliability of reception at the destination and the ratio of the useful energy to the total energy. The useful energy is the energy spent on information (or payload) bits and the total energy is the energy spent on the payload bits, overhead bits, encoding, decoding, idle mode, and start-up mode. Using this model, for a given radiated power at the transmitting nodes, the impact of parameters (such as payload length) on the energy efficiency is investigated. As will be shown, the energy consumption and efficiency models are, among other parameters, functions of node locations in a network. Finding an optimal location of the partner that maximizes these metrics will be one of the points to be investigated and gives additional insight on partner node selection for cooperation.

5.1.2. Literature survey

Liu et al. analyzed the energy efficiency of MIMO transmissions in WSNs considering the tradeoff between diversity and multiplexing gains [73]. The results show that, with proper design, the energy efficiency of MIMO can be higher than that of the traditional Single-Input Single-Output (SISO) transmissions and the optimal energy efficiency is usually achieved when both the diversity gain and the multiplexing gain are jointly considered. An energy consumption model for a MIMO system with Alamouti diversity codes is developed in [72] and the best modulation and transmission strategy to minimize the total energy consumption is analyzed. The results show that BPSK-modulated SISO is more energy-efficient than MIMO systems when transmission distance is short. However, by allowing the constellation size to be optimally chosen, the energy efficiency of MIMO systems can be drastically increased. Sadek et al. developed an analytic framework to study the energy consumption for a class of relaying schemes based on ARQ [74]. The results show that for small separation between the source and destination, point-to-point transmission is more energy-efficient than relaying; moreover, an equal power allocation performs as good as optimal power allocation for some scenarios. The effects of the relay location and the number of employed relays on energy efficiency are also investigated. Simic et al. developed a heuristic for optimal partner choice and power allocation in a selection decode-and-forward relaying [75, 76] protocol. The authors presented that the partner-destination and the inter-user channels have roughly equal influence on optimal energy consumption.

5.2. Wireless Sensor Networks

In this section, we briefly present a block diagram of a WSN transceiver circuit and link layer data packet structure.

5.2.1. WSN Transceiver circuits

The block diagram of a typical WSN transceiver circuit is shown in Figure 5.3. The transmitter circuit includes DAC, channel encoder, filters, mixers, frequency synthesizer, local oscillator, and power amplifier. Similarly, the receiver circuit is composed of filters, LNA, local oscillator, mixers, ADC, and decoder. P_{te}, P_{amp}, $P_{rad,s}$, and P_{rec} represent the power spent in the transmitter circuit, power amplifier, radiated power, and the receiver circuit, respectively. The radiated power $P_{rad,s}$ is usually varied to fulfill the Quality of Service (QoS) requirement (e.g., BER, Packet Error Rate (PER)) at a receiving node.

Assume that the transceiver circuitry works on a *multi-mode* basis, i.e, when there is a signal to transmit all circuits work in *active mode*, when there is no signal to transmit they work in *idle mode*, when the major circuit components are turned off in *sleep mode*, and when switching from sleep mode to active mode there is a *start-up mode*. This multi-mode operation provides a significant saving of energy when sleep mode is employed [77]. Note that the transition from active mode to sleep mode is short enough to be negligible, however, the start-up process may be slow due to the finite Phase-Lock Loop (PLL) settling time in a frequency synthesizer.

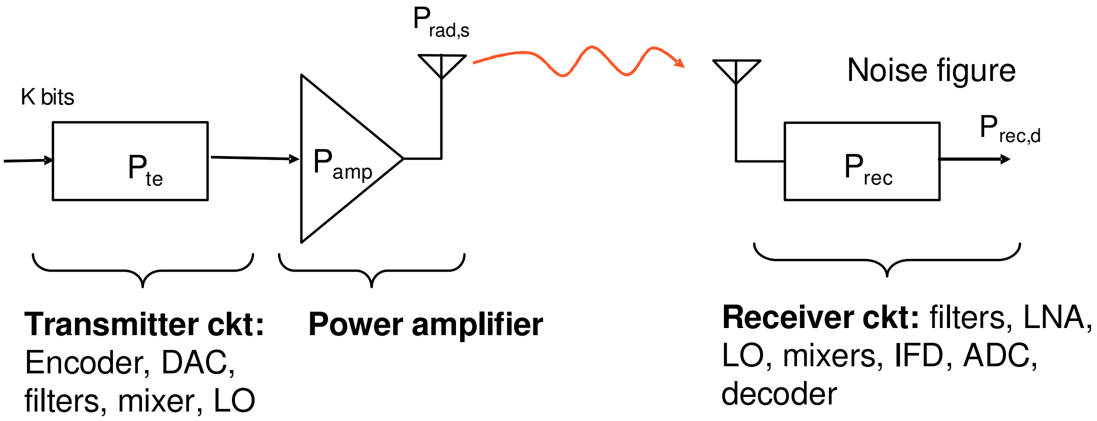

Figure 5.3.: Block diagram of a typical WSN transceiver circuit.

5.2.2. Packet structure in point-to-point transmission

Consider the link layer data packet structure shown in Figure 5.4. It consists of φ header bits, l payload bits, and τ trailer bits. The header field generally includes the current segment number, higher layer packet identifier, and the source and destination identifiers.[1] However, for typical WSN applications φ is expected to be only few bytes. The payload contains information and CRC bits, and the trailer is composed of parity bits and is used for error control of both the header and payload bits.

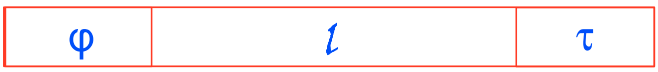

Figure 5.4.: The link-layer packet format.

5.3. Energy Consumption of Point-to-Point Transmission

In the following, we develop a relationship between the energy dissipated in the amplifier and received SNR at the receiving node. The strength of the received SNR determines the error rate (i.e, outage probability or PER) at the receiving node. The bit rate, symbol rate (i.e., after channel coding), and information rate (also called code rate) are represented as R_b

[1] In network-coded cooperation, the header field may contain bits required to set up cooperation (e.g., partner selection) and to implement network coding.

[bits/s], R_s [symbol/s], and R_c [bit/symbol], respectively. The three rates are related to each other as $R_b = R_s R_c$.

5.3.1. Power amplifier calibration

Consider the transceiver circuit shown in Figure 5.3, where the source transmits to the destination. For the power amplifier at the source, a typical power consumption model is given as [72]

$$P_{amp} = \frac{\zeta}{\eta} P_{rad,s} \tag{5.1}$$

where P_{amp} is the consumed power, $P_{rad,s}$ is the radiated power, ζ is the drain efficiency of the RF power amplifier, and η is the peak-to-average ratio, which depends on the modulation scheme used.[2] The radiated power determines the error rate at the receiving node.

The wireless channel between the source and destination is assumed to be slow fading and Rayleigh distributed with AWGN. Following the link budget analysis approach of [3] and the model developed in Chapter 3, the average received power $P_{rec,d}$ at the output of the destination is related to the transmit power $P_{rad,s}$ according to the path-loss model

$$\begin{aligned} P_{rec,d} &= G_t G_r \left(\frac{\lambda}{4\pi}\right)^2 E\left\{|h_{s,d}|^2\right\} \left(\frac{1}{N_f}\right)\left(\frac{1}{d_{s,d}^{\alpha}}\right) P_{rad,s} \\ &= \xi \left(\frac{1}{d_{s,d}^{\alpha}}\right) P_{rad,s} \end{aligned} \tag{5.2}$$

where λ is the carrier wavelength, $d_{s,d}$ is the distance between the source and destination, $2 \leqslant \alpha < 5$ is the channel path-loss exponent, N_f is the receiver noise figure that accounts for attenuation in the receiver, G_t and G_r are the transmitter and receiver antennas' gains, and $E\left\{|h_{s,d}|^2\right\}$ is the expected value of the fading coefficient. The term ξ in (5.2) represents

$$\xi = G_t G_r \left(\frac{\lambda}{4\pi}\right)^2 \left(\frac{1}{N_f}\right) E\left\{|h_{s,d}|^2\right\}. \tag{5.3}$$

The average received SNR at the destination, represented by $\Gamma_{s,d}$, is written in terms of the radiated power as

$$\left(\frac{P_{rec,d}}{N}\right) = \xi \left(\frac{1}{kT_oB}\right)\left(\frac{1}{d_{s,d}^{\alpha}}\right) P_{rad,s} = \Gamma_{s,d} \tag{5.4}$$

[2] An alternative power consumption model is $P_{amp} = \alpha_{amp} + \beta_{amp} P_{rad,s}$, where α_{amp} and β_{amp} are constants depending on process technology and amplifier architecture [77].

where $N = kT_oB$ is the noise power in Watt, $k = 1.38X10^{-23}$Joule/Kelvin is Boltzman's constant, T_o the noise temperature in Kelvin, and B the bandwidth in Hz. The power spectral density of the receiver noise is given as $N_o = kT_o$ Watt/Hz. Substituting Equation (5.1) into Equation (5.4) and rearranging terms, the dissipated power at the amplifier circuit is written as

$$\begin{aligned} P_{amp} &= \frac{\zeta}{\eta}\frac{1}{\xi}\,kT_oB\;d_{s,d}^{\alpha}\left(\frac{P_{rec,d}}{N}\right) \\ &= L_{total}^{'}\,\Gamma_{s,d}\,d_{s,d}^{\alpha} \end{aligned} \tag{5.5}$$

where $L_{total}^{'} = \frac{\zeta}{\eta}\,\frac{1}{\xi}\,kT_oB$ is an attenuation factor. Equation (5.5) relates the dissipated power at the power amplifier to the received SNR and distance between the source and destination. In Equation (3.25), it was shown that the received SNR given in (5.4) is approximately related to the outage probability, $P_{out,ppt}$, as

$$\Gamma_{s,d} \;\approx\; \frac{g(R)}{P_{out,ppt}} \tag{5.6}$$

where $g(R) = 2^{R_c} - 1$ and R_c is the code rate. This equation helps us to compute the required SNR at the destination for a given outage probability which represents the required QoS. If a closed form expression relating PER and $\Gamma_{s,d}$ exists, then the PER Equation (5.6) can be used instead of outage probability. Substituting (5.6) into (5.5), we get

$$P_{amp} \;\approx\; \frac{g(R)}{P_{out,ppt}}\,L_{total}^{'}\,d_{s,d}^{\alpha} \tag{5.7}$$

Equation (5.6) and (5.7) show that, for a given location of nodes and information rate R_c, the dissipated power at the source can be varied to meet the desired outage probability. This expression will be used when energy efficiency is computed in the next section.

5.3.2. Energy consumption formulation

Network-coded cooperation involves the transmission of both the source's and partner's packet to the destination. To make a fair comparison, in point-to-point transmission we consider that both the source and partner directly transmit to the destination, instead of a single transmission from either the source or partner. The consumed energy per payload bit can be formulated following the approach in [78]. In Table 5.1, the energy spent per information bit in the transmitter circuitry of the source and partner, given as E_t, is modeled as

$$\begin{aligned} E_t &= \frac{1}{2l}\left[(2P_{te} + P^d_{amp,s} + P^d_{amp,p})(\frac{\varphi + l + \tau}{R_s}) + 2P_{idle}T_{idle} + 2P_{start}T_{start} + 2E_{enc}\right] \\ &= \frac{1}{l}\left[(P_{te} + \frac{1}{2}(P^d_{amp,s} + P^d_{amp,p}))(\frac{\varphi + l + \tau}{R_s}) + P_{idle}T_{idle} + P_{start}T_{start} + E_{enc}\right] \end{aligned} \tag{5.8}$$

where $P^d_{amp,s}$ and $P^d_{amp,p}$ are the power dissipated in the amplifier circuitry of the source and partner node. P_{start} and T_{start} are the startup power and time in the transmitter circuit; E_{enc} is the energy to encode the packet. Moreover, P_{idle} and T_{idle} are the power and time spent in idle mode. E_{enc} depends on the type of code and the implementation of the encoding algorithm (i.e, hardware or software) and is zero if no encoding is used. In Equation (5.8), the term inside the square brackets represents the total energy to send two packets of $\varphi + l + \tau$ bits. This energy is divided by $2l$, the total number of information bits of the source and partner, to get the energy spent per information bit. Similarly, the total receiving energy per payload bits is given as

$$\begin{aligned} E_r &= \frac{1}{2l}\left[2P_{rec}(\frac{\varphi + l + \tau}{R_s}) + P_{start}T_{start} + 2E_{dec}\right] \\ &= \frac{1}{l}\left[P_{rec}(\frac{\varphi + l + \tau}{R_s}) + \frac{1}{2}P_{start}T_{start} + E_{dec}\right] \end{aligned} \tag{5.9}$$

where E_{dec} is the energy to decode a packet. The energy to communicate (i.e, transmit and receive) one bit of information, E_{ppt}, is then

$$\begin{aligned} E_{ppt} &= E_t + E_r \\ &= \underbrace{k_1^{'}}_{\text{useful energy}} \underbrace{+ k_1^{'}\frac{(\varphi + \tau)}{l} + \frac{k_2^{'} + E_{enc} + E_{dec}}{l}}_{\text{additional energy}} \end{aligned} \tag{5.10}$$

where $k_1^{'}$ and $k_2^{'}$ are constants for a given transceiver and symbol rate R_s. For the RFM-TR1000 transceiver circuit, $k_1{'}$ and $k_2{'}$ were calculated to be 1.85μJ/bit and 24.86μJ, respectively. The contribution of $k_2{'}$ is high at high data rate and short payload [78].

The "*useful energy*", $k_1^{'}$, refers to energy spent to communicate an information bit, while "*additional energy*" refers to the energy spent because of circuit inefficiency (i.e, start-up energy), link-layer protocol design (i.e, idle mode), and error control (i.e, encoding and decoding). If we further assume that $P^d_{amp,s} = P^d_{amp,p} = P^d_{amp}$, then

$$\begin{aligned} k_1^{'} &= \frac{1}{R_s}\left[P_{te} + \frac{1}{2}(P^d_{amp,s} + P^d_{amp,p}) + P_{rec}\right] = \frac{1}{R_s}\left[P_{te} + P^d_{amp} + P_{rec}\right] \\ k_2{'} &= P_{start}T_{start} + \frac{1}{2}P_{start}T_{start} + P_{idle}T_{idle} = \frac{3}{2}P_{start}T_{start} + P_{idle}T_{idle}. \end{aligned} \quad (5.11)$$

If the outage probability is fixed, then P^d_{amp} can be computed from (5.7) (provided separation distance and L'_{total} are known) and substituted into (5.10) to get the total *energy consumption*. Using this approach, the total consumed energy that fulfills the outage probability requirement (or QoS specified by the BER or PER at the destination) can easily be computed.

5.3.3. Energy efficiency formulation

The energy consumption formulated above specifies the total energy per information bit for a given QoS. Sometimes, instead of fixing the error rate and computing the energy consumption, we may be interested to know the joint "*optimal*" energy consumption and error rate of the system. The *energy efficiency* metric that takes percentage of energy spent to communicate the payload and reliability of reception at the destination is defined as

$$\eta = \left[\frac{k_1' l}{k_1'(\varphi + l + \tau) + k_2{'} + E_{enc} + E_{dec}}\right](1 - P_{out,ppt}) \quad (5.12)$$

where $\frac{k_1' l}{k_1'(\varphi + l + \tau) + k_2{'} + E_{enc} + E_{dec}}$ accounts for the energy throughput and $(1 - P_{out,ppt})$ is the packet success rate which accounts for the transmission reliability. Energy efficiency expresses the ratio of the energy to communicate the payload bits to the total energy spent and conditions this energy ratio on the probability of successfully receiving a packet. Using this formula, for various values of radiated power, the percentage of energy spent to communicate the information bit is computed. Considering a black box where the total energy is the input and the energy to communicate the payload as the output, the term in the square bracket of Equation (5.12) accounts for the ratio of the output energy to the input energy, hence the name energy efficiency.

5.4. General Assumptions in Network-Coded Cooperation

In this section, we present general assumptions that will be used for energy consumption formulation in network-coded cooperation. Let the radiated powers in cooperative transmission be $P^{c,i}_{amp,j}$, with phase $i \in \{1,2\}$ and node $j \in \{s,p\}$. The total power spent in the two phases is given as $P^{c}_{amp,j} = P^{c,1}_{amp,j} + P^{c,2}_{amp,j}$. Similarly, in the point-to-point transmission $P^{d}_{amp,j}$ represents the radiated power from node j. For the analysis in the following section, let us make the following general assumptions.

- *System or user level:* In system level, we compute the energy consumption/efficiency considering the successful reception of both users' packets. On the other hand, in user level we will be concerned with one user only and compute the consumption/efficiency of that user.

- *Same bit rate and code rate in both the point-to-point transmission and the two phases of cooperation:* The total time spent to send one packet in cooperation is twice that of the point-to-point transmission (see Figure 5.5). This assumption benefits cooperation as there is no loss in spectral efficiency (i.e, same coding rate like in point-to-point) because of cooperation. In Chapter 4, the cooperative scheme was designed to operate at twice the code rate so that the total time in the two phases was the same as the point-to-point (so that same radiated energy in both cooperation and point-to-point).

  We note in Figure 5.5 that same time T is required to send one packet in each phase of cooperation or the point-to-point transmission. The radiated energy in point to point transmission is given as $E^{d}_{amp,s} = \mathrm{T}P^{d}_{amp,s}$. Similarly, the radiated energy in cooperation is $E^{c}_{amp,s} = E^{c,1}_{amp,s} + E^{c,2}_{amp,s} = \mathrm{T}(P^{c,1}_{amp,s} + P^{c,2}_{amp,s}) = \mathrm{T}P^{c}_{amp,s}$, where $E^{c,1}_{amp,s}$ and $E^{c,2}_{amp,s}$ are the radiated energy in phase one and two, respectively. We see that the sum of radiated energy in the two phases is given by the sum of the radiated power in the two phases times the packet duration.

- *Reciprocal inter-user channel*: In Chapter 4, it was defined that in reciprocal inter-user channels the received *instantaneous* SNR at the source and partner are the same. When the channel attenuations are the same, the reciprocal channels case happens if the radiated power of the two users, in each phase, are the same, i.e, $P^{c,i}_{amp,s} = P^{c,i}_{amp,p} =$

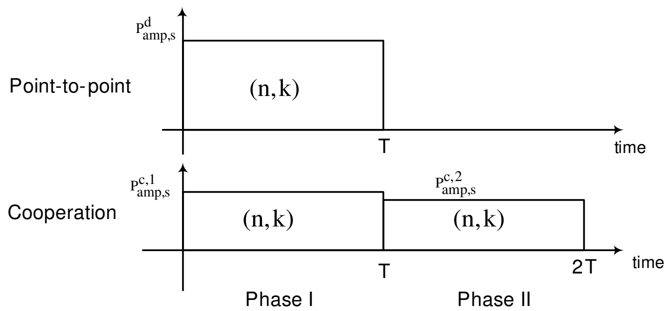

Figure 5.5.: Timing diagram of cooperative and point-to-point transmissions. (n, k) represent the number of symbols, n, and the informations bits, k, in one packet.

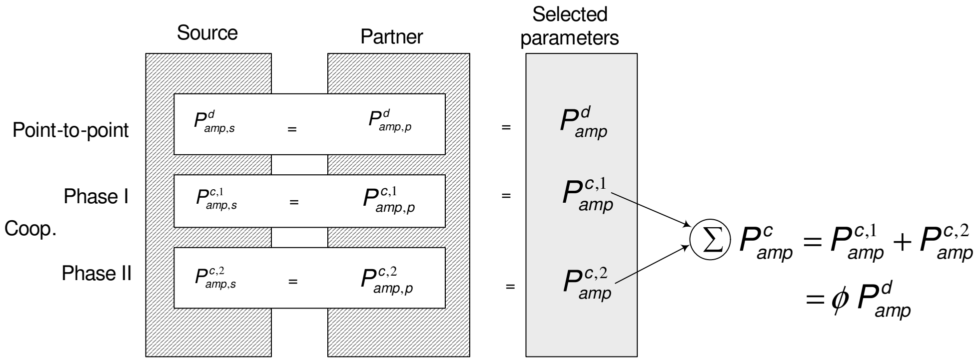

Figure 5.6.: Summary of parameters used in energy allocation.

$P^{c,i}_{amp}$. Because of this assumption, we will have only two cases for analysis, i.e, either both nodes cooperate or both will not cooperate.

In general, we can allocate the same or different radiated energy in cooperation and point-to-point transmission. Moreover, within the cooperation scheme, the same or different radiated energy can be used in the two phases. The following two points explain the energy allocation in cooperation vs. point-to-point and phase one vs. phase two.

- *Proportion of the total radiated energy in cooperation and point-to-point*: We consider that $E^c_{amp} = \phi E^d_{amp}$. Substituting for E^c_{amp} and E^d_{amp}, we get the relationship $P^c_{amp} = P^{c,1}_{amp} + P^{c,2}_{amp} = \phi P^d_{amp}$, where $0 \leqslant \phi = \frac{E^c_{amp}}{E^d_{amp}} = \frac{P^c_{amp}}{P^d_{amp}} < \infty$ is the proportion of the total energy (or power) in cooperation and point-to-point transmissions. When $\phi = 1$, i.e, the radiated energy in both cooperation and point-to-point transmissions is the same, the PER at the destination will be different for the two schemes. Alternatively, one can set $\phi \neq 1$, i.e., different energy allocation in the two schemes, and the transmit powers in the two schemes are varied to deliver the same PER at the destination.

- *Energy allocation per phase*: Assume β of the total available energy is used in phase one and $(1-\beta)$ in the second phase. This implies that

$$P^{c,i}_{amp} = \begin{cases} \beta\phi P^d_{amp} & \text{phase one, i.e. i =1;} \\ (1-\beta)\phi P^d_{amp} & \text{phase two, i.e. i =2.} \end{cases} \qquad (5.13)$$

  A value of $\beta = 0.5$ means equal allocation in the two phases, which is usually assumed in the conventional cooperative approaches.

To conclude, the three parameters of interest are the radiated powers in the point-to-point transmission, P^d_{amp}, phase one of cooperation, $P^{c,1}_{amp}$, and phase two of cooperation, $P^{c,2}_{amp}$.

Figure 5.6 summarizes these parameters. These parameters are related to each other as

$$\begin{aligned} P^{c,1}_{amp} &= \beta\phi P^{d}_{amp} \\ P^{c,2}_{amp} &= (1-\beta)\phi P^{d}_{amp}. \end{aligned} \tag{5.14}$$

5.5. Energy Consumption in Network-Coded Cooperation

In this section, the energy consumption of the network-coded cooperation protocol will be formulated. Based on the above assumptions, the reciprocal inter-user channels suggest that there are two cases to consider, i.e, when the inter-user channels are good or bad so that transmissions are either correctly or incorrectly decoded.

5.5.1. Good inter-user channel

In the case of good inter-user channel, there is *full cooperation* between the source and partner. Figure 5.7 shows states of the source, partner, and destination in the two phases. Referring to Table 5.1, the total energy per information bit, taking transmissions of both the source and partner into account, is formulated as

$$\begin{aligned} E_{coop,1} &= \frac{1}{2l}\left[(4P_{te}+2\phi P^{d}_{amp}+6P_{rec})\left(\frac{\varphi+l+\tau}{R_s}\right)+3P_{st}T_{st}\right]+ \\ &\quad \frac{1}{2l}\left[2P_{id}T_{id}+4E_{enc}+5E_{dec}\right] \\ &= k_1+\underbrace{k_1+2k_1\left(\frac{\varphi+\tau}{l}\right)+\frac{P_{rec}-\phi P^{d}_{amp}}{R_s}\left(\frac{\varphi+l+\tau}{l}\right)+k_2}_{\text{Energy due to add. tx, rx, and processing}} \end{aligned} \tag{5.15}$$

where $k_1=\frac{P_{te}+\phi P^{d}_{amp}+P_{rec}}{R_s}$ and $k_2=\frac{k_2^{'}+2E_{enc}+\frac{5}{2}E_{dec}}{l}$. P_{te}, P_{amp}, P_{rec}, and R_s are the power, in Watts, spent at the transmitter circuit, power amplifier, receiving circuit, and symbol rate in symbols/s, respectively. Also P_{st} and T_{st} are the power and time at the startup mode. For $\phi=1$, the term $k_1^{'}=k_1$ where $k_1^{'}$ is defined in Equation (5.11). The term $2\phi P^{d}_{amp}$ in Equation (5.15) is the sum of the radiated power from the source and partner over the two phases, i.e., $2\phi P^{d}_{amp}=P^{c,1}_{amp,s}+P^{c,2}_{amp,s}+P^{c,1}_{amp,p}+P^{c,2}_{amp,p}$.

From Equation (5.15), we note that extra power is spent on cooperation because of redundant transmissions, overhead and error correction bits, encoding and decoding, circuit inefficiency

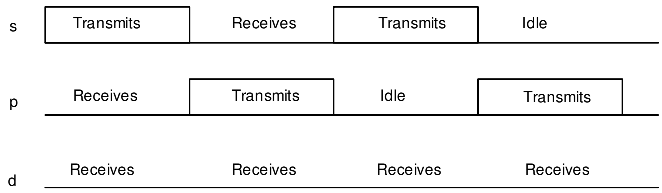

Figure 5.7.: Diagram showing the states of the source, partner, and destination when good-quality inter-user channel exist.

(start up), and idle mode (to avoid collision). Equation (5.15) can also be written as

$$E_{coop,1} = E_{ppt} + \underbrace{\left(k_1^{'} + \frac{P_{rec} - P_{amp}^{d}(2-\phi)}{R_s}\right)\left(\frac{\varphi + l + \tau}{l}\right) + \frac{E_{enc} + \frac{3}{2}E_{dec}}{l}}_{\text{Loss due to cooperation}} \tag{5.16}$$

where E_{ppt} is the energy needed for point-to-point transmission defined in Equation (5.10). From this equation, we see the loss of energy by using cooperation instead of point-to-point transmission. On the other hand, we exploit diversity by cooperation which can be used to reduce PER and BER.

System-level outage probability

The energy consumption formulated in Equation (5.15) is valid when the inter-user channels are of good quality. The probability that good quality inter-user channels occur (or the probability that the consumed energy is given by Equation (5.15)) is obtained from the outage probability of the inter-user transmissions following an identical approach as in Chapter 4. The system-level outage event occurs when the destination is unable to decode the transmissions both from the source and partner. At the destination, let $\overline{X_s}$, $\overline{X_p}$, and $\overline{X_s \oplus X_p}$ represent the outage events of the source's packet, partner's packet, and network-coded packet, respectively. The overall outage event is written as

$$\overline{X_s \wedge X_p} \cong \left[\overline{X_s} \wedge [\overline{X_p} \vee \overline{X_s \oplus X_p}]\right] \vee \left[\overline{X_p} \wedge \overline{X_s \oplus X_p}\right] \tag{5.17}$$

where bars indicate the outage of an event; $\wedge$ and $\vee$ are logical 'AND' and 'OR' operations, respectively. The probability that the event in Equation (5.17) occurs, given by $P(\overline{X_s \wedge X_p}) = P_{sys,1}$, is then

$$P_{sys,1} = P(\overline{X_s})[P(\overline{X_p}) + P(\overline{X_s \oplus X_p})] + P(\overline{X_p})P(\overline{X_s \oplus X_p})(1 - 2P(\overline{X_s})). \tag{5.18}$$

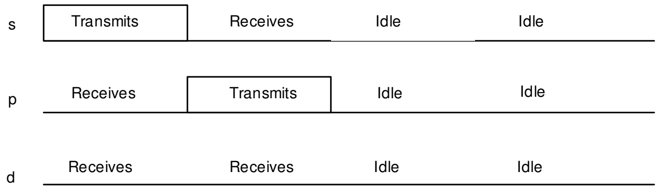

Figure 5.8.: Diagram showing the states of the source, partner, and destination when bad-quality inter-user channel exist.

At high SNR, this outage probability is approximated as

$$P_{sys,1} \approx \left[\frac{k_3}{P^d_{amp}}\right]^2 d^{\alpha}_{s,d} d^{\alpha}_{p,d}. \tag{5.19}$$

The term $k_3 = \left[\frac{g(R)}{\beta\phi} L^{'}_{total}\right]$ where $L^{'}_{total}$ and $g(R)$ are given in Equations (5.5) and (5.6).

User-level outage probability

For the case of good inter-user channels, the approximation in Equation (5.19) also applies to user-level outage probability, denoted as $P_{use,1}$.

5.5.2. Bad inter-user channel

Following an identical approach as in the case of good-quality inter-user channel and assuming that nodes remain silent during the second phase when decoding fails, only two packets will be transmitted, i.e., one by the source and one by the partner (see Figure 5.8 and also Table 5.1). This case of no cooperation is similar to the point-to-point transmission except that there are two additional receptions and decodings as well as 4 idle times. The destination tries to recover the source's and partner's packets from the first phase transmissions and remains idle in the second phase. The total energy in this case is then given as

$$E_{coop,2} = k_1^{'} + \underbrace{k_1^{'}\left(\frac{\varphi+\tau}{l}\right) + \left(\frac{P_{rec} - P^d_{amp}(1-\beta\phi)}{R_s}\right)\left(\frac{\varphi+l+\tau}{l}\right) + k_2^{''}}_{\text{Energy due to overhead, add. rx., and processing}} \tag{5.20}$$

with new constant $k_2'' = \frac{k_2{}' + 2P_{idle}T_{idle} + E_{enc} + 2E_{dec}}{l}$. To see the energy loss by using cooperation instead of point-to-point transmission, we can re-write Equation (5.20) as

$$E_{coop,2} = E_{ppt} + \underbrace{\left(\frac{P_{rec} - P_{amp}^{d}(1-\beta\phi)}{R_s}\right)\left(\frac{\varphi + l + \tau}{l}\right) + \frac{2P_{idle}T_{idle} + E_{dec}}{l}}_{\text{Energy loss b/c of cooperation}}. \tag{5.21}$$

System-level outage probability

In the case of bad inter-user channel, the system level outage probability, $P_{sys,2}$, can be calculated as follows. The outage event $\overline{X_s \wedge X_p}$ in this case is written as

$$\overline{X_s \wedge X_p} \cong \overline{X}_s \wedge \overline{X_p} \tag{5.22}$$

and the corresponding probability is then

$$P_{sys,2} \approx \left[\frac{k_3}{P_{amp}^{d}}\right]^2 d_{s,d}^{\alpha} d_{p,d}^{\alpha}. \tag{5.23}$$

where k_3 is defined in Equation (5.19). Note that this approximation is identical to that of good inter-source channels case because, at high SNR values, the first-phase transmissions dominates the outage behavior.

User-level outage probability

The user-level outage probability, as computed in the previous chapters, is given by

$$P_{use,2} \approx \left[\frac{k_3}{P_{amp}^{d}}\right] d_{s,d}^{\alpha}. \tag{5.24}$$

5.5.3. Average energy consumption

The average energy consumption is the weighted sum of the energy consumptions calculated for the two cases above, where the weighing coefficients are the probabilities that the cases occur (which in turn depend on inter-user channel condition). The average consumed energy,

$E_{c,t}$, is given as

$$\begin{aligned} E_{c,t} &= E_{coop,1}(1-P_1) + E_{coop,2}P_1 \\ &= E_{coop,1} + P_1(E_{coop,2} - E_{coop,1}) \\ &= E_{coop,1} - P_1\left[\left(k_1 - \frac{P^d_{amp}(1-\phi(1-\beta))}{R_s}\right)\left(\frac{\varphi+l+\tau}{l}\right) + \frac{k_2 - k_2''}{l}\right] \end{aligned} \tag{5.25}$$

where the outage probability $P_1 = \left[\frac{k_3}{P^d_{amp}}\right] d^{\alpha}_{s,p}$ shows failure of the inter-user transmission, $E_{coop,1}$ and $E_{coop,2}$ are given by Equations (5.15) and (5.20), and $k_2 - k_2'' = E_{enc} + \frac{1}{2}E_{dec} - 2P_{idle}T_{idle}$. Replacing for $E_{coop,1}$ and $E_{coop,2}$, we get

$$\begin{aligned} E_{c,t} &= E_{ppt} + \underbrace{\left(\frac{\varphi+l+\tau}{l}\right)\left(k_1(1-P_1) + \frac{P_{rec} - P^d_{amp}\left[(2-P_1) + \phi P_1(1-\beta) - \phi\right]}{R_s}\right)}_{\text{Because of cooperation}} \\ &\quad + \underbrace{\frac{E_{enc}(1-P_1) + \frac{1}{2}E_{dec}(3-P_1) + 2P_{idle}T_{idle}P_1}{l}}_{\text{Because of add. processing}}. \end{aligned} \tag{5.26}$$

To simplify Equation (5.26) further, we assume that the energy ratio $\phi = 1$ and $\beta P_1 << 1$. Replacing the point-to-point transmission from Equation (5.10) into Equation (5.26), we get

$$E_{c,t} = \left(\frac{P^d_{amp}(1-P_1) + P_{te}(2-P_1) + P_{rec}(3-P_1)}{R_s}\right)\left(\frac{\varphi+l+\tau}{l}\right) + k_4 \tag{5.27}$$

where the constant $k_4 = \frac{\frac{3}{2}P_{st}T_{st} + P_{idle}T_{idle}(1+2P_1) + E_{enc}(2-P_1) + \frac{1}{2}E_{dec}(5-P_1)}{l}$. Equation (5.27) approximates the average total energy consumed in network-coded cooperation and it is, among other variables, a function of the amplifier power, P^d_{amp}. In the next section, P^d_{amp} will be expressed as a function of the average outage probability at the destination. The resulting expression is substituted into Equation (5.26) (or Equation (5.27)) so that the total energy consumption will become a function of the outage probability.

5.5.4. Average outage probability

The average outage probability (like the average energy consumption discussed above) is the weighted sum of the outage probabilities calculated for the two cases, where the weighing coefficients are the probabilities that the cases occur. The system level average outage probability, denoted as $P_{out,sys}$, is given by

$$\begin{aligned} P_{out,sys} &= (1-P_1)P_{sys,1} + P_1 P_{sys,2} \\ &\approx \left[\frac{k_3}{P^d_{amp}}\right]^2 d^\alpha_{s,d} d^\alpha_{p,d} \end{aligned} \tag{5.28}$$

where k_3 is defined in Equation (5.19). We see that the case for full cooperation dominates at high SNR values. If this outage probability is required to be less than a threshold value, given by P^*_{out}, then we get the condition

$$P^d_{amp} \geqslant k_3 \sqrt{\frac{(d_{s,d}\, d_{p,d})^\alpha}{P^*_{out}}}. \tag{5.29}$$

Once the P^*_{out} is fixed, P^d_{amp} is readily computed from Equation (5.29). Then using knowledge of P^d_{amp}, the inter-user outage probability P_1 is calculated next and then finally all substituted into Equation (5.26) to get the average consumed energy per information bit. The same approach can be followed for the user level.

Similarly, the user-level average outage probability is approximated as

$$\begin{aligned} P_{out,use} &= (1-P_1)P_{use,1} + P_1 P_{use,2} \\ &\approx \left[\frac{k_3}{P^d_{amp}}\right]^2 d^\alpha_{s,d} d^\alpha_{p,d} \left[\left(1 - \frac{k_3}{P^d_{amp}} d^\alpha_{s,p}\right) + \left(\frac{d_{s,p}}{d_{p,d}}\right)^\alpha\right]. \end{aligned} \tag{5.30}$$

If the desired outage probability, P^*_{out}, is specified, then identical threshold expression as in Equation (5.29) can be computed. Finally, we note that the total consumed energy formulated in Equation (5.29) as well as the amplifier power P^d_{amp} computed from either Equation (5.29) or Equation (5.30) are functions of many variables. If we specifically consider their relation with node separations, there is a non-linear relationship between $d_{s,d}$ and $d_{p,d}$ and the average energy consumption and outage probability. In the following, we will pick $d_{s,d}$ and $d_{p,d}$ as variables and study their impact on energy consumption.

5.5.5. Numerical results

In this section, numerical results of the energy consumption model are presented. The parameter values used for numerical results are: symbol rate R_s = 250 Kbit/s, code rate R_c = 0.8, $\eta = 0.2$, $\xi = 1$, $\alpha = 4$, $G_t.G_r$=-10 dB, $\beta = 0.65$, P_{te} = 5.75 mW, P_{rec} = 11.13mW, P_{start} = 60mW, T_{start} = 466 μs, and $P_{idle} = 100$ mW. The source and destination are separated by 30

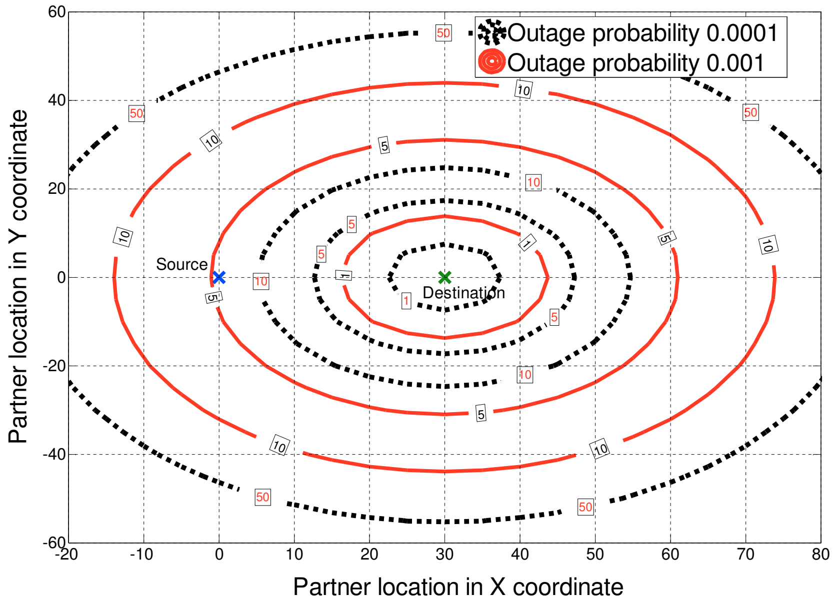

Figure 5.9.: Transmit power P^d_{amp}, in milliwatt, contours for various locations of the partner node.

meters; however, the partner is located anywhere in the network and its position is specified by its coordinate on the two axes. The output power from the amplifiers is variable.

For a propagation environment specified by k_3 and the network geometry specified by a range of potential partner locations, Figure 5.9 depicts the contour plot of the optimal transmit power P^d_{amp}, in milliwatt, for system-level outage probability of either 0.0001 or 0.001. A given contour specifies that, to operate at a fixed outage probability, the required transmit power is the same for a range of locations defined by the contour. We see that the optimal transmit power generally increases the further away the partner is from the source and destination nodes and the contours are concentric around the location of the destination. Thus, higher energy savings are expected from network-coded cooperation when the partner is in the vicinity of the destination.

For each location of the partner node whose optimal transmit power is shown Figure 5.9, we next compute and plot the average energy consumption per information bit at each point. Figure 5.10 is a three-dimensional plot of this average consumed energy, given by Equation (5.27), when the system-level outage probability is fixed to 0.0001. Examining this energy plot, we can draw the following conclusions.

Firstly, Figure 5.10 confirms that the closer a potential partner is to both the source and destination, the smaller the consumed energy from network-coded cooperation. Moreover, the energy consumption contours on the horizontal axis approximately form circles centered about the destination. This implies that from energy consumption perspective, the source

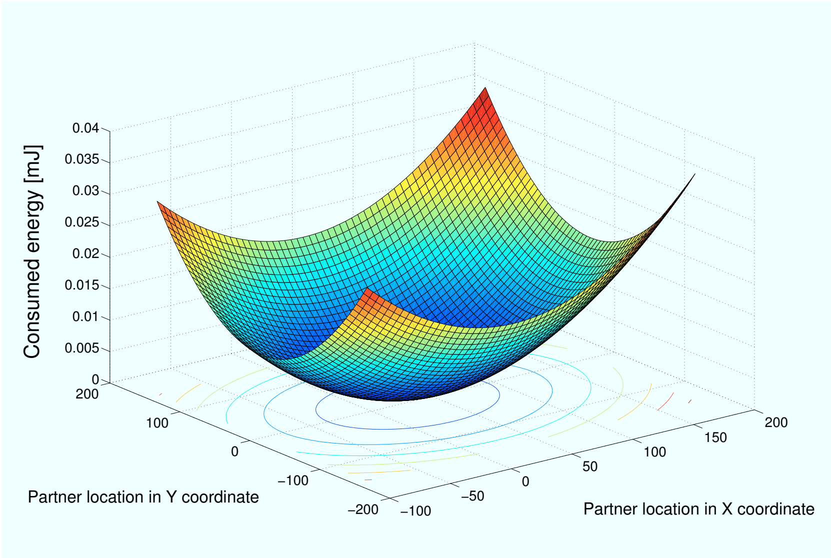

Figure 5.10.: Average consumed energy, in mill Joule, for system-level outage probability of 0.0001.

should pick a partner located in the vicinity of the destination. Secondly, we observe that the lower the consumed energy, the smaller the diameter of the corresponding contour in Figure 5.10. This indicates that the partner choice region is smaller for a lower energy budget.

Finally, in a special case of linear topology where the partner is located in a straight line from the source to the destination, Figure 5.11 depicts the energy consumption for two values of the outage probability. We note that the consumption is minimal when the partner is located in the vicinity of the destination. And if the reliability of transmission is low, e.g., outage probability of 0.001, the consumption is also lower.

5.5.6. Formulation as an optimization problem

The energy consumption in Equation (5.26) can be formulated as an optimization problem. As an example, the optimal amplifier power P_{amp} that fulfills the outage probability requirement at the destination and also minimizes the total energy consumption, can be formulated as

$$\min_{P^d_{amp},\beta,\phi} E_{c,t}(P^d_{amp},\beta,\phi) \quad \text{subject to} \quad P_{out,sys} \leqslant P^*_{out}. \tag{5.31}$$

For presentation convenience, let the amplifier power $x = P^d_{amp}$, the inter-user outage probability $P_1 = \frac{k_3 d^\alpha_{s,p}}{P^d_{amp}} = \frac{h_1}{x}$, the source-destination outage probability $\frac{k_3 d^\alpha_{s,d}}{P^d_{amp}} = \frac{h_2}{x}$, and also

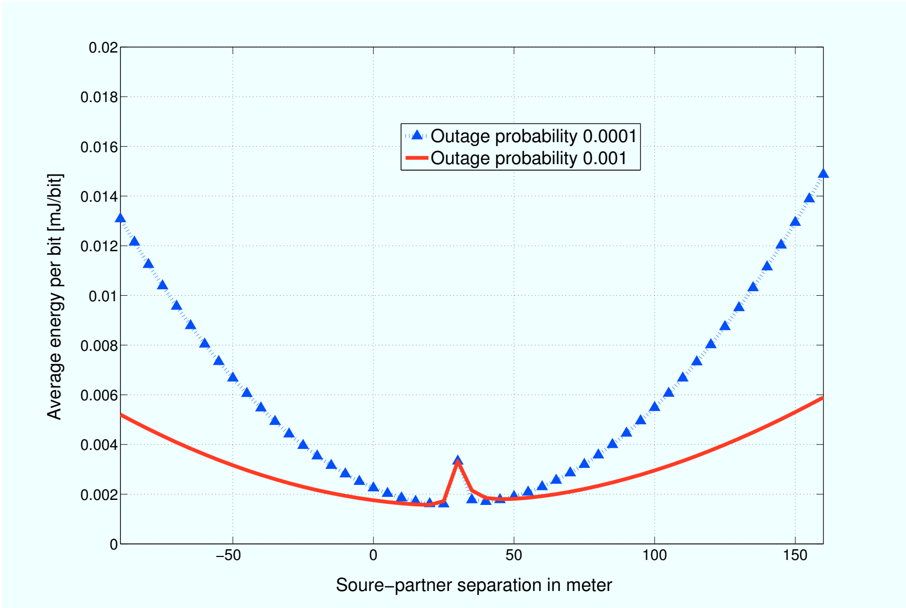

Figure 5.11.: Average consumed energy in mill Joules for system-level outage probability of 0.0001.

the partner-destination transmission $\frac{k_3 d_{p,d}^{\alpha}}{P_{amp}^{d}} = \frac{h_3}{x}$. Substituting for P_1, the objective function defined by Equation (5.27) can be re-arrange into

$$E_{c,t}(x) \approx \frac{1}{R_s}\left[(2P_{te} + 3P_{rec} - h_1) + x(2 - \phi^{'}) - \frac{h_1}{x}(P_{te} + P_{rec})\right]\left(\frac{\varphi + l + \tau}{l}\right) + k_3. \tag{5.32}$$

Similarly, the outage probability constraint of the user level can be written in the form

$$P_{out,use} \approx \frac{h_2 h_3}{(P_{amp}^{d})^2}(1 - \frac{h_1}{P_{amp}^{d}}) + \frac{h_1 h_2}{(P_{amp}^{d})^2} = \frac{1}{x^2}(h_1 h_2 + h_2 h_3) - \frac{h_1 h_2 h_3}{x^3} \leqslant P_{out}^{*}. \tag{5.33}$$

This is a *constrained optimization* problem in variable x. Introducing a Lagrangian function $\Lambda(x, \lambda) = E_{c,t}(x) + \lambda(P_{out,use} - P_{out}^{*})$, where λ is a constant and setting the derivative $d\Lambda(x, \lambda) = 0$ yields two system of equations. Introducing a change of variables $\frac{1}{x} = y$, the first system of equation is given by

$$\frac{\partial \Lambda(x, \lambda)}{\partial \lambda} = (h_1 h_2 + h_2 h_3)y^2 - (h_1 h_2 h_3)y^3 - P_{out}^{*} = 0. \tag{5.34}$$

Likewise, the second system of equation is computed as

$$\frac{\partial \Lambda(x, \lambda)}{\partial x} = \frac{\partial E_{c,t}}{\partial x} + \lambda \frac{\partial (P_{out,user} - P_{out}^{*})}{\partial x} = 0 \tag{5.35}$$

where the derivatives of the total power consumption $E_{c,t}$ and the outage probability $P_{out,use}$ with respect to the transmit power x are given by

$$\frac{\partial E_{c,t}}{\partial x} = \frac{1}{R_s}\left[(2-\phi^{'}) + \frac{h_1}{x^2}(P_{te}+P_{rec})\right]\left(\frac{\varphi+l+\tau}{l}\right), \tag{5.36}$$

and

$$\frac{\partial (P_{out,use}-P_{out}^{*})}{\partial x} = \frac{-2(h_1h_2+h_2h_3)}{x^3} + \frac{3h_1h_2h_3}{x^4}. \tag{5.37}$$

Substituting Equations (5.36) and (5.37) into Equation (5.35), we get the following equation in polynomial form

$$\frac{2-\phi^{'}}{R_s}\left(\frac{\varphi+l+\tau}{l}\right) - \frac{h_1}{R_s}(P_{te}+P_{rec})\left(\frac{\varphi+l+\tau}{l}\right)y^2 - 2\lambda(h_1h_2+h_2h_3)y^3 + 3\lambda(h_1h_2h_3)y^4 = 0 \tag{5.38}$$

which can be written in a compact form as

$$\boxed{g_0 - g_2y^2 - \lambda g_3y^3 + \lambda g_4 y^4 = 0} \tag{5.39}$$

under the outage constraint from Equation (5.34)

$$\boxed{(h_1h_2+h_2h_3)y^2 - (h_1h_2h_3)y^3 \leqslant P_{out}^{*}.} \tag{5.40}$$

This constraint equation is a polynomial of order three, so it may be solvable depending on the constants h_1, h_2, and h_3. In general, this optimization problem is nonlinear with respect to the decision variable y and numerical optimization techniques are required.

5.6. Energy Efficiency Formulation

The energy formulation in the above section focuses on the consumed energy for a given error rate. If we want to study the effect of certain system parameters, for example size of redundancy bits for error protection, on energy consumption, we need to define a metric that takes into account both energy and robustness of the system. In this section, we will formulate energy efficiency of network-coded cooperation and study the consumed energy over a range of error rates instead of a single error rate. We focus on system-level energy efficiency.

5.6.1. Good inter-user channel

In the case of good quality inter-user channel, the energy efficiency, based on information bits only, is computed from Equation (5.15) as

$$\eta_1 = \left[\frac{k_1 l}{2k_1 (\varphi + l + \tau) + \frac{P_{rec} - \phi P_{amp}^d}{R_s} (\varphi + l + \tau) + k_2 l} \right] (1 - P_{sys,1}) \tag{5.41}$$

where $P_{sys,1}$ is the system-level outage probability computed from Equation (5.17). The term within the square brackets is the ratio of the energy spent for the payload bits only versus the total energy, whereas the term $(1 - P_{sys,1})$ on the right accounts for the reliability of reception. If we stick to the size of payload example above, the more redundancy bits we send for error protection, the smaller the energy ratio becomes, but the larger the reliability of the system gets. Therefore, we need to know the optimal payload size that maximizes the energy efficiency. The same can be said about using cooperation instead of point-to-point transmission. The additional transmission, processing, and reception decrease the energy ratio but increases the reliability. To illustrate this point, the energy efficiency can also be defined according to Equation (5.16) as

$$\eta_1 = \left[\frac{E_{ppt}}{E_{ppt} + \left(\frac{\varphi + l + \tau}{l}\right) \left(k_1 + \frac{P_{rec} - P_{amp}^d (2 - \phi)}{R_s}\right) + \left(\frac{E_{enc} + \frac{3}{2} E_{dec}}{l}\right)} \right] (1 - P_{sys,1}). \tag{5.42}$$

5.6.2. Bad inter-user channel

In the case of bad inter-user channels, the energy efficiency is computed from Equation (5.20) and we get

$$\eta_2 = \left[\frac{k_1{}'}{\left(k_1{}' + \frac{P_{rec} - P_{amp}^d (1 - \beta\phi)}{R_s}\right) \left(\frac{\alpha + l + \tau}{l}\right) + k_2''} \right] (1 - P_{sys,2}) \tag{5.43}$$

5.6.3. Average energy efficiency

Following the same approach as in Section 5.5.3 above, the average energy efficiency is the weighted sum of the energy consumptions calculated for the two cases above, where the

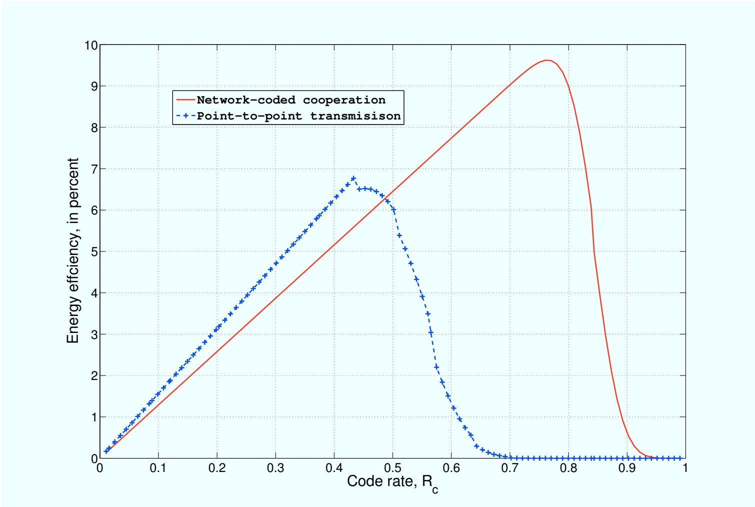

Figure 5.12.: Energy efficiency versus code rate.

weighing coefficient is the probability with which each case occurs. Defining η_{avg} as the average efficiency, we can write it in the form

$$\begin{aligned} \eta_{avg} &= (1 - P_1)\eta_1 + P_1\eta_2 \\ &= \eta_1 + P_1(\eta_2 - \eta_1) \end{aligned} \tag{5.44}$$

where the probability P_1 shows the outage of the inter-user channel transmission. A closed-form expression for η_{avg} can be found by substituting Equations (5.42) and (5.43) into Equation (5.44). Note that the average energy efficiency is a function of transmission power, the energy consumption at the transmitter, receiver, power amplifier, encoding/decoding circuits, node location/separation, and the packet size including overhead bits.

5.6.4. Numerical Results

In this section, we present numerical energy efficiency results for network-coded cooperation. Results are based on (n, k, t) linear block codes with t error correcting capability. Parameter values used for computation are the same as in Section 5.5.5. The effect of payload size on energy efficiency is investigated first. The code rate is varied from $0 < R_c = \frac{k}{n} \leqslant 1$. Figure 5.12 shows energy efficiency, in percent, vs. the code rate. We note that as the rate increases, the energy efficiency increases because more energy is spent on the payload. On the other hand, as the rate approaches 1, the efficiency falls down to 0 as the error rate of the system increases. For the given choice of parameters, the maximum efficiency is around 9 percent for a code rate in the range of 0.7.

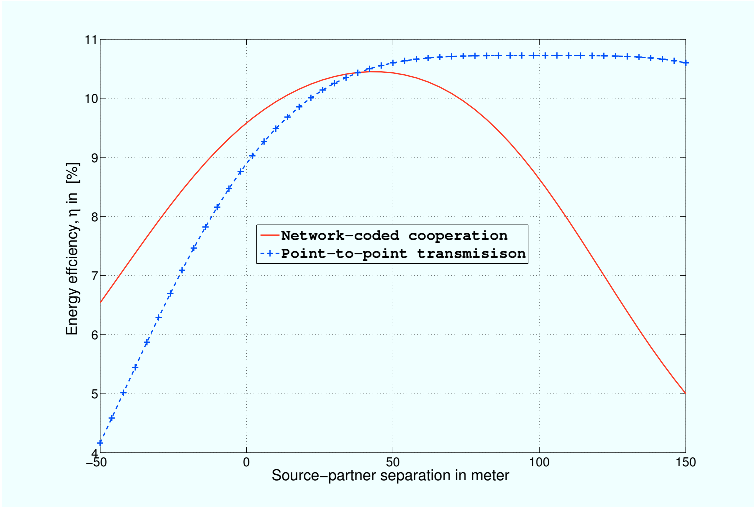

Figure 5.13.: Energy efficiency versus source-partner separation.

Next, for linear network geometry where the partner is located between the source and destination that are separated by 100 meters, we investigate the effect of partner location on energy consumption in Figure 5.13. We see that the energy efficiency becomes optimal when the partner is located halfway between the source and partner and it decreases as the partner approaches either the source or destination, unlike for energy consumption where the partner should be placed in the vicinity of the destination.

5.7. Summary

In this chapter, we investigated the energy consumption of network-coded cooperation in WSN. We defined the energy consumption model as a performance measure that computes the average energy per information bit and for a given error rate at the destination. Based on this model, the energy consumption was studied and the impact of network geometry on the consumption was investigated. From the numerical results, the energy consumption is optimal when the partner is located in the vicinity of the destination. Section 5.5 presents the details of the assumptions, model formulation, and numerical results. Next, we studied the energy efficiency taking both the energy consumption and reliability of reception into account; this is presented in Section 5.6. From the numerical results, we have found out that for an optimal energy efficiency, the partner should be located halfway in between the source and destination. To sum it up, optimal energy consumption requires the partner to be located near the destination; however, optimal energy efficiency requires the partner to be located halfway between the source and destination.

6. Incremental Redundancy Network-Coded Cooperation

This chapter studies the outage behavior of *incremental redundancy network-coded cooperation*, which is an extension of the network-coded cooperation discussed in Chapter 4. To differentiate the two network-coding-based cooperations, in the following, we call the network-coded cooperation *conventional network-coded cooperation*. In the incremental network-coded cooperation, the source, for example, after receiving the partner's codeword in the first phase, combines its own and its partner's *parity symbols* using network coding and transmits these bits to the destination in the second phase. These *network-coded parity symbols* are *code combined* with codeword(s) received in the first phase to form a stronger codeword (i.e., are used as incremental information at the destination and this is unlike the conventional network-coded cooperation where these symbols are decoded on their own).[1] The principle of incremental redundancy network-coding is that the network coding is embedded into the channel coding such that the redundancy in the network code is used to support the channel code for better error protection [29, 30].

Two decoding approaches are proposed in this chapter: *joint network-channel* decoding and *individual network-channel* decoding. In the former approach, one 'big' codeword is formed from all symbols received in the two phases and the information bits of both source and partner are obtained from a single decoding of this big codeword; however, in the latter approach the source's information bits are recovered by decoding a codeword which is formed from code combining of its codeword received in the first phase and the network-coded parity symbols; the same approach is used to decode the partner's information bits [13, 31].

We first study the outage behavior of the incremental network-coded cooperation assuming quasi-static Rayleigh fading channels, orthogonal transmission, and half-duplex constraints. The outage results show that this scheme also achieves full diversity order of two. Then, using the outage result, we investigate 'optimal' *rate* and *energy* allocations that minimize

[1]This is similar to coded cooperation where the source transmits incrementally redundant symbols for the partner, and vice versa.

the outage probability. The results show that outage performance is more sensitive to the energy allocation than to the rate allocation.

Section 6.1 describes the system model and the joint network-channel encoding/decoding operations. The outage probability of the incremental network-coded cooperation is derived in Section 6.2; here, the same four cases approach as in Chapter 4 is followed. In Section 6.3, numerical results are presented, discussions based on the results are made, and conclusions on the general outage behavior and rate and power allocation are drawn. Finally, summary of the chapter is presented in Section 6.4.

6.1. System Model, Joint Network-Channel Coding

6.1.1. System Model

The system diagram of the incremental network-coded cooperation scheme is shown in Figure 6.1. Like other cooperative schemes, each source encodes k of its information bits[2] into a codeword of $N_1 = \alpha N$ symbols, where $0 < \alpha \leqslant 1$ is the *cooperation level* and N is the number of symbols per codeword if the point-to-point transmission were used.[3] The source and partner, using two time slots, transmit their own codeword to the destination and simultaneously decode each other's codeword. If the source, for example, decodes the partner's k bits, then it independently generates parity symbols for its own bits as well as for the partner's bits, each of length $N_2 = (1-\alpha)N$. Then it linearly combines (i.e network codes) these two groups of parity symbols to form *network-coded parity symbols* which are sent to the destination in the second phase. The network-coded parity symbols can also be generated by network coding the information bits of the source and partner first, and then channel coding the resulting network-coded bits by using a different codebook, i.e., performing network coding followed by channel coding. If decoding fails, then the source transmits additional N_2 symbols for its own data or remains silent. The cooperation level can be adjusted to control the number of parity symbols, and the control can be based on various requirements, e.g., inter-user channel condition.

One basic difference between the conventional and incremental network-coded cooperation is the following: in the former, the network-coded codeword is independently channel decoded at the destination and then, depending on the successful recovery of one of the two users' information bits, network decoding follows, i.e., there are *two levels of decoding*:

[2]In this chapter, information bits and symbols refer to bits before and after channel coding, respectively.

[3]A cooperation levels of $\alpha = 0.5$ and $\alpha = 1$ corresponds to the conventional cooperation and point-to-point transmission, respectively.

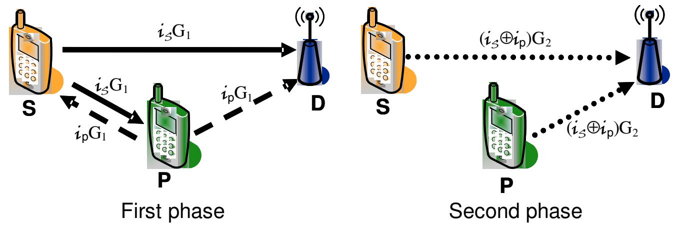

Figure 6.1.: System diagram of incremental network-coded cooperation. Solid, dashed, and dotted lines show the transmission of the source (**S**), partner (**P**), and the network-coded parity symbols, respectively.

channel decoding followed by network decoding. However, in the latter the network-coded parity symbols are jointly decoded with codewords received in the first phase, i.e., there is *one level of decoding* as the network decoding is embedded into the channel decoding. Another difference is if we compare them from a layering perspective: the joint network-channel decoding behavior forces the implementation of incremental network-coded cooperation down to the physical layer as the network decoding is part of the channel decoding. On the other hand, in the conventional network-coded cooperation, the channel decoding is done at the physical layer and the network decoding is done at a layer above the physical (usually network layer) making its implementation a cross layer one.

Finally, the energy per symbol relationship in the incremental network-coded cooperation is as follows. Let $E_T = NE_s$ be the energy budget to transmit N symbols in the point-to-point transmission, where E_s is the radiated energy per symbol in point-to-point transmission; also let β be the fraction of E_T allocated in phase one, where $0 < \beta \leqslant 1$. If we designate $\beta_k E_s$ to represent the energy per symbol in phase $k \in \{1, 2\}$, and the energy allocation term, β_k, takes on value

$$\beta_k = \begin{cases} \frac{\beta}{\alpha} & \text{if } k = 1; \\ \frac{1-\beta}{1-\alpha} & \text{if } k = 2. \end{cases} \tag{6.1}$$

Note that the energy allocation term in Equation (6.1) is different from the allocation term of the conventional network-coded cooperation given in Equation (4.1) which is not a function of α. Moreover, identical allocation by both the source and partner is assumed in this work. The SNR relationship in each phase of the incremental network-coded cooperation and the point-to-point transmission follow the same relation as the energy per symbol relationships given in Equation (6.1). The SNR in the two phases are function of the energy allocation and cooperation level, where the latter controls the information rate in the two phases. One way to optimize performance of this cooperative scheme, e.g., outage probability, is by *joint optimization* of the energy and cooperation level (rate).

6.1.2. Joint network-channel encoding

In this subsection, the implementation of joint network-channel encoding is explained. In general, the focus of this chapter is to study the outage behavior of the incremental network-coded cooperation, and the following discussion is only to mention the existence of codes that perform the joint network-channel encoding and decoding. Designing efficient encoding and decoding schemes is beyond the scope of this work. Moreover, for ease of exposition, discussions are based on binary linear block codes.

In a linear block coding scheme at the transmitting node l and phase k, where $l \in \{s, p\}$, let i_l represent n information bits, $\mathbf{G}_{l,k}$ be the generator matrix of node l, and $X_{l,k}$ be the resulting codeword/symbols. Assume that the source and partner have obtained each other's information bits from first-phase transmissions. The joint network-channel encoding at the source is given as

$$\begin{aligned} [X_{s,1}, X_{s,2}] &= [i_s, i_p] \begin{bmatrix} G_{s,1} & G_{s,2} \\ 0 & G_{p,2} \end{bmatrix} \\ &= [i_s G_{s,1}, (i_s G_{s,2} \oplus i_p G_{p,2})] \,. \end{aligned} \tag{6.2}$$

In a case that identical encoding matrices are used at the source and partner, $G_{s,1} = G_{p,1} = G_1$ and $G_{s,2} = G_{p,2} = G_2$ and Equation (6.2) takes the following form

$$\begin{aligned} [X_{s,1}, X_{s,2}] &= [i_s, i_p] \begin{bmatrix} G_1 & G_2 \\ 0 & G_2 \end{bmatrix} = [i_s, i_p]\, G_s \\ &= [i_s G_1, (i_s \oplus i_p)\, G_2] \,. \end{aligned} \tag{6.3}$$

where G_s is a matrix made of G_1 and G_2. In Equation (6.3), $[i_s G_1]$ represents the systematic and channel-coded parity symbols (achieved by designing the encoding matrix G_1 such that $[i_s G_1]$ contains both systematic and parity symbols for the source), whereas $[(i_s \oplus i_p)\, G_2]$ represents the network-coded parity symbols (as these parity symbols are made of the information bits of both the source and partner). The generator matrices G_1 and G_2 can be designed[4] to have dimensions $n \times \alpha N$ and $n \times (1 - \alpha)N$, respectively. This means that the cooperation level α can be controlled by changing the size of the generator matrices. If the source fails to decode the partner's codeword, i_p in Equation (6.3) will be a zero vector and $[i_s G_1, i_s G_2]$ results from the encoding and the source transmits incremental information for its own. Another option is for the source to remain silent during decoding failure such that

[4]Though it may not be realistic to design these matrices for an arbitrary value of the cooperation level, α.

$[i_s G_1]$ will be transmitted. Similarly, the encoding operation at the partner is

$$\begin{aligned}
[X_{p,1}, X_{p,2}] &= [i_s, i_p] \begin{bmatrix} 0 & G_2 \\ G_1 & G_2 \end{bmatrix} = [i_s, i_p]\, G_p \\
&= [i_p G_1, (i_s \oplus i_p)\, G_2]\,. \qquad (6.4)
\end{aligned}$$

Note that the codewords in Equations (6.3) and (6.4) contain information bits of both the source and partner, and if the size of G_1 and G_2 are fixed, then the joint encoding results in a *higher rate* codeword compared to $[i_s G_1, i_s G_2]$ and $[i_p G_1, i_p G_2]$. The property that we have parity bits from both the channel coding and network coding will be exploited in the decoding procedure. For the design of the joint decoder, let us assume that there exists a *'virtual' joint encoder* that generates the codewords in Equations (6.2) and (6.4), and then design the corresponding parity check matrix. The joint encoding can be represented as:

$$\begin{aligned}
\left[i_s G_1, (i_s \oplus i_p)' \, G_2, i_p G_1\right] &= [i_s, i_p] \begin{bmatrix} G_1 & G_2 & 0 \\ 0 & G_2 & G_1 \end{bmatrix} \\
&= I G_{sp} \qquad (6.5)
\end{aligned}$$

where $I = [i_s, i_p]$ and G_{sp} is the generalized generator matrix. A parity check matrix, H_{sp}, can be designed to fulfill $G_{sp} H_{sp}^T = 0$ and using H_{sp}, a single decoding will give both i_s and i_p. The prime notation in Equation (6.5) shows the possible MRC on the network-coded symbols and this will be apparent when the decoding operation in the next section is discussed. If G_1 is a systematic generator matrix, i.e., $G_1 = [I_n, P_1]$ where I_n is $n \times n$ identity matrix and P_1 is $n \times (\alpha N - n)$ matrix. If we also let the parity matrix $G_2 = P_1$, substituting G_1 and G_2 into Equation (6.5) and re-arranging columns, the joint encoder matrix G_{sp} will take the form

$$G_{sp} = \begin{bmatrix} I_n & 0 & \vdots & P_1 & 0 & P_1 \\ 0 & I_n & \vdots & 0 & P_1 & P_1 \end{bmatrix} = [I_{2n},\ P] \qquad (6.6)$$

where I_{2n} is an identity matrix of dimension $2n \times 2n$ and P is the new parity matrix of dimension $3(\alpha N - n)$.

Example 6.1.2.1. To illustrate the above points, let us consider the $(2, 5)$ systematic code with generator matrix G_1 given by

$$G_1 = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix} = [I_2, P_1] \qquad (6.7)$$

where I_2 is a 2×2 identity matrix. The encoding matrix at the source, G_s, is constructed from G_1 and G_2 as follows

$$G_s = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}. \tag{6.8}$$

We see that 5 and 3 symbols will be sent by the source in phases one and two, respectively, such that $N = 8$ and the condition $\alpha N = 5$ and $(1 - \alpha)N = 3$ should be fulfilled. Solving for α, we get that the cooperation level should be $\frac{5}{8}$. The joint encoding matrix G_{sp} is then formed as

$$G_{sp} = \left[\begin{array}{cccc|ccc|ccc|ccc} 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ \cline{5-13} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \end{array}\right] = [I_4,\ P]\,. \tag{6.9}$$

The encoding matrix G_{sp} is in systematic form and it also models the encoding (distributed) operation at the two nodes as a single encoding operation. Suppose the information bits of the source and partner are $i_s = [u_{11}, u_{12}]$ and $i_p = [u_{21}, u_{22}]$, respectively, and if these bits are jointly encoded using the generator matrix given in Equation (6.9), we get the codeword $[i_s, i_p]\, G$ as

$$\Big[\underbrace{u_{11}, u_{12}, u_{21}, u_{22}}_{\text{systematic bits}}, \underbrace{u_{11} \oplus u_{12}, u_{12}, u_{11}, u_{21} \oplus u_{22}, u_{22}, u_{21},}_{\text{channel-coded parity symbols}}$$
$$\underbrace{u_{11} \oplus u_{12} \oplus u_{21} \oplus u_{22}, u_{12} \oplus u_{22}, u_{11} \oplus u_{21}}_{\text{network-coded parity symbols}}\Big]\,. \tag{6.10}$$

The above codeword consists of systematic bits, channel-coded bits (i.e., coding bits from the same sources), and network-coded bits (i.e coding bits from the two users). In the case that the source and partner fail to decode each others' codewords and if nodes remain silent during decoding error, then part of the channel-coded parity symbols and the entire network-coded parity symbols will vanish from the codeword. In actual implementation, there are more efficient designs of the coding and parity check matrices e.g., using Low Density parity check (LDPC) codes or modifying Turbo codes [29].

6.1.3. Joint network-channel decoding

The destination collects codeword/symbols received from the source and partner and performs joint network-channel decoding. First, using MRC the destination combines the network-coded parity symbols, which are received over the two phases from the source and partner. Second, a 'big' codeword $\left[i_sG_1, (i_s \oplus i_p)' G_2, i_sG_1\right]$ is formed by code combining of all the received symbols, where the prime notation shows the MRC on the network-coded parity symbols [13, 31]. For the generator matrix $G_{sp} = [I_{2n}|P]$, the parity check matrix can be designed as $H_{sp} = [P^T|I_{N-n}]$ such that $G_{sp}H_{sp}^T = 0$, where T stands for the transpose of a matrix. Returning to the encoding matrix example given in Equation (6.9), the corresponding parity check matrix is given as

$$H_{sp} = \left[\begin{array}{cccc|ccccccccc}
1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{array}\right] = \left[P^T,\, I_9\right]. \tag{6.11}$$

Error checking is done by multiplying the codeword in Equation (6.10) with the transpose of the parity check matrix in Equation (6.11) and we see that the product yields a zero vector if no error exists in each bit. A non zero vector means an error exists in one or more bit positions.

Alternatively, the source and partner information bits can be recovered through *individual network-channel* decoding. In individual decoding, the information bits of the source and partner are decoded separately (i.e., two decodings are required) and two approaches can be followed

1. In approach one, the source's information bits are recovered by decoding a codeword which is formed by code combining of the source's codeword received in the first phase and the network-coded parity symbols. This approach considers the partner's information bits, which are contained in the network-coded parity symbols, as if they were transmission errors on these parity bits, and tries to decode the source's bits by exploiting the redundancy in the channel-coded and network-coded parity symbols. To explain the above point, consider a codeword given by Equation (6.3) is transmitted

and received at the destination as

$$[i_sG_1 \oplus e_1, (i_s \oplus i_p)\, G_2 \oplus e_2] \;=\; \left[i_sG_1 \oplus e_1, i_sG_2 \oplus e_2^{'}\right] \qquad (6.12)$$

where e_1, e_2, and $e_2^{'} = i_pG_2 \oplus e_2$ are errors because of the channel fading and noise. This codeword can be assumed as if it were transmitted from the source where the codeword at the time of transmission was $i_s\,[G_1, G_2] = i_sG_s$, where G_s is the encoding matrix. The parity check matrix can be designed accordingly.

This decoding, from the outset, appears to perform inferior as the decoder is attempting to decode information bits of one user without knowledge of the information of the other user (or tries to decode the information from higher rate codeword), but as will be seen from the numerical results, it can still achieve full diversity gain. Moreover, the outage probability computation of this approach is relatively simpler than the joint decoding and it also helps to investigate the behavior of incremental network-coded cooperation. Hence, the outage analysis in the next section is based on this approach.

2. The second approach is to decode the source's information bits by removing the partner's information bits from the network-coded parity symbols first, and then utilize the redundant symbols from the first and second phase transmissions. This assumes that the partner information bits are decoded a priori from its first-phase transmission. In the case that knowledge of the partner's information bits is not available, the source's information bits are decoded from the first-phase transmission only. The decoding requirement that "*the destination decodes the partner's information bits*" can be fulfilled if, for example, the partner is located closer to the destination such that the destination can decode the partner's codeword.

6.2. Outage Behavior of Incremental Redundancy Network-Coded Cooperation

In this section, we compute the outage probability of incremental network-coded cooperation assuming individual decoding at the destination. The analysis of joint decoding can be done in the same manner except that it is less tractable. The same four-cases analysis, based on success of inter-user channels transmissions, will be followed.

6.2.1. Inter-user transmission

Consider the transmission from the source to the partner in phase one. The source transmits its codeword of αN symbols carrying n information bits, such that the codeword rate is $\frac{n}{\alpha N} = \frac{R}{\alpha}$, where $R = \frac{n}{N}$ is the code rate if the point-to-point transmission were used. The Shannon capacity of this channel is given by $C_s^{(s,p)} = \log_2(1+\beta_1\gamma_{s,p})$, where $\gamma_{s,p} = |h_{s,p}|^2 \frac{P_t}{N}$ is the instantaneous SNR of the channel with the transmit SNR $\frac{P_t}{N}$. Using the notation of Chapter 4, the outage event of this source-partner transmission is

$$\overline{X}_s^{(s,p)} \cong \left\{ C_s^{(s,p)} < \frac{R}{\alpha} \right\} \tag{6.13}$$

and, for $|h_{s,p}|^2$ exponentially distributed, the probability that this event occurs is known to be

$$P\left(\overline{X}_s^{(s,p)}\right) \;=\; 1 - \exp\left(-\frac{2^{R/\alpha}-1}{\beta_1\Gamma_{s,p}}\right) \approx \frac{2^{R/\alpha}-1}{\beta_1\Gamma_{s,p}} \tag{6.14}$$

where $\Gamma_{s,p}$ is the average of $\gamma_{s,p}$ and the last term in (6.14) is the result of a high-SNR-regime approximation. Equation (6.14) is also valid for the transmission from the partner to the source with $\Gamma_{s,p}$ replaced by $\Gamma_{p,s}$. In the case of symmetrical inter-user channels, i.e., when $\gamma_{s,p} \neq \gamma_{p,s}$ but $\Gamma_{s,p} = \Gamma_{p,s}$, the source-partner and partner-source transmissions will have the same outage probability.

6.2.2. Uplink transmission

The success of decoding at the destination depends on the four transmissions in the two phases, which in turn depend on the quality of the inter-user channels. To determine the outage, we need to identify parts of the received codewords/symbols that will be used for decoding. In the following, we perform outage analysis for the source only; by symmetry the same approach holds true for the partner. Depending on inter-user transmission success, here are the four cases to consider.

- **Case 1:** Both the source and partner decode each others' codewords correctly. The destination will receive each user's codeword from the first phase and the network-coded parity symbols, $(i_s \oplus i_p)G_2$, in the second phase (see Figure 6.2). For the individual decoding, a codeword $[i_sG_1, (i_s \oplus i_p)'G_2]$ of length N is formed, where the prime denotes MRC. As this codeword contains $2n$ information bits, its code rate is $\frac{2n}{N} = 2R$. The two parts of this codewords can be viewed as if they were received

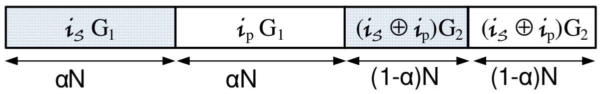

Figure 6.2.: Codewords sent by the source (shaded) and partner in the two phases of **Case 1**.

from parallel channels whose capacities add up [10]. The outage event for the source, in terms of the Shannon capacity, is written as

$$\begin{aligned} \overline{X}_s &\cong \{\alpha \log_2(1+\beta_1\gamma_{s,d}) + (1-\alpha)\log_2(1+\beta_2(\gamma_{s,d}+\gamma_{p,d})) < 2R\} \\ &\cong \left\{\log_2\left[(1+\beta_1\gamma_{s,d})^{\alpha}(1+\beta_2(\gamma_{s,d}+\gamma_{p,d}))^{(1-\alpha)}\right] < 2R\right\} \quad . \end{aligned} \tag{6.15}$$

The sum of capacities $\alpha \log_2(1+\beta_1\gamma_{s,d})$ and $(1-\alpha)\log_2(1+\beta_2(\gamma_{s,d}+\gamma_{p,d}))$ in Equation (6.15) reflects the code combining (i.e., incremental redundancy symbols) and the term $\beta_2(\gamma_{s,d}+\gamma_{p,d})$ that adds SNR values accounts for the MRC on the network-coded parity symbols [12, 10]. The first phase uses a fraction α of the total N allocated symbols, while the second frame uses $1-\alpha$, in other words the cooperation level α denotes the fraction of time that the network is in the first phase; it is hence used in Equation (6.15) to reflect that [12, 10]. The outage probability, corresponding to the event in Equation (6.15), is then

$$\begin{aligned} P\left(\overline{X}_s\right) &= P\left\{(1+\beta_1\gamma_{s,d})^{\alpha}(1+\beta_2(\gamma_{s,d}+\gamma_{p,d}))^{(1-\alpha)} < 2^{2R}\right\} \\ &= \int\int_A \frac{1}{\Gamma_{s,d}\Gamma_{p,d}} \exp\left(-\frac{\gamma_{s,d}}{\Gamma_{s,d}} - \frac{\gamma_{p,d}}{\Gamma_{p,d}}\right) d\gamma_{s,d}d\gamma_{p,d} \end{aligned} \tag{6.16}$$

where $P\left(\overline{X}_s\right)$ is the outage probability of the source, which actually is a function of $\gamma_{s,d}, \gamma_{p,d}, \alpha$, and β. The region A is defined by

$$A \equiv \left\{(1+\beta_1\gamma_{s,d})^{\alpha}(1+\beta_2(\gamma_{s,d}+\gamma_{p,d}))^{(1-\alpha)} < 2^{2R}\right\} \quad . \tag{6.17}$$

In Equation (6.16), the exponential term represents the joint probability density function (PDF) of the random variables $\gamma_{s,d}$ and $\gamma_{p,d}$ and this joint PDF is integrated over A, which defines all possible values of $\gamma_{s,d}$ and $\gamma_{p,d}$. Following similar approaches to [10] and using a combination of Taylor's series and the high-SNR approximation of the exponential function, it can be shown that the outage probability in (6.16) can be approximated as

$$P\left(\overline{X}_s\right) \approx \frac{(2^{2R/(1-\alpha)}-1)^2}{\Gamma_{s,d}\Gamma_{p,d}}\frac{(1-\alpha)^2}{(1-\beta)}. \tag{6.18}$$

Derivation of this probability is shown in Appendix A.1. The overall outage probability under this case is the product of the probability given in (6.18) and the proba-

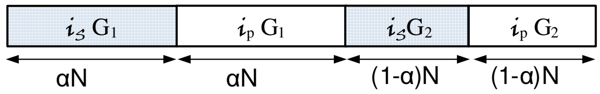

Figure 6.3.: Codewords sent by the source (shaded) and partner in two phases of **Case 2**.

bility of occurrence of **case 1**, which in turn is the product of the success probabilities of the inter-user channels from Equation (6.14) which are given as $P\left(X_s^{(s,p)}\right) = 1 - P\left(\overline{X}_s^{(s,p)}\right)$ and $P\left(X_p^{(p,s)}\right) = 1 - P\left(\overline{X}_p^{(p,s)}\right)$. The overall outage probability at the destination, defined as $P\left(\overline{X}_{(s,1)}\right)$, where '1' indicates the case number, is then given as

$$P\left(\overline{X}_{(s,1)}\right) \approx \underbrace{\left[1 - \frac{2^{R/\alpha}-1}{\beta_1 \Gamma_{s,p}}\right]\left[1 - \frac{2^{R/\alpha}-1}{\beta_1 \Gamma_{p,s}}\right]}_{\text{inter-user txs.}} \underbrace{\left[\frac{(2^{2R/(1-\alpha)}-1)^2}{\Gamma_{s,d}\Gamma_{p,d}} \frac{(1-\alpha)^2}{(1-\beta)}\right]}_{\text{up-link txs.}}. \quad (6.19)$$

For symmetrical inter-user channels, i.e., $\Gamma_{s,p} = \Gamma_{p,s}$ so that $P\left(X_s^{(s,p)}\right) = P\left(X_p^{(p,s)}\right)$, Equation (6.19) reduces to the form

$$P\left(\overline{X}_{(s,1)}\right) \approx \left[1 - \frac{2^{R/\alpha}-1}{\beta_1 \Gamma_{s,p}}\right]^2 \left[\frac{(2^{2R/(1-\alpha)}-1)^2}{\Gamma_{s,d}\Gamma_{p,d}} \frac{(1-\alpha)^2}{(1-\beta)}\right]. \quad (6.20)$$

If we re-parametrize $\Gamma_{i,j} = \Gamma_T \Gamma_{i,j}$ as in Equation (4.33) and for $\beta_1 \Gamma_{s,p} >> 1$, the outage in Equation (6.20) will become

$$P\left(\overline{X}_{(s,1)}\right) \approx \frac{1}{\Gamma_T^2} \left[\frac{(2^{2R/(1-\alpha)}-1)^2}{\Gamma_{s,d}\Gamma_{p,d}} \frac{(1-\alpha)^2}{(1-\beta)}\right]. \quad (6.21)$$

From this outage probability approximation, we observe the following points:

1. When the inter-user channels are good quality, the outage-probability decay is proportional to the square of Γ_T, the transmit SNR; hence full diversity order of two is achievable using the incremental network-coded cooperation.

2. Controlling both the cooperation level α and the energy allocation term β is one way to optimize the performance of this cooperative scheme.

- **Case 2:** Neither the source nor partner decodes each other's codeword correctly. Each user transmits additional symbols for its own (see Figure 6.3). At the destination, a codeword $[i_s G_1, i_s G_2]$ is formed, and the rate of this codeword is R as it contains the information bits of the source only. The outage event in this case is then

$$\begin{aligned}\overline{X}_s &\cong \{\alpha \log_2(1+\beta_1\gamma_{s,d}) + (1-\alpha)\log_2(1+\beta_2\gamma_{s,d}) < R\} \\ &\cong \left\{\log_2\left[(1+\beta_1\gamma_{s,d})^{\alpha}(1+\beta_2\gamma_{s,d})^{(1-\alpha)}\right] < R\right\}. \end{aligned} \tag{6.22}$$

The corresponding outage probability is then

$$\begin{aligned}P\left(\overline{X}_s\right) &= P\left\{(1+\beta_1\gamma_{s,d})^{\alpha}(1+\beta_2\gamma_{s,d})^{(1-\alpha)} < 2^R\right\} \\ &= \int_B \frac{1}{\Gamma_{s,d}} \exp\left(-\frac{\gamma_{s,d}}{\Gamma_{s,d}}\right) d\gamma_{s,d} \end{aligned} \tag{6.23}$$

where B is the contour defined by

$$B \equiv \left\{(1+\beta_1\gamma_{s,d})^{\alpha}(1+\beta_2\gamma_{s,d})^{(1-\alpha)} < 2^R\right\} \quad . \tag{6.24}$$

The term $(1+\beta_1\gamma_{s,d})^{\alpha}(1+\beta_2\gamma_{s,d})^{(1-\alpha)}$ that defines the contour can be approximated using Taylor's series in one variable. Taking the first order terms, it reduces to the form $1+\gamma_{s,d}$ such that the contour becomes $B \equiv \left\{\gamma_{s,d} < 2^R - 1\right\}$. The probability in Equation (6.24) can be easily computed. The high SNR approximation of the overall outage probability in this case is then approximated as

$$\begin{aligned}P\left(\overline{X}_{(s,2)}\right) &\approx \left[\frac{2^{R/\alpha}-1}{\beta_1\Gamma_{s,p}}\right]\left[\frac{2^{R/\alpha}-1}{\beta_1\Gamma_{p,s}}\right]\left[\frac{2^R-1}{\Gamma_{s,d}}\right] \\ &= \left[\frac{2^{R/\alpha}-1}{\beta_1\Gamma_{s,p}}\right]^2\left[\frac{2^R-1}{\Gamma_{s,d}}\right] \quad . \end{aligned} \tag{6.25}$$

In Equation (6.25), the first two terms in the box bracket show the outage for the inter-user transmissions, whereas the the third term is the probability of the uplink transmission obtained from Equation (6.23). The equation in the second line in (6.25) results when $\Gamma_{s,p} = \Gamma_{p,s}$. From this outage probability approximation, we observe the following points:

1. When the inter-user channels are unreliable, the outage-probability decay is proportional to $\Gamma_{s,d}$ (not to the square of $\Gamma_{s,d}$ as in case 1 above); hence, even at high SNR values, no diversity gain is obtained in such a case.

2. The outage probability is sensitive to variation of the inter-user channels' quality.

3. The outage of the uplink transmission is the same as that of the point-to-point transmission and is independent of α and β. If we allow nodes to *remain silent* when they fail to decode their partner's codeword, the outage probability in Equation (6.23) would have reduced to $\frac{2^{R/\alpha}-1}{\beta_1\Gamma_{s,d}}$. If we further set $\alpha = \beta$, then this

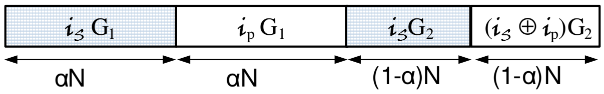

Figure 6.4.: Codewords sent by the source (shaded) and partner in two phases of **Case 3**.

probability becomes $\frac{2^{R/\alpha}-1}{\Gamma_{s,d}}$, which is greater than $\frac{2^{R}-1}{\Gamma_{s,d}}$ and still a function of α. So, unlike the conventional network-coded cooperation, sending extra parity symbols in the case that nodes fail to decode partner's transmission helps to reduce outage probability (actually this is an inherent behavior of incremental redundancy coding systems).

- **Case 3:** The partner can correctly decode the source's codeword, but not vice versa. In this case, the partner helps the source but the source transmits for its own (see Figure 6.4), and the parity symbols received from the source and partner differ in content and will be used as different incremental information. The source's information bits are decoded from a codeword $[i_sG_1, (i_s \oplus i_p)G_2, i_sG_2]$ which is of length $(2-\alpha)N$ and code rate $\frac{2R}{(2-\alpha)}$. The outage event, in terms of the Shannon capacity, is defined as

$$\begin{aligned} \overline{X}_s &\cong \left\{\alpha \log_2(1+\beta_1\gamma_{s,d}) + (1-\alpha)\log_2\left[(1+\beta_2\gamma_{s,d})(1+\beta_2\gamma_{p,d})\right] < \frac{2R}{(2-\alpha)}\right\} \\ &\cong \left\{\log_2\left[(1+\beta_1\gamma_{s,d})^{\alpha}\left[(1+\beta_2\gamma_{s,d})(1+\beta_2\gamma_{p,d})\right]^{(1-\alpha)}\right] < \frac{2R}{(2-\alpha)}\right\} \end{aligned} \tag{6.26}$$

where the capacities $\alpha\log_2(1+\beta_1\gamma_{s,d})$, $(1-\alpha)\log_2(1+\beta_2\gamma_{s,d})$, and $(1-\alpha)\log_2(1+\beta_2\gamma_{p,d})$ account for receptions from the three channels. The outage probability is

$$P\left(\overline{X}_s\right) = \int\int_C \frac{1}{\Gamma_{s,d}\Gamma_{p,d}} \exp\left(-\frac{\gamma_{s,d}}{\Gamma_{s,d}} - \frac{\gamma_{p,d}}{\Gamma_{p,d}}\right) d\gamma_{s,d}d\gamma_{p,d} \tag{6.27}$$

where C is the region defined by

$$C \equiv \left\{(1+\beta_1\gamma_{s,d})^{\alpha}\left[(1+\beta_2\gamma_{s,d})(1+\beta_2\gamma_{p,d})\right]^{(1-\alpha)} < 2^{2R/(2-\alpha)}\right\} . \tag{6.28}$$

Following identical steps and approximations as shown in the Appendix A.2, it can be shown that

$$P\left(\overline{X}_s\right) \approx \frac{(1-\alpha)^2}{(1-\beta)\Gamma_{s,d}\Gamma_{p,d}}\left[2^{2R/[(2-\alpha)(1-\alpha)]} - 1\right]^2 . \tag{6.29}$$

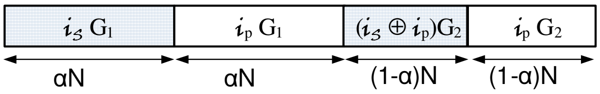

Figure 6.5.: Codewords sent by the source (shaded) and partner in two phases of **Case 4**.

and the total probability in this case is gven as

$$P\left(\overline{X}_{(s,3)}\right) \approx \left[1-\frac{2^{R/\alpha}-1}{\beta_1\Gamma_{s,p}}\right]\left[\frac{2^{R/\alpha}-1}{\beta_1\Gamma_{p,s}}\right]\frac{(1-\alpha)^2}{(1-\beta)\Gamma_{s,d}\Gamma_{p,d}}\left[2^{2R/[(2-\alpha)(1-\alpha)]}-1\right]^2 \quad . \tag{6.30}$$

For the case that $\beta_1\Gamma_{s,p} >> 1$ and $\Gamma_{s,d} = \Gamma_{p,d}$, Equation (6.30) becomes

$$P\left(\overline{X}_{(s,3)}\right) \approx \left[\frac{2^{R/\alpha}-1}{\beta_1\Gamma_{p,s}}\right]\left[\frac{(1-\alpha)^2}{(1-\beta)}\right]\left[2^{2R/[(2-\alpha)(1-\alpha)]}-1\right]^2\left[\frac{1}{\Gamma_{s,d}}\right]^2. \tag{6.31}$$

From the outage probability result in Equation (6.31), we conclude the following points:

1. Like **Case 1**, diversity order of two is achievable as the outage probability decay is proportional to the square of $\Gamma_{s,d}$.

2. For *asymmetric cooperation* (i.e., one node cooperates but not the other), the outage probability depends on the two inter-user channels quality.

3. Like the above two cases, the rate and energy allocation terms play an important role in controlling the outage probability.

- **Case 4:** The source correctly decodes the partner's codeword, but not vice versa. In this case, the source helps the partner but the partner transmits additional symbols for its own (See Figure 6.5) and a codeword $[i_sG_1, (i_s \oplus i_p)G_2]$ of code rate $2R$ is formed at the destination, where the additional network-coded parity symbols are received from the source. It can be shown that the total outage even under this case is given as

$$\begin{aligned}\overline{X}_s &\cong \{\alpha\log_2(1+\beta_1\gamma_{s,d}) + (1-\alpha)\log_2(1+\beta_2\gamma_{s,d}) < 2R\} \\ &\cong \left\{(1+\beta_1\gamma_{s,d})^{\alpha}(1+\beta_2\gamma_{s,d})^{(1-\alpha)} < 2^{2R}\right\}\end{aligned} \tag{6.32}$$

and the corresponding probability is

$$P\left(\overline{X}_{(s,4)}\right) \approx \left[1-\frac{2^{R/\alpha}-1}{\beta_1\Gamma_{p,s}}\right]\left[\frac{2^{R/\alpha}-1}{\beta_1\Gamma_{s,p}}\right]\left[\frac{2^{2R}-1}{\Gamma_{s,d}}\right] \quad . \tag{6.33}$$

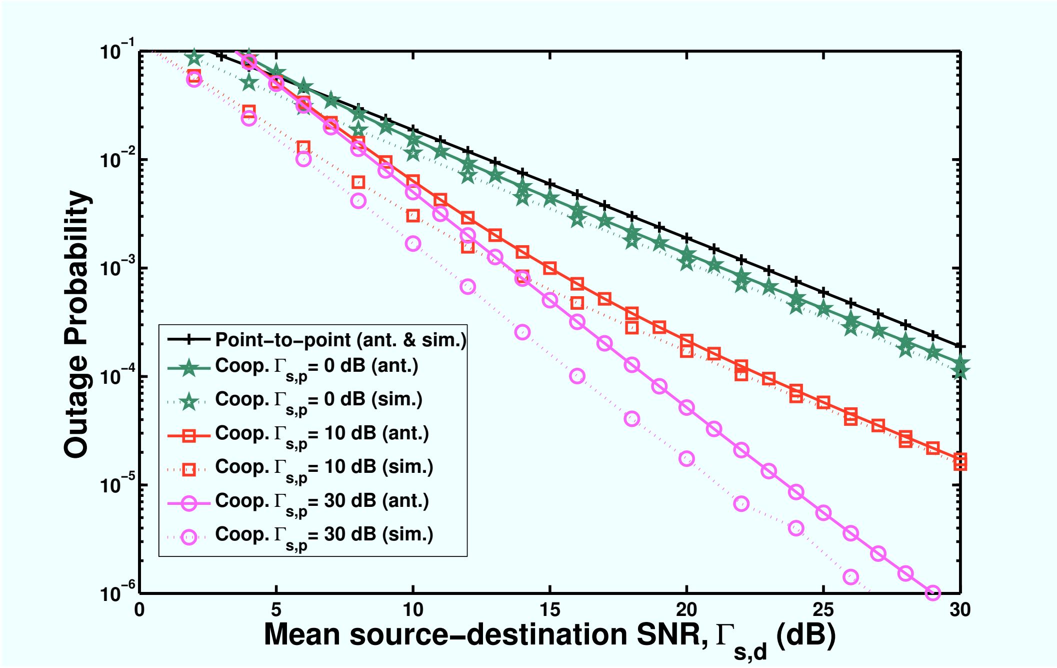

Figure 6.6.: Outage probability vs. uplink SNR $\Gamma_{s,d}$ results when the inter-user SNR $\Gamma_{s,p} =$ 0, 10, and 30 dB. Solid lines are for uncorrelated (analysis) and dashed lines for autocorrelated channels (simulation). For the point-to-point, both the analytic and simulated results match; coop, ant. and sim. stands for cooperation, analytical, and simulation, respectively.

We note from Equation (6.33) that, unlike the case of conventional network-coded cooperation, there is no diversity gain in this case as both receptions are from the source over a time-correlated channel (because of the block fading channel assumption).

The total outage probability, $P\left(\overline{X}_{(s,T)}\right)$, is the sum of the overall probabilities given in the four cases above. Following the parametrization stated above, i.e., $\Gamma_{i,j} = \Gamma_{i,j} \cdot \Gamma_T$, the total outage probability is shown to be

$$P\left(\overline{X}_{(s,T)}\right) \approx \frac{1}{\Gamma_T^2}\left[\left[\frac{2^{R/\alpha}-1}{\beta_1 \Gamma_{s,p}}\right]\left[\frac{2^{2R}-1}{\Gamma_{s,d}}\right] + \frac{(2^{2R/(1-\alpha)}-1)^2(1-\alpha)^2}{\Gamma_{s,d}\Gamma_{p,d}(1-\beta)}\right] + O\left(\frac{1}{\Gamma^3}\right) \tag{6.34}$$

where $O\left(\frac{1}{\Gamma^3}\right)$ denotes the higher-order terms. We can infer from (6.34) that the outage probability decays proportional to the inverse square of Γ_T; hence at higher SNR, full diversity order of two can be obtained. Also note that, in the high-SNR regime, the dependence of outage probability on the inter-user channels $\Gamma_{s,p}$ and $\Gamma_{p,s}$ also appears in the second-order terms, and this is in contrast to the conventional network-coded cooperation.

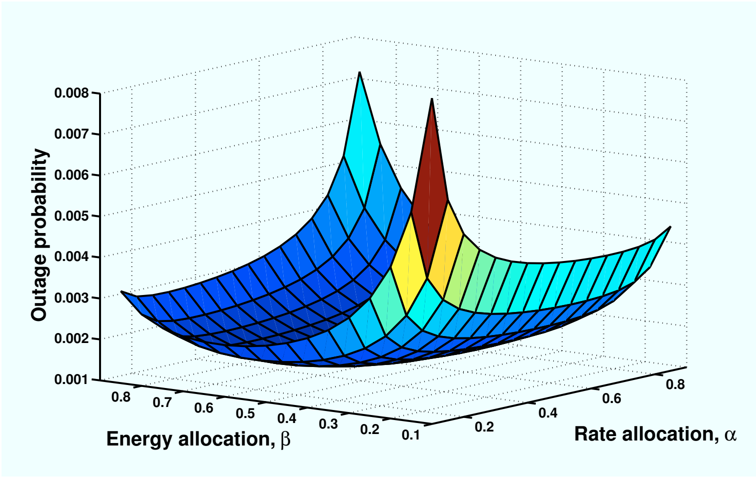

Figure 6.7.: Three dimensional plot of the outage probability taking α and β as the other variables. Results are based on $R = 0.25$ (bits/s/Hz) and all channels having identical average SNR, i.e $\Gamma_{s,d} = \Gamma_{s,p} = \Gamma_{p,d} = 10$ dB.

6.3. Result and Discussion

In this section, numerical and simulation results based on $R = 0.25$ (bits/s/Hz), symmetrical inter-user, and symmetrical uplink (i.e., $\Gamma_{s,d} = \Gamma_{p,d}$) channels are presented. The point-to-point transmission is plotted to serve as a baseline for comparison.

Figure 6.6 depicts the outage probability vs. $\Gamma_{s,d}$ (dB) for $\alpha = \beta = 0.5$ and three inter-user channel values, namely $\Gamma_{s,p} = 0$, 10, and 30 dB. We see that at lower $\Gamma_{s,p}$ values (example 0 dB), which represent a relatively poor channel, the performance of the incremental network-coded cooperation approaches that of the point-to-point; and at higher $\Gamma_{s,p}$ values (example 30 dB), the performance improves substantially and a diversity order of two, taking the point-to-point transmission as a baseline, is achieved. Hence, like other cooperative schemes, this incremental network-coded cooperation also requires good quality inter-user channels to achieve full diversity.

Shown in Figure 6.7 is the three-dimensional plot of the outage probability, with α and β taking the other-two axises and $\Gamma_{s,d} = \Gamma_{p,d} = \Gamma_{s,p} = 10$ dB. We see that the outage results are more sensitive to β variation than to α variation as the outage curve changes significantly for a slight change in the energy allocation term β. Moreover, $\beta = 0.5$ is a sufficient choice over a range of α values. The worst performance happens as both parameters approach zero and unity, where the latter substantiates the benefit of cooperation. The lesser sensitivity to α variation could also mean that this parameter employed to study the effect of other system parameters such as location of nodes in the networks, where α could be varied in accordance with nodes locations. Choosing $0.4 < \alpha < 0.7$ practically minimizes outage. As a simple

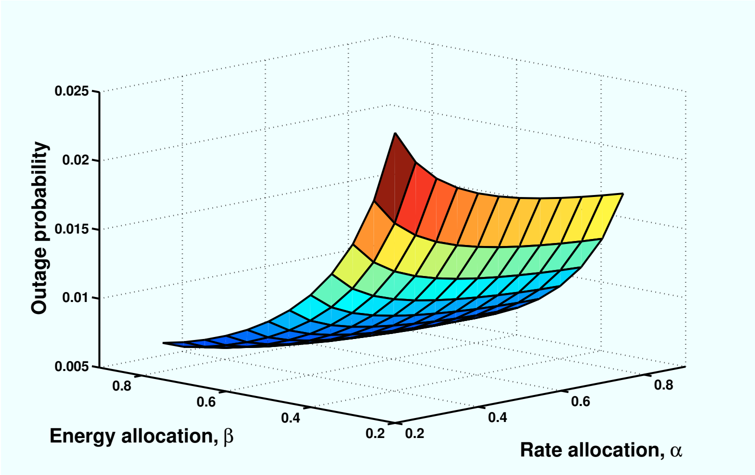

Figure 6.8.: Three dimensional plot of the outage probability taking α and β as the other variables. Results are based on $R = 0.25$ (bits/s/Hz), $\Gamma_{s,d} = \Gamma_{p,d} =$ 10 dB, and $\Gamma_{s,p} =$ 0 dB, which represents a relatively poor link.

rule of thumb, when all the channels are on average identical, $\alpha = \beta = 0.5$ (similar to the conventional schemes) could be employing.

Finally shown in Fig. 6.8 is the outage plot when $\Gamma_{s,p} = 0$ db (i.e relatively poor inter-user channels). The result shows that larger β values (more energy allocated in the first phase) benefits the source, as at these inter-user channel conditions, the benefit of cooperation is less significant (or $\beta > 0.5$ strengthens the inter-user channels).

6.4. Summary

In this chapter, we have introduced the incremental redundancy network-coded cooperation protocol. This protocol embeds network coding into channel coding such that the redundancy in the network code is used to support the channel code for better error protection. Description of the system model and the joint network-channel coding was presented in Section 6.1. We derived the closed-form outage probability expression of this protocol in Section 6.2 and presented numerical results in Section 6.3. The numerical results show that the performance of the protocol mostly depends on the quality of the two inter-user channels. Based on the outage result, energy and rate allocation are also investigated. In the case that the average quality of all channels are identical, an allocation strategy that equally distributes these resources in the two phases is sufficient. Moreover, the sensitivity of the system to energy allocation varies significantly in comparison to the rate allocation.

7. Conclusion

This thesis has presented two types of network-coded cooperation and analyzed their outage probabilities. The main contributions that have been achieved in the course of this work as well as direction for future research are described in the next sections.

7.1. Contribution of the Thesis

Most previous work in the area of network-coded cooperation is based on error-free inter-user channels assumption; however, this is not a realistic assumption in wireless networks. Moreover, a thorough performance analysis was missing and simulation was usually used to investigate protocols. The motivation of this thesis was to perform outage probability analysis of network-coded cooperation protocols based on more realistic assumptions and investigate the outage behavior of the protocols. Using the analysis, the next objective of the thesis was to study other features of network-coded cooperation, e.g., diversity-multiplexing tradeoff. The contributions of the thesis are summarized and presented below according to their order of appearance in the thesis.

7.1.1. Conventional network-coded cooperation

The outage behavior of the conventional network-coded cooperation was studied in Chapter 4. The main observations are presented as follows.

1. We proposed an implementation of an adaptive network-coded cooperation protocol. The adaptiveness is required in a case that inter-user channels are erroneous and a cooperating node has to take decisions (adapt) on what to do next. Then, the outage behavior of this protocol is examined by deriving its outage probability. To make the outage probability analysis more tractable and convenient for exposition, block fading Rayleigh distributed channels, orthogonal transmission, and half-duplex constraints are assumed. Approximating the outage result at high SNR values, we showed that this protocol achieves full diversity order of two.

2. The outage behavior was investigated for various inter-user and uplink channel qualities; we compared various cooperative protocols based on the inter-user channels. In terms of the outage results, network-coded cooperation protocols are found to be superior when the inter-user channels are lower quality; when the inter-user channels are good, protocols without network coding perform better.

3. The outage results are further used to study the diversity-multiplexing tradeoff and the coverage area in cooperative networks. Based on the coverage area results, a static network-coded cooperation protocol is more appropriate when the partner is closer to the destination. With repetition coding, a node closer to the source (or in the center) should be selected as the partner.

7.1.2. Energy consumption of network-coded cooperation

In large networks where a source has potential partners in its surrounding to choose from, metrics that provide insight on how to select a partner are required. One option would be to select a partner node that minimizes the energy spent in the network. With energy minimization in mind, energy consumption of network-coded cooperation was investigated in Chapter 5. The used energy consumption metric computes the average consumed energy per information bit for a fixed error rate at the destination. Using the energy metric, we investigated the energy consumption behavior and impact of network geometry in WSNs that use network-coded cooperation. The main observations are presented as follow.

1. The average energy consumption is optimal when the partner node is located in the vicinity of the destination rather than the source node.

2. A second metric, called energy efficiency, that takes into account both the energy consumption and reliability of reception, was investigated next. From the numerical results, we found out that for an optimal energy efficiency, the partner should be located halfway between the source and destination.

7.1.3. Incremental redundancy network-coded cooperation

The outage behavior of another variant of incremental redundancy network-coded cooperation was investigated in Chapter 6. The results are summarized as follows.

1. We proposed two decoding approaches, namely joint network-channel decoding and individual network-channel decoding, and demonstrated their implementation using linear block coding.

2. Considering individual network-channel decoding at the destination, we provided a framework for computing the outage probability for quasi-static Rayleigh fading channels, orthogonal transmission, and half-duplex constraints. Then the outage behavior was studied and the results show that this protocol also achieves full diversity order.

3. Using the outage result, the rate and energy allocations that minimize the outage probability were studied. The numerical results show that outage performance is more sensitive to the energy allocation than to the rate allocation.

7.2. Recommendations for Future Research

There are plenty of open questions that need to be addressed before any potential deployment of network coding in real networks. In the following, we propose a few promising research areas.

1. The cooperative scenario considered in this thesis was based on three-nodes cooperation. As a future work, protocol design and performance study in large scale networks is one potential research direction. In this regard, [79] can be taken as a good start. In our study, the network coding was on a Galois field of $\mathbb{F}_2 = \{0, 1\}$. However, in large networks one may require larger field sizes and studying the impact of field size in network performance needs to be addressed.

2. In this thesis, we considered the decode-and-forward relaying strategy where decisions are based on hard information bits. The use of soft-bit information in network-coded cooperation is an interesting area to explore, e.g., [21]. Soft-information facilitates the design of an amplify-and-forward-like protocol with network coding, where the coding is done by superposition of soft-bit information of locally generated and received messages.

3. In our analysis, all channels were assumed to be block-fading and Rayleigh distributed. In block fading channels cooperation does not benefit from time diversity. In a case of fast fading channels, channel fading coefficients vary within a packet and the block-fading assumption does not hold. Therefore, performance of network-coded cooperation in fast fading channel environments could be one area of future study [80].

4. A cross-layer study of network-coded cooperation that includes PHY, data link, MAC, and network layers is worth investigating.

5. In Chapter 6, we used linear block codes to demonstrate the joint network-channel coding. Designing more efficient network-channel codes, e.g., using convolutional codes, that improve performance is another future research direction.

6. In our approach, the partner discards a packet that is received in error even if parts of this packet are received correctly. Soft-bit information and symbol-level network coding may be used to filter out correctly received parts of a packet [32, 81]. A detailed study in this area is our final recommendation.

A. Outage Probability Approximation

The outage probability approximations of the incremental redundancy network-coded cooperation are presented as follows.

A.1. Case 1

The outage probability expression given in Equation (6.18) on page 122 is derived next. Re-writing Equation (6.16) once again

$$\begin{aligned} P\left(\overline{X}_s\right) &= P\left\{(1+\beta_1\gamma_{s,d})^{\alpha}(1+\beta_2(\gamma_{s,d}+\gamma_{p,d}))^{(1-\alpha)} < 2^{2R}\right\} \\ &= \int\int_A \frac{1}{\Gamma_{s,d}\Gamma_{p,d}} \exp\left(-\frac{\gamma_{s,d}}{\Gamma_{s,d}} - \frac{\gamma_{p,d}}{\Gamma_{p,d}}\right) d\gamma_{s,d} d\gamma_{p,d}. \end{aligned} \tag{A.1}$$

The term inside the braces is a function of $\gamma_{s,d}$ and $\gamma_{p,d}$ and this function is compared to the threshold value 2^{2R}. This probability can be computed by integrating the joint probability density function (PDF) of the two random variable on a region A defined by

$$A \equiv \left\{(1+\beta_1\gamma_{s,d})^{\alpha}(1+\beta_2(\gamma_{s,d}+\gamma_{p,d}))^{(1-\alpha)} < 2^{2R}\right\}. \tag{A.2}$$

To determine the range of the two variables, we extract $\gamma_{p,d}$ from A and get the range

$$0 < \gamma_{p,d} < \frac{1}{\beta_2}\left[\frac{2^{2R/(1-\alpha)}}{(1+\beta_1\gamma_{s,d})^{\alpha/(1-\alpha)}} - (1+\beta_2\gamma_{s,d})\right]. \tag{A.3}$$

As the term in bracket in Equation (A.3) should be greater than zero, $\gamma_{s,d}$ must satisfy

$$(1+\beta_2\gamma_{s,d})(1+\beta_1\gamma_{s,d})^{\alpha/(1-\alpha)} < 2^{2R/(1-\alpha)} \tag{A.4}$$

It is difficult to solve for $\gamma_{s,d}$ and $\gamma_{p,d}$ from (A.3) and (A.4) directly. Using Taylor's series in two variables and taking up to the first order terms in Equations (A.3), we get the ranges

$$0 < \gamma_{p,d} < \frac{1}{\beta_2}\left[2^{2R/(1-\alpha)} - 1\right] - \left[2^{2R/(1-\alpha)}\frac{\beta}{1-\beta} + 1\right]\gamma_{s,d} \ . \tag{A.5}$$

The right side of Equation (A.5) is a linear function of the variable $\gamma_{s,d}$. Similarly, Equation (A.4) can be approximated as

$$0 < \gamma_{s,d} \quad < \quad \left[2^{2R/(1-\alpha)} - 1\right](1-\alpha) \ . \tag{A.6}$$

Once these ranges are known, the integral in (A.1) can be evaluated and

$$P\left(\overline{X}_s\right) \quad \approx \quad \frac{(2^{2R/(1-\alpha)} - 1)^2}{\Gamma_{s,d}\Gamma_{p,d}} \frac{(1-\alpha)^2}{(1-\beta)}, \tag{A.7}$$

which is the outage probability approximation given in Equation (6.18), is obtained.

A.2. Case 2

Next the outage approximation given in Equation (6.29) is derived. Given the region C where the variables $\gamma_{s,d}$ and $\gamma_{p,d}$ are defined as

$$C \quad \equiv \quad \left\{(1+\beta_1\gamma_{s,d})^{\alpha}\left[(1+\beta_2\gamma_{s,d})(1+\beta_2\gamma_{p,d})\right]^{(1-\alpha)} < 2^{2R/(2-\alpha)}\right\} \tag{A.8}$$

and extracting $\gamma_{p,d}$ from C, we get the range

$$0 < \gamma_{p,d} < \frac{1}{\beta_2}\left[\frac{2^{2R/[(2-\alpha)(1-\alpha)]}}{(1+\beta_1\gamma_{s,d})^{\alpha/(1-\alpha)}(1+\beta_2\gamma_{s,d})} - 1\right]. \tag{A.9}$$

As the right side of Equation (A.9) should be greater than zero, $\gamma_{s,d}$ must fulfill the following condition.

$$(1+\beta_1\gamma_{s,d})^{\alpha/(1-\alpha)}(1+\beta_2\gamma_{s,d}) < 2^{2R/[(2-\alpha)(1-\alpha)]}. \tag{A.10}$$

Using again Taylor's series in two variables and taking up to the first order terms, Equation (A.9) can be approximated as

$$\begin{aligned} 0 < \gamma_{p,d} \quad &< \quad \frac{(1-\alpha)}{(1-\beta)}\left[2^{2R/[(2-\alpha)(1-\alpha)]} - 1\right] - \left[2^{2R/[(2-\alpha)(1-\alpha)]}\frac{1}{1-\beta}\right]\gamma_{s,d} \\ &= \quad a - b\gamma_{s,d} \end{aligned} \tag{A.11}$$

where $a = \frac{(1-\alpha)}{(1-\beta)}\left[2^{2R/[(2-\alpha)(1-\alpha)]} - 1\right]$ and $b = \left[2^{2R/[(2-\alpha)(1-\alpha)]}\frac{1}{1-\beta}\right]$. We see in Equation (A.11) that, for given values of α and β, the term $a - b\gamma_{s,d}$ is a linear function of $\gamma_{s,d}$. Similarly, Equation (A.10) can be approximated as

$$\gamma_{s,d} \quad < \quad \left[2^{2R/(2-\alpha)} - 1\right] \ . \tag{A.12}$$

Once the ranges of $\gamma_{s,d}$ and $\gamma_{s,d}$ are known, the integral in Equation (6.27) can easily be evaluated and the following result is obtained.

$$P\left(\overline{X}_s\right) \approx \frac{(1-\alpha)^2}{(1-\beta)\Gamma_{s,d}\Gamma_{p,d}}\left[2^{2R/[(2-\alpha)(1-\alpha)]}-1\right]^2. \tag{A.13}$$

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