Models, Methods and Applications of Group Multiple-criteria Decision Analysis in Complex and Uncertain Systems

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Abstract: Much of existing decision making research focuses on algorithms designed to measure the preferences of different experts on a finite set of alternatives with regard to multiple conflicting criteria and trade them off alongside objective problem-relevant data. Moreover, a number of analytical methods have been developed to handle informational gaps and uncertainty. However, practical decision structures and problem requirements are versatile and more complex than the existing single-method approaches assume. Therefore, the issue of constructing user-friendly decision support procedures able to capture complexity and produce valid solutions is a relevant and unresolved problem.

This thesis addresses the challenges of developing system-oriented decision support models and methods for complex multi-criteria group decision problems involving uncertainty. In essence, this research builds upon multi-criteria decision analysis (MCDA) theory and system analytical concepts. First of all, the limitations and advantages of different MCDA techniques are analyzed and new integrated methodologies are developed to benefit from the strengths of individual methods while avoiding their weaknesses. Furthermore, this thesis studies the existing empirical and analytical approaches to group decision making and proposes novel methods to aggregate the opinions of different experts. In particular, the focus is on the aspects of group structuring and responsibilities definition, as well as on measuring and analyzing subjective expert estimates' reliability and discordance of opinions in groups. Finally, the developed methods are capable of considering informational uncertainties by using the tools of fuzzy set theory and its generalizations. Additionally, various methods from the broader science of operations research and management as well as visualization techniques are employed to precisely model the practical systems.

While each paper of this cumulative thesis makes its own contribution theoretically and in a particular decision problem, from a broad viewpoint, the contribution of this work is to create more constructive relationships between decision-making-related disciplines to produce accurate, effective and efficient solutions of multi-criteria problems.

Keywords: Multi-criteria Decision Analysis, Uncertainty Modelling, Complex Systems, Group Decision Making, Applications of Operations Research.

Academic dissertation

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Publications

This thesis consists of the present summary article and the following papers:

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[II] Sodenkamp, M. (2012). Modelling synergies in vendor selection problems with application to agricultural commodity trade. *DS & OR Working Papers*, No. 1302, University of Paderborn.

[III] Sodenkamp, M. and Suhl, L. (2012). A Multicriteria Multilevel Group Decision Method for Supplier Selection and Order Allocation. *International Journal of Strategic Decision Sciences*, Vol. 3, No. 1, pp. 81-105.

[IV] Tavana, M. and Sodenkamp, M.A. (2010). A Fuzzy Multi-criteria Decision Analysis Model for Advanced Technology Assessment at Kennedy Space Centre, *Journal of the Operational Research Society*, Vol. 61, No. 10, pp.1459-1470.

[V] Saaty, T.L. and Sodenkamp, M. (2008). Making decisions in hierarchic and network systems, *International Journal of Applied Decision Sciences*, Vol. 1, No. 1, pp. 24-79.

[VI] Tavana, M., Sodenkamp, M. and Suhl, L. (2010). A Soft Multi-Criteria Decision Analysis Model with Application to the European Union Enlargement. *Annals of Operations Research*, Vol. 181, No. 1, pp. 393-421.

[VII] Tavana, M., Sodenkamp, M.A. and Pirdashti, M. (2010). A Fuzzy Opportunity and Threat Aggregation Approach in Multicriteria Decision Analysis. *Fuzzy Optimization and Decision Making*, Vol. 9, No. 4, pp.455-492.

[VIII] Tavana, M., Bourgeois, B.S. and Sodenkamp, M. (2009). Fuzzy Multiple Criteria Base Realignment and Closure (BRAC) Benchmarking System at the Department of Defense. *Benchmarking: An International Journal*, Vol. 16, No. 2, pp. 192-221.

Contribution of the author in the papers

Sodenkamp was single author of Papers [I] and [II]. Sodenkamp was the main contributor to Papers [III], [IV], [VI], [VII] and [VIII], for which Sodenkamp proposed and developed the algorithms and implemented them for the applications. Sodenkamp was the secondary author of Paper [V]. The detailed authors' contribution to the papers written in co-authorship has been as described below;

[III] The methodology development, its application, data collection and analysis presented in the paper were carried out by Sodenkamp. Suhl provided state-of-the-art materials concerning the application area. The results were collaboratively analyzed by the author and the co-author.

[IV] The model introduced in this paper was developed by Sodenkamp, it advances the method previously presented by Tavana in 2002. Tavana provided data for the case study. Sodenkamp performed the case study analysis. The results were collaboratively analyzed by both authors.

[V] The paper is based on the methods developed by Saaty and his examples. Sodenkamp combined the materials provided by Saaty.

[VI] The framework and algorithm was developed by Sodenkamp. Sodenkamp performed the case study analysis. The idea of application belongs to Tavana. Suhl coordinated work of students participating in the case study. The results were collaboratively analyzed by the author and the co-authors.

[VII] The decision support model presented and the method of problem analysis were introduced by Sodenkamp. Sodenkamp made principal calculations for the case study. Tavana initialized the idea of the application and coordinated work of the co-authors. Pirdashti made surveying of the case study participants and delivered practical data. The results were collaboratively analyzed by the author and the co-authors.

[VIII] The model of the benchmarking system presented and the analysis method were developed by Sodenkamp. Sodenkamp applied the method to analyze the pilot study. Tavana initialized the idea of the application and coordinated the authors' work. Bourgeois provided problem requirements and data for the case study. The results were collaboratively analyzed by all co-authors.

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1. Introduction and motivation

Decision making, being a fundamental human activity and the core element of all organizational and social processes, is often challenged by extremely complex and uncontrollable structures and processes. On the other hand, global movement toward sustainable democratic governance on micro and macro levels requires innovative approaches to further strengthen collaborative and transparent decision analysis and policy making. A large contemporary literature provides evidence that the subject of deliberate and responsible decision making has become a topic of increasing importance to both private and public sectors.

On 9 August 2012, United Nations in partnership with 27 leading world organizations [i1], announced the urgent launch of the new global research network which is needed "… to accelerate joint learning and help to overcome the compartmentalization of technical and policy work by promoting integrated "systems" approaches to addressing the complex economic, social and environmental challenges… requiring a new generation of problem solving…" [i2]. "Vision of the whole", i.e. systems analysis, is a prerequisite for adequate high-quality decisions leading to success in strategy achievement (Drucker, 1955; Hammond et al., 1998; Simon, 1960; White, 1971). It implies rigorous and methodologically sound system description and decomposition, study of subsystems from different perspectives and interdisciplinary viewpoints, and integration/aggregation of results, which as a rule cannot be achieved unaided (Drucker, 1955; Kiker et al., 2005; Sokolova and Caballero, 2012).

In light of decision making, multilateral problems of complexity have three main aspects: first, structural complexity is related to a number of decision elements, such as potential options, multiple objectives, opposing groups of interests and other (intermediate) objects, as well as various hierarchical and/or feedback dependencies between the elements; second, functional complexity assumes the diversity of elements' competences, often uncertain, and the dynamics of change in the elements; and, third, modeling complexity concerns system formalization and calculation of results (Goodwin and Wright, 2004; Ivanov and Sokolov, 2013; Mackinnon and Wearing, 1980; Montibeller and Franco, 2010, Saaty, 2008).

During the last years, guidance on empirical and analytical approaches to decision making under high degrees of complexity and uncertainty has arisen in every conceivable field (Polasky et al., 2011; Saaty, 2006). The empirical research is conducted qualitatively within such disciplines as economics, management, psychology, philosophy, sociology and political science. It employs assumptions, observations, experiments and reasoning to discover how people actually solve problems and why the process of making decision can go wrong (Corso and Löbler, 2011; Yukl et al., 2002; Vroom and Yetton, 1973). Empirical methods provide general qualitative guidance to managerial decision making, but are not capable to encompass all objective information and specifics of complex problems in details, and therefore do not assure credibility of the outcome, nor decision process transparency. Ever since analytical decision making was introduced in 1961 by Howard Raiffa and Robert Schlaifer, a succession of mathematically sound and computationally efficient techniques has revolutionized decision theory (Goodwin and Wright, 2004). However, the quantitative analytical methods are either too difficult to implement due to their technical and procedural complexity, or are restricted to a narrow conception of decision analysis which often makes them unusable in practical situations.

Reported experiences from practical applications suggest that process-oriented, straightforward, collaborative and transparent approaches which rely upon the integration of both empirical evidence and analytics, consider multiple objectives of different stakeholders, accommodate uncertain information, and provide support for the whole process rather than some particular phases only are more likely to yield better decisions and be accepted by practitioners (Abaza and Baranzini, 2002; Hobday et al., 2004; Wahoff et al., 2012; Liesiö et al., 2007; Mustajoki, 2006).

This thesis applies to decision making at the level of the individual humans and communities of individuals in complex multi-criteria problems involving uncertain characteristics of decision alternatives that are discrete and finite in number. Contribution of this research is twofold. First, it provides novel integrated process-oriented models and methods for handling collaborative decision situations in dynamic and changing environments with the following key characteristics: (1) large amount of conflicting tangible quantitative and intangible qualitative objectives (all Papers); (2) different stakeholders' circumstances, values, needs and purposes (all Papers); (3) controllable and uncontrollable parameters (Papers [VI]-[V]); (4) interconnections and mutual dependencies among system elements (Papers [II]-[VIII]); (5) synergy effects and non-linear performance measures (Paper [II]); (6) uncertain and unreliable data about current and projected vulnerabilities (all Papers); (7) individual judgments and preferences of involved participants (all Papers); (8) political, cultural, environmental, geographical, social, economical, technological and legal issues (Papers [II]-[VIII]); (9) different effect horizons (Papers [II]-[V], [VIII]); and, (10) multi-source/multi-spectra fuzziness of performance characteristics (Paper [I]).

Second contribution of this research is that usefulness and applicability of the developed formal decision frameworks is demonstrated for several real problems and case studies. The concerned application industries and sectors include aerospace and engineering (Paper [IV]), defence (Paper [VIII]), energy and utilities (Paper [VII]), consumer products and trade (Papers [II]-[III]), regional policy (Papers [VI]-[V]), among others (Paper [V]). Addressed task areas are capital planning (Papers [IV] and [VIII]), innovation, research and development (Paper [IV]), performance analysis (Papers [II], [III], [VI] and [VIII]), strategic planning (Papers [V]-[VII]), as well as portfolio management and resource allocation, particularly vendor selection and procurement (Papers [II]-[III]).

The proposed integrated frameworks utilize a series of existing techniques such as the Analytic Hierarchy Process (AHP) (Papers [I]-[VI]), the Analytic Network Process (ANP) (Papers [V], [VII] and [VIII]), benefits-opportunities-costs-risks (BOCR) analysis (Paper [V]), brainstorming (papers [VI] and [VII]), defuzzification (Papers [I], [IV] and [VI]-[VIII]), deneutrosophication (Paper [I]), entropy (paper [IV] and [VI]-[VIII]), Euclidean metric (Papers [I], [III], [IV] and [VI]-[VIII]), fuzzy sets (Papers [I], [IV] and [VI]-[VIII]), graph theory (Paper [II]), linear optimization (Paper [II]), level-2 fuzzy sets (Papers [VII] and [VIII]), the Displaced Ideal (DI) method (Papers [IV] and [VI]-[VIII]), neutrosophic sets (Paper [I]), normalization (all Papers), numerical and linguistic scoring (all Papers), Pros and Cons analysis (Paper [III]), sensitivity analysis (Paper [V]), strengths-weaknesses-opportunities-threats (SWOT) analysis (Paper [VI]), value-focused thinking (VFT) (Paper [II]), alternative-focused thinking (AFT) (Paper [II]), visualization in Euclidean and polar coordinate systems (Papers [IV] and [VI]-[VIII]) and weighed sum method (all Papers).

The approaches developed in this thesis are aimed at helping stakeholder teams to align their decisions with objectives, achieve consensus and create outcome dependability via uncertainty consideration and transparent audit trail of the entire decision process.

The rest of this summary article is structured as follows. Section 2 provides insight into theoretical foundations of multi-criteria decision analysis (MCDA) and group decision making as well as related aspects of uncertainty modelling. Section 3 reports the key methodological contributions of the Papers and their practical implications. Finally, general conclusions and directions for future research are presented in Section 4.

2. Methodological foundations

2.1. Multi-criteria decision analysis

MCDA is a rapidly growing branch of operations research (OR) and management science (MS) extensively used for evaluating options which involve the achievement of multiple (incommensurate) objectives in face of uncertainty and complexity (Franco and Montibeller, 2009; Hahn, 2003; Schuwirth et al., 2012; Zopoundis and Pardalos, 2010). Multi-criteria decision problem takes place when a decision maker chooses one (or a subset) of a set of feasible discrete alternatives evaluated on the basis of two or more attributes, quantitative and/or qualitative, and acts to maximize a utility or value function that depends on the attributes (Wallenius et al., 2008; Clemen, 1996). Often, decision criteria cannot be condensed into a monetary value, partly because stakeholders' concerns often involve ethical and moral principles that may not be related to any economic use or value (Kiker et al, 2009). MCDA methods can address three types of problems: ranking of a finite set of alternatives, choosing the best alternative(s) and clustering alternatives in similarity groups (Paper [IV]). MCDA process includes five typical stages: defining the problem, identifying and structuring stakeholders, alternatives and criteria; assigning criteria weights, evaluating alternatives against criteria; selecting the evaluation model; and finally, executing the model and interpreting results with possible re-iteration of the process (Mateo, 2012; Saaty, 2008; Tervonen et al. 2009).

It has been widely acknowledged that almost all problems and decisions are multi-criteria in nature, which has inspired reflection of thinkers since ancient times (Belton and Stewart, 2002; Davenport, 2009; Drucker, 1955; Figueira et al., 2005; Keeney and Raiffa, 1976; Norton and Kaplan, 1992; Saaty, 2006; Turskis and Zavadskas, 2011; Zopoundis and Doumpos, 2002). The first known formal decision approach called Pros and Cons was described by Benjamin Franklin in 1772 (Fortemps and Slowinsky, 2002). As a separate science MCDA originates from the 1950s under the influence of von Neumann-Morgenstern's Expected Utility Theory (EUT) (1944), Drucker's "management by objectives" (1954), Savage's utility theory based on subjective probabilities (1954) and Schlaifer's statistics with subjective probabilities (1959). For more MCDA history see Köksalan et al. (2011).

MCDA approaches differ in the way how elements are ranked and the priority information is synthesized into a decision. Brucker et al. (2013) distinguish between three types of aggregation procedures adopted in MCDA, namely: compensatory aggregation (as applied in the AHP, the multi-attribute utility/value theory (MAUT/MAVT) and MACBETH), outranking methods (OMs) (as applied in PROMETHEE and ELECTRE) and non-compensatory approaches (as applied in dominance and lexicographic methods). According to Wallenius et al. (2008), the topmost among approaches to discrete alternative analysis is the AHP by Saaty (1980), followed by Keeney and Raiffa's (1976) MAUT/MAVT, and OMs ELECTRE by Roy (1968) and PROMETHEE by Brans et al. (1986).

In the AHP and its generalization the Analytic Network Process (ANP) criteria and alternatives are compared in pairs within homogeneous clusters using a *ratio scale*. Quantified dominance information is inserted into square matrices, and the right eigenvector method or the geometric mean are employed to derive priorities of all elements (see Paper [V]). However, due to humans' cognitive limitations and the large number of required comparisons the AHP is rather acceptable for small-scale problems or new exploratory decisions (Miller, 1956; Saaty and Vargas, 2012).

MAUT/MAVT offers a normative model for decision making in view of multiple objectives and mutually exclusive options (Keeney and Raiffa, 1976). Criteria weights and performances of the alternatives are established using utility scores on an *interval scale*; the overall alternatives' values are yielded by summarizing the weighed scores. Unlike pairwise comparisons, scores can be used for ranking an unlimited number of elements. For a detailed comparative study of the AHP and MAUT/MAVT see Henriksen (1997).

OMs employ pairwise comparisons based on concordance and discordance indices (Yoe, 2002). OMs can best deal with *ordinal* information, but interpretation of the results is often difficult (Bouyssou and Pirlot, 2005; Kangas et al., 2001). Moreover, OMs are often criticized for their *ad hoc* basis and lack of axiomatic foundations (Bouyssou, 2001). Comparison of the AHP/ANP, MAUT and OMs with respect to various structuring and measuring indicators can be found in Saaty and Vargas (2002).

Location MCDA models, including the displaced ideal (DI) method (Zeleny, 1974), TOPSIS (Hwang and Yoon, 1981) and VIKOR (Opricovic and Tzeng, 2004) are likely to be efficient if the set of alternatives contains a well-known benchmark solution or if the best/worst consequences can be identified. DI involves measurement of options' geometric proximity to the best ("ideal") solution where the objective function is taken to be the minimized total distance (Carling et al., 2012). In TOPSIS criteria-specific performances are compared with both the best ("ideal") and the worst ("anti-ideal") consequences at once. VIKOR employs Manhattan and

Chebyshev distances to maximize utility for the majority of decision makers and, at the same time, minimize individual regrets.

As a matter of fact, although implementation of MCDA tools has gradually become more common in practice, weaknesses of the methods hamper their use. Nevertheless, explicit and structured MCDA-based techniques have a great potential because they lead to more effective and efficient decision process as compared with the bias and intuition-driven practices that organizations are often accused of using in decision making (Kiker et al., 2005).

2.2. Group decision making

In today's globalization era organizations and communities operate in a value network wherein collaboration and group decision making are believed to be the core of long-term advantage (Dennis et al., 2010; Im and Workman, 2004; Agarwal and Selen, 2009; McGee, 1998). Involving all relevant policy makers, experts and stakeholders is acknowledged to be sufficient for conflict resolution, implementation of selected practices and sustainable development (Watson and Johnson, 1972; Priem et al., 1995; Mohr and Speckmann, 1995; Johnson et al., 2012; A/CONF.216/L.1, 2012; Dyer et al. 2008).

While individual human minds are naturally limited, collaboration is inevitable to tackle complex problems in the real world (Fischer and Sugimoto, 2006; Bonabeau, 2009; Forsyth, 2010). Groups are better at choosing, judging, estimating, and problem solving than individuals (Stasser and Dietz-Uhler, 2001); groups form more accurate perception than do individuals (Ruscher and Hammer, 2006; Glick and Staley, 2007), and groups can find information they need faster than single individuals (Lazonder, 2005). Although groups usually take longer to make decisions than do individuals, they are more creative and make fewer errors (Shaw, 1932; Hinsz, 1990).

Supporting a decision making process becomes intensely difficult due to the dynamic and ill-structured environment, as well as presence of conflicting groups of interests each with their own perceptions on the way the problem should be managed (Jelassi et al., 1990; Matsatsinis et al., 2005; Morais et al., 2012). An effective group decision processes should be straightforward and transparent where the experts, on the one hand, fully utilize their knowledge, creative potential and resources (Johnson and Johnson, 2003), and on the other hand, do not become confused with providing information and making tradeoffs (Belton and Pictet, 1997). Importantly is that participants should avoid polarization and groupthink (Forsyth, 2010; Surowiecki, 2005).

Mitchell et al. (1997) suggest that identifying salient stakeholders at any point in time largely remains an empirical question.

Bonabeau (2009) found out that humans perform better in groups when they generate new ideas, whereas evaluation should be made individually and, where possible, facilitated. The aim, though, is to develop group decision tools that ensure shorter response times, accurate results and more exploration of potential opportunities.

In the sense of decision support, collaboration can be seen from at least three following perspectives. First, collaboration concerns the structuring of decision group. Group structuring implies, on the one hand, drawing clear distinctions among the decision makers' professional expertise domains, e.g. financial, political, social, technological, legal, etc.; and, on the other hand, articulating individuals' responsibilities and tasks in terms of establishing value systems or detecting characteristics of the elements (Paper [III]). Second perspective concerns the group work organization, i.e. construction of appropriate procedures for holding meetings and workshops, engaging experts in discussions and surveying. Finally, the analytical group decision support perspective regards the choice of methods for the elicitation, representation and quantification of judgments, assessment of the estimates reliability and finally, for finding representative (compromise) group solutions.

Numerous recent studies of the existing approaches for arriving at a collective decision affirm that consensus is the best rule for producing innovative, creative, and high-quality decision that (i) all members will be committed to implementing; (ii) uses the resources of all group members; and (iii) increase the future decision-making effectiveness of the group (Johnson and Johnson, 2003; Kaner et al. 2007; Song, 2009). Consensus means a cooperative process in which group members develop and agree to support a decision in the best interest of the whole; it embraces individual perspectives, honouring each person's piece of the truth, while emphasizing the sense of the meeting through a creative search (Orsi and Kassan, 2012; Bressen, 2007; Avery et al., 1981).

Forman and Peniwati (1998) suggested that two most useful consensus support methodologies are the aggregation of individual judgments (AIJ) and the aggregation of individual priorities, choice among which depends on whether the group is assumed to act together as a unit or as separate individuals. In either case (weighed) arithmetic or (weighed) geometric mean operators can be applied to derive representative group values if these are given on an interval scale. Azcel and Roberts (1989) claimed that arithmetic mean is not acceptable for the AIJ strategy if judgments are expressed using ratio scale.

Construction of reliable consensus-based decision support methodologies and their amalgamation with cutting-edge communication and computer technologies into a group decision support system is required not only to significantly facilitate the formulation and solution of complex problems in an effective and efficient manner, but also to increase users' satisfaction with the decision process and their commitment to implementing the decision (DeSanctis and Gallupe, 1987; Fan and Shen, 2011; Alonso et al. 2013). In spite of the significant progress achieved in the enhancement of group multi-criteria decision analysis (GMCDA) during the last 30 years, this problem remains far from being ultimately resolved. In particular, not enough attention has been paid to the issues of (i) detecting and taking into consideration individuals' judgments dependability, consistency and errors; (ii) defining and structuring multitasking decision groups with multivariate stakeholders' roles/responsibilities, and adequately correlating such flexible structures with the context and methodology of MCDA; (iii) assigning and quantifying voting power in open and closed decision groups; (iv) measuring and analyzing the degree of members' disagreement; and, (v) integrating techniques for handling the above-mentioned problems within holistic coherent frameworks.

2.3. Uncertainty modeling in multi-criteria decision analysis

The development of models and methods for representing and handling uncertainty in decision theory started initially from the axiomatization of objectivists' probability by Andrey Kolmogorov in the early 20th century as formalization of deterministic randomness. In the classical sense, probability means relative frequency or chance of an outcome that is repeated many times and yielded through experiments (Kangas et al., 2008; Tversky and Kahneman, 1983). EUT (von Neuman and Morgenstern, 1944) was the first and one of the most important twentieth-century theories describing decision under uncertainty, according to which the decision maker chooses between perspectives by comparing their utility values multiplied by the respective probabilities. However, determining objective probabilities is often impossible due to the absence of a reference set, e.g. in the case of non-repeatable events (Lecoutre et al., 2006; Chavas, 2012).

Extensive research on people's ability to perceive randomness and uncertainty that started in early 1950s resulted in the development of subjective likelihood concept and extension of EUT to personal EUT (Savage, 1954). Subjective probability reflects the degree of belief (or confidence) in the occurrence of an event (Lecoutre et al., 2006). Formally, both objective and subjective likelihood distribution functions must satisfy the axioms of probability calculus. In the 1970s, the idea to utilize personal description of a state of knowledge about probabilities was incorporated in the MAUT, a multi-criteria formulation of (subjective) EUT. However, since development of cognitive psychology in the 1960s, a number of systematic violations of normative EUT axioms were reported over the years. The main critique refers to human "biases" and "errors" in decision making under risk and uncertainty (Tversky and Kahneman 1981). This critique had a response from the perspective of MCDA that seeks to extend the utility model so that the resulting model is able to accommodate behaviour violating the axioms of EUT (Durbach and Stewart, 2012). Thus, during recent years, MCDA and GMCDA continued its growth through new techniques that generalize the basic approaches by incorporating tools for handling preferential and informational uncertainties inherent in real world complex domains.

Molodtsov (1999) pointed out that there are three theories which can be considered as mathematical tools for dealing with uncertainties: theory of probability, theory of fuzzy sets (FSs) and interval mathematics. In contrast to probability theory that treats stochastic (aleatoric) uncertainty, FSs and fuzzy logic (FL) (Zadeh, 1965) deal with informational (epistemic) vagueness. Probability refers to the likelihood that something is true, whereas FL establishes the degree of truthfulness through the membership grade. At a most fundamental level, mathematical difference between probabilities of mutually exclusive perspectives and membership grades of fuzzy sets is that probabilities always add up to 1, whereas the sum of membership degrees can be smaller or greater than 1. FSs and FL are sometimes referred to as the stems of possibility theory (Dubois and Prade, 1993; Zadeh, 1999). Dubois and Prade (1993) analyzed the correlation between FSs and probability theory, and found that fuzziness cannot be reduced to randomness. Quantitative measurements may be unpredictable – random (stochastic) – or noisy (fuzzy), while qualitative information is imprecise (fuzzy) on its nature (Mendel and Wu, 2010; Roy and Maji, 2007). The two main motivations for using probabilities in MCDA are to account for the chances that alternatives will occur, and to allow consideration of the event-driven factors (i.e. scenarios – see Stewart et al., 2013). FSs, in turn, enable formalization and reasoning of intangible internal characteristics, typically natural language-based and visual image information, as well as incomplete, unreliable, imprecise and vague performance and priority data. Intervals is another non-probabilistic uncertainty formulation employed in MCDA, where decision makers' preferences, criteria weights and performance values of alternatives are represented by the data ranges (Weber, 1987; Mustajoki et al., 2005; Sugihara et al., 2004; Stanujkic et al., 2012; Yao, 2010).

The MCDA and GMCDA techniques that extend EUT models to permit handling non-probabilistic vagueness include not only the elements of interval and FS theory, but also their modifications and generalizations: rough sets and granular computing (Pawlak, 1991; Greco et al., 2001; Chakhar and Saad, 2012; Bargiela and Pedrycz, 2003; Slowinski et al., 2009), perceptual computing and computing with words (Zadeh, 2012; Mendel and Wu, 2010; Herrera et al., 2009; Martinez et al., 2010) and intuitionistic sets (Atanassov, 2012; Vahdani et al., 2013; Chen, 2011; Smith, 2012) among others.

From the point of view of practical decision aid, the contemporary literature provides sufficient evidence of both probabilistic and non-probabilistic uncertainty measurement concepts as being extremely important for risky and complex domains (Stewart, 2005; Aven and Zio, 2011; Hanafizadeh et al., 2011). Thus, some recent attempts have been made toward the conjoint application of probability theory and non-probabilistic techniques in multi-criteria financial modelling (Liu et al., 2011; Capotori and Barbenera, 2012; Ma et al., 2013), environmental management (Fanghua and Guanchun, 2010), marketing (Utkin and Zhuk, 2013), engineering (Sobral and Ferreira, 2013), supply chain management (Büyüközkan et al., 2012; Deng and Chan, 2011), medicine (Chen et al., 2006) etc.

2.4. Integrated systems approaches to complex multi-criteria problems

In spite of the variety of existing techniques, application of any single-method research approach to the entire system analysis and decision support, as a rule, does not ensure the sufficient degree of comprehensiveness, accuracy and reliability of the outcome. There are three main reasons for this shortage: limitations of the individual methods (Robinson et al., 2012; Kiker et al., 2005); diversity, specificity and multilateral complexity of real-life systems; and multidisciplinary knowledge bases drawn on most decisions (Kiker et al, 2005). As a consequence, there is no unique fundamental concept for addressing all kinds of situations and decisions, so the problem of theoretically substantiated decision making in real-world contexts remains relevant and open, where the process of MCDA is likely to contribute if it is understood in an integrated manner. At this juncture, the integration implies development of hybrid frameworks in order to bring appropriate insights at different phases of the analysis and policy making, as well as combination of MCDA with other structuring and evaluation methods from the broader sciences of OR, MS, psychology, mathematics, etc. (Belton and Stewart, 2002; Großweile et al., 2013; Bottero et al. 2013). The integration across MCDA methods and of MCDA with group decision and uncertainty modeling techniques, as well with optimization modeling, simulation, systems analysis, statistics, and others, underpins the mainstream of the last decade of research on decision analysis.

Combined MCDA-optimization methods are commonly used in portfolio selection problems, where MCDA serves for the evaluation and ranking of alternatives, and a 0/1 knapsack optimization algorithm allows selection of the most effective feasible subset (see, for instance, Liesiö et al., 2007, Wang and Hwang, 2007 or Ghasemzadeh and Archer, 2000). Moreover, some critical tasks of portfolio management, e.g. resource and order allocation, scheduling, staff assignment, performance analysis etc., can be effectively solved using integrated methods of MCDA, linear and non-linear optimization and simulation (see Tiryaki and Ahlatcioglu, 2009; Ballestro et al., 2007; Carazo et al., 2010; and Polyashuk, 2005).

Computation of objective criteria weights, as well as measuring preferences or alternatives' performance in multi-criteria problems is frequently made using statistical analysis/simulation methods if the required reference data is available (see Leskinen and Kangas, 2005; Deng et al., 2000; Liu and Wang, 2007; Araz, 2005).

DEA is an LP based method for the evaluation of efficiency frontiers of decision making units, which is sometimes seen as competing with the MCDA approach. However, there are many ways in which DEA and MCDA can be used complementarily, since DEA focuses on objective historical data about decision making units for monitoring and control, whereas MCDA seeks to elicit, understand and treat value judgments to support planning or choice (Sinuany-Stern et al., 2000; Belton and Stewart, 1999).

Recently, successful efforts were made toward the conjunction of MCDA with data mining for addressing the problems of incidents prediction and management (Peng et al., 2011), association rule prioritization with incorporation of decision makers' preferences (Choi et al., 2005), customers clustering and marketing recommendations production (Liu and Shih, 2005) etc.

Together all the existing studies send a strong message that in spite of the recent advances, the discourse on the topic of integrated analysis for system-oriented decision support is still in its infancy and researchers are faced with an acute practical need to develop transparent, comprehensive, mathematically sound and user-friendly methods, models and tools based on the rapidly evolving state-of-the-art concepts and methodologies.

3. Contributions of this thesis

The fact that multiple goals, different interest groups and vulnerabilities are natural for real life decision situations requires studying such problems in an integrated manner. The main purpose of this thesis is to develop systematic and coherent aids for practice-oriented decision situations in complex multi-objective, multi-person and ambiguous settings. The contributions are obtained in the following three research areas: (a) development and application of integrated MCDA methods and models based on systems analysis and problem structuring techniques; (b) development of GMDCA methodologies to support finding compromise solutions within groups of stakeholders with different (conflicting) interests and unequal authorities, (c) development of approaches to the improvement of MCDA/GMCDA outcome reliability via consideration of informational uncertainties.

3.1. Contributions of Paper [I]

Paper [I] addresses research areas (b) and (c). The newly introduced approach to modelling and aggregation of multiple expert opinions in multi-criteria problems is based on neutrosophic set theory. Reliability of personal priorities assigned to decision elements (criteria and alternatives) is given by the triples of independent metrics: inconsistency or error of judgments underlying the priorities, voting power of the responsible decision makers, and the level of experts' confidence in the plausibility of own statements.

Elicitation and precise analysis of the estimates' reliability is crucial for their adequate account on the group opinions aggregation stage. Hereby, an important characteristic of the individuals' judgments quality is their inconsistency which, in fact, reflects the falsity-degree of the yielded priorities. Usually the inconsistency indices employed in the ratio scale-based compensatory approach are compared with a threshold value in order to specify whether the judgments are satisfactory or should be revised. In Paper [I] this traditional view is broadened to treat the inconsistency magnitudes or errors as continuous arguments of the functions returning the falsity-degrees. On the other hand, authority of the participating experts also reflects reliability of their estimates, and in turn can be quantified and formulated in terms of trustworthiness-degrees of the individual priorities. Finally, decision makers' self-confidence in the validity of their own statements represents the indeterminacy of the associated numeric priorities. In general, the more reliable are the individual values the higher is their impact on the representative group opinion. The ideal estimates reliability is characterized by the maximum possible truth-grades as well as minimum possible falsity- and indeterminacy-grades simultaneously. Such comprehensive formulation of priorities in GMCDA best matches with a model of discrete SVNSs with inherent to them triples of truth-, falsity- and indeterminacy-membership functions.

So far the scientific literature has not provided well-formalized analytical techniques for making operations on SVNSs and neutrosophic numbers needed for the handling of neutrosophic group values in accordance with the rules and procedures underlying compensatory MCDA methods. In Paper [I] is proposed a novel analytical approach for deriving representative group priorities that helps to overcome this methodological gap. First, all constructed ternary membership grades of SVNSs are reduced to single scalar numbers based on the Euclidean metric in three-dimensional space within neutrosophic cube. This step is followed by converting the resulting numbers into membership grades of the respective fuzzy priorities. Ultimately, the constructed fuzzy sets of group evaluations can be synthesized and analyzed using the method introduced in Paper [IV].

The developed method permits not only to effectively synthesize the information uncertain in a multivariate manner, but also to ensure transparent audit trail of subjective judgments credibility already on the stage following their elicitation. For this, the respective truth-, falsity- and indeterminacy-membership grades are mapped as points into the three-dimensional single-valued neutrosophic cube, which is divided into the areas of high, tolerable and unacceptable reliability. Depending on the positions of points within the cube, the stakeholders can be warned about lack of trust to their expressions and recognize the directions of judgments quality improvement.

The developed model and method enable representation, accumulation and analysis of several independent multi-source/multi-spectra imprecision metrics of expert judgments within one coherent GMCDA framework. This approach leads to substantial improvement in the accuracy and dependability of the decision analysis outcome and serves as a learning tool for the decision makers.

3.2. Contributions of Paper [II]

3.2.1. Theoretical contributions of Paper [II]

Paper [II] presents a new integrated method for the problem of supplier selection and order allocation (SSOA) in a Just-in-Time purchasing environment under multiple conflicting stakeholders' objectives. Important is that the model considers synergies of the alternative suppliers with respect to different qualitative and quantitative criteria. The main contributions are made in research areas (a) and (b) (see Section 3, page 12).

Primarily, Paper [II] extends the existing models of SSOA that utilize a combined MCDA and linear programming (LP) approach by providing a technique that enables analysis of positive and negative suppliers' performance synergies that can emerge in the case of multiple sourcing.

Currently existing approaches to SSOA enable ranking of individual suppliers using an appropriate MCDA formulation. An optimization procedure is usually applied for building a set of candidates satisfying the demand and imposed constraints, and ensuring the maximum total value of purchasing (TVP). However, the existing methodologies do not produce valid solutions if several suppliers selected in a combination exhibit an improved or degraded overall performance when compared to summarised individual performances. To date, the issue of taking into account the positive and negative synergy effects of suppliers occurring in the case of multiple sourcing has not been considered in MCDA context.

To prevent misleading results caused by suppliers' interactions, in Paper [II] it is proposed to classify decision criteria into *synergistic* and *non-synergistic* and treat them differently. Non-synergistic factors characterize alternatives as individual and independent units. Total performance of the alternatives selected in different combinations with respect to non-synergistic criteria equals to the sum of the individual performances of these alternatives. On the contrary, the aggregate alternatives' performance with respect to synergistic factors may differ from the sum of their individual performance measures if these alternatives are chosen to act together. During the decision process, all feasible supplier combinations are formed and undergo separate assessment with respect to synergistic and non-synergistic factors. Hereby, quantitative parameters are evaluated using objective data sources; qualitative assessments remain under the charge of responsible experts. Construction of multi-objective value functions for alternative combinations is followed by a series of LP procedures needed to efficiently allocate order quantities within each alternative. The optimized supplier sets are ranked based on their TVP values. The combination with maximum TVP represents the global optimum.

Another essential aspect addressed in Paper [II] concerns strategies to criteria identification. In MCDA, in order to create a full picture of the problem it is necessary to identify factors relevant for different stakeholders, which is a difficult task in practice. In Paper [II] it is argued that the efficacy of the criteria identification process can be enhanced by employing the value-focused thinking (VFT) approach in the interviews with organizational strategists. The alternative-focused thinking (AFT) should be utilized by purchasing and operational executives, as well as suppliers. The mixed VFT-AFT approach is most appropriate for ultimate customers. To summarize, practical value of any GMCDA model can be gained from deliberate deployment of VFT and AFT strategies.

Furthermore, the procedure of priority setting suggested in Paper [II] takes into account the limitations and advantages of direct scoring and the AHP concerning the number of elements to be ranked within one homogeneous cluster. The dedicated algorithm relies upon graph theory to represent dependencies between decision criteria as ordered rooted tree with variable depth and cardinality of its nodes. This kind of representation enables flexible and consequent assignment of subjective priorities in either linguistic or numerical terms during top-down tree traversal based on the ratio or interval scale, depending on the cluster size.

3.2.2. Practical implications of Paper [II]

In general, this research seeks to make a three-fold practical impact: to anchor the concept of suppliers' performance synergy in multi-sourcing SSOA problems, to shape the common flexible and integrated decision framework for all crop SSOA traders and to provide a reference set and a structure of decision criteria specific to the industry. This research was conducted in cooperation with Raiffeisen Westfalen Mitte e.G., one of the largest trading companies selling crops, animal feed, fertilizers and fuel oils in Germany.

The first implication concerns the identification, definition and structuring of the decision criteria relevant to the large crop traders operating within a Just-in-Time purchasing environment. Based on the analysis of scholar and business literature, as well as upon the interviews with Raiffeisen' representatives, a mass of conflicting views were generated, summarized and translated into a common value system for the company and its customers. The collective value system was modelled by means of rooted ordered tree-graph with four levels and 31 SSOA objectives on its leaves. The objectives are conflicting in nature, differ in sense of effect horizon, and transmit tangible and intangible information. The defined criteria and their structure can be easily adopted by the other firms of the branch.

The second practical contribution is much broader than the first one and relates to the method of modelling and capturing suppliers' synergies with respect to different qualitative and quantitative criteria in vendor selection and SSOA problems. The developed model and method of treating synergies add value to the general integrated framework which is confirmed by a study of a series of practical decisions at Raiffeisen taken with the decision aiding tool.

Under reasonable adjustments, the developed method can be adopted to support purchasing decisions in trading companies selling different kinds of goods or services.

3.3. Contributions of Paper [III]

3.3.1. Theoretical contributions of Paper [III]

Paper [III] focuses on research areas (a) and (b) (see Section 3, page 12).

On the one hand, Paper [III] extends the standard approach of Saaty to hierarchical structuring of decision elements that implies their arrangement at the levels of decision goals, alternative choices and evaluation criteria, by adding a fourth level containing auxiliary elements. This paper first introduces the notion of a*uxiliary decision objects* (ADOs) that relate to the alternatives but are described in terms of own performance attributes called *indirect criteria*. In a hierarchy, auxiliary elements should be placed at the level intermediate between the criteria and alternatives. Traditional MCDA structures cannot cope with ADOs. Paper [III] describes a modified procedure of synthesizing decision elements within the extended four-level hierarchy. Basically, handling of ADOs can be considered as a MCDA sub-problem, solution of which is similar to the ordinary alternatives evaluation. Crucial hereby is the methodology of transferring the data on the indirect criteria to the alternatives through ADOs.

On the other hand, the paper attempts to shed more light on the problem of responsibilities definition and tasks formalization within distributed expert groups during a MCDA process. The new Multilevel Group Decision Making framework first introduces notions of the α-, β- and γ-level DMs responsible for a value system establishment, alternatives assessment and ADOs evaluation, respectively. Experts can belong to either one or several task areas simultaneously. α-, β- and γ-voting power indices assigned to the individual DMs or communities depend on their competence and authority. α-voting power stands for the DM's impact on defining criteria importance. β-voting power reflects the DM's ability to estimate performance of alternatives on the subjective direct criteria. Similarly, γ-voting power reflects expert competences to evaluate the ADOs. β- and γ-voting power are vector quantities, because an individual's authority may be unequal with respect to various alternatives and ADOs. The three introduced indices are deployed for scaling the priorities obtained from subjective judgments.

The suggested 16-phase algorithm is appropriate for both multiple and single-sourcing decision strategies. The number of alternatives to choose must be defined explicitly. The real-life systems-analysis-based framework developed during the research reported in Paper [III] relies upon a consistent bulk of well-established methods aimed to improve decision effectiveness and transparency. Moreover, this framework serves as a tool for experts' learning and continuous decisions monitoring.

3.3.2. Practical implications of Paper [III]

The described case study was conducted in cooperation with Raiffeisen Westfalen Mitte e.G., complementary to the research reported in Paper [II]. One practical contribution is in the identification, definition and structuring of decision criteria relevant for Just-in-Time fuel oil acquisition by trading companies. Another contribution relates to the development of a systematic ready-to-use SSOA methodology capturing heterogeneous tangible and intangible data provided by the distributed, dynamic and multiple-tasking expert groups.

In the case study, the decision group was comprised of strategy managers, including one board member, purchasing executives and customers. Based on state-of-the-art literature and interactive facilitated workshops with DMs, the key factors of SSOA in fuel oils trading industry were first detected, defined and systematised. This set includes both direct and indirect criteria.

Whilst modelling the purchasing system at Raiffeisen, it was noticed that the three-level hierarchical structure of decision elements traditionally employed in hierarchical MCDA models is not sufficient for taking into account the third parties. The approach presented in Paper [III] allows incorporating characteristics of ADOs into suppliers' profiles. In the pilot study, a set of ADOs included loading stations in North Rhine-Westphalia having many-to-many relationships with suppliers. Alternative vendors underwent the evaluation process several times. The results were compared with real unaided decisions. The similarities and differences uncovered were analysed and a number of fundamental pitfalls of unaided decisions were reported to the company's management. Specialized software, including Super Decisions, Microsoft Excel and Matlab, were used to support the evaluation process.

With the developed decision support methodology, Raiffeisen is able to balance its needs against supplier strategic and tactical capabilities, and to ensure that demanded fuel oil can be procured cost-effectively through the most capable suppliers. The demonstrated approach can be easily adopted by other trading companies dealing with different types of acquired products.

3.4. Contributions of Paper [IV]

3.4.1. Theoretical contributions of Paper [IV]

In Paper [IV] contributions are made into research areas (a), (b) and (c) (see Section 3, page 12). Paper [IV] develops a GMCDA framework called *Fuzzy Euclid* that builds on the strengths of the weighed-sum MCDA *Euclid* model of Tavana (2002) while eliminating its weaknesses. Contrary to the Euclid model, Fuzzy Euclid captures informational uncertainty caused by differences in DMs' subjective assessments by utilizing concepts of fuzzy set theory. Fuzzy Euclid also provides a new methodology of visualization and interpretation of the decision analysis outcomes in a polar coordinate system.

More specifically, the novelty of *Fuzzy Euclid* with regard to research area (a) is in promoting an approach for improved ranking and classification of alternatives in MCDA. Rather than relying upon the Euclidean distance of alternatives from the ideal point in Cartesian space as implied by Tavana's Euclid, in Paper [IV] the advantage is gained from describing the synthesized alternatives performances in terms of circular coordinates in the space of opportunities and threats. Thus, not only alternatives characterized by smaller vectors from the ideal point should be chosen, but also those with smaller angles between these vectors and the axis of opportunities.

Moreover, in Paper [IV] it is proposed to classify the alternatives plotted on a plane based on their polar coordinates into the four following zones: Exploitation Zone represents most promising projects with little threats and great opportunities; Challenge Zone exhibits very risky projects with significant opportunities; Discretion Zone represents projects without meaningful potential but with little threats; and finally, Desperation Zone includes projects that demonstrate a great deal of threats and little potential.

Contributions of Paper [IV] to research areas (b) and (c) intersect. The Paper first represents and handles a group opinion comprised of individual estimates as an uncertain variable. The proposed decision analysis procedure implements fuzzy logic for project evaluation and selection. An integrated compensatory MCDA and defuzzification process leads to the fusion of importance weights of divisions and decision factors with subjective probabilities of factors occurrences into one set of crisp representative values for the entire group of DMs.

Another significant contribution of the presented research to GMCDA is in utilization of the fuzzy entropy by De Luca and Termini (1972) for calculating the level of experts' opinions discordance. Projects with smaller fuzzy entropy are preferred to those with a higher level of uncertainty reflecting DMs’ disagreement.

In the developed method, final prioritization of projects can be performed using their position in the partitioned decision space. Corrections in obtained rankings can be made by taking into account the fuzzy entropy measures. The proposed visualization scheme helps DMs to get better insight into the characteristics of each alternative project from different points of view.

3.4.2. Practical implications of Paper [IV]

This research was conducted within a project for advanced technologies assessment for shuttle management and operations at Kennedy Space Centre (KSC), National Aeronautic Space Administration (NASA) in the USA. The main data source that the methodology of Paper [IV] refers to is the survey summaries provided by the division chiefs for Safety, Reliability, and Operations at NASA-KSC.

The practical value of Fuzzy Euclid is three-fold. First, it provides a novel consistent, systematic and transparent framework for supporting advanced technology projects evaluation, prioritization and selection at KSC. Second, Fuzzy Euclid helps to determine optimal compromise solutions by considering conflicting qualitative and quantitative DMs’ objectives, as well as by measuring the level of disagreement between the experts involved in the decision process. Finally, along with precise mathematical analysis, the new visualization capabilities allow the DMs from KSC to immediately understand the potential ramifications of changing priorities of the evaluated projects and decision strategies.

Fuzzy Euclid enables evaluation of feasible advanced technology projects with anticipated expenditures against a large number of qualitative and quantitative key performance indicators, including both opportunities and threats, by the field experts of three KSC divisions. In case of dissimilarities in both the DMs’ subjective estimates of projects’ performance values and the judgments regarding importance of decision objectives, the representative rankings of the projects calculated using the developed integrated method of fuzzy MCDA can be revisited and refined. Furthermore, Fuzzy Euclid enables classification of all projects into four similar groups according to the relationships between their overall opportunities and threats. As a result, KSC can align its strategic objectives with the advanced project portfolio selection decisions in a time-efficient, cost-effective and conflict-mitigating manner.

3.5. Contributions of Paper [V]

The main contributions of Paper [V] relate to research area (a) (see Section 3, page 12). Paper [V] was written in collaboration with Professor Thomas Saaty – the creator of the Analytic Hierarchy Process (AHP) and the Analytic Network Process (ANP). Theoretical contribution of this paper is in providing the first overview of all fundamental concepts of the contemporary AHP theory and its generalisation to dependence and feedback, the ANP, within a single-article format.

Thousands of papers and hundreds of books have been written to address different aspects of the AHP and ANP. Paper [V] describes the core vision of the AHP and ANP theories from the creator's point of view in a compact but comprehensive way. This work explains how the AHP and ANP provide a structure and mathematics to elicit and quantify human judgments based on pairwise comparisons and derive priorities for intangible factors, as well as how to incorporate measurements for tangible criteria, with the aim to help individuals to take the best decision.

Paper [V] consequently represents the main theoretical ideas and processes of: (1) constructing the AHP/ANP-based multi-criteria decision hierarchies/networks, (2) ensuring homogeneity of decision elements within the hierarchy/network clusters, (3) eliciting individuals' pairwise comparison judgments, (4) testing consistency of subjective estimates, (5) ranking alternatives one at a time using the AHP-based absolute measurement; (6) synthesizing human judgments expressed using both the ratio and ranking scales within the entire hierarchy/network, in order to find global priorities of decision criteria and alternatives, and (7) analyzing sensitivity of the global priorities and final rankings to changes in the initial judgments.

Along with the basic AHP and ANP concepts, Paper [V] provides an overview of their extension to group decision making in two cases: first, when individuals' pairwise judgments are aggregated immediately after their elicitation, and second, when the construction of a group choice is made from individual choices. Substantial attention is paid to the analysis of Benefits, Opportunities, Costs and Risks within large decision networks. At the end, Paper [V] discusses the recognised issue of rank reversal and preservation. All theoretical aspects presented in Paper [V] are illustrated and validated using several examples from different application domains, including construction project management, defense, marketing and healthcare, among others.

Paper [V] exhibits the present-day condition of the AHP and ANP theories, and is the main starting point for future research in this direction.

3.6. Contributions of Paper [VI]

3.6.1. Theoretical contributions of Paper [VI]

Contributions of Paper [VI] are made into research areas (a), (b) and (c) (see Section 3, page 12). Paper [VI] proposes a new GMCDA model called *Soft SWOT* that builds upon the *Fuzzy Euclid* framework developed in Paper [IV]. *Soft SWOT* is conceptualized to support strategic decision and policy making by considering heterogeneous factual and expert information in a methodological way. The strategic analysis framework developed in this paper is based upon a combination of strengths (S) - weaknesses (W) – opportunities (O) - threats (T) analysis with a properly adapted *Fuzzy Euclid* method. The presented approach seeks to enhance the accuracy and feasibility of traditional SWOT by using up-to-date analytical concepts of fuzzy GMCDA.

Except supporting the evaluation of options' utility to a value-driven organization, *Soft SWOT* does help to recognize managerial measures toward the alternatives' progress in meeting organizational expectations. This is achieved through consistent integration of SWOT analysis with the main ideas and processes of *Fuzzy Euclid*. On the one hand, *Soft SWOT* ranks and classifies decision options that are discrete and finite in number based on multiple objectives that have to be either maximized or minimized, as implied in *Fuzzy Euclid*. On the other hand, *Soft SWOT* extends *Fuzzy Euclid* as it ascertains the potential of options under consideration to conform to organizational standards or goals by considering all key performance indicators in the view of their controllability.

The fundamental difference of *Soft SWOT* from *Fuzzy Euclid* is in reference to the four principal problem areas (S, W, O and T) instead of two (O and T). Methodologically, such extended formulation requires a new procedure for calculating global priorities of decision alternatives, because the aspects of criteria controllability have to be taken into account. Thus, after the integration of parameters referring to each addressed problem area, the standard SWOT plane with four quadrants is deployed instead of the plane in a polar coordinate system. Coordinates of points related to the alternatives on the partitioned SWOT chart are derived based on the pairs of balanced strengths and weaknesses, as well as opportunities and threats. The final rankings are obtained using three metrics: first, Cartesian distance of the alternative to the ideal point; second, SWOT quadrant the alternative belongs to and third, fuzzy entropy measure reflecting discordance of DMs' opinions within and across the groups.

3.6.2. Practical implications of Paper [VI]

This paper explains the ambiguous political approach used by the European Council for the evaluation of the candidates seeking membership in the European Union (EU) and presents the first structured system-analytical model of this approach in the form of a GMCDA framework. Given this, the main practical contribution of this paper is to propose a response to the need for a meaningful and robust analysis concerning a large number of competing and conflicting Copenhagen criteria, established by the European Council and to demonstrate advantages of the operationalized policy making approach.

More specifically, Paper [VI] makes two practical contributions. On the one hand, it presents a generic strategic decision support model for complex group multi-objective problems that embrace qualitative and quantitative internal strengths and weaknesses as well as external opportunities and threats of the alternatives that are discrete and finite in number. On the other hand, this paper proposes the first integrated analytical method that deals with the problem of the EU enlargement by decomposing the complexity associated with the evaluation of the candidates into manageable, consistent and transparent steps without overly simplifying the real process.

The key elements of the provided analysis are: (1) the measurement of the candidates' performance based on uncertain and hard to quantify information described verbally in the official candidate states' progress reports; (2) the classification of the 169 Copenhagen criteria into SWOT groups; (3) the uncovering of the Copenhagen criteria importance weights; (4) the quantification of cross-expert disagreement levels in subjective evaluations; (5) the calculation of the ratings for all candidates and potential candidates to the EU; and (6) the graphical representation of the obtained GMCDA results on a SWOT chart.

The pilot study was conducted at the University of Paderborn based on the data collected and analyzed by six interdisciplinary student teams in 2009. The findings suggested that Croatia is the most prospective candidate. In fact, Croatia finished accession negotiations in 2011 and is set to become an EU member in 2013. The other results were consistent with the official EU's classification of "candidate" and "potential candidate" states, which indicates the model validity. Practically, *Soft SWOT* promotes consistent and systematic strategic decision making throughout the organization. This research is likely to be of interest to a wide range of non-academic users, including both governmental departments and non-governmental organizations faced with a need to improve their practices of evaluating, prioritizing and planning multiple strategic alternatives in highly complex and conflicting group settings.

3.7. Contributions of Paper [VII]

3.7.1. Theoretical Contributions of Paper [VII]

Contributions of Paper [VII] are made into research areas (a)-(c) (see Section 3, page 12). Paper [VII] extends the *Fuzzy Euclid* GMCDA model presented in Paper [IV] in four important ways. First, it incorporates ambiguity of the individuals' assessments. Second, the developed model takes into account feedback dependencies between the decision criteria. Third, this paper first introduces the notion of a *transient factor* to treat criteria with variable impact direction. And fourth, Paper [VII] considers the case when each major alternative may be divided into independent sub-alternatives that have to be separately explored and properly combined.

The case when experts cannot provide estimates with confidence is considered. The numerical expressions associated with the subjective probabilities are vague and are not restricted to a single value or interval of values with sharp boundaries. Such expressions are frequently modelled using triangular fuzzy numbers. However, a problem arises when a single ambiguous judgment for the alternative on a criterion must be modelled for the entire group of DMs. Paper [VII] shows that this situation causes *g*-fuzzification and first proposes to use level-2 fuzzy sets to model the superposition of two potential fuzziness types: vague individual judgments (continuous fuzzy functions) and fuzzy group estimates or opinions (discrete fuzzy function). Furthermore, paper [VII] suggests a novel two-phase defuzzification algorithm to synthesize the dually ambiguous performance variables and to translate them into the overall representative crisp rankings of the alternatives.

The second extension of *Fuzzy Euclid* described in Paper [VII] is the capability to capture the interdependencies between the opportunities and threats by utilizing the Analytic Network Process (ANP). Important is that the presented model does not use ANP conventionally to determine the relative importance of each alternative in terms of the decision factors. Instead, subjective probabilities are used to capture the relative performance of each alternative in the form of fuzzy scores.

The third novelty of Paper [VII] involves the introduction of the notion of a *transient factor* to represent the criteria with positive or negative impact on the goal achievement, depending on the perception of the DM. Note that traditionally decision criteria are considered to be either positive or negative. Paper [VII] explains how to handle transient factors.

Finally, the fourth theoretical contribution of Paper [VII] lies in the possibility to analyze complex decision options comprised of discrete, independent and finite in number sub-alternatives within the integrated fuzzy GMCDA framework. The alternatives' complexity was not foreseen in *Fuzzy Euclid* and its other extensions and modifications.

Most of the utilized modeling, measurement techniques and concepts already exist, are widely recognized and have been implemented for solving a variety of decision problems. However, the systems-oriented analysis based on the presented integrated and operationalized framework is needed to treat real-world GMCDA problems more effectively and efficiently, since this approach utilizes advantages of the individual methods while avoiding their drawbacks.

3.7.2. Practical implications of Paper [VII]

The research presented in Paper [VII] has many practical implications for the next generation of decision support systems dealing with large-scale quantitative data, complex but finite in number discrete options, uncertain or qualitative opinions of multiple experts and intricate interdependencies among the problem parameters.

The usefulness of the proposed methodology is affirmed for practical application described in the case study section of this paper, namely pipeline route planning in the Caspian Sea region. The study was conducted for a multinational oil and natural gas producer established with the objective of the exploration, development, production, marketing and sales of crude oil and natural gas. Within the pilot study, five routes for transporting the oil and gas out of the Caspian Sea region to the world markets going through 14 countries were examined by multiple experts from all relevant points of view. An important practical contribution of the demonstrated approach and the case study is in the identification and analysis of a comprehensive set of the pipeline routes evaluation criteria. Altogether, 79 pipeline route selection factors were first identified and structured as a network with feedback dependencies. The environmental scanning process, investigation of the macro environment of each country and definition of all relevant opportunities, threats and transient evaluation factors related to the political (P), economical (E), socio-cultural (S), technological (T), environmental, legal and geographical impacts on the decision was enhanced by encapsulating the recognized PEST analysis template.

The presented model could be applied to different national and multinational organizations when assessing and planning transmission pipelines or other kinds of routes.

3.8. Contributions of Paper [VIII]

3.8.1. Theoretical contributions of Paper [VIII]

Contributions of Paper [VIII] are made into research areas (a) and (c) (see Section 3, page 12). More specifically, the intellectual contribution of Paper [VIII] can be seen along three dimensions. First and most broadly, it provides a new, integrated and operationalized system for benchmarking, policy making and performance management. The originality of this benchmarking system is in that it is: (1) suitable for large-scale complex objects (e.g. organizational units under consideration) with the plethora of performance indicators involved; (2) open not only to the traditionally tapped objective benefits and costs underpinning the objects' performance, but also to inter-disciplinary and qualitative expert opinions; (3) effectively a basis for communication and compromise finding between the conflicting stakeholders; (4) a modular methodology, with modules that are based on tried and tested methods, processes and practices.

Second contribution of Paper [VIII] is that a numeric measure called *survivability index* is introduced and used to help policy makers to identify the alternatives' strengths and weaknesses by learning from "best-in-class" and other competing alternatives on the hit list. The *survivability index* is used to identify each alternative as either *Efficient*, with high benefits and low costs; *Active*, with high benefits and high costs; *Inactive* with low benefits and low costs; and *Inefficient* with low benefits and high costs.

The theoretical basis for the third contribution of Paper [VIII] lies in the ambiguous nature of preference and performance information at an individual level, as well as uncertainty inherent to objectively characterized indicators. In contrast to the methodologies presented in other papers of this thesis, the framework presented in Paper [VIII] is able to incorporate vague performance of the alternatives on the objective indicators, such as potential costs or expected payback period. The uncertain quantitative values are represented by means of LR-type fuzzy sets. Each alternative is estimated one at a time with respect to each objective parameter, and by a set of DMs' triangular fuzzy scores with respect to each qualitative metric. All arithmetic operations on fuzzy sets provided within the developed framework rely upon Zadeh's Extension Principle (Zadeh, 1965). Another innovation of this method is that it first builds the aggregate benefits and costs relations based on the Cartesian product of fuzzy numbers. The main advantage of this approach is that it does not reduce objective and subjective informational uncertainty on the intermediate stages of the algorithm and carefully transmits it through the entire benchmarking process, which ensures high precision of the final results.

The framework presented in Paper [VIII] affords a measure to provide a detailed and multilateral performance analysis and helps to institutionalize complex benchmarking processes with many people involved. To the best of the author’s knowledge, no such performance management methodology exists to date.

3.8.2. Practical implications of Paper [VIII]

This research was supported by the US Naval Research Laboratory. Practical motivation for this research was the need for a sound theoretical framework to structure and model the decision-making process concerning the base realignment and closure (BRAC) at the Department of Defense (DoD). BRAC decisions are part of a national strategy and are adopted by the US Government. They intend to resolve the military, economic and political issue of excess base capacity. The US Congress has chartered the BRAC Commission to consider employment, environmental, financial, strategic, and tactical impacts of BRAC decisions.

This study was conducted at a naval facility in the USA with seven naval experts. The participating officers provided their expertise to identify factors and sub-factors that influence the BRAC decision. Based on this information the criteria network was constructed and criteria weights were identified using the Analytic Network Process. A total of 52 US military bases were examined with respect to a set of objectively and subjectively assessed criteria. The numerous legal, strategy, policy and planning documents were used to define the military value of the installations on the DoD hit list. The BRAC Commission utilized the *survivability indices* first presented in Paper [VIII] to arrive at a ranking for each of the military bases. The commanding officers of the military bases can use the proposed four-quadrant classification approach to understand the overall benefits and costs by learning from “best-in-class” and other competing bases. Thus, the introduced step-by-step methodology decomposes the complex BRAC policy making into manageable steps and enables systematic, transparent and institutionalized decision support.

This benchmarking method is flexible in that it can easily be adapted by other governmental or commercial organizations, for example, by substituting other factors from those listed here or adding additional factors. Alternatively, some modules of this framework can be changed or adjusted to other GMCDA problem formulations, making this model accessible to a wider range of situations in benchmarking, performance management and strategic planning.

4. Conclusions and future research directions

This thesis consists of eight separate papers that focus on investigation, development and application of models and methods to solve complex decision-making problems involving a finite set of discrete alternatives, multiple evaluation parameters, different active groups influencing the choice and imprecise parameter values.

The fundamental idea of this research is committed to utilizing the principles of systems analysis theory to: 1) describe practical decision problems and decompose them into manageable and interrelated sub-problems; 2) select the most appropriate models and methods to treat each sub-problem; and 3) design new coherent integrated decision frameworks based on properly combined individual methods.

Effectiveness of the developed decision support procedures is achieved by deliberate and meaningful composition of behavioural and analytical aspects. In particular: (a) Multi-criteria Decision Analysis is utilized to articulate decision makers' values and amalgamate them with interdisciplinary subjective and objective performance data; (b) theory of fuzzy sets is employed to consider non-probabilistic informational uncertainty; (c) group decision techniques are tailored to facilitate conflict resolution and compromise finding; and finally, (d) a number of methods from other disciplines are implemented to fully satisfy formulations and requirements of particular problems. Moreover, value of the introduced methods is added by new analytical capabilities gained from the integration of different methods.

However, since subjective human opinions are an integral component of the developed models, the effectiveness of produced solutions depends on the experts' cognitive abilities to provide sound judgments. Therefore, these methods should be used very carefully. And although this thesis provides a new way to measure reliability of subjective opinions, further research is needed in this direction.

Other important avenues for future research are related to the need to make more empirical studies and tests of the developed methods, as well as to extend and modify these models using constantly developing mathematical and decision-making-related theories for addressing miscellaneous practical problems.

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Group multiple criteria decision analysis based on single-valued neutrosophic sets

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Abstract

Neutrosophic set theory is a formal framework that has been recently proposed as a generalization of the ordinary, fuzzy and intuitionistic sets to deal with multi-source and multi-spectra imprecision of information. This paper introduces a new approach for aggregating opinions of multiple experts in multi-criteria problems and a neutrosophic environment. Priorities of the objects (criteria and alternatives) are represented using single-valued neutrosophic sets that reflect the degree of experts' estimates reliability based on the measures of voting power, judgment inconsistency or error, and the level of decision makers' self-confidence in the provided assessments. On the first step of the proposed algorithm each triple of the estimates truth-, falsity- and indeterminacy-membership grades is translated into a fuzzy membership grade using the three-dimensional Euclidean metric. Then, a representative crisp value is found for each fuzzy set of group estimates. Finally, all representative crisp data is synthesized across multiple criteria, and alternatives are prioritised. Accounting for independent multi-source measures of the estimates reliability assures maximum dependability of the decision outcome.

Keywords: Group multiple criteria decision analysis, Uncertainty modelling, Estimates reliability, Single-valued neutrosophic set, Fuzzy set.

1. Introduction

Group decision making is among the most important and frequently encountered processes within public and private sector. Most decision making problems in humanistic systems have multi-criteria and conflicting nature. In order to find a compromise solution the decision makers provide qualitative and quantitative evaluations of the potential options with respect to the relevant criteria, as well as estimate impacts of the criteria on the goal in the context of considered situation. The information expressed by different decision makers is often ambiguous and contradicting, which significantly complicates construction of knowledge-based rules and establishment of decision support procedures. Six ways in which such vagueness can occur are: 1) the words that are used in antecedents and consequents of evaluation rules can mean different things to different people (Mendel, 1999); 2) consequences obtained by polling a group of experts are often different for the same rule or statement because the experts are not necessarily in agreement (Liang and Mendel, 2000; Jelassi et al., 1990); 3) decision groups are often heterogeneous due to the different extent of its members' expertise, knowledge and experience (Ölçer and Odabasi, 2005); 4) expert estimates of criteria importance or performance of alternatives with respect to intangible parameters are not always consistent (Saaty and Vargas, 1984); 5) information provided by individuals is usually incomplete or ill-defined (Jain, 1977; Anagnostopoulos et al., 2008); and, 6) decision makers are not always confident about correctness of their own reasoning (Schanteau, 1992).

Uncertainty is an attribute of information (Zadeh, 1965). Presence of multiple different vague measures in the group multi-criteria decision making can be utterly sophisticated; problems associated with comprehensive modelling and rigorous handling of ambiguous information in group multiple criteria decision analysis (GMCDA) are still not adequately resolvable. In the sense of Ackoff (1974) the problem having ill-defined goals, ill-defined procedures or ill-defined data is a *mess*. Several theories emerged during the last 50 years that generalize probability and are more appropriate to the range of not-probabilistic information formats in which evidence about uncertainty appears include Chiquet's theory of capacities, random set theory, evidence theory, possibility theory, Walley's theory of imprecise probabilities, fuzzy set theory, rough set theory, intuitionistic set theory and neutrosophic set theory, among others (Hall and Anderson, 2002; Van Leekwijck and Kerre, 1999; Pawlak, 1982; Smarandache, 2005). The most commonly used methodology for representing and manipulating imprecise and uncertain information in multi-criteria decision systems is the theory of fuzzy sets (FSs). However, while focusing on the membership grade (i.e., truthfulness or possibility) of vague parameters or events, FSs fail to consider falsity and indeterminacy magnitudes of measured responses. In practical terms, the problem of projecting multi-source and multivariate group decisions uncertainty through mathematical models remains intractable even in terms of fuzzy theoretic sets. In the late 90s Atanassov introduced and developed the idea of intiotionistic fuzzy sets (IFS), intuitionistic logic and intuitionistic algebra that realize more complex mental constructs and semantic uncertainties. Additionally to the membership grade IFSs consider non-membership level. However, IFSs can not handle all uncertainty cases, particularly paradoxes. Neutrosophic set (NS) theory is the cutting edge concept first introduced by Florentin Smarandache in the late 90s and developed the 21 century. NSs generalize FSs and IFSs. Elements of NSs and their specific sub-class of single-valued neutrosophic sets (SVNSs) are characterized by the three independent membership magnitudes (falsity, truth and indeterminacy). Such formulation enables modelling of the most general ambiguity cases, including paradox.

In this paper is proposed a new approach to represent multi-source/multi-spectra uncertainty of estimates provided by various domain experts in multi-criteria decision making problems, and a methodology of integrating these measures within one decision support procedure with the aim to increase reliability, coherence and dependability of the produced outcome. The proposed formulation is based on the assumption of alternatives' performance independency, i.e. synergy effects do not occur with respect to the alternatives' joint performance. Non-linear dependencies between criteria, in sense of their importance for achievement of the overall problem objective, are not concerned in the proposed formulation. The following basic notation is adopted in the paper:

$\boldsymbol{DM} = \{\boldsymbol{DM}^1, \boldsymbol{DM}^2, ..., \boldsymbol{DM}^m, ..., \boldsymbol{DM}^M\}$ is the set of domain experts involved in the decision process;

$\boldsymbol{A} = \{\boldsymbol{A}_1, \boldsymbol{A}_2, ..., \boldsymbol{A}_i, ..., \boldsymbol{A}_I\}$ is the set of alternatives under consideration;

$\boldsymbol{C} = \{\boldsymbol{C}_1, \boldsymbol{C}_2, ..., \boldsymbol{C}_j, ..., \boldsymbol{C}_J\}$ is the set of criteria used for evaluating the alternatives.

Discussion of the assessment of the discrete alternatives' overall priorities will be made using the technology of SVNSs and FSs in relation to the following problems:

Problem 1. Let $\boldsymbol{I}$ alternatives be given and let $\boldsymbol{M}$ experts have to estimate the alternatives with respect to $\boldsymbol{J}$ criteria. Synthesize multi-group multi-person expert judgments to yield the overall rankings of the criteria and alternatives.

Problem 2. Let the committee of $\boldsymbol{M}$ experts be heterogeneous. Let experts' estimates of the objects ($\boldsymbol{I}$ alternatives and $\boldsymbol{J}$ criteria) be affected by the three following facts: first, the experts have different credibility (i.e., voting power); second, the local object priorities that are derived using relative comparison judgments are characterized by an inconsistency or an error measure; third, due to the lack of information or scare experience some experts feel not confident about their own judgments. Incorporate these diverse uncertainty metrics into a coherent ranking model in order to increase dependability of the group decision outcome.

The paper is structured as follows: in section 2 the general formulation of group multi-criteria decision problem is given, in section 3 some basic definitions and concepts of fuzzy, intuitionistic and neutrosophic sets are described, section 4 presents a novel method of GMCDA in neutrosophic environment. An illustrative example of the proposed methodology is given in section 5. Finally, section 6 provides conclusions.

2. Group multi-criteria decision support

Complex problem solving is associated with gathering of interest groups or experts to discuss the critical issues, such as for conflict resolution, for planning and design, for policy formation or for plan brainstorming (Ragade, 1976; Turban, 1988). Multi-criteria decision-making methods are an important set of tools for addressing challenging business decisions because they allow the manager to better proceed in the face of uncertainty, complexity, and conflicting objectives (Hahn, 2003). Group multi-criteria decision support systems emerged in the late 80s. They rely on the four elementary stages (Matsatsinis and Samaras, 2001): 1) an initialisation stage, where the general rules of the process to follow are determined; 2) a preference elicitation stage, where each individual DM expresses her/his estimates of criteria and alternatives local weights; 3) A group preference aggregation stage, where an analytical and synthesizing mechanism is used in order to derive a tentative collective decision; and, 4) A conflict-resolution stage, where an effort to reach consensus or at least attempt to reduce the amount of conflict is performed.

Let there be $\boldsymbol{I}$ discrete alternatives available for the selection, $\boldsymbol{J}$ intangible evaluation criteria, and $\boldsymbol{M}$ experts are responsible for assigning importance weights to the criteria, as well as for estimating performance of the alternatives with respect to the criteria using a methodology that relies on the comparison principle, such as the Analytic Hierarchy Process (AHP) (Saaty and Vargas, 1984) or the Analytic Network Process (ANP) (Saaty, 2006). Each expert $\boldsymbol{DM^m}$ constructs square $\boldsymbol{n \times n}$ matrices $\boldsymbol{B^m}$ of pair-wise comparison ratios for the set of criteria and alternatives as shown below:

$$\boldsymbol{B^m} = \begin{pmatrix} \mathbf{1} & \frac{w_1^m}{w_2^m} & \dots & \frac{w_1^m}{w_l^m} & \dots & \frac{w_1^m}{w_n^m} \\ \frac{w_2^m}{w_1^m} & \mathbf{1} & \dots & \frac{w_2^m}{w_l^m} & \dots & \frac{w_2^m}{w_n^m} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \frac{w_k^m}{w_1^m} & \frac{w_k^m}{w_2^m} & \dots & \frac{w_k^m}{w_l^m} & \dots & \frac{w_k^m}{w_n^m} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \frac{w_N^m}{w_1^m} & \frac{w_N^m}{w_2^m} & \dots & \frac{w_N^m}{w_l^m} & \dots & \mathbf{1} \end{pmatrix} \qquad (1)$$

where $\boldsymbol{w_k^m}$ is priority of the $\boldsymbol{k}$ -th element (criterion or alternative) with respect to the higher level element estimated by the $\boldsymbol{m}$ -th expert. For the set of alternatives $\boldsymbol{n = I}$, and for the set of criteria $\boldsymbol{n = J}$.

After applying one of the existing relative measurement methods (Grzybowski, 2012), eigenvectors of ordered elements $\boldsymbol{w_k^m}$ are derived for matrices $\boldsymbol{B^m}$. $\boldsymbol{\overline{W}^m = (w_1^m, w_2^m, ..., w_k^m, ..., w_l^m, ... w_n^m)^T}$ are transposed vectors of normalized priorities. $\boldsymbol{W^m = \{w_k^m\} (k = 1,..., n)}$ are the sets of priorities of the elements for the $\boldsymbol{m}$ -th expert. $\boldsymbol{W_k = \{w_k^m\}\ (m = 1,..., M)}$ are the sets of derived experts' priorities of the $\boldsymbol{k}$ -th element. Moreover, each comparison matrix must undergo consistency check in order to confirm transitivity and reciprocity of the judgment data. Homogeneity of the compared elements is another sufficient condition needed to assure good consistency (Saaty and Sodenkamp, 2009). For the detailed discussion of the relative measurement consistency issues an interested reader is referred to Grošelj and Stirn (2012), Ishizaka and Lusti (2004), Cavallo and D'Apuzzo (2009), Saaty (2003), Saaty (2006) and Grzybowski (2012). Accordingly to Saaty (1980) the inconsistency measure of the comparison matrices $\boldsymbol{B^m}$ is defined as the consistency ratio

($\boldsymbol{C.R.}$): $\boldsymbol{C.R.}_{B^m} = \dfrac{\boldsymbol{C.I.}_{B^m}}{\boldsymbol{R.I.}_n}$, where $\boldsymbol{C.I.}_{B^m} = \dfrac{\lambda_{B^m,\max} - n}{n-1}$ is the consistency index; $\boldsymbol{R.I.}_n$ is the random inconsistency; $\lambda_{B^m,\max}$ is the largest eigenvalue of B^m. In general, $\boldsymbol{C.R.}$ should be 10% or less. In some cases the $\boldsymbol{C.R.}$ is required to be less than 8% (for $n = 4$) or 5% (for $n = 3$). Otherwise, inference quality should be improved. The higher $\boldsymbol{C.R.}$ is, the less consistent and plausible expert evaluations are (Kuo et al., 2006).

The set of alternatives' global priorities W^G can be elicited by synthesizing individual priorities w_k^m. To accomplish this, Azcel and Alsina (1986) suggest applying the weighed geometric mean method. Tavana and Sodenkamp (2010) applied FSs for modeling and evaluation of group judgments. Once all synthesized values w_k are revealed, the global priorities of alternatives ($w_i^G \in W^G$) can be derived using the weighed-sum approach:

$$w_i^G = \sum_{j=1}^{J} w_j \cdot w_i^j \ , \ \forall i = 1,\dots, I \qquad (2)$$

The higher the weight w_i^G, the more preferable the alternative A_i. For the best alternative ($A*$) is valid: $A* \mid w_{A*}^G = \max_{\forall i=1,..,I} [w_i^G]$.

3. Modelling uncertainties with fuzzy, intuitionistic and neutrosophic sets

There are two main types of uncertainties: external and internal. First, *external (or stochastic) uncertainty* implies that the events or statements are well defined, but the state of the system or environmental conditions lying beyond the control of the decision maker might not be known completely. Second, *internal uncertainty (or fuzziness)* refers to the vagueness concerning the description of the semantic meaning of the event, phenomena, or statements themselves, including uncertainties about decision maker preferences, imprecise judgments and ambiguity of information (Zimmermann, 2001; Durbach and Stewart, 2012). Zadeh (1973, p.28) wrote: "As the complexity of a system increases, our ability to make precise and yet significant statements about its behaviour diminishes until a threshold is reached when precision and significance (relevance) become almost mutually exclusive characteristics." Therefore, precise quantitative analysis is not likely to have much relevance of problems which involve humans either as individuals or in groups. In the rest of this section the foundations and differences of the most significant formal theories dealing with vague and imprecise information (primarily of the "internal" type) in humanistic decision making systems are discussed. Furthermore, an overview of the applications of these theories to group multi-criteria decision support problems is made.

3.1. Fuzzy set

FS theory was first introduced by Zadeh in 1965 in order to formalize the gradedness in class membership, in connection with the representation of human knowledge ("linguistic" uncertainty) and uncertainty about facts (Coletti and Scozzafava, 2004). Since then, FS have been relevant in the three types of information-driven tasks: decision-making problems, classification and data analysis, and approximate reasoning, for measuring degrees of similarity, preference and uncertainty (Dubois and Prade, 1997). FSs generalize ordinary (crisp, non-ambiguous) sets as an element can partially belong to the set where the membership function defines actually the degree of "belonging".

Definition 1 (Fuzzy set). Let X be a universal space of points (objects), with a generic element of X denoted by x. Thus, $X = \{x\}$. A ***fuzzy set*** $\tilde{\boldsymbol{E}} \subset \boldsymbol{X}$ is characterized by a membership function $\boldsymbol{\mu}_{\tilde{E}}(\boldsymbol{x})$ which associates with each point in X a real number in the unit interval [0, 1], with the value of $\boldsymbol{\mu}_{\tilde{E}}(\boldsymbol{x}) \in \mathfrak{R}$ at x representing the "grade of membership" of x in $\tilde{\boldsymbol{E}}$ (Zadeh, 1965; Leekwijck and Kerre, 1999).

General continuous FS has the following view: $\tilde{\boldsymbol{E}} = \int\limits_{\boldsymbol{x}} \mu_{\tilde{E}}(\boldsymbol{x})/\boldsymbol{x}, \boldsymbol{x} \in \boldsymbol{X}$. Finite FS can be presented by the ordered pairs: $\tilde{\boldsymbol{E}} = \{x_1 | \mu_{\tilde{E}}(x_1), x_2 | \mu_{\tilde{E}}(x_2), ..., x_M | \mu_{\tilde{E}}(x_M)\}$, $\forall x \in X$ where $\boldsymbol{M}$ denote the number of elements in $\tilde{\boldsymbol{E}}$.

Property 1. $\int\limits_{x \in X} \mu_{\tilde{E}}(x)dx$ is non-negative and can be smaller or larger than unit. In discrete case, $\boldsymbol{0} \leq \sum\limits_{m=1}^{M} \mu_{\tilde{E}}(\boldsymbol{x}) \leq \boldsymbol{M}$. Mathematically, this property signifies the fundamental difference between the membership grade $\boldsymbol{\mu}_{\tilde{E}}(\boldsymbol{x})$ (sometimes also called possibility) and statistical probability $\boldsymbol{p}_E(\boldsymbol{x})$ of the element or event $\boldsymbol{x} \in \boldsymbol{X}$, since $\int\limits_{x \in X} p_E(x)dx = \boldsymbol{1}$.

In the context of human reasoning (particularly, decision making), uncertainty is seen as possibilistic (i.e. fuzzy) rather than probabilistic one. Statistical data are handled in probabilistic terms, whilst imprecise measurements concerning expert judgments are primarily handled by means of FSs or their generalizations.

Property 2. $\mu_{\tilde{E}}(x) = \begin{cases} \mathbf{1}, \text{ if } x \in \tilde{E}, \\ \mathbf{0}, \text{ if } x \notin \tilde{E}; \end{cases}$ means that the element $x \in X$ completely belongs to the FS $\tilde{E}$ if and only if $\mu_{\tilde{E}}(x) = 1$ (x is ***kernel*** of $\tilde{E}$), and the element $x \in X$ does not belong to $\tilde{E}$ if and only if $\mu_{\tilde{E}}(x) = 0$. For $\forall x \in X$ if $0 < \mu_{\tilde{E}}(x) < 1$ then x partially belongs to $\tilde{E}$. If [$\mu_{\tilde{E}}(x) = 0$ or $\mu_{\tilde{E}}(x) = 1$] for $\forall x \in X$ then E is an ordinary (crisp) set.

Definition 2 (Defuzzification). ***Defuzzification*** is a process of mapping a FS $\tilde{E}$ into a single crisp output $e^* \in X$. If $\tilde{E}$ is discrete then vector of pairs $\{(x, \mu_{\tilde{E}}(x)) | x \in X\}$ is reduced to a single scalar quantity e^*, which is a representative value of $\tilde{E}$. As a result of defuzzification the obtained non-fuzzy (crisp) value the best represents the possibility distribution of an inferred fuzzy action so that certain concepts become clear, certain goals and constraints are considered more relevant (Sethukkarasi and Kannan, 2012; Ragade, 1976).

Leekwijck and Kerre (1999) made a survey of the existing defuzzification techniques and classified them into the three classes: the maxima methods, the distribution methods, and the area methods. The best known and a highly practical defuzzification operator is center of gravity (COG):

$$COG(\tilde{E}) = \frac{\int_{x \in X} x \cdot \mu_{\tilde{E}}(x) dx}{\int_{x \in X} \mu_{\tilde{E}}(x) dx}; \qquad COG(\tilde{E}) = \frac{\sum_{x \in X} x \cdot \mu_{\tilde{E}}(x)}{\sum_{x \in X} \mu_{\tilde{E}}(x)} \tag{3}$$

(for continuous sets) **(for discrete sets)**

Centroid is a general distribution method which computes the center of gravity of the area under the membership function. It can only be used for FSs on $\mathfrak{R}^4$.

FSs are widely used in describing intangible information because they can effectively represent the gradual changes of people's recognition to a concept in a certain context (Dalalah et al., 2011). During the last decades FSs have been used by dozens of researches and practitioneries for representing uncertain preferences of the individual decision makers and rules of their aggregation based on fuzzy logic and fuzzy algebra. Recently, Wang and Lin (2003) proposed to apply the concept of fuzzy majority in the group MCDA for determining the group acceptability of the final rankings. Tavana and Sodenkamp (2010) represented estimates of multiple experts in heterogeneous teams using FSs where the degree of agreement was measured by De Luca and Termini's fuzzy entropy.

3.2. Intuitionistic set

IFSs (Atanassov, 1999) extend FSs as for each imprecise value, event or function along with a membership grade is defined a corresponding not-membership. Important is that both membership and not-membership grades are interconnected and that IFSs can not handle indeterminate information.

Definition 3 (Intuitionistic fuzzy set). Let a set $\boldsymbol{X}$ be fixed. An ***intuitionistic fuzzy set*** $\tilde{\tilde{\boldsymbol{E}}}$ in $\boldsymbol{X}$ is an object of the following form: $\tilde{\tilde{\boldsymbol{E}}} = \{\boldsymbol{x}, \mu_{\tilde{\tilde{E}}}(\boldsymbol{x}), \boldsymbol{v}_{\tilde{\tilde{E}}}(\boldsymbol{x}) \mid \boldsymbol{x} \in \boldsymbol{X})\}$, where the functions $\mu_{\tilde{\tilde{E}}}(\boldsymbol{x}):\ \boldsymbol{X} \to [0,1]$ and $\boldsymbol{v}_{\tilde{\tilde{E}}}(\boldsymbol{x}):\ \boldsymbol{X} \to [0,1]$ determine the degree of membership and the degree of not-membership of the element $\boldsymbol{x} \in \boldsymbol{X}$, respectively (Atanassov et al., 2005).

Property 3. For $\forall \boldsymbol{x} \in \boldsymbol{X}$ on IFS $\tilde{\tilde{\boldsymbol{E}}}$ is valid: $\boldsymbol{0} \le \mu_{\tilde{\tilde{E}}}(\boldsymbol{x}) + \boldsymbol{v}_{\tilde{\tilde{E}}}(\boldsymbol{x}) \le \boldsymbol{1}$. Indeterminacy is defined as $\boldsymbol{1} - \mu_{\tilde{\tilde{E}}}(\boldsymbol{x}) - \boldsymbol{v}_{\tilde{\tilde{E}}}(\boldsymbol{x})$ by default.

Atanassov et al. (2005) demonstrated an application of IFSs to multi-criteria multi-person and multi-measurement decision making. They assigned sets of two-valued reliability scores to the experts in the following interpretation: the relative number of correct estimates of the alternatives with respect to the given attributes provided in the past corresponded to the membership value, whereas the relative number of falsely made prognoses was reflected by the non-membership grade.

3.3. Neutrosophic set

Netrosophic set (NS) is a formal framework proposed by Smarandache in 1999. NS is a part of neutrosophy which studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra (Smarandache, 1999). It generalizes the concept of the ordinary set, FS and IFS. In contrast to IFS, in NS, indeterminacy is quantified explicitly, and truth-membership, falsity-membership and indeterminacy-membership are independent. This assumption is very important in a lot of situations such as information fusion when we try to combine the data from different, possibly conflicting sources, e.g. sensors (Vsanda Kandasamy, 2003; Smarandache et al., 2005; Wang et al., 2010).

Definition 4 (Neutrosophic set). Let X be a universal space of points (objects), with a generic element of X denoted by x. A ***neutrosophic set*** $\mathcal{N} \subset X$ is characterized by a truth-membership function $T_{\mathcal{N}}(x)$, a falsity-membership function $F_{\mathcal{N}}(x)$ and an indeterminacy-membership function $I_{\mathcal{N}}(x)$. $T_{\mathcal{N}}(x)$, $F_{\mathcal{N}}(x)$ and $I_{\mathcal{N}}(x)$ are real standard or non-standard subsets of $]0^-,1^+[$, so that neutrosophic components $T_{\mathcal{N}}(x) \to\ ]0^-,1^+[$, $F_{\mathcal{N}}(x) \to\ ]0^-,1^+[$ and $I_{\mathcal{N}}(x) \to\ ]0^-,1^+[$. The set $I_{\mathcal{N}}(x)$ may represent not only indeterminacy but also vagueness, uncertainty, imprecision, error, contradiction, undefined, unknown, incompleteness, redundancy etc. (Rivieccio, 2008; Ghaderi et al., 2012). Moreover, the indeterminacy $I_{\mathcal{N}}(x)$ can be split into subcomponents, such as “contradiction”, “uncertainty”, “unknown” etc., in order to better catch vague information (Smarandache, 2005).

Property 4. Since $T_{\mathcal{N}}(x)$, $F_{\mathcal{N}}(x)$ and $I_{\mathcal{N}}(x)$ are independent, $0^- \leq \sup T_{\mathcal{N}}(x) + \sup F_{\mathcal{N}}(x) + \sup I_{\mathcal{N}}(x) \leq 3^+$ (Wang et al., 2010).

Definition 5 (Single-valued neutrosophic set). Let X be a universal space of points (objects), with a generic element of X denoted by x. A ***single-valued neutrosophic set*** (SVNS) $\tilde{\mathcal{N}} \subset X$ is characterized by a truth-membership function $T_{\tilde{\mathcal{N}}}(x)$, a falsity-membership function $F_{\tilde{\mathcal{N}}}(x)$ and an indeterminacy-membership function $I_{\tilde{\mathcal{N}}}(x)$ with $T_{\tilde{\mathcal{N}}}(x), F_{\tilde{\mathcal{N}}}(x), I_{\tilde{\mathcal{N}}}(x) \in [0,1]$ for $\forall x \in X$.

SVNS is an instance of NS proposed by Wang et al. in 2010 which can be used in real scientific and engineering application.

Property 5. In a SVNS $\tilde{\mathcal{N}}$ $0 \leq \sup T_{\tilde{\mathcal{N}}}(x) + \sup F_{\tilde{\mathcal{N}}}(x) + \sup I_{\tilde{\mathcal{N}}}(x) \leq 3$ for $\forall x \in X$.

When X is continuous, a SVNS $\tilde{\mathcal{N}}$ can be written as

$$\tilde{\mathcal{N}} = \int_{x} \left\langle T_{\tilde{\mathcal{N}}}(x), F_{\tilde{\mathcal{N}}}(x), I_{\tilde{\mathcal{N}}}(x) \right\rangle / x, \forall x \in X .$$

When X is discrete, a SVNS $\tilde{\mathcal{N}}$ can be written as

$$\tilde{\mathcal{N}} = \sum_{m=1}^{M} \left\langle T_{\tilde{\mathcal{N}}}(x), F_{\tilde{\mathcal{N}}}(x), I_{\tilde{\mathcal{N}}}(x) \right\rangle / x, \forall x \in X .$$

By analogy with FS, general SVNS has the following view: $\tilde{\mathcal{N}} = \{(x \mid T_{\tilde{\mathcal{N}}}(x), F_{\tilde{\mathcal{N}}}(x), I_{\tilde{\mathcal{N}}}(x)) \big| x \in X\}$. Finite SVNS set can be presented by the ordered tetrads:

$$\tilde{\mathcal{N}} = \{x_1 /(T_{\tilde{\mathcal{N}}}(x_1), F_{\tilde{\mathcal{N}}}(x_1), I_{\tilde{\mathcal{N}}}(x_1)),..., x_M /(T_{\tilde{\mathcal{N}}}(x_M), F_{\tilde{\mathcal{N}}}(x_M), I_{\tilde{\mathcal{N}}}(x_M))\},\ \forall x \in X$$

A unique feature of neutrosophy is that it can be used for modeling paradoxes. “The paradox is the only proposition true and false in the same time in the same world, and indeterminate as well” (Schumann and Smarandache, 2007, p.13).

Definition 6 (Deneutrosophication). We define ***deneutrosophication*** of a SVNS $\tilde{\mathcal{N}}$ as a process of mapping $\tilde{\mathcal{N}}$ into a single crisp output $\eta^* \in X$. If $\tilde{\mathcal{N}}$ is discrete then vector of tetrads $\{(x \mid T_{\tilde{\mathcal{N}}}(x), F_{\tilde{\mathcal{N}}}(x), I_{\tilde{\mathcal{N}}}(x)) \big| x \in X\}$ is reduced to a single scalar quantity η^*, which is a representative value of $\tilde{\mathcal{N}}$. As a result of deneutrosophication the obtained crisp value the best represents the aggregate distribution of three membership measures $\langle T_{\tilde{\mathcal{N}}}(x), F_{\tilde{\mathcal{N}}}(x), I_{\tilde{\mathcal{N}}}(x) \rangle$ of an inferred neutrosophic element.

A number of NS applications have been developed during the last ten years. Ansari et al. (2011) applied NS and neutrosophic inference to knowledge based systems in medical domain. NSs proofed to be an effective concept for image segmentation problems (Sengur and Guo, 2011; Cheng et al., 2011; Zhang et al., 2010), for integrating Geographic Information System data (Kraipeerapun et al., 2005), for binary classification problems (Kraipeerapun and Fung, 2009), among others. However, application of the NSs to GMCDA has not been presented in literature yet. In the next section is described a SVNS-based approach for modelling and account for multi-source/multi-spectra uncertainties in GMCDA problems. The GMCDA problem formulation in terms of SVNSs enable capturing semantic complexity of uncertainties that reflect reliability of experts’ estimates, and contribute to the improvement of quality and dependability of the produced outcome.

4. The proposed NS-based GMCDA approach

The following notation is adopted for referring Problem 2 stated in section 1:

δ_k^m is the credibility (or voting power) of the m-th expert in assessing priority of the k-th element (criterion or alternative);

$\boldsymbol{\varepsilon_k^m}$ is the measure of inconsistency (or error) of the $\boldsymbol{m}$ -th expert's assessment of the $\boldsymbol{k}$ -th element;

$\boldsymbol{\theta_k^m}$ is the level of the $\boldsymbol{m}$ -th expert's confidence in her/his estimates of the $\boldsymbol{k}$ -th element.

Confidence is a measure of experts' beliefs about their own appraisals, judgments, skills and abilities to evaluate the criteria importance and performance of the alternatives with respect to the given soft criteria. Reliability of each individual estimate can be expressed by a triad of independent magnitudes: $\left\langle \boldsymbol{\delta_k^m}, \boldsymbol{\varepsilon_k^m}, \boldsymbol{\theta_k^m} \right\rangle$. Until recently, modeling and handling of independent multi-source uncertainties inherent to a single information unit was challenging due to the lack of appropriate formal tools. With development of the NS and SVNS concepts the problem of simultaneous handling different ambiguity indicators of one variable can be resolved by converting values $\boldsymbol{\delta_k^m}$, $\boldsymbol{\varepsilon_k^m}$ and $\boldsymbol{\theta_k^m}$ into the truth-, falsity- and indeterminacy-membership grades of the corresponding priorities $\boldsymbol{w_k^m}$. Thus, discrete SVNSs of $\boldsymbol{M}$ elements have the following view:

$$\tilde{\mathcal{N}}_k = \{(w_k^m \mid T_{\tilde{\mathcal{N}}_k}(w_k^m), F_{\tilde{\mathcal{N}}_k}(w_k^m), I_{\tilde{\mathcal{N}}_k}(w_k^m)) \big| w_k^m \in \bar{w}^m\} \tag{4}$$

where

$$\begin{cases} T_{\tilde{\mathcal{N}}_k}(w_k^m) = f(\delta_k^m), \\ F_{\tilde{\mathcal{N}}_k}(w_k^m) = f(\varepsilon_k^m), \quad \textbf{for } \forall w_k^m \in \overline{W}^m \\ I_{\tilde{\mathcal{N}}_k}(w_k^m) = f(\theta_k^m), \end{cases} \tag{5}$$

and $\left\langle T_{\tilde{\mathcal{N}}_k}(w_k^m), F_{\tilde{\mathcal{N}}_k}(w_k^m), I_{\tilde{\mathcal{N}}_k}(w_k^m) \right\rangle \in [0,1]^3$ is mapping of $\boldsymbol{w_k^m}$ into the neutrosophic space.

$\boldsymbol{\varepsilon_k^m = 0}$ means that the $\boldsymbol{m}$ -th expert is fully consistent or unerring in her/his judgments regarding the relative weight of the $\boldsymbol{k}$ -th element. For priorities computation based on the right eigenvalue method employed in the AHP, the judgments are fully consistent if and only if $\boldsymbol{C.R.(B^m) = 0}$. In this case, the neutrosophic falsity-grade of respective $\boldsymbol{w_k^m}$ is zero: $\boldsymbol{F_{\tilde{\mathcal{N}}_k}(w_k^m) = 0}$. In the AHP / ANP inconsistency exceeding 10% is not acceptable, therefore all priority vectors derived for inconsistent matrices are not relevant: if $\boldsymbol{C.R.(W^m) \geq 0.1}$ then the falsity-grade is unit: $\boldsymbol{F_{\tilde{\mathcal{N}}_k}(w_k^m) = 1}$. If inconsistency $\boldsymbol{0 < C.R.(W^m) < 0.1}$ then the respective estimates are relevant for consideration in the decision making process, however the closer the inconsistency is to 10%, the less credible the values w_k^m are, so their falsity-grade grows.

Neutrosophic reliabilities given by (5) can be rewritten as follows:

$$\begin{cases} T_{\tilde{\mathcal{N}}_k}(w_k^m) = \dfrac{VP_k^m}{\max(VP_k)}, \\ F_{\tilde{\mathcal{N}}_k}(w_k^m) = (\dfrac{\varepsilon_k^m}{\varepsilon'_k} \text{ if } \varepsilon_k^m \le \varepsilon'_k) \cup (1 \text{ if } \varepsilon_k^m > \varepsilon'_k), \\ I_{\tilde{\mathcal{N}}_k}(w_k^m) = \dfrac{\max(SC_k) - SC_k^m}{\max(SC_k)}, \end{cases} \tag{6}$$

where VP_k $(\max(VP_k))$ is the (maximum possible) expert voting power for estimating weight of the k-th element; ε'_k is the maximum acceptable inconsistency/error in the assessments regarding priority of the k-th element; SC_k^m $(\max(SC_k^m))$ is the (maximum possible) level of the m-th expert's self-confidence degree in estimating priority of the k-th element.

Figure 1 illustrates the single-valed neutrosophic cube (SVNC) $Q = Q_1Q_2Q_3Q_4Q_5Q_6Q_7Q_8$ for the problem of expert judgment reliability estimation in the group decision making. The general neutrosophic cube was first introduced by Dezert in 2002.

The areas of estimates reliability in Q are: unacceptable, high and tolerable.

Definition 7 (Unacceptable neutrosophic estimates reliability). The area of ***unacceptable neutrosophic estimates reliability*** χ is represented by the three sides of Q: $\chi_1 = Q_2Q_6Q_7Q_3$, $\chi_2 = Q_5Q_6Q_7Q_8$ and $\chi_3 = Q_3Q_4Q_8Q_7$; $\chi = \chi_1 \cup \chi_2 \cup \chi_3$. In this area, the estimates w_k^m are characterized by 0% truth-, 100% falsity- and 100% indeterminacy-degrees respectively. $\mathcal{W}_k$ is a subset of set W_k that includes all estimates w_k^m with zero truth-membership, unit falsity-membership or unit indeterminacy-membership:

$$\mathcal{W}_k = \{w_k^m \in W_k \,\big|\, [T_{\tilde{\mathcal{N}}_k}(w_k^m) = 0] \wedge [F_{\tilde{\mathcal{N}}_k}(w_k^m) = 1] \wedge [I_{\tilde{\mathcal{N}}_k}(w_k^m) = 1\}.$$

$\forall w_k^m \in \mathcal{W}_k$ should be excluded from the decision process.

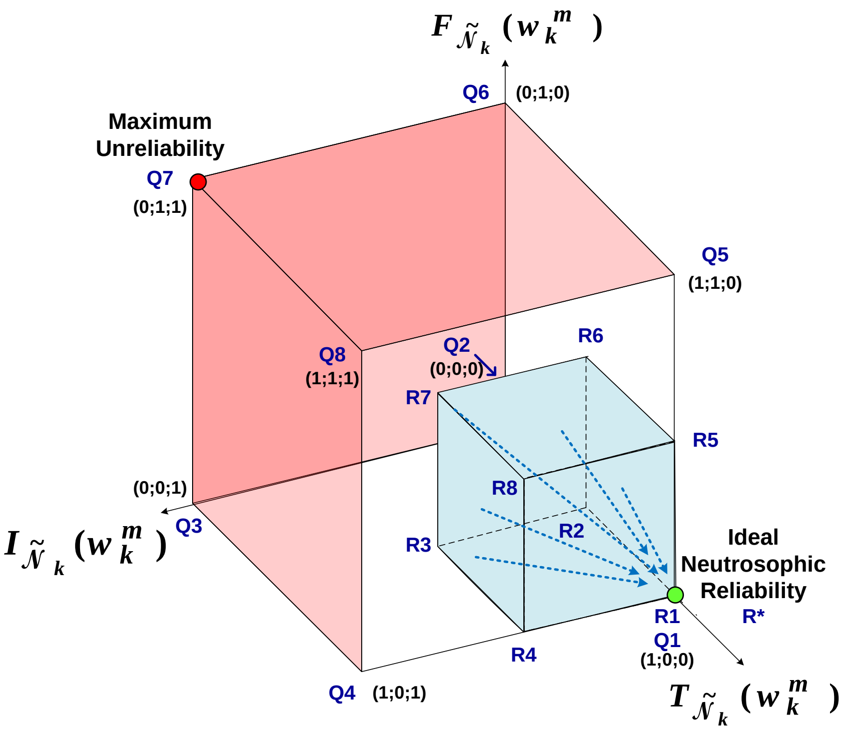

Figure 1: Cube of neutrosophic membership grades reflecting estimates reliability

Definition 8 (High neutrosophic estimates reliability). The sub-cube $\boldsymbol{R} \subset \boldsymbol{Q}$ represents the area of ***high neutrosophic estimates reliability***. Vertices of $\boldsymbol{R}$ are defined as $\boldsymbol{R_1 = (1;0;0)}$, $\boldsymbol{R_2 = (0,5;0;0,5)}$, $\boldsymbol{R_3 = (0,5;0;0,5)}$, $\boldsymbol{R_4 = (1;0;0,5)}$, $\boldsymbol{R_5 = (1;0,5;0)}$, $\boldsymbol{R_6 = (0,5;0,5;0)}$, $\boldsymbol{R_7 = (0,5;0,5;0,5)}$ and $\boldsymbol{R_8 = (1;0,5;0,5)}$. $\boldsymbol{W_k}^{\boldsymbol{R}}$ is a subset of set $\boldsymbol{W_k}$ that includes all estimates $\boldsymbol{w_k^m}$ with above average truth-membership, below average falsity-membership and below average indeterminacy-membership:

$$\boldsymbol{W_k}^{\boldsymbol{R}} = \{\boldsymbol{w_k^m} \in \boldsymbol{W_k} \,\big|\, [\,0,5 \le \boldsymbol{T}_{\tilde{\mathcal{N}}_k}(\boldsymbol{w_k^m}) \le 1] \vee [0 \le \boldsymbol{F}_{\tilde{\mathcal{N}}_k}(\boldsymbol{w_k^m}) \le 0,5] \vee [0 \le \boldsymbol{I}_{\tilde{\mathcal{N}}_k}(\boldsymbol{w_k^m}) \le 0,5]\}.$$

$\forall \boldsymbol{w_k^m} \in \boldsymbol{W_k}^{\boldsymbol{R}}$ contribute extensively to the group decision.

Definition 9 (Tolerable neutrosophic estimates reliability). $\boldsymbol{\Theta = Q \cap \neg R \cap \neg \chi}$ is the area of ***tolerable neutrosophic estimates reliability***. $\boldsymbol{W_k}^{\boldsymbol{\Theta}}$ is a subset of set $\boldsymbol{W_k}$ that includes all estimates $\boldsymbol{w_k^m}$ with below average truth-membership, above average falsity-membership or above average indeterminacy-membership:

$$\boldsymbol{W_k}^{\boldsymbol{\Theta}} = \{\boldsymbol{w_k^m} \in \boldsymbol{W_k} \,\big|\, [\,0 < \boldsymbol{T}_{\tilde{\mathcal{N}}_k}(\boldsymbol{w_k^m}) < 0,5] \wedge [0,5 < \boldsymbol{F}_{\tilde{\mathcal{N}}_k}(\boldsymbol{w_k^m}) < 1] \wedge [0,5 < \boldsymbol{I}_{\tilde{\mathcal{N}}_k}(\boldsymbol{w_k^m}) < 1]\}.$$

$\forall w_k^m \in W_k^{\Theta}$ have a minor impact on the group decision.

DM'_k is a subset of DM that includes all experts with acceptable reliability of the k-th element estimates: $DM'_k = \{DM_k^m \in DM \mid w_k^m \notin W_k\}$. M'_k designates the number of elements in set DM'_k.

Definition 10 (Ideal neutrosophic estimates reliability). The point R^* in the neutrosophic space $\langle T, F, I \rangle$ reflecting 100% truth-, 0% falsity- and 0% indeterminacy-grade of judgments in the decision making process is called ***ideal neutrosophic estimates reliability.*** In the SVNC Q $R^* = Q_1 = R_1 = (1;0;0)$.

Once all SVNSs $\tilde{\mathcal{N}}_k$ of group estimates are constructed, these values must be aggregated across M individuals in order to find a compromise priority of the k-th element for the committee. In terms of NSs, all $\tilde{\mathcal{N}}_k$ must be deneutrosophied, and comparable / operable representative values $\eta^*_k \in \Re^1$ need to be elicited. The proposed deneutrosophication procedure includes two steps: first, conversion of all SVNSs $\tilde{\mathcal{N}}_k$ into FSs $\tilde{E}_k$, and second, defuzzification of sets $\tilde{E}_k$.

Deneutrosophication of group estimates: Step 1 of 2

SVNSs $\tilde{\mathcal{N}}_k = \{w_k^m \mid T_{\tilde{\mathcal{N}}_k}(w_k^m), F_{\tilde{\mathcal{N}}_k}(w_k^m), I_{\tilde{\mathcal{N}}_k}(w_k^m)\}$ are converted into FSs $\tilde{E}_k = \{w_k^m \mid \mu_{\tilde{E}_k}(w_k^m)\}$ as follows. Expert estimates w_k^m remain invariant. For $\forall w_k^m \notin W_k$, the triads of neutrosophic truth-, falsity- and indeterminacy-membership grades are translated into scalar fuzzy membership grades $\mu_{\tilde{E}_k}(w_k^m) \in [0,1]^1$ based on the Euclidean distance between the point $\langle T_{\tilde{\mathcal{N}}_k}(w_k^m), F_{\tilde{\mathcal{N}}_k}(w_k^m), I_{\tilde{\mathcal{N}}_k}(w_k^m) \rangle \in [0,1]^3$ and the ideal neutrosophic reliability $P^* \in [0,1]^3$. Since priorities w_k^m from the set W_k are not reliable, they are assigned a zero fuzzy membership grade:

$$\mu_{\tilde{E}_k}(w_k^m) = \begin{cases} 1 - \sqrt{(1 - T_{\tilde{\mathcal{N}}_k}(w_k^m))^2 + F_{\tilde{\mathcal{N}}_k}(w_k^m)^2 + I_{\tilde{\mathcal{N}}_k}(w_k^m)^2}, & \text{for } \forall w_k^m \notin W_k \\ 0, & \text{for } \forall w_k^m \in W_k \end{cases} \qquad (7)$$

Thus, for any $w_k^m, w_k^m{}' \in W_k$ and $m \in DM$, $\mu_{\tilde{E}_k}(w_k^m) > \mu_{\tilde{E}_k}(w_k^m{}')$ means that w_k^m is *more reliable* than $w_k^m{}'$, where $w_k^m, w_k^m{}'$ and $\mu_{\tilde{E}_k}(w_k^m), \mu_{\tilde{E}_k}(w_k^m)$ are given by (1) and (7) respectively.

Deneutrosophication of group estimates: Step 2 of 2

FSs $\tilde{E}_k$ that represent uncertain group assessments with corresponding reliability grades are defuzzified using the centroid method for discrete case as defined in formula (2). As a result, one representative compromise crisp value $e^*_k = \eta^*_k \in \Re^1$ is obtained for each k:

$$\eta^*_k = \frac{\sum_{m=1}^{M} w_k^m \cdot \mu_{\tilde{E}_k}(w_k^m)}{\sum_{m=1}^{M} \mu_{\tilde{E}_k}(w_k^m)} \tag{8}$$

Normalization of representative opinions η^*_k is needed to reflect their relative weights within the sets these opinions belong to:

$$\eta'^*_k = \eta^*_k \Big/ \sum_{k=1}^{K} \eta'^*_k \tag{9}$$

The overall priorities of the alternatives are derived using the weighted additive aggregation:

$$w_i^G = \sum_{j=1}^{J} \eta'^*_j \cdot \eta'^{*j}_i, \text{ for } \forall i = 1,..., I \tag{10}$$

The higher w_i^G, the better A_i meets the collective group objectives.

5. An illustrative example

Let four domain experts with unequal voting power be responsible for assessment of three alternatives with respect to five criteria ($M = 4, I = 3, J = 5$). First, each expert builds pairwise comparison matrices as shown in expression (1) and applies the right eigenvalue method in order to reveal the weights of the criteria (w_j^m) and the priorities of the alternatives with respect to each criterion ($w_i^{(j,m)}$). Consistency ratio $C.R.(B_{J\times J}^m)/C.R.(B_{I\times I}^{(j,m)})$ is calculated for each comparison matrix $B_{J\times J}^m / B_{I\times I}^{(j,m)}$. The maximum acceptable consistency ratio is 10% ($\varepsilon' = 10\%$ for $\forall i, j, m$). The voting power ranges from 0 to 100

($VP(w_j^m), VP(w_i^{(j,m)}) \in [0,100], \max(VP_j) = 100$; $\max(VP_i^j) = 100$), with higher scores standing for greater decision makers' influence: $VP(w^{DM^1}) = 100$, $VP(w^{DM^2}) = 50$, $VP(w^{DM^3}) = 85$ and $VP(w^{DM^4}) = 75$ for $\forall i, j$. Moreover, the level of confidence in the correctness of the derived priorities is expressed by each committee member on a scale from 0 to 100, with lower scores designating greater hesitation: $SC_j^m, SC_i^{(j,m)} \in [0,100]$, $\max(SC_j^m) = 100$, $\max(SC_i^{(j,m)}) = 100$. Tables 1 and 2 presents the derived priorities of decision criteria and alternatives, respective consistency ratios, as well as confidence scores.

Table 1: Experts' criteria weights, consistency ratios and confidence scores

Criteria, C_j	Experts, DM^m							
	DM^1		DM^2		DM^3		DM^4	
	$w_j^{DM^1}$	$SC_j^{DM^1}$	$w_j^{DM^2}$	$SC_j^{DM^2}$	$w_j^{DM^3}$	$SC_j^{DM^3}$	$w_j^{DM^4}$	$SC_j^{DM^4}$
C_1	0.289	75	0.350	90	0.158	95	0.400	65
C_2	0.124	80	0.176	90	0.234	95	0.150	50
C_3	0.204	90	0.150	90	0.015	92	0.200	50
C_4	0.312	60	0.150	90	0.314	90	0.145	50
C_5	0.071	95	0.174	90	0.279	95	0.105	60
Inconsistency, $C.R.(B_{J \times J}^m)$	7.5%		3.01%		1.74%		5.2%	

Table 2: Experts' estimates of alternatives, consistency ratios and confidence scores

Experts, DM^m	Criteria C_j	Alternatives, A_i						Inconsistency, $C.R.(B_{I \times I}^{(j,DM^m)})$
		A_1		A_2		A_3		
		$w_{A_1}^{(j,DM^m)}$	$SC_{A_1}^{(j,DM^m)}$	$w_{A_2}^{(j,DM^m)}$	$SC_{A_2}^{(j,DM^m)}$	$w_{A_3}^{(j,DM^m)}$	$SC_{A_3}^{(j,DM^m)}$	
DM^1	C_1	0.289	90	0.524	80	0.187	100	8.50%
	C_2	0.465	50	0.117	60	0.418	50	26.00%
	C_3	0.332	75	0.482	80	0.186	85	4.00%
	C_4	0.049	95	0.385	95	0.566	90	2.00%
	C_5	0.455	70	0.240	90	0.305	80	6.50%
DM^2	C_1	0.349	40	0.341	60	0.310	40	36.50%
	C_2	0.200	60	0.250	75	0.550	80	3.07%
	C_3	0.028	80	0.552	75	0.420	65	2.40%
	C_4	0.266	0	0.333	0	0.401	0	15.00%

	C_5	0.417	70	0.200	80	0.383	75	5.00%
DM^3	C_1	0.090	100	0.097	75	0.813	85	2.60%
	C_2	0.333	95	0.080	80	0.587	60	5.30%
	C_3	0.245	70	0.300	85	0.455	85	1.25%
	C_4	0.300	60	0.500	70	0.200	90	4.00%
	C_5	0.358	100	0.369	100	0.273	100	0.01%
DM^4	C_1	0.208	75	0.515	80	0.277	85	5.80%
	C_2	0.252	33	0.300	50	0.448	33	6.50%
	C_3	0.486	0	0.202	0	0.312	0	36.00%
	C_4	0.568	95	0.252	99	0.180	90	2.40%
	C_5	0.450	0	0.386	0	0.164	0	19.00%

Each set of group assessments $w_j^m / w_i^{(j,m)}$, and inherent to them measures of voting power $\delta(w_j^m)/\delta(w_i^{(j,m)})$, inconsistency $\varepsilon_j^m / \varepsilon_i^{(j,m)}$ and confidence $\theta_j^m / \theta_i^{(j,m)}$ is represented as a SVNS $\tilde{\mathcal{N}}_j / \tilde{\mathcal{N}}_i^j$ based on formulations (4) and (6).

$$\tilde{\mathcal{N}}_j = \{w_j^m \mid T_{\tilde{\mathcal{N}}_j}(w_j^m), F_{\tilde{\mathcal{N}}_j}(w_j^m), I_{\tilde{\mathcal{N}}_j}(w_j^m)\} \text{ where}$$

$$\begin{cases} T_{\tilde{\mathcal{N}}_j}(w_j^m) = \dfrac{VP^m}{100}, \\ F_{\tilde{\mathcal{N}}_j}(w_j^m) = (\dfrac{C.R.(B_{J\times J}^m)}{10\%} \text{ if } C.R.(B_{J\times J}^m) \le 10\%) \cup (1 \text{ if } C.R.(B_{J\times J}^m) > 10\%), \\ I_{\tilde{\mathcal{N}}_j}(w_j^m) = \dfrac{100 - SC_j^m}{100}. \end{cases}$$

$$w_j^m \in [0,1], \quad \sum_{j=1}^{J} w_j^m = 1.$$

$$\tilde{\mathcal{N}}_i^j = \left\{ w_i^{(j,m)} \mid T_{\tilde{\mathcal{N}}_i^j}(w_i^{(j,m)}), F_{\tilde{\mathcal{N}}_i^j}(w_i^{(j,m)}), I_{\tilde{\mathcal{N}}_i^j}(w_i^{(j,m)}) \right\} \text{ where}$$

$$\begin{cases} T_{\tilde{\mathcal{N}}_i^j}(w_i^{(j,m)}) = \dfrac{VP^m}{100}, \\ F_{\tilde{\mathcal{N}}_i^j}(w_i^{(j,m)}) = (\dfrac{C.R.(B_{I\times I}^{(j,m)})}{10\%} \text{ if } C.R.(B_{I\times I}^{(j,m)}) \le 10\%) \cup (1 \text{ if } C.R.(B_{I\times I}^{(j,m)}) > 10\%), \\ I_{\tilde{\mathcal{N}}_i^j}(w_i^{(j,m)}) = \dfrac{100 - SC_i^{(j,m)}}{100}, \end{cases}$$

$$w_i^{(j,m)} \in [0,1], \quad \sum_{i=1}^{I} w_i^{(j,m)} = 1.$$

The SVNSs representing uncertain group priorities of the alternatives are listed in Table 3.

Table 3: Neutrosophic group estimates of alternatives with respect to criteria

Alter-natives, A_i	**Crite-ria,** C_j	**SVNSs of alternatives'estimates,** $\tilde{\mathcal{N}}_{A_i}^j = \left\{ w_{A_i}^{(j,m)} \mid T_{\tilde{\mathcal{N}}_{A_i}^j}(w_{A_i}^{(j,m)}), F_{\tilde{\mathcal{N}}_{A_i}^j}(w_{A_i}^{(j,m)}), I_{\tilde{\mathcal{N}}_{A_i}^j}(w_{A_i}^{(j,m)}) \right\}$
A_1	C_1	{(0.289 | 1, 0.85, 0.1), (0.349 | 0.5, 1, 0.6), (0.09 | 0.85, 0.46, 0), (0.208 | 0.75, 0.58, 0.25)}
	C_2	{(0.465 | 1, 1.0, 0.5), (0.2 | 0.5, 0.31, 0.4), (0.333 | 0.85, 0.51, 0.05), (0.252 | 0.75, 0.65, 0.67)}
	C_3	{(0.332 | 1, 0.04, 0.25), (0.028 | 0.5, 0.24, 0.45), (0.245 | 0.85, 0.13, 0.3), (0.486 | 0.75, 1, 1)}
	C_4	{(0.049 | 1, 0.2, 0.5), (0.266 | 0.5, 1, 1), (0.3 | 0.85, 0.4, 0.4), (0.568 | 0.75, 0.14, 0.05)}
	C_5	{(0.455 | 1, 0.55, 0.05), (0.417 | 0.5, 0.5, 0.1), (0.358 | 0.85, 0, 0), (0.45 | 0.75, 1, 1)}
A_2	C_1	{(0.524 | 1, 0.85, 0.2), (0.341 | 0.5, 1, 0.4), (0.097 | 0.85, 0.46, 0.25), (0.515 | 0.75, 0.58, 0.45)}
	C_2	{(0.117 | 1, 1, 0.4), (0.25 | 0.5, 0.31, 0.25), (0.08 | 0.85, 0.51, 0.05), (0.3 | 0.75, 0.65, 0.5)}
	C_3	{(0.482 | 1, 0.04, 0.45), (0.552 | 0.5, 0.24, 0.45), (0.3 | 0.85, 0.13, 0.15), (0.202 | 0.75, 1, 1)}
	C_4	{(0.385 | 1, 0.2, 0.35), (0.333 | 0.5, 1, 1), (0.5 | 0.85, 0.46, 0.45), (0.252 | 0.75, 0.14, 0.01)}
	C_5	{(0.240 | 1, 0.55, 0.1), (0.2 | 0.5, 0.5, 0.1), (0.369 | 0.85, 0, 0.25), (0.386 | 0.75, 1, 1)}
A_3	C_1	{(0.187 | 1, 0.85, 0), (0.31 | 0.5, 1.0, 0.6), (0.813 | 0.85, 0.46, 0.15), (0.277 | 0.75, 0.58, 0.15)}
	C_2	{(0.418 | 1, 1, 0.5), (0.55 | 0.5, 0.31, 0.2), (0.587 | 0.85, 0.51, 0.4), (0.448 | 0.75, 0.65, 0.67)}
	C_3	{(0.186 | 1, 0.04, 0.15), (0.42 | 0.5, 0.24, 0.35), (0.455 | 0.85, 0.13, 0.15), (0.312 | 0.75, 1, 1)}
	C_4	{(0.566 | 1, 0.2, 0.1), (0.401 | 0.5, 1, 1), (0.2 | 0.85, 0.4, 0.1), (0.180 | 0.75, 0.14, 0.1)}
	C_5	{(0.305 | 1.0, 0.55, 0.2), (0.383 | 0.5, 0.5, 0.3), (0.273 | 0.85, 0, 0), (0.164 | 0.75, 1, 1)}

The graphical illustration of the SVNS $\tilde{\mathcal{N}}_{A_1}^{C_1}$ representing the group performance estimates of the alternative A_1 with respect to the criterion C_1 space is given in Figure 2, where each value from the set $\{w_{A_1}^{(C_1,m)}\}_{[0,1]}$ has a mapping into the SVNC $Q_{[0,1]^3}$.

Estimates of all individuals can be classified according to their reliability based on the position within the SVNC Q.

1) The values with *unacceptable neutrosophic reliability* must be excluded from further decision process. These values are:

$w_{A_1}^{(C_2,DM_1)}, w_{A_1}^{(C_1,DM_2)}, w_{A_2}^{(C_2,DM_1)}, w_{A_2}^{(C_1,DM_2)}, w_{A_3}^{(C_2,DM_1)}, w_{A_3}^{(C_1,DM_2)} \in \chi_3 \subset W_i^j$ - the estimates characterized by 100% falsity degree;

$w_{A_1}^{(C_4,DM_2)}, {}_{A_1}^{(C_3,DM_4)}, {}_{A_1}^{(C_5,DM_4)}, w_{A_2}^{(C_4,DM_2)}, {}_{A_2}^{(C_3,DM_4)}, {}_{A_2}^{(C_5,DM_4)}, w_{A_3}^{(C_4,DM_2)}, {}_{A_3}^{(C_3,DM_4)}, {}_{A_3}^{(C_5,DM_4)} \in (\chi_3 \cup \chi_3) \subset W_i^j$ - the estimates characterized by 100% falsity- and 100% indeterminacy degrees.

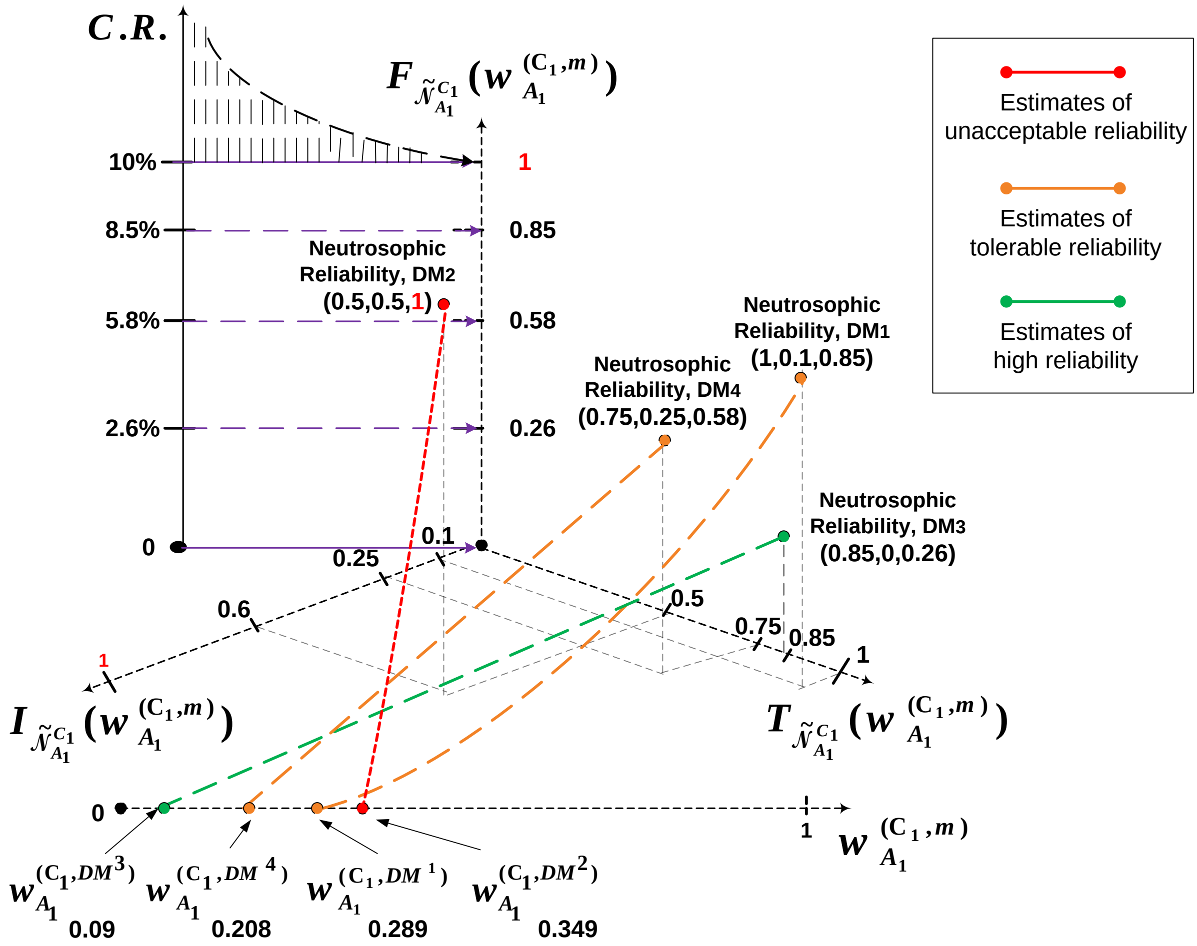

Figure 2: Example of single-valued neutrosophic estimates

2) The estimates with *high neutrosophic reliability* that imply strong impact on the decision include:

$$w_{A_i}^{(C_3,DM_1)}, w_{A_i}^{(C_4,DM_1)}, w_{A_i}^{(C_2,DM_2)}, w_{A_i}^{(C_3,DM_2)}, w_{A_i}^{(C_5,DM_2)}, w_{A_i}^{(C_1,DM_3)}, w_{A_i}^{(C_3,DM_3)}, w_{A_i}^{(C_4,DM_3)}, w_{A_i}^{(C_5,DM_3)},$$
$$w_{A_i}^{(C_4,DM_4)} \in W_i^{jR} \text{ for } \forall i = 1,...,3.$$

3) The estimates with *tolerable neutrosophic reliability* that imply weak to average impact on the decision are:

$$w_{A_i}^{(C_1,DM_1)}, w_{A_i}^{(C_5,DM_1)}, w_{A_i}^{(C_2,DM_3)}, w_{A_i}^{(C_1,DM_4)}, w_{A_i}^{(C_2,DM_4)} \in W_i^{j\Theta} \text{ for } \forall i = 1,...,3.$$

As suggested in section 4, derivation of the compromise group assessments represented as SVNSs can be accomplished using the deneutrosophication process where all triads $\left\langle T_{\tilde{N}_j}(w_j^m), F_{\tilde{N}_j}(w_j^m), I_{\tilde{N}_j}(w_j^m) \right\rangle$ and $\left\langle T_{\tilde{N}_i^j}(w_i^{(j,m)}), F_{\tilde{N}_i^j}(w_i^{(j,m)}), I_{\tilde{N}_i^j}(w_i^{(j,m)}) \right\rangle$ are first converted into fuzzy membership grades $\mu_{\tilde{E}_j}(w_j^m)$ and $\mu_{\tilde{E}_i^j}(w_i^{(j,m)})$, respectively, and then the representative crisp value η^*_j / η^{*j}_i is calculated for each obtained FS. Conversion of SVNSs into FSs relies upon the Euclidean metric in three-dimensional space for all reliable w_j^m / $w_i^{(j,m)}$ and is made as defined by the set of equations (7). Tables 4 and 5 contain the resulting FSs of the criteria and alternatives' group estimates.

Table 4: Fuzzy group estimates of criteria weights

Criteria, C_j	**FSs of criteria importance weights, $\tilde{E}_j = \{w_j^m \mid \mu_{\tilde{E}_j}(w_j^m)\}$**
C_1	{0.289 | 0.672, 0.35 | 0.360, 0.158 | 0.717, 0.4 | 0.543}
C_2	{0.124 | 0.718, 0.176 | 0.369, 0.234 | 0.717, 0.15 | 0.493}
C_3	{0.204 | 0.822, 0.15 | 0.369, 0.015 | 0.717, 0.2 | 0.493}
C_4	{0.312 | 0.552, 0.15 | 0.369, 0.314 | 0.716, 0.145 | 0.493}
C_5	{0.071 | 0.888, 0.174 | 0.369, 0.279 | 0.717, 0.105 | 0.527}

Table 5: Fuzzy group estimates of alternatives with respect to criteria

Alternatives, A_i	**Criteria, C_j**	**FSs of alternatives'estimates, $\tilde{E}_i^j = \{w_i^{(j,m)} \mid \mu_{\tilde{E}_i^j}(w_i^{(j,m)})\}$**
A_1	C_1	{0.289 | 0.1, 0.349 | 0, 0.09 | 0.55, 0.208 | 0.23}
	C_2	{0.465 | 0, 0.2 | 0.2, 0.333 | 0.34, 0.252 | 0.02}
	C_3	{0.332 | 0.6, 0.028 | 0.2, 0.245 | 0.5, 0.486 | 0}
	C_4	{0.049 | 0.34, 0.266 | 0, 0.3 | 0.27, 0.568 | 0.56}
	C_5	{0.455 | 0.33, 0.417 | 0.2, 0.358 | 0.72, 0.45 | 0}
A_2	C_1	{0.524 | 0.09, 0.341 | 0, 0.097 | 0.47, 0.515 | 0.16}
	C_2	{0.117 | 0, 0.25 | 0.26, 0.08 | 0.34, 0.3 | 0.1}
	C_3	{0.482 | 0.41, 0.552 | 0.2, 0.3 | 0.61, 0.202 | 0}
	C_4	{0.385 | 0.45, 0.333 | 0, 0.5 | 0.24, 0.252 | 0.57}
	C_5	{0.240 | 0.32, 0.2 | 0.2, 0.369 | 0.56, 0.386 | 0}
A_3	C_1	{0.187 | 0.103, 0.31 | 0, 0.813 | 0.52, 0.277 | 0.25}
	C_2	{0.418 | 0, 0.55 | 0.27, 0.587 | 0.24, 0.448 | 0.02}
	C_3	{0.186 | 0.711, 0.42 | 0.25, 0.455 | 0.61, 0.312 | 0}
	C_4	{0.566 | 0.632, 0.401 | 0, 0.2 | 0.38, 0.18 | 0.55}
	C_5	{0.305 | 0.3, 0.383 | 0.17, 0.273 | 0.72, 0.164 | 0}

Calculation of the representative crisp estimates for each set of uncertain group members' values is enabled by using the centroid method and formulas (8)-(9). The relative compromise importance weights of the criteria are: $\eta'^*_{C_1} = \mathbf{0.26}$, $\eta'^*_{C_2} = \mathbf{0.195}$, $\eta'^*_{C_3} = \mathbf{0.098}$,

$\eta'^*_{C_4} = \mathbf{0.248}$ and $\eta'^*_{C_5} = \mathbf{0.199}$. The normal representative values for the alternatives are given in Table 6.

Table 6: Relative representative priorities of alternatives and the overall rankings

Criteria, C_j	Alternatives, A_i		
	A_1	A_2	A_3
C_1	0.143	0.241	0.585
C_2	0.282	0.174	0.562
C_3	0.252	0.403	0.327
C_4	0.356	0.348	0.341
C_5	0.393	0.299	0.297
Overall priorities, w_i^G	**0.2834**	**0.2819**	**0.4376**
Rankings	2	3	1

The global alternatives' priorities are calculated using formula (10). As can be seen from Table 6, alternative A_3 (Rank 1) has the highest value and, therefore, it is the best one; alternatives A_1 and A_2 have close priority values, although A_1 (Rank 2) is slightly better than A_2 (Rank 3). For a comparison, the global priorities of the alternatives under consideration for the same experts' estimates w_j^m and $w_i^{(j,m)}$ ($\forall \boldsymbol{i} = \mathbf{1,...,3;}\ \forall \boldsymbol{j} = \mathbf{1,...,5;}\ \forall \boldsymbol{m} = \mathbf{1,...,4}$) but calculated using the geometric mean of group members' opinions and without taking into consideration uncertainty measures δ, ε and θ are: $w_{A_1}^G = \mathbf{0.2823}$ (Rank 3), $w_{A_2}^G = \mathbf{0.3321}$ (Rank 2) and $w_{A_3}^G = \mathbf{0.3856}$ (Rank 1). It is evident from the comparative results that uncertainty data is sufficient and affect the decision outcome.

For more precise analysis, sensitivity of the results to changes in experts' opinions can be tested. Special attention should be paid to the estimates that fall into the area of unacceptable neutrosophic reliability.

6. Conclusions

This paper proposes a novel method of modelling and handling multi-source uncertainty measures reflecting reliability of experts' assessments in GMCDA problems based on SVNSs. In spite of controversies surrounding the NS theory, we demonstrated that SVNSs can serve as a good basis for representing various and conflicting experts' estimates that are characterized by independent tangible and intangible metrics of the individuals' voting power, judgment inconsistency or error and the level of confidence in the obtained local priorities. Atanassov et al. (2003) said about neutrosophy that “these ideas, once properly formalized, will have a profound impact on our future dealings with imprecision”. Rivieccio (2008) wrote that although a variety of new theories have been developed on the basic principles of neutrosophy, neutrosophic formalism should be further extended in many directions. Since the contemporary scholar literature does not contain information about arithmetic operations on SVNSs needed for synthesis of quantified group opinions in multi-criteria decision making, in the proposed method the triples of neutrosophic truth-, falsity- and indeterminacy grades are converted into single scalar values using the Euclidean metric in three-dimensional space, and the obtained values are considered to be the membership grades of the respective experts' assessments. FSs are well investigated and one of the ways to find representative group estimates is to apply a defuzzification operator, such as centre of gravity. The representative crisp parameters can then be synthesized and analyzed in the context of state-of-the art multi-criteria techniques. The comparative analysis of the results obtained by using the geometric mean approach for combining group estimates, and by following the proposed SVNS-based process that takes into account the neutrosophic independent multi-source reliability measures showed that uncertainty consideration affects the overall ranking and is sufficient for taking credible decisions.

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Working Papers

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Modelling synergies in vendor selection problems with application to agricultural commodity trade

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Abstract

This paper proposes a novel meta-approach to support collaborative multi-objective supplier selection and order allocation (SSOA) decisions that integrates techniques of multicriteria decision analysis (MCDA) and linear programming (LP). The model accounts for suppliers' performance synergy effects, multi-level structure and interdependencies among the decision criteria, encompasses heterogeneous objective data and subjective judgments of the decision makers (DMs) representing various groups of interests and possessing different voting power, as well as maximizes the total value of purchasing (TVP) by optimizing order quantity assignment to suppliers taking into consideration their synergies encountered in different time horizons. Application of the model to contractor selection and order quantity assignment by agricultural commodity trading companies (ACTCs) maximizes both customer and supplier strategic importance, minimizes risks, increases grade of cooperation between trading partners on all levels of supply chain integration, enhances transparent knowledge sharing and aggregation, and supports collaborative decision making in a methodological way. The method promotes dynamic monitoring of suppliers' strategic value for the trading firm and entire supply chain; as well as supports taking daily Just-in-Time (JIT) purchasing decisions based on constantly changing commodity prices.

Keywords: Multi-objective Decision Making (MODM); Supplier Selection and Order Allocation (SSOA); Synergy of Alternatives; Collaboration; Value Focused Thinking (VFT); Alternative Focused Thinking (AFT); Agricultural Commodity Trade (ACT).

Disclaimer: The data set presented in the case study has been significantly changed due to confidentiality constraints. For the same reason, some parts of analysis have been omitted.

1. Introduction

Rapid globalization process, economic growth and substantial scientific and technological progress have resulted in enormous competition in international trading (Engau, 2010). The gap between product quality and performance is closing with intensifying competition in the global market (Chang et al., 2011). As business is becoming more and more competitive, purchasing and supply chain management have been increasingly recognized by top managers as key business drivers and have become the foundation for operations management and the core of the enterprise management in the 21th century (Van Weele, 2009; Gunasekaran & Ngai, 2012). For companies who spend a high percentage of their sales revenue on supplies, savings from vendors are of particular importance (Karpak et al., 2001). A great deal of previous research, in supplier evaluation and selection, emphasizes need for a methodology that is simple to use and understand, but yet it shall produce reasonably accurate results (Ha & Krishnan, 2008). Especially, there is a strong need for a systematic approach to purchasing decision making in the area of identifying appropriate suppliers and assigning orders among them (Aissaoui et al., 2007; Weber et al. 1991; Vonderembse & Tracey, 1999; Tempelmeier, 2002).

Difficulties associated with SSOA by trading companies in JIT environment result from several facts: (1) suppliers may be interdependent in terms of resource sharing or synergistic performance, (2) decisions must take into account multiple objectives and opinions of different supply chain participants, (3) the objectives are usually conflicting, (4) vendor assessment criteria can result from DMs' values (VFT), or be based upon a simple comparison of supplier's factual performance (AFT), (5) decision factors can be quantitative and qualitative, (6) some criteria characterize vendors indirectly, via intermediate objects, such as external facilities or third-party service providers, (7) decisions are made frequently and rely upon suppliers' performance history, measure of their strategic value and operational characteristics, (7) in case of multiple sourcing the set of vendors needs to be balanced in terms of criteria importance, (9) the number of feasible solutions is often enormous, and (10) uncertainties can affect the decision outcome.

The supplier selection and order quantity assignment decision is fairly structured when only independent on all criteria vendors are examined and can be directly evaluated. In this situation, the goal is to choose the set of the most effective suppliers at the lowest level of costs subject to demand restriction and other additional requirements of single vendors to the buyer or buyer's requirements to the single vendors. Such decision can be accomplished using hybrid MCDA - optimization approaches where supplier's individual priorities are calculated using multi-criteria analysis tools, and an appropriately constructed optimization procedure serves to find optimal order quantities for all feasible sets of vendors among which the final choice is made. Attention has to be paid to the feasibility of potential solutions, e.g., their capability to satisfy all problem constraints. Restrictions in the model can refer to the buyer or customer needs, as well as to the offers of individual bidders or their groups. For instance, suppliers' interdependency that is based upon resource sharing should be taken into account when several bidders offer a commodity from the same stock of limited capacity and the sum of maximum offered quantities of the individual vendors exceeds the quantity available in stock. However, suppliers' interaction can not always be expressed by constraints. If joint performance of several vendors on a criterion differs from their summarized individual performances on this criterion, positive or negative synergy of the bidders takes place. In MCDM the issue of alternatives' synergy was discussed in the context of project portfolio selection. Sanathanam and Kyparisis (1996) classified interdependencies among information system (IS) projects into resource, benefit and technical interdependencies. Later, Lee and Kim (2001) advocated necessity to consider interdependencies among criteria and alternatives in IS project selection. The existing MODM methods of SSOA fail to take account of interactions among different vendors that entail positive or negative performance synergy on the relevant decision criteria that affect decision outcome. Whereas, jointly selected suppliers can offer additional benefits or opportunities for the trading firm and its customers, or in contrast, cause larger losses or sharper risks. For example, cost savings can be achieved by coordinating the transportation of commodities purchased from several suppliers in a given period. On the contrary, bigger risks may be associated with selecting contractors who purchase from the same source, particularly if its merchandise gets out of stock or in case of delivery difficulties. In the complex supply chains multiple positive and negative synergies of vendors' performance can emerge simultaneously. A modelling technique and a trade-off mechanism are needed to enable synthesis of all suppliers' individual *not-synergistic* and group *synergistic* performance characteristics.

Order quantity allocation is another vital aspect of multiple-sourcing procurement decisions. The problem of order allocation occurs when more than one supplier should be selected and distribution of the demanded quantity of a product must be made so as to maximize the overall value of the purchase. The article presents a new process that enables trade-off of synergistic and not-synergistic supplier characteristics simultaneously. First, combinations of suppliers are formed. These combinations represent alternative problem solutions among which the final choice should be made. Once all combinations of suppliers are generated, each combination undergoes assessment with respect to synergistic criteria. Then, all single suppliers are estimated with respect to not-synergistic criteria. Finally, supplier estimates on the synergistic and not-synergistic criteria are aggregated within each combination in order to derive their total expected values of purchasing (TVPs) and enable ranking of the alternatives.

In general, this research pursues two main objectives: to develop a structural collaborative approach for support of complex multi-objective vendor selection and order allocation decisions involving suppliers' synergism, and to demonstrate application of this methodology to supplier selection and order quantity assignment in agricultural commodity trading firms.

More specifically, the first objective of this study is to present an integrated empirical and technical framework for multi-objective SSOA decision support in complex collaborative environments with the following five key characteristics: (a) the flexible structure of decision criteria should be based upon the compound value system of different decision making and interest groups and utilize both AFT and VFT approaches for criteria identification, (b) all relevant objective data and subjective DMs' judgments regarding importance of decision factors and performance values of the discrete alternatives on intangible strategic and operational criteria must be incorporated within one consistent methodology, (c) the framework should allow for flexible structure of decision options constructed by taking into consideration possible effects of suppliers' synergism in case of multiple sourcing, (d) optimization of order allocation within feasible discrete sets of potential suppliers should maximize the TVP, and (e) the configuration of decision committee needs to be clearly provided to enable articulation of the responsibilities and impact of its members

The second objective of this study is fourfold: (a) to reveal the required variables in measuring utility of agricultural commodity vendors, including criteria with respect to which suppliers' synergism is possible, (b) to apply the developed model for generation of feasible combinations of commodity suppliers and their evaluation, (c) to optimize order quotes to be assigned to suppliers within each feasible combination, and (d) to select the best appropriate set of vendors with optimally distributed order quantities.

The proposed framework was implemented for international agricultural commodity vendor selection and order allocation in one of the largest agricultural corporations in Germany. The methodology is exemplified using a case study of purchasing wheat.

This paper is organized into five sections. The next section presents motivation and background for the proposed collaborative decision support framework and its application to suppler selection and order allocation in commodity trading industry. Section 3 illustrates the formal MCDA-LP model to multi-objective SSOA problems involving synergy among alternatives with demonstration of its practical application for an agricultural commodity trading corporation. Section 4 outlines limitations of the model and points out future research directions. Section 5 provides summary and conclusions.

2. Motivation and background

Before to present the approach and the model, the paper outlines development trends in purchasing management, specifies problems of collaborative decision making, and makes a brief overview of SSOA methods described in the contemporary literature.

2.1. Trends in purchasing management

When buying materials from suppliers, companies traditionally focused on short-term transactional purchases primarily based on cost considerations where vendor assessment was to eliminate the unwanted suppliers rather than developing reliable and acceptable suppliers (Karpak et al., 2001; Lamming et al., 1996). In 1996 Lamming et al. introduced a concept of "lean supply" within "relationship assessment program (RAP)" underscoring the need for effective management of supply networks, including identification of supplier selection criteria, supplier selection decisions, and monitoring of supplier performance (Karpak et al., 2001). With recognition of the need to develop sustainable long-term relations with vendors and focusing on customer needs, concepts of Supplier Relationship Management (SRM) and Customer Relationship Management (CRM) have become attributes of successful purchasing activity. Sheth et al. (2009) argue that integration between purchasing and marketing should be taken into account when choosing vendors. In such case, generation of market intelligence creates superior value for the firm's customers, promotes superior company performance and sustainable competitive advantage in various contexts and industries (Day, 1994; Gatignon & Xuereb, 1997; Hätönen & Ruokonen, 2010; Li et al., 2010; Narver & Slater, 1990). Degree of market orientated activity may vary within different value chains and depending on the managerial decision making activities undertaken by an organization (Grunert et al., 2010). Market orientation predetermines supply chain integration (SCI) strategy that consists of internal integration of different functions within a company and external integration with trading partners (Li et al. 2010, Zhao, 2011). External SCI includes strategic orientation toward competitors and customers. A customer oriented supplier selection decision optimizes the trade-off between the total costs that a supplier causes in the buying firm and the revenues generated by the supplier (Wouters et al., 2005). Internal SCI is orientation toward interfunctional departments of an enterprise and is necessary for alignment of purchasing strategies and the development of synchronized processes aimed to improve and sustain competitive position and fulfil customer requirements (Flynn et al., 2009; Hayes and Wheelwright, 1984; Pagell, 2004). The main purchasing planning and vendor selection problems appear due to structural complexity, multiple conflicting viewpoints and objectives of parties on both external and internal SCI levels.

During the interviews with purchasing executives and top managers of the investigated corporation in Germany it was revealed that choosing between multiple- and single-sourcing strategies is practically an integral part of the whole problem of SSOA in agricultural commodity trade and the traders usually do not specify whether *a-priori* one or another strategy should be followed. ACT is a day-by-day activity and success of the firms depends on consistently profitable operations. On the one hand, selection of one or several vendors depends on the real-time signals and endeavours (i.e., operational metrics) such as bid price, delivery terms, political or weather conditions, quality of commodities etc. On the other hand, suppliers' strategic capabilities, such as management practices, reliability, long-run risks, and relationship potential, affect the decision. The practical requirement for an effective SSOA methodology in ACT is a capability to encompass and synthesize strategic and operational variables, as well as to determine an optimal sourcing strategy for each particular order that maximizes the TVP for each particular transaction. In the developed model and the case study diverse supplier combinations are formed and evaluated, demand quantity is optimally allocated within each combination in order to maximize the TVPs associated with each alternative, all single- and multiple-sourcing options are ranked according to their utility and, finally, the top-ranked alternative is the suggested most appropriate solution.

2.2. Collaborative decision making

Today's organizations operate in a value network on a global basis wherein organizations partner with suppliers, customers, and other stakeholders in pursuit of a sustainable competitive advantage (Agarwal & Selen, 2009). Meaning of the term "collaboration" used in the context of this research is twofold. First, *structural collaboration*, or relationship management, is defined as "a firms set of relationships with other organizations" (Perez Perez

& Sanchez, 2002, p. 261), includes the establishment and maintenance of relationships with supply chain partners (SCPs) such as suppliers, customers, and other key stakeholders (Agarwal & Selen, 2009) and is aimed at improvement of their individual and total effectiveness. Barratt (2004) reported fundamentals and difficulties of structural SCM collaboration. Second, *managerial collaboration*, can be defined as an organized interaction of decision makers representing different links of supply chain for definition, promotion, control and improvement of collaborative activity. It is a managerial capability and a skill that largely reflects knowledge sharing, communication, and the learning ability of the firm (Agarwal & Selen, 2009; Slater, 1995; Dyer & Singh, 1998). Despite the provided distinction among relationship management and decision making collaboration, activities entailed by both concepts are not-separable. Collaborative relationship management requires implementation of group decisions. Regardless of the form of collaboration, it always gives an opportunity to attain positive business synergy.

Collective decision making and learning using multiple soft sources such as information, skills, and knowledge is believed to be the core of competitive advantage for firms (Im & Workman, 2004; Kohli & Jaworski, 1990; Narver & Slater, 1990). When important decision is required, a team is usually formed to make it or to advise the individual decision maker, because a team has more resources, knowledge, and political insight than any one individual working alone (Dennis et al., 2010; Hackman & Kaplan, 1974). Surowiecki (2004) examined dozens of practical cases of group decision making and concluded that amalgamated views of a crowd reach a more accurate conclusion than the single experts in this group do. However, transparent and structured procedures are needed to avoid groupthink. Zollow and Winter (2002) have proposed that deliberate learning efforts articulate and codify collective knowledge, which translate into higher-order managerial skills in pursuit of greater effectiveness and improved efficiency. An efficient way to cope with increasing information complexity is to create coordinated multidisciplinary and multi-stakeholders working groups in order to have diverse perspectives on the problem, reveal alternative approaches for problem solving and use different individual skills and group knowledge (Beers et al., 2006; Shum et al., 2011). In complex decision situations is necessary participation of the decision analyst or/and facilitator to assist the decision group. Montibeller and Franco (2010) suggested a facilitated MCDM modelling. Success and sustainable development of supply chain depend on the structured and transparent collaborative decision making. The entire knowledge sharing coordination for supplier selection includes (Ordoobadi & Wang, 2011): (1) standardization of supplier selection models and concepts, (2) criteria of supplier selection, (3) multiple perspectives of these selection models, (4) coordinated synthesis process of multiple perspectives, and (5) transparent alternative analysis.

2.3. Review of the state-of-the-art SSOA methods

A number of methods have been proposed during the last decades to support SSOA decisions. However, a great deal of previous research emphasizes that empirical and analytical techniques may suffer from certain shortcomings, such as being mathematically too complex, which is undesirable for practitioners, or too subjective. The majority of the existing analytical vendor evaluation approaches are based on functional criteria like quality, price, delivery time, etc., and does not consider the repercussions of the company strategy on the decisions by taking into account soft criteria such as risks, flexibility, responsiveness, innovation, motivation, agility etc. (Muralidharan et al. 2002). On the contrary, most MCDM approaches consider only DM’s subjective judgments, whereas the objective data can play a crucial role (Wang and Lee, 2009). In fact, selection of an appropriate existing technique may be a challenging task when facing complex decision problems and integration of several mathematically sound methods may be beneficial to address problem requirements (Tavana, 2006).

Ho et al. (2010) provided an extensive review of decision methods, reported in literature, for supporting supplier evaluation and selection process. The authors distinguished between a wide range of single MCDM approaches, such as the Analytic Hierarchy Process (AHP), Analytic Network Process (ANP), case-based reasoning (CBR), Data Envelopment Analysis (DEA), fuzzy set theory, Technique for Order Preference by Similarity to Ideal Solution (TOPSIS), genetic algorithm (GA), mathematical programming (including integer linear programming (LP) and non-linear programming (NLP), goal programming (GP) and multi-objective programming (MOP)), and simple multi-attribute rating technique (SMART).

Recently, Özgen et al. (2008) reviewed existing vendor selection and order allocation methods and found (Özgen et al., 2008, p.487): “…there are few researchers who have paid attention to order allocation, and there are only a few studies that integrate the supplier evaluation and order allocation concepts in the same methodology.” Since solution of SSOA problem is only possible by applying hybrid methods, the further literature review is limited to the works where hybrid approaches are discussed.

Sodenkamp and Suhl (2012) developed a multilevel group decision approach that considers auxiliary decision objects (ADOs), such as external service providers associated with vendors. The DMs are divided into the three classes (α, β and γ) to define importance magnitudes of the objectives, estimate suppliers' performance on intangible criteria and rate ADOs according to their contribution. Vendor priorities are calculated based on their Euclidean distance (ED) to the benchmark solution and order quantities are distributed among the suppliers proportionally to their ED measures.

Amin et al. (2011) proposed to apply linguistic variables translated into fuzzy numbers for identification of criteria weights. Ranking of suppliers is made based on their position on the strength-weaknesses-opportunities-threats (SWOT) chart. Finally, order allocation is made using fuzzy LP.

Zouggari and Benyoucef (2011) presented a group framework implementing fuzzy AHP, simulation and knowledge management for dynamical ranking of suppliers. Then closeness coefficients to the best alternatives calculated using fuzzy TOPSIS and order quantities are assigned proportionally to these coefficients.

Mafakheri et al. (2011) ranked suppliers with the AHP and allocated orders among the best ones using a bi-objective dynamic optimization formulation (maximize total value of purchasing and minimize total cost of purchase).

Rezaei and Davoodi (2011) presented two multi-objective mixed integer non-linear models for multi-period lot-sizing problems involving multiple sourcing and multiple products.

Faez et al. (2009) integrated CBR for identification and evaluation of suitable vendors with fuzzy logic to encompass suppliers' vague values, and LP for finding optimal order quantities.

Özgen et al. (2008) developed a SSOA methodology based on the AHP and a two-phased multi-objective possibilistic LP that provides ability to express imprecise data in a logical way.

Demitras and Üstün (2008) presented an integration of the ANP and multi-objective mixed integer linear programming (MOMILP). They applied ANP to calculate supplier's priorities and solved MOMILP program by using ε-constraint method and a reservation level driven Tchebycheff procedure to find optimal order quantities.

Sanayei et al. (2008) proposed to use multi-attribute utility theory (MAUT) for rating and choosing the best suppliers by multiple DMs and built a LP model to identify the optimum order quantities.

Several SSOA decision support systems (DSS) have been developed over the last years. The DSS by Choi and Chang (2006) is based on a two phased optimization that semantically builds a goal program through model identification and candidate supplier screening by a set of predefined rules for applying in a business to business (B2B) e-procurement environment. However, in their DSS only quantitative parameters can be captured. Later, Erdem and Göçen (2012) developed an improved DSS called "SEOA" that implements the integrated AHP-GP approach that takes qualitative criteria into consideration.

Methods and DSSs described in contemporary SSOA literature fail to encompass effects of synergistic supplier performance that may occur in multiple sourcing cases.

3. The proposed model

The approach is proposed to select the best set of vendors with optimally allocated order quantities in complex multi-objective problems involving supplier's synergy.

Here is considered a situation where multiple DMs represent different groups of interest within the agricultural supply chain; their expertise areas are various and voting authorities are unequal. First, the DMs formulate decision objectives, define finite set of alternative suppliers that are available for selection and identify finite set of decision criteria. Second, the DMs estimate criteria importance weights and suppliers' performance on the set of qualitative criteria and collect quantitative objective data. If synergy effects of supplier joint performance encounter with respect to the part of criteria, combinations of the interacting suppliers have to be analysed. The goal is to select the best combination of suppliers so as to maximize the overall value of each combination by optimal adjustment of the decision variables (order quantities) within each combination while satisfying the given constraints and taking into account suppliers' performance synergies.

The 11 crucial stages of the process are isolated and described below. Process application is demonstrated on the case study of grain supplier selection in one of the largest agricultural commodity trading companies in Germany.

(1) Defining the problem

It has been widely acknowledged that problem formulation and structuring is a critical activity in organisational decision making (Eden, 1986; Franco et al., 2007; Lyles, 1981; Montibeller et al., 2009; Nutt, 1992). Participation of facilitators/analysts can firstly help the DMs to explicitly articulate their individual interpretations of the problem to jointly produce the model that adequately captures their complexity and, secondly, to improve conflict management within the group during the decision process (Montibeller et al., 2009; Keeney, 1992; Phillips & Phillips, 1993). ACTCs have two main approaches to vendor selection. If one supplier can meet the demand and all stated requirements it may be the single selected one (single-sourcing). Otherwise, an appropriate combination of suppliers should be formed (multiple-sourcing) and the demand quantity must be split within the set of the best suppliers.

The goal of the analysis is to generate and evaluate all single- and multiple-sourcing purchasing alternatives that vary in a large number of conflicting synergistic and not-synergistic criteria, optimize distribution of the demand quantity among the suppliers within each set, and choose the best vendor combination with optimally allocated order quantities by taking into consideration objectives and values of the firm representatives and the customers.

(2) Forming decision group

Decision group is a team brought together to achieve a shared goal (Johnson, 2002). A set of decision makers $\{\boldsymbol{DM}\}$ is formed to participate in the decision process. The A DMs are arranged into M divisions. Each division m is composed out of N^m clusters of the DMs who represent different areas of expertise, such as political, economical, social, technological and legal (PESTL). The assumption is that each DM belongs to only one division and one cluster of experts, so the total number of the DMs $A = \sum_{m=1}^{M}\sum_{n=1}^{N^m}\sum_{a=1}^{A^{mn}} DM^{mna}$.

Purchasing teams include committees, taskforces, or groups of people to achieve a common supply management-related goal, such as supplier selection, standardization of raw material inputs or quality improvements for purchased materials and services (Ellram & Pearson, 1993). Commodity purchasing teams have to be formed when a commodity represents a significant annual expense, its acquisition is viewed as complex, and it is regarded as critical to the firm’s success (Johnson at al., 2002). Commodity teams can consist of personnel from a variety of functional areas and may include representatives from outside the organization, such as suppliers or customers (Leenders and Fearon, 1997; Trent & Monczka, 1998; Carter & Narasimhan, 1996). Given the importance of customer preferences in demand-driven supply chains, purchasers should integrate customer preferences in their supplier selection decisions.

The decision committee in the case study was composed of two divisions ($M = 2$). The first division ($m = 1$) included two clusters of experts ($N^{m=1} = 2$): a cluster of top managers ($n^{m=1} = 1$) represented by only one person ($A^{m=1,n=1} = 1$, DM^{111}), and a cluster of purchasing executives ($n^{m=1} = 2$), also represented by a single member ($A^{m=1,n=2} = 1$, DM^{121}). The second division ($m = 2$) was composed of two clusters of customer ($N^{m=2} = 2$), each one represented by a single member ($A^{m=2,n=1} = 1$ and $A^{m=2,n=2} = 1$). Therefore, $A = 4$ in the case study.

(3) Identifying decision criteria

A decision may not be appropriately made without fully considering its context and all criteria (Tavana & Zandi, 2012). The criteria in a given problem must encompass all the relevant areas of concern to represent the decision factors as thoroughly as possible. Two main approaches can be followed for identifying criteria (Keeney, 1992) are alternative-focused thinking (AFT) and value-focused thinking (VFT). In AFT criteria are defined from the characteristics that distinguish options, which can be beneficial when the problem and options are well-defined (Montibeller et al. 2009). According to the VFT the evaluation criteria should reflect the organisation's values and strategic objectives; measurement of alternatives' performance indicates how much each option contributes to the achievements of organisational objectives. A combined VFT-AFT approach can be efficient for identification and structuring of I commodity supplier selection criteria within large agricultural supply chains when criteria have to reflect collaborative opinions of different interest groups.

Based on the analysis of scholar and business literature, as well as on the interviews with representatives of the investigated commodity trading company in Germany, a mass of conflicting views was generated, summarized and translated into a common value system for the companies and their customers defined by 31 ($I = 31$) SSOA factors listed in Table 1. All of these criteria except "Financial costs" characterize suppliers' individual performance in both single and multiple sourcing cases; these 30 factors are *non-synergistic*. Financial costs are not linear due to the transportation cost component. For optimal delivery of the purchased commodities several suppliers' logistics centres have to be covered by the vehicles. Joint freight from multiple vendors is usually cheaper than independent delivery from single suppliers. Hence, suppliers' synergy arises with respect to transportation costs in case of multiple sourcing.

Table 1: Criteria for evaluation of single wheat suppliers

No.	Criteria description	Units	Criteria	Concerned decision makers	
				Buyer	Custo-mers
1	Number of business hours the loading terminals are open, per day	Hours	Loading Hours	●	
2	Other companies' bad experience with the supplier, including breaches of contracts through the supplier's fault	Scores	Bad Experience (E)	●	
3	Closeness of relationship with the supplier within and beyond the business	Scores	Closeness	●	
4	Category of the offered commodity according to the given standard	Rating scale	Product Category	●	●
5	Supplier's product recalls in the past	Scores	Recalls	●	●
6	Bad experience with the supplier in the past, except the items No. 2, 5, 10, 21	Scores	Bad Experience (I)	●	●
7	Conventional / Organic product	Rating scale	Production Method	●	●
8	Country of commodity origin	Rating scale	Country of Origin		●
9	Financial costs associated with the purchase	Euros	Financial Costs	●	●
10	Late available orders in the past	Integer number	Delays	●	●
11	Not transparent inspection of the offered product	Scores	Improper Inspection	●	●
12	Number of logistics centres related to the supplier	Integer number	Number of LCs	●	
13	Number of other commodity categories purchased from the supplier during the reference period	Integer number	Number of Items	●	
14	Number of producers that compound the offered lot of commodity	Integer number	Composition		●
15	Number of contact persons authorized to take orders and reply to inquiries	Integer number	Contact Persons	●	
16	Orders per internet, phone, fax	Scores	Multimedia	●	
17	Quantities of other commodities purchased from the supplier during the reference period	Euro	Past Businesses II	●	
18	Quantities of the product at hand purchased from the supplier during the reference period	Euro	Past Businesses I	●	●
19	Maximum allowed payment period	Days	Terms of Payment	●	
20	Probability of delivery difficulties	Subjective probability	Delivery Difficulties	●	●
21	Orders rejected by the supplier in the past	Integer number	Rejected Orders	●	●
22	Rush order processing and supply on order capabilities	Scores	Order Processing	●	●

23	Slow speed of inquiry processing	Scores	Inquiry Processing	●	
24	Friendly and individual treatment by the supplier's contact persons	Scores	Attitude	●	
25	Supplier's desire and attempts to build a sustainable partnership based on trust and commitment	Scores	Desire to Cooperate	●	
26	Supplier's attempts to contribute to environmental protection	Scores	Environmental Management		●
27	Supplier's honesty, fairness and equity in professional and interpersonal relationships	Scores	Ethical Behaviour	●	●
28	Well organized loading process, modern equipment and logistics training programs	Scores	Logistics Facilities	●	
29	Supplier's office hours, per week	Hours	Office Hours	●	
30	Supplier's acting in advance	Scores	Proactiveness	●	
31	Sustainable relations with the supplier	Scores	Sustainability	●	

(4) Structuring decision criteria

For any decision problem or class of problems identified criteria have to be logically arranged and thoroughly classified in order to provide comprehensive analysis of all relevant aspects. In this section is presented a generalized scheme for arranging criteria within a hierarchy with variable number of sub-criteria and criteria levels using notions of graph theory.

Described in a natural way hierarchy can be formally defined in terms of the graph theory as *ordered rooted tree*. Detailed notations of such trees are given in the works of Gossett (2009), Knuth (1997), Knuth (2006), Lu (1984), Valentive (2002) etc. Generally, hierarchy is a tree structure with nodes, leaves and a root which assumes the existence of a unique path from the root to any other node (Chidamber & Kemerer, 1994; Valentive, 2002). For the hierarchy of criteria it means that each group of factors can include only unique attributes. Put differently, each attribute can belong to only one superior group of factors. An efficient method to represent ordered trees of large complexity is Dewey decimal notation (Knuth, 1997; Lu, 1984). Indices in the Dewey decimal notation (*D*-notation) will be further used to speak about unique "addresses" of elements within the criteria hierarchy and to specify their membership to the higher level groups of attributes. Formal representation of criteria hierarchy with *L* levels is given below.

Let T be a tree of decision criteria. The root node of this tree is a decision goal ($Goal_{v0}$). The number of *levels* vL in criteria hierarchy corresponds to the *depth* of the tree. *Degree* of the criterion means the number of sub-criteria this criterion includes. A criterion is called *leaf* of the tree (or *leaf criterion*) if it is not divided into sub-criteria, and its degree is "0" ($c^*_{i_{v1}\ldots i_{vl}} = 0$). A criterion that includes sub-criteria is called *interior criterion*, its degree is equal or larger than two ($I^{i_{v1}...i_{vl}} \geq 2$). Both depth of the goal (root of the tree) and its *D*-notation are “0”. In multi-criteria analysis the goal is always connected to several criteria, therefore it is an interior element. Criteria within each group are ordered lexicographically according to their *D*-notation. If an interior criterion $\hat{c}_{i_{v1}...i_{vl}}$ with the depth vl in T has $I^{i_{v1}...i_{vl}}$ immediate descendants (sub-criteria) then their *D*-notations are: $i_{v1}...i_{vl}1_{v(l+1)}$, $i_{v1}...i_{vl}2_{v(l+1)}$, …, $i_{v1}...i_{vl}I^{i_{v1}...i_{vl}}_{v(l+1)}$. General view of the criteria tree with *L* levels using Dewey indexation system of its nodes is displayed in Figure 1.

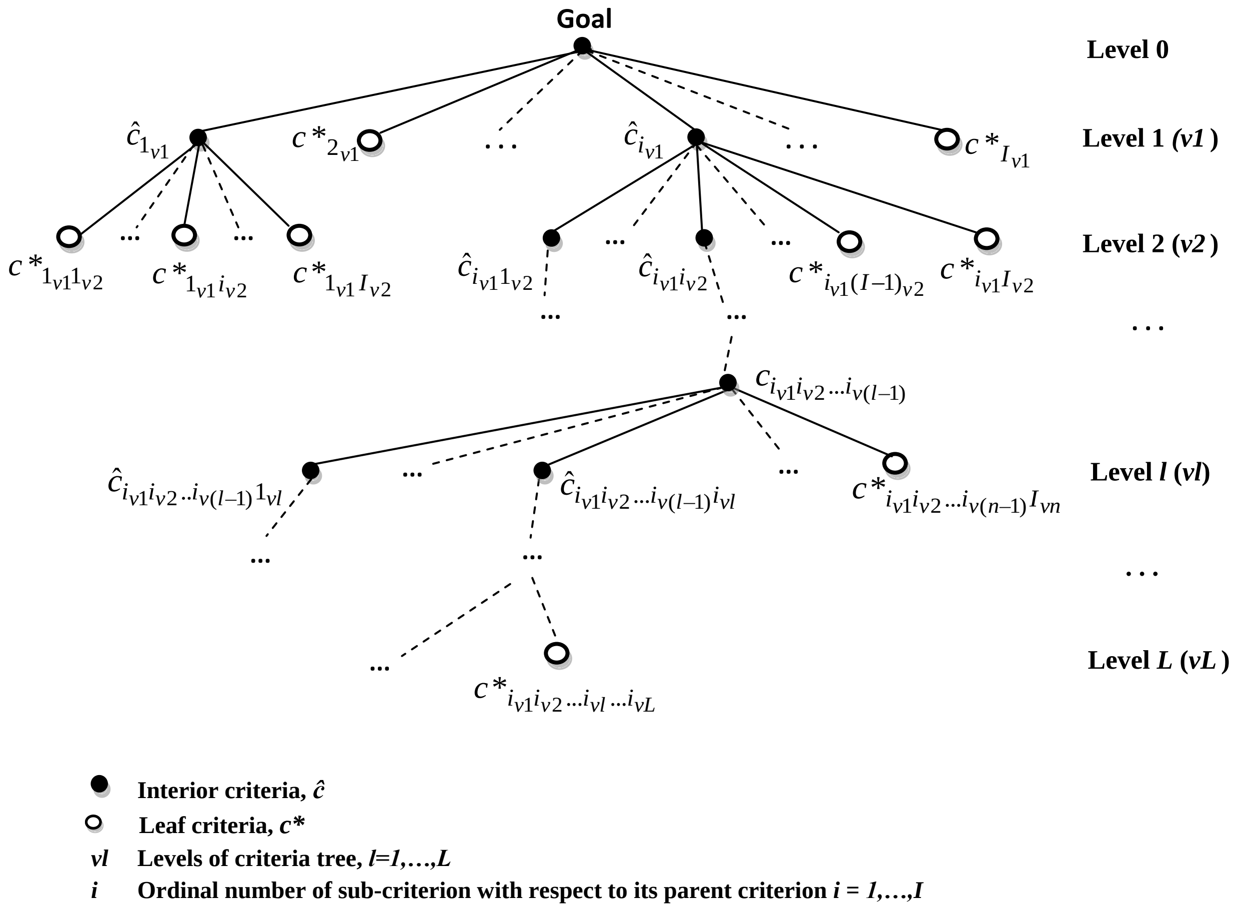

Figure 1: *L*-level tree of decision criteria

The criteria tree for evaluation of international grain suppliers in the investigated case study is exhibited in Figure 2, where all criteria are indexed using *D*-notation.

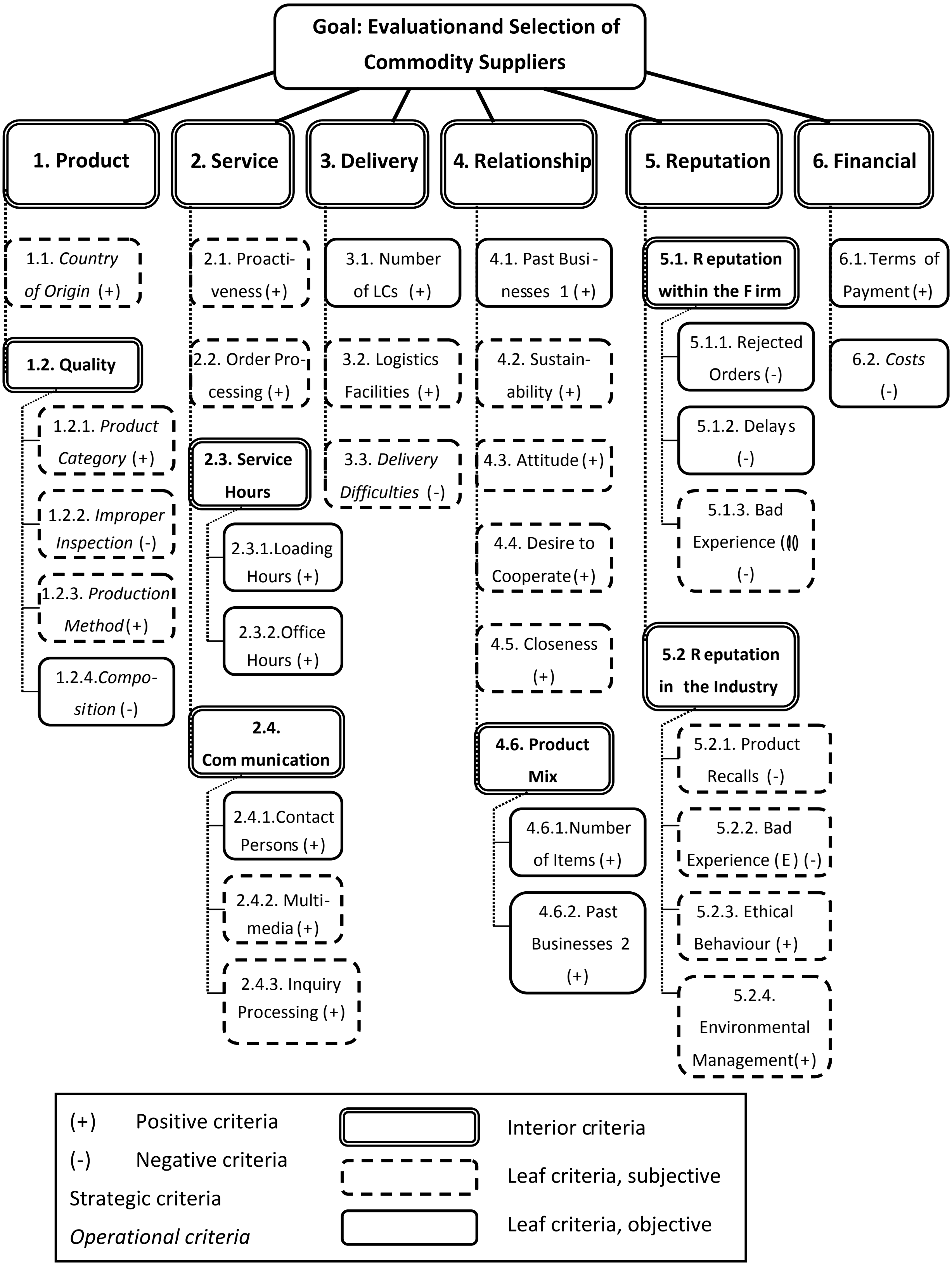

Figure 2: Structure and classification of decision criteria for evaluation of crop suppliers

(5) Determining the DMs' α-voting power

Credibility of members who act as a cooperative decision making community, affects the degree of accuracy of the decision made (Parsa & Parand, 2007). If credibility of decision-makers are equal, then the group of decision-makers is deemed a homogeneous one. Otherwise, the group is deemed a heterogeneous (non-homogeneous) group (Chou et al., 2008). Parsa and Parand (2007) proposed an approach to estimate the credibility of DMs based upon the opinion of other members of DMs' community within a dynamic environment. DM's relative ability to estimate criteria according to their importance is reflected by the α-voting power $w^{\alpha}(DM^{mna}, c_{i_{v1}\dots i_{vl}})$ (Sodenkamp & Suhl, 2012). Relative importance weights of the m-th division $(w^{\alpha}(DM^{m}, c_{i_{v1}\dots i_{vl}}))$ signifies level of contribution of this arrangement to the estimation of criteria weights. Local importance weight of the mn-th cluster of experts $(w^{\alpha}(DM^{mn}, c_{i_{v1}\dots i_{vl}}))$ defines relative impact of this cluster within its divisions in sense of estimating importance of the criterion $c_{i_{v1}\dots i_{vl}}$. The local voting power reflects the relative credibility of a DM within the group of experts he/she belongs to. The global voting power represents DMs' relative influence within the committee and can be calculated using Equation (1):

$$\boldsymbol{w^{\alpha}{}_{G}(DM^{mna}, c_{i_{v1}\dots i_{vl}}) = w^{\alpha}(DM^{m}, c_{i_{v1}\dots i_{vl}}) \cdot w^{\alpha}(DM^{mn}, c_{i_{v1}\dots i_{vl}}) \cdot w^{\alpha}(DM^{mna}, c_{i_{v1}\dots i_{vl}})} \quad (1)$$

where

$$\boldsymbol{w^{\alpha}{}_{G}(DM^{mna}, c_{i_{v1}\dots i_{vl}}), w^{\alpha}(DM^{m}, c_{i_{v1}\dots i_{vl}}), w^{\alpha}(DM^{mn}, c_{i_{v1}\dots i_{vl}}), w^{\alpha}(DM^{mna}, c_{i_{v1}\dots i_{vl}}) \in [0,1]}$$

$$\boldsymbol{\sum_{m=1}^{M} w^{\alpha}(DM^{m}, c_{i_{v1}\dots i_{vl}}) = 1}, \quad \boldsymbol{\sum_{n=1}^{N^{m}} w^{\alpha}(DM^{mn}, c_{i_{v1}\dots i_{vl}}) = 1}, \quad \boldsymbol{\sum_{a=1}^{A^{mn}} w^{\alpha}(DM^{mna}, c_{i_{v1}\dots i_{vl}}) = 1},$$

$$\sum_{a=1}^{A} w^{\alpha}{}_{G}(DM^{mna}, c_{i_{v1}\dots i_{vl}}) = 1 \text{ for } \forall m, n, a, c_{i_{v1}\dots i_{vl}}.$$

If cluster $\boldsymbol{\{DM^{mn}\}}$ is homogeneous, then $w^{\alpha}(DM^{mna}, c_{i_{v1}\dots i_{vl}}) = \frac{1}{A^{mn}}$ for $\forall DM^{mna}$.

In the crop SSOA case study, the decision committee was heterogeneous, relative α-voting power values of its divisions and clusters of experts were defined as follows: $\boldsymbol{w\{DM^{m=1}\} = 0.80}$, $\boldsymbol{w\{DM^{m=2}\} = 0.20}$, $\boldsymbol{w\{DM^{m=1,n=1}\} = 0.75}$, $\boldsymbol{w\{DM^{m=1,n=2}\} = 0.25}$,

$\boldsymbol{w\{DM^{m=2,n=1}\} = 0.60}$, $\boldsymbol{w\{DM^{m=2,n=2}\} = 0.40}$. Since each cluster $\boldsymbol{\{DM^{mn}\}}$ was comprised of a single DM, $w(DM^{mna}) = 1$ for $\forall m, n, a$. The global $\boldsymbol{\alpha}$-voting power weights were derived using Equation (1) and are given in Table 2.

Table 2: Global α-voting power of the DMs

DMs, DM^{mna}	Top-manager, DM^{11}	Purchasing Manager, DM^{12}	Customer 1, DM^{21}	Customer 2, DM^{22}
Global $\boldsymbol{\alpha}$-voting power, $w^{\alpha}{}_G(DM^{mna}, c_{i_{v1}\dots i_{vl}})$	0.60	0.20	0.12	0.08

(6) Setting criteria importance weights

Weights of criteria reflect their gravity to the DMs. In multi-criteria analysis they usually have subjective nature. In commodity trade SSOA criteria reflect purchasing strategy of the buying firm. Methods of weighting include indifference (Rausser & Yassour, 1981; Delforce & Hardaker, 1985), SMART (Simple Multi-Attribute Rating Technique; Edwards, 1977; von Winterfeldt & Edwards, 1986), preference programming (Liesiö et al., 2007), SWING (von Winterfeldt & Edwards, 1986), AHP (Analytic Hierarchy Process; Saaty & Sodenkamp, 2010) and others. Hayashi (2000) and Mustajoki et al. (2005) discussed about difficulties in weighting and compared different methods. Hayashi (2000) stressed that selection of weight assessment procedure depends on the interpretation of weights. Magnitude of criteria effects should be estimated by the $\boldsymbol{\alpha}$-level DMs with $w^{\alpha}{}_G(DM^{mna}, c_{i_{v1}\dots i_{vl}}) > 0$. Once all criteria weights are revealed, distributive normalization of these weights is carried out in order to ensure the conformity of all measurement magnitudes. The normalized local criteria weights are calculated using Equation (2):

$$w^{mna}{}'(c_{i_{v1}\dots i_{vl} i_{v(l+1)}}) = \frac{w^{mna}(c_{i_{v1}\dots i_{vl} i_{v(l+1)}})}{\sum_{i_{v(l+1)}=1}^{I^{i_{v1}\dots i_{vl}}} w^{mna}(c_{i_{v1}\dots i_{v} i_{v(l+1)}})} \tag{2}$$

where $\sum_{i_{v(l+1)}=1}^{I^{i_{v1}\dots i_{vl}}} w^{mna}{}'(c_{i_{v1}\dots i_{v(l+1)}}) = 1$ for $\forall DM^{mna}, \hat{c}_{i_{v1}\dots i_{vl}}$.

Figure 3 illustrates importance weights of four top-level decision criteria estimated by four α-level DMs in the international crop SSOA case study.

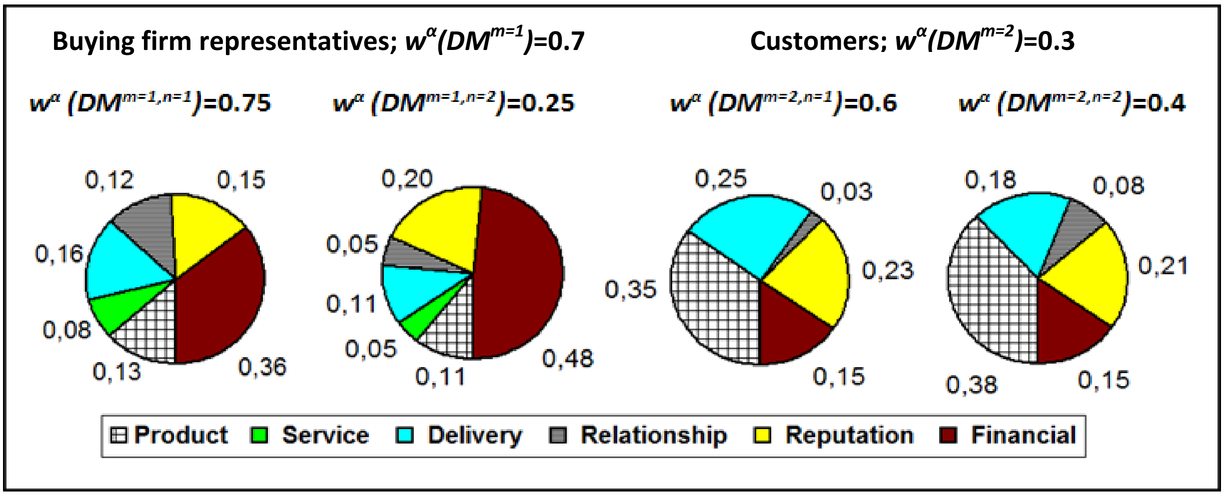

Figure 3: Weights of the top-level decision criteria assigned by the α-level DMs

Criteria weights for the group of all responsible α-level DMs can be obtained by Formula (3).

$$w(c_{i_{v1}\dots i_{vl}}) = \sum_{DM^{mna}} w^{mna}(c_{i_{v1}\dots i_{vl}}) \cdot w^{\alpha}{}_G(DM^{mna}, c_{i_{v1}\dots i_{vl}}) \tag{3}$$

The global weight of leaf criteria are calculated using Equation (4). Global weight represents the portion of the decomposition of unity that criterion receives within the hierarchy, or in other words, criterion importance within the whole value system.

$$w_G(c^*_{i_{v1}\dots i_{vl}}) = \prod_{vl} w(c_{i_{v1}\dots i_{vl}}) = w(\hat{c}_{i_{v1}}) \cdot w(\hat{c}_{i_{v1}i_{v2}}) \cdot \ldots \cdot w(c^*_{i_{v1}\dots i_{vl}}) \tag{4}$$

Global leaf criteria weights in the commodity SSOA case study are shown in Table 3.

(7) Identifying candidate suppliers and generating their combinations

Generally, in MCDM there are two types of alternatives. The first is defined as a single element, and the second is defined as combination of elements (Hayashi, 2000). In SSOA problem these types of alternatives correspond to a single and a multiple sourcing scenarios. On this step, the DMs first identify *D* suppliers available for selection. In a case of multiple sourcing suppliers selected to act jointly may give better of worse performance results on one or several parameters than vendors selected independently. That is, if a demand should be fulfilled by several suppliers a positive and negative synergy that can occur among them must be taken into consideration. In order to enable assessment of suppliers' joint and individual performance, supplier combinations (G_j) that represent alternative solutions have to be formed and evaluated. The final choice is then made among these combinations. The number of generated solutions in general case is determined by Formula (5):

$$J = \sum_{d^*=1}^{D} \frac{D!}{(d^*)!(D-d^*)!} \qquad (5)$$

In Figure 4 is shown the scheme of alternatives generation, and account for the synergistic and not-synergistic criteria.

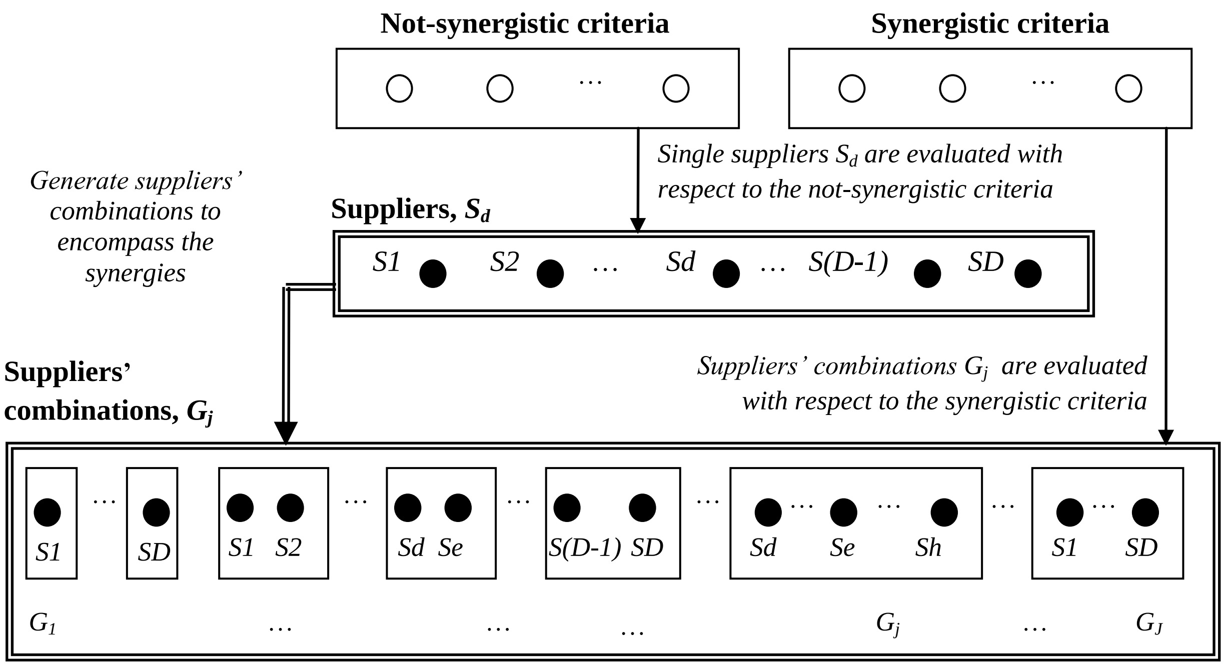

Figure 4: Generation and evaluation of the alternatives in SSOA problems involving suppliers' synergy

In the investigated case study suppliers S_1, S_2 and S_3 ($D=3$) were available for the selection. Then, alternative solutions (G_j) were generated in order to encompass suppliers' synergetic performance on the criterion "Costs". According to (5), the number of generated supplier combinations is $J = \frac{3!}{1!(3-1)!} + \frac{3!}{2!(3-2)!} + \frac{3!}{3!(3-3)!} = 7$. The discrete alternatives

(G_j; $j=1,...,7$) are: $\boldsymbol{G_1=\{S_1\}}$, $\boldsymbol{G_2=\{S_2\}}$, $\boldsymbol{G_3=\{S_3\}}$, $\boldsymbol{G_4=\{S_1,S_2\}}$, $\boldsymbol{G_5=\{S_1,S_3\}}$, $\boldsymbol{G_6=\{S_2,S_3\}}$ and $\boldsymbol{G_7=\{S_1,S_2,S_3\}}$.

(8) Determining β-voting power of the DMs

Level of DM's expertise in estimating alternatives' performance on the subjective criteria is called β-voting power (Sodenkamp & Suhl, 2012). The DMs who participate in the evaluation of alternatives are called β-level DMs. DM's β-voting power for estimating suppliers' performance on subjective not-synergistic leaf criteria $c*^{Sbj,\bar{\sigma}}_{i_{v1}...i_{vl}}$ can be expressed by the set $\boldsymbol{\{w^{\beta}(DM^{mna},S_d)\}^{c*^{Sbj,\bar{\sigma}}_{i_{v1}...i_{vl}}}}$. β-voting power is a value from the interval from 0 to 1: $\boldsymbol{w^{\beta}(DM^{mna},c*^{Sbj,\bar{\sigma}}_{i_{v1}...i_{vl}},S_d)\in[0,1]}$. All DMs' credibility indices have to be normalized in order to assure conformity of their magnitudes: $\sum_{DM^{mna}} w^{\beta}(DM^{mna},c*^{Sbj,\bar{\sigma}}_{i_{v1}...i_{vl}},S_d)=1$. $w^{\beta}(DM^{mna},c*^{Sbj,\bar{\sigma}}_{i_{v1}...i_{vl}},S_d)=0$ means that the mna-th DM is not empowered to estimate performance of the d-th supplier with respect to the $i_{v1}...i_{vl}$-th subjective not-synergistic leaf criterion.

DMs' β-voting power for assessing suppliers S_1, S_2 and S_3 with respect to the criteria "Sustainability", "Country of Origin" and "Product Recalls" in the crop SSOA case study is demonstrated Table 3. The first customer (DM^{21}) does not estimate sustainability of relations between the buyer and supplier S_1, because he is not informed about this issue, therefore, $w^{\beta}(DM^{21},c*_{45},S_1)=0$. In spite of this, DM^{21} is concerned about this factor which is reflected by his α-voting power.

For each synergistic criterion $c*^{Sbj,\sigma}_{i_{v1}...i_{vl}}$ credibility values of the responsible for supplier evaluation DMs are expressed by the set $\boldsymbol{\{w^{\beta}(DM^{mna},G_j)\}^{c*^{Sbj,\sigma}_{i_{v1}...i_{vl}}}}$, where $\boldsymbol{w^{\beta}(DM^{mna},c*^{Sbj,\sigma}_{i_{v1}...i_{vl}},G_j)\in[0,1]}$ and $\sum_{DM^{mna}} w^{\beta}(DM^{mna},c*^{Sbj,\sigma}_{i_{v1}...i_{vl}},G_j)=1$.

$w^{\beta}(DM^{mna}, c*^{Sbj,\sigma}_{i_{v1}...i_{vl}}, G_j) = 0$ means that the mna-th DM does not estimate the j-th combination with respect to the $i_{v1}...i_{vl}$-th subjective synergistic criterion.

(9) Assessing the TVPs of suppliers' combinations

Once all relevant criteria and alternatives are identified by the DMs and structured, collection of the objective data and subjective DMs' performance estimates must be made. For the estimation of how well objectives are being achieved by the alternatives criteria measures have to be developed (Hamilton & Chervany, 1981). If quantitative data is available, it should be used to assess alternatives on the objective criteria. Otherwise, expert judgments have to be provided to extract soft performance information, which are then quantified. Measurement of supplier performance usually includes heterogeneous scales and units. Criteria units in the SSOA case study are shown in Table 1, column "Units".

(9.1) Measuring performance of suppliers and their combinations

On this step four types of performance measures must be provided under condition that measurement units are identical for all suppliers or their groups when making evaluations on each criterion. First, quantitative data describing performance of D individual suppliers on each objective not-synergistic criterion has to be defined and represented by a single numerical value $p(S_d, c*^{Obj,\bar{\sigma}}_{i_{v1}...i_{vl}})$. Second, supplier's performance measures on all objective synergistic criteria ($p(G_j, c*^{Obj,\sigma}_{i_{v1}...i_{vl}})$) must be revealed. Third, DMs' judgments using absolute or relative measurement scale must be provided to estimate suppliers' performance on all subjective not-synergistic criteria: $p^{mna}(S_d, c*^{Sbj,\bar{\sigma}}_{i_{v1}...i_{vl}})$ for $\forall DM^{mna} \middle| w^{\beta}(DM^{mna}, c*^{Sbj,\bar{\sigma}}_{i_{v1}...i_{vl}}, S_d) > 0$. Fourth, DMs' estimates reflecting performance of suppliers' sets on subjective synergistic factors must be provided and quantified: $p^{mna}(G_j, c*^{Sbj,\sigma}_{i_{v1}...i_{vl}})$ for $\forall DM^{mna} \middle| w^{\beta}(DM^{mna}, c*^{Sbj,\sigma}_{i_{v1}...i_{vl}}, G_j) > 0$.

Table 3 provides performance values of the three individual crop suppliers on the objective not-synergistic criteria "Composition", "Past businesses 1" and "Terms of Payment", as well as their estimates on the subjectively examined not-synergistic criteria "Sustainability", "Country of Origin" and "Product recalls" by the β–level DMs with various voting power.

Table 3: Suppliers' data on some not-synergistic criteria in the crop SSOA case study

Suppliers, S_d $d=1,2,3$	β-level DMs, DM^{β}	DMs' voting power, $w^{\beta}(DM^{mna},c_i,S_d)$ / DMs' estimates, $p^{mna}(S_d,c_i)$	Subjective not-synergistic criteria, $c*_i^{Sbj,\overline{\sigma}}$			Objective not-synergistic criteria, $c*_i^{Obj,\overline{\sigma}}$		
			Sustain-ability, $c*_{45}$	Country of Origin, $c*_{11}$	Product Recalls, $c*_{521}$	Compo-sition, $c*_{124}$	Past Businesses1, $c*_{41}$	Terms of Payment, $c*_{61}$
S_1	DM^{11}	$w^{\beta}(DM^{11},c_i,S_1)$	0,7	0,25	0,3	20	850.002,77	14
		$p^{11}(S_1,c_i)$	10	0,4	2			
	DM^{12}	$w^{\beta}(DM^{12},c_i,S_1)$	0,3	0,25	0,7			
		$p^{12}(S_1,c_i)$	6	0,38	2			
	DM^{21}	$w^{\beta}(DM^{21},c_i,S_1)$	0	0,3	0			
		$p^{21}(S_1,c_i)$	0	0,5	0			
	DM^{22}	$w^{\beta}(DM^{22},c_i,S_1)$	0	0,2	0			
		$p^{22}(S_1,c_i)$	0	0,34	0			
S_2	DM^{11}	$w^{\beta}(DM^{11},c_i,S_2)$	0,4	0,25	0	30	1.521.561,15	21
		$p^{11}(S_2,c_i)$	6	0,25	0			
	DM^{12}	$w^{\beta}(DM^{12},c_i,S_2)$	0,6	0,25	1			
		$p^{12}(S_2,c_i)$	5	0,32	1			
	DM^{21}	$w^{\beta}(DM^{21},c_i,S_2)$	0	0,3	0			
		$p^{21}(S_2,c_i)$	0	0,2	0			
	DM^{22}	$w^{\beta}(DM^{22},c_i,S_2)$	0	0,2	0			
		$p^{22}(S_2,c_i)$	0	0,33	0			
S_3	DM^{11}	$w^{\beta}(DM^{11},c_i,S_3)$	0,7	0,25	0,8	17	7.645.000,95	14
		$p^{11}(S_3,c_i)$	8	0,35	3			
	DM^{12}	$w^{\beta}(DM^{12},c_i,S_3)$	0,3	0,25	0,2			
		$p^{12}(S_3,c_i)$	7	0,3	1			
	DM^{21}	$w^{\beta}(DM^{21},c_i,S_3)$	0	0,3	0			
		$p^{21}(S_3,c_i)$	0	0,3	0			
	DM^{22}	$w^{\beta}(DM^{22},c_i,S_3)$	0	0,2	0			
		$p^{22}(S_3,c_i)$	0	0,33	0			

Table 4 presents performance of supplier combinations on the synergistic criterion "Costs". The prices offered by the suppliers in Euro per ton of the crop in the case study are: $p(S_1,c*^{Obj,\overline{\sigma}}_{PricePerTon})=95$, $p(S_2,c*^{Obj,\overline{\sigma}}_{PricePerTon})=94$ and

$p(S_3, c*^{Obj,\bar{\sigma}}{}_{PricePerTon}) = 97$. Crop price for an order is the sum of suppliers offering prices per ton multiplied by the order quantities:

$$CP(G_j) = \sum_{d=1}^{D_j} p(S_d, c*^{Obj,\bar{\sigma}}{}_{PricePerTon}) \cdot x_{G_j}^{S_d} \tag{6}$$

The problem of optimal route planning and estimation of the associated freight cost can be solved by one of the shortest path algorithms (Fu et al., 2006). In general, the transportation cost is a function of the order quantity $x_{G_j}^{S_d}$: $CD(G_j) = f(x_{G_j}^{S_d})$. In the case study, expected costs of delivery for each combination G_j are given for $Y = 700$ and are shown in Table 4.

Table 4: Suppliers' data on the criterion "Costs" in the crop SSOA case study

Combi-nations, G_j	G_1	G_2	G_3	G_4	G_5	G_6	G_7
Suppliers, S_d	S_1	S_2	S_3	$\{S_1, S_2\}$	$\{S_1, S_3\}$	$\{S_2, S_3\}$	$\{S_1, S_2, S_3\}$
Crop price, $CP(G_j)$	$95 \cdot x_{G_1}^{S_1}$	$94 \cdot x_{G2}^{S_2}$	$97 \cdot x_{G_3}^{S_3}$	$95 \cdot x_{G_4}^{S_1} + 94 \cdot x_{G_4}^{S_2}$	$95 \cdot x_{G_5}^{S_1} + 97 \cdot x_{G_5}^{S_3}$	$94 \cdot x_{G6}^{S_2} + 97 \cdot x_{G_6}^{S_3}$	$95 \cdot x_{G_7}^{S_1} + 94 \cdot x_{G_7}^{S_2} + 97 \cdot x_{G_7}^{S_3}$
Cost of delivery, $CD(G_j)$	820	1050	970	1150	1100	1200	1250

The overall financial costs associated with each supplier combination are the sum of the commodity price and the cost of delivery:

$$p(G_j, c*^{Obj,\sigma}{}_{Costs}) = CP(G_j) + CD(G_j) \tag{7}$$

(9.2) Aggregating suppliers' values on subjective criteria within the group

In order to find combined DMs' estimates of suppliers' performance on subjective not-synergistic and synergistic criteria, Equations (8) and (9) should be used respectively:

$$p(S_d, c*^{Sbj,\overline{\sigma}}_{i_{v1}\dots i_{vl}}) = \sum_{DM^{mna}} p^{mna}(S_d, c*^{Sbj,\overline{\sigma}}_{i_{v1}\dots i_{vl}}) \cdot w^{\beta}(DM^{mna}, c*^{Sbj,\overline{\sigma}}_{i_{v1}\dots i_{vl}}, S_d)$$
$$\text{for } \forall DM^{mna} \Big| w^{\beta}(DM^{mna}, c*^{Sbj,\overline{\sigma}}_{i_{v1}\dots i_{vl}}, S_d) > 0 \qquad (8)$$

$$p(G_j, c*^{Sbj,\sigma}_{i_{v1}\dots i_{vl}}) = \sum_{DM^{mna}} p^{mna}(G_j, c*^{Sbj,\overline{\sigma}}_{i_{v1}\dots i_{vl}}) \cdot w^{\beta}(DM^{mna}, c*^{Sbj,\sigma}_{i_{v1}\dots i_{vl}}, G_j)$$
$$\text{for } \forall DM^{mna} \Big| w^{\beta}(DM^{mna}, c*^{Sbj,\sigma}_{i_{v1}\dots i_{vl}}, G_j) > 0 \qquad (9)$$

DMs' estimates of the first supplier with respect to the criterion "Country of origin" is calculated as follows: $p(S_1, c*_{c_{11}}) = 0{,}25 \cdot 0{,}4 + 0{,}25 \cdot 0{,}38 + 0{,}3 \cdot 0{,}5 + 0{,}2 \cdot 0{,}34 = 0{,}413$. The other group suppliers' estimates from Table 4 are: $p(S_1, c*_{45}) = 8{,}8$, $p(S_1, c*_{521}) = 2$, $p(S_2, c*_{45}) = 5{,}4$, $p(S_2, c*_{11}) = 0{,}269$ $p(S_2, c*_{521}) = 1$, $p(S_3, c*_{45}) = 7{,}7$, $p(S_3, c*_{11}) = 0{,}319$ $p(S_3, c*_{521}) = 2{,}6$.

(9.3) Normalizing suppliers' performance values

In order to combine suppliers' data transmitted by different criteria and expressed using various measurement units, and calculate suppliers' total effectiveness, all performance values must be represented in commensurate terms. The most common method called distributive normalization translates numerical values into dimensionless view and relies on the ratio scale. Normalization of suppliers' performance on not-synergistic and synergistic criteria can be made using Equations (10) and (11) respectively:

$$p'(S_d, c*^{\overline{\sigma}}_{i_{v1}\dots i_{vl}}) = \frac{p(S_d, c*^{\overline{\sigma}}_{i_{v1}\dots i_{vl}})}{\sum_{d=1}^{D} p(S_d, c*^{\overline{\sigma}}_{i_{v1}\dots i_{vl}})} \qquad (10)$$

$$p'(G_j, c*^{\sigma}_{i_{v1}\dots i_{vl}}) = \frac{p(G_j, c*^{\sigma}_{i_{v1}\dots i_{vl}})}{\sum_{j=1}^{J} p(G_j, c*^{\sigma}_{i_{v1}\dots i_{vl}})} \qquad (11)$$

where $0 \le p'(S_d, c^{*\bar{\sigma}}_{i_{v1}\dots i_{vl}}) \le 1$, $0 \le p'(G_j, c^{*\sigma}_{i_{v1}\dots i_{vl}}) \le 1$, $\sum_{d=1}^{D} p'(S_d, c^{*\sigma}_{i_{v1}\dots i_{vl}})$

and $\sum_{j=1}^{J} p'(G_j, c^{*\sigma}_{i_{v1}\dots i_{vl}})$.

Table 5 shows normalized performance values of the vendors from Table 3. Similarly, normalization of the rest of performance data in SSOA case study was made.

Table 5: Normalized suppliers' data on some criteria in the crop SSOA case study

Suppliers, S_d $d=1,2,3$	Subjective not-synergistic criteria, $c^{*Sbj,\bar{\sigma}}_i$			Objective not-synergistic criteria, $c^{*Obj,\bar{\sigma}}_i$		
	Sustainability, c^*_{45}	Country of Origin, c^*_{11}	Product Recalls, c^*_{521}	Composition, c^*_{124}	Past Businesses1, c^*_{41}	Terms of Payment, c^*_{61}
	$w_G(c^*_{45})$	$w_G(c^*_{11})$	$w_G(c^*_{521})$	$w_G(c^*_{124})$	$w_G(c^*_{41})$	$w_G(c^*_{61})$
	0,040	0,017	0,026	0,019	0,019	0,028
S_1	0,402	0,413	0,357	0,299	0,085	0,286
S_2	0,247	0,269	0,179	0,448	0,152	0,429
S_3	0,352	0,319	0,464	0,254	0,763	0,286

Normalized financial costs depend on the order quantities. For instance, total normalized costs associated with the fourth alternative are: $p'(G_4, c^*_{62}) = \dfrac{95 \cdot x^{S1}_{G4} + 94 \cdot x^{S2}_{G4} + 1150}{\sum_{j=1}^{7}(CP(G_j) + CD(G_j))}$, where

$\sum_{j=1}^{7} CD(G_j) = 820 + 1050 + 970 + 1150 + 1100 + 1200 + 1250 = 7540$ and

$$\sum_{j=1}^{7} CP(G_j) = 95 \cdot (x^{S1}_{G1} + x^{S1}_{G4} + x^{S1}_{G5} + x^{S1}_{G7}) + 94 \cdot (x^{S2}_{G2} + x^{S2}_{G4} + x^{S2}_{G6} + x^{S2}_{G7}) + 97 \cdot (x^{S3}_{G3} + x^{S3}_{G5} + x^{S3}_{G6} + x^{S3}_{G7})$$

(9.4) Trading-off suppliers' characteristics within each combination

After the identification of suppliers' performance values with respect to all individual factors and their normalization, all parameters should be trade-off for each combination G_j. The

TVP for each alternative solution G_j is composed out of two main components: the overall utility of individual suppliers on not-synergistic criteria and the amalgamated suppliers' value on synergistic criteria. The overall utility of each alternative solution on *not-synergistic criteria* is a sum of differences between summarized weighed positive and summarized weighed negative performances of the individual suppliers composing the combination, multiplied by order quantities assigned to these suppliers:

$$p^{\bar{\boldsymbol{\sigma}}}(G_j) = \sum_{d=1}^{D_j} p^{\bar{\boldsymbol{\sigma}}}(S_d) \cdot x_{G_j}^{S_d} \tag{12}$$

where

$$p^{\bar{\boldsymbol{\sigma}}}(S_d) = p^{Pro,\bar{\boldsymbol{\sigma}}}(S_d) - p^{Con,\bar{\boldsymbol{\sigma}}}(S_d)$$

$$p^{Pro,\bar{\boldsymbol{\sigma}}}(S_d) = \sum_{c*^{\bar{\boldsymbol{\sigma}}Pro}} w_G(c*^{Pro,\bar{\boldsymbol{\sigma}}}{}_{i_{v1}\ldots i_{vl}}) \cdot p'(S_d, c*^{Pro,\bar{\boldsymbol{\sigma}}}{}_{i_{v1}\ldots i_{vl}}) \text{ and}$$

$$p^{Con,\bar{\boldsymbol{\sigma}}}(S_d) = \sum_{c*^{\bar{\boldsymbol{\sigma}},Con}} w_G(c*^{Con,\bar{\boldsymbol{\sigma}}}{}_{i_{v1}\ldots i_{vl}}) \cdot p'(S_d, c*^{Con,\bar{\boldsymbol{\sigma}}}{}_{i_{v1}\ldots i_{vl}})$$

The amalgamated value of the alternative G_j on *synergistic criteria* is a difference between the summarized weighed positive and summarized weighed negative performance values of this combination:

$$p^{\boldsymbol{\sigma}}(G_j) = \sum_{G_j} p^{Pro,\boldsymbol{\sigma}}(G_j) - \sum_{G_j} p^{Con,\boldsymbol{\sigma}}(G_j) \tag{13}$$

where

$$p^{Pro,\boldsymbol{\sigma}}(G_j) = \sum_{c*^{Pro,\boldsymbol{\sigma}}} w_G(c*^{Pro,\boldsymbol{\sigma}}{}_{i_{v1}\ldots i_{vl}}) \cdot p'(G_j, c*^{Pro,\boldsymbol{\sigma}}{}_{i_{v1}\ldots i_{vl}}) \text{ and}$$

$$p^{Con,\bar{\boldsymbol{\sigma}}}(G_j) = \sum_{c*^{Con,\boldsymbol{\sigma}}} w_G(c*^{Con,\boldsymbol{\sigma}}{}_{i_{v1}\ldots i_{vl}}) \cdot p'(G_j, c*^{Con,\boldsymbol{\sigma}}{}_{i_{v1}\ldots i_{vl}})$$

The TVP of suppliers' combinations is the sum of their overall normalized and weighed performance values on all not-synergistic and synergistic criteria:

$$TVP(G_j) = p^{\bar{\boldsymbol{\sigma}}}(G_j) + p^{\boldsymbol{\sigma}}(G_j) \tag{14}$$

In the wheat SSOA case study, the TVP for the fourth alternative becomes: $TVP(G_4) = p^{Pro,\bar{\sigma}}(G_4) - p^{Con,\bar{\sigma}}(G_4) - p^{Con,\sigma}(G_4)$, where

$p^{Pro,\bar{\sigma}}(G_4) = 0{,}112 \cdot x_{G4}^{S1} + 0{,}088 \cdot x_{G4}^{S2}$, $p^{Con,\bar{\sigma}}(G_4) = 0{,}130 \cdot x_{G4}^{S1} + 0{,}128 \cdot x_{G4}^{S1}$ and $p^{Con,\sigma}(G_4) = 0{,}312 \cdot \dfrac{95 \cdot x_{G4}^{S1} + 94 \cdot x_{G4}^{S2} + 1150}{\sum_{j=1}^{7}(CP(G_j) + CD(G_j))}$.

Similarly, the TVPs for the rest six alternatives were derived.

(10) Setting problem constraints and indentifying feasible solutions

In any MODM problem the most appropriate solution ($G*$) has to be selected from the set of all feasible problem solutions. Feasibility implies satisfaction of the alternative solution to the set of U imposed constraints (t_u):

$$G^* \in G_{feasible},\ G_{feasible} \in \{G_j / q_u(G_j) \le t_u,\ u = 1,2,..U\} \tag{15}$$

System constraints are not directly under the company's control while *policy constraints* can be directly influenced by the firm. Internal policy constraints exist either implicitly or explicitly in the buying process for such matters as the number of vendors to employ, minimum and maximum order quantities, the use of minority vendors, etc. (Weber et al., 2000). Similarly, suppliers may impose constraints on the buying process such as their own minimum order quantities or a maximum order quantity based on their production capacity or their willingness to do business with a particular firm (Weber et al., 2000). In the investigated crop SSOA case study, the set of constraints defined by Equations (21) through (25) was sufficient for identifying all feasible solutions ($U = 4$). First, each supplier imposed a constraint on the offered quantity of wheat in tons based on the buyer's credit limit. Equation (16) checks the ability of each combination of suppliers to satisfy the demand under the credit limit constraint:

$$MaxQ_{CL}(G_j) \ge Y \tag{16}$$

where $MaxQ_{CL}(G_j) = \sum_{d=1}^{D_j} MaxQ_{CL}(S_d)$ and $MaxQ_{CL}(S_d) = \dfrac{CL(S_d)}{p(S_d, c*_{PricePerTon})}$

The second (17) and third (18) constraints define the ability of supplier combinations to satisfy the demand under the minimum and maximum offered product quantity restrictions:

$$MinQ_{offered}(G_j) \leq Y \tag{17}$$

where $MinQ_{offered}(G_j) = \sum_{d=1}^{D_j} MinQ_{offered}(S_d)$.

$$MaxQ_{offered}(G_j) \geq Y \tag{18}$$

where $MaxQ_{offered}(G_j) = \sum_{d=1}^{D_j} MaxQ_{offered}(S_d)$.

Finally, the ability to satisfy the demand expressed in tons of wheat, taking into account a shared resource constraint was the fourth requirement for each combination:

$$MaxQ_{shared}(G_j) \geq Y \text{ for } \forall G_j \big| D_j \geq 2 \tag{19}$$

Equations (16), (18) and (19) can be rewritten as follows:

$$MaxQ(G_j) \geq Y \tag{20}$$

where $MaxQ(G_j) = Min\{MaxQ_{CL}(G_j); MaxQ_{Offered}(G_j)\}$ for $\forall G_j \big| D_j = 1$

and $MaxQ(G_j) = Min\{MaxQ_{CL}(G_j); MaxQ_{Offered}(G_j); MaxQ_{shared}(G_j)\}$

for $\forall G_j \big| D_j \geq 2$.

The credit limits defined by each vendor under consideration were: $CL(S_1)$ =31.500,00 Euro, $CL(S_2)$ =33.250,00 Euro and $CL(S_3)$ =40.000,00 Euro. The minimum and maximum offered quantities were: $MinQ_{offered}(S_1)$ =200 Tons, $MinQ_{offered}(S_2)$ =0 Tons, $MinQ_{offered}(S_3)$ =250 Tons, $MaxQ_{offered}(S_1)$ =550 Tons, $MaxQ_{offered}(S_2)$ =450 Tons and $MaxQ_{offered}(S_3)$ =500 Tons. Finally, the maximum available shared resource quantities were: $MaxQ_{shared}(G_4)$ = 800 Tons, $MaxQ_{shared}(G_5)$ = 1000 Tons, $MaxQ_{shared}(G_6)$ = 950 Tons and $MaxQ_{shared}(G_7)$ = 1300 Tons. The demand was $Y = 700$ Tons. Table 6 provides information about feasibility of the alternatives based on the constraints above.

Table 6: Feasibility of decision alternatives in the crop SSOA case study

Combinations, G_j	Suppliers, S_d	$MaxQ(G_j)$	$MinQ(G_j)$	Feasible (F) /Not feasible (NF)
G_1	$\boldsymbol{S_1}$	331,58	200	NF
G_2	$\boldsymbol{S_2}$	353,72	0	NF
G_3	$\boldsymbol{S_3}$	412,37	250	NF
G_4	$\boldsymbol{\{S_1, S_2\}}$	681,58	200	NF
G_5	$\boldsymbol{\{S_1, S_3\}}$	752,63	450	F
G_6	$\boldsymbol{\{S_2, S_3\}}$	779,26	250	F
G_7	$\boldsymbol{\{S_1, S_2, S_3\}}$	1102,63	450	F

Alternatives G_1, G_2, G_3 and G_4 did not satisfy condition (20). Therefore, they were not feasible and had to be excluded from further consideration. G_5, G_6 and G_7 satisfied all constraints and therefore composed a set of feasible alternatives: $\{G_5, G_6, G_7\} \in G_{feasible}$.

(11) Optimizing feasible alternatives and selecting the best option

Defining the best solution is a two-stage procedure. At the first stage, order quantities have to be optimally allocated within each alternative combination, i.e., sub-optimal solutions have to be found. At the second stage, the single best alternative with the maximal TVP, which corresponds to the global optimum G^*, is selected. One of the most commonly used approaches to solve multi-objective optimization problems is weighting method (Weber & Current, 1993; An et al., 2010). Other methods include the ε-constraint method, the goal-attainment method and multi-objective genetic algorithms. For a detailed description of multi-objective solution techniques the interested reader is referred to Cohon (1978), Alves and Climaco (2004) and Dimitras and Üstin (2008). The TVP of alternative supplier combinations has to be maximized.

(11.1) Finding sub-optimal solutions

The sub-optimal solutions are all feasible supplier combinations with optimally allocated order quantities $x_{S_d}^{G_j}$. The order allocation procedure is defined by Model (22).

$TVP(G_j) \rightarrow Maximize$ for $\forall G_j \middle| G_j \in G_{feasible}$, subject to

$$
\begin{aligned}
& x_{S_d}^{G_j} \geq MinQ_{offered}(S_d); \quad && x_{S_d}^{G_j} \leq MaxQ_{offered}(S_d) \\
& x_{S_d}^{G_j} \leq MaxQ_{CL}(S_d); && x_{S_d}^{G_j} \geq 0 \\
& \sum_{d=1}^{D_j} x_{S_d}^{G_j} = Y; && x_{S_d}^{G_j} \text{ - integer}
\end{aligned}
\tag{22}
$$

Restrictions in Model (22) are valid for $\forall S_d \in G_j$ and $\forall G_j \in G_{feasible}$. The TVPs of supplier combinations are mutually dependent due to the component defined by Equation (11). Search of sub-optimal solutions can be made iteratively using the method of convergence. The initial order quantities $x_{S_d}^{G_j} = \frac{Y}{D_j}$ are assigned within each feasible combination and the optimization procedure described by Model (22) is iteratively repeated until $x_{S_d}^{G_j} \rightarrow \lim$. Table 7 presents sub-optimal solutions in the investigated case study.

Table 7: Feasible sub-optimal alternatives in the crop SSOA case study

Feasible alternatives, $G_j \in \{G_{feasible}\}$	Suppliers, S_d	Total value of purchasing, $TVP(G_j)$	Financial costs of purchasing, $p(G_j, c_{Costs})$	Decision variables (Order quantities), $x_{S_d}^{G_j}$	Maximum offered quantities, $MaxQ_{Offered}(S_d)$	Credit limits expressed in tons of crop, $MaxQ_{CL}(S_d)$	Minimum offered quantities, $MinQ(S_d)$
G_5	S_1	0,3467	71.738,00	331,00	550	331,58	200
	S_3			369,00	500	412,37	250
G_6	S_2	0,2950	69.841,00	353,00	450	353,72	0
	S_3			347,00	500	412,37	250
G_7	S_1	0,3484	71.881,00	331,00	550	331,58	200
	S_2			119,00	450	353,72	0
	S_3			250,00	500	412,37	250

(11.2) Finding the global optimum

The global optimum is a combination of vendors with the highest TVP value among all feasible solutions:

$$G^* = G_j \Big| \max_{G_j \in G_{feasible}} TVP(G_j) \tag{23}$$

In the crop SSOA case study $G^* = G_j \Big| \max_{G_j \in G_{feasible}} (0{,}3467; 0{,}2950; 0{,}3484) = G_7$.

Therefore, the order of 700 Tons had to be distributed among the three suppliers S_1, S_2 and S_3 that made up the combination G_7, with the quantities 331, 119 and 250 Tons respectively.

4. Limitations and scope for future work

The presented approach and its application have several shortcomings described below.

1. In the case study, only one synergistic parameter - Cost of delivery is considered, whereas real problems may face with multivariate positive and negative synergism of alternatives. The proposed formal model enables account for performance synergies of alternatives on multiple criteria simultaneously.

2. This research fails to represent and treat uncertainty of the individual and group DMs' judgments regarding criteria importance and suppliers' vague measures on intangible parameters. Moreover, suppliers' ambiguous objective data, such as costs, delivery times, demand, among others, can present. The developed model can be advanced further by representing vague measures using fuzzy numbers (Fu, 2008; Chou et al., 2008; Tavana et al., 2010) or neutrosophic values (Arora & Biswas, 2010), by applying fuzzy AHP (van Laarhoven & Pedrycz, 1983; and Burckley, 1985) or estimating correlation of vague group estimates (Tavana et al., 2009; Tavana & Sodenkamp, 2009; and Aldian & Taylor, 2005).

3. The method does not consider incomplete or unknown data which may influence reliability of the outcome. For future research, I suggest study and development of synergistic SSOA under uncertainty.

4. If conditions entailing non-linearity of suppliers' individual performance, such as account for suppliers' volume discounts, (Kokangul & Susuz, 2009; Ghodsypour & O'Brien, 2001), must be taken into consideration, the linear mathematical programming Model (22) must be modified respectively.

5. The developed set of decision criteria is based on the analysis of scholar and business literature, as well as on the interviews with representatives of the investigated agricultural trading company in Germany. This set can be extended or reduced; the structure of criteria interconnections can be modified and adjusted to the firms' requirements. Future research can

be turned to extension of the presented meta-model by taking into account the above listed shortcomings. Additionally, sensitivity analysis can be performed in order to validate robustness of the outcome.

5. Conclusions

The size and complexity of real-life problems together with their ill-defined nature call for a true synergy between the power of computational techniques and the human capabilities to analyze, envision, reason, and deliberate (Andrienko et al. 2007). The framework is designed to guide and assist in the process of international supplier evaluation and selection by ACTCs within complex supply chains in multi-objective collaborative Just-in-Time environment. Both single- and multiple-sourcing strategies are considered as potential solutions; selection of the appropriate sourcing strategy is situational, it depends on the data set and focuses on the TVP maximization. It is a dynamic method, able to adapt easily to each particular transaction: the strategic and tactical criteria and the data can be actualized which entails revision of the sourcing strategy, choice of vendors and assigned order quantities. Synergy effects that encounter in multiple-sourcing case are systematically incorporated into the evaluation process. Number of DMs and depth of criteria hierarchy are variable. The approach relies on rigorous mathematical methods that utilize objective data and subjective expert judgments. Although the structuring concept of the model is generic, in this paper the multi-criteria choice and allocation problem is customized to the vendor selection and order allocation within agricultural commodity trading industry. This innovative approach can directly assist purchasing managers in taking their day-to-day supplier selection and order quantities assignment. Value of the developed approach and its application expands further as it provides a unified procedure for all trading and manufacturing companies to exploit the results in order to assist DMs in taking transparent and traceable purchasing decisions concerning all types of merchandise and services and acts as an improvement tool for potential vendors.

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NOMENCLATURE

a	DM in the cluster of experts; ($a = 1,..., A^{nm}$)
A^{mn}	Total number of DMs in the n-th cluster of experts of the m-th division;
$\hat{c}$	Interior decision criterion;
$c*$	Leaf decision criterion;
$CD(G_j)$	Cost of delivery associated with the j-th alternative;
$c_{i_{v1}...i_{vl}}$	$(i_{v1}...i_{vl})$-th decision criterion; ($i_{vl} = 1,..., I_{vl}$)
$CL(S_d)$	Credit limit imposed by the d-th supplier;
c^{Obj} (c^{Sbj})	Objective (subjective) decision criterion;
$CP(G_j)$	Total commodity price associated with the j-th alternative;
c^{Con} (c^{Pro})	Negative (positive) decision criterion;
c^{σ} ($c^{\overline{\sigma}}$)	Synergistic (not-synergistic) decision criterion;
D	Total number of suppliers available for the selection;
D_j	Number of suppliers within the j-th combination;
DM^{mna}	a-th decision maker from the n-th cluster of experts of the m-th division of decision committee;
G_j	Alternative supplier combination; ($j = 1,..., J$)
$G_{feasible}$	Feasible alternative;
$Goal_{v0}$	The root of criteria tree, i.e. decision goal;
$G*$	The best alternative;
I	Total number of decision criteria;
I^{Con} (I^{Pro})	Total number of negative (positive) decision criteria;
I_{vl}	Total number of sub-criteria of the $(i_{v1}...i_{v(l-1)})$-th criterion; ($i_{v(l-1)} = 1,..., I_{v(l-1)}$)
$i_{v1}...i_{vl}$	D-notation of a criterion on the l-th level of the tree T;
J	Total number of alternative supplier combinations;

l	Level of criteria tree; ($l = 1,..., L$)
L	Total number of levels within the tree of criteria;
m	Division of the decision committee; ($m = 1,..., M$)
M	Total number of divisions in the committee;
$MaxQ(G_j)$	Maximum quantity available for order from the j-th supplier set;
$MaxQ_{CL}(G_j)$	Maximum available order quantity for the j-th supplier combination based on the credit limit;
$MaxQ_{CL}(S_d)$	Maximum available order quantity for the d-th supplier based on the credit limit;
$MaxQ_{offered}(G_j)$	Maximum quantity available for order from the j-th supplier set, based on the maximum offered quantities of individual suppliers;
$MaxQ_{offered}(S_d)$	Maximum quantity offered by the d-th supplier;
$MaxQ_{shared}(G_j)$	Maximum quantity available for order from the j-th supplier combination, based on the shared resource quantities;
$MinQ_{offered}(G_j)$	Minimum overall order quantity offered by the suppliers from the j-th combination;
$MinQ_{offered}(S_d)$	Minimum order quantity offered by the d-th supplier;
n	Cluster of experts in the division; ($n = 1,..., N^m$)
N^m	Total number of clusters of experts in the m-th division;
$p(G_j, c*^{Sbj,\sigma}_{i_{v1}...i_{vl}})$	Performance of the j-th supplier combination on the $(i_{v1}...i_{vl})$–th subjective synergistic criterion estimated by the decision committee;
$p(S_d, c*^{Sbj,\overline{\sigma}}_{i_{v1}...i_{vl}})$	Performance of the d-th supplier on the $(i_{v1}...i_{vl})$–th subjective not-synergistic criterion estimated by the decision committee;
$p^{Con,\sigma}(G_j)$	Performance of the j-th alternative on all negative synergistic criteria;
$p^{Con,\overline{\sigma}}(S_d)$	Performance of the d-th supplier on all negative not-synergistic criteria;
$p^{mna}(S_d, c*^{Sbj,\overline{\sigma}}_{i_{v1}...i_{vl}})$	Performance of the d-th supplier on the $(i_{v1}...i_{vl})$–th subjective not-synergistic criterion estimated by the mna-th DM;

$p^{mna}(G_j, c*^{Sbj,\sigma}_{i_{v1}...i_{vl}})$ Performance of the j-th supplier combination on the $(i_{v1}...i_{vl})$–th subjective synergistic criterion estimated by the mna-th DM;

$p^{Pro,\sigma}(G_j)$ Performance of the j-th alternative on all positive synergistic criteria;

$p^{Pro,\overline{\sigma}}(S_d)$ Performance of the d-th supplier on all positive not-synergistic criteria;

$p'(S_d, c*^{\overline{\sigma}}_{i_{v1}...i_{vl}})$ Normalized performance of the d-th supplier on the $(i_{v1}...i_{vl})$–th not-synergistic criterion;

$p'(G_j, c*^{\sigma}_{i_{v1}...i_{vl}})$ Normalized performance of the j-th combination on the $(i_{v1}...i_{vl})$–th synergistic criterion;

$p^{\sigma}(G_j)$ Performance of the j-th alternative on all synergistic criteria;

$p^{\overline{\sigma}}(G_j)$ Performance of the j-th alternative on all not-synergistic criteria;

$p^{\overline{\sigma}}(S_d)$ Performance of the d-th supplier on all not-synergistic criteria;

$q_u(G_j)$ Characteristics of the j-th combination checked for conformity to the u-th constraint;

S_d Supplier available for the selection; ($d = 1,..., D$)

T Tree of decision criteria;

t_u Problem constraint; ($u = 1,..., U$)

$TVP(G_j)$ Total value of purchasing for the j-th alternative supplier combination;

vL Depth of criteria tree;

$w(c_{i_{v1}...i_{vl}})$ Local importance of the $(i_{v1}...i_{vl})$-th criterion for the decision committee;

$w_G(c_{i_{v1}...i_{vl}})$ Global importance of the $(i_{v1}...i_{vl})$-th criterion for the decision committee;

$w^{mna}(c_{i_{v1}...i_{vl}})$ Importance of the $(i_{v1}...i_{vl})$-th criterion assigned by the mna-th DM;

$w^{\alpha}(DM^m, c_{i_{v1}...i_{vl}})$ Local α-voting power of the m-th division for estimating importance of the $(i_{v1}...i_{vl})$-th criterion;

$w^{\alpha}(DM^{mn}, c_{i_{v1}...i_{vl}})$ Local α-voting power of the n-th cluster of expert within the m-th division for estimating importance of the $(i_{v1}...i_{vl})$-th criterion;

$w^{\alpha}(DM^{mna}, c_{i_{v1}...i_{vl}})$ Local α-voting power of the mna-th DM for estimating importance of the $(i_{v1}...i_{vl})$-th criterion;

$w^{\alpha}{}_{G}(DM^{mna}, c_{i_{v1}...i_{vl}})$ Global α-voting power of the mna-th DM for estimating importance of the $(i_{v1}...i_{vl})$-th criterion;

$w^{\beta}(DM^{mna}, c*^{Sbj,\sigma}{}_{i_{v1}...i_{vl}}, G_j)$ β-voting power of the mna-th DM for estimating performance of the j-th supplier combination with respect to the $(i_{v1}...i_{vl})$–th subjective synergistic criterion;

$w^{\beta}(DM^{mna}, c*^{Sbj,\bar{\sigma}}{}_{i_{v1}...i_{vl}}, S_d)$ β-voting power of the mna-th DM for estimating performance of the d-th supplier with respect to the $(i_{v1}...i_{vl})$–th subjective not-synergistic criterion;

$x_{G_j}^{S_d}$ Optimal order quantity to be purchased from the d-th supplier considered within the j-th combination;

Y Demand of commodity under consideration.

A Multicriteria Multilevel Group Decision Method for Supplier Selection and Order Allocation

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ABSTRACT

Supplier selection is an integral part of supply chain management (SCM). It plays a prominent role in the purchasing activity of manufacturing and trading companies. Evaluation of vendors and procurement planning requires simultaneous consideration of tangible and intangible decision factors, some of which may conflict. A large body of analytical and intuitive methods has been proposed to trade off conflicting aspects of realism and optimize the selection process. In the large companies the fields of decision makers' (DMs) expertise are highly distributed and DMs' authorities are unequal. On the other hand, the decision components and their interactions are very complex. These facts restrict the effectiveness of using the existing methods in practice. The authors present a multicriteria decision analysis (MCDA) method which facilitates making supplier selection decisions by the distributed groups of experts and improves quality of the order allocation decisions. A numerical example is presented and applicability of the proposed algorithm is demonstrated in the Raiffeisen Westfalen Mitte, eG in Germany.

Keywords: *Multi-Criteria Decision Analysis (MCDA), Multilevel Group Decision Making (MLGDM), Order Allocation, Supplier Selection, Supply Chain Management*

INTRODUCTION

The formalization of complex decision problems requires comprehensive and accurate modeling of the problem environment, its elements and their interactions. Selection of the valid solution methods for such problems is a very challenging task. Fictitious simplifications of decision situations lead to management debacles and loss of profits. To avoid this, the research efforts should be focused on the *flexible decision aiding framework* which could enable problem-oriented modularization of the decision processes, their exhaustive analysis by a set of appropriate and consistent methods and generation of robust solutions. A variety of empirical studies have been conducted to improve decision making in teams. Still, the complex nature of decision groups has been left without proper attention in the analytical decision science. To fill this gap we first introduce notions of Multilevel Group Decision Making (MLGDM) to distinguish between the α, β and γ decision makers (DMs). α-voting power is

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proposed to elicit DMs' contribution to criteria prioritization; β-voting power is used to measure experts ability to evaluate performance of alternatives with respect to the set of direct decision criteria; γ-voting power index reflects the DMs' expertise in evaluation of auxiliary decision components on indirect criteria.

Presented in this paper; the case study was completed in cooperation with Raiffeisen Westfalen Mitte eG (also referred to as Raiffeisen), an agricultural cooperative society operating in Germany, Nord-Rhein Westfalen, since the 18th century, with annual turnover exceeding 275 Mio. Euro in 2010. One of the largest trading companies of crops, animal feed and fertilizers also selling fuel oils is a significant aspect of the company's strategy. We have developed a structured, multilevel group MCDA framework to aggregate multiple objective factors and group subjective expert judgments to enable the strategic evaluation of fuel oil suppliers and optimizing purchasing activity by aligning strategic priorities of the DMs with their daily decisions.

PROPOSED ALGORITHM AND ITS APPLICATION

Taking complex multicriteria decision, including purchasing, is a consequent multistage process. We designed the algorithm that includes 16 steps summarized below.

1. Identify Overall Purpose of the Decision

A first step of MCDA is to establish a clear goal pursued. Generally decision theory deals with three main types of problems: choice, ranking and classification (Zopounidis, 2002). Choice is selection of the most appropriate alternative or set of alternatives. Generally, organizations have two approaches to supplier selection. The first approach is to select the best single supplier, which can meet all the requirements (single sourcing). The second approach is to select an appropriate combination of suppliers (multiple sourcing) (Sanayei et al., 2008). Ranking of suppliers is ordering of alternatives based on measuring of their contribution to the achievement of the stated decision objectives. Classification is division of alternatives into predefined homogeneous classes which are not necessarily ordered, on the other hand in sorting problems groups are ordered from the best to worst (Zopounidis, 2002). The proposed multilevel group framework is aimed at performing the following analytical functions:

- Derive consensus based rankings of suppliers in accordance with their strategic performance. Rankings serve as a legitimate and transparent foundation for establishment of partnership strategies, selection of long-term contractors and stimulation of supplier development.
- Classification of vendors into the groups reflecting their relative competitive advantages and disadvantages.
- Support Just-In-Time (JIT) purchasing decisions for trading activity based on market-rate prices taking into consideration compound strategic weights of vendors.

2. Form Decision Making Group

Once the goal is stated, a group comprised of the people responsible for the successful implementation of the decision must be formed. Zeleny (2010) asserts that any DM makes a decision either for himself or for others, therefore a distinction between the *decision producer* (or provider) and *decision consumer* (or customer) has to be drawn. According to the Crown copyright Multi-criteria analysis manual (Crown, 2009) there are two main types of DMs: *stakeholders* whose organization's values should find expression in the decision, and *key players* who can make a useful and significant contribution to the MCDA and represent important perspectives on the subject of the analysis. Numerical reviews in the field of decision making have concluded that groups learn faster, make fewer errors, recall better, make better decisions, and are more productive, with a higher-quality product than individuals

(Baron et al., 1992; Davis, 1969; Johnson & Johnson, 2003; Laughlin & Early, 1982). The decision group may include:

- Board members and CEOs having clear understanding of the organization's strategy;
- Subject matter experts at various levels having insight to evaluate organization environment, functional design of the company and specific areas of its activity, including purchasing;
- Representatives who can properly define what suppliers should be considered in the decision process;
- Experts who can provide reasonable estimates for proposed suppliers; and
- Operative managers who are deeply involved with the issue at hand and will implement the decision.

Group decision making does not mean that all team members have to be involved in every aspect of a decision; instead they are expected to process data and apply their individual expertise to contribute to the outcome (Saaty & Peniwati, 2008).

In the proposed decision analysis framework is considered a group of K DMs ($k = 1, 2, \ldots, K$). In the study conducted for Raiffeisen was organized a decision group including a Raiffeisen's board member and managers from purchasing department ($K = 3$).

3. Define, Describe and Structure a Finite Set of Decision Criteria

In modern management, one needs to consider many factors with the aim of developing a long-term supplier relationship (Muralidharan et al., 2006). Choosing the right suppliers involves much more than scanning a series of price lists, and choices will depend on a wide range of factors which involve both quantitative and qualitative (Ho et al., 2010). The multi-criteria decision models allow the integration of both objective and subjective criteria to produce an aggregate performance measure (Akarte et al., 2001). In the numerous scientific publications it is clearly indicated that vendor selection has a multi-objective nature implying that multiple conflicting criteria need to be considered in the supplier evaluation and selection process (Dickson, 1966; Weber et al., 1991). These criteria must be defined according to the corporate strategies and the company's competitive situation (Sanayei et al., 2008). According to Bouyssou (1990), the criteria set must have two key qualities; be readable (i.e., include a number of criteria restricted enough so that it is possible to reason on this basis and eventually to model the inter and intra-criteria information required to perform an aggregation procedure) and be operational (i.e., be acceptable as a working basis for the study). Even Swaps method (Hammond et al., 1998) can be applied to simplify the complex decision and reduce the number of objectives in the consequences table.

In multi-criteria analysis decision factors can be grouped into contradictory categories. First classification approach for making trade-offs among various indicators was outlined by Benjamin Franklin in 1772 in his "Moral of prudential algebra" and is known as method of Pros and Cons (Hammond et al., 1998). Other classification schemes include opportunities and threats for evaluation of strategic courses of action (Tavana & Sodenkamp, 2010), division into benefits and costs (Triantaphyllou & Baig, 2005), internal strength and weaknesses along with external opportunities and threats (SWOT) (Tavana et al., 2010; Ghazinoory et al., 2011) or alternatively, consideration of existing benefits and opportunities and potential costs and risks (BOCR) (Saaty & Sodenkamp, 2010). Performance of alternatives on positive criteria has to be maximized and on negative criteria minimized. When the number of factors is large, typically more than a dozen, they may be arranged hierarchically (Saaty, 1977; Triantaphyllou, 2000) or as a feedback network (Saaty, 1996). Such structures allow for a systematic grouping of metrics in complex decision problems.

Proposed here, the supplier evaluation model is based on the *Pros&Cons* classification, where each class contains a three-level hierarchy of criteria. Let us define:

M The total number of groups of factors; ($m = 1,2,...,M$)
N The total number of decision criteria; ($n = 1,2,...,N$)
L The total number of sub-criteria; ($l = 1,2,...,L$)
C^{Pros} (C^{Cons}) The cluster "*Pros*" ("*Cons*") including subjective and objective positive (negative) factors;
C_m^{Pros} (C_m^{Cons}) The *m*-th group of factors within the *Pros* (*Cons*) cluster; ($m = 1,2,...,M^{Pros}$ ($m = 1,2,...,M^{Cons}$))
M^{Pros} (M^{Cons}) The number of groups of factors within the *Pros* (*Cons*) cluster;
N^{Pros} (N^{Cons}) The number of attributes within the *Pros* (*Cons*) cluster;
L^{Pros} (L^{Cons}) The number of sub-criteria within the *Pros* (*Cons*) cluster;
N^{Obj} (N^{Sbj}) The number of objective (subjective) decision criteria; ($n = 1,2,...,N$)
L^{Obj} (L^{Sbj}) The number of objective (subjective) sub-criteria; ($l = 1,2,...,L$)
C_{mn}^{Pros} (C_{mn}^{Cons}) The *n*-th factor within the m-th group of the Pros (Cons) cluster; ($m = 1,2,...,M^{Pros}$; $n = 1,2,...,N_m^{Pros}$ ($m = 1,2,...,M^{Cons}$; $n = 1,2,...,N_m^{Cons}$))
C_{mnl}^{Pros} (C_{mnl}^{Cons}) The *l*-th sub-factor of factor *n* within the *m*-th group of the *Pros* (*Cons*) cluster; ($m = 1,2,...,M^{Pros}$; $n = 1,2,...,N_m^{Pros}$; $l = 1,2,...,L_{mn}^{Pros}$; ($m = 1,2,...,M^{Cons}$; $n = 1,2,...,N_m^{Cons}$; $l = 1,2,...,L_{mn}^{Cons}$)

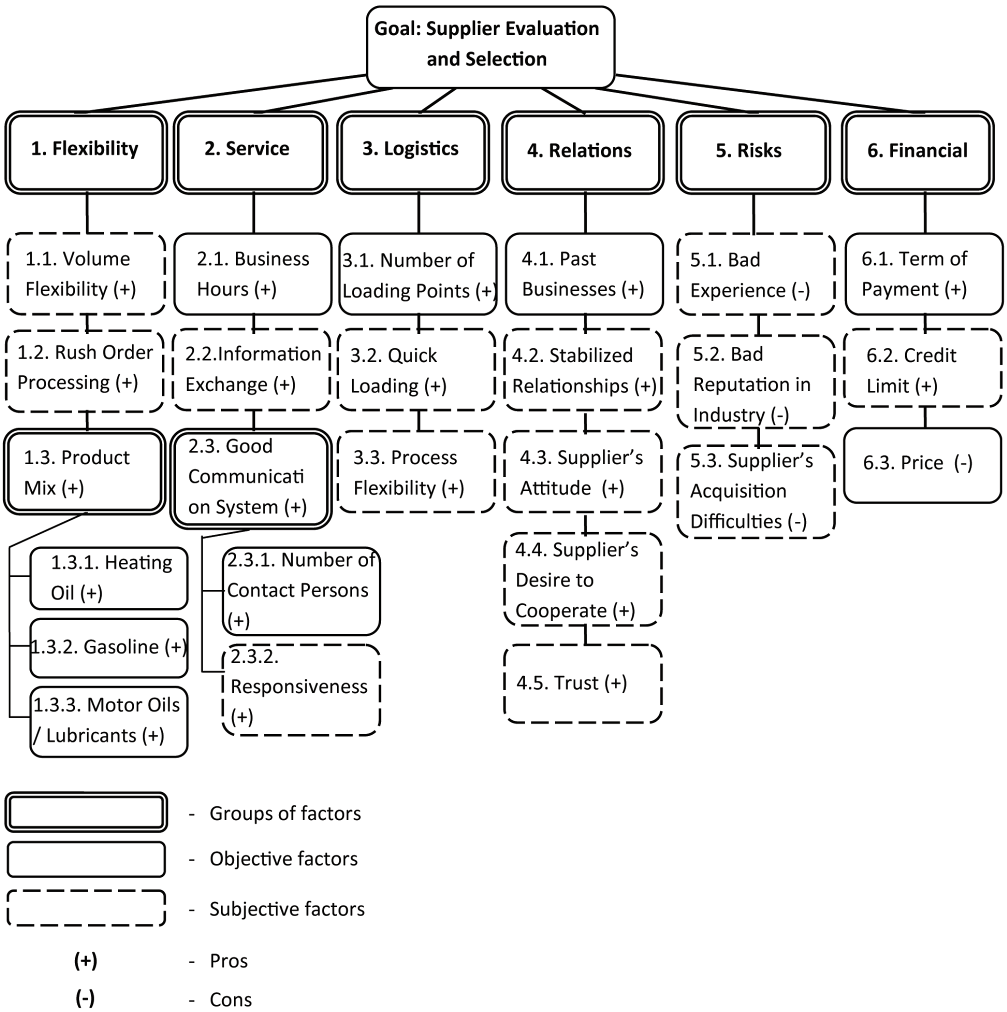
Based on the reviews of vendor selection criteria (Dickson, 1966; Weber et al., 1991; Sen et al., 2008; Inemak & Tuna, 2009) and interviews with Raiffeisen representatives we identified 20 criteria ($N = 20$) including 5 sub-factors ($L = 5$) categorized into 6 groups ($M = 6$) and arranged them into the hierarchy. The *Pros* category included 17 strategic criteria allocated among the six groups ($M^{Pros} = 6$). The first group; Flexibility included three criteria ($N_{Flexibility}^{Pros} = 3$) one of which was comprised of three sub-criteria ($L_{Flexibility\ ProductMix}^{Pros} = 3$). The second group; Service included three criteria ($N_{Service}^{Pros} = 3$) one of which was divided into two sub-criteria ($L_{Service\ GoodCommunicationSystem}^{Pros} = 2$). The other three groups included 2 to 5 criteria each one ($N_{Logistics}^{Pros} = 3$, $N_{Relations}^{Pros} = 5$, $N_{Financial}^{Pros} = 2$) without further division into sub-criteria. The Cons category included one tactical negative attribute and three strategic criteria allocated among the two groups ($M^{Cons} = 2$), strategic criteria in the group of Risks ($N_{Risks}^{Cons} = 3$) and the tactical criterion Price belonged to the group Financial ($N_{Financial}^{Cons} = 1$).

The hierarchy of decision criteria for Raiffeisen's fuel suppliers is visualized in Figure 1 and description of individual criteria is given in Table 1.

4. Define Decision Alternatives

The contemporary supply chain management is to maintain long term partnership with suppliers, and use fewer but reliable suppliers (Ho et al., 2010). Aissaoui et al. (2007) stated that today's logistics environment requires a low number of suppliers as it is very difficult to manage high numbers. Therefore, inefficient candidates should be not included into the evaluation process. In Just-In-Time environment the majority of companies prefer to follow a strategy of a single supplier or at least with few suppliers (Ansari & Modarresss, 1986). Quarly (1998) presented the factors of determining the policy of a single or multi supplier selection. The elimination method is a useful approach for suppliers pre-selection. The idea is that suppliers who do not satisfy the minimum level of key criteria are not accepted for further consideration. Hammond et al. (1998) stated that one may simplify a complex decision by looking for the practical dominance in the consequences table. This method reduces the

Figure 1. Hierarchy of Raiffeisen's supplier selection criteria



number of alternatives and helps to focus only on highly competitive options.

We consider a set $\{A_i\}$ of discrete elements A_i denoting alternatives ($i = 1, 2, \ldots, I$; $I \geq 2$).

The DMs from Raiffeisen selected eight suppliers ($I = 8$) for the evaluation process: *A_1=GRG, A_2=Atrian, A_3=Certyoil, A_4=Naronaft, A_5=Vetic, A_6=Petrolium Nord, A_7=West Petrol Group* and *A_8=POSF*.

5. Build Decision Hierarchy

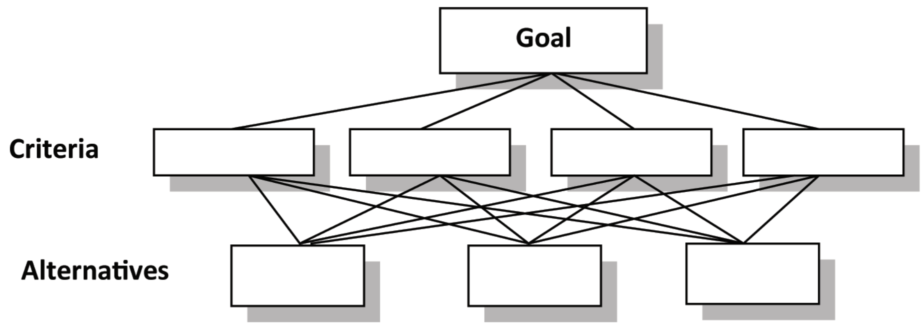
Hierarchy is a fundamental tool of human thinking and the most common way to organize decision problems (Saaty & Peniwati, 2008). Saaty (1994) suggests using the hierarchy containing three basic levels of elements connected from the top to the bottom; goal on the top, decision criteria on the intermediate level and alternatives on the bottom, as shown in Figure 2.

Table 1. Decision criteria and their description

Criterion	Description	Measurement Unit
Volume Flexibility	Supplier's capabilities and readiness to increase/decrease the ordered quantities at short notice	Subjective scale (SS)
Rush Order Processing	Possibility to purchase and deliver the products within a short time	SS
Product Mix	Purchased quantities of other products during the period	Thousand litres
Business Hours	Number of business hours during the week	Hours
Information Exchange	Information and forecasts regarding situation in the industry, market trends and other relevant information e.g., advertising tactic	SS
Number of Contact Persons	Number of contact persons authorized to take orders and to reply to inquiries	Integer number
Responsiveness	Speed of reaction and professionalism of contact persons	SS
Number of Loading Points	Number of disposable loading stations	Integer number
Quick Loading	Possibility to load the goods quickly on the supplier's terminals	SS
Process Flexibility	Well organized loading process on the stations; training programs for the drivers	SS
Past Businesses	Quantities of the product at hand purchased during the period	Euro
Stabilized Relationship	Long lasting relationships without pronounced negative incidents or contradictions in the past	SS
Supplier's Attitude	Friendly and individual treatment; relationships beyond the business	SS
Supplier's Desire to Cooperate	Supplier's attempts for sustainable partnership	SS
Trust	The expectations that the supplier's future behaviour will remain within the framework of common values and moral obligations	SS
Bad Experience	Negative incidents in the past, such as breaches of contracts or supplier defaults	SS
Bad Reputation in Industry	The supplier is not respected by its customers, suppliers or other groups of interests	SS
Supplier's Acquisition Difficulties	Probability of lack of product in stock e.g., due to bad weather (frost, drought or flood)	SS
Terms of Payment	Maximal provided payment period	Days
Credit Limit	The amount guaranteed by the supplier is high enough to satisfy your demand for its product	SS
Price	Bid price of the product for 100 Litres	Euro

Real decision problems usually have a more complex hierarchical structure than is depicted by Saaty and involve criteria that characterize alternatives not directly, but through some external objects. The main reason why such objects may have to be considered as a part of decision hierarchy is the impossibility in some cases to provide direct assessment of alternatives, according to a set of defined decision criteria due to reflection by those

Figure 2. A three level hierarchy



criteria of different aspects of the engaged services, facilities or other external objects and not of alternatives themselves. We shall further call these external elements ***auxiliary decision objects (ADOs)***. The indicators specifying performance of the ADOs will be called ***indirect criteria*** ($C\,*$). The attributes describing alternatives immediately will be referred to as ***direct criteria***. In Figure 3 is shown a decision hierarchy including (*M-1*) direct criteria connected to the alternatives, one indirect criterion describing the ADOs and relationships between the alternatives and the ADOs.

Correspondence between the ADOs and alternatives must be established in accordance with Table 2 to enable tracking the influences of indirect criteria on alternatives.

Raiffeisen's suppliers do not own centralized facilities for warehousing and shipment of fuels. Instead, each supplier leases space on the large loading terminals. Moreover, several competing suppliers can use service of the same loading stations. The eight suppliers under consideration share services of five ($T = 5$) loading terminals situated in Dortmund, Gelsenkirchen, Hamm, Lünnen and Üntrop. These stations differ on two criteria from the group *Logistics*: *Quick loading* ($C_{3\ 2}^{Pros}\,*$) and *Process flexibility* ($C_{3\ 3}^{Pros}\,*$). The layout of suppliers on the loading stations is profiled in Table 3.

Evaluated suppliers, shared by them external facilities, criteria and dependencies among these elements yield the decision hierarchy exhibited in Figure 4.

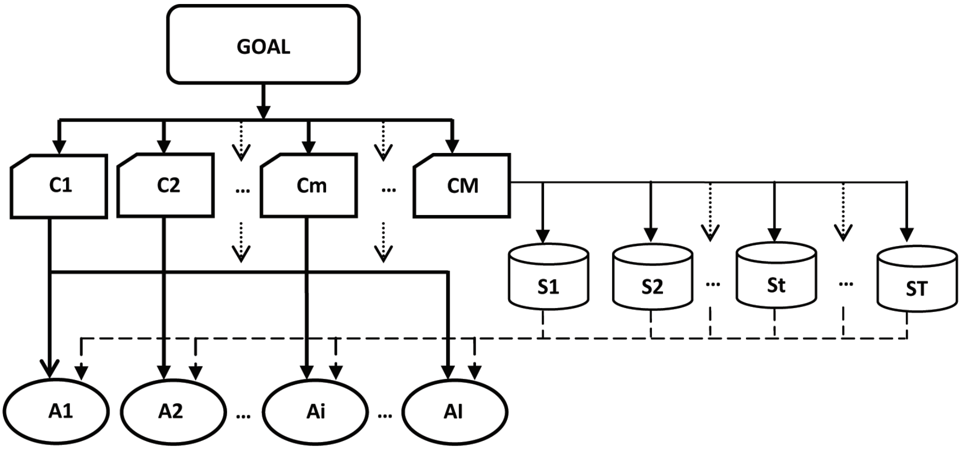
6. Identify the DM's Alpha-Voting Power for Assessment of Criteria

We assume that some DMs have more authority, expertise, knowledge, or skills. Therefore each voting member of the decision team is assigned a voting power which is meant to reflect his or her potential ability to influence the decision outcome. Bodily (1979) indicated that these weights may be assigned either through mutual agreement of the decision team members or by a "super decision maker" (benevolent dictator).

Top managers, CEOs or board members may not be too deeply involved with the daily (purchasing) decisions and evaluation of alternatives (vendors). But they usually have better vision of strategic priorities and objectives of their organization and its functional units than lower level employees.

We introduce a coefficient α standing for ***Alpha voting power*** to designate the relative DMs' impacts on the establishment of strategic priorities expressed by importance weights of the goals, objectives and criteria. The DMs responsible for criteria evaluation will be further called ***α-level DMs***.

Figure 3. Hierarchy including set of auxiliary decision objects



{Cm} – decision criteria

{Ai} – decision alternatives

{St} – auxiliary decision objects

C1,..., CM-1 – direct criteria

CM* – indirect criterion

We consider K^α *α-level DMs*, each with a positive *α-voting power* index $\pm_{k^{\pm}}$, where $\sum_{k^\alpha=1}^{K^\alpha} a_{k^\alpha} = 1$ ($k^\alpha = 1,2,...,K^\alpha$).

In the study conducted for Raiffeisen the strategy group responsible for evaluation of criteria included three *α-level DMs* ($K^\alpha = 3$) with following *α-voting power* coefficients: $\alpha_1 = 0,5$, $\alpha_2 = 0,3$ and $\alpha_3 = 0,2$.

7. Elicit Importance Weights of Criteria

This step includes the assessment of the relative importance of identified criteria by the group of *α-level DMs*. The weight elicitation problem in general is one of the most difficult problems in MCDA, because MCDA methods are supported by mathematical models and therefore the preferences need to be expressed in mathematical

Table 2. Matrix of connections between the alternatives and ADOs

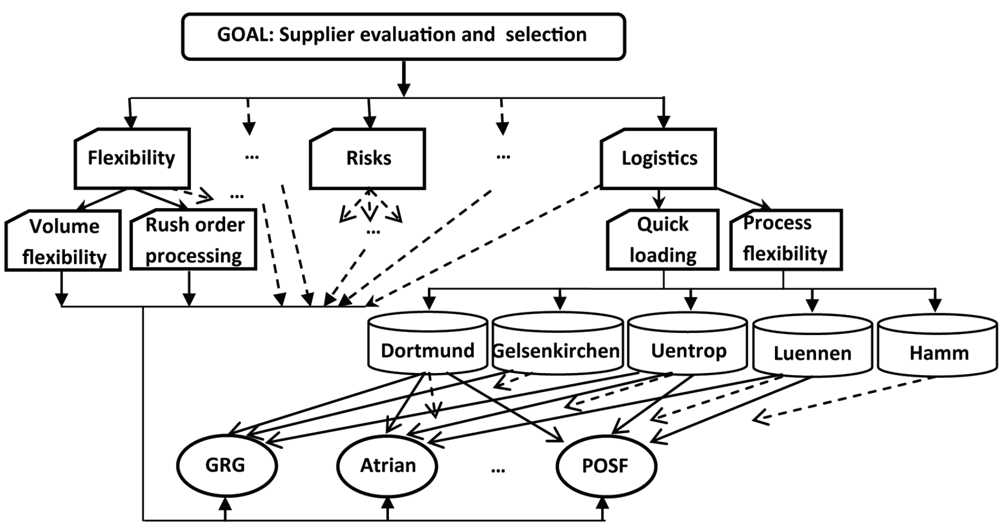
ADO, *St*	**Set of linked alternatives, *{Ai(St)}***
S1	*{Ai(S1)}*
…	…
St	*{Ai(St)}*
…	…
ST	*{Ai(ST)}*

Table 3. Allocation of suppliers among the loading stations

Loading Stations, *St*	Suppliers, *{Ai(St)}*
Üntrop, *t=1*	Atrian, Certyoil, GRG, Petrolium Nord, POSF
Hamm, *t=2*	GRG, Vetic
Lünnen, *t=3*	Certyoil, GRG, Naronaft, Petrolium Nord, POSF
Dortmund, *t=4*	All
Gelsenkirchen, *t=5*	Atrian, GRG, West Petrol Group

terms (Tervonen et al., 2007). Three factors are usually considered to obtain the weights; the variance degree of criteria, the independency of criteria and the subjective preference of the DMs (Wang et al., 2009). A number of approaches have been proposed to define criteria weights. Equal weights method (Dawes & Corrigan, 1974) requires minimal knowledge of the DM's priorities and minimal DM's input and treat all criteria as equally important. The simple multi-attribute rating technique (SMART) (Edwards, 1977) is based on the idea of ranking the importance of the changes in criteria from the lowest to the highest (best) levels, Edwards and Barron (1994) presented an improved version of this method called SMARTER which uses centroid method to find the final rankings. SWING (von Winterfeld & Edwards, 1986) is a direct algebraic decomposed procedure based on the ranking and scoring of criteria on a 100-point scale. Simos (Figueira & Roy, 2002) is a method where the user associates a "playing card" with each criterion, then the user ranks the cards in ascending order, according to the importance he/she wants to ascribe to the criteria; the white cards are used to determine the distance between successive criteria, from which the numerical attribute values are derived. The objective weighting methods use the distance metrics and include TOPSIS (Technique for order preference by similarity to ideal solution) (Hwang & Yoon, 1981), entropy (Srdjevic et

Figure 4. Hierarchy for evaluation of Raifieisen's fuel oil suppliers



al., 2004), principal component analysis etc. Combination of the objective and subjective weights is implemented in the additive and multiplicative synthesis (Wang et al. 2009).

The AHP is a subjective weighting method that relies on the pairwise comparisons to determine the weights of every decision criterion. The AHP was proposed by Saaty (1977). In the AHP pairwise comparisons are performed using 1 to 9 Fundamental Scale (Saaty & Sodenkamp, 2008).

Based on the pairwise comparison, matrices weights of criteria can be derived. The geometric aggregation rule should be used to avoid the controversies associated with rank reversal (Dyer, 1990; Harker & Vargas, 1990; Saaty, 1990b). After that, Consistency Ratio must be calculated to assure accuracy and logicality of provided subjective judgments.

The tree of attributes together with criteria weights reflects the DMs' value system for decision at hand. Figure 5 illustrates a formal scheme of assigning criteria weights by the *α-level DMs*.

Three *α-level DMs* in the Raiffeisen study used the AHP and Super Decisions software / (http://www.superdecisions.com) independently to derive individual weights of criteria groups ($w_m^{k^\alpha}$), criteria ($w_{mn}^{k^\alpha}$) and sub-criteria ($w_{mnl}^{k^\alpha}$). Tables 4 and 5 profile criteria importance weights for two DMs on objective and subjective criteria respectively.

8. Identify DMs' Beta-Voting Power for Evaluation of Alternatives on Subjective Direct Criteria

Once the set of decision alternatives is generated, the DMs' will make their assessment based upon subjective direct criteria should be selected and differentiated according to their ability to evaluate the alternatives. Performance values of alternatives on the objective criteria do not depend on the DMs' opinion. We propose to call the DMs responsible for evaluation of alternatives on the subjective direct criteria ***β-level DMs***. The DMs may vary in the sense of knowing decision alternatives to different extents and having experience to evaluate them rationally. In contrast to the *Alpha-voting power* indices a_{k^α} that indicate relative authority or influence of the DMs in the process of objectives or criteria weighting and establishment of strategic priorities, the ***Beta-voting power*** $\beta^{i}_{k^\beta}$ ($\sum_{k^{2}=1}^{K^{2}} {}^{2}{}^{i}_{k^{2}} = 1$; $k^\beta = 1,2,\ldots,K^\beta$; $i = 1,2,\ldots,I$; $I \geq 2$) specifies the DMs' relative ability to assess performance of alternatives on the direct subjective criteria. Methods of awarding the *Alpha voting power* indices a_{k^α} can be also implemented to establish the *Beta voting power* components $\beta^{i}_{k^\beta}$.

Numerical values $\beta^{i}_{k^\beta}$ for two Raiffeisen's *β-level DMs* ($K^\beta = 2$) are reflected in Table 6.

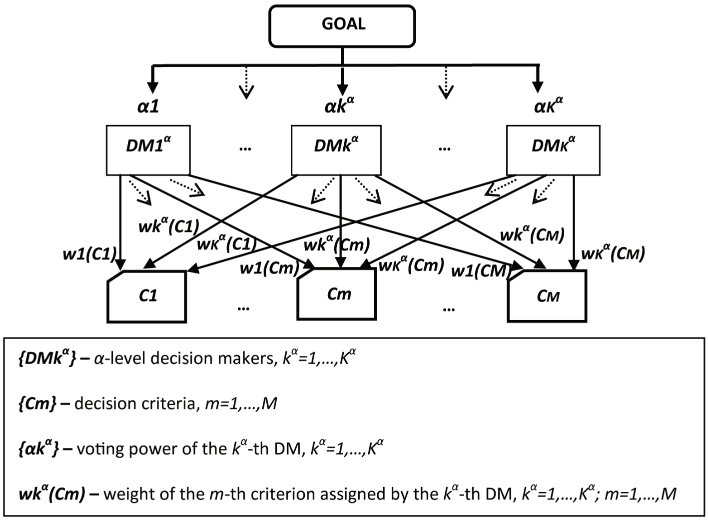
9. Collect Objective Data

The next step is collection of the hard data describing performance of alternatives on qualitatively and objectively measurable criteria. The performance values for the set of objectively measurable factors do not depend on the DM's judgments and are equal for each individual. The units of measurement have to be identical for all the alternatives with respect to the same criterion. Let's denote;

$p(Pros)_{mnl}^{Obj\ i}$ ($p(Cons)_{mnl}^{Obj\ i}$) Performance value of the i-th alternative on the objectively measurable *Pros* (*Cons*) *l*-th sub-criterion of the *n*-th criterion within the *m*-th group; ($i = 1,2,\ldots,I$; $l = 1,2,\ldots,L_{mn}$; $m = 1,2,\ldots,M$; $n = 1,2,\ldots,N_m$)

$p(Pros)_{mn}^{Obj\ i}$ ($p(Cons)_{mn}^{Obj\ i}$) Performance value of the i-th alternative on the objectively

Figure 5. Assignment of criteria weight by the α-level DMs



measurable n-th criterion within the m-th group, if n does not contain sub-criteria; ($i = 1,2,\ldots,I$; $m = 1,2,\ldots,M$; $n = 1,2,\ldots,N_m$)

Performance values of Raiffeisen's suppliers on the objective criteria are shown in Table 7.

10. Develop Scoring System for Subjective Criteria and Assign Scores to Alternatives on Direct Soft Criteria

Intangible (soft) criteria can be defined in several ways with scales varying both in definition and in number of options. The type of scale, with one definition on each end of the scale, gives the respondent space for subjective judgment while a scale with clearly defined alternatives can result in more objective answers according to the predefined alternatives. (Hartley & Betts, 2010) A Likert scale is commonly used in questionnaires to measure qualitative facts. Rensis Likert invented the scale with the purpose of using it within psychology and it can be designed as a 5-, 7- or a 10-point scale. Typical for a Likert scale is that the respondents specify their level of agreement to a statement. By using the Likert scale, the respondents can express their strength of feeling on a scale consisting of response categories.

Muralidharan et al. (2002) suggest guidelines for comparing supplier attribute. That is a five-point rating scale with predefined descriptions of each alternative. Judging whether a supplier has met the company's expectations, or not is not always an easy task if there are no clear statements declaring what the company's expectations are (Muralidharan, Anantharaman, & Deshmukh, 2002). We adopted this scale to the 10 grading points which is shown in Table 8.

The rating scale in Table 8 is appropriate for the *Pros* criteria where the higher values are preferred to the lower ones. For the *Cons* criteria the scale values should be inverted so that 1 is considered as the best value with minimal negative impact and 10 as the worst value with maximal negative impact, as specified in Table 9.

The scale grades for *Pros* should be maximized and for *Cons* minimized. The performance scores for the set of subjectively

Table 4. Weights of objective criteria derived with the AHP

Groups, C_m	**Group weights,** $w_m^{k^\alpha}$		**Criteria,** C_{mn}	**Citeria weights,** $w_{mn}^{k^\alpha}$		**Sub-criteria,** C_{mnl}	**Sub-criteria weights,** $w_{mnl}^{k^\alpha}$	
	DM 1	DM 2		DM 1	DM2		DM 1	DM 2
1.Flexibitity	0,100	0,060	1.3. Product Mix	0,249	0,194	1.3.1. HeatingOil	0,583	0,385
						1.3.2. Gasoline	0,240	0,399
						1.3.3.Motor Oils / Lubricants	0,177	0,216
2. Service	0,090	0,070	2.3.Good Communication System	0,302	0,181	2.3.1. Number of Contact Persons	0,311	0,244
			2.1. Business Hours	0,172	0,170			
3. Logistics	0,060	0,050	3.1.Number of Loading Pionts	0,241	0,146			
4. Relations	0,130	0,080	4.1. Past Businesses	0,146	0,103			
6. Financial	0,310	0,360	6.1. Term of Payment	0,320	0,300			
			6.3. Price	0,560	0,630			

Table 5. Weights of subjective criteria derived with the AHP

Groups, C_m	**Group weights,** $w_m^{k^\alpha}$		**Criteria,** C_{mn}	**Citeria weights,** $w_{mn}^{k^\alpha}$		**Sub-criteria,** C_{mnl}	**Sub-criteria weights,** $w_{mnl}^{k^\alpha}$	
	DM 1	DM 2		DM 1	DM 2		DM 1	DM 2
1.Flexibility	0,100	0,060	1.2.Rash Order Processing	0,342	0,455			
			1.1.Volume Flexibility	0,409	0,351			
2.Service	0,09	0,070	2.3.Good Communication System	0,302	0,181	2.3.2.Responsiveness	0,689	0,756
			2.2.Information Exchange	0,526	0,649			
3.Logistik	0,060	0,050	3.2.Quick Loading	0,446	0,541			
			3.3.Process Flexibility	0,313	0,313			
4.Relations	0,130	0,080	4.2.Stabilized Relations	0,209	0,262			
			4.3.Supplier's Attitude	0,187	0,118			
			4.4.Supplier's desire to cooperate	0,165	0,131			
			4.5. Trust	0,293	0,386			
5.Risks	0,310	0,380	5.1. Bad Experience	0,566	0,614			
			5.2.Bad Reputation in Industry	0,183	0,119			
			5.3.Supplier's Acquisition Difficulties	0,251	0,267			
6.Financial	0,310	0,360	6.1.Credit Limit	0,076	0,04			

Table 6. Beta voting power of Raiffeisen DMs'

β-level DMs	**Suppliers**							
	GRG, *i=1*	Atrian, *i=2*	Certyoil, *i=3*	Naronaft, *i=4*	Vetic, *i=5*	Petrolium Nord, *i=6*	West Petrol Group, *i=7*	POSF, *i=8*
DM1	0,59	0,67	0,50	0,10	0,50	0,83	0,67	0,67
DM2	0,41	0,33	0,50	0,90	0,50	0,17	0,33	0,33

Table 7. Objective performance values of alternatives

Criteria	**Sub-criteria**	GRG	Atrian	Certyoil	Naronaft	Vetic	Petrolium Nord	West Petrol Group	POSF
1.3. Product Mix	1.3.1. Heating Oil	4.354.915	1.579.919	1.098.650	122.618	760.851	0	252.841	3.301.832
	1.3.2. Gasoline	245.176	0	0	0	2.484.775	0	0	0
	1.3.3. Mineral Oils/ Lubricants	26.920,28	0,00	90.219,65	0,00	0,00	0,00	0,00	456,45
2.1. Good Communication System	2.1.1. Number of Contact Persons	5	2	2	2	5	1	1	3
2.3. Business Hours		45	39	45	45	45	45	39	50
3.1. Number of loading points		5	3	3	2	2	3	2	3
4.1. Past businesses		5.701.542	942.060	2.564.668	32.388	1.992.446	63.739	282.345	2.556.490
6.2. Payment period		20	20	20	20	20	20	30	20

measurable attributes are different for the K^{β} *β-level DMs*. Lets denote:

$p(\mathrm{Pr}\,os)_{mnl}^{Sbj\ ik^{\beta}}$ ($p(Cons)_{mnl}^{Sbj\ ik^{\beta}}$) Performance score of the i-th alternative on the subjectively measurable *l*-th *Pros* (*Cons*) sub-criterion of the *n*-th criterion within the *m*-th group assigned by K^{β} the -th β-level DM; ($i = 1, 2, ..., I$; $k^{\beta} = 1, 2, ..., K^{\beta}$; $l = 1, 2, ..., L_{mn}$; $m = 1, 2, ..., M$; $n = 1, 2, ..., N_m$)

$p(\mathrm{Pr}\,os)_{mn}^{Sbj\ ik^{\beta}}$ ($p(Cons)_{mn}^{Sbj\ ik^{b}}$)Performance score of the i-th alternative on the subjectively measurable *n*-th *Pros* (*Cons*) criterion within the *m*-th group, if *n* does not contain sub-criteria, assigned by the k^{β} -th β-level DM; ($i = 1, 2, ..., I$; $k^{\beta} = 1, 2, ..., K^{\beta}$; $m = 1, 2, ..., M$; $n = 1, 2, ..., N_m$)

In Figure 6 is illustrated the process of assigning subjective scores to alternatives on one direct criterion by the β-level DM.

11. Evaluate Auxiliary Decision Objects on Indirect Criteria

Analogously to the *direct criteria* that reflect to what extent alternatives meet the requirements expressed by means of quantitative and qualitative decision factors, *indirect criteria* help to

Table 8. Rating scale for supplier evaluation on the Pros criteria

Point	Grade	Description
10	Exceptional	Demonstrates substantially excellent performance,and has been at least in the excellence category for last 12 months
7	Excellence	Exceeds company's and customers' expectations, demonstrates extra effort
5	Good	Meets the company's expectations
3	Acceptable	Meets company's minimum requirements
1	Poor	Does not meet company's and customers' minimum acceptable level
2,4,6,8,9		Annectent grades

Table 9. Rating scale for supplier evaluation on the Cons criteria

Point	Grade	Description
10	Poor	Does not meet company's and customers' minimum acceptable level
7	Acceptable	Meets company's minimum requirements
5	Good	Meets the company's expectations
3	Excellence	Exceeds company's and customers' expectations, demonstrates extra effort
1	Exceptional	Demonstrates substantially excellent performance,and has been at least in the excellence category for last 12 months
2,4,6,8,9		Annectent grades

distinguish between the ADOs. The same as *direct criteria,* indirect ones can be objective (factual) or subjective (judgmental) merits. Factual data characterizing the ADOs has to be identified uniquely for each ADO and does not depend on the DMs' judgments. To formulate this step algebraically lets define;

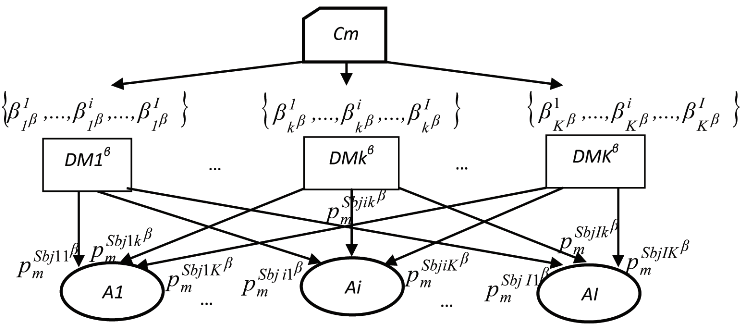
$p(_{Pros})^{Obj\ t}_{mnl\ *}$ ($p(_{Cons})^{Obj\ t}_{mnl\ *}$)Performance value of the t-th ADO on the objectively measurable *l*-th *Pros* (*Cons*) sub-criterion of the *n*-th criterion within the *m*-th group ($t = 1,2,...,T$; $l = 1,2,...,L_{mn}$; $m = 1,2,...,M$; $n = 1,2,...,N_m$)

$p(_{Pros})^{Obj\ t}_{mn\ *}$ ($p(_{Cons})^{Obj\ t}_{mn\ *}$)Performance value of the t-th ADO on the objectively measurable *n*-th *Pros* (*Cons*) criterion within the *m*-th group, if *n* does not contain sub-criteria; ($t = 1,2,...,T$; $m = 1,2,...,M$; $n = 1,2,...,N_m$)

In contrast to the tangible characteristics, intangible *indirect indicators* reflect the DMs' opinions. Decision team responsible for evaluation of the ADOs should include individuals having appropriate expertise and knowledge. Members of this group do not necessarily have to be criteria evaluators (*α-level DMs*), nor alternative assessors (*β-level DMs*). The DMs responsible for estimation of the ADOs on subjective indirect criteria will be called ***γ-level DMs***. *γ-level DMs* may vary in the sense of experience or authority to evaluate performances of particular ADOs. The relative ability of *γ-level DMs* to assess performance of the ADOs on indirect subjective criteria will be called ***γ-voting power***. We consider K^{γ} *γ-level DMs*, each with a positive *γ-voting power* index $\gamma^t_{k^{\gamma}}$, where

$$\sum_{k^{\gamma}=1}^{K^{\gamma}} \gamma^{t}_{k^{\gamma}} = 1 \ (k^{\gamma} = 1,2,...,K^{\gamma};\ t = 1,2,...,T \ ).$$

Figure 6. Estimation of alternatives on the subjective criteria by the β-level DMs



DMk^β	β-level DM; $k^\beta = 1,2,...,K^\beta$
Cm	decision criterion; $m = 1,2,...,M$
$\beta^i_{k^\beta}$	voting power of the **k^β**-th β-level DM for evaluation of the i-th alternative; $i = 1,2,...,I$; $k^\beta = 1,2,...,K^\beta$
$p_m^{Sbj\,ik^\beta}$	performance score of the *i*-th alternative on subjectively measurable *m*-th criterion assigned by the k^β-th β-level DM; $i = 1,2,...,I$; $k^\beta = 1,2,...,K^\beta$; $m = 1,2,...,M$

In order to evaluate the ADOs on subjective *Pros* and *Cons* a scoring system has to be developed. For this purpose a 10-point verbal scale with respective numerical values given in Table 8can be applied for *Pros*, and an inverse 1 to 10 scale from the Table 9 can be adapted for indirect *Cons*. Once the scale for subjective scores has been defined evaluation of the ADOs can begin. To formalize the process let us define:

$p(Pros)_{mnl\,*}^{Sbj\ tk^\gamma}$ ($p(Cons)_{mnl\,*}^{Sbj\ tk^\gamma}$)Performance score of the t-th ADO on the subjectively measurable *l*-th *Pros* (*Cons*) indirect sub-criterion of the *n*-th criterion within the m-th group assigned by the k^γ-the γ-level DM;($t = 1,2,...,T$; $k^\gamma = 1,2,...,K^\gamma$; $l = 1,2,...,L_{mn}$; $m = 1,2,...,M$; $n = 1,2,...,N_m$)

$p(Pros)_{mn\,*}^{Sbj\ tk^\gamma}$ ($p(Cons)_{mn\,*}^{Sbj\ tk^\gamma}$) Performance score of the t-th ADO on the subjectively measurable *n*-th *Pros* (*Cons*) indirect criterion within the *m*-th group, if *n* does not contain sub-criteria, assigned by the k^γ-th γ-level DM ; ($t = 1,2,...,T$; $k^\gamma = 1,2,...,K^\gamma$; $m = 1,2,...,M$; $n = 1,2,...,N_m$)

The group of two purchasing managers (γ-level DMs, $K^\gamma = 2$) was formed to evaluate five shared loading stations of Raiffeisen's suppliers on the subjective indirect criteria *Quick loading* ($C_{3\ 2}^{Pros}*$) and *Process flexibility* ($C_{3\ 3}^{Pros}*$). The *γ-voting power* indices and performance scores assigned to the loading points are given in Table 10.

12. Normalize All Objective Performances and Subjective Scores to Obtain Identical Measurement Units

Next, we normalize variables with multiple measurement scales to assure uniformity. The literature reports on several normalization methods. The selection of a specific normalization method must be based on the problem characteristics and model requirements (Tavana & Sodenkamp, 2010). In this study, we use the

Table 10. Voting power and scores of Raiffeisen γ-level DMs

	Shared loading facilities, *t*									
	Dortmund, t=1		Gelsenkirchen, t=2		Hamm, t=3		Lünnen, t=4		Üntrop, t=5	
γ-level DMs, k^γ	*DM1*, $k^\gamma=1$	*DM2*, $k^\gamma=2$	*DM1*, $k^\gamma=1$	*DM2*, $k^\gamma=2$	*DM1*, $k^\gamma=1$	*DM2*, $k^\gamma=2$	*DM1*, $k^\gamma=1$	*DM2*, $k^\gamma=2$	*DM1*, $k^\gamma=1$	*DM2*, $k^\gamma=2$
γ-voting power	0,5	0,5	0,7	0,3	0,4	0,6	0,75	0,25	0,5	0,5
Quick loading, C_{32*}	10	10	10	5	7	5	4	7	5	8
Process flexibility, C_{33*}	10	10	3	5	7	9	7	8	6	8

approach where the normalized value is the quotient of the initial value divided by the sum of the values of all alternatives/ADOs on that criterion. Normalized objective performances on direct, as well as on indirect *Pros* and *Cons* criteria can be defined by the expressions (1) - (4):

$$p'(Pros)_{mnl}^{Obj\ i} = \frac{p(Pros)_{mnl}^{Obj\ i}}{\sum_l p(Pros)_{mnl}^{Obj\ i}} \tag{1.a}$$

$$p'(Pros)_{mn}^{Obj\ i} = \frac{p(Pros)_{mn}^{Obj\ i}}{\sum_n p(Pros)_{mn}^{Obj\ i}} \tag{1.b}$$

$$p'(Cons)_{mnl}^{Obj\ i} = \frac{p(Cons)_{mnl}^{Obj\ i}}{\sum_l p(Cons)_{mnl}^{Obj\ i}} \tag{2.a}$$

$$p'(Cons)_{mn}^{Obj\ i} = \frac{p(Cons)_{mn}^{Obj\ i}}{\sum_n p(Cons)_{mn}^{Obj\ i}} \tag{2.b}$$

$$p'(Pros)_{mnl\ *}^{Obj\ t} = \frac{p(Pros)_{mnl\ *}^{Obj\ t}}{\sum_l p(Pros)_{mnl\ *}^{Obj\ t}} \tag{3.a}$$

$$p'(Pros)_{mn\ *}^{Obj\ t} = \frac{p(Pros)_{mn\ *}^{Obj\ t}}{\sum_n p(Pros)_{mn\ *}^{Obj\ t}} \tag{3.b}$$

$$p'(Cons)_{mnl\ *}^{Obj\ t} = \frac{p(Cons)_{mnl\ *}^{Obj\ t}}{\sum_l p(Cons)_{mnl\ *}^{Obj\ t}} \tag{4.a}$$

$$p'(Cons)_{mn\ *}^{Obj\ t} = \frac{p(Cons)_{mn\ *}^{Obj\ t}}{\sum_n p(Cons)_{mn\ *}^{Obj\ t}} \tag{4.b}$$

Normalized subjective scores on direct and on indirect *Pros* and *Cons* criteria can be defined by the expressions (5) - (8):

$$p'(\Pr os)_{mnl}^{Sbj\ ik^\beta} = \frac{p(\Pr os)_{mnl}^{Sbj\ ik^\beta}}{\sum_l p(\Pr os)_{mnl}^{Sbj\ ik^\beta}} \tag{5.a}$$

$$p'(\Pr os)_{mn}^{Sbj\ ik^\beta} = \frac{p(\Pr os)_{mn}^{Sbj\ ik^\beta}}{\sum_n p(\Pr os)_{mn}^{Sbj\ ik^\beta}} \tag{5.b}$$

$$p'(Cons)_{mnl}^{Sbj\ ik^\beta} = \frac{p(Cons)_{mnl}^{Sbj\ ik^\beta}}{\sum_l p(Cons)_{mnl}^{Sbj\ ik^\beta}} \tag{6.a}$$

$$p'(Cons)_{mn}^{Sbj\ ik^\beta} = \frac{p(Cons)_{mn}^{Sbj\ ik^\beta}}{\sum_n p(Cons)_{mn}^{Sbj\ ik^\beta}} \tag{6.b}$$

$$p'(\Pr os)_{mnl*}^{Sbj\ tk^\gamma} = \frac{p(\Pr os)_{mnl*}^{Sbj\ tk^\gamma}}{\sum_l p(\Pr os)_{mnl*}^{Sbj\ tk^\gamma}} \tag{7.a}$$

$$p'(\Pr os)_{mn*}^{Sbj\ tk^\gamma} = \frac{p(\Pr os)_{mn*}^{Sbj\ tk^\gamma}}{\sum_n p(\Pr os)_{mn*}^{Sbj\ tk^\gamma}} \tag{7.b}$$

$$p'(Cons)_{mnl*}^{Sbj\ tk^{\gamma}} = \frac{p(Cons)_{mnl*}^{Sbj\ tk^{\gamma}}}{\sum_{l} p(Cons)_{mnl*}^{Sbj\ tk^{\gamma}}} \quad (8.a)$$

$$p'(Cons)_{mn*}^{Sbj\ tk^{\gamma}} = \frac{p(Cons)_{mn*}^{Sbj\ tk^{\gamma}}}{\sum_{n} p(Cons)_{mn*}^{Sbj\ tk^{\gamma}}} \quad (8.b)$$

Normalized values of Raiffeisen's suppliers on objective factors are calculated using formula (1) and are presented in Table 11.

Normalized scores of the Raiffeisen's loading points are calculated using equation (4) and are demonstrated in Table 12.

14. Integrate All Voting Power Indices, Criteria Weights, Tangible and Intangible Estimates

After the normalization process, we use an integration procedure to combine the following elements into one pair of values for *Pros* and *Cons* of each decision alternative;

- Voting power coefficients $\alpha_{k^{\alpha}}$, $\beta_{k^{\beta}}^{i}$ and $\gamma_{k^{\gamma}}^{t}$;
- $K^{\pm}$ sets of M criteria group weights ($w_m^{k^{\alpha}}$), N attribute weights ($w_{mn}^{k^{\alpha}}$) and L sub-criteria weights ($w_{mnl}^{k^{\alpha}}$);
- N^{Obj} and L^{Obj} normalized performances of I alternatives and T ADOs on the direct and indirect objective *Pros* and *Cons* criteria ($p'(Pros)_{mnl}^{Obj\ i}$, $p'(Pros)_{mn}^{Obj\ i}$, $p'(Pros)_{mnl}^{Obj\ t}$, $p'(Pros)_{mn}^{Obj\ t}$, $p'(Cons)_{mnl}^{Obj\ i}$, $p'(Cons)_{mn}^{Obj\ i}$, $p'(Cons)_{mnl}^{Obj\ t}$ and $p'(Cons)_{mn}^{Obj\ t}$);
- K^{β} sets of normalized scores of I alternatives on the direct subjective *Pros* and *Cons* criteria ($p'(\Pr os)_{mnl}^{Sbj\ ik^{\beta}}$, $p'(\Pr os)_{mn}^{Sbj\ ik^{\beta}}$, $p'(Cons)_{mnl}^{Sbj\ ik^{\beta}}$ and $p'(Cons)_{mn}^{Sbj\ ik^{\beta}}$); and
- K^{γ} sets of normalized scores of T ADO's on the indirect subjective *Pros* and *Cons* criteria ($p'(\Pr os)_{mnl*}^{Sbj\ tk^{\gamma}}$, $p'(\Pr os)_{mn*}^{Sbj\ tk^{\gamma}}$, $p'(Cons)_{mnl*}^{Sbj\ tk^{\gamma}}$ and $p'(Cons)_{mn*}^{Sbj\ tk^{'}}$).

14.1. Combination of the Group Criteria Weights

For the first step of the integration procedure it is necessary to find combined among the α-level DM's, weights of criteria groups (w_m), criteria (w_{mn}) and sub-criteria (w_{mnl}).

$$w_m = \sum_{k^{\alpha}=1}^{K^{\alpha}} (\alpha_{k^{\alpha}} \cdot w_m^{k^{\alpha}}) \quad (9.a)$$

$$w_{mn} = \sum_{k^{\alpha}=1}^{K^{\alpha}} (\alpha_{k^{\alpha}} \cdot w_{mn}^{k^{\alpha}}) \quad (9.b)$$

$$w_{mnl} = \sum_{k^{\alpha}=1}^{K^{\alpha}} (\alpha_{k^{\alpha}} \cdot w_{mnl}^{k^{\alpha}}) \quad (9.c)$$

14.2. Prioritization of the ADOs

In the second step of the integration procedure we calculate the group rankings of the ADO's in order to incorporate this information into the decision matrix later for evaluation of the alternatives.

Aggregated group *Pros* and *Cons* of the each ADO must be derived taking into account γ-voting power indices $\gamma_{k^{\gamma}}^{t}$ using formulas (10)-(11) respectively:

$$p(\Pr os)_{mnl*}^{Sbj\ t} = \sum_{k^{\gamma}=1}^{K^{\gamma}} \gamma_{k^{\gamma}}^{t} \cdot p'(\Pr os)_{mnl*}^{Sbj\ tk^{\gamma}} \quad (10.a)$$

$$p(\Pr os)_{mn*}^{Sbj\ t} = \sum_{k^{\gamma}=1}^{K^{\gamma}} \gamma_{k^{\gamma}}^{t} \cdot p'(\Pr os)_{mn*}^{Sbj\ tk^{\gamma}} \quad (10.b)$$

$$p(Cons)_{mnl*}^{Sbj\ t} = \sum_{k^{\gamma}=1}^{K^{\gamma}} \gamma_{k^{\gamma}}^{t} \cdot p'(Cons)_{mnl*}^{Sbj\ tk^{\gamma}} \quad (11.a)$$

$$p(Cons)_{mn*}^{Sbj\ t} = \sum_{k^{\gamma}=1}^{K^{\gamma}} \gamma_{k^{\gamma}}^{t} \cdot p'(Cons)_{mn*}^{Sbj\ tk^{\gamma}} \quad (11.b)$$

We use weighed-sum aggregation method and equations (12)-(13) to calculate the total

Table 11. Normalized performances of alternatives on objective criteria

Groups of criteria, C_m	Criteria, C_{mn}	Sub-criteria, C_{mnl}	Alternative suppliers, A^i							
			GRG	Atrian	Certyoil	Naronaft	Ventic	Petrolium Nord	West Petrol Group	POSF
1.Flexibility	1.3. Product Mix	1.3.1. Heating Oil	0,380	0,138	0,096	0,011	0,066	0,000	0,022	0,288
		1.3.2. Gasoline	0,090	0,000	0,000	0,000	0,910	0,000	0,000	0,000
		1.3.3.Motor Oils/ Lubricants	0,229	0,000	0,767	0,000	0,000	0,000	0,000	0,004
2. Service	2.1. Business Hours		0,238	0,095	0,095	0,095	0,238	0,048	0,048	0,143
	2.3. Good Communication System	2.3.1. Number of Contact Persons	0,127	0,110	0,127	0,127	0,127	0,127	0,110	0,142
3. Logistics	3.1. Number of Loading Points		0,217	0,130	0,130	0,087	0,087	0,130	0,087	0,130
4. Relations	4.1. Past Businesses		0,403	0,067	0,181	0,002	0,141	0,005	0,020	0,181
6. Financial	6.1. Term of Payment		0,125	0,125	0,125	0,125	0,125	0,125	0,125	0,125
	6.3. Price		0,126	0,121	0,122	0,128	0,129	0,125	0,122	0,127

Pros ($p(Pros)^t_*$) and *Cons* ($p(Cons)^t_*$) scores of *T* ADOs.

$$p(Pros)^t_{mn*} = w_{mn} \cdot (w_{mnl}(\sum_l p'(Pros)^{Obj\ t}_{mnl\ *} + \sum_l p(Pros)^{Sbj\ t}_{mnl\ *}) + \sum_n p'(Pros)^{Obj\ t}_{mn\ *} + \sum_n p(Pros)^{Sbj\ t}_{mn\ *}) \quad (12)$$

$$p(Cons)^t_{mn*} = w_{mn} \cdot (w_{mnl}(\sum_l p'(Cons)^{Obj\ t}_{mnl\ *} + \sum_l p(Cons)^{Sbj\ t}_{mnl\ *}) + \sum_n p'(Cons)^{Obj\ t}_{mn\ *} + \sum_n p(Cons)^{Sbj\ t}_{mn\ *}) \quad (13)$$

In the Raiffeisen study integrated group rankings of the five shared loading terminals were obtained using formulas (10) and (12), the results are shown in Table 13.

14.3. Derive Values of the Alternatives on the Indirect Criteria

Once positive and negative ratings of all ADO's have been calculated, impacts of indirect criteria on the decision alternatives have to be measured taking into consideration correspondence between the ADO's and alternatives as defined in Table 2. To calculate performance level of decision alternatives *Ai* on the indirect criteria *C** for contradictory classes *Pros* and *Cons* formulas 14(a) and 14(b) can be applied

Table 12. Normalized scores of the loading stations

Indirect criteria, C_{mn}*	**Shared loading facilities**, *t*									
	Dortmund, t=1		Gelsenkirchen, t=2		Hamm, t=3		Lünnen, t=4		Üntrop, t=5	
	DM1	DM2	DM1	DM2	DM1	DM2	DM1	DM2	DM1	DM2
Quick loading, C_{32}*	0,278	0,286	0,278	0,143	0,194	0,143	0,111	0,200	0,139	0,229
Process flexibility, C_{33}*	0,303	0,250	0,091	0,125	0,212	0,225	0,212	0,200	0,182	0,200

Table 13. Group ratings of the loading stations

Loading terminals, *St*	Dortmund, *t=1*	Gelsenkirchen, *t=2*	Hamm, *t=3*	Lünnen, *t=4*	Üntrop, *t=5*
Weights of terminals, $p(Pros)^t$	0,226	0,134	0,150	0,143	0,153

respectively. The numbers obtained can then be incorporated into the process of alternatives evaluation together with direct criteria.

$$p(Pros)^i_{mn*} = \frac{\sum_{t=1}^{T^i} p(Pros)^t_{mn*}}{T^i} \quad (14.a)$$

$$p(Cons)^i_{mn*} = \frac{\sum_{t=1}^{T^i} p(Cons)^t_{mn*}}{T^i} \quad (14.b)$$

where T^i stands for number of ADOs related to alternative i.

Raiffeisen's loading point information and suppliers' integrated group performance scores on the indirect criteria are collected in Table 14.

14.4. Calculate Group Weights of the Alternatives on Each Criterion

The normalized subjective group scores on *direct factors* must be merged with *β-voting power* magnitudes to derive consensus bases rankings of alternatives.

$$p(\Pr os)^{Sbj\ i}_{mnl} = \sum_{k^\beta=1}^{K^\beta} \beta^i_{k^\beta} \cdot p'(\Pr os)^{Sbj\ ik^\beta}_{mnl} \quad (15.a)$$

$$p(\Pr os)^{Sbj\ i}_{mn} = \sum_{k^\beta=1}^{K^\beta} \beta^i_{k^\beta} \cdot p'(\Pr os)^{Sbj\ ik^\beta}_{mn} \quad (15.b)$$

$$p(Cons)^{Sbj\ i}_{mnl} = \sum_{k^\beta=1}^{K^\beta} \beta^i_{k^\beta} \cdot p'(Cons)^{Sbj\ ik^\beta}_{mnl} \quad (16.a)$$

$$p(Cons)^{Sbj\ i}_{mn} = \sum_{k^\beta=1}^{K^\beta} \beta^i_{k^\beta} \cdot p'(Cons)^{Sbj\ ik^\beta}_{mn} \quad (16.b)$$

14.5. Combine All Direct Pros and Cons Weights For Each Alternative

Then we apply the weighed-sum aggregation to calculate combined objective and subjective *Pros* ($p(Pros)^i$) and *Cons* ($p(Cons)^i$) of *I* alternatives on the *direct criteria*.

$$p(Pros)^i_{mn} = w_{mn} \cdot (w_{mnl} (\sum_l p'(Pros)^{Obj\ i}_{mnl} + \sum_l p(Pros)^{Sbj\ i}_{mnl}) + \sum_n p'(Pros)^{Obj\ i}_{mn} + \sum_n p(Pros)^{Sbj\ i}_{mn}) \quad (17)$$

$$p(Cons)^i_{mn} = w_{mn} \cdot (w_{mnl} (\sum_l p'(Cons)^{Obj\ i}_{mnl} + \sum_l p(Cons)^{Sbj\ i}_{mnl}) + \sum_n p'(Cons)^{Obj\ i}_{mn} + \sum_n p(Cons)^{Sbj\ i}_{mn}) \quad (18)$$

14.6. Find a Pair of Total Pros and Cons for Each Alternative

On the final step of our integration procedure the total *Pros* ($p(Pros)^i$) and *Cons* ($p(Cons)^i$) values are calculated for each alternative as added weighed estimates on the direct and indirect criteria:

$$p(Pros)^i = \sum_{m=1}^{M} (w_m \cdot (p(Pros)^i_{mn} + p(Pros)^i_{mn*})) \quad (19)$$

$$p(Cons)^i = \sum_{m=1}^{M} (w_m \cdot (p(Cons)^i_{mn} + p(Cons)^i_{mn*})) \quad (20)$$

The *Pros* and *Cons* of Raiffeisen's suppliers are presented in Table 15.

Table 14. Suppliers' loading stations information

Suppliers, *Ai*	GRG, i=1	Atrian, i=2	Certyoil, i=3	Naronaft, i=4	Vetic, i=5	Petrolium Nord, i=6	West Petrol Group, i=7	POSF, i=8
Number of Loading Stations, T^i	5	3	3	3	2	3	2	3
Loading stations, *St*	Dortmund, Gelsenkirchen, Hamm, Lünnen, Üntrop	Dortmund, Gelsenkirchen, Üntrop	Dortmund, Lünnen, Üntrop	Dortmind, Hamm, Lünnen	Dortmund, Hamm	Dortmund, Lünnen, Üntrop	Dortmund, Gelsenkirchen	Dortmund, Lünnen, Üntrop
Total scores on indirect criteria, $p(Pros)^i_{mn*}$	0,161	0,171	0,174	0,173	0,188	0,174	0,180	0,174

15. Rank Alternatives Based on their Euclidean Distance to the Ideal Point

Zeleny (1982) suggested using the Euclidean measure to compare alternatives among themselves and with the ideal one. This approach was implemented in numerous researchers (Tavana & Sodenkamp, 2010; Tavana et al., 2010) and by practitioners. The ideal *Pros* value ($p*(Pros)$) is the highest *Pros* weight among the set of $\{p(Pros)^i\}$ and ideal *Cons* value ($p*(Cons)$) is the lowest *Cons* weight among the set of $\{p(Cons)^i\}$. To find the Euclidean distance of each alternative from the ideal one we extract the quadratic root of summarized squared differences between the ideal and the i-th indices of the *Pros* and *Cons*. Lets define:

$E(Pros)^i$ ($E(Cons)^i$) The distance from the ideal positive (negative) merit for the i-th alternative; ($i = 1, 2, \ldots, I$)

E^i Euclidean distance from the ideal point for the i-th alternative; ($i = 1, 2, \ldots, I$)

$\bar{E}$ Mean Euclidean distance for the alternatives;

$$p*(Pros) = Max\{p(Pros)^i\} \tag{21.a}$$

$$p*(Cons) = Min\{p(Cons)^i\} \tag{21.b}$$

$$E^i = \sqrt{(p*(Pros) - p(Pros)^i)^2 + (p*(Cons) - p(Pros)^i)^2} \tag{22}$$

We then sort alternatives A^i from the best to the worst one based on the values E^i and form a set $\{A^{Ord}\}$. A^i_r indicates that i-th alternative has r-th rank in the ordered set $\{A^{Ord}\}$, where $r = 1, \ldots, I$.

The highest *Pros* value among the set of Raiffeisen's suppliers is $p*(Pros) = 0{,}056$, whereas the lowest *Cons* value is $p*(Cons) = 0{,}015$. The Raiffeisen fuel oil suppliers' distances to the ideal point, together with ranks based on Euclidean distance are shown in Table 16.

16. Choose the Optimal Alternative(s) and Assign Order Quantities

To solve the choice decision problem the maximal efficiency can be achieved if the alternative $A^i_{r=1}$ with the highest rank will be selected. In purchasing management this kind of supplier selection is called single sourcing and it is used if one supplier can satisfy all the buyer's needs.

If decision goal is formulated in terms of selection of several best options, the required number Q ($Q \leq I$) of alternatives must be defined by the decision group. The desire of purchasing managers to split orders among

Table 15. Suppliers' overall Pros and Cons

Suppliers, *Ai* **Merits**	GRG, i=1	Atrian, i=2	Certyoil, i=3	Naronaft, i=4	Vetic, i=5	Petrolium Nord, i=6	West Petrol Group, i=7	POSF, i=8
Pros, $p(_{Pros})^i$	0,056	0,037	0,040	0,029	0,050	0,036	0,036	0,049
Cons, $p(_{Cons})^i$	0,015	0,020	0,043	0,016	0,016	0,037	0,018	0,029

vendors may arise for a variety of reasons, including inability of suppliers to satisfy all of the buyer's requirements or intentionally creating an environment of competitiveness. In such case, Q first elements from the ordered set { A^{Ord} } must be selected to assure highest efficiency of Q alternatives. The selected alternatives are A_q^i with $q = 1,...,Q$.

Raiffeisen follows the policy of risks minimization and multiple sourcing. To purchase fuels the DMs select three $Q = 3$ vendors with highest ranks r. These are GRG ($A_{q=1}^{i=1}$), Vetic ($A_{q=2}^{i=5}$) and POSF ($A_{q=3}^{i=8}$). Then order quantities o^q ($q = 1,..Q$) have to be allocated among the selected suppliers proportionally to the normalized relative weights w'_q of selected alternatives within set { Q }.

$$w_q = 1 - \frac{E_q^i}{\sum_{q=1}^{Q} E_q^i} \tag{23}$$

where E_q^i is Euclidean distance for the q-th selected alternative and $E_q^i = E^i$.

$$w'_q = \frac{w_q}{\sum_{q=1}^{Q} w_q} \tag{24}$$

Assuming that d is demand of the product to purchase, the order quantities for vendors are:

$$o^q = w'_q \cdot d \tag{25}$$

Normalized weights of the suppliers selected by Raiffeisen, and assigned to them order quantities for $d = 72\ 000$ (*liters)* are presented in Table 17.

CONCLUSION

The research presented in this study promotes explicit and comprehensive modeling of extremely complex decisions and systematic evaluation and selection of alternatives based on their contribution made throughout the organization. When real decision processes do not fit into the typical hierarchy "goals-criteria-alternatives" due to involvement of the external

Table 16. Suppliers' distances to the ideal and final ranks

Supplier, ***Ai***	**GRG, i=1**	**Atrian, i=2**	**Certyoil, i=3**	**Naro-naft, i=4**	**Vetic, i=5**	**Petrolium Nord, i=6**	**West Petrol Group, i=7**	**POSF, i=8**
Euclidean Distance, E^i	0,000	0,019	0,032	0,026	0,006	0,029	0,019	0,015
Rank, *r*	1	4 (tied)	8	6	2	7	4 (tied)	3

Table 17. Selected suppliers and order quantities

Selected suppliers, A_q^i	GRG, $q=1$	Vetic, $q=2$	POSF, $q=3$
Euclidean distances, E_q^i	0,000	0,006	0,015
Normalized weighs, w'_q	0,500	0,357	0,143
Order quantities, o^q	36 000	25 704	10 296

services or other intermediate units, connections between the decision elements become more intricate and standard methods are no longer applicable. We demonstrate a supplier selection problem including indirect influences of decision criteria on the vendors and formulate appropriate step-by-step assessment framework. All relevant objective information, together with the expert judgments regarding criteria importance; performance scores of alternatives and auxiliary objects are captured consistently in the evaluation procedure. Principal distinction is drawn between the three types of DMs; strategy determination group (α-level DMs), alternatives evaluation group (β-level DMs) and ADOs assessment group (γ-level DMs). Moreover, different grades of DMs influence inherent to real decision teams are expressed by voting power coefficients and then included into the aggregation procedure, aimed to reveal consensus based supplier priorities. Sensitivity analysis can be performed in order to understand impacts of particular parameters on the final result and to examine robustness of the proposed solution.

Systematic holding of non-anonymous assessment sessions with our method makes significant contributions to the decision process transparency. Moreover, the MLGDM process can be used as a tool for DMs' learning and dynamic monitoring of strategic and tactical purchasing decisions on different organizational layers.

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A fuzzy multi-criteria decision analysis model for advanced technology assessment at Kennedy Space Center

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The rapid development of computer and information technology has made project evaluation and selection a difficult task at the Kennedy Space Center (KSC) Shuttle Project Engineering Office. Decision Makers (DMs) are required to consider a vast amount of intuitive and analytical information in the decision process. *Fuzzy Euclid* is a Multi-criteria Decision Analysis (MCDA) model that captures the DMs' beliefs through a series of intuitive and analytical methods such as the analytic hierarchy process (AHP) and subjective probability estimation. A defuzzification method is used to obtain crisp values from the subjective judgments provided by multiple DMs. These crisp values are synthesized with Entropy and the theory of displaced ideal to assist the DMs in their selection process by plotting the alternative projects in a four-zone graph based on their Euclidean distance from the 'ideal choice'.
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1. Introduction

The massive explosion of information and rapid development of technology have focused critical attention on government agencies that support information and technology development. The public is concerned with the governance of these agencies and demands maximum return from investment in technology. Public awareness and pressure has forced Congress to mandate the National Aeronautic and Space Administration (NASA) to be more accountable to the taxpayers. The demand for accountability and the increasing complexity of advanced technology projects has made project assessment and selection an extremely difficult task at NASA.

A large body of scoring, economic and portfolio methods have evolved over the last several decades to assist decision makers (DMs) in project evaluation. Scoring methods use algebraic formulas to produce an overall score for each project (Moore and Baker, 1969; Osawa and Murakami, 2002; Osawa, 2003). Economic methods use financial models to calculate the monetary pay-off of each project (Mehrez, 1988 and Graves and Ringuest, 1991). Portfolio methods evaluate the entire set of projects to identify the most attractive subset (Lootsma *et al*, 1990; Girotra *et al*, 2007; Mojsilović *et al*, 2007; Wang and Hwang, 2007). Cluster analysis, a more specific portfolio method, groups projects according to their support of the strategic positioning of the firm (Mathieu and Gibson, 1993). Decision analysis methods compare various projects according to their expected value (Hazelrigg and Huband, 1985; Thomas, 1985). Finally, simulation, a more specific decision analysis method, uses random numbers and simulation to generate a large number of problems and picks the best outcome (Mandakovic and Souder, 1985; Abacoumkin and Ballis, 2004; Paisittanand and Olson, 2006).

Most of these methods are used to evaluate research and development projects (Osawa and Murakami, 2002; Osawa, 2003; Girotra *et al*, 2007; Wang and Hwang, 2007), information systems projects (Muralidhar *et al*, 1990; Schniederjans and Santhanam, 1993; Santhanam and Kyparisis, 1995; Paisittanand and Olson, 2006; Mojsilović *et al*, 2007) and capital budgeting projects (Mehrez, 1988; Graves and Ringuest, 1991). Although these methods have made great strides in Multi-criteria Decision Analysis (MCDA), the intuitive models lack a structured framework and the analytical models do not capture intuitive preferences.

Project evaluation problems are group MCDA problems that embrace both qualitative and quantitative criteria. MCDA methods provide a structured framework for information exchange among the group members and thus reducing the unstructured nature of the problem. The obvious obstacle when multiple persons are involved in a group decision problem is the fact that each group member has his/her own perception of the problem that accordingly affects the decision outcome. MCDA frameworks permit group members

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to explore their value system from multiple viewpoints and modify their perceptions by obtaining knowledge of the other group members' preference structure and beliefs. A number of decision methodologies in the group decision-making context have been presented in the MCDA literature. A comprehensive survey can be found in Hwang and Lin (1987). Iz and Gardiner (1993) review formal group decision-making models and describe some examples of conceptual frameworks and actual implementations of group decision-making models. A comprehensive collection of research devoted to synthesis and analysis of group support frameworks and procedures can be found in Jessup and Valacich (1993). When facing such multiple criteria problems, the literature and research show that the following difficulties may be encountered:

(a) DMs often use verbal expressions and linguistic variables for subjective judgments, which lead to ambiguity in human decision making (Poyhonen *et al*, 1997). Furthermore, the subjective assessment process is intrinsically imprecise and may involve two types of judgments: comparative judgment and absolute judgment (Saaty, 2006).
(b) DMs often provide imprecise or vague information due to lack of expertise, unavailability of data, or time constraint (Kim and Ahn, 1999).
(c) Meaningful and robust aggregation of subjective and objective judgments causes problems during the evaluation process (Valls and Torra, 2000).

A decision may not be properly made without fully taking into consideration all criteria in MCDA (Belton and Stewart, 2002; Yang and Xu, 2002). Recently, researchers working on project evaluation and selection have focused on MCDA models to integrate the intuitive preferences of multiple DMs into structured and analytical frameworks (Bailey *et al*, 2003; Costa *et al*, 2003; Hsieh *et al*, 2004; Tavana, 2006; Liesiö *et al*, 2007).

MCDA problems involve the ranking of a finite set of alternatives in terms of a finite number of conflicting decision criteria. More often, decision criteria can be grouped into two contradictory categories, called the 'opportunities' and the 'threats'. Alternatively, opportunities may be called 'benefits' or 'returns' and threats may be called 'costs' or 'risks'. Higher alternative scores are preferred for opportunities and lower alternative scores are preferred for threats. In practice, two aggregation techniques are used to compute two aggregated indexes and evaluate the alternatives when criteria are divided into the opportunities and threats. The first approach is the opportunities to threat ratio approach (Tavana and Banerjee, 1995) and the second is the opportunities minus threat approach (Tavana, 2004). The former approach is a ratio scale and the latter approach is an interval scale.

Fuzzy Euclid is a MCDA model that captures the DMs' beliefs through a series of intuitive and analytical methods such as the analytic hierarchy process (AHP) and subjective probabilities. The concept of fuzzy sets is often used to reflect the inherent subjectivity and imprecision involved in the evaluation process (Zadeh, 1965). Fuzzy numbers have been widely used in decision problems where the information available is subjective or imprecise (Zimmermann, 1996). We use a defuzzification method to obtain crisp values from the subjective judgments provided by multiple DMs. These crisp values are synthesized with Entropy and the theory of displaced ideal to assist the DMs in their selection process. Two aggregated opportunity and threat indexes are used to plot the alternative projects in a four-zone graph based on their Euclidean distance from the ideal project.

A decision-making committee of three division chiefs at the Shuttle Project Engineering Office is responsible for the evaluation and selection of advanced-technology projects at the Kennedy Space Center (KSC). The proposed projects are independent and non-additive requests for engineering changes to the space shuttle that are generally initiated by the contractors or different divisions within KSC. *Fuzzy Euclid*, developed at KSC, considers the importance of each project relative to the longevity of the space-shuttle program and enhances the committee's decision quality and confidence. The next section presents a detailed explanation of the mathematical model and procedure followed by a case study and conclusion in Sections 3 and 4.

2. Mathematical model and procedure

The evaluation process begins with a preliminary review of M advanced-technology projects submitted to KSC for funding. The Shuttle Project Engineering Office selects I divisions to participate in the evaluation process. K division chiefs, called DMs in this study, are responsible for the evaluation of the advanced-technology projects. Initially, DMs use AHP independently to weight their importance of the participating divisions. Next, the DMs collectively decide what criteria (opportunities and threats) should be considered in the evaluation process. Once the DMs agree on a set of opportunities and threats, they use AHP independently to weigh their importance of the opportunities and threats. Then, the DMs consult with the experts and specialists within their divisions to assign probabilities of occurrence to the opportunities and threats. Next, a defuzzification method is used to obtain crisp values from the subjective judgments and estimates provided by the K DMs for the M projects. These crisp values are then synthesized in an MCDA model to produce an overall performance score for each of the M projects under consideration.

MCDA techniques require the determination of weights that reflect the relative importance of various competing objectives. Several approaches such as point allocation, paired comparisons, trade-off analysis, and regression estimates could be used to specify these weights (Kleindorfer *et al*, 1993). *Fuzzy Euclid* utilizes AHP developed by Saaty (1977, 1990a) to estimate the importance weight of the opportunities (u_{ij}^k) and their divisions ($w_i^k(U)$) for the I participating divisions and the K DMs. The advantage of AHP is its capability to elicit judgments and scale them uniquely using a procedure that measures the consistency of these scale values (Shim, 1989; Saaty, 1977; Vaidya and Kumar, 2006; Ho, 2008). The process is simplified by confining the estimates to a series of pairwise comparisons. The measure of inconsistency provided by AHP allows for the examination of inconsistent priorities. One of the advantages of AHP is that it encourages DMs to be consistent in their pairwise comparisons. Saaty (1977) suggests a measure of consistency for the pairwise comparisons. When the consistency ration is unacceptable, the DM is made aware that his or her pairwise comparisons are logically inconsistent, and he or she is encouraged to revise them. A similar procedure is used to find the relative importance weight of the threats (t_{ij}^k) and their divisions ($w_i^k(T)$) for the I divisions and the K DMs.

Traditionally, AHP is used to estimate the relative importance weight of the criteria and the relative performance of the alternatives in MCDA problems. However, in *Fuzzy Euclid*, we only use AHP to determine the importance weight of the opportunities and threats. Instead of using AHP to find the relative performance of the alternatives (projects) on each criterion, we use subjective probabilities of occurrence to capture these scores. These probabilities are used to identify 'ideal probabilities' and the 'ideal project' discussed later.

Fuzzy Euclid is a normative MCDA model with multiple factors representing different dimensions from which the projects are viewed. When the number of factors is large, typically more than a dozen, they may be arranged hierarchically (Saaty, 1977; Triantaphyllou and Mann, 1995; Triantaphyllou, 2000). *Fuzzy Euclid* assumes a hierarchical structure by initially identifying the divisions at NASA who are responsible for the evaluation of the advanced technology projects. Following this identification, each division is asked to identify the relevant factors in their decision-making process and group them into opportunities and threats. This hierarchical structure allows for a systematic grouping of decision factors in large problems. The classification of different factors is undoubtedly the most delicate part of the problem formulation (Bouyssou, 1990) because all different aspects of the problem must be represented while avoiding redundancies. Roy and Bouyssou (1987) have developed a series of operational tests that can be used to check the consistency of this classification. The K DMs use AHP to estimate their importance weight of the opportunities and threats for the I divisions.

There has been some criticism of AHP in the operations research literature. Harker and Vargas (1987) show that AHP does have an axiomatic foundation, the cardinal measurement of preferences is fully represented by the eigenvector method, and the principles of hierarchical decomposition and rank reversal are valid. On the other hand, Dyer (1990) has questioned the theoretical basis underlying AHP and argues that it can lead to preference reversals based on the alternative set being analysed. In response, Saaty (1990b) explains how rank reversal is a positive feature when new reference points are introduced. There are several methods for estimating the local importance weights from pairwise comparison matrices in AHP. We employ the row geometric mean method to determine the local priorities and avoid the controversies associated with rank reversal (Dyer, 1990; Harker and Vargas, 1990; Saaty, 1990b). In this procedure, which Saaty (1990b) calls the approximate method; the local priority of each criterion is obtained by the normalization of the row geometric mean associated with this criterion in the pairwise comparison matrices. The row geometric mean method eliminates the undesired rank reversals caused by the traditional arithmetic mean method (Barzilai and Golany, 1994; Aguarón and Moreno-Jiménez, 2000; Xu, 2000; Escobar *et al*, 2004; Leskinen and Kangas, 2005).

Next, the K DMs estimate the subjective probabilities of occurrence of the opportunities ($p_{ij}^{km}(U)$) and threats ($p_{ij}^{km}(T)$) for the M projects. Subjective probabilities are commonly used in strategic decision making because they require no historical data (observation of regularly occurring events by their long-run frequencies) (De Kluyver and Moskowitz, 1984; Weigelt, 1988; Vickers, 1992; Schoemaker, 1993; Schoemaker and Russo, 1993; Tavana, 1995, 2002). Subjective probabilities can be measured by asking a DM for the odds on an event. If the DMs are familiar with probability concepts, they can be asked directly for the required probability. If not, some sort of measuring instrument is required. Some researchers suggest using verbal phrases such as 'likely,' 'possible,' 'quite certain,' and etc., to elicit the required information and then converting them into numeric probabilities (Budescu and Wallsten, 1985; Brun and Teigen, 1988; Tavana *et al*, 1997). Other commonly used approaches include reasoning (Koriat *et al*, 1980), scenario construction (Schoemaker, 1993) and cross-impact analysis (Stover and Gordon, 1978). In this study, verbal probabilistic phrases were used to elicit numeric probabilities as suggested by Tavana *et al* (1997). Alternatively, the DM may use numeric probabilities instead of the probabilistic phrases. Merkhofer (1987) and Spetzler and Stael von Holstein (1975) review some probability elicitation procedures that are used in practice.

The probabilities associated with the opportunities and threats are assumed to be binomial. Binomial probabilities are commonly used in strategic decision making so that the DM can simplify the problem by analyzing possible outcomes as either occurring or not occurring. For example, Schoemaker (1993) assigns binomial probabilities to factor such as 'Dow Jones Industrial Average falling below 1500 mark by 1990'. Vickers (1992) assigns binomial probabilities to similar factor such as 'Japanese car manufacturers gain at least 30% of the European market share'. Tavana and Banerjee (1995) also assign binomial probabilities to similar factor such as 'Reduction of staff by 2%'. The main motivation for using the binomial probabilities is to reduce the complexity of the model and allow DMs to use event-driven factors. Next, we use a defuzzification method to obtain crisp values from the subjective judgments and estimates provided by multiple DMs.

A series of weights and probabilities are used in *Fuzzy Euclid* to estimate the importance weight of the selection criteria and their probabilities of occurrence for each alternative. Decision-making theory generally deals with three types of uncertainty: stochastic uncertainty, subjective uncertainty and informational uncertainty. Stochastic uncertainty is treated by probability theory and subjective and informational uncertainties are the target of fuzzy set and fuzzy logic theory.

Although fuzzy logic and probability theory are similar, they are not identical. Probability refers to the likelihood that something is true and fuzzy logic establishes the degree to which something is true. Probability is not a special case of fuzziness, but leads us to consider probability of fuzzy events. Dubois and Prade (1993) provide an analysis of correlation between fuzzy sets and probability theory. They argue that the existence of mathematical objects in probability theory does not suggest that fuzziness is reducible to randomness and it is possible to approach fuzzy sets and possibility theory without any probability considerations. Their study emphasizes on the interpretation multiplicity of probability and fuzzy set theories and shows that fuzzy set theoretic operations can be categorized according to their membership in the upper probability, the one-point coverage of a random set, or a likelihood function.

The research on the conjoint application of fuzzy sets and probability theory reports on several studies including marine and offshore safety assessment (Eleye-Datubo *et al*, 2008), financial modelling (Muzzioli and Reynaerts, 2007), information systems (Intan and Mukaidono, 2004), auditing (Friedlob and Schleifer, 1999), manufacturing cost estimation (Jahan-Shahi *et al*, 1999), and water quality management (Julien, 1994). We use fuzzy logic for project evaluation and selection at NASA and apply a defuzzification process to integrate I sets of division weights ($w_i^k(U)$ and $w_i^k(T)$), factors weights (u_{ij}^k and t_{ij}^k) and subjective probabilities ($p_{ij}^{km}(U)$ and $p_{ij}^{km}(T)$ into one set of crisp values for the entire group of K DMs. Consider fuzzy sets A_{ij}^m represented by the pairs:

$$A_{ij}^m = \{(p_{ij}^{km}, \mu_{A_{ij}^m}(p_{ij}^{km}))\}, \quad \forall p_{ij}^{km} \in P_{ij}^m \tag{1}$$

where: P_{ij}^m= The set of DMs' judgments on criterion j in the ith division given the choice of the mth project; ($i = 1, 2, \ldots, I$; $j = 1, 2, \ldots, J_i$; $m = 1, 2, \ldots, M$); p_{ij}^{km}= The judgment of the kth DM on criterion j in the ith division given the choice of the mth project; ($i = 1, 2, \ldots, I$; $j = 1, 2, \ldots, J_i$; $k = 1, 2, \ldots, K$; $m = 1, 2, \ldots, M$); $\mu_{A_{ij}^m}(p_{ij}^{km})$= The discrete membership function; ($i = 1, 2, \ldots, I$; $j = 1, 2, \ldots, J_i$; $k = 1, 2, \ldots, K$; $m = 1, 2, \ldots, M$).

Defuzzification is the translation of linguistic or fuzzy values into numerical, scalar, and crisp representations. The process of condensing the information captured by fuzzy sets into numerical values is similar to that of transformation of uncertainty-based concepts into certainty-based concepts. Intuitively speaking, the defuzzification process in *Fuzzy Euclid* is similar to an averaging procedure. Many defuzzification techniques have been proposed in the literature. The most commonly used method is the Center of Gravity (COG). Other methods include: random choice of maximum, first of maximum, last of maximum, middle of maximum, mean of maxima, basic defuzzification distributions, generalized level set defuzzification, indexed center of gravity, semi-linear defuzzification, fuzzy mean, weighted fuzzy mean, quality method, extended quality method, center of area, extended center of area, constraint decision defuzzification, and fuzzy clustering defuzzification. Roychowdhury and Pedrycz (2001) and Dubois and Prade (2000) provide excellent reviews of the most commonly used defuzzification methods.

The literature reports on several aggregation functions (Runkler, 1996; Van Leekwijk and Kerre, 1999; Ali and Zhang, 2001; Roychowdhury and Pedrycz, 2001). The selection of a specific aggregation function must be based on the problem characteristics and model requirements. Although the selection of an aggregation operation is context dependent, it is recommended to consider the criteria suggested by Klir and Yuan (1995). We use COG, also referred to as the center of area method, in *Fuzzy Euclid*. This method is highly popular and is often used as a standard defuzzification method. COG calculates the centroid of a possibility distribution function using Equation (2) for discontinuous cases:

$$COG(N) = \frac{\sum_{i=1}^{k} xi\,\mu(xi)}{\sum_{i=1}^{k} \mu(xi)} \tag{2}$$

The procedure for converting the fuzzy numbers in *Fuzzy Euclid* into a set of crisp values can be divided into the following three steps:

1. Evaluation of the membership functions related to the subjective probabilities of occurrence for opportunities ($\mu_{ij}^k(U)$) and threats ($\mu_{ij}^k(T)$):

$$\mu_{ij}^k(U) = w_i^k(U) \cdot u_{ij}^k \tag{3}$$

$$\mu_{ij}^k(T) = w_i^k(T) \cdot t_{ij}^k \tag{4}$$

   where: $w_i^k(U)(w_i^k(T))$= The ith division importance weight for the opportunities (threats) defined for the kth DM; ($i = 1, 2, \ldots, I; k = 1, 2, \ldots, K$); $u_{ij}^k(t_{ij}^k)$= Importance of the jth opportunity (threat) in the ith division for the kth DM; ($i = 1, 2, \ldots, I^U(I^T)$; $j = 1, 2, \ldots, J_i^U(J_i^T)$; $k = 1, 2, \ldots, K$); $I^U(I^T)$= The number of divisions for the group of opportunities (threats); $J_i^U(J_i^T)$= The number of opportunities (threats) in the ith division.
2. Calculation of the overall weighted subjective probabilities of opportunities ($f_{ij}^m(U)$) and threats ($f_{ij}^m(T)$) for the M projects as the summed product of the probabilities on their

grades of membership:

$$f_{ij}^{m}(U)=\sum_{k=1}^{K}\mu_{ij}^{k}(U)\cdot p_{ij}^{km}(U) \tag{5}$$

$$f_{ij}^{m}(T)=\sum_{k=1}^{K}\mu_{ij}^{k}(T)\cdot p_{ij}^{km}(T) \tag{6}$$

where: $p_{ij}^{km}(U)(p_{ij}^{km}(T))=$ Subjective probability of occurrence of the jth opportunity (threat) for the ith division given the choice of the mth project by the kth DM; ($m=1,2,\ldots,M;\ i=1,2,\ldots,I^{U}(I^{T});\ j=1,2,\ldots,J_i^{U}(J_i^{T});\ k=1,2,\ldots,K$).

3. On the final step of the defuzzification process, we divide the overall weighted subjective probabilities of opportunities and threats by their summed membership grades. These calculations result in M vectors of non-fuzzy values characterizing opportunities and threats for M projects:

$$COG(U_{ij}^{m})=\frac{f_{ij}^{m}(U)}{\mu_{ij}(U)} \tag{7}$$

$$COG(T_{ij}^{m})=\frac{f_{ij}^{m}(T)}{\mu_{ij}(T)} \tag{8}$$

where $\mu_{ij}(U)$ and $\mu_{ij}(T)$ define the membership functions for opportunities and threats with aggregated results for all DMs:

$$\mu_{ij}(U)=\sum_{k=1}^{K}\mu_{ij}^{k}(U) \tag{9}$$

$$\mu_{ij}(T)=\sum_{k=1}^{K}\mu_{ij}^{k}(T) \tag{10}$$

Next, we find the defuzzified importance weights for the opportunities and threats, as well as of total defuzzified opportunities (U^m) and threats (T^m) for all projects under consideration:

$$U^{m}=\sum_{i=1}^{I^{U}}\sum_{j=1}^{J_i^{U}}COG(U_{ij}^{m}) \tag{11}$$

$$T^{m}=\sum_{i=1}^{I^{T}}\sum_{j=1}^{J_i^{T}}COG(T_{ij}^{m}) \tag{12}$$

Finally, we revise the importance weight of the opportunities and threats determined through the defuzzification process with the entropy concept. Each opportunity or threat is an information source; therefore, the more information an opportunity or threat reveals, the more relevant it is to the decision analysis. The level of entropy $e(P)$ as a measure of fuzziness, indicates the variance of the assigned preference relation. The concept of entropy, originated in physics and statistical mechanics, has become increasingly popular in computer science and information theory. Shannon (1948) has defined the entropy of a probability distribution in which the total probability for all elements must add up to 1. However, Luca and Termini (1972) show that this restriction is unnecessary. They define a fuzzy entropy formula on a finite universal set $X=\{x_1,\ldots,x_n\}$ as:

$$\begin{aligned}e_{LT}(A)=&-\beta\sum_{i=1}^{n}[\mu_A(x_i)\ln\mu_A(x_i)\\&+(1-\mu_A(x_i))\ln(1-\mu_A(x_i))],\quad \beta>0\end{aligned} \tag{13}$$

where $\beta>0$ is a normalization constant, *ln* is the natural logarithm, $\mu_A(x_i)$ is a membership function for each preference intensity.

An entropy value of 1 indicates that all factors are biased by the maximum fuzziness and a lack of distinction is apparent in the preference relations. The fuzziness of the membership functions has its highest grade at the 'crossover value' ($\mu=0.5$). An entropy value of 0 indicates that the preference relations are definitely credible or definitely non-credible. Maximal distinctness is reached when $\mu=0$ and $\mu=1$.

The more different the probabilities of occurrence of an opportunity or threat are, the larger is the contrast intensity of the opportunity or threat, and the greater is the amount of information transmitted by that opportunity or threat. Assuming that vector $P_{ij}^{m}(U)=\{p_{ij}^{km}(U)\}$ characterizes the set of weighed probabilities in terms of the jth opportunity for the ith division (*ij*th opportunity) given the choice of the mth project, the entropy measure of the *ij*th opportunity is:

$$\begin{aligned}e(A_{ij}^{m}(U))=&-\beta\sum_{k=1}^{K}[\mu(p_{ij}^{km}(U))\cdot\ln\mu(p_{ij}^{km}(U))\\&+(1-\mu(p_{ij}^{km}(U)))\cdot\ln(1-\mu(p_{ij}^{km}(U)))]\end{aligned} \tag{14}$$

where $0\leqslant\mu(p_{ij}^{km}(U))\leqslant 1$, and $e(A_{ij}^{m}(U))\geqslant 0$. The smaller $e(A)$ is, the more information the jth opportunity transmits, and the larger $e(A)$ is, the less information it transmits. In addition, the total entropy of opportunities for project m is defined as $E_u^m=\sum_{i=1}^{I^U}\sum_{j=1}^{J_i^U}e(A_{ij}^{m}(U))$. Similar to the opportunities, the entropy measure of the *ij*th threat is:

$$\begin{aligned}e(A_{ij}^{m}(T))=&-\beta\sum_{k=1}^{K}[\mu(p_{ij}^{km}(T))\ \ln\mu(p_{ij}^{km}(T))\\&+(1-\mu(p_{ij}^{km}(T)))\ \ln(1-\mu(p_{ij}^{km}(T)))]\end{aligned} \tag{15}$$

where $0\leqslant\mu(p_{ij}^{km}(T))\leqslant 1$, $e(A_{ij}^{m}(T))\geqslant 0$ and the total entropy of threats for project m is defined as $E_T^m=\sum_{i=1}^{I^T}\sum_{j=1}^{J_i^T}e(A_{ij}^{m}(T))$.

Fuzzy Euclid is a weighted-sum MCDA model with opportunities and threats as conflicting criteria. Triantaphyllou (2000) has discussed the mathematical properties of weighted-sum MCDA models. Many weighted-sum models have been developed to help DMs deal with the strategy evaluation process (Leyva-López and Fernández-González, 2003; Gouveia *et al*, 2008). Triantaphyllou and Baig (2005) have examined the use of four key weighted-sum MCDA methods when benefits and costs (opportunities and threats) are used as conflicting criteria. They compared the simple weighted-sum model, the weighted-product model, and AHP along with some of its variants, including the multiplicative AHP. Their extensive empirical analysis revealed some ranking inconsistencies among the four methods, especially, when the number of alternatives was high. Although, they were not able to show which method results in the 'correct' ranking, they did prove multiplicative AHP is immune to ranking inconsistencies.

The weighted-sum scores in *Fuzzy Euclid* are used to compare potential projects among themselves and with the *ideal project*. The concept of ideal choice, an unattainable idea, serving as a norm or rationale facilitating human choice problem is not new (Tavana, 2002). See for example the stimulating work of Schelling (1960), introducing the idea. Subsequently, Festinger (1964) showed that an external, generally non-accessible choice assumes the important role of a point of reference against which choices are measured. Zeleny (1974, 1982) demonstrated how the highest achievable scores on all currently considered decision criteria form this composite ideal choice. As all choices are compared, those closer to the ideal are preferred to those farther away. Zeleny (1982, p 144) shows that the Euclidean measure can be used as a proxy measure of distance.

Using the Euclidean measure suggested by Zeleny (1982), *Fuzzy Euclid* synthesizes the results by determining the ideal opportunity and threat values. The ideal opportunity (U^*) is the highest defuzzified importance weight of the opportunities among the set U^m and the ideal threat (T^*) is the lowest defuzzified importance weight of the threats among the set T^m. The ideal opportunity and threat values form the ideal project. We then find the Euclidean distance of each project from the ideal project. The Euclidean distance is the sum of the quadratic root of squared differences between the ideal and the mth indices of opportunities and threats. To formulate *Fuzzy Euclid* algebraically, let us assume:

D_U^m (D_T^m)	Total Euclidean distance from the ideal opportunity (threat) for the mth project; ($m = 1, 2, \ldots, M$).
D^m	Overall Euclidean distance of the mth project; ($m = 1, 2, \ldots, M$).
$\bar{D}$	Mean Euclidean distance.
$U^m(T^m)$	The total defuzzified opportunity (threat) value of the mth project; ($m = 1, 2, \ldots, M$).
$U^*(T^*)$	The ideal defuzzified opportunity (threat) value.
$E_U^*(E_T^*)$	The entropy of the ideal opportunity (threat).
$DE_U^m(DE_T^m)$	The Euclidean distance from the entropy of the ideal opportunity for the mth project; ($m = 1, 2, \ldots, M$).
DE^m	Overall Euclidean distance of the entropy for the mth project; ($m = 1, 2, \ldots, M$).
$N_i^U(N_i^T)$	Number of opportunities (threats) for the ith division ($i = 1, 2, \ldots, I^U(I^T)$).

$$D^m = \sqrt{(D_U^m)^2 + (D_T^m)^2} \tag{16}$$

$$\bar{D} = \sum_{m=1}^{M} D^m \Big/ M \tag{17}$$

$$DE^m = \sqrt{(DE_U^m)^2 + (DE_T^m)^2} \tag{18}$$

$$U^* = Max\{U^m\} \tag{19}$$

$$T^* = Min\{T^m\} \tag{20}$$

$$E_U^* = Min\{E_U^m\} \tag{21}$$

$$E_T^* = Min\{E_T^m\} \tag{22}$$

where

$$D_U^m = U^* - U^m$$

$$D_T^m = T^m - T^*$$

$$DE_U^m = E_U^m - E_U^*$$

$$DE_T^m = E_T^m - E_T^*$$

Next, we plot the alternative projects on a plane using a polar coordinate system (sometimes also referred to as 'circular coordinates') in which each point is determined by a distance and an angle. The x-axis is represented by the total Euclidean distance from the ideal opportunity (D_U^m) and the y-axis is represented by the total Euclidean distance from the ideal threat (D_T^m). The position of the point corresponding to project m with Cartesian coordinates (D_U^m, D_T^m) on the graph is determined by its Euclidian distance from the coordinate origin (D^m) with an angle of φ_m between vector $(\overline{D_U^m, D_T^m})$ and the x-axis, where:

$$tg(\varphi_m) = D_T^m / D_U^m \tag{23}$$

We use the mean Euclidean distance ($\bar{D}$) and the angle ($\bar{\varphi}$) to divide the graph into four decision zones. In the case of a tie ($D_T^m = D_U^m$), $\varphi_m = 45°$ and $tg(\bar{\varphi}) = 1$. Projects with smaller D^m are closer to the ideal project and are preferred to projects with larger D^m. Furthermore, projects with smaller φ_m and DE^m are preferred to projects with larger φ_m and DE^m. Projects with equal D^m lie on the same circle (sphere). The following assertion is valid for projects lying on the same sphere: with growth of φ_m, the distance to the ideal opportunity decreases ($U^m \to$ min) and the distance to the ideal threat increases

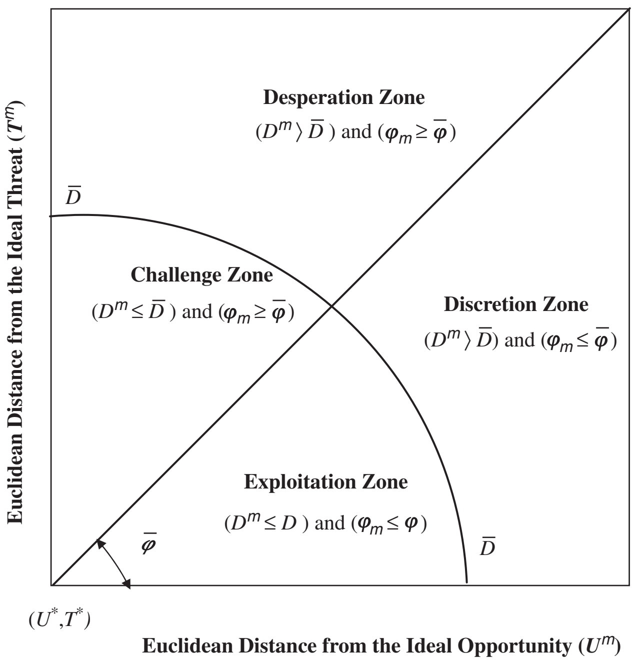

Figure 1 The four zones and their characteristics.

($U^m \rightarrow$ max). Therefore, projects with $\varphi_m \leqslant \bar{\varphi}$ are less risky and at the same time have little potential (Figure 1).

We also consider the overall Euclidean distance of the entropy for the mth project (DE^m). Projects with smaller DE^m (smaller measure of uncertainty) are preferred to those with larger DE^m (larger measure of uncertainty). With the *ideal project* (U^*=0, T^*=0) as the origin, the mean Euclidean distance ($\bar{D}$) and angle ($\bar{\varphi}$) divide the graph into *Exploitation*, *Challenge*, *Discretion*, and *Desperation* Zones:

- *Exploitation Zone*: In this zone $D^m \leqslant \bar{D}$ and $\varphi_m \leqslant \bar{\varphi}$. This area represents little threats and a great deal of opportunities. Projects falling into this zone are close to the *ideal project* ($U^* = 0$, $T^* = 0$) at the origin. These projects are considered very attractive because they have little risk but demonstrate tremendous potentials.
- *Challenge Zone*: In this zone $D^m \leqslant \bar{D}$ and $\varphi_m \geqslant \bar{\varphi}$. This area represents a great deal of threats and a great deal of opportunities. Projects falling into this zone are close to the *ideal project* ($U^* = 0$, $T^* = 0$) at the origin. These projects are considered challenging because they are very risky but also exhibit tremendous potential. This zone requires full use of the organization's capabilities and resources.
- *Discretion Zone*: In this zone $D^m > \bar{D}$ and $\varphi_m \leqslant \bar{\varphi}$. This area represents little threats and little opportunities. Projects falling into this zone are far from the *ideal project* ($U^* = 0$, $T^* = 0$) at the origin. These projects are considered discretionary because they are not risky and do not demonstrate meaningful potential. This zone represents the area where the DMs have freedom or power to act or judge on their own.
- *Desperation Zone*: In this zone $D^m > \bar{D}$ and $\varphi_m \geqslant \bar{\varphi}$. This area represents a great deal of threats and very little opportunities. Projects falling into this zone are far from the *ideal project* (U^*=0, T^*=0) at the origin. These projects should be undertaken as a last resort because they are very risky and do not exhibit significant potential.

Final prioritization of projects can be performed using their position in the decision space described above. However, DMs can make corrections in obtained ranking by taking into account the entropy measures of projects. Generally, DMs can make trade-offs between the distance measure and the entropy measure for the final prioritization of projects by specifying subjective threshold values. DMs might be willing to trade-off higher uncertainty levels for lower Euclidean distance or lower uncertainly levels for higher Euclidean distance.

Once the model is developed, sensitivity analyses can be performed to determine the impact on the ranking of projects for changes in various model assumptions. Some sensitivity analyses that are usually of interest are on the weights and probabilities of occurrence. The weights representing the relative importance of the divisions, opportunities, and threats are occasionally a point for discussion among the various DMs. In addition, probabilities of occurrence that reflect the degree of belief that an uncertain event will occur are sometimes a matter of contention.

3. A case study[1]

We illustrate the application of *Fuzzy Euclid* to a disguised actual case study at NASA—KSC. In this case, the DMs are a committee of three division chiefs for Safety, Reliability, and Operations considering requests for funding 10 advanced technology projects. The following are the projects and anticipated expenditures: Hubble ($1,778,000), Photovoltaic ($1,908,000), Airlock ($1,515,000), Babaloon ($1,949,000), Planet-Finder ($1,266,000), Nebula ($1,348,000), Solar ($1,176,000), Truss ($1,347,000), Centrifuge ($1,790,000) and Tether ($961,000). A budget of $15,038,000 is needed to fund all 10 projects. However, budgetary constraints limit spending to $10 million.

The process began with an initial meeting of the three DMs. They used Expert Choice (Expert Choice, 2006) to weight the importance of each division. Next, the DMs worked with their divisions to identify a set of opportunities and threats to be used in the evaluation process. Each division held separate meetings and developed their set of opportunities and threats. Then, they used Expert Choice to weight these opportunities and threats. The DMs recorded their consistency ratios and made sure it was below 0.10 as suggested by Saaty (1977).

The Safety division identified seven opportunities and seven threats, the Reliability division identified eight opportunities

[1] All the project names presented in this paper are changed to protect the anonymity of the projects. In addition, the data presented in this study is significantly reduced to allow a meaningful illustration of the model.

Table 1 The divisions and their opportunities and threats

	Opportunities
	Safety
1.	Ability to decrease ascent catastrophic risk
2.	Ability to decrease orbital and entry/landing catastrophic risk
3.	Ability to detect and eliminate process variability and uncoordinated changes
4.	Supporting protection from exposure to hostile environment
5.	Ability to control in-flight anomalies
6.	Ability to accommodate process deviations
7.	Ability to minimize unsafe inspection discrepancies
	Reliability
1.	Improving mean time to repair
2.	Improving identification/fault isolation
3.	Providing for a simpler system
4.	Improving access for maintenance tasks
5.	Increasing mean time between failures
6.	Reducing support equipment, special tools, and special training requirements
7.	Providing for the use of standard commercial of-the-shelf parts
8.	Providing for equipment interchangeability
	Operations
1.	Meeting the safety, launch, and landing requirements
2.	Meeting the time-sensitive implementation requirements
3.	Meeting the proposed costs
4.	Meeting the proposed schedule
5.	Meeting the advanced technology requirements
6.	Supporting program for near-term requirements
7.	Ability to use less people
8.	Ability to reduce time
9.	Ability to reduce hardware/materials expended during processing
10.	Supporting multi-system configurations
	Threats
	Safety
1.	Possibility of death or serious injury
2.	Possibility of loss of flight hardware, facility, or ground support equipment
3.	Possibility of personal injury and/or flight hardware, facility, or ground support damage
4.	Possibility of a serious violation of safety, health, or environmental federal/state regulation
5.	Possibility of a deminius violation of safety, health, or environmental federal/state regulation
6.	Possibility of failure propagation to other components or systems
7.	Possibility of critical single failure points
	Reliability
1.	Possibility of cascade failures
2.	Possibility of common cause failures
3.	Possibility of common mode failures
4.	Possibility of dependent failures
5.	Possibility of independent failures
	Operations
1.	Possibility of launchslippage
2.	Possibility of reliance on identified obsolete technology
3.	Possibility of interference in implementation (window of opportunity)
4.	Possibility of flight manifest changes
5.	Possibility of equipment and occupational hazards
6.	Possibility of non-support activity occurrences
7.	Possibility of site-specific restrictions

and five threats, and the Operations division identified 10 opportunities and seven threats to be included in the evaluation process (Table 1).

The importance weight of the three divisions along with the importance weight of the opportunities and threats and the subjective probabilities of occurrence are all integrated using the defuzzification process described earlier. Table 2 presents the total defuzzified opportunity (U^m) and threat values (T^m) associated with the 10 projects under consideration. $U^* = 13.849$ and $T^* = 4.307$ are the total defuzzified

Table 2 Project opportunity and threat values and their Euclidean distances

Project	U^m	T^m	D_u^m	D_t^m	D^m
Airlock	12.215	4.855	1.634	0.548	1.724
Hubble	13.075	5.894	0.774	1.587	1.766
Nebula	12.538	5.545	1.311	1.238	1.803
Planet-Finder	12.068	4.973	1.782	0.666	1.902
Babaloon	12.831	5.989	1.018	1.682	1.966
Centrifuge	11.521	4.788	2.328	0.481	2.378
Solar	11.443	4.307	2.406	0.000	2.406
Photovoltaic	13.849	7.841	0.000	3.534	3.534
Truss	10.289	4.678	3.560	0.371	3.580
Tether	8.670	5.780	5.179	1.473	5.384

Table 3 Project entropies and their Euclidean distances

Project	E_u^m	E_t^m	DE_u^m	DE_t^m	DE^m
Airlock	6.552	2.560	1.134	0.263	1.164
Planet-Finder	6.210	3.234	0.792	0.937	1.227
Solar	6.492	3.133	1.074	0.836	1.361
Nebula	6.692	3.073	1.274	0.776	1.492
Hubble	6.847	3.127	1.429	0.831	1.653
Babaloon	6.963	3.568	1.546	1.271	2.001
Photovoltaic	7.638	4.603	2.221	2.306	3.201
Tether	5.417	3.459	5.417	3.459	6.428
Truss	6.265	2.297	6.265	2.297	6.673
Centrifuge	6.712	2.637	6.712	2.637	7.212

opportunity and threat values for the ideal project. Next, we use Equation (16) to calculate the Euclidean distances (D^m) of the 10 projects from the ideal project presented in Table 2.

The entropy was calculated to evaluate the level of uncertainty in the DMs' estimations. The entropies for the opportunities (E_U^m) and threats (E_T^m)are shown in Table 3. $E_U^* = 5.417$ and $E_T^* = 2.297$ are the ideal entropy of the opportunities and threats for the ideal project. Next, we use Equation (18) to calculate the Euclidean distances of the entropies (DE^m) of the 10 projects from the ideal project presented in Table 3.

A Graphical representation of the results is shown in Figure 2. Project Airlock has the best Euclidean distance as it lies on the orbit, which is the closest to the ideal project. In addition, Airlock has the smallest entropy indicating the DMs agreement regarding this project. Project Photovoltaic has very strong threats and minimal opportunities and should be excluded from the consideration. Projects Centrifuge, Truss and Tether do not have a very high threat, their opportunities are far from the ideal and their large entropy indicates the DMs contradictions concerning these projects. Projects Truss and Tether both lie in the discretion zone. Using the classification scheme introduced earlier, we identified the position of each project in the four zones. Projects Airlock, Planet-Finder and Solar, and a major part of Nebula and Centrifuge lie in the *Exploitation Zone*. Hubble, Babaloon and a minor part of Nebula lie in the *Challenge Zone*. Truss, Tether and a minor part of Centrifuge lie in the *Discretion Zone*. Photovoltaic lies in the *Desperation Zone*.

Table 4 further shows the sorted results for the 10 projects based on their Euclidean distance from the ideal. Given the \$10 million spending limit; Airlock, Hubble, Nebula, Planet-Finder, Babaloon and Centrifuge could be considered for funding. However, although the priorities of Centrifuge and

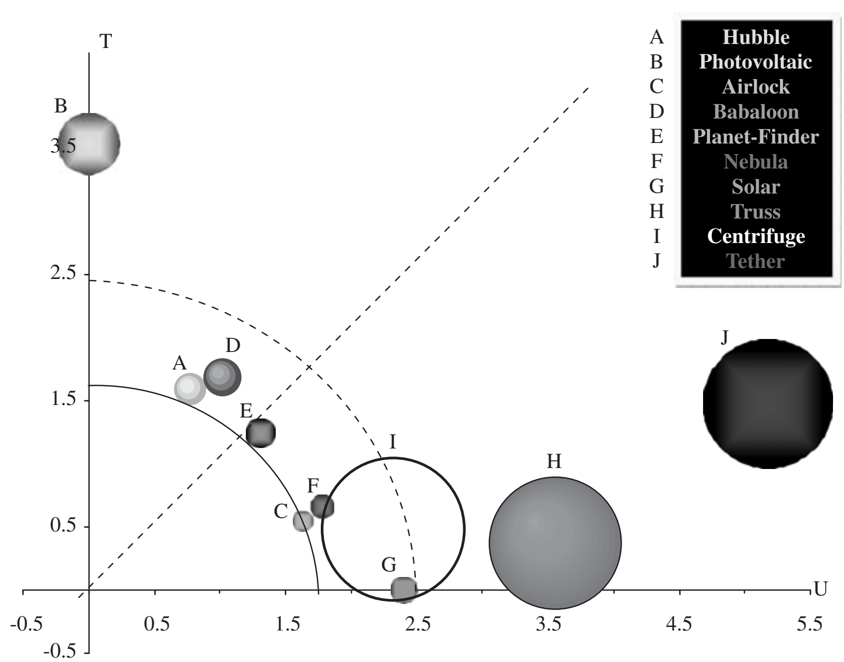

Figure 2 A graphical representation of the project scores and entropies.

Table 4 Project priorities and cumulative costs

Project	D^m	*Priority*	*Zone*		*Cost*	*Cumulative Cost*
			Major	*Minor*		
Airlock	1.724	1	Exploitation	—	1,778,000	1,778,000
Hubble	1.766	2	Challenge	—	1,908,000	3,686,000
Nebula	1.803	3	Exploitation	Challenge	1,515,000	5,201,000
Planet-Finder	1.902	4	Exploitation	—	1,949,000	7,150,000
Babaloon	1.966	5	Challenge	—	1,266,000	8,416,000
Centrifuge	2.378	6	Exploitation	Discretion	1,348,000	9,764,000
Solar	2.406	7	Exploitation	—	1,176,000	10,940,000
Photovoltaic	3.534	8	Desperation	—	1,347,000	12,287,000
Truss	3.580	9	Discretion	—	1,790,000	14,077,000
Tether	5.384	10	Discretion	—	961,000	15,038,000

Solar are close, six and seven respectively, the entropy for Solar is considerably less than the entropy for Centrifuge. This indicates a higher level of consistency in the DMs opinion for Solar compared with Centrifuge. In addition, Centrifuge lies in both the *Exploitation* and *Discretion Zones* while the entire Solar lies in *Exploitation Zone.* Considering this additional information, we recommended to replace Centrifuge with Solar. Ultimately, projects Airlock, Hubble, Nebula, Planet-Finder, Babaloon and Solar were selected for funding at KSC.

4. Conclusions

Global competition and the rapid development of computer and information technology have made strategic decision making more complex than ever. *Fuzzy Euclid* is a MCDA model that uses AHP, subjective probabilities, defuzzification, entropy, and the theory of displaced ideal to reduce these complexities by decomposing the project evaluation process into manageable steps. This decomposition is achieved without overly simplifying the evaluation process.

Fuzzy Euclid promotes consistent and systematic project evaluation and selection throughout the organization. Judgments captured as separate importance weights and probabilities of occurrence are used uniformly across all projects in the evaluation process. In the absence of separate value judgments, it is difficult to apply a set of importance weights and probabilities of occurrence consistently among the opportunities and threats when evaluating projects. *Fuzzy Euclid* provides a consistent combination of all assessments among all the projects. Whether the assessments faithfully represent real world circumstances depends on the competence and degree of effort the DMs exert in making the assessments.

Fuzzy Euclid is also useful in examining how sensitive the overall Euclidean scores are to changes in the portfolio of selected projects. *Fuzzy Euclid* also addresses questions about the sensitivity of the portfolio of selected projects to changes in the relative importance of the organizations, the relative importance of the opportunities and threats, and the probabilities of occurrence.

Fuzzy Euclid is not intended to replace human judgment in project evaluation and selection at KSC. In fact, human judgment is the core input in the process. *Fuzzy Euclid* helps the DMs to think systematically about complex project selection problems and improves the quality of their decisions. It is almost impossible to obtain objective data on the complex advanced-technology projects because of inherent uncertainties. However, experienced DMs can often make fairly accurate estimates of values. *Fuzzy Euclid* decomposes the project evaluation process into manageable steps and integrates the results to arrive at a solution consistent with managerial goals and objectives. This decomposition encourages DMs to carefully consider the elements of uncertainty.

Using a structured framework like *Fuzzy Euclid* does not imply a deterministic approach to project evaluation and selection. Although *Fuzzy Euclid* enables DMs to crystallize their thoughts and organize their beliefs, it should be used very carefully. Managerial judgment is an integral component of *Fuzzy Euclid*; therefore, the effectiveness of the model relies heavily on the DM's cognitive abilities to provide sound judgments. As with any decision analysis model, the researchers and practicing managers must be aware of the limitations of subjective estimates and use them carefully.

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Making decisions in hierarchic and network systems

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Abstract: This paper summarises a mathematical theory of the measurement of both tangible and intangible factors, the Analytic Hierarchy Process (AHP) and its generalisation to dependence and feedback, the Analytic Network Process (ANP) and illustrates their application to making complex multicriteria decisions.

Keywords: intangibles; judgements; pairwise comparisons; importance; likelihood; preference; priorities; synthesis; decision; Analytic Hierarchy Process; AHP; Analytic Network Process; ANP.

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1 Introduction

The Analytic Hierarchy Process (AHP) and its generalisation to dependence and feedback, the ANP are psychophysical theories of measurement. This means that they make the assumption that our judgements are subjective and that we acquire our experience and understanding about things using several distinct functions of the mind such as awareness, sensation, perception, thought and others in an inseparable mass. Our brains respond to intensities of occurrence, such as the varying intensities of sight, sound and smell. These intensities fall in different threshold intervals of just noticeable differences because we are unable to detect change in intensity until a stimulus is increased by a noticeable amount. Judgements must reflect not only knowledge about influences, but also the strengths with which these influences occur. These strengths are expressed in terms of relative priorities by experts who have experienced the complexity with which they are concerned and are then synthesise with respect to different criteria needed to make a decision. When desired, in the AHP/ANP known measurements from ratio scales can be used directly as they are in relative form and without interpretation. Derived numerical relative priorities can be validated in those cases where we have measurement so that we can improve our confidence in the applicability of our quantified judgements when applied to intangibles where measurements are unknown and unavailable.

In science, measurements of factors with different ratio scales are combined by means of formulas. The formulas apply within structures involving variables and their relations under natural law. The scales have a zero and an arbitrary unit applied uniformly in all measurements of that scale but its meaning remains elusive and becomes better understood through much practice. The meaning and use of the outcome is then interpreted according to the judgement of an expert as to how well it meets understanding and experience or satisfies laws of nature that are always there. Science derives results objectively, but interprets their significance subjectively. Because of the diversity of influences with which decision making is concerned, there are no set laws to characterise in fine details the structures in which relations are predetermined for every decision. Understanding is needed to structure a problem and then also to use judgements to represent importance, preference or likelihood quantitatively so that a best outcome can be derived by combining and trading off different factors or attributes according to given rules of composition thus reducing a multidimensional scaling problem to a one-dimensional scale of priorities. In decision making the priority scales can only be derived objectively after subjective judgements are made, which reflects the importance of influences on which our actions are based. The process is the opposite of what we do in science. All this tells us that it is not enough to advocate the use of a theory with numbers as a justifiable way to make decisions because judgements are subjective anyway. There has to be validation of the process through a variety of examples to make it a science based on reason, quantity and mathematics. Otherwise it would be as the saying goes "Garbage in garbage out".

To make complex risky decisions we need not only judgements but also structures that represent our best understanding of the flow of influences. The basic structure in doing this is an influence network of clusters and nodes contained within the clusters for the ANP and a hierarchy for the AHP. Priorities are established in the ANP and its particular case – the AHP – using pairwise comparisons and judgement. Many decision problems cannot be structured hierarchically because they involve the interaction and dependence of higher-level elements such as objectives and criteria in a hierarchy on lower-level elements. Not only does the importance of the criteria determine the importance of the alternatives as in a hierarchy, but also the importance of the alternatives themselves determines the importance of the criteria as in a network. Feedback enables us to factor the future into the present to determine what we have to do to attain a desired future.

The feedback structure does not have the top-to-bottom form of a hierarchy but looks more like a network without specifying levels, with cycles connecting its components of elements, and with *loops* that connect a component to itself. It also has sources and sinks. A *source* node is an origin of *paths of influence* (importance) and never a destination of such paths. A *sink* node is a destination of paths of influence and never an origin of such paths. A full network can include source nodes; intermediate nodes that fall on paths from source nodes, lie on *cycles*, or fall on paths to sink nodes; and finally sink nodes. Some networks can contain only source and sink nodes. Still others can include only source and cycle nodes or cycle and sink nodes or only cycle nodes. A decision problem involving feedback arises frequently in practice (Saaty and Özdemir, 2005). It can take on the form of any of the networks just described. The challenge is to structure the problem, to determine the priorities of the elements in the network and in particular the alternatives of the decision and even more to justify the validity of the outcome. Because feedback involves cycles, and cycling is an infinite process, the operations needed to derive the priorities become more demanding than it is with hierarchies.

2 Paradigm case: pairwise comparisons, the fundamental scale, eigenvectors, consistency and homogeneity

How to measure intangibles in a credible and 'valid' way is a main concern of the mathematics of the AHP/ANP. That is why it is an effective approach to multi-criteria decisions, because of their many intangible factors. At the end we must fit our entire experience into our system of priorities if we need to understand it in both its details and its general workings. As we said above, the AHP/ANP reduces a multidimensional problem into a unidimensional one. Decisions can be determined by a vector of priorities that gives an ordering of the different possible outcomes or by a single number for the best outcome. If a group wish to cooperate to agree on a single decision, it can combine the individual judgements into a representative group judgement or combine their individual final choices into a representative group final choice.

The development of the theory for analysing impacts of different levels of hierarchy, representing subsystems of a system, has called for a new and general method of measurement yielding absolute scales. This method has been successfully tested by way of validation in optics, heat, the measurement of distances, the weights of objects, the relative consumption of drinks by a population, the amount of electricity consumed by different devices and similar phenomena. It has also been applied in economics to estimate the relative wealth of nations (correlating closely with their GNP's), in politics to determine a measure of influence of nations as well as in a number of similar problems, whenever possible, validated against existing measurements.

2.1 Pairwise comparisons

Suppose we are given a set of objects that are all sufficiently light and can be lifted by hand. We wish to estimate their relative weights. One way would be to directly guess the weight of each object in pounds for example, by lifting it (perhaps using the lightest one as the standard), comparing the whole class, and then dividing the weight of each by the total to get its relative weight. The danger here is that we have no good idea about how much a pound weighs and make poor and arbitrary estimates. Another method which utilises more of the available information in the experiment is to compare the objects in pairs, such as lifting one and then lifting another and then back to the first and then again the second and so on until we have formulated a judgement as to the relative weight (ratio) of each pair of objects. The problem then is to determine the relative values of these objects. The second process has the advantage of focusing on two objects at a time and on how they relate to each other. It also uses redundant information since each object is methodically compared with every other. Unlike estimating weights one at a time using pounds for measurement, paired comparisons is a process of using judgements first in order to derive priority measurements from them second.

It may be useful to reinterpret what we just said. To make *tradeoffs* among the many objectives and criteria of a decision, which cannot be made simply by using words and logic, the judgements that are usually made in qualitative terms must be expressed numerically. To do this, rather than simply assign a seemingly arbitrary score out of a person's memory that appears reasonable, one must make pairwise comparisons in a carefully designed scientific way. In paired comparisons the smaller or lesser element is used as the unit, and the larger or greater element is estimated as a multiple of that unit with respect to the common property or criterion for which the comparisons are made. The unit element then has the reciprocal value when compared with the larger element. In this sense measurement with judgements is made more scientifically than assigning numbers more or less arbitrarily.

In decision making, pairwise comparisons as to dominance of one element over another with regard to an attribute, property or criterion they share, generally occur in three basic ways: *importance*, *preference* and *likelihood*. Likelihood means that probabilities can be estimated as priorities obtained from a pairwise comparison process.

From all the paired comparisons, one derives a scale of relative values for the priorities. As we shall see below, due to inevitable inconsistency among the judgements it is mathematically *necessary* to derive the priorities in the form of the principal eigenvector of the matrix of paired comparisons.

We learn from making paired comparisons in the AHP that if A is five times larger than B and B is three times larger than C, then A is 15 times larger than C and A dominates C 15 times. Thus, in decision making, dominance rather than closeness is the essential property and we need the topology of order and not the usual metric topology that is prevalent in the physical sciences. In accordance with metric topology, if A has 5 dollars more than B and B has 3 dollars more than C then A has 8 dollars more than C. Order measurement says 15 times more, whereas metric measurement says 8 dollars more. The first is dimensionless while the second requires a unit of measurement. One attaches measurement to an object, the other attaches dominance between a pair of objects.

2.2 The fundamental scale

We now turn to a question of what numerical scale to use in the pairwise comparison matrices. Whatever problem we deal with we must use numbers that are sensible. By using them, particularly when they are inconsistent, the eigenvalue process would provide a scale of relative values called priorities. The best argument in favour of a scale is if we can be use it to reproduce results already known in physics, economics or whatever area that there is a scale. But that is not enough because for intangibles we have no easy way of validation except perhaps when there are frequencies and probabilities to use for validation.

Our choice of scale hinges on the following observation. Roughly, the scale should satisfy the requirements:

- It should be possible to represent people's differences in feelings when they make comparisons. It should also represent all distinct shades of feeling that people have.
- If we denote the scale values by $x_1, x_2, \ldots, x_n$ then it would be desirable that $x_{i+1}/x_1 = i$, $i = 1, \ldots, n$. The reasons for this are:
  - We need uniformity to make sure that the scale covers all judgements. We require that the subject must be aware of all objects at the same time.
  - We agree with the psychological experiments Miller (1956), later we validated with mathematics, which show that an individual cannot simultaneously compare more than seven objects (±2) without being confused (see later).
  - We can only compare things that are closely similar. When their differences are great, we put them in clusters and compare the clusters of similar elements first, and then compare the elements in each cluster.

The Fundamental Scale used for the judgements applied to compare homogeneous (close) elements is shown in Table 1. Judgements are first given verbally as indicated in the scale and then a corresponding number is associated with that judgement.

We have assumed that an element with weight zero is eliminated from comparison because zero can be applied to the whole universe of factors not included in the discussion. Reciprocals of all scaled ratios that are ≥1 are entered in the transpose positions.

The foregoing integer-valued scale of response used in making paired comparison judgements can be derived mathematically from the well-known psychophysical logarithmic response function of Weber-Fechner (Fechner, 1966). For a given value of the stimulus, the magnitude of response remains the same until the value of the stimulus is increased sufficiently large in proportion to the value of the stimulus, thus preserving the proportionality of relative increase in stimulus for it to be detectable for a new response. This suggests the idea of just noticeable differences (jnd), well known in psychology.

Table 1 The fundamental scale of absolute numbers

Intensity of importance	*Definition*	*Explanation*
1	Equal importance	Two activities contribute equally to the objective
2	Weak or slight	
3	Moderate importance	Experience and judgement slightly favour one activity over another
4	Moderate plus	
5	Strong importance	Experience and judgement strongly favour one activity over another
6	Strong plus	
7	Very strong or demonstrated importance	An activity is favoured very strongly over another; its dominance demonstrated in practice
8	Very, very strong	
9	Extreme importance	The evidence favouring one activity over another is of the highest possible order of affirmation
1.1–1.9	When activities are very close a decimal is added to 1 to show their difference as appropriate	A better alternative way to assigning the small decimals is to compare two close activities with other widely contrasting ones, favouring the larger one a little over the smaller one when using the 1–9 values
Reciprocals of above	If activity *i* has one of the above nonzero numbers assigned to it when compared with activity *j*, then *j* has the reciprocal value when compared with *i*	A logical assumption
Measurements from ratio scales		When it is desired to use such numbers in physical applications. Alternatively, often one estimates the ratios of such magnitudes by using judgement

To derive the values in the scale starting with a stimulus s_0 successive magnitudes of the new stimuli take the form:

$$s_1 = s_0 + \Delta s_0 = s_0 + \frac{\Delta s_0}{s_0} s_0 = s_0(1+r)$$
$$s_2 = s_1 + \Delta s_1 = s_1(1+r) = s_0(1+r)^2 \equiv s_0\alpha^2$$
$$\vdots$$
$$s_n = s_{n-1}\alpha = s_0\alpha^n \; (n = 0, 1, 2, \ldots).$$

We consider the responses to these stimuli to be measured on a ratio scale ($b = 0$). A typical response has the form $M_i = a \log \alpha^i$, $i = 1, \ldots, n$, or one after another they have the form:

$$M_1 = a\log\alpha, M_2 = 2a\log\alpha, \ldots, M_n = na\log\alpha.$$

We take the ratios $M_i = M_1$, $i = 1, \ldots, n$ of these responses in which the first is the smallest and serves as the unit of comparison, thus obtaining the *integer* values 1, 2, …, n of the fundamental scale of the AHP. It appears that the positive integers are intrinsic to our ability to make comparisons, and that they were not an accidental invention by our primitive ancestors. In a less mathematical vein, we note that we are able to distinguish ordinally between high, medium and low at one level and for each of them in a second level below that also distinguish between high, medium and low giving us nine different categories. We assign the value one to (low, low) which is the smallest and the value nine to (high, high) which is the highest, thus covering the spectrum of possibilities between two levels, and giving the value nine for the top of the paired comparisons scale as compared with the lowest value on the scale. As we mentioned above, we do not need to keep in mind more than 7 ± 2 elements because of increase in inconsistency when we compare more than about seven elements. Finally, we note that the scale just derived is attached to the importance we assign to judgements. If we have an exact measurement such as 2.375 and want to use it as it is for our judgement without attaching significance to it, we can use its entire value without approximation. It is known that small changes in judgement lead to small changes in the derived priorities. Note that if we use actual measurements in the pairwise comparisons, the eigenvector approach recovers these values in relative form.

Judgements that represent dominance belong to an absolute scale of numbers which unlike interval and ratio scales that can be transformed to other interval or ratio scales respectively and yield different numbers that mean the same thing, an absolute scale is invariant under the identity transformation that is its numbers cannot be changed to other numbers and mean the same thing. From such numbers priorities can be derived which also belong to an absolute scale of relative numbers whose total sum is equal to one.

In the judgement matrix A, instead of assigning two numbers w_i and w_j that belong to a prior ratio scale of and forming the ratio w_i/w_j we assign a single number drawn from the Fundamental Scale of absolute numbers to represent the ratio $(w_i/w_j)/1$. It is a nearest integer approximation to the ratio w_i/w_j. The derived scale will reveal what w_i and w_j are. This is a central fact about the relative measurement approach. It needs a fundamental scale to express numerically the relative dominance relationship.

Very early in the history of the subject, Saaty and Khouja did the following exercise on an airplane in 1973. They simply used their common knowledge about the relative influence and standing of these countries in the world and without referring to any specific economic data related to GNP values. The two results are close and demonstrate that the general understanding an interested person has about a problem can be used to advantage to make fairly good estimates through paired comparisons.

Table 2 gives the judgements using the AHP 1–9 scale to represent the dominance of the elements on the left side of the matrix with respect to wealth over those at the top of the matrix and Table 3 provides the derived priorities, the actual and relative GNP values (how many times richer is the country on the left than a country at the top?).

Table 2 Paired comparisons of the relative dominance in wealth of seven nations

	USA	*USSR*	*China*	*France*	*UK*	*Japan*	*W. Germany*
USA	1	4	9	6	6	5	5
USSR	1/4	1	7	5	5	3	4
China	1/9	1/7	1	1/5	1/5	1/7	1/5
France	1/6	1/5	5	1	1	1/3	1/3
UK	1/6	1/5	5	1	1	1/3	1/3
Japan	1/5	1/3	7	3	3	1	2
W. Germany	1/5	1/4	5	3	3	1/2	1

Table 3 Outcome of estimated relative wealth and the actual and relative values

	Normalised eigenvector	*Actual GNP* (*1972*)	*Normalised GNP values*
USA	0.427	1,167	0.413
USSR	0.23	635	0.225
China	0.021	120	0.043
France	0.052	196	0.069
UK	0.052	154	0.055
Japan	0.123	294	0.104
W. Germany	0.094	257	0.091

The general eigenvalue formulation is obtained by perturbation of the following consistent formulation:

$$Aw = \begin{array}{c} \\ A_1 \\ \vdots \\ \\ A_n \end{array} \begin{array}{c} \begin{array}{ccc} A_1 & \cdots & A_n \end{array} \\ \begin{bmatrix} \dfrac{w_1}{w_1} & \cdots & \dfrac{w_1}{w_n} \\ \vdots & & \vdots \\ \dfrac{w_n}{w_1} & \cdots & \dfrac{w_n}{w_n} \end{bmatrix} \end{array} \begin{bmatrix} w_1 \\ \vdots \\ w_n \end{bmatrix} = n \begin{bmatrix} w_1 \\ \vdots \\ w_n \end{bmatrix} = nw.$$

where A has been multiplied on the right by the transpose of the vector of weights $w = (w_1, \ldots, w_n)$. The result of this multiplication is nw. Thus, to recover the scale from the matrix of ratios, one must solve the problem $Aw = nw$ or $(A - nI)w = 0$. This is a system of homogeneous linear equations. It has a nontrivial solution if and only if the determinant of $A - nI$ vanishes, that is, n is an eigenvalue of A. Now A has unit rank since every row is a constant multiple of the first row. Thus all its eigenvalues except one are zero. The sum of the eigenvalues of a matrix is equal to its trace, that is, the sum of its diagonal elements. In this case the trace of A is equal to n. Thus n is an eigenvalue of A, and one has a nontrivial solution. The solution consists of positive entries and is unique to within a multiplicative constant.

2.3 Consistency

Assume that n activities are being considered by a group of interested people. We assume that the group's goals are:

a to improve judgements on the relative importance of these activities

b to insure that the judgements are quantified to an extent which also permits a quantitative interpretation of the judgements among all activities.

Clearly, goal b will require appropriate technical assistance.

Our goal is to describe a method for deriving, from the group's quantified judgements (i.e., from the relative values associated with *pairs* of activities), a set of weights to be associated with individual activities and these weights should reflect the group's quantified judgements. What this approach achieves is to put the information resulting from a and b into usable form without deleting information residing in the qualitative judgements. It becomes important for us to know how inconsistent we are and which are the most inconsistent judgements and how they can be changed to improve the consistency. But our knowledge may not be adequate to correct our inconsistency as needed. If the inconsistency remains very high despite the changes we make that are compatible with out understanding, we cannot make a decision.

The priority weights are obtained directly by adding and normalising to one the sum of the rows of the matrix, or any of its columns. The intransitivity of influences (how much A dominates B and how much B dominates C and then how much C dominates A) cannot occur when the judgements are consistent for then $A^k = n^{k-1}A$ for all k. However, when the judgements are inconsistent $a_{ij}\, a_{jk} = a_{ik}$ for all i, j, k no longer holds and the sum of dominances along different paths leds to different outcomes. It is known that the different order transitivity of influences can be measured by raising the matrix to different powers. Each power of the matrix yields a set of priorities obtained as the normalised sum of its rows. It is not difficult to show that the average priority of the all these priority vectors is their Cesaro sum that leads to taking the limiting power of the matrix. Perron's theory about positive matrices tells us that this limit is the principal eigenvector of the matrix thus requiring us to solve the principal eigenvalue problem for our positive matrix. This shows that the principal eigenvector is a necessary condition for deriving priorities from inconsistent judgements. Alternatively, the priorities can be used to weight the judgements in each row of the pairwise comparisons matrix and add them to get these priorities in normalised from back. This means that we must solve the eigenvalue problem $Aw = cw$. It can be shown that $c = \lambda_{\max}$ the principal eigenvalue of the matrix.

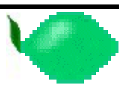
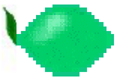
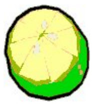
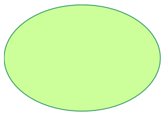
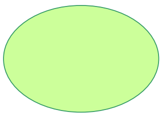
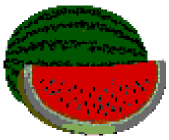
Associated with the weights is an inconsistency index. The consistency index of a matrix is given by $\mathrm{CI} = \lambda_{\max} - n / n - 1 \equiv \mu.$ The Consistency Ratio (CR) is obtained by forming the ratio of CI and the appropriate one of the following set of numbers shown in Table 4, each of which is an average random consistency index computed for $n \leq 10$ for very large samples. They create randomly generated reciprocal matrices using the scale 1/9, 1/8, …, 1/2, 1, 2, …, 8, 9 and calculate the average of their eigenvalues. This average is used to form the Random Consistency Index RI. Table 5 shows the values obtained from one set of such simulations and also their first order differences, for matrices of size 1, 2, …, 15. However, as we mentioned above, we do not recommend comparing more than seven items in any single matrix because of the findings below. They show that beyond seven elements the inconsistency increases so slowly, that it is not possible to tell which element causes the greatest inconsistency.

Figure 1 is a plot of the first two rows of Table 4. It shows the asymptotic nature of random inconsistency.

Table 4 Random index

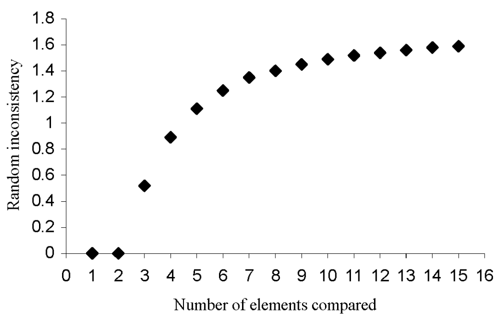
Order	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*
RI	0	0	0.52	0.89	1.11	1.25	1.35	1.40	1.45	1.49	1.52	1.54	1.56	1.58	1.59
First order differences		0	0.52	0.37	0.22	0.14	0.10	0.05	0.05	0.04	0.03	0.02	0.02	0.02	0.01

Table 5 Comparisons according to volume

	0.07		0.28		0.65
Cherry Tomato		*Small Green Tomato*		*Lime*	
	0.08		0.22		0.70
Lime		*Grapefruit*		*Honeydew*	
$\frac{0.08}{0.08}=1$		$\frac{0.22}{0.08}=2.75$		$\frac{0.70}{0.08}=8.75$	
$0.65\times 1=0.65$		$0.65\times 2.75=1.79$		$0.65\times 8.75=5.69$	
	0.10		0.30		0.60
Honeydew		*Sugar Baby Watermelon*		*Oblong Watermelon*	
$\frac{0.10}{0.10}=1$		$\frac{0.30}{0.10}=3$		$\frac{0.60}{0.10}=6$	
$5.69\times 1=5.69$		$5.69\times 3=17.07$		$5.69\times 6=34.14$	

This means that 34.14/.07.487.7 cherry tomatoes are equal to the oblong watermelon.

Figure 1 Plot of random inconsistency



Since it would be pointless to try to discern any priority ranking from a set of random comparison judgements, we should probably be uncomfortable about proceeding unless the consistency index of a pairwise comparison matrix is very much smaller than the corresponding random index value in Table 4. The *Consistency Ratio* (CR) of a pairwise comparison matrix is the ratio of its consistency index ~ to the corresponding random index value in Table 4. The notion of order of magnitude is essential in any mathematical consideration of changes in measurement. When one has a numerical value say between 1 and 10 for some measurement and one wishes to determine whether change in this value is significant or not, one reasons as follows: A change of a whole integer value is critical because it changes the magnitude and identity of the original number significantly. If the change or perturbation in value is of the order of a percent or less, it would be so small (by two orders of magnitude) and would be considered negligible. However if this perturbation is a decimal (one order of magnitude smaller) we are likely to pay attention to modify the original value by this decimal without losing the significance and identity of the original number as we first understood it to be. Thus in synthesising near consistent judgement values, changes that are too large can cause dramatic change in our understanding, and values that are too small cause no change in our understanding. We are left with only values of one order of magnitude smaller that we can deal with incrementally to change our understanding. It follows that our allowable consistency ratio should be not more than about 0.10 for a matrix larger than 5×5, 8% for a 4×4 matrix and 5% for a 3×3 matrix. This requirement cannot be made smaller such as 1% or 0.1% without trivialising the impact of inconsistency.

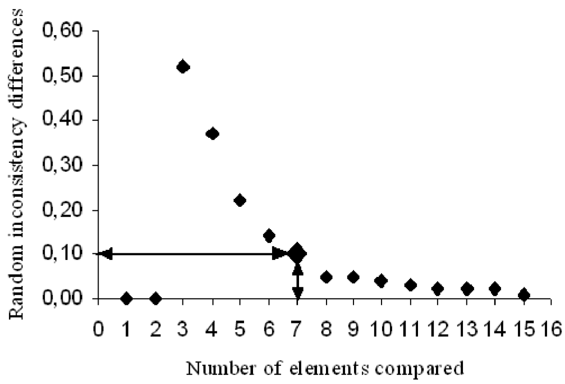
Inconsistency itself is important because without it, new knowledge that changes preference cannot be admitted. Assuming that all knowledge should be consistent contradicts experience that requires continued revision of understanding. If the CR is larger than desired, we do three things:

- Find the most inconsistent judgement in the matrix (for example, that judgement for which $\varepsilon_{ij} = a_{ij}w_j/w_i$ is largest).
- Determine the range of values to which that judgement can be changed corresponding to which the inconsistency would be improved.
- Ask the judge to consider, if he can, change his judgement to a plausible value in that range. If he is unwilling, we try with the second most inconsistent judgement and so on. If no judgement is changed the decision is postponed until better understanding of the stimuli is obtained.

The third row of Table 4 gives the differences between successive numbers in the second row. Figure 2 is a plot of these differences and shows the importance of the number seven as a cutoff point beyond which the differences are less than 0.10 where we are not sufficiently sensitive to make accurate changes in judgement on several elements simultaneously. A similar argument plot be made by using the ratios of the numbers in the third row of Table 4 for $n \geq 3$.

We do not insist that judgements be consistent and, hence, they need not be transitive. An interesting illustration is afforded by tournaments regarding inconsistency or lack of transitivity of preferences. A team C_1 may lose against another team C_2 which has lost to a third team C_3; yet C_1 may have won against C_3. Thus, team behaviour is inconsistent – a fact which has to be accepted in the formulation, and nothing can be done about it.

Figure 2 Plot of first differences in random inconsistency



2.4 Homogeneity

Because human beings are limited in size and the firings of their neurons are limited in intensity, it is clear that there is a limit on their ability to compare the very small with the very large. Homogeneity as an important concept to ensure consistency in the paired comparisons requires the elements to be of the same order of magnitude. It is precisely for this reason that pairwise comparisons are made on elements or alternatives that are close or homogeneous and the more separated they are, the more need there is to put them in different groups and link these groups with a common element from one group to an adjacent group of slightly greater or slightly smaller elements. In this way one can gradually compare grains of sand of varying sizes increasing to small pebbles and larger stones. When done properly, the largest element in one group (the pivot) is used as the smallest one in the next group. The weights of the elements in the second group are divided by the priority of the pivot in that group and then multiplied by the priority of the same pivot element from the first group, making them comparable with the first group. The process is then continued.

Table 5 shows how this process works in comparing a cherry tomato with a water melon, which appears to be two orders of magnitude bigger in size, by introducing intermediate objects in stages.

For a given positive reciprocal matrix $A = [a_{ij}]$ and a given pair of distinct indices $k > l$, define $A(t) = [a_{ij}(t)]$ by $a_{kl}(t) \equiv a_{kl} + t$, $a_{lk}(t) \equiv (a_{lk} + t)^{-1}$, and $a_{ij}(t) \equiv a_{ij}$ for all $i \neq k$, $j \neq l$, so $A(0) = A$. Let $\lambda_{\max}(t)$ denote the Perron eigenvalue of $A(t)$ for all t in a neighbourhood of $t = 0$ that is small enough to ensure that all entries of the reciprocal matrix $A(t)$ are positive there. Finally, let $v = [v_i]$ be the unique positive eigenvector of the positive matrix A^T that is normalised so that $v^T w = 1$. Then a classical perturbation formula tells us that

$$\left.\frac{\mathrm{d}\lambda_{\max}(t)}{\mathrm{d}t}\right|_{t=0} = \frac{v^T A'(0) w}{v^T w} = v^T A'(0) w = v_k w_l - \frac{1}{a_{kl}^2} v_l w_k .$$

We conclude that

$$\frac{\partial \lambda_{\max}}{\partial a_{ij}} = v_i w_j - a_{ji}^{\;2} v_j w_i \quad \text{for all} \quad i, j = 1, \ldots, n.$$

Because we are operating within the set of positive reciprocal matrices, $\partial\lambda_{\max} / \partial a_{ji} = -\partial\lambda_{\max} / \partial a_{ij}$ for all i and j. Thus, to identify an entry of A whose adjustment within the class of reciprocal matrices would result in the largest rate of change in $\lambda_{\max}$ we should examine the $n(n - 1)/2$ values $\{v_i w_j - a_{ji}^2 v_j w_i\}$, $i > j$ and select (any) one of largest absolute value.

3 Hierarchies and their priorities

Although the notion of a hierarchy is old, our method of measurement in hierarchical structures is new. It cannot be compared with any analytical macro models because so far all such modelling manages to pull all its variables into a single level. What we need to do is apply our analysis to problems whose hierarchical structure is carefully defined and note the results for their relevance and validity.

Any system is a large matrix of interactions between its components in which most of the entries are (close to) zero. Ordering those entries according to their orders of magnitude, a distinct hierarchic structure, is discerned. In fact, this arrangement of the elements of a system in an incidence type matrix can be used to identify the levels of a hierarchy.

The laws characterising different levels of a hierarchy are generally different. The levels differ both in structure and function. The proper functioning of a higher level depends on the proper functioning of the lower levels. The basic problem with a hierarchy is to seek understanding at the highest levels from interactions of the various levels of the hierarchy rather than directly from the elements of the levels.

At this stage of development of the theory the choice of levels in a hierarchy generally depends on the knowledge and interpretation of the observer. Rigorous methods for structuring systems into hierarchies are gradually emerging in the many areas of the natural and social sciences and, in particular, in general systems theory as it relates to the planning and design of social systems.

What is a Hierarchy?

Definition 1: An ordered set is any set S with a binary relation $\leq$ which satisfies the reflexive, anti-symmetric, and transitive laws:

Reflexive: For all x, $x \leq x$;

Anti-symmetric: If $x \leq y$ and $y \leq x$, then $x = y$;

Transitive: If $x < y$ and $y < z$, then $x < z$.

For any relation $x < y$ (read, y includes x) of this type, we may define $x < y$ to mean that $x \leq y$ and $x \neq y$. y is said to cover (dominate) x if $x < y$ and if $x < t < y$ is possible for no t.

Ordered sets with a finite number of elements can be conveniently represented by a directed graph. Each element of the system is represented by a vertex so that an arc is directed from a to b if $b < a$.

Definition 2: A simply or totally ordered set (also called a chain) is an ordered set with the additional property that if $x, y \in S$ then either $x \leq y$ or $y \leq x$.

Definition 3: A subset E of an ordered set S is said to be bounded from above if there is an element $s \in S$ such that $x \leq s$ for every $x \in E$. The element s is called an *upper bound* of E. We say E has a supremum or least upper bound in S if E has upper bounds and if the set of upper bounds U has an element u_1 such that $u_1 \leq u$ for all $u \in U$. The element u is unique and is called the supremum of E in S.

There are many ways of defining a hierarchy. The one which suits our needs best here is the following:

We use the notation $x^- = \{y \mid x \text{ covers } y\}$ and $x^+ = \{y \mid y \text{ covers } x\}$, for any element x in an ordered set.

Definition 4: Let H be a finite set with largest element b.

H is a hierarchy if it satisfies the conditions:

a There is a partition of H into sets L_k, $k = 1, \ldots, h$ where $L_1 = \{b\}$.

b $x \in L_k$ implies $x^- c\ L_{k+1}$ $k = 1, \ldots, h - 1$.

c $x \in L_k$ implies $x^+ c\ L_{k-1}$ $k = 1, \ldots, h$.

For each $x \in H$, there is a weighting function w_x: $x^- \rightarrow [0,1]$ such that $\Sigma(y \in x^-)\ w_x(y) = 1$ (The weighting function is determined for each problem individually.)

The sets L_i are the *levels* of the hierarchy, and the function w_x is the *priority function* of the elements in one level with respect to the objective x. We observe, that even if $x^- \neq L_k$ (for some level L_k), w_x may be defined for all of L_k, by setting it equal to zero for all elements in L_k not in x^-.

A hierarchy is *complete* if, for all $x \in L_k$, $x^+ = L_{k-1}$, for $k = 2, \ldots, h$.

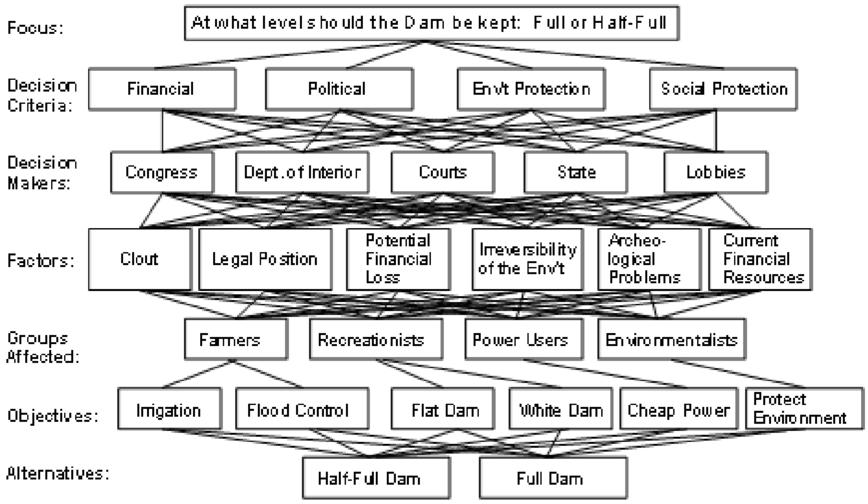
Let us consider a basic problem in less technical terms. Given a social (or economic) system with a major objective b, and the set L_h of basic activities, such that the system can be modelled as a hierarchy with largest element b and lowest level L_h. What are the priorities of the elements of L_h with respect to b?

We assume that the elements in each level may belong to more than a single hierarchy. To measure their priority they must be regarded as *independent* within a given hierarchy.

The mathematical basis of priority vector derivation was described in Saaty (1979) but in much more detail in Saaty (2005, 2006). The following observation holds for a complete hierarchy and it is also useful in general. The priority of an element in a level is the sum of its priorities in each of the subsets to which it belongs, each weighted by the fraction of elements of the level which belong to that subset and by the priority of that subset. The resulting set of priorities of the elements in the level is then normalised by dividing by its sum. The priority of a subset in a level is equal to the priority of the dominating element in the next level.

The decision on what the level of a dam should be kept can be as elaborate as shown in Figure 3. This is an example of a structure where different actors are included in the hierarchy. One group of actors is decision makers which is located in the third level of the hierarchy because of their different concerns that affect the outcome of a decision. The other actors are groups who are affected by the decision located in fifth level of the model. The level of the dam need to be determined based on which would serve them the best.

Figure 3 Hierarchy for level of a dam: full or half-full



4 Structuring, composition and synthesis

4.1 Structuring

Structuring a complex decision is one of the most important tasks along with the process of prioritisation. Experience has shown that one can prescribe guidelines for structuring a hierarchy. Here are some suggestions for an elaborate design of a hierarchy:

1. Identify the overall goal. What are you trying to accomplish? What is the main question?
2. Identify the subgoals of the overall goal. If relevant, identify time horizons that affect the decision.
3. Identify criteria that must be satisfied to fulfill the subgoals of the overall goal.
4. Identify subcriteria under each criterion. Note that criteria or subcriteria may be specified in terms of ranges of values of parameters or in terms of verbal intensities such as high, medium, low.
5. Identify the actors involved.
6. Identify the actors’ goals.
7. Identify the actors’ policies.
8. Identify the people affected by the decision.
9. Identify the objectives of these people.

10 Identify options or outcomes to take that serve people's objectives best.

11 For restricted yes-no decisions, take the most preferred outcome and compare the benefits and costs of making the decision with those of not making it.

12 Do a benefit/cost (BOCR) analysis using total priority values and marginal values.

13 Perform sensitivity analysis on the outcome to determine its stability to changes in the judgements. If desired, include a criterion in each hierarchy called 'other' or 'the unknown' for which appropriate priority values may be derived from paired comparisons. Sensitivity testing with respect to such a criterion can determine the impact of the unknown on the outcome to the best of an experienced person's understanding. It must be understood that such a factor cannot be included to cover up for total ignorance about a decision. Only the wise should use it.

Sometimes people have assigned criteria different weights when they are measured in the same unit. Others have used different ways of synthesis than multiplying and adding. An example should clarify what we must do. Synthesis in the AHP involves weighting the priorities of elements compared with respect to an element in the next higher level, called a parent element, by the priority of that element and adding over all such parents for each element in the lower level. Consider the example of two criteria C_1 and C_2 and three alternatives A_1, A_2 and A_3 measured in the same scale such as dollars. If the criteria are each assigned the value 1, then the weighting and adding process produces the correct dollar value as in Table 6.

Table 6 Calculating returns arithmetically

Alternatives	*Criterion C_1 unnormalised weight = 1.0*	*Criterion C_2 unnormalised weight = 1.0*	*Weighted sum unnormalised*	*Normalised or relative values*
A_1	200	150	350	350/1300 = 269
A_2	300	50	350	350/1300 = 269
A_3	500	100	600	600/1300 = 462
Column totals	1000	300	1300	1

However, it does not give the correct outcome if the weights of the criteria are normalised, with each criterion having a weight of 0.5. Once the criteria are given in relative terms, so must the alternatives also be given in relative terms. A criterion that measures values in pennies cannot be as important as another measured in thousands of dollars. In this case, the only meaningful importance of a criterion is the ratio of the total money for the alternatives under it to the total money for the alternatives under both criteria. By using these weights for the criteria, rather than 0.5 and 0.5, one obtains the correct final relative values for the alternatives.

What is the relative importance of each criterion? Normalisation indicates relative importance. Relative values require that criteria be examined as to their relative importance with respect to each other. What is the relative importance of a criterion, or what numbers should the criteria be assigned that reflect their relative importance? Weighting each criterion by the proportion of the resource under it, as shown in Table 7, and multiplying and adding as in the additive synthesis of the AHP, we get the same correct answer. For criterion C_1 we have

$$(200 + 300 + 500)/[(200 + 300 + 500) + (150 + 50 + 100) = 1000/1300$$

and for criterion C_2 we have

$$(150 + 50 + 100)/[(200 + 300 + 500) + (150 + 50 + 100) = 300/1300.$$

Here the criteria are automatically in normalised form, and their weights sum to one. We see that when the criteria are normalised, the alternatives must also be normalised to get the right answer. For example, if we look in Table 7 we have 350/1300 for the priority of alternative A_1. Now if we simply weight and add the values for alternative A_1 in Table 8 we get for its final value (200/1000) (1000/1300) + (150/300) (300/1300) = 350/1300 which is the same as in Table 7. It is clear that if the priorities of the alternatives are not normalised one does not get the desired outcome.

Table 7 Normalised criteria weights and normalised alternative weights from measurements in the same scale (additive synthesis)

Alternatives	*Criterion C_1 Normalised weight = 1000/1300= 0.7692*	*Criterion C_2 Normalised weight = 300/1300 = 0.2308*	*Weighted sum*
A_1	200/1000	150/300	350/1300 = 0.2692
A_2	300/1000	50/300	350/1300 = 0.2692
A_3	500/1000	100/300	600/1300 = 0.4615

We have seen in this example that in order to obtain the correct final relative values for the alternatives when measurements on a measurement scale are given, it is essential that the priorities of the criteria be derived from the priorities of the alternatives. Thus when the criteria depend on the alternatives we need to normalise the values of the alternatives to obtain the final result. This procedure is known as the distributive mode of the AHP. It is also used in case of functional (real life not paired comparison) dependence of the alternatives on the alternatives and of the criteria on the alternatives. The AHP is a special case of the ANP. The dominant mode of synthesis in the ANP with all its interdependencies is the distributive mode. The ANP automatically assigns the criteria the correct weights, if one only uses the normalised values of the alternatives under each criterion and also the normalised values for each alternative under all the criteria without any special attention to weighting the criteria.

4.2 Benefits, opportunities, costs and risks

The process of decision-making requires us to analyse a decision according to Benefits (B), the good things that would result from taking the decision; Opportunities (O), the potentially good things that can result in the future from taking the decision; Costs (C), the pains and disappointments that would result from taking the decision; and Risks (R), the potential pains and disappointments that can result from taking the decision. We then identify control criteria (criteria in terms of which we make the comparisons) and subcriteria or even a network of criteria under each and develop a subnet and its connection for each control criterion in order to make and focus judgements in terms of each criterion separately. Next we determine the best outcome for each control criterion and combine the alternatives in what is known as the ideal form for all the control criteria under each of the BOCR merits. Then we take the best alternative under B and use it to think of benefits and the best alternative under O, which may be different than the one under C, and use it to think of opportunities and so on for costs and risks. Finally we must rate these four top alternatives with respect to the strategic criteria (criteria that underlie the evaluations of the merits of all the decisions we make) using the ratings mode of the AHP to obtain priority ratings for B, O, C, and R. We then normalise and use these weights to combine the four vectors of outcomes for each alternative under BOCR to obtain the overall priorities. We add the weighted benefits and opportunities and subtract the weighted costs and risks If the corresponding weights obtained from the ratings with respect to the strategic criteria are b, o, c, r and if the ranking vectors of alternatives are B, O, C, R, then we have for the overall weights of the alternatives $b\mathrm{B} + o\mathrm{O} - c\mathrm{C} - r\mathrm{R}$. We can also form the ratio BO/CR which does not need the BOCR ratings to obtain marginal overall outcomes. If we use the weights b, o, c, r in this formula, we would know in general whether or not the advantages of the decision dominate its disadvantages.

5 Rating alternatives one at a time

People are able to make two kinds of comparisons – absolute and relative. In absolute comparisons, people compare alternatives with a standard in their memory that they have developed through experience. In relative comparisons, they compared alternatives in pairs according to a common attribute, as we did throughout the hospice example.

People use ratings to rank independent alternatives one at a time in terms of rating intensities for each of the criteria. An intensity is a range of variation of a criterion that enables one to distinguish the quality of an alternative for that criterion. An intensity may be expressed as a numerical range of values if the criterion is measurable or defined in qualitative terms.

For example, if ranking students is the objective and one of the criteria on which they are to be ranked is performance in mathematics, the mathematics ratings might be: excellent, good, average, below average, poor; or, using the usual school terminology, A, B, C, D and F. Relative comparisons are first used to set priorities on the ratings themselves. If desired, one can fit a continuous curve through the derived intensities. This concept may go against our socialisation. However, it is perfectly reasonable to ask how much an A is preferred to a B or to a C. The judgement of how much an A is preferred to a B might be different under different criteria. Perhaps for mathematics an A is very strongly preferred to a B, while for physical education an A is only moderately preferred to a B. So the end result might be that the ratings are scaled differently. For example one could have the scale values for the ratings as shown in Table 8.

Table 8 Examples of scale values for ratings

	Math	*Physical education*
A	0.50	0.30
B	0.30	0.30
C	0.15	0.20
D	0.04	0.10
E	0.01	0.10

The alternatives are then rated or ticked off one at a time using the intensities. We will illustrate ratings with an example. A firm evaluates its employees for raises. The criteria are dependability, education, experience, and quality. Each criterion is subdivided into intensities, standards, or subcriteria (Figure 4). The managers set priorities for the criteria by comparing them in pairs. They then pairwise compare the intensities according to priority with respect to their parent criterion (as in Table 9) or with respect to a subcriterion if they are using a deeper hierarchy. The priorities of the intensities are divided by the largest intensity for each criterion (second column of priorities in Figure 4).

Figure 4 Hierarchy with absolute measurement

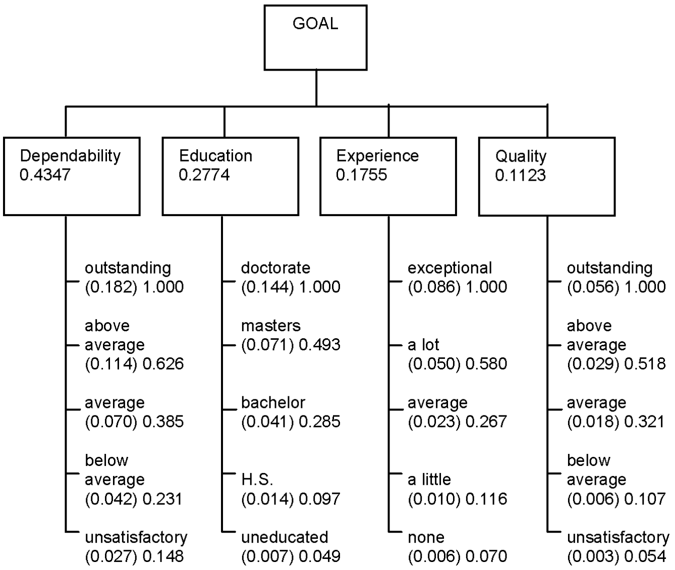


Table 9 shows a paired comparison matrix of intensities with respect to dependability. The managers answer the question: which intensity is more important and by how much with respect to dependability. The priorities of the intensities for each criterion are divided by the largest one and multiplied by the priority of the criterion. Finally the managers rate each individual (Table 10) by assigning the intensity rating that applies to him or her under each criterion. The scores of these intensities are each weighted by the priority of its criterion and summed to derive a total ratio scale score for the individual (shown on the right of Table 10). These numbers belong to an absolute scale, and the managers can give salary increases precisely in proportion to the ratios of these numbers. Adams gets the highest score and Kesselman the lowest. This approach can be used whenever it is possible to set priorities for intensities of criteria; people can usually do this when they have sufficient experience with a given operation. This normative mode requires that alternatives be rated one by one without regard to how many there may be and how high or low any of them rates on prior standards. Some corporations have insisted that they no longer trust the normative standards of their experts and that they prefer to make paired comparisons of their alternatives. Still, when there is wide agreement on standards, the absolute mode saves time in rating a large number of alternatives.

Table 9 Ranking intensities: Which intensity is preferred most with respect to dependability and how strongly?

Dependability	*Outstanding*	*Above average*	*Average*	*Below average*	*Unsatisfactory*	*Priorities*	*Idealised priorities*
Outstanding	1.0	2.0	3.0	4.0	5.0	0.419	1.000
Above avg.	1/2	1.0	2.0	3.0	4.0	0.263	0.628
Average	1/3	1/2	1.0	2.0	3.0	0.160	0.382
Below avg.	1/4	1/3	1/2	1.0	2.0	0.097	0.232
Unsatisfactory	1/5	1/4	1/3	1/2	1.0	0.062	0.148

CR = 0.015.

Table 10 Rating alternatives

Employees	*Dependability 0.4347*	*Education 0.2774*	*Experience 0.1775*	*Quality 0.1123*	*Total*
V. Adams	Outstanding	Bachelor	A Little	Outstandin	0.646
L. Becker	Average	Bachelor	A Little	Outstandin	0.379
F. Hayat	Average	Masters	A Lot	Below	0.418
S. Kessel	Above	H.S.	None	Average	0.369
K. O’Shea	Average	Doctorate	A Lot	Above	0.605
T. Peters	Average	Doctorate	A Lot	Average	0.583
K. Tobias	Above	Bachelor	Average	Above	0.456

6 A full BOCR example for a hierarchy: AHP analysis of strategies towards Iran

The threat of war in Iran is a complex and controversial issue, involving many actors in different regions and several possible courses of action. Nearly 40 people were involved in the exercise. They were divided into groups of 4 or 5 and each of these groups worked out the model and derived results for a designated merit: benefits, opportunities, costs or risks. In the end there were two outcomes for each merit which were combined using the geometric mean as described in the section on group decision making and then the four resulting outcomes were combined into a single overall outcome as described below. It should be understood that this is only an exercise to illustrate use of the method and no real life conclusions should be drawn from it primarily because it did not involve political expert and negotiators from all the interested parties. Its conclusions should be taken as hypotheses to be further tested.

6.1 Creating the model

A model for determining the policy to pursue towards Iran seeking to obtain weapon grade nuclear material was designed using a benefits, opportunities, costs, and risks model. The benefits model shows which alternative would be most beneficial, the opportunities model shows which alternative has the greatest potential for benefits, the costs model (costs may include monetary, human and intangible costs) shows which alternative would be most costly and the risks model shows which alternative has the highest potential costs.

Strategic Criteria

Strategic Criteria are used to evaluate the BOCR merits of all decisions by a decision maker. They are the overriding criteria that individuals corporations or governments use to determine which decision to make first, and what are the relative advantages and disadvantages of that decision.

For policy towards Iran, the BOCR model structured by the group is evaluated using the strategic criteria of *World Peace* (0.361), *Regional Stability* (0.356), *Reduce Volatility* (0.087) and *Reduce Escalation of Middle East Problem* (0.196). The priorities of the strategic criteria indicated in parentheses next to each, are obtained from a pairwise comparisons matrix with respect to the goal of long term peace in the world.

Control Criteria

The BOCR model is evaluated using the control criteria: *Economic*, *Political*, *Rule of Law* and *Security*. They are the criteria for which we are able to represent the different kinds of influences that we are able to perceive which later need to be combined into an overall influence using AHP/ANP calculations.

Actors

The countries mainly concerned with this problem are: the *USA, Iran, Russia and China, Middle East* countries and *Israel.*

Alternatives

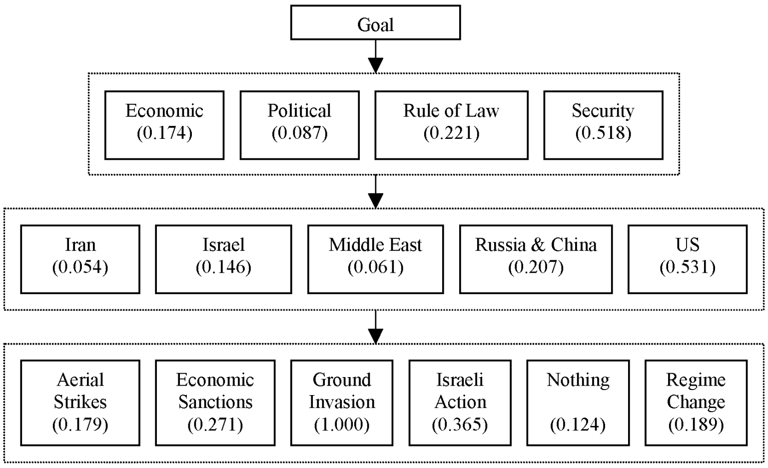
The group identified six alternatives:

- It is reasonable to undertake *Aerial Strikes* towards Iran
- *Economic Sanctions* should be applied against Iran
- The actors should carry out *Ground Invasion* of Iran
- Israeli Action towards Iran
- To do *Nothing*, leaving everything so as it is
- To make efforts to make a *Regime Change.*

6.2 BOCR models

With a view to saving space we do not give all the hierarchies and their matrices of judgements. In this exercise it was determined to keep the structure simple by using the same structure for all four merits (see Figure 5) albeit with different judgements. In particular for the costs and risks one asks the question which is more (not less) costly or risky, and in the end subtract the corresponding values from those of benefits and opportunities. The analysis derives four rankings of the alternatives, one for each of the BOCR merits. Following that one must obtain priorities for the BOCRs themselves in terms of the strategic criteria and use the top ranked alterantive for each merit in order to think about that merit and then use those priorities to weigh and synthesise the alternatives. The priorties of the alternatives are proportional to the priority of the top ranked alternative, thus they would all be multiplied by the same number that is the priority of the merit.

Figure 5 Costs hierarchy to choose the best strategy towards Iran



It is important to note again that usually for a general decision problem each merit would have a different structure than the other merits. However, for the sake of expediency in this decision, the group decided to use the same structure with the appropriate formulation of the questions to provide the judgements.

6.3 Judgements and comparisons

As previously mentioned, a judgement is an expression of opinion about the dominance (importance, preference or likelihood) of one element over another. It is done every day through verbal expression that has some quantitative significance that we need to use to combine the many dominance judgements involved in a decision. The set of all such judgements in making comparisons with respect to a single property or goal can be represented by means of a square matrix in which the set of elements is compared. It is a way of organising all the judgements with respect to some property to be processed and synthesised along with other matrices of comparisons involved in the decision. Each judgement represents the dominance of an element in the left column of the matrix over an element in the row on top. It reflects the answers to two questions: which of the two elements is more important with respect to a higher level criterion, and how strongly.

As usual with the AHP, in the models of benefits, opportunities, cost, and risks the group compared the criteria and subcriteria according to their relative importance with respect to the parent element in the adjacent upper level. For example, the entries in the matrix shown in Table 11 are responses to the question: which control criterion is more important with respect to choosing the best strategy towards Iran and how strongly? Here economic costs are moderately more important than political costs and are assigned the absolute number 3 in the (1, 2) or first-row second-column position. Three signifies three times more. The reciprocal value is automatically entered in the (2, 1) position, where political costs on the left are compared with economic costs at the top. Similarly a

5, corresponding to strong dominance or importance, is assigned to security costs over political costs in the (4, 2) position, and a 2, corresponding to weak or slight dominance, is assigned to the costs of rule of law over political costs in the (3, 2) position with corresponding reciprocals in the transpose positions of the matrix.

Table 11 Judgement matrix for the control criteria of the costs hierarchy

Choosing best strategy towards Iran (costs)	*Economic*	*Political*	*Rule of law*	*Security*	*Normalised priorities*
Economic	1	3	1/2	1/3	0.173
Political	1/3	1	1/2	1/5	0.087
Rule of Law	2	2	1	1/3	0.222
Security	3	5	3	1	0.518

CR = 0.049.

Judgements in a matrix may not be consistent. In eliciting judgements, one makes redundant comparisons to improve the validity of the answer, given that respondents may be uncertain or may make poor judgements in comparing some of the elements. Redundancy gives rise to multiple comparisons of an element with other elements and hence to numerical inconsistencies. The group first made all the comparisons using semantic terms from the fundamental scale and then translated them to the corresponding numbers.

For example, where we compare security costs with economic costs and with political costs, we have the respective judgements 3 and 5. Now if $x = 3y$ and $x = 5z$ then $3y = 5z$ or $y = 5/3z$. If the judges were consistent, economic costs would be assigned the value 5/3 instead of the three given in the matrix. Thus the judgements are inconsistent. In fact, we are not sure which judgements are the accurate ones and which are the cause of the inconsistency. However, these can be determined in a systematic way and improved by interrogating the decision maker.

The process is repeated for all the matrices by asking the appropriate dominance or importance question. For example, the entries in the judgement matrix shown in Table 12 are responses to the question: which party is more committed to ensure security?

Table 12 Judgement matrix of subcriteria with respect to security costs

Security costs	*Iran*	*Israel*	*Middle East*	*Russia and China*	*USA*	*Normalised priorities*
Iran	1	1/8	1/3	1/6	1/9	0.029
Israel	8	1	4	1/2	1/7	0.138
Middle East	3	1/4	1	1/5	1/7	0.054
Russia and China	6	2	5	1	1/6	0.182
USA	9	7	7	6	1	0.597

CR = 0.1.

Here US security costs are regarded as extremely more important than the security costs for Iran, and 9 is entered in the (5, 1) position and 1/9 in the (1, 5) position.

In comparing the six strategies (alternatives) towards Iran, we asked members of the decision group which strategy in their opinion would be more costly for each of the actors. For example, for the USA, we obtained a matrix of paired comparisons in Table 13 in which Ground Invasion is the most expensive strategy. On the contrary, regime change and doing nothing are the least costly ones. In this example the criteria (here the different countries) are assumed to be independent of the alternatives and hence the priorities of alternatives are given in ideal form by dividing by the largest priority among them. Here ground invasion would be the most costly.

Table 13 Relative costs of the strategies for the USA

Strategies costs for the USA	*Aerial strikes*	*Economic sanctions*	*Ground invasion*	*Israeli action*	*Nothing*	*Normalised priorities*	*Idealised priorities*
Aerial strikes	1	1/3	1/7	1/2	3	0.087	0.164
Economic sanctions	3	1	1/6	2	3	0.160	0.301
Ground invasion	7	6	1	6	7	0.533	1.000
Israeli action	2	1/2	1/6	1	3	0.122	0.229
Nothing	1/3	1/3	1/7	1/3	1	0.058	0.108
Regime change	1/3	1/3	1/6	1/4	1/3	0.040	0.075

CR = 0.08.

Tables 14–16 give the priorities obtained from all the comparisons for the BOCR.

Each column in Table 14 gives in bold face the priorities of the control criteria with respect to which the comparisons are made. For example economic has the value 0.047 under opportunities obtained by comparing it in a matrix with Political, Rule of Law and security whose priorties are also shown in bold face. These priorities sum to one. Similarly under opportunities costs and risks. The priorties of the actors are given under the priority of each of the control criteria in the same column. At the bottom of Table 14 are given the overall priorties of the actors with respect to each of the BOCR obtained by weighting by the priority of the control criteria and adding in the same column. We do not yet combine numbers in the same row but only in the same column. Similarly in Table 15 for the actors and the alternatives. At the bottom of Table 15 we have the overall idealised weights of the alternatives for each of the BOCR. In Table 16 we rate the top alternative for each BOCR merit with respect to each of the strategic criteria using a rating scale derived from comparisons. Usually and for greater precision one should develop a different rating scale for each criterion or subcritierion, but we have simplified the analysis here by adopting a single scale for all the strategic criteria. We then weight the ratings by the priorities of the strategic criteria and add to obtain a weight for each BOCR merit. Finally we normalise these four values to obtain the priorities b, o, c, and r. We then use these priorities in Table 17 to synthesise the idealised weights of the alternatives according to the marginal formula that represents short term solution and the total formula that represents long term solution to the problem.

Next, we rate the top outcome for each of the BOCR against the strategic criteria using the five-level ratings scale obtained from paired comparisons: The synthesised Rating Results are shown in Table 17. We want to evaluate or rate the top alternative for Benefits and that for opportunities against the stratedic criteria as to how they help with respect to each criterion. We also want to rate the top alternatives for the costs and risks as to how much they hurt or damage with respect to each criterion. This yields the priorities of the BOCR before normalisation.

Table 14 Priorities of the actors with respect to control criteria of BOCR groups

		Benefits	*Opportunities*	*Costs*	*Risks*
Control criteria	*Actors*				
Economic		*0.047*	*0.626*	*0.174*	*0.209*
	Iran	0.031	0.066	0.129	0.321
	Israel	0.259	0.041	0.036	0.126
	Middle East	0.166	0.233	0.087	0.306
	Russia and China	0.095	0.120	0.425	0.075
	USA	0.449	0.540	0.324	0.173
Political		*0.128*	*0.156*	*0.087*	*0.033*
	Iran	0.043	0.201	0.082	0.506
	Israel	0.311	0.125	0.226	0.145
	Middle East	0.133	0.494	0.067	0.248
	Russia and China	0.079	0.090	0.152	0.045
	USA	0.433	0.090	0.483	0.056
Rule of law		*0.246*	*0.043*	*0.222*	*0.066*
	Iran	0.031	0.114	0.044	0.317
	Israel	0.347	0.415	0.218	0.118
	Middle East	0.132	0.169	0.060	0.443
	Russia and China	0.101	0.051	0.119	0.059
	USA	0.389	0.251	0.559	0.063
Security		*0.579*	*0.175*	*0.518*	*0.692*
	Iran	0.068	0.131	0.029	0.200
	Israel	0.115	0.298	0.138	0.200
	Middle East	0.165	0.308	0.054	0.200
	Russia and China	0.407	0.106	0.181	0.200
	USA	0.245	0.156	0.597	0.200
Overall					
	Iran	*0.031*	*0.100*	*0.054*	*0.243*
	Israel	*0.259*	*0.115*	*0.204*	*0.177*
	Middle East	*0.166*	*0.284*	*0.153*	*0.240*
	Russia and China	*0.095*	*0.110*	*0.275*	*0.159*
	USA	*0.449*	*0.390*	*0.314*	*0.180*

Table 15 Priorities of the alternatives with respect to the actors in BOCR groups

		Benefits	*Opportunities*	*Costs*	*Risks*
Actors	*Alternatives*				
Iran		*0.031*	*0.100*	*0.054*	*0.243*
	Aerial strikes	1.000	0.078	0.115	0.701
	Economic sanctions	1.000	0.452	0.260	1.000
	Ground invasion	1.000	0.057	1.000	0.294
	Israeli action	1.000	0.142	0.149	0.762
	Nothing	1.000	1.000	0.068	0.239
	Regime change	1.000	0.220	0.362	0.882
Israel		*0.259*	*0.115*	*0.204*	*0.177*
	Aerial strikes	0.214	0.359	0.212	0.240
	Economic sanctions	0.463	0.069	0.120	0.209
	Ground invasion	0.128	0.228	0.355	0.385
	Israeli action	0.070	0.104	1.000	1.000
	Nothing	0.177	0.062	0.210	0.153
	Regime change	1.000	1.000	0.130	0.052
Middle East		*0.166*	*0.284*	*0.153*	*0.240*
	Aerial strikes	0.095	0.168	0.257	0.186
	Economic sanctions	0.357	0.676	0.132	0.073
	Ground invasion	0.159	0.196	1.000	0.328
	Israeli action	0.131	0.062	0.483	1.000
	Nothing	1.000	0.371	0.111	0.068
	Regime change	0.702	1.000	0.175	0.045
Russia and China		*0.095*	*0.110*	*0.275*	*0.159*
	Aerial strikes	0.778	0.141	0.114	0.190
	Economic sanctions	0.825	0.259	0.215	0.057
	Ground invasion	0.331	0.174	1.000	0.379
	Israeli action	1.000	0.122	0.160	1.000
	Nothing	0.559	1.000	0.071	0.072
	Regime change	0.303	0.154	0.411	0.086
USA		*0.449*	*0.390*	*0.314*	*0.180*
	Aerial strikes	0.167	1.000	0.164	0.133
	Economic sanctions	0.231	0.094	0.301	0.076
	Ground invasion	0.130	0.285	1.000	0.388
	Israeli action	0.165	0.050	0.229	1.000
	Nothing	1.000	0.150	0.108	0.079
	Regime change	0.130	0.417	0.075	0.038
Overall					
	Aerial strikes	*0.402*	*0.891*	*0.179*	*0.259*
	Economic sanctions	*0.586*	*0.496*	*0.271*	*0.221*
	Ground invasion	*0.258*	*0.378*	*1.000*	*0.368*
	Israeli action	*0.414*	*0.136*	*0.365*	*1.000*
	Nothing	*1.000*	*0.666*	*0.124*	*0.112*
	Regime change	*0.651*	*1.000*	*0.189*	*0.165*

Table 16 Ratings of strategic criteria for BOCR merits

	World peace (0.362)	*Regional stability (0.356)*	*Reduce volatility (0.087)*	*Reduce escalation of the Middle East conflict (0.196)*	*Priorities*	*Normalised priorities*
Benefits	Very High	High	High	Medium	*0.710*	*0.254*
Opportunities	Medium	Low	Medium	Medium	*0.330*	*0.118*
Costs	Very High	Very High	Very High	Medium	*0.878*	*0.314*
Risks	Very High	Very High	Very High	Medium	*0.878*	*0.314*

Very High (1), High (0.619), Medium (0.381), Low (0.238), Very Low (0.143).

Table 17 Synthesis of the alternatives' overall priorities for the four BOCR merits

	Benefits b = 0.254	*Opportunities o = 0.118*	*Costs c = 0.314*	*Risks r = 0.314*	*BO/CR*	*BB + oO – cC – rR*
Aerial strikes	0.402	0.891	0.179	0.259	7.711	0.069
Economic sanctions	0.586	0.496	0.271	0.221	4.841	0.053
Ground invasion	0.258	0.378	1.000	0.368	0.265	–0.319
Israeli action	0.414	0.136	0.365	1.000	0.155	–0.308
Nothing	1.000	0.666	0.124	0.112	*48.077*	*0.258*
Regime change	0.651	1.000	0.189	0.165	20.814	0.172

Benefits and Opportunities are positive merits, whereas Costs and Risks are negative. The overbalance of weights is negative for Ground Invasion and Israeli Action and is positive for Aerial Strikes, Economic Sanctions, doing Nothing and Regime Change. As a result, in the current situation doing Nothing turns out to be the best alternative and Ground Invasion is the worst.

6.4 Sensitivity analysis

There are many ways of doing sensitivity analysis, we show one of them here. Sensitivity graphs for BOCR groups are shown in Figures 6–9 respectively. From the software program Superdecisions we see that the results obtained by perturbing the priorities of each of the benefits and opportunities, costs and risks are stable. The model is sensitive to changes of priorities in the BOCR merits. As the priority of Costs increases, the alternative 'Israeli Action' becomes more preferred than 'Ground Invasion' and 'Aerial Strikes' becomes more important than 'Economic Sanctions'. On the other hand, as the priority of Risks increases, the last two alternatives 'Israeli Action' and 'Ground Invasion' trade places in the overall order of ranking. Results obtained for Benefits and Opportunities are stable and 'Nothing' remains the best alternative.

Figure 6 Sensitivity analysis for benefits

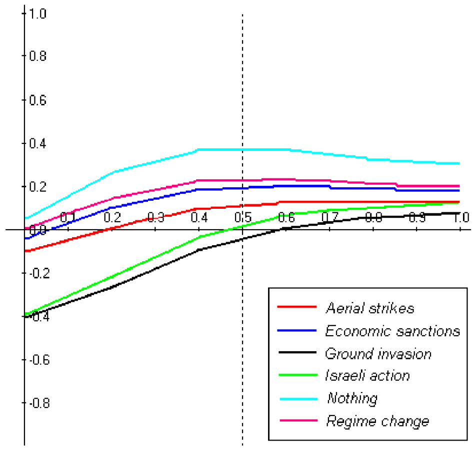


Figure 7 Sensitivity analysis for opportunities

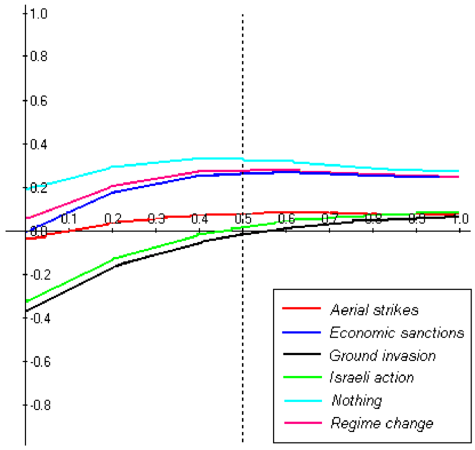


Figure 8 Sensitivity analysis for costs

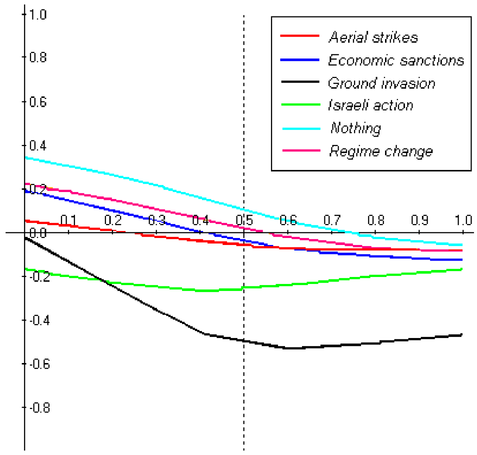
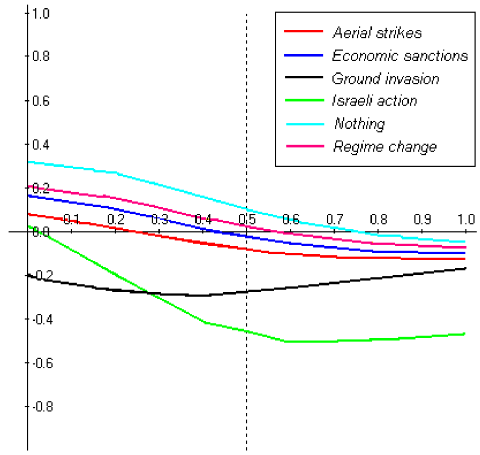


Figure 9 Sensitivity analysis for risks



7 Networks, dependence and feedback

Many decisions cannot be structured hierarchically because they involve the interaction and dependence of higher-level elements on lower-level elements. Not only does the importance of the criteria determine the importance of the alternatives as in a hierarchy, but also the importance of the alternatives themselves determines the importance of the criteria. Two bridges, both strong, but the stronger is also uglier, would lead one to choose the strong but ugly one unless the criteria themselves are evaluated in terms of the bridges, and strength receives a smaller value and appearance larger value because both bridges are already strong. Feedback enables us to factor the future into the present to determine what we have to do to attain a desired future.

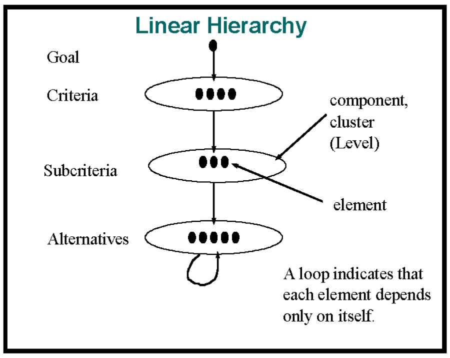
The feedback structure does not have the linear top-to-bottom form of a hierarchy but looks more like a network, with cycles connecting its clusters of elements, which we can no longer call levels, and with loops that connect cluster to itself. A decision problem involving feedback arises often in practice. It typically has many interactions, which in the limit converge determine the goal. Our minds need a tool to manage this complexity.

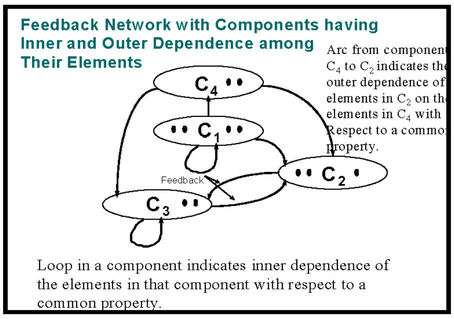
At present, in their effort to simplify and deal with complexity, people who work in decision making use mostly very simple hierarchic structures consisting of a goal, criteria, and alternatives. Yet, not only are decisions obtained from a simple hierarchy of three levels different from those obtained in a multilevel hierarchy, but also decisions obtained from a network can be significantly different from those obtained from a complex hierarchy.

We cannot collapse complexity artificially into a simplistic structure of two levels, criteria and alternatives, and hope to capture the outcome of interactions in the form of highly condensed judgements that correctly reflect all that goes on in the world. We must learn to decompose these judgements through more elaborate structures and organise our reasoning and calculations in sophisticated but simple ways to serve our understanding of the complexity around us. Experience indicates that it is not very difficult to do this although it takes more time and effort. Indeed, we must use feedback networks to arrive at the kind of decisions needed to cope with the future.

In Figure 10, we exhibit a hierarchy and a network. A hierarchy is comprised of a goal, levels of elements and connections between the elements. These connections are oriented only to elements in lower levels. A hierarchy is authoritarian. It passes the word down from higher up. It describes our commitments, what is important to us and what we prefer even if we imagine it all. A hierarchy is a special case of a network. In a hierarchy connections go only in one direction. In the view of a hierarchy such as that shown in Figure 10 the levels correspond to clusters in a network. A network has clusters of elements, with the elements in one cluster being connected to elements in another cluster (outer dependence) or the same cluster (inner dependence). A network is concerned with all the influences from people and from nature that can affect an outcome. It is a model of continual change because everything affects everything else and what we do now can change the importance of the criteria that control the evolution of the outcome.

Figure 10 How a hierarchy compares with a network





Speaking of loops, a good example is that of a family of father mother and baby with their interdependencies. Using pairwise comparisons, we attempt to answer the question of how much each of them depends on the others for survival. We take for example the baby as a criterion and compare all three in pairs as to who influences the baby's survival the most, the father or the mother, the father or the baby itself, the mother or the baby itself. In this case the baby is not so important in contributing to its own survival as its parents are. But if we take the mother and ask the same question as to who contributes to her survival more, herself or her husband, herself would probably be larger, or herself and the baby, again herself. Another example of inner dependence is making electricity. To make electricity a company needs steel turbines, and fuel. So we have the electric industry, the steel industry and the fuel industry. Who does the electric industry depend on more to make electricity, itself or the steel industry? The steel industry is more important at first to provide materials to make turbines; itself or the fuel industry? The fuel industry is much more important; the steel or fuel industry? Fuel is more important. The electric industry does not need its own electricity to make electricity. It needs fuel. Its electricity is only used to light the rooms, which it may not even need.

If we think about it carefully everything can be seen to influence everything else according to many criteria including each thing by itself. The world is more interdependent than we know how to deal with using our ways of thinking and taking action. The ANP is our logical way to deal with dependence.

However, not every element of a component need impact an element in another component. In that case, those elements that make no impact are given a zero value for their contribution. The resulting matrix of components with their elements displayed vertically on the left side of the matrix and horizontally at the top of the matrix must be stochastic (each column sums to one) to obtain meaningful limiting results. To ensure that this matrix, called the supermatrix, is stochastic, we need to compare the components themselves (rather than their elements) that are on the left with respect to their impact on each component at the top according to some attribute represented in a separate control hierarchy for that system. The resulting priorities of the components are then each used to weight the corresponding block of column vectors. Each block of column vectors defines an entry of the supermatrix.

Thus, all the column vectors in a block are multiplied by the single priority of the corresponding component on the left. The columns of the supermatrix corresponding to the impacts on the elements of the component at the top now sum to one. The resulting supermatrix is column stochastic. What is desired, if it exists, is the long-run or limiting priority of impact of each element on every other element.

Contributions to this impact can be obtained in many ways. They can be obtained directly from the matrix or indirectly for any two elements by taking the impact of the first on some third element and then multiplying it by the impact of that element on the second. One must consider every such possibility of a third element. All such possibilities are obtained from the square of the matrix. Again the impact can be obtained by considering a third element that impacts a fourth element, which in turn impacts the second element. All such impacts are obtained from the cubic power of the matrix, and so on. Thus we have an infinite number of impact matrices: the matrix itself, its square, its cube, etc. If we sum all these matrices and take the average of each entry, does the result converge to a limit? Does the limit exist, and how do we compute it to obtain the desired priorities? The supermatrix may not be positive and may have zeros in certain positions where there is no direct impact of an element on another. Alternatively, the matrix may be positive or may become positive after raising it to powers. What theory do we have to deal with this problem? Note that if the matrix is positive or if, after raising it to some power, it becomes positive, it turns out that one can obtain a unique answer. But when no power of the matrix is strictly positive, we need to examine what happens closely because even in those situations where every element can be reached from every other element, we may not have a unique limit. For example, powers of the matrix may oscillate, and different limits are obtained. Also, if it is not possible to reach every element from every other, then the graph representing the connections of the components and even the elements themselves may be divided into subgraphs, in some of which every element can be reached from every other but not in others. How then do we obtain the desired results? The graph of a decision system must always be connected. It cannot be divided into two or more disjoint parts. We should note that when the criteria do not depend on the alternatives, the latter may be kept out of the supermatrix and evaluated according to the performance (ideal) or dominance (distributive) modes after the limiting priorities of the criteria are obtained from the limiting supermatrix. Otherwise, if some criterion depends on the alternatives or if there is inner dependence among the alternatives, they must be included in the supermatrix.

A supermatrix along with an example of one of its general entry matrices is shown in Figure 11. The component C_i in the supermatrix includes all the priority vectors derived for nodes that are 'parent' nodes in the C_i cluster. Figure 12 gives the supermatrix of a hierarchy along with the kth power that yields the principle of hierarchic composition in its (k, 1) position.

Figure 11 The supermatrix of a network and detail of a component in it

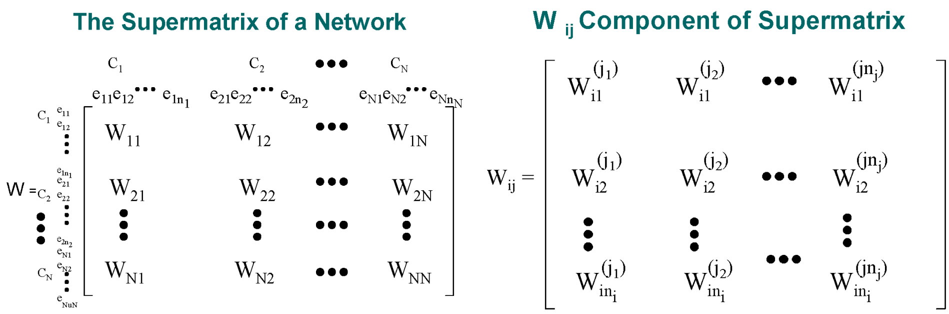
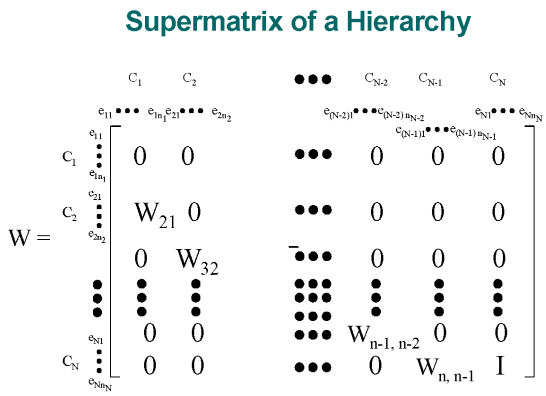


Figure 12 The supermatrix of a hierarchy with the resulting limit matrix corresponding to hierarchical composition



$$W^k = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 0 \\ W_{n,n-1}W_{n-1,n-2}\cdots W_{32}W_{21} & W_{n,n-1}W_{n-1,n-2}\cdots W_{32} & \cdots W_{n,n-1}W_{n-1,n-2} & W_{n,n-1} & I \end{bmatrix}$$

The (n, 1) entry of the limit supermatrix of a hierarchy as shown in Figure 12 gives the hierarchic composition principle.

8 Compatibility Index

Let us show first that the priority vector $w = (w_1, \ldots, w_n)$ is completely compatible with itself. Thus we form the matrix of all possible ratios $W = (w_{ij}) = (w_i/w_j)$ from this vector. This matrix is reciprocal, that is $w_{ji} = 1/w_{ij}$. The Hadamard product of a reciprocal matrix W and its transpose W^T is given by:

$$A \circ A^T = \begin{pmatrix} w_1/w_1 & \cdots & w_1/w_n \\ \vdots & & \vdots \\ w_n/w_1 & \cdots & w_n/w_n \end{pmatrix} \circ \begin{pmatrix} w_1/w_1 & \cdots & w_n/w_1 \\ \vdots & & \vdots \\ w_1/w_n & \cdots & w_n/w_n \end{pmatrix}$$

$$= \begin{pmatrix} 1 & \ldots & 1 \\ \vdots & & \vdots \\ 1 & \ldots & 1 \end{pmatrix} = \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} (1 \cdots 1) \equiv ee^T$$

The sum of the elements of a matrix A can be written as $e^T Ae$. In particular we have $e^T AoA^T e = n^2$ for the sum of the elements of the Hadamard product of a matrix and its transpose. The index of compatibility is the sum resulting from the Hadamard product divided by n^2. Thus a vector is completely compatible with itself as $(n^2/n^2) = 1$. Now we have an idea of how to define a measure of compatibility for two matrices A and B. It is given by $(1/n^2)e^T AoB^T e$. Note that a reciprocal matrix of judgements that is inconsistent is not itself a matrix of ratios from a given vector. However, such a matrix has a principal eigenvector and thus we speak of the compatibility of the matrix of judgements and the matrix formed from ratios of the principal eigenvector. We have Theorem 1 for a reciprocal matrix of judgements and the matrix W of the ratios of its principal eigenvector:

Theorem 1: $\frac{1}{n^2} e^T AoW^T e = \frac{\lambda_{\max}}{n}$.

Proof: From $Aw = \lambda_{\max} w$ we have

$$\sum_{j=1}^{n} a_{ij} w_j = \lambda_{\max} w_i \quad \text{and} \quad \frac{1}{n^2} e^T AoW^T e = \frac{1}{n^2} \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} \frac{w_j}{w_i} = \frac{\lambda_{\max}}{n}.$$

We want this ratio to be close to one or in general not much more than 1.01 and be less than this value for small size matrices. It is in accord with the idea that a 10% deviation is at the upper end of acceptability.

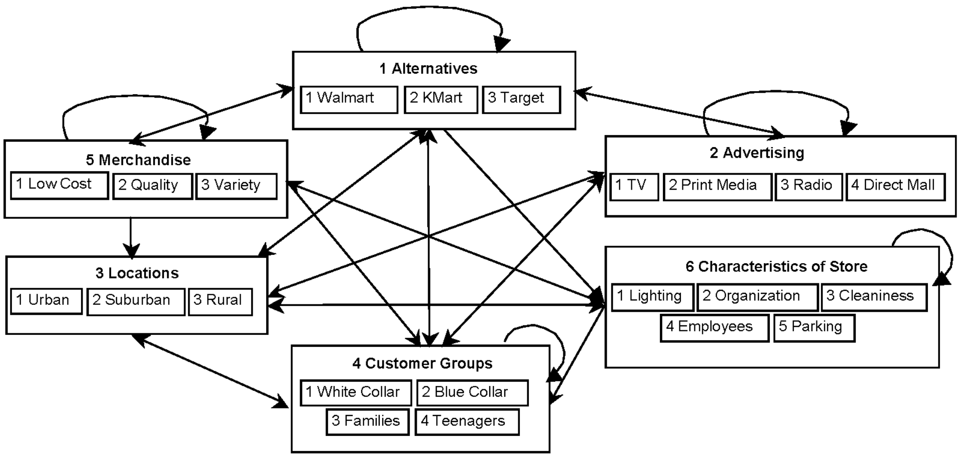
We will give an example of market share estimation showing the models and results.

9 Actual relative market share based on sales

9.1 Estimating the relative market share of Walmart, Kmart and Target

The object is to estimate the market share of Walmart, Kmart, and Target without using dollar values in the estimate. The network for the ANP model shown in Figure 13 describes well the influences that determine the market share of these companies. We will not use space in this paper to describe the clusters and their nodes in great detail.

Figure 13 The clusters and nodes of a model to estimate the relative market share of Walmart, Kmart and Target



9.2 The unweighted supermatrix

The unweighted supermatrix is constructed from the priorities derived from the different pairwise comparisons. The nodes, grouped by the clusters they belong to, are the labels of the rows and columns of the supermatrix. The column of priorities for a node at the top of the supermatrix includes the priorities of the nodes on the left side of the matrix that have been pairwise compared as to their influence with respect to market share on that node. The sum of these priorities is equal to one. The supermatrix of the network of Figure 13 is shown in Table 18. We have broken the matrix into two parts because it is wider than the page we are writing on.

It is clear that this matrix is not column stochastic and its blocks under each cluster at the top need to be weighted by the priorities of the influence with respect to market share of the clusters at the left on the cluster to which they fall under.

Table 18 The unweighted supermatrix, displayed in two parts

		Alternatives			*Advertising*				*Locations*		
		Walmart	*KMart*	*Target*	*TV*	*Print Media*	*Radio*	*Direct Mail*	*Urban*	*Suburban*	*Rural*
Alternatives	Walmart	0.000	0.833	0.833	0.687	0.540	0.634	0.661	0.614	0.652	0.683
	KMart	0.750	0.000	0.167	0.186	0.297	0.174	0.208	0.268	0.235	0.200
	Target	0.250	0.167	0.000	0.127	0.163	0.192	0.131	0.117	0.113	0.117
Advertising	TV	0.553	0.176	0.188	0.000	0.000	0.000	0.000	0.288	0.543	0.558
	Print Media	0.202	0.349	0.428	0.750	0.000	0.800	0.000	0.381	0.231	0.175
	Radio	0.062	0.056	0.055	0.000	0.000	0.000	0.000	0.059	0.053	0.048
	Direct Mail	0.183	0.420	0.330	0.250	0.000	0.200	0.000	0.273	0.173	0.219
Locations	Urban	0.114	0.084	0.086	0.443	0.126	0.080	0.099	0.000	0.000	0.000
	Suburban	0.405	0.444	0.628	0.387	0.416	0.609	0.537	0.000	0.000	0.000
	Rural	0.481	0.472	0.285	0.169	0.458	0.311	0.364	0.000	0.000	0.000

Table 18 The unweighted supermatrix, displayed in two parts (continued)

		Alternatives			*Advertising*				*Locations*		
		Walmart	*KMart*	*Target*	*TV*	*Print Media*	*Radio*	*Direct Mail*	*Urban*	*Suburban*	*Rural*
Cust. groups	White Collar	0.141	0.114	0.208	0.165	0.155	0.116	0.120	0.078	0.198	0.092
	Blue Collar	0.217	0.214	0.117	0.165	0.155	0.198	0.203	0.223	0.116	0.224
	Families	0.579	0.623	0.620	0.621	0.646	0.641	0.635	0.656	0.641	0.645
	Teenagers	0.063	0.049	0.055	0.048	0.043	0.045	0.041	0.043	0.045	0.038
Merchandise	Low cost	0.362	0.333	0.168	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	Quality	0.261	0.140	0.484	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	Variety	0.377	0.528	0.349	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Characteristic	Lighting	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	Organisation	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	Cleanliness	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	Employees	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	Parking	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

		Custom. groups				*Merchandise*			*Characteristics of store*				
		White Collar	*Blue Collar*	*Families*	*Teens*	*Low cost*	*Quality*	*Variety*	*Lighting*	*Organis.*	*Clean*	*Employees*	*Park*
Alternat.	Walmart	0.637	0.661	0.630	0.691	0.661	0.614	0.648	0.667	0.655	0.570	0.644	0.558
	KMart	0.105	0.208	0.218	0.149	0.208	0.117	0.122	0.111	0.095	0.097	0.085	0.122
	Target	0.258	0.131	0.151	0.160	0.131	0.268	0.230	0.222	0.250	0.333	0.271	0.320
Advertis.	TV	0.323	0.510	0.508	0.634	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	Print Med.	0.214	0.221	0.270	0.170	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	Radio	0.059	0.063	0.049	0.096	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	Direct Mail	0.404	0.206	0.173	0.100	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Locations	Urban	0.167	0.094	0.096	0.109	0.268	0.105	0.094	0.100	0.091	0.091	0.111	0.067
	Suburban	0.833	0.280	0.308	0.309	0.117	0.605	0.627	0.433	0.455	0.455	0.444	0.293
	Rural	0.000	0.627	0.596	0.582	0.614	0.291	0.280	0.466	0.455	0.455	0.444	0.641
Cust. groups	White Col.	0.000	0.000	0.279	0.085	0.051	0.222	0.165	0.383	0.187	0.242	0.165	0.000
	Blue Collar	0.000	0.000	0.649	0.177	0.112	0.159	0.165	0.383	0.187	0.208	0.165	0.000
	Families	0.857	0.857	0.000	0.737	0.618	0.566	0.621	0.185	0.583	0.494	0.621	0.000
	Teenagers	0.143	0.143	0.072	0.000	0.219	0.053	0.048	0.048	0.043	0.056	0.048	0.000
Merchand.	Low Cost	0.000	0.000	0.000	0.000	0.000	0.800	0.800	0.000	0.000	0.000	0.000	0.000
	Quality	0.000	0.000	0.000	0.000	0.750	0.000	0.200	0.000	0.000	0.000	0.000	0.000
	Variety	0.000	0.000	0.000	0.000	0.250	0.200	0.000	0.000	1.000	0.000	0.000	0.000
Character.	Lighting	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.169	0.121	0.000	0.250
	Organis.	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.251	0.000	0.575	0.200	0.750
	Cleanli.	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.673	0.469	0.000	0.800	0.000
	Employee	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.308	0.304	0.000	0.000
	Parking	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.075	0.055	0.000	0.000	0.000

9.3 The cluster matrix

The cluster themselves must be compared to establish their relative importance and use their priorities to weight the supermatrix to make it column stochastic. A cluster impacts another cluster with respect to market share when it is linked from it, that is, when at least one node in the source cluster is linked to nodes in the target cluster. The clusters linked from the source cluster are pairwise compared for the importance of their impact on it with respect to market share, resulting in the column of priorities for that cluster in the cluster matrix. The process is repeated for each cluster in the network to obtain the priorities shown in Table 19. An interpretation of the priorities in the first column is that Merchandise (0.442) and Locations (0.276) have the most impact on Alternatives, (the three competitors) with respect to market share.

Table 19 The cluster matrix

	Alternatives	*Advertising*	*Locations*	*Customer groups*	*Merchandise*	*Characteristics of store*
Alternatives	0.137	0.174	0.094	0.057	0.049	0.037
Advertising	0.091	0.220	0.280	0.234	0.000	0.000
Locations	0.276	0.176	0.000	0.169	0.102	0.112
Customer groups	0.054	0.429	0.627	0.540	0.252	0.441
Merchandise	0.442	0.000	0.000	0.000	0.596	0.316
Characteristics of store	0.000	0.000	0.000	0.000	0.000	0.094

9.4 The weighted supermatrix

The weighted supermatrix shown in Table 20 is obtained by multiplying each entry in a block of the component at the top of the supermatrix by the priority of influence of the component on the left from the cluster matrix in Table 19. For example, the first entry, 0.137, in Table 19 is used to multiply each of the nine entries in the block (Alternatives, Alternatives) in the unweighted supermatrix shown in Table 18. This gives the entries for the (Alternatives, Alternatives) component in the weighted supermatrix of Table 20. Each column in the weighted supermatrix has a sum equal to 1, and thus the matrix is column stochastic and converges to a single vector or is periodic in which case the average is usually used.

The limit supermatrix is not shown here to save space. It is obtained from the weighted supermatrix by raising it to powers until all columns are identical to within a certain decimal place. From the top part of the first column of the limit supermatrix we get the priorities we seek for Alternaives and normalise them.

Table 20 Weighted supermatrix

		Alternatives			*Advertising*				*Locations*		
		Walmart	*KMart*	*Target*	*TV*	*Print Media*	*Radio*	*Direct Mail*	*Urban*	*Suburban*	*Rural*
Alternatives	Walmart	0.000	0.114	0.114	0.120	0.121	0.110	0.148	0.058	0.061	0.064
	KMart	0.103	0.000	0.023	0.033	0.066	0.030	0.047	0.025	0.022	0.019
	Target	0.034	0.023	0.000	0.022	0.037	0.033	0.029	0.011	0.011	0.011
Advertising	TV	0.050	0.016	0.017	0.000	0.000	0.000	0.000	0.080	0.152	0.156
	Print Media	0.018	0.032	0.039	0.165	0.000	0.176	0.000	0.106	0.064	0.049
	Radio	0.006	0.005	0.005	0.000	0.000	0.000	0.000	0.016	0.015	0.014
	Direct Mail	0.017	0.038	0.030	0.055	0.000	0.044	0.000	0.076	0.048	0.061
Locations	Urban	0.031	0.023	0.024	0.078	0.028	0.014	0.022	0.000	0.000	0.000
	Suburban	0.112	0.123	0.174	0.068	0.094	0.107	0.121	0.000	0.000	0.000
	Rural	0.133	0.130	0.079	0.030	0.103	0.055	0.082	0.000	0.000	0.000
Cust. groups	White Collar	0.008	0.006	0.011	0.071	0.086	0.050	0.066	0.049	0.124	0.058
	Blue Collar	0.012	0.011	0.006	0.071	0.086	0.085	0.112	0.140	0.073	0.141
	Families	0.031	0.033	0.033	0.267	0.356	0.275	0.350	0.411	0.402	0.404
	Teenagers	0.003	0.003	0.003	0.021	0.024	0.019	0.023	0.027	0.028	0.024
Merchandise	Low cost	0.160	0.147	0.074	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	Quality	0.115	0.062	0.214	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	Variety	0.166	0.233	0.154	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Characteristic	Lighting	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	Organisation	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	Cleanliness	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	Employees	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	Parking	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

		Custom groups				*Merchandise*			*Charact. of store*				
		White Collar	*Blue Collar*	*Families*	*Teens*	*Low cost*	*Quality*	*Variety*	*Lighting*	*Organ.*	*Clean*	*Employees*	*Pkg.*
Alternat.	Walmart	0.036	0.038	0.036	0.040	0.033	0.030	0.032	0.036	0.024	0.031	0.035	0.086
	KMart	0.006	0.012	0.012	0.009	0.010	0.006	0.006	0.006	0.004	0.005	0.005	0.019
	Target	0.015	0.007	0.009	0.009	0.006	0.013	0.011	0.012	0.009	0.018	0.015	0.049
Advertising	TV	0.076	0.119	0.119	0.148	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	Print Med.	0.050	0.052	0.063	0.040	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	Radio	0.014	0.015	0.012	0.023	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	Direct Mail	0.095	0.048	0.040	0.023	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Table 20 Weighted supermatrix (continued)

		Custom groups				*Merchandise*			*Charact. of store*				
		White Collar	*Blue Collar*	*Families*	*Teens*	*Low cost*	*Quality*	*Variety*	*Lighting*	*Organ.*	*Clean*	*Employees*	*Pkg.*
Locations	Urban	0.028	0.016	0.016	0.018	0.027	0.011	0.010	0.016	0.010	0.015	0.018	0.031
	Suburban	0.141	0.047	0.052	0.052	0.012	0.062	0.064	0.071	0.051	0.074	0.073	0.135
	Rural	0.000	0.106	0.101	0.098	0.063	0.030	0.029	0.076	0.051	0.074	0.073	0.295
Cust. groups	White Col.	0.000	0.000	0.151	0.046	0.013	0.056	0.042	0.247	0.082	0.156	0.107	0.000
	Blue Collar	0.000	0.000	0.350	0.096	0.028	0.040	0.042	0.247	0.082	0.134	0.107	0.000
	Families	0.463	0.463	0.000	0.398	0.156	0.143	0.157	0.119	0.257	0.318	0.400	0.000
	Teenagers	0.077	0.077	0.039	0.000	0.055	0.013	0.012	0.031	0.019	0.036	0.031	0.000
Merchandise	Low Cost	0.000	0.000	0.000	0.000	0.000	0.477	0.477	0.000	0.000	0.000	0.000	0.000
	Quality	0.000	0.000	0.000	0.000	0.447	0.000	0.119	0.000	0.000	0.000	0.000	0.000
	Variety	0.000	0.000	0.000	0.000	0.149	0.119	0.000	0.000	0.316	0.000	0.000	0.000
Charact.	Lighting	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.016	0.017	0.000	0.097
	Organis.	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.035	0.000	0.079	0.027	0.290
	Cleanli.	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.092	0.044	0.000	0.110	0.000
	Employee	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.029	0.042	0.000	0.000
	Parking	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.010	0.005	0.000	0.000	0.000

9.5 Synthesised results from the limit supermatrix and actual relative market share

The relative market shares of the alternatives Walmart, Kmart and Target from the limit supermatrix are: 0.057, 0.024 and 0.015. When normalised they are 0.599, 0.248 and 0.154 (Table 21).

Table 21 The synthesised results for the alternatives

Alternatives	*Values from limit supermatrix*	*Actual values July13, 1998* (billion $)	*Normalised values from supermatrix*	*Actual market share as dollar sales normalised*
Walmart	0.057	58	0.599	54.8
KMart	0.024	27.5	0.248	25.9
Target	0.015	20.3	0.154	19.2

The object was to estimate the market share of Walmart, Kmart, and Target. The normalised results from the model were compared with sales as reported in the Discount Store News of July 13, 1998, p.77, of $58, $27.5 and $20.3 billions of dollars respectively. Normalising the dollar amounts shows their actual relative market shares to be 54.8, 25.9 and 19.2. The relative market share from the model was compared with the sales values by constructing a pairwise comparisons matrix from the results vector in column 2 and a pairwise comparisons matrix from the results vector in column 4 and computing the compatibility index. The index value is equal to 1.016. As that is about 1.01 the ANP results may be said to be close to the actual relative market share.

10 A complete BOCR example: the Ford explorer case

10.1 Introduction/background

The analysis in this section is based on a study done in June 2001 by J.P. Alberio and S. Mulani. On August 9, 2000 the companies Firestone and Ford announced a recall of 6.5 million tyres that contained a safety-related defect. The recall was the result of an abnormally high rate of tread separations that caused catastrophic rollover crashes that maimed and killed drivers and passengers.

In May 2001, the Ford Motor Company also announced a new recall of 13 million tyres from the Ford Explorer models and the termination of the business relationship with Firestone. It also announced in March 2001 that the company would redesign the Explorer model (creating the new Explorer) adding a wider body and incorporating some 'rollover' features. We investigate here whether that was the right decision?

There are several key players in the tyre separation tread case. The first is the company that designed and manufactured the tyres: Firestone. The second is the company that designed and manufactured the vehicles: Ford Motor Company. The third is the governmental regulation agency: the National Highway Safety Administration (NHTSA).

10.2 Creating the model

The model for finding the optimal decision for the Ford Motor Company regarding the Explorer/Firestone conflict was designed using BOCR model. No Opportunities were included because it was thought that the decision should be corrective. The Benefits indicate advantages obtained from the decision, whereas Costs and Risks reflect current and potential negative effects. There are different clusters defined under Benefits, Costs and Risks. In the models of Benefits and Risks, the control criteria are social and economic, whereas Costs model has an additional political control criterion. Although the clusters and the specific elements assigned to each network vary due to their interactions, the following general definitions apply to all.

Alternatives

The alternative choices cluster includes:

- *Discontinue Explorer* production
- *Redesign the Model*
- *Maintain Current Model*
- *Maintain* the production of Explorer *Model,* but *change the* tyre *Supplier*.

Stakeholders

The stakeholders include people or groups that would be impacted by the alternative decisions made by the Ford Motor Company. The elements in this cluster are:

- *Customers*: current and potential buyers
- *Community*: people who may not be a customer but could be affected by the alternative decisions

- *Employees*: the Ford Motor Company employees, including labour and management
- *Nation's Highway Safety Agency*: government agency.

Tyre Suppliers

This cluster considers current and potential tyre suppliers for the Ford Motor Company. The elements in this cluster are: *Firestone, Goodyear, Michelin*, and *Other Tyre Suppliers*.

Competition

The competition cluster includes other SUV brands and models owned by the Ford Motor Company and other companies. The nodes of this cluster are:

- *Ford's other SUV brands* (e.g., Escape)
- *Ford affiliates' SUV brands* (e.g., Land Rover)
- *Other companies' SUV brands* (e.g., GM, Honda, Lexus, Dodge, etc.).

Public Relation

This cluster considers elements that would impact relationships between the company and the stakeholders. The elements in this cluster are:

- *Image*: image of the company in public
- *Trust*: reliability of the name of the company
- *Accountability*: how the company reacts to community threats caused by its products
- *Legal matters*: current and potential lawsuits filed against the company.

Brand Image

The Brand Image cluster describes major aspects of the products that would impact image of the company. The elements in this cluster are:

- *Quality, Safety, Prestige*, and *Service*.

Cost of resources

The cost of resources refers to those costs that the Ford Motor Company may incur when choosing the alternative decisions. The nodes of this cluster are:

- *Layoff costs*: the cost that the company would incur in case it decides to reduce the number of employees.
- *Launching costs*: the cost that the company would incur in case it decides to launch a new product.
- *Write-off costs*: the cost that the company would incur in case it decides to reduce the inventory of discontinued products
- *Production costs*: the cost that the company incurs during the production stage

Resources

- Resources cluster includes *Revenues, Production Capacity*, and *Market Share*.

Procedure

In order to rate the Benefits, Costs and Risks in the decision, three strategic criteria were considered: *Domestic Issues, International Relations* and *Human Well-Being*. For Domestic Issues, the subcriteria were:

- Ford Motor Company's reputation
- Car's industry reputation
- US government's reputation.

In the case of International Relations, the subcriteria used were:

- Relationship with customers in other countries
- Relationship with suppliers in other countries
- Relationship with other countries' governments.

Finally, in the case of Human Well-Being, the subcriteria were:

- Future safety factors
- Confidence in government agencies
- Confidence in the Justice system.

10.3 Benefits model

Benefits in our model are gains and advantages from making a given decision, partitioned into two categories: economic and social. Economic benefits refer to a decision's positive effect on stakeholders, tyre suppliers, competition and resources. Social benefits describe a decision's positive effect on stakeholders, tyre suppliers, competition and resources.

The dependencies of clusters in the Economic Benefits network are shown in Figure 14.

The Stakeholders cluster, obviously, refers to the people or group of people who could potentially benefit economically, based on different decision alternatives taken by the Ford Motor Company. This cluster also affects the Competition cluster, because the decisions made may drive the stakeholders to provide economic benefit to anyone of the competitors. The Stakeholders cluster also affects the Resources cluster. The Resources cluster refers to the internal resources that the company has. For example, revenue of the company would be impacted by some of the actions taken by the stakeholders.

The Tyre Suppliers cluster refers to tyre companies that may gain economically based on the decision alternatives taken by Ford. This cluster would also affect the Public Image cluster; more specifically, legal matters.

The Stakeholders and Tyre Suppliers clusters have more inter-links than the other clusters. This is due to the nature of the network of Economic Benefits, which usually has more impact on a person or a group of persons.

In this particular network, there is no inner-dependence in any of the clusters.

Table 22 shows the result for the Economic Benefits network. It is computed along the lines of the market share example worked out in detail in the previous section.

Figure 14 Economic benefits network

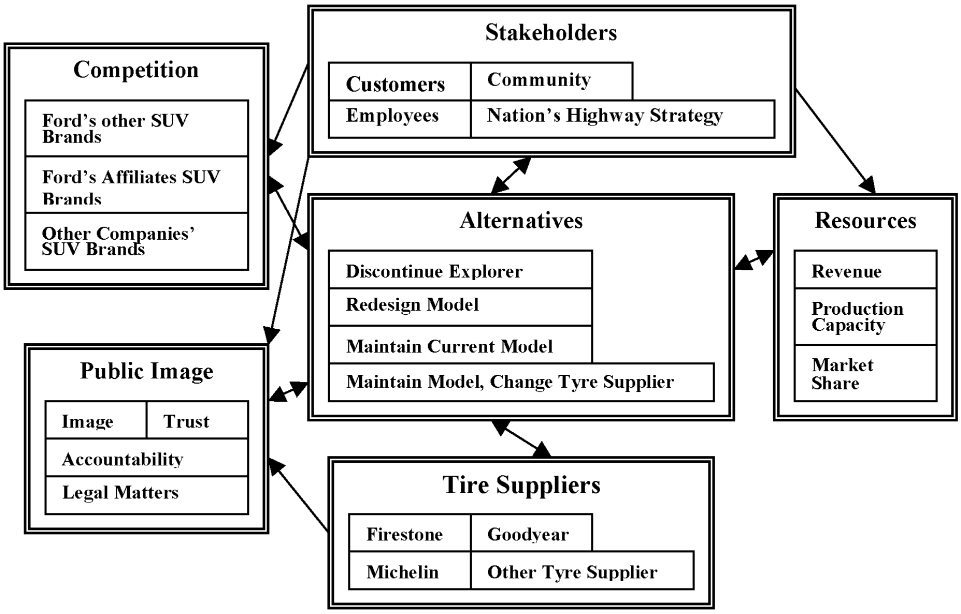


Table 22 Final result from the economic benefits network

Graphics	*Alternatives*	*Priority*	*Ranking*
	Discontinue Explorer	0.48	1
	Redesign Model	0.18	3
	Maintain Current Model	0.06	4
	Maintain Model, Change Tyre Supplier	0.28	2

The dependencies of clusters in the Social Benefits model are shown in Figure 15. Table 23 summarises the results obtained from the Social Benefits network.

Synthesis of priorities in the Benefits model

Both networks in the Benefits model have independent results that feed into the higher-level network (the overall benefits network). The combined results from Economic Benefits and Social Benefits networks are shown in Table 24.

This result indicates that the alternative decision of discontinuing the Explorer gives the highest benefits, both from the economic and social standpoints.

Another observation is that the overall priority 0.49 for the first ranked alternative, i.e., to discontinue the Explorer, is significantly larger than 0.25 the priority of the next alternative to Maintain the Model but change the Tyre Suppliers.

Figure 15 Social benefits network

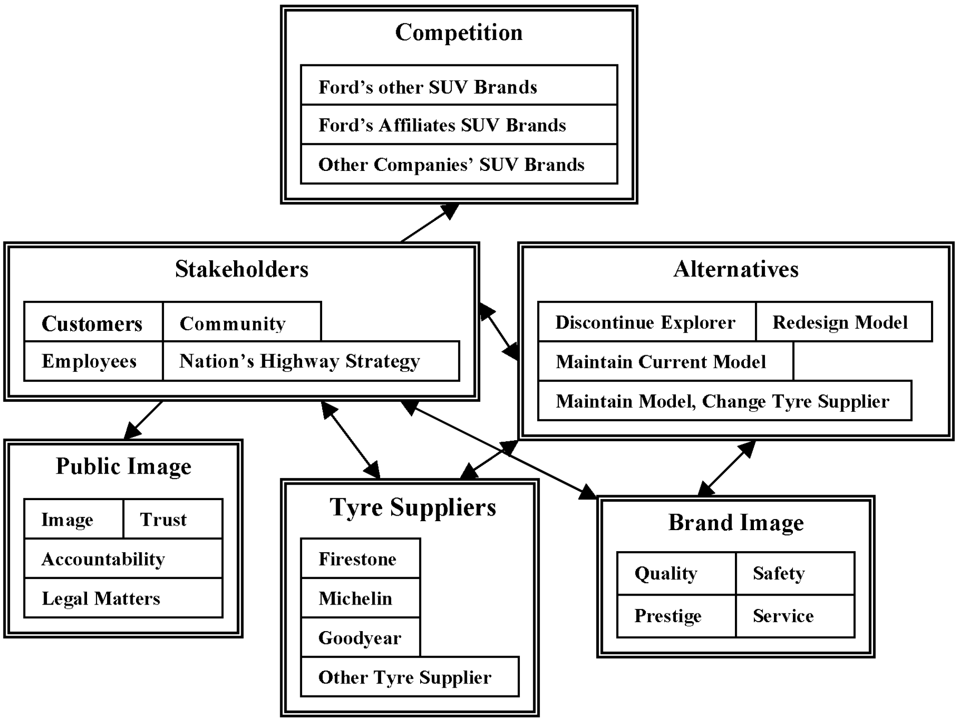


Table 23 Final result from the social benefits network

Graphics	*Alternatives*	*Priority*	*Ranking*
	Discontinue Explorer	0.55	1
	Redesign Model	0.32	2
	Maintain Current Model	0.01	4
	Maintain Model, Change Tyre Supplier	0.12	3

Table 24 Synthesised judgements in the benefit model

Alternative	*Economic benefit priority* (*0.80*)	*Social benefit priority* (*0.20*)	*Overall priority*	*Overall ranking*
Discontinue explorer	0.48	0.55	0.49	1
Maintain model, change tyre supplier	0.28	0.12	0.25	2
Redesign model	0.18	0.35	0.21	3
Maintain current model	0.06	0.01	0.05	4

10.4 Costs model

The costs are divided into Economic, Political and Social, which comprise the control criteria for this model. Economic Costs are the costs in which a monetary value can be assigned to the production and advertising costs involved on the redesign of the Ford Explorer. Political Costs can be defined as the intangible costs due to the decision taken, such as breaking the long standing relationship between Ford and its tyre supplier. Social Costs are defined as the expense (pain) to society in terms of stakeholder exposure to decisions made regarding the Ford Explorer.

The dependencies of clusters in the economic costs model are shown in Figure 16.

Figure 16 Economic costs network

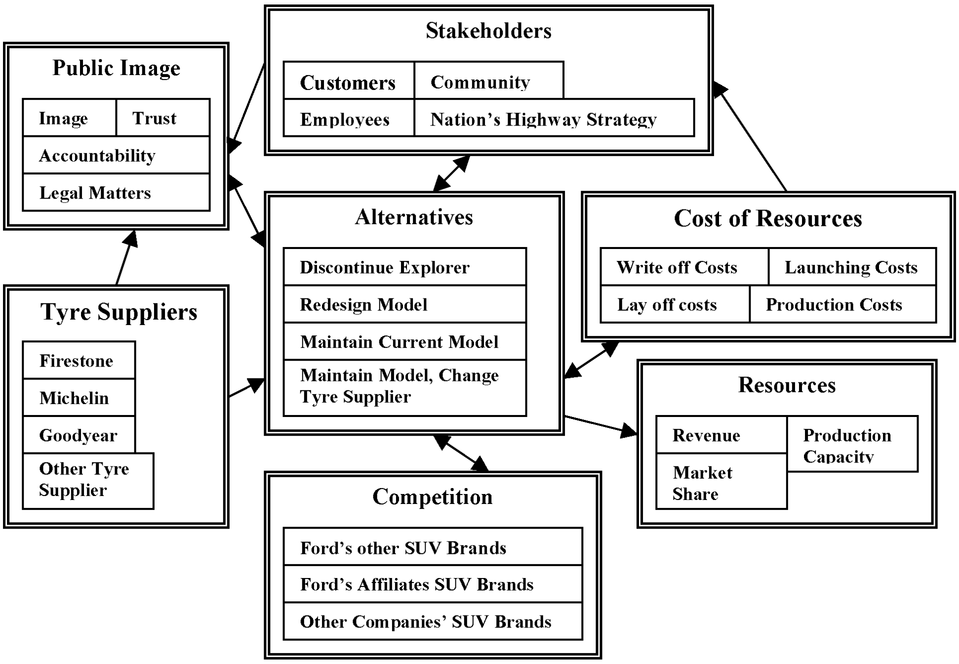


Table 25 shows the result from the Economic Costs network.

Table 25 Final results from economic costs network

Graphics	*Alternatives*	*Priority*	*Ranking*
	Discontinue Explorer	0.14	4
	Redesign Model	0.37	1
	Maintain Current Model	0.32	2
	Maintain Model, Change Tyre Supplier	0.17	3

The dependencies of clusters in the Political Costs model are shown in Figure 17.

Figure 17 Political costs network

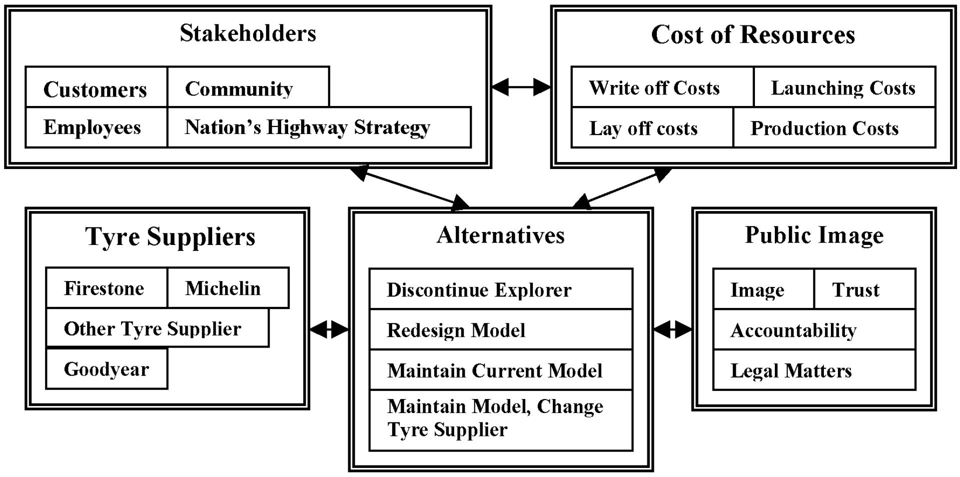


Table 26 summarises the results from the network of Political Costs.

Table 26 Final results from the political costs network

Graphics	*Alternatives*	*Priority*	*Ranking*
	Discontinue Explorer	0.08	3
	Redesign Model	0.00	4
	Maintain Current Model	0.10	2
	Maintain Model, Change Tyre Supplier	0.82	1

Social Costs Clusters

The dependencies of clusters in the economic risks model are shown in Figure 18.

Figure 18 Social costs network

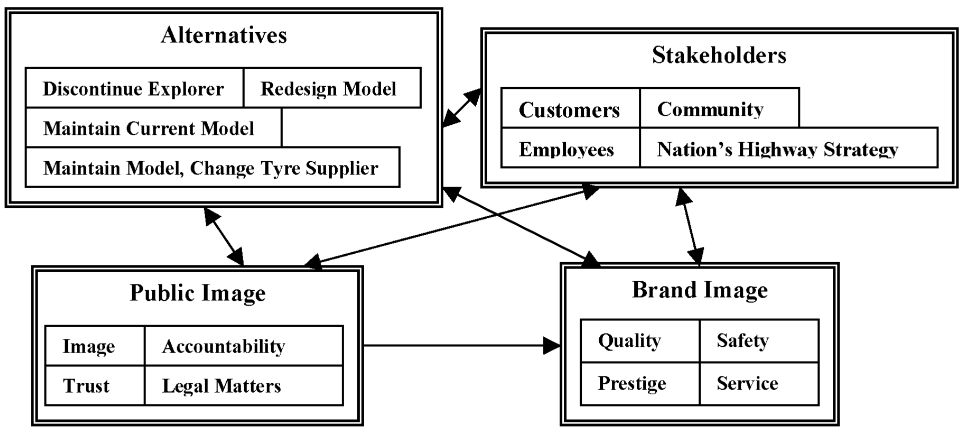


Table 27 summarises the results from the network of Social Costs.

Table 27 Final result in social costs network

Graphics	*Alternatives*	*Priority*	*Ideal*	*Ranking*
	Discontinue Explorer	0.052	0.10	4
	Redesign Model	0.54	1.00	1
	Maintain Current Model	0.18	0.33	3
	Maintain Model, Change Tyre Supplier	0.23	0.43	2

Synthesis of judgements in the Costs Model

The combined results from Economic Cost, Political Cost and Social Cost networks can be seen in Table 28.

Table 28 Synthesised results for costs

Alternative	*Economic costs priority (0.66)*	*Political costs priority (0.08)*	*Social costs priority (0.26)*	*Overall priority*	*Overall ranking*
Discontinue explorer	0.14	0.08	0.05	0.45	1
Maintain model, change tyre supplier	0.17	0.82	0.23	0.24	2
Maintain current model	0.32	0.10	0.18	0.19	3
Redesign model	0.37	0.00	0.54	0.11	4

This result indicates that from the Costs Model point of view, the alternative decision of discontinuing the Explorer gives the highest cost for the Ford Company, and the Redesign alternative would have the smallest impact on the company's costs.

10.5 Risks model

Unlike the Benefits and Costs models, the Risks model is slightly different. Risks are defined as the negative uncertainties in the decisions taken by the Ford Motor Company regarding the Ford Explorer/Firestone matters.

Risks are classified into two categories, Economic and Social. Economic Risks refer to financial risks that may be incurred as a result of the decisions taken by the Ford Motor Company. For example, if the decision is to discontinue the Explorer, there is a risk that Ford would jeopardise its relationship with Firestone which may impact Firestone's relation on other Ford brands. Social Risks describe other than financial risks that may be incurred as a result of the decision taken by Ford. For example, if the decision is to maintain the current Explorer model, there is a risk that the number of accidents happening to customers who drive this car would increase.

Networks of the Economic and Social Risks are exhibited in Figures 19 and 20 respectively. Tables 29 and 30 summarise the results from the Economic and Social Risks networks.

Figure 19 Economic risks network

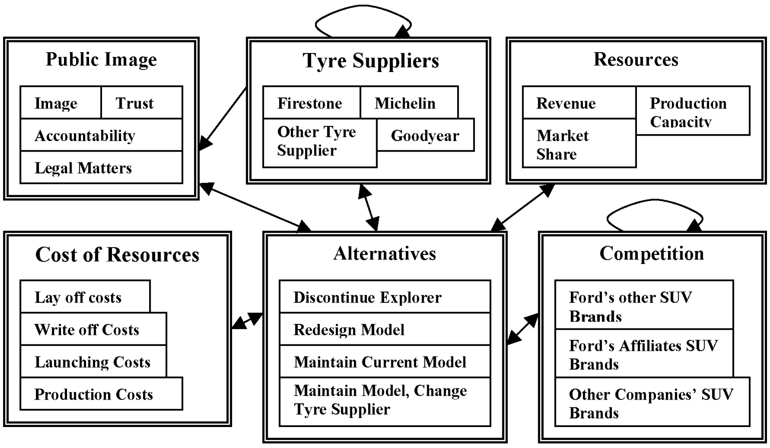


Figure 20 Social risks network

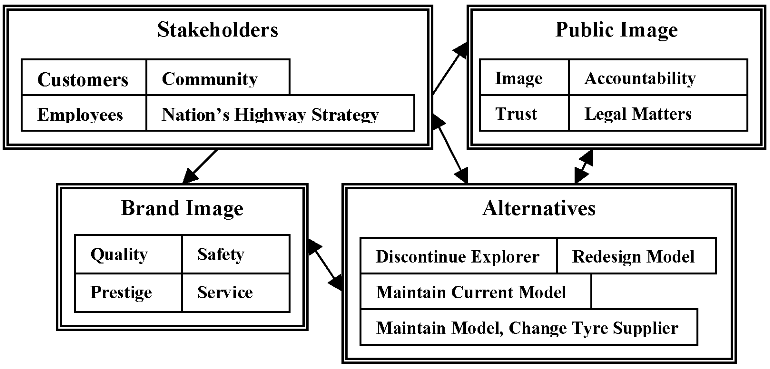


Table 29 Final result in economic risks network

Graphics	*Alternatives*	*Priority*	*Ranking*
	Discontinue explorer	0.19	4
	Redesign model	0.35	1
	Maintain current model	0.20	3
	Maintain model, change tyre supplier	0.25	2

Table 30 Final result in social benefits network

Graphics	*Alternatives*	*Priority*	*Ranking*
	Discontinue explorer	0.10	3
	Redesign model	0.60	1
	Maintain current model	0.05	4
	Maintain model, change tyre supplier	0.24	2

Synthesis of judgements in the Risks Model

The combined results from Economic Risks and Social Risks networks are shown in Table 31.

Table 31 Synthesised judgements from the risks model

Alternative	*Economic risks priority (0.25)*	*Social risks priority (0.75)*	*Overall priority*	*Overall ranking*
Maintain current model	0.20	0.05	0.49	1
Discontinue explorer	0.20	0.10	0.29	2
Maintain model, change tyre supplier	0.25	0.24	0.15	3
Redesign model	0.35	0.60	0.08	4

The least risky alternative would be to redesign the model, with overall priority of 0.08. In this alternative, the driven force is the Social Risks whose contribution is 0.60 which is in contrast with the Economic Risks contribution of 0.35, and with Social Risks having nearly twice the influence than Economic Risks.

10.6 Strategic criteria and final priorities of the alternatives

The strategic criteria in this case have a hierarchical structure shown in Figure 21. The priorties of these criteria are shown in Table 32.

Figure 21 Strategic criteria and subcriteria

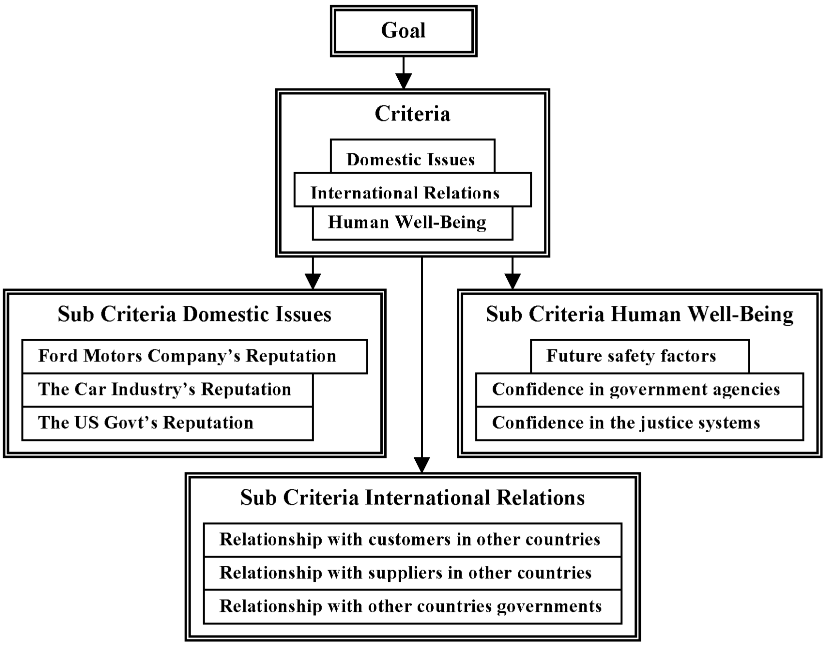


Table 32 Ratings with respect to strategic criteria for BCR merits

	Strategic criteria priorities	*Domestic issues* (*0.22*)			*International relations* (*0.07*)			*Human well-being* (*0.71*)		
	Strategic sub-criteria local priorities	Ford Motor Company's Reputation (0.73)	The Car Industry's Reputation (0.08)	The US Govt's Reputation (0.19)	Relationship with customers in other countries (0.64)	Relationship with suppliers in other countries (0.10)	Relationship with other countries governments (0.26)	Future safety factors (0.73)	Confidence in government agencies (0.19)	Confidence in the justice system (0.08)
	Strategic sub-criteria global priorities	Ford Motor Company's Reputation (0.16)	The Car Industry's Reputation (0.02)	The US Govt's Reputation (0.04)	Relationship with customers in other countries (0.04)	Relationship with suppliers in other countries (0.01)	Relationship with other countries governments (0.02)	Future safety factors (0.52)	Confidence in government agencies (0.13)	Confidence in the justice system (0.06)
Benefits	*0.49*	Very High	High	Medium	Medium	Low	Medium	Very High	Very High	High
Costs	*0.40*	Very High	Medium	Low	High	Very Low	Low	Very High	Very Low	Very Low
Risks	*0.11*	Medium	Low	Very Low	Low	Medium	Low	Medium	Medium	Very Low

Very High (0.50), High (0.29), Medium (0.12), Low (0.06), Very Low (0.03).

To rate the top outcome for each of the BCR against the strategic criteria we use the five-level ratings scale obtained from paired comparisons shown in Table 32.

To obtain priorities of all the alternatives we use the formula: bB-cC-rR. Here, $b = 0.49$, $c = 0.40$, $r = 0.11$ are used to weight the vectors B, C and R for the alternatives shown in Table 33 and obtain the overall priorities of the alternatives.

The final priorities of the alternatives are shown in Table 34. Benefits is a positive merit, whereas Costs and Risks are negative ones.

Table 33 Vectors of priorities for the alternatives for B, C and R

	B	*C*	*R*
Discontinue explorer	0.21	0.18	0.14
Redesign model	0.08	0.05	0.04
Maintain current model	0.02	0.08	0.24
Maintain model, change the supplier	0.11	0.10	0.07

Table 34 Final priorities of the alternatives

	B/CR	*bB-cC-rR*
Discontinue explorer	8.21	0.01
Redesign model	48.03	0.02
Maintain current model	1.17	–0.05
Maintain model, change the supplier	15.45	0.01

The second alternative, i.e., Redesign Model has the highest ranking with overall priority of 0.02 in the right column of Table 34. The benefit of this alternative is not very attractive with an overall value close to zero. In fact, the benefit is the second lowest among the alternatives. However, both its costs and risks are extremely low and offset the also low benefits and contribute to the end result, which drive this alternative to be the best option for the Ford Motor Company to take.

As affirmed by the result of this model using the standard formula (*bB-cC-rR*), the worst alternative, i.e., Maintain Current Model has the least benefit (0.02) and the highest risk (0.24).

For marginal analysis, the formula B/CR was used. Here we obtain the same results, i.e., Redesign Model has the highest priority and Maintain Current Model – the smallest.

10.7 Sensitivity analysis

In order to determine when different alternatives become preferred, sensitivity analysis was made by varying different weights and ratings in the model.

Sensitivity graphs for Benefits, Costs and Risks are shown in Figures 22–24 respectively. The order of the curves from top to bottom is as in the last column of Table 34.

Figure 22 Sensitivity graph for benefits

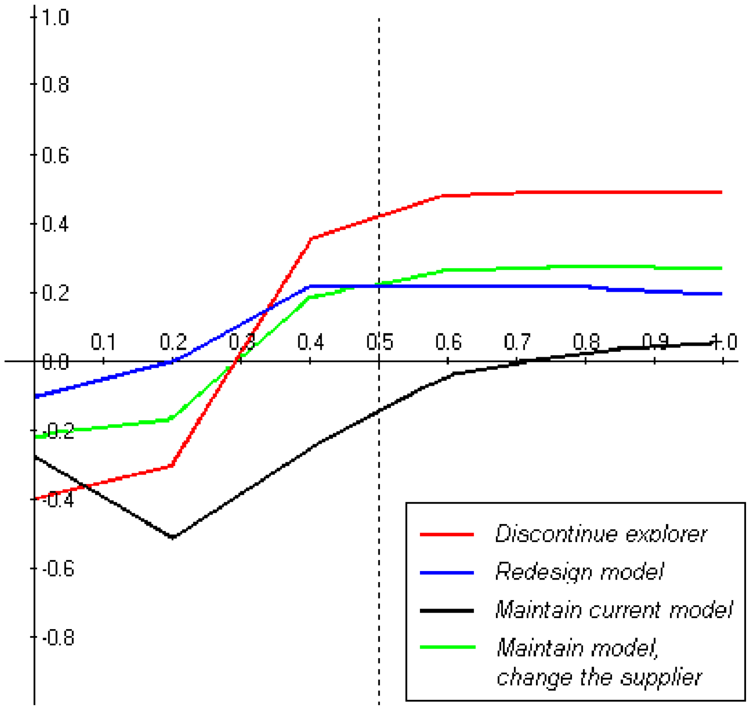


Figure 23 Sensitivity graph for costs

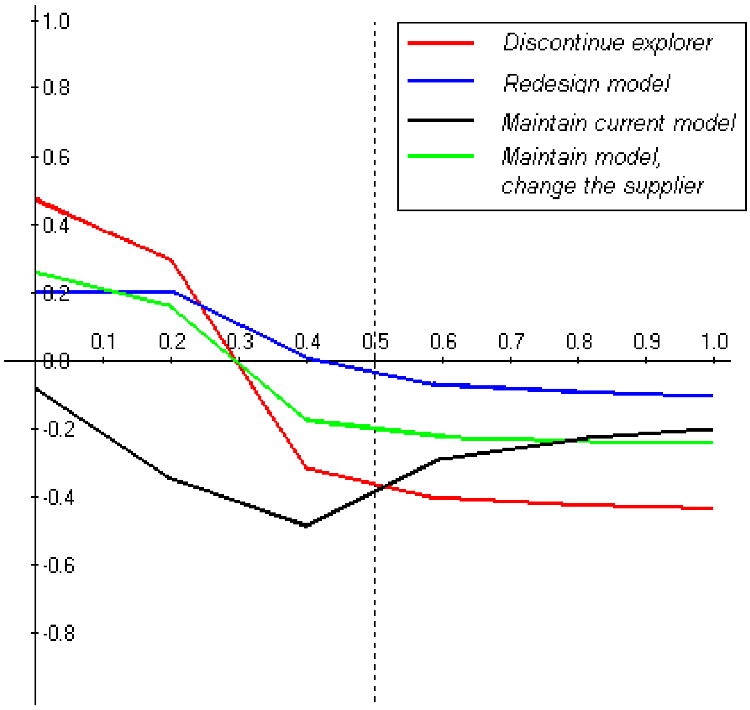
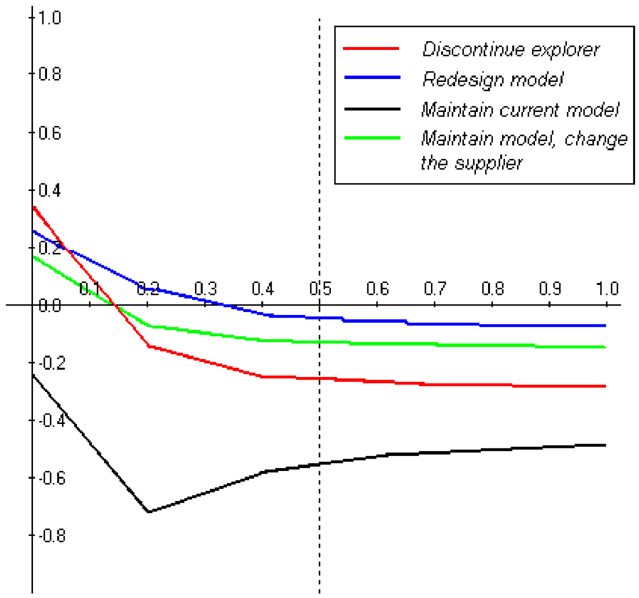


Figure 24 Sensitivity graph for risks



11 Group decision making

Here we consider two issues in group decision making. The first is how to aggregate individual judgements into a representative group judgement, and the second is how to construct a group choice from individual choices (Saaty and Peniwati, 2008). In reality group decisions should not go by consensus because not all people feel the same about things. A minority can have very strong commitments to a cause and can give rise to disruptions that the majority feels lukewarm about. There is no hiding from this issue in the real world. The reciprocal property plays an important role in combining the judgements of several individuals to obtain a judgement for a group. Judgements must be combined so that the reciprocal of the synthesised judgements must be equal to the syntheses of the reciprocals of these judgements. It has been proved that the geometric mean is the unique way to do that. If the individuals are experts, they may not wish to combine their judgements but only their final outcome from a hierarchy. In that case one takes the geometric mean of the final outcomes. If the individuals themselves have different priorities of importance their judgements (final outcomes) are raised to the power of their priorities and then the geometric mean is formed.

11.1 How to aggregate individual judgements

Let the function $f(x_1, \ldots, x_n)$ for synthesising the judgements given by n judges, satisfy the

- *Separability condition (S)*: $f(x_1,\ldots,x_n) = g(x_1)\ldots g(x_n)$, for all $x_1, \ldots, x_n$ in an interval P of positive numbers, where g is a function mapping P onto a proper interval J and is a continuous, associative and cancellative operation. [(S) means that the influences of the individual judgements can be separated as above.]

- *Unanimity condition (U)*: $f(x, \ldots, x) = x$ for all x in P. [(U) means that if all individuals give the same judgement x, that judgement should also be the synthesised judgement.]
- *Homogeneity condition (H)*: $f(ux_1, \ldots, ux_n) = uf(x_1, \ldots, x_n)$ where $u > 0$ and x_k, ux_k ($k = 1, 2, \ldots, n$) are all in P. [For ratio judgements (H) means that if all individuals judge a ratio u times as large as another ratio, then the synthesised judgement should also be u times as large.]
- *Power conditions (P_p)*: $f(x_1^p, \ldots, x_n^p) = f^p(x_1, \ldots, x_n)$. [($P_2$) for example means that if the kth individual judges the length of a side of a square to be x_k, the synthesised judgement on the area of that square will be given by the square of the synthesised judgement on the length of its side.]

Special case ($R = P_{-1}$):

$$f\left(\frac{1}{x_1}, \ldots, \frac{1}{x_n}\right) = 1/f(x_1, \ldots, x_n).$$

[(R) is of particular importance in ratio judgements. It means that the synthesised value of the reciprocal of the individual judgements should be the reciprocal of the synthesised value of the original judgements.] Aczel and Saaty (1983) proved the Theorem 2:

Theorem 2: *The general separable (S) synthesising functions satisfying the unanimity (U) and homogeneity (H) conditions are the geometric mean and the root-mean-power. If moreover the reciprocal property (R) is assumed even for a single n-tuple ($x_1, \ldots, x_n$) of the judgements of n individuals, where not all x_k are equal, then only the geometric mean satisfies all the above conditions.*

In any rational consensus, those who know more should, accordingly, influence the consensus more strongly than those who are less knowledgeable. Some people are clearly wiser and more sensible in such matters than others, others may be more powerful and their opinions should be given appropriately greater weight. For such unequal importance of voters not all g's in (S) are the same function. In place of (S), the Weighted Separability property (WS) is now: $f(x_1, \ldots, x_n) = g_1(x_1) \ldots g_n(x_n)$ [(WS) implies that not all judging individuals have the same weight when the judgements are synthesised and the different influences are reflected in the different functions $(g_1, \ldots, g_n)$.]

In this situation, Aczel and Alsina (1986) proved the Theorem 3:

Theorem 3: *The general weighted-separable (WS) synthesising functions with the unanimity (U) and homogeneity (H) properties are the weighted geometric mean* $f(x_1, x_2, \ldots, x_n) = x_1^{q_1} x_2^{q_2} \ldots x_n^{q_n}$ *and the weighted root-mean-powers* $f(x_1, x_2, \ldots, x_n) = x_2^{q_2} \ldots x_n^{q_n} = \sqrt[\gamma]{q_1 x_1^\gamma + q_2 x_2^\gamma + \cdots + q_n x_n^\gamma}$, *where* $q_1 + \ldots + q_n = 1$, $q_k > 0$, $k = 1, \ldots, n$, $\gamma > 0$, *but otherwise* $q_1, \ldots, q_n$, γ *are arbitrary constants.*

If f also has the reciprocal property (R) and for a single set of entries ($x_1, \ldots, x_n$) of judgements of n individuals, where not all x_k are equal, then only the weighted geometric mean applies. We give the Theorem 4 which is an explicit statement of the synthesis problem that follows from the previous results:

Theorem 4: *If $x_1^{(i)},\ldots,x_n^{(i)}$ $i = 1, \ldots, m$ are rankings of n alternatives by m independent judges and if a_i is the importance of judge i developed from a hierarchy for evaluating the judges, and hence*

$$\sum_{i=1}^{m} a_i = 1, \quad then \left(\prod_{i=1}^{m} x_1^{a_i}\right),\ldots,\left(\prod_{i=1}^{m} x_n^{a_i}\right),$$

are the combined ranks of the alternatives for the m judges.

The power or priority of judge i is simply a replication of the judgement of that judge (as if there are as many other judges as indicated by his or her power a_i), which implies multiplying his or her ratio by itself a_i times, and the result follows.

The first requires knowledge of the functions which the particular alternative performs and how well it compares with a standard or benchmark. The second requires comparison with the other alternatives to determine its importance.

11.2 On the construction of group choice from individual choices

Given a group of individuals, a set of alternatives (with cardinality greater than 2), and individual ordinal preferences for the alternatives, Arrow proved with his Impossibility Theorem that it is impossible to derive a rational group choice (construct a social choice function that aggregates individual preferences) from ordinal preferences of the individuals that satisfy the following four conditions, i.e., at least one of them is violated:

- *Decisiveness.* The aggregation procedure must generally produce a group order.
- *Unanimity.* If all individuals prefer alternative A to alternative B, then the aggregation procedure must produce a group order indicating that the group prefers A to B.
- *Independence of irrelevant alternatives.* Given two sets of alternatives which both include A and B, if all individuals prefer A to B in both sets, then the aggregation procedure must produce a group order indicating that the group, given any of the two sets of alternatives, prefers A to B.
- *No dictator.* No single individual preferences determine the group order.

Using Fundamental Scale of absolute numbers of the AHP, it can be shown that because now the individual preferences are cardinal rather than ordinal, it is *possible* to derive a rational group choice satisfying the above four conditions. It is possible because:

- individual priority scales can always be derived from a set of pairwise cardinal preference judgements as long as they form at least a minimal spanning tree in the completely connected graph of the elements being compared
- the cardinal preference judgements associated with group choice belong to an absolute scale that represents the relative intensity of the group preferences (Saaty and Vargas, 2003).

12 Conclusions

The AHP/ANP is a useful way to deal with complex decisions that involve dependence and feedback analysed in the context of benefits, opportunities, costs and risks. It has been applied literally to hundreds of examples both real and hypothetical. What is important in decision making is to produce answers that are valid in practice. The AHP/ANP has also been validated in many examples. People often argue that judgement is subjective and that one should not expect the outcome to correspond to objective data. But that puts one in the framework of garbage in garbage out without the assurance of the long term validity of the outcome. In addition, most other approaches to decision making are normative. They say, "If you are rational you do as I say". But what people imagine is best to do and what conditions their decisions face after they are made can be very far apart fro what can happen in the real world. That is why the framework of the AHP/ANP is descriptive as in science rather than normative and prescriptive giving license to unrealistic assumptions like insisting on transitivity of preferences when we know that people are intransitive and inconsistent. The AHP/ANP produce outcomes that are best not simply according to the decision maker's values, but also to the external risks and hazards faced by the decision.

It is unfortunate that there are people who use fuzzy sets without proof to alter the AHP when it is known that fuzzy applications to decision making have been ranked as the worst among all methods. Buede and Maxwell write about their findings,

> "These experiments demonstrated that the MAVT and AHP techniques, when provided with the same decision outcome data, very often identify the same alternatives as 'best'. The other techniques are noticeably less consistent with MAVT, the Fuzzy algorithm being the least consistent." (Buede and Maxwell, 1995)

The fundamental scale used in the AHP/ANP to represent judgements is already fuzzy. To fuzzify it further does not improve the outcome as Saaty and Tran (2007) have shown through numerous examples. The intention of fuzzy seems to be to perturb the judgements in the AHP. It is already known in mathematics that perturbing the entries of a matrix perturbs the eigenvector by a small amount but not necessarily in a more valid direction.

The SuperDecisions software used to analyse complex decisions is named after the supermatrix. It can be downloaded free from creativedecisions.net and is available on the internet along with a manual and with numerous applications to enable the reader to apply it to his or her decision. Alternatively, go to www.superdecisions.com/~saaty and download the SuperDecisions software. The installation file is the exe file in the software folder. The serial number is located in the .doc file that is in the same folder. The important thing may be not the software but the models which are in a separate folder called models. The military are constantly involved in making complex decisions and appear to like using the ANP and investing in its development. Why do we do all this with so much effort? Because we believe strongly in the creativity of the human race and hope that our world will become increasingly more rational in making its decisions and in resolving its conflicts.

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A soft multi-criteria decision analysis model with application to the European Union enlargement

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Abstract This paper proposes a new multi-criteria decision analysis (MCDA) model that uses a series of existing intuitive and analytical methods to systematically capture both objective and subjective beliefs and preferences from a group of decision makers (DMs). A defuzzification method that combines entropy and the theory of displaced ideal synthesizes crisp values from the DMs' subjective judgments. This approach assists the DMs in their selection process by plotting alternatives in a four quadrant graph and considering their Euclidean distance from the "ideal" choice. A pilot study illustrates the details of the proposed method. The DMs were a group of graduate students from the University of Paderborn in Germany. The pilot study concerned the addition of new members into the European Union (EU), a decision that has profound economic and political effects on both the entering and existing members of the Union. The DMs were required to consider a large number of internal strengths and weaknesses and external opportunities and threats in assessing the decision to enlarge the EU. Although the pilot study was not performed by actual DMs from the EU, it was an excellent platform for testing the proposed model.

Keywords Multi-criteria decision analysis · Soft computing · Fuzzy systems · SWOT · Analytic hierarchy process · European Union enlargement · Defuzzification · Entropy and theory of displaced ideal

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1 Introduction

A large body of intuitive and analytical models has evolved over the last several decades to assist decision makers (DMs) in multi-criteria decision analysis (MCDA). While these models have made great strides in multi-criteria decision-making, the intuitive models lack a structured framework and the analytical models do not capture intuitive preferences. The literature and research show that the following difficulties may be encountered in MCDA:

(a) A decision may not be properly made without fully taking into consideration all subjective and objective criteria (Belton and Stewart 2002; Yang and Xu 2002).
(b) DMs often use verbal expressions and linguistic variables for subjective judgments which lead to ambiguity in human decision-making (Poyhonen et al. 1997).
(c) DMs often provide imprecise or vague information due to lack of expertise, unavailability of data, or time constraint (Kim and Ahn 1999).
(d) Meaningful and robust aggregation of subjective and objective judgments causes problems during the evaluation process (Valls and Torra 2000).

We propose a new MCDA model that uses a series of existing intuitive and analytical methods to systematically capture both objective and subjective beliefs and preferences from a group of DMs. A defuzzification method that combines entropy and the theory of displaced ideal is used to synthesize crisp values from the subjective judgments. This approach assists the DMs in their selection process by plotting alternatives in a four quadrant graph and considering their Euclidean distance from the "ideal" choice. We present the results of a pilot study to elucidate the details of the proposed method. The pilot study concerned the addition of new members into the European Union (EU), a decision that has profound economic and political effects on both the entering and existing members of the Union. In Sect. 2 we review the relevant literature and in Sect. 3 we present a detailed explanation of the mathematical model. In Sect. 4 we discuss the results of the pilot study and in Sect. 5 we present our concluding remarks and future research directions.

2 Literature review

The literature on MCDA contains hundreds of methods, including scoring methods, economic methods, portfolio methods, and decision analysis methods. Scoring methods use algebraic formulas to produce an overall score for each alternative (Osawa and Murakami 2002). Economic methods use financial models to calculate the monetary payoff of each alternative (Graves and Ringuest 1991). Portfolio methods evaluate the entire set of alternatives to identify the most attractive subset (Girotra et al. 2007; Lootsma et al. 1990; Mojsilovi et al. 2007; Wang and Hwang 2007). Cluster analysis, a more specific portfolio method, groups alternatives according to their support of the strategic positioning of the firm (Mathieu and Gibson 1993). Finally, decision analysis and simulation methods use random numbers and simulation to generate a large number of problems and pick the best outcome (Abacoumkin and Ballis 2004; Paisittanand and Olson 2006). Recently, strategic management researchers have focused on MCDA models to integrate the intuitive preferences of multiple DMs into structured and analytical frameworks (Bailey et al. 2003; Costa et al. 2003; Hsieh et al. 2004; Liesiö et al. 2007; Tavana 2006).

MCDA techniques require the determination of weights that reflect the relative importance of various competing criteria. Several approaches such as point allocation, paired comparisons, trade-off analysis, and regression estimates could be used to specify these weights (Kleindorfer et al. 1993). *Soft SWOT* utilizes the analytic hierarchy process (AHP) developed by Saaty (1977) to estimate the importance weight of the criteria. The process is simplified by confining the estimates to a series of pairwise comparisons. The measure of inconsistency provided by the AHP allows for the examination of inconsistent priorities. One of the advantages of the AHP is that it encourages DMs to be consistent in their pairwise comparisons. Saaty (1990a) suggests a measure of consistency for the pairwise comparisons. When the consistency ratio is unacceptable, the DMs are made aware that their pairwise comparisons are logically inconsistent, and they are encouraged to revise them. The AHP has been a very popular technique for determining weights in MCDA problems (Ho 2008; Vaidya and Kumar 2006; Saaty and Sodenkamp 2008). The advantage of the AHP is its capability to elicit judgments and scale them uniquely using a procedure that measures the consistency of these scale values (Saaty 1989).

There has been some criticism of the AHP in the operations research literature. Harker and Vargas (1987) show that the AHP does have an axiomatic foundation, the cardinal measurement of preferences is fully represented by the eigenvector method, and the principles of hierarchical decomposition and rank reversal are valid. On the other hand, Dyer (1990a) has questioned the theoretical basis underlying the AHP and argues that it can lead to preference reversals based on the alternative set being analyzed. In response, Saaty (1990b) explains how rank reversal is a positive feature when new reference points are introduced. In *Soft SWOT*, the geometric aggregation rule is used to avoid the controversies associated with rank reversal (Dyer 1990a, 1990b; Harker and Vargas 1990; Saaty 1990b).

Decision making in complex MCDA problems requires judgments on a large number of uncertain criteria. However, the estimation of uncertain criteria in MCDA is often very challenging. To address this issue, some researchers use fuzzy AHP to determine the weighting of subjective judgments for each criterion and to derive fuzzy synthetic utility values of alternatives. In *Soft SWOT* we do not use fuzzy AHP for the following reason. Fuzzy set theory has been introduced into AHP mainly to deal with uncertainty associated with pairwise comparison judgments. However, as Saaty and Tran (2007, p. 968) point out, "fuzzy AHP guarantees nothing and can foul up the outcome of a decision." The point is to use fuzzy numbers as a suitable tool to represent uncertain judgments. Judgments in the AHP are fuzzy themselves and additional fuzzifying of the numerical judgments will result in further fuzzification of the outcome. "Making good judgments gives good (valid) answers with the AHP and fuzzifying these judgments is simply a perturbation that leaves the results where they are without producing uniformly better outcomes" (Saaty and Tran 2007, p. 973).

Among the many tools and techniques in the strategic management literature, the strengths, weaknesses, opportunities, and threats or SWOT analysis has been widely used by both researchers and practitioners during the last several decades. SWOT is used to segregate the environmental factors and forces into internal strengths and weaknesses and external opportunities and threats (Valentin 2001; Duarte et al. 2006). Since its inception in the 1950s, SWOT has gained increasing success as a strategic management tool (Panagiotou 2003). SWOT is still alive and well as the popular framework for classifying environmental factors (Hitt et al. 2000; Anderson and Vince 2002). Despite its popularity, SWOT has remained a conceptual framework with limited prescriptive power for practice and minor significance for research (Novicevic et al. 2004).

Over the past few years, there has been an increasing application of the integrated SWOT with AHP (Ho 2008). Kurttila et al. (2000) proposed the combined SWOT-AHP approach to aid the decision-making in a Finnish forestry. Kajanus et al. (2004) proposed the combined approach to investigate the role of culture in rural tourism. Their approach was similar to the framework presented previously in Kurttila et al. (2000) where the AHP was used to measure the relative importance weightings of the individual SWOT factors. Shrestha et al. (2004) used the integrated framework to analyze the possibilities for silvopasture adoption in south-central Florida. Their approach was similar to those adopted by (Kurttila et al. 2000) and (Kajanus et al. 2004). Shrestha et al. (2004) applied the integrated framework to agricultural planning and Masozera et al. (2006) adopted the same approach to assess the suitability of community-based management method to the Nyungwe forest reserve in Rwanda. Shinno et al. (2006) presented the combined AHP–SWOT approach to analyze the global competitiveness of Japanese manufacturers of machine tools.

Recently, fuzzy concepts and theories have been applied to strategic management and SWOT analysis because of their complex and fuzzy nature (Buyukozkan and Feyzioglu 2002). Lin and Hsieh (2003) used fuzzy weighted average to defuzzify the industry attractiveness-business strength matrix. Pap et al. (2000) employed a fuzzy rule-base to handle the growth-share matrix. Ghazinoory et al. (2007) used the fuzzy approach to evaluate quantitative and qualitative factors and strategies in a SWOT matrix. Sodenkamp (2005) applied fuzzy sets to the network structured SWOT factors to measure their relative importance weights. More recently, the MCDA research community has extended their interest in fuzzy set theory into soft computing (Zadeh 1998). The integration of MCDA with soft computing for handling uncertainty is of major interest, both from a research and practical perspective (Kaliszewski 2006; Zopounidis and Doumpos 2001). Soft computing systematically applies the approximate "soft" treatments to MCDA methodologies to reduce the problems' computational complexity. Soft computing, unlike conventional (hard) computing, achieves tractability, robustness, low solution cost, and close resemblance with human like decision-making by exploiting the tolerance for imprecision, uncertainty, approximate reasoning and partial truth.

Soft SWOT is a new MCDA model that captures the DMs' beliefs through a series of intuitive and analytical methods such as the analytic hierarchy process (AHP) and SWOT analysis. A defuzzification method is used to obtain crisp values from the subjective judgments provided by multiple decision groups (DGs). These crisp values are synthesized with entropy and the theory of displaced ideal to assist the DMs in their selection process. Two aggregated opportunity/threat and strength/weakness indexes are used to plot the candidate states in a four quadrant graph based on their Euclidean distance from the ideal state. We do not introduce additional fuzziness to the problem. On the contrary, we use fuzzy sets as a tool for aggregating and analyzing uncertain group judgments. Representation of group estimates by means of fuzzy sets is not new and it was used by Tavana and Sodenkamp (2009) to facilitate advanced technology assessment by multiple DGs at the Kennedy space center.

Soft SWOT is a normative MCDA model with multiple criteria representing different dimensions from which the candidate states are viewed. When the number of criteria in a MCDA problem is large, they may be arranged hierarchically (Saaty 2003; Triantaphyllou 2000). Six groups of DMs, all from the University of Paderborn in Germany, were selected to participate in the EU expansion pilot study. We held several brainstorming sessions within and between groups to classify the 169 Copenhagen criteria into internal and external categories. Internal criteria reflect domestic affairs of a candidate state and external criteria are the environmental factors that influence the entire EU membership. Internal criteria are essentially controllable and external criteria are uncontrollable. The internal and external criteria include political, economic, and community standards. Each standard was divided into several criteria and each criterion was further divided into multiple sub-criteria.

Next, we used a scoring system to capture the intensity of each criterion and further classify the internal criteria into strengths and weaknesses and the external criteria into opportunities and threats. According to this scoring system, the DMs assigned a score from -10 to $+10$ to each internal and external criterion. A positive score to an internal criterion indicated strength and a negative score indicated weakness. In addition, a positive score to an external criterion indicated opportunity and a negative score indicated threat. Higher scores were preferred to lower scores for both internal and external criteria. In practice, two aggregation techniques are used to compute two aggregated indexes and evaluate the alternatives when criteria are divided into positive and negative forces. The first approach is the positive to negative ratio approach (Tavana and Banerjee 1995) and the second is the positive minus negative approach (Tavana 2004). The former approach is a ratio scale and the latter approach is an interval scale.

Soft SWOT is a weighted-sum MCDA model with strengths, weaknesses, opportunities and threats as conflicting criteria. Triantaphyllou (2000) has discussed the mathematical properties of weighted-sum MCDA models. Many weighted-sum models have been developed to help DMs deal with the strategy evaluation process (Gouveia et al. 2008; Leyva-Lopez and Fernandez-Gonzalez 2003). Triantaphyllou and Baig (2005) have examined the use of four key weighted-sum MCDA methods when benefits and costs (opportunities and threats) are used as conflicting criteria. They compared the simple weighted-sum model, the weighted-product model, and the analytic hierarchy process (AHP) along with some of its variants, including the multiplicative AHP. Their extensive empirical analysis revealed some ranking inconsistencies among the four methods, especially, when the number of alternatives was high. Although they were not able to show which method results in the "correct" ranking, they did prove multiplicative AHP is immune to ranking inconsistencies.

The weighted-sum scores in *Soft SWOT* are used to compare potential candidate states among themselves and with the *ideal state*. The concept of ideal choice, an unattainable idea, serving as a norm or rationale facilitating human choice problem is not new (Tavana 2002). See for example the stimulating work of Schelling (1960), introducing the idea. Subsequently, Festinger (1964) showed that an external, generally non-accessible choice assumes the important role of a point of reference against which choices are measured. Zeleny (1974, 1982) demonstrated how the highest achievable scores on all currently considered decision criteria form this composite ideal choice. As all choices are compared, those closer to the ideal are preferred to those farther away. Zeleny (1982, p. 144) shows that the Euclidean measure can be used as a proxy measure of distance.

3 Mathematical model and procedure

The evaluation process begins with a preliminary review of M candidate states. The DMs from the K DGs consider H set of standards for the screening and evaluation of the candidate states. A series of weights and scores are used in *Soft SWOT* to estimate the importance weight of the selection criteria and their performance degree for each alternative. Initially, DMs use the AHP independently to weight their importance of the standards (q_h^k). We do not describe the technical details of the AHP because the procedure is well-documented in numerous research papers and literature sources (e.g., see Saaty 2006 or Saaty and Sodenkamp 2008 for a detailed discussion of pairwise comparisons and priority derivations in the AHP).

Next, each DG independently decides what internal and external criteria should be considered in the evaluation process. Both internal and external criteria have hierarchical structures with L levels. Once the DGs agree on a hierarchical structure, they define their importance of the internal and external criteria and sub-criteria ($w_{u_{hi}}^k$, $w_{v_{hi}}^k$, u_{hij}^k and v_{hij}^k).

Miller (1956) has shown that an individual can simultaneously compare not more than seven criteria (± 2). In agreement with this proposition, we use the AHP in *Soft SWOT* for clusters with nine or less criteria. It is a common practice to divide clusters with more than nine criteria into smaller ones. When it is not justified to change the criteria structure, the DMs can assign direct priorities ($w'^k_{u_{hi}}$, $w'^k_{v_{hi}}$, u'^k_{hij} and v'^k_{hij}) to the criteria on a 1 to 10 scale and normalize those using (1) and (2) to unify their dimensions with the priority dimensions derived through the AHP.

$$\begin{aligned} w^k_{u_{hi}} &= \frac{w'^k_{u_{hi}}}{\sum_{i=1}^{Ih} w'^k_{u_{hi}}} \\ w^k_{v_{hi}} &= \frac{w'^k_{v_{hi}}}{\sum_{i=1}^{Ih} w'^k_{v_{hi}}} \end{aligned} \tag{1}$$

$$\begin{aligned} u^k_{hij} &= \frac{u'^k_{hij}}{\sum_{j=1}^{Jhi} u'^k_{hij}} \\ v^k_{hij} &= \frac{v'^k_{hij}}{\sum_{j=1}^{Jhi} v'^k_{hij}} \end{aligned} \tag{2}$$

where:

q^k_h = the hth standard importance weight for the kth DG; ($h = 1, 2, \ldots, H$; $k = 1, 2, \ldots, K$),

$w^k_{u_{hi}}(w^k_{v_{hi}})$ = the importance weight of the ith group of internal (external) criteria within the standard h for the kth DG; ($h = 1, 2, \ldots, H$; $i = 1, 2, \ldots, I_h$; $k = 1, 2, \ldots, K$),

$u^k_{hij}(v^k_{hij})$ = the weight of the jth internal (external) criterion in the ith group of the standard h for the kth DG; ($h = 1, 2, \ldots, H$; $i = 1, 2, \ldots, I_h$; $j = 1, 2, \ldots, J_{hi}$; $k = 1, 2, \ldots, K$),

$w'^k_{u_{hi}}(w'^k_{v_{hi}})$ = the importance weight of the ith group of internal (external) criteria within the standard h for the kth DG; ($h = 1, 2, \ldots, H$; $i = 1, 2, \ldots, I_h$; $k = 1, 2, \ldots, K$; $I_h > 9$),

$u'^k_{hij}(v'^k_{hij})$ = the weight of the jth internal (external) criterion in the ith group of the standard h for the kth DG; ($h = 1, 2, \ldots, H$; $i = 1, 2, \ldots, I_h$; $j = 1, 2, \ldots, J_{hi}$; $k = 1, 2, \ldots, K$; $J_{hi} > 9$).

Next, the DMs provide their judgment on the intensity of the internal and external factors for each candidate state using a 1 to 10 scale. Positive scores represent strengths for the internal criteria and opportunities for the external criteria. Negative scores represent weaknesses for the internal criteria and threats for the external criteria. In those situations where the DMs are not sure about their estimates or cannot render judgments with full confidence, triangular or trapezoidal fuzzy numbers could be used in lieu of crisp scores. For example, Tavana et al. (2009) used triangular fuzzy numbers instead of exact judgments when evaluating alternative military bases in a base realignment and closure problem at the Department of Defense. However, using triangular or trapezoidal fuzzy numbers requires additional information about the dispersion of the scores (spreads of fuzzy numbers) and ultimately increasing the total number of judgments required from the DMs. The real-world decision problems can include hundreds of criteria. In order to use triangular fuzzy numbers, each DG has to make two additional judgments for each candidate on each criterion signifying the left and right spreads. In other words, three sets of judgments are required in comparison with one set of judgments required by *Soft SWOT*. As we shall see in the pilot study presented in Sect. 4, each DG has to judge 8 candidates on 169 criteria which in our case generates 1352 estimates. Whereas, for the scores expressed with triangular fuzzy numbers, a total of 2704 additional judgments are required from each DG. Furthermore, in order to use trapezoidal fuzzy numbers, each DG has to assign their scores as intervals, along with uncertainty spreads. The trapezoidal representation demands 5408 judgments by each DG in our pilot study. Therefore, in large problems with many uncertainties, it is not justified to introduce additional fuzziness and increase the number of DMs' judgments. Alternatively, a verbal judgment scale might be used to rate the alternatives where each verbal term is expressed with a fuzzy number. In this case, the DMs have to use predefined linguistic terms and cannot assign their own confidence bounds (left and right spreads of the fuzzy numbers or intervals). Such an approach supposes an equal uncertainty level for each DG when evaluating an alternative on a criterion, which sufficiently restricts the DMs' rights to decide on their own.

We then use a defuzzification method to obtain crisp values from the subjective judgments and estimates provided for the H standards and the M candidate states. These crisp values are then synthesized in a MCDA model to produce an overall performance score for each of the M countries under consideration. Table 1 presents the mathematical notations of a defuzzified decision matrix for a general problem.

Decision theory generally deals with three types of uncertainty: stochastic uncertainty, subjective uncertainty and informational uncertainty. Stochastic uncertainty is treated by probability theory and subjective and informational uncertainties are treated by fuzzy logic theory. Although fuzzy logic and probability theory are similar, they are not identical. Probability refers to the likelihood that something is true while fuzzy logic establishes the degree to which something is true. Probability is not a special case of fuzziness, but leads us to consider the probability of fuzzy events. Dubois and Prade (1993) provide an analysis of correlation between fuzzy sets and probability theory. They argue that the existence of mathematical objects in probability theory does not suggest that fuzziness is reducible to randomness and it is possible to approach fuzzy sets and possibility theory without any probability considerations. Their study emphasizes the interpretation multiplicity of probability and fuzzy set theories and shows that fuzzy set-theoretic operations can be categorized according to their membership in the upper probability, the one-point coverage of a random set, or a likelihood function.

The research on the conjoint application of fuzzy sets and probability theory reports on several studies including marine and offshore safety assessment (Eleye-Datubo et al. 2008), financial modeling (Muzzioli and Reynaerts 2007), information systems (Rolly Intan and Mukaidono 2004), auditing (Friedlob and Schleifer 1999), manufacturing cost estimation (Jahan-Shahi et al. 1999), and water quality management (Benoit 1994).

We use fuzzy logic in a defuzzification process to collapse H sets of standard weights, criteria weights and subjective scores into one set of crisp values for K groups of DMs. Consider discrete fuzzy sets A^m_{hij} represented by the pairs:

$$A^m_{hij} = \{(p^{km}_{hij}, \mu_{A^m_{hij}}(p^{km}_{hij}))\}, \quad \forall p^{km}_{hij} \in P^m_{hij} \tag{3}$$

where:

P^m_{hij} = the set of judgments of the DGs on criterion j in the ith group of standard h given the choice of the mth candidate; ($h = 1, 2, \dots, H$; $i = 1, 2, \dots, I_h$; $j = 1, 2, \dots, J_{hi}$; $m = 1, 2, \dots, M$),

Table 1 The defuzzified decision matrix

h	q_h^k	i_h	w_{hi}^k	j_{hi}	$u_{hij}^k(v_{hij}^k)$	μ_{hij}	EU membership candidate, m						
							1		2		…	M	
1	$\{q_1^k\}$	1	$\{w_{11}^k\}$	1	$\{u_{111}^k\}(\{v_{111}^k\})$	μ_{111}	$\{p_{111}^{k1}\}$	r_{111}^1	$\{p_{111}^{k2}\}$	r_{111}^2	…	$\{p_{111}^{kM}\}$	r_{111}^M
				2	$\{u_{112}^k\}(\{v_{111}^k\})$	μ_{112}	$\{p_{112}^{k1}\}$	r_{112}^1	$\{p_{112}^{k2}\}$	r_{112}^2	…	$\{p_{112}^{kM}\}$	r_{112}^M
				…	…	…	…	…	…	…	…	…	…
				J_{11}	$\{u_{11J_{11}}^k\}(\{v_{111}^k\})$	$\mu_{11J_{11}}$	$\{p_{11J_{11}}^{k1}\}$	$r_{11J_{11}}^1$	$\{p_{11J_{11}}^{k2}\}$	$r_{11J_{11}}^2$	…	$\{p_{11J_{11}}^{kM}\}$	$r_{11J_{11}}^M$
		2	$\{w_{11}^k\}$	1	$\{u_{121}^k\}(\{v_{121}^k\})$	μ_{121}	$\{p_{121}^{k1}\}$	r_{121}^1	$\{p_{121}^{k2}\}$	r_{121}^2	…	$\{p_{121}^{kM}\}$	r_{121}^M
				2	$\{u_{122}^k\}(\{v_{122}^k\})$	μ_{122}	$\{p_{122}^{k1}\}$	r_{122}^1	$\{p_{122}^{k2}\}$	r_{122}^2	…	$\{p_{122}^{kM}\}$	r_{122}^M
				…	…	…	…	…	…	…	…	…	…
				J_{12}	$\{u_{12J_{12}}^k\}(\{v_{111}^k\})$	$\mu_{12J_{12}}$	$\{p_{12J_{12}}^{k1}\}$	$r_{12J_{12}}^1$	$\{p_{12J_{12}}^{k2}\}$	$r_{12J_{12}}^2$	…	$\{p_{12J_{12}}^{kM}\}$	$r_{12J_{12}}^M$
		…	…	…	…	…	…	…	…	…	…	…	…
		I_1	$\{w_{1I_1}^k\}$	1	$\{u_{1I_11}^k\}(\{v_{1I_11}^k\})$	μ_{1I_11}	$\{p_{1I_11}^{k1}\}$	$r_{1I_11}^1$	$\{p_{1I_11}^{k2}\}$	$r_{1I_11}^2$	…	$\{p_{121}^{kM}\}$	$r_{1I_11}^M$
				2	$\{u_{1I_12}^k\}(\{v_{1I_12}^k\})$	μ_{1I_12}	$\{p_{1I_12}^{k1}\}$	$r_{1I_12}^1$	$\{p_{1I_12}^{k2}\}$	$r_{1I_12}^2$	…	$\{p_{1I_12}^{kM}\}$	$r_{1I_12}^M$
				…	…	…	…	…	…	…	…	…	…
				J_{1I1}	$\{u_{1I_1J_{12}}^k\}(\{v_{1I_1J_{12}}^k\})$	$\mu_{1I_1J_{12}}$	$\{p_{12I_1J_{12}}^{k1}\}$	$r_{1I_1J_{12}}^1$	$\{p_{1I_1J_{12}}^{k2}\}$	$r_{1I_1J_{12}}^2$	…	$\{p_{1I_1J_{12}}^{kM}\}$	$r_{1I_1J_{12}}^M$
2	$\{q_2^k\}$	1	$\{w_{21}^k\}$	1	$\{u_{111}^k\}(\{v_{111}^k\})$	μ_{111}	$\{p_{111}^{k1}\}$	r_{111}^1	$\{p_{111}^{k2}\}$	r_{111}^2	…	$\{p_{111}^{kM}\}$	r_{111}^M
				2	$\{u_{112}^k\}(\{v_{112}^k\})$	μ_{112}	$\{p_{112}^{k1}\}$	r_{112}^1	$\{p_{112}^{k2}\}$	r_{112}^2	…	$\{p_{112}^{kM}\}$	r_{112}^M
				…	…	…	…	…	…	…	…	…	…
				J_{11}	$\{u_{11J_{11}}^k\}(\{v_{111}^k\})$	$\mu_{11J_{11}}$	$\{p_{11J_{11}}^{k1}\}$	$r_{11J_{11}}^1$	$\{p_{11J_{11}}^{k2}\}$	$r_{11J_{11}}^2$	…	$\{p_{11J_{11}}^{kM}\}$	$r_{11J_{11}}^M$
		2	$\{w_{22}^k\}$	1	$\{u_{221}^k\}(\{v_{221}^k\})$	μ_{121}	$\{p_{121}^{k1}\}$	r_{121}^1	$\{p_{121}^{k2}\}$	r_{121}^2	…	$\{p_{121}^{kM}\}$	r_{121}^M
				2	$\{u_{222}^k\}(\{v_{222}^k\})$	μ_{122}	$\{p_{122}^{k1}\}$	r_{122}^1	$\{p_{122}^{k2}\}$	r_{122}^2	…	$\{p_{122}^{kM}\}$	r_{122}^M
				…	…	…	…	…	…	…	…	…	…
				J_{22}	$\{u_{12J_{22}}^k\}(\{v_{11I_{22}}^k\})$	$\mu_{12J_{12}}$	$\{p_{12J_{12}}^{k1}\}$	$r_{12J_{12}}^1$	$\{p_{12J_{12}}^{k2}\}$	$r_{12J_{12}}^2$	…	$\{p_{12J_{12}}^{kM}\}$	$r_{12J_{12}}^M$
		…	…	…	…	…	…	…	…	…	…	…	…

Table 1 (*Continued*)

h	q_h^k	i_h	w_{hi}^k	j_{hi}	$u_{hij}^k(v_{hij}^k)$	μ_{hij}	EU membership candidate, m						
							1		2		...	M	
		I_2	$\{w_{2I_2}^k\}$	1	$\{u_{2I_21}^k\}(\{v_{2I_21}^k\})$	μ_{2I_21}	$\{p_{2I_21}^{k1}\}$	$r_{2I_21}^1$	$\{p_{2I_21}^{k2}\}$	$r_{2I_21}^2$	...	$\{p_{2I_21}^{kM}\}$	$r_{2I_21}^M$
				2	$\{u_{2I_22}^k\}(\{v_{2I_22}^k\})$	μ_{2I_22}	$\{p_{2I_22}^{k1}\}$	$r_{2I_22}^1$	$\{p_{2I_22}^{k2}\}$	$r_{2I_22}^2$	...	$\{p_{2I_22}^{kM}\}$	$r_{2I_22}^M$
				...	...	...	...	...	...	...	...	...	...
				J_{2I_2}	$\{u_{2I_2J_{12}}^k\}(\{v_{2I_2J_{12}}^k\})$	$\mu_{2I_2J_{I22}}$	$\{p_{2I_2J_{22}}^{k1}\}$	$r_{2I_2J_{22}}^1$	$\{p_{2I_2J_{22}}^{k2}\}$	$r_{2I_2J_{22}}^2$	...	$\{p_{2I_2J_{22}}^{kM}\}$	$r_{2I_2J_{22}}^M$
...	...	...	...	...	...	...	...	...	...	...	...	...	...
H	$\{q_H^k\}$	1	$\{w_{H1}^k\}$	1	$\{u_{111}^k\}(\{v_{111}^k\})$	μ_{111}	$\{p_{111}^{k1}\}$	r_{111}^1	$\{p_{111}^{k2}\}$	r_{111}^2	...	$\{p_{111}^{kM}\}$	r_{111}^M
				2	$\{u_{112}^k\}(\{v_{112}^k\})$	μ_{112}	$\{p_{112}^{k1}\}$	r_{112}^1	$\{p_{112}^{k2}\}$	r_{112}^2	...	$\{p_{112}^{kM}\}$	r_{112}^M
				...	...	...	...	...	...	...	...	...	...
				J_{H1}	$\{u_{11J_{11}}^k\}(\{v_{1J_{11}}^k\})$	$\mu_{11J_{11}}$	$\{p_{11J_{11}}^{k1}\}$	$r_{11J_{11}}^1$	$\{p_{11J_{11}}^{k2}\}$	$r_{11J_{11}}^2$	...	$\{p_{11J_{11}}^{kM}\}$	$r_{11J_{11}}^M$
		2	$\{w_{H2}^k\}$	1	$\{u_{H21}^k\}(\{v_{H21}^k\})$	μ_{H21}	$\{p_{121}^{k1}\}$	r_{121}^1	$\{p_{121}^{k2}\}$	r_{121}^2	...	$\{p_{121}^{kM}\}$	r_{121}^M
				2	$\{u_{H22}^k\}(\{v_{H22}^k\})$	μ_{H22}	$\{p_{122}^{k1}\}$	r_{122}^1	$\{p_{122}^{k2}\}$	r_{122}^2	...	$\{p_{122}^{kM}\}$	r_{122}^M
				...	...	...	...	...	...	...	...	...	...
				J_{H2}	$\{u_{H2J_{22}}^k\}(\{v_{H1I_{H2}}^k\})$	$\mu_{H2J_{H2}}$	$\{p_{12J_{12}}^{k1}\}$	$r_{12J_{12}}^1$	$\{p_{12J_{12}}^{k2}\}$	$r_{12J_{12}}^2$	...	$\{p_{12J_{12}}^{kM}\}$	$r_{12J_{12}}^M$
		...	...	...	...	...	...	...	...	...	...	...	...
		I_H	$\{w_{HI_H}^k\}$	1	$\{u_{HI_{H1}1}^k\}(\{v_{HI_H1}^k\})$	μ_{HI_H1}	$\{p_{HI_H1}^{k1}\}$	$r_{HI_H1}^1$	$\{p_{HI_H1}^{k2}\}$	$r_{HI_H1}^2$	...	$\{p_{HI_H1}^{kM}\}$	$r_{HI_H1}^M$
				2	$\{u_{HI_H2}^k\}(\{v_{HI_H2}^k\})$	μ_{HI_H2}	$\{p_{HI_H2}^{k1}\}$	$r_{HI_H2}^1$	$\{p_{HI_H2}^{k2}\}$	$r_{HI_H2}^2$	...	$\{p_{HI_H2}^{kM}\}$	$r_{HI_H2}^M$
				...	...	...	...	...	...	...	...	...	...
				J_{HI1}	$\{u_{HI_HJ_{12}}^k\}(\{v_{HI_HJ_{12}}^k\})$	$\mu_{HI_HJ_{I22}}$	$\{p_{H2I_HJ_{22}}^{k1}\}$	$r_{HI_HJ_{22}}^1$	$\{p_{HI_HJ_{22}}^{k2}\}$	$r_{HI_HJ_{22}}^2$	...	$\{p_{HI_HJ_{22}}^{kM}\}$	$r_{HI_HJ_{22}}^M$
							$U^1(V^1)$		$U^2(V^2)$		...	$U^M(V^M)$	

p_{hij}^{km} = the judgment given by the kth DG on criterion j in the ith group of standard h given the choice of the mth candidate; ($h = 1, 2, \ldots, H$; $i = 1, 2, \ldots, I_h$; $j = 1, 2, \ldots, J_{hi}$; $k = 1, 2, \ldots, K$; $m = 1, 2, \ldots, M$),

$\mu_{A_{hij}^m}(p_{hij}^{km})$ = the membership grade of judgment of the kth DG; ($h = 1, 2, \ldots, H$; $i = 1, 2, \ldots, I_h$; $j = 1, 2, \ldots, J_{hi}$; $k = 1, 2, \ldots, K$; $m = 1, 2, \ldots, M$).

Defuzzification is the translation of linguistic or fuzzy values into numerical, scalar, and crisp representations. The process of condensing the information captured by fuzzy sets into numerical values is similar to that of transformation of uncertainty-based concepts into certainty-based concepts. Intuitively speaking, the defuzzification process in *Soft SWOT* is similar to an averaging procedure. Special defuzzification methods can be used to increase the numerical efficiency and transparency of the computations. Many defuzzification techniques have been proposed in the literature. The most commonly used method is the center of gravity (COG). Other methods include: random choice of maximum, first of maximum, last of maximum, middle of maximum, mean of maxima, basic defuzzification distributions, generalized level set defuzzification, indexed center of gravity, semi-linear defuzzification, fuzzy mean, weighted fuzzy mean, quality method, extended quality method, center of area, extended center of area, constraint decision defuzzification, and fuzzy clustering defuzzification. Roychowdhury and Pedrycz (2001) and Dubois and Prade (2000) provide excellent reviews of the most commonly used defuzzification methods.

The literature reports on several aggregation functions (Ali and Zhang 2001; Roychowdhury and Pedrycz 2001; Runkler 1996, Van Leekwijk and Kerre 1999). The selection of a specific aggregation function must be based on the problem characteristics and model requirements. While the selection of an aggregation operation is context dependent, it is recommended to consider the criteria suggested by Klir and Yuan (1995). We use COG, also referred to as the center of area method, in *Soft SWOT*. This method is highly popular and is often used as a standard defuzzification method. COG calculates the centroid of a possibility distribution function using (4) for continuous cases and (5) for discontinuous cases:

$$\mathrm{COG}(N) = \frac{\int_{-\infty}^{\infty} x\mu N(x)dx}{\int_{-\infty}^{\infty} \mu N(x)dx} \tag{4}$$

$$\mathrm{COG}(N) = \frac{\sum_{k=1}^{K} x_k \mu(x_k)}{\sum_{k=1}^{K} \mu(x_k)} \tag{5}$$

The procedure for converting the fuzzy numbers into a set of crisp values in *Soft SWOT* can be divided into the following three steps:

Step 1 Evaluation of the membership functions related to the subjective scores for the internal ($\mu_{u_{hij}}^k$) and external criteria ($\mu_{v_{hij}}^k$):

$$\begin{aligned} \mu_{u_{hij}}^k &= q_h^k \cdot w_{u_{hij}}^k \cdot u_{hij}^k \\ \mu_{v_{hij}}^k &= q_h^k \cdot w_{v_{hij}}^k \cdot v_{hij}^k \end{aligned} \tag{6}$$

We should note that, even though we assume an equal voting power for all the DGs, alternatively, different voting weights could be assigned to different DGs in the model.

Step 2 Calculation of the overall weighted scores of the strengths ($r_{hij}^m(S)$), weaknesses ($r_{hij}^m(W)$), opportunities ($r_{hij}^m(O)$) and threats ($r_{hij}^m(T)$) for the M candidate countries as the summed product of the scores on their grades of membership:

$$
\begin{aligned}
r^m_{hij}(S) &= \sum_{k=1}^{K} \mu^k_{U_{hij}} p^{km}_{hij}(S) \\
r^m_{hij}(W) &= -\sum_{k=1}^{K} \mu^k_{U_{hij}} p^{km}_{hij}(W) \\
r^m_{hij}(O) &= \sum_{k=1}^{K} \mu^k_{V_{hij}} p^{km}_{hij}(O) \\
r^m_{hij}(T) &= -\sum_{k=1}^{K} \mu^k_{V_{hij}} p^{km}_{hij}(T)
\end{aligned}
\tag{7}
$$

where:

$p^{km}_{hij}(S) =$ the intensity of the jth strength for the ith group of the hth standard given the choice of the mth candidate state by DG k; $(h = 1, 2, \dots, H^S; i = 1, 2, \dots, I^S_h; j = 1, 2, \dots, J^S_{hi}; k = 1, 2, \dots, K; m = 1, 2, \dots, M)$,

$p^{km}_{hij}(W) =$ the intensity of the jth weakness for the ith group of the hth standard given the choice of the mth candidate state by DG k; $(h = 1, 2, \dots, H^W; i = 1, 2, \dots, I^W_h; j = 1, 2, \dots, J^W_{hi}; k = 1, 2, \dots, K; m = 1, 2, \dots, M)$,

$p^{km}_{hij}(O) =$ the intensity of the jth opportunity for the ith group of the hth standard given the choice of the mth candidate state by DG k; $(h = 1, 2, \dots, H^O; i = 1, 2, \dots, I^O_h; j = 1, 2, \dots, J^O_{hi}; k = 1, 2, \dots, K; m = 1, 2, \dots, M)$,

$p^{km}_{hij}(T) =$ the intensity of the jth threat for the ith group of the hth standard given the choice of the mth candidate state by DG k; $(h = 1, 2, \dots, H^T; i = 1, 2, \dots, I^T_h; j = 1, 2, \dots, J^T_{hi}; k = 1, 2, \dots, K; m = 1, 2, \dots, M)$.

and:

$H^S(H^W, H^O, H^T) =$ the number of standards in the cluster of strengths (weaknesses, opportunities, threats),

$I^S_h(I^W_h, I^O_h, I^T_h) =$ the number of criteria groups in the hth standard of strengths (weaknesses, opportunities, threats),

$J^S_{hi}(J^W_{hi}, J^O_{hi}, J^T_{hi}) =$ the number of criteria in the ith group of the hth standard of strengths (weaknesses, opportunities, threats).

Step 3 On the final step of the defuzzification process, we divide the overall weighted scores of the internal and external factors by their summed membership grades. These calculations result in M vectors of non-fuzzy values characterizing strengths, weaknesses, opportunities and threats for M countries:

$$
\begin{aligned}
\mathrm{COG}(S^m_{hij}) &= \frac{r^m_{hij}(S)}{\mu_{u_{hij}}} \\
\mathrm{COG}(W^m_{hij}) &= \frac{r^m_{hij}(W)}{\mu_{u_{hij}}} \\
\mathrm{COG}(O^m_{hij}) &= \frac{r^m_{hij}(O)}{\mu_{v_{hij}}} \\
\mathrm{COG}(T^m_{hij}) &= \frac{r^m_{hij}(T)}{\mu_{v_{hij}}}
\end{aligned}
\tag{8}
$$

where $\mu_{u_{hij}}$ and $\mu_{v_{hij}}$ define the membership functions for the internal and external criteria with aggregated results for all DGs:

$$
\begin{aligned}
\mu_{u_{hij}} &= \sum_{k=1}^{K} \mu_{u_{hij}}^{k} \\
\mu_{v_{hij}} &= \sum_{k=1}^{K} \mu_{v_{hij}}^{k}
\end{aligned}
\tag{9}
$$

Next, we find the defuzzified value of the importance weights of the strengths, weaknesses, opportunities and threats, as well as the total defuzzified values of the internal (U^m) and external (V^m) criteria for all countries under consideration:

$$
\begin{aligned}
S^m &= \sum_{h=1}^{H^S} \sum_{i=1}^{I_h^S} \sum_{j=1}^{J_{hi}^S} \mathrm{COG}(S_{hij}^m) \\
W^m &= \sum_{h=1}^{H^W} \sum_{i=1}^{I_h^W} \sum_{j=1}^{J_{hi}^W} \mathrm{COG}(W_{hij}^m) \\
O^m &= \sum_{h=1}^{H^O} \sum_{i=1}^{I_h^O} \sum_{j=1}^{J_{hi}^O} \mathrm{COG}(O_{hij}^m) \\
T^m &= \sum_{h=1}^{H^T} \sum_{i=1}^{I_h^T} \sum_{j=1}^{J_{hi}^T} \mathrm{COG}(T_{hij}^m)
\end{aligned}
\tag{10}
$$

$$
\begin{aligned}
U^m &= S^m - W^m \\
V^m &= O^m - T^m
\end{aligned}
\tag{11}
$$

Finally, we revise the importance weight of the strengths, weaknesses, opportunities and threats determined through the defuzzification process with the entropy concept. Each internal or external criterion is an information source; therefore, the more information each criterion reveals, the more relevant it is to the decision analysis. The level of entropy $e(P)$ as a measure of fuzziness, indicates the variance of the assigned preference relation. The concept of entropy originated in physics and statistics and has become increasingly popular in computer science and information theory. Shannon (1948) has defined the entropy of a probability distribution where the total probability for all elements must add up to 1. However, De Luca and Termini (1972) show that this restriction is unnecessary. They define a fuzzy entropy formula on a finite universal set $X = \{x_1, \ldots, x_n\}$ as:

$$
e_{LT}(A) = -\beta \sum_{i=1}^{n} [\mu_A(x_i) \ln \mu_A(x_i) + (1 - \mu_A(x_i)) \ln(1 - \mu_A(x_i))] \tag{12}
$$

where $\beta > 0$ is a normalization constant, ln is the natural logarithm and $\mu_A(x_i)$ is the membership function for each preference intensity.

The larger the difference between the subjective weights and scores of criteria The more different the subjective weights and scores of criteria are, the larger is the contrast intensity of the strength, weakness, opportunity or threat, and the greater is the amount of information transmitted by that criteria. Assuming that the vector $P^m_{hji} = \{P^{km}_{hji}\}$ characterizes the set of weighted scores in terms of the jth factor for the ith group of standard h (hijth factor) given the choice of the mth candidate state, the entropy measure of the hijth factor is:

$$e(A^m_{hij}) = -\beta \sum_{k=1}^{K} [\mu(p^{km}_{hij}) \ln \mu(p^{km}_{hij}) + (1-\mu(p^{km}_{hij})) \ln(1-\mu(p^{km}_{hij}))] \tag{13}$$

where $0 \le \mu(p^{km}_{hij}) \le 1$ and $e(A^m_{hij}) \ge 0$. The smaller $e(A^m_{hij})$ is, the more information the hijth criterion transmits, and the larger $e(A^m_{hij})$ is, the less information it transmits. Taking $\beta = 0$, the entropy measure of the hijth strength, weakness, opportunity or threat is:

$$\begin{aligned}
e(A^m_{hij}(S)) &= -\sum_{k=1}^{K} [\mu^k_{hij}(S) \ln \mu^k_{hij}(S) + (1-\mu^k_{hij}(S)) \ln(1-\mu^k_{hij}(S))] \\
e(A^m_{hij}(W)) &= -\sum_{k=1}^{K} [\mu^k_{hij}(W) \ln \mu^k_{hij}(W) + (1-\mu^k_{hij}(W)) \ln(1-\mu^k_{hij}(W))] \\
e(A^m_{hij}(O)) &= -\sum_{k=1}^{K} [\mu^k_{hij}(O) \ln \mu^k_{hij}(O) + (1-\mu^k_{hij}(O)) \ln(1-\mu^k_{hij}(O))] \\
e(A^m_{hij}(T)) &= -\sum_{k=1}^{K} [\mu^k_{hij}(T) \ln \mu^k_{hij}(T) + (1-\mu^k_{hij}(T)) \ln(1-\mu^k_{hij}(T))]
\end{aligned} \tag{14}$$

The total entropies of strengths (E^m_S), weaknesses (E^m_W), opportunities (E^m_O) and threats (E^m_T) for candidate m are defined as $E^m_S = \sum_{h=1}^{H^S} \sum_{i=1}^{I^S_h} \sum_{j=1}^{J^S_{hi}} e(A^m_{hij}(S))$, $E^m_W = \sum_{h=1}^{H^W} \sum_{i=1}^{I^W_h} \sum_{j=1}^{J^W_{hi}} e(A^m_{hij}(W))$, $E^m_O = \sum_{h=1}^{H^O} \sum_{i=1}^{I^O_h} \sum_{j=1}^{J^O_{hi}} e(A^m_{hij}(O))$ and $E^m_T = \sum_{h=1}^{H^T} \times \sum_{i=1}^{I^T_h} \sum_{j=1}^{J^T_{hi}} e(A^m_{hij}(T))$, respectively.

In order to calculate the overall entropy in the groups of internal and external criteria, we add up the entropies of strengths and weaknesses, and the entropies of opportunities and threats:

$$\begin{aligned}
E^m_U &= E^m_S + E^m_W \\
E^m_V &= E^m_O + E^m_T
\end{aligned} \tag{15}$$

Using the Euclidean measure suggested by Zeleny (1982), *Soft SWOT* synthesizes the results by determining the ideal internal and external criteria values. The ideal overbalance of strengths and weaknesses (U^*) is the highest defuzzified importance weight of the internal criteria among the set U^m and the ideal overbalance between opportunities and threats (V^*) is the lowest defuzzified importance weight of the external criteria among the set V^m. We then find the Euclidean distance of each candidate state from the ideal state. The Euclidean distance is the sum of the quadratic root of squared differences between the ideal and the mth indices of the internal and external characteristics. To formulate *Soft SWOT* algebraically, let us assume:

D_U^m = total Euclidean distance from the ideal internal characteristic for the mth candidate state; $(m = 1, 2, \ldots, M)$,
D_V^m = total Euclidean distance from the ideal external characteristic for the mth candidate state; $(m = 1, 2, \ldots, M)$,
D^m = overall Euclidean distance of the mth candidate state; $(m = 1, 2, \ldots, M)$,
$U^*(V^*)$ = the ideal defuzzified value of the internal (external) criteria,
$E_U^*(E_V^*)$ = the entropy of the ideal internal (external) characteristic,
$DE_U^m(DE_V^m)$ = the Euclidean distance from the entropy of the ideal internal (external) characteristic for the mth candidate state; $(m = 1, 2, \ldots, M)$,
D^m = overall Euclidean distance of the entropy for the mth candidate state; $(m = 1, 2, \ldots, M)$.

$$D^m = \sqrt{(D_U^m)^2 + (D_V^m)^2} \tag{16}$$

$$DE^m = \sqrt{(DE_U^m)^2 + (DE_V^m)^2} \tag{17}$$

$$\begin{aligned} U^* &= \mathrm{Max}\{U^m\} \\ V^* &= \mathrm{Max}\{V^m\} \end{aligned} \tag{18}$$

$$\begin{aligned} E_U^* &= \mathrm{Min}\{E_U^m\} \\ E_V^* &= \mathrm{Min}\{E_V^m\} \end{aligned} \tag{19}$$

where:

$$\begin{aligned} D_U^m &= U^* - U^m \\ D_V^m &= V^m - V^* \\ DE_U^m &= E_U^m - E_U^* \\ DE_V^m &= E_V^m - E_V^* \end{aligned}$$

Candidate states with smaller D^m are closer to the ideal state and are preferred to candidate states with larger D^m which are further away from the ideal state. Next, we plot the candidate state on a graph where the x-axis is represented by the overbalance between strengths and weaknesses (U) and the y-axis is represented by the overbalance between opportunities and threats (V). The position of the point corresponding to candidate state m has Cartesian coordinates (U^m, V^m) on the graph.

Soft SWOT also considers the overall Euclidean distance of the entropy for the mth candidate state (DE^m). States with smaller Euclidean distance of entropy (minimal measure of uncertainty) are preferred to states with larger Euclidean distance of entropy (higher measure of uncertainty). The resulting SWOT graph contains four quadrants: *exploitation*, *challenge*, *discretion*, and *desperation*:

- *Exploitation quadrant*: In this quadrant, the EU membership candidate state has a positive overbalance of strengths over weaknesses and opportunities over threats. This area represents the greatest possible advantage for the EU. States falling into this quadrant should be considered seriously.

- *Challenge quadrant*: In this quadrant, the state has prevalence of strengths over weaknesses within internal scope. However, the condition of its external issues is unsatisfactory for the EU acceptance since threats exceed opportunities. A state falling in this quadrant should utilize their strengths to reduce their vulnerability to external threats. This quadrant requires full use of the EU's abilities and resources.
- *Discretion quadrant*: In this quadrant, the state has a high level of the EU integration and cooperation which is reflected by a positive overbalance of opportunities over threats. Strong weaknesses in this quadrant point to a negative internal situation. An improvement plan must be developed for reducing weaknesses and increasing strengths where possible. This quadrant represents the area where the EU has freedom or power to act or judge on its own.
- *Desperation quadrant*: This is the most risky quadrant. States falling into this quadrant have negative results with respect to the internal and external evaluation criteria. These should be considered as a last resort since they are characterized by high weaknesses and threats.

Once the model is developed, sensitivity analyses can be performed to determine the impact on the ranking of projects for changes in various model assumptions. Some sensitivity analyses that are usually of interest are on the weights and scores. The weights representing the relative importance of the standard and the internal and external evaluation criteria are occasionally a point for discussion among the various DMs. In addition, scores which reflect the degree of performance of an uncertain criterion are sometimes a matter of contention.

4 The pilot study

The European Union (EU) is a geo-political and economic community covering a large portion of the European continent. It was founded upon numerous treaties and has undergone several expansions taking it from its six founder states to twenty-seven member states. To join the EU, a country must go through an extensive screening process and conform to a series of fairly demanding criteria established by the European Council in Copenhagen. The Copenhagen criteria require a stable democracy which respects human rights and the rule of law (political standard); a functioning market economy capable of competition within the EU (economic standard); and the acceptance of the obligations of membership, including EU law (community standard). Each standard has several criteria and each criterion is comprised of multiple sub-criteria. The screening process is intended to determine how well a candidate state is prepared to join the EU. The Commission issues a report to the Council on the screening of the political, economic and community standards. This leads to open negotiations or a requirement that the candidate state must first meet these benchmarks. It is the responsibility of the European Council to determine whether the candidate state has fulfilled the Copenhagen criteria. The current selection process is ambiguous and unstructured. One problem is that the accession criteria adopted at the Copenhagen summit of EU leaders in 1993 are not all quantitative and precise.

We illustrate the application of *Soft SWOT* in a pilot study conducted for the European Commission to screen candidates for membership in the EU. The pilot study involved 30 graduate students from the University of Paderborn in Germany. Six groups of five DMs with equal voting power were formed. Each group included at least one economist, one political scientist, one social scientist, and one business student. Decisive thresholds and benchmarks were used to assess the candidate states and their progress towards fulfilling the accession requirements. Currently, accession negotiations are underway with eight countries identified by the European Commission as either "candidate" or "potential candidate" states. Candidate states which include Croatia, Macedonia and Turkey have already applied for membership and are involved in current negotiations. Potential candidate states which include Albania, Bosnia and Herzegovina, Kosovo, Montenegro and Serbia are interested in membership and are promised a prospect of EU membership. To join the EU, each country must conform to a series of criteria established by the European Council in Copenhagen. These criteria are combined in three large groups ($L = 3$) of political, economic and community standards. In our study, the political, economic and community standards have been considered to be of equal importance. Hence, we could operate with the three respective hierarchies independently where the weight of the top-element is 1. Altogether, 38 criteria divided into 169 sub-criteria were considered in this study.

The process began with an initial meeting of the six groups of DMs who used Expert Choice (Expert Choice 2006) to weight the importance of each standard. The DMs worked within and between groups to classify 169 Copenhagen sub-criteria into internal criteria which were believed somewhat controllable by the candidate states, and external criteria which were less controllable depending on the environmental conditions and forces. Internal and external criteria were further classified into favorable and unfavorable categories. Favorable internal criteria were characterized as "strengths" and favorable external criteria as "opportunities." Unfavorable internal criteria were characterized as "weaknesses" and unfavorable external criteria as "threats." Such classification allows for the simultaneous inclusion of favorable and unfavorable internal and external criteria in an integrated model. Tables 2 and 3 present selected Copenhagen criteria along with their associated classification. A complete listing of the criteria and their descriptions are provided in the official progress reports published by the European Commission and presented in Appendix. The criteria used in this study were identical to those listed in the progress reports.

Next, the DGs held separate meetings to weight 38 criteria and 169 sub-criteria. Theoretically, $N(N-1)/2$ pairwise comparisons are needed for each set of N criteria. However, the psychological experiments conducted by Miller (1956) have shown that an individual cannot simultaneously compare more than seven criteria (± 2) without being confused. In agreement with Miller (1956), we used the AHP and Expert Choice (Expert Choice 2006) for sets with nine or less criteria. The DMs recorded their consistency ratios and made sure it was below 0.10 as suggested by Saaty (1977, 1980). The Copenhagen criteria classification is strictly defined by the European Council. Therefore, we did not break down the large groups of standards to operationalize the AHP pairwise comparison process. Instead, we used the scoring and normalization process described in the previous section for clusters with more than nine criteria. Altogether, each decision-making group made 365 judgments. However, we were able to effectively manage the pairwise comparison process by dividing the judgments into manageable groups and assigning them to those with relevant background and expertise in economics, political science, social science, and business.

Next, the six DGs identified their performance score of each internal and external criterion for each of the eight candidate states using the data provided in the official progress report (see Appendix). They used a -10 to -1 scale to assign a score to those internal and external criteria which had a negative performance and used a $+1$ to $+10$ scale to assign a score to those internal and external criteria which had a positive performance. The scoring of the criteria resulted in the identification of the strengths, weaknesses, opportunities and threats. Higher scores were preferred to lower scores for both internal and external criteria.

The importance weight of the six standards along with the importance weight of the criteria and sub-criteria and the performance scores were all collapsed using discrete fuzzy

Table 2 The selected internal evaluation criteria

Standard	Criterion	Sub-criterion
Political	Guarantee of democracy and rule of law	Parliament Public administration Anti-corruption policy ...
	Guarantee of human right and respect for and protection of minorities	Civil and political rights Economic and social rights Minority rights, cultural rights and protection of minorities ...
	...	...
Economic	Functioning market economy	Legal system Financial sector development ...
	Ability to cope with the pressure of competition and the market forces at work inside the Union	Existence of a functioning market economy Human and physical capital Sector and enterprise structure State influence on competitiveness ...
	...	...
Acceptance of the Community *Acquis*	Free movement of goods	Administrative capacity Metrology ...
	Freedom of movement for workers	Access to the labour market Coordination of social security systems ...
	Company law	Company law Corporate accounting Auditing ...
	Competition policy	State aid/state aid enforcement Fiscal aid ...
	Financial services	Banks and financial conglomerates Financial market infrastructure ...
	Agriculture	Rural development Organic farming ...
	Food safety, veterinary and phytosanitary policy	Veterinary checks Phytosanitary issues Animal welfare ...
	...	...

Table 3 The selected external evaluation criteria

Standard	Criterion	Sub-criterion
Political	Guarantee of human right and respect for and protection of minorities	Observance of international human rights law . . .
	Regional issues and international obligations	Dayton/Paris and Erdut peace agreements Bilateral relations with other enlargement countries and neighboring member states . . .
	. . .	. . .
Economic	Functioning market economy	Market entry and exit . . .
	Ability to cope with the pressure of competition and the market forces at work inside the Union	Economic integration with the EU . . .
	. . .	. . .
Acceptance of the Community *Acquis*	Freedom of movement for workers	Participation in the EURES network European health insurance card . . .
	Free movement of capital	Capital movements AML directives/standards of the financial action task force . . .
	Agriculture	Integrated administration and control system (IACS)/Land parcel identification system (LPIS) Common market organizations . . .
	Energy	Gas market Energy community treaty Nuclear safety and radiation protection . . .
	Trans-European networks	Development of the trans-European networks Conformity with TEN guidelines . . .
	Justice, freedom and security	Schengen and external borders Police cooperation Organized crime/terrorism . . .
	Science and research	EC framework programmes Integration into the European research area . . .
	. . .	. . .

Table 4 A membership grade example

A. The criterion weights for *guarantee of democracy and rule of law*

Standard	Criterion	Criterion weights					
		DG 1	DG 2	DG 3	DG 4	DG 5	DG 6
Political	Guarantee of democracy and rule of law	0.50	0.40	0.40	0.45	0.40	0.38

B. The sub-criteria weights for *guarantee of democracy and rule of law*

Criterion	Sub-criteria	Sub-criteria weights					
		DG 1	DG 2	DG 3	DG 4	DG 5	DG 6
Guarantee of democracy and rule of law	Parliament	0.30	0.20	0.15	0.20	0.15	0.23
	Public administration	0.20	0.25	0.20	0.20	0.20	0.22
	Anti-corruption policy	0.10	0.10	0.10	0.10	0.10	0.13
	...	...	...	...	...	...	...

C. The sub-criteria scores for Albania

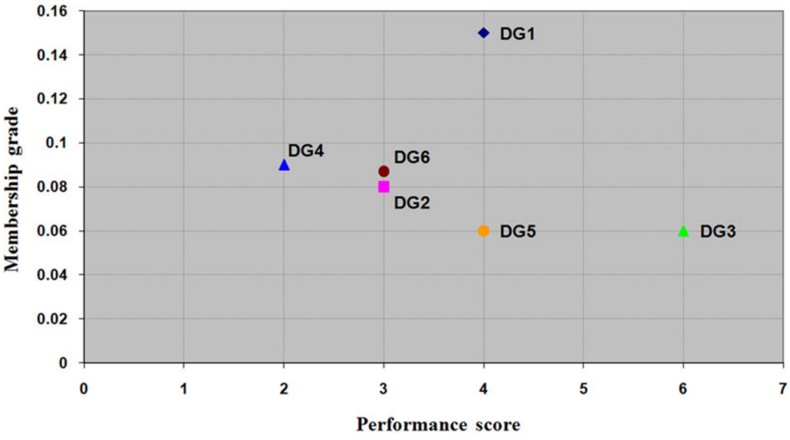
Criterion	Sub-criteria	Scores for Albania					
		DG 1	DG 2	DG 3	DG 4	DG 5	DG 6
Guarantee of democracy and rule of law	Parliament	4	3	6	2	4	3
	Public administration	6	4	3	3	2	4
	Anti-corruption policy	4	4	2	2	3	4
	...	...	...	...	...	...	...

D. The sub-criteria weights for Albania

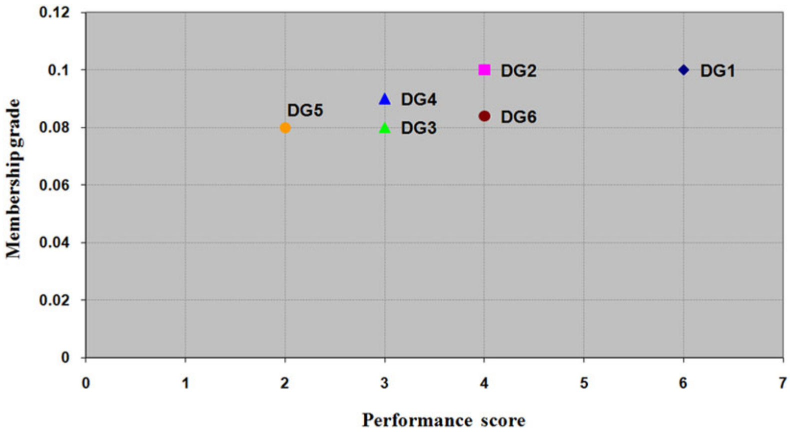
Criterion	Sub-criteria	Grades of membership					
		DG 1	DG 2	DG 3	DG 4	DG 5	DG 6
Guarantee of democracy and rule of law	Parliament	0.15	0.08	0.06	0.09	0.06	0.087
	Public administration	0.10	0.10	0.08	0.09	0.08	0.084
	Anti-corruption policy	0.05	0.04	0.04	0.045	0.04	0.05
	...	...	...	...	...	...	...

sets with a [0, 1] membership function range. The fuzzy sets are composed of criteria and sub-criteria where their intensities are assigned by the DMs and their grades of membership are obtained from (5) and (6). Let us exemplify this idea through a set of sample judgments provided by the DMs from the six DGs for the sub-criteria *Parliament*, *public administration* and *anti-corruption* for Albania. These sub-criteria belong to the *guarantee of democracy and rule of law* criterion and *political* standard. The results are presented in Table 4.

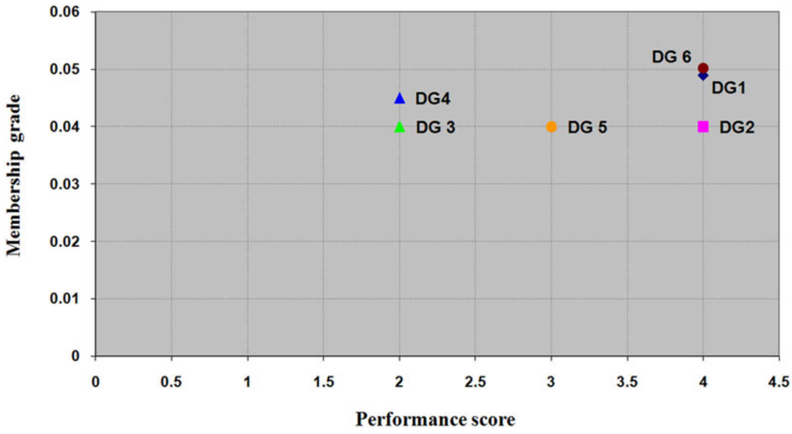
Table 4A shows the importance weights of the guarantee *of democracy and rule of law* criteria group provided by the DMs from the six DGs. Table 4B shows the sub-criteria weights for *Parliament*, *public administration* and *anti-corruption policy*. Table 4C shows the sub-criteria scores provided by the DMs from the six DGs for Albania. The mem-



(a)



(b)



(c)

Fig. 1 The fuzzy set results of judgments for Albania on criterion (**a**) Parliament, (**b**) public administration, (**c**) anti-corruption policy

Table 5 A tabular representation of the opportunities/threats and strengths/weaknesses

Country	Croatia	Turkey	Macedonia	Serbia	Montenegro	Albania	B&H	Kosovo
Strengths	252.54	206.43	156.9	122.55	112.73	116.43	78.18	124.81
Weaknesses	42.57	65.28	97.31	123.3	132.17	137.7	213.69	249.76
Opportunities	288.26	200.43	183.4	132.02	102.46	115.69	120.92	67.05
Threats	48.09	82.73	105.13	132.42	133.11	162.91	152.03	211.62

bership grades for the *Parliament*, *public administration* and *anti-corruption policy* sub-criteria, shown in Table 4D, were calculated using (6). A graphical representation of the results for this example is presented in Fig. 1. The fuzzy sets for the three sub-criteria are presented in Figs. 1A, 1B and 1C. The fuzzy judgments of the DM groups for Albania on the three sub-criteria are presented in Fig. 1A, 1B and 1C. For example, the fuzzy set of the DGs' judgments for Albania on the criterion *Parliament*, *Public Administration* and *Anti-corruption Policy* is depicted on Fig. 1A and can be formulated as $\tilde{A}^{Albania}_{Parlament} = \{2/0.09, 3/0.08, 3/0.087, 4/0.06, 4/0.15, 6/0.06\}$. The set is composed of six pairs that represent the scores for the DGs and their corresponding membership grades calculated using (6). Each pair corresponds to a point on the graph. Intuitively, such a set can be interpreted as the vague score of the country on a criterion treated using fuzzy sets. The aim is to integrate the discrete elements of the scores, defuzzify them and extract additional characteristics assisting in the decision making process (see Dubois and Prade 2000 for further notations on fuzzy sets).

After computations within each hierarchy, the outcomes were integrated into the SWOT groups for each candidate using (10) and (11) to obtain the total values, as well as the strengths/weaknesses and opportunities/threats overbalances. Table 5 presents the opportunities/threats and strengths/weaknesses of the eight candidate states. As it is shown in this table, Croatia has the highest opportunities and strengths and the lowest threats and weaknesses. Contrary to Croatia, Kosovo has low opportunities and strengths and the highest threats and weaknesses. This information is also depicted in the radar diagram presented in Fig. 2 where Croatia is shown in green and Kosovo in red. Similarly, other candidate states could be analyzed through this table and figure.

Table 6 presents the Euclidean distances (D^m) of the eight candidate states from the ideal state using (16). Croatia with a Euclidean distance of 0.00 was closest to the ideal state followed by Turkey and Macedonia. Kosovo was judged farthest away from the ideal state by our DMs.

The entropy was calculated next to evaluate the level of uncertainty in the DMs' estimations. $E^*_U = 10.494$ and $E^*_V = 8.092$ were the ideal entropies of the internal and external characteristics for the ideal state. We then used (17) to calculate the Euclidean distances of the entropies (DE^m) for the eight candidate states from the ideal state. The entropies of the strengths (E^m_S), weaknesses (E^m_W), opportunities (E^m_O) and threats (E^m_T), and the Euclidean distances of the entropies (DE^m) are shown in Table 7. Croatia, Turkey and Macedonia had the largest entropies indicating the disagreement among the DMs for these candidate states while Montenegro, Serbia and Kosovo had the smallest entropies indicating the agreement among the DMs.

Next, we present the final results in Fig. 3. Each bubble in this figure represents a candidate state and the size of the bubble is directly proportional to its entropy level. Croatia has the maximum overbalance of strengths and weaknesses, as well as, opportunities and

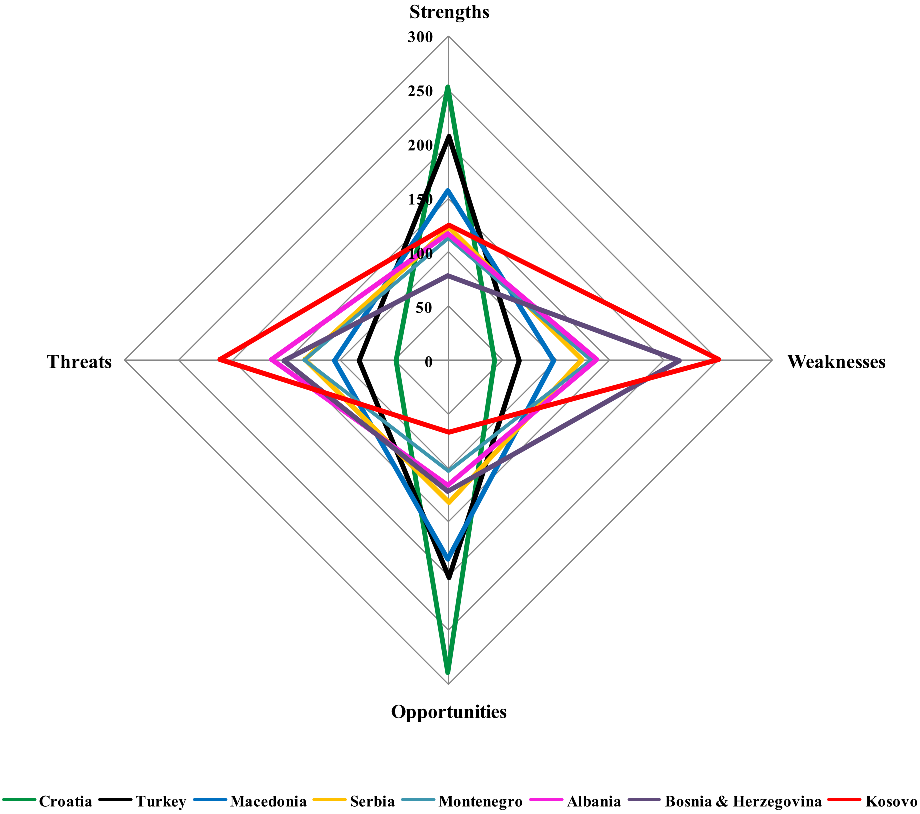


Fig. 2 A graphical representation of the opportunities/threats and strengths/weaknesses

Table 6 The internal and external characteristics and the Euclidean distances

	Albania	B&H	Croatia	Kosovo	Macedonia	Montenegro	Serbia	Turkey	Ideal	
U^m	−2.66	−16.94	26.25	−15.62	7.45	−2.43	−0.09	17.64	U^*	26.2
V^m	−5.90	−3.89	30.02	−18.07	9.78	−3.83	−0.05	14.71	V^*	30.0
D^m	5.76	6.86	0.00	7.97	3.45	5.55	5.00	2.20		

Table 7 The entropies of the candidate states

	Albania	B&H	Croatia	Kosovo	Macedonia	Montenegro	Serbia	Turkey
E_S^m	14.991	10.025	25.706	5.821	16.772	15.623	13.736	22.264
E_W^m	12.452	10.962	4.522	22.736	8.758	11.991	10.740	7.204
E_O^m	10.034	10.081	19.560	6.088	14.006	10.042	10.531	13.385
E_T^m	8.269	8.066	2.513	11.360	5.173	6.142	7.215	4.829
DE^m	13.002	12.911	19.721	12.888	14.931	11.772	12.766	14.230

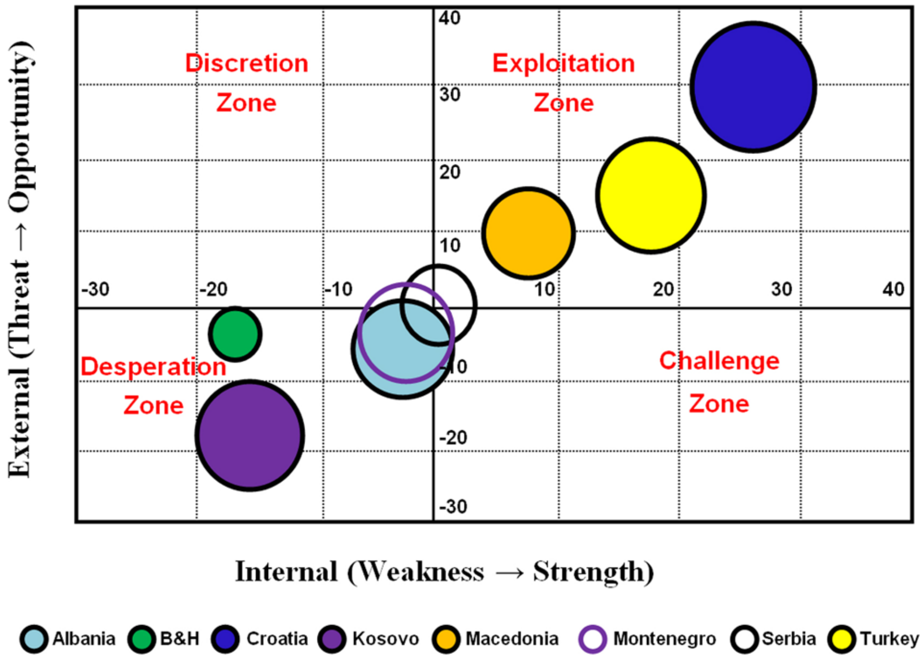


Fig. 3 A graphical representation of the candidate states and their scores

Table 8 The final ranking and classification of the candidate states

Candidate state	Priority	Quadrant			
		Exploitation	Discretion	Challenge	Desperation
Croatia	1	Completely			
Turkey	2	Completely			
Macedonia	3	Completely			
Serbia	4	Partially	Partially	Partially	Mainly
Montenegro	5		Partially	Partially	Mainly
Albania	6		Partially	Partially	Mainly
Bosnia & Herzegovina	7				Completely
Kosovo	8				Completely

threats. However, Croatia has large entropy indicating the DMs' disagreement for this candidate state. Turkey is also characterized by a positive outcome for the internal and external criteria. The overbalance of negative and positive internal and external factors is favourable for Macedonia. This overbalance is not as strong for Croatia and Macedonia.

Finally, we present the overall rankings of the candidate states in Table 8. The top-three countries of Croatia, Turkey and Macedonia all lie in the exploitation quadrant which means they should be considered seriously for the EU membership. It should be noted that these countries also had large entropies indicating the DMs' discords of their opinion. Albania, Bosnia and Herzegovina, Kosovo, Montenegro and Serbia all had prevalence of weaknesses over strengths and prevalence of threats over opportunities. These states lie in the desperation quadrant. Serbia had the best internal and external characteristics while Kosovo has the worst characteristics. These results were consistent with the EU's classification of "candidate" and "potential candidate" states described earlier.

5 Conclusions

The addition of new members into the EU is a strategic problem with profound economic and political effects on both the entering and existing members of the Union. The EU enlargement problem is a complex MCDA problem that embraces qualitative and quantitative internal strengths and weaknesses as well as external opportunities and threats. Candidates seeking membership in the EU must conform to a large number of quantitative and qualitative Copenhagen criteria established by the Copenhagen European Council. The current selection process is ambiguous and lacks structure. *Soft SWOT* w*as* developed in response to the need for a meaningful and robust aggregation of subjective and objective judgments concerning a large number of competing and conflicting criteria. *Soft SWOT* uses the AHP, subjective probabilities, defuzzification, entropy, and the theory of displaced ideal to reduce these complexities by decomposing the evaluation process into manageable steps. This decomposition is achieved without overly simplifying the process.

The results from this pilot study shows that Croatia, Turkey and Macedonia should be considered seriously for the EU membership. These countries had a positive overbalance of strengths over weaknesses and opportunities over threats. The results also reveals that Albania, Bosnia and Herzegovina, Kosovo, Montenegro and Serbia all had prevalence of weaknesses over strengths and prevalence of threats over opportunities. These countries are risky and should be avoided for the time being since they are characterized by high weaknesses and threats. The results of this study are consistent with the EU's current classification of "candidate" and "potential candidate" states.

Soft SWOT promotes consistent and systematic alternative evaluation and selection throughout the organization. Judgments captured as separate importance weights and performance scores are used uniformly across all alternatives in the evaluation process. In the absence of separate value judgments, it is difficult to apply a set of importance weights and performance scores consistently among the strengths, weaknesses, opportunities and threats when evaluating alternatives. *Soft SWOT* provides a consistent combination of all assessments among all the alternatives. Whether the assessments faithfully represent real-world circumstances depends on the competence and degree of effort the DMs exert in making the assessments.

Soft SWOT is also useful in examining how sensitive the overall Euclidean scores are to changes in the portfolio of selected alternatives. *Soft SWOT* also addresses questions about the sensitivity of the portfolio of selected alternatives to changes in the relative importance of the organizations, the relative importance of the strengths, weaknesses, opportunities and threats, and the performance scores.

Soft SWOT is not intended to replace human judgment in alternative evaluation and selection. In fact, human judgment is the core input in the process. *Soft SWOT* helps the DMs to think systematically about complex MCDA problems and improves the quality of the decisions. It is almost impossible to obtain objective data on the complex strategic problems because of inherent uncertainties. However, experienced DMs can often make fairly accurate estimates of values. *Soft SWOT* decomposes the alternative evaluation process into manageable steps and integrates the results to arrive at a solution consistent with managerial goals and objectives. This decomposition encourages DMs to carefully consider the elements of uncertainty.

Using a structured framework like *Soft SWOT* does not imply a deterministic approach in MCDA. While *Soft SWOT* enables DMs to crystallize their thoughts and organize their beliefs, it should be used very carefully. Managerial judgment is an integral component of *Soft SWOT*; therefore, the effectiveness of the model relies heavily on the DM's cognitive abilities to provide sound judgments. As with any decision analysis model, the researchers and practicing managers must be aware of the limitations of subjective estimates and use them carefully.

Finally, although the pilot study was not carried out with real DMs from the European Commission, it was an excellent platform for testing the practicality of the proposed framework with student subjects. The use of student subjects has been opposed by some researchers who argue that students may not be representative of real DMs. A central question regarding the legitimacy of using student subjects is whether the research findings obtained from student subjects can be generalized to actual organizational settings? In other words, do the findings obtained from the student subjects have external validity? Critics of student subject research argue that students are different from practicing managers and the dissimilarity between them precludes the validity of student subject research. This reasoning implies two broad assumptions. First, it assumes that external validity is the most important determinant of the value of the research project. Second, it assumes that practicing managers and students are different and that any differences between them will influence the results of the research. Below we scrutinize these two assumptions:

- The first assumption can be challenged by the fact this study is more concerned with the application process rather than the application outcome. The application process required the DMs to classify 169 Copenhagen sub-criteria into strengths, weaknesses, opportunities and threats, weigh these sub-criteria and determine their performance scores. This process was effectively simulated by a group of graduate students from the University of Paderborn in Germany with a good grasp of economics, political, social and business issues. Another aspect of the process required a series of numerical calculations involving a large number of equations aggregating a large number of weights with defuzzified subjective and objective sores. We argue this study is not concerned with the particular outcome of the case but rather with the process and its effective implementation with a group of mature and knowledgeable students who played the role of the DMs.
- The second assumption assumes there could be differences between the practitioners and the students and this difference could significantly affect the results of the research. We acknowledge the differences between the practitioners and the student subjects could change the particular judgments and potentially the final outcome. However, we argue these dissimilarities are not relevant in this study. These differences would most likely derive from the fact that practitioners may have more experience and better access to additional information. While these differences might change the particular results, they do not decrease the overall validity of the process.

We also argue there might be sufficient similarity between the students and the practicing managers which allows the students to be a representative sample from the intended population. In conclusion, we contend the use of students is a valid way to carry out a test of the model and is comparable with the role playing exercises that are generally carried out in graduate management classes to simulate decision making processes. These points are also clarified by Walters-York and Curatola (2000):

> Research relying on student subjects is likely no less valid than research relying on non-student subject groups; student samples provide no greater threat to external validity than typical real-world samples. The customary real-world sample can be placed

under the same scrutiny for lack of formal representativeness and atypicality as the customary student sample. Moreover, even when the sample is formally representative, real-world subjects in experimental settings are likely to be poor surrogates for real-world subjects in the real world due to a lack of experimental realism. As such, there is no universally valid basis for automatically privileging real-world samples and dismissing student samples as inherently inferior."

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Appendix: The European Commission enlargement strategy and progress reports

- The main page:
  http://ec.europa.eu/enlargement/press_corner/key-documents/reports_nov_2008_en.htm

The links to the progress reports for the candidate states are available at:

- Croatia:
  http://ec.europa.eu/enlargement/pdf/press_corner/key-documents/reports_nov_2008/croatia_progress_report_en.pdf
- The former Yugoslav Republic of Macedonia:
  http://ec.europa.eu/enlargement/pdf/press_corner/key-documents/reports_nov_2008/the_former_yugoslav_republic_of_macedonia_progress_report_en.pdf
- Turkey:
  http://ec.europa.eu/enlargement/pdf/press_corner/key-documents/reports_nov_2008/turkey_progress_report_en.pdf

The links to the progress reports of the potential candidate states are available at:

- Albania:
  http://ec.europa.eu/enlargement/pdf/press_corner/key-documents/reports_nov_2008/albania_progress_report_en.pdf
- Bosnia and Herzegovina:
  http://ec.europa.eu/enlargement/pdf/press_corner/key-documents/reports_nov_2008/bosnia_herzegovina_progress_report_en.pdf
- Montenegro:
  http://ec.europa.eu/enlargement/pdf/press_corner/key-documents/reports_nov_2008/montenegro_progress_report_en.pdf
- Serbia:
  http://ec.europa.eu/enlargement/pdf/press_corner/key-documents/reports_nov_2008/serbia_progress_report_en.pdf
- Kosovo under UNSCR 1244/99:
  http://ec.europa.eu/enlargement/pdf/press_corner/key-documents/reports_nov_2008/kosovo_progress_report_en.pdf

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A fuzzy opportunity and threat aggregation approach in multicriteria decision analysis

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Abstract Economic expansion in developed countries coupled with dramatically growing economies in countries such as China and India have precipitated a steady increase in demand for oil and natural gas. The Caspian Sea region holds large quantities of both oil and natural gas. Because the Caspian Sea is landlocked and the region's nations are distant from the largest energy markets, transportation must at least begin by pipeline. While some lines currently exist, pipelines with the capacity of transporting larger amounts of energy resources must be constructed to meet the global demand. This study is conducted for a multinational oil and natural gas producer to develop a multicriteria decision analysis (MCDA) framework for evaluating five possible pipeline routes in the Caspian Sea region. The proposed MCDA model considers a large number of conflicting criteria in the evaluation process and captures decision makers' (DMs') beliefs through a series of intuitive and analytical methods such as the analytic network process and fuzzy scoring. A defuzzification method is used to obtain crisp values from the subjective judgments and estimates provided by multiple DMs. These crisp values are aggregated and synthesized with the concept of entropy

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and the theory of the displaced ideal. The alternative routes are plotted on a diagram in a polar coordinate system and a classification scheme is used along with the Euclidean distance to measure which alternative is closer to the ideal route.

Keywords Multi-criteria decision analysis · Group decision making · Analytic network process · Fuzzy scoring · Level-2 fuzzy sets · Defuzzification · Entropy · Theory of displaced ideal

1 Introduction

Economic expansion in developed countries coupled with dramatically growing economies in countries such as China and India have precipitated a steady increase in demand for energy, especially oil and natural gas. The Caspian Sea region in Central Asia and Caucasus holds large quantities of both oil and natural gas that could help meet the increasing global demand for energy resources. However, many of the countries surrounding the Caspian Sea are landlocked. Only Iran and Russia have access to open water and existing pipeline networks that can effectively transport oil and natural gas resources to world markets both in Asia and in the West. Thus, unlike Iran and Russia, The Central Asia and Caucasus Republics must transport their resources across at least one international border, possibly two. Kazakhstan, Turkmenistan, and Azerbaijan appear to contain most of the regions energy resources. However, since none of these countries has access to seaports, they must rely on international pipelines to export their valuable resources. While some lines currently exist, pipelines with the capacity of transporting larger amounts of energy resources must be constructed to meet the global demand, but political disagreements have made this a complicated issue (Klare 2003).

This study is conducted for a multinational oil and natural gas producer to develop a multicriteria decision analysis (MCDA) framework for evaluating five alternative pipeline routes in the Caspian Sea region. The proposed MCDA model must consider a large number of factors before deciding on the appropriate route. Naturally, the financial obligations associated with each of the proposed pipeline routes carry much weight. However, since this region of the globe is a hot bed for political controversy and instability, the company is concerned about encountering many potential complications. Terrorist attacks, illegal tapping of the pipelines, and having the line be in jeopardy of being shut off by the governing country are among some of the factors that must first be considered by any investor (Klare 2003). Strategic assessment of the key opportunities and threats must be carefully analyzed before making any decision. Hasty decisions made solely on the basis of financial assessment of the proposed alternatives may result in outcomes that are less desirable. While a perfect, problem-free solution does not exist; all facets of this issue must be weighed cautiously before selecting the best possible alternative.

The MCDA framework proposed in this study is intended to facilitate evaluation of the alternative regions and countries for the purpose of pipeline construction. Each country is characterized by a large number of subjective and objective conflicting criteria with mutual dependencies. We capture multiple decision makers' (DMs') beliefs through a series of intuitive and analytical methods such as the analytic network process (ANP) and fuzzy scoring. A defuzzification method is used to obtain crisp values from the subjective judgments and estimates provided by multiple DMs. These crisp values are synthesized with the concepts of entropy and the theory of the displaced ideal to assist the DMs in their selection process. Two aggregated opportunity and threat indices are used to plot the alternative routes based on their position with respect to the "ideal" route. Our proposed method has several unique features:

(a) In reality, the distinction between an opportunity and threat is not always clear. We allow the DMs to designate each factor as an opportunity, a threat, or a transient factor. Opportunities have positive impact and threats have negative impact on the goal achievement. Transient factors represent those criteria with positive or negative impact, depending on the perception of the DM. If an alternative's impact on a transient factor is perceived to be positive, this factor will be designated as an opportunity and if the alternative's impact on a transient factor is perceived to be negative, that factor will be designated as a threat.
(b) We use a defuzzification process to translate ambiguous group judgments into crisp values.
(c) We use level-2 fuzzy sets because of the superposition of two potential fuzzinesses: vague individual judgments (continuous fuzzy functions); and, fuzzy group estimates or opinions (discrete fuzzy function).
(d) We use entropy to evaluate the level of uncertainty in multiple DMs' judgments.
(e) We complement the Euclidean distance analysis with a comprehensive classification scheme to gain additional insight on each alternative.

This paper is organized as follows. In the next section, we review the relevant literature followed by the proposed methodology in Sect. 3. In Sect. 4, we discuss the results of the case study and in Sect. 5, we present our conclusions and future research directions.

2 Review of relevant literature

A large body of intuitive and analytical models has evolved over the last several decades to assist DMs in project evaluation. While these models have made great strides in project evaluation, the intuitive models lack a structured framework, whereas the analytical models do not capture subjective preferences. The literature on project selection contains hundreds of models, including scoring methods, economic methods, portfolio methods, and decision analysis methods. Scoring methods use algebraic formulas to produce an overall score for each project (Moore and Baker 1969; Cooper 1992; Osawa and Murakami 2002; Osawa 2003). Economic methods use financial models to calculate the monetary payoff of each project (Graves and Ringuest 1991; Mehrez 1988). Portfolio methods evaluate the entire set of projects to identify the most attractive subset (Cooper et al. 1999; Girotra et al. 2007; Lootsma et al. 1990; Mojsilovi et al. 2007; Vepsalainen and Lauro 1988; Wang and Hwang 2007). Cluster analysis, a more specific form of a portfolio method, groups projects according to their support of the strategic positioning of the firm (Mathieu and Gibson 1993). Decision analysis models compare various projects according to their expected value (Hazelrigg and Huband 1985; Thomas 1985). Finally, simulation, a special form of decision analysis, uses random numbers and simulation to generate a large number of problems and pick the best outcome (Abacoumkin and Ballis 2004; Mandakovic and Souder 1985; Paisittanand and Olson 2006).

MCDA considers three types of problems: ranking of a finite set of alternatives, choosing the best alternative and clustering alternatives in similarity groups in terms of a finite number of objective and subjective criteria. More often, decision criteria can be grouped into two contradictory categories of factors having positive and negative effects on the goal achievement. Selecting or ranking projects with respect to multiple, often conflicting criteria in a fuzzy environment is usually referred to as fuzzy multicriteria analysis (Chen and Hwang 1992).

Recently, fuzzy multicriteria analysis methods are frequently used to improve project selection in businesses (Chen and Cheng 2009). Chou et al. (2008) proposed a MCDA method that used fuzzy set theory and simple additive weighting to integrate objective and subjective judgments under group decision making. Huang et al. (2008) presented a fuzzy analytic hierarchy process (AHP) that utilized crisp judgment matrices to evaluate subjective expert judgments in research and development project selection. Yang and Hsieh (2009) introduced a fuzzy MCDA method that incorporated a hierarchical criteria evaluation process with Delphi to solve project selection problems. Yeh and Chang (2009) proposed a fuzzy MCDA approach for solving project selection problems involving subjective judgments made by a group of DMs. They used a pairwise comparison process and a linguistic rating method to help individual DMs make comparative judgments. To reflect the inherent imprecision of subjective judgments, they aggregated individual assessments as a group assessment using triangular fuzzy numbers. Deng (2009) presented an excellent overview of the developments in fuzzy MCDA and analyses the existing approaches from four different perspectives for facilitating a better understanding of the recent developments in this domain.

The aforementioned methods use fuzzy sets to integrate objective and subjective judgments under group decision making and generate project rankings based on an additive weighting method. However, these methods fail to (1) separate the decision criteria with positive and negative impacts on the goal achievement. In our model, we identify a set of opportunities with positive impact and a set of threats with negative impact on the overall goal. We also introduce a new criteria category called transient factors which are either opportunities or threats depending on the DM's perception; (2) consider the interdependencies among the opportunities and threats. In our model, we use ANP to capture the interdependencies among the opportunities and threats; (3) estimate the level of vagueness (inconsistency) of the DMs' judgments on the opportunities and threats. In our model, we use entropy to capture the uncertainty of the DM's opinions; and (4) provide a classification scheme showing the similarities and differences among the alternative projects. In our model, we plot the alternatives on a diagram in a polar coordinate system and use a comprehensive classification scheme along with the Euclidean distance to group similar alternatives and measure which alternative is closer to the ideal choice.

3 The proposed methodology

Group MCDA involves a group of DMs, or persons responsible for decision making, who are faced with the problem of evaluating a set of alternatives measured by a family of criteria. One of the basic features of the proposed methodology is the effort to impose a structure on this formulation, evaluation and selection process. The proposed framework consists of 11 steps depicted in Fig. 1.

3.1 Identify DMs and their voting power

We consider K DMs each with a voting power index, a_k; $(k = 1, 2, \ldots, K)$. Power indices are meant to assess the power that a voting rule confers a priori to each of the DMs who use it. Voting forms the basis of group decision-making and is usually taken as a fair method. We accept the fact that some DMs have more authority, expertise, knowledge, or skills. For this reason, each DM is granted with a voting power which reflects his or her ability to influence the decision outcome. This type of weighted scheme is frequently used in practical group decision making (Laruelle and Widgren 2000; Uno 2003; Felsenthal and Machover 2004).

3.2 Identify a finite set of alternatives

Alternatives are the set of potential means by which the previously identified objectives may be attained. There must be a minimum of two mutually exclusive alternatives in the set to permit a choice to be made (Zeleny 1982). The "choice problem" is the fundamental purpose of MCDA. Alternatives indeed are rarely simple, and are often multiple alternatives or even whole scenarios involving sub-alternatives. In this case, the lower-level alternatives are evaluated and the results are aggregated for their groups. We consider a set $\{x_i\}$ where the elements x_i denote alternatives $(i = 1, 2, \ldots, I; I \geq 2)$. All the alternatives are combined into J groups, so we have a set $\{E_j\}$ and the elements E_j denote the groups of alternatives; $(j = 1, 2, \ldots, J; J \geq 2)$. Each element x_i belongs to one group E_j. x_i^j represents the ith alternative belonging to the jth group; $(i = 1, 2, \ldots, I; j = 1, 2, \ldots, J)$.

3.3 Identify relevant factors/sub-factors and group them into clusters

The criteria selection process is a crucial step in MCDA. According to Bouyssou (1990), the criteria set must have two key qualities: be readable (i.e., include a number of criteria restricted enough so that it is possible to reason on this basis and eventually to model the inter- and intra-criteria information required to perform an aggregation procedure) and be operational (i.e., be acceptable as a working basis for the study).

One way to formalize this process is to use an *effective* and *coherent* set of criteria (Hites et al. 2006). To be effective, the criteria set must be directional (i.e., can distinguish between minimization, maximization or otherwise optimization), concise

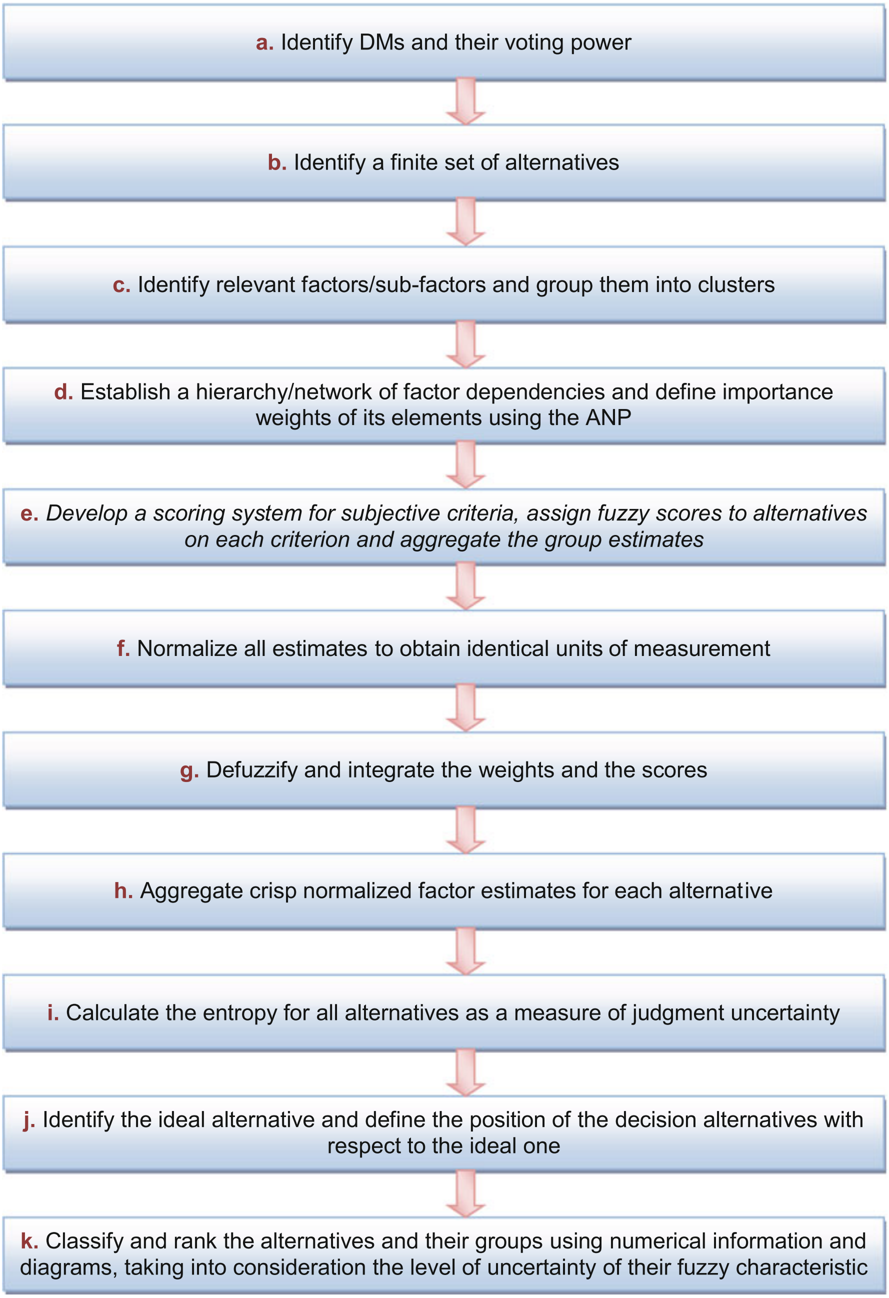

Fig. 1 The proposed framework

(i.e., provide the smallest number of measures that allows all significant impacts to be assessed), complete (i.e., cover all aspects of success so that no significant impact goes unmeasured) and clear (i.e., define how measurements are to be made whether in quantitative or qualitative terms) (Yoe 2002). To be coherent, the criteria set must be exhaustive (i.e., no attribute of significance to the DMs is left out), consistent (i.e., no hidden or unexpected preferences) and non-redundant (i.e., avoid double counting) (Roy 1985). See Roy (1975, 1985), Bouyssou (1989) and Roy and Bouyssou (1993) for a discussion of the qualities of a good system of criteria.

More often, decision criteria can be grouped into contradictory categories, such as, opportunities and threats, or alternatively, benefits or costs. Higher scores are preferred for positive criteria and lower scores are preferred for negative criteria. The classification of different factors is a delicate part of the problem formulation because all different aspects of the problem must be represented while avoiding redundancies (Bouyssou 1990). Roy and Bouyssou (1987) have developed a series of operational tests that can be used to check the consistency of this classification. Let us define:

$Cl =$ The lth cluster of factors; $(l = 1, 2, \ldots, L)$
$L =$ The number of clusters;
$Cl_m =$ The mth group of factors within the lth cluster; $(l = 1, 2, \ldots, L; m = 1, 2, \ldots, Ml)$
$Ml =$ The number of groups of factors within the lth cluster; $(l = 1, 2, \ldots, L)$
$Cl_n^m =$ The nth factor within the mth group of the lth cluster; $(l = 1, 2, \ldots, L; m = 1, 2, \ldots, Ml; n = 1, 2, \ldots, N_m^l)$
$N_m^l =$ The number of factors within the mth group of the lth cluster; $(l = 1, 2, \ldots, L)$

3.4 Establish a hierarchy/network of factor dependencies and define importance weights of its elements using the ANP

The proposed methodology is a normative MCDA model with multiple factors representing different dimensions from which the alternatives are viewed. MCDA techniques require the determination of weights that reflect the relative importance of various competing factors. When the number of factors is large, typically more than a dozen, they may be arranged hierarchically (Saaty 1977; Triantaphyllou 2000; Triantaphyllou and Mann 1995). Such structure allows for a systematic grouping of decision factors in large problems. We use the ANP, generalization of the AHP developed by Saaty (Saaty 2001; Saaty and Sodenkamp 2008). The AHP assumes unidirectional hierarchical relationships among the decision elements in a problem. However, in many real-life problems, there are mutual dependencies among the elements in a hierarchy.

The ANP is a more general form of the AHP that does not require independence and allows for decision elements to "influence" or "be influenced" by other elements in the model. Our model does not use ANP conventionally to determine the relative importance of each route in terms of the decision factors. Instead, probabilities of occurrence are used to capture the relative performance of each route. DMs are asked to provide a series of pairwise comparisons of the factors at each level of the hierarchy with respect to a control factor. This is the fundamental requirement for developing the super-matrix in the ANP (Saaty 2001). The Super-matrix is composed of multiple matrices of pairwise comparisons. The pairwise comparisons for the factors at one level with respect to the control factor at another level are expressed in a preference matrix with a 1–9 scale.

Let us assume that we have a system of L clusters whereby the factors in each cluster interact or have an impact on or are influenced by some or all of the factors of another cluster with respect to a property governing the interactions of the entire system. The impacts of the defined factors in a cluster on another factor in the system are represented by a ratio scale priority vector derived from paired comparisons. Each such priority vector is introduced in the appropriate position as a column vector in a super-matrix of impacts displayed as follows:

$$
W = \begin{array}{cc}
 & \begin{array}{cccc} C1 & C2 & \cdots & CL \\ C1_1 C1_2 \ldots C1_{M1} & C2_1 C2_2 \ldots C2_{M2} & & CL_1 CL_2 \ldots CL_{ML} \end{array} \\
\begin{array}{cc} C1 & \begin{array}{c} C1_1 \\ C1_2 \\ \cdots \\ C1_{M1} \end{array} \\ C2 & \begin{array}{c} C2_1 \\ C2_2 \\ \cdots \\ C2_{M2} \end{array} \\ \cdots & \cdots \\ CL & \begin{array}{c} CL_1 \\ CL_2 \\ \cdots \\ CL_{ML} \end{array} \end{array} &
\begin{bmatrix} W_{11} & W_{12} & \cdots & W_{1L} \\ W_{21} & W_{22} & \cdots & W_{2L} \\ \cdots & \cdots & \cdots & \cdots \\ W_{L1} & W_{L2} & \cdots & W_{LL} \end{bmatrix}
\end{array}
$$

The i, jth block of the above super-matrix is given by the following matrix:

$$
W_{ij} = \begin{bmatrix} w_{i1}^{(j1)} & w_{i1}^{(j2)} & \ldots & w_{i1}^{(jMj)} \\ w_{i2}^{(j1)} & w_{i2}^{(j2)} & \ldots & w_{i2}^{(jMJ)} \\ \ldots & \ldots & \ldots & \ldots \\ w_{iMi}^{(j1)} & w_{iMi}^{(j1)} & \ldots & w_{iMi}^{(j1)} \end{bmatrix}
$$

where each column is a principal eigenvector that represents the impact of all the factors in the ith cluster on each cluster of the jth cluster.

The super-matrix is then converged to obtain a long-term stable set of weights. For convergence to occur, the super-matrix needs to be column stochastic. In other words, the sum of each column of the super-matrix needs to be one. Saaty (2001) suggests raising the weighted super-matrix to the power until it reaches a limit state. This new matrix is called the limit super-matrix. By normalizing each block of the limit super-matrix, the final importance weights of all the elements in the matrix can be obtained. For complete treatment, see Saaty (2001) and Saaty and Ozdemir (2005). The final priorities of the clusters (or any set of factors in a cluster) are obtained by normalizing the corresponding values in the appropriate columns of the limit matrix that will be further used with the following designations:

w_l^k = The weight of the lth cluster of factors given by the kth DM; ($l = 1, 2, \ldots, L$)

w_{lm}^{k} = The weight of the mth group of factors within the lth cluster given by the kth DM; $(l = 1, 2, \ldots, L; m = 1, 2, \ldots, Ml)$

w_{lmn}^{k} = The weight of the nth factor within the mth group of the lth cluster given by the kth DM; $(l = 1, 2, \ldots, L; m = 1, 2, \ldots, Ml; n = 1, 2, \ldots, N_m^l)$

3.5 Develop a scoring system for subjective criteria, assign fuzzy scores to alternatives on each criterion and aggregate the group estimates

This step begins by asking the DMs to provide their judgments and assign scores to the alternatives using a 0 to S scale for all the criteria. These scores are called subjective probabilities. Subjective probabilities are commonly used in strategic decision making because they require no historical data (observation of regularly occurring events by their long-run frequencies) (De Kluyver and Moskowitz 1984; Schoemaker 1993; Schoemaker and Russo 1993; Tavana and Banerjee 1995; Vickers 1992; Weigelt and Macmillan 1988). The difficulties in subjective probability estimation arise when the experts cannot provide probability estimates with confidence. In those cases, the numerical expressions associated with the subjective probabilities are often vague and are not restricted to a single value or interval of values with sharp boundaries. Fuzzy set theory (Zadeh 1965) can play a significant role in this kind of decision situation. In our model, the individual judgments are represented by triangular fuzzy numbers $(\tilde{p}_{lmn}^{ki})$ defined on the discrete universe set Z_S, $s = 0, 1, 2, \ldots, S$. Zadeh (1996) characterizes the triangular fuzzy numbers by a triple $z = (z_1, z_2, z_3)$ where $\mu(z_1) = \mu(z_3) = 0$ and $\mu(z_2) = 1$ or z_2 is considered to be most reliable. The triangular fuzzy numbers assigned to each alternative are:

$$\begin{aligned}\tilde{p}_{lmn}^{ki} = \Big[& p_{lmn}^{ki}(left)/\mu\left(p_{lmn}^{ki}(left)\right), p_{lmn}^{ki}(top)/\mu\left(p_{lmn}^{ki}(top)\right), \\ & p_{lmn}^{ki}(right)/\mu\left(p_{lmn}^{ki}(right)\right)\Big], \end{aligned} \tag{1}$$

where

$\tilde{p}_{lmn}^{ki}$ = The fuzzy judgment of the kth DM on the $n-$th criterion in the mth group of cluster l given the choice of the ith alternative; $(i = 1, 2, \ldots, I; k = 1, 2, \ldots, K; l = 1, 2, \ldots, L; m = 1, 2, \ldots, Ml; n = 1, 2, \ldots, N_m^l)$.

$p_{lmn}^{ki}(top)$ = The value of the subjective probability assigned by the kth DM to the ith alternative on the nth criterion in the mth group of cluster l on $\alpha = 1$; $(i = 1, 2, \ldots, I; k = 1, 2, \ldots, K; l = 1, 2, \ldots, L; m = 1, 2, \ldots, Ml; n = 1, 2, \ldots, N_m^l)$.

$p_{lmn}^{ki}(left)\left(p_{lmn}^{ki}(right)\right)$ = The boundary of the left (right) deviation of the value $p_{lmn}^{ki}(top)$ on $\alpha = 0$; $(i = 1, 2, \ldots, I; k = 1, 2, \ldots, K; l = 1, 2, \ldots, L; m = 1, 2, \ldots, Ml; n = 1, 2, \ldots, N_m^l)$.

$\mu\left(p_{lmn}^{ki}(top)\right)$, $\mu\left(p_{lmn}^{ki}(left)\right)$ and $\mu\left(p_{lmn}^{ki}(right)\right)$ = The membership grades

$0 \leq p_{lmn}^{ki} \leq S, \forall i, k, l, m, n$

$\mu\left(p_{lmn}^{ki}(top)\right) = 1; \mu\left(p_{lmn}^{ki}(left)\right) = 0; \mu\left(p_{lmn}^{ki}(right)\right) = 0.$

The expression (1) can then be rewritten as follows:

$$\tilde{p}^{ki}_{lmn} = \left[p^{ki}_{lmn}(left)/0,\, p^{ki}_{lmn}(top)/1,\, p^{ki}_{lmn}(right)/0 \right]. \tag{2}$$

Alternatively, the DMs can assign intervals instead of points on α-level = 1 for the most reliable judgments and their left and right spreads. Such judgments can be represented by trapezoidal fuzzy numbers. The trapezoidal fuzzy numbers need an additional judgment for each criterion on each alternative. In case of I alternatives and N criteria, each DM needs to provide $I \times N$ more estimates with trapezoidal fuzzy numbers. In this study, the triangular fuzzy numbers are used to ease the additional computational burden of the trapezoidal fuzzy numbers.

Another problem arises when a single judgment for the alternative on a criterion must be modeled for the entire group of DMs. Saaty and Shang (2007) have shown that the geometric mean is the most rational method for combining individual judgments into a group judgment. An alternative approach for synthesizing individual judgments into a group judgment was introduced by Tavana and Sodenkamp (2009). They considered multiple sets of group judgments by means of discrete fuzzy sets of multiple scores ($\tilde{A}^i_{lmn}$) represented by the pairs:

$$\tilde{A}^i_{lmn} = \left\{ \left(p^{ki}_{lmn}, \mu_A(p^{ki}_{lmn}) \right) \right\}, \quad \forall p^{ki}_{lmn} \in P^i_{lmn}, \tag{3}$$

where

$P^i_{lmn} =$ The discrete set of DMs' judgments for the nth criterion in the mth group of cluster l given the choice of the ith alternative; ($i = 1, 2, \ldots, I$; $l = 1, 2, \ldots, L$; $m = 1, 2, \ldots, Ml$; $n = 1, 2, \ldots, N^l_m$).

$\mu_A(p^{ki}_{lmn}) =$ The membership grade of the kth DM judgment; ($i = 1, 2, \ldots, I$; $k = 1, 2, \ldots, K$; $l = 1, 2, \ldots, L$; $m = 1, 2, \ldots, Ml$; $n = 1, 2, \ldots, N^l_m$).

Since the judgments p^{ki}_{lmn} in our model are given in the form of fuzzy scores defined in (1), we can rewrite (3) as follows:

$$\tilde{A}^i_{lmn} = \left\{ \left(\tilde{p}^{ki}_{lmn}, \mu_A\left(\tilde{p}^{ki}_{lmn} \right) \right) \right\}, \quad \forall \tilde{p}^{ki}_{lmn} \in \tilde{P}^i_{lmn}$$

where

$\tilde{P}^i_{lmn} =$ The discrete set of DMs' judgments for the nth criterion in the mth group of cluster l given the choice of the ith alternative; ($i = 1, 2, \ldots, I$; $l = 1, 2, \ldots, L$; $m = 1, 2, \ldots, Ml$; $n = 1, 2, \ldots, N^l_m$).

$\mu_A(\tilde{p}^{ki}_{lmn}) =$ The membership grade of the kth DM judgment; ($i = 1, 2, \ldots, I$; $k = 1, 2, \ldots, K$; $l = 1, 2, \ldots, L$; $m = 1, 2, \ldots, Ml$; $n = 1, 2, \ldots, N^l_m$).

Therefore, we deal with fuzzy sets of fuzzy sets or a level-2 fuzzy set (Dubois and Prade 1980). Originally, level-2 fuzzy sets were presented by Zadeh (1971). In Zadeh's notation, $A = \int_x \mu_A(x)/x$ where $x \in X$, $\mu_A(x) \in [0, 1]$. When $\mu_A(x)$ becomes fuzzy, A becomes a type 2 fuzzy set. This transformation of an ordinary fuzzy set into a type 2 fuzzy set by blurring the grades of membership is called

g-fuzzification. When x is blurred into a fuzzy set $\tilde{x}$ on X, A is said to be a level-2 fuzzy set. A level-2 fuzzy set can be viewed as a two-level hierarchy of fuzzy sets.

If fuzzy set A is defined on a discrete set $y_r, r = 1$ to$R$, and y_r are presented by ordinary fuzzy sets defined on a discrete universe set $z_q, q = 1$ to Q, then, A is a level-2 fuzzy subset defined by the following expressions: $A = \{y_r/\mu_A(y_r)\}$, $y_r = \{z_q/\mu_r(z_q)\}$ (Dimova et al. 2006). Recent applications of level-2 fuzzy sets in the literature include object-oriented database modeling (De Tré and De Caluwe 2003), investment projects assessment (Dimova et al. 2006), and steel material selection (Sevastjanov and Figat 2007).

3.6 Normalize all estimates to obtain identical units of measurement

Next, we normalize variables with multiple measurement scales to assure uniformity. The literature reports on several normalization methods. The selection of a specific normalization method must be based on the problem characteristics and model requirements. In this study, we use the approach where the normalized value is the quotient of the initial value divided by the sum of the values of all alternatives on that criterion:

$$d_i{}' = \frac{d_i}{\sum_{i=1}^{n} d_i}. \tag{4}$$

Since the DMs may have different importance weights, it is reasonable to multiply their estimates by the voting power index (a_k). Then, the normalized representation of Eq. (2) is:

$$\tilde{p}_{lmn}^{ki}{}' = \left[p_{lmn}^{ki}{}'(left),\, p_{lmn}^{ki}{}'(top),\, p_{lmn}^{ki}{}'(right)\right]. \tag{5}$$

And its components are defined by the following expressions:

$$\begin{aligned} p_{lmn}^{ki}{}'(top) &= a_k \frac{p_{lmn}^{ki}(top)}{\sum_n p_{lmn}^{ki}(top)} \\ p_{lmn}^{ki}{}'(left) &= a_k \frac{p_{lmn}^{ki}(left)}{\sum_n p_{lmn}^{ki}(left)}. \\ p_{lmn}^{ki}{}'(right) &= a_k \frac{p_{lmn}^{ki}(right)}{\sum_n p_{lmn}^{ki}(right)} \end{aligned} \tag{6}$$

3.7 Defuzzify and integrate the weights and the scores

We use a two-step defuzzification process to integrate M sets of criteria group weights $\left(w_{lm}^{k}\right)$, N factor weights $\left(w_{lmn}^{k}\right)$, and the set of fuzzy subjective probabilities $\left(\tilde{p}_{lmn}^{ki}\right)$ into one set of crisp values for the entire group of K DMs for each alternative and group of alternatives.

Defuzzification is the translation of linguistic or fuzzy values into numerical, scalar and crisp representations. Many defuzzification techniques have been proposed in the literature. The most commonly used method is the Center of Gravity (COG). Roychowdhury and Pedrycz (2001) and Dubois and Prade (2000) provide excellent reviews of the most commonly used defuzzification methods. We use COG to calculate the centroid of a possibility distribution function using Eq. (7) for continuous cases and Eq. (8) for discrete cases:

$$COG(N) = \frac{\int_{-\infty}^{\infty} z\mu_N(z)dz}{\int_{-\infty}^{\infty} \mu_N(z)dz}. \tag{7}$$

$$COG(N) = \frac{\sum_{q=1}^{Q} z_q \mu(z_q)}{\sum_{q=1}^{Q} \mu(z_q)}. \tag{8}$$

In the first step of the defuzzification process, we convert the fuzzy scores defined in the expression (4) into a set of crisp scores using formula (7). When fuzzy scores belong to the triangular type of fuzzy functions, formula (9) can be used instead of formula (7):

$$COG(\Delta) = \frac{\sum_{q=1}^{3} z_q}{3}. \tag{9}$$

where z_q are the values on the vertices of a triangle. For the sets $\tilde{p}_{lmn}^{ki}{}'$ defuzzified values are:

$$\mathscr{D}1_{lmn}^{ki} = COG(\tilde{p}_{lmn}^{ki}{}') = \frac{p_{lmn}^{ki}{}'(left) + p_{lmn}^{ki}{}'(top) + p_{lmn}^{ki}{}'(right)}{3}. \tag{10}$$

In the second step of the defuzzification process, the discrete fuzzy sets of the K DMs' scores are brought to the crisp view for I alternatives with respect to N criteria using the center of gravity defuzzification function (8) for the discrete case:

$$COG\left(\tilde{A}_{lmn}^{i}\right) = \frac{\sum_{i=1}^{k} \tilde{p}_{lmn}^{ki} \cdot \mu_A\left(\tilde{p}_{lmn}^{ki}\right)}{\sum_{i=1}^{k} \mu_A\left(\tilde{p}_{lmn}^{ki}\right)}. \tag{11}$$

Here, instead of fuzzy function $\tilde{p}_{lmn}^{ki}$ we write its defuzzified value obtained in (10), and the membership grade $\mu_A\left(\tilde{p}_{lmn}^{ki}\right)$ is calculated in accordance with (12):

$$\mu_A(\tilde{p}_{lmn}^{ki}) = a_k \cdot w_l^k \cdot w_{lm}^k \cdot w_{lmn}^k. \tag{12}$$

Then, (11) can be rewritten as follows:

$$\mathscr{D}2_{lmn}^{ki} = COG\left(\tilde{A}_{lmn}^{i}\right) = \frac{\sum_{k=1}^{K}\left(COG\left(\tilde{p}_{lmn}^{ki}{}'\right) \cdot a_k \cdot w_l^k \cdot w_{lm}^k \cdot w_{lmn}^k\right)}{\sum_{k=1}^{K}\left(a_k \cdot w_l^k \cdot w_{lm}^k \cdot w_{lmn}^k\right)}. \tag{13}$$

3.8 Aggregate crisp normalized factor estimates for each alternative

We find the total defuzzified weights of the L contradictory classes for all alternatives and their groups under consideration by using a two-dimensional factor classification for two sets of positive $\{v_{xi}(l^+)\}$ and negative $\{v_{xi}(l^-)\}$ factors.

$$v_{xi}(l) = \sum_{m=1}^{Ml} \sum_{n=1}^{N_m^l} \mathscr{D}2_{lmn}^{i}, \tag{14}$$

$$v_{Ej}(l) = \sum_{i=1}^{I} v_l\left(x_i^j\right), \tag{15}$$

where

$v_{xi}(l) =$ The total defuzzified value of the alternative x_i on the lth merit; $(i = 1, 2, \ldots, I; l = 1, 2, \ldots, L)$.
$v_{Ej}(l) =$ The total defuzzified value of the group of alternatives E_j on the lth merit; $(j = 1, 2, \ldots, J; l = 1, 2, \ldots, L)$.
$v_l(x_i^j) =$ The total defuzzified value of the alternative x_i^j on the lth merit; $(i = 1, 2, \ldots, I; j = 1, 2, \ldots, J; l = 1, 2, \ldots, L)$.

3.9 Calculate the entropy for all alternatives as a measure of judgment uncertainty

We revise the weights of the alternatives and their groups determined through the defuzzification process with the entropy concept. Entropy is a measure of uncertainty used to estimate the level of vagueness (inconsistency) of the DMs' judgments. Shannon (1948) has defined the entropy of a probability distribution where the total probability of the elements adds up to 1. However, De Luca and Termini (1972) showed that this restriction is unnecessary. They defined a fuzzy entropy formula on a finite universal set $X = \{x_1, \ldots, x_n\}$ as:

$$e_{LT}(A) = -\beta \sum_{i=1}^{n} [\mu_A(x_i) \ln \mu_A(x_i) + (1 - \mu_A(x_i)) \ln(1 - \mu_A(x_i))]. \tag{16}$$

An entropy value of 1 indicates that all factors are biased by the maximum fuzziness and a lack of distinction is apparent in the preference relations. The fuzziness of the membership functions has its highest grade at the "crossover value" ($\mu = 0.5$). An entropy value of 0 indicates that the preference relations are credible or non-credible. The maximal distinctness is reached when $\mu = 0$ and $\mu = 1$.

Assuming that the vector $\{\tilde{p}_{lmn}^{ki}{}'\}$ characterizes the set of weighted defuzzified scores in terms of the nth factor for the mth group of cluster l (lmnth factor) given the choice of the ith alternative, the entropy measure of the lmnth factor is:

$$e_{LT}\left(A^i_{lmn}\right) = -\beta \sum_{k=1}^{K} \left[\mu\left(\mathscr{D}1^{ki}_{lmn}\right) \ln \mu\left(\mathscr{D}1^{ki}_{lmn}\right) + \left(1-\mu\left(\mathscr{D}1^{ki}_{lmn}\right)\right) \ln\left(1-\mu\left(\mathscr{D}1^{ki}_{lmn}\right)\right)\right], \tag{17}$$

where $0 \le \mu\left(\mathscr{D}1^{ki}_{lmn}\right) \le 1$ and $e_{LT}\left(A^i_{lmn}\right) \ge 0$.

The total entropies of positive and negative merits for alternative x_i are defined as $E^i_{l(+)} = \sum_{m=1}^{Ml} \sum_{n=1}^{N^l_m} e_{LT}\left(A^i_{l(+)mn}\right)$ and $E^i_{l(-)} = \sum_{m=1}^{Ml} \sum_{n=1}^{N^l_m} e_{LT}\left(A^i_{l(-)mn}\right)$ respectively, where (+) and (−) designate whether a cluster includes criteria having positive or negative impact on the achievement of the decision goal.

3.10 Identify the ideal alternative and define the position of the decision alternatives with respect to the ideal one

The weighted-sum scores in the proposed approach are used to compare the potential alternatives among themselves and with the *ideal alternative*. Using the Euclidean measure suggested by Zeleny (1982), we synthesize the results by determining the ideal values for the positive and negative classes of criteria. The ideal value for positive merit $(v^*(l^+))$ is the highest defuzzified weight of that merit among the set of $\{v_{xi}(l^+)\}$ and the ideal value for negative merit $(v^*(l^-))$ is the lowest defuzzified weight of that merit among the set $\{v_{xi}(l^-)\}$. Next, we find the Euclidean distance of each alternative from the ideal one. The Euclidean distance is the sum of the quadratic root of squared differences between the ideal and the ith indices of the positive and negative merits. Finally, we replicate these operations for the set of alternative groups. To formulate this step algebraically, let us assume:

$D^{xi}_{l(+)}\left(D^{xi}_{l(-)}\right) =$ The distance from the ideal value for the cluster of positive (negative) criteria for the alternative x_i; $(i = 1, 2, \ldots, I)$.

$D^{Ej}_{l(+)}\left(D^{Ej}_{l(-)}\right) =$ The distance from the ideal value for the cluster of positive (negative) criteria for the group of alternatives E_j; $(i = 1, 2, \ldots, I)$.

$D^{xi} =$ The overall Euclidean distance of the alternative x_i; $(i = 1, 2, \ldots, I)$.

$D^{Ej} =$ the overall Euclidean distance of the group of alternatives E_j; $(j = 1, 2, \ldots, J)$.

$\bar{D}^I =$ The mean Euclidean distance for the alternatives.

$\bar{D}^J =$ The mean Euclidean distance for the groups of alternatives.

$v^*(l^+)(v^*(l^-)) =$ The ideal value on the positive (negative) merit for the alternatives.

$v^*J(l^+)(v^*J(l^-)) =$ The ideal value on the positive (negative) merit for the groups of alternatives.

$e^*(l^+)\left(e^*(l^-)\right) =$ The entropy of the ideal positive (negative) merit for the alternatives.
$e^*J(l^+)\left(e^*J(l^-)\right) =$ The entropy of the ideal positive (negative) merit for the groups of alternatives.
$De^{xi}_{l(+)}\left(De^{xi}_{l(-)}\right) =$ The distance from the entropy of the ideal positive (negative) merit for the ith alternative; $(i = 1, 2, \ldots, I)$.
$De^{Ej}_{l(+)}\left(De^{Ej}_{l(-)}\right) =$ The distance from the entropy of the ideal positive (negative) merit for the jth group of alternatives; $(j = 1, 2, \ldots, J)$.
$De^{xi} =$ The overall Euclidean distance of the entropy for the alternative x_i; $(i = 1, 2, \ldots, I)$.
$De^{Ej} =$ The overall Euclidean distance of the entropy for the group of alternatives E_j; $(j = 1, 2, \ldots, J)$.

$$D^{xi} = \sqrt{\left(D^{xi}_{l(+)}\right)^2 + \left(D^{xi}_{l(-)}\right)^2}, \tag{18}$$

$$D^{Ej} = \sqrt{\left(D^{Ej}_{l(+)}\right)^2 + \left(D^{Ej}_{l(-)}\right)^2}, \tag{19}$$

$$\bar{D}^I = \sum_{i=1}^{I} D^{xi} \Big/ I, \tag{20}$$

$$\bar{D}^J = \sum_{j=1}^{J} D^{Ej} \Big/ J, \tag{21}$$

$$De^{xi} = \sqrt{\left(De^{xi}_{l(+)}\right)^2 + \left(De^{xi}_{(-)}\right)^2}, \tag{22}$$

$$De^{Ej} = \sqrt{\left(De^{Ej}_{l(+)}\right)^2 + \left(De^{Ej}_{(-)}\right)^2}, \tag{23}$$

$$\begin{aligned} v^*(l^+) &= Max\left\{v_{xi}(l^+)\right\} \\ v^*(l^-) &= Min\left\{v_{xi}(l^-)\right\}, \end{aligned} \tag{24}$$

$$\begin{aligned} v^*J(l^+) &= Max\left\{v_{Ej}(l^+)\right\} \\ v^*J(l^-) &= Min\left\{v_{Ej}(l^-)\right\}, \end{aligned} \tag{25}$$

$$\begin{aligned} e^*(l^+) &= Min\left\{e^{xi}_{l(+)}\right\} \\ e^*(l^-) &= Min\left\{e^{xi}_{l(-)}\right\}, \end{aligned} \tag{26}$$

$$\begin{aligned} e^*J(l^+) &= Min\left\{e^{Ej}_{l(+)}\right\} \\ e^*J(l^-) &= Min\left\{e^{Ej}_{l(-)}\right\}, \end{aligned} \tag{27}$$

where

$$\begin{aligned} D^{xi}_{l(+)} &= v^*(l^+) - v_{xi}(l^+); & D^{Ej}_{l(+)} &= v^*J(l^+) - v_{Ej}(l^+). \\ D^{xi}_{l(-)} &= v_{xi}(l^-) - v^*(l^-); & D^{Ej}_{l(-)} &= v_{Ej}(l^-) - v^*J(l^-). \\ De^{xi}_{l(+)} &= e^{xi}_{l(+)} - e^*(l^+); & De^{Ej}_{l(+)} &= e^{Ej}_{l(+)} - e^*J(l^+). \\ De^{xi}_{l(-)} &= e^{xi}_{l(-)} - e^*(l^-); & De^{Ej}_{l(-)} &= e^{Ej}_{l(-)} - e^*J(l^-). \end{aligned}$$

3.11 Classify and rank the alternatives and their groups using numerical information and diagrams, taking into consideration the level of uncertainty of their fuzzy characteristic

In the final step, we plot the alternative projects on a plane using a polar coordinate system in which each point is determined by a distance and an angle. The x-axis is represented by the total Euclidean distance from the ideal positive merit $\left(D^{xi}_{l(+)}\left(D^{Ej}_{l(+)}\right)\right)$ and the y-axis is represented by the total Euclidean distance from the ideal negative merit $\left(D^{xi}_{l(-)}\left(D^{Ej}_{l(-)}\right)\right)$. The position of the point corresponding to alternative x_i (group E_j) with Cartesian coordinates $\left(D^{xi}_{l(+)}, D^{xi}_{l(-)}\right)\left(\left(D^{Ej}_{l(+)}, D^{Ej}_{l(-)}\right)\right)$ on the graph is determined by its Euclidian distance from the coordinate origin $\left(D^{xi}\left(D^{Ej}\right)\right)$ with an angle of $\phi_{xi}(\phi_{Ej})$ between vector $\left(\overline{D^{xi}_{l(+)}, D^{xi}_{l(-)}}\right)$ $\left(\left(\overline{D^{Ej}_{l(+)}, D^{Ej}_{l(-)}}\right)\right)$ and the x-axis, where

$$tg(\varphi_{xi}) = D^{xi}_{l(+)}/D^{xi}_{l(-)}, \tag{28}$$

$$tg(\varphi_{Ej}) = D^{Ej}_{l(+)}/D^{Ej}_{l(-)}. \tag{29}$$

We use the mean Euclidean distance $\left(\bar{D}^I\left(\bar{D}^J\right)\right)$ and the angle ($\bar{\phi} = 45°$) to divide the graph into four decision zones. In the case of a tie, $\left(D^{xi}_{l(+)} = D^{xi}_{l(-)}\left(D^{Ej}_{l(+)} = D^{Ej}_{l(-)}\right)\right)$, $\phi_{xi} = 45°(\phi_{Ej} = 45°)$ and $tg(\bar{\varphi}) = 1$. As depicted in Fig. 2, alternatives (groups) with smaller $D^{xi}(D^{Ej})$ are closer to the ideal alternative (group) and are preferred to alternatives (groups) with larger D^{xi}. Furthermore, alternatives (groups) with smaller $\phi_{xi}\left(\phi_{Ej}\right)$ and $De^{xi}\left(De^{Ej}\right)$ are preferred to alternatives (groups) with larger $\phi_{xi}\left(\phi_{Ej}\right)$ and $De^{xi}\left(De^{Ej}\right)$. Alternatives (groups) with equal $D^{xi}\left(D^{Ej}\right)$ lie on the same circle (sphere). The following assertion is valid for alternatives (groups) lying on the same sphere: with the growth of $\phi_{xi}\left(\phi_{Ej}\right)$, the distance to the ideal positive merit decreases:$v_{xi}(l^+) \to \min\left(v_{Ej}(l^+) \to \min\right)$ and the distance to the ideal negative merit also decreases: $v_{xi}(l^-) \to \min\left(v_{Ej}(l^-) \to \min\right)$. Therefore, alternatives (groups) with $\phi_{xi} \leq \bar{\phi}(\phi_{Ej} \leq \bar{\phi})$ are less unfavorable and at the same time have little potential.

We also consider the overall Euclidean distance of the entropy for the ith alternative (group) $De^{xi}\left(De^{Ej}\right)$. Alternatives (groups) with smaller $De^{xi}\left(De^{Ej}\right)$ (smaller measure of uncertainty) are preferred to those with larger $De^{xi}\left(De^{Ej}\right)$

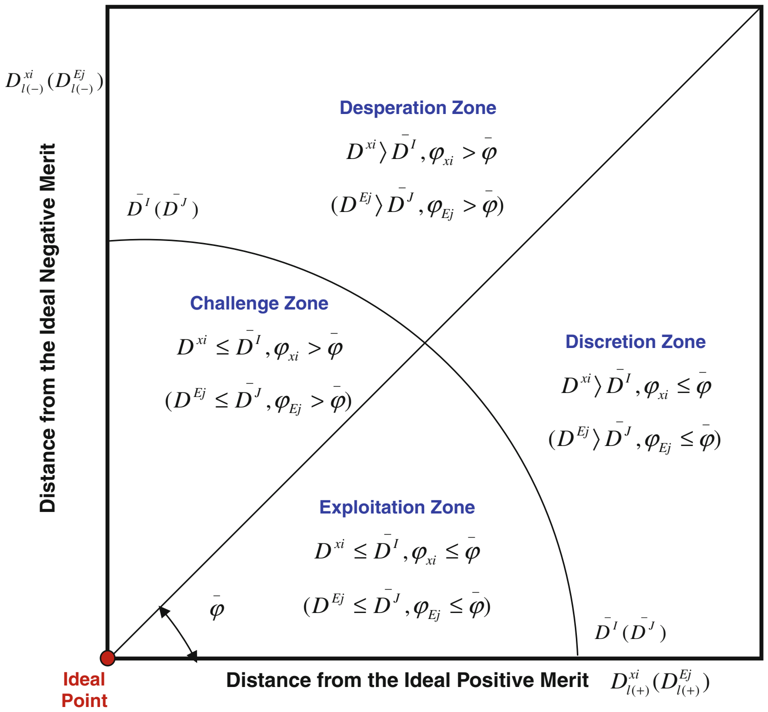

Fig. 2 The four zones and their characteristics

(larger measure of uncertainty). With the *ideal alternative* (group of alternatives) $\left(v^*(l^+) = 0, v^*(l^-) = 0\right)$ $\left(v^*J(l^+) = 0, v^*J(l^-) = 0\right)$ as the origin, the mean Euclidean distance $\left(\bar{D}^I(\bar{D}^J)\right)$ and angle $(\bar{\phi})$ divide the graph into four zones:

- *Exploitation Zone*: In this zone $D^{xi} \leq \bar{D}^I \left(D^{Ej} \leq \bar{D}^J\right)$ and $\varphi_{xi} \leq \bar{\varphi} \left(\varphi_{Ej} \leq \bar{\varphi}\right)$. This area represents a small amount of negative merits and a great deal of positive merits. Alternatives falling into this zone are close to the *ideal alternative* (group of alternatives) at the origin. These alternatives (groups) are considered very attractive because they have little risk but demonstrate tremendous potentials.
- *Challenge Zone*: In this zone $D^{xi} \leq \bar{D}^I (D^{Ej} \leq \bar{D}^J)$ and $\varphi_{xi} > \bar{\varphi} \left(\varphi_{Ej} > \bar{\varphi}\right)$. This area represents a great deal of both positive and negative merits. Alternatives (groups) falling into this zone are close to the *ideal alternative* (group of alternatives) at the origin. These alternatives (groups) are considered challenging because they are very risky but also exhibit tremendous potential. This zone requires full use of the organization's capabilities and resources.
- *Discretion Zone*: In this zone $D^{xi} > \bar{D}^I \left(D^{Ej} > \bar{D}^J\right)$ and $\varphi_{xi} \leq \bar{\varphi} \left(\varphi_{Ej} \leq \bar{\varphi}\right)$. This area represents a small amount of positive and negative merits. Alternatives (groups) falling into this zone are far from the *ideal alternative* (group of alternatives) at the origin. These alternatives (groups) are considered discretionary because they are not very risky and do not demonstrate any meaningful potential. This zone represents the area where the DMs have freedom or power to act or judge on their own.

- *Desperation Zone*: In this zone $D^{xi} > \bar{D}^{I} \left(D^{Ej} > \bar{D}^{J}\right)$ and $\varphi_{xi} > \bar{\varphi} \left(\varphi_{Ej} > \bar{\varphi}\right)$. This area represents a large amount of negative merits and very little positive merits. Alternatives (groups) falling into this zone are far from the *ideal alternative* (group of alternatives) at the origin. These alternatives (groups) should be considered as a last resort because they are very risky and do not exhibit significant potential.

4 Pipeline route evaluation case study

The following study was conducted for Horizon Oil Company,[1] a multinational oil and natural gas producer established with the objective of the exploration, development, production, marketing and sales of crude oil and natural gas. The study was intended to develop a structured framework to aggregate multiple objective and subjective judgments for pipeline route planning in the Caspian Sea region. We followed the procedure described in the previous section to assess five alternative routes in this region.

4.1 Identify DMs and their voting power

Five groups were selected by Horizon to participate in this study ($K = 5$).

4.2 Identify a finite set of alternatives

Five routes ($J = 5$) were identified by Horizon as feasible options for transporting the oil and gas out of the Caspian Sea region to the world markets going through 14 countries ($I = 14$). Each country is included only in one region.

- *Northern Route* ($E1$) (*Ukraine*, x_1^1 *and Russia*, x_2^1)
- *The Western Route* ($E2$) (*Azerbaijan*, x_3^2, *Georgia*, x_4^2, *Armenia*, x_5^2 *and Turkey*, x_6^2)
- *The Southern Route* ($E3$) (*Iran*, x_7^3)
- *The Eastern Route* ($E4$) (*Kazakhstan*, x_8^4, *Uzbekistan*,x_9^4, *Kyrgyzstan*, x_{10}^4 *and Tajikistan*, x_{11}^4)
- *The Southeastern Route* ($E5$) (*Turkmenistan*, x_{12}^5, *Afghanistan*, x_{13}^5 *and Pakistan*, x_{14}^5)

4.3 Identify relevant factors/sub-factors and group them into clusters

Initially, the group of five experts (DMs) appointed by Horizon used brainstorming to establish 79 potential factors ($N = 79$) that could influence route selection. Subsequently, the DMs met again as a group and classified the 79 factors into eight groups

[1] The name of the company and some details of the study are changed to protect the anonymity of the company.

($M = 8$). Following these group meetings, all DMs individually categorized each of the 79 potential factor into the opportunity (O) ($C1$), threat (R) ($C2$), and transient (T) ($C3$) clusters ($L = 3$). The opportunity, threat, and transient factors along with their individual and group weights for the five DMs are shown in Tables 1, 2 and 3, respectively. The weights of criteria $\left(w^k_{lmn}\right)$ and their groups $\left(w^k_{lm}\right)$ are obtained through the ANP described next.

4.4 Establish a hierarchy/network of factor dependencies and define importance weights of its elements using the ANP

Next, we met with the DMs and constructed a network model presented in Fig. 3. There were two different kinds of dependencies in the network – the first was between the elements within each group of the opportunities, threats, and transient factors; the second was between cluster dependencies. The directions of the arrows signified dependence. An example of a between cluster dependency was the dependency between ability to expand current pipelines and earthquake activity; and an example of a within cluster dependency was the interdependency between the level of export in the region/pipeline countries and society openness.

The clusters of opportunities, threats and transient factors had equal importance weights $\left(w^k_l = 1, \forall k\right)$. Tables 1, 2 and 3 show the weights of criteria and their groups obtained through the ANP by the five DMs.

4.5 Develop scoring system for subjective criteria and assign fuzzy scores to alternatives on each criterion

Subjective judgments were obtained from our five expert DMs who considered three clusters of factors, altogether including 79 criteria. The DMs provided their judgments independently and assigned fuzzy scores to the alternative routes on the scale from 0 to 100 ($S = 100$) on all the criteria. These scores reflected expert opinions about the intensity of the opportunities and threats. Higher alternative scores were preferred for opportunities and lower alternative scores were preferred for threats. The fuzzy numbers of triangular type on two α-levels of 0 and 1 were used to treat the DMs' judgments. Table 4 shows the fuzzy scores of the economical threats for Ukraine assigned by the five DMs (cluster $l = 1$ (threats) and group $m = 1$ (economical)) and Fig. 4 shows an example graphical representation of the scores assigned by one DM to Ukraine on four different criteria from the opportunity, threat, and transient clusters.

4.6 Normalize all estimates to obtain identical units of measurement

In this study we used a unique dimensionless measurement scale from 0 to 100 for all the subjective estimates. These scores $\left(p^{ki}_{lmn}\right)$ were normalized using Eq. (6). We used distributive normalization to make sure that all the normalized scores add up to one

Table 1 The opportunity factors ($l = 1$)

Criteria groups (m)	Weights of criteria groups $\left(w_{lm}^{k}\right)$					Criteria (n)	Criteria weights $\left(w_{lmn}^{k}\right)$				
	DM 1 $k=1$	DM 2 $k=2$	DM 3 $k=3$	DM 4 $k=4$	DM 5 $k=5$		DM 1 $k=1$	DM 2 $k=2$	DM 3 $k=3$	DM 4 $k=4$	DM 5 $k=5$
O.1. Economical	0.20	0.21	0.20	0.18	0.24	O.1.1. Financial support of the international community	0.15	0.14	0.15	0.17	0.17
						O.1.2. Availability of investment tax credits for oil and gas explorations in the region/pipeline countries	0.11	0.10	0.10	0.11	0.10
						O.1.3. High ROI potentials	0.12	0.12	0.15	0.14	0.12
						O.1.4. Financial support of the region/pipeline countries for oil and gas explorations	0.12	0.11	0.10	0.12	0.14
						O.1.5. Availability of cheap labor in the region/pipeline countries	0.05	0.05	0.06	0.05	0.05
						O.1.6. High level of export in the region/pipeline countries	0.09	0.12	0.12	0.10	0.12
						O.1.7. Potential for high and stable energy demand in the region/pipeline countries	0.15	0.15	0.09	0.16	0.14
						O.1.8. High level of GDP in the region/pipeline countries	0.21	0.21	0.23	0.15	0.16
O.2. Political	0.25	0.30	0.30	0.27	0.26	O.2. 1. Political support of the neighboring countries for the project	0.35	0.35	0.40	0.35	0.45
						O.2.2. Political support of the international community for the project	0.65	0.65	0.60	0.65	0.55
O.3. Technological	0.18	0.18	0.15	0.15	0.15	O.4. 1. Ability to maintain and repair current pipelines	0.50	0.50	0.50	0.45	0.45
						O.4.2. Ability to expand current pipelines	0.40	0.40	0.45	0.45	0.35
						O.4.3. Ability to convert natural gas to liquid gas	0.10	0.10	0.05	0.10	0.20

Table 1 continued

Criteria groups (m)	Weights of criteria groups $\left(w_{lm}^{k}\right)$					Criteria (n)	Criteria weights $\left(w_{lmn}^{k}\right)$				
	DM 1 $k=1$	DM 2 $k=2$	DM 3 $k=3$	DM 4 $k=4$	DM 5 $k=5$		DM 1 $k=1$	DM 2 $k=2$	DM 3 $k=3$	DM 4 $k=4$	DM 5 $k=5$
O.4. Social	0.13	0.10	0.15	0.15	0.10	O.5. 1. Open society	0.25	0.50	0.50	0.40	0.40
						O.5.2. Availability of jobs and public assistance programs	0.40	0.30	0.30	0.35	0.25
						O.5.3. Educated and trained workers	0.35	0.20	0.20	0.35	0.25
O.5. Geographical	0.30	0.21	0.20	0.25	0.25	O.6.1. Accessibility to open sea and oceans	0.15	0.20	0.20	0.10	0.15
						O.6.2. Suitable beaches with calm waves	0.05	0.10	0.10	0.10	0.15
						O.6.3. Shorter distance	0.80	0.70	0.70	0.80	0.70

Table 2 The threat factors ($l = 2$)

Criteria groups (m)	Weights of criteria groups $\left(w_{lm}^{k}\right)$					Criteria (n)	Criteria weights $\left(w_{lmn}^{k}\right)$				
	DM 1 $k=1$	DM 2 $k=2$	DM 3 $k=3$	DM 4 $k=4$	DM 5 $k=5$		DM 1 $k=1$	DM 2 $k=2$	DM 3 $k=3$	DM 4 $k=4$	DM 5 $k=5$
R.1. Economical	0.20	0.25	0.19	0.20	0.23	R.1.1. High tax rate in the region/pipeline countries	0.12	0.13	0.10	0.12	0.12
						R.1.2. High cost of building and maintaining pipelines in the region/pipeline countrie	0.25	0.20	0.24	0.19	0.18
						R.1.3. High level of tariffs and commissions in the region/pipeline countries	0.13	0.13	0.11	0.13	0.12
						R.1.4. High cost of oil and gas transportation and transfer in the region/pipeline countries	0.17	0.16	0.17	0.17	0.18
						R.1.5. High oil and gas drilling and exploration expenses in the region/pipeline countries	0.21	0.20	0.19	0.17	0.18
						R.1.6. Negative effect of pipelines on other industries such as tourism and fishing	0.07	0.08	0.09	0.11	0.09
						R.1.7. Economic dependency of the region/pipeline countries to other countries	0.05	0.10	0.10	0.11	0.13
R.2. Political	0.25	0.30	0.23	0.20	0.25	R.2.1. Threat of terrorism in the region/pipeline countries	0.80	0.60	0.85	0.80	0.60
						R.2.2. Nuclear proliferation initiatives in the region/pipeline countries	0.05	0.10	0.04	0.05	0.10
						R.2.3. Foreign oil and gas dependency of the region/pipeline countries	0.06	0.10	0.05	0.10	0.15
						R.2.4. Possibility of Russian control of the pipeline	0.09	0.20	0.06	0.05	0.15

Table 2 continued

Criteria groups (m)	Weights of criteria groups $\left(w_{lm}^{k}\right)$					Criteria (n)	Criteria weights $\left(w_{lmn}^{k}\right)$				
	DM 1 $k=1$	DM 2 $k=2$	DM 3 $k=3$	DM 4 $k=4$	DM 5 $k=5$		DM 1 $k=1$	DM 2 $k=2$	DM 3 $k=3$	DM 4 $k=4$	DM 5 $k=5$
R.3. Legal	0.12	0.10	0.15	0.15	0.13	R.3.1. Oil and gas reserve ownership disputes in the region/pipeline countries	1.00	1.00	1.00	1.00	1.00
R.4. Environmental	0.15	0.15	0.13	0.15	0.16	R.4.1. Pollution of the sea surface	0.15	0.14	0.15	0.14	0.15
						R.4.2. Pollution of the sea bottom	0.10	0.11	0.12	0.12	0.12
						R.4.3. Pollution of the beaches	0.13	0.12	0.14	0.14	0.12
						R.4.4. Pollution of the water sources	0.14	0.15	0.13	0.13	0.15
						R.4.5. Pollution of the water destinations	0.12	0.11	0.12	0.12	0.11
						R.4.6. Pollution of the rivers and water canals	0.11	0.13	0.11	0.12	0.13
						R.4.7. Pollution caused by nuclear activities	0.05	0.08	0.05	0.08	0.08
						R.4.8. Availability of underground water sources along the route	0.20	0.16	0.18	0.15	0.14

Table 3 The transient factors ($l = 3$)

Criteria groups (m)	Weights of criteria groups $\left(w_{lm}^{k}\right)$					Criteria (n)	Criteria weights $\left(w_{lmn}^{k}\right)$				
	DM 1 $k = 1$	DM 2 $k = 2$	DM 3 $k = 3$	DM 4 $k = 4$	DM 5 $k = 5$		DM 1 $k = 1$	DM 2 $k = 2$	DM 3 $k = 3$	DM 4 $k = 4$	DM 5 $k = 5$
I.1. Economical	0.18	0.20	0.17	0.16	0.18	I.1.1. Investment security in the region/pipeline countries	0.15	0.13	0.20	0.15	0.13
						I.1.2. The quality and productivity of the labor force in the region/pipeline countries	0.10	0.1	0.09	0.10	0.10
						I.1.3. Economic stability of the region/pipeline countries	0.15	0.14	0.15	0.15	0.13
						I.1.4. Non-oil and gas import/export level in the region/pipeline countries	0.05	0.1	0.05	0.08	0.11
						I.1.5. Oil and gas quality	0.15	0.15	0.11	0.12	0.15
						I.1.6. Current oil and gas supply	0.20	0.19	0.20	0.20	0.19
						I.1.7. Forecasted oil and gas supply	0.20	0.19	0.20	0.20	0.19
I.2. Political	0.20	0.20	0.20	0.21	0.18	I.2.1. Security concerns in the region/pipeline countries	0.40	0.3	0.40	0.35	0.35
						I.2.2. Political stability of the region/pipeline countries	0.40	0.45	0.40	0.35	0.40
						I.2.3. Military stability of the region/pipeline countries	0.20	0.25	0.20	0.30	0.25
I.3. Legal	0.14	0.15	0.14	0.13	0.15	I.3.1. Strict import/export laws and regulations in the region/pipeline countries	0.35	0.3	0.32	0.35	0.30
						I.3.2. Strict foreign investment rules and regulations in the region/pipeline countries	0.40	0.45	0.38	0.35	0.40
						I.3.3. Availability and stability of insurance industry in the region/pipeline countries	0.25	0.25	0.30	0.30	0.30

Table 3 continued

Criteria groups (m)	Weights of criteria groups $\left(w_{lm}^{k}\right)$					Criteria (n)	Criteria weights $\left(w_{lmn}^{k}\right)$				
	DM 1 $k=1$	DM 2 $k=2$	DM 3 $k=3$	DM 4 $k=4$	DM 5 $k=5$		DM 1 $k=1$	DM 2 $k=2$	DM 3 $k=3$	DM 4 $k=4$	DM 5 $k=5$
I.4. Technological	0.12	0.15	0.13	0.13	0.15	I.4.1. Scientific and technological foundation of the society	0.20	0.16	0.20	0.25	0.15
						I.4.2. Adequacy of the oil and gas refineries	0.11	0.1	0.11	0.11	0.10
						I.4.3. Adequacy of oil and gas transportation infrastructure	0.26	0.26	0.30	0.25	0.25
						I.4.4. Adequacy of the railroad infrastructure	0.09	0.1	0.06	0.05	0.10
						I.4.5. Adequacy of roads with proper surface and foundation	0.09	0.1	0.09	0.09	0.10
						I.4.6. Adequacy of ports for oil and gas transportation	0.11	0.12	0.11	0.15	0.14
						I.4.7. Adequacy of technologically advanced oil and gas tankers	0.08	0.08	0.06	0.05	0.08
						I.4.8. Adequacy of technologically advanced oil and gas trucks	0.08	0.08	0.07	0.05	0.08
I.5. Cultural	0.13	0.15	0.12	0.13	0.15	I.5.1. Common language in the region/pipeline countries	0.30	0.27	0.28	0.25	0.25
						I.5.2. Common religion in the region/pipeline countries	0.14	0.16	0.14	0.11	0.14
						I.5.3. Common religious sect in the region/pipeline countries	0.20	0.2	0.21	0.14	0.18
						I.5.4. Common race in the region/pipeline countries	0.09	0.1	0.09	0.23	0.15

Table 3 continued

Criteria groups (m)	Weights of criteria groups $\left(w_{lm}^{k}\right)$					Criteria (n)	Criteria weights $\left(w_{lmn}^{k}\right)$				
	DM 1 $k=1$	DM 2 $k=2$	DM 3 $k=3$	DM 4 $k=4$	DM 5 $k=5$		DM 1 $k=1$	DM 2 $k=2$	DM 3 $k=3$	DM 4 $k=4$	DM 5 $k=5$
						I.5.5. Common culture and customs in the region/pipeline countries	0.10	0.09	0.10	0.09	0.09
						I.5.6. Common national identity in the region/pipeline countries	0.08	0.09	0.09	0.09	0.10
						I.5.7. Common history in the region/pipeline countries	0.09	0.09	0.09	0.09	0.09
I.6. Social	0.04	0.05	0.04	0.05	0.05	I.6.1. Familiarity of the society with oil and gas industries	0.55	0.6	0.50	0.55	0.60
						I.6.2. Traffic obstacles	0.45	0.4	0.50	0.45	0.40
I.7. Geographical	0.19	0.10	0.20	0.19	0.14	I.7.1. Soil condition and quality	0.20	0.25	0.30	0.40	0.35
						I.7.2. Accessibility and availability of oil and gas reserves in the region/pipeline countries	0.80	0.75	0.70	0.60	0.65

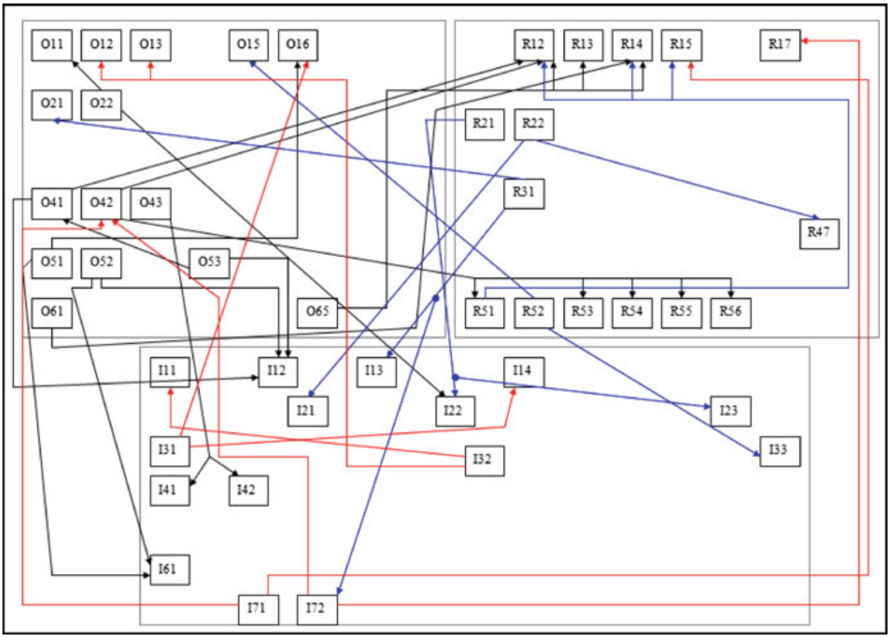

Fig. 3 Some network dependencies among the decision criteria

making it possible to see the contribution of each criterion to the total performance of the country. It was important that the scores lie in the 0 to 1 interval because they were used to calculate the uncertainty of the DMs' opinions (entropy) and had to satisfy the conditions of logarithmical operations.

4.7 Defuzzify and integrate the weights and the scores

Next, we applied a two-step defuzzification process described in the previous section to integrate eight sets of criteria group weights ($M = 8$), 79 factor weights ($N = 79$), and set of fuzzy normalized subjective probabilities into one set of crisp values for the entire group of five DMs ($K = 5$) for the 14 producing/transiting countries ($I = 14$) and the five pipeline routes ($J = 5$).

The graphical representation of the level-2 fuzzy set of the estimates for Ukraine on the criterion financial support of the international community (O.1.1) is given in Fig. 5. The five triangles are discrete parts of the fuzzy set of the DMs' judgments, where each triangle is a fuzzy function itself. After normalization and the first step of the defuzzification process for the judgments depicted on Fig. 5, we obtain a fuzzy set of discrete numbers which is shown in Fig. 6.

In the final stage of the defuzzification process, we used Eq. (13) for discrete fuzzy sets after a single number was obtained for each country with respect to each criterion within the classes of opportunities, threats, and transient factors $\left(\mathscr{D}2^i_{Omn}, \mathscr{D}2^i_{Rmn}, \mathscr{D}2^i_{Tmn}\right)\quad i = 1, \ldots, 14;\quad m = 1, \ldots, 5;\quad n = 1, \ldots, 79.$

Table 4 The fuzzy scores of the economical threats for Ukraine

Criteria (n)	Ukraine (i)														
	Left bound ($\mu = 0$) $p_{lmn}^{ki}(left)$					Score ($\mu = 1$) $p_{lmn}^{ki}(top)$					Right bound ($\mu = 0$) $p_{lmn}^{ki}(right)$				
	DM1 ($k = 1$)	DM2 ($k = 2$)	DM3 ($k = 3$)	DM4 ($k = 4$)	DM5 ($k = 5$)	DM1 ($k = 1$)	DM2 ($k = 2$)	DM3 ($k = 3$)	DM4 ($k = 4$)	DM5 ($k = 5$)	DM1 ($k = 1$)	DM2 ($k = 2$)	DM3 ($k = 3$)	DM4 ($k = 4$)	DM5 ($k = 5$)
R.1.1. High tax rate in the region/pipeline countries	55	40	50	55	50	60	70	60	60	60	70	80	75	70	70
R.1.2. High cost of building and maintaining pipelines in the region/pipeline countries	60	40	50	55	55	70	70	65	65	60	80	80	75	75	70
R.1.3. High level of tariffs and commissions in the region/pipeline countries	60	45	55	60	55	75	75	70	70	60	85	85	80	80	70
R.1.4. High cost of oil and gas transportation and transfer in the region/pipeline countries	60	40	55	60	55	75	75	75	75	60	85	85	85	80	70
R.1.5. High oil and gas drilling and exploration expenses in the region/pipeline countries	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
R.1.6. Negative effect of pipelines on other industries such as tourism and fishing	25	0	10	30	35	30	5	25	45	45	45	10	40	50	55
R.1.7. Economic dependency of the region/pipeline countries to other countries	30	20	25	40	25	45	30	35	45	30	50	40	45	55	40

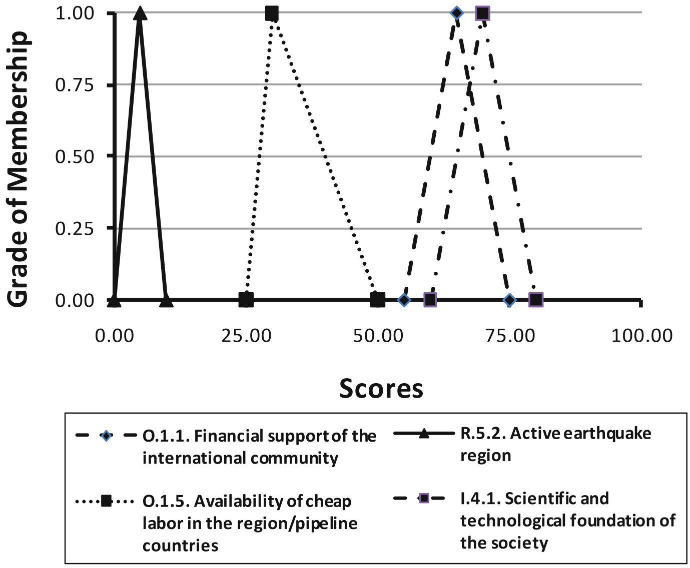

Fig. 4 The triangle fuzzy numbers for the first DM's estimates on different criteria

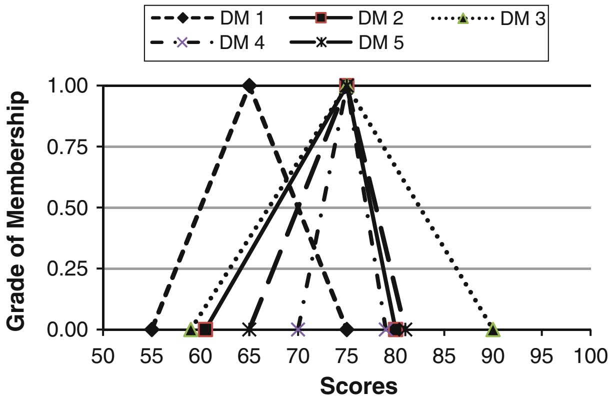

Fig. 5 The level-2 fuzzy set of the DMs' judgments of Ukraine on the criterion O.1.1

4.8 Aggregate crisp normalized factor estimates for each alternative

The defuzzified and integrated total weights and scores for the countries ($v_{xi}(l)$) and alternative routes ($v_{Ej}(l)$) were obtained using Eqs. (14) and (15) and are represented in Tables 5 and 6, respectively. The tables also represent the priorities normalized using the ideal mode when all the values are distributed in the interval [0,1]. The data on the transient factors ($l = 3$) has been distributed among the groups of opportunities ($l = 1$) and threats ($l = 2$), subject to their positive/negative performance for each particular alternative.

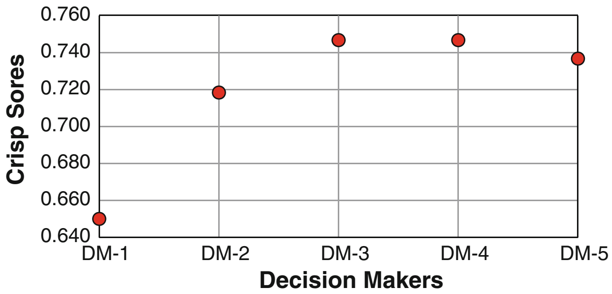

Fig. 6 The fuzzy set of the DMs' crisp scores for Ukraine on the criterion O.1.1

Table 5 The defuzzified priorities for the producing/transiting countries

Country	Opportunities	Normalized opportunities	Threats	Normalized threats
Ukraine	27.27	0.75	11.44	0.03
Russia	32.66	1.00	13.82	0.32
Azerbaijan	22.57	0.53	18.46	0.88
Georgia	18.00	0.31	16.86	0.68
Armenia	12.69	0.06	15.38	0.51
Turkey	26.04	0.69	13.02	0.22
Iran	24.64	0.62	17.50	0.76
Turkmenistan	20.14	0.41	15.78	0.55
Afghanistan	11.40	0.00	11.19	0.00
Pakistan	22.70	0.53	11.94	0.09
Kazakhstan	20.70	0.44	19.49	1.00
Uzbekistan	18.13	0.32	17.43	0.75
Kyrgyzstan	11.80	0.02	13.58	0.29
Tajikistan	11.53	0.01	12.49	0.16

Table 6 The defuzzified priorities of the regions

Region	Opportunities	Normalized opportunities	Threats	Normalized threats
Northern Region	59.93	0.65	25.26	0.17
Western Region	79.31	1.00	63.71	1.00
Southern Region	24.64	0.00	17.50	0.00
Eastern Region	62.15	0.54	38.91	0.46
Southeastern Region	54.23	0.69	63.00	0.98

4.9 Calculate the entropy for all alternatives as a measure of judgment uncertainty

In this step, we first calculated the entropy of the DMs' judgments for each alternative country on each criterion separately. Next, we aggregated these characteristics for the

Table 7 The normalized entropies of the producing/transiting countries

Country	Opportunity	Threat
Ukraine	0.106	0.076
Russia	0.113	0.092
Azerbaijan	0.104	0.090
Georgia	0.100	0.082
Armenia	0.079	0.068
Turkey	0.101	0.084
Iran	0.112	0.092
Turkmenistan	0.116	0.070
Afghanistan	0.093	0.048
Pakistan	0.102	0.070
Kazakhstan	0.112	0.093
Uzbekistan	0.116	0.081
Kyrgyzstan	0.107	0.059
Tajikistan	0.101	0.054

Table 8 The normalized entropies of the regions

Region	Opportunities	Threats
Northern region	0.110	0.084
Western region	0.096	0.081
Southern region	0.112	0.092
Eastern region	0.109	0.072
Southeastern region	0.104	0.063

groups of criteria and for the entire opportunity, threat, and transient classes. Since the initial estimates were given in the form of fuzzy numbers and we considered discrete fuzzy sets of subjective scores and weights (sets of five triangular fuzzy numbers $K = 5$), it was reasonable to use the formula for fuzzy entropy proposed by De Luca and Termini (1972) given in Eq. (18), where the normalization constant $\beta = 1$. The normalized entropies of the alternative countries and regions are shown in Tables 7 and 8.

4.10 Identify the ideal alternative and define the position of the decision alternatives with respect to the ideal one

In order to rank the 14 producing/transiting countries, we first identified the ideal country with maximum opportunities ($v^*(l^+) = 32.66$) and minimum threats ($v^*(l^-) = 11.19$). We then used Eq. (18) to find the Euclidean distance of the producing/transiting countries from the ideal country. Russia, with a Euclidean distance of 2.63, was the closest country to the ideal country and Afghanistan, with a

Euclidean distance of 21.26, was the furthest country from the ideal country. The Euclidean distances of the remaining countries were: Ukraine (5.39), Turkey (6.86), Pakistan (9.99), Iran (10.21), Azerbaijan (12.43), Turkmenistan (13.33), Kazakhstan (14.56), Georgia (15.71), Uzbekistan (15.81), Armenia (20.40), Kyrgyzstan (20.99) and Tajikistan (21.17).

Next, we identified the ideal region with maximum opportunities $(v^* J(l^+) = 79.31)$ and minimum threats $\left(v^* J(l^-) = 17.50\right)$. We then used Eq. (19) to find the Euclidean distance of the five regions from the ideal region. The Northern region with a Euclidean distance of 20.87 was the region closest to the ideal region and the Southern region with a Euclidean distance of 54.67 was the region furthest from the ideal region. The Euclidean distances of the remaining regions were: Southeastern region (32.97), Western region (46.21) and Eastern region (48.62).

Next, we considered the entropy information by finding the ideal entropy for the opportunities $(e^*(l^+) = 11.87)$ and the ideal entropy for the threats $(e^*(l^-) = 7.59)$. We then used Eq. (22) to find the Euclidean distances of the producing/transiting countries from the ideal one (De^{xi}) as: Afghanistan (1.29), Tajikistan (2.22), Kyrgyzstan (3.13), Armenia (5.67), Georgia (7.70), Turkmenistan (8.47), Uzbekistan (8.67), Pakistan (9.26), Kazakhstan (10.48), Turkey (10.66), Azerbaijan (11.74), Ukraine (11.56), Iran (11.96), and Russia (16.17).

Finally, we found the ideal entropy (minimum) for both opportunities and threats among the regions. The Southern region with $e^* J(l^+) = 23.05$ and $e^{*J}(l^-) = 11.86$ had the ideal entropy. We used Eq. (23) and found the Euclidean distances of the five regions from the ideal region as: Southern region (0), Northern region (29.06), Southeastern region (34.74), Eastern region (53.92), and Western region (63.24).

4.11 Classify and rank the alternatives and their groups using numerical information and diagrams, taking into consideration the level of uncertainty of their fuzzy characteristic

We used the scheme introduced in Fig. 2 to plot the 14 countries on the diagram shown in Fig. 7. In this figure, the bubbles represent the alternative countries. The centers of the bubbles are positions of the alternatives relative to the ideal point. The sizes of the bubbles reflect the distances of the entropy of the alternatives from the ideal entropy. For example, Russia fell into the challenge zone while Ukraine and Turkey fell into the exploitation zone.

Next, we considered the entropy information provided in Fig. 7 to rank the 14 countries based on their Euclidean distance from the ideal country (D^{xi}). Table 9 shows the overall rankings before and after calibration for risky and risk-averse scenarios.

Russia, in spite of its high risk level, fell into the challenge zone because of its minimal Euclidean distance. Ukraine, Turkey and Pakistan with little opportunities and minimal threats fell into the exploitation zone. Ukraine was ranked first followed by Turkey. Pakistan with its low threats and low entropy fell primarily into the exploitation zone. Considering a risky scenario, Russia could drop to the third position leaving Pakistan in the fourth position. However, if a risk-averse scenario is considered,

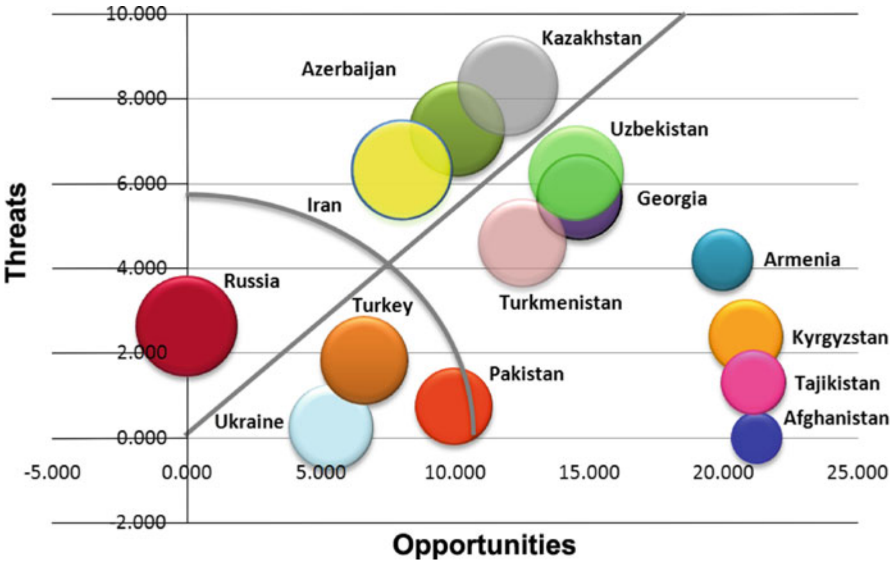

Fig. 7 A graphical classification of the producing/transiting countries

Table 9 The overall rankings before and after calibration for risky and risk-averse scenarios

Country	Priority			Zone	
	Before calibration	After calibration (Risky scenario)	After calibration (Risk-Averse scenario)	Major	Minor
Russia	1	3	4	Challenge	–
Ukraine	2	1	1	Exploitation	–
Turkey	3	2	2	Exploitation	–
Pakistan	4	4	3	Exploitation	Discretion
Iran	5	12	12	Desperation	–
Azerbaijan	6	13	13	Desperation	Discretion
Turkmenistan	7	7	9	Discretion	–
Kazakhstan	8	14	14	Desperation	Discretion
Georgia	9	8	10	Discretion	–
Uzbekistan	10	9	11	Discretion	–
Armenia	11	13	8	Discretion	–
Kirgizstan	12	12	7	Discretion	–
Tajikistan	13	11	6	Discretion	–
Afghanistan	14	10	5	Discretion	–

Russia and Pakistan will switch their rankings. Similar analysis was conducted for the remaining countries.

Next, we classified and ranked the alternative regions (pipeline routes). As depicted in Fig. 8, the Northern region (Russia and Ukraine) was the best alternative route as it fell into the exploitation zone. This high ranking was enforced by the high rankings of

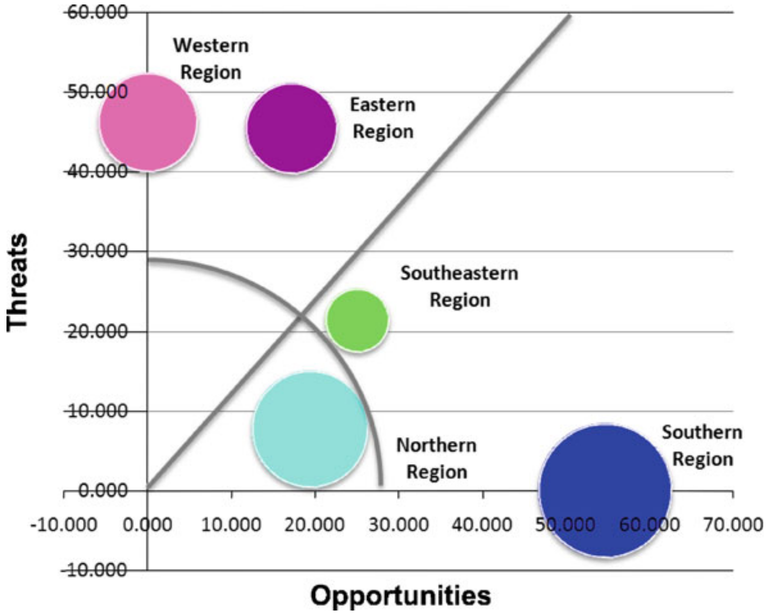

Fig. 8 A graphical classification of the alternative regions/routes

Russia and Ukraine independently. The Southeastern region (Pakistan, Turkmenistan, and Afghanistan) with the second best distance from the ideal region and the lowest entropy fell into the discretion zone and was ranked second. Pakistan, Turkmenistan, and Afghanistan had moderately positive characteristics. Pakistan fell primarily in the exploitation zone while Turkmenistan and Afghanistan fell in the discretion zone. The Southern region with the largest Euclidean distance (but not much different from the Western and Eastern regions) and an ideal threat was ranked third. The fourth and fifth rankings were the Eastern and Western routes respectively as they fell into the desperation zone.

5 Conclusions and future research directions

The method proposed in this study promotes consistent and systematic alternative evaluation and selection throughout the organization. Judgments captured as separate importance weights and performance scores are used uniformly across all alternatives in the evaluation process. In the absence of separate value judgments, it is difficult to apply a set of importance weights and performance scores consistently among the opportunities and threats when evaluating alternatives. Our method provides a consistent combination of all assessments among all the alternatives. Whether the assessments faithfully represent real-world circumstances depends on the competence and degree of effort the DMs exert in making the assessments.

Our method is also flexible in that it can easily be adapted by other organizations, for example, by substituting other factors from those listed here or adding additional factors. Another advantage of our method is the ease with which it places both inherently subjective criteria (e.g., common culture or religious freedom in the region) and more objective criteria (e.g., oil and gas production in the region) on a common measuring scale. Sensitivity analyses of the results can be performed in order to understand the impact of certain decision parameters (e.g. individual preference structure or the voting power of the DMs) on the final result. Sensitivity analyses can also address questions about the sensitivity of final results to changes in the relative importance of the opportunities and threats, and the performance scores.

Using a structured, step-by-step approach like the method proposed in this study is not intended to imply a deterministic approach to MCDA. While our framework enables DMs to crystallize their thoughts and organize data by placing both inherently subjective criteria and more objective criteria on a common measuring scale, it should be used very carefully. As with any decision analysis model, the researchers and practicing managers must be aware of the limitations of subjective estimates. Our approach should not be used blindly to plug-in numbers and crank-out solutions. The effectiveness of the model relies heavily on the ability and willingness of DMs to provide sound judgments. Potentially, DMs could make poor judgments as they do with any approach. As Russo and Schoemaker (1989) note, considerable research indicates that DMs can maximize their chances of making the best choice(s) if they find a systematic way to evaluate all the evidence favorable or unfavorable to each alternative (i.e., use a subjective linear model such as the one described here). Still, in most applied settings, it is not possible to demonstrate the accuracy of subjective linear models. In contrast, where the same decision is made repeatedly, data on the outcomes of past decisions are available, and one expects the future to resemble the past, objective linear models (e.g., multiple regression) can be used to determine the optimal set of predictors (e.g., the set that accounts for the most variance in the outcome being predicted). But for many decisions, including the one described here, there are no objective outcomes of past decisions. For example, there is no objective index that can be used to evaluate whether the solution resulting from our approach is optimal. In such situations, rigorous subjective linear models such as the framework illustrated here are likely to provide the best hope for optimizing the quality of decisions and the acceptability of those decisions to organizational stakeholders.

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Fuzzy multiple criteria base realignment and closure (BRAC) benchmarking system at the Department of Defense

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Abstract

Purpose – The US Government adopted the base realignment and closure (BRAC) to resolve the military, economic and political issue of excess base capacity. There have been five rounds of BRAC since 1988, and more are expected to come in the years ahead. The complexity of the closure and realignment decisions and the plethora of factors that are often involved necessitate the need for a sound theoretical framework to structure and model the decision-making process. This paper aims to address the issues.

Design/methodology/approach – The paper presents a multiple criteria benchmarking system that integrates the employment, environmental, financial, strategic, and tactical impacts of the closure and realignment decisions into a weighted-sum measure called the "survivability index." The proposed index is used to determine whether the returns generated by each military base on the Department of Defense (DoD) hit list meet a sufficient target benchmark.

Findings – There is a significant amount of evidence that intuitive decision making is far from optimal and it deteriorates exponentially with problem complexity. The benchmarking system presented in this study helps decision makers (DMs) crystallize their thoughts and reduce the environmental complexities inherent in the BRAC decisions. The presented model is intended to create an even playing field for benchmarking and pursuing consensus not to imply a deterministic approach to BRAC decisions.

Originality/value – An iterative process is used to consistently analyze the objective and subjective judgments of multiple DMs within a structured framework based on the analytic network process and fuzzy logic. This iterative and interactive preference modeling procedure is the basic distinguishing feature of the presented model as opposed to statistical and optimization decision-making approaches.

Keywords Benchmarking, Decision theory, Fuzzy logic, United States of America, Government agencies

Paper type Research paper

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1. Introduction

It is best, Sun Tzu said, to prepare for war in peace and to prepare for peace in war. The Department of Defense (DoD) adopted the base realignment and closure (BRAC) process as a national strategy to resolve the military, economic, and political issue of excess base capacity created by the collapse of the former Soviet Union. As forces were drawn down, excess base capacity was created. BRAC was brought together to evaluate the USA population of bases on certain criteria and set forth a recommendation to the Secretary of Defense to close some bases and realign others. The strategic and financial impacts of BRAC are immense. When bases are closed or realigned, the community is dramatically affected by losing/gaining jobs and environmental affects. Economic issues in terms of costs and savings are of great importance in BRAC. People, communities, and environmental impacts are direct consequences of the closure and realignment efforts. The immediate fears of base closures are the loss of jobs in the adjacent communities. Directly tied to the future reuse of closed military installations are the cleanup of known environmental contamination. Beginning in 1988, Congress authorized the DoD to conduct five rounds of BRC including the recent round in 2005. At the completion of all five rounds, the DoD had 130 fewer major bases, 84 major realignments and hundreds of other smaller facilities realigned (United States Government Accountability Office, 2007). Table I provides a general overview of BRAC Activities since its initiation in 1988.

Legislation authorizing BRAC has stipulated that closure and realignment decisions must be based upon selection criteria, a current force structure plan and infrastructure inventory developed by the Secretary of Defense. The criteria historically included employment, environmental, financial, strategic and tactical impacts. BRAC is essentially a multi-criteria capital budgeting problem where the Commission is charged to determine whether the military bases on the hit list should be left alone, realigned or closed. Ideally, the Commission should pursue those military bases that enhance shareholder (American public) value. A large body of intuitive and analytical multi-criteria capital budgeting models has evolved over the last several decades to assist decision makers (DMs) in strategic decision making. While these models have made great strides, the intuitive models lack a structured framework and the analytical models do not capture intuitive preferences.

We present a structured multi-criteria benchmarking framework that processes objective and subjective estimates provided by a group of DMs with the analytic network process (ANP) and fuzzy logic. The proposed framework provides a set of performance measurements that could be utilized for benchmarking or BRAC decisions. The remainder of the paper is organized as follows. The next section presents the state of the art in multi-criteria decision analysis (MCDA) and

BRAC	Major base closures	Major base realignments	Minor closures and realignments	Costs ($billion)	Annual recurring savings ($billion)
1988	16	4	23	2.7	0.9
1991	26	17	32	5.2	2.0
1993	28	12	123	7.7	2.6
1995	27	22	57	6.5	1.7
2005	33	29	775	31.0	4.0

Table I.
History of BRAC rounds

benchmarking followed by a description of the hierarchical and network model in Section 3. In Section 4, we demonstrate the procedural steps of the model along with the results of a study conducted by the US Navy. Section 6 presents the conclusions and future research directions.

2. State of the art in MCDA and benchmarking

The state of the art in multi-criteria capital budgeting contains hundreds of methods, including scoring methods, economic methods, portfolio methods, and decision analysis methods. Scoring methods use algebraic formulas to produce an overall score in capital budgeting (Osawa and Murakami, 2002; Osawa, 2003). Economic methods use financial models to calculate the monetary payoff of alternative projects (Graves and Ringuest, 1991; Huang, 2008; Kamrad and Ernst, 2001; Lotfi *et al.*, 1998). Portfolio methods evaluate the entire set of projects to identify the most attractive subset (Cooper *et al.*, 1999; Girotra *et al.*, 2007; Mojsilovi *et al.*, 2007; Wang and Hwang, 2007). Cluster analysis, a more specific portfolio method, groups projects according to their support of the strategic positioning of the firm (Mathieu and Gibson, 1993). Decision analysis methods compare various projects according to their expected value (Hazelrigg and Huband, 1985; Thomas, 1985). Finally, simulation, a more specific decision analysis method, uses random numbers and simulation to generate a large number of problems and pick the best outcome (Abacoumkin and Ballis, 2004; Mandakovic and Souder, 1985; Paisittanand and Olson, 2006).

Most of these methods are used to evaluate research and development projects (Coffin and Taylor, 1996; Girotra *et al.*, 2007; Osawa and Murakami, 2002; Osawa, 2003; Wang and Hwang, 2007), information systems projects (Mojsilovi *et al.*, 2007; Paisittanand and Olson, 2006) and capital budgeting projects (Graves and Ringuest, 1991; Mehrez, 1988). Recently, researchers working on project evaluation and selection have focused on MCDA models to integrate the intuitive preferences of multiple DMs into structured and analytical frameworks (Costa *et al.*, 2003; Hsieh *et al.*, 2004; Liesiö *et al.*, 2007; Tavana, 2006). MCDA has also been applied to important military applications involving complex alternatives, conflicting quantitative and qualitative objectives, and major uncertainties. Parnell (2006) compared 10 single-decision applications and 14 portfolio decision value model applications. Ewing *et al.* (2006) developed a similar model to determine the military value of 63 army installations. Additional multi-criteria portfolio decision models used by the military include Archer and Ghasemzadeh (1999); Stummer and Heidenberger (2003).

Finding the "best" MCDA framework is an elusive goal that may never be reached (Triantaphyllou, 2000). Pardalos and Hearn (2002) discuss the importance of exploring ways of combining criteria aggregation methodologies to enable the development of models that consider the DM's preferential system in complex problems. Belton and Stewart (2002) also argue the need for integrating frameworks in MCDA. We propose a multi-criteria BRAC model for benchmarking at the DoD. The model solves complex and judgmental multi-criteria problems by carefully combining a set of well-known and proven techniques in MCDA. This integration allows for the objective data and subjective judgments to be collected and used side-by-side in a weighted sum model (Triantaphyllou, 2000). The proposed MCDA model systematically considers a series of hierarchical and networked factors in a structured framework to develop a measure to determine whether the returns generated by each military base on DoD hit list meet a sufficient target benchmark.

Benchmarking is the systematic comparison of performance elements in an organization against those best practices of relevant organizations and obtaining information that will help the observing organization to identify and implement improvement (Lau *et al.*, 2001). While a number of benchmarking definitions can be found in the literature, they all essentially share the same theme. Benchmarking is a framework within which indicators and best practices are examined in order to identify areas where performance can be improved. Public sector benchmarking has been the subject of numerous studies (Magd and Curry, 2003; Tavana, 2004, 2008; Triantafillou, 2007; Vagnoni and Maran, 2008; Wynn-Williams, 2005). The benchmarking system developed in this study uses a numeric measure called the "survivability index" to help policy makers and the commanding officers of the military bases on the DoD hit list identify their strengths and weaknesses by learning from "best-in-class" and other competing bases on the list. The survivability index is used to identify each military base on the hit list as either efficient, with high benefits and low costs; active, with high benefits and high costs; inactive with low benefits and low costs; and inefficient with low benefits and high costs.

3. The hierarchical and network multi-criteria model

The US Congress has chartered the BRAC Commission to consider employment, environmental, financial, strategic, and tactical impacts of BRAC decisions.

Employment impacts are measured by three sub-factors: direct job changes, indirect job changes and total job changes as a percentage of area employment. Direct job changes are comprised of military, civilian and contractor jobs that are either gained or lost in a certain location due to the change recommended by the Commission. Indirect job changes are those jobs changes that would be indirectly affected (gained or lost) by the recommendation set forth by the Commission.

Environmental impacts are used to measure the impact of the military base on the surrounding environment. For example, the closure of a military chemical depot would require an extensive and costly cleanup. Other examples include the clean up required due to fuel spills on an air force base or weapons disposal at an army munitions depot. Conversely, there are also many instances such as a medical center or a guard station where there is minimal to no environmental impact. Several military bases have already begun an environmental restoration. The costs to complete the environmental restoration as well as the cost that have already been incurred are considered by the Commission. It should be noted that several bases do not have any environmental restoration costs.

Financial impacts are measured by one-time costs, payback period, six year net savings, annual recurring savings and 20 year net present value (NPV) savings. One-time costs are those costs associated with closing a particular base. The Secretary of Defense initially submits an estimated cost, which is then reviewed by the BRAC Commission. Upon the Commissions approval, a final one-time cost is determined. Since federal cost savings is the main driver behind BRACs, the one-time cost plays a very important factor in determining whether to close a base or not. Payback period is the time period it would take to recuperate the one time closing costs through savings incurred by closing the base. The range of payback periods varies but the majority of the bases fall somewhere between 1 and 20 years. Twenty year NPV is the present value of 20 years worth of savings for closing a military base.

Strategic impacts are non-monetary impacts that usually cannot directly be assigned with a value but greatly sway the BRAC decisions. The post cold war era has changed the strategic significance of several bases located in the USA. During the cold war, military installations were placed to defend or attack against the Soviet Union. Depots were maintained at high levels in order to support any conflicts that would arise. With the end of the cold war, the primary purpose of several installations became obsolete. These bases were given new roles, and in some instances, these roles were just as important as their cold war era roles. Strategic impacts are measured using a sliding scale (0 = unimportant to 10 = extremely important).

Tactical impacts are measured by community support, commercial, and residential use of land. Community support for base closures or realignments has generally been low. Military installations have typically benefited the surrounding community by providing jobs, boosting local economies and attracting visitors who may not have otherwise come to town. A military presence also provides a sense of pride for the community, knowing that their community is playing a role in the defense of the USA. The ability for land to be re-used after a closure is also an important factor. The communities affected by a closure need to be able to re-claim the land for either commercial or residential purposes. Some bases are obviously more suited for commercial use. For example, an air base can easily be converted to a private or regional airfield. A naval base can be converted to a commercial ship yard, or due to it is proximity to water; it may be attractive to real estate development. Army bases, depending on location, may also be viable for other uses. As noted earlier, sites with higher environmental clean up costs may not be attractive for any future use as a high clean up cost would indicate some type of on-site contamination. Tactical impacts are also measured using a sliding scale (0 = unimportant to 10 = extremely important).

This study was conducted at a naval facility in the USA with seven naval logistic experts. The expert officers contributed their professional experience to identify factors and sub-factors that influence the BRAC decision and constructed the network presented in Figure 1 based on document reviews and stakeholder analysis. Numerous legal, strategy, policy and planning documents were used to define the military value of the installations on the DoD hit list. The solid lines in this diagram represent the hierarchical dependencies and the dotted arrows represent influence and interdependencies among the BRAC factors and sub-factors.

4. The procedure and results

We use a nine-step procedure to systematically evaluate the bases by plotting them in a 4D space based on their "survivability index." The survivability index is the Euclidean distance from the ideal alternative. Ideal alternative is an unattainable choice that serves as a norm or rationale facilitating a human choice problem. Using the theory of displaced ideal to grasp the extent of the emerging conflict between means and ends, the DM explores the limits attainable with each benefit and cost. As all alternatives are compared, those closer to the ideal are preferred to those farther away. Zeleny (1982, p. 144) shows that the Euclidean measure can be used as a proxy measure of distance. The nine steps used in our model are:

(1) Consider a set of military bases for realignment and benchmarking.

(2) Identify the relevant objective and subjective factors and sub-factors and define their importance weights using the ANP.

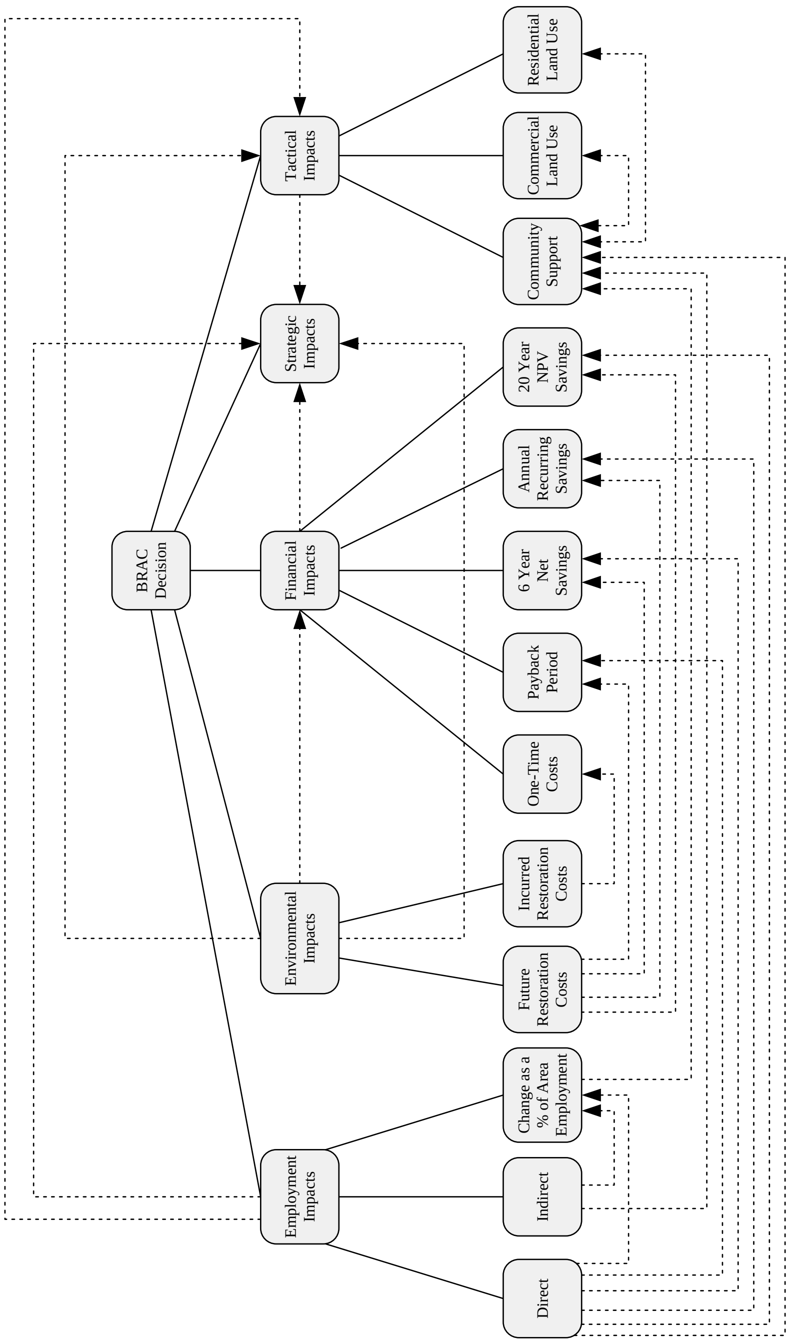

Figure 1.
BRAC hierarchical and network interdependencies

(3) Develop scores for the subjective factors and identify values of the objective factors on each alternative.
(4) Group all the factors into benefit and cost factors.
(5) Normalize all estimates to obtain identical units of measurement.
(6) Aggregate subjective and objective factor estimates for the costs and benefits on each alternative for each DM.
(7) Find combined fuzzy group ratings for the alternative benefits and costs.
(8) Identify the ideal alternative and calculate the total Euclidean distance of each base.
(9) Rank the bases using visual and numerical information, taking into consideration the level of uncertainty of their fuzzy characteristics.

4.1 Consider a set of military bases for realignment and benchmarking

Alternatives are the set of potential means by which the previously identified objectives may be attained. Assuming that there are m alternatives ($m = 1,2,\ldots, M$), there must be a minimum of two mutually exclusive alternatives in the set to permit a choice to be made (Zeleny, 1982). A total of 52 US military bases comprised of 16 Air Force, 19 Army, and 17 Navy bases from 27 states and the District of Columbia were assessed in this study.

4.2 Identify the relevant objective and subjective factors and sub-factors and define their importance weights using the analytic network process (ANP)

The ANP is a more general form of the analytic hierarchy process (AHP) used in MCDA. Saaty (1980) developed the AHP to capture the intuitive judgments in multi-criteria decision problems. AHP assumes unidirectional hierarchical relationships among the decision elements in a problem. However, in many real-life problems, there are dependencies among the elements in a hierarchy. ANP does not require independence and allows for decision elements to "influence" or "be influenced" by other elements in the model. Both processes have been widely used on a practical level and numerous applications have been published in literature (Saaty, 1996). The hierarchical model presented in Figure 1 was used in this study. There are two different kinds of dependencies in a hierarchy, within level or between levels dependencies. The directions of the arrows (or arcs) signify dependence (or influence). An example of a between level dependency (or outer dependency) is the dependency between direct employment impacts and community support and an example of a within level dependency (or inner dependency) is the interdependency between future restoration costs and payback period. With such interactions, the hierarchical structure becomes a network and a matrix manipulation approach developed by Saaty and Takizawa (1986) is used to measure the relative importance or strength of the impacts on a given element in the network using a ratio scale similar to AHP (Saaty, 1996).

According to Saaty (2005), the ANP comprises four main steps:

(1) problem structuring;
(2) pairwise comparisons;
(3) super-matrix formation; and
(4) selection of best alternatives.

In ANP, similar to AHP, DMs are asked to provide a series of pairwise comparisons of the elements at each level of the hierarchy with respect to a control element. The control element can be an element at the upper or lower levels of the hierarchy. This is the fundamental requirement for developing the super-matrix in the ANP (Saaty, 2001). The pairwise comparison for the elements at one level with respect to the control element at another level is expressed in a matrix form (A) with Saaty's 1-9 scale shown in Table II.

A reciprocal value is assigned to the inverse comparison; that is, $a_{ij} = 1/a_{ji}$, where $a_{ij}(a_{ji})$ represents the importance weight of the ith (jth) element. Once the pairwise comparisons are completed, the local priority vector w is computed as the unique solution to $A \times w = \lambda_{\max} w$ where A is the matrix of pairwise comparison, w is the eigenvector, and $\lambda_{\max}$ is the largest eigenvalue of A. There are several algorithms available for approximating the vector w (Saaty and Takizawz, 1986). We use a two-stage algorithm proposed by Meade and Sarkis (1998) for averaging normalized columns and approximating the vector w:

$$w_i = \frac{\left(\sum_{j=1}^{n}\left(A_{ij}/\sum_{i=1}^{n} A_{ij}\right)\right)}{n} \quad \text{for } i = 1, \ldots, n \tag{1}$$

The deviation from consistency of the pairwise comparisons must be addressed in the assessment process. Saaty (1980) provides a consistency index (CI) defined as $\mathrm{CI} = (\lambda_{\max} - n)/(n-1)$ for this test in which λ_{max} is approximated by $\sum_{i=1}^{n}[(Aw_i)/w_i]/n$. The acceptable consistency index is $\mathrm{CI} \leq 0.10$.

Next, the super-matrix is formed. The super-matrix concept is similar to a Markov chain process (Saaty, 1996). The local priority vectors developed earlier are entered in the appropriate columns of a matrix to obtain global priorities in a problem with interdependencies. As a result, a partitioned matrix called a super-matrix is created,

Intensity of importance	Definition	Explanation
1	Equal importance	Two activities contribute equally to the objective
2	Weak or slight	
3	Moderate importance	Experience and judgment slightly favor one activity over another
4	Moderate plus	
5	Strong importance	Experience and judgment strongly favor one activity over another
6	Strong plus	
7	Very strong or demonstrated importance	An activity is favored very strongly over another; its dominance demonstrated in practice
8	Very, very strong	
9	Extreme importance	The evidence favoring one activity over another is of the highest possible order of affirmation

Table II. The fundamental scale used in AHP and ANP where each matrix segment represents a relationship between two elements in the model. When there is an interrelationship between the elements of a component or two components, zeros can be replaced by a matrix in the super-matrix. Let the components of a decision system be C_i, $i = 1, \ldots, N$; each component i is assumed to have u_i elements, denoted by $e_{i1}, e_{i2}, \ldots, e_{Nu_N}$. The standard form of a super-matrix and a W_{ij} component matrix proposed by Saaty (1996) are shown in Figures 2(a) and (b) (Saaty, 1996).

The super-matrix is then converged to obtain a long-term stable set of weights. For convergence to occur, the super-matrix needs to be column stochastic. In other words, the sum of each column of the super-matrix needs to be one. Saaty (1996) suggests raising the weighted super-matrix to the power of $2k + 1$, where k is an arbitrarily large number, to achieve a convergence on the importance weights. This new matrix is called the limit super-matrix. The limit super-matrix has the same form as the weighted super-matrix but all the columns of the limit super-matrix are the same. By normalizing each block of the limit super-matrix, the final importance weights of all the elements in the matrix can be obtained. The limit may not converge unless the matrix is column stochastic (i.e. each of its columns sums to one). Note that $\lambda_{\max}(T) = 1$ for the super-matrix. Since $\mathrm{Max}\sum_{j=1}^{n} a_{ij} \geq \sum_{j=1}^{n} a_{ij}\frac{w_j}{w_i} = \lambda_{\max}$ for max w_i and

$$
\begin{array}{ccc}
 & & \begin{array}{cccc} C_1 & C_2 & \cdots & C_N \\ e_{N1}e_{N2}\ldots e_{Nn_N} & e_{N1}e_{N2}\ldots e_{Nn_N} & \cdots & e_{N1}e_{N2}\ldots e_{Nn_N} \end{array} \\
W = & \begin{array}{cc} C_1 & \begin{array}{c} e_{11} \\ e_{12} \\ \vdots \\ e_{1n_1} \end{array} \\ C_2 & \begin{array}{c} e_{21} \\ e_{22} \\ \vdots \\ e_{2n_2} \end{array} \\ \cdots & \cdots \\ C_N & \begin{array}{c} e_{N1} \\ e_{N1} \\ \vdots \\ e_{Nu_N} \end{array} \end{array} & \begin{bmatrix} W_{11} & W_{12} & \cdots & W_{1N} \\ W_{21} & W_{22} & \cdots & W_{2N} \\ \cdots & \cdots & \cdots & \cdots \\ W_{N1} & W_{N2} & \cdots & W_{NN} \end{bmatrix}
\end{array}
$$

(a)

$$
W = \begin{bmatrix} W_{i1}^{(j_1)} & W_{i1}^{(j_1)} & \cdots & W_{i1}^{(jn_j)} \\ W_{i2}^{(j_1)} & W_{i2}^{(j_1)} & \cdots & W_{i2}^{(jn_j)} \\ \cdots & \cdots & \cdots & \cdots \\ W_{in_i}^{(j_1)} & W_{in_i}^{(j_1)} & \cdots & W_{in_i}^{(jn_j)} \end{bmatrix}
$$

(b)

Figure 2.
Standard super-matrix and W_{ij} componenet (a) super-matrix and (b) W_{ij} component of supermatrix

$\text{Min}\sum_{j=1}^{n} a_{ij} \leq \sum_{j=1}^{n} a_{ij}\frac{w_j}{w_i} = \lambda_{\max}$ for min w_i, the eigenvalue of the matrix ($\lambda_{\max}$), lies between its largest and smallest column sums ($1 = \text{Min}\sum_{j=1}^{n} a_{ij} \leq \lambda_{\max} \leq \text{Max}\sum_{j=1}^{n} a_{ij} = 1$). When the eigenvalues of the matrix W are distinct then the power series expansion of $f(x)$ converges for all finite values of x with x replaced by W:

$$f(W) = \sum_{i=1}^{n} f(\lambda_i)Z(\lambda_i), Z(\lambda_i) = \frac{\prod_{j\neq i}(\lambda_j I - A)}{\prod_{j\neq i}(\lambda_j - \lambda_i)}, \quad \sum_{i=1}^{n} Z(\lambda_i) = I, Z(\lambda_i)Z(\lambda_j) = 0, \tag{2}$$

$$Z^2(\lambda_i) = Z(\lambda_i)$$

where I and 0 are the identity and the null matrices, respectively.

A similar expression is also available when some or all of the eigenvalues have multiplicities. When $f(W) = W^k$, then $f(\lambda_i) = \lambda_i^k$ and as $k \to \infty$, the only terms that give a finite nonzero value are those for which the modulus of λ_i is equal to one. The priorities of the clusters (or any set of elements in a cluster) are obtained by normalizing the corresponding values in the appropriate columns of the limit matrix. For complete treatment, see Saaty (2001) and Saaty and Ozdemir (2005). Let us further define:

Gn = the nth cluster of factors; $(n = 1, 2, \ldots, N,\ 2 \leq n \leq N)$

Gn^i = the ith sub-factor within the nth cluster of factors; $(n = 1, 2, \ldots, N, i = 1, 2, \ldots, I, 1 \leq i \leq N)$

W_{Gn} = the importance weight of the nth cluster; $(n = 1, 2, \ldots, N, 2 \leq n \leq N)$

W_{Gn^i} = the importance weight of the nth sub-factor; $(n = 1, 2, \ldots, N, i = 1, 2, \ldots, I, 1 \leq i \leq N)$

k = the kth DMs, $(k = 1, 2, \ldots, K, k \geq 0)$

W_{Gnk} = the kth DM weight for the nth cluster; $(n = 1, 2, \ldots, N,\ 2 \leq n \leq N, k = 1, 2, \ldots, K, k \geq 0)$

W_{Gn^ik} = the kth DM weight for the nth sub-factor; $(n = 1, 2, \ldots, N, i = 1, 2, \ldots, I, k = 1, 2, \ldots, K, k \geq 0)$

W_{VGnk} = the kth DM weight for the nth cluster of objective criteria (V); $(n = 1, 2, \ldots, N, 2 \leq n \leq N, k = 1, 2, \ldots, K, k \geq 0)$

W_{VGn^ik} = the kth DM weight for the nth sub-factor of objective criteria (V); $(n = 1, 2, \ldots, N, i = 1, 2, \ldots, I, k = 1, 2, \ldots, K, k \geq 0)$

W_{UGnk} = the kth DM weight for the nth cluster of subjective criteria (U); $(n = 1, 2, \ldots, N, 2 \leq n \leq N\ k = 1, 2, \ldots, K\ k \geq 0)$

W_{UGn^ik} = the kth DM weight for the nth sub-factor of subjective criteria (U); $(n = 1, 2, \ldots, N, i = 1, 2, \ldots, I, k = 1, 2, \ldots, K, k \geq 0)$.

The general views of the factor and sub-factor weights for the K DMs are given in Table III.

	$k=1$	$k=2$	...	$k=K-1$	$k=K$
Factor weights					
G_1	$W_{G_1 1}$	$W_{G_1 2}$	...	$W_{G_1(K-1)}$	$W_{G_1 K}$
G_2	$W_{G_2 1}$	$W_{G_2 2}$	...	$W_{G_2(K-1)}$	$W_{G_2 K}$
...	...	...	...	...	...
$G_{N\text{-}1}$	$W_{G_{N-1} 1}$	$W_{G_{N-1} 2}$	...	$W_{G_{N-1}(K-1)}$	$W_{G_{N-1} K}$
G_N	$W_{G_N 1}$	$W_{G_N 2}$	...	$W_{G_N(K-1)}$	$W_{G_N K}$
Sub-factor weights					
G_1^1	$W_{G1^1 1}$	$W_{G1^1 2}$	...	$W_{G1^1(K-1)}$	$W_{G1^1 K}$
G_1^2	$W_{G1^2 1}$	$W_{G1^2 2}$	...	$W_{G1^2(K-1)}$	$W_{G1^2 K}$
...	...	...	...	...	...
G_1^{I1}	$W_{G1^{I1} 1}$	$W_{G1^{I1} 2}$	...	$W_{G1^{I1}(K-1)}$	$W_{G1^{I1} K}$
G_2^1	$W_{G2^1 1}$	$W_{G2^1 2}$	...	$W_{G2^1(K-1)}$	$W_{G2^1 K}$
G_2^2	$W_{G2^2 1}$	$W_{G2^2 2}$	...	$W_{G2^2(K-1)}$	$W_{G2^2 K}$
...	...	...	...	...	...
G_2^{I2}	$W_{G2^{I2} 1}$	$W_{G2^{I2} 2}$	...	$W_{G2^{I2}(K-1)}$	$W_{G2^{I2} K}$
...	...	...	...	...	...
G_N^1	$W_{GN^1 1}$	$W_{GN^1 2}$	...	$W_{GN^1(K-1)}$	$W_{GN^1 K}$
G_N^2	$W_{GN^2 1}$	$W_{GN^2 2}$	...	$W_{GN^2(K-1)}$	$W_{GN^2 K}$
...	...	...	...	...	...
G_N^{IN}	$W_{GN^{IN} 1}$	$W_{GN^{IN} 2}$	...	$W_{GN^{IN}(K-1)}$	$W_{GN^{IN} K}$

Table III.
Factor and sub-factor weight notations

The expert DMs participating in this study provided their independent pairwise comparison matrices. The local priority vectors are then calculated and entered in the appropriate columns of a matrix for each DM to obtain global priorities in a problem with interdependencies. A super-matrix was created for each DM. Normalizing each block of the limit super-matrix resulted in the importance weights of the factors and sub-factors presented in Table IV.

4.3 Develop scores for the subjective factors and identify values of the objective factors on each alternative

The decision criteria in this study were divided into two groups: objective (such as monetary, physical or statistical) and subjective (such as beliefs, likeliness or judgments). Data on objective factors were obtained from financial, statistical, and economic reports. Subjective judgments were obtained from our seven expert DMs who considered five groups of factors divided into 14 sub-factors. The sub-factors were further grouped into three clusters. One cluster included 10 objective factors (employment, environmental and financial impacts) and the other two clusters included subjective strategic and tactical factors. The objective factors and their respective uncertainty levels (distribution) are presented in Table V.

Objective factors are treated as fuzzy numbers and their values are defined as:

$$\tilde{v}_{m_n^i} = \left\{(x, \mu_{m_n^i}(x)) | x \in R\right\} \tag{3}$$

where $\tilde{v}_{m_n^i}$ is the set of fuzzy objective values for the ith objective sub-factor within the nth cluster on alternative m represented by pairs $(x, \mu_{m_n^i}(x))$ with membership functions of LR-type; $(n = 1, 2, \ldots, N,\ i = 1, 2, \ldots, I,\ 1 \le i \le N,\ m = 1, 2, \ldots, M)$. $\mu_{m_n^i}(x) \in [0, 1]$ represents the interval from which the membership functions take on

Table IV. Factor and sub-factor weights for the seven DMs in this study

	DM-1	DM-2	DM-3	DM-4	DM-5	DM-6	DM-7
Factor weights							
Criteria							
1 Employment impacts	0.291	0.243	0.346	0.256	0.32	0.258	0.27
2 Environmental impacts	0.122	0.095	0.087	0.102	0.111	0.166	0.15
3 Financial impacts	0.412	0.352	0.372	0.365	0.312	0.42	0.291
4 Strategic impacts	0.144	0.212	0.110	0.228	0.180	0.090	0.179
5 Tactical impacts	0.031	0.098	0.085	0.049	0.077	0.066	0.11
Sub-factor weights							
Sub-criteria							
1.1 Direct	0.063	0.072	0.054	0.090	0.108	0.029	0.078
1.2 Indirect	0.054	0.044	0.065	0.034	0.033	0.044	0.035
1.3 Changes as a percent of area employment	0.092	0.107	0.112	0.097	0.104	0.088	0.123
2.1 Future environmental restoration costs	0.073	0.067	0.078	0.082	0.071	0.068	0.054
2.2 Incurred environmental restoration costs	0.056	0.032	0.044	0.056	0.049	0.093	0.042
3.1 One-time costs	0.112	0.131	0.091	0.121	0.122	0.104	0.128
3.2 Payback period (years)	0.084	0.064	0.077	0.053	0.091	0.043	0.054
3.3 Six year net savings	0.074	0.087	0.069	0.065	0.056	0.077	0.021
3.4 Annual recurring savings	0.143	0.142	0.126	0.178	0.189	0.168	0.176
3.5 20 year NPV savings	0.055	0.034	0.066	0.051	0.032	0.055	0.065
5.1 Community support for closure	0.064	0.072	0.066	0.054	0.071	0.032	0.045
5.2 Commercial land use	0.073	0.063	0.073	0.076	0.045	0.044	0.066
5.3 Residential land use	0.056	0.085	0.079	0.043	0.029	0.156	0.113

Table V. Objective criteria and their uncertainty levels

Factor	Uncertainty level (percent)	Normalized uncertanty level
Direct	± 1.42	0.0142
Indirect	± 3.69	0.0369
Changes as a percent of area employment	± 0.45	0.0045
Future environmental restoration costs	± 5.75	0.0575
Incurred environmental restoration costs	No deviation	0.0000
One-time costs	± 0.75	0.0075
Payback period (years)	± 7.78	0.0778
Six year net savings	± 2.65	0.0265
Annual recurring savings	± 4.25	0.0425
20 year NPV savings	± 12.50	0.1250

their values and $\alpha \in [0, 1]$ is a representation of $\tilde{v}_{m_n^i}$ by the set of α-levels (α-cuts). Interval representations of the fuzzy objective values $\tilde{v}_{m_n^i}$ on α-levels are:

$$\tilde{v}_{m_n^i} = \left\{ v_{m_n^i}^{\alpha} = \left[v_{m_n^i}^{\alpha L}, v_{m_n^i}^{\alpha R} \right] \right\} \tag{4}$$

where $v_{m_n^i}^{\alpha L}$ and $v_{m_n^i}^{\alpha R}$ are the left (L) and right (R) bounds on α-cuts of fuzzy value $\tilde{v}_{m_n^i}$. The graphical representation of a fuzzy set and its characteristics are depicted in Figure 3.

In this study, the values of the objective factors are considered as triangular fuzzy numbers on two α-levels of 0 and 1. According to Zadeh (1996), triangular fuzzy numbers are characterized by a triple $x = (x_1, x_2, x_3)$ in which x_1, x_2, and x_3 are the

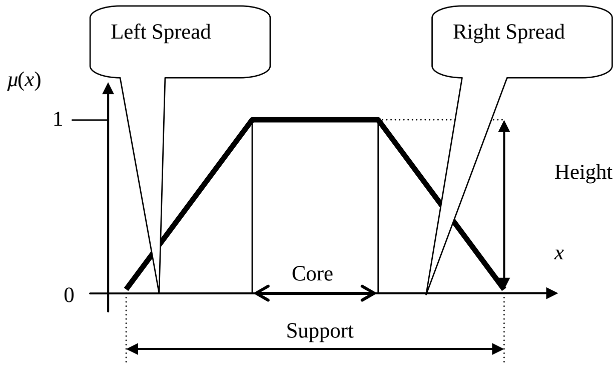

Figure 3.
Fuzzy set and its characteristics

abscissae of the three vertices of the triangle [i.e. $\mu(x_1) = \mu(x_3) = 0, \mu(x_2) = 1$]. The graphical representation of a triangular fuzzy number is shown in Figure 4.

A triangular fuzzy number with center x_2 is a fuzzy quantity where "x is approximately equal to x_2." The deviations given in Table V showed maximal and minimal possible spreads for the most reliable values given on $\alpha = 1$. The closer the objective value is to the left-hand side or the right-hand side boundaries (defined by the uncertainty level), the less reliable the value. Consequently, those values smaller than the left-hand side boundary or larger than the right-hand side boundary are considered impossible (unreliable). On α-cut $= 1$, the left bound value will coincide with the right bound value and on α-cut $= 0$, the left and right bounds will be calculated as shown in the following equations:

$$v_{m_n^i}^{\alpha=1L} = v_{m_n^i}^{\alpha=1R} = v_{m_n^i}^{\alpha=1} \tag{5a}$$

$$v_{m_n^i}^{\alpha=0L} = v_{m_n^i}^{\alpha=1} - \left(v_{m_n^i}^{\alpha=1} \cdot s_{G_n^i}\right) \tag{5b}$$

$$v_{m_n^i}^{\alpha=0R} = v_{m_n^i}^{\alpha=1} + \left(v_{m_n^i}^{\alpha=1} \cdot s_{G_n^i}\right) \tag{5c}$$

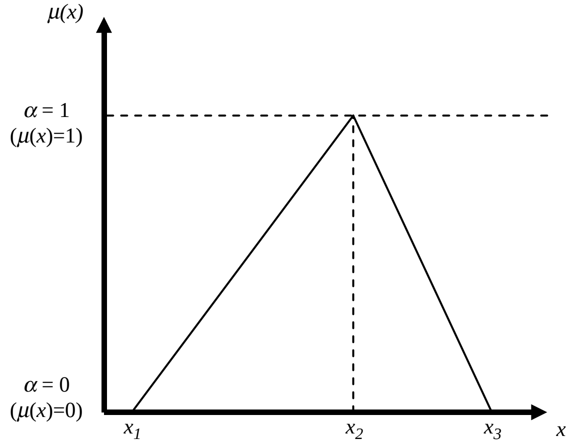

Figure 4.
Triangular fuzzy number

where $s_{G_n^i}$ is the normalized spread (deviation) of the objective value G characterizing the ith sub-factor within the nth cluster; $(n = 1, 2, \ldots, N,\ \ i = 1, 2, \ldots, I,\ \ 1 \leq i \leq N)$.

An equal scoring scale of 0-10 is used for all subjective factors. Seven DMs ($K = 7$) evaluated the bases independently on the subjective factors. The scores of the subjective factors are represented by $u_{mk_n^i}$, the intensity of the ith subjective sub-factor within the nth cluster on alternative m for kth DM; $(m = 1, 2, \ldots, M,\ \ n = 1, 2, \ldots, N,\ \ i = 1, 2, \ldots, I,\ \ k = 1, 2, \ldots, K,\ \ k \geq 0)$.

4.4 Group all the factors into benefit and cost factors

Next, the DMs analyzed all relevant objective and subjective factors and classified them into benefits and costs. While most factors were either a benefit or a cost, some were classified into both groups, depending on their values. The employment impact values for direct, indirect and changes as a per cent of area employment were considered costs if negative and benefits if positive. The environmental impact values for future and incurred environmental restoration costs were considered costs. The financial impact values for one-time costs, six year net savings, annual recurring savings and 20 year NPV savings were considered benefits if negative and costs if positive. However, the financial impact values for the payback period were considered costs since a shorter payback period was more desirable than a longer payback period. The strategic and tactical impact values were all considered benefits.

Variables defined in steps (ii) and (iii) can be rewritten for benefits and costs as follows:

Gn_b^i = the ith benefit sub-factor within the nth cluster of factors; $(n = 1, 2, \ldots, N,$ $i = 1, 2, \ldots, I, 1 \leq i \leq N, b = 1, 2, \ldots, B, b \leq i)$.

Gn_C^i = the ith cost sub-factor within the nth cluster of factors; $(n = 1, 2, \ldots, N,$ $i = 1, 2, \ldots, I, 1 \leq i \leq N, c = 1, 2, \ldots, C, c \leq i)$.

Subsequently, equation (3) can be rewritten for benefits and costs as:

$$\tilde{v}_{m_{nb}^i} = \left\{ \left((x, \mu_{m_{nb}^i}(x) \right) | x \in R \right\} \tag{6a}$$

$$\tilde{v}_{m_{nc}^i} = \left\{ \left((x, \mu_{m_{nc}^i}(x) \right) | x \in R \right\} \tag{6b}$$

Equations (6a) and (6b) define sets of fuzzy objective values of the ith benefit (cost) sub-factors within the nth clusters of objective factors on alternative m which are represented by pairs $(x, \mu_{m_{nb}^i}(x))$ and $(x, \mu_{m_{nc}^i}(x))$ with membership functions of LR-type, $\mu_{m_{nb}^i}(x) \in [0, 1]$ and $\mu_{m_{nc}^i}(x) \in [0, 1]$ are the intervals from which the membership functions for benefits and costs take on their values $(n = 1, 2, \ldots, N,$ $i = 1, 2, \ldots, I, 1 \leq i \leq N, m = 1, 2, \ldots, M)$.

According to equation (4), the interval representations of the fuzzy objective values $\tilde{v}_{m_{nb}^i} (\tilde{v}_{m_{nc}^i})$ on α-levels are:

$$\tilde{v}_{m_{nb}^i} = \left\{ v_{m_{nb}^i}^{\alpha} = \left[v_{m_{nb}^i}^{\alpha L}, v_{m_{nb}^i}^{\alpha R} \right] \right\} \tag{7a}$$

$$\tilde{v}_{m_{nc}^i} = \left\{ v_{m_{nc}^i}^{\alpha} = \left[v_{m_{nc}^i}^{\alpha L}, v_{m_{nc}^i}^{\alpha R} \right] \right\} \tag{7b}$$

where $v^{\alpha L}_{m^i_{nb}}$ and $v^{\alpha R}_{m^i_{nb}}$ ($v^{\alpha L}_{m^i_{nc}}$ and $v^{\alpha R}_{m^i_{nc}}$) are the left (L) and right (R) bounds on α-cuts of fuzzy value $\tilde{v}_{m^i_{nb}}$ ($\tilde{v}_{m^i_{nc}}$).

Analogous to equations (5a)-(5c), we derive function values on the left and right bounds of α-levels for benefits and costs. Values on $\alpha = 1$ and $\alpha = 0$ for benefits are:

$$v^{\alpha=1L}_{m^i_{nb}} = v^{\alpha=1R}_{m^i_{nb}} = v^{\alpha=1}_{m^i_{nb}} \tag{8a}$$

$$v^{\alpha=0L}_{m^i_{nb}} = v^{\alpha=1}_{m^i_{nb}} - \left(v^{\alpha=1}_{m^i_{nb}} \cdot s_{G^i_n}\right) \tag{8b}$$

$$v^{\alpha=0R}_{m^i_{nb}} = v^{\alpha=1}_{m^i_{nb}} + \left(v^{\alpha=1}_{m^i_{nb}} \cdot s_{G^i_n}\right) \tag{8c}$$

Values on $\alpha = 1$ and $\alpha = 0$ for costs are:

$$v^{\alpha=1L}_{m^i_{nc}} = v^{\alpha=1R}_{m^i_{nc}} = v^{\alpha=1}_{m^i_{nc}} \tag{9a}$$

$$v^{\alpha=0L}_{m^i_{nc}} = v^{\alpha=1}_{m^i_{nc}} - \left(v^{\alpha=1}_{m^i_{nc}} \cdot s_{G^i_n}\right) \tag{9b}$$

$$v^{\alpha=0R}_{m^i_{nc}} = v^{\alpha=1}_{m^i_{nc}} + \left(v^{\alpha=1}_{m^i_{nc}} \cdot s_{G^i_n}\right) \tag{9c}$$

The scores of the subjective factors for benefits and costs are represented by $u_{mk^i_{nb}}$ ($u_{mk^i_{nc}}$), the intensity of the ith sub-factor within the nth cluster of subjective benefits (costs) factors on alternative m for the kth DM.

4.5 Normalize all estimates to obtain identical units of measurement

Next, we normalize variables with multiple measurement scales to assure uniformity. The literature reports on several normalization methods. The selection of a specific normalization method must be based on the problem characteristics and model requirements. In this study, we use the approach where the normalized value is the quotient of the initial value divided by the sum of the values of all alternatives on that criterion:

$$d_i{}' = \frac{d_i}{\sum_{i=1}^{n} d_i} \tag{10}$$

Using the above normalization procedure, the normalized values for the objective benefits are:

$$\tilde{v}'_{m^i_{nb}} = \left\{ v'^{\alpha}_{m^i_{nb}} = \left[v'^{\alpha L}_{m^i_{nb}}, v'^{\alpha R}_{m^i_{nb}} \right] \right\} \tag{11}$$

where:

$$\nu'^{\alpha=1}_{m^i_{nb}} = \frac{\nu'^{\alpha=1}_{m^i_{nb}}}{\sum_{m=1}^{M} \nu'^{\alpha=1}_{m^i_{nb}}}$$

is the normalized fuzzy value of alternative m on sub-criterion i from the group of benefit factors n on α-level $= 1$;

$$v'^{\alpha=0L}_{m^i_{nb}} = \frac{v'^{\alpha=0L}_{m^i_{nb}}}{\sum_{m=1}^{M} v'^{\alpha=0L}_{m^i_{nb}}}$$

is the normalized fuzzy value of alternative m on sub-criterion i from the group of benefit factors n on the left bound of α-level $= 0$; and:

$$v'^{\alpha=0R}_{m^i_{nb}} = \frac{v'^{\alpha=0R}_{m^i_{nb}}}{\sum_{m=1}^{M} v'^{\alpha=0R}_{m^i_{nb}}}$$

is the normalized fuzzy value of alternative m on sub-criterion i from the group of benefit factors n on the right bound of α-level $= 0$.

Using equation (10), we obtain the normalized values of the objective costs as:

$$\tilde{v}'_{m^i_{nc}} = \left\{ v'^{\alpha}_{m^i_{nc}} = \left[v'^{\alpha L}_{m^i_{nc}}, v'^{\alpha R}_{m^i_{nc}} \right] \right\} \tag{12}$$

where:

$$v'^{\alpha=1}_{mc} = \frac{v'^{\alpha=1}_{m^i_{nc}}}{\sum_{m=1}^{M} v'^{\alpha=1}_{m^i_{nc}}}$$

is the normalized fuzzy value of alternative m on sub-criterion i from the group of cost factors n on α-level $= 1$;

$$v'^{\alpha=0L}_{m^i_{nc}} = \frac{v'^{\alpha=0L}_{m^i_{nc}}}{\sum_{m=1}^{M} v'^{\alpha=0L}_{m^i_{nc}}}$$

is the normalized fuzzy value of alternative m on sub-criterion i from the group of cost factors n on the left bound of α-level $= 0$; and:

$$v'^{\alpha=0R}_{m^i_{nc}} = \frac{v'^{\alpha=0R}_{m^i_{nc}}}{\sum_{m=1}^{M} v'^{\alpha=0R}_{m^i_{nc}}}$$

is the normalized fuzzy value of alternative m on sub-criterion i from the group of cost factors n on the right bound of α-level $= 0$.

The normalized scores for the subjective benefits and costs are:

$$u'_{mk^i_{nb}} = \frac{u_{mk^i_{nb}}}{\sum_{m=1}^{M} u_{mk^i_{nb}}} \tag{13a}$$

$$u'_{mk^i_{nc}} = \frac{u_{mk^i_{nc}}}{\sum_{m=1}^{M} u_{mk^i_{nc}}} \tag{13b}$$

4.6 Aggregate subjective and objective factor estimates for the costs and benefits on each alternative for each DM

After the normalization process, we calculate the fuzzy characteristics of each alternative military base for K DMs. Zadeh's Extension Principle (1965, 1975) is widely used technique to perform arithmetic operations with fuzzy values represented by functions having pointwise arguments on level-cuts. The main interest of the level-cut representation is to be very handy when extending set-theoretic notations of fuzzy sets. Any usual point-to-point function can be lifted to a fuzzy-set-to-fuzzy-set function on this basis. See DeBaets and Kerre (1994) for a survey of fuzzy concepts defined via cuts – the main application of the Extension Principle is fuzzy interval analysis. In order to apply an operation f to fuzzy values A and B, it is necessary to apply f to the values $a \in A_\alpha$ and $b \in B_\alpha$ of fuzzy sets A and B on all α-levels. Since we treat our objective values as fuzzy triangular numbers, we can apply arithmetic operations to them on the given (0 and 1) α-cuts in accordance with the Extension Principle. Weighted objective benefits and costs values on the mth alternative for the kth DM on $\alpha = 1$ are calculated as follows:

$$V_{mkb}^{\alpha=1} = \sum_{n=1}^{N} \sum_{i=1}^{I} W_{VGnk} \cdot W_{VGn^ik} \cdot v'^{\alpha=1}_{m^i_{nb}} \tag{14a}$$

$$V_{mkc}^{\alpha=1} = \sum_{n=1}^{N} \sum_{i=1}^{I} W_{VGnk} \cdot W_{VGn^ik} \cdot v'^{\alpha=1}_{m^i_{nc}} \tag{14b}$$

$$(n = 1, 2, \ldots, N, \;\; k = 1, 2, \ldots, K, \;\; k \geq 0, \;\; i = 1, 2, \ldots, I, \;\; m = 1, 2, \ldots, M).$$

Using the Extension Principle, we calculate the weighted objective benefits and costs values on the mth alternative for kth DM on the left and right bounds of a zero-α-level using equations (15a), (15b), (16a), and (16b), respectively:

$$V_{mkb}^{\alpha=0L} = \sum_{n=1}^{N} \sum_{i=1}^{I} W_{VGnk} \cdot W_{VGn^ik} \cdot v'^{\alpha=0L}_{m^i_{nb}} \tag{15a}$$

$$V_{mkb}^{\alpha=0R} = \sum_{n=1}^{N} \sum_{i=1}^{I} W_{VGnk} \cdot W_{VGn^ik} \cdot v'^{\alpha=0R}_{m^i_{nb}} \tag{15b}$$

$$V_{mkc}^{\alpha=0L} = \sum_{n=1}^{N} \sum_{i=1}^{I} W_{VGnk} \cdot W_{VGn^ik} \cdot v'^{\alpha=0L}_{m^i_{nc}} \tag{16a}$$

$$V_{mkc}^{\alpha=0R} = \sum_{n=1}^{N} \sum_{i=1}^{I} W_{VGnk} \cdot W_{VGn^ik} \cdot v'^{\alpha=0R}_{m^i_{nc}} \tag{16b}$$

$$(n = 1, 2, \ldots, N, \;\; k = 1, 2, \ldots, K, \;\; k \geq 0, \;\; i = 1, 2, \ldots, I, \;\; m = 1, 2, \ldots, M).$$

The weighted subjective benefit values on the mth alternative for the kth DM on α-cuts of 1 and 0 are:

$$U_{mkb}^{\alpha=1} = U_{mkb}^{\alpha=0L} = U_{mkb}^{\alpha=0R} = \sum_{n=1}^{N}\sum_{i=1}^{I} W_{UGnk} \cdot W_{UGn^ik} \cdot u'_{mk^i_{nb}} \quad (17a)$$

$$U_{mkc}^{\alpha=1} = U_{mkc}^{\alpha=0L} = U_{mkc}^{\alpha=0R} = \sum_{n=1}^{N}\sum_{i=1}^{I} W_{UGnk} \cdot W_{UGn^ik} \cdot u'_{mk^i_{nc}} \quad (17b)$$

$$(n = 1, 2, \ldots, N, \;\; k = 1, 2, \ldots, K, \;\; k \geq 0, \;\; i = 1, 2, \ldots, I, \;\; m = 1, 2, \ldots, M).$$

The overall (aggregated) fuzzy benefit characteristic for the mth alternative and the kth DM is:

$$B_{mk} = \left\{B_{mk}^{\alpha=0L}, B_{mk}^{\alpha=1}, B_{mk}^{\alpha=0R}\right\} \quad (18)$$

where $B_{mk}^{\alpha=0L} = V_{mkb}^{\alpha=0L} + U_{mkb}^{\alpha=0L}$, $B_{mk}^{\alpha=0R} = V_{mkb}^{\alpha=0R} + U_{mkb}^{\alpha=0R}$, and $B_{mk}^{\alpha=1} = V_{mkb}^{\alpha=1} + U_{mkb}^{\alpha=1}$. The overall aggregated fuzzy cost characteristic for the mth alternative and the kth DM is:

$$C_{mk} = \left\{C_{mk}^{\alpha=0L}, C_{mk}^{\alpha=1}, C_{mk}^{\alpha=0R}\right\} \quad (19)$$

where $C_{mk}^{\alpha=0L} = V_{mkc}^{\alpha=0L} + U_{mkc}^{\alpha=0L}$,$C_{mk}^{\alpha=0R} = V_{mkc}^{\alpha=0R} + U_{mkc}^{\alpha=0R}$, and $C_{mk}^{\alpha=1} = V_{mkc}^{\alpha=1} + U_{mkc}^{\alpha=1}$.

4.7 Find combined fuzzy group ratings for the alternative benefits and costs

We use arithmetic mean to collapse the fuzzy values obtained for multiple DMs on the previous step and find a single fuzzy rating for each alternative in the benefit and cost groups. Lets define $B_m = \{B_m^{\alpha=0L}, B_m^{\alpha=1}, B_m^{\alpha=0R}\}$ as the fuzzy rating of alternative m in the group of benefits, ($m = 1,2, \ldots, M$), and $C_m = \left\{C_m^{\alpha=0L}, C_m^{\alpha=1}, C_m^{\alpha=0R}\right\}$ as the fuzzy rating of alternative m in the group of costs ($m = 1, 2, \ldots, M$).

Equations (20a), (20b), (21a), and (21b) are used to calculate the spreads of the above fuzzy ratings. The left and right spreads of the fuzzy number characterizing benefits for the mth alternative are:

$$B_m^L = B_m^{\alpha=1} - B_m^{\alpha=0L} \quad (20a)$$

$$B_m^R = B_m^{\alpha=0R} - B_m^{\alpha=1} \quad (20b)$$

Analogously, the left and right spreads of fuzzy number characterizing costs for the mth alternative are:

$$C_m^L = C_m^{\alpha=1} - C_m^{\alpha=0L} \quad (21a)$$

$$C_m^R = C_m^{\alpha=0R} - C_m^{\alpha=1} \quad (21b)$$

where:

$$B_m^{\alpha=1} = \frac{\sum_{k=1}^{K} B_{mk}^{\alpha=1}}{K},$$

$$B_m^{\alpha=0L} = \frac{\sum_{k=1}^{K} B_{mk}^{\alpha=0L}}{K},$$

$$B_m^{\alpha=0R} = \frac{\sum_{k=1}^{K} B_{mk}^{\alpha=0R}}{K},$$

$$C_m^{\alpha=1} = \frac{\sum_{k=1}^{K} C_{mk}^{\alpha=1}}{K},$$

$$C_m^{\alpha=0L} = \frac{\sum_{k=1}^{K} C_{mk}^{\alpha=0L}}{K}, \quad \text{and}$$

$$C_m^{\alpha=0R} = \frac{\sum_{k=1}^{K} C_{mk}^{\alpha=0R}}{K}.$$

4.8 Identify the ideal alternative and calculate the total Euclidean distance of each base

The weighted-sum fuzzy values in this study are used to compare potential military bases among themselves and with the ideal base. The concept of ideal choice, an unattainable idea, serving as a norm or rationale facilitating human choice problem is not new (Tavana, 2002). See for example the stimulating work of Schelling (1960), introducing the idea. Subsequently, Festinger (1964) showed that an external, generally non-accessible choice assumes the important role of a point of reference against which choices are measured. Zeleny (1974, 1982) demonstrated how the highest achievable scores on all currently considered decision criteria form this composite ideal choice. As all choices are compared, those closer to the ideal are preferred to those farther away. Zeleny (1982, p. 144) shows that the Euclidean measure can be used as a proxy measure of distance.

Using the Euclidean measure suggested by Zeleny (1982), we synthesize the results by determining the ideal benefits and costs values. The ideal benefit (B^*) is the highest value among the set B_m on $\alpha = 1$, and the ideal cost (C^*) is the lowest value among the set C_m on $\alpha = 1$. We then find the Euclidean distance of each military base from the ideal base. The Euclidean distance is the sum of the quadratic root of squared differences between the ideal and the mth indices of the benefits and costs. To formulate the described model algebraically, let us assume:

D_B^m = total Euclidean distance from the ideal benefit for the mth alternative military base; ($m = 1,2, \ldots, M$)

D_C^m = total Euclidean distance from the ideal cost for the mth alternative military base; ($m = 1,2, \ldots, M$)

D^m = overall, Euclidean distance of the mth alternative military base; ($m = 1,2, \ldots, M$)

$$D^m = \sqrt{(D_B^m)^2 + (D_C^m)^2} \tag{22}$$

where:

$$B^* = \mathrm{Max}\left\{B_m^{\alpha=1}\right\}$$

$$C^* = \mathrm{Min}\left\{C_m^{\alpha=1}\right\}$$

and:

$$D_B^m = B* - U_m^{\alpha=1}$$

$$D_C^m = C_m^{\alpha=1} - C*.$$

Alternative military bases with smaller D^m are closer to the ideal base and are preferred to alternative bases with larger D^m which are further away from the ideal base.

Fuzzy relations play an important role in the theory of fuzzy sets. A fuzzy relation is a fuzzy subset R of a Cartesian product of sets. Fuzzy relations obtained by combining fuzzy sets offer a general setting for multi-factorial evaluation. A particular case of fuzzy relation is a fuzzy Cartesian product. It is presumed that R has projections on all the axes. An example of Cartesian product of two triangular fuzzy sets A and B is shown on Figure 5.

Let us further define triangular fuzzy benefits ($\tilde{B}_m$) and costs ($\tilde{C}_m$) estimates for the mth alternative that compose the survivability index:

$$\begin{aligned} \tilde{B}_m &= \left\{ D_B^{\alpha=0L}, D_B^m, D_B^{\alpha=0R} \right\} \\ \tilde{C}_m &= \left\{ D_C^{\alpha=0L}, D_C^m, D_C^{\alpha=0R} \right\} \end{aligned} \tag{23}$$

where $D_B^{\alpha=0L} = D_B^m - B_m^L$ and $D_B^{\alpha=0R} = D_B^m + B_m^R$ are the left and right boundaries of the fuzzy benefits component of the survivability index for the m −th alternative, and; $D_C^{\alpha=0L} = D_C^m - C_m^L$ and $D_C^{\alpha=0R} = D_C^m + C_m^R$ are the left and right boundaries of the fuzzy costs component of survivability index for the mth alternative.

Next, we evaluate the Cartesian product of the benefit and cost components of the fuzzy survivability index for each of the 52 alternative military bases. A general view of the Cartesian product for the mth alternative is given in Figure 6.

The numerical designations of the alternative military bases and the survivability indices and their components are presented in Tables VI and VII.

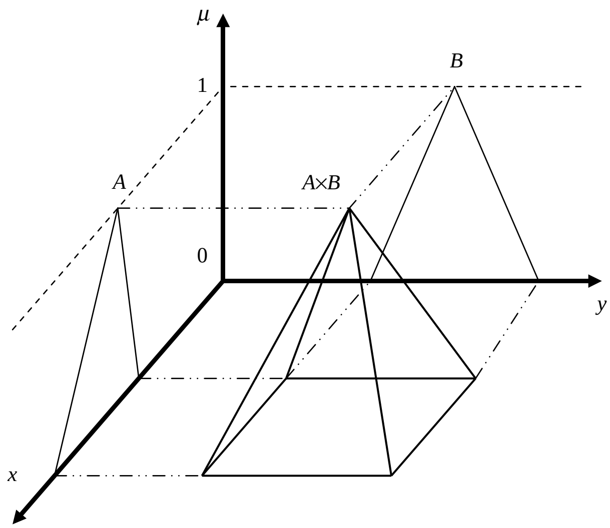

Figure 5.
The Cartesian product of two triangular fuzzy sets

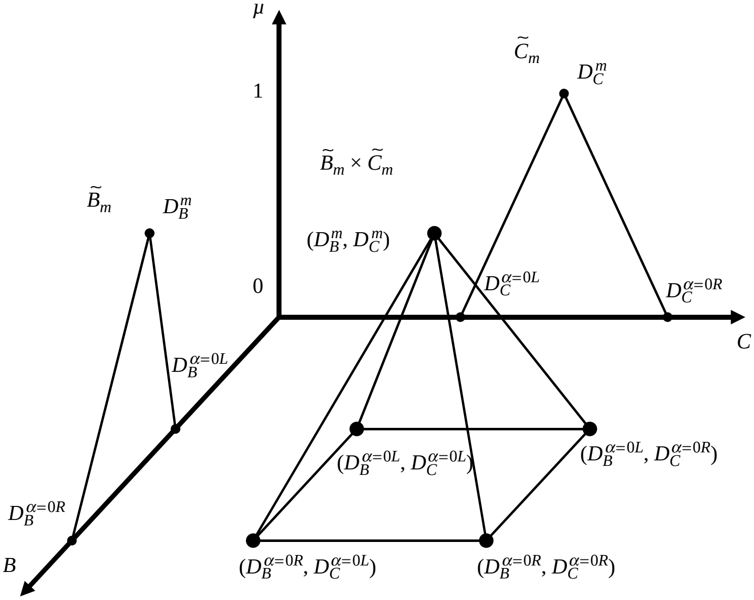

Figure 6.
The Cartesian product of fuzzy survivability index components

4.9 Rank the bases using visual and numerical information, taking into consideration the level of uncertainty of their fuzzy characteristics

The computations described earlier result in 52 pyramids for our set of the alternative military bases. In order to compare the results, we first consider the most reliable values given on α-level $= 1$. Table VIII provides the ranking of each alternative military base according to it Euclidean distance from the *ideal base*.

Next, we plot the alternative military bases on a graph where the x-axis is represented by the benefits (B) and the y-axis is represented by the costs (C). Figure 7 shows the alternative arrangement on $\alpha = 1$. The position of the point corresponding to alternative military base m has the Cartesian coordinates (D_B^m, D_C^m). We have excluded military bases 3, 12, 17, 44, 48, and 52 from this figure to zoom on the area where the majority of the bases are located. The 52 military bases fell into efficient, active, inactive or inefficient quadrants. The efficient zone with $D_B^m < \underset{m}{\mathrm{MAX}}\{D_B^m\}/2$ and $D_C^m < \underset{m}{\mathrm{MAX}}\{D_C^m\}/2$ or high benefits and low costs included military base 2, 4, 7-9, 11, 14, 20, 22, 28, 32, 36, 38, and 40 (plus 12 and 44 excluded from the graph). The active zone with $D_B^m < \underset{m}{\mathrm{MAX}}\{D_B^m\}/2$ and $D_C^m > \underset{m}{\mathrm{MAX}}\{D_C^m\}/2$ or high benefits and high cost included military base 10 (plus 3 excluded for the graph). The inactive zone with $D_B^m > \underset{m}{\mathrm{MAX}}\{D_B^m\}/2$ and $D_C^m < \underset{m}{\mathrm{MAX}}\{D_C^m\}/2$ or low benefits and low costs included military bases 1, 5, 6, 13, 15, 16, 18, 19, 21, 23-27, 29, 30, 31, 35, 37, 39, 41-43, 45-47, 49, 50, and 51. Finally, the inefficient zone with $D_B^m > \underset{m}{\mathrm{MAX}}\{D_B^m\}/2$ and $D_C^m > \underset{m}{\mathrm{MAX}}\{D_C^m\}/2$ or low benefits and high costs included military base 33 and 34 (plus 17, 48, and 52 excluded from the graph).

In the final step of the process, we compare the fuzzy attributes of the M alternatives to see if the uncertainty levels could influence the rankings. In general, alternatives with close Euclidean distance and varying uncertainty levels could swap their rankings.

Code	Military base
1	Army Reserve Personnel Center St. Louis
2	Brooks City Base
3	Cannon Air Force Base
4	Desecret Chemical Depot
5	Eielson, AFB
6	Elmendorf AFB
7	Fort Gillem
8	Fort Knox
9	Fort McPherson
10	Fort Monmouth
11	Fort Monroe
12	Ft Eustis
13	Gen Mitchell International Airport ARS
14	Grand Forks AFB
15	Kansas Amunition Plant
16	Kulis Air Guard Station
17	Lackland AFB
18	Lone Star Army Ammunition Plant
19	Marine Corps Logistics Barstow
20	McChord AFB
21	Mississippi Army Ammunition Plant
22	Mountain Home AFB
23	NAS Corpus Chisti
24	NAS Oceana
25	NAS Pensacola
26	Naval Air Station Atlanta
27	Naval Air Station Brunswick
28	Naval Air Station Williow Grove
29	Naval Base Coranado
30	Naval Base Ventura City
31	Naval District Washington DC
32	Naval Medical Center Portsmouth
33	Naval Medical Center San Diego
34	Naval Station Great Lakes
35	Naval Station Ingleside
36	Naval Station Pascagoula
37	Naval Suport Activity, New Orleans
38	Naval Support Activity Crane
39	Naval Weapons Stations Seal Beach Concord Detachment
40	Newport Chemical Depot
41	Niagara Falls International Airport Air Guard Station
42	Onizuka Air Force Station
43	Otis Air National Guard Base
44	Pope AFB
45	Red River Army Depot
46	Riverbank Army Ammunition Plant
47	Rock Island Arsenal
48	Selfridge Army Activity
49	Sheppard AFB
50	Umatilla Army Depot
51	W.K. Kellogg Air Force Guard Station
52	Walter Reed National Military Medical Center

Table VI. Alternative military bases and their numerical designations

Base m	Survivability Index: Benefits ($\tilde{B}_m$) $D_B^{\alpha=0L}$	D_B^m	$D_B^{\alpha=0R}$	Costs ($\tilde{C}_m$) $D_C^{\alpha=0L}$	D_C^m	$D_C^{\alpha=0R}$
1	0.05897	0.05905	0.05913	0.00099	0.00102	0.00105
2	0.05464	0.05487	0.05510	0.00461	0.00468	0.00476
3	0.04617	0.04688	0.04760	0.00857	0.00863	0.00869
4	0.05680	0.05690	0.05700	0.00141	0.00149	0.00156
5	0.06036	0.06039	0.06043	0.00113	0.00115	0.00118
6	0.06089	0.06092	0.06095	0.00313	0.00323	0.00333
7	0.05799	0.05810	0.05821	0.00114	0.00118	0.00121
8	0.05757	0.05763	0.05769	0.00296	0.00303	0.00309
9	0.05472	0.05494	0.05516	0.00301	0.00307	0.00312
10	0.05603	0.05634	0.05664	0.01262	0.01286	0.01311
11	0.05542	0.05561	0.05579	0.00213	0.00218	0.00223
12	0.00000	0.00000	0.00000	0.00019	0.00021	0.00023
13	0.05886	0.05887	0.05888	0.00097	0.00102	0.00107
14	0.05617	0.05633	0.05649	0.00246	0.00249	0.00252
15	0.05823	0.05825	0.05828	0.00156	0.00161	0.00166
16	0.06076	0.06079	0.06082	0.00282	0.00291	0.00301
17	0.06161	0.06162	0.06162	0.03332	0.03479	0.03626
18	0.06134	0.06138	0.06142	0.00044	0.00045	0.00046
19	0.06041	0.06047	0.06053	0.00001	0.00002	0.00002
20	0.05783	0.05787	0.05792	0.00000	0.00000	0.00000
21	0.05840	0.05841	0.05842	0.00066	0.00071	0.00075
22	0.05799	0.05804	0.05809	0.00407	0.00412	0.00417
23	0.05867	0.05882	0.05898	0.00183	0.00187	0.00190
24	0.06004	0.06008	0.06012	0.00649	0.00664	0.00678
25	0.05966	0.05966	0.05967	0.00099	0.00104	0.00108
26	0.05893	0.05905	0.05917	0.00045	0.00047	0.00048
27	0.05855	0.05875	0.05895	0.00544	0.00555	0.00567
28	0.05621	0.05640	0.05659	0.00232	0.00236	0.00241
29	0.06099	0.06100	0.06101	0.00000	0.00002	0.00005
30	0.06098	0.06100	0.06102	0.00115	0.00119	0.00123
31	0.06036	0.06041	0.06045	0.00062	0.00064	0.00066
32	0.05403	0.05433	0.05464	0.00321	0.00325	0.00330
33	0.05910	0.05910	0.05910	0.01211	0.01337	0.01464
34	0.06077	0.06077	0.06077	0.00869	0.00941	0.01012
35	0.05805	0.05820	0.05835	0.00268	0.00273	0.00278
36	0.05496	0.05514	0.05533	0.00102	0.00103	0.00104
37	0.06048	0.06050	0.06051	0.00195	0.00202	0.00210
38	0.05686	0.05692	0.05698	0.00306	0.00308	0.00310
39	0.05995	0.06000	0.06006	0.00120	0.00125	0.00130
40	0.05800	0.05804	0.05807	0.00019	0.00020	0.00020
41	0.06189	0.06189	0.06189	0.00444	0.00472	0.00500
42	0.05927	0.05933	0.05938	0.00151	0.00156	0.00161
43	0.05860	0.05868	0.05876	0.00407	0.00424	0.00441
44	0.04291	0.04365	0.04438	0.00389	0.00394	0.00399
45	0.06003	0.06008	0.06013	0.00301	0.00312	0.00323
46	0.06019	0.06021	0.06022	0.00097	0.00100	0.00103
47	0.06085	0.06086	0.06087	0.00157	0.00164	0.00171
48	0.05919	0.05919	0.05919	0.03008	0.03233	0.03458
49	0.05980	0.05984	0.05987	0.00417	0.00426	0.00435
50	0.06025	0.06034	0.06042	0.00115	0.00117	0.00118
51	0.05991	0.05991	0.05991	0.00043	0.00047	0.00050
52	0.05603	0.05631	0.05658	0.01773	0.01809	0.01846

Table VII.
The survivability indices and their components

Rank	Alternative	Military base	Euclidean Distance
1	12	Ft Eustis	0.000210
2	44	Pope AFB	0.043825
3	3	Cannon Air Force Base	0.047671
4	32	Naval Medical Center Portsmouth	0.054431
5	9	Fort McPherson	0.055027
6	2	Brooks City Base	0.055071
7	36	Naval Station Pascagoula	0.055153
8	11	Fort Monroe	0.055649
9	14	Grand Forks AFB	0.056383
10	28	Naval Air Station Williow Grove	0.056451
11	4	Desecret Chemical Depot	0.056920
12	38	Naval Support Activity Crane	0.057007
13	8	Fort Knox	0.057708
14	10	Fort Monmouth	0.057786
15	20	McChord AFB	0.057871
16	40	Newport Chemical Depot	0.058040
17	7	Fort Gillem	0.058114
18	22	Mountain Home AFB	0.058187
19	35	Naval Station Ingleside	0.058265
20	15	Kansas Amunition Plant	0.058273
21	21	Mississippi Army Ammunition Plant	0.058416
22	43	Otis Air National Guard Base	0.058829
23	23	NAS Corpus Chisti	0.058853
24	13	Gen Mitchell International Airport ARS	0.058878
25	27	Naval Air Station Brunswick	0.059009
26	26	Naval Air Station Atlanta	0.059050
27	1	Army Reserve Personnel Center St. Louis	0.059058
28	52	Walter Reed National Military Medical Center	0.059142
29	42	Onizuka Air Force Station	0.059347
30	25	NAS Pensacola	0.059673
31	51	W.K. Kellogg Air Force Guard Station	0.059911
32	49	Sheppard AFB	0.059987
33	39	Naval Weapons Stations Seal Beach Concord Detachment	0.060016
34	45	Red River Army Depot	0.060163
35	46	Riverbank Army Ammunition Plant	0.060214
36	50	Umatilla Army Depot	0.060348
37	5	Eielson, AFB	0.060404
38	31	Naval District Washington DC	0.060409
39	24	NAS Oceana	0.060445
40	19	Marine Corps Logistics Barstow	0.060470
41	37	Naval Suport Activity, New Orleans	0.060531
42	33	Naval Medical Center San Diego	0.060595
43	16	Kulis Air Guard Station	0.060857
44	47	Rock Island Arsenal	0.060881
45	29	Naval Base Coranado	0.061000
46	6	Elmendorf AFB	0.061007
47	30	Naval Base Ventura City	0.061014
48	18	Lone Star Army Ammunition Plant	0.061378
49	34	Naval Station Great Lakes	0.061498
50	41	Niagara Falls International Airport Air Guard Station	0.062067
51	48	Selfridge Army Activity	0.067442
52	17	Lackland AFB	0.070764

Table VIII.
The rankings according to the Euclidean distance from the ideal military base

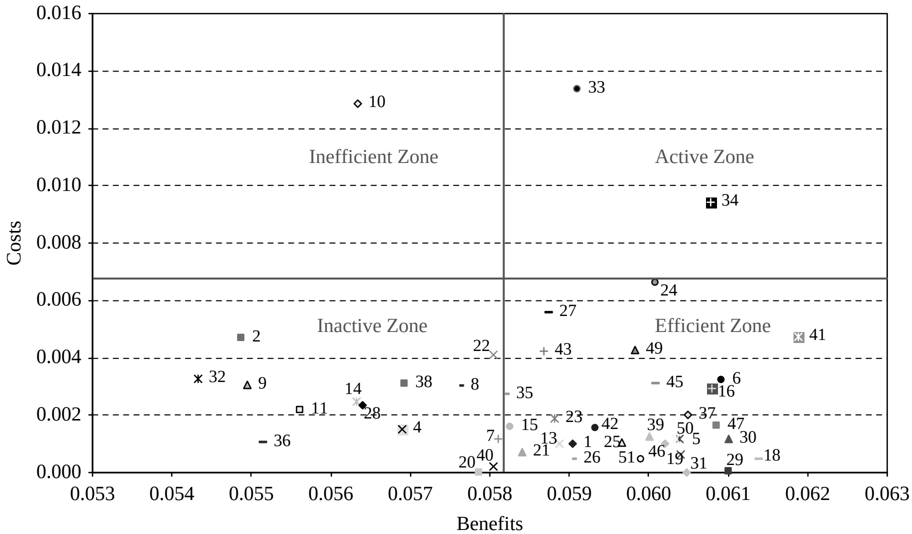

Figure 7.
Alternatives arrangement on α-level = 1

Given two sets A and B, we were interested in the following questions: Is the intersection between A and B empty or not? Is A a subset of B? Are A and B equal? These questions can be answered using a fuzzy extension of the Boolean inclusion index proposed by Bandler and Kohout (1980). Dubois and Prade (1982) have proposed a framework for building fuzzy comparison indices where three types of indices are considered: overlap indices (called partial matching), inclusion indices and similarity indices (evaluating equality between fuzzy sets). The comparison of fuzzy sets could also be described by means of a fuzzy-valued compatibility index introduced by Zadeh (1978). However, there is a gap in the literature on the comparison methods for multidimensional fuzzy relations. To compare our alternative military bases we must take into consideration the Cartesian product of their benefits and costs constituents. Consequently, we considered calculating the volume of each resulting pyramid as the characteristics of an alternative fuzziness degree. The volume of a pyramid is equal to one third of the product of the area of pyramid basis and the length of its height:

$$V = \frac{1}{3} S_{\mathrm{b}} \cdot H \tag{24}$$

where S_{b} is the area of pyramid basis and H is the length of the pyramid height.

In our case, $H = 1$ and S_{b} is a rectangle. Using our variables, equation (24) can be reformulated as:

$$V_m = \frac{1}{3}\left(D_{Bm}^{\alpha=0R} - D_{Bm}^{\alpha=0L}\right) \cdot \left(D_{Cm}^{\alpha=0R} - D_{Cm}^{\alpha=0L}\right) \tag{25}$$

On the basis of these uncertainty levels, it is reasonable to swap the rankings for some of the alternative military bases, namely 2 and 36, 4 and 38, 10 and 20, 35 and 15, 43 and 23, 27 and 26, 45 and 46, 24 and 19, 16 and 47, 6 and 30, 18 and 34. The alternative rankings are presented in Table IX.

Rank	Alternative	Base
1	12	Ft Eustis
2	44	Pope AFB
3	3	Cannon Air Force Base
4	32	Naval Medical Center Portsmouth
5	9	Fort McPherson
6	36	Naval Station Pascagoula
7	2	Brooks City Base
8	11	Fort Monroe
9	14	Grand Forks AFB
10	28	Naval Air Station Williow Grove
11	38	Naval Support Activity Crane
12	4	Desecret Chemical Depot
13	8	Fort Knox
14	20	McChord AFB
15	10	Fort Monmouth
16	40	Newport Chemical Depot
17	7	Fort Gillem
18	22	Mountain Home AFB
19	15	Kansas Amunition Plant
20	35	Naval Station Ingleside
21	21	Mississippi Army Ammunition Plant
22	23	NAS Corpus Chisti
23	43	Otis Air National Guard Base
24	13	Gen Mitchell International Airport ARS
25	26	Naval Air Station Atlanta
26	27	Naval Air Station Brunswick
27	1	Army Reserve Personnel Center St. Louis
28	52	Walter Reed National Military Medical Center
29	42	Onizuka Air Force Station
30	25	NAS Pensacola
31	51	W.K. Kellogg Air Force Guard Station
32	49	Sheppard AFB
33	39	Naval Weapons Stations Seal Beach Concord Detachment
34	46	Riverbank Army Ammunition Plant
35	45	Red River Army Depot
36	50	Umatilla Army Depot
37	5	Eielson, AFB
38	31	Naval District Washington DC
39	19	Marine Corps Logistics Barstow
40	24	NAS Oceana
41	37	Naval Suport Activity, New Orleans
42	33	Naval Medical Center San Diego
43	47	Rock Island Arsenal
44	16	Kulis Air Guard Station
45	29	Naval Base Coranado
46	30	Naval Base Ventura City
47	6	Elmendorf AFB
48	34	Naval Station Great Lakes
49	18	Lone Star Army Ammunition Plant
50	41	Niagara Falls International Airport Air Guard Station
51	48	Selfridge Army Activity
52	17	Lackland AFB

Table IX.
The revised rankings

5. Conclusions and future research directions

The benchmarking system presented in this study helps DMs crystallize their thoughts and reduce the environmental complexities inherent in the BRAC decisions. The BRAC Commission can utilize the survivability indices to arrive at a ranking of the military bases on the DoD hit list. Moreover, the commanding officers of the military bases can use the four-quadrant classification approach to identify their strengths and weaknesses by learning from "best-in-class" and other competing bases.

Our model is intended to create an even playing field for benchmarking and pursuing consensus not to imply a deterministic approach to BRAC decisions. The BRAC is a very complex problem requiring compromise and negotiation within various branches of government and public. The analytical methods in the proposed benchmarking system help DMs decompose complex MCDA problems into manageable steps, making this model accessible to a wide variety of situations. These methods are not developed through a straightforward sequential process where the DM's role is passive. On the contrary, the iterative process is used to analyze the objective and subjective judgments of multiple DMs and represent them as consistently as possible in an appropriate structured framework. This iterative and interactive preference modeling procedure is the basic distinguishing feature of our model as opposed to statistical and optimization decision making approaches.

MCDA and fuzzy sets are useful tools for handling inherent uncertainty and imprecision in rapidly changing environments. There are many facets of MCDA in fuzzy environment which require more thorough investigation. The model developed in this study can be extended to a multi-stage model with probabilistic outcomes. BRAC decisions are generally long-term and could be considered in stages over a period of time. A multi-stage model under fuzziness, involving objective and subjective aspects, could assess potential impact on different stakeholders over a period of time. The model could focus not only on which bases should be closed, but how closure and realignment should take place in stages.

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