DOCTORAL THESIS

Team-oriented Airline Crew Scheduling and Rostering:
Problem Description, Solution Approaches,
and Decision Support

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To my
loving parents
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Abstract

Airline crew scheduling is a comparably well-studied field in Operations Research, and most cost-relevant factors have been optimized to the greatest extent. As a result, usually second-class or casual attention is paid to the consideration of the teams involved that form the mandatory flight personnel. It is well-known that misunderstandings and disharmonies among the crew due to unfamiliar colleagues lead to a negative crew satisfaction which has a negative effect on the airline’s customers. More importantly, their occurrence is even safety-critical with regard to the operating cockpit crew.

In this work a first interpretation of the Team-oriented Scheduling Problem (ToSP) is presented. Irrespective of the assignment approach chosen (bidline systems, personalized rostering or preferential bidding systems), current approaches do not account for frequently occurring changes within daily or day-by-day team compositions. So crew members rarely know in advance the strengths and weaknesses of their team-mates they are scheduled to work with. Therefore, the realization of enhanced team stability should be highly appreciated by the crew as well as by the airline itself. In this document a special emphasis is put on the personalized rostering in the so-called Team-oriented Rostering Problem (ToRP).

Tailored to the requirements for the cockpit crew, namely captains and first officers, several mathematical programming-based optimization models are discussed. Based on a case study of a European tourist airline, a set of solution approaches is presented. When the new approach is embedded into a decision support system, the implied trade-off between additional operational cost and the selected evaluation criteria for team orientation, e.g., the amount of team changes, is examined along with further computational results.

Keywords: airline, cockpit crew, crew scheduling problem (CSP), crew pairing problem (CPP), crew rostering problem (CRP), crew assignment problem (CAP), decision support system (DSS), set partitioning problem (SPP), team orientation, team-oriented rostering problem (ToRP), team-oriented scheduling problem (ToSP)
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Chapter 1
Introduction

The motivation for this work is explained in section 1.1, followed by the thesis’
goals and their demarcation in 1.2. A presentation of the proceeding structure in
1.3 concludes this first chapter.

1.1 Motivation

Already since many years airline companies for passenger traffic operate in a
highly competitive global market. During last years also continental and domes-
tic air traffic markets in Europe have become very competitive. Therefore cost
minimization on the one hand, as well as customer satisfaction on the other, are
the dominant driving forces for their current business and future development.

Numerous factors influence the performance of an airline company. After fuel,
the second highest expense known is personnel. Since the chances for fuel re-
duction are restricted to basically physical constraints, especially the great pro-
portion of cost imposed by the aircrew turned out to be an excellent starting
point for further investigation. Due to the fact that the resulting general Crew
Scheduling Problem (CSP) is fairly easy to formulate, researchers from Opera-
tions Research (OR) and related fields hence examined a variety of optimization
models, instances, and other approaches. The assignment of flights to crew
members is acknowledged to be a combinatorial challenge, particularly because
of the NP-completeness of the problem class. This comes along with the grow-
ing size of instances that occur in practice. Nevertheless, great progress has
been achieved during the past decades whereas the overall focus remained on
an emphasized consideration of cost aspects for the CSP.

The onboard crew is composed by several individuals. Thus let us define a
team (in the context of airlines) for the moment as “a group of people that is
scheduled to fly together”. They directly interact with each other, but also with
the valuable customers of the company, especially during the operation of a
flight. This expresses the necessity to pay more attention to the perspective of social and organizational psychology, where the interaction and dynamics inside and among groups are studied. Prof. Hackman from the Harvard University reported in his book “Leading Teams” (Hackman 2002) the enabling conditions for efficient, but moreover, highly effective teams. Based on several case studies including the sector of airline crew scheduling, he determines several factors in order to obtain an increased likelihood for a better team performance.

Returning to the field of business administration, another domain involved in the scheduling of aircrew is a dedicated area of Human Resource Management (HRM). Apart from generating crew schedules at minimum cost, researchers as well as practitioners in this field concentrate more on recruiting and training of staff, but, among others, also on employee satisfaction, since this may be greatly influenced by the tasks assigned to the crew.

While traveling as a passenger on a long journey, everyone may have already experienced how stressful it can be to change the aircraft or even the airport. Now imagine the following commonly experienced scenario for flight personnel:

A pilot is stationed in Frankfurt. According to his work plan, he starts his four day working period in Cologne. He has to take the train about four hours ahead of his first flight to be serviced. There are three flights that he has to execute today: Cologne to Hamburg, Hamburg to Palma de Mallorca, and Palma to Hamburg. A short break takes place between the first two flights, and he has to directly change the aircraft in Palma after his second break. At the end of his working day he will stay overnight in a hotel in Hamburg, from where he will depart on the next day.

The setting described above is the conventional single-person perspective as it is experienced in a similar way by a passenger traveling on his/her own. But the aircrew has to work during this time, and all the flights assigned to this exemplary pilot during the day may imply to work with different colleagues. Both members of the cockpit crew (captain and first officer) have to maneuver the aircraft hand in hand as a team. Not only that they risk their own life, but for public transportation, passengers and additional flight personnel are massively endangered as well in case of inconsistent or mistaken decisions encountered on the flight deck. Concerning the cabin crew, usually, their team members experience a similar effect during the day: Servicing passengers and being in a hurry to get on/off board right in time for their next scheduled duty, again working with different colleagues and mostly without enough time to relax or to overcome the recent, sometimes negative, incidents encountered. Of course, all crew members successfully attended intensive education and training programs on the aircraft type (or fleet) they are working on. Further activities, such as simulator or specific trainings on emergency situations are compulsory, and valid certificates prove the knowledge required to be employed by the airline.
Already in 1994, the National Transportation Safety Board (NTSB) conducted a study on the circumstances for cockpit crews which experienced major accidents over a time span of almost 13 years, see (NTSB 1994): According to their findings, 73% of all incidences took place during the crew’s first day, and 44% occurred even during the initial flight of newly formed crews. Additionally, other researchers emphasized the higher on-time performance of lasting teams, see (Rand 1998). Keeping this in mind, how sensible does a normal job advertisement appear in which a high degree of flexibility is required, e.g., “be prepared to work on any day of the year at any time of the day” (Easyjet 2004). If the same requirement is valid for all other employees as well as for the new applicant, they may also demand that you should be prepared to work with potentially everyone else. Additional personal restrictions are not appreciated, since they are in conflict with the desired adaptability of the personnel as a regular resource. But if such frequently changing crews must excellently perform without exception, also interpersonal aspects always have to be coped with, such as disharmonies and communication problems.

In the author’s opinion, an intense regard of those rather psychological aspects as described above are weakly investigated in the context of OR science and literature on the scheduling of personnel. It has to be acknowledged that – apart from general availability requirements with regard to the requested quantity and quality of an aircrew – important team aspects, such as its stability, are mostly neglected so far. Thus, the overall question of this work can be expressed as:

How to actively consider teams while scheduling flight personnel?

Our business partner, a European tourist airline has multiple home bases located in Germany where their crew members are stationed. Most of the more than 30 destination airports are spread all over Europe, especially those resorts which attract great amounts of travelers every year. Therefore, passengers usually spend some time at a destination and return afterwards. An effect of such a characteristic is that a large portion of the flights can be observed as round trips. With regard to the CSP, a typical schedule of a crew member may be a trip to one city, coming back to Germany within the same day if he or she has only two flights to serve. Frequently, the crew member may not fly back directly afterwards because the next flight heads to another location. However, it is usual that there are other flights going back to the original departure airport while the referred flight is operated by another crew member. The planning period considered is usually half a month, within which the number of crew members of one crew position (e.g., first officer) ranges from around 50 to nearly 200 individuals to be scheduled. Depending on the aircraft fleet, the amount of flights involved may be greater than 2,000 for half a month.
For our cooperation partner the consideration of teams – as it is evident so far – brings up another advantage: The dominant positive effect reveals itself when the airline’s specific operational cost structure is realized: Since all home bases are located near to each other, the aircrew can be transferred directly to them by public transport, namely by taxi. Based on the contracts arranged, transfer cost may then be saved due to non-seat dependant transit expenses. However, in their current approach with separated crew functions and without team orientation, taxis are seldom shared up to their full capacity.

1.2 Goals of the Thesis and Demarcation

The aim of this work is to examine selected aspects that answer the key question introduced above. Therefore, the enclosed list of topics is covered in the upcoming chapters:

- A state-of-the-art analysis on the airline CSP has to be given in order to present the operations research perspective of the problem, recent advances, solution approaches and their limitations with respect to the current knowledge of research.

- A brief introduction to team work aspects from the socio psychological and HRM perspective should be presented which gives some important insights regarding the relevance of the team orientation potentially considered for the scheduling of aircrew.

- As a result, the combination of both fields should allow an interpretation of the Team-oriented Scheduling (ToS) with special focus on the OR perspective and an analysis of its impact on the business application.

- Optimization models and solution approaches have to be developed, applied and evaluated for the assignment of crews as far as possible.

- The Team-oriented Scheduling should be integrated into a Decision Support System (DSS) while different input and output scenarios are considered and their results are reviewed.

- For the proof of concept, a selected subset of the ideas and methods mentioned above should be applied to and discussed for practical instances of our business partner.

The emphasis of this thesis lies on the operations research perspective of the problem. Therefore especially socio-psychological as well as HRM-related ideas and methods are briefly introduced when necessary. Furthermore, this work mainly concentrates on the personalized rostering approach for the rostering
step (see subsections 2.1.2 and 2.1.3) due to the fact that a general applicability of the so-called Team-oriented Rostering (ToR) approach is retained and all major changes occur at this point of the scheduling process by the proposed proceeding. In addition, all concepts described later on are especially designed for the cockpit crew. This focus is chosen because the cockpit crew is usually the investigated test case, and the consideration of teams for cabin crews is much more complex, see subsection 3.3.1.

1.3 Structure of the Thesis

According to the thesis goals mentioned, this work is structured as follows:

In chapter 2 a detailed state-of-the-art on the CSP for airlines is introduced and discussed. This includes a description of the task environment, basic definitions and scheduling approaches applied all over the world during the past decades. Since the CSP usually consists of two consecutive optimization steps, namely the Crew Pairing Problem (CPP) and the Crew Rostering Problem (CRP), both are introduced and examined in further subchapters, as well as their integration. This chapter ends with a brief summary.

Chapter 3 deals with the Team-oriented Scheduling (ToS). Firstly, the HRM perspective of team work is examined where the term team and its concepts are also analyzed regarding their occurrence in the airline sector. Later on, the valuable influence of team work on the companies’ safety and performance is presented as indicated by several airline case studies. This is followed by the author’s understanding of the Team-oriented Scheduling Problem (ToSP) in general and the Team-oriented Rostering Problem (ToRP) in particular. Hence problem characteristics are examined, and possible alternative objectives are discussed for their later investigation. To achieve a general applicability of the new approach, certain strategies on the modified prerequisites for the ToRP are included, also given as an extension to the network formulation usually applied for the partially integrated scheduling approach developed at the Decision Support & OR Lab. This chapter is concluded by a summary as well.

In chapter 4 a more formal description to the ToRP is given by a set of optimization models that apply mathematical programming for their solving. All models are designed to work on the cockpit crew. Besides the proposed quadratic model, two integer programming formulation and one mixed integer programming formulation are presented. They are summarized and evaluated in comparison with a classical CSP model regarding their usability for the ToRP in all its particulars in the final section.
Several solution methods of one of the models proposed in Chapter 4 are examined in chapter 5 using test data from our business partner. Here some implementation details on the necessary extensions with respect to the newly defined ToR approach are shown. A variety of tests on different instances demonstrate its application for this business setting.

The integration of the ToR approach extracted above into a decision support system is specified in chapter 6. Therefore, the scheduling processes of airlines are embedded in a general system architecture, where a set of modifications pays attention to the special requirements implied by the ToRP. In addition a list of features and system enhancements regarding the redefined rostering step is proposed for the input, evaluation and visualization scheme of the DSS.

In conclusion, in chapter 7 a summary of the results achieved in this work is presented and an outlook on useful directions for further research is given.

An overview of the complete proceeding structure is depicted in Fig. 1-1, where the gray arcs indicate the structure followed in this document. Further arcs show the interdependency throughout chapter 3 to chapter 6.
Chapter 2
Airline Crew Scheduling

In this chapter, an overview of the CSP is given from the OR perspective. First, the scheduling process at airlines is described, followed by general characteristics, goals and definitions in section 2.1. A more formal description of the CSP is presented in 2.2 by mathematical programs for the subdivided scheduling steps, namely crew pairing and crew assignment. After that, a state-of-the-art literature review on solution approaches is presented in section 2.3. In addition to the previously discussed classical two-phase scheduling for flight personnel, our partially integrated approach on the CSP is described in section 2.4. As an excursion in 2.5, the operational CSP, also known as crew recovery problem, is briefly introduced. Finally, this chapter is summarized in its last section.

2.1 Basics

The basis of the further content of this document is provided by this section: Firstly, the airline’s scheduling steps are presented in subsection 2.1.1, followed by a more detailed problem description and related definitions in 2.1.2. Commonly applied assignment approaches are introduced in subsection 2.1.3, while a classification of work rules and regulations is given in 2.1.4.

2.1.1 Scheduling Processes

The main business of an airline is to offer and execute flights in such a way that profit is maximized. In order to “produce” such a kind of service, there is a set of underlying scheduling tasks that need to be fulfilled. In general, it is required to determine:

- which flights are offered,
- when and how often such flights should take place,
which resources are allocated at which scheduling step, and

how the schedule execution is controlled and unpredictable events are handled.

In Fig. 2-1 all general scheduling steps at airlines are presented: The first step, block and ground time estimation, evaluates the specific time requirements for boarding, loading/unloading, refueling, cleaning, and catering. It may vary by airline, pattern, and airport. Demand estimation examines the market in order to extract the potential needs of the customers while considering, e.g., competitors outside the airline industry. During network planning, it is defined whether destinations are approached via a hub-and-spoke (as usual in the US) or a point-to-point network (often encountered in Europe). Capacity planning decides on the required amount of seats as well as the flight’s frequency.

Subsequent to capacity estimation, fleet assignment considers, e.g., capacity, range of aircraft and operational cost (fuel, maintenance, etc.) while covering all requirements defined during earlier steps by selecting the appropriate fleet to each individual flight to be serviced. Aircraft routing works at this stage on logical vehicles ensuring that regularly scheduled ground maintenance cycles remain feasible in such a way that the final flight scheduling is economically useful and can be published afterwards. Crew scheduling enables the execution of all flights by the available flight personnel, whereas physical aircraft scheduling assigns dedicated (physical) aircraft to the defined rotations (including vehicle-dependant maintenance requirements). Closely prior to the flight’s execution, ground operation scheduling determines details like gates, ground personnel, aviation security, catering, etc.

Operational time rescheduling as the final scheduling step handles all unexpected disruptions of the normal operation by minimizing cost and impact of those events. For further details on the entire scheduling process at airlines it is referred to, e.g., (Etschmaier and Mathaisel 1985; Barnhart et al. 2003; Suhl 1995).

All steps above are interrelated very much, e.g., the capacity of a flight may change by the exact departure time which is finally fixed during one of the successive steps. Each step itself receives the deliverables from the earlier ones, and its results are processed during the subsequent ones. Following the circle that is depicted in Fig. 2-1, the results of the scheduling steps get more concrete until all details are determined for the day of the schedule’s realization. Due to the high economic influence of the airline’s flight schedule, there are several alternative strategies how major scheduling steps above are sequenced and/or combined. It is referred to (Antes 1997) for a comparison among those strategies.
According to the airline scheduling steps introduced above, the consideration of onboard crews for the aircraft – as highlighted in Fig. 2-1 – takes place during two phases:

- Crew scheduling in planning phase, and
- Crew rescheduling during operations control phase.

2.1.2 Task Characteristics and Definitions

A general formulation for the airline crew scheduling problem (CSP) can be paraphrased as follows: Given the published flight schedule of an airline, the key task is to assign all necessary crew members of a cockpit and a cabin crew in such a way that the airline is able to operate all its flights at minimal expense for personnel. This assignment has to consider all restrictions imposed by governmental regulations, union agreements, and company-specific rules. In addition, individual time- and location-dependent crew availability has to be accounted for, especially in a setting where the personnel is partitioned to multiple airports (called home bases).

As for all personnel assignment problems, labor demand is calculated over long and short forecast periods. Regarding the overall accepted definition of the crew scheduling problem in research, here, issues like day-off scheduling, training
and vacation scheduling, etc. are explicitly neglected, since they are usually determined ahead of the CSP. Also the later scheduling of the standby crew (being available to substitute immediately a crew member) and reserve crew (available after a certain time interval) are not considered. For their application, see, e.g., (Strauss 2001; Abdelghany et al. 2004).

The cost associated with flight personnel in the CSP is determined by two components: crew salary and (planned) operational cost. Whereas the crew salary at most European airlines is handled as a gradual linear function (fixed salary for about 2/3 of the contracted flight hours, higher hourly rate(s) for the rest, if needed), airlines in North America apply a system called pay-and-credit. It refers to the difference between the number of hours that a crew member is paid for and the actual hours of flying (see Gershkoff 1989). Besides, operational cost has to be minimized, such as expenses for hotel stays and for transit of crew members from/to their current/next scheduled location.

The general CSP is recognized to be very difficult to solve (NP hard) due to its combinatorial complexity, see, e.g., (Barnhart et al. 2003; Suhl 1995). Thus, it is usually decomposed into several sub-problems and even sub-steps: Firstly, the cockpit and the cabin crew are separated, commonly even to the level of their crew functions. As an example, for the cockpit crew, this results in a dedicated CSP for the captains and one for the first officer position.

Each of these crew scheduling instances is subdivided again into

- the Crew Pairing Problem (CPP), see, e.g., (Andersson et al. 1997; Barnhart et al. 2003) for an overview, and

- the Crew Assignment Problem (CAP) or Crew Rostering Problem (CRP), see, e.g., (Kohl and Karisch 2004) for an overview.

Even such problem instances belongs the class of NP hard problems. They are usually solved sequentially as shown in Fig. 2-2.
**Basic definitions**

The basic terminology with respect to the CSP applied throughout this work is defined as follows:

A *flight leg* is a non-stop air transit from a departure airport to its corresponding destination airport.

A *flight* may consist of several flight legs.

A *flight duty* is a series of flight legs that can be serviced by one crew member within a workday (24 hours). Such a flight duty is surrounded (before and after) by rest periods, whereas the off-time duration depends, for instance, on the start of the first flight leg and the number of flights serviced.

A *transit* occurs if the crew members’ time-dependent location does not equal to the next scheduled location. A *pre-transit* is necessary in the case that this relocating is directly required in advance of the next flight leg scheduled, a *post-transit* for its direct succeeding occurrence. Those *transits* (or *taxing*) are usually realized via public transportation (e.g., bus, taxi or train), or by flying as a passenger of the examined airline, called *deadheading*.

The next aggregation level is a *pairing* which starts from and returns to the crew member’s home base. Its maximum duration is limited by a given upper bound, e.g., five working days. Therefore, *hotel stays* become necessary, if flight personnel have to spend their daily rest period outside their home base.

*Pre-scheduled activities* like vacation, requested-and-granted off-periods, office, simulator, training, medical examinations, etc. represent activities that a crew member has to undertake without exception. They form the so-called *fingerprint*
(Mellouli 2001) or skeleton roster (Barnhart et al. 2003) for each crew member. Since those activities are usually determined prior to the scheduling process, overlapping flight duties are not permitted.

A roster or line-of-work (LOW) represents a potential crew schedule for a dedicated crew member of the planning periods of usually two or four weeks. It consists of assigned flight duties being compatible with the individual skeleton roster, besides incorporating with all the governmental, union and company rules as well as with the crew member’s individual work history and remaining contracted flight/work duration (in hours). A null-roster is defined as a roster without any flight legs assigned.

As an example a crew roster with the above defined components is arranged in Fig. 2-3: For the crew member CM1 a roster with three pairings (P1, P2, P3) is given. Note that CM1 is blocked on day3, since there is a simulator activity (SIM) already scheduled. The execution of P2 requires a hotel stay (H) as well as a transit (T) back to the home base of CM1 in order to operate P3 on the next day. The pairing P2 lasts for two working days, and as a result, it consists of two consecutive flight duties (FD1, FD2). FD1 itself can be decomposed again into the flights F1, F2 and F3 which may consume a total of less than 14 working hours without daily rest. The examined flight F3 starts from Düsseldorf (DUS) and heads to Palma de Mallorca (PMI) with a stopover in Munich (MUC). Therefore, flight legs are the lowest aggregation level for the trips from DUS to MUC as well as from MUC to PMI.

Figure 2-3: Exemplary crew roster and its components
Finally, a crew schedule consists of the optimal set of individual rosters. It may combine, e.g., home bases, crew functions, etc. and is published to the flight personnel a few days before the considered scheduling period starts.

### 2.1.3 Assignment Approaches

Even though a detailed formulation of the airline’s individual objectives may be stated in various ways, the main characteristic of the scheduling problem for crews focusing on the minimization of cost remains the same. In addition, airlines want to enhance or maximize the quality-of-life criteria for their personnel which can be – at least in parts – contradictory to cost minimization. Such quality-of-life criteria are addressed, e.g., by considering crew requests or their personal preferences during the scheduling process. This leads to the three assignment approaches known in practice and literature: Bidlines approach, personalized rostering, and preferential bidding, see also Fig. 2-2.

**Bidlines approach**

As widely applied in North America, the rostering task is performed in a two-step procedure: First a set of anonymous LOWs or bidlines is created that covers all flights at the minimum of operational cost. Once those have been determined, the bidding process starts: All crew members draw up their favorable rosters which are granted or rejected based on their seniority – the accumulated time span they have worked for the company.

Finally no employee has a less preferred LOW than any other less senior colleague. Additionally, the individual crew member also knows exactly the roster assigned to him or her, since it was directly pre-selected by him or her before. On the other hand, it is mostly impossible for less senior employees to get any of their favorable rosters during early stages of the bidding process or their range of rosters offered is already limited right at the beginning.

The bidlines approach is encountered in many airlines, e.g., in (Jarrah and Diamond 1997) for a major US airline, in (Christou et al. 1999) for Delta Airlines, in (Anbil et al. 1991) for American Airlines, and in (Campbell et al. 1997) at FedEx.
Personalized rostering

In Europe, Australia and New Zealand, personalized rostering, also known as fair-and-equal share, is more commonly used. It is focused on fairness for sharing workload among crew members, which replaces the seniority principle completely, sometimes only to a certain extent, see (Ryan 1992). Therefore, the system accepts or rejects crew requests (as pre-scheduled activities) and assembles the optimal crew schedule automatically while considering a high degree of expressed preferences and fair-share. For instance, if a certain crew member has been assigned some difficult night duties during the last rostering periods, he or she might receive “easier” trips for the next period, see (Jarrah and Diamond 1997). In this approach crew members have no further influence on the LOW assigned to them.

There is intensive research conducted on the personalized rostering in, e.g., (Ryan 1992; Day and Ryan 1997; Butchers et al. 2001) for the rostering at Air New Zealand, in (Mellouli 2001; Guo et al. 2003; Guo and Thiel 2004) it is applied on instances of our cooperation partner, and in (Gamache et al. 1999) at Air France. The latter also provide an overview on earlier solution approaches for the personalized rostering.

The ToR approach – as it will be discussed in succeeding chapters of this work – is based on personalized rostering. It extends the conventional personalized rostering approach in order to increase the degree of teamwork based on longer existing crew compositions which will be noticed by the crews during the execution of the crew schedule.

Preferential bidding

During the last decade, a third approach – also known as personalized rostering with preferences – has become more popular since it bypasses the drawbacks of the earlier concepts. Preferential bidding considers crew preferences at a certain level during the scheduling process, for instance, regularly pre-scheduled weekly rest periods or working with specific colleagues, etc. Such individual preferences may interfere with other crew members’ interests, sometimes even with company interests. Conflicts are then solved by the application of the (strict) seniority principle while crew members can only express their preferences without knowing to which extent such first choices are satisfied by the roster finally assigned.

Preferential bidding systems (PBS) are examined, e.g., in (Gamache et al. 1998) at Air Canada, and, e.g., in (Byrne 1988; Rouse 2001) for applications on technical aircrew.
2.1.4 Governmental Work Rules and Union Agreements

As already mentioned above, crew schedules have to obey a set of rules and regulations in order to be considered as legal. Those detailed restrictions imposed by the FAA were created in order to protect the security in air traffic. Additionally, there might also be further agreements among employee unions and the airline which have to be followed. Over all, there are often more than several hundreds of rules to be rigidly fulfilled as *hard constraints*. Apart from this, there are also *soft constraints* which may be violated, although it is in general undesirable to do so.

According to (Kohl and Karisch 2004) such constraints can be classified into *horizontal, vertical, and artificial rules* as follows:

*Horizontal rules*

The majority of rules are applied only to characteristics of a single roster instead of considering several rosters or a roster combination. Typically, such rules restrict the general compatibility of a crew member, which is based on the task and the time setting for the scheduling instance examined. Furthermore, it includes regulations, such as rest time between flight legs, flight duties, prescheduled activities, etc. Off-day patterns for weekly rest periods are imposed as well, e.g., after a maximum of up to five working days that can be filled with flight legs, flight duties or pairings, a weekly rest period of two complete off-days (midnight to second next midnight) or 36 consecutive hours is required.

Besides, accumulated values are commonly used to restrict, e.g., the maximum number of flying hours, working hours or take-offs. Such restrictions are based on the recent individual work history of the employee, e.g., if limitations for yearly or quarterly values have to be maintained.

*Vertical rules*

The second group of rules addresses at least more than one roster in the solution. A typical application for this kind of restriction is the *crew complement* for a certain flight leg. Even though the crew rostering problem is usually divided up by fleet and crew function (see subsection 2.1.2), some activities may require crew complements with dedicated characteristics, such as (*qualification, type*) *constraints*. Examples for this kind of constraints are the handling of inexperienced flight personnel, must-fly-together requests, interpersonal incompatibilities, language qualifications, but also flying-below-rank (or *downgrading*).
Moreover, _global constraints_ regarding all rosters in the solution (instead of subsets) are realized by vertical rules, too. They may set, e.g., an upper bound on the overall cost in the solution or guarantee a certain level of roster satisfaction for the individual crew member.

The major differences between all kinds of rules introduced so far are visualized in Fig. 2-4.

![Figure 2-4: Horizontal, vertical and global work rules](image)

**Artificial rules**

In addition to the explicitly required constraints, artificial rules may be applied in order to enhance the solution quality. They can ensure aspects like _robustness_ against disruptions during the operational phase, e.g., by penalizing valid, but very short rest periods. Also solution methods can be supported by certain penalization strategies that prevent the consideration of valid, but unfavorable solutions, such as the ToR approach discussed later in this work.

### 2.2 Model Formulations

As given in subsection 2.1.2, the CSP usually consists of the two sequential scheduling steps: The CPP and the CAP/CRP. Their specific objectives and a general mathematical formulation are described in this subsection.

#### 2.2.1 The Crew Pairing Problem

The CPP is the first step of the solution process for the CSP. It aims at finding a set of pairings that covers all flight legs in the considered planning period at the
minimum of cost. Whereas those pairings themselves have to be compliant to a multitude of regulations (see subsection 2.1.4), they are still anonymously built without consideration of flight personnel’s individual requirements or preferences. Typically, crew pairing models are formulated as set partitioning problems (SPP) where every flight leg is covered exactly once, such as follows:

Let \( F \) be the set of flight legs, and \( P \) represents the set of all feasible pairings. Let the binary decision variable \( x_p \) equal to 1, if a pairing \( p \in P \) is included in the solution, 0 otherwise. Every pairing itself is given as a vector (column in the constraint matrix) with 0/1-entries, indicating in row \( i \) whether a flight leg \( f \in F \) is included or not. The resulting generalized CPP is given below.

\[
\begin{align*}
\min & \quad \sum_{p \in P} c_p x_p \\
\text{Subject to:} & \quad \sum_{p \in P} x_p = 1 \quad \forall f \in F \\
& \quad x_p \in \{0,1\} \quad \forall p \in P
\end{align*}
\]

The objective function (2.1) minimizes the sum over all operational cost for all pairings selected in the solution, whereas restriction (2.2) ensures that each flight leg \( f \in F \) is included in exactly one pairing \( p \in P \). All decision variables are defined as binary in (2.3).

### 2.2.2 The Crew Assignment Problem

The second step of the CSP is called crew assignment or rostering. In contrast to the first step, the CAP/CRP is solved for individual crew members: The set of pairings created during the CPP is assigned in a way that considers all governmental rules, union and company agreements as well as the fingerprint imposed by pre-scheduled activities, see 2.1.2.

All flight legs have to be properly staffed with the requested crew complement. The assignment is also realized on decomposed sub-instances of the CAP, e.g., by crew types (cockpit, cabin), crew functions (captain, first officer, etc.), and fleet, see (Ryan 1992). Based on (Gamache et al. 1998) the CRP can be formulated as a set covering problem (SCP) as follows:
Let $K$ be the set of crew members of the considered problem instance and let $S^k$ be the set of feasible rosters for the crew member $k \in K$. Let $P$ represent the set of all feasible pairings determined during the CPP. The minimum of required crew members for pairing $p \in P$ is given by $n_p$. In addition, $\gamma_s^p$ equals 1, if pairing $p \in P$ is included in roster $s$, 0 otherwise. The binary decision variable $x_s^k$ is set to 1, if roster $s \in S^k$ is assigned to employee $k \in K$, and 0 otherwise. Roster cost is given by $c_s^k$ which stands for real operational cost which may depend on the individual crew members’ characteristics: The same set of pairings results in different roster cost due to the special requirements for hotel stays and/or crew transits regarding the individual crew, also known as base variants (Kohl and Karisch 2004). Alternatively those roster costs may also express the degree of individually satisfied crew requests (Barnhart et al. 2003), since the minimum cost is already achieved by the CPP.

The generalized optimization model is given below.

\[
\text{min } \sum_{k \in K} \sum_{s \in S^k} c_s^k x_s^k \tag{2.4}
\]

Subject to:

\[
\sum_{k \in K} \sum_{s \in S^k} \gamma_s^p x_s^k \geq n_p \quad \forall p \in P \tag{2.5}
\]

\[
\sum_{s \in S^k} x_s^k = 1 \quad \forall k \in K \tag{2.6}
\]

\[
x_s^k \in \{0, 1\} \quad \forall s \in S^k, \forall k \in K \tag{2.7}
\]

In the objective function (2.4) the sum over all (employee-dependant) operational cost is minimized for all rosters selected in the solution. In restrictions (2.5) all pairings $p \in P$ are at least covered by the pairing-dependant amount of crew members $n_p$, and (2.6) ensures that every crew member $k \in K$ receives exactly one valid roster $s \in S^k$. Binary decision variables are defined for every crew-dependant roster in (2.7).
2.3 Solution Approaches

Basically, the simplest approach for solving a combinatorial problem is a complete enumeration and evaluation of all feasible solutions, followed by the selection of the best alternative. Although such a proceeding is possible in theory, it is not applicable to problems of practical size because of the combinatorial explosion.

As given in Fig. 2-5, in this section the most common solution approaches in literature are discussed with regard to their application for the CSP, mainly in the airline sector.

2.3.1 Constructive Heuristics

Despite the fact that the optimum is rarely obtained, simple but fast heuristic algorithms are important to get a feasible solution quickly. Frequently they may provide a good starting point for more complex computational efforts, or their results are sufficient with regard to the (sub-)task to be solved, see (Ernst et al. 2004a).

Constructive heuristics first decompose the problem into numerous smaller sub-problems, and then build a solution based on certain given strategies. For example, in the case of the CRP, a heuristic approach such as the day-by-day method can be applied as it constructs the schedule sequentially from first to last day, see, e.g., (Nicotte 1975; Glanert 1984), or the pilot-by-pilot method considering seniority among crew members when creating their rosters, see, e.g., (Moore et al. 1978; Byrne 1988). In (Glanert 1984) the CRP is solved by assigning higher priority tasks to crew members with higher priority (here: seniority). It is referred to (Teodorovic and Lucic 1998; Gamache et al. 1999) for an overview of such approaches.
More recently, constructive heuristics are also applied in (Strauss 2001) within their application, the so-called SWIFTROSTER, and in (Kharrazilha et al. 2003; Kohl and Karisch 2004) for the rostering at Carmen Systems (Carmen 2005). In (Guo et al. 2003) several multi-weight based assignment strategies similar to the day-by-day and pilot-by-pilot concepts were implemented tailored to the special needs of the CRP in a partially integrated CSP approach (see subsection 2.4).

2.3.2 Mathematical Programming

Both the CPP and the CRP are usually formulated as set partitioning or covering problems. In a SPP every activity to be scheduled is assigned exactly once, whereas over-coverage is allowed in a SCP (denoted as deadheads, see 2.1.2). The most common models can be distinguished as integer programming (IP), binary integer programming (BIP, or 0/1-IP), mixed-integer programming (MIP), and linear programming (LP). Here the main difference lies in the feasible area of the solution space with regard to their classification, and dedicated solving methods are available accordingly. Such models are then solved by state-of-the-art optimizers, such as ILOG CPLEX (ILOG 2005) and MOPS (Suhl 1994, 2005).

Real instances often have to cope with a high number of variables, e.g., 800 flights may easily result in billions of possible pairings, see (Vance et al. 1997a; Hoffmann and Padberg 1993), where each of them is represented by a column in the model (similar for rosters in the CRP). Besides, the data representation and the (sometimes huge set of) vertical and artificial constraints (see subsection 2.1.4) are given as additional rows.

As a common solving approach, constraints are relaxed. The remaining relaxed problems are easier to solve, but they require further steps in order to obtain a feasible solution to the original problem, e.g., a SPP can be relaxed to a SCP which makes it necessary to resolve overcoverage of activities afterwards. For IP models, the LP relaxation removes the integer constraint on the decision variables, and Lagrangean relaxation eases the computational burden by removing some constraints and incorporating them by penalized functions into the formulation of the objective function. For the latter, best bounds are determined by iteratively updating the objective function’s penalty coefficients. For an overview it is referred to, e.g., (Barnhart et al. 2003; Wolsey 1998).

Commonly known solving approaches for huge mathematical programs are branch-and-bound, column generation, branch-and-cut and branch-and-price,
as briefly introduced below together with some selected applications in literature.

**Branch-and-Bound**

Branch-and-bound follows the idea of divide-and-conquer. Customized schemes are usually applied to solve LP relaxations. Then, for a fractional variable in a given optimal LP solution, two sub-problems are generated (branching), and lower bounds for minimization problems (as in our case) are created that determine on which branch to continue according to defined strategies.

Since branch-and-bound is a standard strategy, it is rarely explicitly explained for the CSP. It is employed, e.g., by (Ryan 1992; Day and Ryan 1997) for a massive generalized CRP and for the duty rostering in short-haul operations, as well as in (Desaulniers et al. 1997) to solve a column generation (see below) sub-problem. In (Gamache et al. 1999) a partial exploration of the branch-and-bound tree is described. (Butchers et al. 2001) apply a generalized SPP model using LP and branch-and-bound to solve an application at Air New Zealand.

**Column Generation**

A further commonly used computational technique for solving large-scale integer programs (or linear programs) is column generation (CG): In CG, a restricted LP master problem, a relaxed reformulation based on the Dantzig-Wolfe decomposition, see (Dantzig and Wolfe 1960), is solved on a subset of columns (rosters), and dual variables are calculated to price out new columns. The sub-problem (often a constrained shortest path problem) checks whether any new columns with negative reduced cost exist. If so, they are added to the current restricted LP master problem and optimized again, otherwise the current LP solution is already optimal. For IP and MIP models, the later application of a branch-and-bound or a branch-and-price might be required which may still lead to a situation where there is no integer solution available, see (Wolsey 1998; Gamache et al. 1999).

The CG approach is examined in detail by (Lavoie et al. 1988; Gamache et al. 1999) for an application at Air France. In (Yan et al. 2002) optimal crew pairings are generated by a CG-based algorithm for a Taiwan airline. A constraint programming-based CG is applied among other strategies for the CAP in (Sellmann et al. 2000; Fahle et al. 2002). A CG is also implemented for the new formulation and decomposition proposed in (Vance et al. 1997b). For an overview and recent advances on CG it is referred to, e.g., (Desaulniers et al. 2002; Borndörfer et al. 2001; Lübbecke and Desrosiers 2002).
**Branch-and-Cut**

Branch-and-cut usually applies customized branch-and-bound schemes in conjunction with *cutting planes*. With regard to the convex hull of feasible solutions, classes of valid inequalities (constraints) remain excluded in the LP relaxation in order to reduce the size of the problem. If an LP relaxation does not comply with all constraints, a separation problem tries to identify invalid equalities which can be added (cutting off the solution space) to the original problem for its next optimization run. If there are no additional constraints violated, branching occurs on fractional values of the solution to the LP relaxation, see (Barnhart et al. 1998).

(Hoffmann and Padberg 1993) find optimal integer solutions for problems with a maximum of 300,000 pairings using a branch-and-cut algorithm while base constraints are explicitly considered. It is based on a heuristic method to obtain good integer-feasible solutions quickly while a cut generation procedure tightened the linear relaxation. In (Fischetti et al. 2001) a polyhedral approach to a simplified CSP and vehicle scheduling problem is examined, in which heuristics are incorporated into an exact branch-and-cut algorithm.

**Branch-and-Price**

Whereas branch-and-cut works on a LP relaxation with fewer rows (constraints), branch-and-price follows a similar strategy regarding columns (variables). Here, a sub-problem, the *pricing problem*, is solved to identify additional columns for the next optimization run of the LP relaxation. Branching is indicated according to branch-and-cut, when no additional columns are found, or the LP solution does not satisfy integrality conditions. Branch-and-price allows CG to be applied everywhere in the branch-and-bound-tree, see (Barnhart et al. 1998).

(Vance et al. 1997b) present a heuristic branch-and-price approach for the CPP which solved the CG approximately together with a node selection of the branch-and-bound tree. In (Barnhart et al. 1998) classes of problems are presented, such as the CSP, for which implicit pricing strategies are implemented to achieve tighter LP relaxations. The paper of (Freling et al. 2001) reports of a decision support system for crew planning which is built on a flexible branch-and-price algorithm. (Makri and Klabjan 2003) work on a CG approach for the CSP which incorporates with non-linear pricing strategies, subdivided into approximate and exact pruning rules. In (Hansen and Liden 2005) cost and pattern cuts are applied during a CG method applied for their cabin crew problem.
2.3.3 Network-based Models

The complete scheduling task or sub-problems of the CPP and CRP can also be formulated as network flows. In literature they are divided into two different kinds: trip-as-node networks and trip-as-arc networks, where the term trip usually stands for a flight leg, flight duty or pairing in the CSP setting. Furthermore, such networks can be transformed to equivalent SPP or SCP models with respect to the coverage constraints required.

**Trip-as-node networks**

Trip-as-node networks put emphasis mainly on the linkage between flights legs. In such connection-based networks, trips are given as nodes, and arcs represent possible connections, e.g., based on the rules for valid rosters, in between the nodes. Sometimes nodes are also decomposed into a set of two nodes connected by a fixed flight arc, see, e.g., (Yan and Chang 2002).

As shown in (Nicoletti 1975) already early papers describe the application of connection-based networks for the CRP. More recently, in (Yan and Tu 2002; Yan and Chang 2002) they are deployed to solve cockpit and cabin crew problems at China Airline, whereas in (Ozdemir and Mohan 2001) they are embedded in a Genetic Algorithm (see 2.3.4) on the CSP. Trip-as-node networks are also in use for the CPP in order to solve an underlying k-shortest path problem such as in, e.g., (Galia and Hjörring 2003).

**Trip-as-arc networks**

Alternatively, a trip-as-arc network exploits the characteristics of time and space in the problem, also known as time-space networks (TSN) or duty/flight based networks, see also (Desaulniers 1997; Klabjan and Schwan 2001). As already given by its name, arcs represent trips or crew activities, e.g., a flight leg or a rest period, while nodes are used to indicate station locations at dedicated times, such as arrival or departure events. In order to ensure a flow among those nodes, additional arcs, such as waiting arcs, as well as source and sink nodes are introduced, see (Mellouli 2003).

In (Kress and Golany 1994) a TSN dealt with military airlift operations determining the minimum number of crew members required, and in (Desaulniers et al. 1997) it is applied for solving the CPP at Air France. Further applications can be found in (Cappanera and Gallo 2003), where a 0/1-multi-commodity flow problem addressed the CRP, and in (Vance et al. 1997a; Klabjan and Schwan 2001).
such kind of network is deployed on different levels (flight leg, flight duty) for the CPP. In the latter approach a parallel solving procedure is described.

A formulation of an aggregated TSN flow model for a partially integrated CSP approach as given in (Mellouli 2001; Guo et al. 2003) is explained in detail in section 2.4.

2.3.4 Meta-Heuristics

In contrast to the above-mentioned constructive heuristics in subsection 2.3.1, the following quite popular algorithms are motivated by the simulation of naturally-occurring processes. Most common meta-heuristics are: Genetic algorithm, simulated annealing, and tabu search. See also, e.g., (Reeves 1993; Michalewicz and Fogel 2000) for an overview.

Genetic Algorithms

A genetic algorithm (GA) procedure involves the creation of a set of individuals (called population) where each of them represents a – not necessarily feasible – solution to the problem. While the population is updated by defined operators and mutations that occur at a certain probability, existing individuals are replaced by their offspring either randomly or at a certain probability. A so-called fitness value which implies the probability of being chosen for the reproduction is attached to each individual, see (Holland 1975; Ernst et al. 2004a).

In (El Moudani et al. 2001) a bi-criterion model is presented for the CRP which considers operational cost together with crew satisfaction aspects like total flown time, realized pairing preferences, etc.. (Chang 2002) introduces a GA application of short-haul (domestic) routes for a Taiwan airline, and (Ozdemir and Mohan 2001) apply a flight graph based GA for the CSP.

The approaches proposed in, e.g., (Kerati et al. 2002; Guo and Thiel 2004) handle the CPP and CAP in an integrated way, whereas both consider comparably short scheduling periods, and the latter works on the crew recovery problem, see also section 2.5.

Simulated Annealing

Adapted from the energy minimizing process for slow physical cooling of metals, simulated annealing (SA) algorithms accept at early stages worse solutions
to escape from local optima. As temperature declines the probability of accepting bad solutions converges to zero, see (Reeves 1993).

The bidline generation for FedEx is optimized by a SA approach as proposed in (Campbell et al. 1997) in which line purity (no mixtures of international and domestic, AM and PM work within flight duties) are aimed at. In (Lucic and Teodorovic 1999) a multi-objective aircrew rostering problem is addressed by SA.

**Tabu Search**

Due to the intended avoidance of getting trapped in local optima, a tabu list is maintained and updated by the tabu search (TS) approach. It prevents or penalizes already recently visited areas of the solution space, see (Glover 1989, 1990; Reeves 1993).

As an example, in (Bellanti et al. 2004) a TS and an iterative local search (ILS) procedure are compared in their application for a nurse scheduling problem in comparison to manually generated plans.

2.3.5 Further Approaches

Apart from the previously examined approaches there is a variety of further concepts to address the CSP or one of its sub-problems, such as: local optimization, constraint logic programming, and dynamic programming.

**Local Optimization**

Local optimization works on a dedicated area of a solution found instead of covering the complete model at the same time. This approach is used as an improvement strategy for existing solutions achieved by, e.g., heuristics or relaxed constraints.

In (Gershkoff 1989) subsets of pairings of the currently best solution are decomposed and recombined into new legal pairings. If their evaluation determines their superiority to the original pairings, they should be exchanged. A further example of this proceeding can be found in (Anbil et al. 1991). In (Graves et al. 1993) a crew schedule optimization model is presented which allows overcoverage and undercoverage of flights in the initial solution by penalties. A local optimization method is then applied to find potential improvements. In
(Guo et al. 2004) a local improvement is applied for the generation of recovered crew rosters subsequently to the application of a genetic algorithm.

**Constraint logic programming**

The constraint logic programming technology is able to solve highly combinatorial problems, where – in contrast to mathematical programming – the problem is represented by *domain variables* (instead of inequalities). Therefore, each variable is defined for its feasible region. Constraint logic programming (CLP) is applicable whenever feasible, but not necessarily optimal solutions are required, see, e.g., (Ernst et al. 2004a; Caprara et al. 1998).

A CLP approach is applied in (Guerinik and Van Caneghem 1995) on the dataset of a French airline, in (Christodoulou and Stamatopoulos 2002) on Olympic Airways, and in (Kakas and Michael 1999) for Cyprus Airways showing the realization of the concept. In, e.g., (Caprara et al. 1998; Sellmann et al. 2000; Fahle et al. 2002; Yunes et al. 2000) the integration of mathematical programming and CLP is outlined to be beneficial for hard combinatorial problems in comparison to the pure application of OR techniques. (Hansen and Liden 2005) apply CLP for the generation of optimal subgroups for their team-oriented cabin crew problem.

**Dynamic Programming**

*Dynamic Programming* (DP) works for optimization problems, in which the optimal solution value for a dedicated problem can be calculated recursively from the optimal values obtained previously for their sub-problems, see, e.g., (Wolsey 1998).

DP has been implemented in many CG approaches, such as in (Yunes et al. 2000), or to solve constrained shortest path problems in (Desrochers et al. 1992). In (Beasley and Cao 1998) DP is deployed for the calculation of a lower bound on the CSP search tree, which enabled the calculation of relatively large problems to their optimality.

It would be beyond the scope of this section to cover all solution approaches published over the last decades. Further approaches, such as combinations of the above, can be found, for instance in (Wedelin 1995) who presents an approximation algorithm employed by *Carmen Systems* that was a combination of LP relaxation, dynamic programming and a greedy algorithm for the CSP. In
addition, several more technically oriented papers have been published, e.g., (Atamtürk et al. 1996), examine a combined Lagrangian relaxation, linear programming and heuristics based approach to address large-scale set partitioning problems.

Hence, for a general classification and a state-of-the-art of the application areas and solution methods for personnel scheduling and rostering it is referred to (Ernst et al. 2004a, 2004b). An early and well-known survey on the airline CSP is given in (Arabeyre et al. 1969), while a more recent overview on instances solved is presented in, e.g., (Vance et al. 1997a; Beasley and Cao 1998).

In the following section the partial integration of CPP and CRP is proposed.

2.4 Partially Integrated Crew Scheduling

As given in subsection 2.1.2, the classical procedure to address the CSP follows the strictly sequential solving of its sub-problems: First CPP, CRP second. Both are handled separately due to their combinatorial complexity as well as their distinct objectives, see section 2.2. However, such a sequential procedure has some drawbacks; especially in the case that pairings generated in the CPP are “inconvenient” or even impossible to be assigned in the later CRP step due to already known unavailability of personnel. Such occurrences may impose additional cost for the pairing reconstruction and reassignment.

Although many publications highlight the need to solve the CSP (see section 2.3), rather few of them focus on the complete problem with joint scheduling steps: For instance, (Freling et al. 2001) presented an approach in which they solved the daily CSP first, whereas requests were not granted a priori, but assigned during the optimization process for a weekly CSP. Furthermore, in (Kerati et al. 2002; Guo et al. 2003) dedicated GAs are proposed which handled the CSP during the operational phase, and therefore, comparably short planning periods (for schedule recovery) were considered.

As described in 2.1.3, our previous research on the airline CSP is based on the personalized rostering approach, see, e.g., (Mellouli 2003; Guo et al. 2003). The author’s colleagues and himself proposed a partially integrated procedure to solve the airline crew scheduling problem for the complete planning period, thus making a contribution towards an exact optimal solution of the fully integrated CSP. As a first step we investigated models that generate not just pairings, but pairing chains taking guaranteed individual pre-scheduled activities of crew members into account.
To compare the different approaches on the CSP during the planning phase, our research regards the accumulation of skeleton rosters by summing up all pre-scheduled activities being assigned to the aircrew ahead of the examined rostering of flight legs. The classical sequential approach (see sections 2.1 to 2.3) and the partially integrated CSP approach in this section are visualized in Fig. 2-6. As indicated, their main difference can be seen in the earlier consideration of pre-scheduled activities in the integrated approach.

Therefore, this section is organized as follows: In 2.4.1 an aggregated TSN formulation for the pairing chain generation is presented, and a combination of constructive heuristics for the remaining rostering step is given in 2.4.2.

### 2.4.1 Aggregated Network Formulation for the Crew Pairing Chain Problem

In contrast to the well-known approaches for the CPP, see section 2.3, we focused on solving the above airline crew pairing chain problem (CPCP, a strict extension of the standard CPP) in the first step. A pairing chain is a sequence of pairings which covers the scheduled time period, incorporating weekly rest periods such that all valid rules and regulations have been taken into account. The underlying basic TSN structure is briefly introduced as follows:

Let $T$ be a timetable with a given set of flights, involving a set of airports $A$. For each airport $k \in A$, let $E^k$ be the list of all arriving flight legs of $T$ at airport $k$, and $S^k$ the list of all departing flight legs of $T$ from airport $k$. Hence, each flight leg $i$ of $T$ given as an end event (arriving flight leg) in $E^{\text{dest}(i)}$ and as a start event (departing flight leg) in $S^{\text{orig}(i)}$. 
Without the loss of generality, end and start nodes at each airport are aggregated to reduce the size of the network. All flight legs in the lists $E^k$ and $S^k$ are chronologically sorted with regard to their start and end times at each airport $k \in A$. Furthermore, all $E^k$ and $S^k$ are partitioned into end blocks and start blocks, respectively, such that end times of $E^k$ flights are less than or equal to start times of $S^k$ flight legs which in turn are strictly less than end times of $E^{k+1}$ flights. (Hereby, let us assume that the end times include the required minimum turnover time as well.) Since this partitioning is unique, let $w_k$ be the resulting number of end blocks, which is equal to the number of start blocks, at airport $k \in A$. Each end block $E^k_l$ ($l = 1, \ldots, w_k$) precedes the corresponding start block. Only $E^k_1$ and/or $S^{w_k}$ may be empty for some airports $k$.

Based on the flight leg lists, a network can be built according to the following structure: For each airport $k \in A$ and $l = 1, \ldots, w_k$, each arriving flight leg of $E^k_l$ is compatible with each of the departing flight legs of $S^k_l$. Representing flight legs by arcs and crew members by flow units, this compatibility is established through a connection node $n^k_l$, see Fig. 2-7. Observe that each arriving flight leg of $E^k_l$ is also compatible with each departing flight leg of $S^k_{l'}$ for each $l' > l$, because the flow units may follow the connection nodes $n^k_l, n^k_{l+1}, \ldots, n^k_{l'}$ along horizontal waiting arcs on the time axis.

For each airport $k \in A$, a connection line $CL^k$ is defined as the line constituted by the connection nodes together with the waiting arcs. It represents all possible flight leg connections at airport $k$. In addition, it is distinguished between direct connections (for aggregated arriving flight legs and aggregated connection flight legs) through one connection node, and indirect connections which include several connection nodes linked by waiting arcs. For indirect connections, crew members at the same airport are aggregated that are waiting at the same time for indirect connections between two connection nodes.

![Figure 2-7: Structural view of an aggregated time-space network](image-url)
The aggregation proposed does not automatically deliver the intended result of a flow on this network. After having computed such a flow on the TSN, the freedom becomes clear: As an example, let a computed optimal flow deliver the flow values $X_0^k = 3$, $X_1^k = 2$, and $X_2^k = 0$ for airport $k \in A$ for arriving and departing flight legs as shown in Fig. 2-7. Starting at node $n_1^k$, three crew members are available at airport $k$ at the beginning, and two additional ones enter airport $k$ through arriving flight legs of $E_1^k$. Therefore, in total five crew members are available for the set of three departing flight legs in $S_1^k$: they either just arrived or used the waiting arc of the node considered. The remaining two together with two new crew members arriving through the flight legs in $E_2^k$ have to service all four departing flight legs in $S_2^k$.

In conjunction with further application-dependent enhancements, such as multiple home bases, crew states (ensuring a weekly rest period after a maximum of five consecutive working days), even workload balancing (realized by penalized deviations from accumulated target values for each home base), etc., additional layers for each home base and each crew state are introduced, as well as hard and soft cover constraints, see subsection 2.1.4. The result can also be formulated as an optimization model for a state-expanded multi-commodity aggregated time-space network flow as it is published in (Guo et al. 2003).

### 2.4.2 Constructive Heuristics for the Rostering

The above-described CPCP approach for the pairing generation step already provides a set of pairings (decomposed from pairing chains) which considers crew availabilities, pre-scheduled activities and crew requests together with workload balancing for home bases. For the remaining second step, the CRP, the potential need to restructure pairings is significantly reduced, since the most cost-intensive work of generating pairings at minimum cost as well as their appropriate partitioning in terms of workload balancing among home bases was already done during the pairing chain generation phase. However, crew capacities were calculated anonymously, and balancing among crew members is not fulfilled until now.

Thus, a situation-based heuristic is proposed which includes three phases sequentially carried out: initial assignment, global balancing over all home bases, and local balancing of each modified domicile. Within the initial assignment step, the main task is to allocate all partitioned activities (assigned in pairing chains to a specific home base) among all available individual crew members in terms of “best fitting” by a multi-weight-based heuristic which combines several constructive heuristics:
Firstly, for each home base all pre-scheduled activities are linked to their corresponding crew members. Then several multi-weight-based selection strategies are adopted while each of them aims at mapping the most promising pairing $P_1, \ldots, P_n$ (not pairing chain) to the most promising available crew member. At this time, the airline CRP work rules must be completely fulfilled in the context of the individual crew member, see horizontal rules in subsection 2.1.4. Two queues dynamically store all remaining pairings and available crew members for those items on both sides, which can be interpreted as the set for linking mutual candidates, see Fig. 2-8. Two alternative evaluation criteria to sort and process the queue of pairings were realized: By start time and by duration. Focusing on start time fills the generated crew schedule from the first to the last day of the planning period, whereas sorting based on duration follows the idea of assigning long pairings first because of their low likelihood in finding later a suitable free time slot when more pairings have already been assigned.

The available crew member list is dynamically updated since permanent cross-checking has to ensure the consistence of the already assigned rosters of all given crew members at the examined home base. For this queue several weighting strategies are applied as well, such as sorting by decreasing remaining contracted flight hours, or choosing the crew member having less working days left than others for a comparable amount of remaining flight hours. After selection and assignment of the best mutual candidates, the new situation enforces an update of the queues' composition and resorting by the chosen strategies. Since this simple, but sufficient greedy heuristic works even for larger instances tested, various strategy combinations for queue sorting on both
items can be performed in order to choose the best solution. The term “best” refers here to, e.g., a more harmonious balancing of workload, since cost differences accumulated by crew salary are not to be expected for the airline as long as all crew members work on an equal level of workload (see subsection 2.1.2). In the rarely occurring case that few pairings still remain unassigned due to the set of work rules together with pre-scheduled activities, a 2-opt procedure with backtracking tries to merge the remaining pairings into the corresponding home base.

After handling all home bases as described above, phase two for global rebalancing is required wherever there are still unassigned pairings left: It operates on a comparable 2-opt procedure which tries to shift all remaining pairings to other home bases. In this step we pay attention not only to cost minimization (shifting a pairing to another home base introduces additional cost for transit and hotel expenses) but also to workload balancing, e.g., whether there is enough contractual or individual crew capacity available with regard to the already assigned pairings and pre-scheduled activities. Because of this, lower and upper bounds for contract usage are created (based, e.g., on flight time, off-days) for each crew member, and their violation is penalized, if necessary. Before shifting to another home base a procedure is applied which aims at decreasing the imposed cost of such an operation.

Finally, a local balancing procedure is applied for all crew domiciles that have been modified in the previous phase two. This time it runs on the new (slightly changed) set of allocated pairings for each considered home base. A local improvement procedure for these schedules is conducted in order to enhance the local balance of workload, which can use either swapping flights among crew members or re-executing the mentioned multi-weight-based heuristic method locally.

It has to be emphasized that the above-presented constructive rostering scheme for multiple home bases greatly benefits from the requirements of the previous work conducted during the pairing chain generation. Its application to the general CRP setting (see subsection 2.2.2) is not suggested since corresponding IP models may find a superior solution.

2.5 Excursus: The Operational Crew Scheduling Problem

In practice, schedules are seldom operated exactly as planned; they are rather constantly disrupted by irregular events during day-to-day operations, such as aircraft mechanical problems, severe weather conditions, crew unavailability, and air congestion. Consequently, disturbances of normal operations change
the planned schedule completely or at least partly. More importantly, tremendous costs may have to be paid in order to recover from them.

In contrast to the scheduling tasks introduced above, operational crew scheduling, also known as crew rescheduling or crew recovery, handles disrupted situations in which original crew schedules require several, sometimes major modifications to keep the airline’s operations running after their unplanned occurrences.

Among the scheduling steps of an airline, operational crew scheduling belongs to the final step: operational time rescheduling (see subsection 2.1.1). Hence it is performed by the coordinators of the airline’s Operations Control Center (OCC), also known as Operations Control (Ops) instead of the planning department.

2.5.1 Rescheduling Processes

When disruptions occur, a series of flights have to be delayed and even cancelled, and often standby and reserve crew members (see subsection 2.1.2) and additional flights are required in order to cover all the flights originally scheduled that need to be operated. The main task of rescheduling is to reassign disrupted flights as well and as quickly as possible, and substantially assist airline coordinators in evaluating the updated plan. Basically, three kinds of resources must be recovered during a disrupted time period: Aircraft, crew and passengers.

Each resource greatly impacts the new flight schedule. For example, a shortage of aircraft may cause not only unexpected delays and cancellations, see, e.g., (Jarrah et al. 1993), but also some additional difficulties to the later crew rescheduling, for instance, the crew may lose their connections or get stuck at an unfavorable airport. Due to its complexity, the complete recovery problem is usually decomposed into a sequence of sub-problems, each of them solved independently. The aircraft recovery is solved first to restore a flight schedule with respect to all company rules and maintenance requirements. The impact of disruptions upon passengers is reduced as much as possible by minimizing their inconvenience, missing connections and further delays. Finally, a crew has to be rescheduled under the updated situation. The way to decompose the entire recovery problem differs from airline to airline because of heterogeneous company rules. One reason for applying a sequential approach relies on the fact that a complete integration of the three phases is unrealistic from a practical point of view.
However, better overall solutions may be achieved in a way that allows collaboration between the three steps of the sequential approach, and the first research results on this type of integrated disturbance measurement have appeared recently, see, e.g., (Stojkovic et al. 2002; Davis et al. 2002). In Fig. 2-9 the process of operational time rescheduling with its interrelations is shown. It is usually based on an earlier disruption classification which evaluates the impact, and a strategy mapping to select appropriate, mostly predefined, actions regarding the resource(s) affected.

![Diagram of processes during operational time rescheduling](Source: Guo 2005)

### 2.5.2 Model Formulation

In the following, it is assumed that the aircraft recovery problem has been solved. In other words, there exists a modified flight schedule where some flights may be cancelled, delayed, rerouted or added. All those flights are defined as affected flights. The objective of the airline crew recovery is now to provide an adequate crew for each (affected or not affected) flight, such that the updated crew rosters, with minimal variation from previously planned rosters, produce as little additional cost as possible. Hereby, the current situation of the crew has to be considered, such as partially flown pairings, the crew’s current location, their individual work history, and so on.

Similar to the airline CSP, the crew recovery problem can be mathematically formulated as a SPP, where a set of affected flights caused by disruptions needs to be assigned or reassigned exactly once. These disrupted flights grouped with previously planned flights are chained into a huge amount of rosters, which represent all possible individual schedules for crew members within the recovery period. (The recovery period is the time interval for which the rescheduling is carried out. It usually starts from the earliest disruption and ends...
on the same day or some days later.) Therefore, finally each crew member will be potentially assigned to one revised roster for the examined time period with respect to all regulations and rules.

In our approach, see also (Guo and Thiel 2004), the concept of integration is applied again: The problem is solved in an integrated way instead of addressing the pairing generation prior to the assignment phase. Rosters for individual crew members are generated directly from the level of flight legs. This is only possible because the recovery period is normally much shorter than the period examined in the planning phase. The problem is treated as a SPP, where a set of rosters is given and needs to be assigned to a certain number of individual crew members, by which all the flights are covered exactly once. Different crew functions are considered separately here as well, see subsection 2.1.2, as it may reduce the size of the problem without influencing the quality of the final result. The crew recovery model can be formulated as follows:

Let \( F \) be the set of flight legs, \( S^k \) denotes the set of feasible sub-rosters, and let \( K \) be the set of crew members of the considered problem instance. Operational roster cost are given by \( c^k_s \) for crew member \( k \in K \). Two different types of penalties are introduced: \( u_f \) for covering unassigned flights by standby and reserve crew, and \( v_s^k \) for the degree of deviation from the originally planned schedule. In addition, \( \gamma^k_s \) equals \( 1 \), if flight leg \( f \in F \) is included in roster \( s \), \( 0 \) otherwise, and \( \beta^k_s \) equals \( 1 \) if roster \( s \) belongs to crew member \( k \), \( 0 \) otherwise. Binary decision variables are \( x^k_s \) indicating whether roster \( s \in S^k \) is assigned to crew member \( k \in K \), and \( x_f \) equals to \( 1 \), if flight leg \( f \in F \) needs to be assigned to standby and reserve crew, \( 0 \) otherwise. The resulting crew recovery model is formulated below.

\[
\min \sum_{k \in K} \sum_{s \in S^k} (c^k_s + v^k_s) x^k_s + \sum_{f \in F} u_f y_f
\]

Subject to:

\[
\sum_{k \in K} \sum_{s \in S^k} \gamma^k_s x^k_s + y_f = 1 \quad \forall f \in F \tag{2.9}
\]

\[
\sum_{s \in S^k} \beta^k_s x^k_s \leq 1 \quad \forall k \in K \tag{2.10}
\]

\[
x^k_s \in \{0,1\} \quad \forall s \in S^k, \forall k \in K \tag{2.11}
\]

\[
x_f \in \{0,1\} \quad \forall f \in F \tag{2.12}
\]
The first part of the objective function (2.8) denotes minimizing the total operational cost \(c^k_s\), together with the effect of the disturbances inflicted on the crew \(v^k_s\) (as it is realized by expressing the changes in a monetary sense). Therefore, \(v^k_s\) equals zero, if the corresponding roster is identical with an original roster.

For those flights which cannot be assigned to any crew member in service, reserve and standby crews are required, which imposes additional cost \(u_f\). Constraints (2.9) guarantee that all flight legs \(f \in F\) are covered exactly once, while constraints (2.10) ensure that each crew member \(k \in K\) takes at most one roster \(s \in S^k\).

### 2.5.3 Solution Approaches

Some preliminary work was done to solve the above model with several instances. However, results indicate that only relatively small real-life problems are suitable to be solved directly by standard integer optimizers. Medium- and large-sized instances require too much computational time. This is apparently impractical in reality, especially in the case that the recovery period is comparatively long, such that it often takes more than 30 minutes to be solved. Due to the extremely large number of possible rosters, common methods apply efficient roster generation procedures to build a model. There is also a great variety of methods available to solve such kind of optimization problems, compare with section 2.3.

Nevertheless, it is an overall shared opinion that today the work in solving the crew recovery problem is only at the beginning: In (Wei et al. 1997) an optimization model for handling disruptions was proposed, while in (Yan and Lin 1997) the rescheduling problem caused by the closure of airports was described. Further systematic studies of the crew recovery problem were conducted in (Stojkovic et al. 1998; Stojkovic and Soumis 2001), in which they solved such a problem as an integer non-linear multi-commodity network flow model with time windows and additional constraints. Also a Dantzig-Wolfe decomposition combined with a branch-and-bound method was described in detail. Furthermore, in (Lettovsky 1997) and (Lettovsky et al. 2000) a pairing generation method working together with special branching strategies was proposed. In (Yu et al. 2003) an award-winning real-life application employed by Continental Airlines in the US was presented, in which the problem was treated as a set covering problem and a so-called generate-and-test heuristic was applied to generate rosters. Moreover, some studies about airline irregular operations were also discussed in (Irrgang 1995) and (Rosenberger et al. 2003).
Even recently, in (Abdelghany et al. 2004) a proactive recovery decision support tool was introduced which works with a rolling approach that solves a sequence of mixed integer optimization assignment problems. In addition, in (Guo and Thiel 2004) a genetic algorithm approach with local improvement strategies for the crew recovery problem was implemented. It was based on dedicated chromosome encoding and mutation operators. Finally, in (Nissen and Haase 2004) the application of a special duty-period-based network model was presented which solves the single-day operation recovery problem.

Rather than addressing complex recovery problems during the operational phase, aspects like robustness of crew schedules are already treated earlier during the planning phase, see, e.g., (Schaefer et al. 2001; Klabjan et al. 2001; Chebalov and Klabjan 2002) for schedule robustness, and (Penz et al. 2002) for a related sensitivity analysis.

2.6 Summary

The crew scheduling is one among eleven scheduling processes in a commercial scheduled airline. It aims at assigning flights to a limited number of available crew members. This should be achieved at minimal cost while satisfying external and internal working rules, individually pre-scheduled activities and work history.

The Crew Scheduling Problem is usually decomposed into two sub-problems, namely Crew Pairing Problem and Crew Assignment Problem/ Crew Rostering Problem which were introduced in a formal way. There are three common assignment approaches for the later CRP, known as bidline systems, preferential bidding, and personalized rostering. They differ with regard to the chances how preference and requests of a crew are handled. Further basics, such as a classification of work rules are introduced.

There has already been a great variety of instances and approaches applied to address the general CSP, more precisely, each of its sub-problems over the past decades: On both individual tasks, constructive heuristics, mathematical programming, and network models are often applied, meanwhile also meta-heuristics as well as other additional concepts given in the state-of-the-art of this chapter.

The partially integrated approach on the CSP, as proposed in previous papers of the author and his colleagues, was presented as well. As one of the first research groups, we focused on the modified requirements for a CPCP – an extension of the original CPP. This enabled an easier assignment step, especially
for instances with multiple home bases due to the changed generation scheme for pairings.

Beside the planning phase in advance of operation, a crew has to be (re)scheduled on a real-time basis as part of the operational time rescheduling. Here a disruption forces a sometimes even massive change of the originally planned schedule that need to be bridged at minimal cost and little deviation from the scheduling obtained during earlier steps A formal representation and several state-of-the-art solution approaches are discussed.

With regard to the setting of an airline, the upcoming chapter will place emphasis on the consideration of scheduling the aircrew as a group of personnel instead of requesting quantified and qualified individuals. Tailored to the needs of personalized rostering, chapter 3 defines and examines a variety of aspects on the Team-oriented Scheduling approach which will be investigated in detail in the subsequent chapters.
In contrast to the state-of-the-art on airline crew scheduling examined in the previous chapter 2, this part of the work presents an extension of the classical CSP with special regard to the importance of team aspects.

Teams and team work become necessary whenever people have to work together. Both are greatly influenced by HRM aspects, such as employee satisfaction and team structures. In combination with socio-cultural findings, team dynamics are considered in section 3.1. Here, HRM functions as “a set of processes, which – through the recruitment, training, motivation, appraisal, reward, and development of individuals (and teams; added by author), and through the effective handling of industrial relations – translates strategy into action” (Holloway 1998, 17).

After all, this leads to the important task of characterizing and defining the so-called Team-oriented Scheduling and the Team-oriented Rostering. They are examined in detail in section 3.2. Further considerations when applying the ToRP are discussed in 3.3, followed by a summary on the new approach in section 3.4.

3.1 Scheduling of Aircrew Teams

The first subsection 3.1.1 provides a more concrete, but still generally applicable description of a team and its main characteristics while in 3.1.2 the most common team structures of flight personnel are pointed out. The major impacts of team consideration for the scheduling of aircrew are evaluated in subsection 3.1.3.
3.1.1 Teams and Team Processes in General

According to (Argyle 1975; Morgan et al. 1986; Wiendieck 1992) a team can be specified by the following criteria:

A team

- consists of two or more people,
- shares a common goal or objective,
- interacts due to intensive internal working correlations,
- follows a pronounced community spirit, and
- encounters a specific division of tasks and labor.

Furthermore, when examining a team, (Wiendieck 1992) proposes several essential characteristics, such as size, duration, performance orientation, style of working, and cohesion:

- **Size**: Each team consists of several members while face-to-face communication is limited to comparably small teams.

- **Duration**: Usually, teams are considered which persist at least over a certain time period. It is distinguished between lasting teams (e.g., executive boards) and temporary ones (e.g., quality circles, project teams).

- **Performance (output) orientation**: Besides the overall agreed values and goals, a certain commitment about the output of the group has to be defined, whereas certain aspects, such as individual and/or collective experiences, are mostly neglected.

- **Style of working**: In a team, competent experts in dedicated areas have to cooperate with each other. In conjunction with proper coordination as a further fundamental requirement, the team can be expected to succeed.

- **Cohesion**: The cohesion is the (invisible) power that keeps team members from leaving the group. A high cohesion is generally agreed on to enhance the performance of the team.

In, e.g., (Knebel 1995) a comparison among a traditional group and a team with regard to the capacity of teamwork and its performance evaluation is presented. For more details and further classifications, it is referred to, e.g., (Badke-Schaub 1993; Becker and Mathieu 2003; von Rosenstiel 1994, 1999).
**Team processes**

In order to be recognized as a team, the above-mentioned set of characteristics develops with regard to the time span the team members remain together. Such a dynamic process is called **team building** or **team development**, where socio-emotional relations, interaction and behavioral rules are incorporated. There are several phases within the development of teams (or groups), see (Lacoursiere 1980; Kobi 1994; Schneider 1996):

- **Orientation (initial) phase**: Team members get to know each other, initially identify common values, objectives and expectations. They start to evaluate their position in the team.

- **Dissatisfaction (confrontation) phase**: Differences in team members’ opinion, behavior, and values become recognizable. The team position is established; also cliques can be created that prevent isolating a single team member in case of confronting objectives and ideas.

- **Resolution (organization) phase**: Earlier problems are resolved while cliques are relaxed. The main focus is directed back towards the task assigned to the team which gets more organized and sets up team-specific rules and behavior patterns.

- **Production (integration) phase**: All team members work highly productive and willing. They operate as one unit based on overall agreed standards, behavior patterns, and objectives.

In Fig. 3-1 two different curves are visualized over all four team phases introduced above: First is the team’s **productivity (competence) index** which grows permanently except for the stagnation during the transition between the second and the third phase. It reaches the highest level in the final production phase. Second is the simultaneous development of the team’s **moral (commitment) index** as drawn. Although it starts already on a high level, it turns out that the rock-bottom is hit at the end of the dissatisfaction phase while it increases again to a high level after this point, see (Lacoursiere 1980).

Note that every new team composition usually imposes massive changes among the developed idiosyncratic spirit among its members in such a way that the team’s development process basically requires to be redone. This consumes time and energy of all individuals involved, see, e.g., (Lacoursiere 1980; Hackman 2002).
3.1.2 Team Structures

Because of the dedicated tasks to be performed by an aircrew, team structures appear to significantly differ from the ones examined in other organizations.

From now on the term team is synonymously applied when referring to a dedicated crew – as a composition of personnel operating a specific flight or set of flights. Firstly, crew positions (crew functions) are introduced, followed by a set of crew compositions that constitute the teams examined later in this work.

Crew positions

A brief job description of all working positions of an aircrew is given in hierarchical descending order from captain to cabin attendant as follows:

- The captain (CP) is responsible for flight safety and for the crew as well as the passengers. He or she prepares flights by plotting the flight route and calculating the fuel consumption, and checks weather conditions and circumstances for the flight route and the destination airport. Before the take-off, the captain performs several technical tests and coordinates with the cabin crew. The captain is an obligatory crew member.
- The co-pilot or first officer (FO) supports the captain navigating and operating the aircraft. He is an obligatory part of every commercial flight as well.
- Some large, older aircraft have a third cockpit-member: the flight engineer (FE) or second officer. Their job is to assist the other pilots by operating and watching several instruments and systems. They are able to conduct minor in-flight repairs. Nowadays, flight engineers have become rare as new technology can perform many of these flight tasks automatically. New aircraft fly
with solely two pilots, usually CP and FO, who are supported by computer-ized controls.

- The senior cabin attendant, purser (PU) or chef-de-cabin (CDC) (depending on the airline and fleet type examined) manages service, galley (kitchen of the aircraft) and the entire cabin on board of the aircraft in a professionally and commercially responsible way with regard to his or her duty as a service provider. He or she also links communication between the cockpit and the cabin crew. Members of these crew functions supervise the other cabin crew not only in emergency cases. Before the flight they brief the cabin crew on the course of the service, the booking numbers, care of special guests (people with medical disabilities, families with small children but also frequent flyers who need special attention) and any specialties of the flight. On a commercial flight there has always to be one PU or CDC in charge. However, depending on the size of the aircraft, an additional purser can be scheduled to coordinate and supervise a particular area of the aircraft. Taken Lufthansa as an example, there are always two pursers on an inter-continental flight: A Purser I who is usually in charge of the economy class and a Purser II who has the overall responsibility. However, there is no Purser II on a continental flight.

- Cabin attendants (CA) sometimes are referred to as flight attendants, stewards or hostesses. They are responsible for the safety of the passengers and, in the manner of a service provider, for their well-being. They have to check the state and completeness of the cabin equipment, especially the emergency equipment, before every aircraft boarding. In addition they have to help when the catering is delivered. Preparing for take-off, cabin attendants have to make sure that safety standards imposed by the aviation authorities, such as the FAA in the US, are followed. This comprises the assurance that hand luggage is stored safely, seatbelts are fastened and emergency procedures are explained. Moreover, they inform about the course of the flight and the appropriate as well as required behavior in emergency situations. During the flight, cabin attendants provide flight guests with beverages and/or food and magazines. Depending on the flight destination, they also offer duty-free articles for sale. Each flight attendant is assigned to a certain work area and to specific in-flight duties. A cabin attendant is usually trained to work on more than one fleet to enhance their applicability.

- For airlines having only one home base or handling home bases separately like at Lufthansa, a so-called floater is a flight attendant who is not assigned to a specific team throughout a pairing, but who will change working areas and even the aircraft to fill up positions that are required for the flight. Set-
tings with multiple home bases like at our business partner do not have this crew position so far, since they aim at covering the complete problem instance at once.

The corresponding crew is formed out of the above-introduced crew positions.

*Downgrading* (or flying-below rank) is another important concept briefly introduced already in subsection 2.1.4. It is applied in the airline crew scheduling quite frequently. According the clear hierarchy among the crew functions described above, a captain could also serve as a first officer, but not vice versa. A similar interpretation is used for the cabin crew.

**Crew compositions**

Usually aircrews are composed in such a way that all the flights examined are adequately staffed (based on the crew complement, see 2.1.4). For commercial flights, a cockpit crew is constituted by one CP and one FO. In some cases an additional FE becomes necessary. The cabin crew is composed of at least one CDC or PU and a certain number of flight attendants which depends on the number of passengers and on the aircraft type. There must be one flight attendant per 50 passengers on commercial flights according to the FAA regulations, see (FAAR 2004).

As an example, at our business partner there are the following typical settings for a crew complement, see Table 3-1. Setting (A) shows the common crew request of a flight operated on the available Airbus fleet, instance (B) is the appropriate demand of aircrew for the corresponding Boeing fleet, while (C) may be applied to very short domestic flights without any onboard service.

<table>
<thead>
<tr>
<th>Crew position</th>
<th>(A)</th>
<th>(B)</th>
<th>(C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CP</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>FO</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>PU</td>
<td>1</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>CDC</td>
<td>--</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>CA</td>
<td>≤5</td>
<td>≤3</td>
<td>--</td>
</tr>
</tbody>
</table>
Other airlines, such as Lufthansa, subdivide their crew roughly by the region they operate on. Thus, it is possible to take advantage of special language requirements as well as area or destination specific knowledge regarding visa regulations. Furthermore, information about local places of interest can be directly offered to the customer for international locations. At our business partner, all crew members are trained to service all destinations, usually without any further exception.

With regard to the crew positions introduced above, a further sub-classification can be made when the operating crew is composed: Due to their distinct job focus it makes sense to divide up the above introduced crew compositions by their crew type (see 2.2.2) into the staff of the cockpit and the cabin crew. As described earlier in this subsection both crew types have different tasks and competencies on board, and they follow an explicit and clear top-down hierarchy.

3.1.3 Impacts of Team Orientation

Based on the discussion above, it is obvious that team orientation throughout the complete scheduling process may impose many types of advantages to an airline. As described in this subsection, especially aspects such as better security, higher crew satisfaction, and a simplified rescheduling may be advantageous, but sometimes at a higher cost.

Team stability enables superior cockpit security

In subsection 3.1.1 the duration of the team composition is identified to be an important factor for its development process, and as a result, it can be decisive for the team’s performance. Hackman states: “Teams with stable membership perform better than those that constantly have to deal with the arrival of new members and the departure of old ones” (Hackman 2002, 55).

How does this affect the scheduling of aircrew? As already mentioned in the introduction it is known that the majority of major accidents is caused directly or at least worsened by mistakes of human beings. Recapitulating the study on such accidents recorded from 1978 to 1990 in (NTSB 1994) it is revealed that:

- 73% of all major accidents of US air carriers occurred during the first day of the crew flying together.
- 44% of those accidents happened even during the very first flight of the operating flight team.
As a further example, an earlier study from a project of NASA-Ames Research Center (NASA 2005) concerns the effects of fatigued pilot behavior. According to their findings, fatigued pilots showed a certain degree of tiredness when they returned from a pairing lasting several days. This condition resulted in more mistakes in comparison to newly deployed crew members. But in addition, they concluded that such exhausted cockpit crews (as unchanged teams) caused significantly fewer mistakes than those newly composed ones of rested pilots that had not yet flown together, see (Foushee et al. 1986).

To summarize, in particular for a cockpit crew, security aspects indicate the importance of teams when scheduling the aircrew. Nevertheless, the consideration of working as teams is left to the moment where the crew schedule is executed. Flight personnel gets to know each other directly before the flight, sometimes even not earlier than in the aircraft. And team compositions change quite frequently. This may include the cockpit crew, but it is also common for the cabin crew to encounter a modified team setting during short layovers from time to time. Furthermore, teams with such fluid memberships are very hard to evaluate in terms of team performance, see (Becker and Mathieu 2003).

**Improved crew satisfaction is the key to customer satisfaction**

For both crew types, employee satisfaction is recognized as one of the very important influencing factors with regard to the well-being of an airline. Similar to all service-centered organizations, the staff works as the company’s direct interface to the customer. It is natural that the behavior of staff members may leave a durable impression in a customer’s mind, and thus, influence future decisions on booking or purchasing a service or flight.

There are several case studies available that underline this assumption: For example, (Yeung and Berman 1997) observed an impact cycle at Eastman Kodak in which the connection between employee, customer and shareholder satisfaction was discovered. Another study was carried out at Continental Airlines. Their management achieved a critical turnaround starting in 1994 based on elaborated human resource practices with respect to an increased level of employee satisfaction. (Carrig 1997) reported the fundamental four-step strategy of their successful change program:

1. **Fly to win.** Improvement of revenue and delivery of profits by downsizing/eliminating unprofitable services.
2. **Fund the future.** Reduction of debts and unfreezing of accumulated company assets.
3. **Make reliability a reality.** Improvement of service to attract more passengers and to achieve a high level of personnel's proudness when representing the company.

4. **Working together.** Creation of an environment formed by dignity and respect. Make employees feel valued and enjoy their work.

Although the first stage imposed a strict reduction of the staffing level of the airline, especially the last two stages directly addressed aspects of the crew's satisfaction. The link between employee satisfaction and profit is visualized in Fig. 3-2: Similar to the findings in service-driven companies presented in (Schlesinger and Heskett 1991), the airline's internal service environment enables employee competence and satisfaction which itself results in employees' retention and productivity. Both together, they form a high external service value which positively influences customer satisfaction, and subsequently, an increased customer loyalty raises the revenue as well as the company's profitability.

![Figure 3-2: Link between employee satisfaction and profitability (based on (Carrig 1997))](image)

In general, employee satisfaction is influenced by a set of conditions. In (Trost 1997) the author distinguishes between subjective and objective conditions. A combination of both conditions delivers the individual level of contentment for the person examined while the surrounding people (colleagues, team-mates, etc.) are recognized to affect all of them directly, at least indirectly, over time. As mentioned in the introduction (see section 1.1), especially team changes have been identified to have a negative impact on the subjective crew satisfaction, e.g., being left alone at a non-domicile site after work or giving up harmonizing working teams as scheduled or rescheduled.
Higher team stability may impose additional operational cost

The main drawback considering teams can be seen in the fact that a crew schedule which realizes a certain degree of team stability may impose more operational cost in comparison to the one computed on the CSP’s classical objectives, such as the pure minimization of cost. Generally speaking, there is a notable trade-off between both distinct objectives: Emphasizing rosters with a higher team stability results at in outweighing operationally less expensive rosters rather than those with higher team orientation. From the practical perspective, neither the one nor the other extreme is preferred. Hence, decision support systems are usually applied since more complex sets of weighed objectives are available in order to create a collection of optimal alternatives (based on distinct parameter settings). The alternatives can be evaluated according to the user’s needs afterwards. For more details, it is referred to chapter 6.

Note that some problem instances may have several optimal solutions from the pure cost perspective, but they can vary with regard to their teams’ stability. Furthermore, the comparison between two alternatives relies on the assumption that both teams, with and without team orientation, perform in the same way. If not, additional virtual cost should be added to the classical objective function in order to remain comparable.

Referring back to the introduction, our business partner benefits from the internal cost structure especially when team orientation is realized: The integrated consideration of different crew functions together with a prevented lack of team stability may save the airline expensive transits due to mostly underutilized taxis when transferring flight personnel inside of Germany. As a result of the teams’ stability, transits are shared by the whole crew instead of granting individual route requirements due to separated crew members or sub-teams.

Team-oriented behavior must be adopted by all crew members

Further possible obstacles arising from the socio-cultural perspective on the enforcement of teams can be confronted as well: In long lasting teams the members may tend to become too comfortable with one another. As a result, important standard procedures may be given up or they may get too forgiving on team-mates’ mistakes, see (Hackman 2002). Individual disharmonies among directly interacting team members may get worse over time as well. Of course, everyone knows about his or her personal consequences when not fully committing to the FAA regulations. But such aspects previously mentioned require usually intensive training programs arranged by the HRM department. Especially team leaders, here the captains and pursers or chef-de-cabins, have to be
trained on team-oriented briefings when introducing changes, as well as on sensible decision making in case of interpersonal issues.

**Team stability simplifies rescheduling processes**

Apart from the consideration of the planning phase, team orientation implies a positive effect for the operational time rescheduling. Hackman proposes “… on any given trip they [cockpit crew; added by the author] should fly the same aircraft and work with the same cabin crew.” (Hackman 2002, 56). When this suggestion is implemented, unavailable crew members, e.g., due to sickness, need to be replaced by floaters as before (subsection 3.1.2). However, the impact of delay propagation could be significantly reduced since it may only affects the team involved and may not spread out to other teams. Thus, the aircrew rescheduling problem itself could be simplified by this, but in practice it may also turn out to be disadvantageous, e.g., in a situation where the aircraft and the crew are coupled, and by this, both of them would have to be replaced instantly.

**3.1.4 Operations Research-related Literature Review**

In general, aircrews as teams are rarely considered throughout the OR literature. Relevant aspects, such as crew satisfaction, are usually viewed on a single person perspective, e.g., based on crew functions. Most articles imply that a high level of crew satisfaction is granted by the implementation of the assignment approach agreed-upon (such as bidline systems, personalized rostering or preferential bidding), by which the processing of crew requests and/or preferences is defined, see subsection 2.1.3. Among the few publications that address crew satisfaction explicitly are (Kohl and Karisch 2004; Nissen and Haase 2004; Guo and Thiel 2004) where the latter two papers work on the minimization of changes during crew rescheduling (during operations).

Usually, limited attention is paid to specific details apart from those directly measurable in the operations research perspective of the CSP. For instance, without direct consideration, in (Hoffmann and Padberg 2003) it is reported that during their work they experienced a higher crew satisfaction. This was reasoned by the fact that the newly generated crew schedules consider more requests and prevent generally undesirable flight connections or long waiting times in comparison to the former system in operation. Besides, in (El Moudani and Mora-Camino 2001; El Moudani et al. 2001) the authors estimated the degree of crew satisfaction by individual satisfaction levels as well as by a global level (average over all individuals) which is embedded into the objective function.
When teams are examined, two contrary but simple alternatives have been presented for the creation of crew compositions:

- **Fixed crew compositions**, e.g., as *must-fly-together* restrictions, see (Kohl and Karisch 2004). They are usually applied to train inexperienced or new crew members, also for married couples, if desired. Commonly, those fixed crew compositions are constructed ahead of the assignment step, see also (Hansen and Liden 2005) for cabin teams. By this, crew members share the complete planning horizon — usually without any exception.

- **Prevented crew compositions** rely on the experience or crew’s request that particular working teams are known ahead of operations to disharmonize, see, e.g., (Strauss 2001; Kohl and Karisch 2004). Their occurrence is hence avoided or penalized during the assignment step, at least manually resolved afterwards.

For preferential bidding systems, there is a third alternative already mentioned (see subsection 2.1.3):

- **Requested crew compositions**, such as, e.g., a *buddy request*, see (Doerner et al. 2002). Crew members explicitly express their preferences for a single or several colleagues that he or she would like to work with while granting such a request is incorporated into the objective function, see (Gamache et al. 1998).

Analyzing the above realizations for building crews, it is obvious that — although their application indicates high team stability — they still comprise certain drawbacks: On the one hand, the realization of completely fixed teams eases the scheduling process, since there is only one assignment required for all crew functions involved (using a representative crew member instead of individuals). By this the model size remains similar to an instance for a single crew function. On the other hand, since crew members of such a team may have non-overlapping pre-scheduled activities, different rule sets, etc., their fixed composition may turn out to be quite inefficient, since the complete team is blocked due to a single team-mate, e.g., having a pre-scheduled activity like a day off.

With regard to the requested crew compositions in a Preferential Bidding System, the fact is neglected that such crew compositions do not necessarily require a continuous duration. As an example in Fig. 3-3, two crew members (CP1 and FO3) prefer to work with each other, and in their rosters both share three of five flight duties while the remaining two are scheduled to work with other colleagues. As significantly different levels of team stability, but equal alternatives in the preferential bidding approach, both crew members could pos-
sibly work together for a multiple day block, e.g., from day1 until day3 in alternative A, or on day1, day3 and day5 as visualized in alternative B.

![Figure 3-3: Teams composed by a preferential bidding system](image)

Even though the consideration of working in teams may have a recognizable positive impact on the airline’s performance (as discussed in the earlier subsection 3.1.3), it is concluded that current work on the CSP either neglects the subject or proposes a quite rigid implementation, namely by fixing or preventing teams. Usually, a preferential bidding system does not implement stable teams due to its inherent scheduling philosophy: In a PBS, individuals express their preferences independently, which are then granted or rejected while the system considers also the colleagues’ preferences according to their seniority.

### 3.2 Task Characteristics and Definitions

In this section the task of the Team-oriented Scheduling is concretized. First of all in 3.2.1, the Team-oriented Scheduling Problem is introduced with regard to its effects on the underlying optimization problems. Especially in subsection 3.2.2, the Team-oriented Rostering Problem is addressed in detail, since this provides the basis for the rest of this work. Possible ways to build aircrew teams are presented in subsection 3.2.3, while changes in the team composition and their penalization are examined more closely in 3.2.4. In subsection 3.2.5 potential objectives of the ToS approach and their measurement are discussed.
Chapter 3 – Team-oriented Scheduling and Rostering

3.2.1 The Team-oriented Scheduling Problem

Repeating the general objective of the CSP given in subsection 2.1.2, the aircrew required is assigned to a flight schedule in a work rule-compliant, but moreover, in a cost minimizing way. Furthermore, particularly team stability appears to be an important factor on the performance of teams, see 3.1.3. Since this team stability is directly or indirectly in the hands of the crew schedule planners, a team-oriented approach demands for a changed proceeding with regard to the methods originally applied for the CSP.

Because of this, the overall goal of the Team-oriented Scheduling Problem can be expressed in the airline context as follows:

The Team-oriented Scheduling Problem (ToSP) is the scheduling of flight personnel by which a certain degree of team orientation (or teaming) is demonstrated.

This team orientation intends to grant higher crew satisfaction in terms of quality-of-life criteria on the one hand, and it has also a positive effect on the risk of human mistakes due to unfamiliar people working together on the other hand. (Hackman 2002) explains that by team orientation a work team is enabled to develop its own idiosyncratic ways, e.g., people learn to deal with individual strengths and weaknesses. They even begin to care for each other.

Regular optimization instances of the CSP are usually subdivided with regard to not only the fleet, but also to each of the crew positions involved, see 2.1.2. In combination with individual pre-scheduled activities, teams working together may get separated quite frequently. Therefore, already on the level of pairings a modified CPP is an important prerequisite in order to allow higher team stability. Several strategies are discussed in the upcoming subsections 3.3.2 and 3.3.3.

Especially the second step, the assignment of rosters to the aircrew (or CRP), is responsible for the team stability. Keeping teams together may interfere with traditionally grown organizational structures and practices, e.g., by changing the way how people are used to express their preferences on schedules and their assignment. The impact of team orientation is basically different according to the three common assignment approaches as discussed below:

For the bidlines approach, where the assignment decisions are solely based on the seniority of the flight personnel, any kind of team consideration is very hard to implement. For this assignment method, junior crew members know about their limited chance to get a desired roster. Already during an early stage of the assignment process even marginally less senior flight personnel would experience a comparatively small selection of rosters offered to bid for: Because of the strictly sequential assignment procedure and the roster selections already
made, only few alternatives granting a certain team orientation would be available for them. In their eyes, they would have to give up most of the advantages imposed by their individual seniority status.

In preferential bidding systems crew requests are considered in addition to the personnel’s seniority, and individual preferences are incorporated into the objective function. As shown in 3.1.4, usually a PBS does not consider team stability. It requires only little changes with respect to the specification of the objective function which are similar to those necessary for the third assignment approach presented below. In spite of this, any team consideration greatly suffers due to the effects on the underlying seniority principle.

The impact of team orientation on the personalized rostering is discussed in detail in the upcoming section.

### 3.2.2 The Team-oriented Rostering Problem

The personalized rostering as the basis for the Team-oriented Rostering Problem is the assignment approach chosen in this study. As stated in subsection 2.1.3, personalized rostering is characterized by two major ideas: Firstly, crew members are treated in a more equal way than in the other approaches (due to absentness of the seniority principle); secondly, crew schedules are generated almost autonomously by the crew planners.

Therefore, in addition to the objectives mentioned earlier (see subsection 2.1.2 and 2.2.2), the basic idea of Team-oriented Rostering is the consideration of team orientation by, e.g., the avoidance of frequent changes in the composition of the servicing or operating onboard team. It is defined as follows:

The **Team-oriented Rostering Problem** (ToRP) is the search for an appropriate set of individual rosters (one roster for each crew member) that all flights given are covered properly while certain team stability is achieved.

For personalized rostering, changes on the assignment strategies embedded in the scheduling system do not affect any of the rules the flight personnel is already used to. The employees may only notice the result in such a way that “somehow” there is higher team stability than before. Because of this, the deployment of the new approach can be undertaken without any significant protest from the personnel’s side.

When comparing the different assignment approaches with regard to teaming presented in the previous and this subsection, the personalized rostering itself is obviously the only assignment approach for the scheduling of aircrew that
encourages the application of team concepts. Although it can be regarded as highly beneficial especially with regard to safety aspects, it may imply unacceptable changes to the employees’ way of understanding how “fairness” and preferences are granted to them.

Therefore, from now on the upcoming work will particularly focus on application of the ToRP based on the personalized rostering assignment approach as defined above.

3.2.3 Team Construction Concepts

Because of the obvious advantages, airlines are often interested in team orientation. However, it can be understood and implemented in many different ways. In the following, three basic interpretations of team orientation for cockpit teams are introduced and discussed: Pre-defined teams, pre-selected teams, and free team building. It is common to these approaches that teams remain together for the complete planning period. These principles are introduced as a basis, but there are other ways for defining team orientation in a more flexible way, as it will be discussed in subsection 3.2.4. To simplify matters, only pairs of flight personnel are examined, here: the distinct crew functions of the cockpit crew.

- **Pre-defined teams:** A pre-defined team is fixed a priori for the complete planning period and it remains together during the complete period. This basic team composition is commonly applied when company policies require stable teams, e.g., for married couples or inexperienced crew members. The scheduling task is reduced to the standard CSP for a single crew function, whereas it turns out to be unfavorable for settings with non-overlapping pre-scheduled activities and/or disjoint individual preferences, see also 3.1.4. It may result in many left-over duties, and crew members are likely to experience an unequal workload due to the situation-dependant cockpit pairs chosen.

  In Fig. 3-4 the interrelation of the rosters assigned to several cockpit crews is depicted: The set of rosters is constrained by each pair’s totalized preferences and pre-scheduled activities, and both crew members receive exactly the same roster determined during the assignment step. Consequently, since there are more captains than first officers, two of them get no roster assigned. (They can be used, e.g., to work as reserve or standby crew.)

- **Pre-selected teams:** With respect to the staff’s individual availability patterns (fingerprint), a pre-selection among potential pairs can be carried out. Pairs of employees with identical or similar fingerprints are selected as possible teams. Such possible teams are able to cover most activities together, and
the number of potential crew pairs to be considered is limited. In contrast to the pre-defined teams, the actual selection is carried out during the scheduling process, so that the teams which really fly together are not known a priori. When the remaining activities are assigned in a subsequent step, teams are separated again as a consequence.

Fig. 3-5 shows potential crew pairs, e.g., (CP2, FO2), (CP2, FO3), (CP2, FO4) for which rosters are generated. Out of those rosters one is chosen for the combination for (CP2, FO4), whereas all the other first officers involved (FO2 and FO3) work together with CP4 and CP5 respectively.

- **Free team building**: The most desired, but also most difficult approach to teaming is when teams are composed during the optimization. As a result, rosters on all combinations have to be considered, since none of the possible crew pairs is excluded in advance.

In the example of Fig. 3-6 the same set of rosters is chosen as in Fig. 3-5. Here, rosters are generated for all combinations among captains and first officers. The amount of rosters generated is by far the highest among the alternative team settings described here.
For the cabin crew, the team setting is significantly more complex than that of the cockpit crew with only two crew functions being examined. Besides the inherent differences of this crew type as already discussed in 3.1.2, the team setting differs in particular with regard to the number of crew positions, e.g., there is a variable demand for cabin attendants on a level of flight legs and the special downgrading structure may include different fleets. Therefore, such a clear separation into two crew functions is only applicable for the cabin crew, if cabin attendants themselves remain in fixed teams, downgrading is neglected, and there is only one leading position (PU or CDC) to be considered.

3.2.4 Team Changes and Their Penalization

So far, team settings defined above are fixed over the planning period which leads to a high team stability. In (Hackman 2002, 56) it is argued that “in commercial aviation... crews should be trained together and then remain intact for a considerable period of time, giving members the opportunity to develop themselves into the best-performing unit that they are able to become”. In contrast to this, the competitive situation in the airline industry in combination with individual preferences and pre-scheduled activities reinforce a team stability that lasts only over a considerable time period instead of fixing those teams for several weeks or even months.

This section discusses more sophisticated ways to measure team stability which are applicable also in cases when teams do not remain together for the complete planning period. In some sense it would be desirable to minimize the total number of team changes, or better, the total “impact” of team changes within a planning period.

Therefore, the term team change is introduced as follows:

A team change occurs if at least one crew member is scheduled to service the next flight activity together with a different team composition (other colleagues).

If teams are not fixed a priori, such team changes occur due to the given rule sets (e.g., a crew member has reached his or her maximum of daily working hours), or by very strict fair-and-equal share of workload; however, the main reason for frequent team changes so far lies in the fact that (due to the dominating cost minimization objective) they are simply not considered at all.

In Fig. 3-7 team changes among selected rosters are visualized, e.g., FO1 works together with CP1, but also with CP3, while FO4 and F05 experience no team change with their corresponding captain.
For the ToRP emphasis is placed on roster combinations instead of single rosters, because all team members have to fulfill their individual roster assigned when the team changes occur.

In Fig. 3-8, some roster combinations among a selected captain roster (CP1 R1) and several first officers and their rosters (FO1, R4), (FO2, R9), (FO3, R7) are given. (For better understanding flight duties are used in this example.) As defined above, whenever a shared time period is terminated, a team change takes place; this is a case between day2 and day3, day3 and day4, etc. On day8, there is a team change after the weekly rest period (two consecutive OFF-days). Team changes are counted for all crew members as shown in the example. Therefore, captain CP1 experiences a total of five team changes (including the initial one) over the given time span of ten days.

When the complete crew is considered for the evaluation of team stability, especially changes among the different crew functions as well as crew types (cockpit and cabin crew) should be considered.

Besides counting the number of team changes for given roster combinations, there are other means in measuring the impact of various types of team changes when considering complete crews.

For example, it can be differentiated between different team change types. This aspect is evaluated according to the time and the location of their occurrence: Team changes may be encountered, e.g., during a flight duty, at its beginning and/or end. They may take place at any of the destination airports as well as at the airlines’ crew domiciles or the crew member’s home base. As given in Fig. 3-8, a change in the crew composition after the weekly rest period is common, too.
In the above example, team changes between complete duties are considered. Any kind of team instability during the day (within a duty) is even more undesired, especially in combination with an outside location, e.g., transferring from one gate to another can be very stressful when being in a hurry due to some previously propagated delays.

Regarding the cabin crew, a team usually consists of more than two members. Therefore, the ranking of team changes gets more complex. The *degree of a team change* can be determined as follows:

Three cabin crew members may get separated in exactly two dedicated ways: Firstly, everyone involved becomes part of a new team resulting in a (1:1:1) team change. Secondly, two of the three crew members may remain together for the next crew building, and by this a (2:1) team change is encountered. For a team of five crew members, it may result in a separation of (4:1), (3:2), (3:1:1), (2:2:1), (2:1:1:1), and (1:1:1:1:1) team changes.

For that reason a huge set of different team changes with regard to their degree is produced which requires a consistent penalty ranking, e.g., by the

- *Crew functions being separated*, e.g., if the crew considered is composed by one CP, one FO, and one CDC, a (2:1) team change can result in a separation of the cockpit crew. According to the earlier line of argumentation, this can be regarded as less favorable than the alternative of keeping their sub-team intact and exchanging the CDC associated with them.

- *Amount of the separated sub-teams*, e.g., for the above introduced cabin crew with five team members, the following ranking for the penalty $p$ may be the result: $p^{4:1} < p^{3:2} < p^{3:1:1} < p^{2:2:1} < p^{2:1:1:1} < p^{1:1:1:1:1}$. 

![Figure 3-8: Team changes between roster combinations](image-url)
It is not obvious how to measure the total impact of team changes belonging to different types during a planning period. In the following, some ideas are proposed how to extend the standard objective function of crew rostering with various team-oriented aspects. Their implementation within a mathematical optimization model will be discussed in the next chapter.

The basic idea is to introduce penalties into the objective function in case of team changes. The penalty may be hierarchically structured in order to reflect the relative importance of different team change types as discussed above.

In general, such penalties are usually chosen as positive values to incorporate in the objective function of the CRP. In contrast to this, negative team change penalties (or team change bonuses) can be applied for evaluating teams that remain together for at least a certain period of time.

By this, different kinds of team changes and a proposed weighting via team change penalties are defined as the basis for their assessment in the next subsection.

### 3.2.5 Alternative Objectives

In this section a set of distinct objectives for the Team-oriented Scheduling is presented. They have to be incorporated in the objective function of the CSP and/or its underlying steps, here, the Team-oriented Rostering.

Let \( R \) represent the overall number of rosters among all crew members for both cockpit crew positions. Without loss of generality, it is assumed that this set of rosters is sorted in a way that the first \( r_{\text{CP}} \) rosters belong to the captains, while the remaining roster indices from \( r_{\text{CP}} + 1 \) to \( R \) are dedicated to the first officers. Additionally, let \( x_{r_1,r_2} \in \{0;1\} \) indicate, if the roster combination \((r_1, r_2)\) with \( r_1, r_2 \in \{1, \ldots, R\} \) is chosen (e.g., when applying \( x_{r_1}, x_{r_2} \) as decision variables for single rosters, \( x_{r_1,r_2} = 1 \) is valid only for \( x_{r_1} = 1 \land x_{r_2} = 1 \), 0 otherwise).
Some potential objectives are introduced and discussed below:

(A) **Minimize the weighted amount of team changes:** The crew schedule’s evaluation can be based on the number of team changes weighted by the type of their occurrence. Let $c_{r1,r2}$ indicate the team change penalties of the chosen roster combination $(r1, r2)$. It is computed by multiplying the number of team changes by given constant values (regarding the different kinds of team changes, see previous subsection 3.2.4). The objective is then given by:

$$\min \sum_{r1=1}^{C^P} \sum_{r2=r_{r1}+1}^{R} c_{r1,r2} x_{r1,r2}$$

(B) **Maximize the (minimum or average) duration of the teams:** In order to avoid that some crew members (in contrast to the majority) experience a rather high number of (weighted) team changes during the planning period, the duration aspect of stable (sub)teams can be emphasized by this objective. Let $d_{r1,r2}$ express the minimal (average) duration of the teams which implicitly execute the chosen roster combination. This results in:

$$\max \sum_{r1=1}^{C^P} \sum_{r2=r_{r1}+1}^{R} d_{r1,r2} x_{r1,r2}$$

(C) **Minimize the (maximum or average) number of different team compositions:** So far, there is no distinction made between newly composed teams and those that work together again after a certain period of time. But it may make sense to favor a team that has worked together at an earlier time instead of being newly composed. Based on indicators for roster combinations $x_{r1,r2}$, let $Y_{r1,r2}^{Comp}$ be an additional indicator for the realized different team compositions in the solution among all crew members $k \in K$

Let $Y_{r1,r2}^{Comp}=1$, if there is a roster combination $(r1, r2)$ such that $x_{r1,r2}=1$ and $r1$ is a roster of $k1$ and $r2$ is a roster of $k2$. The value of $Y_{r1,r2}^{Comp}=1$ can be ranked by another (e.g., quadratic) evaluation function $g(x)$ giving:

$$\min g(\sum_{k1 \in K} \sum_{k2 \in K} Y_{k1,k2}^{Comp})$$
(D) Minimize the (maximum or average) amount of different teammates someone works with: In particular for the cabin crew, the evaluation of some common team-oriented objectives, e.g., the number of team changes as given above may turn out to be less appropriate. Therefore, the experienced number of exchanged teammates can be a better indicator with respect to team orientation. The resulting objective function is derived analogously to the previous one, multiplied by \( h(x) \) as weighting function. The objective is expressed by:

\[
\min h(\sum_{k_1 \in K} \sum_{k_2 \in K} Y_{k_1,k_2}^{\text{Mates}})
\]

Reviewing the above-introduced team-oriented objectives of the CSP, the first two options are interdependent: In (A) a low (weighted) amount of team changes results in few, but not necessarily equally divided team changes among crew members. On the other hand, in (B) an increased minimal (or average) duration over all teams may lead to more harmonious team periods lengths, but also in total more team changes. Strongly dependent on the penalty setting chosen, it is also possible that a small set of massive team changes might be preferred rather than a higher amount of team changes with less impact on the team composition. When applying one of the latter two alternative objectives (C) and (D), there is a high risk of computing crew schedules with team compositions that frequently alternate during the planning horizon. Since this significantly undermines the general idea of how the author understands a stable team throughout this work, alternative (A) or (B) should be chosen, if possible. Furthermore, it can be useful to globally constrain certain variables, such as the amount of team changes or the amount of team compositions within the time period to be scheduled. All of the above-mentioned objectives impose the application of additional artificial rules (see subsection 2.1.4) to the CSP.

As introduced previously in subsection 3.1.4, (Hansen and Liden 2005) aim at minimizing the amount of exceptions to handle through assigning the given pairings to so-called subgroups corresponding to the pre-defined teams constructed earlier. Each exception can be regarded as a team change which is manually resolved afterwards.

In this work, the minimization of team changes given in (A) is the objective chosen for subsequent parts, including chapters 5, 6 and chapter 7. All other objectives mentioned above are thinkable as well, while (A) and (B) are comparatively easy to incorporate into a decision support system as it is presented in chapter 6. The implementation of both alternatives (C) and (D) requires many more indicators which is usually not applicable, see subsection 3.3.2. In addition, it may hinder the readability if all those alternatives are specified repeat-
edly. Since the focus lies on the cockpit crew, it is applied without any further weighting as suggested in subsection 3.2.4.

### 3.3 Further Considerations

In this section, additional important aspects of the Team-oriented Scheduling are brought into the discussion: Subsection 3.3.1 covers certain aspects of multi-objective optimization which are implied by the rostering problem examined. Next is subsection 3.3.2 which deals with the impact of team orientation on the handling of roster combinations and the resulting penalization strategies being considered in the ToRP. In 3.3.3 the so-called shared flight activities are introduced and their generation is analyzed. Finally, the extensions required for the CPCP are introduced in subsection 3.3.4.

#### 3.3.1 Multi-Objective Decision Making and Optimization Problems

As denoted above, the nature of the Team-oriented Rostering Problem implies a multi-objective optimization instance: Some of the criteria involved can be in conflict, e.g., improving one of them is only achievable by worsening another. In this subsection, a brief overview on the main solution approaches with regard to the decision making process and the solution’s quality evaluation is offered since both are subsequently applied throughout this thesis.

For further details on multi-objective optimization and decision making, it is referred to, e.g. (Gandibleux et al. 2004). Especially in (Landa Silva et al. 2004) a detailed overview on multi-objective meta-heuristics for scheduling and timetabling is presented.

#### Search and Decision Making

The search and decision making processes have to be arranged and determined in advance, see also (Steuer 1986):

- **Decision making – then search:** In this a priori approach all preferences or parameters for each of the objectives are set in advance by the decision makers. The best possible solution(s) has or have to be found afterwards satisfying mostly the set of preferences determined earlier.

  - **Search – then decision making:** Here the decision makers select the most adequate solution *a posteriori* out of the various solutions com-
puted. This approach is usually applied when there is a trade-off between the objectives considered.

- **Interactive search and decision making:** If it is possible to adjust preferences during the optimization, decision makers may like to intervene during the search process in order to guide it towards promising solutions.

The concept applied in this work for the search and decision making process embedded a decision support system can be seen as a combination of the a priori and the a posteriori approach, see chapter 6.

**Quality evaluation**

According to (Coello Coello et al. 2002) multiple objectives can be evaluated in several different ways as given below:

- **Combined objectives:** A multiple-criteria optimization problem is converted into a single-criteria one, e.g., by aggregating the various criteria into a single function or value, while the different components can be weighted.

- **Alternated objectives:** For many years it was common to optimize one criterion at a time while imposing constraints on the others. In this case the solution quality highly depends on the chosen execution order which is mostly hard to determine in advance.

- **Pareto-based evaluation:** In this approach, the concept of dominance is applied in such a way that a set of solutions has to be explored where there is no other solution available without the expense of any of the other criteria examined (known as non-dominated or non-inferior solutions).

With regard to the quality evaluation of the ToRP, from now on classical crew scheduling objectives (such as operational cost, workload balancing, etc.) are combined together with the team-oriented objectives discussed in section 3.2.5, namely the minimization of team changes.

### 3.3.2 Penalization Strategies for Roster Combinations

From a more conceptual view, penalization strategies are addressed in this subsection. Whenever teeming is emphasized, it is necessary to evaluate roster combinations intensively. In this section, the set of implicitly examined roster combinations is underlined: As an example, a problem instance consists of 10
captains and 10 first officers, while each of them has 100 valid rosters as the combination basis. In total, there are 1,000 CP rosters, and 1,000 FO rosters. Combining both roster sets, 1,000,000 (= 1,000 times 1,000) roster combinations are obtained. If the number of rosters for each crew member is doubled, already four times as many roster combinations have to be considered.

Moreover, if the instance examined on cockpit crews allows downgrading, 1,000 CP rosters have to be combined not only with all the FO rosters, but also with the CP rosters themselves (equals a total of 2,000 CP rosters). Thus, there are directly two million roster combinations in this instance, and even eight million of them, if 200 rosters for each crew member are included.

Unlike the classical formulations of the CSP sub-problems given in subsections 2.2.1 and 2.2.2, it is not enough to ensure that all the basic restrictions are satisfied. Here, those constraints are implicitly applied on groups of roster combinations, e.g., by ensuring that the roster combination chosen covers all flight legs or pairings exactly once. For the Team-oriented Rostering Problem such roster combinations have to be addressed mostly explicitly via binary indicator variables (columns) whenever a penalty or weighting occurs, see subsections 2.3.2, 3.2.4, and 3.2.5. (For more details on these aspects, it is referred to chapter 4.) Also note that for alternatives (C) and (D) of subsection 3.2.5 all binary indicators are necessary as intermediate decision variables for the objective function.

As a result, when continuing with alternative (A) as defined in subsection 3.2.5, the amount of team change penalties to be incorporated in the model is linked to its applicability on real-life instances due to the dimensions of the model. Apart from the different penalty types and their implication on the corresponding penalty value assigned, there are only two major aspects to consider: Firstly, out of the huge amount of potential roster combinations only those “undesired” ones require a further investigation, for which a team change is encountered. Secondly, the team change should be only penalized according to the penalization strategy defined.

Because of this, it may be beneficial to adjust the penalization strategy in such a way that a comparatively small subset of the considered roster combinations requires a further treatment. This helps greatly to avoid overgrowing model sizes. For instance, our business partner agreed on a penalization strategy which ignores team changes that occur after the weekly rest. (It also includes the consideration of the previous planning period.) Even more, team changes among experienced flight personnel of the same home base are neglected, since especially these people know each other already very well over the years.
Considering teaming for the cabin crew is much more complex, since particularly for CAs combinations of multiple, e.g., up to five, rosters have to be evaluated. However, the cabin crew scheduling problem involves fingerprints with less fixed days than the pilot scheduling problem (because of missing simulation and office days, etc.). As already indicated by the variety of possible team changes (see subsection 3.2.4) imposed by the potential exponentially higher amount of roster combinations, it remains insolvable for real-life instances. As a result, stable cabin teams in combination with floaters (for over-demands of crew) are proposed.

### 3.3.3 Shared Flight Activities and Their Generation

In this section a further key term for the Team-oriented Scheduling is defined. The impact on two distinct generation strategies is discussed afterwards.

**A shared flight activity (SFA)** is regarded as the smallest unit that is considered by the team-oriented scheduling approach: Such an activity or *piece-of-work* (POW) is serviced by a team without any team change.

With respect to the ToRP for several crew functions, a SFA can be a single flight leg or flight duty, but also a set of flight legs or flight duties; even a complete pairing is possible. Such a POW is extracted from the pairings generated in the CPP.

The overall reason, why pairings are not applicable any longer, was already introduced in subsection 2.2.2: Regular optimization instances on the CSP are usually subdivided with regard to the fleet, but also to each of the crew positions involved. In combination with individual pre-scheduled activities, teams working together get separated rather frequently.

Also by our computational experiments, the ToRP approach should be based on the assignment of SFAs, not pairings. The necessity of a clear distinction between both terms is depicted in Fig. 3-9: Although there is a pair of (one-day) pairings for both cockpit crew functions, the assignment of those pairings as normally executed during the CRP (see subsection 2.2.2) already results in four unavoidable team changes among the cockpit crew within this single day.

As a result, directly assigning pairings at this point of the scheduling process is not appropriate for the minimization of team changes on the one hand, since only changes in the team composition between pairings can be taken into account, whereas their occurrence inside of pairings is kept as unchangeable. However, this approach has a main drawback which can be seen in the resulting set of probably small POWs that cover only a single flight leg. Thus, already
optimized pairings of the CPP are given up at the expense of team orientation. As a further consequence the amount of valid rosters grows exponentially, including again a huge set of rosters which had been implicitly excluded by the pairings determined before in the pairing generation phase. (In the example in Fig. 3-9, there are now six SFAs over both crew functions examined instead of two separate assignment problems with only two pairings each.)

Although it is theoretically possible to solve the CPP for several crew functions simultaneously, the computational effort is usually not applicable, see 2.1.2. Especially for instance settings with more than a single home base and work-load balancing among the flight personnel, the pairing generation procedure may require some modifications in order to address the team stability aspects appropriately.

Three different alternative approximation approaches to the generation of the SFA set are presented below:

- **Directly extract shared flight activities from the CPP:** As a generally applicable approach to any kind of pairing generation process, the SFAs can be directly determined by the comparison of one crew function’s pairings with those of the other one. In the case of a team change, the af-
fected pairings must be cut into pieces in such a way that afterwards all crew functions share the same set of shared flight activities. Such a generation process is easy to compute, but there is a risk of many small pieces of work with the implications as described above.

- **Adopt pairings of a selected crew function**: The pairings of one crew function are assigned as far as possible to all others. Whenever this is not feasible due to flight personnel’s individual availability, the referred pairing has to be broken up or, for instances with several home bases, it has to be assigned to another crew domicile via base variants (see 2.2.2). The first alternative may impose additional operational cost and the potential for new team changes, whereas the second avoids at least team changes inside of earlier formed pairings.

- **Build shared flight activities applying a sequential procedure**: In contrast, a sequential procedure for the SFA generation addresses some of the drawbacks mentioned above. Here, the pairings of one crew function are taken as the basis for all others. The main disadvantage is given by the fact that such a sequential procedure may increase operational cost imposed by the new set of pairings in comparison to the separately generated ones of a classic CPP solving procedure. Nevertheless, it can be impossible to decide in advance on the best crew function to start with.

The later approach is applied in the next subsection dealing with the generation of shared flight activities for the CPCP.

### 3.3.4 Excursus: Extensions for the Crew Pairing Chain Problem

In this subsection a model extension to the time-space-network for the CPCP presented in subsection 2.4.1 is briefly introduced as published in (Thiel et al. 2004).

As presented so far, the pairing chain generation works independently for each crew function which results in a set of pairings with little team stability, see 3.2.2. According to the above-introduced sequential procedure for the CPP, the information obtained from the generated pairing chains of the first crew function should be utilized within the optimization of the remaining crew functions.

In the context of the proposed network structure, such a hierarchical procedure is enabled by the introduction of additional bonus arcs representing the avoidance of team changes, e.g., within a flight duty or pairing.

As given in subsection 3.2.4, such bonuses (as negative penalties) should be granted based on the team change classification described, e.g. team changes
within the day, over night, outside the home base, and so on. Such bonus values should increase by the duration of the granted synchronous (or team-stable) (sub)pairings, nevertheless they should not exceed a certain value at which the sum of bonuses outbalances the selection of optimal pairing chains for the examined crew function. (Such high bonuses may absorb undesirable additional operational cost, e.g., for hotel and taxiing expenses, see 3.1.3)

Additional bonus arcs that cover more than one pairing are not required since all chains will be decomposed into pairings in the rostering phase. Fig. 3-10 shows the application of such an additional bonus arc. It represents the connection between three airports by two flight legs and one waiting arc.

As pointed out in Fig. 3-10 the compliance with the inflow-outflow requirement for each connection node requires one additional restriction at the start and the end of each bonus arc. It has to ensure that, in case the bonus arc is chosen,
3.4 Summary

In this chapter the Team-oriented Scheduling was introduced. It is a distinct extension to the requirements imposed by the classical CSP described in the previous chapter 2.

Generally speaking, a team is defined as a group of at least two people that share common goals and/or objectives, and that interact based on their working correlations. From the HRM point of view, work is shared within a team and every member follows the pronounced community spirit. Important factors are: Team size, duration, performance, style of working and cohesion. Such a team gradually develops certain dynamics by following a life-cycle which consists of the four phases: Orientation, dissatisfaction, resolution, and production. All these phases greatly impact the team’s productivity and moral. Such phases are not easy to identify, but they require at least a certain period of time in order to take place which is again related to the team’s duration. For an aircrew, common team structures are presented based on the crew’s job description and their exemplary occurrence in a European tourist airline.

The term team orientation in the context of this work is expressed by the concept of scheduling aircrew teams which remain stable over a considerable time, e.g., several days. By analyzing the impact of team orientation and its effect on crew satisfaction, it can be concluded that team stability leads to a superior level of cockpit safety. It may also improve crew satisfaction which is known as the key to the customers’ satisfaction. Therefore, team-oriented behavior must be adopted by all crew members. Besides this, additional operational cost may be imposed when higher team stability is aimed at, whereas it may simplify rescheduling processes due to the lower impact when delays are propagated.

A review on the up-to-date operations research literature on the crew scheduling problem reveals that teams and team stability is so far not addressed rather than those trivial cases: Fixing or preventing dedicated teams together with the allowance of buddy requests. Especially the personalized rostering approach offers the environment to work on the consideration of stable teams. Here, automated schedules have to be computed among equally regarded crew members. Like before, each of them receives a roster with equal workload that realizes in addition a certain stability of the so far implicitly constructed work teams. In fact, he or she does not directly notice the change, since it does not cut off any of the flight personnel’s preferences or request behavior.

In order to enhance the understanding of team orientation several terms such as team change and team change penalty are introduced and some of their characteristics are examined for their application on the cockpit and the cabin crew. In addition, several alternative objectives on the Team-oriented Rostering
Problem are examined. The most common ones are (1) minimize the amount of team changes, (2) maximize the duration of teams, (3) minimize the amount of different team compositions, and (4) minimize the amount of different teammates a member works with. Such team-oriented objectives in combination with classical CSP objectives form a multi-objective optimization problem, for which a weighted and aggregated objective function can be chosen.

In contrast to the usage of pairings (as the usual rostering entity), shared flight activities form the smallest unit that have to be considered. Several strategies for their generation are introduced, followed by a possible extension to the aggregated time-space network for the CPCP.

Further aspects on the Team-oriented Scheduling and the Team-oriented Rostering are still thinkable. Therefore, the above-introduced aspects do not form an exclusive list of characteristics on the definition and evaluation of team orientation as it is presented in this chapter.

Based on the interpretation of the Team-oriented Scheduling presented above, a formal representation of the Team-oriented Rostering Problem is addressed by several optimization models in chapter 4.
Chapter 4

Model Formulations for Team-oriented Rostering

In the previous chapter 3 the Team-oriented Scheduling and its main characteristics were defined to a great extent while special attention was paid to the application of the personalized rostering approach in the Team-oriented Rostering Problem. This part of the work examines the problem formulation by exact optimization for the important case of open team compositions in which teams are solely determined during the rostering process itself, see subsection 3.2.3.

Recapitulating the key difference of the ToS and the classical CSP, roster combinations have to be evaluated in a much more detailed manner beyond the pure coverage of SFAs as applied in the conventional approaches. Since the set of roster combinations chosen influences safety and quality-of-life criteria of flight personnel receiving this crew schedule, each particular roster selected for a dedicated crew member may greatly impact the selection of rosters for one or more other crew members, e.g., for the complementary crew position in the cockpit team. The resulting complexity of teaming is by far higher than that of the corresponding setting for the regular CRP with separately treated crew functions.

Due to the fact that there is no previous work existing which handles the above characteristic for aircrews, the Team-oriented Rostering approach is formalized via mathematical programming in this chapter, see 2.3.2. In order to provide a first reference point, a simultaneous rostering process among the crew functions involved is chosen instead of a sequential proceeding (in analogy to 3.2.3 and 3.3.3). As there is already great knowledge on the CSP available, especially to develop and to solve optimization models, it can be exploited in working more specifically on this dedicated problem. Furthermore, state-of-the-art optimizers are available for its solving process.

As discussed before, in the following crew function pairs are examined, here the cockpit crew. Such pairs are usually composed by one captain and one first officer. They form the sub-team considered as part of the overall crew, see 3.1. In
addition, all models presented below would lose their readability, if extensions with more than two crew functions are presented.

Therefore, it is referred to a basic formulation of a *rostering model for crew function pairs* as the overall starting point for the later investigation on modeling the ToRP throughout this chapter. Basically, it is a combined version for the two cockpit crew functions. Any kind of team-change consideration is not embodied in this reference model. All required variables throughout this work are introduced in section 4.1. The basic model formulation is explained in 4.2. The next sections propose and examine different optimization programs, such as a quadratic formulation (MIQP) in 4.3, and two distinct BIP models in 4.4 and 4.5 as presented in (Thiel 2004), followed by a MIP formulation in 4.6 which is based on the LP relaxation of the BIP presented in 4.4. A comparison and review among the models introduced is given in the section 4.7, followed by a summary in 4.8.

### 4.1 Notation

Before discussing any optimization model to address the ToRP, commonly used variables are defined for the remaining part of this work as follows:

Let

\[ F \] represent the number of shared flight activities (SFA) \( f \in \{1,...,F\} \) to be serviced, see subsection 3.3.2,

\[ K \] indicate the total number of crew members. Captains are numbered starting from 1 to \( k_{\text{CP}} \) and first officers have the indices from \( k_{\text{CP}} + 1 \) to \( K \),

\[ R_k \] express the total number of rosters for crew member \( k \) being considered in the model, \( k \in \{1,...,K\} \),

\[ R = \sum_{k=1}^{K} R_k \] gives the overall number of all rosters among all crew members \( k \in \{1,...,K\} \), where \( r_{\text{CP}} = \sum_{k=1}^{k_{\text{CP}}} R_k \) is the number of all captain rosters, first officer rosters have the indices from \( r_{\text{CP}} + 1 \) to \( R \),

\( r^k \) be the index of the first roster for crew member \( k \in \{1,...,K\} \) with \( r^1 = 1 \) and \( r^k = \sum_{i=1}^{k-1} R_i + 1 \) \( \forall k \in \{2,...,K\} \). The special case \( k = K + 1 \), see section 4.5, is defined as \( r^{K+1} = R + 1 \),
$c_r$ represent the overall considered cost for roster $r \in \{1,\ldots,R\}$,

$c_{r1,r2}$ indicate team change penalties of the chosen roster combination $(r1, r2)$ $r1, r2 \in \{1,\ldots,R\}$, see subsection 3.2.4 and 3.3.1.

$a_{r,f}^{CP}$ and $a_{r,f}^{FO}$, each equal 1, if a SFA $f \in \{1,\ldots,F\}$ is included in roster $r \in \{1,\ldots,R\}$ as a captain or first officer activity, 0 otherwise,

$x_r \in \{0;1\}$ equal 1, if roster $r \in \{1,\ldots,R\}$ is chosen, 0 otherwise,

$x_{r1,r2} \in \{0;1\}$ equal 1, if a specific roster combination $(r1, r2)$, $r1, r2 \in \{1,\ldots,R\}$, is chosen by $x_{r1}=1 \land x_{r2}=1$, 0 otherwise,

$x_f^{ECP}$, $x_f^{UFO}$ $\in \{0;1\}$ equal 1, if a SFA $f \in \{1,\ldots,F\}$ is unassigned, 0 otherwise, and let

$c_f^E$ point out the (virtual) cost for any unassigned SFA $f \in \{1,\ldots,F\}$.

Further explanations are added below:

- $c_r$ is characterized by operational cost (here: hotel and crew transfer expenses) as well as deviation penalties from planned flight time or contract usage for the individual crew member to facilitate the fair-and-equal share of workload (as required in a personalized rostering system).

- $c_f^E$ is introduced to guarantee feasibility even if the roster set selected in the solution is not capable to fully cover all SFAs.

4.2 Basic Rostering Model for Crew Function Pairs

The basic formulation of a rostering model for the simultaneous assignment of two crew functions is expressed as a binary IP model as given by (4.2.1) to (4.2.6). Note that with regard to the newly introduced numbering and separation of the roster set for each crew member involved, it is possible to address the rosters of an individual $k$ directly via their indices that constitute the range from $r^k$ to $r^k + R^k -1$. (In contrast, in the basic rostering model given in subsection 2.2.2, this information is not explicitly known, and as a consequence, e.g., a further sum over all crew members in the objective function is required.)
\[
\min \sum_{r=1}^{R} c_r x_r + \sum_{f=1}^{F} c_f^E (x_f^{ECP} + x_f^{EFO})
\]  \hspace{1cm} (4.2.1)

Subject to:

\[
\sum_{r=1}^{R} x_r = 1 \quad \forall k \in \{1, \ldots, K\}
\]  \hspace{1cm} (4.2.2)

\[
\sum_{r=1}^{R} a_{r,f}^{CP} x_r + x_f^{ECP} = 1 \quad \forall f \in \{1, \ldots, F\}
\]  \hspace{1cm} (4.2.3)

\[
\sum_{r=1}^{R} a_{r,f}^{FO} x_r + x_f^{EFO} = 1 \quad \forall f \in \{1, \ldots, F\}
\]  \hspace{1cm} (4.2.4)

\[
x_r \in \{0,1\} \quad \forall r \in \{1, \ldots, R\}
\]  \hspace{1cm} (4.2.5)

\[
x_f^{ECP} \in \{0,1\}, \quad x_f^{EFO} \in \{0,1\} \quad \forall f \in \{1, \ldots, F\}
\]  \hspace{1cm} (4.2.6)

As stated in (4.2.1), the objective function consists of the sum over all operational cost \(c_r\) that is associated to rosters selected in the solution of the model by the decision variable \(x_r\). Furthermore, uncovered SFAs are caught by the second part of the objective function, which assigns high virtual cost \(c_f^E\), if such dummy usage given by \(x_f^{ECP}\) and \(x_f^{EFO}\) becomes necessary. In analogy to subsection 2.2.2, restriction (4.2.2) assures that each crew member \(k\) is assigned exactly one roster out of his or her individually generated roster set. This implies that a null roster (see subsection 2.1.2) is required for each crew member that needs to be included in the individual set of valid rosters. In restriction (4.2.3) a SPP formulation is given such that each SFA \(f \in \{1,\ldots,F\}\) for captains has to be covered exactly once, either by one of the captain rosters, ranging from indices \(r\) to \(r^{CP}\), while restriction (4.2.4) does the same for the coverage of SFAs for first officers (\(r^{CP+1}\) to \(R\)), respectively. Hereby, the model is defined. All decision variables are defined as binaries in (4.2.5) and (4.2.6).

For an example see Fig. 4-1: Let us assume five SFAs (\(F=5\)) to be covered by three rosters for each of the two captains and the two first officers (\(R=12\), \(r^{CP}=6\), \(r^1=1\), \(r^2=4\), \(r^3=7\), \(r^4=10\), \(K=4\), \(k^{CP}=2\)). All rosters proposed are defined by \(R_1 = R_4 = R_7 = R_{10} = \{\emptyset\}\), \(R_2 = \{f_1, f_2, f_3\}\), \(R_3 = \{f_1, f_4, f_5\}\), \(R_5 = \{f_2, f_3\}\), \(R_6 = \{f_2, f_4\}\), \(R_7 = \{f_2, f_5\}\), \(R_8 = \{f_3, f_4\}\), \(R_9 = \{f_3, f_5\}\), \(R_{11} = \{f_4, f_5\}\), \(R_{12} = \{f_1, f_2\}\), while operational roster cost \(c^R\) is expressed by \(c_1 = c_4 = c_7 = 0\),
For the subsequent sections, team change penalties \( c_{i,j,2} \) among roster combination \((r_1, r_2)\) are defined by \( c_{2,8} = c_{2,11} = c_{3,12} = c_{6,8} = 2, c_{2,12} = c_{3,11} = c_{5,8} = c_{5,9} = c_{5,12} = 1, \) and \( c_{6,9} = -1. \)

For this case, the set of rosters presented allows a limited combination to satisfy the cover constraints: According to (4.2.3) captain rosters can be combined as either \((R_2, R_6)\) or \((R_3, R_5)\), whereas first officers with respect to (4.2.4) should be combined as \((R_8, R_{11})\) or \((R_9, R_{12})\). (Of course, it would be also possible to replace one or even both of the rosters above by null rosters, whereas each unassigned SFA is covered by one of the expensive id entity matrix columns on the right side.) Such subsets of rosters (as columns) open the solution space of the model. Regarding the objective of minimizing operational cost, the optimal solution is \( c^R = c_2 + c_6 + c_8 + c_{11} = 4 + 6 + 5 + 4 = 19, \) as highlighted in Fig. 4-1. Note that even null rosters may impose certain cost, see \( c_{i_0} \) for \((FO_2 R_1)\), since prescheduled activities (which are not indicated explicitly in the model) require, e.g., necessary transits for their execution.

<table>
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<th>CP2</th>
<th>FO1</th>
<th>FO2</th>
<th>Identity Matrix</th>
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Figure 4-1: Basic rostering model for two crew functions
Chapter 4 – Model Formulations for Team-oriented Rostering

4.3 MIQP Formulation

After examining the key characteristic of the ToRP in comparison to the reference model described above, it becomes obvious that this time a multi-criteria optimization and decision making has to be embodied for the team-oriented scheduling task. As already discussed in subsection 3.3.1, a simple and intuitive approach appears: Adding a term to the objective function which penalizes team changes among roster combinations. For the ease of simplicity both objectives are summed (minimizing operational cost and minimization of team change penalty here. Due to the discrete nature of the CSP and its sub-problems, a pairing, roster or SFA is either completely covered (=1) or not (=0), the occurrence of such team change penalties can be expressed by the product of binary decision variables \( (x_{r1}, x_{r2}) \) which is assessed by a defined penalty factor \( (c_{rl,r2}) \), see also alternative (A) in subsection 3.2.5. As a result, team changes of an entire schedule can be evaluated for all given roster combinations among a captain roster \((r1)\) and a first officer roster \((r2)\) by the term (4.3.1):

\[
\sum_{r1=1}^{c} \sum_{r2=1}^{R} c_{rl,r2} x_{r1} x_{r2} 
\]

(4.3.1)

In literature, the Quadratic Assignment Problem (QAP) is introduced and examined, see, e.g., (Pardalos and Wolkowicz 1994). Quadratic programs (QP) are capable to solve squared constructs in the objective function, such as given by cost (or penalties) that appear only, e.g., if a combination of two decision variables are given in the solution vector of the model. With regard to a binary formulation of the QAP, a so-called mixed integer quadratic program (MIQP) can be defined, also known as 0/1 Quadratic Program.

As described above the MIQP model formulation for the ToRP is derived from the general CRP for two crew functions by adding the non-linear term to the objective function (4.3.2) while all other linear restrictions (4.3.3) to (4.3.7) remain unchanged.
\[ \min \sum_{r=1}^{R} c_r x_r + \sum_{r=1}^{CP} \sum_{l=r+1}^{R} c_{r,l} x_r x_l + \sum_{f=1}^{E} C_f \left( x_f^{ECP} + x_f^{EFO} \right) \]  
\hspace{1cm} (4.3.2) 

Subject to:

\[ \sum_{r=1}^{r} x_r = 1 \ \forall k \in \{1, ..., K\} \]  
\hspace{1cm} (4.3.3)

\[ \sum_{r=1}^{CP} a_{r,f} x_r + x_f^{ECP} = 1 \ \forall f \in \{1, ..., F\} \]  
\hspace{1cm} (4.3.4)

\[ \sum_{r=1}^{R} a_{r,f} x_r + x_f^{EFO} = 1 \ \forall f \in \{1, ..., F\} \]  
\hspace{1cm} (4.3.5)

\[ x_r \in \{0, 1\} \ \forall r \in \{1, ..., R\} \]  
\hspace{1cm} (4.3.6)

\[ x_f^{ECP}, x_f^{EFO} \in \{0, 1\} \ \forall f \in \{1, ..., F\} \]  
\hspace{1cm} (4.3.7)

**Realization of MIQP**

In order to solve the MIQP, a reformulation of (4.3.2) becomes necessary, where \( \bar{x} \) is a vector containing all decision variables of the model, \( \bar{x}^T \) is the corresponding transpose, \( Q \) is a symmetric and positive semi-definite matrix containing the coefficients for the penalty (or cost) multiplications, and \( \bar{c}^T \) is the transpose of the cost vector of the model. This results in the term (4.3.8).

\[ \text{Min } \left( \frac{1}{2} \bar{x}^T Q \bar{x} + \bar{c}^T \bar{x} \right) \]  
\hspace{1cm} (4.3.8)

Furthermore it is crucial for the objective function to be either convex or concave: While a convex objective function is required for minimization problems, and a concave for maximization problems. Contrary to that the restriction graph must have a shape opposite to the one of the objective function: Convex, if the objective function is concave, and vice versa. Only in this case a local minimum, respectively a local maximum, is also the global extreme point, see e.g., (Zimmermann 1995).

Summed up, to solve a minimization problem like the ToRP with the help of QP techniques the following requirements have to be assured, since only in this case a global optimum can be found and the problem is solved:
(a) The restriction graph is concave.

(b) The objective function is convex.

(c) The matrix Q is symmetric and positive semi-definite.

The matrix $Q$ is created by transferring the non-linear term of the objective function into a matrix form. It is referred to as the $QMatrix$ according to the term used in the context of QP and MIQP.

When formulating a QMatrix like $Q$, the multiplier $0.5$ of (4.3.8) can be calculated into the matrix. Taking this into account a definition of the corresponding symmetric n-square QMatrix $Q$ for the MIQP model is given below.

$$
Q = \begin{pmatrix}
0 & \frac{c_{1,2}}{2} & \ldots & \frac{c_{1,R-1}}{2} & \frac{c_{1,R}}{2} \\
\frac{c_{1,2}}{2} & 0 & \ldots & \ldots & \frac{c_{2,R}}{2} \\
\ldots & \ldots & \ldots & \ldots & \ldots \\
\frac{c_{1,R-1}}{2} & \ldots & \ldots & 0 & \frac{c_{R-1,R}}{2} \\
\frac{c_{1,R}}{2} & \frac{c_{2,R}}{2} & \ldots & \frac{c_{R-1,R}}{2} & 0
\end{pmatrix}
$$

First, it is shown that both terms, (4.3.1) and the QMatrix proposed, are equal:

$$
\bar{x}^T Q \bar{x} = \begin{pmatrix} x_1 & x_2 & \ldots & x_{R-1} & x_R \end{pmatrix} \cdot \begin{pmatrix}
0 & \frac{c_{1,2}}{2} & \ldots & \frac{c_{1,R-1}}{2} & \frac{c_{1,R}}{2} \\
\frac{c_{1,2}}{2} & 0 & \ldots & \ldots & \frac{c_{2,R}}{2} \\
\ldots & \ldots & \ldots & \ldots & \ldots \\
\frac{c_{1,R-1}}{2} & \ldots & \ldots & 0 & \frac{c_{R-1,R}}{2} \\
\frac{c_{1,R}}{2} & \frac{c_{2,R}}{2} & \ldots & \frac{c_{R-1,R}}{2} & 0
\end{pmatrix} \cdot \begin{pmatrix} x_1 \\
x_2 \\
\ldots \\
x_{R-1} \\
x_R
\end{pmatrix}
$$
\[
\begin{align*}
&= \left( x_1 \ x_2 \ \ldots \ x_{R-1} \ x_R \right) \\
&= \left( \begin{array}{c}
0 + \frac{c_{1,2}}{2} x_2^2 + \frac{c_{1,3}}{2} x_3^2 + \ldots + \frac{c_{1,R-1}}{2} x_{R-1}^2 + \frac{c_{1,R}}{2} x_R^2 \\
+ \frac{c_{1,2}}{2} x_1 x_2 + \frac{c_{1,3}}{2} x_1 x_3 + \ldots + \frac{c_{1,R-1}}{2} x_1 x_{R-1} + \frac{c_{1,R}}{2} x_1 x_R \\
+ \ldots \\
+ \frac{c_{1,R-1}}{2} x_{R-1} x_{R-1} + \frac{c_{2,2}}{2} x_{R-1} x_R + \ldots + \frac{c_{R-1,R}}{2} x_{R-1} x_R + 0 \\
+ \frac{c_{1,R}}{2} x_R x_{R-1} + \frac{c_{2,R}}{2} x_R x_R + \ldots + \frac{c_{R-1,R}}{2} x_R x_R + 0 \\
\end{array} \right) \\
&= \sum_{r_1=1}^{R-1} \sum_{r_2=r_1+1}^{R} c_{r_1,r_2} x_{r_1} x_{r_2} \\
&= \sum_{r_1=1}^{c_{r_1}} \sum_{r_2=r_1+1}^{c_{r_2}} c_{r_1,r_2} x_{r_1} x_{r_2} + \sum_{r_1=1}^{c_{r_1}} \sum_{r_2=r_1+1}^{R} c_{r_1,r_2} x_{r_1} x_{r_2} + \sum_{r_1=1}^{R-1} \sum_{r_2=r_1+1}^{R} c_{r_1,r_2} x_{r_1} x_{r_2} \\
&= (4.3.9)
\end{align*}
\]

The term in (4.3.9) is given by three addends through which all penalties among (CP, CP), (CP, FO), and (FO, FO) roster combinations are represented. In (4.3.1) only roster combinations for (CP, FO) pairs are considered which is a subset of the combinations that are covered by the definition of the QMatrix. (They are therefore implicitly set to 0.)

To achieve solvability of the MIQP, all constraints (a) to (c) introduced above have to be obeyed in order to find a global optimum. For a complete proof, it is referred to, e.g., (Zimmermann 1995).
Typical applications for the QAP are efficient wiring problems (e.g., Steinberg Wiring problem) or layout problems for hospitals and production lines, see (Commander 2003) for further examples. To the knowledge of the author, there is no application reported for personnel scheduling so far, since instances with more than 10,000 binary variables (here: rosters) are still almost impossible to solve today by MIQP techniques, see (Anstreicher et al. 2002; Caprara 2003; Pardalos and Wolkowicz 1994).

By this restriction the application for most real-life instances of the ToRP with easily several thousands of rosters being considered in a combined model remains impossible, see 3.3.1. Therefore, the deployment of more conventional alternatives is examined, such as an IP formulation for the problem in the next sections.

### 4.4 BIP Formulation (I): The Extended Rostering Model

The key concept of the Extended Rostering Model can be depicted as a strict extension of the basic SPP model given in section 4.2 in such a way that it handles penalties for team changes via additional rows and columns. In this model \( x_{r_1,r_2} \) is used as an indicator variable. The resulting model is formulated as follows:

\[
\min \sum_{r=1}^{R} c_r \cdot x_r + \sum_{r=1}^{R} C_{r_1,r_2} \cdot x_{r_1,r_2} + \sum_{f=1}^{F} C_{f}^{ECP} (x_{f}^{ECP} + x_{f}^{EFO}) 
\]

Subject to:

\[
\sum_{r=r_f}^{R-1} x_r = 1 \quad \forall k \in \{1, \ldots, K\} 
\]

\[
\sum_{r=1}^{r_f} a_{r_f}^{CP} \cdot x_r + x_r^{ECP} = 1 \quad \forall f \in \{1, \ldots, F\} 
\]

\[
\sum_{r=r_f+1}^{R} a_{r_f}^{FO} \cdot x_r + x_r^{EFO} = 1 \quad \forall f \in \{1, \ldots, F\}
\]

\[
x_{r_1} + x_{r_2} - x_{r_1,r_2} \leq 1 \quad \forall r_1 \in \{1, \ldots, R^{CP}\} \quad \forall r_2 \in \{r^{CP} + 1, \ldots, R\}
\]
The objective function (4.4.1) consists of three parts: The first addend of the minimization function summarizes the required operational roster cost, whereas the second one covers the corresponding team change penalties when captain rosters \( r_1 \) and first officer rosters \( r_2 \) are combined. The third part ensures the solvability by treating unassigned SFAs with special cost \( E \).

Restrictions (4.4.2) to (4.4.4) remain unchanged. In (4.4.5) all required team change penalties for a roster combination \((r_1, r_2)\) occur only in the case that both rosters are chosen. Restrictions (4.4.9) and (4.4.10) become necessary, if negative team change penalties (or bonuses) are set up. Due to the minimizing objective function such bonuses should be only selected in the solution, if both rosters themselves are chosen, otherwise 0.

The schematic view on the Extended Rostering Model is given in Fig. 4-2. In comparison to Fig. 4-1 on the general CRP for two crew functions, further columns and rows are added. In this example, only nonzero team change penalties \( c_{r_1,r_2} \neq 0 \) are included, since all others have no influence on the objective function. (They are directly removed during the preprocessing when solving the model otherwise). Also note that not all team change penalties in this example are positive, see \((CP2 R3, FO1 R3)\) given as \((R6, R9)\), therefore restrictions (4.4.9) and (4.4.10) are necessarily included as described above.

Furthermore, due to the explicit consideration of penalties by the newly introduced constraints, a different optimal roster set (solution) is chosen. The associated minimal operational cost \( c^R \) together with team change penalties \( c^T \) are

\[
 c^R + c^T = c_2 + c_6 + c_9 + c_{12} + c_{22} + c_{6,9} = 4 + 6 + 4 + 7 + 1 - 1 = 21.
\]

Although this roster set is less favorable with respect to solely operational cost (they equal 21, which is inferior to 19 achieved by the former roster set in section 4.2), team change
penalties outbalance the earlier optimal solution due to several undesirable changes encountered in the team compositions. Therefore, the original roster set of the former solution in section 4.2 would be penalized according the penalty setting of this section with $c^T = c_{2,8} + c_{2,11} + c_{6,8} = 2 + 2 + 2 = 6$, such that $19+6=25$ is determined as the new total objective value for this roster combination, see Fig. 4-2).

\[ T_{cc} = 2,8,11,6,2226 \]

\[ 19+6=25 \]

\[ \text{Figure 4-2: Schematic view on the Extended Rostering Model} \]

4.5 BIP Formulation (II): The Roster Combination Model

As an alternative to the formulation in the previous section, the Roster Combination Model follows the idea of considering directly roster combinations instead of single rosters for each individual crew member.

Therefore all columns in this model represent a roster combination for two crew members of different crew functions, independently of whether they share any
SFA or not. Such roster combinations are based on all available rosters for each individual crew member. For a direct comparison between this model and the Extended Rostering Model described in the previous section, let us define

\[
\tilde{c}_{ri} = \frac{c_{ri}}{K - k_{CP}}, \text{ and } \tilde{c}_{rj} = \frac{c_{rj}}{k_{CP}} \quad \forall r1 \in \{1, \ldots, r_{CP}\} \quad \forall r2 \in \{r_{CP} + 1, \ldots, R\}
\]

By this, operational cost for a captain roster is divided by the number of first officers and vice versa. This is done because each captain roster is combined with all first officer rosters, and each first officer roster is combined with all captain rosters, respectively. Since roster combinations (as potential cockpit teams) are already considered, here \(x_{r1,r2}\) is used as the decision variable (and not as an indicator for the occurrence of team change penalties like in section 4.4).

The resulting model can be formulated as follows:

\[
\begin{align*}
\min \sum_{r=1}^{CP} \sum_{r2=r+1}^{R} (\tilde{c}_{ri} + \tilde{c}_{rj} + c_{r1,r2}) x_{r1,r2} + \sum_{f=1}^{F} c_{rf} (x_{r1}^{ECP} + x_{r1}^{EFO}) \\
\text{Subject to:} \\
\sum_{r1=rf}^{a_{r1}^{k1}} \sum_{r2=r+1}^{a_{r2}^{k2}} x_{r1,r2} = 1 \quad \forall k1 \in \{1, \ldots, k_{CP}\} \\
\sum_{k2=r_{CP}}^{k_{CP}+1} x_{r1,r2} = 1 \quad \forall k2 \in \{k_{CP} + 1, \ldots, K\}
\end{align*}
\]

\[
\begin{align*}
\sum_{r1=1}^{a_{r1}^{k1}} a_{r1,f}^{CP} x_{r1,r2} + (K - k_{CP}) x_{r1}^{ECP} &= K - k_{CP} \\
\text{Subject to:} \\
\sum_{r2=r+1}^{a_{r2}^{k2}} a_{r2,f}^{FO} x_{r1,r2} + k_{CP} x_{r1}^{EFO} &= k_{CP} \\
\sum_{r2=r+1}^{a_{r2}^{k2}} - \sum_{r2=r+1}^{a_{r2}^{k2}} x_{r1,r2} &= 0
\end{align*}
\]

\[
\forall (k1,k2) : k1 \in \{1, \ldots, k_{CP}\} \\
k2,k2' \in \{k_{CP} + 1, \ldots, K\} : k2 \neq k2' \\
r1 \in \{r^{k1}, \ldots, r^{k1+1} - 1\}
\]
As mentioned above, this model already considers roster combinations \((r_1, r_2)\). Here, operational roster cost and team change penalties are processed simultaneously within the objective function (4.5.1), whereas the second part summarizes all unassigned SFAs. A special characteristic of this modeling approach is the fact that every selected captain roster of the solution is combined with all selected first officer rosters of the solution. As a consequence, in order to remain consistent with the objective value of the Extended Rostering Model, all operational cost factors for each captain roster \(c_{r_1}\) are divided by the number of first officers \(K - k_{CP}\), the same for first officer roster cost and the utilization of the accordingly assessed identity matrix for unassigned SFAs.

All constraints satisfy the consistency of the chosen solution: Out of every \((CP, FO)\) combination exactly one corresponding roster combination \((r_1, r_2)\) has to be selected by (4.5.2). That is the reason why in (4.5.3) all captain SFAs have to be assigned exactly as often as there are first officers in the model. (Every SFA is still covered exactly once by a single captain; but (as there are \(K - k_{CP}\) combinations) every SFA needs to be covered as often as first officers are available.) In (4.5.4) all SFAs for first officers are treated accordingly.

In the solution a set of roster combinations is selected; each roster combination implies that a specific captain executes a selected roster \(r_1\), the same for a designated first officer with \(r_2\). Since all possible roster combinations among captains and first officers are considered, restriction (4.5.5) ensures that the chosen captain roster is selected within all other chosen roster combinations among this captain and all other first officers; whereas restriction (4.5.6) assured the equivalent treatment for the determined roster of each first officer.

In Fig. 4-3 the structure of the Roster Combination Model is illustrated. Every column represents a roster combination \((r_1, r_2)\) for each possible \((CP, FO)\)
cockpit team followed by columns that handle unassigned SFAs (similar to the Extended Rostering Model via the multiplied identity matrix). The first row shows the objective values of each column which are built as the combination of operational cost for the two rosters together with team change penalties, see above.

As an example for \((CP1 \ R2, \ FO \ R2)\) as given by the roster combination \((R2, \ R8)\) the objective value is calculated by

\[
\tilde{c}_{2,8} = \tilde{c}_2 + \tilde{c}_8 + c_{2,8} = \frac{c^{R}_2}{K-k^{CP}} + \frac{c^{R}_8}{k^{CP}} + c_{2,8} = \frac{4}{4-2} + \frac{5}{4-2} + 2 = 2 + 2.5 + 2 = 6.5.
\]

Restrictions (4.5.2) to (4.5.4) are realized in each block of rows, while the synchronous arrangements in the lower half of the figure implement the set of restrictions for (4.5.5) and (4.5.6) for a consistent selection of roster combinations in the solution. Since this and the previous formulation can be considered to be equal, the optimal roster set in the solution remains the same. As a result they share the optimal objective value given by

\[
\tilde{c}_{2,9} + \tilde{c}_{2,12} + \tilde{c}_{6,9} + \tilde{c}_{6,12} = 4 + 6.5 + 4 + 6.5 = 21
\]
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<th>CP2, F02</th>
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4.6 MIP Formulation

As discussed in subsection 2.3.2, in order to solve an IP, its LP formulation is solved first leading to a lower bound of the optimal IP solution. The LP relaxation of the Extended Rostering Model (see section 4.4) typically includes many fractional variables, as shown in Fig. 4-4, so that usually an extensive branch-and-bound phase is required.

![Figure 4-4: Partial solution of LP relaxation on BIP formulation](image)

As a matter of fact, in the application proposed most decision variables appear with fractional values in the LP solution, e.g., crew member CP16 receives five rosters encoded as 00000, 01121, 01196, 02930, and 04985, see Fig. 4-4. In the optimal solution of the LP relaxation such fractional rosters accumulate to one for each crew member as required in (4.4.2). The full coverage of each SFA as defined in (4.4.3) and (4.4.4) is achieved as well. The equation (4.4.1) allows selecting rosters in such a way that there is no team change penalty encountered (as indicated by the cryptic encoding below the CP rosters, e.g., 000gjer8h and all lines below with the value 0.00000 assigned).

Thus, in the formulation (4.4.1) - (4.4.10) it does not cost anything to divide a roster in several pieces and combine it with pieces of different rosters of employees belonging to another required group. This means that the consideration of penalties is not satisfactory in the LP solution, leading to a very long branch-and-bound phase, so that mid-size or large practical problems cannot be solved to optimality, if at all. The penalty consideration in (4.4.1) is hence correct for the IP model but it does not help in finding useful LP relaxations.
Another way to model penalties is the following:

\[ x_{r_1} + x_{r_2} - 2x_{r_1, r_2} \leq 0 \quad \forall r_1 \in \{1, \ldots, r^{CP}\} \]

\[ \forall r_2 \in \{r^{CP} + 1, \ldots, R\} \]

The idea behind the constraint in (4.6.1) is to penalize roster combinations by the average of their occurrence. Exchanging (4.4.5) by (4.6.1), the solution of the refined model sets almost all teaming cost decision variables to 0.5, whereas in addition the decision variables representing rosters are still set to fractional values. However, team change penalties are applied incorrectly, because in the frequently occurring case of a roster combination in which one roster is selected and the other is not, the corresponding team change penalties variable equals to 0.5. It has to be set to 0 though, because no team change penalties should arise between these two rosters.

In the following, a model extension is introduced which retains the consideration of penalties in the LP solution according to (4.6.1), but it models them correctly in the case that a roster combination is selected in the optimal solution. This is ensured through additional logical rules to be applied only in cases that a combination is selected for the solution. Starting point of their deduction is the summary in Table 4-1 which demonstrates the cases where the team change penalty is required.

<table>
<thead>
<tr>
<th>Roster combination case</th>
<th>r1 is chosen</th>
<th>r2 is chosen</th>
<th>Team change penalty should occur</th>
</tr>
</thead>
<tbody>
<tr>
<td>(I)</td>
<td>False</td>
<td>False</td>
<td>False</td>
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<tr>
<td>(II)</td>
<td>True</td>
<td>False</td>
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<tr>
<td>(III)</td>
<td>False</td>
<td>True</td>
<td>False</td>
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<tr>
<td>(IV)</td>
<td>True</td>
<td>True</td>
<td>True</td>
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Since constraint (4.6.1) violates especially the penalty occurrence cases (II) and (III), the following additional binary indicators are introduced:
$y_{r1}$ equal 1, if roster $r1$ is chosen.

$y_{r2}$ equal 1, if roster $r2$ is chosen.

$y_{r1,r2}$ equal 1, if team change penalties should be applied to a roster combination $(r1, r2)$; in other words: $y_{r1,r2}$ is set to 1, only if both rosters of the examined combination are chosen in the solution. This can be formulated as the logic constraints below:

$$y_{r1} \wedge y_{r2} \Rightarrow y_{r1,r2}$$

Incorporating this logic constraint into (4.6.1) prevents the misleading occurrence of team change penalties. Therefore, the new constant $M$ is included which relaxes the constraint for all cases where there is no penalization necessary:

$$x_{r1} + x_{r2} - 2x_{r1,r2} + My_{r1,r2} \leq M \quad \forall r1 \in \{1, ..., r^{CP}\}$$

$$\forall r2 \in \{r^{CP} + 1, ..., R\} \quad (4.6.2)$$

Furthermore, a set of seven additional restrictions becomes necessary which assures that all indicators $y_{r1}$, $y_{r2}$, $y_{r1,r2}$ are set in the right way. For $\forall r1 \in \{1, ..., r^{CP}\}$, $\forall r2 \in \{r^{CP} + 1, ..., R\}$ it is defined:

$$x_{r1} - y_{r1} \leq 0 \quad (4.6.3)$$

$$y_{r1} - Mx_{r1} \leq 0 \quad (4.6.4)$$

$$x_{r2} - y_{r2} \leq 0 \quad (4.6.5)$$

$$y_{r2} - Mx_{r2} \leq 0 \quad (4.6.6)$$

Restrictions (4.6.3) and (4.6.4) make sure that when $x_{r1}$ is greater than 0, $y_{r1}$ is set to 1, otherwise 0. The same effect is achieved for $x_{r2}$ and $y_{r2}$ through restrictions (4.6.5) and (4.6.6) accordingly.
Chapter 4 – Model Formulations for Team-oriented Rostering

\[-y_{r_2} + y_{r_1,r_2} \leq 0 \quad (4.6.7)\]
\[-y_{r_1} + y_{r_2} + y_{r_1,r_2} \leq 1 \quad (4.6.8)\]
\[y_{r_1} + y_{r_2} - y_{r_1,r_2} \leq 1 \quad (4.6.9)\]

Restrictions (4.6.7) to (4.6.9) set \( y_{r_1,r_2} \) to the value corresponding to the chosen rosters of a combination as indicated by the variables \( y_{r_1}, y_{r_2} \). The correctness is easy to see as given for all four cases in Tab. 4-2.

<table>
<thead>
<tr>
<th>Case</th>
<th>Indicators</th>
<th>Restriction</th>
<th>Impact</th>
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<tbody>
<tr>
<td>(I)</td>
<td>( y_{r_1} = 0 ) ( y_{r_2} = 0 )</td>
<td>( 0 + y_{r_1,r_2} \leq 0 ) ( 0 + 0 + y_{r_1,r_2} \leq 1 ) ( 0 + 0 - y_{r_1,r_2} \leq 1 )</td>
<td>Influence, ( y_{r_1,r_2} = 0 ) none</td>
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<td>(II)</td>
<td>( y_{r_1} = 1 ) ( y_{r_2} = 0 )</td>
<td>( 0 + y_{r_1,r_2} \leq 0 ) ( -1 + 0 + y_{r_1,r_2} \leq 1 ) ( 1 + 0 - y_{r_1,r_2} \leq 1 )</td>
<td>Influence, ( y_{r_1,r_2} = 0 ) none</td>
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<tr>
<td>(III)</td>
<td>( y_{r_1} = 0 ) ( y_{r_2} = 1 )</td>
<td>( -1 + y_{r_1,r_2} \leq 0 ) ( 0 + 1 + y_{r_1,r_2} \leq 1 ) ( 0 + 1 - y_{r_1,r_2} \leq 1 )</td>
<td>none Influence, ( y_{r_1,r_2} = 0 ) none</td>
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<tr>
<td>(IV)</td>
<td>( y_{r_1} = 1 ) ( y_{r_2} = 1 )</td>
<td>( -1 + y_{r_1,r_2} \leq 0 ) ( -1 + 1 + y_{r_1,r_2} \leq 1 ) ( 1 + 1 - y_{r_1,r_2} \leq 1 )</td>
<td>none none Influence, ( y_{r_1,r_2} = 1 )</td>
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To conclude, the resulting MIP model is presented below. The objective function and all other restrictions remain unchanged and hence can be taken from the LP relaxation of the Extended Rostering Model in section 4.4. By this all decision variables are continuous, see (4.6.14) to (4.6.16), while indicators are defined as 0/1 variables in (4.6.17).
\[ \min \sum_{r=1}^{R} c_r x_r + \sum_{r=1}^{R} \sum_{r=2=r_{CP}+1}^{R} c_{r,r_2} y_{r_1,r_2} + \sum_{f=1}^{F} c^F (x_f^{ECP} + x_f^{EFO}) \]  \hspace{1cm} (4.6.10)

Subject to:

\[ \sum_{r=r^r-1}^{r^r} x_r = 1 \quad \forall k \in \{1, \ldots, K\} \]  \hspace{1cm} (4.6.11)

\[ \sum_{r=1}^{r_{CP}} a_{r,f}^{CP} x_r + x_f^{ECP} = 1 \quad \forall f \in \{1, \ldots, F\} \]  \hspace{1cm} (4.6.12)

\[ \sum_{r=r^r}^{R} a_{r,f}^{FO} x_r + x_f^{EFO} = 1 \quad \forall f \in \{1, \ldots, F\} \]  \hspace{1cm} (4.6.13)

\[ x_{r_1} + x_{r_2} - 2y_{r_1,r_2} + My_{r_1,r_2} \leq M \quad \forall r_1 \in \{1, \ldots, r^{CP}\} \]
\[ \quad \forall r_2 \in \{r^{CP} + 1, \ldots, R\} \]  \hspace{1cm} (4.6.2)

\[ x_{r_1} - y_{r_1} \leq 0 \quad \forall r_1 \in \{1, \ldots, r^{CP}\} \]  \hspace{1cm} (4.6.3)

\[ y_{r_1} - Mx_{r_1} \leq 0 \quad \forall r_1 \in \{1, \ldots, r^{CP}\} \]  \hspace{1cm} (4.6.4)

\[ x_{r_2} - y_{r_2} \leq 0 \quad \forall r_2 \in \{r^{CP} + 1, \ldots, R\} \]  \hspace{1cm} (4.6.5)

\[ y_{r_2} - Mx_{r_2} \leq 0 \quad \forall r_2 \in \{r^{CP} + 1, \ldots, R\} \]  \hspace{1cm} (4.6.6)

\[ -y_{r_2} + y_{r_1,r_2} \leq 0 \quad \forall r_1 \in \{1, \ldots, r^{CP}\} \]
\[ \quad \forall r_2 \in \{r^{CP} + 1, \ldots, R\} \]  \hspace{1cm} (4.6.7)

\[ -y_{r_1} + y_{r_1,r_2} \leq 1 \quad \forall r_1 \in \{1, \ldots, r^{CP}\} \]
\[ \quad \forall r_2 \in \{r^{CP} + 1, \ldots, R\} \]  \hspace{1cm} (4.6.8)

\[ y_{r_1} + y_{r_2} - y_{r_1,r_2} \leq 1 \quad \forall r_1 \in \{1, \ldots, r^{CP}\} \]
\[ \quad \forall r_2 \in \{r^{CP} + 1, \ldots, R\} \]  \hspace{1cm} (4.6.9)

\[ x_r \in [0,1] \quad \forall r \in \{1, \ldots, R\} \]  \hspace{1cm} (4.6.14)
\[ x_{r_1,r_2} \in [0,1] \quad \forall r_1 \in \{1,\ldots,r^{CP}\} \quad \forall r_2 \in \{r^{CP}+1,\ldots,R\} \quad (4.6.15) \]

\[ x_f^{ECP}, x_f^{EFO} \in [0,1] \quad \forall f \in \{1,\ldots,F\} \quad (4.6.16) \]

\[ y_{r_1}, y_{r_2}, y_{r_1,r_2} \in \{0,1\} \quad \forall r_1 \in \{1,\ldots,r^{CP}\} \quad \forall r_2 \in \{r^{CP}+1,\ldots,R\} \quad (4.6.17) \]

If \( c_{r_1,r_2} < 0 \), then include also

\[ x_{r_1,r_2} \leq x_{r_1} \quad \forall r_1 \in \{1,\ldots,r^{CP}\} \quad \forall r_2 \in \{r^{CP}+1,\ldots,R\} \quad (4.6.18) \]

\[ x_{r_1,r_2} \leq x_{r_2} \quad \forall r_1 \in \{1,\ldots,r^{CP}\} \quad \forall r_2 \in \{r^{CP}+1,\ldots,R\} \quad (4.6.19) \]

The constraints (4.6.2) can also be only activated for all \( x_r \) variables which have fractional values in the LP solution. In such a way the model tries to avoid fractional values of \( x_r \), because (4.6.3), (4.6.5) and (4.6.8) imply that the full penalty is imposed for each fractional \( x_r \) value.

The MIP model introduced in this section can be implemented with a branch-and-cut-scheme, thus adding the seven new cuts for one combination with fractional variables after each other, see also subsection 5.3.1.

### 4.7 Model Comparison

In section 4.2 a reference model for the classical rostering was set up. The basic CRP model covers two crew functions independently, such that team changes are not considered. The model size with regard to columns (as rosters) is constituted by the number of rosters and the number of SFAs which may be left uncovered for both crew functions. As a result the term \( R + 2F \) expresses all necessary columns. The corresponding set of rows is \( K + 2F \) with one row for each crew member and SFA of each crew function.

The first model that deals with the ToRP requirements is the MIQP formulation given in 4.3. Here, team change penalties are considered in the objective function via an external QMatrix. The model itself does not change in size compared to the one in 4.2. However, due to the high computational efforts for solving quadratic programs, especially MIQP, there are still limitations regarding the size of the restrictively defined matrix. Also negative entries may easily lead to its infeasibility.
The next model is the Extended Rostering Model formulated as a binary IP model in section 4.4. It penalizes each roster combination considered by additional columns and rows. The number of those possible combinations increases dramatically with regard to the number of crew members and their rosters, see subsection 3.3.1. The implementation of all possible combinations outranges rather soon the computable limitations for generating and especially solving the model. Therefore, it is important to choose an appropriate penalization strategy which results in a comparably small subset of possible roster combinations with \( c_{r_1 r_2} \neq 0 \), and by this, only few additional columns and rows in the model. Regarding the model size, such roster combinations require (in addition to the model presented in 4.2) a single additional restriction (row) in order to be applied properly, but in case of negative penalties, two further rows become necessary which may lead to a tremendous growth of the amount of rows for the model. For this reason the model size increases almost proportionally to the number to penalized roster combinations.

On the other hand, the Roster Combination Model in section 4.4 considers team change penalties simultaneously with operational cost. Since the second BIP-model builds explicitly all possible roster combinations, its proposed size remains hence fixed, such that it is independent from the chosen penalization strategy. For comparably small instances where all \( c_{r_1 r_2} < 0 \) (as the worst case for the Extended Rostering Model), this model demonstrates great advantages because the identical problem can be expressed by a much smaller model, e.g., for an instance of thirteen SFAs with five captains with a sum of 763 rosters and six first officers with totally 468 rosters, both models are almost equal with respect to the number of columns (around 350,000), but the Extended Rostering Model requires more than 1 million rows whereas all restrictions of the Roster Combination Model demand only around 5,700 rows. The maximum number of rows required equals the number (CP, FO) combinations \( k_{CP} (K - k_{CP}) \) plus a row for each CP roster which is combined with all first officers \( r_{CP} (K - k_{CP} - 1) \), and vice versa for all first officer rosters \( (R - r_{CP})(k_{CP} - 1) \).

The final MIP model in section 4.6 is formed out of the LP relaxation of the Extended Rostering Model. In order to achieve a feasible solution to the ToRP, a set of indicators for each roster and for each roster combination is required, which needs to be combined by seven additional restrictions (rows) for each roster combination. Once this model formulation is fully implemented, the resulting model size is comparatively huge by what already the model generation becomes mostly impossible for real-life instances. However, this modeling approach will be considered again in the later section 5.3 where a basic branch-and-cut procedure is applied.
A further practical requirement is downgrading (see section 3.1.3) where for the cockpit crew a captain operates one or multiple SFAs in the function of a first officer. For all modeling approaches which are based on the basic rostering model (such as the Extended Rostering Model and its MIP formulation) the realization of downgrading is comparatively easy to implement by inserting additional columns: Any additional valid roster is created in such a way that a subset of the included SFAs are shifted to the position of first officer SFAs. Solvability is not endangered by this action since the former roster (without any downgraded SFAs) is still in the model. But in order to consider now also team changes among two captains (CP, CP'), several modifications become necessary for the range of the sums in the objective function and the affected restrictions of the models in section 4.4 and 4.6. For the more compressed Roster Combination Model it is very hard to realize downgrading without restructuring the complete formulation while the QMatrix proposed for the MIPQ model already handles all possible combinations among crew members, independently from their job position. An overall comparison of all modeling approaches and their characteristics is given in Table 4-3.
### Table 4-3: Comparison among modeling approaches for the ToRP

<table>
<thead>
<tr>
<th>Model Characteristic</th>
<th>Basic rostering model for two crew functions (Section 4.2)</th>
<th>MIQP (Section 4.3)</th>
<th>Extended Rostering Model (Section 4.4)</th>
<th>BIP</th>
<th>Roster Combination Model (Section 4.5)</th>
<th>MIP (Section 4.6)</th>
</tr>
</thead>
</table>
| **Basic idea**       | • Reference model while team change penalties are NOT considered  
                      • Columns represent individual rosters  
                      • QMatrix handles team change penalties  
                      • Columns represent rosters and penalized combinations  
                      • Separation of operational cost and penalty  
                      • Columns represent roster combinations  
                      • Aggregation of operational cost and penalty  
                      • Modified LP relaxation of Extended Rostering Model with indicator variables  
                      • See Extended Rostering Model |
| Maximum number of columns | \( R + 2F \)  
<br>(\(+ n\)-squared QMatrix)  
<br>\(? c_{1,2} \neq 0 \)  
<br>\( R + r^{CP}(R - r^{CP}) + 2F \)  
<br>\( r^{CP}(R - r^{CP}) + 2F \)  
<br>For \( c_{1,2} \neq 0 \)  
<br>\( 2F + 2r^{CP}(R - r^{CP}) + 2F \) |
| Maximum number of rows | \( K + 2F \)  
<br>(\(+ n\)-squared QMatrix)  
<br>\( K + 2F \)  
<br>\(? c_{1,2} < 0 \)  
<br>\( K + 2F + 3r^{CP}(R - r^{CP}) \)  
<br>\( K^{CP}(K - K^{CP}) + 2F \)  
<br>\( K^{CP}(K - K^{CP}) + 2F \)  
<br>For \( c_{1,2} < 0 \)  
<br>\( K + 2F + 10r^{CP}(R - r^{CP}) \)  
<br>\( K + 2F + 10r^{CP}(R - r^{CP}) \) |
| Model growth due to teaming consideration | --- (not applied)  
<br>Fixed, because of the \( n\)-squared QMatrix  
<br>Per penalized roster combination:  
<br>• For \( c_{1,2} = 0 \),  
<br>1 column, 1 row  
<br>• For \( c_{1,2} < 0 \),  
<br>2 additional rows  
<br>Fixed, because penalties are aggregated together with operational cost  
<br>Per penalized roster combination:  
<br>• See Extended Rostering Model  
<br>• Additionally, 1 column for roster and their combination, 7 rows. |
| Application of negative penalties | --- (not applied)  
<br>Only yes, if QMatrix still positive semidefinite  
<br>Yes, but two additional rows are required per penalized roster combination  
<br>Yes  
<br>Yes, but two additional rows are required per penalized roster combination |
| Application of downgrading | Yes  
<br>Yes, all possible combinations are directly considered  
<br>Yes, but modification of index ranges becomes necessary  
<br>No, model structure is not capable for this  
<br>Yes, but modification of index ranges becomes necessary |
| Decisive disadvantages | Impractical since model ignores ToRP requirements  
<br>Very slow solving and limitations on number of decision variables  
<br>Model size remains greatly influenced by penalizing strategy  
<br>Impractical model sizes due to explicit enumeration of roster combinations  
<br>Full model is inferior to Extended Rostering Model in terms of model size and solving times |
4.8 Summary

In the previous sections of this chapter different model formulations for the Team-oriented Rostering Problem are presented. Therefore, a common notation was introduced. It is followed by a basic model for teams based on crew pairs at the example of cockpit crews.

Due to the fact that the ToRP requires the direct evaluation of decision variables chosen, here: roster combinations, the nature of the problem is hence quadratic. A corresponding quadratic assignment problem is offered.

Furthermore, two binary integer programming formulations are discussed: The Extended Rostering Model and the Roster Combination Model. The first is a clear extension to the basic CRP model for two crew functions, where team changes are penalized as they occur among the roster combination examined. The second combines those penalties and operational cost since it directly incorporates roster combinations instead of single rosters.

The last alternative model formulation applies mixed-integer variables, which requires further indicators for the proper penalty consideration.

With respect to the model comparison in the previous section, it has to be acknowledged that the Extended Rostering Model is by far the best alternative to conduct the further solving process for the Team-oriented Rostering Problem. As mentioned above, the main drawback of its applicability still remains the high dependence on carefully chosen penalization strategies, see subsection 3.3.2. In cases when all possible roster combinations are penalized, it remains unsolvable due to the model’s total size in terms of several millions of columns and rows. Besides, also the model characteristics may impact the solvability: The Roster Combination Model is much harder to compute, since the roster combination dependencies are tighter in comparison with those associated with the Extended Rostering Model.

As described earlier in subsection 2.1.2, crew scheduling problems are widely known as NP hard problems. The same is as well valid for the dedicated model formulations introduced in the subsections of this chapter. In the following chapter 5, several approximation concepts and their combination are presented for the Extended Rostering Model in order to avoid impractically huge model sizes that occur even for regular business instances.
Chapter 5
Solution Approaches
for Team-oriented Rostering

While in the preceding chapter 4 several dedicated mathematical models on the Team-oriented Rostering Problem were presented and analyzed, this chapter elaborates several ways on how to approach the solution process for real world problems of this type.

Since the greatest effort on the CSP in the OR literature has been spent on mathematical programming, this chapter addresses solving the Team-oriented Rostering Problem by several methods and techniques of this domain as presented in section 2.3.2. In order to overcome the great number of considerable combinations, e.g., implied by the Extended Rostering Model (see section 4.4), several approximation concepts and their combination will be presented in the following sections. It is often the case that several, at first sight, “unfavourable” rosters and their combinations are required to obtain a good overall solution; however, it is usually not practical to enumerate all of them or choose the right ones a priori. Therefore, the resulting approximation models do not fully grant optimality with regard to the original complete scheduling instance. Also appropriate penalization strategies proposed in section 3.3.2 have to be paid attention to.

Please note again that the number of flights of the instances chosen is not important here for the evaluation with regard to the applicability of the described ToR approaches. This is due to the fact that the flights considered are aggregated in blocks of shared flight activities (in analogy to pairings for the conventional CRP), see section 3.3.3.

As mentioned before, instances of regular size (e.g., with around 200 cockpit members) – sometimes even much smaller ones – may already impose this many restrictions, and especially combinatorial complexity. As a result, the application of most models compared in section 4.7 turns out to be unsolvable and therefore unrealistic to be considered. Consequently, this chapter is structured as follows: In section 5.1 the challenge of handling a huge number of legal ros-
ters is addressed. Therefore, a generation algorithm and its filtering extensions are presented. In the subsequent section 5.2 a concept for the reduction of roster combinations is proposed. So far, both sections end with some computational results based on the setting of a European tourist airline. In section 5.3 several elaborated concepts are discussed: The first aims at the reduction of the huge amount of roster combination constraints via the application of a basic branch-and-cut procedure. Further aggregation concepts and extensions for the rostering of the cabin crew, as well as a procedure for the entire scheduling process over multiple crew functions are introduced. The chapter ends with a summary in 5.4.

5.1 Reduction of Rosters

First of all, before the actual optimization for the Team-oriented Rostering is initiated, the set of crew member-specific valid rosters has to be created. In order to fully guarantee that the optimal solution can be found, all such rosters should be included which turns out to be impossible even for comparatively small CSP instances.

As already described in subsection 2.3.2, a column generation approach is applied quite frequently for the classical Crew Rostering Problem. Due to the fact that the Team-oriented Rostering Problem has to consider roster combinations explicitly, a standard column generation approach may easily turn out to deliver inappropriate columns, since there is no one-dimensional evaluation scale (like reduced cost) applicable for a roster anymore. This results in, e.g., rosters characterized by a high team orientation with respect to some of the existing ones in the model, but there is no guarantee that exactly one among those combinations is chosen afterwards. Nevertheless, the realization of a customized CG approach for the ToRP appears to be an interesting research direction, as proposed again in the outlook in section 7.2.

Instead, this section concentrates on the roster generation process on the one hand, and its pre-selection options on the other hand: In subsection 5.1.1 an underlying algorithm is presented, followed by multiple conventional roster elimination measures in subsection 5.1.2. This section ends with some computational results based on the data from our business partner in 5.1.4.
5.1.1 Roster Generation and Pre-Selection Algorithm

For the algorithm implemented some formal notations are given in analogy to those in section 4.1. It defines the set of necessary variables prior to the algorithm description.

Notation

Let

\[ F \] represent the set of all shared flight activities (SFA) \( f \in \{1, \ldots, |F|\} \) to be serviced, see subsection 3.3.2,

\[ K \] indicate the set of all of crew members, with \( K^{CP} \subseteq K \) as the set of captains, and \( K^{FO} \subseteq K \) for the first officers, as disjoint subsets of \( K \) \((K^{CP} \cap K^{FO} = \emptyset)\),

\[ T \] be the considered time period with \( t = \{1, \ldots, |T|\} \) for the set of days,

\[ R \] express the set of all valid (legal) rosters within \( T \), and

\[ Q \] represent the set of rules to be obeyed by \( Q = Q^l \cup Q^p \). It can be distinguished between rules imposed by law \( Q^l \subseteq Q \), such as governmental work rules and union agreements (see subsection 2.1.4), and planner rules \( Q^p \subset Q \), by which so far legal rosters are excluded due to more restrictive filtering as it may be intended by the responsible decision makers.

Furthermore, let

\[ R^k \subseteq R \] define the subset of valid rosters for crew member \( k, k \in K \), called the individual roster set of \( k \),

\[ F^k \subseteq F \] be the subset of all SFAs which are compatible to \( Q \) for the roster generation of crew member \( k, k \in K \), (forming the combination basis for \( k \)'s valid rosters),

\[ j \in F^k \] denote a SFA \( j \) of the combination basis of crew member \( k \),

\[ F_j^k \subseteq F^k \] be the set of SFAs that are still applicable for crew member \( k \) when the assignment of \( j \in F^k \) already took place, and

\[ F_{\text{Assigned}}^k \subseteq F^k \] represent the set of SFAs that are already assigned to crew member \( k \).
Algorithm Description

The resulting rostering task can then be verbally explained as follows: A set of valid rosters $R^k$ for crew member $k$ has to be generated with regard to his or her individual roster combination basis $R^k$, together with the applied rule sets in $Q$. (Due to the fact that $Q^k$ is already fixed by law, only additional planner rules in $Q^p$ may show their influence during the roster generation.)

To conclude, the number of elements of the individual roster set is given by function $f$ which depends on the previously introduced parameters.

$$|R^k| \rightarrow f(|F^k|, Q^p).$$

The roster generation can be executed via the preceding recursive algorithm. (It is proposed in analogy to the approach presented in (Kohl and Karisch 2004) which is applied by Carmen Systems.):

```plaintext
createRosters (k, j, FAssigned)
  add (j, FAssigned)
  r = newRoster (FAssigned)
  addSchedAct (k, r)
  addProcAndHotel (k, r)
  if ( isAcceptable(r, Q^k) )
    { if ( isAcceptable(r, Q^p) )
      add (r, R^k)
    else
      delete (r)
    for ( ∀i ∈ F^k )
      createRosters (i, FAssigned)
  } else
    delete (r)
  remove(j, FAssigned)
```

The initial call of the function `createRosters` starts with $j = \text{null}$ and $F^{\text{Assigned}} = \emptyset$ for a given crew member $k$. By this, a null-roster is generated, see subsection 2.1.2, which remains the basis for the upcoming roster generation process. The `add`-function includes the parameter value of SFA $j$ in the set of already as-
signed SFAs in $F_{\text{Assigned}}$ for which a new roster is built. By the functions $\text{addSchedAct}$ and $\text{addProcAndHotel}$, it is assured that all hotel and transit activities are included where and when they are necessary for the examined crew member. This has to be done prior to the roster validity check, but also in order to get the appropriate result for the subsequent calculations, see 5.1.2.

As a next step, the rosters compatibility to the rule set $Q$ has to be determined. It is important to distinguish whether the roster examined is already valid according to the planners criteria expressed in $Q^P$ or whether it has at least the opportunity to become a valid roster defined in $Q^L$. (The latter is achieved, e.g., by adding further SFAs — also known as the illegal sub-chain problem, see (Kohl and Karisch 2004)). If both criteria are fulfilled, this roster set is added to $R^k$, otherwise deleted.

After that, the recursive call of $\text{createRosters}$ tries to insert all potential successors of $j$ into $F_{\text{Assigned}}$, whereas on its return, the $\text{remove}$ function handles their corresponding deletion from $F_{\text{Assigned}}$.

Regarding the algorithm presented above, the rules defined in $Q^L$ prevent the occurrence of all illegal rosters, and therefore, the total number of potential rosters is greatly declined. But a more restrictive setting of $Q^L$ could turn out to be necessary for solving larger instances as discussed in the subsequent section 5.1.2. It has to be set prior to the generation process.

### 5.1.2 Conventional Roster Evaluation Measures

It is obvious that the roster pre-selection that takes place to filter so-called “good” rosters out of valid ones may greatly influence the quality of the solution. Especially when this elimination step turns out to be too restrictive, the likelihood of excluding parts of the optimal solution increases significantly. Because of the extremely high number of possible rosters, the usage of a variety of pre-selection strategies remains mostly unavoidable; however, they have to be chosen very carefully.

There are basically three common types of roster evaluation measures:

- **Cost-based measures**,
- **Frequency-based measures**, and
- **Quality-based measures**.
Cost-based measures

Each roster is assigned with its corresponding operational cost (value) in terms of hotel expenses and crew’s (per seat) transfer cost, see section 4.1. When restricting the amount of rosters considered with respect to the implied cost, a variety of measures are available, such as:

- *Upper limit for crew transfer cost*: Applying only cost imposed by crew transits helps to avoid transferring a crew member over long (expensive) distances within the airline’s network and/or to prevent frequent crew transits during the rostering period from or to another airport base outside of his or her own home base. However, setting this value very low may disable crew transfers even if such transits would make sense, and, thus, this easily leads to a temporary lack of personnel for a certain (time, location)-dependency. Therefore, the upper limit should cover at least the cost for a single pre-transit and a single post-transit from or to the surrounding airports, if necessary. For instance, by this an external shared flight activity or pairing, etc. can be served that is originally pre-assigned to the crew of another home base during the pairing generation phase of a multiple home base setting.

- *Upper limit for hotel expenses*: Only overnight cost is considered for this roster evaluation. The difficulty for this measure is given by a scenario which has to allow overnights required even at expensive locations, as well as to restrict the number of hotel stays at cheaper locations. For similar hotel rates, frequency-based measures (see below) appear also to be sufficient, otherwise a combination cost-based and frequency-based measures ensures a more appropriate pre-selection among the legal rosters.

- *Upper limit for a roster’s total operational cost*: The basic idea behind this measure is the fact that from the optimizer’s point of view the likelihood of a high cost roster to be found in the optimal solution is very low. Following this type of measure, in general, there are three types of rosters to be found:

  1. No or low transfer expenses — no or low hotel expenses,
  2. High transfer expenses — no or low hotel expenses, and
  3. No or low transfer expenses — high hotel expenses.

As given above, an upper limit for the operational cost of each roster restricts the maximum for transit expenses as well as the maximum of hotel expenses, e.g., enabling higher transit cost also requires setting higher operational cost respectively.
On the one hand, for the general crew scheduling problem, the optimizer aims to find a cost minimal solution, favouring rosters with comparatively low cost. But on the other hand, the team-oriented rostering approach can also require slightly more crew transits and hotel stays in order to avoid unnecessary team changes. For that reason, the executing planner has to estimate and/or evaluate the impact of the upper limit values since they are decisive with regard to the set of rosters pre-selected for the subsequent optimization procedure. As described above, even “unfavourable” rosters are sometimes necessary in order to obtain the optimal (occasionally even feasible) solution for the problem instance examined.

**Frequency-based measures**

In addition to the cost perspective on the roster evaluation, each roster can also be evaluated with respect to certain quantitative criteria, such as the amount of hotel stays and/or crew transits for the crew member examined. Most common measures are as follows:

- **Upper limit for the amount of crew transfers**: Setting a limit to the cost of crew transfers automatically restricts the number of transfers as well. However, because of practical reasons it is required from time to time that the general cost limit is set not too low, which may impose too many transfers in cases where the cost of a single crew transit is fairly low. In order to avoid this drawback, it is possible to restrict also the number of crew transits for each roster independent of the corresponding cost implications.

- **Upper limit for the amount of hotel stays**: In analogy to the crew’s transit cost, by restricting the cost for hotel stays, their occurrence is implicitly limited. But sometimes a certain hotel contingent becomes necessary in order to receive a promising set of rosters for the later assignment, especially when the crew is not allowed to return to their home base to stay overnight, such as during a pairing or SFA which lasts for multiple days.

- **Upper limit for the roster’s total usage of transits and hotel stays**: Similar to the cost-based measures above, the combination of both quantitative criteria delivers a set of rosters which shows comparable characteristics. It has to be acknowledged that most crew transits come along with hotel stays, especially when such external multi-day-SFAs (or pairings) occur, see above.

Occasionally, a combination of frequency-based and cost-based roster evaluation measures has to be applied: As given above, especially for those cases
where expenses (independent of their origin during the operational phase) differ very much among the alternatives, setting them to very low upper limits actually prohibits their occurrence. In order to prevent this drawback, the resulting high limit of maximum cost may easily lead to a high amount of hotel stays and/or transits which are comparatively cheap. To avoid such phenomena, additional frequency-based measures are regarded to be beneficial for a better roster selection process.

Quality-based measures

Besides the pure consideration of operational cost, a variety of quality-based measures are available as well:

- Deviation limit of a crew member’s target flight hours: Individually calculated target flight hours can be applied for the roster pre-selection, e.g., in order to assure a higher degree of fair-and-equal share assignments for the personalized rostering, see also section 4.1. According to the overall workload of the period(s) examined, every member of the flight personnel should then receive a dedicated proportion of the flights, whereas the individual work history, current pre-scheduled activities, and the contractual situation are considered, too.

A crew member $k$’s target flight hours ($TFH^k_T$), $k \in K$, for time period $T$ can be calculated as the combination of the introduced factors below:

Let

- $CFH^k_T$ give the contracted flight hours (CFH) for a crew member $k$,
- $FCH^k_T$ represent the flight compensation hours (FCH) for crew member $k$, FCH is used to balance out a lack of workload induced by the pre-scheduled activities, e.g., four flight compensation hours are calculated as “virtual” flight hours for one vacation day. By this, having pre-scheduled activities fixed during the rostering period does not result in the regular (two weeks’) workload for the remaining working days,
- $FH^f$ express the flight hours (FH) of SFA $f \in \{1, \ldots, |F|\}$,
- $\gamma_T$ indicate the workload factor for all crew members $k$ over time period $T$ with

$$\gamma_T = \frac{\sum_{f \in F} FH^f + \sum_{k \in K} FCH^k_T}{\sum_{k \in K} CFH^k_T}$$
By this, $\gamma_T$ equals to the sum over all flight hours together with the sum over all flight compensation hours which is divided by the overall contracted flight hours.

After this preparation, the individual target flight hours of crew member $k$ for period $T$ ($TFH^k_T$) is the sum of his or her individual contracted flight hours ($CFH^k_T$) weighted by the workload factor $\gamma_T$ minus the flight compensation hours due to pre-scheduled activities ($FCH^k_T$), as follows:

$$TFH^k_T = \gamma_T CFH^k_T - FCH^k_T$$

A special case is the null roster: Although this roster (without any SFA its flight hours equals to 0) can show a huge deviation from the crew member’s specific target flight hours computed, it is still required since it ensures the solvability of the model, see 2.1.2 and 4.2.

Most rosters do not fit exactly to the calculated $TFH^k_T$ value. Therefore, a certain acceptance threshold has to be defined by the planner, e.g., as a deviation of three hours (absolute value) or a deviation by less than 10% (relative value). In the case of a negative amount of target flight hours ($TFH^k_T < 0$), this parameter is set to 0.

- **Acceptance limit for non-robust rosters:** Furthermore, the generated rosters may not be desirable for either the crew or the crew planers with respect to their robustness evaluation. Short connection times between SFAs are critical, especially when there is a delay. A lot of effort has to be made for the immediate recovery, see section 2.5. Therefore, a set of criteria can restrict their occurrence in the optimization model.

- **Acceptance limit of undesired flight connections and/or hotel stays:** Planners may also prefer to exclude certain SFA (or flight) combinations and/or hotel locations which were put into practice during earlier crew schedules or their generation. Due to their experience and/or crews’ request or recommendation rosters identified by those criteria should be left out. Their occurrence can also be penalized, see subsection 2.1.4.

Those additional restrictions or their combinations can be applied for $Q^P$ within pre-defined rule sets that will be explained in subsection 5.1.3. Due to the fact that instance characteristics may vary dramatically, such rule sets are not applicable in general. In other words, they should not be applied with further re-evaluation at least for some of the instances available or thinkable.

All of the above measures presented can (maybe even should) be individualized, e.g., for each home base, scheduling (sub-)period and/or crew member
according to the actual instance examined, which obviously requires experienced planners. See also section 6.2, where a prototypical realization in a decision support system is presented.

Note that – in addition to the static roster evaluation measures – it may also make sense to pay attention to the aggregated crew capacity and even more to their availability: It changes significantly for each home base over the time period being scheduled. For instance, although a crew member is able to serve one of several dozens or hundreds of SFAs on a specific day, their entire consideration in the roster generation process will generate several millions of valid rosters. They may still be acceptable with regard to the evaluation measures above, but the crew member’s individual situation (e.g., the work history, like available OFF-days that have to be assigned, and so on) should be paid attention to as well. This is emphasised because the combination basis $F^k$ can be reduced dramatically by the latter mentioned. For multi-home base environments, the situation is even more obvious: In case of a temporary lack of personnel at one home base, those SFAs being already assigned to that home base (e.g., by the CPCP in section 2.4) should be prioritized when the local crew member’s roster generation is started.

5.1.3 Computational Results

The aim of this subsection is to show the impact of the Team-oriented Rostering approach, especially with regard to the solution approaches proposed above. Since this work is based on real-life data, our business partner is briefly introduced with the most interesting characteristics. In addition, the experimental environment is explained together with some of the important attributes for the set of exemplary instances chosen. (This general part is also valid for subsection 5.2.2.)

In this subsection all tests presented were conducted only after applying all the roster reduction steps described earlier. Again, this was necessary because of the sheer outgrowing model size. In such a way those reduction steps became a prerequisite for applying the ToRP in most cases examined.

The tests conducted later on allow analyzing the impact of team change penalties, and the application of restrictive rule sets.
Our business partner

The concepts described above were successfully applied to solve real-life crew management problems. In general, they may handle medium-sized airline crew scheduling problems efficiently. All instances presented originate from a medium-sized European tourist airline, where its flight plan is executed on a mixture of a point-to-point and a hub-and-spoke type of network (in contrast to the clear hub-and-spoke structure adopted in the US, and the usual point-to-point networks in Europe, see subsection 2.1.1). Within such a network, multiple home bases are located in Germany, while many other airports are spread out around Europe. The airports outside Germany are normally those resorts which attract great amount of travellers every year. Passengers, therefore, usually spend some days at a destination and come back few days or weeks later. A simplified version of the business partner’s flight network in year 2002 is depicted in Fig. 5-1 where the home bases are given as white points, destinations as filled ones.

![Simplified flight network of our business partner](image)

Since the airline sector of our business partner is engaged in the German tourism market, and schools’ vacations are state-dependant, there is a temporary, but notable, location-based demand shift. These result in high demand seasons for the summer and the winter break based on the regional holiday regulations.
setting. Moreover, an effect of the network architecture given is that a large portion of the flights can be observed as round trips. In other words, a typical schedule for one crew member may be a trip to one city, and come back within the day if he or she has only two flights to serve. Sometimes the crew member may not fly back directly after he or she arrives at the destination, and goes somewhere else instead. This may happen, because his or her next flight heads for other places. However, it is usual that there are other flights going back to his or her departure airport, and the flight is operated by someone else.

The planning period is usually half or one month, within which the number of crew members with one crew position (e.g., first officer) ranges from around 50 to nearly 250. Depending on fleets, the number of flights involved may grow to more than 2,000 for half a month.

Optimization environment and test instances

All experiments were realized on a PC with an Intel Pentium IV, 2.26 GHz CPU with 2.0 GB RAM, operating Microsoft Windows XP Professional. The prototype is implemented with Visual C++ 6.0 and considers only newly computed valid rosters for each individual crew member. All models were solved using CPLEX, version 9.0 (see CPLEX, 2003). Time measurements are given in CPU seconds. The instances examined below are based on two typical holiday periods which represent average and high demand periods of the year 2002. (More recent data was not available due to the actual highly competitive market situation.)

Each instance below is described by the time period chosen, the number of home bases (HB), the amount of captains (CP), first officers (FO), and considered shared flight activities (SFA). Further parameters that have been considered are the maximum number of elements in the roster combination basis, the maximum number of disposable working days within the period, the chosen rule set, an indicator whether other airports are serviced, the penalty value for a single team change (TP), the number of loops regarding the step-wise rostering procedure (RL) (introduced in section 5.2), and an indicator whether downgrading is considered in this model or not.

The resulting Integer Programming models have been implemented following description of the Extended Rostering Model in section 4.4. The model characteristics include the number of rosters generated for captains (RGCP) and first officers (RGFO), the time for their generation (RGT), the number of rosters included in the model (RCP and RFO), the model size in rows and columns, the number on non-zeros (NZ), the duration for solving (ST). Furthermore, the operational cost (OC), and the number of team changes (TC) are given. (All pa-
rameters above which are not mentioned in the tables remain unchanged for all examined instances.)

Tests on Team change penalties

The consideration and appropriate setting of team change penalties are among the major aspects within the Team-oriented Rostering approach presented. Therefore a set of test runs was conducted with different penalty values for team changes (100, 200, 300, 500 and 1,000) on the same instance in comparison to the conventional approach without any penalization. The results are documented in Table 5-1.

For all instances the conventional approach (with $TP = 0$) offers the cheapest solution in terms of operational cost (OC), but with very frequent team changes. In contrast to that, a tendency for a monotonously slightly increasing operational cost for all listed ToR variants ($TP \neq 0$) is observed which can be explained by the amplified trade-off between operational cost and increasing team change penalties. As the instances proved, simply applying the Team-oriented Rostering approach manages to dramatically enhance the realization of team orientation, here by reducing the number of team changes for the new crew schedules at the expense of slightly higher operational cost. All instances were solved with the same amount of unassigned SFAs. In Table 5-1, the significant difference regarding model size between the conventional and ToR variants becomes quite obvious. It is caused by the additional columns and rows for team change penalization as discussed earlier, e.g., in subsection 4.4.
Table 5-1: Results for different team change penalties

| Period       | HB  | CP | FO | SFA | TP  | RGCP | RGFO | RGT | RCP  | RFO  | Rows | Cols | NZ  | ST  | OC  | TC  |
|--------------|-----|----|----|-----|-----|------|------|-----|------|------|------|-----|-----|-----|-----|
| Jul 1-15, 2002 | 2   | 8  | 10 | 31  | 0   | 11103| 12803| 11:06| 11103| 20   | 80   | 11185| 69232| 00:01| 4685| 10  |
|              |     |    |    |     | 100 | 34851| 45956| 208316|      |      | 01:05| 5110 | 3   | 4925| 4   |
|              |     |    |    |     | 200 |      |      |      | 01:05| 5110 | 3   | 4925| 4   |
|              |     |    |    |     | 300 |      |      |      | 01:19| 5386 | 2   | 3683| 2   |
|              |     |    |    |     | 500 |      |      |      | 03:10| 5386 | 2   | 3683| 2   |
|              |     |    |    |     | 1000|      |      |      | 01:49| 5386 | 2   | 3683| 2   |

| Period       | HB  | CP | FO | SFA | TP  | RGCP | RGFO | RGT | RCP  | RFO  | Rows | Cols | NZ  | ST  | OC  | TC  |
|--------------|-----|----|----|-----|-----|------|------|-----|------|------|------|-----|-----|-----|-----|
| Jul 1-15, 2002 | 4   | 24 | 22 | 78  | 0   | 126504| 169680| 2:36:17| 126576| 42   | 202  | 126774| 802220| 01:04| 13649| 30  |
|              |     |    |    |     | 100 | 543205| 669777| 23974232| 240:25*| 14084| 11  | 110  | 15  |
|              |     |    |    |     | 200 | 240:19*| 13875| 14090 | 13   |
|              |     |    |    |     | 300 | 240:23*| 14421| 14200 | 11   |
|              |     |    |    |     | 500 | 240:19*| 14200| 14200 | 11   |
|              |     |    |    |     | 1000| 480:24**| 14200| 14200 | 11   |

| Period       | HB  | CP | FO | SFA | TP  | RGCP | RGFO | RGT | RCP  | RFO  | Rows | Cols | NZ  | ST  | OC  | TC  |
|--------------|-----|----|----|-----|-----|------|------|-----|------|------|------|-----|-----|-----|-----|
| Jul 1-15, 2002 | 6   | 29 | 27 | 99  | 0   | 102281| 121177| 2:23:32| 103375| 52   | 254  | 103625| 629153| 00:34| 19196| 51  |
|              |     |    |    |     | 100 | 433512| 536883| 2362185| 240:15*| 19529| 34  | 19735| 32  |
|              |     |    |    |     | 200 | 240:11*| 19735| 19735 | 32   |
|              |     |    |    |     | 300 | 240:11*| 19787| 19787 | 33   |
|              |     |    |    |     | 500 | 240:11*| 19787| 19787 | 33   |
|              |     |    |    |     | 1000| 240:15*| 19603| 19603 | 32   |

| Period       | HB  | CP | FO | SFA | TP  | RGCP | RGFO | RGT | RCP  | RFO  | Rows | Cols | NZ  | ST  | OC  | TC  |
|--------------|-----|----|----|-----|-----|------|------|-----|------|------|------|-----|-----|-----|-----|
| Dec 16-31, 2002 | 6   | 33 | 26 | 44  | 0   | 14163| 10904| 20:40| 14163| 48   | 147  | 14299| 72973| 00:01| 10532| 9   |
|              |     |    |    |     | 100 | 36604| 50756| 218801| 02:01| 10828| 0   | 10828| 0   |
|              |     |    |    |     | 200 | 01:05| 10828| 0   |
|              |     |    |    |     | 300 | 01:10| 10828| 0   |
|              |     |    |    |     | 500 | 00:55| 10828| 0   |
|              |     |    |    |     | 1000| 01:25| 10828| 0   |

* Optimization was terminated after 240 minutes. Usage of the best IP-solution found.
** Optimization was terminated after 480 minutes. Usage of the best IP-solution found.
In Fig. 5-2, the decreasing amount of team changes is visualized for the different team change penalties, where two basis configurations for each first officer are considered ($RL = 2$, see subsection 5.2.1) and downgrading is enabled.

![Figure 5-2: Impact of different team change penalties](image)

Note that for the variant with $TP = 0$ the same roster reduction steps were applied in order to identify the impact of the team orientation more clearly. Due to the different aggregation level (pairings vs. shared flight activities), the calculated operational cost are not necessarily close to the optimal solution which can be found by the application of a column generation approach for the same instance with separately scheduled crew functions. This is also the reason why there is no final qualitative (precise) comparison between the conventional crew assignment and the Team-oriented Rostering approaches introduced.

**Tests on the application of restrictive rule sets**

In subsection 5.1.2 several dedicated conventional roster evaluation measures were introduced in detail. In the following, they are combined as rule sets which are given in Table 5-2. The first three measures are cost-based, the next are two frequency-based and one quality-based one as defined previously.
Table 5-2: Definition of rule sets for conventional roster evaluation

<table>
<thead>
<tr>
<th>Rule set</th>
<th>Max. total operational cost</th>
<th>Max. crew transit cost</th>
<th>Max. hotel expenses</th>
<th>Max. amount of crew transfers</th>
<th>Max. amount of hotel stays</th>
<th>Max. deviation from target flight minutes</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>400</td>
<td>200</td>
<td>200</td>
<td>2</td>
<td>2</td>
<td>200</td>
</tr>
<tr>
<td>R2</td>
<td>700</td>
<td>400</td>
<td>400</td>
<td>4</td>
<td>4</td>
<td>300</td>
</tr>
<tr>
<td>R3</td>
<td>1000</td>
<td>600</td>
<td>600</td>
<td>6</td>
<td>6</td>
<td>400</td>
</tr>
<tr>
<td>R4</td>
<td>1300</td>
<td>800</td>
<td>800</td>
<td>8</td>
<td>8</td>
<td>500</td>
</tr>
<tr>
<td>R5</td>
<td>1600</td>
<td>1000</td>
<td>1000</td>
<td>10</td>
<td>10</td>
<td>600</td>
</tr>
</tbody>
</table>

The rules sets chosen with increasing limits on cost, frequencies and qualities demonstrate their notable impact on the amount of rosters generated, but also on the solution quality of the problem instance based on the selection of remaining rosters. See Table 5-3 for the results of the first and the last instance examined earlier.

Table 5-3: Results for the application of different rule sets

<table>
<thead>
<tr>
<th>Period</th>
<th>HB</th>
<th>CP</th>
<th>FO</th>
<th>SFA</th>
<th>DG</th>
<th>RS</th>
<th>RGCP</th>
<th>RGFO</th>
<th>RGT</th>
<th>RCP</th>
<th>RFO</th>
<th>Rows</th>
<th>NZ</th>
<th>ST</th>
<th>OC</th>
<th>TC</th>
<th>UA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jul 1-15, 2002</td>
<td>2</td>
<td>8</td>
<td>10</td>
<td>31</td>
<td>true</td>
<td>R1</td>
<td>2996</td>
<td>3087</td>
<td>02:59</td>
<td>2996</td>
<td>20</td>
<td>6616</td>
<td>11614</td>
<td>52001</td>
<td>00:07</td>
<td>5073</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>R2</td>
<td>11103</td>
<td>12803</td>
<td>11:04</td>
<td>11103</td>
<td>20</td>
<td>34851</td>
<td>45956</td>
<td>208316</td>
<td>03:14</td>
<td>5386</td>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>R3</td>
<td>21527</td>
<td>32679</td>
<td>22:36</td>
<td>21527</td>
<td>20</td>
<td>77172</td>
<td>98701</td>
<td>447548</td>
<td>06:38</td>
<td>5148</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>R4</td>
<td>32018</td>
<td>55534</td>
<td>29:35</td>
<td>32018</td>
<td>20</td>
<td>120359</td>
<td>152379</td>
<td>690205</td>
<td>07:02</td>
<td>5084</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>R5</td>
<td>38673</td>
<td>69188</td>
<td>31:45</td>
<td>38673</td>
<td>20</td>
<td>149760</td>
<td>188435</td>
<td>852421</td>
<td>08:39</td>
<td>5084</td>
<td>0</td>
<td>0</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Dec 16-31, 2002</td>
<td>6</td>
<td>33</td>
<td>26</td>
<td>44</td>
<td>true</td>
<td>R1</td>
<td>2947</td>
<td>2067</td>
<td>04:13</td>
<td>3230</td>
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<td>6954</td>
<td>10169</td>
<td>43146</td>
<td>00:02</td>
<td>1408721</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>R2</td>
<td>14163</td>
<td>10904</td>
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<td>48</td>
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<td>01:11</td>
<td>10828</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>R3</td>
<td>38920</td>
<td>30246</td>
<td>46:13</td>
<td>38920</td>
<td>48</td>
<td>102820</td>
<td>141729</td>
<td>620706</td>
<td>04:14</td>
<td>10253</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>R4</td>
<td>66102</td>
<td>51125</td>
<td>57:05</td>
<td>66102</td>
<td>48</td>
<td>197306</td>
<td>263397</td>
<td>1159045</td>
<td>15:14</td>
<td>10207</td>
<td>0</td>
<td>0</td>
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</tr>
<tr>
<td></td>
<td>R5</td>
<td>88328</td>
<td>71776</td>
<td>59:50</td>
<td>88328</td>
<td>46</td>
<td>280265</td>
<td>368580</td>
<td>1626637</td>
<td>08:34</td>
<td>9957</td>
<td>0</td>
<td>0</td>
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<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Plenty of additional tests were executed and analyzed on those instances together with many other available parameters. Interestingly, so far all limitations of the individual valid combination basis $F^k$ turned out to be a quite unsuccessful approach. In other words: It is impossible to identify and eliminate further “unfavourable” rosters by the evaluation of their team orientation in terms of
their combinability to all other rosters. Especially some of those rosters are obviously required very frequently. Once the roster generation process is finished, they have to be included into the model.

Already in section 4.4, where the Extended Rostering Model was described, the problems caused by too many constraints (implying a vast amount of additional rows and columns) was highlighted. Consequently, the next two sections will address this challenge in different ways.

## 5.2 Reduction of Roster Combinations

As introduced above, the consideration of roster combinations turns out be the second challenge to deal with. Therefore, certain strategies have to be identified and tested for their deployment in the Team-oriented Rostering process. They have to reduce the number of considered roster combinations, but simultaneously, they are not allowed to interfere perceivably with the overall solution quality again.

After the analysis of several instances and reinforced by the need to solve larger problem instances, a stepwise rostering procedure turned out to be an appropriate approach. It is introduced in subsection 5.2.1, while a set of instances will demonstrate its realization by some computational results in the remaining subsection 5.2.2.

### 5.2.1 A Stepwise Rostering Procedure

The basic idea behind the great reduction of the model size achieved below can be explained as follows: Since not all roster combinations are realistic to be considered, a small subset of the rosters for at most one of the two cockpit crew functions may turn out to be beneficial. All crew members of the complementary crew function have to keep all their rosters (remaining rosters, see subsection 5.1.2) for the later combination in the Extended Rostering Model.

As a result, the crew function with the reduced roster set has to be determined. Here, downgrading shows its strengths: Since for cockpit crew only captains have the permission for downgrading, see subsection 3.1.3, the opposite crew function has to be the in the first position. (Only in this order, unassigned SFAs can be covered later on by the captains, if necessary.)

The next task is to identify those rosters for the first officers that should be inserted into the model for the later combination with all captain rosters. There-
fore, a loop is executed which solved the basic crew rostering problem, while at least parts of the solution previously found are prohibited for the next optimization run. The counter \( r_l \) is incremented until the upper bound \( RL \) is reached. All rosters found are then the pre-defined ones for all first officers during the subsequent optimization model composed for both crew cockpit functions. The corresponding flowchart of this process can be found in Fig. 5-3.

5.2.2 Computational Results

The stepwise rostering procedure introduced above is now applied on the same Team-oriented Rostering instances that were examined earlier in the subsection 5.1.3. By this, only few first officer rosters were pre-selected instead of including all of them in the model to be solved. The ToR variants chosen differ by the number of iterations (RL) which is set to values from 1 to 5. Due to the model size, only the two small instances (July, 1-15, with one home base and December, 16-31, with two home bases) have been solved without the stepwise rostering procedure as a reference for a better comparison.

Figure 5-3: Stepwise rostering procedure for cockpit crew
As shown in Table 5-4, the application of this simplified selection procedure turned out to be quite valuable for both: solution quality and model size. Instead of many hundreds or all rosters generated only those pre-defined rosters have been chosen, and the gap in terms of operational cost between their complete consideration in the model and an obviously appropriate pre-selection ($RL = 1, 2, ..., 5$) appears to be notably low. E.g., in the first July instance with one home base, the minimum of operational cost is achieved already with $RL = 2$). It has to be acknowledged that the number of team changes tend to decrease with a higher number of pre-defined rosters.

At the same time, a significant reduction of model size is accomplished of even more than 90 percent, indicated by the comparison rate (MR) expressing the proportions of model sizes with and without the application of the stepwise rostering procedure. (All instances were computed with $TP = 300$ and downgrading enabled.)

### Table 5-4: Results of stepwise rostering procedure

<table>
<thead>
<tr>
<th>Period</th>
<th>HB</th>
<th>CP</th>
<th>FO</th>
<th>SFA</th>
<th>DG</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jul 1-15, 2002</td>
<td>1</td>
<td>4</td>
<td>3</td>
<td>16</td>
<td>true</td>
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<table>
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<th>FO</th>
<th>SFA</th>
<th>DG</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jul 1-15, 2002</td>
<td>2</td>
<td>8</td>
<td>10</td>
<td>31</td>
<td>true</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Period</th>
<th>HB</th>
<th>CP</th>
<th>FO</th>
<th>SFA</th>
<th>DG</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dec 16-31, 2002</td>
<td>2</td>
<td>10</td>
<td>9</td>
<td>12</td>
<td>true</td>
</tr>
</tbody>
</table>

As shown in Table 5-4, the application of this simplified selection procedure turned out to be quite valuable for both: solution quality and model size. Instead of many hundreds or all rosters generated only those pre-defined rosters have been chosen, and the gap in terms of operational cost between their complete consideration in the model and an obviously appropriate pre-selection ($RL = 1, 2, ..., 5$) appears to be notably low. E.g., in the first July instance with one home base, the minimum of operational cost is achieved already with $RL = 2$). It has to be acknowledged that the number of team changes tend to decrease with a higher number of pre-defined rosters.

At the same time, a significant reduction of model size is accomplished of even more than 90 percent, indicated by the comparison rate (MR) expressing the proportions of model sizes with and without the application of the stepwise rostering procedure. (All instances were computed with $TP = 300$ and downgrading enabled.)
Table 5-4: Results of stepwise rostering procedure (cont.)

<table>
<thead>
<tr>
<th>Period</th>
<th>RB</th>
<th>CP</th>
<th>FO</th>
<th>SFA</th>
<th>DG</th>
<th>Rows</th>
<th>Cola</th>
<th>NZ</th>
<th>ST</th>
<th>OC</th>
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</tr>
</thead>
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</table>

In the case that the stepwise rostering procedure proposed is still not sufficient to solve the Team-oriented Rostering model, e.g., due to its extremely large size, the next section suggests further solution approaches.

5.3 Further Concepts

In this section, some additional concepts for solving the Team-oriented Rostering Problem are discussed: In subsection 5.3.1 a basic branch-and-cut procedure is introduced. A so-called Hybrid IP Model is presented in subsection 5.3.2 in which redundant roster and crew information is aggregated. The next subsection 5.3.3 focuses on several necessary extensions for handling the more complex cabin crew scenario, and the final subsection 5.3.4 deals with two alternative proceedings regarding the Team-oriented Rostering of the entire flight crew.

5.3.1 A Basic Branch-and-Cut Procedure

So far, the roster set as well as the total amount of roster combinations considered are reduced. However, following the procedures described in the previous sections of chapter 5 leads to the exclusion of “unfavourable” rosters on the one hand, as well as the avoidance on “unnecessary” roster combinations on the other. It is acknowledged that — although parameters are assumed to be chosen carefully — this has mostly a negative impact on the overall solution quality encountered.

In contrast to this, the concept presented in this section will not affect the solution quality negatively at all. Moreover, some of the necessary approximation steps above are not required any more, and even larger instances become solvable. This is due to the fact that the third challenge of dealing with the huge
amount of additional constraints for all roster combinations considered is addressed. The main drawback of this approach is the time consumed for the roster generation and the potentially high number of necessary iterations encountered during many problem instances.

Getting back to the idea of branch-and-cut as introduced in subsection 2.3.2, huge IP models can be solved by branch-and-bound schemes in conjunction with cutting planes as follows: At the beginning, a reduced LP relaxation of the original IP model with only few constraints is solved. If this becomes infeasible, a further sub-set of the constraints (out of those that were removed at the beginning of the process) is inserted again for the next optimization run. Once all constraints are satisfied, the branching on fractional values is continued with.

In order to transfer this to the solving of the Team-oriented Rostering models a similar proceeding is proposed: It is suggested to leave out all additional constraints that are imposed by the team orientation. By this, the initial model to start with is the LP relaxation of the basic rostering model for crew function pairs (or the standard CRP for two crew functions) as explained in section 4.2. The resulting solution may then indicate two major issues to work on:

1. **Team orientation is not completely considered.** Therefore, team change constraints are added to the current version of the model in order to encourage a logically (not mathematically) feasible solution, see, e.g., section 4.4.

2. **The solution is fractional.** This means that the full coverage of each SFA is ensured by a combination of several partially chosen rosters, as already encountered in the MIP model presented in section 4.6.

Then the new or the updated MIP model for the Team-oriented Rostering is solved again, until none of the two obstacles is experienced any more. As a result, branching is internally done by the optimizer, and the optimal solution is obtained after several iterations, as depicted in Fig. 5-4.

The issue presented in (2) for fractional solution values can be realized in different ways. Exemplarily, two distinct strategies are explained as follows:

1. **Change all continuous decision variables with fractional values to binary integer variables:** This approach follows the Extended Rostering Model presented in section 4.4, and the branching is enforced explicitly.

2. **Add binary indicator variables and constraints to all continuous decision variables with fractional values:** By this, a sub-set of binary indicator variables and constraints becomes necessary as proposed in the MIP model in section 4.6. For this strategy, the branching takes place more implicitly during the optimization process due the additional constraints included.
5.3.2 Aggregation of Identical Roster Sets and Potential Teams

According to the author’s experimental experience, it has to be acknowledged that – once the Team orientation feature is activated – most crew members receive exactly identical rosters. Furthermore, the majority of the staff (up to more than 70%) never experiences any team change in the final crew schedule. But by the way how the mathematical models are constructed, identical (redundant) rosters are inserted several times, and the flexibility of generating teams during the optimization process is obviously not of that high importance in order to re-
main close to the optimum. Simultaneously, this aggregation dealt with the main
drawbacks experienced earlier on the correlation between problem size and the
limitations for its computational solvability.

While the set of solution approaches discussed in the previous sections of this
chapter, a smaller but still notable advantage is neglected when it comes to the
model generation: The avoidance of redundant information.

Returning to both BIP models (the Extended Rostering Model in section 4.4 and
the Roster Combination Model in 4.5) the aggregation of identical rosters as
well as the aggregation of pre-defined teams are proposed and combined into a
Hybrid IP model that is able to prevent most of the shortcomings described for
practical instances.

Aggregation of identical rosters

In this step, identical rosters are identified ahead of their insertion into the
model. In order to avoid inconsistencies that may result from this first aggrega-
tion dimension, the following conditions have to be met:

(a1) Crew members belong to the same crew function. (If not, team
changes, downgrading, etc. could not be considered appropriately.)

(a2) Crew members are stationed at the same home base. (Base variants
with different operational cost would be ignored otherwise.)

(a3) Crew members (should) have the same (or similar) fingerprints, con-
tracted flight hours $CFH^k$, individual working history, etc. regarding the
rostering period examined. By this, their subset of valid SFAs $F^k$ is the
same, see subsection 5.1.1.

Only if all requirements from (a1) to (a3) are fully satisfied, the resulting valid
roster set $R^k$ is the same for all crew member $k$ that constitute the subgroup
examined.

Aggregation of pre-defined teams

The second aggregation dimension is applicable for (CP, FO)-teams without
team changes: The resulting requirements are the following:

(b1) Crew members are not necessarily stationed at the same home base.
(In contrast to (a2), base variants are considered as the sum over the
implied operational cost.)
(b2) Crew members (should) have the same (or similar) fingerprints, contracted flight hours $CFH^k_T$, individual working history, etc., see (a3).

Hybrid IP-model

Combining both aggregation concepts, the resulting Hybrid IP model is depicted in Fig. 5-5 by following the upcoming steps:

1. Determine and select a set of pre-defined teams according to the requirements defined above by (b1) to (b2).

2. Generate identical rosters for these cockpit teams. The coverage of comparatively difficult assignable SFAs is preferred, e.g., long-lasting ones or short connections outside the home base first.

3. Include these rosters into the model following the style of the Roster Combination Model (as shown on the left half of Fig. 5-5). Since there are no team change penalties (due to the absence of team changes), every column in this part of the Hybrid IP model represents a dedicated roster for a (CP,FO)-team and their combined operational cost.

4. Only for the remaining crew members: Determine the set of subteams that are compliant to the prerequisites of (a1) to (a3).

5. Generate and include rosters for those subteams based on the remaining SFAs by implementing the Extended Rostering Model, see also right side of Fig. 5-5. Columns represent here SFAs and operational cost for CP and FO rosters, or team change indicators and penalties, as introduced in section 4.4. Furthermore, a little coverage overlapping might be necessary in order to prevent unassigned SFAs from the first group of pre-defined teams.
The application of the Hybrid IP model introduced requires both: a customized pre-processing procedure (for the aggregation step) as well as a post-processing procedure (for the reverse task) in addition to the general solution procedure (input – processing – output).

The entire model, especially the remaining Extended Rostering Model, is much smaller than the original one. However, this approach requires a certain experience on the problem examined for setting up carefully all parameters and adjusting them appropriately. In addition, it can only be expected to perform well if the crew characteristics as well as the requested crew complements do not differ in too many details, such that a great proportion of the columns and rows is aggregated in advance of the solving process.

### 5.3.3 Extensions for Cabin Crew

When introducing the concepts for team construction, the higher degree of combinatorial freedom for cabin crew was already outlined in comparison to the emphasized and previously discussed cockpit crew setting, see subsection 3.2.3. Their adoption to the cabin crew is briefly presented. Also two cabin crew construction concepts are introduced: *micro teams*, and *macro teams*.

Quite similar concepts are recently presented in (Meehan 2005). The author calls them *spouses* (for micro teams) and so-called *POD’s* (company-specific abbreviation for macro teams). They are reported as part of the latest long haul cabin crew initiatives at Air New Zealand.
As introduced above, cockpit crews can be achieved by pre-defined teams, pre-selected teams, as well as by free team building. They can be realized for teams of a single crew function (several cabin attendants) or mixed cabin teams consisting of a CDC and/or PU and several CAs. Once such homogenous teams are formed, the resulting Team-oriented Rostering Problem among CDC or PU and CAs can be dealt with in a similar way to the one for cockpit crew. For the heterogeneous case, the setting may require one of the following cabin team construction concepts:

**Micro teams**

The basic idea of a micro team can be seen as the implementation of married couples: Mirrored lines-of-work, sometimes even subparts (e.g., a labour week of five days) are assigned to those pairs of crew members (or larger subteams of a size less or equal to the requested one) that are not planned to be separated during this time period. The complete cabin crews operating the SFAs are then formed out of the defined micro teams. Here, also individual preferences (buddy requests) can optionally be taken into consideration.

Changes in the team compositions can be evaluated by the degree of the team changes encountered, see subsection 3.2.4. For a team’s overcapacity on CDC or PU personnel the crew members in charge should be exchanged after a while by one of the downgraded team mates.

As implicitly said above, the main prerequisite for micro teams is the same (= overlapping) or at least comparatively similar pre-scheduled activity (sub-) patterns or fingerprints. In practice, only a certain percentage of crew members should be tightly linked up in advance in order to keep a higher flexibility for the subsequent rostering step.

**Macro teams**

In contrast to this, macro teams are built out of crew members with (at best) complementary pre-scheduled activities. By this a permanently high availability level is achieved within each macro team together with team composition constraints. As a result, macro teams are characterized by redundant crew capacities, since their crew supply is normally greater than (at worst: equal to) the demand requested. This becomes significantly noticeable when the teams’ usability is compared with other approaches. Regarding team orientation, changes inside the macro team with respect to the currently operating crew are not considered as team changes. (Those people of a macro team usually know each other already in advance, e.g., because they belong to the same home base.)
Nevertheless, such macro teams should be regarded as soft constrains for the rostering: Due to the time and location dependant over-capacity, exchange and/or cooperation among the macro teams defined may become economically inevitable. Also the macro team’s home base and (where applicable) the consideration of the destination regions covered by the airline examined should be taken into account when constructing them.

The Fig. 5-6 illustrates again the differences between both concepts applied for the team construction of cabin crews: Each team composed consists of at least one PU and five CAs, see also the typical (Airbus) crew complement given in subsection 3.1.2. Those groups are fixed ahead of the Team-oriented Rostering optimization according to the defined team construction. For the approach applying micro teams, loosely coupled crew pairs are symbolized by the dotted connection links. In the macro team approach, a certain redundancy in the team composition is imposed. Although they might be required, there should be a very limited amount of inter-macro team groups in order to keep the approach consistent to its initial idea. Besides this, also a combination of both, micro and macro teams, is thinkable, e.g., inseparable micro teams as parts of a dedicated macro team.

![Figure 5-6: Construction of micro and macro teams for cabin crew](image)

Figure 5-6: Construction of micro and macro teams for cabin crew
5.3.4 Scheduling the Entire Flight Crew

Finally, when it comes to schedule the cockpit and the cabin personnel together as the entire flight crew corresponding to the of the airline’s fleets, the set of solution approaches and concepts introduced above should be combined to achieve team orientation. However, the simultaneous solving of such an optimization problem is unrealistic in practice.

Therefore, the following sequential alternative procedures are exemplarily proposed. Note also that the set of “favourable” shared flight activities have to be generated in advance, e.g., for cockpit crew (as given in subsection 3.3.3).

*Alternative (A)*

1. Apply the Team-oriented Rostering solution approaches for the cockpit crew (CP and FO), as described previously in this chapter (except for subsection 5.3.3).

2. Apply the Team-oriented Rostering solution approaches for the leader of cabin crew (CDC or PU, depending on the aircraft type or fleet) together with the above determined cockpit teams (CP and FO).

3. Define micro and/or macro teams only for the cabin attendants, see subsection 5.3.3.

4. Apply the Team-oriented Rostering solution approaches for the leader of cabin crew (CDC/PU) together with the cabin attendant teams, including downgrading of CDC/PU.

*Alternative (B)*

1. Apply the Team-oriented Rostering solution approaches for the cockpit crew (CP and FO), see chapter 5 (except for subsection 5.3.3).

2. Define micro and/or macro teams for the complete cabin crew (CA, CDC and PU), see subsection 5.3.32.

3. Apply the Team-oriented Rostering solution approaches for the defined cockpit teams together with the cabin crew teams determined above (CA, CDC/PU), including downgrading.
The general procedure for both alternatives is illustrated in Fig. 5-7, where for each alternative the Team-oriented Rostering steps discussed have to be carried out from the inside to the outside of the onion model.

Like most sequential procedures, such approaches show their drawbacks in terms of finding the overall optimal solution, since they only combine optimally solved sub-problems. Moreover, such sub-problems themselves are mostly approximation problems with respect to the solution approaches and their parameters selected. In general, the resulting advantages and disadvantages are similar to those of the classical vs. the partially integrated crew scheduling problem, see section 2.4, and therefore, they will not be discussed here again. In spite of this, both alternatives have their dedicated impacts on the way how the team orientation is regarded:

On the one hand, alternative (A) handles the Team-oriented Rostering at first for cockpit crew, then combined together with the CDC or PU. By this, changes between the operating flight deck and the chief of the cabin crew are considered ahead of the Team-oriented Rostering for the remaining cabin attendants, and the special relationship for the important communication linkage between both crew types is emphasized, see subsection 3.1.2.

On the other hand, for alternative (B) the ToR process follows a different sequence. Since the cockpit crew and the entire cabin crew are composed separately as dedicated optimization instances, they are combined in a subsequent step. As a result, both crew types may experience a higher team orientation inside their teams, whereas there is less team orientation to be expected among cockpit and cabin crew, i.e., notable as more team changes.
5.4 Summary

Once the Team-oriented Rostering Problem for cockpit crew defined earlier is implemented, e.g., by the Extended Rostering Model, one is directly confronted with two major problems:

1. the vast amount of valid rosters available, and
2. the implications from the huge number of roster combinations, expressed by additional constraints to be considered.

Both are known to easily result in unsolvable optimization models simply due to the mathematical model’s sheer size – even for comparatively small real-life instances. Having this in mind, a set of solution approaches were proposed and discussed in this chapter.

The first challenge was addressed by a variety of common roster evaluation measures that led to a subset among the valid rosters for each crew member.

“Team-oriented rosters” often require a second, third or even more rosters of the corresponding crew function. As a result, their appearance in the solution afterwards is quite seldom identified in advance which leads to discard the application of such filtering approaches. Nevertheless, as underlined by the computational results, too restrictive rule sets imply quite negative effects in terms of unassigned SFAs, unnecessary team changes, and/or higher operational cost. Such filtering should also consider the overall crew capacities and availabilities, and by this, it should limit their roster combination basis when indicated.

The second challenge regarding the combinations of rosters and their possible penalization was dealt with in two directions: On the one hand, a stepwise rostering procedure was introduced which pre-selects a set of rosters for one crew function, namely the first officers in the approach presented. Those rosters are then combined with the second crew function, here the captains due to their chance for covering unassigned FO activities via downgrading. By this, a dramatic model reduction could be achieved; whereas the overall solution quality was in general not significantly reduced as the experiments conducted indicate. On the other hand, if the stepwise rostering procedure is not sufficient because of some computational limitations implied by the still huge amount of constraints, a basic branch-and-cut approach was introduced which enhances the optimized model only when and where it is required. As a result, so far unsolvable medium-sized optimization models for the Team-oriented Rostering can be expected to be solved. The computational results indicate their current limitations for instances of even less than 100 unevenly distributed cockpit crew members. The reason for this can be found in the SFA generation strategy cho-
sen and the resulting huge set of valid rosters that do still remain after facing both challenges discussed previously.

Later in this chapter, two further concepts were briefly introduced. The first one worked on the so far ignored aggregation of identical crew rosters, also in combination with pre-defined teams where applicable. Other subsections addressed the scheduling of cabin crew as well as a procedure to schedule the entire flight crew of an airline, including multiple crew functions and home bases.

As discussed earlier, each approximation presented affects the overall solution quality that is also greatly influenced by numerous decisions on procedures, solution approaches and their parameters. However, their application enables the crew planners to think of an alternative scheduling process which is characterized by a certain degree of team orientation. This complex task is facilitated by a decision support system like the one introduced in the upcoming chapter 6. The presented solution approaches, especially their combination as solution methods, may provide a great variety of insights on how the crew members’ needs can be balanced against economical obligations.
Chapter 6

Decision Support for Team-oriented Scheduling

Today’s business environment becomes more and more complex in such a way that it is almost impossible for human decision makers to decide appropriately on the overwhelming set of available data within a usually limited time span.

While the previous chapters dealt with the definition and realization of the Team-oriented Scheduling, especially the Team-oriented Rostering of cockpit crew, in this part the facilitating impact of Decision Support (DS) is emphasized. Moreover, the application of computer-aided optimization which utilizes a Decision Support System (DSS) is examined regarding the previously described approaches and concepts.

Following a very wide definition of decision support, one can say that every time when (human) decision makers are not left on their own while they determine something; there is already a certain kind of decision support going on. In the end, the more complex the examined situation is, the greater is the likelihood to be aided by decision support – even for non managerial tasks to perform.

According to (Turban 1993, 87), a decision support system is defined as follows:

“A DSS is an interactive, flexible and adaptable CBIS (computer-based information system; added by the author), specially developed for supporting the solution of a particular management problem for improved decision making. It utilizes data, it provides easy user interface, and it allows for the decision maker’s own insights. (…)

Furthermore, a DSS should provide accurate decisions based on the full set of available information at the time of a service request. It has to grant the objectiveness of a decision which enhances the decision maker’s capability to cope with the overwhelming input of data together with his or her emotional involvement, e.g., lack of concentration due to stress. Due to the notable shorter processing time for generating alternative solutions, several different strategies can
be tested before the final decision making conducted by the human being. Finally, a DSS should always be interpreted in a way that it supports, but not replaces, the user in front of the system.

This chapter is structured as follows: A DSS architecture for the airline crew management processes is proposed in section 6.1. It covers the planning and the operational phase of the personnel-related scheduling tasks. Subsequently in 6.2, the general architecture is enhanced by a set of detailed requirements visualized by several screenshots. They show the Team-oriented Scheduling as it is implemented exemplarily at our business partner for real-life instances. It is concluded with a summary in subsection 6.3.

6.1 Decision Support System for Airline Crew Management

It is well-known that different kinds of decision support systems are deployed in airlines today – especially in the area of crew scheduling. Although it is theoretically thinkable to handle the entire scheduling process including planning and operational phase, such systems are still quite rarely applied in practice, where mostly the recovery process is still manually conducted. As reported in (Yu et. al 2003), a tool called CrewSolver by Caleb Technologies Corp. (Caleb 2005) is successfully set up at Continental Airlines. Further important vendors are Carmen Systems AB (Carmen 2005), Sabre Airline Solutions (Sabre 2005), and Lufthansa Systems (Lufthansa 2005).

Integrating all activities of the airline crew management would be desirable in the future. Thus, in this section a dedicated architecture for the overall crew management task is presented by linking the underlying crew scheduling processes and tasks within airlines as introduced already in chapter 2. Therefore, a multitude of requirements according the considered scheduling phases are derived in subsection 6.1.1. They constitute the fundamental basis for a later construction of a DSS architecture and its components in subsection 6.1.2, and the remaining subsection 6.1.3 discusses their specific functionality.

6.1.1 Requirements Description

It is generally agreed on the way how a DSS should be applied. Due to the distinct purpose of the two examined scheduling phases, planning and operations control, their requirements should be determined separately as follows.
Planning phase

As described in section 2.1.2, the crew scheduling determines the assignment of flight legs to crew members during the planning phase. For this problem all input information given is fixed, such as the flight schedule, aircraft rotations, crews’ individual pre-scheduled activities, payment and cost structures, as well as work rules.

Therefore, a DSS for this problem setting has to combine the complex data set above for the solution processing. It needs to set up and coordinate dedicated solution methods in combination with appropriate, sometimes alternative, optimization strategies. These require still intensive testing and evaluation, whereas results are necessarily presented in an understandable way to the users: Firstly to the planning experts, secondly to the operating air crew by official publications after the final crew schedule is determined. In a subsequent step, statistics may help to uncover unbalanced workload situations or strategic decisions regarding expert rules, etc.

Operational phase

The input for the crew rescheduling problem differs from the CSP, see also section 2.5: In addition to the actually operated schedule, the CRP has to recover from disruptions for crew members as soon as possible. (Note that the given schedule might be already different to the scheduled one of the CSP because of earlier disruptions.)

Hence a DSS for crew management in the operational phase has to model not only the situation scheduled (until this point of time), but also more actual events caused by delays, cancellations, absentness and so on, as long as they are related to the crew. (Other objectives related to aircraft and passengers are not considered explicitly here.) Therefore, it is especially important for Operations (Ops) personnel to reveal a disruption’s impact on the remaining schedule and to act quickly and appropriately towards the full schedule recovery.

In this case the user needs support to handle the vast amount of input data, an appropriate environment to address the disruptions by dedicated solution approaches and their evaluation for quick and efficient decision making. Hereby, it is favourable to “learn” from earlier recovery operations, also in order to prevent disruption’s occurrence proactively.
6.1.2 System Architecture and Its Components

After the requirements for a crew management system are discussed, in this subsection the following set of components of a DSS is suggested. The architecture can be observed by identifying three tiers: users, core components, and input data.

Users and user interface

The graphical user interface (GUI) allows its users to interact with the system. For airline crew management, it is differentiated between planners (or dispatchers) for the planning phase and coordinators (or operators) for the operational phase. Both groups are usually working for different departments at the airline. The interface covers their needs, especially, in order to support their jobs effectively by utilizing components efficiently.

Core components of a DSS

There are in total at least nine fundamental components listed below that have to be developed and combined into the system:

1. **DSS configuration**: The central component is the configuration which constitutes the basis of the DSS. A bundle of functions and settings should be enabled or disabled when and where necessary. Therefore, it is essential to group and to set up a complete definition of the company’s objectives. (It has to be decided how to set the parameters which are associated with achieving the intended goal(s) within all further components.)

2. **Data communication**: This component works on the exchange of data between the DSS and the necessary databases and other data. There is a high similarity of input data for the crew management tasks. Therefore a fully shared component is proposed.

3. **Solution methods**: Based on the objectives of the task and the given parameters, a set of solution methods is provided. This includes set partitioning (or covering) models for integer programming, their LP relaxation, network flow models, branch-and-bound approach, column generation, constructive or modern meta-heuristics, e.g., genetic algorithms, simulated annealing, etc. These solution methods might be combined or sequenced in a defined order to compute the desired result(s).
4. **What-if analysis:** This component enables the handling of different parameter settings within the DSS. Even for well-experienced experts it is sometimes very hard to decide in advance all details of the “best” values for certain parameters. As a result, examining further alternatives might present valuable insights rather than what a single intuitive approach is able to provide.

5. **Simulation:** Since the operation of an airline is not a deterministic system, the application of a simulation component is very useful. By this it is possible to evaluate more stochastic characteristics of crew schedules, e.g., robustness against delays or delay propagations, and their impacts on the entire system.

6. **Visualization:** Both crew management tasks, crew scheduling and rescheduling, are based on a huge amount of data. A proper visualization helps the user to understand the underlying information more easily in a way that over-demanding or misinterpretable output is prevented, see also GUI.

7. **Evaluation:** This component is responsible for the complex evaluation of alternative solutions, through which it supports the final decision making, e.g., based on comparisons. After several solutions are generated by a single or multiple components introduced above (e.g., solution methods, what-if-analysis, simulation), an evaluation scheme has to be defined which determines the pros and cons among the alternatives and provides a concrete suggestion on how to react in the current situation.

8. **Publication and Notification:** All decisions that are made during the crew management need to be published which has to be done in a way that all individual crew members affected by the updated information are informed accordingly by print-outs or only via terminal stations at their current location.

9. **Statistics:** In order to improve the solution quality, the consideration of tracked data may produce additional benefits. Derived from the experienced problems, expert rules can be extracted and, therefore, the repeated occurrence of similar disruption in the future might become avoidable to a certain extent.
Input data

There are two major categories of data, static data, rules and expertise:

- **Static data:** For the crew management tasks the input data is provided by several databases: They include information of crew members (e.g., individually contracted flying hours, vacations, pre-scheduled activities and home bases), the flight plan (e.g., flights with arrival and departure times, requested crew qualifications and fleet requirements, etc.), and airport information (such as landing capacities, turnaround times, hotel availability). Furthermore, a tracking database provides the real-time data as it is executed during operations which monitors also delays, cancellations, crew sickness or absentness, etc. A knowledge container (or expert system) covers all less structured information, such as general guidelines and concepts, knowledge access points (or yellow pages) with contact information, and so on.

- **Rule sets and expertise:** In addition, more or less structured regulations based on governmental instances, union agreements or company internal rules have to be instantly satisfied during the crew scheduling and rescheduling. Besides this, expert rules are usually considered that may define even more restrictive (sometimes simpler) constraints on the problem setting or – based on the users’ experience – they may turn out to speed up the solution process.

As given previously, the architecture is given by three tiers (users, core components and input data), which are visualized from the left side to the right side in Fig. 6-1. The upper third of this illustration represents the crew scheduling during the planning phase, including different alternatives, such as sequential or integrated approaches with or without team orientation (see also Fig. 2-6 in section 2.4). In contrast to this, the lower third of the figure highlights the rescheduling processes encountered in the operational phase as discussed in section 2.5, see Fig. 2-9. In the center – and, therefore, embedded in-between both scheduling phases – the core components of the DSS can be found. It consists of the components introduced above and arranges them according the architecture proposed.
6.1.3 DSS Functionality

Despite the fact that all components above are commonly shared among both scheduling phases, some of them require a diverse functionality set as revealed earlier in subsection 6.1.1. For example, regarding components like the configuration, what-if, evaluation, databases and rule set, there are no obvious differences apart from the general purpose of the considered scheduling phase as
described in subsection 6.2.2. For all remaining components, Table 6-1 compares the functionality requested and exemplifies on their specifications.

<table>
<thead>
<tr>
<th>DSS Component</th>
<th>DSS Functionality</th>
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<tbody>
<tr>
<td><strong>Graphical User Interface</strong></td>
<td>• Preparation of crew schedules by planning department</td>
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<td></td>
<td>• Monitoring and handling of unexpected events by airline OCC</td>
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<td></td>
<td>• Providing multiple options of strategies and their configuration</td>
</tr>
<tr>
<td><strong>Data Communication</strong></td>
<td>• Access to planning information of current and earlier scheduling periods</td>
</tr>
<tr>
<td></td>
<td>• Access to static data regarding crew, airports, aircraft, etc.</td>
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<tr>
<td></td>
<td>• Access to detected disruptions</td>
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<tr>
<td></td>
<td>• Access to current crew schedule, planning information of current and earlier time periods, and other static data</td>
</tr>
<tr>
<td><strong>Solution Methods</strong></td>
<td>• Dedicated models and heuristics regarding the DSS configuration on complete planning period</td>
</tr>
<tr>
<td></td>
<td>• Optimal importance more important than computational time</td>
</tr>
<tr>
<td><strong>Simulation</strong></td>
<td>• Evaluation of crew schedules, e.g., regarding robustness, bottlenecks, etc.</td>
</tr>
<tr>
<td></td>
<td>• Analysis of impacts regarding current and/or future events</td>
</tr>
<tr>
<td><strong>Visualization</strong></td>
<td>• Overview on the planning task, especially the results achieved in different scheduling steps</td>
</tr>
<tr>
<td></td>
<td>• Overview on scheduled and actual scheduling situation</td>
</tr>
<tr>
<td></td>
<td>• Visualization of variations from originally planned schedules</td>
</tr>
<tr>
<td><strong>Publication &amp; Notification</strong></td>
<td>• Regular publication of crew schedule information days ahead of execution to all crew members</td>
</tr>
<tr>
<td></td>
<td>• Early notifications only of affected crew members</td>
</tr>
<tr>
<td></td>
<td>• Consideration of crew’s current location</td>
</tr>
<tr>
<td><strong>Statistics</strong></td>
<td>• Generation of accumulated or calculated schedule quality indicators, e.g., workload balancing, scheduled activities, hotels stays, proceedings, operational cost, etc.</td>
</tr>
<tr>
<td></td>
<td>• Computing quality indicators for the operated schedule, but also review on rescheduling strategies, impact analysis, detection of disruption regularities, etc.</td>
</tr>
</tbody>
</table>
A more detailed requirement specification would lead to an inappropriate amount of too many details here. Instead, it is referred to, e.g., (Turban and Aronson 2000) for a general description of a DSS over all business areas.

### 6.2 Enhancements for Team-oriented Scheduling

Applying team orientation for the scheduling of aircrews is followed by many changes during the processing step as discussed in the previous chapters. After a general DSS architecture is presented in section 6.1, the impact of the Team-oriented Scheduling on such a system is briefly studied in this section.

To a certain degree, most of the components described in subsections 6.1.2 and 6.1.3 have to be modified in order to comply with the new approach. However, the greatest adjustments and enhancements can be found the following components which are examined hereafter:

- DSS configuration,
- solution methods,
- visualization, and
- evaluation.

All screenshots of this section are taken from a basic prototype. During this work it was enhanced by several Team-oriented Scheduling methods, which were implemented and tested. In the meantime, a continuous improvement and feedback process under the supervision of Prof. Dr. Taïeb Mellouli, University of Halle, Germany, leads to the current version of the DSS which is not considered in this work. At present, it is deployed at the crew planning department at our business partner, and its commercialization is planned.

#### 6.2.1 DSS Configuration

The first component discussed is the DSS configuration unit. Besides pure configuration, it controls the whole scheduling process as well which is hereby organized, e.g., as a sequence of several steps. While the tracking database traces all events, the DSS configuration component initiates and monitors them permanently. However, the mainly visible function of this component is emphasized in the following.
In Fig. 6-2 the main DSS configuration window is given: Besides the regular
determination of the most relevant factors for a crew scheduling problem, it re-
quires certain enhancements in order to address team orientation as well.
Hence, on the left side, the load environment factors such as the airline setting,
the applied working rules, the fleet selected, and the scheduling period have to
be chosen. In addition, some basic optimization parameters are arranged. Note
that those steps for the execution of the scheduling process may be combined
into batches allowing their realization sequentially or on different machines in
parallel, e.g., over night. Therefore, a time limit is usually set up in advance for
the solution time. The crew functions to be examined are presented on the right
side, while a separated downgrading among both crew types remains optional.

The team orientation parameters are realized in another section of the DSS
configuration window: One of the alternative ToRP objectives can be selected,
see subsection 3.2.5. Furthermore, it has to be determined if the chosen objec-
tive implies team change penalties, or alternatively, to which extent team orien-
tation is dominated by cost consideration or vice versa.

In one of the following steps the roster evaluation setting has to be initiated as
well. According to the specification described in subsection 5.1.2, cost-based,
frequency-based and quality-based evaluation measures are determined together with further important parameters that are required for the optimization process.

6.2.2 Solution Methods

The emphasis of the work is reflected in the solution methods’ component: The set of solution approaches presented in the previous chapter 5 were combined into dedicated solution methods. For instance, the Team-oriented Rostering applied to the cockpit crew can be realized by enabled or disabled roster filtering (see subsection 5.1.2), the stepwise rostering procedure (see 5.2.1) and/or the application of the basic branch-and-cut procedure introduced in subsection 5.3.1. Besides this, additional distinct parameter settings may become necessary, if one of the alternative mathematical models given in chapter 4 is chosen.

In Fig. 6-3 such a solution method created earlier is given. The input of the method is well-defined by a so-called input scheduling state for which the solution method selected is applied. It can be finished without any stop. Though, sometimes even sub-steps show interesting issues encountered. This may encourage a more detailed view on preliminary results if desired by the crew planner, e.g., in the given dialog box.
Afterwards, the result can be visualized (see subsection 6.2.3) and/or evaluated (see 6.2.4). However, this output may form the input scheduling state for another subsequent solution method, if desired.

### 6.2.3 Visualization

The visualization component briefly discussed in this subsection is also essential for the DSS. As described previously, the entire scheduling process is embedded in the DSS by following different steps. Each of them, even earlier phases prior to the Team-oriented Rostering, require their dedicated visualizations.

In Fig. 6-4 a set of such graphical representations (without team orientation) is given: In the upper third part of the figure, the aircraft rotations are presented as the input of the CSP, followed by the so-called *flight duty trees* which represent the potential sequences of flights during the pairing chain generation (see subsection 2.4.2). Finally, this results in anonymous lines-of-work given as rows in the remaining lower part of Fig. 6-4.

![Figure 6-4: Visualization of different scheduling steps (Source: Mellouli 2003)](image-url)
As already mentioned, the pieces-of-work considered for the Team-oriented Rostering Problem are shared flight activities, see subsection 3.3.2. Furthermore, the basic idea behind the team orientation concept is to avoid the “impact” of team changes, e.g., by their prevention, which is only possible in-between those SFAs.

In Fig. 6-5, an exemplary team-oriented captain crew schedule for the home base Düsseldorf (DUS) is depicted: Each column represents the day inspected, and each row shows the corresponding crew member with his or her flight duties to serve. For instance, the captain TON is scheduled for July 1st to fly from DUS to AYT (Antalya, Turkey) and back to DUS. As printed in his or her line-of-work, the crew member will depart and return to his home base each single working day. Assuming that there is no team change during such a flight duty, they can be interpreted as well as SFAs for this captain. As a consequence, several team changes during the time period examined are possible, but as the figure indicates he or she will serve his first four days together with first officer DEE.

Note that the final captain mentioned in this crew schedule, namely PAL, arrives on the first day in CGN (Cologne). Together with the first officer GHB (who is assumed to be stationed as well in DUS) both will transit by a shared taxi back to their home base (DUS).
6.2.4 Evaluation

The final subsection deals with the evaluation of the Team-oriented Scheduling. There are many evaluations thinkable. Usually, they are presented as a table filled with evaluation measurement values whose combination leads to a clearly ranked position in the hierarchy computed and proposed among all alternatives.

In the following, a more powerful evaluation is presented: One major challenge is finding a graphical display that allows to evaluate simultaneously multiple decisive characteristics of a generated alternative crew schedule: Apart from the conventional pure cost consideration also team-oriented aspects have to be taken into account which implies at least a two dimensional evaluation. Those essential crew schedule characteristics can be analogously defined to the conventional roster evaluation measures described in subsection 5.1.2. For the evaluation of team orientation the alternative objectives presented in subsection 3.2.5 are applicable.

It is the aim of such a visualization to assist the user of the DSS who has to compare and evaluate several alternative crew schedules computed by the system together with manually modified or hand-made ones. Additionally, the planner is usually also interested in more than the highest and lowest values of those characteristics among the alternatives examined.

Based on the “theory for data graphics” worked out in (Tufte 1983) the following visualization method can be applied as an example. In Fig. 6-6 two schedules are compared with each other. The criteria examined are average (or absolute) values for the planned assignment cost, the number of team changes, the connection time, and the realized workload. The interpretation of such a diagram can be conducted as follows for the first two criteria: Schedule 1 is associated a comparatively moderate amount of operational cost together with a moderate amount of team changes in average. In contrast to this, schedule 2 is characterized by a very high amount of operational cost, and the lowest amount of team changes. The same can be done for the average connection time and the workload, whereas the first should be maximized and the latter minimized to indicate a good performance.
The advantage of this visualization is that the trained viewer of such a diagram may get very quickly an impression of how “good” a schedule is compared with even dozens of other schedules.

Further improvements can be achieved as follows: Firstly, all scales should be normalized in a way that low (or high) values indicate a better performance evaluation of that specific criterion: Since most of the criteria examined have to be minimized (such as operational cost, team changes, etc.), their interpretation is more consistent, and therefore, much easier in comparison to a mixed structure with additional maximizing criteria. Secondly, the importance of each criterion has to be emphasized, because the lowest amount of team changes observed over all alternatives may have the highest operational cost associated, which is usually of higher interest. Thirdly, the absolute values have to appear as well before a final decision can be made: Knowing that a specific criterion is on the worst level among all alternatives examined does not necessarily mean that its performance is unacceptably low, especially if the overall range is comparatively small. However, the individual preferences of a decision maker for setting up such evaluation scales and diagrams may interfere with the commonly agreed ones in the company. As a major risk, this can easily lead to misinterpretation, if someone is not completely familiar with the chosen evaluation style.
6.3 Summary

Elaborated decision support embedded within a computer system is of high interest for researchers as well as for business users. For the highly complex data which is attached to the crew scheduling problem, dedicated decision support systems become necessary in order to fulfil the planners and/or coordinators monthly to daily tasks appropriately.

In this chapter, the impact of the Team-oriented Rostering on the implementation of such a decision support system was shown. Firstly, an integrated system architecture was described for the entire crew management process with regard to the planning and the operations phase. Therefore, the set of distinct requirements was discussed together with the implications for the corresponding components. Four out of those DSS components were exemplarily acknowledged as being influenced by the team orientation approaches newly introduced. Thus, the DSS configuration, the solution methods, the visualization, and the evaluation component were examined in detail.

Independent from the set of parameters chosen, their modification usually requires a highly qualified expert which is capable to forecast the impact of that change. Hence, well-experienced crew planners and/or coordinators will always be necessary. As a matter of fact, somehow “human intelligence” is still required in order to fully analyze and to understand the results presented even by one of the best DSS available. Especially those seldom inherent drawbacks hidden in some of the unexamined or new problem (sub)instances will point out this aspect. In the end, considering permanently changing requests on such a system results in a situation where there will never be a something like a “perfect” DSS that covers all issues in advance of their occurrence proactively.
Chapter 7

Conclusion

At the end of this work on the Team-oriented Scheduling of flight personnel, a set of conclusions can be drawn: In section 7.1 a thesis review is presented which covers all major achievements and scientific contributions being framed again into the overall context. In addition to the research already conducted, the final section 7.2 suggests some potential further directions and relevant research areas in the outlook.

7.1 Thesis Review

Already in the introductory section 1.1 the overall key question was posed: “How to actively consider teams while scheduling flight personnel?” Apart from randomly formed teams (of people scheduled as co-workers), it was the aim of this work to address and analyse this question from different perspectives, but with a strong focus on the operations research view.

Therefore a proceeding was established which covers the most important topics: Starting with an introduction in chapter 1 together with a general problem description and literature review on the airlines’ dedicated crew scheduling problems in chapter 2, the newly introduced Team-oriented Scheduling Problem itself was defined step-by-step and later examined in great detail in chapter 3. In addition, chapter 4 followed with a set of mathematical model formulations for the resulting Team-oriented Rostering step which was enhanced by several solution approaches for their implementation regarding the real-life application intended in chapter 5. The complete approach is then proposed for integration into a general decision support system architecture presented in chapter 6, summarized by the conclusions and outlook of this chapter.

In this work, the new Team-oriented Rostering Problem was examined for the context of airlines – an approach within the crew assignment phase for the onboard crew scheduling. The Team-oriented Rostering itself – as part of the per-
sonalized rostering (with fair-and-equal share of workload, not seniority) – was mainly focused on the minimization of team changes within the cockpit crew (while a variety of other potential objectives and instances were discussed theoretically). Based on a setting with time and location dependent crew availabilities, numerous strategies (mathematical models as well as solution methods) and their combination addressing the high combinatorial complexity were analysed and implemented. For this problem type, roster combinations are accounted for instead of single rosters (as usual for the general CSP).

Several distinct mathematical formulations were given to realize the ToR approach, whereas for real-life instances the so-called Extended Rostering Model turned out to be more applicable than the Roster Combination Model defined among the mathematical models presented. Although some problem characteristics are literally shared by the widely examined quadratic assignment problems, the proposed IP models are comparably easier to solve in terms of size and time.

One of the key accomplishments of this work lies on the effects of the Team-oriented Rostering: It is shown that the ToR results in a trade-off between the operational cost on the one hand, and the number of team changes on the other (if this objective function is chosen for the evaluation of team orientation). In general it is concluded that team orientation itself is responsible for only comparatively small raises of operational cost, whereas the indicators considered for team orientation illustrate the measurable impacts of the ToRP approach significantly, e.g., in terms of weighted team changes, team durations, and so on.

It is acknowledged that only relatively small instances (with less than 1,000 flight legs) are solved within an acceptable time window so far. The reason for this can be found in the great model size, which is highly influenced by the set of additional constraints required. Therefore, further research will especially concentrate on this drawback of the current approach: Firstly, it is suggested to apply appropriate penalization strategies, since their setting is tightly linked to the mostly critical size of the model. By this, even the subsequent application of a Branch-and-Cut approach turns out to be suitable. Secondly, a great benefit will arise when defining SFAs properly already during the pairing generation phase: Although this requirement implies a modification of the models and techniques applied currently for the CPP, the generation of thousands of potential rosters (consisting of short SFAs) is prevented and, as a result, larger instances can be solved.

Another option appeared after analyzing the computational results: Is was noticed that (1) several crew members share exactly identical sets of rosters and (2) the majority of staff (>70%) never experiences any team change in the final
crew schedule. Therefore, a great model reduction can be achieved by grouping crew members with identical rosters, and, if possible, by building “pre-defined” groups already for (potential) teams, where there will be no team change at all. This results in the hybrid IP model presented, where the residual problem can be solved, e.g., by the Extended Rostering Model as described previously. Especially the pre-selection of good rosters for one crew function – here: first officers due to downgrading – turned out to result in high model size reduction rates without a notable lack of solution quality.

Subsequently, all concepts and solution approaches were embedded in a general system architecture of a decision support system for crew scheduling and rescheduling. Here, certain input and output interfaces were exemplarily shown that indicate the application among the usual setting of an airline. Regarding the evaluation of inherent multi-criteria decision making settings, also a more elaborated visualization scheme was explained.

By following up with this proceeding and including the details discussed, all major goals of the thesis can be regarded as being met in a short, but still comprehensive way.

### 7.2 Outlook

In this final section further developments of the Team-oriented Scheduling approach are proposed as follows:

On the one hand, especially with regard to the great variety of methods and techniques that origin from, e.g., computer science and/or operations research; there are many more alternatives available to be considered rather than the ones implemented when it comes to address the ToRP. Most likely, the application of a dedicated column generation approach may become very interesting to examine, also the set of meta-heuristics, e.g. Genetic Algorithms or Simulated Annealing, can turn out to be quite valuable to investigate.

On the other hand, the airline-centred deployment of the Team-oriented Scheduling and Rostering Problems as defined may be too tight: According to, e.g., (Hackman 2002, 59), such team-oriented approaches are as well decisive for the realization of project task forces, but also for management, sales and even for virtual teams inside the organizations. Although the problem setting itself differs — partially, sometimes significantly — from the one examined above, synergies achieved by proper team compositions deliver notable results to the companies which are applying them. Even outside the business environment, e.g., when staffing an orchestra or a sports team, intra-team relationships are
unquestioned regarding the team’s performance. Of course, the emphasis of this work lies more on the technical business case analysis and its potential realization from the OR point of view. For a closer look at the supportive Human Resource Management aspects (e.g., real team spirit, compelling direction, enabling structure, expert coaching, etc.), it is referred again to (Hackman 2002).

Besides this, the computational power available has been significantly improved over the last decades, e.g., by hardware development, more elaborated solvers, and so on. Nevertheless the Team-oriented Scheduling Problem introduced above still belongs to the NP-hard solvable problem class. Even more, already for two crew functions, the Team-oriented Rostering turns out to be a mathematical problem of quadratic nature. And as long as there is no really notable breakthrough in this area of optimization, it implies — maybe even for the next decade(s) — the fact that the particular problem type discussed will remain a great challenge to fully cope with when the exact optimal solution for real life instances is aimed at.
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Glossary

Acronyms / Abbreviations

AI       Artificial Intelligence
AYT     Antalya (city in Turkey)
BIP     Binary Integer Program
CA      Cabin attendant
CAP    Crew Assignment Problem (same to CRP)
CBIS   Computer-based Information System
CDC    Chef-de-Cabin
CLP    Constraint Logic Programming
CM     Crew member
Cols   Columns
CP     Captain
CPCP   Crew Pairing Chain Problem
CPP    Crew Pairing Problem
CRP    Crew Rostering Problem (The same abbreviation is also applied for the Crew Recovery Problem in literature.)
CSP    Crew Scheduling Problem
DG     Downgrading
DP     Dynamic Programming
DS     Decision support
DSS    Decision support system
<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>DUS</td>
<td>Düsseldorf (city in Germany)</td>
</tr>
<tr>
<td>FAA</td>
<td>Federal Aviation Administration</td>
</tr>
<tr>
<td>FD</td>
<td>Flight duty</td>
</tr>
<tr>
<td>FE</td>
<td>Flight engineer</td>
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<tr>
<td>FO</td>
<td>First officer</td>
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<tr>
<td>GA</td>
<td>Genetic algorithm</td>
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<tr>
<td>HB</td>
<td>Home base</td>
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<tr>
<td>HRM</td>
<td>Human Resource Management</td>
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<tr>
<td>ILS</td>
<td>Iterative local search</td>
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<tr>
<td>IP</td>
<td>Integer Program</td>
</tr>
<tr>
<td>LoW</td>
<td>Line-of-Work</td>
</tr>
<tr>
<td>LP</td>
<td>Linear Program</td>
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<tr>
<td>MIP</td>
<td>Mixed-Integer Program</td>
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<tr>
<td>MR</td>
<td>Model reduction</td>
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<tr>
<td>NZ</td>
<td>Non-zero</td>
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<tr>
<td>OC</td>
<td>Operational cost</td>
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<tr>
<td>OCC</td>
<td>Operations Control Center</td>
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<tr>
<td>Ops</td>
<td>Operations Control</td>
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<tr>
<td>OR</td>
<td>Operations Research</td>
</tr>
<tr>
<td>PBS</td>
<td>Preferential Bidding System</td>
</tr>
<tr>
<td>POD</td>
<td>Concept name introduced by Air New Zealand representative (no abbreviation)</td>
</tr>
<tr>
<td>PoW</td>
<td>Piece-of-Work</td>
</tr>
<tr>
<td>PU</td>
<td>Purser</td>
</tr>
<tr>
<td>RG</td>
<td>Rosters generated</td>
</tr>
<tr>
<td>RGCP</td>
<td>Rosters generated for Captains</td>
</tr>
<tr>
<td>RGFO</td>
<td>Rosters generated for first officers</td>
</tr>
<tr>
<td>Acronym</td>
<td>Description</td>
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<td>---------</td>
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</tr>
<tr>
<td>RGT</td>
<td>Roster Generation Time span</td>
</tr>
<tr>
<td>RL</td>
<td>Rostering loop</td>
</tr>
<tr>
<td>RS</td>
<td>Rule set</td>
</tr>
<tr>
<td>SA</td>
<td>Simulated Annealing</td>
</tr>
<tr>
<td>SCP</td>
<td>Set Covering Problem</td>
</tr>
<tr>
<td>SFA</td>
<td>Shared flight activity</td>
</tr>
<tr>
<td>SO</td>
<td>Second officer</td>
</tr>
<tr>
<td>SPP</td>
<td>Set Partitioning Problem</td>
</tr>
<tr>
<td>ST</td>
<td>Solving Time span</td>
</tr>
<tr>
<td>TC</td>
<td>Team change</td>
</tr>
<tr>
<td>ToR</td>
<td>Team-oriented Rostering</td>
</tr>
<tr>
<td>ToRP</td>
<td>Team-oriented Rostering Problem</td>
</tr>
<tr>
<td>TP</td>
<td>Team change penalty</td>
</tr>
<tr>
<td>TS</td>
<td>Tabu Search</td>
</tr>
<tr>
<td>TSN</td>
<td>Time Space Network</td>
</tr>
<tr>
<td>UA</td>
<td>Unassigned flight activity</td>
</tr>
</tbody>
</table>
## Key Concepts and Terms

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Airport</td>
<td>Location where a →flight is scheduled to start and end</td>
</tr>
<tr>
<td>Assignment</td>
<td>Application of the →crew assignment problem</td>
</tr>
<tr>
<td>Bidline systems</td>
<td>A common →crew scheduling approach in the US where the →crew member’s individual →seniority is accounted for during the →assignment</td>
</tr>
<tr>
<td>Binary Integer Programming</td>
<td>→Integer Programming with 0/1-variables</td>
</tr>
<tr>
<td>Binary IP model</td>
<td>Mathematical model which applies →Binary Integer Programming</td>
</tr>
<tr>
<td>Cabin</td>
<td>Part of the aircraft where passengers are seated during the flight is executed, and where the →cabin crew works</td>
</tr>
<tr>
<td>Cabin attendant</td>
<td>Lowest →crew position of the →onboard crew</td>
</tr>
<tr>
<td>Cabin crew</td>
<td>→Crew which is in charge of dealing with the passengers, it may consist of several →crew functions, e.g., →cabin attendants, several →chefs-de-cabin, or →pursers</td>
</tr>
<tr>
<td>Captain</td>
<td>Person in charge for flight safety, the →crew as well as the passengers while steering the aircraft from departure to arrival →airport</td>
</tr>
<tr>
<td>Chef-de-Cabin</td>
<td>Highest crew position in a Boeing fleet, for which a →downgrading to a →cabin attendant is possible</td>
</tr>
<tr>
<td>Cockpit</td>
<td>Front part of the aircraft where the →cockpit crew works</td>
</tr>
<tr>
<td>Cockpit crew</td>
<td>Onboard crew which is in charge of steering the aircraft; it may consist of at least two →crew functions: →captain, →first officer, and (optionally) →flight engineer</td>
</tr>
<tr>
<td>Contracted flight duration</td>
<td>Amount of hours that a →crew member is contracted to fly, e.g., 90 or 99 hours per month</td>
</tr>
<tr>
<td>Term</td>
<td>Definition</td>
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<tr>
<td>-------------------------------------------</td>
<td>---------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Co-pilot</td>
<td>Same as → first officer</td>
</tr>
<tr>
<td>Crew</td>
<td>Group of flight personnel that consists of → cockpit crew and → cabin crew</td>
</tr>
<tr>
<td>Crew Assignment Problem</td>
<td>Second → crew scheduling step of the → Crew Scheduling Problem (after → Crew Pairing Problem) during which → pieces-of-work such as → flight activities are assigned</td>
</tr>
<tr>
<td>Crew function</td>
<td>For → cockpit crew there are → captain, and → first officer; a → cabin crew may consist of, e.g., → cabin attendants, several → chefs-de-cabin, or → pursers</td>
</tr>
<tr>
<td>Crew member</td>
<td>A person to be scheduled during the → Crew Scheduling Problem; he or she will receive a → roster or → line-of-work afterwards as his or her individual → crew schedule</td>
</tr>
<tr>
<td>Crew Pairing Chain Problem</td>
<td>In addition to the → crew pairing problem time and location dependent crew availabilities are considered already during the first → crew scheduling step while the workload to be assigned is partitioned evenly among all → home bases</td>
</tr>
<tr>
<td>Crew Pairing Problem</td>
<td>First → crew scheduling step of the → crew scheduling problem (before → crew assignment problem) during which → pieces-of-work are created out of → flight legs; output are anonymous → lines-of-work for the → assignment step</td>
</tr>
<tr>
<td>Crew planner</td>
<td>A person of the crew planning department who has to schedule the crew by solving → crew scheduling problems</td>
</tr>
<tr>
<td>Crew position</td>
<td>Same as → crew function</td>
</tr>
<tr>
<td>Crew Recovery Problem</td>
<td>Handles disrupted situations in which original → crew schedules require several, sometimes major modifications to keep the airline’s operations running after their unplanned occurrence</td>
</tr>
<tr>
<td>Crew Rescheduling Problem</td>
<td>Same as → crew recovery problem</td>
</tr>
<tr>
<td>Crew Rostering Problem</td>
<td>Same as → crew assignment problem</td>
</tr>
<tr>
<td>Crew schedule</td>
<td>Output of the → crew assignment problem which expresses the assignment of → flight legs and → pre-scheduled activities to → crew members</td>
</tr>
</tbody>
</table>
Crew scheduling approaches | Different ways how the crew scheduling problem, especially the crew assignment problem is conducted in practice; approaches are bidline systems, preferential bidding systems, and personalized rostering.

Crew Scheduling Problem | Given the published flight schedule of an airline, the key task is to assign all necessary crew members of a cockpit crew and a cabin crew in such a way that the airline is able to operate all flights at minimal expense for personnel; the assignment has to consider all restrictions imposed by governmental regulations, union agreements, and company-specific rules; in addition, individual time- and location-dependent crew availability has to be accounted for, especially in a setting where the personnel is partitioned to multiple home bases; due to its NP hardness, it is decomposed into two crew scheduling steps.

Crew scheduling steps | The crew scheduling problem is usually decomposed into crew pairing problem, and subsequently the crew assignment problem.

Crew transit | Same as transit.

Crew type | Each crew member belongs to the cabin crew or to the cockpit crew where he or she fulfils a certain crew function.

Deadheading | A crew member flies as a passenger of the airline examined, whereas non-deadheading air transits are operated by another company.

Decision support | Any kind of method or tool that extends the users’ ability (here crew planners and crew coordinators) during their work.

Decision support system | An interactive, flexible and adaptable CBIS, specially developed for supporting the solution of a particular management problem for improved decision making; it utilizes data, it provides an easy user interface, and allows for the decision maker’s own insights.

Downgrading | Applied in the airline crew scheduling quite frequently. According to a clearly defined hierarchy among the crew functions, e.g., a captain could also serve as a first officer, but not vice versa, similar as for the cabin crew.
<table>
<thead>
<tr>
<th>Glossary</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Fingerprint</strong></td>
<td>→Line-of-work where only →pre-scheduled activities are assigned; it shows the availability of a →crew member over the →planning period</td>
</tr>
<tr>
<td><strong>First officer</strong></td>
<td>Mandatory second highest →crew position in the →cockpit crew</td>
</tr>
<tr>
<td><strong>Flight</strong></td>
<td>A sequence of →flight legs offered by the airline, e.g., from one departure →airport via another to the destination airport</td>
</tr>
<tr>
<td><strong>Flight activity</strong></td>
<td>Implies all kinds of →pieces-of-work to be considered during the →rostering, e.g., →flight legs, →flight duties; for the →Team-oriented Rostering: →shared flight activities</td>
</tr>
<tr>
<td><strong>Flight crew</strong></td>
<td>Same to →onboard crew</td>
</tr>
<tr>
<td><strong>Flight duty</strong></td>
<td>Represents a series of →flight legs that can be serviced by one →crew member within a workday (24 hours); it is surrounded (before and after) by →rest periods</td>
</tr>
<tr>
<td><strong>Flight engineer</strong></td>
<td>Third →crew position of the →cockpit crew which is rarely applied nowadays</td>
</tr>
<tr>
<td><strong>Flight leg</strong></td>
<td>Represents a non-stop air transit from a departure →airport to its corresponding destination airport</td>
</tr>
<tr>
<td><strong>Fly-below-rank</strong></td>
<td>Same as →downgrading</td>
</tr>
<tr>
<td><strong>Free team building</strong></td>
<td>A →team construction approach by which teams are freely composed while the optimization model is solved, not ahead</td>
</tr>
<tr>
<td><strong>Home base</strong></td>
<td>→Airport where →crew members are stationed; usually this is close to their home domicile; each →crew member has only one home base where all →pairings start and end</td>
</tr>
<tr>
<td><strong>Hub-and-spoke network</strong></td>
<td>Network topology usually found in the US where smaller →airports are linked to major airport (so-called hubs); hubs are then connected directly, like in a →point-to-point network</td>
</tr>
<tr>
<td><strong>Integer Programming</strong></td>
<td>Mathematical programming with integer variables</td>
</tr>
<tr>
<td><strong>Linear Programming</strong></td>
<td>Mathematical programming with linear variables</td>
</tr>
<tr>
<td><strong>Line-of-work</strong></td>
<td>Represents a potential crew schedule for a dedicated crew member of the planning periods of usually two or four weeks. It consists of assigned →flight duties being compatible with the individual →skeleton roster, besides incorporating with all the governmental, union and company rules as well as with the</td>
</tr>
</tbody>
</table>
Glossary

crew member’s individual work history and remaining contracted flight/work duration

Mixed Integer Programming Mathematical programming with integer and fractional variables

Mixed Integer Quadratic Programming Mathematical programming with a quadratic objective function and integer and fractional variables

NP-hard A problem class defined by the following criteria: Once there is an algorithm available for solving a single nondeterministic polynomial time (NP)-problem, it can be translated into one for solving any other problem of such type; NP-hard therefore means at least as hard as any NP-problem, although it might in fact be harder, see (Weisstein 2005)

Null roster A roster without any flight legs assigned that is still compliant to the fingerprint of the crew member

Off-day Day inside the roster where there is no flight activity assigned

Onboard crew Same as crew

Operational cost Composed by crew salary, hotel overnights, and expenses for crew transits

Operations phase The crew recovery problem has to be solved for the recovery period directly of the occurrence of a disruption, e.g., absentness of crew, etc.

Pairing A piece-of-work which starts from and returns to the crew member’s home base. Its maximum duration is limited by a given upper bound, e.g., five working days

Pairing generation Applying the crew pairing problem

Personalized rostering A common crew scheduling approach in Europe where the crew member’s individual rosters are built based of a fair-and-equal share of workload without any seniority during the assignment

Piece-of-work The smallest unit that is considered by the problem examined, e.g., a flight leg for the crew pairing problem, a pairing for the crew assignment problem, and a shared flight activity for the Team-oriented Rostering Problem
Pilot  Same as →captain

Planning period  Time period being considered during the →crew scheduling; usually 14 days to one month

Planning phase  The →crew scheduling problem is solved for the →planning period several weeks or days ahead of its execution

Point-to-point network  Network topology usually found in Europe where theoretically every →flight legs are possible among all →airports involved

Pre-defined teams  A →team construction approach by which teams are fixed ahead of the →crew scheduling problem

Preferential bidding systems  A common →crew scheduling approach in the US and Europe where the →crew member’s individual preferences are considered together with their →seniority during the →assignment

Pre-scheduled activities  Represent activities that a crew member has to undertake without exception, such as vacation, requested-and-granted off-periods, office, simulator, training, medical examinations, etc.

Pre-selected teams  A →team construction approach by which teams are pre-selected ahead of the →crew scheduling problem; the optimal team is chosen out of those limited combinations of →crew members

Purser  Highest crew position in an Airbus fleet. There is a →downgrading possible to work also as a →chef-de-cabin, as well as to a →cabin attendant

Recovery period  Time period being considered during the →crew recovery problem; usually much shorter than the →planning period, e.g., several hours or days

Rest periods  Time period to be spent at least in-between to consecutive →flight activities; the off-time duration depends, for instance, on the start of the first flight leg and the number of flights serviced

Roster  Same as →line-of-work

Roster combination  Two (or more) →rosters are considered simultaneously for the →Team-oriented Rostering Problem

Rostering  Same as →assignment
<table>
<thead>
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<tr>
<td>Rostering loop</td>
<td>Describes a repeated execution of a rostering procedure proposed for addressing the →<em>Team-oriented Rostering</em></td>
</tr>
<tr>
<td>Schedule</td>
<td>Same as →<em>crew schedule</em></td>
</tr>
<tr>
<td>Scheduled activities</td>
<td>Same as →<em>pre-scheduled activities</em></td>
</tr>
<tr>
<td>Scheduling period</td>
<td>Same as →<em>planning period</em></td>
</tr>
<tr>
<td>Seniority</td>
<td>A general indicator for the time span that a →<em>crew member</em> spent on working for the airline; it can also be influenced by, e.g., language or regional knowledge</td>
</tr>
<tr>
<td>Set Covering Problem</td>
<td>Mathematical program formulation which defines that all constraints are at least covered, here: all →<em>flight activities</em> are covered, while →<em>deadheading</em> is allowed</td>
</tr>
<tr>
<td>Set Partitioning Problem</td>
<td>Mathematical program formulation which insists that all flights constraints are exactly covered as required, here: all →<em>flight activities</em> are partitioned among all →<em>crew members</em></td>
</tr>
<tr>
<td>Shared Flight Activity</td>
<td>A →<em>piece-of-work</em> considered for the →<em>Team-oriented Rostering Problem</em> which is serviced by a →<em>team</em> of →<em>crew members</em> without any →<em>team change</em></td>
</tr>
<tr>
<td>Skeleton roster</td>
<td>Same as →<em>fingerprint</em></td>
</tr>
<tr>
<td>Target flight duration</td>
<td>Calculated time period that should be assigned to →<em>crew members</em> in order to achieve a fair-share of workload</td>
</tr>
<tr>
<td>Team</td>
<td>A group of two or more people, here: →<em>crew members</em></td>
</tr>
<tr>
<td>Team change</td>
<td>Occurs if at least one →<em>crew member</em> is scheduled to serve the next →<em>flight activity</em> together with a different →<em>team composition</em> (other colleagues)</td>
</tr>
<tr>
<td>Team change penalty</td>
<td>Tries to evaluate a dedicated →<em>roster combination</em> examined; according to the →*team change type, and the →<em>team change degree</em> encountered; it can be calculated by, e.g., summing up the products of fixed penalty values times their occurrence</td>
</tr>
<tr>
<td>Team change type</td>
<td>A →<em>team change</em> can be categorized according to the time (during the day, between →<em>flight duties</em>, etc.) and location (e.g., outside the →<em>home base</em>) of its occurrence</td>
</tr>
<tr>
<td>Team change degree</td>
<td>For more than two →<em>crew members</em>, a →<em>team change</em> can be categorized according to the separation of this group</td>
</tr>
<tr>
<td>Glossary</td>
<td></td>
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<td>------------------------------------------------------------------------</td>
<td>---------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td><strong>Team composition</strong></td>
<td>It is formed as a set of →crew functions which (among other requirements) represent a →team that has to serve together for the airline</td>
</tr>
<tr>
<td><strong>Team construction approach</strong></td>
<td>A way how →teams are built; there are three kinds mentioned: →pre-defined teams, →pre-selected teams, and →free team building</td>
</tr>
<tr>
<td><strong>Team orientation</strong></td>
<td>Emphasizes on the consideration of →team stability for the crew scheduling problem; the expected major benefits are higher security and crew satisfaction</td>
</tr>
<tr>
<td><strong>Team stability</strong></td>
<td>Achievable by keeping the same →team composition together for a certain time period, usually several days; it can be enforced by, e.g., minimizing the amount of →team changes encountered during the →scheduling period</td>
</tr>
<tr>
<td><strong>Teaming</strong></td>
<td>Same as →team orientation</td>
</tr>
<tr>
<td><strong>Team-oriented Rostering</strong></td>
<td>Application of the →Team-oriented Rostering Problem</td>
</tr>
<tr>
<td><strong>Team-oriented Rostering Problem</strong></td>
<td>Search for an appropriate set of individual →rosters (one roster for each crew member) such that all flights given are covered properly while certain →team stability is achieved</td>
</tr>
<tr>
<td><strong>Team-oriented Scheduling</strong></td>
<td>Application of the →Team-oriented Scheduling Problem</td>
</tr>
<tr>
<td><strong>Team-oriented Scheduling Problem</strong></td>
<td>→Crew Scheduling Problem of flight personnel by which a certain degree of →team orientation is demonstrated</td>
</tr>
<tr>
<td><strong>Time-space network</strong></td>
<td>Network modelling technique which considers time and location of the elements examined</td>
</tr>
<tr>
<td><strong>Transfer</strong></td>
<td>Same as →crew transit</td>
</tr>
<tr>
<td><strong>Transit</strong></td>
<td>Occurs if a →crew member’s time-dependent location does not equal to the next scheduled location; a pre-transit is necessary in the case that this relocating is directly required in advance of the next flight leg scheduled, a post-transit for its direct succeeding occurrence; it is usually realized via public transportation (e.g., bus, taxi or train), or →deadheading</td>
</tr>
<tr>
<td><strong>Weekly rest time</strong></td>
<td>Rest time after a working week, e.g., five consecutive days</td>
</tr>
</tbody>
</table>