Numerical approximation of the Stokes equation using artificial boundary conditions in a system of pipes

Stephan Blazy

Abstract

In this thesis we consider the numerical approximation of the Stokes equation in three dimensional domains Ω with several cylindrical outlets to infinity.

Boundary value problems of this type require also asymptotic conditions at infinity, either hidden in the choice of function spaces for the solution, or in an explicit form, for example, by prescribing the flux or the pressure drop in each outlet. Computational work, of course, can only be done on finite domains. Therefore we cut every outlet of the domain. The resulting domain is named Ω_R. We assume that the original problem is preserved on Ω_R, at least an additional boundary condition has to be imposed on $\Gamma = \partial\Omega_R \backslash \partial\Omega$.

In chapter 1 we introduce the geometry of the domain, notations of function spaces and we recall the main results for the Stokes problem in the unbounded domain. Next we derive existence and uniqueness results for a class of mixed boundary value problems in the bounded domain Ω_R. At the end of chapter 1 we prove an error estimation for $|u - u_R|$, where u is the solution of the Stokes system in Ω and u_R in Ω_R.

In chapter 2 we describe the discretization of the domain Ω_R using tetrahedrons and show some interpolation properties. Then we consider the weak formulation of the problem and define some function spaces. Here we use the mini element approach for the numerical approximation, i.e. we use a P1-P1 element (linear for the pressure and velocity), where the velocity function space is enhanced by a bubble function on each tetrahedron. To show existence and uniqueness for the discrete problem, we prove the Babuska-Brezzi condition. Next we give an error estimation for the discrete problem. At the end of this chapter we describe the emerging linear systems of equations.

In chapter 3 we present the numerical simulation method, model problems and numerical results. The implemented algorithms for the discrete system are based on the parallel data-structure PadFEM (**P**arallel **ad**aptive **F**inite **E**lement **M**ethod). After an introduction to PadFEM, we present numerical computations, where different boundary conditions are compared on model problems. Finally we show some examples for numerical standard tests.