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Routing and scheduling for home care services

Solution approaches for static and dynamic settings

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Contents

Abbreviations

ADL	activities of daily living
ALNS	adaptive large neighborhood search
B&P	branch-and-price
CG	column generation
conVRP	consistent vehicle routing problem
CP	constraint programming
DARP	dial-a-ride problem
DSS	decision support system
EA	evolutionary algorithm
ETPHD	employee timetabling problem with high diversity of shifts
EU	European Union
GA	genetic algorithm
GGA	gender-based genetic algorithm
GRASP	greedy randomized adaptive search procedure
HHCP	home health care problem
IADL	instrumental activities of daily living
LNS	large neighborhood search
LP	linear program
MIP	mixed-integer program
mTSPTW	multiple traveling salesman problem with time windows

NRP	nurse rostering problem
OR	operations research
OR/MS	operations research and management sciences
PSO	particle swarm optimization
RCSPP	resource-constrained shortest path problem
RVNS	reduced variable neighborhood search
SA	simulated annealing
TS	tabu search
TSP	traveling salesman problem
TSPTW	traveling salesman problem with time windows
VND	variable neighborhood descent
VNS	variable neighborhood search
VRP	vehicle routing problem
VRPTW	vehicle routing problem with time windows
WHO	World Health Organization

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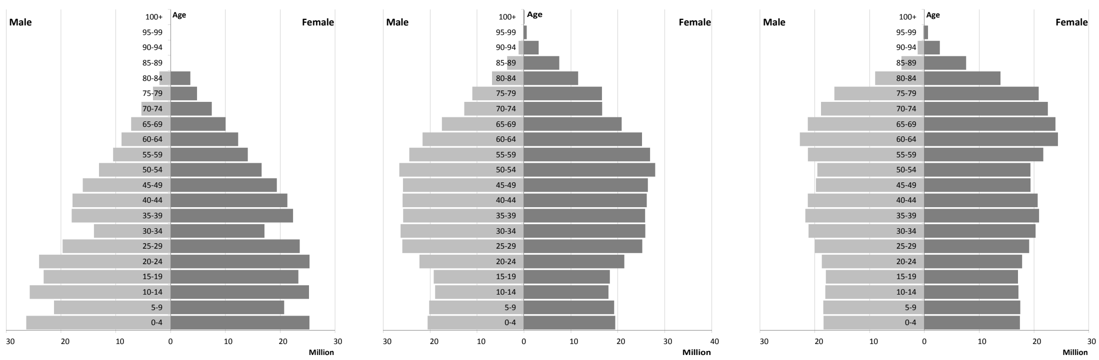
1. Introduction

Long-term care supports people in need of care with their activities of daily living [Colombo et al., 2011, p.39]. It has a great importance in society today and the demand is expected to grow in the next years and decades [European Commission Economic and Financial Affairs, 2015, p. 144]. One reason is the demographic change that leads to an ageing society [Colombo et al., 2011, p.62]. The age distribution in Europe for the years 1950, 2015 and 2050 is shown in the age pyramids in Figure 1.1. In 1950 only a small part of the populations was aged over 70 years whereas in 2015 the entire distribution is shifted upwards with a clearly higher portion of people aged over 70 years. The projection for the year 2050 shows that this trend will continue. With the ageing society also the demand for long-term care grows [Colombo et al., 2011, p.62].

In the provision of long-term care, home care is an alternative to residential care in many cases. Home care is any type of care service provided to persons in their own homes consisting of assistance with daily living, household and nursing activities [Genet et al., 2012, p.9-10]. In connection with a growing demand for long-term care, the demand of home care is also increasing. Additionally, there are several reasons why home care in particular will become more important in the near future. Amongst others, these are technology advancement in medicine and the preference of people



(a) 1950 (b) 2015 (c) 2050

Figure 1.1.: Demographic change in Europe. Age distribution in Europe for the years 1950, 2015 and 2050 (Data source: United Nations - Department of Economic and Social Affairs - Population Division [2015])

staying home as long as possible [Tarricone and Tsouros, 2008]. Furthermore, many OECD countries encourage home care to face the growing demand for long-term care [Colombo et al., 2011, p.39]. In most of the OECD countries the share of recipients receiving home care exceeded those in residential care facilities in 2008 [Colombo et al., 2011]. Smaller family sizes, women participating in the labor market and greater mobility of family members promote a shift from care by relatives to formal care providers [Tarricone and Tsouros, 2008].

In comparison to residential care, the daily management and planning in home care is more complex because of the geographical dispersion of clients [Milburn, 2012]. Not only the duty plan of nurses, but also the driving routes need to be determined. During the planning many requirements need to be respected. The complexity increases with the number of clients, employees and regulations and lead to a challenging task. Quantitative approaches from operations research (OR) have the capability to ensure efficient and legal plans as well as consider many requirements at the same time. In many countries home care is funded by public authorities [Colombo et al., 2011], and more efficiency could reduce the cost pressure. Furthermore, modeling preferences of clients and nurses improves the service quality, satisfaction and work-life-balance.

1.1. Scope of the thesis

Many planning problems arise in the context of home care planning for public and private providers, whereas an important one is addressed in this thesis: Support the home care providers by proposing solution approaches for the operational planning that has to be performed on a regular basis. This thesis focuses on the routing and scheduling task originating from the geographically dispersed locations of the clients. There is no differentiation between home care and home health care because for both the routes of the care givers need to planned. Furthermore, the methods are suitable for private and public providers. In both cases the scheduling of appointments for clients and routing of employees arises. During the planning many restrictions have to be considered. These are time restrictions on the services (e.g. time windows due to medical reasons) and working regulations (e.g. labor law regulations and work contracts).

The home health care problem in literature considers the routing and scheduling problem in this context to determine the daily plans for the nurses and client. The task combines components from the well-known vehicle routing problem with duty scheduling problems from the health care sector. Additionally, there are home care related constraints such as the qualifications of nurses that must be considered to assign the routes to nurses. The routing problem in home care services has received broad attention in the literature [Sahin and Matta, 2014]. However, the consideration of working regulations, e.g., labor laws, work contracts or personal agreements, is essential for acceptance in practice. Furthermore, the integration is crucial to ensure compliance with the regulations. Many publications consider constraints such as the maximum daily working time and simple break rules, but abstract from further working regulations.

Therefore, the first part of this thesis contributes to the state-of-the-art by integrating legal working regulations and work contracts into the operational planning of home care providers. To achieve this, the following research goals are defined:

1. Integrate relevant working regulations to the routing and scheduling for home care providers in a static multi-day setting
2. Solve the integrated routing and scheduling for real-world sized problem instances
3. Evaluate the influence of working regulations on working hours and compliance with labor law regulations

To achieve these goals, quantitative solution approaches taking those restrictions into account are developed. The methods adapt and integrate many regulations and add new components resulting from practical requirements. The first step is the formalization of the problem setting as a mixed-integer program resulting in an exact solution approach. Afterwards, three heuristic approaches are developed and compared to provide a solution method that is capable of computing feasible and efficient schedules in a reasonable computation time. All methods are evaluated on artificial test datasets, which are published online, and instances provided in literature. A detailed numerical analysis determines the performance of the methods in comparison to each other and to the exact approach. Based on the capability of the methods to consider extended working regulations, they are used for an evaluation of the influence of those restrictions on the working time and feasibility of the schedules.

The methods mentioned above solve the routing and scheduling of home care providers for a static setting. This means that the clients and nurses as well as their demands and requirements do not change in the considered planning horizon. However, in practice the operational planning task needs to be solved periodically due to the dynamic environment of changes in clients' health statuses and newly admitted clients. At the point of planning the current situation cannot be seen isolated from the previous and future periods. The past decisions have major implications on the quality and capacities in the future. On the one hand, working regulations like maximum working times and rest time requirements span often more than one planning horizon. On the other hand, for the mainly elderly clients, the continuity of services is important. Thus, the number of different nurses should be minimized and changes in visit times avoided. These quality indicators need to be ensured across planning horizons to improve satisfaction. The second main contribution of this thesis is to support the planning of home care providers in a dynamic setting. Two research goals in this thesis address this setting:

4. Incorporate the feasibility of working regulations and continuity between planning periods
5. Evaluate the influence of the different continuity metrics on the solutions

Several possibilities to model continuity in a dynamic setting are proposed and evaluated. The heuristic solution approach is extended to a rolling horizon approach that takes the changing demands into account while ensuring feasibility and continuity between periods. The numerical results are analyzed concerning the length of the planning interval and different scenarios of fluctuations.

1.2. Structure

This thesis comprises of eight chapters and is structured as follows. Chapter 2 gives an short introduction to home care services in the context of the health care system and shows the development in the sector. Furthermore, an overview of planning problems arising for home care services is provided and the problem setting in this thesis is described. The relevant methods from OR and related planning problems are presented in Chapter 3. The literature review of solution approaches for home care routing and scheduling in static and dynamic settings is given in Chapter 4. The research opportunities and goals of this thesis are derived from the state-of-the-art in Sections 4.4 and 4.5.

The description of how the goals are achieved starts with a mathematical formalization of the problem for a static setting in Chapter 5. The chapter is concluded with a detailed numerical analysis. The heuristic solution approaches are addressed in Chapter 6. Three heuristics based on the metaheuristics large neighborhood search, adaptive large neighborhood search and variable neighborhood search, are described in detail. The parameters for the heuristics are determined with an algorithm configurator. Based on the results of the parameter tuning, the evaluation of the methods is performed in an extensive numerical analysis that takes the observed empirical distributions of results into account. After determining the most suitable heuristic, the chosen method is used for evaluating the impact of introducing working regulations in Section 6.8.

The extension of the heuristic solution approach to a dynamic setting is addressed in Chapter 7. First, the adapted parts of the heuristic are described, followed by the formalization of different metrics to achieve continuity between planning periods. Afterwards, the method is embedded into a rolling horizon approach to provide a method to support the regular planning task of a home care provider. The numerical analysis determines the metrics and configurations that are suitable.

Chapter 8 summarizes the thesis, provides a critical review of the results and achieved goals and an outlook on future research opportunities.

2. Planning home care services

This chapter gives an introduction to the field of home care and shows the opportunities of quantitative solution approaches to assist home care planners in their daily work. Section 2.1 describes home care services from a general perspective to provide background information of the field. In Section 2.2 the development of long-term care is shown to underline the growing importance of home care services. An overview of home care planning problems is given in Section 2.3 followed by the problem description for routing and scheduling in home care services in Section 2.4 that defines the scope of this thesis.

2.1. Definition and description of home care services

In addition to institutional and semi-institutional care, like nursing homes and day care centers, home care services are one possibility for the provision of long-term care [Leichsenring et al., 2013, p.20]. The World Health Organization (WHO) defines long-term care as follows:

> "*Long-term care* is the system of activities undertaken by informal caregivers [...] and/or professionals [...] to ensure that a person who is not fully capable of self-care can maintain the highest possible quality of life, according to his or her individual preferences, with the greatest possible degree of independence, autonomy, participation, personal fulfillment and human dignity." [World Health Organization, 2000, p.6]

Home care is often placed at the intersection of health care and social services, because services range from household and care (social) to medical activities (health care provision) [Tarricone and Tsouros, 2008, p.13]. [Genet et al., 2012, pp.9-10] give an overview of three typical types of activities:

1. *Basic Activities of daily living (ADL)* [Murray, 2008]: ADLs can be defined as "the tasks that are required for a person to be able to live in the community" [Murray, 2008, p.78]. Examples for basic ADLs are eating, bathing and mobility. Basic ADLs are sometimes called personal ADLs and normally required daily [Genet et al., 2012, pp.9-10].

2. *Instrumental activities of daily living (IADL)* [Murray, 2008]: IADLs consists of services that are related to independent living but not necessarily on a daily

basis. Examples are household activities, shopping and support using the telephone.

3. *Nursing activities* [Genet et al., 2012, pp.9-10]: This category includes services concerning medical activities that require trained or qualified staff, e.g., giving injections. Medical activities often need to be prescribed by a physician.

It can be differentiated between the terms *home health care* and *home care*. Castillo-Salazar et al. [2014] distinguish them by the activities carried out and the length of the stay. They state that home health care is often provided after a hospital stay to recover at home assisted by nurses for a limited period. In contrast, they define home care as the assistance to elderly and disabled people in their daily living, where the services include household and care activities and are normally for a long-term period. Madigan [2008] distinguishes home health care and home care not only by the type of service but also by the qualification of the employees. Thus, home health care is delivered by health care professionals whereas home care is delivered by aides. However, in both settings the care givers have to visit clients at their home to provide a service for which they potentially need a qualification [Castillo-Salazar et al., 2014]. The following definition by Genet et al. [2012] combines both and summarizes the term home care services:

> "*Home care* can be conceived of as any care provided behind someone's front door or, more generally, referring to services enabling people to stay living in their home environment." [Genet et al., 2012, p.9]

An overview of the types of care and the classification used in this thesis is given in Figure 2.1. Throughout this thesis the term home care is used for all three types of care activities, i.e., home care includes home health care.

Home care activities can be carried out by informal or formal providers [European Commission Economic and Financial Affairs, 2015, p.146]. Exceptions are the already mentioned nursing activities in home health care that need a special training. Informal carers are not employed to care for a person but are, e.g., spouse, relatives or friends of the person in need of care [Tejada, 2008]. Formal providers are single persons employed by clients or institutions servicing multiple customers [European Commission Economic and Financial Affairs, 2015, p.146], e.g., hospitals or home care providers.

The main costs for providing long-term care, and therefore home care, are the labor costs of the care takers [European Commission Economic and Financial Affairs, 2015, p.147]. But in comparison to inpatient institutions the provision with home care is potentially less expensive [Milburn, 2012, p. 281] [Slotala, 2011, p. 24] [Tarricone and Tsouros, 2008, p.6]. The financing of home care services differs by country.

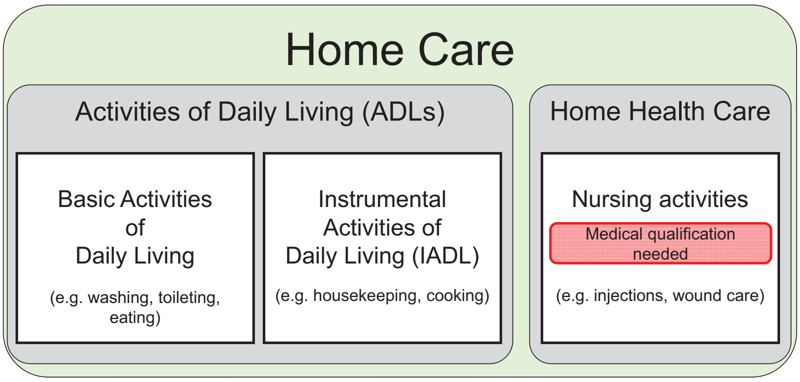


Figure 2.1.: Home care vs. home health care, including types of care (types of care based on [Genet et al., 2012, pp.9-10], [Murray, 2008])

In OECD countries the provision of long-term care is mainly funded by public authorities [Colombo et al., 2011, p.47]. Colombo et al. [2011] cluster the different systems in public funding in the following categories: single programs, means-tested safety-net schemes and mixed systems. The single programs for long-term care can be based on taxes (e.g. Denmark), insurances (e.g. Germany) or the health system (e.g. Belgium). The means-tested safety-net schemes fund long-term care only for people with income or assets below a certain threshold (e.g. USA). Mixed systems have different programs and benefits, sometimes dependent on the recipient or the income (e.g. France). The type of funding for recipients is distinguished mainly in cash benefits, allowances for informal carers or benefits in-kind [Colombo et al., 2011, p.49]. Most countries have both, cash and in-kind benefits, and the eligibility of receiving long-term care services is determined before receiving benefits [Colombo et al., 2011, p.50]. The financial support for long-term care is often based on the ability of a person to perform ADLs [Murray, 2008]. The market for private long-term care funding, e.g., by private insurances, is small and mainly common in the United States because the public funding is only for poor people [Colombo et al., 2011, p.248-249]. In other countries, the private insurances extend the services covered by public funding [Colombo et al., 2011, p.248].

2.2. Development of the home care sector

The home care sector has a major share in the provision of long-term care and is expected to become more important in the future. This section gives an overview of the past development and future projections.

For people in need of care home care is an alternative to inpatient institutions like elderly homes or hospitals. It provides the possibility to a person to stay in

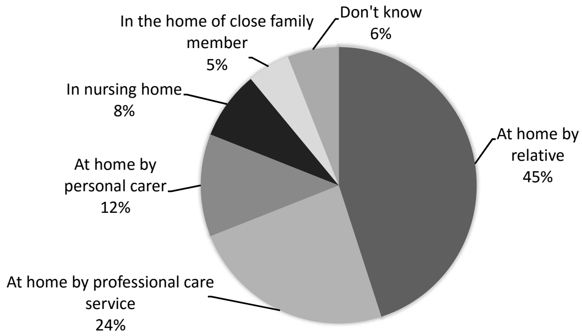
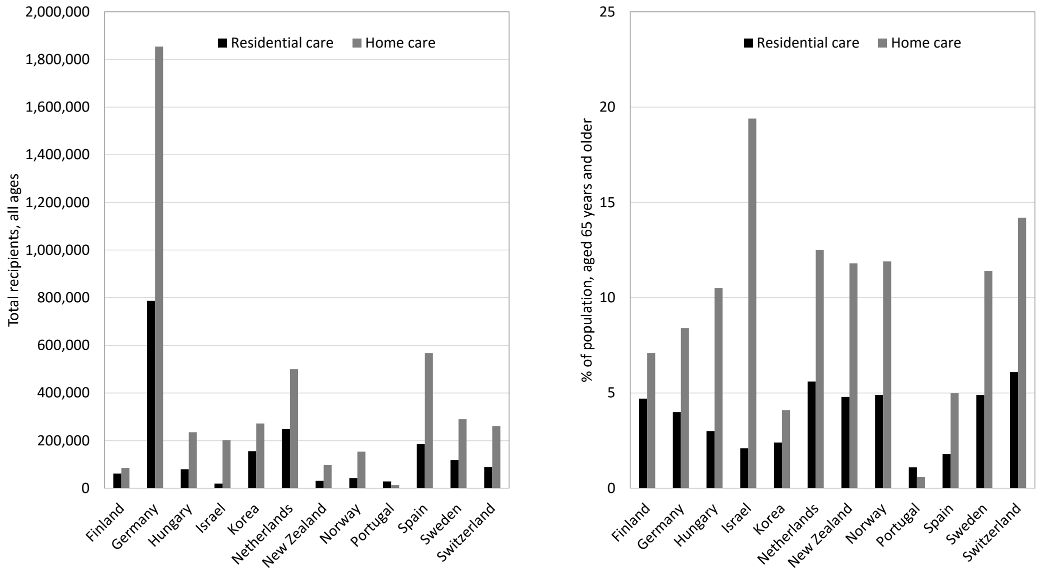


Figure 2.2.: Preferred long-term care arrangement in EU-27 countries based on the survey data of [European Commission, 2007, pp.95-97]

his or her familiar environment that is preferred by most of the people [European Commission, 2007, pp.95-97]. Figure 2.2 shows the results of a survey conducted in the European Union. According to the data provided by [European Commission, 2007, pp.95-97], the three most preferred types of long-term care (81%) all take place in the person's own home. The participants prefer to be cared after in their home by a relative (45%) or a professional home care provider (25%), if they would need a long-term care arrangement. The next preferred care alternative is the employment of a personal carer at home (12%). At last, the options moving to a nursing home (8%) or a close relative (5%) follow. From this results the preference of people staying at their own home is obvious.

Figure 2.3 shows the provision of long-term care split up by residential care and home care for selected OECD countries in 2013. The total and relative number of recipients underline the importance of and preference for the home care sector in the presented countries. In nearly all countries the number of persons receiving home care is substantially higher than those in residential care facilities, with the only exception of Portugal. In Germany more than 1.8 million people received home care in 2013 (Figure 2.3(a)). In proportion to the total population aged 65 years and older, other countries (e.g. Israel, the Netherlands and Switzerland) have a higher rate of long-term care clients, also in home care, than Germany (see Figure 2.3(b)).

The development of the formal home care sector in the last decade is shown in Figure 2.4. In most of the countries, the number of home care recipients increased constantly. Only in the Netherlands the client numbers are volatile. At the same time the number of persons in residential care facilities also increased for the same countries (not shown in the Figure). However, in Figure 2.5 the shares of both types at providing long-term care underline the growing importance of home care. From this figure it can be seen that the home care provision takes an increasing share in long-term care in most of the countries.



(a) Total number of recipients (all ages)

(b) Percentage of population aged 65 years and older receiving long-term care

Figure 2.3.: Formal long-term care provision in 2013 in selected OECD countries by type based on data of OECD [2016] (values for residential care exclude hospitals; home care includes care by non-professionals receiving payments and recipients of cash benefits, excluding only IADL recipients)

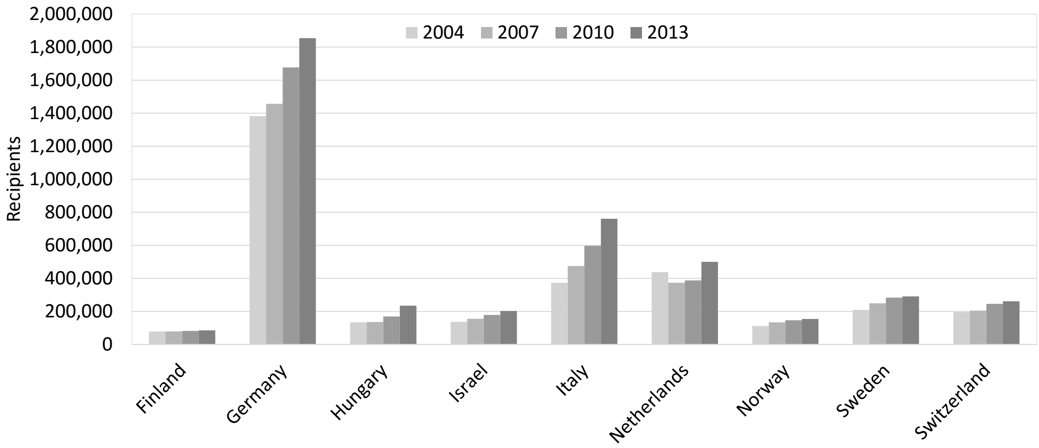


Figure 2.4.: Recipients of formal long-term care at home for different countries and the years 2004, 2007, 2010 and 2013 based on data of OECD [2016] (home care includes care by non-professionals receiving payments and recipients of cash benefits, excluding only IADL recipients)

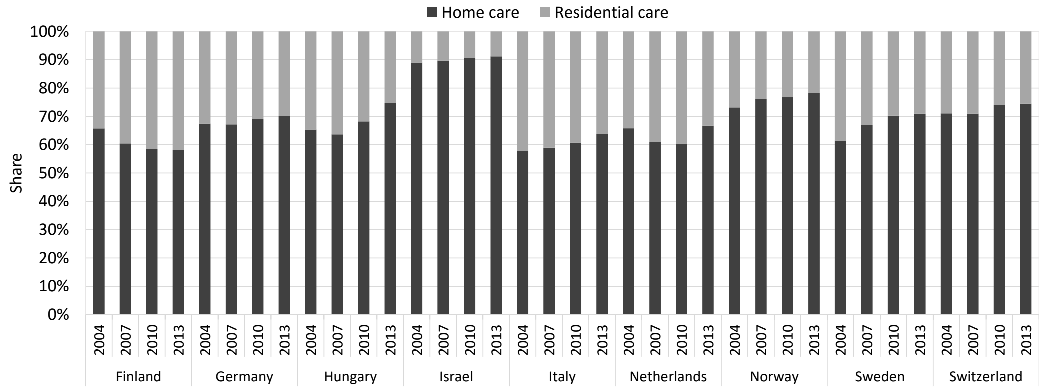


Figure 2.5.: Development of the distribution of residential and home care in the formal long-term care for different countries in the years 2004, 2007, 2010 and 2013 based on data of OECD [2016] (values for residential care exclude hospitals; home care includes care by non-professionals receiving payments and recipients of cash benefits, excluding only IADL recipients)

Experts state several reasons why the demand for formal home care will further increase in the next decades:

1. *Demographic change:* Due to the aging population, the number of people in need of care is expected to grow in the next decades [Tarricone and Tsouros, 2008, p.3][European Commission Economic and Financial Affairs, 2015, p. 144]. As an example, Figure 2.6 shows the projections of the German Statistical Office for people in need of care for the years 2030, 2045 and 2050 in Germany. The projections assume constant care dependency ratios based on the year 2013. However, even with the assumption of higher life expectation and healthier people becoming dependent on care later in their lives, the number is expected to increase in Germany [Statistische Ämter des Bundes und der Länder, 2010].
2. *Preference for home care:* Along with the number of long-term care recipients the number of home care recipients will increase [Tarricone and Tsouros, 2008, p.3]. As mentioned before, most people prefer staying in the own home [European Commission, 2007, pp.95-97] and expect a high quality provision of long-term [Colombo et al., 2011, p.38] and individualized care [Tarricone and Tsouros, 2008, p.5]
3. *Shift from informal to formal home care:* Currently, the greatest portion of home care is provided by informal carers [Fujisawa and Colombo, 2009, p.27] [Statistisches Bundesamt, 2013b, p.7]. The number of informal carers will decrease and more home care will be provided by professionals because of social changes. Families tend to be smaller in the future and therefore less informal

providers are available [Tarricone and Tsouros, 2008, p.3]. Furthermore, the current majority of informal caregivers are women but many of them are working full-time nowadays [Tarricone and Tsouros, 2008, p.3][Colombo et al., 2011, p.38,86] and therefore have less time to care for their relatives. Another reason is the mobility of people leading to longer distances between family members [Tarricone and Tsouros, 2008, p.4].

4. *Policy decisions:* Many countries prioritize and encourage home care over residential care [Colombo et al., 2011, p.39]. Mainly, because home care is less expensive and addresses preferences of the population [Tarricone and Tsouros, 2008, p.6]. Together with the shift from informal care to formal care, this will increase the demand for formal home care.
5. *Research and technology advancements:* Developments in medicine and technology influence the demand for home care. First, new pharmaceuticals and therapies increase life expectancy and lead to an aging population [Tarricone and Tsouros, 2008, p.5]. Second, new technology make home care available in cases where before only a stay in inpatient facilities (e.g. hospitals) was possible [Tarricone and Tsouros, 2008, p.5].

Based on these reasons we can conclude that the demand of formal home care will rise in the next decades. As a result, home care providers must cope with more clients, which introduces new challenges to the operational planning.

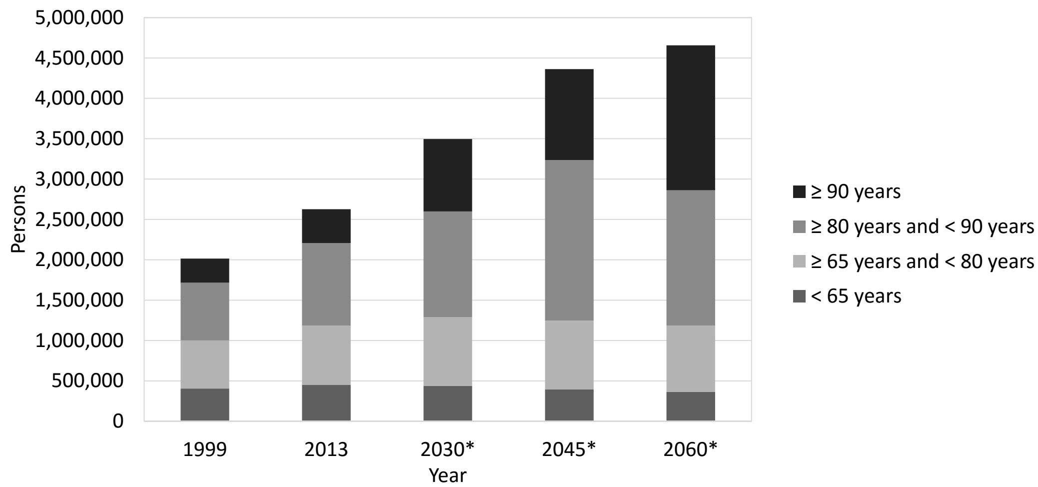


Figure 2.6.: Projections of people in need of care in Germany. Data source: Statistisches Bundesamt [2015] (For the projections (*) constant rates of age- and gender-specific dependency ratio of 2013 are assumed)

2.3. Planning problems of home care providers

In this section an overview of planning problems arising in home care services is given to set the context for this thesis (see Section 2.4). First, the definitions for the terms clients, nurses and providers used for the description of the planning problems are given as follows:

- *Client:* The people in need of care that are serviced by a home care provider are called *clients*. They live at their own homes and request one or more services per week.

- *Nurse:* The employees of a home care provider are called *nurses*. The nurses travel to the clients and perform the requested services at the clients' homes. According to the training of the nurse, he or she can carry out tasks requiring different *qualifications*. For example, medical services can only be fulfilled by a registered nurses. In contrast to housekeeping activities, which can be done by every employee, including registered nurses.

- *Provider:* The home care *provider* employs the nurses to fulfill the requests of the accepted clients. Furthermore, the provider is responsible for the duty planning of the nurses and also for determining the appointments of the clients. Thus, the office of the provider is the place where decision support techniques can be used to assist the person in charge in planning the schedules for the next weeks.

A broad overview of planning problems arising in the health care context is given by Hulshof et al. [2012]. The review focuses on publications in operations research and management sciences (OR/MS) and classifies the relevant literature in six service areas. The resulting categories are depicted in Figure 2.7. The definition for each category by Hulshof et al. [2012] can be summarized as follows: *Ambulatory care services* comprise of health care services where the patient does not stay in an institution after the treatment. This includes primary care and outpatient clinics. In contrast to this, *inpatient care services* consider the health care services provided to clients in institutions like hospitals. *Emergency care services* are handled in urgent situations. Planning problems in this area consider ambulances and emergency departments of hospitals. Planning problems for *surgical care services* consider mainly capacity planning for surgery facilities like the operating rooms. *Residential care services* and *home care services* concentrate on offering assistance with ADLs. The former services are performed in an institution like an elderly home whereas the latter is provided in the patient's homes. The focus in this thesis lies on the sector of home care services.

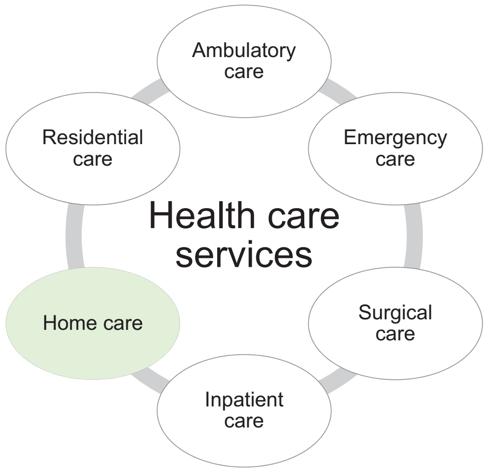


Figure 2.7.: Health care services according to Hulshof et al. [2012]

The terms home health care and home care are sometimes not clearly differentiated in the OR/MS literature on home care planning (see also discussion in Section 2.1), but as the problems incorporate the same components, publications from both settings are reviewed throughout this thesis.

Gutiérrez and Vidal [2013] and Gutiérrez et al. [2013] provide a systematic review of planning problems in the management of home care operations with focus on decisions in logistics. The authors propose three different dimensions with categories to classify the relevant literature:

1. *Planning horizon:* strategic, tactical, operational
2. *Management decisions:* network design, transportation management, staff management, inventory management
3. *Service processes:* medical services, patient services, support services

An overview of the first two dimensions is given in Figure 2.8. The problem setting in this thesis is located in the staff assignment and staff routing categories, because the integrated problem of these two components is considered.

Sahin and Matta [2014] also provide a review on operations management in home care. They give information about the complexity of the processes and summarize relevant literature in the field. The proposed classification scheme is based on the length of the planning horizon: long-term, mid-term, short-term and very-short term decisions. For an overview of the planning problems on the individual levels see Figure 2.9.

The following description of planning problems arising in the home care context is based on the classifications of Gutiérrez et al. [2013] and Sahin and Matta [2014]. Thus, the description is related to the Figures 2.8 and 2.9.

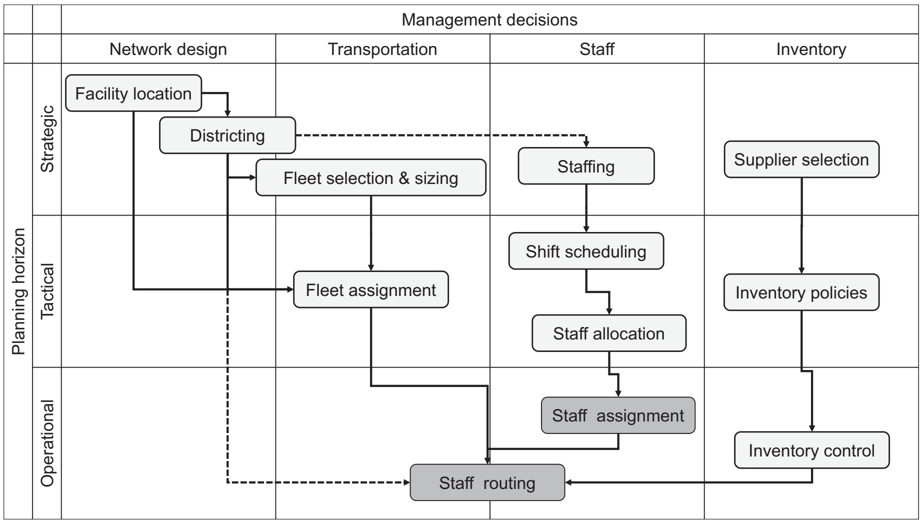


Figure 2.8.: Framework of Gutiérrez et al. [2013] to classify management decisions in home health care logistics (see Figure 1 in Gutiérrez et al. [2013], only dimensions one and two are shown here)

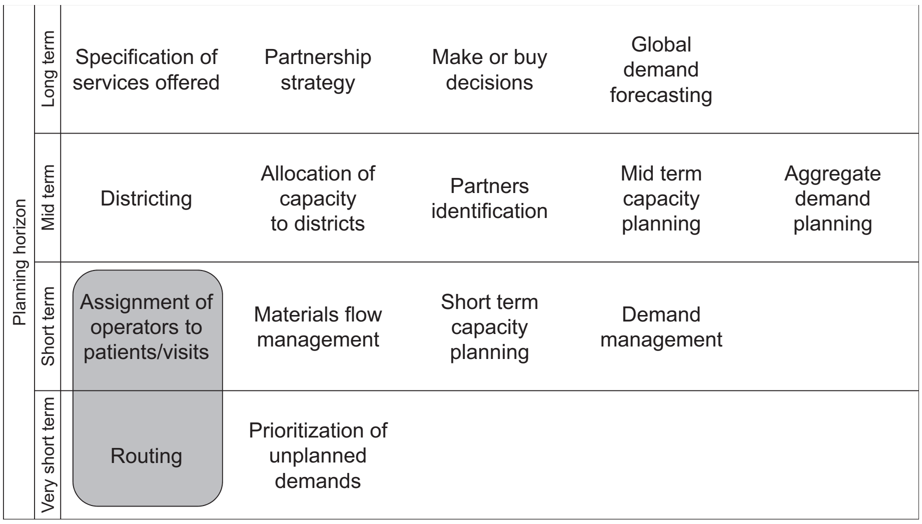


Figure 2.9.: Framework of Sahin and Matta [2014] to classify operations management decisions in home care (see Figure 2 in Sahin and Matta [2014])

2.3.1. Strategic planning problems

On the strategic level home care providers have to face several decisions. One is the determination of the service mix, i.e., which services the provider should offer to his potential clients [Hulshof et al., 2012, Sahin and Matta, 2014]. Furthermore, the location of the provider needs to be determined [Gutiérrez et al., 2013]. Also, the long-term capacity dimensioning takes place on a strategic level and includes staffing and equipment (e.g. vehicles) [Hulshof et al., 2012, Gutiérrez et al., 2013, Sahin and Matta, 2014]. Other problems are the strategic decision on suppliers of materials [Gutiérrez et al., 2013] and partnership decisions with other providers [Sahin and Matta, 2014]. The research on quantitative solutions approaches in strategic planning for home care is scarce. Sahin and Matta [2014] state they found no publication in the strategic area (see Figure 2.9 for their classification). In the review of Gutiérrez et al. [2013] quantitative solutions approaches are also mentioned only on the tactical and operational level.

2.3.2. Tactical planning problems

The decisions on a strategic level are the basis for tactical planning problems. One planning problem is the tactical determination of districts [Milburn, 2012, Sahin and Matta, 2014]. According to Blais et al. [2003] and Benzarti et al. [2013], the districting problem in home care consists of clustering small units of clients into larger units with the goal of balanced workload between districts. Districts are serviced by a team of nurses and thereby reduction of travel distances can be achieved. Benzarti et al. [2013] claim that smaller teams of nurses lead to a less complex planning problems and the number of different nurses assigned to a client can be reduced.

Furthermore, the strategic resource allocations can be refined on a tactical planning level by building care teams [Sahin and Matta, 2014], determining common work shifts or deciding on temporary staff to handle demand peaks [Gutiérrez et al., 2013]. These decisions are made based on the chosen districts [Hulshof et al., 2012, Sahin and Matta, 2014]. One example is the assignment of nurses to districts [Boldy and Howell, 1980]. Additionally, demand forecasting is done for a tactical planning horizon to include the results in the capacity dimensioning [Sahin and Matta, 2014].

2.3.3. Operational planning problems

On the operational level several planning problems arise. The *routing and scheduling* of home care nurses is the most widely studied problem in the OR/MS literature on home care [Gutiérrez et al., 2013, Sahin and Matta, 2014].

The routing and scheduling is characterized by Milburn [2012] as follows: A set of nurses provides services for a set of clients at the clients' homes. A client has several services defined for consecutive weeks. The services have time requirements, i.e., possible days and time windows on a day. The days of the services are not determined a priori but have to be selected during planning. The nurses have qualifications that they need to perform a service and their workday length and availability is given a priori. The outcome of a planning method is the assignment of day, nurse and time for each service at a client.

Common objectives range from economic goals to client and nurse satisfaction. Economic objectives aim at minimizing the travel time [Milburn, 2012] or balancing the workload between nurses (e.g. Cappanera and Scutellà [2014]). The latter also improves the satisfaction of nurses. Additionally, in some methods the preferences of nurses regarding clients are maximized (e.g. Trautsamwieser and Hirsch [2010]). Client satisfaction is achieved by maximizing the preferences of clients (e.g. Trautsamwieser and Hirsch [2010]) or continuity. Two continuity metrics widely used in home care planning are continuity of care and continuity of time. The former minimizes the number of nurses visiting a client during the planning horizon whereas the latter minimizes the deviation in visit times [Milburn, 2012].

The integrated problem of assignment of nurses to clients and routing decisions is often referred to as home health care problem (HHCP) [Cheng and Rich, 1998, Bertels and Fahle, 2006, Trabelsi et al., 2012] or home care routing and scheduling [Mankowska et al., 2014, Morito et al., 2014]. The planning problem and process for the integrated routing and scheduling underlying in this thesis is described in Section 2.4. An extensive literature review of the state-of-the-art in routing and scheduling is given in Chapter 4.

Another planning problem studied in OR literature is the *assignment of new patients* to home care nurses [De Angelis, 1998, Hertz and Lahrichi, 2009, Sahin and Matta, 2014]. Mostly, one so called reference nurse per patient needs to be selected for each new client while balancing the workload among nurses [Hertz and Lahrichi, 2009, Lanzarone and Matta, 2009]. In this way the continuity of care is ensured for each client as the assigned nurses is carrying out all services for the client. In some variants of this problem the assignment is not considered as hard requirement but reassignments are allowed [Lanzarone et al., 2012], the requirement is softened [Lanzarone et al., 2012] or omitted for some of the clients [Carello and Lanzarone, 2014]. The routing of nurses has to be determined in a second stage for each nurse independently [Yalcindag et al., 2012]. In this case, the routing can be solved as a traveling salesman problem (TSP) [Yalcindag et al., 2012] or traveling salesman problem with time windows (TSPTW). The demand of the clients is often assumed to be uncertain [Lanzarone and Carello, 2013, Carello and Lanzarone, 2014]. Methods applied to this problem are mixed-integer programming [Carello and Lanzarone, 2014], heuristics [Levary, 2015], cost policies [Lanzarone and Matta, 2012] and markov decision processes [Koeleman et al., 2012].

Another problem addressed is the *duty planning* of nurses by assigning them to shifts [Hulshof et al., 2012]. In this case, the routes need to be planned afterwards or given as input to the planning like in Wirnitzer et al. [2016]. In this planning problem working regulations for the nurses need to be considered [Wirnitzer et al., 2016].

The integration of inventories and materials like in a *supply chain* is also a potential extension to the routing and scheduling [Milburn, 2012]. An example is the integration of drug production and delivery for the chemotherapy at home in the publication of Chahed et al. [2009]. The specialty in the home care context is that goods can either be delivered to the client's home or the provider's office to be collected by the nurses on their way to the client [Milburn et al., 2012].

2.4. Routing and scheduling home care services

In the following sections the planning tasks considered in this thesis are described in detail. We begin with definitions and describe the planning process and requirements. Afterwards, the context of the problem setting in this thesis is elaborated.

2.4.1. Definitions

First, the terms clients, nurses and shifts are defined for the context of this thesis to provide a consistent usage in the literature review and description of the proposed methods and results.

Clients and jobs. A *client* (or patient) requires one or more home care services during the planning horizon. Throughout this thesis, the term home care is used for services comprising ADLs, IADLs and nursing activities. There is no differentiation between home care and home health care because both are services that need to be fulfilled by an employee of a home care provider. For each service a day and the duration is given. The days are negotiated between client and provider or given due to a medical prescription, e.g., an injection that has to be given daily. The services are represented by *jobs*. A job can bundle several services if they should be carried out by one nurse in one visit. The total duration of the job is determined by the included services.

Nurses. The jobs of clients need to be fulfilled by the set of available *nurses* employed by the home care provider. Each nurse has specific qualifications determined by the professional training the nurse received. For example, a nursing activity can be carried out only by a nurse with medical training whereas help with the household can be performed by any employee. The jobs that can be assigned to a nurse are

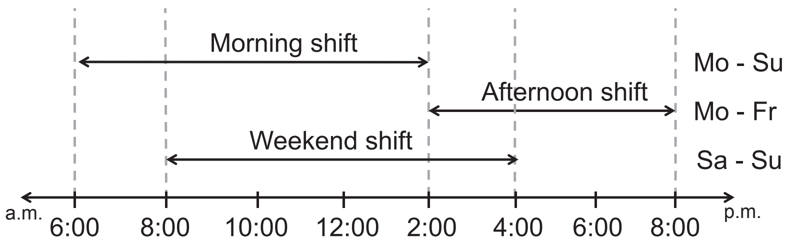


Figure 2.10.: Example for three possible shift types

further restricted due to availability and working regulations. Finally, the start and end location of the routes of the nurse need to be known to take driving times at the beginning and end of tours into account. The location can be either the provider's office or the nurses home.

Shifts. Like in inpatient institutions the workday is often divided into *shifts*, e.g., morning and afternoon shifts, to cover 24 hours service. A shift type is defined by its earliest start time and latest end time on a day. Furthermore, the valid weekdays for a shift type can be defined to allow different shift types on particular days. In practice the weekend is often handled differently than normal weekdays, e.g., there are less employees working longer shifts. An example for shift types is given in Figure 2.10. In our context the defined shift types provide a framework for the route planning and to structure the day. Every nurse works at most one shift per day. The working time is determined by the route and not the shift length.

2.4.2. Planning tasks and requirements

This thesis focuses on the planning process on an operational level at institutions that provide home care and/or home health care. This means the planning takes place at a home care provider or hospital employing several persons to take out care services for geographically dispersed clients. Furthermore, the methods are suitable for private and public providers. In both cases the described planning has to be performed.

The operational planning of a home care provider consists of two planning tasks: routing and nurse scheduling. The planning process is shown in Figure 2.11. In the *routing* task the provider groups the jobs to *tours* and determines the sequences of jobs (called *routes*). The visit times at the clients are defined based on the duration of the jobs in a tour and the driving times between two clients. The *nurse scheduling* determines the duty plan of the nurses. Therefore, the provider decides on the working times for each nurse on each day of the planning horizon. If shift types are defined, the duty plan is determined by the shift type assigned to each nurse.

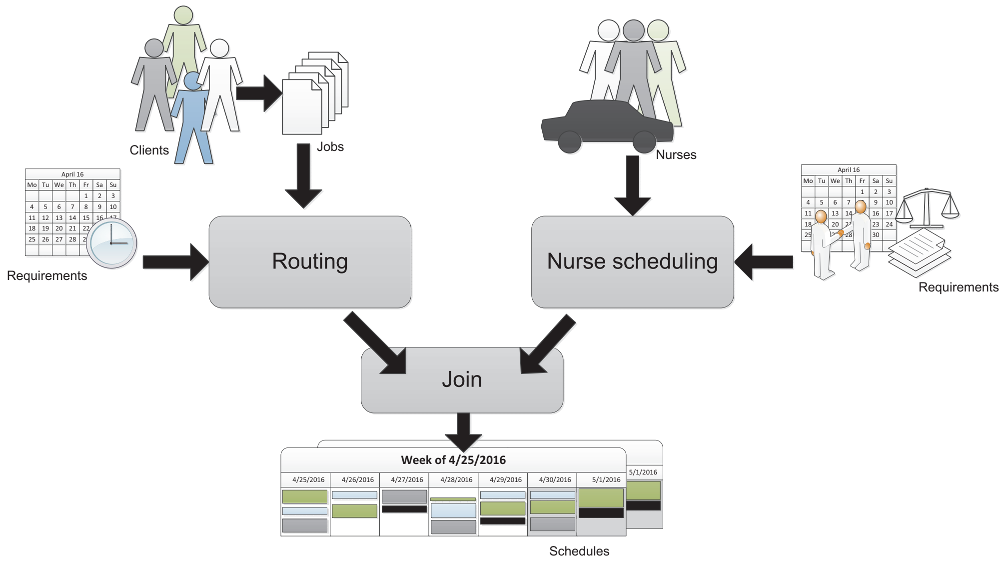


Figure 2.11.: Routing and scheduling planning process

After the tours and nurse schedules are determined, they need to be combined to a joint schedule. Either the routing is performed first and the nurses are assigned to the routes or the duty plan is generated first and the routes are build based on the available nurses. In both planning tasks several requirements must be respected. These are described in the remainder of the section based on the planning task they affect.

Requirements for the routing task

The requirements that have to be considered during route construction are the time windows of jobs and the break assignment inside routes.

Time windows. The start time of a job is restricted by a *time window*. Like the days, the time windows are either negotiated by provider and client or predetermined by a physician due to medical reasons. The latter time windows are often narrower because the nursing activities must ensure the healthiness of the client, e.g., by giving a medication at a specific time. The start times of jobs containing household help do not have narrow time windows but are assignable some time on the day. A time window can also restrict the start time due to preferences of the clients.

Breaks. The adherence to working regulations is important for the application in practice. The labor laws determine whether a schedule is legal or not. Many countries define regulations in labor laws. The European Union (EU) passed a directive to provide a framework of working hour regulations for the member countries [European Parliament and Council of the European Union, 2003]. The same elements are mostly also considered in the national working hours acts. Although the directive of the EU is cited in this section, the regulations from other countries can also be considered even if they state other limits. The rules concerning the routing task are the required breaks during working time. The directive states the following about breaks during the workday:

> "Member States shall take the measures necessary to ensure that, where the working day is longer than six hours, every worker is entitled to a rest break, the details of which, including duration and the terms on which it is granted, shall be laid down in collective agreements or agreements between the two sides of industry or, failing that, by national legislation."
> [European Parliament and Council of the European Union, 2003, §4]

The placement of *breaks* during working times can vary between countries, but most of them follow the same structure: a maximum time span without a break is defined. If this duration is expired, a break is needed. In some countries also stages of breaks are defined based on the working period lengths or the division of a break into smaller breaks is allowed. An example of break rules, derived from the database of the International Labour Organization [2012], is given in Figure 2.12 for four countries. In Germany, the 30-minutes break after six working hours or 45-minutes break after nine working hours can be divided, but each part has to be at least 15 minutes long [Bundesministerium für Arbeit und Soziales, 1994]. Note that breaks are not considered as working time [European Parliament and Council of the European Union, 2003].

Furthermore, the placement of a break in a route must be selected such that the maximum time without a break is not violated. A break placed directly at the beginning or end of the tour is not preferred and would possibly result in an uninterrupted working time after or before the break that is longer than allowed, respectively.

Requirements in nurse scheduling

In the nurse scheduling many requirements resulting from work contracts and working regulations need to be considered. Furthermore, the time windows of shift types and qualifications of nurses are addressed.

Time windows. The shift types define time windows for start and end times of tours because a shift divides the day into smaller units that are used for further working regulations.

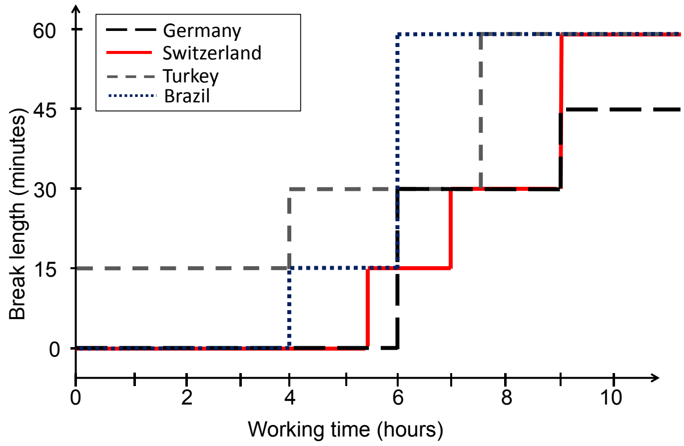


Figure 2.12.: Break rules resulting from the working hours acts of Germany, Switzerland, Turkey and Brazil [International Labour Organization, 2012]

Qualifications. Each nurse has a set of qualifications that determines which jobs he or she can carry out. If a job should be assigned to a particular nurse the requirement must be fulfilled. This aspect has to be considered when assigning nurses to routes or jobs to nurses.

The required *qualification* to perform a job depends on the type of service. Qualifications mostly represent the professional requirements but can also be used for further planning restrictions. If a provider wants to build special routes that are carried out by certain employees, an additional qualification can be added to the problem setting. A possible example is the meals-on-wheels services. No special training is needed to deliver the meals to the clients but the routes are assigned most likely to employees that are recruited for this special task. By adding a separate qualification this can be achieved during automated planning without assigning care takers for this routes and vice versa. A human planner would not explicitly define a new qualification but intuitively separates the set of nurses according to their functions.

Nurse availability. The availability of nurses on specific days and shifts can be limited due to vacations and agreements between provider and employee. Vacation days are known in advance and can be considered during planning. This kind of unavailability is called irregular. Regular shifts off can be due to specific agreements between provider and employee, e.g., every Monday afternoon off. Another example is a group of nurses working only in the morning, which is typical for home care services in Germany [Grabbe et al., 2006, p.37].

Work contracts. The maximum workload of a nurse is defined by the *work contract* closed between nurse and provider. The regulations from work contracts concern the working hours of each employee and should not be violated as overtimes are expensive and lead to lower employee satisfaction. The resulting restrictions are the daily working time as well as weekly or monthly working time. The contracts provide hard limits that have to be considered during planning. The European Parliament and Council of the European Union [2003] states the maximum weekly working time of 48 hours (including overtime) but this is often less restrictive than the working times agreed upon in work contracts. Additional to working times, agreements between provider and employees influence the weekly workdays. For example, there are two possibilities for part-time contracts: a nurse working three days a week for six and a half hours or five days a week only four hours per day. These agreements must be respected when planning the duty schedules.

Rest times. Additional to breaks during working hours, *rest times* between shifts have to be considered to ensure a legal plan. These rest times are again defined by the working hour acts of the governments. The framework for the daily rest time in the EU is addressed in the following article:

> “Member States shall take the measures necessary to ensure that every worker is entitled to a minimum daily rest period of 11 consecutive hours per 24-hour period.“ [European Parliament and Council of the European Union, 2003, §3]

This means every employee should be granted an uninterrupted rest of at least eleven hours before working again. Thus, in some cases the consecutive assignment of an early shift after a late shift is not possible. It is also common to define a weekly rest for uninterrupted duty-off time:

> “Member States shall take the measures necessary to ensure that, per each seven-day period, every worker is entitled to a minimum uninterrupted rest period of 24 hours plus the 11 hours daily rest referred to in Article 3.“ [European Parliament and Council of the European Union, 2003, §5]

Figure 2.13 shows an example of daily and weekly rest times fulfilling the criteria.

Cyclic shift pattern. The definition of shift types can be used to allow cyclic shift patterns to ensure a fair distribution of unpopular shifts among employees. A *cyclic shift pattern* determines the sequences of shifts and limits the shift assignments in the planning periods [De Causmaecker and Vanden Berghe, 2011]. Cyclic shift patterns

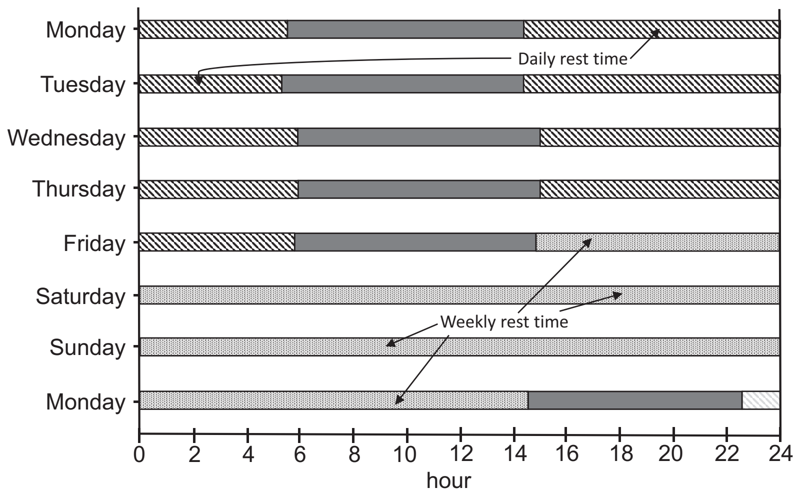


Figure 2.13.: Example for daily (dashed) and weekly rest times (dotted)

Shift rotation:

Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Shift type	M	M	M	M	M	-	-	A	A	A	A	A	W	W

Resulting shift pattern:

Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$p = 1$	M	M	M	M	M	-	-	A	A	A	A	A	W	W
$p = 2$	A	A	A	A	A	W	W	M	M	M	M	M	-	-

Figure 2.14.: Example for a two-week shift rotation and the resulting shift patterns (M = morning shift, A = afternoon shift, W = weekend shift, - = day off, p = pattern id)

are widely used in Germany for full-time workers in home care services. According to Grabbe et al. [2006, p.37], 32.9% of the persons in their study about home care workers in Germany worked in cyclic shift patterns. One typical example is the pattern given in Figure 2.14. The example shows a week of early shifts and weekend off followed by a week of late shifts with an assigned weekend afterwards. In this pattern two nurses are alternating their shifts to fulfill the demands without one nurse working only late shifts.

Further restrictions for duty planning. Further restrictions regarding the duty planning are widely used in inpatient institutions. One example is the maximum number of consecutive workdays [De Causmaecker and Vanden Berghe, 2011]. Furthermore, the weekend is not favored by most of the employees and the avoidance of single weekend shifts, i.e., both days or none should be assigned, improves nurse satisfaction [De Causmaecker and Vanden Berghe, 2011].

2.4.3. Objectives

Like in many planning problems originating from practical applications, there exist different goals to achieve during planning. The main costs for home care providers are the personnel costs for the nurses [European Commission Economic and Financial Affairs, 2015, p.147]. Furthermore, the income of a home care provider is often determined by the number and type of services it performs at clients. For example, in Germany each specific service is compensated with a certain amount and driving time with an allowance independent of the length [Simon, 2010, p.372-373]. Therefore, the economic goal from a provider's point of view is to reduce the driving times of the nurses to have more time to perform jobs at clients [Milburn, 2012]. Furthermore, the waiting time of nurses for a job time window to open is considered as working time and should be avoided to increase efficiency. To incorporate both, the *minimization of the tour lengths* while assigning as many jobs as possible is suitable. Note that the durations of jobs are fixed and not shorted during planning. As the breaks are included in the tour length, the artificial lengthening of tours to insert breaks is prohibited.

In contrast to the economic goal, *client satisfaction* is important for home care providers due to the competition between providers. For example, there is a free choice of providers in Germany [Simon, 2010, p.364] and the competition is mainly based on client satisfaction because most of the prices are fixed [Simon, 2010, p.372-373]. *Continuity of care* is one criterion for client satisfaction that is often used in previous research [Milburn, 2012]. The clients of home care providers prefer to be visited by the same nurse or only a small number of different nurses [Woodward et al., 2004]. This results from the fact that the employees enter the clients home and some of the services can be intimate, e.g., showering [Woodward et al., 2004]. Therefore, the number of different nurses during the planning horizon should be held small. Another criterion to support client satisfaction is the *continuity in time* [Milburn, 2012], i.e., to avoid large fluctuations in start times of jobs. The adherence to usual times is important for the clients to plan their daily life more easily. The times should not change from week to week but stay nearly the same.

The *satisfaction of nurses* is also important for home care providers because there is a high fluctuation of workers in the long-term care sector in OECD countries [Colombo et al., 2011, p.173]. Nurse satisfaction can be ensured by adherence to work contracts and working regulations, because it reduces overtime hours. Furthermore, it is important to incorporate the preferences of the nurses according to shift assignments. The agreements between provider and nurses regarding regular or irregular unavailability must be considered and the avoidance of single weekend shifts improves the satisfaction of nurses. The similarity of duty schedules is another aspect that can increase nurse satisfaction. If the duty schedules differ highly from week

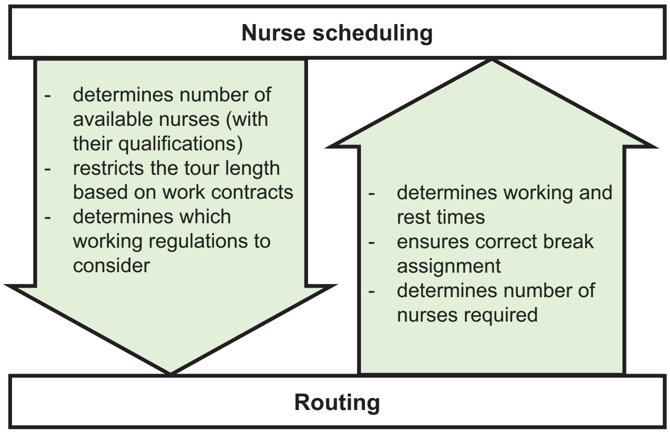


Figure 2.15.: Dependency between routing and nurse scheduling

to week, the nurses cannot plan their everyday life and must adapt to new working times every week. Therefore, the assurance of similarity should be modeled. We call this criterion *continuity of duty schedules*.

2.4.4. Dependency between routing and nurse scheduling

When the routes and duty plans are combined to a joint schedule several aspects have to be considered. It has to be ensured that both are matched with each other without violating any of the restrictions. During this task, the dependencies between both planning tasks become apparent (see Figure 2.15). Depending on how many nurses are planned by the nurse scheduling, the number of tours is limited. The qualifications of the scheduled nurses also determine which jobs can be considered together on the day and in one route. Furthermore, the working time of the nurses limits the length of the tours. If not enough nurses with the required qualifications and working hours are assigned, not all jobs can be considered in the routing. If too many nurses are assigned, the plan is inefficient. The routing also has influences on the nurse scheduling. The exact working times are known only if the routes are determined because the driving times depend on the sequence of jobs. The same holds for the start and end time of a tour which is essential to ensure rest times. Furthermore, the breaks must be scheduled in the route but the requirements of the break depend on the assigned nurse because the break types can differ between nurses, e.g., apprentices under 18 years need earlier breaks than trained employees in Germany [Bundesministerium für Arbeit und Soziales, 1994].

In contrast to perform both tasks isolated from each other, it is beneficial to consider the routing and nurse scheduling in an integrated approach because of these dependencies. In a two-phase approach there may be several revisions necessary until

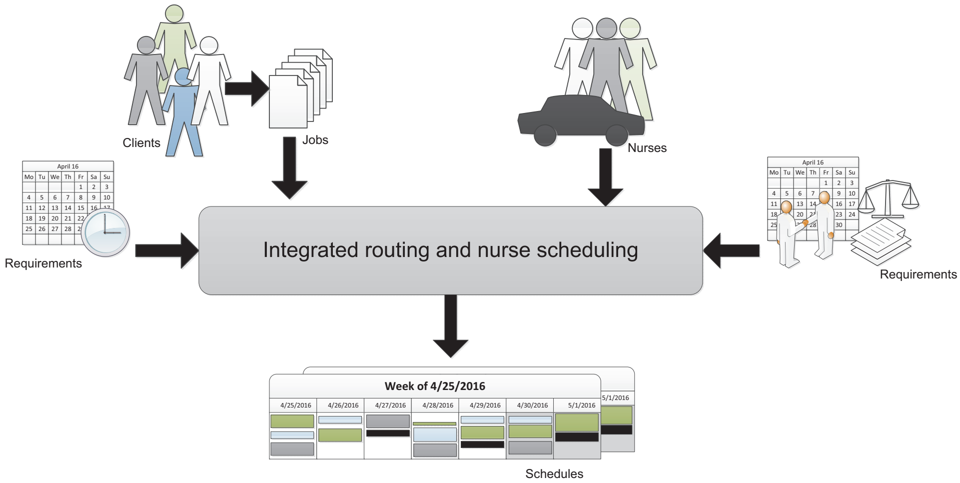


Figure 2.16.: Planning process for integrated routing and scheduling

both plans are coordinated and efficient. An integrated approach that considers all requirements at the same time reduces this effort (see Figure 2.16). Regarding the objectives presented in the previous section, the integrated approach also has several benefits. The continuity of care requirement can be considered directly during planning only if the nurse assigned to a tour is already known. The same holds for the continuity of duty schedules because the similarity depends on the assignment of nurses to tours. Thus, an integrated approach allows us to consider those criteria directly during planning. Besides the benefits of an integrated approach, the integration also increases the complexity and the planning task gets harder to manage for a human planner, especially in planning horizon of several days or weeks.

2.4.5. Problem settings

There are two different problem settings surrounding the integrated planning task that are considered in this thesis. First, the static setting where there are no changes of clients and nurses during a fixed planning horizon. Second, a dynamic setting where the demands of client and capacity provided by nurses can change and the planning has to be performed on a regular basis to approach these changes. Both settings are described in the remainder of this section. Figure 2.17 shows the differences in the planning process.

Static setting. The time horizon for the planning task can vary between providers. The duty planning in inpatient institutions is mostly done for a period of at least four weeks. This has the advantage that the employees know their working hours

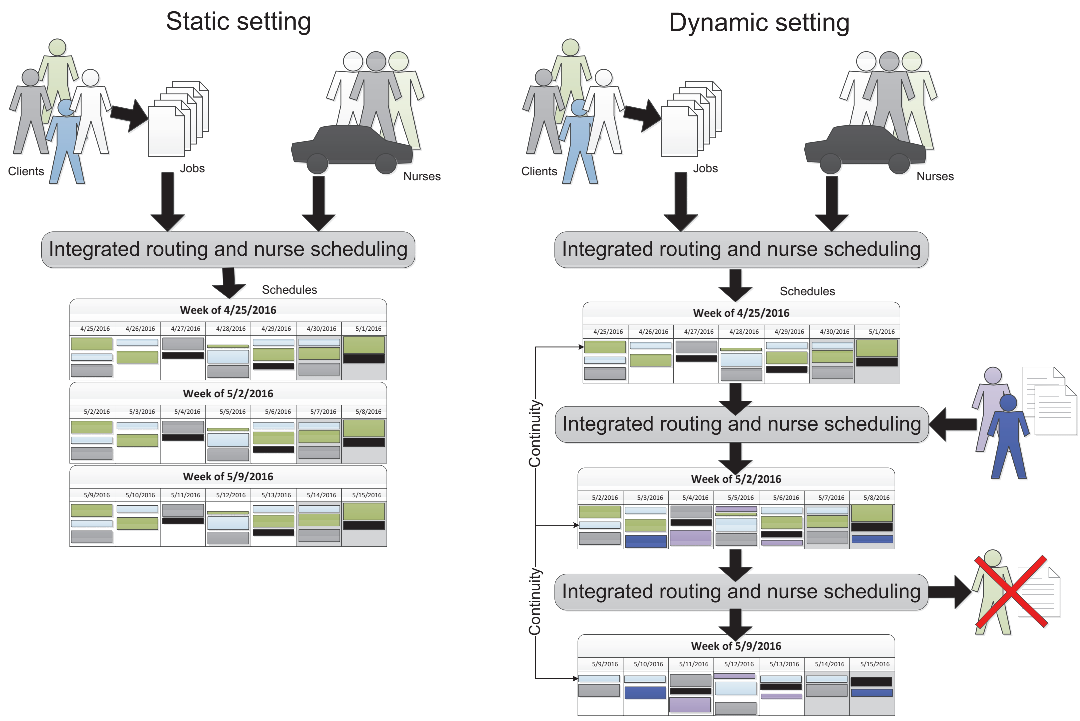


Figure 2.17.: Static vs. dynamic setting (requirements are not shown)

in advance and therefore can plan their everyday life. This is also important for the employees of a home care provider. In contrast to this, in practice the routing decisions are often determined for the next week. To let the providers be flexible with this decision, the planning horizon in this thesis is not fixed, but can be determined by the provider. Note that a planning horizon of at least seven days is necessary to ensure most of the working regulations, e.g., weekly rest times. Therefore, the test cases in this thesis mainly contain a planning horizon of at least a week. The demands of clients and the working hours of nurses are assumed to stay stable for the length of the planning period and the goal is to minimize the tour lengths.

Dynamic setting. In home care several changes of demands and capacities occur over time. New clients arrive or clients do not need service any more. The demand of clients can change to more or less services and new or different time windows or days can be requested. Additionally, new nurses need to be incorporated in the planning or routes canceled due to leaving nurses.

To handle these changes a planning period of several weeks in a one-time planning is not beneficial. The plan needs to be revised because the old plan can be infeasible for the current period. Therefore, the planning problem stated above is not only solved in a static setting of fixed clients and nurses, but also for the dynamic setting with changing demands of clients and capacities of working time. When the planning is carried on a regular basis (e.g. weekly), the changes can be incorporated in the new planning task. Furthermore, the previous period can be considered as input to respect working regulations for a longer period.

The essential part of the planning in a dynamic setting is that the continuity between periods is ensured. At the point of scheduling major changes in assignments of nurses and times must be avoided to ensure the client satisfaction. It is unacceptable to provide totally different visits times for the clients with changing nurses every week. The same holds for the nurses who do not want to have changing working times. Therefore, the objective function in the dynamic setting has to consider, besides the tour length, the continuity criteria leading to client and nurse satisfaction.

3. Basic models and solution approaches

This chapter introduces and describes methods and models that build the basis for this thesis. Section 3.1 provides an overview of operations research methods used for solving the home care routing and scheduling. Section 3.2 reviews planning problems that are related to home care planning, namely vehicle routing, nurse rostering and technician routing.

3.1. Operations research methods

Operations research (OR) aims at planning tasks more efficiently [Eiselt and Sandblom, 2012, p.1]. Therefore, planning problems are abstracted and solved with appropriate quantitative methods [Eiselt and Sandblom, 2012, p.4]. Amongst others, methods that fall in the area of OR are mixed-integer programming and heuristics [Eiselt and Sandblom, 2012].

3.1.1. Mixed-integer programming

Mixed-integer programs (MIPs) allow to formalize and solve planning problems within practical applications [Wolsey, 1998, p.3].

The general definitions in literature are often given for the case of maximization. Note that all definitions in this chapter are transformed to minimization problems for a consistent objective sense throughout this thesis. The following description of a MIP is based on [Wolsey, 1998, p.3]:

$$(MIP) \qquad \min cx + hy \tag{3.1}$$
$$Ax + Gy \leq b \tag{3.2}$$
$$x \in \mathbb{R}^n_+ \tag{3.3}$$
$$y \in \mathbb{Z}^p_+ \tag{3.4}$$

A MIP is defined for the column-vectors x and y representing variables. The variables x can take non-negative continuous values (3.3) whereas the y variables can take only non-negative *integer* values (3.4). If the y variables are limited to the values 0 and 1, they are called binary variables. The lengths of the x-vector and y-vector are n and p, respectively. The feasible values for x and y are restricted by m constraints that are represented by the matrices A (for continuous variables) and G (for integer variables) as well as the right-hand-side b (3.2). The objective is to find such variable values that minimize the linear function (3.1) with c and h being vectors containing the coefficients. All feasible solutions S of the MIP can be described by the following set [Nemhauser and Wolsey, 1999, p.4]:

$$S = \{x \in \mathbb{R}^n_+, y \in \mathbb{Z}^p_+, Ax + Gy \leq b\} \tag{3.5}$$

The feasible solution $(x^*, y^*) \in S$ is an optimal solution, if no other feasible solution has a lower objective value. This property is expressed in the following inequality [Nemhauser and Wolsey, 1999, p.4]:

$$cx^* + hy^* \leq cx + hy \quad \forall (x, y) \in S \tag{3.6}$$

When no integer variables are present in the formulation, the program reduces to a linear program (LP) of the form [Wolsey, 1998, p.3]:

$$(LP) \qquad \min cx \tag{3.7}$$

$$Ax \leq b \tag{3.8}$$

$$x \in \mathbb{R}^n_+ \tag{3.9}$$

The optimal objective value of a MIP can be calculated as $z^* = cx^* + hy^*$. During the solution process primal and dual bounds on this value are obtained. The bounds are defined as follows [Wolsey, 1998, p.24-25]: Every feasible solution $(x, y) \in S$ provides a primal (upper) bound on the optimal objective value, because the optimal solution must be at least as good as this value. A dual (lower) bound can be obtained by relaxation of the original MIP, i.e., replacing the problem by a model that is easier to solve for an approximation on the real objective value. One possibility is to enlarge the set of feasible solutions by dropping the integrality constraint of the y variables MIP, thus resulting in an LP, which is easier to solve. The resulting LP is called LP relaxation of the original MIP.

Amongst other methods, a MIP can be solved using the Branch-and-Bound algorithm. For the description of methods and further details on mixed-integer programming the reader is referred to, e.g., Wolsey [1998] and Nemhauser and Wolsey [1999]. Although, MIP solvers are able to prove the optimality of a solution, for problems with a practical application often many computational resources in terms of computation time and memory are needed.

3.1.2. Metaheuristics

Metaheuristic provide an alternative to exact solution approaches like mixed-integer programming, in particular for complex problems [Gendreau and Potvin, 2010, p.x].

Although heuristics do not guarantee the optimality of a solution, they are able to provide high-quality solutions with less computation time and resources. The term metaheuristic is defined as follows.

> "Metaheuristics [...] are solution methods that orchestrate an interaction between local improvement procedures and higher level strategies to create a process capable of escaping from local optima and performing a robust search of a solution space." [Gendreau and Potvin, 2010, p.ix]

Gendreau and Potvin [2010, p.ix] state that the field of metaheuristics was extended over the years from this definition to any search procedures that can overcome local optima. Typically, the methods contain neighborhood operators or destroy and repair procedures to move from one solution to another [Gendreau and Potvin, 2010, p.ix]. Thus, the neighbors of a solution are all solutions that can be reached by performing a defined change step [Michalewicz and Fogel, 2004, p.41]. A local optimum is a solution that has better or equal objective value than all its neighbors [Michalewicz and Fogel, 2004, p.41-42].

Although metaheuristics cannot prove optimality of solutions, they are often favored to solve real-world problems where they achieve excellent results in less computation time [Gendreau and Potvin, 2010, p.x]. This section gives a short overview of the used metaheuristics in this thesis. For further descriptions of the methods the reader is referred to the mentioned references.

Simulated annealing

In this section first the concept of hill climbing is described because it provides the basis for simulated annealing (SA) [Nikolaev and Jacobson, 2010]. Hill climbing means that the method searching the solution space always accepts a solution that is better in the sense of the objective function [Michalewicz and Fogel, 2004, p.43]. Thus, if the current found neighbor is better the method switches to this solution. The pitfall of this method is that it can run into local optima and is not able to escape them [Michalewicz and Fogel, 2004, p.43], because all other neighbor solutions around it are worse than the current solution.

The following description of SA is based on Nikolaev and Jacobson [2010]. SA is a metaheuristic which originates from the physical annealing process. The method is initialized with a start temperature that is cooled down during the search process according to a cooling schedule based on the current temperature and iteration.

SA can escape local optima by accepting also worse solutions with a certain probability (better solutions are always accepted). The probability depends on the temperature of the current iteration and the distance between the objective functions. Thus, in the beginning worse solutions are accepted with a higher probability than later in the search process. The function in the acceptance procedure is called metropolis criterion. The outline of the SA algorithm is given in Algorithm 1.

Algorithm 1: Simulated annealing [Michalewicz and Fogel, 2004, p.120]

```
Input: initial solution x_0, start temperature T_0;
t ← 0, T ← T_0, x* ← x_0 ;                                    // initialization
repeat
    repeat
        x ← select random neighbor from N(x*) ;
        if (objective(x) < objective(x*)) then
            x* ← x ;                              // new global best solution found
        end
        else if (random[0,1) < e^((obj(x)−obj(x*))/T)) then
            x* ← x;        // accept worse solution depending on probability
        end
    until cooling criterion is reached;
    T ← cooling(T, t) ;                               // cool down temperature
    t ← t + 1 ;
until stop criterion is reached;
return x*;
```

Variable neighborhood search

Variable neighborhood search (VNS) was first proposed by Mladenović and Hansen [1997]. In Hansen and Mladenović [2001] the basic idea of the method is described as follows: Instead of using one neighborhood operator like in local search procedures, VNS performs systematic changes of neighborhoods during the search procedure. A set of neighborhood operators in some order is necessary for the application of this method. The neighborhoods are iterated during the search. In every iteration a random neighbor from the current neighborhood is chosen, called shaking step. This neighbor is improved by a local search method until no better solution is found. If the local search led to a global best solution, the neighborhood iterator is reseted to the start neighborhood. Otherwise the neighborhood is changed to the next level. The method stops when all neighborhoods are searched for new solutions and no improvement was found. Often the neighborhoods are nested so that the search starts with smaller changes to the solutions and increasing to bigger changes during the search [Hansen and Mladenović, 2001]. If the local search procedure includes more than one neighborhood and deterministic changes of neighborhoods are made this step is called *variable neighborhood descent (VND)* [Hansen and Mladenović, 2001]. Hansen et al. [2010b] state that the algorithm is based, amongst others, on the fact that different neighborhoods lead to different local optima and a change of neighborhood can therefore lead to better results and escape local optima.

There exist many extensions and adaptions of the basic VNS that are presented and summarized in Hansen et al. [2010a]. Examples are first or best improvement strategies in the local search step (Basic VNS) or the use of VND instead of local search (General VNS). The variant used in this thesis is the reduced variable neighborhood search (RVNS). Therefore, it is described in more detail here. For further extensions and variants, the reader is referred to Hansen et al. [2010a].

The RVNS does not evaluate all neighbors but relies on the *shaking* step, i.e., it gets a random neighbor in the current neighborhood and moves to it, if the solution is better. The basic outline of RVNS is given in Algorithm 2. The outer loop is repeated until a stop criterion is reached, which is a time limit in our implementation. The inner loop generates new neighbors by applying the *shake* function to the current solution. The function *NeighborhoodChange* is given in Algorithm 3. It determines whether the new solution is better than the current global best solution. If yes, the neighborhood is reseted to the first neighborhood. Otherwise, the next neighborhood is selected. When the last neighborhood is reached ($k = k_{max}$), the entire procedure is repeated until the time limit is reached.

Algorithm 2: RVNS(x^0, k_{max}) (Hansen et al. [2010b])

```
x ← x^0 ;                                              // initial solution
repeat
    k ← 1 ;                                            // neighborhood iterator
    repeat
        x' ← Shake(x, k);                 // random neighbor in neighborhood k
        NeighborhoodChange(x, x', k);
    until k = k_max;
until stop criterion reached;
```

Algorithm 3: NeighborhoodChange(x, x', k) (Hansen et al. [2010b])

```
if obj(x') < obj(x) then
    x ← x';
    k ← 1;
else
    k ← k + 1;
end
```

Large neighborhood search

The metaheuristic large neighborhood search (LNS) was first proposed by Shaw [1998] as a solution method for vehicle routing problems (VRPs).

The concept of LNS is presented by Shaw [1998] as follows: The search process switches between relaxation and re-optimization phases. In this context relaxation means that parts of the solution are removed. In the re-optimization phase they are inserted back to the partial solution resulting in a new solution. This step often relies on heuristics for the problem at hand. In the concept of neighborhood structures, a new neighbor is generated by applying a relaxation and re-optimization step. Thus, large steps in the solution space are possible which explains the name of the method. The basic LNS always moves to solutions that improve the current best known solution.

In Pisinger and Ropke [2010] the developments of the concept are described and reviewed. The authors call the relaxation phase *destroy* and the re-optimization phase *repair*. They state that heuristics with large neighborhood structures achieve good results in many cases, although the search is more time consuming due to the extensive changes to the solution in each iteration. The acceptance of solutions is often extended in recent applications by using the simulated annealing acceptance criterion [Pisinger and Ropke, 2010]. The outline is given in Algorithm 4.

Algorithm 4: Large neighborhood search (Pisinger and Ropke [2010])

```
x ← initial solution, x* ← x ;                        // initialization
repeat
    x' ← repair(destroy(x)) ;                          // new temporary solution
    if accept(x', x) then
        x' ← x ;                                       // update current solution
    end
    if (objective(x') < objective(x*)) then
        x* ← x' ;                                      // new global best solution
    end
until stop criterion is reached;
Return x*;
```

Adaptive large neighborhood search

The metaheuristic adaptive large neighborhood search (ALNS) was first proposed by Ropke and Pisinger [2006] as an extension to the LNS of Shaw [1998]. Both are widely used in transportation and scheduling problems [Pisinger and Ropke, 2010].

Ropke and Pisinger [2006] describe the ALNS process as follows: In contrast to LNS that has one destroy and repair method, ALNS works with a set of *destroy* and *repair* operators. In each iteration one destroy and one repair operator are chosen by a roulette wheel selection. This means that the operators are chosen randomly whereas each operator has a different probability. The probability $prob(i)$ to select a destroy operator i is determined by the weights ρ_j^D of all destroy operators $j \in \Omega^D$ and given in Equation (3.10) that is stated by Pisinger and Ropke [2010] as follows:

$$prob(i) = \frac{\rho_i^D}{\sum_{j \in \Omega^D} \rho_j^D} \tag{3.10}$$

The same formula is applicable to the repair operators. The weights for the roulette wheel depend on the success of the operators in the prior iterations, i.e., the weights are adjusted during the execution. An operator finding a new global solution is rewarded more than an operator finding a worse solution. Normally, four different outcomes are used for weight adaption: new global best, new local best, accepted or worse solution. The acceptance of a neighbor is often decided by the metropolis criterion which is also used in simulated annealing (see Section 3.1.2). The sensitivity of the weight update depends on the *decay* parameter $\lambda \in [0, 1]$ indicating how much of the old weight is kept and how much the adaption factor Ψ (depending on the outcome) influences the weight. The formula for weight adaption is given based on Pisinger and Ropke [2010] in Equation (3.11) for the destroy operator i.

$$\rho_i^D = \lambda \rho_i^D + (1 - \lambda)\Psi \tag{3.11}$$

The outline of ALNS is given in Algorithm 5.

Algorithm 5: Adaptive large neighborhood search (Pisinger and Ropke [2010])

```
x ← initial solution, x* ← x, ρ^D = (1, ..., 1), ρ^R = (1, ..., 1) ;   // initialization
repeat
    select destroy and repair operator based on ρ^D and ρ^R;
    x' ← repair(destroy(x)) ;                        // new temporary solution
    if accept(x', x) then
        x' ← x ;                                      // update current solution
    end
    if (objective(x') < objective(x*)) then
        x* ← x' ;                                     // new global best solution
    end
    Update ρ^D and ρ^R ;                              // update weights
until stop criterion is reached;
Return x*;
```

3.2. Related planning problems

Before the extensive literature review on solution approaches for the home care routing and scheduling in the next chapter, this section provides an overview of related planning problems from other domains. As the problem in this thesis contains a routing subproblem that is basically a vehicle routing problem with time windows (VRPTW), Section 3.2.1 introduces the VRP and in this context relevant variants. The consideration of shifts and working regulations also occurs in the duty planning of inpatient institutions like hospitals. Therefore, a short introduction to the nurse rostering problem (NRP) is given in Section 3.2.2. Section 3.2.3 gives an overview of planning problems arising in technician routing that has similar requirements.

3.2.1. Variants of the vehicle routing problem

The vehicle routing problem (VRP) is a standard problem in logistics that is faced by companies every day [Cordeau et al., 2007]. The first mentioning of this planning problem in OR literature was the *truck dispatching problem* presented by Dantzig and Ramser [1959] as a general extension to the well-known TSP. The problem statement for the VRP is described as follows [Cordeau et al., 2007]: A fleet of vehicles needs to serve a set of geographically dispersed customers with a given demand (each customer exactly once). The capacity of the vehicles is limited to a certain amount. The route of every vehicle starts and ends at the depot of the company and the length of the route must not exceed a maximal length. The underlying network is presented as a graph with customers as nodes and driving connections as arcs. The pairwise distances between customers are often considered symmetric.

Milburn [2012] states the relationship of home care routing and scheduling to standard routing problems depending on whether the days and nurses for services are fixed a priori or not. If the nurse assignment is fixed for each visit the routing can be modeled by extending the multiple traveling salesman problem with time windows (mTSPTW), otherwise a VRPTW is needed. If the days are unfixed, the underlying problem relates to periodic extensions of the problems. The continuity of nurses and time are also considered in the consistent vehicle routing problem (conVRP). These variants are presented in the remainder of this section.

The vehicle routing problem with time windows

The VRPTW extends the VRP by adding time windows for the visits at customers [Desaulniers et al., 2014]. If these time windows are considered as hard constraints, a vehicle arriving early has to wait for the time window to open. Starting service after the time window is prohibited. The VRPTW is the basis for the MIP formulation in this thesis. To show the similarities of the models, the formulation of the VRPTW based on Desaulniers et al. [2014] is stated here.

The problem is formulated for a directed graph $G = (V, A)$. The set V represents the customers including the depot (0 and $n+1$) and N the set of customers without the depot. The set of vehicles K is homogeneous with each a maximum capacity of Q. Each customer $i \in N$ requires a delivery of quantity q_i that takes s_i time units to be finished. The arrival of a vehicle is restricted to the time window $[a_i, b_i]$ at customer i. The traveling time between customers i and j is denoted by t_{ij} and the traveling costs by c_{ij}. The sets $\delta^+(i) \subseteq A$ and $\delta^-(i) \subseteq A$ contain the nodes that are reachable by the outgoing and incoming arcs of node i, respectively.

The decision variables of the problem are the binary variables x_{ijk} and continuous variables z_{ik} with the following definitions:

$$x_{ijk} = \begin{cases} 1, \text{if vehicle } k \text{ travels directly from customer } i \text{ to customer } j \\ 0, \text{otherwise} \end{cases}$$

$$z_{ik} = \text{Start time of the service at customer } i \text{ by vehicle } k$$

The corresponding MIP is stated in Equations (3.12) to (3.20).

$$\text{minimize} \sum_{k \in K} \sum_{(i,j) \in A} c_{ij} x_{ijk} \tag{3.12}$$

$$\text{s.t.} \sum_{k \in K} \sum_{j \in \delta^+(i)} x_{ijk} = 1 \quad \forall i \in N \tag{3.13}$$

$$\sum_{j \in \delta^+(0)} x_{0jk} = 1 \quad \forall k \in K \tag{3.14}$$

$$\sum_{i \in \delta^-(n+1)} x_{i,n+1,k} = 1 \quad \forall k \in K \tag{3.15}$$

$$\sum_{i \in \delta^-(j)} x_{ijk} - \sum_{i \in \delta^+(j)} x_{jik} = 0 \quad \forall k \in K, j \in N \tag{3.16}$$

$$z_{ik} + s_i + t_{ij} - z_{jk} \leq (1 - x_{ijk}) M_{ij} \quad \forall k \in K, (i,j) \in A \tag{3.17}$$

$$a_i \leq z_{ik} \leq b_i \quad \forall k \in K, i \in V \tag{3.18}$$

$$\sum_{i \in N} q_i \sum_{j \in \delta^+(i)} x_{ijk} \leq Q \quad \forall k \in K \tag{3.19}$$

$$x_{ijk} \in \{0, 1\} \quad \forall k \in K, (i,j) \in A \tag{3.20}$$

The objective function (3.12) minimizes the total traveling costs while serving each customer exactly once (3.13). Constraints (3.14) and (3.15) ensure that each vehicle leaving the depot returns to it at the end of the route. Each customer can be visited by at most one vehicle and this vehicle has to leave the customer again (3.16). The variables for the start times are set by constraints (3.17) based on the service and travel times. The time window of each customer is limited by constraints (3.18) and the capacity of the vehicles by constraints (3.19).

In many practical applications the VRP and VRPTW are solved using heuristics due to less computation time [Desaulniers et al., 2014]. Local search, large neighborhood searches and population-based methods are mentioned by Desaulniers et al. [2014] in particular. They state that in general search with large neighborhoods achieves better solutions than small neighborhoods but are more time consuming.

Although heuristic methods are successful, the research on exact solution approaches like Branch-and-Cut-and-Price and Branch-and-Cut also advanced in the last year as for many of the benchmark instance presented in literature no optimal solutions are known yet [Desaulniers et al., 2014].

In summary, there exists a huge amount of exact and heuristic solution approaches for the VRPTW. An overview of route construction and local search algorithms is given in Bräysy and Gendreau [2005a]. For a review on metaheuristics applied to the VRPTW the reader is referred to Bräysy and Gendreau [2005b]. A more recent survey on exact and heuristic approaches is given in Desaulniers et al. [2014].

The VRPTW forms the basis for the home care problem because the nurses can be represented by the vehicles and the time windows for jobs can be modeled. Nevertheless, many requirements for the home care problem are missing in the basic VRPTW such as the qualifications and availability of nurses, a planning horizon of multiple days and working regulations.

The skill vehicle routing problem

Another variant of the VRP, which is related to the routing of home care providers, is the *skill VRP* proposed by Cappanera et al. [2011]. The authors state that this problem arises in the setting of technician routing, which is also shortly presented in Section 3.2.3. The following differences to the standard VRP exist [Cappanera et al., 2011]: Instead of delivering goods, a service has to be fulfilled at the customer's location. For this service a skill level is required that a technician must have to perform a service. Vehicles represent the technicians in this problem setting. The skills are defined hierarchically, i.e., a technician can perform services requiring a skill level equal or less than his or her skill level. The traveling costs are skill dependent, normally with higher costs for higher skill level. The planning horizon of this problem is one day and no time windows for the services are present. Although the qualification (or skill) requirements of the nurses in home care can be modeled with this variant of the VRP, the model formulation needs to be extended by working regulations, availability constraints and time windows to be applicable to the home care setting.

The consistent vehicle routing problem

The *conVRP* was first introduced by Groër et al. [2009] to extend the VRP by considering customer satisfaction during route construction. The practical context in the publication is the small package shipping industry, but the customer satisfaction is also relevant for the home care problem. The following description of the conVRP is based on Groër et al. [2009]. In contrast to the VRP the conVRP has a planning period of multiple days. The customers require deliveries with a certain duration and quantity on one or more days. To ensure similar visit times for each customer, the variation of start times of a customer is limited to a maximum difference (e.g. 15 minutes). Furthermore, the same vehicle has to visit a customer on all days of the planning horizon. With these two additional restrictions the customer satisfaction is assumed to improve, because the customers get used to their driver. Additionally, the customers can adjust to their delivery times due to only small variations.

In Kovacs et al. [2014] the problem formulation is generalized in several aspects. The number of drivers is still limited, but more than one driver per customer is allowed. The difference in visit times per customer is part of the objective function instead of being enforced by constraints. This generalized conVRP considers time windows for delivery (morning and afternoon), and the vehicles do not have to leave the depot directly at the beginning of day.

Routing in long-distance transports

Routing under consideration of driving hour regulations in the EU has been studied for long-distance transports. This topic arose in OR literature due the regulation No. 561/2006 restricting driving hours [European Parliament and Council of the European Union, 2006], and the directive 2002/15/EC concerning working hours for drivers [European Parliament and Council of the European Union, 2002]. Both are settled by the EU for long-distance transports and contain rules restricting the driving hours without breaks and maximum driving hours per week as well as regulating the breaks and rest times for drivers.

The first solution approach considering the VRP with the EU regulation No. 561/2006 was proposed by Goel [2009]. The solution approach combines a labeling algorithm with an LNS heuristic. The heuristic manages the improvement by inserting and deleting jobs for route construction. The labeling algorithm ensures the adherence to regulations and allows early rest times. The planning horizon is a week so that no weekly rest time needs to be considered. In addition to regulation No. 561/2006, Kok et al. [2010] integrate the directive 2002/15/EC into one solution approach. They use a dynamic programming heuristic to plan the routes and driving hours for drivers in a weekly planning period.

The inclusion of driver regulations is still scarce in VRP literature and the problem has been mainly addressed with heuristics due to the complexity [Lahyani et al., 2015]. In Kopfer and Meyer [2010] a MIP has been proposed for the special case of the two mentioned regulations. The model solves instances with up to 10 customers. Lahyani et al. [2015] provide a survey of publications in rich vehicle routing problem also considering driver regulations from different countries.

The difference to routing with working regulations in the home care problem are the distances. In home care planning many clients live in one city and the driving times are short compared to long-distance transports where the customers can be located in different countries. Because of the long driving times the breaks and rest times have to be scheduled mostly on the road interrupting the driving. In home care planning the breaks are scheduled such that the driving is not interrupted. Furthermore, the rest times are calculated between tours and do not have to be inserted in the routes. Additionally, no qualifications are modeled in this problem.

3.2.2. The nurse rostering problem

The nurse rostering problem (NRP) addresses the personnel scheduling for inpatient institutions like hospitals. Burke et al. [2004] describe the following attributes of the NRP. Every day is divided into shift types with known start and end time. The typical case is early, late and night shifts. The nurses have skills and each shift in the planning horizon has a demand for each skill level that needs to be covered. The planning horizon for the NRP is often set to a period of four weeks. During the planning several constraints and working contracts are considered. Sometimes the scheduling is enforced to have cyclical patterns that repeat after a defined number of weeks (see also description in Section 2.4.2). A categorization of NRPs is given by De Causmaecker and Vanden Berghe [2011]. Therefore, they also state typical constraints of the problem area such as:

- Availability of nurses for days and shifts
- Minimum and maximum restrictions on assignments per nurse in the planning horizon or for certain types of shifts (e.g. night shifts)
- Working hours per day, week or month (minimum and maximum)
- Preferences on shift assignments (e.g. day-off)
- Restriction on consecutive workdays or shift types
- Rest times between shifts
- Forbidden consecutive shift types (e.g. early after night shift)

In the extensive literature review of Burke et al. [2004] the variety of solution methods for the NRP becomes apparent. Among others, these are based on mathematical programming, constraint programming and (meta)heuristics. The authors further state that they see metaheuristic approaches as the most suitable for solving the NRP in practical applications with many constraints.

Although many of the working regulations in the NRP are also important for home care planning, the problems differ in several aspects. The shift length in the NRP is fixed because a nurse always works the whole shift. In the home care problem, the length of the shift is determined by the route that is dependent on the assignment of jobs to nurses. Furthermore, the rest time requirements between shifts need to be determined based on the varying tour start and end times, whereas in the NRP these can be calculated due to the fixed start and end of the shift. A working regulation that is not considered in the standard NRP but important in the home care problem, is the insertion of breaks based on the length of the routes. In home care the necessity of a break is unclear before the routing is determined. Furthermore, breaks can be used to reduce waiting time before time windows.

3.2.3. Technician routing and scheduling

A problem that is related to home care planning is the routing and scheduling of technicians who visit customers or locations to perform installation, maintenance or repair activities [Mısır et al., 2014, Cordeau et al., 2010]. Example application areas are telecommunication and energy provision [Bostel et al., 2008].

In this planning problem the skills of the technicians must be matched with the demanded services [Cordeau et al., 2010]. The planning horizon is several days or weeks and sometimes solved in a rolling horizon setting [Bostel et al., 2008]. Furthermore, Cordeau et al. [2010] model interdependencies between services, i.e., predecessor and successor relationships. Another considered feature is the building of teams to perform a service at a customer [Cordeau et al., 2010]. Like in the home care problem availability, maximum daily working time, breaks and rest times need to be respected during optimization [Mısır et al., 2014, Bostel et al., 2008]. The time windows are normally wider than in the home care problem, spanning several hours or even days [Mısır et al., 2014, Bostel et al., 2008]. Furthermore, the client satisfaction and continuity is not as important as in the home care problem because the visit at a customer is only once in a long period and not frequent and recurring like in home care.

Mısır et al. [2014] propose a hyper heuristic that uses the same low-level neighborhood operators to solve the home care planning and technician scheduling problem. They considered all hard constraints as soft constraints, because only then the method is capable of solving both settings. But they consider only simple working regulations. In their analysis it becomes apparent that both problems need individual operators to be solved adequately.

4. State-of-the-art in home care routing and scheduling

This chapter provides an overview of publications related to the routing and scheduling of home care services that is first mentioned in Begur et al. [1997] and Cheng and Rich [1998]. The focus of the literature review in this chapter lies on quantitative solution approaches for the routing and scheduling problem arising in home care services. The chapter is concluded by deriving research opportunities from the current state-of-the-art and stating the goals of this thesis.

The review is divided into three categories dependent on the specific problem addressed in the publication:

1. *Daily routing and scheduling in a static setting*: Section 4.1 reviews publications considering the routing and scheduling of home care nurses for one day. Thus, many constraints like working regulations spanning more than one day (e.g. rest times) and continuity requirements cannot be incorporated. However, the daily working time and breaks are applicable in a daily setting. Some publications decompose the planning problem in routing and assignment of nurses to clients. These are reviewed in Section 4.1.1. An overview of integrated approaches is given in Section 4.1.2.

2. *Multi-day routing and scheduling in a static setting*: Solution approaches for routing and scheduling in a planning horizon of multiple days are reviewed in Section 4.2. The overview is divided into approaches decomposing (Section 4.2.1) and integrating routing and scheduling (Section 4.2.2). As the integration of working regulations is a major part of this thesis we review publications with consideration of additional working regulations in Sections 4.2.3.

3. *Routing and scheduling in a dynamic setting*: The second major part of this thesis addresses the routing and scheduling in a dynamic setting. Publications that have an underlying dynamic setting are reviewed in Section 4.3.

Static setting means that the underlying sets of clients, jobs and nurses are deterministic and do not change. Therefore, the solution approach has to be applied only once. In the dynamic setting, the demand of clients and capacity provided by nurses change over time and therefore a regular planning is necessary to incorporate data changes. Note that the subcategories of the sections are non-exclusive and some of the publication could be categorized in more than one. However, the publication appears in the section where it has the highest similarity with our problem setting.

We consider the basic problem as the integrated routing and scheduling of the nurses. Thus, the solution approach for the basic problem needs to assign jobs to nurses and determine the sequences and start times of the jobs. Furthermore, the following constraints are assumed to be considered in the basic setting.

- Each job needs to be scheduled once on the given day.
- Each job has a hard time window to start in. A nurse arriving early has to wait for the time window to open.
- The assignment of nurses to jobs is limited by the qualification requirement needed for the job and provided by the nurse.
- The availability of the nurse is either restricted by a time window or given on a daily basis. Furthermore, the daily working time is limited by a maximum.

If not indicated otherwise, the publications in Sections 4.1 to 4.3 solve this problem setting. Constraints apart from the above mentioned or constraints omitted are stated explicitly in the text or tables.

4.1. Daily routing and scheduling in static setting

In this section the publications addressing the routing and scheduling for a planning horizon of one day are presented and summarized. The review is divided into decomposed and integrated solution approaches. An overview of all publications is given Table 4.1. The table shows whether the publication uses an *integrated* approach of routing and scheduling and if *shift types* are modeled. Furthermore, the consideration of the following requirements is given: job time windows (*Job TW*), qualifications of nurses (*Qualifications*), break rules (*Breaks*), break placement in the route (*Break TW*), daily working time (*Daily WT*) and availability of nurses at specific times or days (*Availability*). For further details on the constraints see Section 2.4.

4.1.1. Decomposed assignment and routing

The publications in this section decompose the daily problem in two subproblems: the assignment of nurses to jobs and the routing.

In Cire and Hooker [2012] the basic HHCP is solved by a greedy logic-based benders method. The authors decompose the problem in assignment and routing. The master problem assigns the nurses to the jobs in a greedy manner. The subproblem is solved with constraint programming (CP) and provides additional constraints (so called

		Integrated	Shift types	Job TW	Qualifications	Breaks	Break TW	Daily WT	Availability
Decomposed	Cire and Hooker [2012]			h	x	o	x	x	x
	Yalcindag et al. [2012]							x	
	Yalcindag et al. [2013]							x	
	Allaoua et al. [2013]		x	h	x				x
	Jemai et al. [2013]			h	x				x
	Aiane et al. [2015]			h	f				
	Bastos et al. [2015]							x	
	Issaoui et al. [2015]			h	x			x	
Integrated	Begur et al. [1997]	x			h	x			
	Elbenani et al. [2008]	x		h					
	Ben Bachouch et al. [2010]	x	x	h	x				x
	Mısır et al. [2010]	x		s	s				s
	Mankowska et al. [2014]	x		s	x				
	Di Mascolo et al. [2013]	x		h	o				x
	Riazi et al. [2014]	x		h	x				x
	Ait Haddadene et al. [2014]	x		h	x				x
	Laesanklang et al. [2015]	x		h	x				x
	Rasmussen et al. [2012]	x		h					x
	Redjem et al. [2011a]	x		h					x
	Redjem et al. [2011b]	x		h					x
	Redjem et al. [2012]	x		h					x
	Mutingi and Mbohwa [2013a]	x		s					
	Mutingi and Mbohwa [2013b]	x		s					
Integrated + WT	Bertels and Fahle [2006]	x		s/h	x			x	x
	Akjiratikarl et al. [2007]	x		h				x	
	Trabelsi et al. [2012]	x		h	x			x	
	Rendl et al. [2012]	x		h	x			x	x
	Hiermann et al. [2015]	x		s/f	x			x	x
	Mutingi and Mbohwa [2014]	x		h	x			x	x
	Morito et al. [2014]	x		h	x			x	x
	Braekers et al. [2015]	x		h/s	x			x	x
	Yuan et al. [2015]	x		x	x			x	
Integrated + Breaks	Cheng and Rich [1998]	x		h	x	o	x		x
	En-nahli et al. [2015]	x		h	x	o		x	
	Eveborn et al. [2006]	x		h	x	o			x
	Eveborn et al. [2009]	x		h	x	o		x	
	Kergosien et al. [2009]	x		h	x	o			x
	Trautsamwieser and Hirsch [2010]	x		s/h	x	o	x	x	
	Trautsamwieser and Hirsch [2011]	x		s/h	x	o	x	x	
	Trautsamwieser et al. [2011]	x		s/h	x	o	x	x	
	Fikar and Hirsch [2014]	x		h	x	o	x	x	

Table 4.1.: Overview of main constraints in publications considering a daily routing and scheduling (x = considered, o = partially considered, h = hard, s = soft, f = fixed, WT = working time, TW = time windows)

cuts) for the master problem. The constraints considered are the qualifications and time windows of jobs. Working regulations are modeled using time windows for nurses and minimum and maximum daily working time. The authors also implement the assignment of breaks and assurance of a maximum time without a break.

The combination of patient assignment and routing models is investigated in Yalcindag et al. [2012]. The authors provide a two-stage approach to balance the workload between nurses: first new patients are assigned to a reference operator and afterwards the routing problem is solved separately for each day and operator. In the assignment phase two policies are tested (stochastic demand) and a MIP (deterministic demand) of Lanzarone et al. [2012] is used. The routing component is solved by a TSP with limited daily working time. The results show that the policies achieve better results due to the consideration of uncertain demand. Furthermore, the consideration of average travel times achieves nearly the same results as performing an explicit routing. In Yalcindag et al. [2013] a travel time estimator based on regression and historic data is proposed to replace the average travel times. The estimator is tested in a two-stage and an integrated approach (both MIPs). The results show the improvement by using the estimator instead of average travel times. The two-stage approach performs almost as good as the integrated approach on the small instances.

Allaoua et al. [2013] propose two solution approaches for the HHCP: a MIP and a matheuristic. For the matheuristic they decompose the problem into rostering and routing. The routing component is the subproblem for the rostering model and solved as a TSPTW. The set of jobs for the subproblem is determined by different clustering algorithms. The constraints considered are the time windows and qualifications for nurses and jobs and shift types. The goal is to minimize the number of employees needed to serve the clients.

Jemai et al. [2013] model an abstract version of the problem as mTSPTW, where each nurse has one qualification and each client requests one job. Therefore, the authors argue, the problem reduces to a TSPTW for each nurse.

In Aiane et al. [2015] the routing of nurses is modeled as a variant of the mTSPTW. The assignment of nurses to clients is given as input to the model. The difference to other approaches is that the MIP considers one time window for several visits on a day and periods of unavailability of clients. Other constraints like qualifications and working time limitations are not incorporated.

Bastos et al. [2015] develop a web-based optimization tool for minimizing the route lengths for care teams offering home health care. The novelty about the approach is the consideration of aseptic patients that need to be visited before the other patients to reduce infection risk. The routing problem is an extended VRP with maximum route length that is solved as MIP and with an adapted savings heuristic. The solution approaches exclude time windows and qualifications. The second part of the publication describes the architecture and implementation of the software system.

In Issaoui et al. [2015] a three component objective function for the basic problem is optimized with a three-phase heuristic. First, the assignment of nurses and clients is determined. Second, a VNS minimizes the travel distance and unassigned jobs while considering the minimum and maximum working time. In the last phase, the satisfaction of clients (assuring the right qualification of nurses) is improved by local search. The method improves the individual objectives in a lexicographical order.

4.1.2. Integrated routing and scheduling

The integrated routing and scheduling, i.e., assigning jobs and determining routes at the same time, is reviewed in this section. This variant is the one that is studied most in home care routing and scheduling.

Begur et al. [1997] are the first presenting a decision support system (DSS) to support the planning of home health care nurses. The overall process has three stages: assigning clients to weeks, assigning clients to days and assigning nurses to clients including routing. Thus, the routing problem is addressed for each day separately. The days are fixed by the agency beforehand. The time availability of nurses is given as input to the model. The routing problems are solved by a savings heuristic and a nearest neighbor heuristic.

Elbenani et al. [2008] model the HHCP as a VRPTW without qualifications. They further include the scenario of taking blood samples from clients to a hospital two times a day and consider continuity of care. The authors implemented a tabu search (TS) as solution method.

A MIP approach is proposed by Ben Bachouch et al. [2010]. They model the scheduling as a task assignment problem, i.e., they assign tasks to the nurses and each task gets a rank in the sequence. The driving times are included in the task durations to propose a general model which can handle all types of requests. The objective is to balance the workload of the employees.

Mısır et al. [2010] provide a hyper-heuristic with several low-level heuristics (e.g. swap visits, move visits) to solve the HHCP. All of the HHCP constraints (time windows, qualifications, work time windows, several nurses per job, unpreferred clients and nurses) are considered as soft constraints. In Mısır et al. [2014] the problem is considered again with an extended set of low-level heuristics. In this publication, the heuristic is also applied to the related technician routing and routing of security guards. Although the problems are related, the results show that different low-level heuristics are needed to achieve good results on each of the problems.

Mankowska et al. [2014] consider the daily planning with temporal interdependencies (minimum and maximum time distance) and synchronization constraints between two jobs. They formulate a MIP and propose an adaptive VNS to solve the problem. For the heuristic approach they introduce a matrix representation to handle the linked jobs. The neighborhood structures handle linked jobs together to avoid infeasibility. Time windows are considered as soft constraints and the working time of nurses is not limited. The objective function minimizes the sum and maximum of the time window violations as well as the travel distance.

The synchronization of two nurses for a visit is also modeled and investigated by Di Mascolo et al. [2013]. They propose a MIP for the daily planning under consideration of qualifications and time windows for jobs and nurses' working hours. The objective function minimizes the waiting time of the nurses at clients. The results show that with increasing number of synchronized visits the computation time increases.

Riazi et al. [2014] solve the problem with a MIP and different variants of the gossip algorithm. In the gossip algorithm the sets of clients and nurses are divided into smaller subsets. The subproblems are optimized locally. Afterwards, two or more randomly selected subproblems are considered together and also optimized locally. If an improvement is achieved, the solutions are updated. The gossip algorithm outperforms the MIP in computation time with small remaining gaps or even better solutions for larger instances.

Ait Haddadene et al. [2014] propose a MIP and heuristic solution approach for the daily routing problem with time interdependencies between jobs. The heuristic is based on a greedy randomized adaptive search procedure (GRASP). The interdependencies comprise synchronization of two nurses and maximum time between jobs. The objective is to minimize the travel time and avoid preference violations of clients.

Laesanklang et al. [2015] solve a problem setting where different geographical regions are known a priori. In addition to travel costs and client preferences, the objective function assigns the nurses in their preferred regions. To solve the MIP, the authors decompose the problem by region and optimize the resulting subproblems sequentially. The capacity of the nurses is updated after each optimization step. Thus, the results are dependent on the ordering of subproblems. To address this issue, several ordering strategies are developed and analyzed.

Rasmussen et al. [2012] focus on interdependencies between jobs in the HHCP. They model five types of them in a MIP: synchronization, overlap, minimum time difference, maximum time difference as well as minimum and maximum time difference. Assignments are not limited by qualifications in this formulation. The model is solved with a branch-and-price approach. To reduce the computation time, the authors further introduce clustering methods based on the preferences of clients. Thus, the subproblem for determining new routes only contains clients that favor the nurse currently considered. With this clustering the computation time can be reduced with only slightly negative impact on the results compared to the optimal solution [Rasmussen et al., 2012].

In the approach of Redjem et al. [2011b] one or more nurses are preassigned to each client. Thus, no qualifications requirements need to be modeled. The routes of the employees must be coordinated to avoid two visits at the same time and ensure valid sequences of jobs. The authors propose a MIP that they solve with two strategies. First, they limit the waiting time of clients between visits and minimize the traveling and waiting time of nurses. Second, they minimize the waiting time of clients. A similar MIP is subject in Redjem et al. [2011a]. Here two different objectives to minimize the nurses traveling and waiting times are presented and analyzed in different scenarios of underlying data. Redjem et al. [2012] formulate and solve another variant of this problem setting. Here the completion time of visits is compared to the minimization of waiting and traveling times. The authors conclude that the latter is more suitable to minimize the waiting times at clients.

Mutingi and Mbohwa [2013b] consider the daily HHCP with different objectives: holding soft time windows of clients, minimizing traveling distance and adhering to the preferred working hours of nurses. The different objectives are incorporated with a weighted sum. The solution approach is a group-based genetic algorithm (GA) and abstracts from further home care related constraints like qualifications and hard time windows. In Mutingi and Mbohwa [2013a] the same problem as in Mutingi and Mbohwa [2013b] is considered but the multi-objective setting is addressed with a fuzzy satisficing approach. Thus, the designated objective ranges can be given by the nurses and clients and changed overtime in an interactive way.

Consideration of daily working time

In addition to the qualifications and availability of nurses, the consideration of daily working time is essential to adhere to work contracts. The publications considering this constraint are reviewed in this section.

Bertels and Fahle [2006] propose an integrated routing and scheduling with preferences of nurses and clients in one solution approach. They model hard and soft time windows that are due to client preferences. The working time of nurses is limited by a given minimum and maximum. The qualification requirements are partitioned in hard and soft constraints. The preferences of clients are modeled as soft qualification requirement. The problem is solved using different heuristic approaches based on CP, SA and TS as well as a combination of those. They maintain a solution pool to use historical information about already found solutions to guide the search. The determination of start times for a given route in the methods is provided by an LP.

Akjiratikarl et al. [2007] use a particle swarm optimization approach to create routes for home care workers on a daily basis. They abstract from further home care related constraints and therefore the result is a VRPTW with multiple depots for the nurse start locations and maximum tour length.

In Trabelsi et al. [2012] the basic daily planning is solved with a MIP. The authors consider the maximum daily working time and driving distance as well as multiple nurses per job while balancing the workload of nurses.

The applicability of metaheuristics on the daily HHCP is examined by Rendl et al. [2012]. Multi-modality is taken into account by allowing the nurses to use cars and public transport. The authors create an initial solution with a CP approach and apply different metaheuristics afterwards. They implement an evolutionary algorithm (EA), a VNS, a scatter search and a SA hyper-heuristic. The best results on their instances are provided by the evolutionary algorithm. The objective function of all approaches is a weighted sum of different penalty terms which are mainly soft constraint violations. They consider the preferences of clients (time windows and start times) and nurses (working time) in addition to the traveling time. In Hiermann et al. [2015] this work is extended by a formalization of the problem, improvement of the metaheuristics and an extensive numerical analysis. In addition to the economic goals, all constraint violations (time windows, rejections of staff, daily working time) are incorporated in the objective. They argue, this setting is easily adaptable for different providers. The metaheuristics analyzed here are VNS, SA hyper-heuristic, memetic algorithm and scatter search. The memetic algorithm that extends the EA of Rendl et al. [2012] provides the best results.

A particle swarm optimization (PSO) approach for an extended problem of Mutingi and Mbohwa [2013a] is provided by Mutingi and Mbohwa [2014]. The time windows are considered as hard constraints and the nurses working time is limited by a minimum and maximum working time. The goal is to balance the workload among nurses with a fuzzy objective function. The fitness function of the PSO evaluates the solutions using fuzzy set theory.

An exact approach with different modes of transportation for the basic HHCP is proposed by Morito et al. [2014]. The problem considers time windows for jobs and nurses, qualifications and daily working time. The authors solve their MIP with a column generation (CG) approach. The master problem is formulated as a set partitioning problem and the pricing is performed with a resource-constrained shortest path problem (RCSPP). A preprocessing step (reducing time windows and possible assignments) reduces the computational time.

Braekers et al. [2015] consider a bi-objective version of the daily HHCP. The objectives are the traveling and overtime costs versus the client satisfaction (preferences for nurses and time). The authors provide a MIP formulation and a multi-objective metaheuristic approach. The MIP is solved using ϵ-constraint programming. The metaheuristic uses a multi-directional local search framework with an LNS as subsolver. The results show the trade-off between costs and convenience and allows the planner to select his/her preferred solution.

Uncertain service times (normally distributed) in the HHCP are investigated by

Yuan et al. [2015] in a branch-and-price (B&P) approach. The time limitation of a job is modeled with a soft deadline before which the job needs to start. The daily workload is limited by the maximal number of clients per nurse. The pricing problem consists of an adaption of the VRPTW.

Consideration of breaks

Representing breaks is important to obtain legal plans. In contrast to other working regulations, break rules can be considered in a daily setting. All publications in this section provide only basic break rules because the splitting of breaks is not allowed and in some cases the break is fixed to a defined point in time.

One of the first solution approaches for the basic home care routing and scheduling is proposed by Cheng and Rich [1998]. They state two MIP formulations for the HHCP. They consider full- and part-time nurses and schedule breaks in a fixed time window. As the MIP is only solvable for small instances, they also propose a heuristic based on a fix-and-resolve pattern that iteratively fixes variable values and optimizes the remaining variables.

Eveborn et al. [2006] provide a DSS for the daily planning of home care providers. They model a set partitioning formulation and solve it with a repeated matching algorithm. The objective function minimizes the costs of schedules resulting from traveling and working times as well as preference violations. The availability of nurses is given as input to the model. For some of the jobs more than one nurse is needed to carry them out. The schedules consider fixed breaks that cannot be shifted to other times in a tour. The continuity of care is modeled by a set of preferred nurses. In Eveborn et al. [2009] the system is presented again but with focus on challenges of introducing the DSS to organizations and the benefits for the home care providers.

Kergosien et al. [2009] model the HHCP as mTSPTW with additional constraints for qualifications and work time windows. Furthermore, they consider visits of multiple nurses with time interdependencies between them and visits that must be performed by the same nurse. Breaks are considered as preassigned jobs in the solution approaches. The authors formulate a MIP to solve the problem and improve the solution time by adding domain-specific cuts to the formulation.

The daily planning of home care services is solved by Trautsamwieser and Hirsch [2010] with a MIP and a VNS heuristic. In addition to the common constraints, both methods consider the daily working time and break assignment during the scheduling. The breaks are inserted in such a way that the amount of time without break never exceeds a certain threshold. The objective consists of seven components concerning working time and preferences of nurses and clients that are weighted to form a single function. The model is addressed in Trautsamwieser and Hirsch [2011] with a more extensive discussion of numerical results and a sensitivity analysis in which the number of time-critical jobs is altered. The same approach is used in Trautsamwieser et al. [2011] to analyze the influence of natural disasters (e.g. floods, storms, heat-waves) on home care services in Austria. In particular, a real-world flood scenario from 2002 and three possible future scenarios are used to measure the impact of the disrupted street connections on the routing decisions.

A novel setting in the home care context is proposed by Fikar and Hirsch [2014]. They consider the routing as a dial-a-ride problem (DARP) where the nurses share vehicles driven by a driver. The drivers take the nurses to their clients and pick them up after the job is finished. If the distance is below a certain threshold the nurses can also walk from one client to the next. Furthermore, the authors consider the maximum daily working time and break assignments in the routes (with a threshold for allowed working time without break). The problem is solved with a matheuristic consisting of several steps including the creation and selection of walking routes, an adapted savings heuristic and a TS for improvement. The start times inside the heuristic are determined by an LP with prefixed breaks.

An exact approach is provided by En-nahli et al. [2015]. The objective function consists of four components: traveling time, balanced workload, preferences of clients and waiting time. These are normalized and combined in a single objective. The model further considers the basic home care related constraints and the assignment of breaks in routes.

4.2. Multi-day routing and scheduling in static setting

This section reviews the publications considering the routing and scheduling in planning horizon of more than one day. Solution approaches either decompose the routing and scheduling or solve an integrated problem for several days (see Sections 4.2.1 and 4.2.2, respectively). The integrated approaches are divided into publications considering working time restrictions for several days and additional working regulations. An overview of all publications in this section is given in Table 4.2. The table shows whether the publication uses an *integrated* approach of routing and scheduling, if *shift types* are modeled and if the days of the jobs are *fixed* a priori. The same constraints as in the previous section are shown. Furthermore, the consideration of the following additional requirements is given: total working time in the planning horizon (*Total WT*), weekly workdays (*Weekly workdays*), daily rest time (*Daily RT*), weekly rest time (*Weekly RT*), consecutive workdays (*Consecutive workdays*) and special rules for weekends (*Weekend rules*). The table also indicates whether cyclic shift patterns are possible (*Shift rotations*). For further details on the constraints see Section 2.4.

	Publication	Integrated	Shift types	Days fixed	Job TW	Qualifications	Breaks	Break TW	Daily WT	Total WT	Weekly workdays	Daily RT	Weekly RT	Availability	Shift rotations	Consecutive days	Weekend rules
Decomposed	Borsani et al. [2006]					x								x			
	Maya Duque et al. [2015]		x		s	x				x							
	Nguyen et al. [2015]		x	x	h/s	x			x					x			
Integrated	Cattafi et al. [2012]	x		x					x								
	Gamst and Sejr Jensen [2011b]	x			h	x								x			
	An et al. [2012]	x							x								
	Kergosien et al. [2013]	x		o	h	x								x			
	Cappanera and Scutellà [2013a]	x				x								x			
	Cappanera and Scutellà [2013b]	x				x								x			
	Cappanera et al. [2013]	x				x								x			
	Cappanera and Scutellà [2014]	x			h	x			x					x			
	Torres-Ramos et al. [2014]	x			h	x			x								
	Ben Bachouch et al. [2011]	x		x	h	x	o	x	x								
Integrated + working time	Steeg and Schröder [2008]	x	x		h					x				x			
	Gamst and Sejr Jensen [2011a]	x			h	x				x				x			
	Shao et al. [2012]	x		o	f/h	x	o	x		x				x			
	Bard et al. [2012]	x		x	f		o	x		x				x			
	Bard et al. [2014]	x		o	f/h	x	o	x		x				x			
	Luna et al. [2013]	x		x	f				x	x				x			
	Nguyen and Montemanni [2013]	x		x	h/s	x			x	x				x			
	Yuan and Fügenschuh [2015]	x			h	x			x	x				x			
Additional working regulations	Bäumelt et al. [2010b]		x	x	h					x		x		x		x	x
	Trautsamwieser and Hirsch [2014]	x		x	h	x	o	x	x	x		x	x	x			
	Di Gaspero and Urli [2014]	x		x	h	x			x					x		x	
	Wirnitzer et al. [2016]		x	x		x			x	x		x		x			

Table 4.2.: Overview of constraints in publications considering a multi-day setting (x = considered, o = partially considered, h = hard, s = soft, f = fixed, TW = time window, WT = working time, RT = rest time)

4.2.1. Decomposed routing and scheduling

Borsani et al. [2006] solve the weekly planning by decomposing the problem in daily planning intervals. They propose a four-stage solution approach. The paper describes the first two stages, i.e., assigning nurses to clients and determining the visit days for a client. During the assignment continuity of care is respected by selecting a reference nurse for every client. The routing of nurses is neglected. The authors distinguish between full- and part-time workers and their availability on a specific day and hour.

In Maya Duque et al. [2015] each day of the planning horizon (several weeks) is divided into time slots. In a first step, possible visit patterns for each client are determined by assigning clients to time slots. During the process for some of the patterns, the time continuity between weeks is considered. Afterwards, so called schemes are generated from the pattern by assigning nurses to the days (at most two nurses dependent on the number of visits). A set partitioning model selects schemes for each patient by optimizing the preference of clients for time slots and nurses. The traveling distance in each time slot is minimized in a second step by a local search procedure. The process is embedded into a DSS.

A robust solution approach based on a matheuristic is developed by Nguyen et al. [2015]. The uncertainty considered is the absence of nurses (e.g. calling in sick). The shift schedule and therefore the availability of the nurses is input to the model. The metaheuristic used is a GA that optimizes the routes for every day separately. The start times of jobs are determined by an LP. To address the robustness of a solution, the worst case costs for different scenarios are included in the objective function.

4.2.2. Integrated routing and scheduling

The integrated routing and scheduling is the basis for considering the working regulations while planning the routes of the nurses. In this section, first, the publications proposing an integrated approach are reviewed. The subsections address further extensions, namely the working time consideration and working regulations.

Gamst and Sejr Jensen [2011b] propose a solution approach based on Branch-and-Price. The subproblem creates daily schedules for every employee. These are combined in the master problem to form a feasible schedule for the whole planning horizon. The days of jobs are not fixed but a frequency is given. The objective function considers continuity of care and continuity of time (i.e. time differences between appointments). The daily working time of nurses is limited to a time window.

Cattafi et al. [2012] use constraint logical programming to solve a basic problem without qualifications and time windows. They consider the daily working time and minimize the workload unbalance and number of nurses assigned to a client.

In the problem setting of An et al. [2012] the clients are already assigned to the nurses but the visit days are not given a priori. The proposed solution approaches determine the days for jobs and the routes for one nurse, simultaneously. Home care related constraints are omitted except the daily working time of the nurse. The authors formulate a MIP and provide a two-phase heuristic to solve the problem.

The collection of medical test samples at patients' homes is solved by Kergosien et al. [2013] providing three different methods: MIP, TS and VNS. The difference to the HHCP is the drop-off of time critical samples several times during the day. The planning horizon of multiple days is solved day-by-day. For some of the jobs the day is fixed, for others it can be selected from a valid time window of several days. Requirements and preferences of clients are modeled as hard constraints.

The integrated routing and scheduling with flexible service days is conducted by Cappanera and Scutellà [2013a] using a MIP. The authors generate several day patterns for each client and one of those is selected during the solution process. The patterns are generated by three different methods: greedy, taken from the real-world solution or solved with a multi-commodity flow model. The method abstracts from time windows in the home care context, but considers the qualifications and availability of nurses. The proposed model and pattern generation methods are based on the publication of Cappanera and Scutellà [2013b]. Here the authors also perform an extensive analysis regarding symmetry breaking constraints and different objective functions. The model is also used in the study in Cappanera et al. [2013], where two different objectives for balancing the workload (minimize the maximum workload and maximize the minimum workload) are investigated in a deterministic and stochastic setting. The stochastic setting takes the deterministic solution and evaluates it using random variables for the driving and service times. The results show that the *maxmin* approach leads to a better balancing while the *minmax* approach achieves lower costs. The publication of Cappanera and Scutellà [2014] extends the analysis and adds continuity of care by limiting the maximum number of different nurses. They further limit the maximum daily working time and extend their model to time window consideration. The authors provide an extensive analysis of the different pattern generation methods, ways of modeling workload balancing and handling the continuity of care constraints on the extended problem setting.

Jobs considering more than one staff member and unfixed days are incorporated to a MIP formulation for the HHCP by Torres-Ramos et al. [2014]. The number of jobs and period of time between visits is given as input to the model.

Ben Bachouch et al. [2011] model the weekly HHCP as a MIP which minimizes the traveling distance. Continuity of care is ensured by assigning only one reference operator to each client. Furthermore, they include shared visits, where more than one nurse has to care for the client. The lunch breaks of the nurses are ensured by a one-hour job with a time window.

Consideration of working times

To model work contracts not only the consideration of daily working times but also the incorporation of maximum working times in a longer horizon (e.g. a week or month) is essential. This constraint can be modeled in a multi-day planning horizon.

Steeg and Schröder [2008] solve the HHCP for a planning horizon of several shifts. The jobs need to be assigned to shifts and sequenced. The feasible shift combinations are calculated and the availability of nurses regarding shifts is given a priori. Time windows for jobs are considered, but no qualifications. The problem is solved with a combination of CP and ALNS. The CP component ensures the validity of the roster while the ALNS improves the routes. The objective consists of minimizing the nurse costs and traveling distance while maximizing the continuity of care.

Gamst and Sejr Jensen [2011a] propose a branch-and-price approach for constructing a master schedule for the HHCP. The jobs of the clients are given with a specified frequency after which they reoccur. The planning period of the master schedule depends on these frequencies and can span several weeks. The resulting schedule is used as template for future planning, thus every few weeks the schedule repeats. Daily operational changes are incorporated ad hoc on the day, if necessary. The authors take preferences of clients, continuity of care and working hours into account.

Bard et al. [2012] solve the weekly scheduling for traveling therapists. The appointment times of the patients are fixed and cannot be influenced by the routing. The authors assume the decomposition of therapists and clients such that only one qualification is present in each model. The lunch break of a therapist must be scheduled in a given time window. The authors model different work contracts by limiting the availability of therapists to specific time windows. In the publication three different MIP formulations are proposed: for the one therapist case, for the multiple therapist case including breaks and for the multiple therapist case with overtime and mileage reimbursement consideration. For the latter two different relaxations are developed to get a heuristic solution in less computation time. The authors provide an extensive numerical analysis of the results. A modified problem of Bard et al. [2012] is considered in Shao et al. [2012]. Here not all appointment times are fixed but only for a subset of clients. For the remaining clients the days and visit times need to be scheduled by the solution approach. Furthermore, breaks for the therapists only need to be planned if the route is longer than six hours. The authors solve the problem using a parallel GRASP. In the algorithm the problem is decomposed by first assigning days to clients, then clients to therapists and afterwards scheduling the daily routes in parallel. In the second phase of GRASP a neighborhood search is applied for improvement. The solution approach is modified in Bard et al. [2014]. The authors change the parallel GRASP to a sequential GRASP. Routes are constructed one-by-one and the feasibility of solutions can be guaranteed. The difference to the parallel approach is that the days of the clients are not fixed in a first step, but incorporated in the route construction. The sequential approach achieves better results on most of the instances but needs more computation time in some cases.

The weekly planning problem with fixed appointments is considered by Luna et al. [2013]. Thus, the determination of routes has no influence on the start times of jobs. The authors propose a parallelized EA which considers the minimum and maximum daily and weekly working times of nurses as well as their availability. The goal is to minimize the needed employees and the total working time. Qualifications are omitted in this solution approach.

A MIP for the weekly planning with a weighted sum as objective is proposed by Nguyen and Montemanni [2013]. The objective function consists of six parts: minimizing the number of unassigned jobs, overtime, time window violations and waiting time as well as maximize the continuity of care and balanced workload. The availability of nurses is restricted day-wise and by the daily and weekly working time.

Yuan and Fügenschuh [2015] consider the HHCP with unfixed days of jobs and time dependencies between the jobs for a home health care provider in Germany. In the MIP the shift starts of the nurses are fixed and the working time is limited per day and week. The authors propose a greedy construction and local search to reduce the computation time. The nurses can state their preferred workdays and hours (hard constraints). Thus, it is possible use the unfixed visit days to achieve higher satisfaction because the wishes of the nurses are fulfilled.

4.2.3. Consideration of additional working regulations

The publications in this section integrate extended working regulations to the HHCP. This means the publications consider additional working regulations to the maximum working time and simple break assignments, which are part of many publications. These regulations can be labor law regulations, agreements from work contracts or constraints from the NRP.

Bäumelt et al. [2010b] present a heuristic two-phase approach: a routing and a rostering algorithm are iteratively executed. The routing algorithm uses a set of already opened routes for inserting jobs. In each iteration the jobs are released and reinserted. Overtime or the opening of new routes is possible, if no insertion position is found. The duty roster is built based on these routes. The rostering problem is solved as an employee timetabling problem with high diversity of shifts (ETPHD) [Bäumelt et al., 2010a]. The rostering part considers forbidden shift combinations, availability of nurses and rest times between shifts. The objective tries to balance nurses' total and weekend workloads as well as to avoid isolated days-on or -off.

Trautsamwieser and Hirsch [2014] extend the model of Trautsamwieser and Hirsch [2011] to a medium-term planning horizon. This allows the authors to include the rest time requirements for consecutive shifts on different days. Additionally, maximum weekly working times and minimum weekly rest times are ensured. Clients and nurses can state rejections regarding each other to be taken into account. The break rules are the same as in the daily planning model. The more complex model is solved with a Branch-Price-and-Cut algorithm and an extensive numerical analysis is provided. The objective function minimizes the working time of the nurses. The instances from this publication are considered in the numerical analysis of this thesis.

Di Gaspero and Urli [2014] solve the multi-day HHCP using CP and LNS. They model lower and upper bounds for the daily working time. Additionally, their model restricts the maximum number of consecutive workdays and for some jobs more than one nurse is needed. The objective function penalizes unassigned jobs and minimizes a weighted sum of overtime, slack time and travel distance. The LNS heuristic clearly outperforms the CP approach on the instance set.

In Wirnitzer et al. [2016] the assignment of nurses to routes in the HHCP is considered on a monthly basis. The problem is called *home care rostering*. The routing is not part of this publication as a master schedule of routes (e.g. like created in Nickel et al. [2012]) is given as input. The proposed MIP incorporates shift types as well as maximum daily and total working times. Additionally, the legally required rest time between shifts is respected. Clients can reject nurses due to preferences. The goal is to achieve a high degree of continuity of care. Therefore, different metrics are proposed and compared in the computational study.

4.3. Routing and scheduling in a dynamic setting

So far publications have a static setting with neither changes in the demands or work capacities nor consideration of the previous planning period. The publications reviewed in this section consider the planning problem in a dynamic setting that addresses these two issues. All publications ensure continuity between planning periods but in different ways. An overview of the reviewed publications is given in Table 4.3. The table indicates the components that are considered as dynamic in the first three rows (*Dynamic*). A "+" indicates that new clients, jobs or nurses enter the setting and a "-" shows a drop-out. In the next rows the considered continuity types are reviewed (*Continuity*). Finally, the consideration of the same requirements as in the previous section are given (*Constraints*).

Bennett and Erera [2011] are the first to develop a solution approach for the HHCP with multiple days in a rolling horizon setting. They propose two heuristic solution methods. New clients are admitted for service and leave after a certain amount of time. The planning horizon is divided into time intervals of 15 minutes and planning is done twice a day. As they consider only one nurse no duty planning is necessary.

		Bennett and Erera [2011]	Nickel et al. [2012]	Nowak et al. [2013]	Bowers et al. [2014]	Rest and Hirsch [2015]
Dynamic	Clients	+/-	+	+	+	
	Jobs					
	Nurses					
Continuity	Care	x		x	x	x
	Time	x	x			x
	Duty schedules					
Constraints	Job time windows		h			h/s
	Qualifications		x			x
	Breaks					x
	Break time window					x
	Daily working time		x	x	x	x
	Total working time		x			
	Daily rest time					
	Weekly rest time					
	Availability	x	x		x	x
	Shift rotations					
	Consecutive workdays					
	Weekend rules					

Table 4.3.: Overview of dynamic, continuity and constraints in publications for home care planning with a dynamic setting (x = considered, h = hard, s = soft, + = new, - = drop-out)

Continuity of time is ensured because the visits at a patient have to be assigned to the same time slot on the same weekday every week. Common constraints like time windows, breaks and qualifications are omitted in this study. The objective is to assign as many clients as possible.

Nickel et al. [2012] solve different models in the context of the HHCP. First, they model the HHCP for multiple days and solve it with an ALNS. Second, they propose a model for master scheduling. A master schedule is a template plan which is independent of specific weeks and dates. The refinement for a particular week is determined by an operational HHCP, which uses the master schedule as input. The authors consider the continuity of time by minimizing the sum of changes in job start times, while inserting new clients. For the other clients the continuity is supported by the use of the master schedule. The operational model is solved by a combination of CP and TS. Time windows of shift types and availability of nurses are modeled as hard constraints.

Nowak et al. [2013] solve a conVRP to ensure continuity of care in the home care setting. They adapt the record-to-record travel algorithm of Groër et al. [2009] that uses template routes for solution construction. This concept is similar to master scheduling. Besides the continuity of care, none of the home care related constraints is considered. The authors investigate the influences of a week-by-week planning compared to a long-term planning on the traveling distances and number of required nurses. The long-term planning achieves superior results even if the arrival of new clients is assumed uncertain. To address the uncertainty, the authors insert dummy clients in the template routes as place holders for future clients.

Bowers et al. [2014] consider a home care related optimization problem that routes midwifes for postnatal care at the clients' homes. They assume no time windows for visits and all midwifes have the same qualification. The routing is performed by an adapted savings heuristic and executed daily. The arrival of new clients is simulated with a Poisson process. The continuity of care between days is modeled by maximizing the preferences of clients. Different scenarios are analyzed. The input of a shift pattern for the midwifes limits the daily availability and working hours.

A recent publication by Rest and Hirsch [2015] considers a daily planning horizon and is the first allowing to split breaks. The authors consider the daily planning during normal business as well as in times of disaster (e.g. blackouts or floods) using different modes of transport. The travel times are dependent on the time of day because public transport is considered. The working regulations that are considered are the breaks, maximum daily working times as well as time windows for the working time. To ensure continuity of time, the difference to the previous period is taken into account with a reduced time window. Continuity of care is ensured by using teams of nurses that can only be extended by a limited number of nurses. The duty plan is either given as input to the algorithm or flexible working hours are considered. The problem is solved with a time-dependent tabu search. New clients are not handled by the solution approach.

4.4. Research opportunities

This section summarizes the state-of-the-art in routing and scheduling for home care planning and derives the research opportunities in this area. Based on this analysis the research goals of this thesis are defined in the next section. The summary of the state-of-the-art is divided into the static and dynamic context, because in both settings different research opportunities arise.

Static setting

The literature in a static context is reviewed in Sections 4.1 and 4.2 for daily and multi-day planning, respectively.

The daily routing and scheduling for home care providers is a well-studied problem in the OR literature. Almost all solution approaches reviewed in Section 4.1 cover the basic home care constraints needed. These are time windows, qualifications and availability restrictions. Furthermore, many publications propose a MIP and a heuristic because of the complexity of the problem. Many of the working regulations from labor laws and work contracts, like rest times or weekly working times, cannot be respected during optimization because of the limited planning period of one day. Thus, the proposed solution approaches are not suitable for the context of this thesis. However, some of the working regulations that are representable in daily context have been studied, namely break constraints and daily working time restrictions.

The solution approaches considering a planning horizon of several days offer the possibility of considering working regulations relevant for home care planning. These regulations were described in Section 2.4 and often span more than one day, e.g., rest times between shifts, work contracts and maximum consecutive days. The solution approach of Trautsamwieser and Hirsch [2014] considers a problem setting that is closest to the one in this thesis, because they consider rest times and weekly working times. However, some aspects are missing in their approach, namely the splitting of breaks, shift types, weekly workdays and shift rotations. Furthermore, the number of consecutive workdays is not limited and no rules regarding weekends are modeled. These two constraints are modeled by Bäumelt et al. [2010b], but this publication omits other regulations like the weekly rest time and breaks. The splitting of breaks is mentioned by Rest and Hirsch [2015] for the Austrian rules that are not applicable in all countries because they use specific partitions. Another approach considering maximum consecutive workdays is proposed by Di Gaspero and Urli [2014] but the authors omit rest times and breaks.

None of the solution approaches covers all the working regulations described in Section 2.4 in one integrated approach. Furthermore, two practical requirements are missing. First, the maximum number of weekly workdays that allows the incorporation of more complex work contracts. Second, the use of cyclic shift patterns that are widely used by German home care providers. Additionally, the generic modeling of the working regulations is essential for the approaches to be applicable in different countries and providers.

We conclude that the integration of all mentioned labor law regulations in one approach together with the modeling of work contracts and requirements from practice is a research opportunity in home care planning. This is also mentioned in the review publication of Gutiérrez et al. [2013]: “However, key real features [...] as work legal regulations for medical staff have received little attention in the research literature“. The following enumeration summarizes the research opportunities:

1. Model work contracts by including weekly workdays and shift rotations.
2. Integrate the working regulations from labor law regulations and work contracts in one solution approach.
3. Ensure a generic formulation of work contracts and labor law regulations to be applicable in many countries and institutions.
4. Investigate the influence of the working regulations on the quality of solutions.

Dynamic setting

There are only a few publications modeling the routing and scheduling in a dynamic setting. From Section 4.3 we conclude that the consideration of cross-period working regulations is not considered by these publications. The working regulations spanning more than one day are omitted by the approaches, although the rest times and total working times need to be ensured between periods. Additionally, only changes in the set of clients are investigated with the proposed methods. The changes of jobs and nurses also need to be considered, because they arise on a regular basis.

The continuity between planning periods is addressed for continuity of care and/or time. However, there are many possibilities to model continuity and no investigation of different metrics is carried out. This aspect is also mentioned in the review of [Milburn, 2012, p.299]: “Furthermore, a comprehensive routing and scheduling tool with the flexibility to evaluate various consistency policies and their impact on nurse efficiency would provide important information to home health planners as they develop operating policies“. Continuity of duty schedules for the nurses that would be interesting for practical applications is neglected in literature. As a result, several research opportunities arise in the context of the dynamic home care setting:

1. Allowing demand changes for existing clients in the form of new or less job requests and time window changes.
2. Model the entrance or leave of nurses leading to opening of new or closing of existing tours.
3. Ensure the feasibility of working regulations across planning periods.
4. Compare different continuity measures for continuity of care and time from the literature, propose new measures and investigate their influences.
5. Model continuity of duty schedules to improve nurse satisfaction.

4.5. Research goals

The research opportunities derived from the state-of-the-art in the last section lead to the following research question that should be answered in this thesis:

> "How can legal, efficient and continuous plans for the integrated routing and scheduling of home care providers in a dynamic setting be achieved in reasonable computation time?"

To provide an answer for this question, the following goals are defined within in the scope of this thesis:

1. Integrate relevant working regulations to the routing and scheduling for home care providers in a static multi-day setting.
2. Solve the integrated routing and scheduling for real-world sized problem instances.
3. Evaluate the influence of working regulations on working hours and compliance with labor law regulations.
4. Incorporate the feasibility of working regulations and continuity between planning periods.
5. Evaluate the influence of different continuity metrics on the solutions.

The goals and the course of action to achieve them are described in more detail in the remainder of this section.

Goal 1: Integrate relevant working regulations to the routing and scheduling for home care providers in a static multi-day setting.

This goal aims at the integration of the working regulations provided by work contracts and labor law regulations into one solution approach (presented in Section 2.4). Therefore, the problem setting is formalized by a mathematical program, more specifically a mixed-integer program (MIP). To ensure the applicability in different settings, the model formulation should be generic and not limited to a specific case. The solution of this model provides a schedule for a multi-day planning horizon taking into account the current set of clients, nurses and jobs. The efficiency of a plan is measured by the working time needed to perform all the jobs in the planning horizon.

Because the integrated home care routing and scheduling is a combinatorial optimization problem [Bertels and Fahle, 2006], the computation time needed to solve the problem to optimality for real-world sized instances is expected to be too long for an application in practice leading to the second goal. Nevertheless, even if the proof of optimality is not adduced in reasonable time, intermediate results provide lower and upper bounds on the optimal objective value.

Goal 2: Solve the integrated routing and scheduling for real-world sized problem instances.

The second goal is to develop efficient solution approaches that can solve real-world sized problem instances in a reasonable computation time due to the complexity of the problem. To achieve this goal, heuristics have to be designed and implemented to solve the model formulation from Goal 1. The heuristics use domain-specific knowledge of the problem setting to achieve good results. The quality of the solutions computed by the heuristics is evaluated by the bounds on the optimal solution obtained by the MIP. The parameters of the heuristics are determined using an algorithm configurator. Furthermore, the heuristics are compared to each other and the most suitable heuristic is determined with an extensive numerical analysis.

Goal 3: Evaluate the influence of working regulations on working hours and compliance with labor law regulations.

The methods proposed for Goal 1 and 2 extend the state-of-the-art because they integrate all the mentioned working regulations. Therefore, they offer the opportunity to investigate the influence of these regulations on the solution quality in terms of working hours. To perform this analysis, the best heuristic from Goal 2 is used to perform a sensitivity analysis. The experimental setup consists of several scenarios considering different sets of working regulations or none and comparing the results. Furthermore, the necessity of modeling working regulations can be emphasized by analyzing the constraint violations if the regulations are omitted.

Goal 4: Incorporate the feasibility of working regulations and continuity between planning periods.

To model continuity between planning periods and allowing a dynamic setting of demands and capacities, the selected heuristic solution approach for a static setting must be extended with a rolling planning horizon by adapting the instance and solution representation. The working regulations modeled for the static setting have to be ensured between planning periods by the method to obtain legal plans. The approach allows changes in clients, jobs and nurses whereas the current state-of-the-art omits the latter two. The continuity of care and time must be ensured across planning periods by implementing metrics from the literature and proposing new metrics that are incorporated in the objective function. Furthermore, the proposed method is the first considering the continuity of duty schedules between planning periods.

Goal 5: Evaluate the influence of the different continuity metrics on the solutions.

To provide an insight into the influences of the continuity on the quality of solutions, the metrics designed and adapted in Goal 4 are used for an experimental study in a rolling horizon setting of several weeks. The different continuity metrics for care, time and duty schedules are compared regarding their influence on the continuity and working times. Furthermore, different parameters of the rolling horizon approach are analyzed to show how they affect the continuity over several weeks.

All proposed methods have to be tested and analyzed on different datasets to provide reliable results and exclude overfitting for one testset. One instance set is generated to represent the problem setting in this thesis[1]. The instances cover all relevant aspects taking into account real-world statistics and labor law regulations. Two further datasets are provided by Trautsamwieser and Hirsch [2014] and Cappanera and Scutellà [2013a] who published on home care planning. Those are based on real-world data from Austria and Italy, respectively.

[1]The instances are available at `http://hc.guericke.org/`.

5. Exact solution approach for the static setting

In this chapter the formalization of the home care routing and scheduling problem with a MIP is proposed to achieve the first goal mentioned in the research goals. First, the notation is introduced in Section 5.1. The MIP is described in Section 5.2 providing a mathematical formulation of the problem approached in this thesis. The model is solved with a commercial solver to provide optimal solutions and bounds on objective values. The test instances used for this experiments are presented in Section 5.3. The numerical results are analyzed in Section 5.4.

5.1. Problem setting and notation

The notation and symbols for the formalization of the home care routing and scheduling problem with working regulations for a static planning horizon are introduced in this section and commonly used throughout the thesis. An overview of all sets and parameters for the MIP is also given in Section 5.2 (Table 5.1).

The set $\mathcal{J}$ denotes all jobs in the considered planning horizon of $|\mathcal{D}|$ days and is divided into two subsets. Subset $\mathcal{J}^N \subseteq \mathcal{J}$ contains artificial jobs representing start and end locations of nurses. These are described in more detail in Section 5.1.1. Jobs that are services at a client's home are contained in $\mathcal{J}^\mathcal{C} = \mathcal{J} \backslash \{\mathcal{J}^N\}$. The set of clients is denoted by $\mathcal{C}$ and the locations of the clients determine the driving time $driv_{ij}$ (in minutes) between two jobs. Each job i in $\mathcal{J}$ is defined by its day in the planning horizon d_i, time window $[a_i, b_i]$ and duration r_i. The qualification required to perform a job is denoted by q_i.

Each job needs to be assigned to one nurse n in the set $\mathcal{N}$ whereas the assignment is restricted by the qualifications $\mathcal{Q}_n$ of the nurse. The binary parameter $Q_{j,n}$ indicates whether nurse n is assignable to job j by matching available nurses with required job qualifications. Each nurse is not only assigned to jobs but also to a shift type $s \in \mathcal{S}$ on each day of the planning horizon. The route formed for a nurse on a day is restricted by the time window of the selected shift type s, i.e., it has to start and end in the time window $[A_s, B_s]$.

The planning horizon is represented by the set of days $\mathcal{D} = \{0, 1, \ldots, |\mathcal{D}| - 1\}$. Furthermore, the set of complete weeks, i.e., weeks containing the days from Monday to Sunday, are denoted by $\mathcal{W}$. For an example consider a planning horizon of $|\mathcal{D}| = 14$ days starting on a Monday results in the following sets: $\mathcal{D} = \{0, 1, ..., 13\}$ and $\mathcal{W} = \big\{\{0, 1, \ldots, 6\}, \{7, \ldots, 13\}\big\}$.

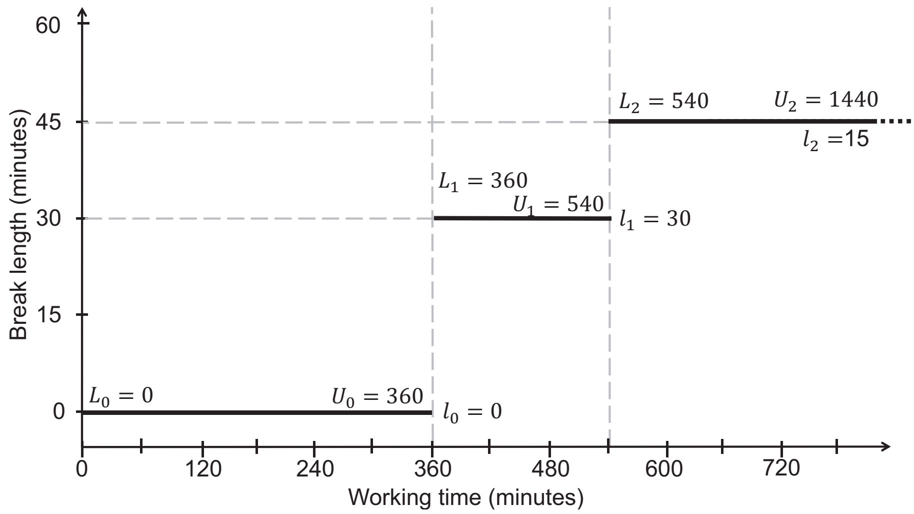


Figure 5.1.: Parameters for the break regulations in Germany

5.1.1. Working regulations

The basis for correct calculation of working and rest times are the start and end times of each nurse's route. Therefore, a set of artificial jobs $\mathcal{J}^N$ representing the start and end of each route is introduced. The set contains two jobs for each nurse on each day of the planning horizon. To determine a specific job in $\mathcal{J}^N$, two functions are defined. The start and end jobs of nurse n on day d are given by $O(n,d)$ and $E(n,d)$, respectively. The duration of all artificial jobs is zero and no qualification is needed. The location of the job depends on the start and end location valid for the nurse.

Work contracts

The formalization of work contracts needs several parameters. Each parameter is given for each nurse separately to allow individual agreements. The first parameter is the maximum daily working time of nurse n denoted by H_n^{D}. The maximum weekly working time per nurse is introduced by H_n^{W}. To differentiate contracts by workdays per week, the maximum number of days in one week is given by D_n^{W}. The availability of a nurse n for a specific shift type s on a day d is denoted by the binary parameter $F_{n,s,d}$ that equals 1 if the nurse is available, and 0 otherwise.

Labor law regulations

The break regulations described in Section 2.4.2 can be represented by a monotonically increasing step function, like given in Figure 5.1 for the German break rules.

For the formalization of the problem setting this function needs to be linearized, which can be achieved by introducing binary variables as described in Section 5.2.1.

The function to determine the required break length is divided into intervals of equal break duration. These intervals are called break levels in this thesis and are given by the set $\mathcal{B}_n$ for a nurse n. The interval of the break level $b \in \mathcal{B}_n$ is defined by $[L_b, U_b]$ where $U_b = L_{b+1}$. The break levels are given in ascending order of L_b. The required break duration is calculated by summing up the parameter l_b of all activated break levels b, i.e., the break duration in level b that is needed in addition to the duration in the lower levels is denoted by l_b. If a splitting of breaks is allowed, the minimum length of each part is represented by the parameter l_n^{min}.

The specific parameters for the German break rules are given in Figure 5.1. In this example we have three break levels with the intervals $[0, 360], [360, 540]$ and $[540, 1440]$. The lowest break level implies no break, therefore, l_0 and L_0 are set to zero. After a working period of six hours ($U_0 = L_1 = 360$) a break of $l_1 = 30$ minutes and after nine hours ($U_1 = L_2 = 540$) additional $l_2 = 15$ minutes are needed leading to a total break length of $30 + 15 = 45$ minutes.

Additional to breaks during working times, minimum rest times between shifts need to be ensured. Thus, at least R_n^{D} minutes must lie between the end of the shift on the previous and the beginning on the current day. Additionally, a minimum weekly rest time has to be fulfilled and is given by parameter R_n^{W}, i.e., once a week the daily rest time must be extended to this value.

Cyclic shift pattern

As mentioned in the problem description in Section 2.4.2 a cyclic shift rotation is given for a predefined time horizon (two weeks in the example in Figure 5.2). A nurse can start to work according to the rotation on every Monday of the given rotation. As a result, there are two feasible shift patterns for a cyclic shift rotation of length two weeks (see Figure 5.2). One pattern starts in the first week, the other in the second week. The generation of feasible patterns is done in a preprocessing step, so the input to the model is the set of feasible shift patterns $\mathcal{P}_n$ for each nurse n.

To incorporate the pattern in the model formulation, the valid shift assignments need to be encoded in parameters. We introduce the binary parameter $a_{p,s,d}$ indicating whether pattern p allows the assignment of shift type s on day d of the planning horizon. This means the pattern is transferred to the entire planning horizon, even if the pattern is shorter. An exemplary cyclic shift rotation with parameters $a_{p,s,d}$ is given in Figure 5.2. The shift rotation results in two possible sets of parameters, one for each pattern, whereas the second pattern is equal to the first pattern shifted by one week. A value of one indicates that the shift type is assignable on the specific day, a zero indicates incompatibility. The solution approach has to decide on the

Shift rotation

Day (d)	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Shift type (s)	M	M	M	M	M	-	-	A	A	A	A	A	W	W

Binary parameters $a_{p,s,d} \in \{0,1\}$

	Day (d) / Shift type (s)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	...
Pattern $p = 1$	M	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0	0	...
	A	0	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	...
	W	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	...
Pattern $p = 2$	M	0	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	...
	A	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0	0	...
	W	0	0	0	0	0	1	1	0	0	0	0	0	1	1	0	0	0	0	0	1	1	0	...

Figure 5.2.: Parameters for an exemplary shift pattern (M = morning shift, A = afternoon shift, W = weekend shift)

pattern for each nurse n in the set $\mathcal{N}^{\mathrm{R}} \subseteq \mathcal{N}$, i.e., determine the start week of each nurse in the cyclic shift rotation.

Further restrictions for duty planning

Additional to the already mentioned parameters and sets, we specify subsets and parameters to model further restrictions on the duty planning of nurses. All nurses contained in the set $\mathcal{N}^{\mathrm{WE}} \subseteq \mathcal{N}$ must be either assigned on both days of a weekend or none. For this restriction we need to know all Sundays $\mathcal{D}^{Su}$ in the planning horizon $\mathcal{D}$. The maximum number of consecutive workdays for nurse n is limited by D_n^{C}.

5.1.2. Network structure

The mathematical formulation in the home care context can be modeled based on a graph $G = (V, \mathcal{A})$. The nodes V represent the route start and end locations of nurses as well as jobs of clients, i.e., $V = \mathcal{J}$. The arcs $\mathcal{A} \subseteq (\mathcal{J} \times \mathcal{J} \times \mathcal{N})$ consist of feasible connections between two nodes, i.e., if and only if $(i, j) \in \mathcal{A}$ job j can directly follow job i in a route assigned to nurse n. The following infeasible arcs can be removed from the set of all arcs $(\mathcal{J} \times \mathcal{J} \times \mathcal{N})$, because they would violate the requirements of the problem description and thus form an infeasible solution:

1. Cyclic arcs of type (i, i, n) because each job is scheduled exactly once.
2. Arcs connecting jobs that can be planned only in separate routes due to different days or shift types.
3. Arcs to and from jobs that need another qualification than the nurse has.
4. Arcs that are excluded by time window constraints.

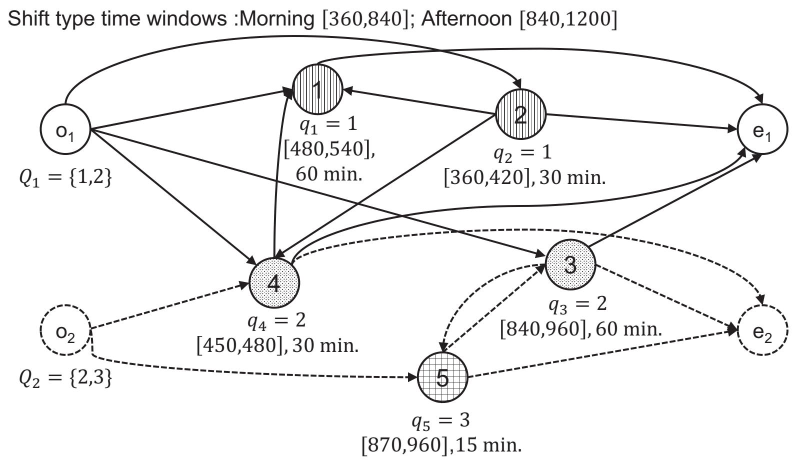


Figure 5.3.: Network structure for example instance with two nurses (qualifications shown) and five jobs (qualification, time window and duration shown) on one day

5. Incoming arcs for shift start nodes and outgoing arcs for shift end nodes because no other jobs can precede or follow those, respectively.

6. Arcs starting or ending at nodes defined for another nurse than the current.

7. Arcs connecting start with end nodes and vice versa.

Figure 5.3 shows a small example network containing two nurses and five jobs to explain the rules in more detail. The shift start and end nodes for the two nurses are labeled o and e, respectively. Additionally, the qualifications of the nurses are given by Q_n. The nodes with numbers from one to five are the jobs at clients labeled with the needed qualification q, time window $[a, b]$ and duration in minutes. The solid arcs are valid for nurse 1 and the dashed arcs are valid for nurse 2.

As we can see from Figure 5.3, the start node o_1 of nurse 1 is only connected to the jobs 1, 2, 3 and 4 because they require qualification 1 (striped) or 2 (dotted) (Rule 3). On the contrary, job 5 (checkered) can be performed only by nurse 2 because it requires qualification 3. Jobs 1, 2 and 3 have no connecting arcs with jobs 4 and 5 due to the shift type definitions (Rule 2). All feasible routes start and end within the assigned shift type and, thus, the jobs cannot be scheduled in the same route. There is an arc from job 2 to 1 but not vice versa because the time window restrictions allow a sequencing only in this order (Rule 4). The same holds for the arcs connecting jobs 4 and 1 as well as jobs 4 and 2. The arcs leading to the shift end nodes of each nurse are starting only at feasible jobs for the respective nurse (Rule 6).

With the above mentioned rules, the reduced sets of incoming $\Omega^-(i)$ (5.1) and outgoing $\Omega^+(i)$ (5.2) arcs for each node $i \in \mathcal{J}$ can be defined.

$$\begin{aligned}\Omega^-(i) =& \{(j,i,n) \in \mathcal{A} | i \neq j \land d_i = d_j \land q_i \in \mathcal{Q}_n \land q_j \in \mathcal{Q}_n \land a_j \leq b_i \\ & \land (i \neq O(n,d_i)) \land \big((i = E(n,d_i) \land j \notin \mathcal{J}^N) \lor (i \notin \mathcal{J}^N)\big)\} \end{aligned} \tag{5.1}$$

$$\begin{aligned}\Omega^+(i) =& \{(i,j,n) \in \mathcal{A} | i \neq j \land d_i = d_j \land q_i \in \mathcal{Q}_n \land q_j \in \mathcal{Q}_n \land a_i \leq b_j \\ & \land (i \neq E(n,d_i)) \land \big((i = O(n,d_i) \land j \notin \mathcal{J}^N) \lor (i \notin \mathcal{J}^N)\big)\} \end{aligned} \tag{5.2}$$

These definitions are used for building sets of reachable nodes $j \in \mathcal{J}$ for each nurse n based on the incoming and outgoing arcs of job i. They are given by the sets $\Delta^+(i,n)$ and $\Delta^-(i,n)$, respectively.

$$\Delta^-(i,n) = \{j \in \mathcal{J} \mid (j,i,n) \in \Omega^-(i)\} \tag{5.3}$$

$$\Delta^+(i,n) = \{j \in \mathcal{J} \mid (i,j,n) \in \Omega^+(i)\} \tag{5.4}$$

The set $\Gamma(i,j)$ defines the set of nurses that are feasible for traveling from i to j.

$$\Gamma(i,j) = \{n \in \mathcal{N} \mid j \in \Delta^+(i,n)\} \tag{5.5}$$

With these definitions the constraints regarding qualifications matchings of nurses and jobs are already satisfied. The overall set of arcs $\mathcal{A}$ is defined based on the sets Ω^+ and Ω^-:

$$\mathcal{A} = \{(i,j,n) | i \in \mathcal{J} \land j \in \mathcal{J} \land \big((i,j,n) \in \Omega^+(i) \lor (i,j,n) \in \Omega^-(j)\big)\} \tag{5.6}$$

5.2. Formalization of the problem setting

This section proposes the formalization of the home care routing and scheduling problem with working regulations by a MIP. First, the decision variables are presented and, afterwards, the objective function and constraints of the model are explained in detail.

5.2.1. Decision Variables

The decision variables $x_{i,j,n}$ model the sequence of jobs in routes and the assignment of nurses to jobs. The binary variable $x_{i,j,n}$ equals 1 if nurse n travels from job i directly to job j. The variables $x_{i,j,n}$ are defined on the set of arcs $(i,j,n) \in \mathcal{A}$ to reduce the number of binary variables as many infeasible combinations are already excluded. The start time of a job i is modeled with the continuous variable z_i and the waiting time before job j due to hard time windows with variable w_j. The duty plan of the nurses is encoded in the binary variables $y_{n,s,d}$ that equals 1 if nurse n

Notation	Definition
$\mathcal{A}$	Set of arcs, i.e., feasible combinations in $(\mathcal{J} \times \mathcal{J} \times \mathcal{N})$
$\mathcal{B}_n$	Set of break levels for nurse n
$\mathcal{D} = \{0, \dots, \lvert\mathcal{D}\rvert - 1\}$	Set of days
$\mathcal{D}^{\text{Su}} \subseteq \mathcal{D}$	Set of Sundays in the planning horizon
$\mathcal{J}$	Set of jobs
$\mathcal{J}^C \subseteq \mathcal{J}$	Set of jobs at clients
$\mathcal{J}^N \subseteq \mathcal{J}$	Set of jobs representing start and end or routes
$\mathcal{J}_d$	Set of jobs on day d
$\mathcal{N}$	Set of nurses
$\mathcal{N}^{\text{R}} \subseteq \mathcal{N}$	Set of nurses working shift rotations
$\mathcal{N}^{\text{WE}} \subseteq \mathcal{N}$	Set of nurses working only complete weekends
$\mathcal{P}_n$	Set of feasible shift patterns for nurse n
$\mathcal{Q}$	Set of qualifications
$\mathcal{Q}_n$	Set of qualifications of nurse n
$\mathcal{S}$	Set of shift types
$\mathcal{W}$	Sets of complete weeks in $\mathcal{D}$
$\Omega^-(i) \subseteq \mathcal{A}$	Incoming arcs of job i
$\Omega^+(i) \subseteq \mathcal{A}$	Outgoing arcs of job i
$\Delta^-(i, n) \subseteq \mathcal{J}$	Set of jobs from which job i can be reached by nurse n
$\Delta^+(i, n) \subseteq \mathcal{J}$	Set of jobs which can be reached from job i by nurse n
$\Gamma(i, j) \subseteq \mathcal{N}$	Set of nurses feasible to travel from job i to job j
$O(n, d) \in \mathcal{J}^N$	Start job for nurse n on day d
$E(n, d) \in \mathcal{J}^N$	End job for nurse n on day d
$a_{p,s,d} \in \{0, 1\}$	Binary parameter, equals 1 if shift type s on day d is assignable in shift pattern p
$[a_i, b_i]$	Time window of job i
$[A_{s,d}, B_{s,d}]$	Time window of shift type s on day d
$d_i \in \mathcal{D}$	Day of job i
$driv_{i,j} \in \mathbb{N}$	Driving time between job i and j (in minutes)
$D_n^{\text{W}} \in \mathbb{N}$	Max. weekly workdays of nurse n
$D_n^{\text{C}} \in \mathbb{N}$	Max. number of consecutive workdays of nurse n
$F_{n,s,d} \in \{0, 1\}$	Binary parameter, equals 1 if shift type s can be performed by nurse n on day d, 0 otherwise
$H_n^{\text{D}} \in \mathbb{N}$	Max. daily working time of nurse n (in minutes)
$H_n^{\text{W}} \in \mathbb{N}$	Max. weekly working time of nurse n (in minutes)
$[L_b, U_b]$	Validity interval of break level b
$l_b \in \mathbb{N}$	Required additional length of break in break level b (in minutes)
$l_n^{min} \in \mathbb{N}$	Minimum uninterrupted break length for nurse n (in minutes)
$q_i \in \mathcal{Q}$	Qualification needed for job i
$Q_{j,n} \in \{0, 1\}$	Binary parameter, equals 1 if job j can be performed by nurse n, 0 otherwise
$r_i \in \mathbb{N}$	Duration of job i (in minutes)
$R_n^{\text{D}} \in \mathbb{N}$	Min. daily rest time of nurse n (in minutes)
$R_n^{\text{W}} \in \mathbb{N}$	Min. weekly rest time of nurse n (in minutes)
$v_n \in \mathbb{N}$	End of shift on last day of previous period ($d = -1$) for nurse n

Table 5.1.: Sets and parameters

Name	Range	Definition
$x_{i,j,n}$	$\in \{0,1\}$	Binary decision variable, equals 1 if job j is performed by nurse n directly after job i, 0 otherwise, $(i,j,n) \in \mathcal{A}$
z_j	$\in \mathbb{R}$	Continuous decision variable, start time of job $j \in \mathcal{J}$ in minutes from the beginning of the planning horizon
w_j	$\in \mathbb{R}^+$	Continuous decision variable, waiting time before job $j \in \mathcal{J}$ in minutes
$y_{n,s,d}$	$\in \{0,1\}$	Binary decision variable, equals 1 if nurse $n \in \mathcal{N}$ works shift type $s \in \mathcal{S}$ on day $d \in \mathcal{D}$, 0 otherwise
$\rho_{n,d,b}$	$\in \{0,1\}$	Binary decision variable, equals 1 if nurse $n \in \mathcal{N}$ needs a break of type $b \in \mathcal{B}_n$ on day $d \in \mathcal{D}$, 0 otherwise
$\beta_{j,n}$	$\in \{0,1\}$	Binary decision variable, equals 1 if a break for nurse $n \in \mathcal{N}$ is assigned before job $j \in \mathcal{J}$, 0 otherwise
$\delta_{j,n}$	$\in \mathbb{R}^+$	Continuous decision variable, duration of the break for nurse $n \in \mathcal{N}$ before job $j \in \mathcal{J}$ in minutes
$k_{n,p}$	$\in \{0,1\}$	Binary decision variable, equals 1 if nurse $n \in \mathcal{N}^{\mathrm{R}}$ works shift pattern $p \in \mathcal{P}_n$, 0 otherwise

Table 5.2.: Decision variables and ranges

works shift type s on day d. The binary variable $k_{n,p}$ is defined for the nurses in $\mathcal{N}^R$ and equals 1 if pattern p is selected for nurse n.

The modeling of break assignments requires three sets of variables. First, the necessity of assigning a break of level b on day d for nurse n is modeled by the binary decision variable $\rho_{n,d,b}$. The placement of the break inside the route is model by the binary decision variable $\beta_{j,n} \in \{0,1\}$ indicating whether a break is inserted before job j for nurse n. The length of the break is given by the continuous variables $\delta_{j,n} \in \mathbb{R}^+$. Table 5.2 gives an overview of all variables and their ranges.

5.2.2. Mixed integer program

In this section we describe our mathematical formulation for home care routing and scheduling with working regulations. The model formulation is based on the VRPTW formulation described in Section 3.2.1 because the routing with time windows is a subproblem of the problem setting in this thesis. The model is a MIP, because it contains integer, more specifically binary, variables and continuous variables. First, we describe the objective function. Afterwards, the required constraints are presented.

$$min \sum_{(i,j,n)\in\mathcal{A}} w_i + \delta_{j,n} + (r_i + driv_{i,j})x_{i,j,n} \tag{5.7}$$

The objective function (5.7) minimizes the total tour length (in minutes) consisting of waiting, break and driving times. Note that job durations are a fixed input and cannot be shortened during optimization. Due to the inclusion of break durations in the objective function, unnecessary break assignments are avoided because they would increase the objective value. Thus, artificial lengthening of routes to include breaks to reduce the overall working time is not possible.

$$\sum_{j \in \Delta^+(O(n,d),n)} x_{O(n,d),j,n} = \sum_{s \in \mathcal{S}} y_{n,s,d} \qquad \forall n \in \mathcal{N}, d \in \mathcal{D} \tag{5.8}$$

$$\sum_{j \in \Delta^-(E(n,d),n)} x_{j,E(n,d),n} = \sum_{s \in \mathcal{S}} y_{n,s,d} \qquad \forall n \in \mathcal{N}, d \in \mathcal{D} \tag{5.9}$$

$$\sum_{j \in \Delta^+(i,n)} x_{i,j,n} = \sum_{j \in \Delta^-(i,n)} x_{j,i,n} \qquad \forall i \in \mathcal{J}^C, n \in \mathcal{N} \tag{5.10}$$

$$\sum_{n \in \mathcal{N}} \sum_{j \in \Delta^+(i,n)} x_{i,j,n} = 1 \qquad \forall i \in \mathcal{J}^C \tag{5.11}$$

$$\sum_{j \in \Delta^+(i,n)} x_{i,j,n} \leq Q_{i,n} \sum_{s \in S} y_{n,s,d_i} \qquad \forall i \in \mathcal{J}, n \in \mathcal{N} \tag{5.12}$$

Constraints (5.8) and (5.9) ensure outgoing and incoming arcs at shift start and end on assigned workdays, respectively. Thus, no empty routes are possible. Constraints (5.10) are the flow balance constraints of the VRP, i.e., each job has the same number of incoming as outgoing arcs. The start and end jobs of nurses are excluded because they have only outgoing or incoming arcs, respectively. The flow balance equality must be held for each nurse separately to ensure individual and connected routes. Constraints (5.11) assign every job of the clients exactly once by setting the number of outgoing arcs to one. If and only if a nurse is assigned to a shift, the routing variable can be activated for jobs on that day (5.12).

$$z_i + r_i + driv_{i,j} + w_j + \sum_{n \in \Gamma(i,j)} \delta_{j,n} \leq z_j + M(1 - \sum_{n \in \Gamma(i,j)} x_{i,j,n}) \quad \forall i \in \mathcal{J}, j \in \Omega^+(i) \tag{5.13}$$

$$z_i + r_i + driv_{i,j} + w_j + \sum_{n \in \Gamma(i,j)} \delta_{j,n} \geq z_j - M(1 - \sum_{n \in \Gamma(i,j)} x_{i,j,n}) \quad \forall i \in \mathcal{J}, j \in \Omega^+(i) \tag{5.14}$$

$$a_i \leq z_i \leq b_i \qquad \forall i \in \mathcal{J}^C \tag{5.15}$$

The start time for each job is set in constraints (5.13) and (5.14) similar to the VRPTW. If job j is scheduled after job i, the start time of job j is calculated using the start time of job i, the duration of job i and the driving time needed to travel from i to j. Additionally, a potential waiting time w_j or a break of length $\delta_{j,n}$ in between is considered. If j is not scheduled after i, the constraints are deactivated using a big-M ($M = 1440|\mathcal{D}|$) formulation. The time windows for jobs at clients are ensured by constraints (5.15).

Working time regulations

The following constraints model the working time regulations defined by the work contracts of the nurses.

$$\sum_{d \in w} \big(z_{E(n,d)} - z_{O(n,d)} - \sum_{b \in \mathcal{B}_n} l_b \rho_{n,d,b}\big) \leq H_n^{\mathrm{W}} \qquad \forall n \in \mathcal{N}, w \in \mathcal{W} \tag{5.16}$$

$$z_{E(n,d)} - z_{O(n,d)} - \sum_{b \in \mathcal{B}_n} (l_b \rho_{n,d,b}) \leq H_n^{\mathrm{D}} \quad \forall n \in \mathcal{N}, d \in \mathcal{D} \tag{5.17}$$

$$\sum_{d \in w} \sum_{s \in \mathcal{S}} y_{n,s,d} \leq D_n^{\mathrm{W}} \qquad \forall n \in \mathcal{N}, w \in \mathcal{W} \tag{5.18}$$

$$\sum_{d=d'}^{d'+D_n^{\mathrm{C}}} \sum_{s \in \mathcal{S}} y_{n,s,d} \leq D_n^{\mathrm{C}} \qquad \forall n \in \mathcal{N}, d' \in \{0, ..., |\mathcal{D}| - D_n^{\mathrm{C}}\} \tag{5.19}$$

Constraints (5.16) limit the weekly working time of nurses, with break time not considered as working time, by summing up the lengths of all tours in one week. Therefore, the difference between end ($z_{E(n,d)}$) and start time ($z_{O(n,d)}$) is calculated and the assigned break duration subtracted from it ($l_b \rho_{n,d,b}$). The weekly working time is determined based on the days in week $w \in \mathcal{W}$. The maximum working time on each day is incorporated by constraints (5.17) and calculated like the weekly working time just for one day. The number of workdays per week is limited by constraints (5.18), and the maximum number of consecutive workdays by constraints (5.19). Both use the sum of shift assignment variables to calculate the number of workdays.

Break assignments

Several constraints are needed to ensure the correct assignment, position and length of breaks.

$$\begin{aligned} z_{E(n,d)} - z_{O(n,d)} - \sum_{b \in \mathcal{B}_n} l_b \rho_{n,d,b} \leq \\ U_0 + \sum_{b \in \mathcal{B}_n \setminus \{|\mathcal{B}_n|\}} \rho_{n,d,b} (U_{b+1} - U_b) + 2R_n^W (1 - \sum_{s \in S} y_{n,s,d}) \quad \forall n \in \mathcal{N}, d \in \mathcal{D} \end{aligned} \tag{5.20}$$

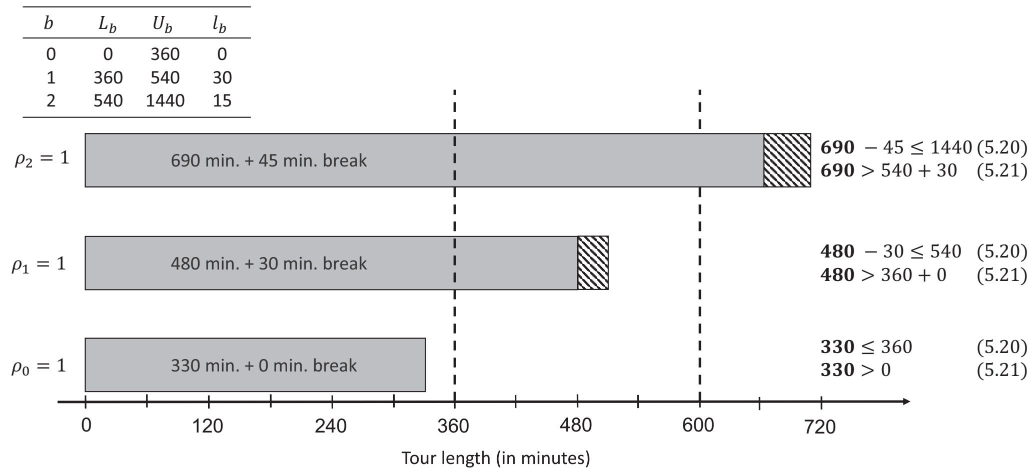


Figure 5.4.: Overview of break activation constraints for break levels 0, 1 and 2 ($\rho_0 = 1, \rho_1 = 1$ or $\rho_2 = 1$). The bars consist of working time (grey) and break time (dashed). The ranges on the right are calculated based on constraints (5.20) and (5.21). The actual working time is printed in bold

$$z_{E(n,d)} - z_{O(n,d)} > 0 + \sum_{b \in \mathcal{B}_n \setminus \{0\}} (L_b - L_{b-1} + l_{b-1})\rho_{n,d,b} \qquad \forall n \in \mathcal{N}, d \in \mathcal{D} \quad (5.21)$$

First, the necessity of a break depending on the working time (without break time) must be determined in constraints (5.20) and (5.21). In (5.20) the correct break level is activated by limiting the working time depending on the break level intervals. Only if a break level is activated, the tour length can be longer than U_0 minutes. Furthermore, a break can be scheduled only if a shift assignment is selected. Otherwise, the constraint is deactivated using $2R_n^W$ as big-M. To avoid unnecessary breaks, the minimum length of a tour based on the selected break level is ensured in constraints (5.21). Figure 5.4 shows the ranges for activation based on the break level for the German rules.

$$\rho_{n,d,b} \leq \sum_{s \in \mathcal{S}} y_{n,s,d} \qquad \forall n \in \mathcal{N}, d \in \mathcal{D}, b \in \mathcal{B}_n \quad (5.22)$$

$$\beta_{i,n} \leq \sum_{j \in \Delta^-(i,n)} x_{j,i,n} \qquad \forall i \in \mathcal{J}, n \in \mathcal{N} \quad (5.23)$$

$$\rho_{n,d,b} \leq \rho_{n,d,b-1} \qquad \forall n \in \mathcal{N}, d \in \mathcal{D}, b \in \{2, ..., \mathcal{B}_n\} \quad (5.24)$$

A break can only be assigned for nurse n on day d, if the nurse is assigned to any shift on day d (5.22). Constraints (5.23) limit the break assignment to insertion before jobs that are scheduled for the respective nurse. If a connection to job j for nurse

n is activated, the break variable for job j can be set to a value different from zero. The consecutive selection of breaks due to the step function is ensured by constraints (5.24).

$$\sum_{j\in\mathcal{J}_d}\delta_{j,n} = \sum_{b\in\mathcal{B}_n} l_b\rho_{n,d,b} \qquad \forall n\in\mathcal{N}, d\in\mathcal{D} \tag{5.25}$$

$$\sum_{j\in\mathcal{J}_d}\beta_{j,n} \leq \frac{1}{l_n^{min}}\sum_{b\in\mathcal{B}_n} l_b\rho_{n,d,b} \qquad \forall n\in\mathcal{N}, d\in\mathcal{D} \tag{5.26}$$

$$l_b^{min}\beta_{j,n} \leq \delta_{j,n} \leq \max_{b\in\mathcal{B}_n}\{l_b\}\beta_{j,n} \qquad \forall j\in\mathcal{J}, n\in\mathcal{N} \tag{5.27}$$

Based on the selected break level, constraints (5.25) ensure that the required break duration is assigned (in one or more breaks). Constraints (5.26) limit the number of breaks to the maximum number of breaks based on the minimum break length. To ensure that a break fulfills the minimum length criteria, the bounds on the duration are set in constraints (5.27). The upper bound limits the break duration before job j to zero, if no break activation variable is set for j.

$$z_{E(n,d)} - (z_j - \delta_{j,n}) - M(1-\beta_{j,n}) \leq U_0 \qquad \forall n\in\mathcal{N}, d\in\mathcal{D}, j\in\mathcal{J} \tag{5.28}$$

$$(z_j - \delta_{j,n} - w_j) - z_{O(n,d)} - M(1-\beta_{j,n}) \leq U_0 \qquad \forall n\in\mathcal{N}, d\in\mathcal{D}, j\in\mathcal{J} \tag{5.29}$$

To plan breaks in the middle of the route and achieve at most U_0 minutes of working time without a break, constraints (5.28) and (5.29) restrict the placement of the break inside the route. Constraint (5.28) limits the time after the break and constraints (5.29) the time before the break to at most U_0 minutes, respectively. Constraints (5.29) have to take the waiting time w_j before job j into account because a break is defined before job j with the variable $\beta_{j,n}$, . If the break is split in smaller breaks, these constraints hold for every part. These constraints work under the assumption that the maximum daily working time is less than $2U_0$, which is the common case.

Rest times

As mentioned in Section 2.4.2, the daily and weekly rest times of the employees are regulated in many countries. The following constraints ensure that these requirements are kept.

$$z_{O(n,d+1)} - z_{E(n,d)} \geq R_n^{\mathrm{D}} - (2 - \sum_{s\in S} y_{n,s,d} - \sum_{s\in S} y_{n,s,d-1})R_n^{\mathrm{D}}$$
$$\forall n\in\mathcal{N}, d\in\{0,...,|\mathcal{D}|-1\} \tag{5.30}$$

$$\max_{d\in\{d'\in w|d'>0\}}\{z_{O(n,d)} - z_{E(n,d-1)}\} \geq R_n^{\mathrm{W}} \qquad \forall n\in\mathcal{N}, w\in\mathcal{W} \tag{5.31}$$

The daily rest time R_n^D between shifts on consecutive days is held by constraints (5.30). If one of the two consecutive days has no shift assignment, the constraints is deactivated by using a big-M formulation, where M is set to R_n^D. The minimum weekly uninterrupted rest time is guaranteed by constraints (5.31). Due to the fact that this constraint has to be fulfilled once a week, only the maximum of all rest times in one week has to be greater equal R_n^W. The minimum weekly rest time R_n^W considered in this problem setting often ranges more than 24 hours. As we calculate the rest time between tour end and start times (5.31), the times at start and end jobs must be allowed to lie outside their current day. Otherwise, rest times of more than 24 hours are not possible. However, the times can be restricted by the start of the current day minus the weekly rest time or the end of the current day plus the weekly rest time, because these are the maximum cases. These ranges are modeled in constraints (5.33) and (5.34).

$$-(1440 - v_n) \leq z_{O(n,0)} \leq z_{E(n,0)} \qquad \forall n \in \mathcal{N}, \tag{5.32}$$

$$(1440d - R_n^W) \leq z_{O(n,d)} \leq z_{E(n,d)} \qquad \forall n \in \mathcal{N}, d \in \{1, \ldots, |\mathcal{D}|\} \tag{5.33}$$

$$(1440(d+1) + R_n^W) \geq z_{E(n,d)} \geq z_{O(n,d)} \qquad \forall n \in \mathcal{N}, d \in \{1, \ldots, |\mathcal{D}|\} \tag{5.34}$$

An exception is the first day of the planning horizon. Here, the time variables cannot lie before the tour end of the last tour in a potential previous planning horizon (5.32) given by the parameter v_n. The constraints (5.32) to (5.34) model the ranges when there is no shift assigned. If a shift is assigned to the nurse, the start and end times of the route have to lie within the time window of the shift. These time window constraints are modeled in (5.35) to (5.38) based on the value of the shift assignment variable $y_{n,s,d}$.

$$\sum_{s \in \mathcal{S}} A_{s,d} y_{n,s,d} - (1 - \sum_{j \in \Delta^+(O(n,d),n)} x_{O(n,d),j,n}) M \leq z_{O(n,d)} \quad \forall n \in \mathcal{N}, d \in \mathcal{D} \tag{5.35}$$

$$\sum_{s \in \mathcal{S}} B_{s,d} y_{n,s,d} + (1 - \sum_{j \in \Delta^+(O(n,d),n)} x_{O(n,d),j,n}) M \geq z_{O(n,d)} \quad \forall n \in \mathcal{N}, d \in \mathcal{D} \tag{5.36}$$

$$\sum_{s \in \mathcal{S}} A_{s,d} y_{n,s,d} - (1 - \sum_{j \in \Delta^-(E(n,d),n)} x_{j,E(n,d),n}) M \leq z_{E(n,d)} \quad \forall n \in \mathcal{N}, d \in \mathcal{D} \tag{5.37}$$

$$\sum_{s \in \mathcal{S}} B_{s,d} y_{n,s,d} + (1 - \sum_{j \in \Delta^-(E(n,d),n)} x_{j,E(n,d),n}) M \geq z_{E(n,d)} \quad \forall n \in \mathcal{N}, d \in \mathcal{D} \tag{5.38}$$

An example of the constraint regarding daily and weekly rest times is given in Figure 5.5 to explain the connections of variables and constraints in more detail. The figure shows shift assignments on the days 0, 1 and 3 for nurse 1. On days 0 and 1 shift type 1 is assigned ($y_{1,1,0} = y_{1,1,1} = 1$) and shift type 2 on day 3 ($y_{1,2,3} = 1$). The

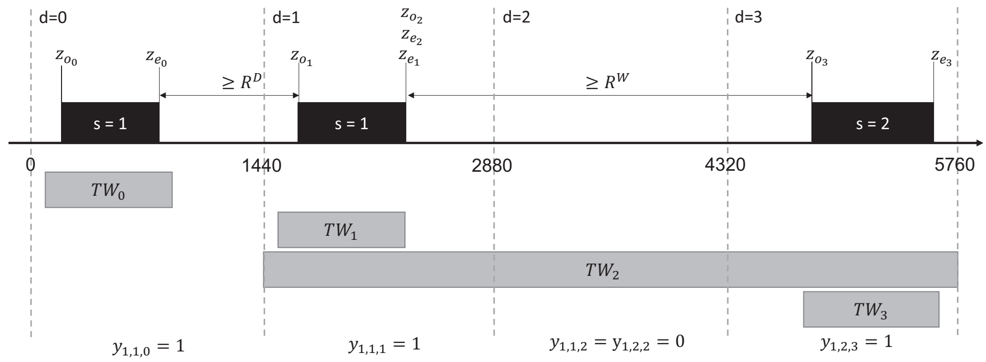


Figure 5.5.: Overview of rest time constraints and variables for an example of four days with one nurse and $R^W = 1440$ minutes (black bars = tours, grey bars = time windows for start and end jobs)

tours inside the shifts are visualized with black bars. On day 2 no shift is assigned and thus the y-variables for both shift types are set to zero. According to these shift assignments, the time windows for the start and end times of the routes are limited to the shift type time windows (depicted as gray bars below the time line). On day 2 no shift assignment is present and therefore, the start job o_2 and end job e_2 have a larger time window to model the weekly rest time. In this example with $R^W = 1440$, the time window equals $[2 \cdot 1440 - 1440, 1440 \cdot (2+1) + 1440] = [1440, 5760]$ (based on constraints (5.33) and (5.34)). Thus, the start times of the jobs o_2 and e_2 can moved to the end of the tour on day 1 and start of the tour on day 3 to fulfill the weekly rest time requirement.

Availability and shift rotations

The assignments of nurses due to rotations and availabilities are modeled by the following constraints. Furthermore, the correct assignment of shift patterns resulting from shift rotations is guaranteed.

$$y_{n,s,d} \leq \sum_{p \in \mathcal{P}_n} a_{p,s,d} k_{n,p} \qquad \forall n \in \mathcal{N}^{\mathrm{R}}, s \in \mathcal{S}, d \in \mathcal{D} \tag{5.39}$$

$$\sum_{p \in \mathcal{P}_n} k_{n,p} \leq 1 \qquad \forall n \in \mathcal{N}^{\mathrm{R}} \tag{5.40}$$

Constraints (5.39) limit the shift assignment of a nurse to the selected shift pattern based on the binary parameters $a_{p,s,d}$. Constraints (5.40) ensure only one pattern per nurse is selected. These constraints are valid for nurses in N^{R} only. The variables $k_{n,p}$ are excluded for the nurses $n \notin N^{\mathrm{R}}$.

$$\sum_{s\in\mathcal{S}} y_{n,s,d} \leq 1 \qquad \forall n \in \mathcal{N}, d \in \mathcal{D} \tag{5.41}$$

$$\sum_{s\in\mathcal{S}} y_{n,s,d} = \sum_{s\in\mathcal{S}} y_{n,s,d-1} \qquad \forall d \in \mathcal{D}^{\text{Su}}, n \in \mathcal{N}^{\text{WE}} \tag{5.42}$$

$$y_{n,s,d} \leq F_{n,s,d} \qquad \forall n \in \mathcal{N}, s \in \mathcal{S}, d \in \mathcal{D} \tag{5.43}$$

At most one shift assignment per day per nurse is enforced by constraints (5.41). Constraints (5.42) assign both days on weekends if it is required for a nurse ($n \in \mathcal{N}^{\text{WE}}$), i.e., Saturday and Sunday or neither (independent of shift types). The shifts from Monday to Friday are not affected by these constraints. Constraints (5.43) restrict the shift assignments to the valid shift types and days for each nurse to model (un)availability of nurses.

Soft constraints

We relax two constraint sets of the above presented MIP, namely the assignment of jobs and the weekend shift assignments.

First, constraints (5.11) ensuring that each job is assigned exactly once are relaxed. This enables the solver to find a feasible solution for every data input. This is essential for practical application because one job that cannot be inserted would lead to an infeasible model and therefore no schedule can be determined from the MIP. When the job assignment constraint is relaxed, the solver is able to leave jobs unassigned if it is not possible to insert them. A home care provider could insert these jobs manually at the end of a tour after the solution process, although it would cause overtime.

Furthermore, the MIP solver is able to find feasible solutions earlier in the solution process and can use the information to potentially speed up the solution process. Nevertheless, unassigned jobs should be avoided and therefore every unassigned job is penalized with high costs. The required modifications of the model are presented in the following. We introduce a new binary variable u_i indicating whether a job is unassigned or not and constraints (5.44) to replace constraints (5.11) by inserting the new variable as compensation for no assignment.

$$u_i = \begin{cases} 1, & \text{if job } i \in \mathcal{J}^C \text{ is unassigned} \\ 0, & \text{otherwise.} \end{cases}$$

$$\sum_{n\in\mathcal{N}} \sum_{j\in\Delta^+(i,n)} x_{i,j,n} = 1 - u_i \quad \forall i \in \mathcal{J}^C \tag{5.44}$$

Second, we relax the constraints regarding the complete weekend assignments because they are no legal or contractual regulations. But as single workdays on weekends are not preferred by nurses, this case is penalized in the objective function. Note that the penalty costs are less than the costs for unassigned jobs, because assigning jobs has a higher priority in practice. For the relaxation of weekend assignments, we introduce a binary variable to model constraints (5.42) as soft constraints (5.45).

$$\gamma_{n,d} = \begin{cases} 1, & \text{if the weekend belonging to Sunday } d \in \mathcal{D}^{\text{Su}} \text{ for nurse } n \in \mathcal{N}^{\text{WE}} \\ & \text{has only one shift assignment} \\ 0, & \text{otherwise.} \end{cases}$$

$$-\gamma_{n,d} \leq \sum_{s \in \mathcal{S}} y_{n,s,d} - \sum_{s \in \mathcal{S}} y_{n,s,d-1} \leq \gamma_{n,d} \quad \forall d \in \mathcal{D}^{\text{Su}}, n \in \mathcal{N}^{\text{WE}} \tag{5.45}$$

Finally, the original objective function (5.7) is extended by the penalty costs ϕ^U (=10^6) for unassigned jobs and ϕ^W (=14400) for single shifts on weekends:

$$min \sum_{(i,j,n) \in \mathcal{A}} \big(w_i + \delta_{j,n} + driv_{i,j} x_{i,j,n}\big) + \phi^U \sum_{j \in \mathcal{J}^C} u_j + \phi^W \sum_{n \in \mathcal{N}^{\text{WE}}} \sum_{d \in \mathcal{D}^{\text{Su}}} \gamma_{n,D} \tag{5.46}$$

In the remainder of this thesis, the original model (5.7) to (5.43) is referred to as model with *hard* constraints whereas the model with the relaxed constraints described in this section is referred to as model with *soft* constraints.

5.2.3. Consideration of Generic Working Regulations

With the above mentioned model formulation different kinds of working regulations and agreements can be modeled. The set of shift types allows a home care provider to define its own set of shift types. If needed, it is possible to define a shift type which is only valid for one day (e.g. public holidays) or the weekend, which allows a high degree of freedom, especially since the weekend is often handled in a different way than the weekdays (e.g. longer shifts). Specific agreements with employees can also be modeled by defining special shift types for them. The number of shift types per day is not limited, but increases the model complexity by the number of $y_{n,s,d}$ variables. However, if the shift type is only valid for one employee, the variables for all other employees can be omitted. Further individual agreements with employees can be incorporated by the definition of unavailability or the usage of shift rotations.

Another advantage is the consideration of different contract types, which can be modeled by the maximum number of weekly workdays as well as daily and weekly working times. Thereby, we can differentiate between a part-time nurse working three days a week the whole day or five days a week for only half of the day. Vacations and regular unavailability of employees are introduced by the availability parameter. The generic modeling of the break rules allows the model to be used in many countries with differences in the labor law restrictions, as long as the break rules can be defined as a monotonically increasing step function.

Since most of the parameters, such as rest times and working times, are defined for a specific nurse, it is further possible to incorporate different rules for different sets of employees. For examples apprentices younger than 18 years versus trained employees. In many countries there are special regulations for young employees. These affect mainly the break and rest times. In Germany apprentices younger than 18 need 12 instead of 11 hours rest time and a breaks is mandatory after 4.5 hours instead of 6 hours [Bundesministerium für Arbeit und Soziales, 1994]. In summary, many important working regulations and agreements used in the everyday business of a home care provider can be used as input to the model.

5.3. Test instances

We use three different sets of test instances to evaluate the MIP formulation. These instances are also the basis for the evaluation of the metaheuristics presented in the next chapter. The first set of instances is artificially generated to have all the relevant data representing the problem setting in this thesis. The second set is provided by Trautsamwieser and Hirsch [2014]. The third set contains the data of Cappanera and Scutellà [2013a] and Cappanera and Scutellà [2014].

Generated set

The instances in this set are artificially generated but with use of information from different sources. These sources are legal texts, publications, statistics and discussions with home care providers. The input parameters of the test data generator are the number of clients, nurses and days. The remaining data is generated using the parameters and methods described in the following.

The distance matrix for clients and starting points of nurses is calculated on uniform distributed geographic coordinates. The ranges of the coordinates are based on three cities in Germany with different sizes (small, medium, large). The driving time is approximated by multiplying the Euclidean distance with a factor of 1.28 to account for the real street network and assuming an average speed of $30\frac{km}{h}$ due to traffic [Mattfeld and Vahrenkamp, 2014].

The probability for a client having a job on a day is 0.7 and for having a second job on the same day 0.3 (conditional on the first job). The duration of a job is assumed

	Nurses				**Jobs**		
Qualification(s)	{1}	{2}	{1,2}	{2,3}	1	2	3
Probability	0.15	0.05	0.5	0.3	0.4	0.5	0.1

Table 5.3.: Qualification probabilities for nurses and jobs. The qualifications for nurses are grouped

to be normal distributed [Trautsamwieser and Hirsch, 2014] with $\mu = 30$ minutes and $\sigma = 20$ minutes. The minimum duration is set to 5 minutes. The possible time window width of a job is $30, 45, 60, 120$ or 240 minutes and the probability is uniform distributed. The position of the time window is determined by first randomly choosing a shift type and a time in this shift type. The start time of the time window is rounded to the next quarter of an hour. The qualification types considered are nursing (3), personal care (2, ADLs) and other activities (1, IADLs). The probabilities for a job requiring one of these qualifications are shown in Table 5.3 (right). The qualifications of nurses are grouped to get one or multiple qualifications. The groups and probabilities are also shown in Table 5.3 (left).

We use three shift types: morning, afternoon and weekend. The morning shift is valid from 6 a.m. to 2 p.m. on all days of the week. The afternoon shift is used for the time from 2 p.m. to 8 p.m. and is valid on Monday to Friday. The weekend shift is assignable on Saturday and Sunday and has a time window of 6 a.m. to 8 p.m. on both days.

The data for labor law regulations are taken from the German Working Hours Act [Bundesministerium für Arbeit und Soziales, 1994]. According to the regulations, an employee has to rest eleven hours between working shifts and the maximum working time per day is eight hours. An employee has to take a 30-minute-break, if he or she is working longer than six hours and 45 minutes, if he or she is working longer than nine hours. The breaks are splittable in several shorter breaks, but every part must be at least 15 minutes. The maximum number of consecutive workdays is set to twelve days, which corresponds to the common shift rotation pattern in Germany. The minimum weekly uninterrupted rest time is set to $24 + 11 = 35$ hours according to the directive proposed by the European Union [European Parliament and Council of the European Union, 2003].

The available work contracts for employees are given in Table 5.4, which shows their attributes and probabilities (ρ_1). The contracts represent common work contracts in Germany. The probabilities are chosen according to the data of the German Federal Statistical Office for the year 2011 concerning employee numbers in German home care providers [Statistisches Bundesamt, 2013a].

In some instances, we added apprentices as they have other requirements that have

Id	**Attributes**				**Probabilities**	
	h/week	h/day	Days	$\in \mathcal{N}^{\text{WE}}$	ρ_1	ρ_2
1	40	8	7	yes	0.28	0.26
2	30	8	5	yes	0.34	0.33
3	20	4	5	yes	0.075	0.055
4	20	8	3	no	0.075	0.055
5	10	4	3	no	0.21	0.19
6	40	8	5	no	-	0.10

Table 5.4.: Attributes and probabilities of work contracts (h/week = weekly working hours, h/day = daily working hours, Days = weekly workdays, $\in \mathcal{N}^{\text{WE}}$ = whether single shifts on weekends are prohibited (yes) or not (no))

	Irregular unavailability				**Regular unavailability**	
	Vacation length					
	0.5 day	1 day	3 days	7 days	Incompatible shift type	Incompatible weekday
Probability	0.05	0.05	0.05	0.025	0.05	0.05

Table 5.5.: Regular and irregular unavailability of nurses and the corresponding probabilities

to be incorporated. Their rest time is twelve hours and the weekly uninterrupted rest time is 48 hours. Apprentices need a 30-minute-break, if they are working longer than 4 hours and 30 minutes and a break of 60 minutes, if they work longer than 6 hours. The work contract attributes are described in the last row of Table 5.4 (Id 6). If we allow apprentices the probabilities ρ_2 are used instead of ρ_1.

The availability of a nurse can be limited due to incompatible shift types and weekdays (regular) as well as vacations (irregular). The considered unavailability is shown in Table 5.5 with according probabilities. In some instances, the nurses with a full-time contract (Id 1 in Table 5.4) work according to shift rotations. We selected the most common cyclic pattern in Germany, i.e., the first week contains morning shifts from Monday to Friday followed by a weekend off, and the second week afternoon shifts from Monday to Friday and a weekend work.

The test instances have different planning horizon lengths of 7, 14 or 28 days. All nurses use the office of the home care providers as start and end locations of the routes on every day. There are no incompatibilities between nurses and jobs due to preferences. In total, there are 135 generated instances whereas the first 30 instances are small instances manually constructed for test purposes. The remaining 105 instances are grouped by the attributes *unavailability*, *shift rotations*, *apprentices* and *city size*.

Set	#	Jobs	Nurses	Days	Unavail.	Rotation	Apprent.	City
small	30	36.9	3.4	1-15	false	true,false	false	misc.
basic-1	10	339.1	18.1	7,14	false	false	false	small
basic-2	10	375.4	17.5	7,14	false	false	false	med
basic-3	10	386.0	20.1	7,14	false	false	false	large
unavail-1	10	328.1	18.1	7,14	true	false	false	small
unavail-2	10	371.4	17.5	7,14	true	false	false	med
unavail-3	10	385.2	20.1	7,14	true	false	false	large
rotation-1	10	661.2	18.1	14,28	true	true	false	small
rotation-2	10	768.6	17.8	14,28	true	true	false	med
rotation-3	10	778.7	20.1	14,28	true	true	false	large
appr-1	5	438.6	19.4	14	true	true	true	small
appr-2	5	510.0	18.6	14	true	true	true	med
appr-3	5	516.8	22.4	14	true	true	true	large

Table 5.6.: Overview of sizes and attributes of generated instances (the numbers of jobs and nurses are averaged)

An overview of these instance groups, their sizes and attributes is given in Table 5.6. The generated instances are available for download at `http://hc.guericke.org/`.

Instances of Trautsamwieser and Hirsch

Trautsamwieser and Hirsch [2014] provide instances that are generated in cooperation with the Austrian Red Cross. The problem setting described is similar to the one in this thesis, therefore many of the attributes are included in the data. Further parameters for the working regulations are added to the instances.

The provided data sets all have a planning horizon of seven days, thus we set the maximum number of consecutive workdays to seven days. As shift types are not used in the formulation of Trautsamwieser and Hirsch, we add a general shift type that is valid for 24 hours on every day of the week. Differences to our problem structure are that the breaks are not splittable and no shift rotations or unavailability are modeled. All nurses have the same type of work contract but start and end their routes at different locations. These locations are set for the start and end jobs of the routes. Due to preferences or languages some jobs are incompatible with some nurses, we add them to the list of incompatible jobs per nurse by setting $Q_{j,n}$ to zero. An overview of the 44 instances is given Table 5.7 based on the instance ids that distinguish them by the number of clients.

Note that the resulting solutions and objective values of our methods are not

Id	#	Clients	Jobs	Nurses	Days
2	1	10	32.0	2	7
3	1	20	76.0	4	7
4	12	30	111.8	6	7
5	2	35	122.0	7	7
6	2	40	153.0	8	7
7	22	45	180.5	9	7
8	2	50	218.0	10	7
9	2	60	255.0	12	7

Table 5.7.: Overview of Trautsamwieser and Hirsch [2014] instances (the number of jobs is averaged)

directly comparable to the results reported in Trautsamwieser and Hirsch [2014]. Due to the fact, that we prohibit the usage of artificial waiting time to achieve the time needed for a break, the resulting plans can differ.

Instances of Cappanera and Scutellà

Cappanera and Scutellà [2013a] and Cappanera and Scutellà [2014] provide anonymized data instances of an Italian home care provider. The data needs to be extended because their solution methods solve the home care planning with unfixed days. Thus, the days of the jobs are not known a priori. However, Cappanera and Scutellà provide the day patterns for each client that are generated by their method together with the test instances. Therefore, one valid pattern for each client is sampled and fixed to know the days before solving. For each instance the day patterns are sampled three times resulting to a total of 78 instances.

Again we need to add attributes for the working regulations. First, a shift type valid on each day is introduced. Second, the break rules of the EU regulations [European Parliament and Council of the European Union, 2003] are assumed, which are the same as in Germany. The number of weekly workdays is set to five and the weekly working time to five times the daily working time. The daily rest time equals eleven and the weekly rest time $24 + 11 = 35$ hours [European Parliament and Council of the European Union, 2003]. All nurses are available on every day. The time windows of jobs are valid the entire day because they are not given in the instances. Cappanera and Scutellà model different municipalities in their solution approach. They assume the driving time to be three minutes within one municipality but different driving times between municipalities. This driving time is transferred to the job level to avoid the necessity to model municipalities. An overview of the 78 instances is given Table 5.8 based on the number of clients.

Id	#	Clients	Jobs	Nurses	Days
0106	18	40	56.8	4	5
0106	18	60	80.7	5	5
0106	3	128	184.0	11	5
0407	18	50	96.7	5	5
0407	18	80	150.7	6	5
0407	3	162	302.0	11	5

Table 5.8.: Overview of Cappanera and Scutellà [2013a] and Cappanera and Scutellà [2014] instances (the number of jobs is averaged)

Set	#	Jobs	Nurses	Days	Attributes		
		Mean	Mean	Sets	Availability	Rotations	Apprentices
G1	12	426.8	20.9	7,14	False	False	False
G2	13	365.0	17.8	7,14	True	False	False
G3	16	655.1	16.4	14,28	True	True	False
G4	5	442.4	18.6	14	True	True	True
all	46	490.4	18.2	7,14,28			

Table 5.9.: Instance attribute and size overview of generated instances in the test set

Training and test set

The combined instance set consists of the generated instances as well as the adapted instances provided by Trautsamwieser and Hirsch [2014], Cappanera and Scutellà [2013a] and Cappanera and Scutellà [2014] resulting in a total number of 257 instances. For the analyses of the exact and heuristic solution proposed in this thesis, the instance set is divided into a training and a test set.

The *small* instances are excluded from the set. From the remaining set 100 instances are randomly sampled to form the test set providing the results used for the analyses in this thesis. The other 128 instances are used for parameter tuning of the heuristic as described in Section 6.6. Throughout this thesis the generated instances are marked with *G*, the instances of Trautsamwieser and Hirsch [2014] with *TH* and the instances of Cappanera and Scutellà [2014] with *CS*. An overview of the instances used as test set is given in Tables 5.9 to 5.11. The instances are grouped based on the attributes availability, rotations and apprentices (G) or the number of jobs (TH and CS). A detailed table with all instances is provided in Table A.2 in Appendix A.

Set	#	Clients	Jobs	Nurse	Days
TH1	7	15-30	110.6	5.6	7
TH2	3	35-40	132.3	7.3	7
TH3	8	45	178.3	9.0	7
TH4	3	50-60	242.7	11.3	7
all	21	39.8	158.3	8.0	7

Table 5.10.: Overview of TH instances in the test set

Set	#	Clients	Jobs	Nurse	Days
CS1	11	40-50	72.1	4.4	5
CS2	11	60	78.7	5.0	5
CS3	7	80	149.7	6.0	5
CS4	4	128-162	213.5	11.0	5
all	33		107.9	5.7	5

Table 5.11.: Overview of CS instances in the test set

5.4. Numerical results

This section presents the numerical results of solving the proposed mathematical formulation with a commercial MIP solver. Therefore, first the small instances from the *small* set are analyzed in Section 5.4.1 as many of them are solvable in less than 12 hours computation time. The results for instances of the test set are given in Section 5.4.2. All results in this section are computed with the solver Gurobi 6.0.5 on Xeon E5 processors with four 2.6 GHz CPUs using 32 GB RAM[1]. If not indicated otherwise the time limit is set to 12 hours (=43200 seconds).

As described in Section 5.2, there are two different mathematical models. First, the model containing only hard constraints and, second, the relaxed model with soft constraints. Furthermore, this section analyzes the influence of giving a start solution to the solver. This is possible only for the model with soft constraints as the heuristic solution may have unassigned jobs or single shifts on weekends. The different settings lead to three MIP solver runs that are indicated in the analysis as *Hard, Soft* and *Soft+Start*. These are defined as follows:

Hard All constraints of the proposed MIP are considered as hard constraints. Thus, the model consists of Equations (5.7) to (5.43).

[1]All the reported results have been computed on resources of the Paderborn Center for Parallel Computing. Gurobi is limited to one thread to reduce memory usage.

Soft The constraints concerning assignment of jobs and weekend shift assignment are relaxed and the resulting soft constraints are penalized in the objective function (see Section 5.2.2). The model in this run is given by Equations (5.8) to (5.10), (5.12) to (5.41) and (5.43) to (5.46).

Soft+Start The same model as in **Soft** is used. Additionally, a heuristic start solution is given as input to the solver. Thus, the solver can use the information of this solution during the process. The start solution is computed by the construction heuristic described in Section 6.2.

5.4.1. Results for *small* instances

The results for the three solver runs on the small instance are given in Table 5.12. The solver is able to solve instances with up to 40 jobs (instances D-01 to D-18) to optimality. The computation time to solve these instances is relatively short in comparison to those for instances D-19 to D-30 that still have a remaining gap after 12 hours. For the instances solved to optimality the model with *hard* constraints needs only 390 seconds whereas the two others need more than 800 seconds on average. For the instances that reach the time limit without proven optimality (D-19 to D30), the solver setting *Soft+Start* achieves the smallest remaining gap on average (13.73%), although the results for *Soft* are only slightly worse (13.77%). The model with *hard* constraints is not beneficial when the instances get more complex because for five of the instances the solver found no feasible solution at all, although the results of the other two settings show that there is a feasible solution without penalty costs that could have been found by the model with hard constraints. Only for instance D-27 there are remaining penalty costs due to unassigned jobs. The smallest remaining gap is achieved by *soft* and *soft+start* for five instances each. Thus, no suggestion for one particular setting is possible and the larger instances from the test set are analyzed in the next section.

5.4.2. Results for test set instances

Figure 5.6 shows the performance of the three solver settings on the 100 instances in the test set. The x-axis represents the remaining gaps after 12 hours of computation time in percent and the y-axis the number of instances having a remaining gap of the value on the x-axis. For example, the model with *hard* constraints solved 30 instances with a remaining gap of equal or less than 15%. The solver setting with results in the upper left corner achieves the best results.

From Figure 5.6 it becomes clear that the solver setting *hard* performs worse than the two others on the test set. For 70 of the 100 instances this setting does not find a feasible solution during the solution process. This can be due to the infeasibility

	Hard			**Soft**			**Soft+Start**		
Name	Obj	Gap	t [s]	Obj	Gap	t [s]	Obj	Gap	t [s]
S-01	328	**0.00**	0.2	328	**0.00**	0.1	328	**0.00**	0.1
S-02	380	**0.00**	0.4	380	**0.00**	0.4	380	**0.00**	0.4
S-03	473	**0.00**	10.5	473	**0.00**	16.6	473	**0.00**	14.4
S-04	749	**0.00**	10.5	749	**0.00**	9.7	749	**0.00**	6.8
S-05	657	**0.00**	57.1	657	**0.00**	58.4	657	**0.00**	48.6
S-06	586	**0.00**	3.8	586	**0.00**	5.9	586	**0.00**	2.1
S-07	1207	**0.00**	0.4	1207	**0.00**	0.5	1207	**0.00**	0.6
S-08	624	**0.00**	2.6	624	**0.00**	1.2	624	**0.00**	1.4
S-09	540	**0.00**	7.5	540	**0.00**	1.4	540	**0.00**	1.4
S-10	637	**0.00**	70.9	637	**0.00**	88.9	637	**0.00**	42.7
S-11	591	**0.00**	32.8	591	**0.00**	33.5	591	**0.00**	34.2
S-12	708	**0.00**	1656.7	708	**0.00**	2806.0	708	**0.00**	7476.2
S-13	516	**0.00**	273.6	516	**0.00**	467.8	516	**0.00**	280.3
S-14	1313	**0.00**	2.3	1313	**0.00**	1.4	1313	**0.00**	1.1
S-15	846	**0.00**	2445.8	846	**0.00**	4268.3	846	**0.00**	4893.4
S-16	1565	**0.00**	695.4	1565	**0.00**	684.8	1565	**0.00**	881.5
S-17	2411	**0.00**	267.8	2411	**0.00**	2181.5	2411	**0.00**	643.9
S-18	1325	**0.00**	1482.9	1325	**0.00**	4120.0	1325	**0.00**	980.0
S-01-18		**0.00**	390.1		**0.00**	819.2		**0.00**	850.5
S-19	930	**5.94**	43200.0	930	6.13	43200.0	930	7.53	43200.0
S-20	902	2.12	43200.0	902	**1.90**	43200.0	902	2.07	43200.0
S-21	884	1.58	43200.0	884	2.10	43200.0	884	**1.32**	43200.0
S-22	1038	3.89	43200.0	1038	**3.11**	43200.0	1038	3.37	43200.0
S-23	2669	**1.51**	43200.0	2669	2.90	43200.0	2669	2.11	43200.0
S-24	-		43200.0	2334	**5.95**	43200.0	2378	8.19	43200.0
S-25	2520	5.28	43200.0	2520	**4.27**	43200.0	2520	4.30	43200.0
S-26	-	-	43200.0	9940	6.68	43200.0	9940	**6.11**	43200.0
S-27	-	-	43200.0	4002535	99.94	43200.0	3002606	**99.92**	43200.0
S-28	-	-	43200.0	14275	12.58	43200.1	14104	**11.56**	43200.1
S-29	-	-	43200.0	3290	**10.88**	43200.0	3290	11.25	43200.0
S-30	1391	6.31	43200.0	1424	8.83	43200.0	1391	**6.98**	43200.0
S-19-30		43.89	43200.0		13.77	43200.0		13.73	43200.0
Average		17.55			5.51			5.49	
Optimal		18			18			18	
Inf./Pen.		5			1			1	

Table 5.12.: MIP results for instances in set *small* (instances without feasible solution are assumed to have a gap of 100%; row *Inf./Pen.* indicates that no feasible solution was found or there are remaining penalty costs)

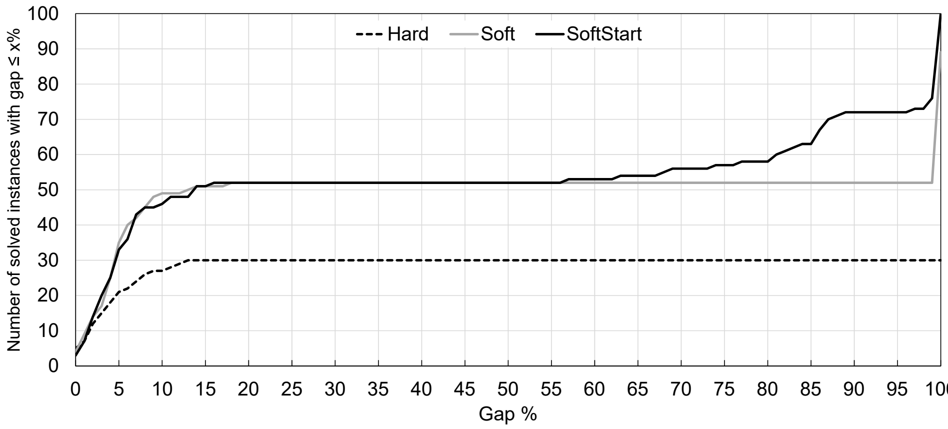


Figure 5.6.: Comparison of different MIP solver settings on the test set[2]

of the instance when only hard constraints are considered or the incapability of the solver to find a feasible solution in the computation time.

Both solver runs considering soft constraints are more successful on the test set. Note that large remaining gaps indicate the presence of penalty costs in the objective function. *Soft* and *soft+start* compute solutions with less than 15% remaining gap on 51 of the 100 instances. For a gap of up to 55% the two settings achieve similar results. On the more complex instances that have a large remaining gap, the input of a start solution (*soft+start*) enables the solver to reduce the remaining gap mainly due to the presence of the input solution. In most cases the heuristic start solution is better than the first solution of the solver resulting in a smaller but still relatively large remaining gap. Note also that the model with *soft* constraints does not find an incumbent solution for 11 instances. On a first glance it can be concluded that the setting *soft+start* is the best for solving the instances of the test set.

Tables 5.13 to 5.15 show the results of the solver settings averaged for the different instance sets to provide an insight for the behavior on particular sets because no instance information is given in Figure 5.6. The detailed results for each instance and setting individually are given in Table B.1 in Appendix B. Note that a gap value of 100% in Tables 5.13 to 5.15 indicates that no feasible solution is found in 12 hours computation time. The column *Inf.* indicates the number of instances without incumbent solution or with penalty costs due to unassigned jobs or single shifts on weekends.

The results for the generated instances in Table 5.13 show that many instances are not solved in 12 hours computation time indicated by penalty costs in the best found solution. The number of instances with penalty costs or infeasible solutions are 42,

[2]The calculation is based on 1% steps of the gap leading to 100 data points.

Set	#	Hard			Soft			Soft+Start		
		Mean	Median	Inf.	Mean	Median	Inf.	Mean	Median	Inf.
G1	12	100.00	100.00	12	92.19	99.99	11	**68.66**	80.05	9
G2	13	69.90	100.00	9	**56.31**	99.19	7	68.73	83.92	10
G3	16	100.00	100.00	16	99.95	99.98	16	**92.73**	93.80	16
G4	5	100.00	100.00	5	99.97	99.98	5	**91.97**	98.63	5
all	46	91.49	100.00	42	85.59	99.98	39	**79.59**	87.07	40

Table 5.13.: Average MIP Gaps [%] for generated instances

Set	#	Hard			Soft			Soft+Start		
		Mean	Median	Inf.	Mean	Median	Inf.	Mean	Median	Inf.
TH1	7	1.52	1.31	0	1.60	1.57	0	**1.29**	1.18	0
TH2	3	0.56	0.25	0	**0.08**	0.00	0	0.39	0.33	0
TH3	8	41.94	11.87	3	**18.80**	8.08	1	19.32	8.58	1
TH4	3	39.10	10.93	1	8.40	8.92	0	**8.33**	8.59	0
all	21	22.15	4.42	4	**8.91**	3.39	1	9.04	4.68	1

Table 5.14.: Average MIP Gaps [%] for TH instances

39 and 40 out of a total of 46 instances for the settings *hard, soft* and *soft+start*, respectively. The best average remaining gap computed over all generated instances is achieved by using the start solution for the solver (79.59%). However, the soft constrained model achieves the best average results on instance set G2, because more instances have a feasible solution leading to a smaller average gap.

The results for the TH instances are given in Table 5.14 and show that these instances are easier to solve for an exact approach. In particular, the solver settings

Set	#	Hard			Soft			Soft+Start		
		Mean	Median	Inf.	Mean	Median	Inf.	Mean	Median	Inf.
CS1	11	38.76	4.77	4	3.36	3.18	0	**3.21**	2.99	0
CS2	11	83.26	100.00	9	22.38	4.67	2	**13.54**	4.40	1
CS3	7	100.00	100.00	7	44.92	4.62	3	**44.92**	5.32	3
CS4	4	100.00	100.00	4	76.30	99.66	3	**6.43**	6.39	0
all	33	74.01	100.00	24	27.36	4.65	8	**15.89**	4.62	4

Table 5.15.: Average MIP Gaps [%] for CS instances

	Hard	**Soft**	**Soft+Start**
Avg. gap [%][3]	71.88	50.27	**43.75**
Sum of instances with no solution	70	11	0
Sum of solutions with penalty cost	-	48	**45**
Sum of unassigned jobs[4]	25729	12687	**395**
Avg. number of unassigned jobs[5]	257.3	126.9	**4.0**

Table 5.16.: Summary of MIP results for 12 hours computation time for the entire test set

with soft constraints are able to solve all instances, except one, with small remaining gaps. On average *soft* (8.91%) slightly outperforms *soft+start* (9.04%) but the difference is small. Some of the instances in set TH1 and TH2 can even be solved to optimality (see Table B.1). All other instances in TH1 and TH2 only have a small remaining gaps leading to small average values. The relatively high average value for set TH3 is caused by the one infeasible solution with a large remaining gap due to the penalty costs of one unassigned job. The setting with *hard* constraints is again outperformed by the two other. By the results for the CS instances, presented in Table 5.15, the advantage of an input solution becomes apparent. The setting *soft+start* achieves the best results regarding average gaps and number of feasible solutions. The average gaps for the sets CS2 and CS3 are biased by the penalty costs for unassigned jobs, which can be seen from the relatively low median values.

To explain the results of the exact approach, the instance information needs to be taken into account (see Section 5.3 for instance description). The largest remaining gaps appear on the generated instances. This can be explained by the size and complexity of the instances. The generated set contains more jobs and working regulations than the TH and SC instances. Thus, it is harder for the solver to find a feasible solution, which explains the number of infeasible solutions and large gaps. The capability of the solver to find good solutions on the TH set follows the same argument, because the instances consider less jobs in a planning horizon of seven days. Furthermore, all nurses have the same working regulations and shift rotations and shift types are not considered. The CS instances have even less jobs, but the remaining gaps after 12 hours are still larger due to the structure of the problem instances. The jobs have no time windows and, thus, more feasible solutions exist and need to be considered during the solution process. Due to equal driving times between all clients in a municipality and the absence of time windows, many solutions with equal objective value exist and the solver needs to evaluate those to proof optimality.

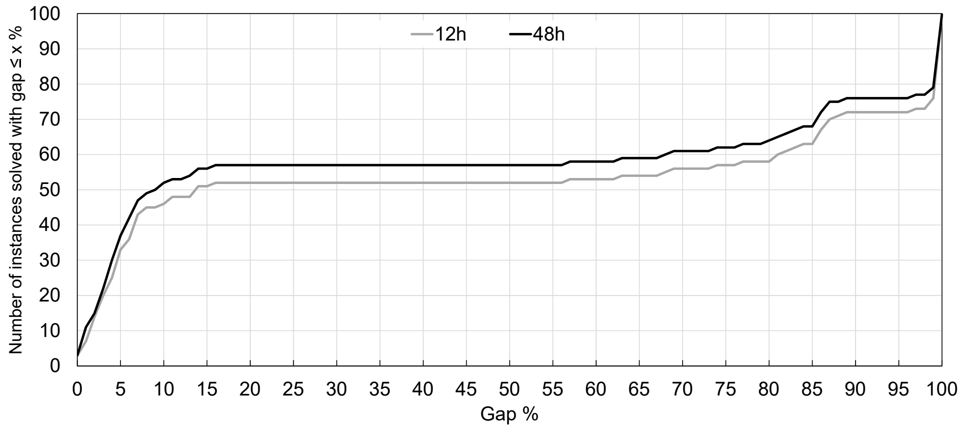


Figure 5.7.: Comparison of 12 hours and 48 hours computation time for MIP with soft constraints and heuristic start solution (*soft+start*). The calculation is based on 1% steps of the gap leading to 100 data points

Table 5.16 gives an overview of the results for all instances. In summary, the exact solution approach is not capable of providing good solutions in a computation time of 12 hours. By looking at the overall average remaining gaps, the solver setting using soft constraints and a heuristic start solution achieves the best result. Additionally, a feasible solution (without penalty costs) is found for more instances than in the other settings *soft* and *hard*. This leads to a substantially lower number of unassigned jobs in comparison to the two other approaches (395 compared to 25729 and 12687). Therefore, the setting *soft+start* is selected to be analyzed with a longer computation time in the next section.

5.4.3. Analysis of increased computation time

Figure 5.7 shows the remaining gaps of the test set instances for the solver setting *soft+start* after 12 hours and 48 hours computation time. The objective and bound values after 48 hours are given in Table C.5 with the results of the heuristics in Appendix C. Naturally, the remaining gaps are smaller after the additional 36 hours of computation. This is indicated by the 48h curve above the 12h curve. However, the improvement is not as considerable as expected. The curve shapes are very similar but the runs with 48 hours reach a higher number of solved instances at a remaining of gap 15%. The difference on the y-axis between the two curves is five at this point,

[3]For instances with no solution a gap of 100% is assumed.

[4]For instances with no solution all client jobs are considered as unassigned. Total number of jobs in all instances is 29446.

[5]See footnote 4.

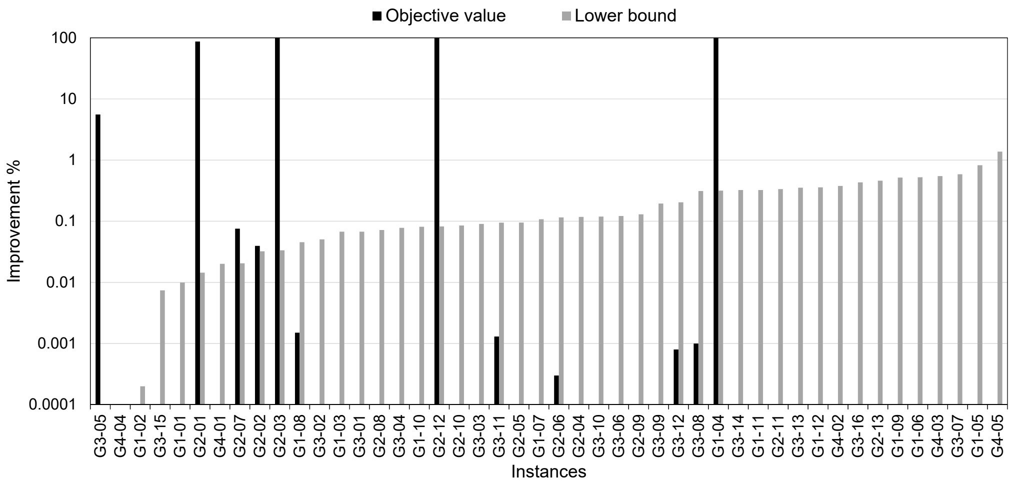


Figure 5.8.: Improvement of bound and objective values from 12 hours to 48 hours computation time for MIP with soft constraints and heuristic start solution on generated instance set

i.e., fives instances with a remaining gap of more than 15% after 12 hours are shifted to the instances with less than 15% gap after the 48 hours computation time.

The background of this observation becomes apparent in Figures 5.8 to 5.10 showing the relative improvement of the objective values and lower bounds after 48 hours compared to the 12 hour values for all three data sets. Note that the improvement is given on a logarithmic scale. The figures show that during the 36 hours additional computation time mainly the lower bounds are improved for the generated (Figure 5.8) and TH instances (Figure 5.9) but not more than 1% in most cases. On the CS instances (Figure 5.10) the objective value is improved more often than the lower bound, which can be explained by the above mentioned high number of feasible solutions possible for those instances.

For all three sets it can be said, that the improvement of the objective value is less than 1% for most of the instances. Exceptions with a large improvement of the objective value are the instances G2-01, G2-03, G2-12 and G1-04 (Figure 5.8) as well as instance TH4-03 (Figure 5.9). The improvement of the objective value for those instances is nearly 100% compared to the 12 hours value, which indicates that the solver succeeded in finding a solution without penalty costs in the additional computation time. The improved results for those instances lead to the differences of the curves in Figure 5.7. For nearly all other instances the improvements of the objective value and bounds are so small that the gap is improved by less than 1% and, thus, showing no difference in Figure 5.7.

The fact that there are mainly gap improvements can be explained by the inca-

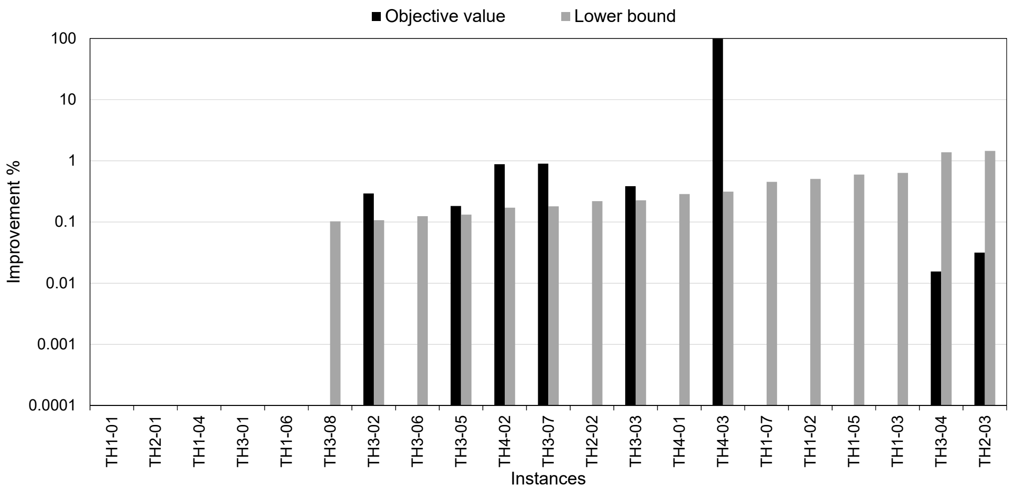


Figure 5.9.: Improvement of bound and objective values from 12 hours to 48 hours computation time for MIP with soft constraints and heuristic start solution on TH instance set

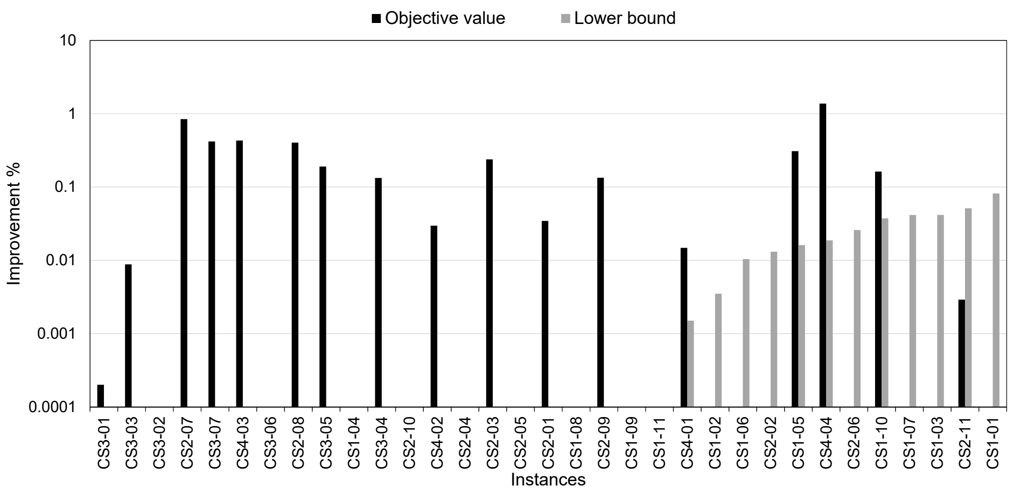


Figure 5.10.: Improvement of bound and objective values from 12 hours to 48 hours computation time for MIP with soft constraints and heuristic start solution on CS instance set

pability of the solver to find new best solutions. Either because the problem is too complex or the current solution is optimal and there exists no better the solution. The former is most likely the case for the generated instances and some of the TH and CS instances with large gaps. The latter may be true for the instances with already a small remaining gap mostly in the sets TH and CS. In this case, the solver has to rely on the results of the LP relaxation to slowly close the gap and, thus, prove optimality. From this results it can be concluded that the additional computation time has no major implications on the outcome for most of the instances.

5.5. Summary

This chapter proposed a formalization for the home care routing and scheduling with working regulations as a MIP. The model formulation is solved with a commercial solver and results for the presented test instances were computed. For each instance three solver runs were performed. First, all constraints were considered as hard constraints. Second, no assignment of jobs and single shifts on weekends were allowed and penalized in the objective function. Third, the latter model was solved with an additional heuristic start solution given as input to the solver. The results indicated the last setting as the most successful. However, the results also showed that an exact approach is not capable of solving the underlying problem setting in a reasonable amount of computation time. The experiments were performed with a 12 hour time limit that is reached in most of the cases. The remaining gaps were large and, therefore, the solution quality is not clearly determinable. Either the solutions are poor and the solver struggles to find better solutions (most likely for instances with large gaps) or the solutions are optimal but not proven to be so (most likely for instances with small gaps). An increase of the computation time to 48 hours showed no major improvement.

For application in practice the computation time of the exact approach is too long and the solution quality (number of unassigned jobs) unsatisfactory. Therefore, the next chapter introduces heuristic solution approaches to find good solutions in less computation time.

6. Heuristic solution approaches for the static setting

Based on the observations in the previous chapter, i.e., very long computation times and many infeasible solutions, this chapter introduces three heuristic solution approaches to solve the home care routing and scheduling for real-world sized instances. The proposed heuristics address goal 2 of this thesis presented in Section 4.5.

Each heuristic is based on one of the following metaheuristics: LNS, ALNS or RVNS. These metaheuristics are selected because of their success on a variety of routing and scheduling problems [Pisinger and Ropke, 2010, Hansen et al., 2010a]. The implementation[1] of the heuristics use shared methods that perform basic operations like the insertion or removal of jobs. These methods and further notation needed to describe the heuristics are introduced in Section 6.1. The construction heuristic to provide a start solution is presented in Section 6.2. The description of the domain-specific components of the LNS heuristic is presented in Section 6.3. Afterwards, the more complex ALNS and RVNS heuristics are described in Sections 6.4 and 6.5, respectively. All heuristics contain several parameters that need to be determined before execution. Therefore, an algorithm configurator is used to determine the parameters. The results of the configuration are presented in Section 6.6. Based on the selected parameters the numerical analysis of the heuristics is given in Section 6.7. The chapter is concluded by addressing goal 3 of thesis, i.e., the analysis of considering working regulations, in Section 6.8 and a summary in Section 6.9.

6.1. Shared subproblems and methods

This section introduces the notation and sets of the solution representation used in the heuristics. The common methods used for job insertion and removal as well as shift assignment are described afterwards. An overview of the notation introduced in this section is given in Table 6.1.

6.1.1. Notation and solution representation

The proposed heuristics all work with the same solution object containing the information about the current tours and assignments. Set T^* is introduced to contain all possible tours. A tour $t \in T^*$ has a related nurse n_t, day d_t and shift type s_t. Not

[1] All heuristics are implemented in C# with .NET framework 4.5. For the evaluation on Unix systems, the programs are compiled with Mono 4.0.0.

	Name		Definition
known a priori	T^*		Set of all possible tours
	n_t	$\in \mathcal{N}$	Nurse of tour t
	s_t	$\in \mathcal{S}$	Shift type of tour t
	d_t	$\in \mathcal{D}$	Day of tour t
	$N(i)$	$\subseteq \mathcal{N}$	Set of nurses compatible with job $i \in \mathcal{J}$
	$S(i)$	$\subseteq \mathcal{S}$	Set of shift types compatible with job $i \in \mathcal{J}$
based on current solution	U	$\subseteq \mathcal{J}$	Set of unassigned jobs in current solution
	T^+	$\subseteq T^*$	Set of all assigned tours in current solution
	T^-	$\subseteq T^*$	Set of all unassigned tours in current solution
	$T(u)$	$\subseteq (T^+ \cup T^-)$	Candidate tours for job u, i.e., all tours compatible with job u in current solution
	$P^*(t)$		Sequence of jobs given by the arcs (i,j) in tour t in solution
	$P(t,u)$		Candidate positions for job u, i.e., all feasible insertion positions (i,j) for job u in tour t in solution
	ϕ_t		Penalty costs of tour $t \in T^+$, if the tour is empty
	$s_{n,d}$	$\subseteq \mathcal{S}$	Shift type s assigned to nurse n on day d

Table 6.1.: Sets and attributes of the solution object

all tours $t \in T^*$ are part of the current solution because each nurse has at most one tour per day. Therefore, we distinguish between *assigned* and *unassigned* tours. An assigned tour creates a work shift for a nurse, i.e., the tour contains jobs that the nurse has to perform. The only exceptions are tours with penalty costs ϕ_t due to single shifts on weekends, which have no jobs but are also assigned to create a penalized shift assignment. *Unassigned* tours are empty and contain only the information about nurse, day and shift. The subsets $T^+ \subseteq T^*$ and $T^- \subseteq T^*$ represent the current solution by containing the assigned and unassigned tours, respectively. The union of both sets represents all possible tours $T^* = T^+ \cup T^-$. Figure 6.1 shows an example for the deviation of tours to those sets. The tour of nurse 1 on Sunday is an empty tour with penalty costs. The currently assigned shift type of nurse n on day d is given by $s_{n,d}$ representing the duty schedule based on T^+.

In addition to the assigned nurse and shift, a tour $t \in T^+$ also has an ordered sequence of jobs $P(t)$ representing the route. All jobs that are not assigned to one of the tours in T^+ are contained in set U of unassigned jobs leading to penalty costs in the objective function. As the heuristics allow unassigned jobs and single shifts on weekends, they resemble the MIP with soft constraints (see Section 5.2.2).

Before describing the determination of candidate tours and positions for job insertion in the next section, two sets containing feasible combinations of jobs and nurses as well as jobs and shift types can be determined a priori because the compatibility does not change during the execution. All nurses that are feasible to assign to job i are given in set $N(i) \subseteq N$. The feasibility is based on the qualifications $\mathcal{Q}_n$ a nurse

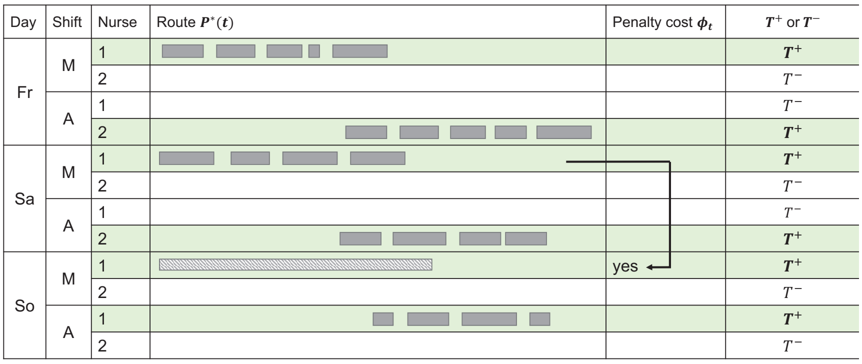


Figure 6.1.: Example for the sets of assigned T^+ and unassigned tours T^- in a solution for the days Friday to Sunday with two nurses and shift types each (solid bars represent jobs and striped bars represent empty tours)

n provides and the required qualification of the job q_i.
The formal statement for this selection criterion is given in Equation 6.1.

$$N(i) = \{n \in \mathcal{N} | q_i \in \mathcal{Q}_n\} \quad \forall i \in \mathcal{J} \tag{6.1}$$

The feasible shift types for a job i can be determined based on the compatibility of the time windows of the shift types $[A_s, B_s]$ and the job $[a_i, b_i]$. They are compatible if the start of the job's time window is early enough to fulfill the job before the end of the shift type time window or the end of the job's time window is late enough to start the job at the earliest possible point in time in the shift type (6.2).

$$S(i) = \{s \in \mathcal{S} | a_i + r_i \leq B_s \vee b_i \geq A_s\} \quad \forall i \in \mathcal{J} \tag{6.2}$$

Note that the driving time to the start and end location of the route cannot be taken into account here as it can depend on the start location of the assigned nurse. Additional criteria for the compatibility of jobs with tours and insertion positions need to be evaluated based on the current solution containing the assigned tours and shifts. Thus, the determination of candidate tours and positions is described in the next section.

6.1.2. Determination of candidate tours and positions for job insertion

All heuristics insert jobs from the set U into routes to create new solutions. To ensure the feasibility of a job insertion, several constraints need to be checked beforehand. The operators of the heuristics decide the job to insert as well as the tour and position in the route. To limit the number of possible insertion points of a job, the candidate tours and positions considered are determined based on the current solution. Thus, some infeasible combinations are excluded to reduce the computation time.

The set of *candidate tours* $T(u)$ for insertion of job u are determined from all tours T^* based on assigned nurses, current shift assignments and workloads. Tour t has to fulfill the following criteria to be considered for insertion.

1. Day d_t of tour t equals day of the job d_u.

2. Assigned nurse n_t has the required qualification for the job, i.e., $n_t \in N(u)$.

3. The tour has a shift type s_t that is compatible with the time window of the job. Thus, s_t needs to be contained in $S(u)$.

4. The tour is already assigned ($t \in T^+$) or the nurse has no other shift assignment on day d_t ($t \in T^-$). In the latter case, the additional assignment must not violate the maximum number of weekly workdays and the shift type of the tour must be valid for the nurse.

5. The current weekly working time is not violated by the additional job duration.

All tours that comply with the five criteria can be considered for insertion. Additionally, the *feasible positions* $P(t, u)$ for job $u \in U$ in a tour $t \in T(u)$ can be determined based on time windows. Let $P^*(t)$ be all insertion positions in tour t. Each position $(i, j) \in P^*(t)$ with i as predecessor and j as successor of the candidate job u can be checked for time window compatibility resulting in the feasible set $P(t, u)$ (6.3).

$$P(t,u) = \{(i,j) \in P^*(t) | (a_i + r_i + driv_{i,u} \leq b_u) \wedge (a_u + r_u + driv_{u,j} \leq b_j)\} \tag{6.3}$$

Job u can be inserted directly after job i, if the time window start a_i of i is early enough to reach job u before the end of its time window b_u after fulfilling job i and driving from one client to the other $driv_{i,u}$. The same condition must be true for the start of the time window of u and the end of time window for the successor j. All positions $(i, j) \in P(t, u)$ allow a feasible insertion of u between i and j without violating the time windows of i, j and u.

Although the candidate tours $t \in T(u)$ and positions $(i, j) \in P(t, u)$ limit the number of possible insertion points for job u, the feasibility is dependent on other factors that are not known prior to insertion. These are rest times, time windows of all jobs in the route (not only predecessor and successor) and further criteria on shift assignments. Therefore, the additional feasibility criteria for insertion are described in the next section.

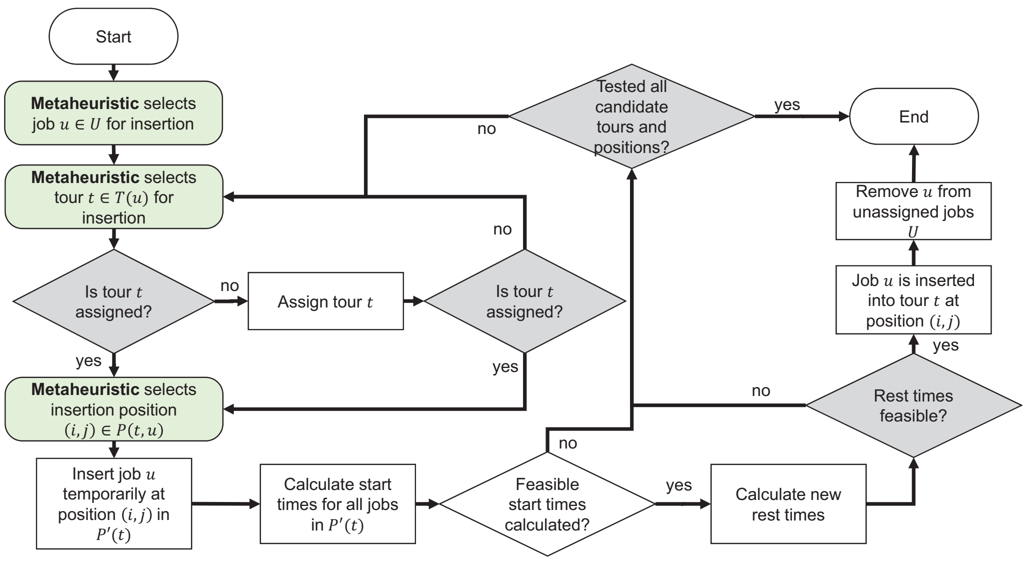


Figure 6.2.: Job insertion process

6.1.3. Insertion of jobs into routes

After a heuristic has decided on the job, the position and tour to insert, the final feasibility of the insertion is determined. Therefore, the following steps are performed:

1. If the considered tour is currently unassigned ($t \in T^-$), the feasibility of a shift assignment for this tour is checked.
2. Start times of all jobs in the tour are determined while the time windows are kept for all jobs. Furthermore, the daily rest times to shifts on surrounding days are ensured and necessary breaks are inserted.
3. The rest times are calculated based on the new start times and the weekly rest time requirement is checked.

The process of job insertion is shown in Figure 6.2. Only if all steps are performed successfully the job is finally inserted to the tour and the heuristic can proceed with the next job. If one of the criteria is not fulfilled, the heuristic determines a new insertion position for consideration or leaves the job unassigned (causing penalty costs). The details of the tour assignment and start time calculation are described in the remainder of this section.

Assignment of tours

The assignment of a tour $t \in T^-$ in step 1 considers the availability of the nurse and maximum weekly and consecutive workdays. If a shift pattern is selected for

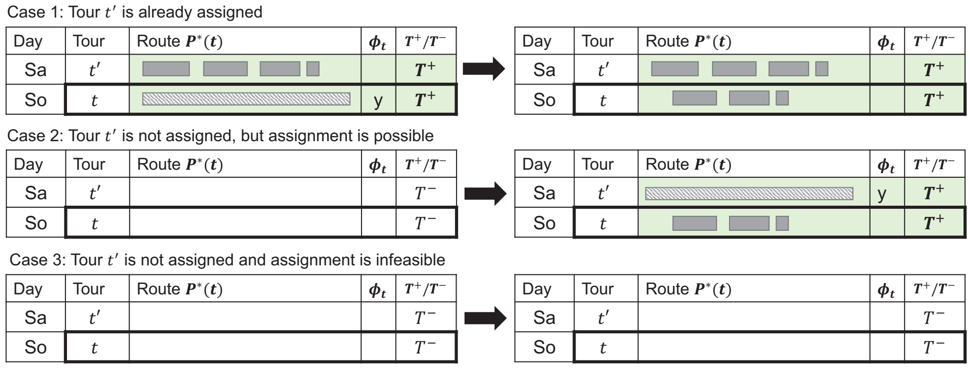


Figure 6.3.: Three cases that can occur for the assignment of tour t on a weekend. The assignment before and after the assignment is shown

the nurse, the assignment needs to comply with restrictions of the pattern. Another constraint to consider is the assignment of both days on a weekend or none. In case of a weekend shift, a tour on the other day on the weekend t' must also be assigned or assignable for nurses for which this rule applies. Furthermore, penalty costs can potentially be removed. There are three cases that can occur for weekend shifts, which are also visualized in Figure 6.3.

1. Tour t is assigned and currently has penalty costs ($\phi_t > 0$), i.e., a tour on the other day t' is already assigned ($t' \in T^+$) and t is assigned with empty route to fulfill the weekend condition. Then the penalty costs of t can be removed and t stays in T^+.

2. The other tour t' is not assigned ($t' \in T^-$) but the additional assignment of it does not violate the maximum weekly or consecutive workdays. Then the penalty costs of t' are set and t and t' are moved to T^+.

3. The other tour t' is not assigned ($t' \in T^-$) and the additional assignment is infeasible due to weekly or consecutive workdays. Then tour t cannot be assigned.

Calculation of start times and insertion of breaks

The calculation of start times of jobs in step 2 is a non-trivial task even if the sequence is determined by the heuristic. This is due to the fact that the start and end times of a tour are not fixed and the jobs can be shifted in their time window bounds. The objective during determination of start times is to minimize waiting time because it is time spend without providing services. The driving time is already fixed due to the sequence given by the heuristic.

	Name		Definition
Sets	$P'(t)$		Temporary sequence of jobs given by the arcs (i, j) in tour t considered for start time calculation
	$J'(t)$		Temporary set of jobs included in tour t, which are considered for start time calculation
	o	$\in J'(t)$	Start job of tour t
	e	$\in J'(t)$	End job of tour t
Parameters	e^{d-1}		End time of the tour for nurse n_t on day $d_t - 1$
	o^{d+1}		Start time of the tour for nurse n_t on day $d_t + 1$
	R^D		Daily rest time of nurse n_t
	H^D		Daily working time of nurse n_t
	l		Required break length
	$driv_{ij}$		Driving time between jobs i and j
	r_i		Duration of job i
Variables	w_j	$\in \mathbb{R}^+$	Waiting time before job j in $J'(t)$
	z_j	$\in \mathbb{R}^+$	Start time of job j in $J'(t)$
	δ_j	$\in \mathbb{R}^+$	Break duration before job j in $J'(t)$
	β_j	$\in \{0, 1\}$	Binary decision variable. Equals 1, if a break is inserted before j in $J'(t)$, 0 otherwise

Table 6.2.: Sets, parameters and variables for the mathematical model determining start times

We use a subproblem of the proposed MIP in Section 5.2 to calculate optimal start times for a given tour. A similar approach is also used by Bertels and Fahle [2006], Fikar and Hirsch [2014] and Nguyen et al. [2015]. The subproblem considers the time windows of jobs, the rest times between consecutive days and break insertion for one tour. The sequence of jobs in the tour is given as input to the MIP and has to be determined by one of the heuristic solution approaches described in the remainder of this chapter. The sequence of jobs in the temporary tour t considered is denoted by $P'(t) = \{(o, j_1), (j_1, j_2), \dots, (j_n, e)\}$ where $P'(t)$ represents a sequence of jobs $j_i \in J'(t) \subseteq \mathcal{J}$ by containing the pairwise connections (i, j) of the jobs. Furthermore, the end time of the tour on the previous and the start of the tour on the next day are given by e^{d-1} and o^{d+1}, respectively. An overview of the notation is given in Table 6.2.

The subproblem (6.4) to (6.11) contains the variables w_i denoting the waiting time before job $i \in J'(t)$ and z_i denoting the start time of job $i \in J'(t)$.

$$\min \sum_{j \in J'(t)} w_i \tag{6.4}$$

$$\text{s.t. } z_i + r_i + driv_{i,j} + w_j + \delta_j = z_j \qquad \forall (i, j) \in J'(t) \tag{6.5}$$

$$a_i \leq z_i \leq b_i \qquad \forall i \in J'(t) \tag{6.6}$$

$$z_o \geq R^D + e^{d-1} \tag{6.7}$$
$$z_e \leq o^{d+1} - R^D \tag{6.8}$$
$$z_e - z_o - l \leq H^D \tag{6.9}$$
$$z_j \geq 0 \qquad \forall j \in J'(t) \tag{6.10}$$
$$w_j \geq 0 \qquad \forall j \in J'(t) \tag{6.11}$$

The objective function (6.4) minimizes the waiting time of the route. The start times of all jobs are calculated based on the input sequence $P'(t)$ in constraints (6.5) while constraints (6.6) ensure time window limits. The validity of the daily rest time requirement to the surrounding days is held by constraints (6.7) and (6.8). Constraint (6.9) restricts the maximum daily working time.

The model (6.4) to (6.11) is an LP and solved for the given sequence of jobs. Afterwards, the resulting working time is checked for a required break denoted by l. The required break level with the length l is iteratively increased until the solution is valid. Thus, first the LP (6.4) to (6.11) with $\delta = 0$ and $l = 0$ is solved and the calculated working time compared with the break requirements. If the working time exceeds the lower bound (L_1) of the next break level, the model is extended to a MIP by the following variables and constraints (6.12) to (6.16) and solved with the required break length l. This process is repeated until the correct break level is determined. The break duration before job j is modeled by the variable δ_j and the insertion position of a break before job j by $\beta_j \in \{0, 1\}$, respectively.

$$l_{min}\beta_j \leq \delta_j \leq l\beta_j \qquad \forall j \in J'(t) \tag{6.12}$$
$$\sum_{j \in J'(t)} \delta_j = l \tag{6.13}$$
$$z_j - w_j - \delta_j - z_o \leq U_0 + (1 - \beta_j)U_0 \tag{6.14}$$
$$z_e - (z_j - \delta_j) \leq U_0 + (1 - \beta_j)U_0 \tag{6.15}$$
$$\beta_j \in \{0, 1\} \qquad \forall j \in J'(t) \tag{6.16}$$

Constraint (6.13) inserts the required break length l in one or more breaks, and constraints (6.12) ensure the minimum break length of each part. The maximum time without a break is limited by constraints (6.14) and (6.15). The only binary variables in this model are the break positioning variables β_j. Thus, if no break assignment is needed inside the route ($l = 0$), the variables β_j and δ_j can be excluded and the model reduces to the LP defined by (6.4) to (6.11).

The heuristics do not check all constraints for a feasible insertion of a job into a route. Thus, the model can be infeasible for a given sequence $P'(t)$. In this case, no start times are returned to the heuristic, which indicates that the current sequence is infeasible regarding time window, daily rest time and/or working time constraints.

The presented MIP is very fast to solve as it contains at most $|J'(t)|$ binary variables, if any. The routes of nurses usually contain not more than ten jobs ($|J'(t)| \leq 10$). Therefore, a commercial solver is able to solve the MIP in less than one millisecond in most of the cases. The determined start times are used as basis for the rest time calculation in step 3.

6.1.4. Removal of jobs from tours

Like the insertion of a job, the removal of jobs from existing tours is an essential operation during execution of the developed heuristics in order to create new solutions. Again, the set of jobs to remove is determined by the heuristic. These sets can consist of single or multiple jobs as well as entire tours. Furthermore, the removal of a shift assignment is necessary to free the nurses working time on a particular day.

Removal of shift assignments

An empty tour t ($J'(t) = \emptyset$) can be removed from the current list of shift assignments T^+ without any implications in most of the cases. Only if the tour is on a weekend and the assigned nurse has the requirement of forbidden single shift assignments on weekends, the tour on the other weekend day t' has also to be checked for removal. Two cases can occur, which are also shown in Figure 6.4:

1. If both tours are empty ($J'(t) = J'(t') = \emptyset$), they can be moved from T^+ to T^- and their penalty costs are set to zero.
2. If the other tour t' contains jobs ($J'(t') \neq \emptyset$), the considered tour t cannot be removed. Thus, the sets stay the same and the penalty costs ϕ_t for tour t are activated.

The shift assignment removal is considered whenever the removal of jobs resulted in an empty tour. The next section addresses the removal of jobs.

Removal of jobs

There are different possibilities for job removal depending on the number and relation of jobs to remove. The differentiation is made to save computation time for start time calculations of the remaining jobs. Thus, the following operations are used:

1. *Removal of a single job*: If only a single job is removed from a tour, the start times of all other jobs are determined after the removal by the model presented in Section 6.1.3. This can also result in the removal of breaks that are not

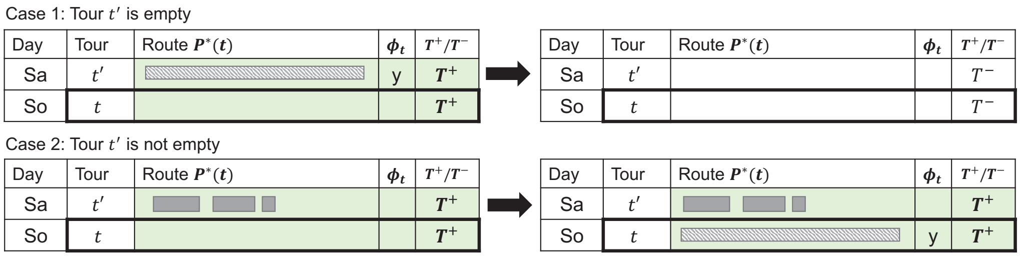


Figure 6.4.: Two cases that can occur for the removal of the shift assignments of tour t on a weekend. The assignment before and after the removal is shown

necessary anymore because the start time calculation starts without break and only inserts one if necessary. Furthermore, the rest times and working times are updated. If the tour is empty after the change, the removal of shift assignment from the previous section is taken into account.

2. *Removal of multiple jobs*: This operation considers the removal of several jobs from one tour. First, all jobs of the set are removed from the tour. Afterwards, the start times of the remaining jobs are recalculated to save computation time for solving the model. Again, rest and working times are updated and the tour is unassigned, if empty.

3. *Removal of entire tour*: In the case that all jobs of a tour are removed, no start time calculation is necessary. Only the removal of the shift assignment takes place.

Each removed job is moved to the set of unassigned jobs U because this set is used by the heuristics to determine the jobs for insertion. The start times of unassigned jobs are set to zero.

6.2. Construction heuristic

All heuristics in this thesis start with an initial solution that has to be provided by a construction heuristic. The time-oriented, nearest-neighbor heuristic proposed by Solomon [1987] is used as basis for initial route construction. Solomon [1987] presents this greedy construction heuristic for the VRPTW. This approach is selected because it is easy adaptable to further criteria than only driving time and time windows, which is essential due to the consideration of available working time based on work contracts and qualification criteria in home care planning. In this section, first, the underlying method proposed by Solomon [1987] and, afterwards, the adaption and extension for home care routing is presented.

Solomon [1987] describes the outline of the time-oriented, nearest neighbor heuristic as follows: The greedy heuristic builds the routes by sequentially appending the “closest“ customer to the end of the route. In our case a customer is a job. The closeness of a customer is measured by considering distance, route duration and time windows. Therefore, Solomon [1987] defines the following components to assess a job for insertion. Note that the symbols are changed to match the notation in this thesis. Let i be the job currently at the end of the route and u the job that it is considered for appending.

$$z_u = \max\{a_u, z_i + r_i + driv_{iu}\} \tag{6.17}$$

$$AddedTime_{iu} = z_u - (z_i + r_i) \tag{6.18}$$

$$RemainingTime_{iu} = b_u - (z_i + r_i + driv_{i,u}) \tag{6.19}$$

$$Score_{iu} = \omega^{Dis} Distance_{iu} + \omega^{AT} AddedTime_{iu} + \omega^{RT} RemainingTime_{iu} \tag{6.20}$$

Equation (6.17) calculates the earliest possible start time z_u of job u when it is inserted after job i by using the start time z_i and duration of r_i of i and the driving time between both. The time added to the tour length by appending job u after i is calculated in Equation (6.18). To address the time windows, Equation (6.19) determines the remaining time until the time window of job u closes. Thus, a job u with a high value of $RemainingTime_{iu}$ is not so urgent in the current state. As jobs with small values are preferred, the jobs with a tight deadline will be inserted first.

The overall decision about which job u to append next is based on the score function in Equation (6.20) that is a weighted sum of all three components. The weights are denoted by $\omega^{Dis}, \omega^{AT}$ and ω^{RT}. The first component represents the distance between i and u. All feasible jobs are considered for insertion and job u with the lowest score will be appended to the current end of the route. When no job is feasible to append but there are still unassigned jobs, the approach opens a new route initialized with the start location.

The difference between home care routing and the VRPTW handled by Solomon [1987] is the heterogeneity of the vehicles that are nurses in our case. In the general VRPTW all vehicles have the same maximum tour length and no incompatibilities between vehicles and customers are given. Furthermore, we have to address several days and qualifications as well as working regulations and shift rotations of nurses during opening of new routes and insertion of jobs. Figure 6.5 shows the steps of the extended time-oriented, nearest neighbor heuristic that works as follows.

The first issue to handle is the *selection of shift patterns* for nurses in set $\mathcal{N}^{\mathrm{R}}$. The construction heuristic starts by iterating through all shift rotations and spreading the shift patterns evenly among nurses valid for the rotation. If the pattern allows more than one shift type per day, the shift type with the least number of assignments until

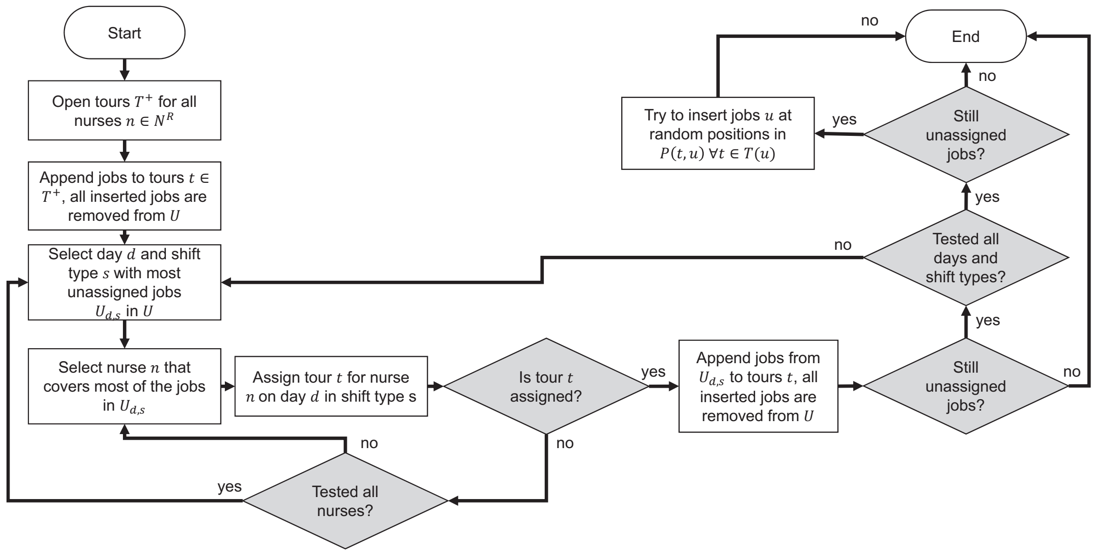


Figure 6.5.: Steps of the construction heuristic

then is selected. In this way, on every day of the planning horizon a shift is assigned to each nurse working according to shift rotations (subject to unavailability). Thus, the heuristic starts with a set of opened routes and these are filled iteratively with jobs according to the score function.

After this step, there are probably still unassigned jobs. These are successively appended to tours, similar to the method proposed by Solomon [1987], i.e., whenever no job is inserted at the end of a route, a new tour is opened. Due to the heterogeneity of the nurses and several days of planning horizon, the next tour to open has to be carefully selected. Here the new tour to open is determined based on the number of candidate jobs. All days and shift types with unassigned jobs to insert are sorted in descending order of jobs they can potentially cover. Afterwards, the nurse to assign is selected in descending order of jobs he or she can fulfill based on the qualifications and availability. If two nurses tie, the nurse with less current weekly working time is selected to balance the workload between nurses. The first successful pair of shift and nurse is selected for a new route and the jobs are inserted based on the score function. This procedure of opening tours is repeated until all jobs are assigned or an iteration passed without any successful insertion.

The score function of Solomon [1987] is extended with a fourth criterion that considers the qualification of a job. It is based on the formula stated in (6.21) that calculates the ratio of nurses providing the required qualification in comparison to all qualifications. The extended score function used in our version of the construction heuristic is given in Equation (6.22).

$$QualificationFrequency_u = \frac{|\{n \in \mathcal{N} | q_u \in \mathcal{Q}_n\}|}{\sum_{q \in \mathcal{Q}} |\{n \in \mathcal{N} | q \in \mathcal{Q}_n\}|} \tag{6.21}$$

$$\begin{aligned} Score_{iu}^{Ext} = {} & \omega^{Dis} Distance_{iu} + \omega^{AT} AddedTime_{iu} + \\ & \omega^{RT} RemainingTime_{iu} + \omega^{Q} QualificationFrequency_u \end{aligned} \tag{6.22}$$

The idea behind this additional criterion is that jobs with a rare qualification are inserted first because other jobs have more possibilities for insertion later in the process as more nurses provide their qualification.

Due to the greedy manner of constructing routes in the proposed heuristic, it is possible to have still unassigned jobs at the end the procedure. All feasible positions for the remaining jobs are tried in random order to insert them while the first feasible position is selected. All jobs that remain unassigned afterwards are left unassigned and penalized with costs in the objective function. The proposed metaheuristics are able to handle unassigned jobs and will try to insert them in the search process to avoid penalty costs. The adapted time-oriented, nearest neighbor heuristic provides the initial solution for all metaheuristics presented in the remainder of this chapter.

6.3. Large neighborhood search

The first heuristic proposed for solving the home care routing and scheduling problem is based on the metaheuristic LNS. A recent survey states that large neighborhoods are often included in successful state-of-the-art methods for the VRPTW [Desaulniers et al., 2014], which is related to our problem. The LNS metaheuristic by Shaw [1998] provides a framework to use a large neighborhood in a simple search process and was applied successfully to different transportation problems [Pisinger and Ropke, 2010]. Furthermore, large neighborhoods are often beneficial in highly restricted solution spaces because it is easier to find feasible solutions than in neighborhoods where small changes most often result in infeasibility [Pisinger and Ropke, 2010]. Due to the time windows and working regulations, the problem setting in this thesis is highly restricted.

The basic outline of LNS is given in Section 3.1.2. In this section the problem specific components and design decisions are described. First, the acceptance procedure of the metaheuristic is explained followed by the large neighborhood operator. An overview of the search process of LNS is given in Figure 6.6. The large neighborhood operator is embedded in the overall process of the heuristic. The other steps show the temperature and solution management of LNS.

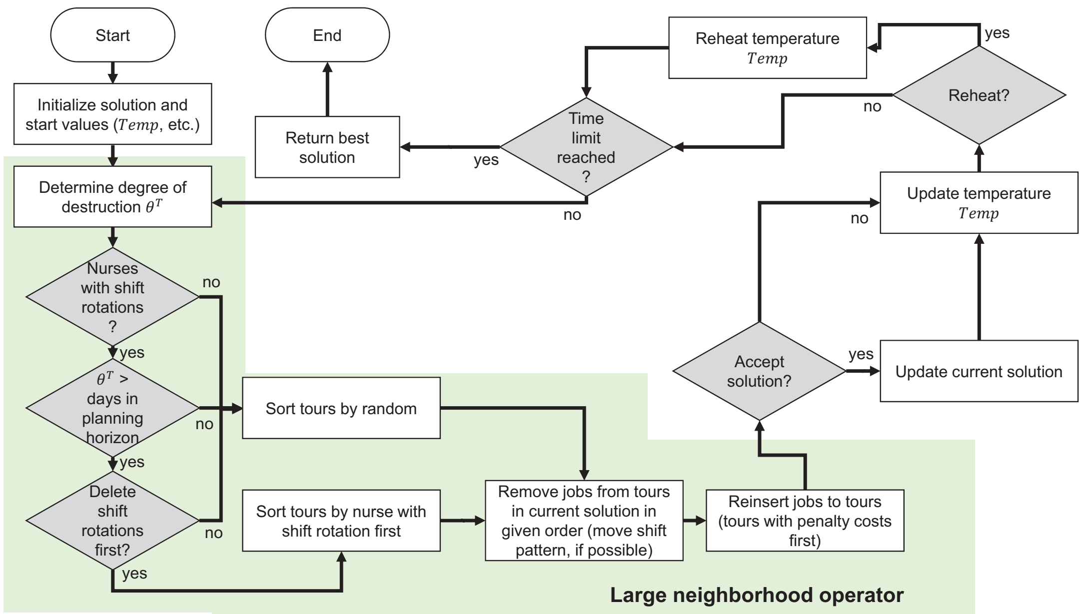


Figure 6.6.: Steps of LNS including large neighborhood operator

6.3.1. Acceptance of solutions

Pisinger and Ropke [2010] state that, although not proposed for the original method, the acceptance criterion of simulated annealing can be used in an LNS implementation to allow the acceptance of worse solutions in the beginning of the search. This can be beneficial to diversify and guide the search [Pisinger and Ropke, 2010]. An outline of the simulated annealing method is given in Section 3.1.2. This approach is also used in this implementation of LNS. The heuristic manages the temperature $Temp$ during the search. The temperature is initialized with the value $Temp^0$ and decreased during search based on a cooling schedule determined by the cooling factor f^{Cool} and number of iterations between cooling it^{Cool}.

To determine the initial temperature $Temp^0$, the approach of Ropke and Pisinger [2006] is chosen, i.e., the initial temperature is selected based on the objective value of the initial solution to provide an instance-dependent value. Ropke and Pisinger [2006] take the objective value x^0 of the initial solution and determine $Temp^0$ such that a solution with an objective value $(1.0 + w^{Start})$ times worse than x^0 is accepted with a probability of 0.5. The parameter w^{Start} has to be determined a priori. They further state that the penalty costs should be removed from this calculation to avoid a deteriorated initial temperature due to the high values of those. The formula to calculate $Temp^0$ is derived from the probability calculation in simulated annealing (6.23) and given in (6.24).

$$e^{-\frac{(1.0+w^{Start})x^0-x^0}{Temp^0}} = 0.5 \tag{6.23}$$

$$\Leftrightarrow \; Temp^0 = \frac{(1.0+w^{Start})x^0-x^0}{-ln(0.5)} \tag{6.24}$$

Every iteration a newly created solution x' is considered for acceptance. If the solution has a value improving the global best solution value x^*, it is always accepted. If the solution has a worse objective value it is only accepted with probability $e^{\frac{x^*-x'}{Temp}}$.

If the method does not find a new best solution for numerous iterations, a reheating of the temperature takes place to achieve bigger steps in the search process because the probability of accepting worse solutions increases again. The reheating depends on the number of iterations without improvement and the current temperature. If the temperature sinks below the minimum $Temp^{Min}$ and it^{Reheat} iterations passed without improvement of the global best solution, the current temperature is increased to $f^{Reheat}Temp^0$ dependent on the reheat factor f^{Reheat}.

6.3.2. Large neighborhood operator

The large neighborhood operator developed to change a solution consists of two phases: destroy and repair. The *destroy* phase removes partial and entire tours from the solution by removing jobs from them. The number of tours to destroy is determined by the degree of destruction η that is dependent on the current temperature $Temp$. In iterations with a high temperature the degree of destruction is higher and vice versa. The degree of destruction is a value in the interval $[0, 1]$ and calculated based on the temperature in Equation (6.25).

$$\eta = \frac{Temp - Temp^{Min}}{Temp^0 - Temp^{Min}} \tag{6.25}$$

The value of η is used as percentage of assigned tours to destroy. Therefore, the current number of assigned tours is multiplied with η.

$$\theta^T = \lfloor \eta |T^*| \rfloor \tag{6.26}$$

The tours to destroy are selected randomly from the current assigned tours. The exception are instances containing nurses working shift rotations. In this case, the operator checks if θ^T exceeds the number of tours needed to remove all tours of one of those nurses. If yes, the shift pattern of this nurse is moved by one with a certain probability and all tours are destroyed. Otherwise, the normal procedure takes place and the tours are selected randomly. Finally, an additional tour is selected and destroyed partially based on the remaining fractional tour count $\eta T^* - \lfloor \eta T^* \rfloor$. This value determines the percentage of jobs to randomly remove from the selected tour.

The *repair* phase of the operator reinserts all unassigned jobs into tours. The candidate tours $T(u)$ and positions $P^*(t, u)$ for the unassigned jobs $u \in U$ are determined by the methods described in Section 6.1. Candidate positions belonging to a tour currently having penalty costs are considered first for insertion. Afterwards, the order of insertion positions is randomly selected. The job is inserted at its first feasible position.

This operator is a simple and easy neighborhood operator that allows to search the solution space. Domain knowledge influences the actions in essential parts. Shift rotations can be changed and penalty costs are avoided. Furthermore, the candidate insertion positions are limited based on domain knowledge to reduce the number of infeasible insertions as described in Section 6.1. Additionally, some parts of the destroy and repair part are randomized to diversify the search, which is important in large neighborhoods according to [Pisinger and Ropke, 2010].

6.4. Adaptive large neighborhood search

The second metaheuristic implemented for the solving home care routing and scheduling is ALNS. As already mentioned in Section 3.1.2 it extends the concept of LNS by allowing several destroy and repair operators with different focuses during search and an adaptive weighting of those operators. Thus, the heuristic changes the weights of the operators based on the success during search and can adapt to different instances [Pisinger and Ropke, 2010], which is promising to improve the performance of LNS. The overall process of ALNS is shown in Figure 6.7. The acceptance of solutions as well as the determination of temperature, reheating and degree of destruction are

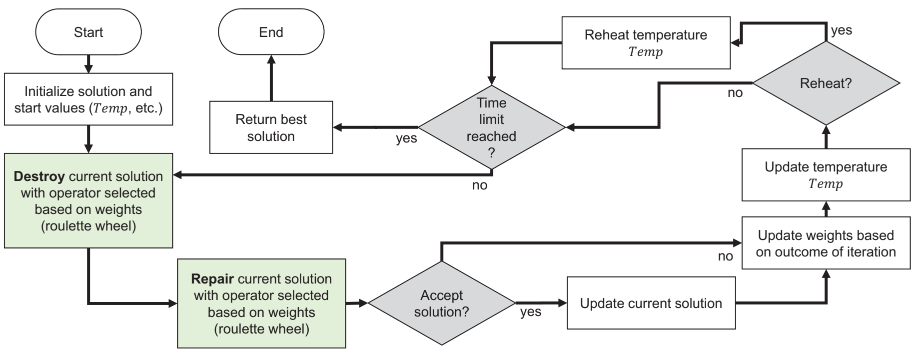


Figure 6.7.: Process of ALNS

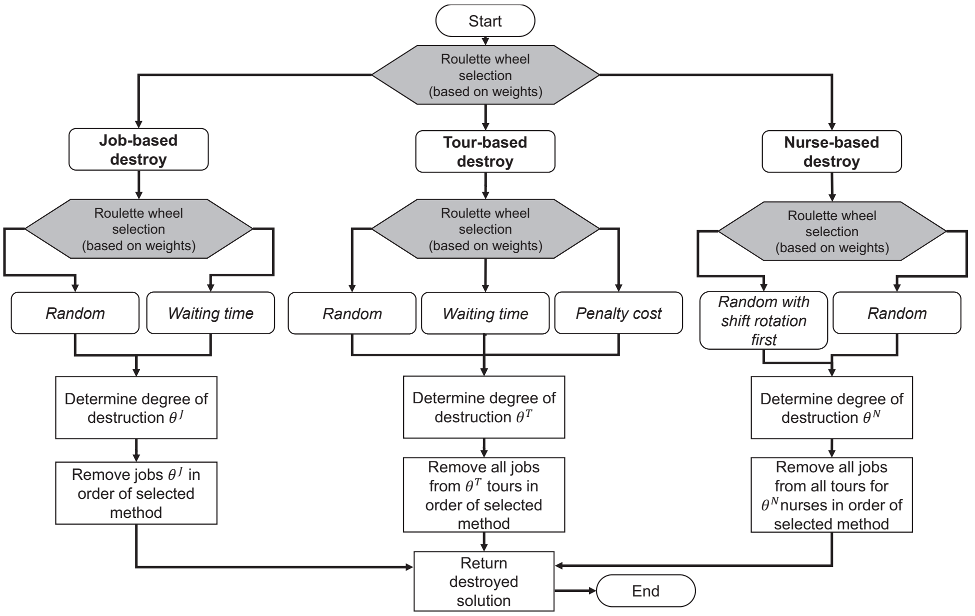


Figure 6.8.: ALNS destroy process

the same as for the LNS presented in the previous section. The description focuses on the destroy and repair operators that are used to create new solutions.

6.4.1. Destroy operators

The presented problem formulation has many different types of constraints that restrict the metaheuristic during search. The removal of only small parts of the solution, can lead to difficult and inefficient repair mechanisms. Therefore, the developed destroy operators change solutions with emphasis on different parts. Common concept is the removal of jobs from routes. The set of jobs depends on the selected destroy operator. We implement three types of destroy operators: job-based, tour-based and nurse-based. Each of this operators has several selection strategies that determine which jobs, tours or nurses to destroy.

Figure 6.8 shows the three destroy operators with their available selection strategies. The decision for an operator in combination with a strategy consists of two levels. On the first level the roulette wheel selection determines the operator for this iteration. In the second phase the selection strategy for jobs, tours or nurses is determined by a second roulette wheel selection considering only the valid strategies. Note a random sorting is included for all three operators to diversify the search. After the operator is selected, the methods from Section 6.1.4 are used for job, tour and shift removal. The details of the destroy operators are described in the remainder of this section.

Job-based destroy

The job-based destroy operator removes a set of jobs from their current tours. The jobs are selected independent of day, client or qualification. The degree of destruction η determines the number of jobs to remove θ^J (6.27).

$$\theta^J = \left\lceil \eta \cdot |\mathcal{J}^C| \right\rceil \tag{6.27}$$

Additionally, there are two strategies to select the θ^J jobs. The first strategy takes *random* jobs until the limit is reached. The second strategy sorts the jobs in descending order of the current *waiting time* inserted before them and takes the first θ^J jobs to reduce inefficiencies. In both approaches the unassigned jobs are excluded from consideration. Finally, the operator iterates through the set and removes one job after the other until the required number is reached. After the removal of jobs, the start times of all other jobs are updated before reinsertion.

Tour-based destroy

The tour-based destroy operator removes all jobs of a subset of currently assigned tours T^+ that are selected independent of day and nurse. The number of tours is calculated by applying the degree of destruction η resulting in θ^T tours taken for destruction (6.28).

$$\theta^T = \lceil \eta |T^+| \rceil \tag{6.28}$$

There are three different sorting methods to determine which tours being destroyed first. The first method sorts the tours in ascending order of *penalty costs* due to single shifts on weekends caused by the assignment. Thus, tours on Sundays that have no related assignment on Saturday and therefore causing a penalty on Saturday are taken first. The same holds for tours on Saturdays without equivalent on Sundays. All other tours are not affected by the penalty consideration and are appended at the end of this list in random order.

The second sorting strategy determines the total *waiting time* inside the tours and prioritizes tours with a high waiting time for destruction. The third strategy sorts the assigned tours *randomly*. The destruction itself removes all jobs from the tours and deletes the according shift assignment of the nurse based on the methods described in Section 6.1.4.

Nurse-based destroy

The nurse-based destroy operator changes shift rotations of nurses by destroying all tours of a nurse. This means, all jobs assigned to a nurse are removed from the tours, leaving the nurse with no tours assigned. The operator proceeds as follows: First, the number of nurses is determined based on the degree of destruction η and number of nurses available (6.29).

$$\theta^N = \lceil \eta |\mathcal{N}| \rceil \tag{6.29}$$

Afterwards, the nurses are sorted to determine the set of nurses considered in this iteration. The first sorting method consists of two parts. First, all nurses working according to *shift rotations* are randomly sorted. Second, the remaining nurses not assigned to shift rotations are appended to the list in random order. The second strategy selects the nurses *randomly*.

The destruction phase iterates through the list of nurses until θ^N nurses are reached. Every tour of a nurse is deleted by removing all jobs and the shift assignment. If the nurse has a pattern assigned, the pattern is moved by one week to enable the method to change the rotation pattern.

6.4.2. Repair operators

The repair operators of ALNS in this thesis are based on greedy construction and insertion heuristics for the VRPTW. They are adapted and extended to handle qualifications of nurses and working regulations, which are essential for feasible job insertion. In total, there are three different repair heuristics. The first is a reduced variant of the time-oriented nearest neighbor heuristic described in Section 6.2. The two other methods are insertion heuristics.

Figure 6.9 gives an overview of the repair step of the ALNS. First, a roulette wheel selection determines which of the three repair operators to apply. If one of the insertion heuristics is determined, the job order has an influence on the result. Thus, the order of the unassigned jobs before executing has to be selected by a second roulette wheel selection. If the reduced variant of the time-oriented, nearest neighbor heuristic is selected no job sorting is necessary, but the method has to be linked to an assignment and a nurse sorting strategy. The details of the heuristics are described in the next sections.

Reduced time-oriented, nearest neighbor heuristic

This repair heuristic extends the proposed construction heuristic based on the time-oriented, nearest neighbor heuristic of Solomon [1987] described in Section 6.2. There

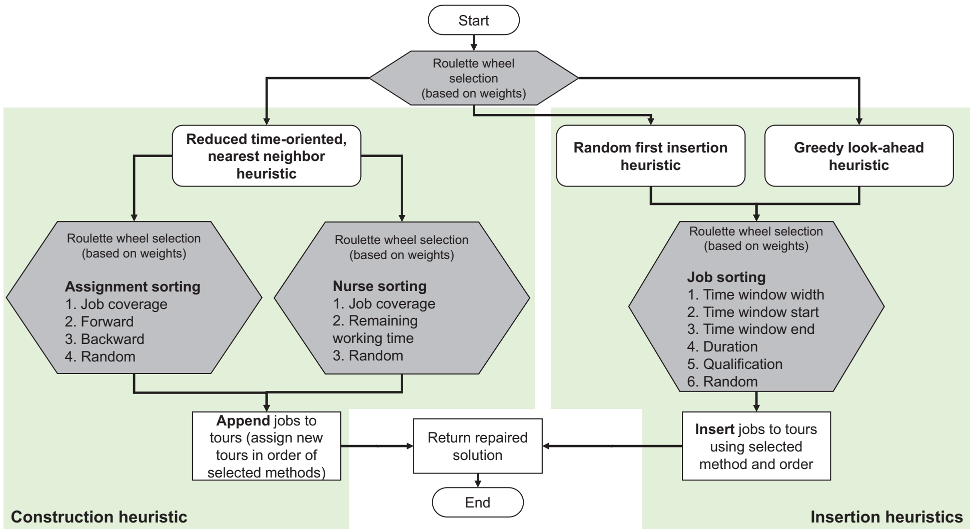


Figure 6.9.: ALNS repair process

are two alterations to the construction heuristic, namely the limitation of jobs considered for insertion and the sorting methods of nurses and shifts to open new routes.

The basic variant of the time-oriented, nearest neighbor heuristic evaluates every candidate job for appending it to the current tour's end. This phase can be time-consuming if there are many candidate jobs, which is the case when the destroy operator has a high degree of destruction, especially because the heuristic is called for repair numerous times during the search process. Therefore, we limit the number of candidate jobs considered for insertion. We distinguish between already existing routes and newly opened routs. The set of candidate jobs for an existing tour are limited to a maximum of τ^{Exist} jobs. The jobs are sampled randomly from the set of candidate jobs and the job with the lowest score (see Section 6.2) is appended to the route's end. Recall that the candidate jobs contain only a subset of all jobs that are compatible with the day and qualification of the nurse (see Section 6.1.2). Furthermore, the number of jobs considered for insertion to a new tour is limited to τ^{New} jobs. This number is higher than the jobs considered for insertion in existing tours, i.e., $\tau^{Exist} \leq \tau^{New}$, because the construction of a new tour has more degrees of freedom and therefore has potential to improve the new solution substantially, if the jobs are selected based on a more extensive evaluation. If ALNS passed several iterations without an improvement and the temperature is reheated, the values of τ^{Exist} and τ^{New} are doubled as long as no new global best solution is found, otherwise they are reseted to their initial values.

As described in Section 6.2, the time-oriented, nearest neighbor heuristic opens new tours, if no insertion to the existing tours is possible. Thus, the heuristic has to decide which nurse to assign on which day for which shift type next. The combination of shift type and day to assign next is determined by one of the following metrics:

1. Number of candidate jobs available for insertion on this day and in this shift type (like in the construction heuristic in Section 6.2).
2. Forward sorting based on day and shift type.
3. Backward sorting based on day and shift type.
4. Random sorting.

The first metric addresses the goal of assigning as many jobs as possible. Therefore, days and shift types with many candidates are selected first. The second and third criterion offer the possibility of shift assignments on consecutive days. The last criterion of random order is used for diversification of the search, reaching solutions that are not possible to reach with the other three sorting methods. After the day and shift type are selected, the nurses that are feasible for assignment for that day and shift type are determined. The decision which nurse to assign is essential to the remaining solution process because it consumes working time of the assigned nurse. No other shift assignment on this day is possible and the weekly working time and workdays are reduced. Therefore, there are several different sorting methods to determine the priorities for nurse assignment.

The first strategy selects the nurse with the highest *job coverage* in the corresponding shift type to aim at full job assignment. The second strategy sorts the nurses in descending order of *remaining weekly working time*. Thus, low utilized nurses are considered first. Besides, the workload is spread more evenly among nurses. The last strategy returns the nurses in *random* order.

The combination of nurse and assignment sorting together with the time-oriented, nearest neighbor heuristic leads to several repair mechanisms that can be called by the ALNS. The probability for sorting methods are equally distributed at the beginning of the search and adapted during search based on the success.

Insertion heuristics

In contrast to the time-oriented nearest neighbor heuristic that only appends jobs, the following two repair operators are based on insertion heuristics that reinsert jobs sequentially to tours by evaluating insertion positions in routes. The order of jobs has an impact on the quality of the resulting solution because a job inserted can block space for a later considered job.

Job sorting methods. ALNS allows us to use several different sorting methods for the unassigned jobs. Each sorting method in combination with one of the repair heuristics described in the remainder of this section comprises an own repair operator that can be handled by ALNS, i.e., each combination has an adaptive weight used for the operator selection. The six implemented insertion orders of unassigned jobs are the following:

1. Time window width $(b_i - a_i)$ of job i (tie-breaking by 2.)
2. Time window start a_i of job i (tie-breaking by 1.)
3. Time window end b_i of job i (tie-breaking by 2.)
4. Duration r_i of job i (tie-breaking by 2.)
5. Qualification q_i of job i (tie-breaking by 2.)
6. Random

All criteria are used to sort the jobs in ascending order of the respective criterion. The time windows in 1. to 3. are considered independent of days, i.e., a time window $[360, 480]$ on day 1 and 2 have the same value. The tie-breaking strategy for each criterion is given in parentheses.

Random first insertion heuristic. The first insertion heuristic acts randomly and iterates through the unassigned jobs in the order determined by one of the job sorting methods. It explores all candidate tours and positions in random order until a feasible insertion position is found. This position is selected and the next job considered.

Greedy look-ahead heuristic. The second insertion heuristic is based on the *greedy look-ahead heuristic* of Ioannou et al. [2001] originally proposed for the VRPTW. This heuristic is also mentioned by Desaulniers et al. [2014] for providing good quality solutions in a relatively short computation time. The proposed scoring function of inserting a job at a position is described by Ioannou et al. [2001] as follows. Note that the notation is changed to match the symbols in this thesis.

Their method evaluates all feasible insertion positions and chooses the position with the least (negative) impact on the current solution. Therefore, it calculates the impact of inserting job u at position (i, j) on itself IS_{ij}, other jobs in the same route IR_{ij} and unassigned jobs IU_{ij}. In this thesis, the positions are determined by the candidate tours $T(u)$ and positions $P(t, u) \forall t \in T(u)$ that are described in Section 6.1.2. Set $P(u)$ resembles all positions $P(t, u)$ of all tours $t \in T(u)$, i.e., $P(u) = \bigcup_{t \in T(u)} P(t, u)$. The insertion process is given in Figure 6.10.

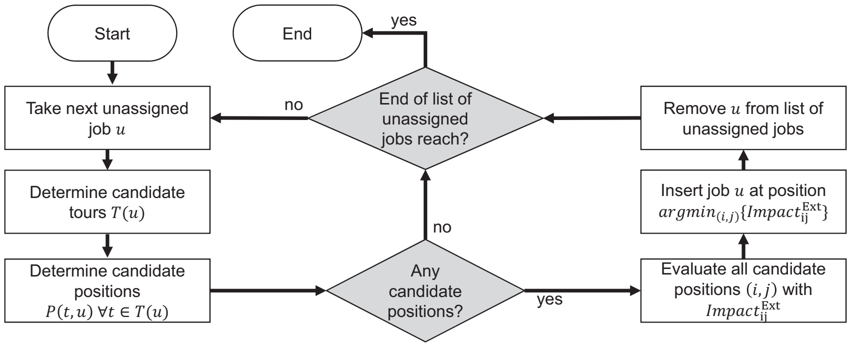


Figure 6.10.: Steps of adapted greedy look-ahead heuristic

The impact of a position $(i, j) \in P(u)$ on job u itself is the time difference to the start of time window given in Equation (6.30).

$$IS_{ij} = z_u - a_u \tag{6.30}$$

Unassigned jobs $U' = \{u' \in U | u' \neq u \wedge d_{u'} = d_u\}$ on the same day are influenced by the insertion of a job u. If they have overlapping time windows, the position may be blocked for another job considered later in the insertion procedure. Thus, the minutes of overlapping are determined and averaged over all unassigned jobs (6.31).

$$IU_{ij} = \sum_{u' \in U'} \frac{max\{(b_{u'} - a_u - driv_{uu'}), (b_u - a_{u'} - driv_{u'u})\}}{|U'|} \tag{6.31}$$

The inner-route IR_{ij} impact is a weighted sum of the increase in driving time (6.32), the remaining time to the end of the time window of the subsequent job (6.33) and the remaining time to the end of the time window of u (6.34). This weighted sum is averaged over all jobs $J(t)$ in route $P(t)$ of tour t.

$$IR_{ij} = \omega_1^{IR}(driv_{i,u} + driv_{u,j} - driv_{i,j}) \tag{6.32}$$
$$+ \omega_2^{IR}\Big[\big(b_j - (z_i + r_i + driv_{i,j})\big) - \big(b_j - (z_u + r_u + driv_{u,j})\big)\Big] \tag{6.33}$$
$$+ \omega_3^{IR}\big(b_u - (z_i + r_i + driv_{i,u})\big) \tag{6.34}$$

The final calculated impact of inserting job u at position (i, j) is the weighted sum of all three criteria given in Equation (6.35).

$$Impact_{ij} = \omega^{IS} IS_{ij} + \omega^{UI} UI_{ij} + \omega^{IR} \frac{IR_{ij}}{|J(t)|} \tag{6.35}$$

Like for the time-oriented, nearest neighbor heuristic, the heuristic is extended by a fourth component QF that incorporates the qualification requirement of the job regarding the offered qualifications of nurses. After the assignment of job u to the route, there is less available working time provided for the qualification q_u on the corresponding day and in the week. We calculate with how many jobs we share the qualification requirement and how rare the qualification is. Thus, jobs with rare qualification requirements are weighted higher. We take a weighted normalized sum of all jobs, which could also be assigned to nurse n (6.36).

$$QF_{ij} = \frac{\sum_{u' \in \{U' | q_u \neq q'_u \wedge q'_u \in \mathcal{Q}_{n_t}\}} (1 - QualificationFrequency_u)}{\sum_{u' \in U'} (1 - QualificationFrequency_{u'})} \tag{6.36}$$

A job is inserted at the position (i, j) with the least costs of the four component weighted sum given in Equation (6.37).

$$Impact^{Ext}_{ij} = \omega^{IS} IS_{ij} + \omega^{UI} UI_{ij} + \omega^{IR} \frac{IR_{ij}}{|J(t)|} + \omega^{QF} QF_{ij} \tag{6.37}$$

All weights for the sums (ω) in this method are parameters and need to be determined a priori. The evaluation of all possible insertion positions for a job can be computational expensive when the solution contains many tours that fit with jobs requirements. Therefore, we restrict the number of tours taken into account by the parameter τ^{Tours} to restrict the computation time per iteration.

6.5. Reduced variable neighborhood search

The third metaheuristic is RVNS. VNS has been applied successfully to the vehicle routing domain [Hansen et al., 2010a] and it provides a different concept than the large neighborhoods. But like ALNS it allows several different operators to search different neighborhoods in the process [Pisinger and Ropke, 2010].

The advantage of the RVNS is that the operators do not evaluate all feasible neighbors in each iteration but perform the shake step with the current valid neighborhood and always accept improving solutions. Hansen et al. [2010b] state that “RVNS is useful in very large instances, for which local search is costly“ [Hansen et al., 2010b, p.373]. The home care planning instances are large and a local search method is computational costly due to several reasons. First, there are many neighbors that need to be evaluated. Second, the evaluation of a new neighbor requires the non-trivial re-computation of start and rest times for each tour which should be saved in as many cases as possible. Third, small changes to solutions often lead to infeasible new solutions due to many requirements.

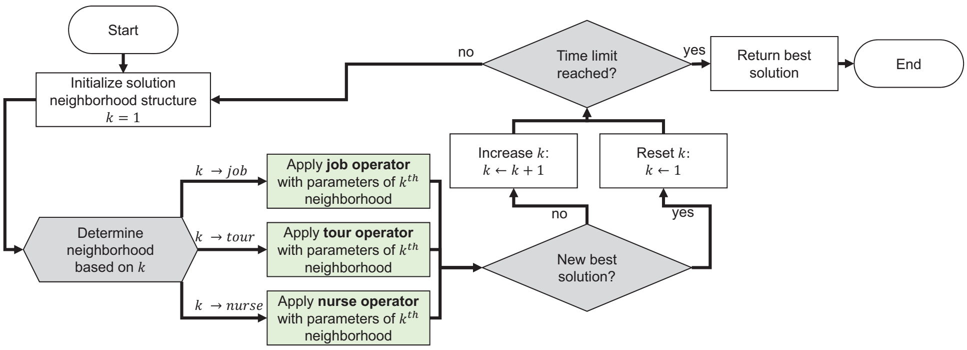


Figure 6.11.: Process of RVNS

The outline of the standard RVNS is given in Section 3.1.2 and for the specific RVNS in this thesis in Figure 6.11. This section describes the implemented operators, neighborhood structure and their parameters.

6.5.1. Neighborhood operators

There are three types of neighborhood operators that are described in this section. The job operator changes the solution by moving jobs in and between tours. The tour operator considers entire tours that are altered in one step. The nurse neighborhood is based on the entire set of tours of one nurse.

Job operator

The job operator removes a set of jobs independent of tours or days from the solution and directly reinserts them to another random position. The feasible positions and tours are calculated based on the methods described in Section 6.1.2.

The number of jobs to remove depends on the current parameter setting for the heuristic because there is no degree of destruction in an RVNS. There are two possibilities implemented. Either the number of jobs is determined relative to the number of all jobs (6.38) or an absolute number of jobs is given (6.39). The calculation is based on the parameter l^J determining the percentage or number depending on the chosen formula.

$$\theta^J = Max\{1, |\mathcal{J}\backslash\{\mathcal{J}^*\}|l^J\}, \qquad \text{with } l^J \in [0,1] \tag{6.38}$$
$$\theta^J = l^J, \qquad \text{with } l^J \in \mathbb{N} \tag{6.39}$$

If the relative or absolute value is used, is defined as a parameter and needs to be determined a priori.

Tour operator

The tour operator is similar to the repair step in the large neighborhood operator of the LNS presented in Section 6.3.2. The number of tours to remove from the current solution is given by parameter l^T, which determines the number of tours relative to the number of assigned tours (6.40) or the absolute value (6.41) depending on the parameter setting.

$$\theta^T = Max\{1, |T|l^T\}, \qquad \text{with } l^T \in [0,1] \tag{6.40}$$
$$\theta^T = l^T, \qquad \text{with } l^T \in \mathbb{N} \tag{6.41}$$

The tours to change are selected randomly from the current assigned tours. The resulting unassigned jobs are reinserted in random order. The candidate tours considered for reinsertion are first the tours with penalty costs due to single shifts on weekends and afterwards all other tours. The jobs are inserted to the first position feasible for insertion.

Nurse operator

The nurse operator removes all tours of a selected nurses from the current solution. If there are nurses working according to shift rotations these are considered first because only by removing all tours the shift pattern can be changed. If there are still nurses left for removal, the remaining nurses are selected for removal.

The number of nurses is determined by the parameter l^N that is again a relative (6.42) or absolute (6.43) value of nurses.

$$\theta^N = Max\{1, |\mathcal{N}|l^N\}, \qquad \text{with } l^N \in [0,1] \tag{6.42}$$
$$\theta^N = l^N, \qquad \text{with } l^N \in \mathbb{N} \tag{6.43}$$

The order of nurses is chosen randomly. If the instance has nurses with shift rotations these are the basis for consideration and only if this set is empty, all other nurses are added. If no shift rotations are present, the entire set of nurses is considered for removal from the beginning. For nurses working shift rotations the selected pattern is changed to the next pattern.

The insertion of jobs works the same way as in the RVNS tour operator, i.e., jobs are reinserted in random order to the first feasible position whereas tours with penalty costs are considered first.

Composition of neighborhoods from parameter settings

As described in Section 3.1.2, RVNS iterates through the k_{max} neighborhoods during search. Therefore, the neighborhood structure has to be determined a priori and is composed by several parameters in this thesis. The parameters $level^J, level^T$ and $level^N$ determine how many levels with different removal rates are used for each operator. Consider a $level^J = 3$ for the job operator, then the operator is used with three removal parameters creating three neighborhoods for RVNS. The lowest level always has the lowest removal rate. The removal rates on the higher level are calculated based on the multiplier parameters mul^J, mul^T and mul^N. For our example of $level^J = 3$, level 1 gets the minimum removal rate, level 2 considers the removal rate of level 1 multiplied by mul^J and level 3 the rate of level 2 multiplied by mul^J. The process of this calculation is given in Algorithm 6 without referring to any specific operator because the procedure is the same for all three operators. The parameters l^J, l^T and l^N are represented by l and mul^J, mul^T and mul^N by mul. Furthermore, the parameter contains one value per *level* indicated by the index in the squared brackets.

Algorithm 6: Creating neighborhood structure

```
l[0] ← l^min;                                  // minimum removal rate
for i = 2, i ≤ level, i = i + 1 do
│  l[1] ← l[i − 1] mul;
end
```

Neighborhood order

The operator order influences the outcome of the algorithm. The operators linked to a lower k are called more often during the search because the neighborhood iterator k is reseted every time a new best solution is found. Therefore, four orders are tested for their results. An overview of the proposed orders for an example parameter setting are given in Figure 6.12.

The first order *Job-Tour-Nurse* represents a nested neighborhood structure because the neighborhood moves from small (single jobs are moved) to large changes (all tours of a nurse are moved). The size of the changes increases when no better solutions are found, although the extent of the change depends also on the parameter setting, but the aforementioned is the most likely case. The second order *Nurse-Tour-Job* represents the first in reverse order to intensify the search when a promising solution is found. In the third case *Tour-Job-Nurse*, the tour operator is the first to apply followed by the job and nurse operator. This order is implemented due to the fact that the tour operator allows shift assignment changes that have a major influence on

Order 1 *Job-Tour-Nurse*	**Order 2** *Nurse-Tour-Job*	**Order 3** *Tour-Job-Nurse*	**Order 4** *Mixed*
Job operator	Nurse operator	Tour operator	**1.** Job operator – Level 1
1. Level 1	**1.** Level 1	**1.** Level 1	**2.** Tour operator – Level 1
2. Level 2	**2.** Level 2	**2.** Level 2	**3.** Nurse operator – Level 1
3. Level 3	Tour operator	**3.** Level 3	**4.** Job operator – Level 2
Tour operator	**3.** Level 1	Job operator	**5.** Tour operator – Level 2
4. Level 1	**4.** Level 2	**4.** Level 1	**6.** Nurse operator – Level 2
5. Level 2	**5.** Level 3	**5.** Level 2	**7.** Job operator – Level 3
6. Level 3	Job operator	**6.** Level 3	**8.** Tour operator – Level 3
Nurse operator	**6.** Level 1	Nurse operator	
7. Level 1	**7.** Level 2	**7.** Level 1	
8. Level 2	**8.** Level 3	**8.** Level 2	

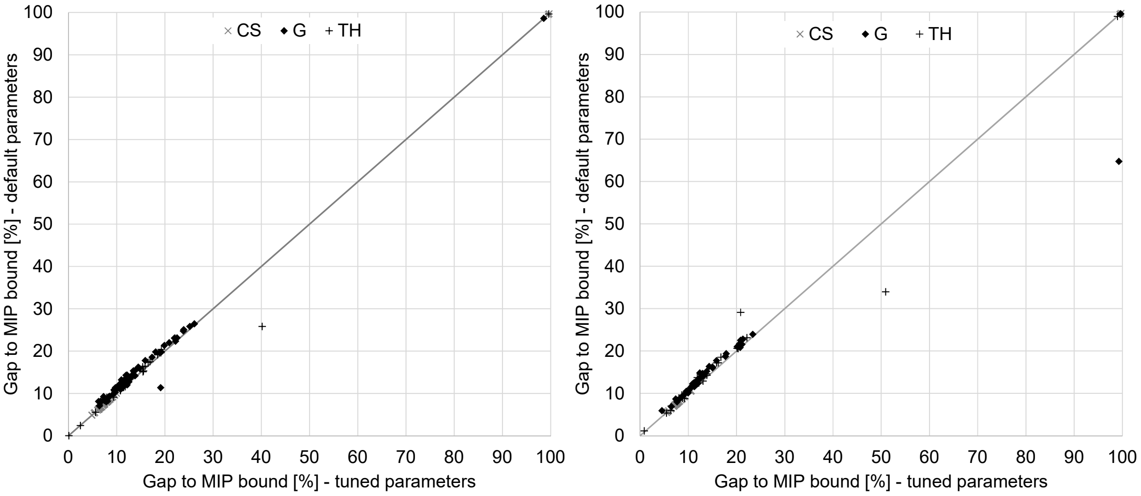
Figure 6.12.: RVNS operator order for a job operator with three levels, a tour operator with three levels and a nurse operator with two levels (the bold figures represent the neighborhood iterator k)

the working time capacities. In contrast to this, the job operator only removes single jobs from a tour and the reassignment of shifts is only possible in the case when all jobs of a tour are removed. The above mentioned first three orders are based on the operator, i.e., the different levels of operators are kept together in ascending order. The fourth operator order *Mixed* alternates the operators on different levels whereas the main order is *Job-Tour-Nurse*. Thus, the search changes between smaller and larger changes. The order to be used in the evaluation is determined by the parameter tuning procedure described in Section 6.6.

6.6. Parameter tuning

The parameters for the construction heuristic and all three metaheuristics are determined by the gender-based genetic algorithm (GGA) configurator developed by Ansótegui et al. [2009], which uses a genetic algorithm to improve the parameters starting with default values. The quality tuning option is used to improve the default parameters of our heuristics. The criterion evaluated for each run is the gap to the best bound found for the MIP after 48 hours (see Section 5.4.3 and Table C.5 in Appendix C) because the evaluation values need to be in the same range for all instances. The original objective function of tour lengths differs between instances, especially if they have different sizes. Let x denote the objective value of the heuristic and $\underline{x}$ the best bound on the objective value found by the MIP, then the gap is calculated by Equation (6.44).

$$Gap^{bound} = 100\frac{x - \underline{x}}{x} \tag{6.44}$$



(a) Training set (b) Test set

Figure 6.13.: Parameter tuning results for LNS

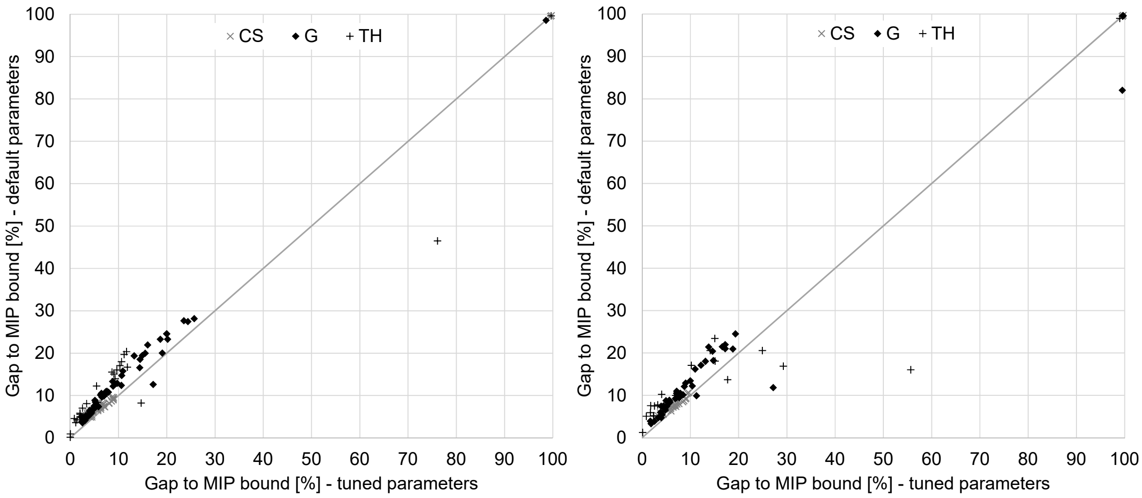
As the bound $\underline{x}$ is always lower than x, the gap lies in the interval $[0, 100]$. Thus, the gap is a normalized evaluation criterion that makes the heuristics results comparable for all instances. The configurator GGA improves the parameters by minimizing the average outcome over all instances.

The configuration was performed for three days wall-clock time with eight cores[2], i.e., GGA runs eight instance evaluations in parallel. The evaluation of a parameter setting for one instance is based on three runs with each three minutes computation time due to the stochasticity of the algorithms. The training set used for configuring the heuristics is different from the test set. The training set consisted of 128 instances sampled from the entire instance set as described in Section 5.3. The parameters of the heuristics that are determined by GGA are given in Tables C.1 to C.4 in Appendix C including the default values, tuned values and allowed assignments for each parameter. After the configuration all determined parameter values are validated on the test set because these are instances that are not considered during configuration.

The improvement for each heuristic on the training and test set is given in Figures 6.13 to 6.15. For this results each instance is evaluated with the default and tuned parameters[3]. The runtime is set to five minutes and the values are average over 10 runs. Each data point represents one instance labeled according to the instance set it belongs to. The x-axis indicates the gap of the heuristic solution to the MIP best bound (6.44) with tuned parameters and the y-axis with default parameters. All

[2]All runs are computed on Xeon E5 processors with four 2.6 GHz CPUs using 16 GB RAM provided by the Paderborn Center for Parallel Computing, using Gurobi 6.0.5 for the start time calculation

[3]Using one CPU of the Xeon E5 processors and 4 GB RAM.



(a) Training set (b) Test set

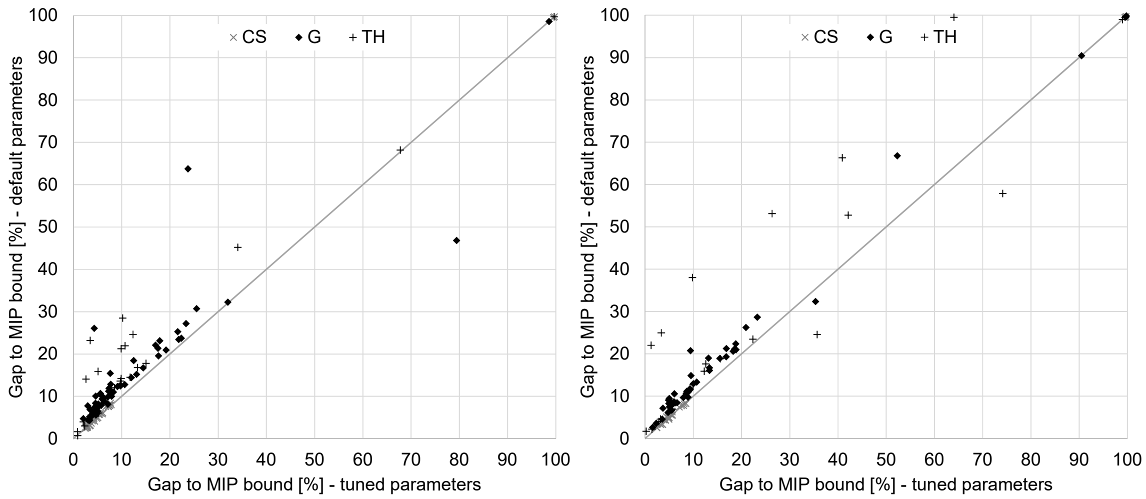
Figure 6.14.: Parameter tuning results for ALNS

data points above the diagonal represent an improvement by the configuration.

The results for the LNS heuristic in Figure 6.13 show that the algorithm configuration resulted in only small improvements for most of the instances of training and test set. There are also some instances where the new parameters resulted in worse solution values (data points below the diagonal), which can be explained by the tuning goal: GGA uses the average outcome of the algorithm. Therefore, a deterioration for single instances is possible.

The configuration of the ALNS heuristic leads to a bigger improvement compared to LNS on training and test set as shown in Figure 6.14. In particular, the generated instances and TH instances show smaller gaps with the tuned parameters. The improvement for the CS instances is very small because the instances are not so restrictive and ALNS reaches good results already with the default parameters. However, several instances show a deterioration after the configuration due to tuning the average outcome. The results before and after configuration for the RVNS in Figure 6.15 shows the best success of GGA because the improvement is clearly visible. Many instances, especially from the TH and generated set, are improved substantially. But the positions of the data points also indicate that even if the improvement is large, the instances have a relatively high remaining gap.

These observations are supported by the average values of gaps and improvements given in Table 6.3. The smallest remaining gaps for training, test and the entire set are achieved by ALNS (13.40) whereas the new parameters for RVNS lead to the highest improvement (-2.82). Looking at the results for each instance set individually, the large improvement on the TH instance set for the RVNS becomes apparent. The



(a) Training set (b) Test set

Figure 6.15.: Parameter tuning results for RVNS

Heur.	**Set**	**Train**			**Test**			**Train+Test**		
		Def.	Tuned	Δ	Def.	Tuned	Δ	Def.	Tuned	Δ
ALNS	**all**	**13.45**	**11.11**	**-2.34**	**18.27**	**16.80**	**-1.47**	**15.15**	**13.40**	**-1.74**
	G	12.88	10.26	-2.62	17.19	14.85	-2.34	14.62	12.27	-2.36
	TH	15.73	13.11	-2.62	20.00	19.70	-0.30	17.73	16.19	-1.54
	CS	11.21	10.13	-1.07	18.674	17.68	-1.00	14.37	13.33	-1.04
LNS	**all**	**15.53**	**15.07**	**-0.47**	**19.86**	**19.76**	**-0.10**	**16.86**	**16.59**	**-0.27**
	G	15.64	14.77	-0.88	19.09	18.91	-0.18	14.58	14.56	-0.01
	TH	17.46	17.66	0.21	23.18	23.13	-0.05	17.15	16.54	-0.62
	CS	11.47	11.46	-0.01	18.82	18.80	-0.02	20.13	20.22	0.09
RVNS	**all**	**14.21**	**11.58**	**-2.63**	**22.14**	**19.12**	**-3.02**	**17.69**	**14.87**	**-2.82**
	G	15.52	12.32	-3.20	19.90	17.02	-2.89	17.44	14.34	-3.10
	TH	20.40	14.67	-5.74	35.51	27.73	-7.79	27.45	20.76	-6.69
	CS	9.20	8.96	-0.23	16.74	16.58	-0.16	12.39	12.18	-0.20

Table 6.3.: Comparison of parameter tuning results for all three heuristics given for training, test as well as training and test set. *Def.* gives the result with default values, *Tuned* after the parameter tuning and Δ the difference of both. All values represent the gap [%] to MIP best bound

default parameters of the RVNS worked poorly on these instances. To summarize the parameter tuning results, the gap to the MIP bound in the test set is reduced by -0.10 up to -3.02 on average. For the remainder of this thesis, the tuned parameters are used for all evaluations.

6.7. Numerical results

This section presents numerical results for the heuristics. If not indicated otherwise, the time limit is set to five minutes per run[4]. Due to the stochasticity of the heuristics, each instance is solved ten times and the solution values are averaged.

In section 6.7.1 the performance of each heuristic is compared to the MIP results. Afterwards, one heuristic for further use in this thesis is selected based on a detailed analysis in Section 6.7.2.

6.7.1. Comparison of exact and heuristic solution approaches

To compare the heuristics with the exact solution approach in Chapter 5, the best found solutions and lower bounds of the MIP with *soft+start* setting for 48 hours are used for evaluation, which provides incumbent solutions for all instances (some containing penalty costs) and constraint violations regarding job assignments and weekend assignments are considered like in the heuristic.

The gap to the best bound of the MIP is calculated by Equation (6.44) in Section 6.6. In contrast to the gap to the best bound, the objective value of the MIP can have worse (larger) values than the heuristic. Let $\overline{x}$ be the objective value of the MIP solution and x the heuristic solution, then the gap to the objective is calculated by Equation (6.45).

$$Gap^{obj} = \begin{cases} 100\frac{x-\overline{x}}{x} & if, x \geq \overline{x} \\ -100\frac{\overline{x}-x}{\overline{x}} & if, x < \overline{x} \end{cases} \tag{6.45}$$

Depending on whether the MIP solution or the heuristic solution is better, the gap is calculated with one of the two formulas resulting in a value in the interval $[-100, 100]$. The resulting value is multiplied by -1, if the heuristic achieved a superior result. Thus, negative values for the gap to the MIP objective in tables and figures in this section indicate the heuristic outperformed the MIP solver.

Table 6.4 shows the results for LNS, ALNS and RVNS, respectively. The detailed results for all instances and heuristics are given in Table C.5 in Appendix C. By looking at the average and median gaps to MIP objective and bound on the entire test

[4]using one 2.6 GHz CPU and 4 GB RAM

Set	**LNS**					**ALNS**					**RVNS**				
	Objective		**Bound**		**Inf.**	**Objective**		**Bound**		**Inf.**	**Objective**		**Bound**		**Inf.**
	Mean	Med.	Mean	Med.		Mean	Med.	Mean	Med.		Mean	Med.	Mean	Med.	
G1	-65.18	-77.75	13.36	10.76	0	-66.49	-78.70	10.46	7.67	1	-62.17	-78.38	16.12	11.17	2
G2	-36.59	-50.67	18.47	11.81	1	-39.28	-53.14	13.25	7.12	1	-36.94	-52.38	14.44	8.56	1
G3	-90.33	-86.66	24.28	12.73	2	-90.33	-87.28	20.22	8.61	3	-90.44	-87.09	20.85	8.88	2
G4	-90.35	-98.41	16.18	15.84	0	-90.79	-98.48	12.31	12.20	0	-90.60	-98.48	13.59	13.60	0
TH1	7.05	8.16	7.97	8.63	0	1.70	1.69	2.68	2.41	0	10.75	2.25	11.69	3.19	2
TH2	41.46	13.81	41.55	13.81	1	34.83	3.73	34.93	4.02	1	35.91	5.40	36.00	5.68	1
TH3	18.85	9.51	30.82	18.42	2	18.80	8.78	30.42	20.02	4	33.93	34.00	44.28	38.28	6
TH4	12.37	12.71	19.58	20.36	0	7.96	5.84	15.63	15.07	1	4.88	5.00	12.72	12.59	0
CS1	3.67	3.42	6.71	6.92	0	1.74	1.69	4.84	4.65	0	0.87	0.79	4.00	3.65	0
CS2	3.56	3.77	16.81	8.19	1	2.26	2.49	15.56	6.72	1	0.94	0.82	14.31	5.37	1
CS3	2.11	2.85	46.84	8.02	3	1.80	2.48	46.54	7.67	3	0.64	0.32	45.44	5.63	3
CS4	2.59	2.60	8.43	8.60	0	2.43	2.48	8.28	8.55	0	0.97	1.08	6.91	7.31	0
all	-26.88	2.88	19.76	10.98	10	-28.51	1.31	16.80	7.14	15	-26.29	0.41	19.12	7.62	18

Table 6.4.: Comparison of the heuristics to MIP bound and objective (Inf. = number of instances with infeasible solutions due to unassigned jobs or single shifts on weekends (causing penalty costs))

set for all three heuristics, the similar outcomes of the heuristics become apparent. The negative gaps to the objective value of the MIP for the instance sets G1 to G4 show that all heuristics are able to improve the solutions in comparison to the MIP solver on the generated instances. However, the gaps to the best bound value are also relatively high for these sets indicating further optimization potential. Taking the instance characteristics into account, the results for the heuristics on the generated set are promising because they outperform the solver using less than 0.2% of the computation time (five minutes versus 48 hours). The large improvement of the objective value is caused by the capability of the heuristics to insert more jobs to the final solution and avoid single shifts on weekends, thus, reducing the penalty costs. There are only three to five instances, depending on the heuristic, in the generated set that have penalty costs at the end of computation (indicated by column *Inf.*). Note that solving the MIP resulted in 36 instances with penalty costs after 48 hours.

For the TH and CS instance sets no improvement of the MIP objective is achieved by the heuristics indicated by the positive gaps to the objective value. However, for the CS instances the distance to the MIP solution is relatively small indicated by gaps of less than 4%, for the RVNS below 1%. The average gap to the best bound for the CS instances ranges up to 47%, but is less in most of the cases as indicated by the median values. The four instances that have remaining penalty costs are the same four instances that have unassigned jobs in the solution of the MIP solver. Thus, it is possible that no full job assignment is possible in those cases, especially because none of the heuristics found a feasible solution for them. The good performance of the heuristics compared to the MIP on the CS instances can be explained by the nature of the instances. Due to the missing time windows for jobs, many solutions are feasible and can be searched quickly by the heuristics.

The worst solutions in comparison to the MIP are found for the TH instances. The gaps to the objective are largest for sets TH2 and TH3 with up to more than 40% but the median values indicating lower values for most of the instances. At the same time the gaps to the bound are also large due to the gaps to the objective value. The TH instances are tight instances that have many jobs in a short time horizon. Therefore, the heuristics have problems of inserting all jobs and, thus, causing penalty costs.

To provide further details on an instance basis, Figures 6.16 to 6.18 show the results of the ALNS for each instance set separately. LNS and RVNS show similar behavior and outcomes and the results are shown in Figures C.1 and C.2 in Appendix C. The two lines show the gap to the best bound and objective of the MIP after 48 hours computation time. Both gap values are shown on the primary y-axis. Constraint violations causing penalty costs in the heuristic solution are depicted for each instance. The values, relating to the secondary y-axis, are either the number of unassigned jobs or the number of weekends with single shifts.

ALNS achieves better results than the MIP for the majority of the generated in-

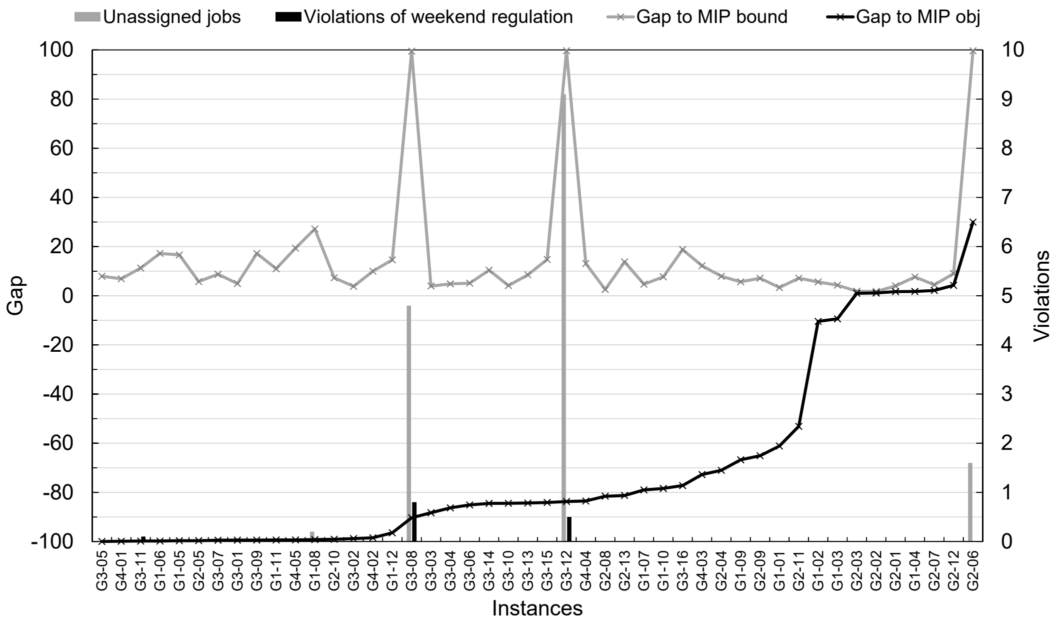


Figure 6.16.: Comparison of ALNS to MIP results for generated instances

stances (Figure 6.16). For 39 of the 46 instances, the heuristic improved the objective value, which is indicated by the negative values of the gap. The gap to the bounds are less than 25% for most of the instances. Only the instances G3-08, G3-12 and G2-06 have a gap to the bound of nearly 100% due to the penalty costs caused by unassigned jobs (G3-12) and single shifts on weekends (all three). Note that G3-08 and G3-12 are also not feasibly solved by the MIP, LNS or RVNS, i.e., all solutions have penalty costs.

For the TH instances (Figure 6.17) the heuristic was not able to improve the MIP results, which can be seen from the positive values for the gap to the MIP objective. The objective gaps is less than 10% for most of the instances. For the three instances on the right (TH03-07, TH3-06 and TH2-01) the gap is relatively high because the heuristic was not able to insert all jobs in contrast to the MIP. The bound gaps are less than 20% for most of the instances. On the CS instances ALNS (Figure 6.18) performs with nearly the same success as the MIP in only less than 0.2% of the computation time. The gap to the MIP objective value is less than 4% for all instances. The gap to the best bound is approximately 100% for the four instances on the left. This value is caused by one unassigned job on average. But as the gap to the objective value is also low for these instances, the MIP also returned a solution with unassigned jobs. For all other instances the gap to the bound is less than 10%.

Note that a computation time of 15 minutes is shortly evaluated for the test set

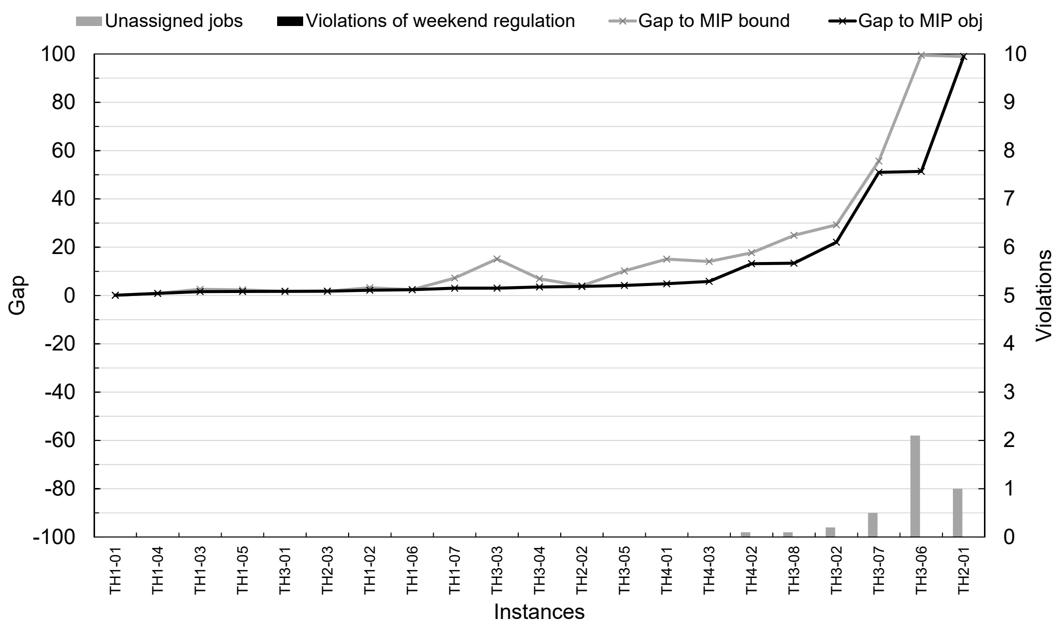


Figure 6.17.: Comparison of ALNS to MIP results for TH instances

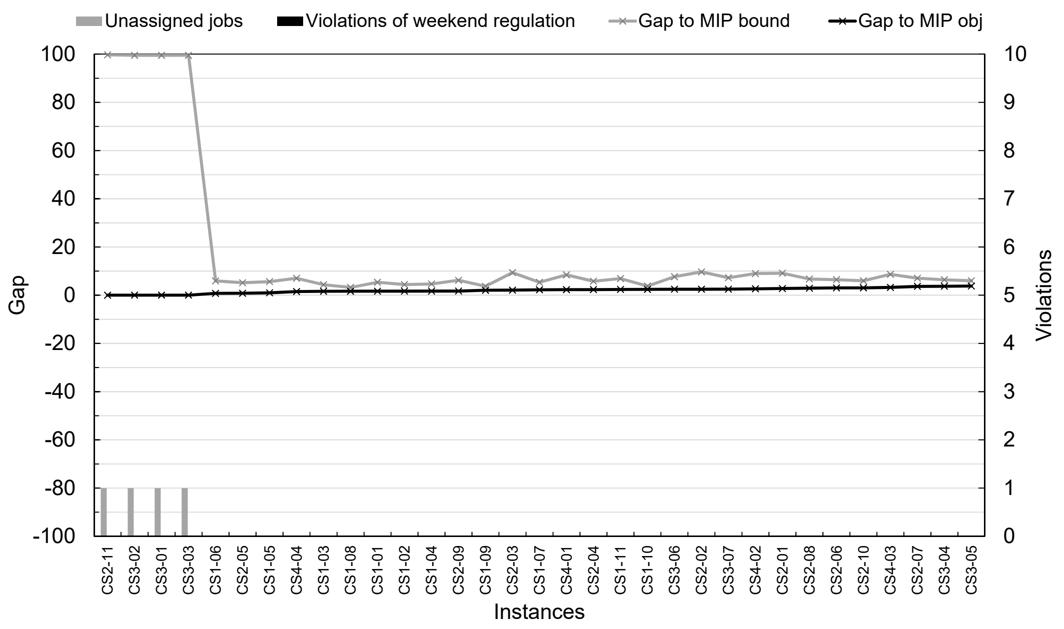


Figure 6.18.: Comparison of ALNS to MIP results for CS instances

instances in Appendix C. Figure C.5 shows that there is no significant improvement for most of the instances by all three heuristics. Only for a few instances the gap could be notably reduced. Based on this observation for the remainder of this thesis a computation time of five minutes is assumed for the heuristics.

6.7.2. Comparison of heuristics

The analyses in the previous sections considered all three heuristics proposed in this thesis. The goal in this section is to determine the “best“ heuristic for the analysis of working regulations in Section 6.8 and as basis for the dynamic setting in Chapter 7. The gap to the MIP bound is used as criterion for the decision because this evaluation criterion is instance and heuristic independent and it allows us to compare the heuristics directly with each other.

Table 6.5 shows the average gaps to the MIP bound for all three heuristics on the entire test set and each instance set separately. Furthermore, the average number of unassigned jobs is given. Note that the unassigned jobs are considered indirectly also in the gaps because they cause penalty costs in the objective function. According to the results in Table 6.5, ALNS achieves the lowest average gap on the instance sets G and TH and RVNS on the CS set. The lowest number of unassigned jobs on all sets is reached by LNS. As the heuristic selected for further consideration should work good on the entire instance set, the overall average gap to the MIP bound is selected as first decision criterion. ALNS with 16.80% has the best result compared to 19.76% of LNS and 19.12% of RVNS and should be selected as preferred heuristic, although the difference is small.

However, because of the small difference and due to the fact that the heuristic with best performance differs by examined instance set, a further analysis of the results is taken into account to determine if ALNS is indeed the best heuristic to consider. Therefore, the difference between the gaps of ALNS and the two other heuristics is calculated for each instance and used as basis. Let x^{ALNS}, x^{LNS} and x^{RVNS} be the

Set	**ALNS**		**LNS**		**RVNS**	
	Gap	$\vert\mathcal{U}\vert$	Gap	$\vert\mathcal{U}\vert$	Gap	$\vert\mathcal{U}\vert$
G	**14.85**	15.70	18.91	12.10	17.02	14.50
TH	**19.70**	4.00	23.13	3.40	27.73	6.10
CS	17.68	4.00	18.80	4.00	**16.58**	4.00
all	**16.80**	23.70	19.76	19.50	19.12	24.60

Table 6.5.: Average gaps to MIP best bound and average unassigned jobs of heuristic results

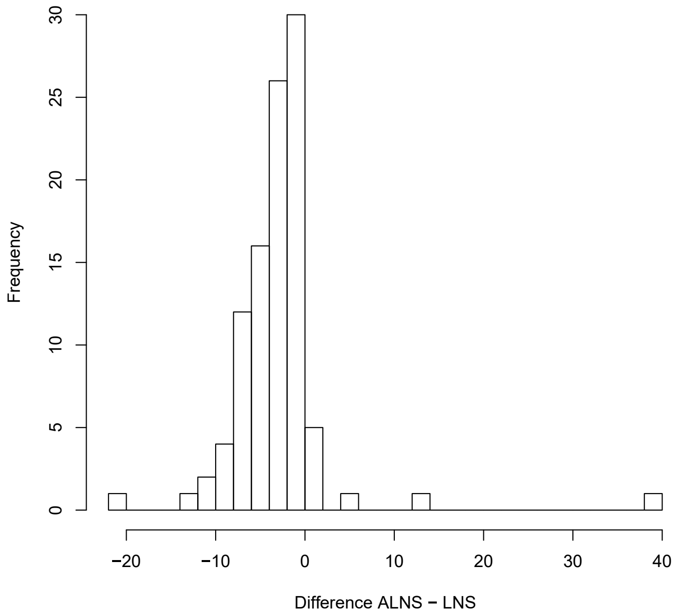


Figure 6.19.: Empirical distribution of the difference between the gaps to the MIP bound of ALNS-LNS

solution values for ALNS, LNS and RVNS, respectively, and $\underline{x}$ the bound of the MIP. Then the differences are calculated by Equations (6.46) to (6.48).

$$\text{(ALNS} - \text{LNS)} := 100\Big(\frac{x^{ALNS} - \underline{x}}{x^{ALNS}} - \frac{x^{LNS} - \underline{x}}{x^{LNS}}\Big) \tag{6.46}$$

$$\text{(ALNS} - \text{RVNS)} := 100\Big(\frac{x^{ALNS} - \underline{x}}{x^{ALNS}} - \frac{x^{RVNS} - \underline{x}}{x^{RVNS}}\Big) \tag{6.47}$$

$$\text{(LNS} - \text{RVNS)} := 100\Big(\frac{x^{LNS} - \underline{x}}{x^{LNS}} - \frac{x^{RVNS} - \underline{x}}{x^{RVNS}}\Big) \tag{6.48}$$

The histograms in figures 6.19 and 6.20 show the empirical distributions of (ALNS-LNS) and (ALNS-RVNS), respectively. Note that the differences of the gaps of different instances are assumed to be independent of each other by looking at the autocorrelation coefficients in Figure C.3 in Appendix C.

Figure 6.19 depicts the empirical distribution of the values (ALNS-LNS). A value below zero indicates that ALNS achieved a better result than LNS and vice versa. The majority of the 100 instances considered as data clearly have a negative difference, although the highest frequency is reached in the interval close to zero. The values in Table 6.6 supports this conclusion because 87% of the instances had a negative and 92% a negative or zero value for the difference (ALNS-LNS). We can conclude that ALNS achieves better results than LNS considering the entire instance set for 87%

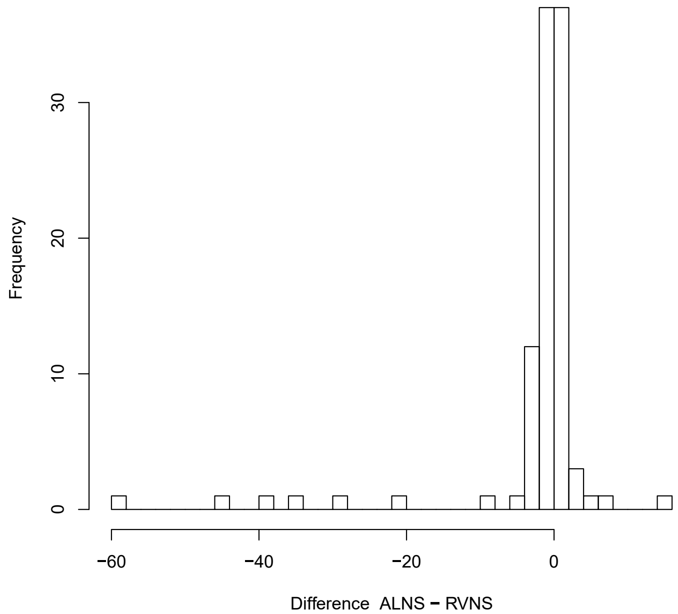


Figure 6.20.: Empirical distribution of the difference between the gaps to the MIP bound of ALNS-RVNS

of the instances.

The empirical distribution of the difference (ALNS-RVNS) is given in Figure 6.20. Although the average gap of RVNS is nearly the same as the one of LNS and we just determined that LNS is outperformed by ALNS in most of the cases, the picture looks different here. The majority of the differences is not as clearly located on the negative side of the x-axis as in Figure 6.19 for LNS. For some instances ALNS achieves clearly better results (values between -60 to -20 on the x-axis) but the highest frequency is achieved equally on both sides of the value zero. Therefore, it cannot be clearly determined which of the heuristics achieves better results. The same can be interpreted by the values in Table 6.6. ALNS determines better solutions only in 52% of the instances reflecting the results in Figure 6.20. It can be concluded, that a decision based on the average gap disguises the good performance of RVNS.

To finally decide which heuristic to use in the remainder of the thesis, the instance

Diff	< 0	≤ 0
ALNS-LNS	87%	92%
ALNS-RVNS	52%	57%

Table 6.6.: Relative frequency of gap differences less than or equal to zero (based on gaps to MIP bound)

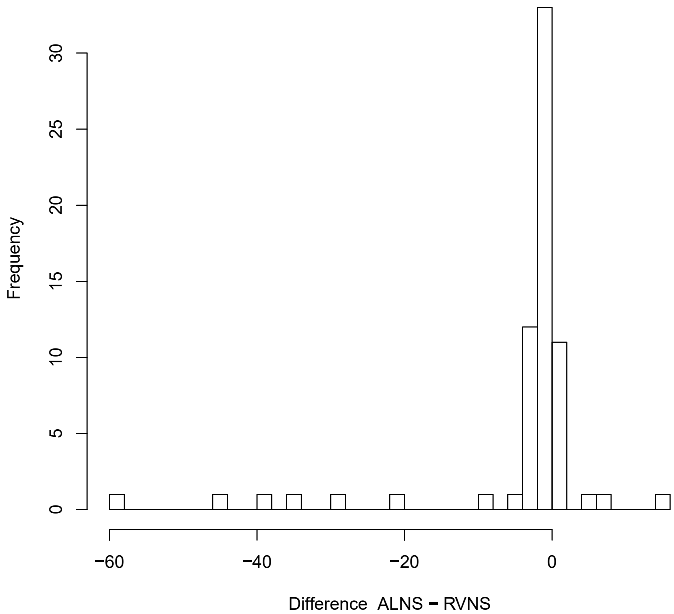


Figure 6.21.: Empirical distribution of the difference between the gaps to the MIP bound of ALNS-RVNS excluding CS instances

properties are taken into account. The average gaps in Table 6.5 indicate that RVNS works particularly good on the CS instances. As described in Section 5.3 the CS instances do not contain time windows. Furthermore, most of the working regulations are not considered in the instances, e.g., weekly rest time, shift rotations and weekend rules. Thus, we can conclude that the instance sets G and TH represent the problem setting in this thesis more accurately. Taking this background into account, Figure 6.21 shows the empirical distribution of (ALNS-RVNS) without CS instances. Now, the entire distribution is shifted slightly towards the negative part of the x-axis. Furthermore, the relative frequency increases to 79.11% of ALNS achieving better results than RVNS.

Thus, ALNS is chosen as heuristic for further consideration in remaining sections. For the completion of the analysis in this section, Figure C.3 and C.4 in Appendix C also show the autocorrelation coefficient and empirical distribution of (LNS-RVNS), respectively. The observation clearly shows RVNS is superior to LNS.

6.8. Impact of working regulations

The impact of introducing working regulations to the problem setting is analyzed in this section relating to goal 3 of the thesis (see Section 4.5). The solutions for a subset of the test instances are investigated in matters of working time changes and

Scenario	Description	Regulations									
		Availability	Daily WT	Breaks	Daily Rest	Weekly Rest	Weekly WT	Weekly WD	Weekend	Consec. Days	Rotations
All	All	x	x	x	x	x	x	x	x	x	x
WR0	None	x	x								
WR1	Legal	x	x	x	x	x					
WR2	Legal+Contracts	x	x	x	x	x	x	x			

Table 6.7.: Scenarios for working regulations sensitivity analysis (x = considered)

constraint violations. CS instances are not presented because they consider only the break rules and daily rest time. Note that the working time is considered for this analysis and not the tour length.

Four different scenarios are defined based on the working regulations considered and shown in Table 6.7. Scenario *All* refers to the basic problem setting in this thesis as described in Section 2.4, i.e., all working regulations are considered. Scenario *WR0* relates to the problem without any of the working regulations. Note that the daily working time and availability of nurses are not excluded, because they are the most common used working regulations in literature. In scenario *WR1* only legally binding constraints are considered, namely breaks, daily and weekly rest time. Scenario *WR2* considers legal working regulations and work contracts (represented by weekly working time and days). Thus, the only constraints left out are the avoidance of single shifts on weekends, consecutive workdays and shift rotations. These three requirements are additional constraints without legal or contractual background.

The ALNS heuristic is executed for each of the four scenarios to get the solutions for evaluating the influences of the respective constraints. The results are averaged over ten runs and the computation time is set to five minutes per run. The following sections investigate each scenario in relation to the basic scenario *All*.

Scenario *WR0* - No working regulations

The results for the first scenario *WR0* with no working regulations are given in Table 6.8. The working time is given in total hours in the entire planning horizon. The constraints violations are given as percentage values to take the instance size into account. The violations of breaks is considered as percentage of tours missing a break. The weekly working time, days and rest time as well as the weekend regulations are considered as percentage of workweeks (nurses multiplied by weeks) with violations.

Instance	Working time [h]			Unassigned		Violations [%]		
	All	WR0	Δ%	All	WR0	Breaks	Daily Rest	Weekly Rest
G1-08	259.04	252.00	-2.72	0.2	0	2.62	0.08	51.11
G2-06	89.95	79.19	-11.97	1.6	0	0.00	0.00	38.33
G2-09	408.48	407.71	-0.19	0	0	3.01	0.16	1.30
G2-11	454.25	454.34	0.02	0	0	1.75	0.43	2.32
G2-13	623.51	627.95	0.71	0	0	9.58	0.24	1.21
G3-02	224.79	223.54	-0.56	0	0	0.25	0.05	11.00
G3-05	344.36	341.36	-0.87	0	0	2.01	0.13	6.18
G3-06	258.82	257.73	-0.42	0	0	1.12	0.33	14.62
G3-09	490.93	487.31	-0.74	0	0	10.76	0.23	3.16
G3-10	588.05	582.37	-0.97	0	0	0.48	0.07	1.38
G4-01	168.70	167.36	-0.80	0	0	3.37	0.45	14.38
G4-04	439.00	432.32	-1.52	0	0	11.55	0.58	1.36
TH1-01	70.76	70.62	-0.19	0	0	15.00	0.00	66.67
TH1-03	119.31	118.67	-0.54	0	0	19.62	0.00	63.33
TH1-07	144.39	136.35	-5.57	0	0	14.52	0.00	100.00
TH2-02	129.17	127.99	-0.91	0	0	12.49	0.00	42.86
TH3-01	185.22	178.96	-3.38	0	0	9.30	0.00	68.89
TH3-06	192.24	178.63	-7.08	2.1	0	9.88	0.00	77.78
TH3-08	217.45	200.11	-7.98	0.1	0	13.32	0.79	58.89
TH4-03	244.45	243.97	-0.19	0	0	24.27	0.00	47.50
Mean	282.64	278.42	-2.29					

Table 6.8.: Average difference in working time and violations of working regulations for scenario WR0 (no working regulations)

The first observation is that the difference in working hours of considering all and no working regulations (Δ) is relatively small. In most cases, the solutions without working regulations have less total working time. This is an expected result because the tours can be built with less restrictions. The working time is reduced by -2.29% in the planning horizon. For the instances G2-11 and G2-13 a small increase in working time is given that can be explained by the replacement of waiting time through breaks or the stochasticity of the heuristic. The greatest reduction in working time are -11.97% (10 hours) for G2-06 and -7.98% (17 hours) for TH3-08. Considering eight hours as a typical working day, this would mean saving one or two workdays, respectively. Compared to the available 42 workdays for G2-06 and 63 workdays in

TH3-08 (9 nurses and planning horizon of 7 days), this reduction is relatively small. In the basic scenario with all working regulations, there are four instances where there are still unassigned jobs in some solutions and for instances G2-06 and TH3-06 in every run. After removing the regulations, the heuristic is able to insert all jobs because the tours are not so restricted anymore and an insertion is easier to achieve.

The average values of constraint violations can be taken into account to explain these observations. The instances with a high reduction in working time have also many workweeks for which the weekly rest time constraint is violated. This indicates that this constraint restricts the heuristics in finding better solutions. The large values for the TH instances are due to the tightness of the instances. There are many jobs for only a few nurses. Thus, ensuring the weekly rest time is hard to achieve and therefore violated often, if not enforced. Furthermore, the average percentage of tours requiring a break is rather high. There are up to 24.27% of tours in instance TH4-03 that miss a break.

In contrast to the violations for break and weekly rest times, the violations of daily rest time are low for all instances. For the generated instances the shift type definitions of morning and afternoon shift indirectly avoid many violations because only tours with a very early start time and a very late end time can violate this constraint. The same holds for the TH instances although there are no shift types. Here the time windows of the jobs lie in the interval $[0, 720]$. Thus, the violation of daily rest time of 720 minutes occurs only, if a job is scheduled at the end of a late time window leading to a tour end after minute 720. Only then the difference to the next day is less than 1440-720=720 minutes.

Note that all solutions presented in Table 6.8 do not comply with the labor law regulations because either the break or the weekly rest time constraints are violated. Thus, even if the consideration of those regulations leads to a small increase in working time hours, the consideration is essential to compute legal schedules.

Scenario *WR1* - Only legal working regulations

The results for scenario *WR1*, i.e, enabling the heuristic to consider only legal working regulations, are given in Table 6.9. The constraints regarding work contracts, weekends, consecutive workdays and rotations are omitted in this scenario. The difference in working time is less than in scenario *WR0* for most of the instance sets. Exceptions are the instances G3-02 and G4-01, where there is a greater reduction than in scenario *WR0*. However, the difference to *WR0* is 0.04 percent and is probably caused by the stochasticity of the heuristic. The number of unassigned jobs can be reduced in this scenario. However, for instance TH3-06 there remain two unassigned jobs, which is two more than in scenario *WR0*. Thus, the insertion of the jobs is limited due to the legal working regulations and not due to the work contracts.

Instance	Working time [h]			Unassigned		Violations [%]		
	All	WR1	Δ%	All	WR1	Weekly WT	Weekly WD	Overtime [h]
G1-08	259.04	252.4	-2.54	0.2	0	5.00	58.33	1.05
G2-06	89.95	79.56	-11.55	1.6	0	25.00	58.33	10.07
G2-09	408.48	408.5	0.01	0	0	3.52	13.15	7.27
G2-11	454.25	455.3	0.23	0	0	14.46	20.89	29.76
G2-13	623.51	628.8	0.84	0	0	8.64	12.12	18.61
G3-02	224.79	223.4	-0.60	0	0	13.33	28.67	13.37
G3-05	344.36	342	-0.68	0	0	5.29	17.94	9.54
G3-06	258.82	257.9	-0.36	0	0	1.54	20.77	0.57
G3-09	490.93	489.9	-0.21	0	0	23.42	25.26	30.43
G3-10	588.05	583.4	-0.79	0	0	7.88	16.75	21.46
G4-01	168.70	167.3	-0.84	0	0	17.50	51.88	10.99
G4-04	439.00	434.1	-1.12	0	0	2.05	7.05	0.80
TH1-01	70.76	70.73	-0.04	0	0	0.00	0.00	0.00
TH1-03	119.31	119.3	-0.02	0	0	0.00	0.00	0.00
TH1-07	144.39	144.4	0.03	0	0	11.67	0.00	2.15
TH2-02	129.17	128.8	-0.26	0	0	0.00	0.00	0.00
TH3-01	185.22	185.1	-0.08	0	0	0.00	0.00	0.00
TH3-06	192.24	190.6	-0.85	2.1	2	2.22	0.00	1.37
TH3-08	217.45	219	0.71	0.1	0	6.67	0.00	1.41
TH4-03	244.45	244.3	-0.07	0	0	8.33	0.00	6.47
Mean	282.64	281.24	-0.91					8.27

Table 6.9.: Average difference in working time and violations of working regulations for scenario WR1 (only legal regulations)

The average reduction of working time is -0.91% compared to -2.29% in scenario *WR0*. This indicates that the legal regulations limit the tour construction more often than the constraints considered in scenario *WR1*, especially for the TH instances where there are many weekly rest time violations in scenario *WR0*. For all generated instances the violations of work contract constraints, namely the weekly working time and particularly the weekly workdays, are high. The solutions of TH instances do not have any violations of the weekly workdays constraints because the value is set to the number of days in the planning horizon and can be never violated. When looking at the mean working time reduction values for the G and TH instances separately the influence of work contract regulations becomes visible. The average working

Instance	Working time [h]			Unassigned		Violations [%]		
	All	WR2	Δ%	All	WR2	Weekend	Consec. Days	Rotations
G1-08	259.04	255.95	-1.19	0.2	0	34.44	0.00	-
G2-06	89.95	87.52	-2.71	1.6	1.6	15.00	0.00	-
G2-09	408.48	407.51	-0.24	0	0	27.41	0.00	-
G2-11	454.25	454.16	-0.02	0	0	29.46	0.00	-
G2-13	623.51	626.94	0.55	0	0	32.12	0.00	-
G3-02	224.79	223.67	-0.50	0	0	30.33	0.00	12.29
G3-05	344.36	341.40	-0.86	0	0	40.59	0.00	13.28
G3-06	258.82	257.71	-0.43	0	0	47.31	0.00	9.56
G3-09	490.93	487.28	-0.74	0	0	26.58	0.00	9.25
G3-10	588.05	582.12	-1.01	0	0	27.88	0.00	20.07
G4-01	168.70	166.99	-1.02	0	0	28.13	0.00	3.93
G4-04	439.00	434.77	-0.96	0	0	33.41	0.00	7.08
Mean	362.49	360.50	-0.76					

Table 6.10.: Average difference in working time and violations of working regulations for scenario WR2 (legal regulations and work contracts)

time reduction for the generated instances is -1.47% whereas the reduction for the TH instances is only -0.07%. Thus, it can be concluded that the work contracts restrict mainly the generated instances, which is also indicated by the high constraint violations. Furthermore, the violations of weekly rest times cause overtime hours for the nurses. The column on the right indicates the sum of overtime hours for all nurses in the instance. There are up to 30 overtime hours in the planning horizon of 14 days for instance G3-09. On average there are eight overtime hours.

The violations of the weekly working time and workdays lead to a lower satisfaction of nurses because some of them work longer than agreed upon. Thus, it is beneficial for the home care provider to include these constraints to increase the satisfaction of nurses. Furthermore, overtime hours can be costly.

Scenario *WR2* - Legal regulations and work contracts

Scenario *WR2* is only analyzed for the generated instances because the TH instances do not contain the constraints regarding weekends, consecutive workdays and shift rotations. The results for the scenario are presented in Table 6.10. Again, the average reduction in working time is less (-0.76%) than in scenario *WR1* (-1.47%). This is the expected result because more constraints are considered in this scenario. On average 1.6 jobs are unassigned in instance G2-06 in contrast to 0.0 in scenario *WR1*, indicating that the insertion is restricted by the consideration of work contracts.

There are no violations of the consecutive workdays requirement due the weekly rest time constraint. As the minimum weekly rest time is 35 hours every week, the limit of 12 consecutive workdays cannot be reached in the presented instances. The avoidance of single shifts on weekends is neglected in many cases (30% of all weekends on average). The heuristic uses the absence of constraints to assign shifts more freely. The same holds for the consideration of shift rotations. The violations are only presented for instances of set G3 and G4, because they consider full-time nurses working according to shift rotations (Table 6.10).

Again the constraint relaxation does not lead to major reductions in the working time. Thus, a home care provider should use them to avoid single shifts on weekends. Furthermore, the consideration of shift rotations enables a provider to model different circumstances that can be considered during optimization. This can be, e.g., the common shift rotation pattern in Germany, as mentioned earlier in the thesis. Furthermore, the patterns facilitates an even distribution of shift types among nurses.

6.9. Summary

In this chapter three heuristics were proposed to solve real world sized instances of the home care routing and scheduling problem. These are based on the metaheuristics LNS, ALNS and RVNS that are adapted with several domain specific neighborhood operators. The parameters for each heuristic were determined by the algorithm configurator GGA to improve the default parameter settings. For all heuristics the parameter configuration led to a slight improvement. Compared to the exact approach presented in Chapter 5 the heuristics showed superior results, because all three of them achieve good solutions in a very small portion of the computation time.

An empirical analysis showed that ALNS and RVNS outperform the LNS heuristic. Although the average results of RVNS were worse than the results of ALNS, a more detailed analysis revealed that this impression is misleading. The empirical distributions of the results indicated no clear difference between ALNS and RVNS. A decision between ALNS and RVNS based on the entire test set could not be clearly made due to advantages on different instance sets. However, ALNS was chosen for further consideration, because it achieves better results on the instances relating better to the problem setting introduced in Section 2.4.

ALNS was used for an analysis of the impact of introducing working regulations to the problem setting. The results showed a slightly negative implication on the working time when respecting working regulations for the instance sets and heuristic in this thesis. However, the consideration of the regulations, especially legal labor law regulations, is essential to comply with the law. Otherwise, many of the regulations will be violated by the computed solutions. Further restrictions from work contracts exclude overtime and, therefore, improve the satisfaction of nurses. Even additional requirements like shift rotations and avoidance of single weekend shifts can be introduced without increasing the working time significantly.

The solution approaches presented so far focus on a static setting. Thus, the planning horizon is considered isolated from previous or future planning horizons and the underlying data does not change over time, i.e., the demands of clients and the entire client set stay the same. In practice, there are frequent alterations by newly admitted clients or due to changes in health condition. To address this issue, the ALNS heuristic is extended to a dynamic setting by using a rolling horizon approach in the next chapter.

7. Heuristic solution approaches for the dynamic setting

The presented solution approaches in the previous chapters consider a static setting, i.e., during the planning there are no changes of demands or in available working hours. However, in practical application those occur on a regular basis, e.g., if a new client requests services. Due to the changes, the solution of the previous period may be infeasible or inefficient for the current period. Therefore, a new schedule has to be determined. As mentioned in the problem description in Section 2.4 the continuity between subsequent periods has to be taken into account to ensure client and nurse satisfaction. Variations between planning periods may lead to dissatisfaction because clients and nurses have to adapt to the new schedule with every period. To approach this issue, this chapter considers the home health care routing and scheduling problem in a dynamic setting.

The ALNS heuristic is used as basis for a solution approach considering a dynamic setting because it is selected as most suitable heuristic by the numerical analysis in Section 6.7.2. The definitions and notation used for the description of the solution approach for the dynamic setting are given in Section 7.1. The adaption is essential for the continuity metrics and modes presented in Section 7.2. To provide an approach for regular weekly planning, the heuristic, continuity metrics and modes are embedded into a rolling horizon setting in Section 7.3.

In Section 7.4 the generation of periodical test instances with a Poisson process is described. Afterwards, the numerical results provide insights in individual aspects of modeling continuity in a dynamic setting. The metrics and rolling horizon approach are investigated for choosing a suitable setting.

7.1. Planning in a dynamic setting

The difference to the planning process in the solution approaches for a static setting is that the methods proposed for a dynamic setting do not necessarily provide a solution for the entire planning horizon. In fact, the planning in a dynamic setting is performed on a regular basis, e.g., every Friday for the next week. In contrast to the static setting, the approaches are able to consider the surrounding weeks. This is essential for ensuring continuity over time. We start by defining relevant terms and sets.

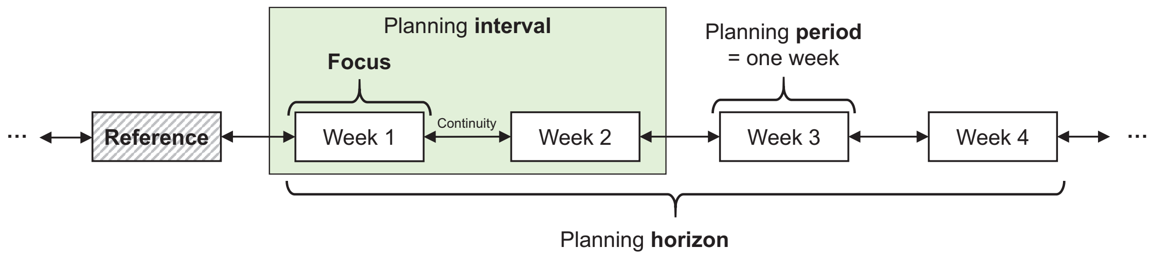


Figure 7.1.: Definitions in a dynamic setting with a weekly planning period

7.1.1. Definitions

The definitions used in the description of the solution approaches in a dynamic setting are given in Figure 7.1. The term *planning horizon* has the same meaning as in the static setting. It is the time interval for which we know or anticipate the jobs that need to be scheduled and the nurses available for assignment. In the dynamic setting, the clients and nurses are not necessarily the same for each week in the planning horizon because the demands and available working hours change over time. The changes, which are considered in this thesis, are the following:

Clients
- New clients request home care services
- Clients do not require home care services anymore

Jobs
- Additional jobs are requested by a client
- A client does not require particular jobs anymore
- The time window of a job changes
- The duration of a job changes

Nurses
- New nurses start employment
- A nurse terminates her or his contract
- The work contract of a nurse is altered

The length of the period that is considered as one indivisible unit in the planning horizon is called *planning period*. For example, it is common in practice to plan the schedule for the entire next week. Then the planning period is one week. Changes for client and nurses, as listed above, only occur between two planning periods. The solution approach is able to solve several planning periods at the same time. The number of planning periods is determined by the planning interval. A *planning interval* has at least the length of a planning period and is at most the entire planning horizon. All periods in the planning interval are considered together in one run of the solution approach. To improve client and nurse satisfaction, the solution approach aims at ensuring the *feasibility* and *continuity* between the periods in the planning interval. How to model the continuity in a dynamic setting is addressed in the next section. The period prior to the planning interval is called *reference* period and important for the continuity. By giving a reference period the past schedule can be taken into account during scheduling the current planning interval. This enables us to avoid an isolated planning where the schedules can differ highly from period to period. This is also reason for having a planning interval that is longer than the planning period. Thus, the demands and capacities of future weeks can already be taken into account for the current period to improve continuity by avoiding solutions that would lead to a poor continuity or *violation of constraints* in the future. The first period in the planning interval, called *focus* period, is the relevant period because this schedule will be implemented in practice next. Furthermore, the schedule for this period will be used as reference in the next planning. The remaining weeks in the planning interval are still allowed to be changed in future planning.

7.1.2. Extension of instance and solution representation

In order to address a multi-period planning horizon, ALNS is adapted by changing the instance and solution representation to explicitly consider nurses, clients and jobs on a weekly basis. All the definitions mentioned in the previous section are modeled in the instance and solution representation by extending the notation from Section 5.1. This is the basis for the definition of the continuity metrics in Section 7.2 and proposing the rolling horizon approach in Section 7.3. An overview of the notation is given in Table 7.1.

The planning interval for the dynamic setting consists of several weeks $\mathcal{W}^* = \{1, \ldots, |\mathcal{W}|\}$. Set $\mathcal{W} \subset \mathcal{W}^*$ contains all weeks of $\mathcal{W}^*$ except the first week and is the basis for continuity calculation. Between the weekly periods changes for the sets of nurses, jobs and clients can occur.

Based on the occurring changes the set of clients $\mathcal{C}_w$ and nurses $\mathcal{N}_w$ are determined to specify the valid weeks for a client c or nurse n, respectively. For example, a client c' requiring services starting from the second week in a four week planning interval is valid only in weeks three and four, i.e., $c' \in \mathcal{C}_3$ and $c' \in \mathcal{C}_4$, whereas $\mathcal{W} = \{2, 3, 4\}$ and $\mathcal{W}^* = \{1, 2, 3, 4\}$. The same example is valid for a nurse n' beginning employment in the second week of the planning interval, i.e., $n' \in \mathcal{N}_3$ and $n' \in \mathcal{N}_4$. For continuity calculation the client nurse assignments are determined on a weekly basis. The set of nurses assigned to a client c in week w in the current solution is denoted by $\mathcal{N}_{c,w}$.

The determination of valid weeks for jobs and which to consider together for continuity calculations is slightly more complex because there can be several jobs per client per day. Therefore, the concept of *job groups* is introduced. All jobs in a job

	Name		Definition
Instance	$\mathcal{W}^*$	$\{1,\ldots,\lvert\mathcal{W}\rvert\}$	Set of all weeks in the planning interval
	$\mathcal{W}$	$\{2,\ldots,\lvert\mathcal{W}\rvert\}$	Set of weeks in the planning interval, excluding first week
	$\mathcal{N}_w$	$\subseteq \mathcal{N}$	Set of valid nurses n for week w, excluding first week
	$\mathcal{C}_w$	$\subseteq \mathcal{C}$	Set of valid clients c for week w, excluding first week
	$\mathcal{J}_w$	$\subseteq \mathcal{C}$	Set of jobs j in week w
	$\mathcal{G}$	$\subseteq \mathfrak{P}(\mathcal{J})$	Set of job groups
Solution	$\mathcal{W}_g$	$\subseteq \mathcal{W}$	Set of valid weeks for job group g, excluding first week and weeks with unassigned jobs
	$\mathcal{G}_w$	$\subseteq \mathcal{G}$	Set of valid job groups in week w, excluding job groups that have an unassigned job in week $w-1$
	$\mathcal{N}_{c,w}$	$\subseteq \mathcal{N}$	Nurses assigned to client c in week w
	$n_{g,w}$	$\in \mathcal{N}$	Nurse assigned to job representing job group g in week w
	$z_{g,w}$	$\in \mathbb{R}^+$	Start time of job representing job group g in week w

Table 7.1.: Sets and attributes of the instances and solutions in a dynamic setting

group are assumed to be recurring jobs of the same client, e.g., a bi-weekly service on Mondays and Thursdays morning that repeats every week would create two job groups, one on Monday and one on Thursday. These jobs should preferably start at approximately the same time and be handled by the same nurse to improve client satisfaction.

Thus, a job group g bundles all jobs that are considered jointly for continuity calculation. A job group $g \in \mathcal{G}$ contains jobs that belong to the same client c on the same weekday wd in different weeks and have overlapping time windows $[a, b]$. More formally, the set of job groups $\mathcal{G}$ contains all subsets of jobs $\mathfrak{p}$ of the power set $\mathfrak{P}(\mathcal{J})\backslash\{\emptyset\}$ fulfilling the following criteria:

$$\forall i, j \in \mathfrak{p} : c_i = c_j \wedge w_i \neq w_j \wedge a_i < b_j \wedge a_j < b_i \wedge wd_i = wd_j \tag{7.1}$$

Figure 7.2 shows an example for job groups of one client where only the days Monday and Thursday are taken into account for a three week planning interval. Jobs 1, 2 and 3 build one job group because they are all on Monday with the same time window. Jobs 4, 5 and 6 on Monday afternoon are also in one job-group, although job 6 has a different time window. However, the time window of job 6 is overlapping with the time windows of job 4 and 5 and, therefore, the same start time is possible for them. The jobs 7, 8 and 9 are divided into two job groups, although they are all on Thursday morning. Jobs 7 and 8 form a job group due to the same time window and job 9 is in another job group because the time window is not compatible with job 7 and 8.

A job j in a job group g is well-defined by the week w because every job group contains only one job per week. A nurse assigned to a job in job group g in week

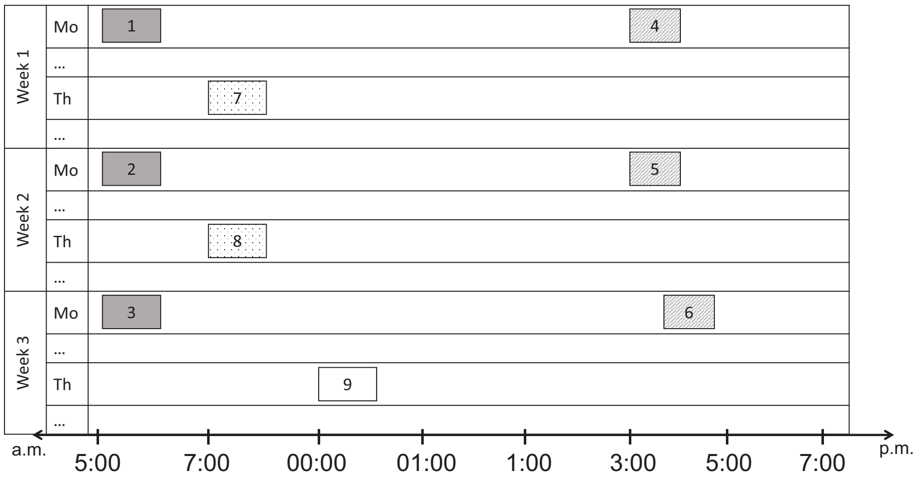


Figure 7.2.: Example for job groups resulting from nine jobs for one client on Mondays and Thursdays in a three week planning interval (the bars represent the time window of the job)

w is denoted by $n_{g,w}$ and the start time of the job is given by $z_{g,w}$. Unassigned jobs are excluded from continuity calculations because they neither have a start time nor an assigned nurse. Set $\mathcal{W}_g$ contains the weeks where there are jobs of job group g assigned in the current w and previous week $w-1$ and $\mathcal{G}_w$ the job groups valid in week w.

7.2. Modeling continuity in a dynamic setting

The introduced weekly solution representation is used for the definition of continuity metrics (Section 7.2.1) that are combined to a single objective function (Section 7.2.2). Furthermore, the focus area for continuity calculation has to be determined, i.e., which week is compared to which other week. Different modes are possible and presented in Section 7.2.3.

7.2.1. Continuity metrics

There are three types of continuity considered in this thesis: continuity of time, continuity of care and continuity of duty schedules. For each of them different metrics are defined and presented in this section. The metrics can be used as part of the objective function to improve the continuity in planning horizon with multiple weeks.

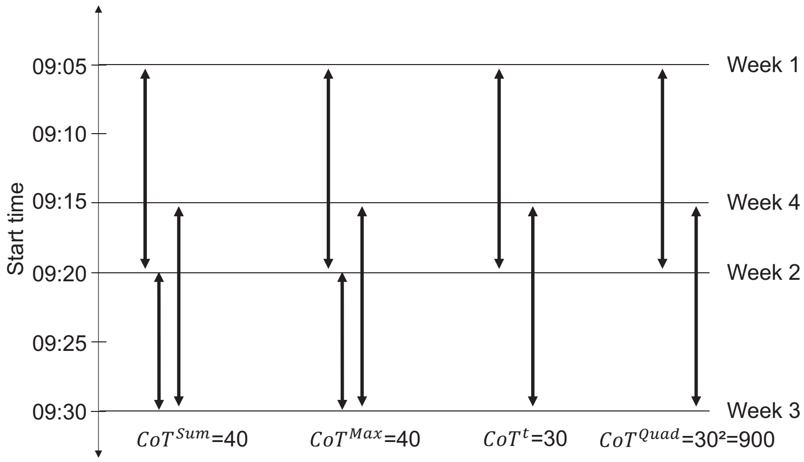


Figure 7.3.: Example for continuity of time metrics for one job group. Arrows indicate the deviations taken into account for calculation ($\theta = 10$ for CoT^t and CoT^{Quad})

Continuity of time

The continuity of time metrics aim at avoiding start time fluctuations of jobs between weeks of the planning interval. The satisfaction of clients is improved when continuity of time is ensured because the clients do not have to adapt to new appointments every week and can keep their daily routine. The start times are defined for each assigned job. Therefore, the metrics presented in this section are based on the set of job groups $\mathcal{G}$. An example for all metrics is visualized for one job group in Figure 7.3.

The first possibility to model continuity of time is to minimize the *sum* of all deviations in start times for consecutive weeks, like in Nickel et al. [2012]. As they consider only two periods the definition of job groups is not necessary. However, the planning interval in this thesis contains several weeks and the definition in Equation (7.2) uses the job groups and weeks as basis. The sum considers all deviations between consecutive weeks for all job groups generated from the set of jobs.

$$CoT^{Sum} = \sum_{w \in \mathcal{W}} \sum_{g \in \mathcal{G}_w} |z_{g,w} - z_{g,w-1}| \tag{7.2}$$

The sum of deviations does not specifically consider the maximum deviation. Thus, it is possible that there are many small deviations and less large deviations leading to the same sum as only medium deviations. To address this issue, the second metric minimizes the weekly *maximum* deviation of start times of all job groups (7.3).

$$CoT^{Max} = \sum_{w \in \mathcal{W}} \max_{g \in \mathcal{G}_w} \left\{ |z_{g,w} - z_{g,w-1}| \right\} \tag{7.3}$$

In practice, small deviations of start times might be tolerated by clients because they can occur due to several external factors, e.g., uncertainty in services times. Therefore, the third metric for continuity of time considers only the sum of deviations *that are larger than the threshold* of θ minutes (7.5). Values less than the threshold are assumed to be zero (7.4).

$$\sigma_{g,w} = \begin{cases} 0, & \text{if } |z_{g,w} - z_{g,w-1}| \leq \theta \\ |z_{g,w} - z_{g,w-1}|, & \text{if } |z_{g,w} - z_{g,w-1}| > \theta. \end{cases} \tag{7.4}$$

$$CoT^t = \sum_{w \in \mathcal{W}} \sum_{g \in \mathcal{G}_w} \sigma_{g,w} \tag{7.5}$$

The last continuity of time metric implicitly combines the second and third metric by minimizing the *quadratic sum* of all deviations above the threshold θ (7.6). By minimizing the quadratic sum large deviations are weighted more than small deviations.

$$CoT^{Quad} = \sum_{w \in \mathcal{W}} \sum_{g \in \mathcal{G}_w} \sigma_{g,w}^2 \tag{7.6}$$

Note that the quadratic sum is possible only because a heuristic solution approach is used. In an exact approach, the quadratic objective function would lead to a non-linear model causing a higher complexity to solve it.

Continuity of care

The continuity of care criterion has the goal of keeping the number of nurse changes for one client small during the planning horizon. This type also increases the client satisfaction because the clients get familiar with the nurses and do not prefer changing visitors. There are two different continuity of care metrics presented in this section.

The first metric minimizes the number of new nurses assigned to client on a weekly basis (similar to Wirnitzer et al. [2016] for a one period). Equation (7.7) calculates the difference of nurses assigned to a client in the current week to the previous week. The number of nurses that were not assigned before are considered in the sum.

$$CoC^{Client} = \sum_{w \in \mathcal{W}} \sum_{c \in \mathcal{C}_w} |\{\mathcal{N}_{c,w}\} \backslash \{\mathcal{N}_{c,w-1}\}| \tag{7.7}$$

In contrast to calculation on a nurse level, the changes in nurse client assignments can be determined on a job level. This introduces even more familiarity, because it aims at assigning the same nurse to one job group. The metric in Equation (7.9)

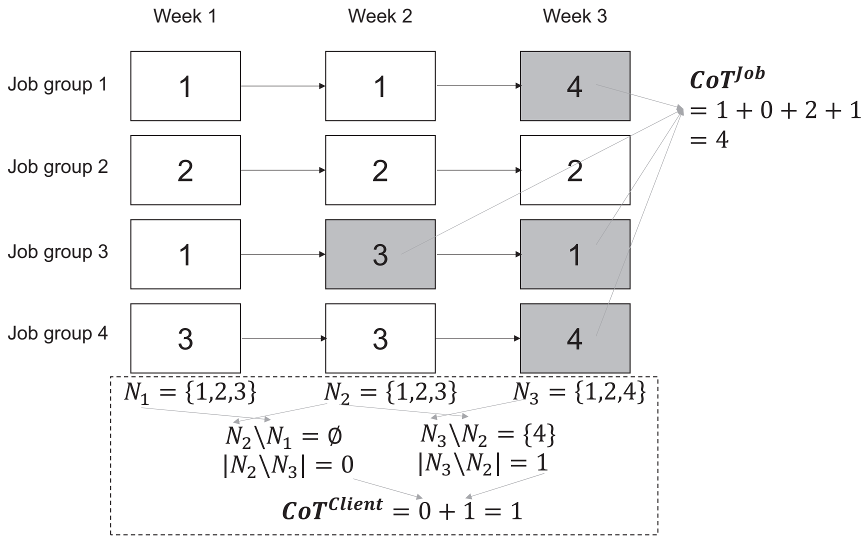


Figure 7.4.: Example for continuity of care metrics for four job groups of one client. The numbers indicate the nurse assigned to the job (marked jobs have a different nurse assigned than in the previous week)

sums up all differences in nurse assignments in consecutive weeks based on the job groups, where (7.8) determines if the nurses in two consecutive weeks are the same.

$$\gamma_{g,w} = \begin{cases} 1, & \text{if } n_{g,w} \neq n_{g,w-1} \\ 0, & \text{otherwise} \end{cases} \tag{7.8}$$

$$CoC^{Job} = \sum_{w \in \mathcal{W}} \sum_{g \in \mathcal{G}_w} \gamma_{g,w} \tag{7.9}$$

Carello and Lanzarone [2014] also use the sum of of reassignments but on a client level. They solve the patient assignment problem; therefore, no routing decisions are included to their problem.

An example for the calculation of the two metrics is given in Figure 7.4. There are four job groups of one client with each having exactly one job per week. The jobs contained in one job group are connected with an arrow. The CoC^{Client} is given below the job groups and only takes the set of nurses for each week as basis for calculation. In contrast to this, CoC^{Job} looks specifically at each job group separately. All jobs that have another nurse assigned than the job in the previous week are marked gray.

Continuity of duty schedules

A new metric introduced to the routing and scheduling of home care providers in this thesis is the continuity of duty schedules to achieve similar shift assignments for

Day	1	2	3	4	5	6	7	
Week 1	1	1	2	1	1	-	-	
Week 2	2	1	1	1	1	-	-	
Week 3	2	1	2	2	1	-	-	
Week 4	2	1	1	1	1	1	1	
CoD^{Type}=	1 +	0 +	3 +	2 +	0 +	1 +	1 +	= 8

Figure 7.5.: Example for continuity of duty schedules metric CoT^{Type} for one nurse in a four week planning interval. The numbers in the cells indicate the shift type id (marked cells show a shift type change compared to the previous week)

nurses each week. This can improve the nurse satisfaction because they can plan their everyday life more easily as they know their usual work shifts. Note that the continuity of duty schedules is only calculated based on nurses that do not work according to shift rotations ($\mathcal{N}\backslash\{\mathcal{N}^{\mathrm{R}}\}$) because shift rotations have changing shift types by definition.

The similarity of shift assignments is achieved on a shift type level or based on the start and end times of tours. The first metric minimizes the sum of shift type changes (7.11) for consecutive weeks (7.10) where $s_{n,d}$ indicates the shift type assigned to nurse n on day d. An example for the calculation is visualized in Figure 7.5.

$$\psi_{n,d} = \begin{cases} 1, & \text{if } s_{n,d} \neq s_{n,d-7} \\ 0, & \text{otherwise} \end{cases} \tag{7.10}$$

$$CoD^{Type} = \sum_{w\in\mathcal{W}} \sum_{n\in\mathcal{N}_w\backslash\{\mathcal{N}^{\mathrm{R}}\}} \sum_{d\in w} \psi_{n,d} \tag{7.11}$$

This metric is reasonable if different shift types are defined per day. In case of only one shift type, determining the differences is not effective. Therefore, a second metric based on the tour start and end times is given in Equation (7.12). The calculation is similar to CoT^{Sum} for continuity of time but here the time deviations in start and end times of tours are calculated instead of start times of jobs in job groups.

$$CoD^{Time} = \sum_{w\in\mathcal{W}} \sum_{n\in\mathcal{N}_w\backslash\{\mathcal{N}^{\mathrm{R}}\}} \sum_{d\in w} \left(|z_{E(n,d)} - z_{E(n,d-7)}| + |z_{O(n,d)} - z_{O(n,d-7)}|\right) \tag{7.12}$$

By measuring the time deviations in start and end times for weekdays in consecutive weeks, the similarity of working times can be improved. Furthermore, even small time deviations in the same shift type are taken into account which is not possible with the first metric.

7.2.2. Overall continuity measurement and trade off

Each of the presented metrics addresses only one particular continuity type. However, usually the goal is to achieve continuity in all three domains. Therefore, the metrics must be combined to a single objective function.

In a weighted sum approach, the desired continuity metrics are combined to a single linear function. Depending on the priorities the components are weighted. Let $\omega^{CoT}, \omega^{CoC}$ and ω^{CoD} be the weights from the interval $[0, 1]$ for each type of continuity whereas the sum of weights equals one ($\omega^{CoT} + \omega^{CoC} + \omega^{CoD} = 1$). A smaller weight indicates a lower priority and vice versa. The weighted sum is given in Equation (7.13) for all three types of continuity without referring to a particular metric.

$$Continuity = \omega^{CoT} CoT + \omega^{CoC} CoC + \omega^{CoD} CoD \tag{7.13}$$

The problem with a weighted sum approach is that if the metrics have different domains, the values need to be normalized to have a consistent domain and weighting. To achieve the normalization for each metric, a normalization factor is calculated based on the worst case value dependent on the instance. The actual metric values are divided by this worst case values on a weekly basis limiting the overall sum to the interval $[0, 1]$ (7.14). The weekly continuity metrics are taken from the sums in the definitions in the previous section.

$$Continuity^{Norm} = \omega^{CoT} \sum_{w \in W} \frac{CoT_w}{CoT_w^{Norm}} + \omega^{CoC} \sum_{w \in W} \frac{CoC_w}{CoC_w^{Norm}} + \omega^{CoD} \sum_{w \in W} \frac{CoD_w}{CoD_w^{Norm}} \tag{7.14}$$

Besides the continuity, the tour length is still an objective to be minimized during optimization. Therefore, the final objective function considered in the heuristic for a dynamic setting integrates the tour length with the combined continuity measure (7.15). The weight for the tour length is denoted by ω^{TL}.

$$min\ \omega^{TL} \frac{Tour\ length}{TL^{Norm}} + Continuity^{Norm} \tag{7.15}$$

The calculation of the worst case values for each metric is described in Appendix D. In short, the maximum deviations per week for each metric are determined and summed up. The worst case for the tour length is approximated by multiplying the start tour length of the construction heuristic by 1.5.

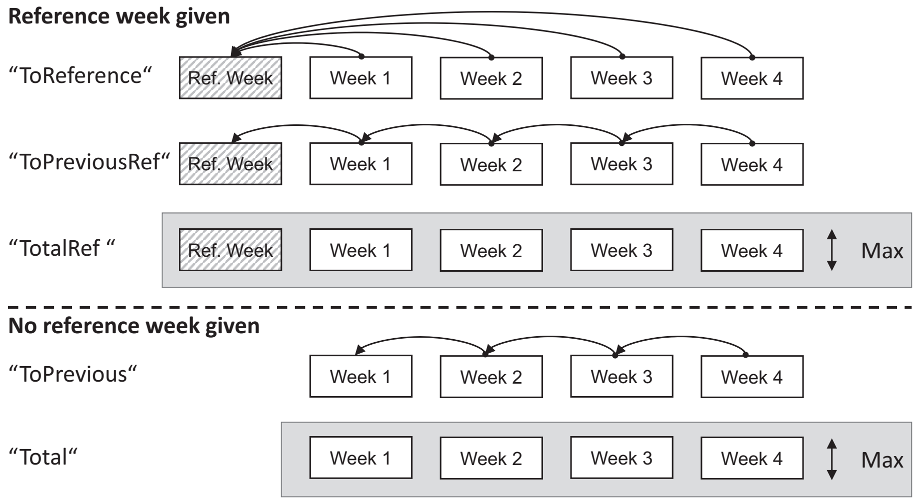


Figure 7.6.: Modes for continuity calculation for a planning interval of four weeks

7.2.3. Modes for continuity calculation

After the definition of continuity types and the corresponding metrics, the relation of the weeks for continuity calculation needs to be determined. There are two cases: continuity calculation with regard to a reference week or without a reference week. The former is a use case for application in practice where there exists a previous schedule that should be altered due to changes of the demand. The latter can be used to achieve continuity in an isolated planning interval, e.g., when the scheduling is started from scratch. A *reference week* is a solution that is determined for a previous week and the continuity of the current planning period to this week should be ensured during the execution. When a reference week is considered it is added as first element to the set $\mathcal{W}$ for continuity metric calculation. The heuristic performs change operations by the neighborhood operators only on the weeks of the current planning interval and leaves tours of the reference weeks untouched. The tours and assignments of the reference week are fixed and their solution values are only used for the calculation of continuity metrics and ensuring the feasibility across periods.

The different settings for the selection of weeks investigated in this thesis are called *continuity modes* and an overview is given in Figure 7.6. In the continuity mode *ToReference* the deviations of times and assignments of all weeks are calculated in comparison to the reference week. In *ToPreviousRef* only the continuity of the first week is considered based on the reference week. All other weeks in the planning interval are aligned to their respective previous week. The same is done in the mode *ToPrevious* but without reference week. Another setting is the continuity calculation based on the entire planning interval. In *TotalRef* and *Total* only the largest deviation per job group, client or nurse is considered in the continuity metric. In *TotalRef* the solution values of the reference week are included for determining the largest deviation. The definitions of the metrics in Section 7.2.1 are valid for the modes *ToPreviousRef* and *ToPrevious*. For the former case, the reference week is added as first element to the set of weeks for continuity calculation. For the other three continuity modes the formulas have to be changed slightly. The changes are discussed for CoT^{Sum}, CoC^{Client} and CoD^{Time} in this section. The formulas for the other metrics are presented in Appendix D.

The original metric for CoT^{Sum} is given in Equation (7.16), i.e., the deviations are calculated between consecutive weeks w and $w-1$. Equation (7.17) takes the deviations of all weeks to the reference week $w=1$ into account to represent the mode *ToReference*. The largest deviations in a job group are the basis for the modes *TotalRef* and *Total* and given in Equation (7.18) by determining the minimum and maximum start time in each job group.

$$\textit{ToPrevious/ToPrevRef:} \quad CoT^{Sum} = \sum_{w \in \mathcal{W}} \sum_{g \in \mathcal{G}_w} |z_{g,w} - z_{g,w-1}| \tag{7.16}$$

$$\textit{ToReference:} \quad CoT^{Sum} = \sum_{w \in \mathcal{W}} \sum_{g \in \mathcal{G}_w} |z_{g,w} - z_{g,1}| \tag{7.17}$$

$$\textit{Total/TotalRef:} \quad CoT^{Sum} = \sum_{g \in \mathcal{G}} \big(\max_{w \in \mathcal{W}_g} \{z_{g,w}\} - \min_{w \in \mathcal{W}_g} \{z_{g,w}\} \big) \tag{7.18}$$

The calculation of CoC^{Client} based on consecutive weeks for *ToPrevious* and *ToPreviousRef* is given in Equation (7.19). When only the deviations to the reference week should be considered (*ToReference*), the changed formula in Equation (7.20) can be used. It sums up the number of new nurses assigned each week which are not known from the reference week. The metric for *TotalRef* is given in Equation (7.21) and similar. Finally, Equation (7.22) minimizes the number of assigned nurses in the entire planning interval without considering a reference solution (*Total*).

$$\textit{ToPrevious/ToPrevRef:} \quad CoC^{Client} = \sum_{w \in \mathcal{W}} \sum_{c \in \mathcal{C}_w} |\{\mathcal{N}_{c,w}\} \setminus \{\mathcal{N}_{c,w-1}\}| \tag{7.19}$$

$$\textit{ToReference:} \quad CoC^{Client} = \sum_{w \in \mathcal{W}} \sum_{c \in \mathcal{C}_w} |\{\mathcal{N}_{c,w}\} \setminus \{\mathcal{N}_{c,1}\}| \tag{7.20}$$

$$\textit{TotalRef:} \quad CoC^{Client} = \sum_{c \in \mathcal{C}} \Big| \bigcup_{w \in \mathcal{W}} (\mathcal{N}_{c,w} \setminus \{\mathcal{N}_{c,1}\}) \Big| \tag{7.21}$$

$$\textit{Total:} \quad CoC^{Client} = \sum_{c \in \mathcal{C}} \Big| \bigcup_{w \in \mathcal{W}} \mathcal{N}_{c,w} \Big| \tag{7.22}$$

The metric CoD^{Time} for the modes *ToPrevious* and *ToPreviousRef* for calculating differences between consecutive weeks is given Equation (7.23). If only the deviation to the reference week (*ToReference*) should be taken into account, Equation (7.24) is used. The day index for the start and end times in the reference weeks is calculated by subtracting $7w$, i.e., seven days for every week lying between the current day d and the day in the reference week. The last metric for the modes *Total* and *TotalRef* determines the maximum and minimum start and end times of the shifts for every weekday $d \in \{1, \ldots, 7\}$. The day in the planning horizon is determined by $d + 7w$.

ToPrevious/ToPreviousRef:

$$CoD^{Time} = \sum_{w \in \mathcal{W}} \sum_{n \in \mathcal{N}_w \setminus \{\mathcal{N}^{\mathrm{R}}\}} \sum_{d \in w} \left(|z_{E(n,d)} - z_{E(n,d-7)}| + |z_{O(n,d)} - z_{O(n,d-7)}| \right) \tag{7.23}$$

ToReference:

$$CoD^{Time} = \sum_{w \in \mathcal{W}} \sum_{n \in \mathcal{N}_w \setminus \{\mathcal{N}^{\mathrm{R}}\}} \sum_{d \in w} \left(|z_{E(n,d)} - z_{E(n,d-7w)}| + |z_{O(n,d)} - z_{O(n,d-7w)}| \right) \tag{7.24}$$

Total/TotalRef:

$$\begin{aligned} CoD^{Time} = & \sum_{n \in \mathcal{N} \setminus \{\mathcal{N}^{\mathrm{R}}\}} \sum_{d \in \{1 \ldots 7\}} \left(\max_{w \in \mathcal{W}_n} \{z_{E(n,d+7w)}\} - \min_{w \in \mathcal{W}_n} \{z_{E(n,d+7w)}\} \right) \\ & + \sum_{n \in \mathcal{N} \setminus \{\mathcal{N}^{\mathrm{R}}\}} \sum_{d \in \{1 \ldots 7\}} \left(\max_{w \in \mathcal{W}_n} \{z_{O(n,d+7w)}\} - \min_{w \in \mathcal{W}_n} \{z_{O(n,d+7w)}\} \right) \end{aligned} \tag{7.25}$$

7.3. Heuristic rolling horizon approach

The continuity metrics and modes are the basis for a rolling horizon approach that can be used for application in practice when a weekly planning is performed. In this case, a previous schedule exists and the continuity between this and the new schedule must be ensured even if the demand of clients and the requirements of nurses changed.

To cover this scenario, the ALNS heuristic is embedded in a rolling horizon approach. The instance and solution representation from Section 7.1.2 are used during the execution of the heuristic. Furthermore, the objective function is a weighted sum of the tour length and the combined continuity metric as presented in Section 7.2.2.

In the proposed rolling horizon approach, the planning is performed for several planning intervals that are shifted forward week-by-week through the entire planning horizon. Only the weeks currently in the planning interval are considered by the heuristic. The first of the planning interval is the week currently in focus, i.e., with

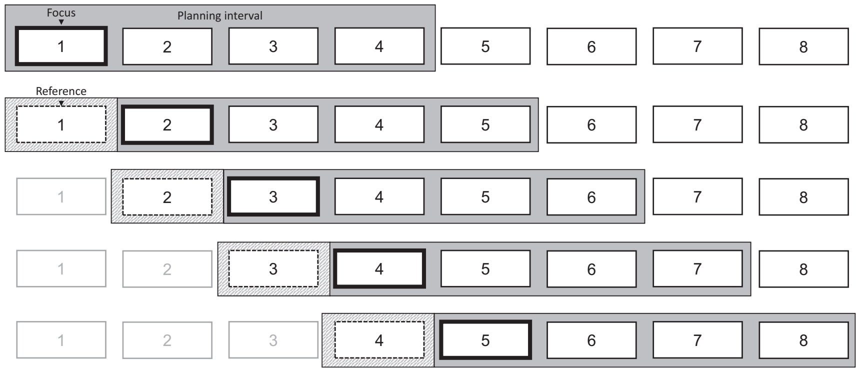


Figure 7.7.: Planning interval, reference and focus week for a planning horizon of eight weeks. The length of the planning interval is set to four weeks ($W^I = 4$)

this iteration of the rolling horizon approach the schedule for this week has to be fixed. The scheduling for the additional weeks can be revised when they are focused in later iterations of the rolling horizon approach. If a reference week is present, it is always the week prior to the focus week. The moving of a planning interval of four weeks through a planning horizon of eight weeks is given in Figure 7.7. In this example, the schedules for weeks 1 to 5 are determined.

Figure 7.8 shows the steps to perform a rolling horizon optimization. The ALNS heuristic is embedded to provide a schedule for the current planning interval of length W^I. When no reference week is given the planning interval spans just the first W^I weeks, e.g., when the optimization is started from scratch. After the planning interval is moved to the next week, the previous focus week is considered as reference week and the planning interval spans the next W^I weeks. The solution from the previous planning interval works as initial solution to start the ALNS heuristic. The initial solution is not complete, because every time the planning interval is forwarded by one week, the jobs and nurses in the last week are not assigned because they were not considered before. Therefore, one repair step of the ALNS heuristic is performed before the actual heuristic solution approach is started. The call of the repair operator considers the nurses and jobs in the last week of the planning interval as unassigned and reinserts them into the initial solution to start the ALNS for the next interval.

When the planning interval spans more than one week, several weeks that are currently not in focus, are considered for continuity calculation. The weeks farther in the future are not as important in the current focus week as the direct subsequent week. Therefore, a time-dependent weighting of weeks is introduced and investigated. The continuity weight between weeks is adjusted based on the point in time, where earlier weeks are weighted higher than later weeks. The time-dependent weights f_w

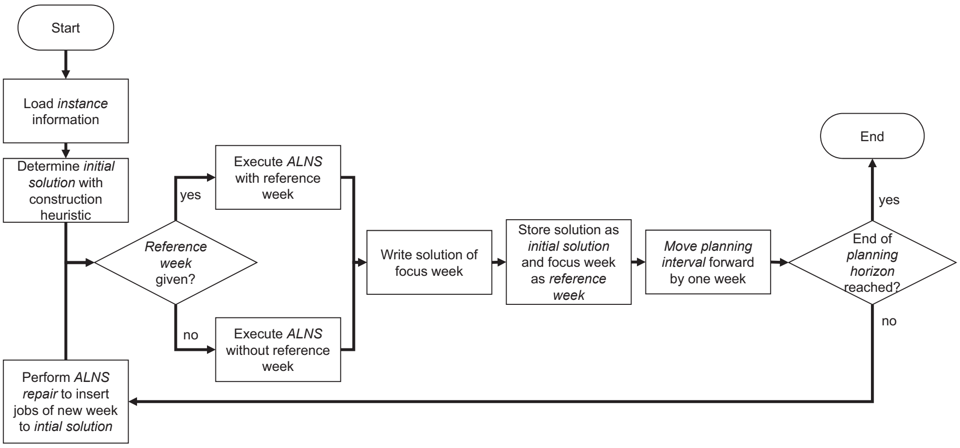


Figure 7.8.: Steps of the rolling horizon approach with embedded ALNS

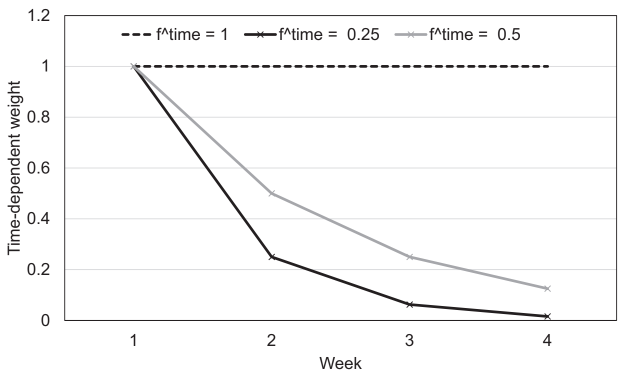


Figure 7.9.: Time-dependent weight for time factor 1.0, 0.5 and 0.25

for each week w in the planning interval are defined in Equation (7.26) based on the given time-factor f^{time}, where $f^{time} \in [0, 1]$. The adjusted continuity metric is given in Equation (7.27). *Metric* is a place holder for any of the presented continuity metrics and $Metric_w$ is the value of the metric in week w.

$$\forall w = 1, .., W^I : f_w = \frac{f'_w}{\sum_{w' \in \{1,...,W^I\}} f'_w} \text{ with } f'_w = \begin{cases} 1.0 & , w = 1 \\ f'_{w-1} f^{time} & , w > 1 \end{cases} \tag{7.26}$$

$$Metric = \sum_{w=\{1,...,W^I\}} f_w Metric_w \tag{7.27}$$

The time-factors used in the analysis of this thesis are 1.0, 0.5 and 0.25, and their resulting time-dependent weights are shown in Figure 7.9. Note that a time factor of 1.0 equals no time-dependent weighting.

7.4. Analysis of results

This section analyzes the proposed continuity metrics, modes and the rolling approach. First the test instances used for evaluation are described in Section 7.4.1. Afterwards the individual metrics for each continuity type are analyzed for their influence on the continuity and their interdependencies (Section 7.4.2). The best continuity mode for a rolling horizon settings is selected in Section 7.4.3. Based on these results, the rolling horizon is investigated in Section 7.4.4.

All results in this section are computed on Xeon E5 processors with one 2.6 GHz CPU using 4 GB RAM[1]. If not indicated otherwise, the time limit is set to five minutes per run. Due to the stochasticity of the heuristics, each instance is solved ten times and the solution values are averaged.

7.4.1. Test instances for a dynamic setting

The test instances used for the analysis of continuity metrics and the rolling horizon approach need to have changes over the weeks to simulate a dynamic setting. Therefore, the instances used for the analysis of working regulations in Section 6.8 are reused as basis instances. The changes over time are simulated with a Poisson process for each type of change like in Bowers et al. [2014].

The following short description of a Poisson process is based on DeGroot and Schervish [2002, p.255-262, p.300-301]. Poisson processes are often used to model the arrival of particular events over time, e.g., the arrival of customers or phone calls. The underlying distribution of arrivals in a fixed time period of length t is the Poisson distribution with an arrival rate λ per time unit. For a Poisson process the average number of arrivals in time interval t is λt. Furthermore, the number of arrivals in two time intervals are independent of each other. The time that elapses between arrivals in a Poisson process is exponential distributed with parameter λ.

In this thesis, the exponential distribution is used to determine the time between two requests. For each type of change an own arrival rate parameter λ is used. The exponential distribution is approximated using uniform distributed random numbers and the logarithm method proposed by Knuth [1981, p.128]. Each of the 20 instances is extended to an eight week planning horizon. Instances initially having a planning horizon longer than one week are shortened to one week. The arrival of requests concerning nurses, clients and jobs is simulated with the Poisson Process by determining the time between two arrivals based on the exponential distribution. For each instance three scenarios are generated: *basic*, *increase* and *extreme*. The arrival rate parameters λ for each scenario are given in Table 7.2.

[1] All the reported results are computed on resources provided by the Paderborn Center for Parallel Computing.

		Scenario		
		Basic	Increase	Extreme
Nurses	New nurse	1/12	1/12	1/6
	Remove nurse	1/15	0	2/15
	Change work contract	1/24	0	1/12
Clients	New client	1/2	1/1	3/1
	Remove client	7/15	1/2	2/1
Jobs	New job	7/24	7/12	1/1
	Remove job	1/4	1/4	1/1
	Change job duration	1/3	0	1/1
	Change job time window	1/3	0	1/1

Table 7.2.: Poisson process parameters: Average arrival rates given as number of occurrences per number of weeks (e.g. 1/2 = one request in 2 weeks)

The generation of dynamic instances uses the static instance as initial state of the first week. The Poisson processes for each type of change simulate for a given planning horizon of eight weeks. The points in time (generated by the Poisson process) are broken down to the corresponding weeks to determine the number of add, remove and change requests. The nurses, jobs and clients to change or remove are randomly chosen. The attributes for changed jobs (time windows and duration) and nurses (work contracts) are determined as described for the instance generation in the static setting in Section 5.3. New nurses and clients are also generated by those methods.

The *basic* scenario has slow changes over time, e.g., by introducing a new client every two weeks on average. Furthermore, the staff changes only slowly over several months. Eventually requests of already known clients arrive, creating or removing jobs as well as changing time windows or durations.

The *increase* scenario focuses on an expanding provider, because only new nurses are added. The arrival rate of new clients is higher than in the basic scenario, with a relatively small number of clients dropping out. The same holds for new or removed jobs of known clients. Changes in work contracts, job durations and time windows are not included in this scenario because it is used to evaluate continuity under growing demands. The *extreme* scenario resembles the basic scenario but with higher arrival rates. Thus, the underlying data sets are more volatile and used to test the metrics under larger fluctuations.

7.4.2. Comparison of continuity metrics

In this section the proposed continuity metrics from Section 7.2.1 are evaluated to determine which to use in a combined continuity metric. The results in this section are computed by the presented approach for the first four weeks of each of the 20 instances in the basic scenario. No rolling horizon setting is used but the continuity is calculated for the first planning interval with length of four weeks. The continuity mode used is *ToPrevious*, i.e., the goal is to achieve continuity between consecutive weeks.

Every continuity metric is evaluated with an objective weight of 1.0 and 0.5. The former only optimizes the continuity metric. The latter has a weight of 0.5 for the tour length and 0.5 for the continuity metric to take both criteria equally into account. The analysis of results is based on three parts. First, a boxplot diagram is generated to show the distribution of the results. Second, the mean, median and standard deviation of the results are given to provide more details in addition to the boxplot diagram. Third, the metrics are compared directly with each other by evaluating the solutions calculated by one metric with all other metrics.

Continuity of time

The influence of the individual continuity of time metrics is shown in the boxplot diagram in Figure 7.10 for an objective weight of 0.5. The data points show the maximum deviation of start times for each job group in the instances. The results when only considering tour length in the objective function is given as reference in the left boxplot.

The first observation is that the metric CoT^{Max} leads to a much larger deviation per job group than the sum-based metrics CoT^{Sum}, CoT^{t} and CoT^{Quad}. The improvement in minimizing the maximum deviation of start times in each job group is only slightly better than just considering the tour length. The sum-based metrics show a clear improvement when considered in the objective function. The median deviation in a job group is reduced from 43.9 minutes (only tour length) to a value less than 30 minutes (Table 7.4.2). CoT^{Sum} and CoT^{t} achieve the lowest deviations measured by mean, median and standard deviation for a weight of 0.5 and 1.0 (Table 7.4.2). When the tour length is omitted in the objective function, i.e., the continuity metric is weighted with 1.0, the heuristic is able to further reduce the deviations resulting in a median of about 14 minutes deviation for the metrics CoT^{Sum} and CoT^{t}. The boxplot diagram for a continuity weight of 1.0 is given in Figure E.1 in Appendix E and shows the same observations for the metrics.

The interdependency of the continuity of time metrics is given in Tables 7.4 and 7.5 for a weight of 0.5 and 1.0, respectively. The rows indicate the metric used in

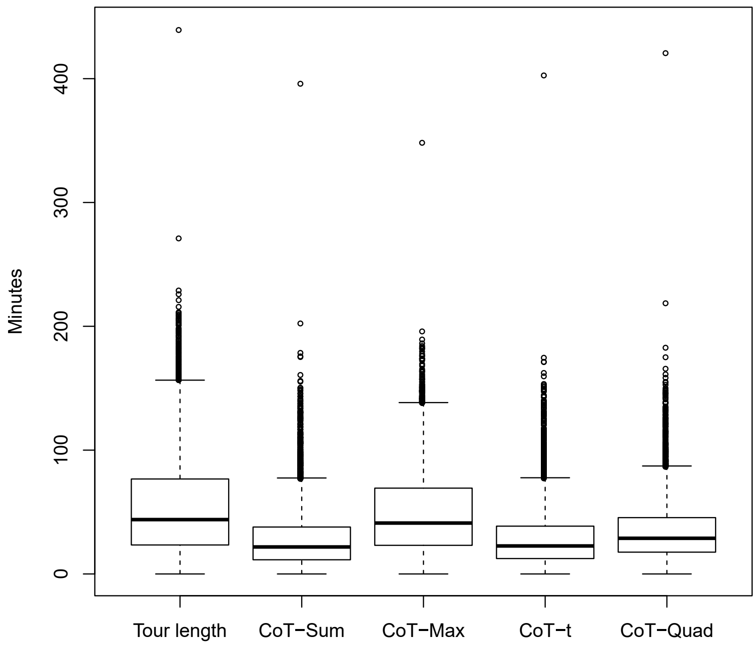


Figure 7.10.: Boxplot diagram for continuity of time metrics with weight 0.5. Data points represent the maximum deviation in a job group

Weight		**Tour length**	CoT^{Sum}	CoT^{Max}	CoT^{t}	CoT^{Quad}
0.5	Mean	54.9	29.0	48.8	29.5	35.4
	Median	43.9	21.8	41.1	22.6	28.8
	Std.Dev.	42.6	26.8	34.5	26.4	27.5
1.0	Mean	54.9	20.8	45.0	21.1	22.3
	Median	43.9	14.3	35.2	14.6	17.5
	Std.Dev.	42.6	24.4	36.7	24.3	22.2

Table 7.3.: Mean, median and standard deviation of maximum start time deviation in job groups for each metric and continuity weights 0.5 and 1.0

the objective function. The values shown are averaged results over all instances and are normalized column-wise. The columns represent the value of the metric when the objective function would have consisted of the continuity metric indicated, i.e., the solution achieved by the metric given in the row is evaluated with the other metrics indicated in the column. The value achieved with the same metric as in the objective function always has the value 1.0. Values greater than 1.0 indicate a worse continuity and values less than 1.0 an improvement. For example, the value 5.97 in row CoT^{Max} and column CoT^{Sum} of Table 7.5 is interpreted as follows: When CoT^{Max} is used in the objective function, the value for CoT^{Sum} is on average 5.97

		Metric				
		CoT^{Sum}	CoT^{Max}	CoT^{t}	CoT^{Quad}	Tour length
Objective	CoT^{Sum}	*1.00*	1.29	0.97	0.85	1.06
	CoT^{Max}	2.25	*1.00*	2.26	2.05	1.05
	CoT^{t}	1.05	1.32	*1.00*	0.90	1.06
	CoT^{Quad}	1.37	1.17	1.35	*1.00*	1.03
	Tour length	3.08	1.69	3.05	3.44	*1.00*

Table 7.4.: Comparison of continuity of time metrics. The rows indicate the metric used in the objective function with weight 0.5. The columns show the relative values of the other metrics evaluated on the solution provided by the selected metric. The tour length is weighted 0.5 in this setting

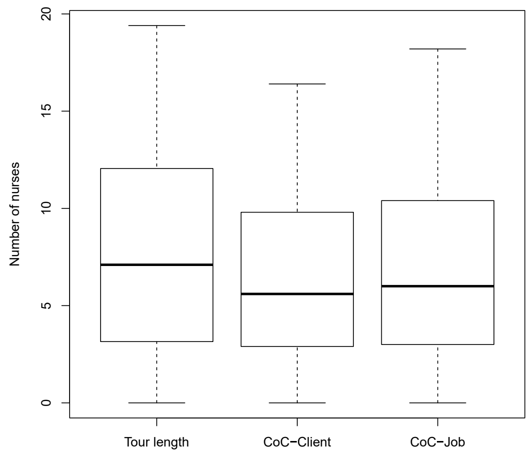
		Metric				
		CoT^{Sum}	CoT^{Max}	CoT^{t}	CoT^{Quad}	Tour length
Objective	CoT^{Sum}	*1.00*	1.01	1.01	1.29	1.33
	CoT^{Max}	5.97	*1.00*	6.73	9.07	1.27
	CoT^{t}	1.06	1.00	*1.00*	1.28	1.33
	CoT^{Quad}	1.17	0.79	1.13	*1.00*	1.32
	Tour length	16.94	1.80	19.50	29.89	*1.00*

Table 7.5.: Comparison of continuity of time metrics. The rows indicate the metric used in the objective function with weight 1.0. The columns show the relative values of the other metrics evaluated on the solution provided by the selected metric. The tour length is weighted 0.0 in this setting

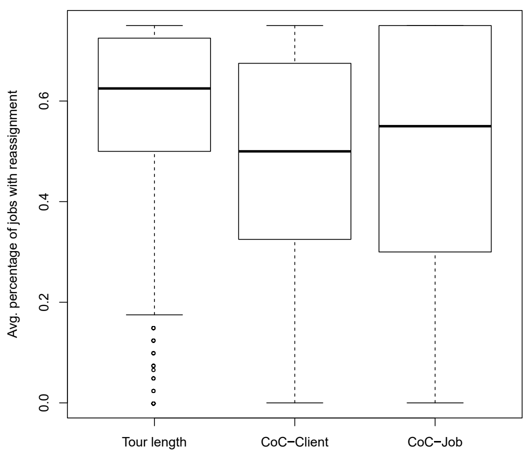
times higher compared to when CoT^{Sum} was used in the objective function.

The values in Tables 7.4 and 7.5 show that using the metric CoT^{Max} leads to a large deterioration of the values for the sum-based metrics, especially when the continuity is weighted width 1.0 (Table 7.5). In contrast to this, all sum-based metrics only lead to a slightly deteriorated value for CoT^{Max} or even an improvement. CoT^{Max} considers only the maximum deviation over all job groups per week into account. If this value is high, e.g., caused by constraints like the working regulation, all other job groups also can have large deviations without negative influence on the objective value. The observation that CoT^{Sum} and CoT^{t} achieve the best results for continuity of time are supported by this comparison. Both work best in comparison with the other two metrics, i.e., they achieve the best values for all four metrics. However, CoT^{Sum} often shows superior results, especially for a weight of 0.5.

The increase in tour length when considering continuity of time in the objective



(a) Continuity of care per client. Data points represent the average number of nurses assigned to a client



(b) Continuity of care per job group. Data points represent the average percentage of jobs per job group that have a reassignment

Figure 7.11.: Boxplot diagrams for continuity of care metrics per client and job group for objective weight 0.5

function is also given in Tables 7.4 and 7.5. When only the continuity is considered, there is an increase in tour length of about 30%. If the continuity is weighted 0.5, the tour length is 3 to 6 % worse than without continuity consideration. The increase in tour length is lowest for CoT^{Max} (Table 7.5) because the heuristic has more flexibility in assigning the jobs without reducing the continuity. CoT^{Sum} is used as continuity of time metric for further analyses because of the observations in this section.

Continuity of care

The continuity of care metrics CoC^{Client} and CoC^{Job} are analyzed and compared in this section. Figure 7.11 shows the boxplot diagrams of the results based on clients and job groups for a continuity weight of 0.5. The data points included in Figure 7.11(a) represent the number of nurses assigned to a client in the planning interval. The same solutions but with focus on the nurse reassignments on a job level are given in Figure 7.11(b). The data points represent the average percentage of jobs in a job group that have a new nurse assigned to them between two consecutive weeks. The results for a continuity weight of 1.0 are shown in Figure E.2 in Appendix E.

In Figure 7.11(a) the improvement of the continuity of care becomes visible when using either one of the two metrics as objective function. When only the tour length is minimized, the median number of nurses per client is 7.1 (Table 7.6). The metric CoC^{Client} and CoC^{Job} both reduce this number to a value between 5 and 6 (Table 7.6). The metric CoC^{Client} achieves the better continuity of care on a client basis,

		Weight 0.5		Weight 1.0	
	Tour length	CoT^{Client}	CoT^{Job}	CoT^{Client}	CoT^{Job}
Mean	7.6	6.2	6.6	5.6	6.2
Median	7.1	5.6	6	5	5.6
Std.Dev.	5.2	4.2	4.4	4.1	4.9

Table 7.6.: Mean, median and standard deviation of number of assigned nurses per client

		Weight 0.5		Weight 1.0	
	Tour length	CoT^{Client}	CoT^{Job}	CoT^{Client}	CoT^{Job}
Mean	0.58	0.48	0.51	0.42	0.46
Median	0.63	0.50	0.55	0.45	0.60
Std.Dev.	0.18	0.22	0.24	0.25	0.30

Table 7.7.: Mean, median and standard deviation of percentage of jobs with a nurse reassignment per job group

especially by looking at the values in Figure 7.11(a). For CoC^{Job} there are more clients that see more than 15 nurses in the planning interval. When the continuity of care is weighted with 0.5 and, thus, the tour length is included into the objective, both metrics still can reduce the number of nurses and the results are only slightly worse than with a full continuity weighting (Table 7.6).

The improvement of continuity of care on a job basis is visualized in Figure 7.11(b). When only the tour length is minimized the median percentage of jobs having a reassignment is more than 60%. This value is clearly reduced by the metric CoC^{Client} although it does not consider a job level. The metric CoC^{Job} has a higher median than the CoC^{Client}, but many job groups have a much smaller value of reassignments. In all cases, CoC^{Client} achieves better results.

Tables 7.8 and 7.9 support this observation and further show the influence on the tour length when continuity is considered. When we select the metric CoC^{Client} with weight 1.0 (Table 7.9), the continuity value of CoC^{Job} is 2.46 times worse compared to the case where CoC^{Job} is used as metric. The other way around, the value of CoC^{Client} is only 1.63 worse when CoC^{Job} is selected as metric. When only the tour length is selected as objective, it has less negative impact on the metric CoC^{Client} than on CoC^{Job}. However, the tour length is significantly higher, when CoC^{Client} is used as metric (32%). When the continuity metric is weighted with 0.5 (Table 7.8), CoC^{Client} could even improve the results for the CoC^{Job} metric. Furthermore, the increase in tour length reduces to only 4%, which is less than for CoC^{Job} (15%).

		Metric		
		CoC^{Client}	CoC^{Job}	Tour length
Objective	CoC^{Client}	*1.00*	0.94	1.04
	CoC^{Job}	1.45	*1.00*	1.15
	Tour length	1.76	1.23	*1.00*

Table 7.8.: Comparison of continuity of care metrics with weight 0.5. The rows indicate the metric used in the objective with weight 0.5. The columns show the relative values of the other metrics evaluated on the solution provided by the selected metric. Tour length is weighted 0.5

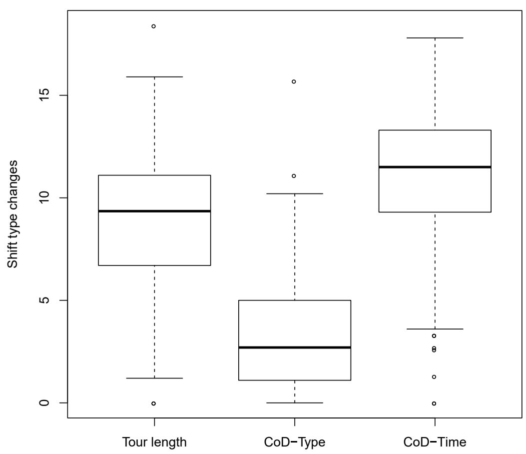
		Metric		
		CoC^{Client}	CoC^{Job}	Tour length
Objective	CoC^{Client}	*1.00*	2.46	1.32
	CoC^{Job}	1.63	*1.00*	1.27
	Tour length	4.19	9.38	*1.00*

Table 7.9.: Comparison of continuity of care metrics with weight 1.0. The rows indicate the metric used in the objective with weight 1.0. The columns show the relative values of the other metrics evaluated on the solution provided by the selected metric. Tour length is weighted 0.0

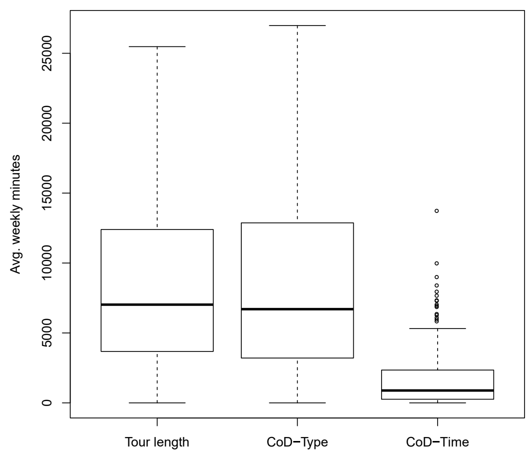
The observations in this section indicate CoC^{Client} as the superior metric to measure continuity of care because it works better in a weighted objective function with the tour length, which will be the common case in practice. This can be explained by the fact that CoC^{Client} allows more flexibility. The nurses assigned to jobs in a job group can be selected from a set of nurses without causing an increase in the objective function. CoC^{Job} is more restrictive and avoids reassignments of known nurses just because they handled another job group in the previous week. Furthermore, the metric CoC^{Client} also has a positive impact on the value of CoC^{Job} in a weighted objective function because the number of nurses is held small directly from the beginning whereas a different nurse is possible for every job group in CoC^{Job} although several job groups can have the same client.

Continuity of duty schedules

The results for both continuity of duty schedule metrics are given in Figure 7.12 for a weight of 0.5. Figure E.3 in Appendix E shows the results for a continuity weight of 1.0. The number of shift type changes per nurse for the continuity of duty schedules



(a) Continuity of duty schedules based on shift types. Data points represent the average sum of shift type changes in the planning interval



(b) Continuity of duty schedules based on time fluctuations. Data points represent the average sum of minutes that the start and ends deviate per week

Figure 7.12.: Boxplot diagrams for continuity of duty schedules metrics with objective weight 0.5

		Weight 0.5		**Weight 1.0**	
	Tour length	CoD^{Type}	CoD^{Time}	CoD^{Type}	CoD^{Time}
Mean	8.96	3.33	11.02	2.84	12.37
Median	9.35	2.70	11.50	2.45	13.10
Std.Dev.	3.10	2.77	3.23	2.52	3.65

Table 7.10.: Mean, median and standard deviation of shift type changes per nurse

metrics is shown in Figure 7.12(a). The metric CoD^{Type} reduces the number of shift type changes drastically. The median is reduced from 9 changes to less than 3 (Table 7.10 and Figure 7.12(a)) for both weights. The metric CoD^{Time} leads to more shift reassignments because partially assigned weekends cannot be considered due to missing start times. The chosen shift types can differ on partially assigned weekends without influencing the objective value. This can lead to even more shift type changes compared to considering only the tour length (Table 7.10 and Figure 7.12(a)).

Looking at the sum of deviations in start and end times of shifts per week in Figure 7.12(b), the time-based metric CoD^{Time} naturally achieves the better results. The metric CoD^{Type} does not take time deviations into account, only the shift types are considered. Therefore, deviations in one shift type are possible without negative

	Tour length	Weight 0.5		Weight 1.0	
		CoD^{Type}	CoD^{Time}	CoD^{Type}	CoD^{Time}
Mean	8352.83	8411.82	1705.31	8229.01	837.75
Median	7024.50	6701.13	884.13	6627.88	193.50
Std.Dev.	5732.96	6046.87	2123.79	5668.88	1399.65

Table 7.11.: Mean, median and standard deviation of average shift start and end times fluctuations per week per nurse

		Metric		
		CoD^{Type}	CoD^{Time}	Tour length
Objective	CoD^{Type}	*1.00*	246.74	1.29
	CoD^{Time}	64.35	*1.00*	1.33
	Tour length	36.40	316.43	*1.00*

Table 7.12.: Comparison of continuity of duty schedules metrics. The rows indicate the metric used in the objective with weight 1.0. The columns show the relative values of the other metrics evaluated on the solution provided by the selected metric. Tour length is weighted 0.0

impact on the objective function. The sum of deviations can be large as it is the case in Table 7.11. Here, the advantage of using CoD^{Time} on similar start and end times becomes clear. CoD^{Time} achieves on average approx. 6700 minutes less start and end time fluctuations than CoD^{Type} for a weight of 0.5 and more than 7300 minutes for a weight of 1.0.

Comparing the normalized values in Tables 7.12 and 7.13 the observation of largely differing results are supported. For an objective weight of 1.0 for the continuity

		Metric		
		CoD^{Type}	CoD^{Time}	Tour length
Objective	CoD^{Type}	*1.00*	6.90	1.03
	CoD^{Time}	10.52	*1.00*	1.03
	Tour length	7.11	6.90	*1.00*

Table 7.13.: Comparison of continuity of duty schedules metrics. The rows indicate the metric used in the objective with weight 0.5. The columns show the relative values of the other metrics evaluated on the solution provided by the selected metric. Tour length is weighted 0.5

metrics (Table 7.12), the value of CoD^{Time} is 246 times worse when using CoD^{Type} as metric compared to when CoD^{Time} is set as objective. However, when selecting CoD^{Time} the impact on CoD^{Type} is also clearly negative but less than vice versa. The increase of tour length is nearly the same for both metrics (approximately 30%).

The major negative impacts on each other reduce when the metrics are used in a weighted objective function with the tour length (Table 7.13). Here the normalized values are 6.9 and 10.5 which is only a portion of the results when a weight of 1.0 is used. The high reduction can be explained by the fact, that the values of CoD^{Time} are much worse in this case and, therefore, the normalization leads to lower values for CoD^{Type}, which can also be seen from Table 7.11.

Concluding this analysis of the continuity of duty schedule metrics, both metrics indicate problems to achieve similar duty schedules. This is due to the presence of working regulations that restrict the shift assignments and regulations spanning more than one week and influencing the possibility of assigning the same shift type again. The metric for the remainder of this thesis is CoD^{Time} due to four reasons. First, this metric also works when no shift types are present and therefore are applicable more generally. Second, the impact on the shift type changes is only slightly worse. Third, the same work times are more preferably than the same shift type because the deviations of start and end times can be large if ignored in the objective function. Fourth, although CoD^{Time} ignores partially assigned weekends, this fact is not as important in practice as similar working times.

7.4.3. Comparison of continuity modes

After CoT^{Sum}, CoC^{Client} and CoD^{Time} are selected as metrics for continuity of time, care and duty schedules, respectively, the different continuity modes presented in Section 7.2.3 are analyzed for their suitability. The ALNS heuristic is executed with a combined continuity metric of these three metrics for each mode on each instance of the basic scenario. There are two different weightings used for the evaluation: First, a weight of 0.5 for the tour length and $\frac{1}{6}$ for each continuity metric and, second, a weight of $\frac{1}{3}$ for each continuity (the tour length is not considered in this case). In the first case, the combined continuity is weighted with 0.5 and in the second case with 1.0. The continuity modes are divided by the attribute whether a reference week is given. If a reference week is needed, the best solution provided by the ALNS in a static setting is used as input because the first week equals to the static instance.

The relative metrics are used as comparison criterion throughout this section due to the different domains of the three continuity metrics . The results for the continuity modes without reference week are given in Tables 7.14 and 7.15 for a continuity weight of 1.0 and 0.5, respectively, and each continuity metric separately. The values are normalized column-wise and the row indicates the used mode during execution.

	$\boldsymbol{CoT^{Sum}}$		$\boldsymbol{CoC^{Client}}$		$\boldsymbol{CoD^{Time}}$	
	ToPrevious	*Total*	*ToPrevious*	*Total*	*ToPrevious*	*Total*
ToPrevious	*1.00*	0.73	*1.00*	1.00	*1.00*	1.10
Total	2.14	*1.00*	1.32	*1.00*	3.92	*1.00*

Table 7.14.: Normalized results for continuity modes without reference week and combined continuity weight of 1.0. The rows indicate the mode used in the solution approach. The columns represent the relative metric values for each other mode evaluated on the solution

	$\boldsymbol{CoT^{Sum}}$		$\boldsymbol{CoC^{Client}}$		$\boldsymbol{CoD^{Time}}$	
	ToPrevious	*Total*	*ToPrevious*	*Total*	*ToPrevious*	*Total*
ToPrevious	*1.00*	0.83	*1.00*	0.99	*1.00*	1.69
Total	1.89	*1.00*	1.31	*1.00*	2.35	*1.00*

Table 7.15.: Normalized results for continuity modes without reference week and combined continuity weight of 0.5. The rows indicate the mode used in the solution approach. The columns represent the relative metric values for each other mode evaluated on the solution

In both cases, the mode *ToPrevious* reaches superior results because it has good results also for the mode *Total*. For the continuity of time it even outperforms the continuity mode *Total*. The metric CoT^{Sum} are 2.14 and 1.89 times worse when using mode *Total* instead of *ToPrevious* with weight of 1.0 and 0.5, respectively. It can be concluded, that measuring the continuity metrics on a week to week basis leads to better results than taking only the worst case of the entire planning interval into account. If the worst case has high deviations all other deviations with a value below it do not influence the objective function leading to superior results using the sum of deviations. *ToPrevious* is selected as mode for instances without reference week.

The results for the three continuity modes considering a reference week are given in Tables 7.16 and 7.17. The column-wise normalized metrics show a similar result for the modes *ToPreviousRef (ToPrevRef)* and *TotalRef* as for the counterparts without reference week. Again, *ToPrevRef* is superior to *TotalRef*, which is, of course, due to the same argument as for the setting without reference week. Comparing the setting *ToReference (ToRef.)* with *ToPrevRef*, *ToReference* reaches slightly better results for CoT^{Time}. We interpret this based on the normalized metrics. The evaluation of the results provided by *ToReference* achieves better results evaluated with *ToPreviousRef* than vice versa (1.17 and 1.18 versus 2.11 and 1.43). For the metrics CoC^{Client} and CoD^{Time} *ToPreviousRef* is superior.

	CoT^{Sum}			CoC^{Client}			CoD^{Time}		
	ToRef.	*ToPrevRef*	*TotalRef*	*ToRef.*	*ToPrevRef*	*TotalRef*	*ToRef.*	*ToPrevRef*	*TotalRef*
ToRef.	*1.00*	1.17	0.79	*1.00*	1.18	1.18	*1.00*	2.36	1.01
ToPrevRef	2.11	*1.00*	0.89	1.25	*1.00*	1.00	2.21	*1.00*	1.09
TotalRef	2.12	1.48	*1.00*	1.17	1.09	*1.00*	2.06	2.10	*1.00*

Table 7.16.: Normalized results for continuity modes with reference week and combined continuity weight of 1.0. The rows indicate the mode used in the solution approach. The columns represent the relative metric values for each other mode evaluated on the solution

	CoT^{Sum}			CoC^{Client}			CoD^{Time}		
	ToRef.	*ToPrevRef*	*TotalRef*	*ToRef.*	*ToPrevRef*	*TotalRef*	*ToRef.*	*ToPrevRef*	*TotalRef*
ToRef.	*1.00*	1.18	0.91	*1.00*	1.14	1.09	*1.00*	1.81	1.07
ToPrevRef	1.43	*1.00*	0.94	1.07	*1.00*	0.98	1.54	*1.00*	1.08
TotalRef	1.46	1.31	*1.00*	0.99	1.04	*1.00*	1.43	1.65	*1.00*

Table 7.17.: Normalized results for continuity modes without reference week and combined continuity weight of 0.5. The rows indicate the mode used in the solution approach. The columns represent the relative metric values for each other mode evaluated on the solution

Based on the knowledge that *ToReference* aligns all following weeks to the reference weeks, the mode *ToPreviousRef* is selected for further evaluation. For long-term use of the rolling horizon approach, this modes is beneficial. Over time clients, nurses and jobs that are present in the reference week drop out and, therefore, they are not included for the continuity calculation in the remaining weeks. Furthermore, new clients, jobs and nurses, have no respective values in the reference week and, thus, their continuity cannot be calculated for the scenario *ToReference*.

7.4.4. Evaluation of rolling horizon approach

Based on the selected continuity metrics and modes, the heuristic rolling horizon is evaluated for a scheduling of five consecutive weeks in the basic scenario, i.e., the planning interval is shifted five times. In the evaluation, the metrics CoT^{Sum}, CoC^{Client} and CoD^{Time} are weighted with $\frac{1}{6}$ each and the tour length with 0.5. The continuity modes used are *ToPrevious* for the first week, because no reference is present at that time, and *ToPreviousRef* for the remaining weeks. For each week the heuristic's runtime is set to five minutes leading to 25 minutes total computation time. The evaluation in this section excludes the results of the first week because it is used just for providing the first reference week for the rolling horizon planning.

First, the influence of a time-dependent weighting on the continuity metrics is evaluated based on a planning interval of four and two weeks. Afterwards, the results using a planning interval of one, two and four weeks are analyzed. Finally, the results of all three instance scenarios are compared based on the continuity metrics.

Influence of time-dependent weighting

The average values of the metrics for the time-factors 0.25, 0.5 and 1.0 in a planning interval of four weeks are given in Table 7.18. The values presented are given for the four focus weeks that are planned with the rolling horizon approach. The metrics are normalized based on the result of the time-factor 1.0. The table shows that a time-factor of 0.25 and 0.5 both improve the continuity metrics. The tour length is only increased slightly. The best continuity results are achieved with a time-factor of 0.25. The continuity of time is only 88% of the value achieved with no time-dependent weighting (time-factor of 1.0). The improvement for the continuity of care and duty schedules is even larger wit results of 0.84 and 0.79, respectively, because the focus week has a higher weight for a time-factor of 0.25.

The results for a planning interval with a length of two weeks are given in Table 7.19. Again the time-factor of 0.25 achieves the best results for continuity of all three types. However, the improvement, compared to no time-dependent weighting, is less than with a four week planning interval.

Focus week	***CoT***			***CoC***			***CoD***			**Tour length**		
	0.25	0.5	1.0	0.25	0.5	1.0	0.25	0.5	1.0	0.25	0.5	1.0
1	0.85	0.92	1.00	0.75	0.84	1.00	0.71	0.82	1.00	1.01	1.01	1.00
2	0.89	0.95	1.00	0.80	0.85	1.00	0.77	0.87	1.00	1.01	1.01	1.00
3	0.87	0.92	1.00	0.79	0.84	1.00	0.76	0.84	1.00	1.01	1.01	1.00
4	0.88	0.93	1.00	0.82	0.87	1.00	0.77	0.83	1.00	1.01	1.01	1.00
Avg.	**0.88**	0.93	1.00	**0.75**	0.84	1.00	**0.79**	0.85	1.00	1.01	1.01	**1.00**

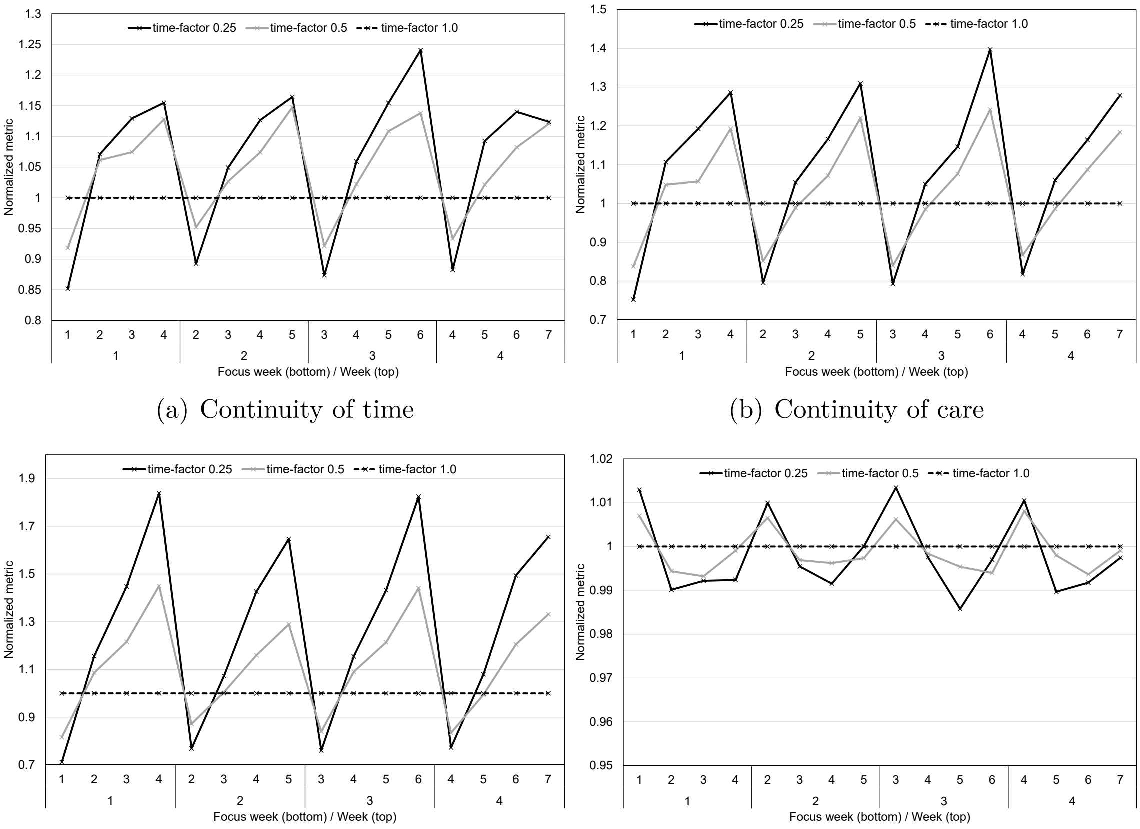
Table 7.18.: Averaged results of metrics for the weeks in the planning horizon with a planning interval of *four* weeks given for time-factors 0.25, 0.5 and 1.0. The values are normalized based on the values for time-factor 1.0

Focus week	***CoT***			***CoC***			***CoD***			**Tour length**		
	0.25	0.5	1.0	0.25	0.5	1.0	0.25	0.5	1.0	0.25	0.5	1.0
1	0.86	0.91	1.00	0.79	0.86	1.00	0.80	0.84	1.00	1.00	1.00	1.00
2	0.90	0.94	1.00	0.80	0.85	1.00	0.84	0.86	1.00	1.00	1.00	1.00
3	0.90	0.94	1.00	0.81	0.89	1.00	0.87	0.91	1.00	1.00	1.00	1.00
4	0.93	0.96	1.00	0.84	0.90	1.00	0.82	0.89	1.00	1.00	1.00	1.00
Avg.	**0.90**	0.94	1.00	**0.81**	0.88	1.00	**0.83**	0.88	1.00	1.00	1.00	1.00

Table 7.19.: Averaged results of metrics for the weeks in the planning horizon with a planning interval of *two* weeks given for time-factors 0.25, 0.5 and 1.0. The values are normalized based on the values for time-factor 1.0

The results presented in the tables so far show only the values for the focus weeks, i.e., the relevant week in the planning interval. Figure 7.13 additionally shows the metric values for the other weeks in the planning interval, which are used for providing information of future planning periods. The focus week and week in the planning interval are indicated on the x-axis. The plan used in practice would be the solution where focus week equals week. For example, the actual plan for the first week is given by the first value of the x-axis. The values for week 2, 3 and 4 are only used in the current planning interval for week 1 and their final solutions are obtained when the planning interval is shifted. Again, the results of time factor $f^{time} = 0.25$ and $f^{time} = 0.5$ are calculated in comparison to the basic setting with no time-dependent weighting and, thus, show the improvement or deterioration of the respective metric.

From the Figures 7.13(a) to 7.13(c) the effect of the time-dependent weighting on the continuity metrics becomes apparent. For the time-factors 0.25 and 0.5, all weeks except the focus week are weighted less in continuity calculation. Therefore,



(a) Continuity of time

(b) Continuity of care

(c) Continuity of duty schedules

(d) Tour length

Figure 7.13.: Influence of time-dependent weighting in in rolling horizon approach with planning interval of four weeks

the values for all continuity metrics are worse than in the basic setting with equal weighting. However, the values for the focus week are always better than in the basic setting. The heuristic can improve the continuity metrics because the highest weight in the objective function lies on the focus week and, therefore, the search leads to a better continuity for this week. Even if continuity values for the remaining weeks in the planning interval are worse if they are not in focus at the moment, the values are clearly improved over the basic setting when the week is focused. The values for the tour length in 7.13(d) show an opposite behavior. The tour length is higher than in the focus weeks because higher continuity weighting leads to longer tours.

The results for the three time-factors for a two week planning interval in a rolling horizon setting lead to the same observations. They are depicted in Figure E.4 in Appendix E.

Analysis of planning interval length

To evaluate the length of the planning interval in the rolling horizon approach, three different settings are tested. First, a four week planning interval and, second, a two week planning interval. Both with a time-factor of 0.25 as selected in the previous section. The last setting, considers only one week as planning interval, i.e, only the focus week and the reference week are considered during optimization. Note that no time-dependent weighting is possible in this case because no weeks for further weighting are present. The planning interval is shifted five times during the execution leading to schedules for five focus weeks.

The comparison is shown in Figure 7.14 where the x-axis indicates the parts of the objective function (continuity metrics and tour length) for each of the four focus weeks consecutively planned in a rolling horizon approach. The value for a planning interval of one week is used as base value to normalize the two other values. All three setting lead to a similar tour lengths. The continuity for the two week and four week planning interval are worse than for the one week planning interval and the results of the two week planning interval are better than for the four week planning interval in nearly all the cases. These observations lead to a possible conclusion why a shorter planning interval leads to better results in our case. First, the heuristic has a bigger focus on the first week in those two settings. Second, a shorter planning interval contains less jobs and nurses that are handled by the heuristic. The repair operators of ALNS need to reinsert less jobs and an iteration is finished faster, which leads to more solutions that are evaluated in five minutes computation time.
The results for the scenarios *Increase* and *Extreme* are given in Figures E.5 and E.6 in Appendix E and look similar, leading to the same conclusion.

Analysis of scenarios

In this section the continuity metrics for all three instance scenarios *Basic*, *Increase* and *Extreme* are compared to each other. The results are computed with the rolling horizon approach for a planning horizon of five weeks and a planning interval of one week. The continuity mode used is *ToPreviousRef.* The continuity metrics CoT^{Sum}, CoC^{Client} and CoD^{Time} are combined with the tour length in one objective function (weight $\frac{1}{6}$ for each continuity metric and 0.5 for the tour length).

The weekly instances resulting from the same initial static instance can differ in clients, job groups and nurses because different parameters of the Poisson processes are used. For the comparison a common basis is needed. Therefore, all clients, job groups and nurses that are valid in all three scenarios are determined. The results for the metrics in this section are calculated based on these common sets of client, job groups and nurses. All others are excluded, because they are not valid from

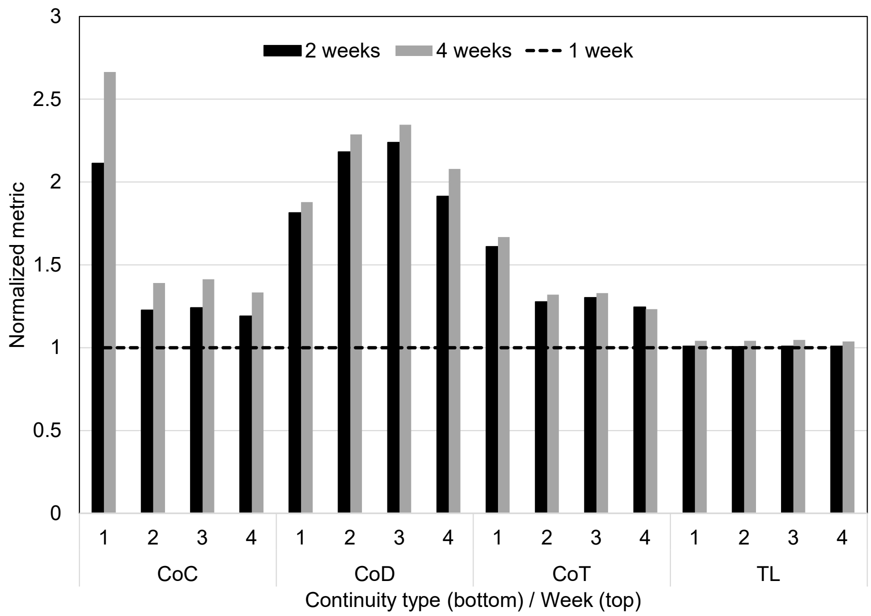


Figure 7.14.: Comparison of planning interval length in a rolling horizon setting for *basic* scenario

Metric		**Scenario**		
		Basic	Increase	Extreme
CoT	Mean	69.22	69.60	80.56
	Median	54.95	55.90	66.20
	Std. Dev.	53.34	52.76	56.95
CoC	Mean	5.34	5.35	5.72
	Median	3.70	3.70	4.00
	Std. Dev.	4.87	4.98	5.10
CoD	Mean	1352.35	1554.54	1965.37
	Median	573.40	695.70	734.40
	Std. Dev.	1684.41	1861.34	2476.42

Table 7.20.: Mean, median and standard deviation for the continuity metrics and all three scenarios. Values show the average sum for all four weeks per metric (CoT = sum of deviations in minutes, CoC = number of nurses, CoD = sum of deviations of start and end time in minutes)

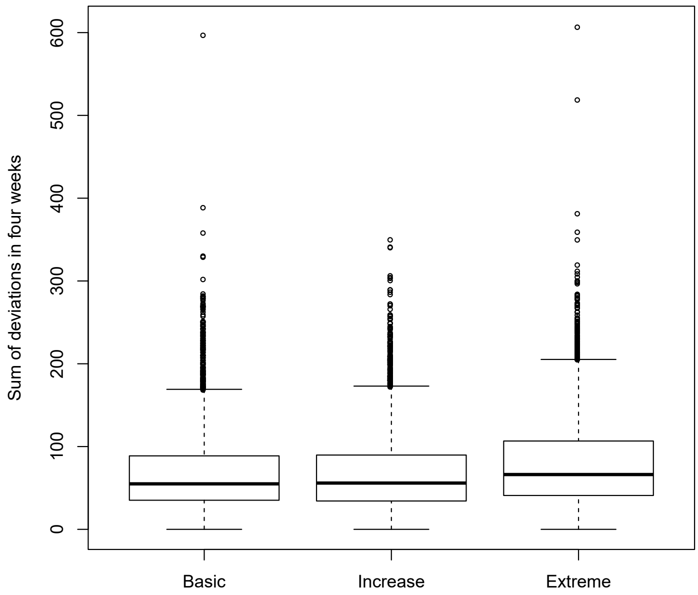


Figure 7.15.: Continuity of time metric for all three scenarios. Data points represent the sum of start time deviations per job group

the beginning of the planning horizon. The continuity of the first weeks cannot be evaluated for them and represent missing values. In the scenario *Extreme* many changes occur over the planning horizon leading to many missing values that bias the overall continuity calculation in favor of better looking results for this scenario. The clients, jobs and nurses present from the beginning are the most important entities in the continuity consideration because they are used to the old schedules. New clients and nurses are more flexible until they get familiar with the schedules.

The mean, median and standard deviations for each continuity type and scenario are given in Table 7.20. Figures 7.15, 7.16 and 7.17 show the boxplot diagrams for the deviations in start times, number of assigned nurses and fluctuations in shift times, respectively. The best continuity metrics for all three types are achieved for the *Basic* scenario with slowly changing demands of clients and working hours of nurses. In contrast to this, the values for the *Extreme* scenario are always the worst for all three types. The deviations of start times in job groups are similar for the scenarios *Basic* (69.22) and *Increase* (69.20) as given Table 7.20. The slowly growing client sets in the scenario *Increase* have no major negative impact on the continuity of time for the jobs of known clients. The new clients can be inserted to the plan without affecting the appointment times of those. The same holds for the number of assigned nurses representing the continuity of care. The values differ only by 0.01 nurses on average (Table 7.20). However, the differences in start and end times of shifts are clearly influenced by the increased number of new clients. They differ by

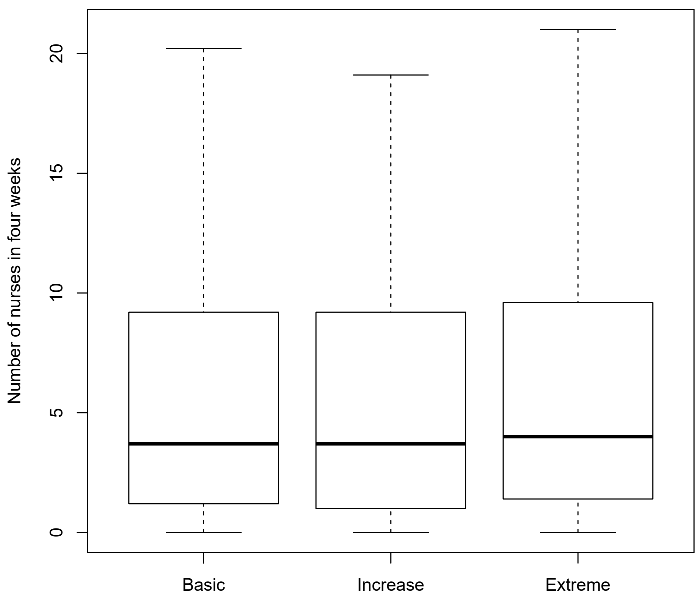


Figure 7.16.: Continuity of care metric for all three scenarios. Data points represent the number of nurses assigned in four weeks per client

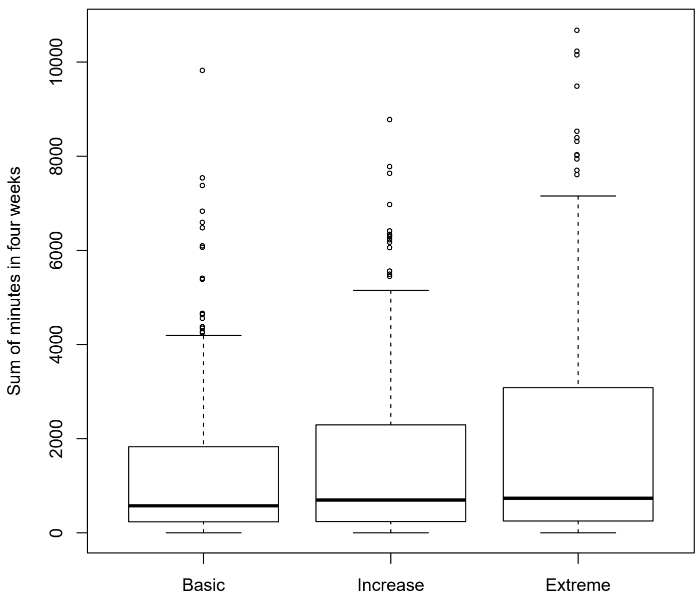


Figure 7.17.: Continuity of duty schedules metric for all three scenarios. Data points represent the sum of start and end time deviations per nurse

about 200 minutes more, on average, than in the *Basic* scenario. The reason for this is the higher number of clients leading to longer tours, which are affected by the working regulations more often than short tours (break and rest times). Therefore, similar shift times are harder to achieve.

The deviations in start times are much higher in the *Extreme* scenario than in the other two. On average they deviate by 10 minutes more (Table 7.20). The distribution of deviations of start times in the boxplot diagram in Figure 7.15 looks very similar for all three scenarios. The median number of assigned nurses in the scenario *Extreme* is only slightly higher than in the two other scenarios (Figure 7.16). This can be explained by the higher number of new nurses over the planning horizon that are assigned to the new clients. The deviations in shift times in scenario *Extreme* is even higher than in the scenario *Increase* due the larger fluctuations during the planning horizon.

To conclude the analysis of the three different scenarios, it can be said that higher fluctuations makes it harder to achieve continuity values. However, for the continuity of time and care the differences are not large. Only the similarity of shift times is clearly negatively affected by the major fluctuations.

7.5. Summary

The extensions proposed in this chapter enable the ALNS heuristic (presented in 6.4) to be used in a dynamic setting. A dynamic setting is present if demands of clients and working hours of nurses change over time. Furthermore, requests arrive or expire. In practice, those changes occur on a regular basis. Therefore, it is important to provide a heuristic to handle this setting. Besides the economic goal of minimizing the tour lengths, the satisfaction of nurses and clients is essential in a dynamic setting. Major changes to the schedules must be avoided so that the nurses and clients do not have to adapt to new appointments, shift times and assignments every week. The satisfaction of clients is addressed by the continuity types time and care whereas the nurse satisfaction is considered by the continuity of duty schedules. In this chapter several different metrics for each of the three continuity types were presented and different possibilities for determining the basis for continuity calculations (continuity mode), i.e., which week to compare to which other week, were proposed.

The continuity metrics and mode were used in the ALNS heuristic so that it is embedded to a rolling horizon approach. The heuristic provides a solution approach for regular recurring planning while considering the surrounding weeks for continuity calculation. The metrics, modes and the rolling horizon approach were evaluated on an instance set for which the changes were simulated by a Poisson process. Based on the results, the metrics and modes to be used in a rolling horizon approach were determined. The analysis of the entire approach showed that a time-dependent weighting of continuity in different weeks is beneficial. Weeks that are farther in the future are considered less for continuity calculation in the current focus week, if a planning interval of several weeks is considered. The analysis also showed that a planning interval of only one week (plus the previous week as reference week) leads to the best results regarding continuity. This is probably because the heuristic needs more time for searching the solution space of a planning interval of several weeks compared to only one week. There exist more possible solutions and the repair mechanisms take longer due to the number of considered jobs and nurses. The analysis of three different scenarios of fluctuations showed that even if there are larger fluctuations the continuity of duty schedules can be improved, although not as much as in the other two scenarios with less fluctuations. For the continuity of time and care the negative impact was only small.

8. Concluding remarks

This chapter summarizes this thesis and provides a critical review of the results and goals achieved. Finally, an outlook on future research opportunities is given.

8.1. Summary and critical review

An introduction to home care services and the development of the sector was given in Chapter 2. The statistics and references used for the description show the growing importance of long-term care and, in particular, home care as alternative to residential care facilities. The projections and reasons mentioned indicate an increasing demand in the near future. To support the management of home care providers, several planning problems are addressed in OR literature providing quantitative solution approaches. An overview of the planning problems on a strategic, tactical and operational level was given in Section 2.3. The chapter is concluded with a description of the problem setting in this thesis, namely the routing and scheduling for home care services in a static and dynamic setting. The planning process and requirements that need to be considered from a practical point of view were presented in detail.

Chapter 3 presents the OR methods used in this thesis, namely MIP and the metaheuristics LNS, ALNS and RVNS. Additionally, an overview of related planning problems from literature, in particular routing problems and duty planning in other health care institutions, is given.

The state-of-the-art in quantitative solution approaches for home care routing and scheduling is reviewed in Chapter 4. The literature was categorized based on the type and length of the considered planning horizon and the working regulations addressed in the solution approaches. The review shows that many publications address the routing and scheduling for one day. The publications considering a planning horizon of multiple days mainly concentrate on simple working regulations, e.g., the maximum daily working time or basic break rules. Additionally, several requirements from practice (e.g. shift rotations), were missing in the literature. Based on the lack of an integrated solution approach considering legal working regulations as well as work contracts, the first three research goals of this thesis were deduced. First, to provide a mathematic formalization of the problem setting including the identified working regulations. Second, to solve the problem setting for real-world sized instances in reasonable computation time. Third, to investigate the influence of working regulations on the schedules. The second finding of the literature review was that only a few publications address solution approaches in a dynamic setting. These focus on the arrival of new clients, provide single continuity metrics for time and care and consider only basic working regulations. Therefore, the remaining two research goals of thesis were derived. Fourth, enabling the heuristic solution approach to ensure feasibility and continuity between planning periods in a dynamic setting, and, fifth, assessing different continuity metrics to select the most suitable.

Chapter 5 proposed a formalization of home care routing and scheduling with working regulations in a static setting to address the first research goal. The result was a MIP that can be solved with a mathematical programming solver. Test instances used for the evaluation were introduced, consisting of artificial instances and data sets provided in other publications. The results computed with a state-of-the-art commercial solver showed that the solver is not capable of providing solutions for real-world sized instances with a high quality in a reasonable computation time. Even with a computation time of 48 hours nearly all of the instances were not solved with proven optimality. Still, the solutions of the MIP offered lower and upper bounds on the optimal solutions that are used to evaluate the results of further solution approaches.

To reduce the computation time and provide solutions with a high quality for real-world sized instances (second research goal), Chapter 6 proposed three heuristic solution approaches based on the metaheuristics LNS, ALNS and RVNS. These were selected due to their success on many routing and scheduling problems. Each metaheuristic uses domain problem specific operators that determine which part of the solution should be changed to create a new solution. The parameters of the heuristics were determined with the algorithm configurator GGA. The tuning of the parameters led to an improvement for all heuristics. In the numerical analysis, the heuristics are compared with each other and to the results of the MIP solver. The analysis showed that the heuristics outperform the MIP solver on most of the generated instances considering many working regulations and jobs. The results on test sets from literature also showed a good performance of the heuristics. Furthermore, the heuristics only needed a very small portion (5 minutes) of the computation time to achieve these results compared to the MIP solver (48 hours). The comparison of the heuristics with each other was carried out afterwards. First, the average performances of the heuristics were analyzed leading to the conclusion that ALNS outperforms LNS and RVNS on the entire instance set. A more detailed analysis of the empirical distribution showed that the average performance is misleading. Although LNS and RVNS achieved the same average results, the empirical distribution indicated that ALNS is clearly superior only to LNS. For RVNS no clear statement could be made based on the entire set because each heuristic was superior to the other on approximately 50% of the instances. Based on the information about the underlying instance sets, ALNS is selected as more suitable and was used for the evaluation working regulations, which relates to research goal three. Scenarios with different working regulations were solved and the effects regarding the constraint violations and changes in working time investigated. On the one hand, the analysis showed that many working regulations are violated if they are omitted from the method. On the other hand, there was only a slight increase in working time for the instances and method used in the analysis. We can conclude that the consideration of regulations was essential to achieve legal plans and the increase in working time was small.

The ALNS heuristic was extended to a dynamic setting in Chapter 7 to address the fourth research goal. Several continuity metrics were defined for the three types continuity of time, continuity of care and continuity of duty schedules. The latter was newly introduced to the context of home care routing and scheduling in this thesis. All metrics can handle several types of changes over time and not only the arrival of new clients. Furthermore, different possibilities to measure continuity based on the selection of weeks to compare were presented. The continuity metrics created a multi-objective optimization that was addressed with to weighted sum approach. On the basis of the metrics, a rolling horizon approach was proposed that can be used for regular weekly planning. The evaluation of the heuristic for a dynamic setting consisted of several analyses. The instances used for the evaluations are based on the static instances extended to a dynamic setting using a Poisson process for simulating the changes over time. First, the most suitable metric for each continuity type was selected based on a four week planning horizon for the proposed instances (fifth research goal). The results showed that most of the metrics have a positive effect on the continuity in a weighted objective function with the tour length. The tour length increased on average by 5.6% when continuity was considered as equally important part of the objective (in addition to the tour length) but the continuity was clearly improved for all types. Second, the rolling horizon approach was investigated regarding the length of the planning interval considered and whether a time-dependent weighting of continuity in the objective is reasonable. The conclusion was that considering only the previous and current week leads to a higher continuity within the same computation time than having a longer planning interval by including future weeks. When several weeks are considered at the same time, weighting the continuity in later periods less than in earlier periods led to an improvement of continuity. Finally, three scenarios with different degrees of fluctuations of demand were analyzed. The results indicated that larger fluctuations make it harder to achieve a good continuity.

To summarize the thesis, we can point out that all research goals derived from the state-of-the-art were successfully addressed in this thesis. The relevant working regulations were integrated into exact and heuristic solution approaches for the routing and scheduling of home care providers. The heuristics provided good results in reasonable computation time for real-world sized instances. The problem was also addressed in a dynamic setting by an extended heuristic solution approach which ensures feasibility and continuity between planning periods to improve client and nurse satisfaction in addition to minimizing the tour lengths. Different continuity metrics were successfully evaluated and appropriate ones chosen for the rolling horizon approach.

The instance sets used for the evaluation in this thesis were artificially generated to represent the real-world application. Additionally, test sets from previous research were used to analyze the performance on different data. To provide further insights into the performance of the proposed methods, the heuristic approaches should be evaluated on real-world data sets from several home care providers. The resulting schedules could be compared with the plans of a human planner. This could reveal the optimization potential and reduction of planning effort compared to a manual planning. Additionally, the solutions methods can be integrated into a decision support system to provide a tool that can used in practical application.

8.2. Opportunities for future research

There are several aspects that offer potential for future research. Other requirements from practice that are already modeled in the literature about home care routing and scheduling, like multiple transportation modes or time interdependencies between jobs, can also be integrated with the working regulations to add further details to the problem setting. The proposed methods in this thesis should be applied to related planning problems, like the technician routing or the skill vehicle routing problem, to ensure working regulations that are also important in these areas.

Different degrees of uncertainty are interesting to investigate, especially in the dynamic setting. The future demands of clients can vary and the fluctuations could be integrated in a stochastic programming or robust optimization approach to take the uncertainty into account. Furthermore, the uncertainty of driving or service times on a daily level should be considered together with the working regulations because they can cause overtime. Stochastic or dynamic routing problems could provide the basis for this scenario.

Regarding the methods proposed in this thesis, several extensions are possible. To solve the proposed MIP formulation for larger instances, the development of advanced exact solution approaches, like Branch-and-Price, is preferable. They can take domain knowledge into account to reduce computation times and improve the solution quality. For the dynamic setting, further advanced continuity metrics can be investigated for their capabilities of ensuring continuity between periods. The currently proposed weighted sum approach could be replaced by a multi-objective solution approach that explicitly considers multiple criteria and enables a Pareto optimization. The result would be a set of indifferent solutions from which the planner can choose the best one regarding his or her preferences.

Finally, the operational planning problem considered in this thesis is influenced by many decisions made on a strategic or tactical level, e.g, resource allocation, staffing decisions or the selection of the geographical area to cover. To evaluate the influences of these decisions on the operational planning problem is another research opportunity.

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Appendix A.

Instance information

This section provides the detailed instance information of the generated instances in the *small* set (Table A.1) and test set (Table A.2). The details on the size of the instances is given by the number of clients, jobs, nurses and days in the planning horizon. Furthermore, the attributes of some considered working regulations are given in the columns *Unavail.*, *Rotations* and *Apprentice*. *Unavail.* indicates whether there are unavailabilities of nurses considered in the instance (e.g. vacations). If *Rotations* is true, some of the full-time nurses in the instance are working according to shift rotations. *Apprentice* indicates whether there are apprentices among the nurses that require different break and rest time regulations.

The instances are available for download at `http://hc.guericke.org/`.

Table A.1.: Test set instances - Set *small*

Name	Clients	Jobs	Nurse	Days	Unavail.	Rotations	Apprentice
S-01	5	5	2	1	False	False	False
S-02	5	7	2	1	False	False	False
S-03	10	10	2	1	False	False	False
S-04	9	10	2	1	False	False	False
S-05	10	12	3	2	False	False	False
S-06	10	11	2	1	False	False	False
S-07	4	16	3	3	False	True	False
S-08	9	10	4	1	False	True	False
S-09	10	11	3	1	False	False	False
S-10	10	11	4	1	False	False	False
S-11	10	10	2	1	False	False	False
S-12	10	15	3	2	False	False	False
S-13	10	13	2	1	False	False	False
S-14	9	19	4	2	False	True	False
S-15	15	16	3	1	False	False	False
S-16	14	28	4	2	False	True	False
S-17	5	40	3	8	False	True	False
S-18	10	27	3	3	False	False	False

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Table A.1 – *Continued from previous page*

Name	Clients	Jobs	Nurse	Days	Unavail.	Rotations	Apprentice
S-19	15	16	3	1	False	False	False
S-20	20	21	4	1	False	False	False
S-21	12	22	4	2	False	False	False
S-22	15	20	4	2	False	False	False
S-23	14	47	4	3	False	True	False
S-24	15	43	3	3	False	False	False
S-25	10	64	4	7	False	False	False
S-26	6	168	6	14	False	True	False
S-27	20	53	4	3	False	False	False
S-28	10	300	6	15	False	True	False
S-29	10	60	5	3	False	True	False
S-30	20	21	4	1	False	False	False

Table A.2.: Test set instances - Sets G1-G4

Name	Clients	Jobs	Nurse	Days	Unavail.	Rotations	Apprentice
G1-01	40	275	18	7	False	False	False
G1-02	40	255	17	7	False	False	False
G1-03	50	320	24	7	False	False	False
G1-04	30	175	10	7	False	False	False
G1-05	40	249	19	7	False	False	False
G1-06	60	395	25	7	False	False	False
G1-07	50	681	26	14	False	False	False
G1-08	30	378	9	14	False	False	False
G1-09	50	640	27	14	False	False	False
G1-10	40	525	21	14	False	False	False
G1-11	40	484	22	14	False	False	False
G1-12	60	744	33	14	False	False	False
G2-01	30	192	16	7	True	False	False
G2-02	25	154	10	7	True	False	False
G2-03	25	156	10	7	True	False	False
G2-04	50	318	24	7	True	False	False
G2-05	60	388	20	7	True	False	False
G2-06	20	126	6	7	True	False	False
G2-07	25	148	10	7	True	False	False
G2-08	25	293	14	14	True	False	False

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Table A.2 – *Continued from previous page*

Name	Clients	Jobs	Nurse	Days	Unavail.	Rotations	Apprentice
G2-09	50	647	27	14	True	False	False
G2-10	40	510	21	14	True	False	False
G2-11	60	741	28	14	True	False	False
G2-12	25	306	13	14	True	False	False
G2-13	60	766	33	14	True	False	False
G3-01	25	316	10	14	True	True	False
G3-02	30	384	15	14	True	True	False
G3-03	40	497	18	14	True	True	False
G3-04	50	647	25	14	True	True	False
G3-05	40	525	17	14	True	True	False
G3-06	30	388	13	14	True	True	False
G3-07	25	300	10	14	True	True	False
G3-08	30	349	10	14	True	True	False
G3-09	40	506	19	14	True	True	False
G3-10	40	991	20	28	True	True	False
G3-11	20	525	8	28	True	True	False
G3-12	30	731	9	28	True	True	False
G3-13	40	1067	22	28	True	True	False
G3-14	60	1508	28	28	True	True	False
G3-15	30	735	17	28	True	True	False
G3-16	40	1013	22	28	True	True	False
G4-01	20	251	8	14	True	True	True
G4-02	25	303	13	14	True	True	True
G4-03	30	393	17	14	True	True	True
G4-04	40	512	22	14	True	True	True
G4-05	60	753	33	14	True	True	True

Appendix B.

Detailed results for the exact approach in a static setting

Table B.1 in this section states the detailed results for the exact approach in a static setting with 12 hours computation time. For each instance the results of the three settings *Hard*, *Soft* and *Soft+Start* are stated. The objective value and remaining gap between lower and upper bound are given in the columns *Obj* and *Gap* (in %), respectively. Note that the objective value in the latter two cases (*Soft* and *Soft+Start*) can contain penalty costs. The objective and gap values are presented for all three settings. In contrast to this, the tour length without penalty costs (*TL*), the number of unassigned jobs ($|\mathcal{U}|$) and single shifts on weekends (*WE*) are stated only for the settings *Soft* and *Soft+Start* because the setting *Hard* does not relax the respective constraints.

The MIP objective and bound for the setting *Soft+Start* and 48 hours computation time are given in Table C.5 together with the results for the heuristics.

Table B.1.: MIP results for entire test set

	Hard		**Soft**					**Soft + Start**				
Name	Gap	Obj	Gap	Obj	TL	$\lvert\mathcal{U}\rvert$	WE	Gap	Obj	TL	$\lvert\mathcal{U}\rvert$	WE
CS1-01	4.77	2164	5.40	2169	2169	0	0	3.84	2142	2142	0	0
CS1-02	2.72	2057	2.43	2051	2051	0	0	2.78	2051	2051	0	0
CS1-03	2.76	2041	2.96	2041	2041	0	0	2.81	2041	2041	0	0
CS1-04	3.05	2036	3.18	2045	2045	0	0	2.99	2040	2040	0	0
CS1-05	4.14	2244	5.96	2281	2281	0	0	4.97	2269	2269	0	0
CS1-06	5.40	2284	5.18	2277	2277	0	0	5.22	2285	2285	0	0
CS1-07	3.55	2345	3.23	2345	2345	0	0	3.21	2345	2345	0	0
CS1-08	-	-	1.60	3369	3369	0	0	1.60	3369	3369	0	0
CS1-09	-	-	1.49	3356	3356	0	0	1.70	3356	3356	0	0
CS1-10	-	-	1.51	3690	3690	0	0	1.60	3696	3696	0	0
CS1-11	-	-	4.05	3853	3853	0	0	4.62	3876	3876	0	0
CS2-01	7.45	2939	99.98	12002512	2512	12	0	6.56	2911	2911	0	0
CS2-02	8.38	2964	9.00	3000	3000	0	0	7.36	2930	2930	0	0
CS2-03	-	-	7.11	2956	2956	0	0	7.65	2942	2942	0	0
CS2-04	-	-	3.95	2872	2872	0	0	3.55	2874	2874	0	0
CS2-05	-	-	3.98	2893	2893	0	0	4.40	2904	2904	0	0
CS2-06	-	-	4.19	2874	2874	0	0	3.54	2867	2867	0	0
CS2-07	-	-	4.65	2969	2969	0	0	4.19	2959	2959	0	0
CS2-08	-	-	4.24	2962	2962	0	0	4.33	2977	2977	0	0
CS2-09	-	-	4.75	2991	2991	0	0	4.68	2989	2989	0	0
CS2-10	-	-	4.67	2994	2994	0	0	3.01	2955	2955	0	0
CS2-11	-	-	99.70	1003194	3194	1	0	99.69	1003269	3269	1	0
CS3-01	-	-	99.50	1005164	5164	1	0	99.50	1005148	5148	1	0

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Table B.1 – *Continued from previous page*

	Hard		**Soft**					**Soft + Start**				
Name	Gap	Obj	Gap	Obj	TL	$\lvert\mathcal{U}\rvert$	WE	Gap	Obj	TL	$\lvert\mathcal{U}\rvert$	WE
CS3-02	-	-	99.49	1005201	5201	1	0	99.49	1005158	5158	1	0
CS3-03	-	-	99.50	1005180	5180	1	0	99.50	1005239	5239	1	0
CS3-04	-	-	3.32	5306	5306	0	0	2.94	5274	5274	0	0
CS3-05	-	-	3.43	5297	5297	0	0	2.45	5255	5255	0	0
CS3-06	-	-	4.62	5453	5453	0	0	5.32	5486	5486	0	0
CS3-07	-	-	4.55	5450	5450	0	0	5.25	5502	5502	0	0
CS4-01	-	-	99.37	1006748	6748	1	0	6.29	6735	6735	0	0
CS4-02	-	-	5.88	6687	6687	0	0	6.49	6731	6731	0	0
CS4-03	-	-	99.95	12006349	6349	12	0	6.02	6735	6735	0	0
CS4-04	-	-	99.99	95007137	7137	95	0	6.93	10987	10987	0	0
G1-01	-	-	99.98	44009725	9725	44	0	62.66	25076	12116	0	1
G1-02	-	-	99.95	19009783	9783	19	0	16.57	10577	10577	0	0
G1-03	-	-	99.98	55011589	11589	55	0	13.35	13026	13026	0	0
G1-04	-	-	6.54	9453	9453	0	0	6.47	9493	9493	0	0
G1-05	-	-	99.99	106008137	8137	106	0	99.71	4030264	17304	4	1
G1-06	-	-	99.99	264007139	7139	264	0	99.86	13023274	23274	13	0
G1-07	-	-	-	-	-	-	-	80.00	112469	34709	0	6
G1-08	-	-	99.92	17015698	15698	17	0	99.95	27044602	18682	27	2
G1-09	-	-	-	-	-	-	-	68.79	69096	30216	0	3
G1-10	-	-	99.99	196014491	14491	196	0	80.09	94773	29973	0	5
G1-11	-	-	99.99	155017703	17703	155	0	99.44	4029109	29109	4	0
G1-12	-	-	-	-	-	-	-	96.98	1086343	47463	1	3
G2-01	3.97	6884	4.78	6938	6938	0	0	87.18	51567	12687	0	3
G2-02	1.38	5094	0.73	5063	5063	0	0	0.68	5062	5062	0	0
G2-03	0.76	5858	0.66	5847	5847	0	0	99.43	1021013	8053	1	1

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Table B.1 – *Continued from previous page*

	Hard		**Soft**					**Soft + Start**				
Name	Gap	Obj	Gap	Obj	TL	$\lvert\mathcal{U}\rvert$	WE	Gap	Obj	TL	$\lvert\mathcal{U}\rvert$	WE
G2-04	-	-	99.99	106009747	9747	106	0	98.89	1043390	17470	1	2
G2-05	-	-	99.99	116011145	11145	116	0	99.66	4030683	17723	4	1
G2-06	-	-	17.30	5734	5734	0	0	16.00	5646	5646	0	0
G2-07	2.65	7946	2.25	7948	7948	0	0	2.42	7952	7952	0	0
G2-08	-	-	7.11	10772	10772	0	0	82.04	55737	16857	0	3
G2-09	-	-	-	-	-	-	-	67.61	70178	31298	0	3
G2-10	-	-	99.99	152013862	13862	152	0	99.15	2035297	22337	2	1
G2-11	-	-	-	-	-	-	-	56.73	58273	32353	0	2
G2-12	-	-	99.19	2017372	17372	2	0	99.77	7047938	22018	7	2
G2-13	-	-	-	-	-	-	-	83.92	200649	58089	0	11
G3-01	-	-	99.63	3011994	11994	3	0	99.46	2057388	18508	2	3
G3-02	-	-	99.97	49012919	12919	49	0	98.85	1131389	27709	1	8
G3-03	-	-	99.98	68018036	18036	68	0	88.75	150769	34129	0	9
G3-04	-	-	99.99	269016229	16229	269	0	86.96	170329	40729	0	10
G3-05	-	-	99.99	181015718	15718	181	0	99.99	181015718	15718	181	0
G3-06	-	-	99.97	50013768	13768	50	0	85.89	104459	26699	0	6
G3-07	-	-	99.96	36014943	14943	36	0	99.51	3105523	27763	3	6
G3-08	-	-	99.96	49032372	19412	49	1	99.96	50033505	20545	50	1
G3-09	-	-	99.99	266014663	14663	266	0	99.52	5119829	42069	5	6
G3-10	-	-	-	-	-	-	-	85.10	226817	58337	0	13
G3-11	-	-	99.79	9020646	20646	9	0	99.84	12152435	35795	12	9
G3-12	-	-	99.98	133024952	24952	133	0	99.95	57116482	38722	57	6
G3-13	-	-	-	-	-	-	-	85.72	263097	68697	0	15
G3-14	-	-	-	-	-	-	-	86.15	381597	96477	0	22
G3-15	-	-	99.99	318023905	23905	318	0	86.47	262033	67633	0	15

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Table B.1 – *Continued from previous page*

	Hard		Soft					Soft + Start				
Name	Gap	Obj	Gap	Obj	TL	$\|\mathcal{U}\|$	WE	Gap	Obj	TL	$\|\mathcal{U}\|$	WE
G3-16	-	-	-	-	-	-	-	81.59	268692	87252	0	14
G4-01	-	-	99.91	11009725	9725	11	0	99.88	8040160	14240	8	2
G4-02	-	-	99.96	38015105	15105	38	0	98.63	1076210	24370	1	4
G4-03	-	-	99.98	120015976	15976	120	0	76.19	82704	30864	0	4
G4-04	-	-	99.99	179017817	17817	179	0	85.68	160553	43913	0	9
G4-05	-	-	-	-	-	-	-	99.47	6232858	64378	6	13
TH1-01	0.00	4331	0.00	4331	4331	0	0	0.00	4331	4331	0	0
TH1-02	1.52	6707	1.95	6707	6707	0	0	1.88	6707	6707	0	0
TH1-03	1.67	7231	1.57	7231	7231	0	0	1.27	7231	7231	0	0
TH1-04	0.00	7568	0.00	7568	7568	0	0	0.00	7568	7568	0	0
TH1-05	1.31	8487	3.02	8487	8487	0	0	1.18	8487	8487	0	0
TH1-06	0.00	8991	0.00	8991	8991	0	0	0.00	8991	8991	0	0
TH1-07	6.12	8659	4.65	8651	8651	0	0	4.68	8659	8659	0	0
TH2-01	0.00	10443	0.00	10443	10443	0	0	0.00	10443	10443	0	0
TH2-02	0.25	7544	0.25	7544	7544	0	0	0.33	7544	7544	0	0
TH2-03	1.42	9492	0.00	9492	9492	0	0	0.83	9492	9492	0	0
TH3-01	0.00	11087	0.00	11087	11087	0	0	0.00	11087	11087	0	0
TH3-02	-	-	8.25	10787	10787	0	0	8.29	10776	10776	0	0
TH3-03	12.68	13327	12.35	13419	13419	0	0	12.60	13486	13486	0	0
TH3-04	4.42	12882	3.39	12857	12857	0	0	5.78	12855	12855	0	0
TH3-05	7.33	11039	5.66	10825	10825	0	0	6.66	10950	10950	0	0
TH3-06	-	-	99.02	1011274	11274	1	0	99.02	1011085	11085	1	0
TH3-07	11.05	13966	7.91	13737	13737	0	0	8.87	13554	13554	0	0
TH3-08	-	-	13.82	12925	12925	0	0	13.35	12823	12823	0	0
TH4-01	10.93	12935	9.76	12824	12824	0	0	10.62	12921	12921	0	0

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Table B.1 – *Continued from previous page*

	Hard		**Soft**					**Soft + Start**				
Name	Gap	Obj	Gap	Obj	TL	$\|\mathcal{U}\|$	WE	Gap	Obj	TL	$\|\mathcal{U}\|$	WE
TH4-02	6.36	15710	6.54	15774	15774	0	0	5.79	15682	15682	0	0
TH4-03	-	-	8.92	14191	14191	0	0	8.59	14141	14141	0	0

Appendix C.

Detailed results for the heuristic approaches in a static setting

C.1. Parameter settings before and after algorithm configuration

Tables C.1 to C.4 show the results of the algorithm configuration by GGA [Ansótegui et al., 2009] for the construction heuristic, LNS, ALNS and RVNS, respectively. The tables contain the following information. The column *Param.* indicates the name of the parameter. The defined ranges of the parameter are given in column *Range*. Furthermore, the value of the parameter before (*Def.* = default) and after (*Tuned*) the tuning is presented. Column *Description* contains a short explanation of the parameter.

Table C.1.: Default and tuned parameters for construction heuristic (time-oriented, nearest neighbor heuristic)

Param.	**Range**	**Def.**	**Tuned**	**Description**
ω^{Dis}	$[0.0, 1.0]$	0.1	0.24758811	Weight for distance
ω^{AT}	$[0.0, 1.0]$	0.4	0.58029951	Weight for added time to tour
ω^{RT}	$[0.0, 1.0]$	0.2	0.05444633	Weight for remaining time until time window end is reached
ω^{Q}	$[0.0, 1.0]$	0.3	0.11766605	Weight for qualification frequency

Table C.2.: Default and tuned parameters for LNS

Param.	**Range**	**Def.**	**Tuned**	**Description**
w^{Start}	$[0.4, 0.8]$	0.5	0.678123	Parameter for calculation of start temperature
f^{Cool}	$[0.5, 0.9999]$	0.9	0.662491	Temperature cooling factor
it^{Cool}	$[1, 10]$	1	4	In every it^{Cool} iterations the temperature is cooled
f^{Reheat}	$[0.0, 1.0]$	0.5	0.432538	Reheating factor (percentage of start temp.)
it^{Reheat}	$\{50, 100, 200, 500\}$	100	200	Number of iterations that pass at minimum temperature until reheating

Table C.3.: Default and tuned parameters for ALNS

Param.	**Range**	**Def.**	**Tuned**	**Description**
w^{Start}	$[0.4, 0.8]$	0.5	0.659006	Parameter for start temperature determination
f^{Cool}	$[0.5, 0.9999]$	0.9	0.562291	Temperature cooling factor
it^{Cool}	$[1, 10]$	1	1	In every it^{Cool} the temperature is cooled
f^{Reheat}	$[0.0, 1.0]$	0.5	0.400035	Reheating factor (percentage of start temperature)
it^{Reheat}	$\{50, 100, 200, 500\}$	100	500	Number of iterations that pass at minimum temperature until reheating
$decay$	$[0.0, 1.0]$	0.9	0.114645	ALNS decay parameter
F^{Best}	$[1, 40]$	7	23	Operator update weight for new global best solution
F^{Better}	$[1, 40]$	4	20	Operator update weight for new local best solution
$F^{Accepted}$	$[1, 40]$	2	35	Operator update weight for accepted solution
$F^{Rejected}$	$[1, 40]$	1	15	Operator update weight for rejected solution
ω^{Dis}_{ALNS}	$[0.0, 1.0]$	0.1	0.279005	Weight for distance in time-oriented nearest neighbor heuristic in repair of ALNS
ω^{AT}_{ALNS}	$[0.0, 1.0]$	0.4	0.058017	Weight for added time to tour in time-oriented nearest neighbor heuristic in repair of ALNS
ω^{RT}_{ALNS}	$[0.0, 1.0]$	0.2	0.276847	Weight for remaining time until time window end is reached in time-oriented nearest neighbor heuristic in repair of ALNS
ω^{Q}_{ALNS}	$[0.0, 1.0]$	0.3	0.386131	Weight for qualification frequency in time-oriented nearest neighbor heuristic in repair of ALNS
τ^{Exist}	$[1, 100]$	5	87	Limit of jobs considered for insertion in existing tour of time-oriented, nearest neighbor heuristic
τ^{Neu}	$[1, 500]$	50	103	Limit of jobs considered for insertion in new tour of time-oriented, nearest neighbor heuristic
ω^{IS}	$[0.0, 1.0]$	0.2	0.175172	Weight for self-impact IS in greedy look-ahead heuristic
ω^{IR}	$[0.0, 1.0]$	0.3	0.407108	Weight for inner-route impact IR in greedy look-ahead heuristic
ω^{IU}	$[0.0, 1.0]$	0.4	0.353553	Weight for impact on unassigned IU in greedy look-ahead heuristic
ω^{QF}	$[0.0, 1.0]$	0.1	0.064167	Weight for qualification frequency in greedy look-ahead heuristic
ω^{IR}_1	$[0.0, 1.0]$	0.2	0.574714	Weight for distance in inner-route impact
ω^{IR}_2	$[0.0, 1.0]$	0.3	0.413988	Weight for time windows of others in inner-route impact
ω^{IR}_3	$[0.0, 1.0]$	0.5	0.011298	Weight for own time window in inner-route impact
τ^{Tours}	$[1, 10]$	3	6	Number of tours considered in greedy look-ahead heuristic

Table C.4.: Default and tuned parameters for RVNS

Param.	**Range**	**Def.**	**Tuned**	**Description**
Order	$\{1, 2, 3, 4\}$	1	1	Operator order
Job remove	$\{relative, absolute\}$	relative	absolute	Relative / absolute removal rate for job operator
Tour remove	$\{relative, absolute\}$	relative	absolute	Relative / absolute removal rate for tour operator
Nurse remove	$\{relative, absolute\}$	relative	relative	Relative / absolute removal rate for nurse operator
$level^J$	$[1, 3]$	3	2	Operator levels of job operator
$level^T$	$[1, 3]$	2	2	Operator levels of tour operator
$level^N$	$[1, 3]$	2	1	Operator levels of nurse operator
mul^J	$[1, 3]$	2	1	Level multiplier for job operator
mul^T	$[1, 3]$	2	2	Level multiplier for tour operator
mul^N	$[1, 3]$	2	2	Level multiplier for nurse operator
l^{minJ}	$[1, 20]$	5	5	Minimum removal rate for job operator
l^{minT}	$[1, 20]$	2	1	Minimum removal rate for tour operator
l^{minN}	$[1, 20]$	1	13	Minimum removal rate for nurse operator

C.2. Averaged results per instance

The results for the three heuristics LNS, ALNS and RVNS are given in Table C.5 for each instance in the test set. The instance name is given in the first column (*Inst.*). The next two columns contain the objective value (*Obj*) and lower bound (*LB*) of the MIP with the setting *Soft+Start* after 48 hours computation time as reference values for the results. For each of the three heuristics the objective value (*Obj*) and the gap to the MIP objective value (*Gap obj*) and lower bound (*Gap LB*) are presented. The results of the heuristics are averaged over 10 runs with a computation time of five minutes for each run.

Table C.5.: Results of the three heuristics for entire test set

Inst.	**MIP**		**LNS**			**ALNS**			**RVNS**		
	Obj	LB	Obj	Gap obj	Gap LB	Obj	Gap obj	Gap LB	Obj	Gap obj	Gap LB
CS1-01	2142	2061.7	2220.6	3.54	7.16	2178.6	1.68	5.37	2170.7	1.32	5.02
CS1-02	2051	1994.1	2113.9	2.98	5.67	2086.3	1.69	4.42	2068.1	0.82	3.58
CS1-03	2041	1984.3	2099	2.76	5.47	2074.8	1.63	4.36	2051.3	0.50	3.27
CS1-04	2040	1979	2129	4.18	7.05	2075.5	1.71	4.65	2054.1	0.68	3.65
CS1-05	2262	2156.5	2329.5	2.90	7.42	2284.8	1.00	5.61	2266.3	0.19	4.84
CS1-06	2285	2165.9	2343.7	2.50	7.58	2302.7	0.77	5.94	2291.7	0.29	5.49
CS1-07	2345	2270.6	2428	3.42	6.48	2398.7	2.24	5.34	2388.7	1.83	4.95
CS1-08	3369	3315	3561.6	5.41	6.92	3425.1	1.64	3.21	3398.2	0.86	2.45
CS1-09	3356	3299	3525	4.79	6.41	3426.1	2.04	3.71	3382.6	0.79	2.47
CS1-10	3690	3638.1	3867	4.58	5.92	3781.1	2.41	3.78	3755.7	1.75	3.13
CS1-11	3876	3697	4008	3.29	7.76	3970.9	2.39	6.90	3899.3	0.60	5.19
CS2-01	2910	2720	3000	3.00	9.33	2992.9	2.77	9.12	2955.1	1.53	7.95
CS2-02	2930	2714.7	3037	3.52	10.61	3005.1	2.49	9.66	2954.5	0.82	8.11
CS2-03	2935	2717	3036	3.33	10.51	2998.6	2.11	9.38	2968.3	1.12	8.46
CS2-04	2874	2772	3003	4.30	7.69	2942.3	2.32	5.78	2893.4	0.67	4.19
CS2-05	2904	2776.2	2999	3.17	7.43	2927.6	0.80	5.16	2890.3	-0.47	3.95
CS2-06	2867	2766	2991	4.15	7.52	2956.8	3.03	6.45	2888	0.73	4.22
CS2-07	2939	2835	3088	4.83	8.19	3050.4	3.65	7.06	3007.3	2.27	5.73
CS2-08	2965	2848.2	3081.3	3.77	7.57	3053.8	2.90	6.72	2996.2	1.04	4.94
CS2-09	2985	2849	3119.7	4.32	8.68	3037.2	1.71	6.19	2992.3	0.24	4.79
CS2-10	2955	2866	3104	4.80	7.67	3048.3	3.05	5.97	3028.7	2.43	5.37
CS2-11	1003240	3061.8	1003356	0.01	99.69	1003304.4	0.01	99.69	1003271.7	0.00	99.69
CS3-01	1005146	5073	1005379	0.02	99.50	1005364.3	0.02	99.50	1005227	0.01	99.50
CS3-02	1005158	5079.5	1005371	0.02	99.49	1005342.6	0.02	99.49	1005259.7	0.01	99.49
CS3-03	1005151	5066.4	1005399	0.02	99.50	1005376.7	0.02	99.50	1005242.7	0.01	99.50
CS3-04	5267	5119	5515	4.50	7.18	5470.6	3.72	6.42	5356.8	1.67	4.44
CS3-05	5245	5126	5471	4.13	6.31	5452.6	3.80	5.99	5335.3	1.69	3.92

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Table C.5 – *Continued from previous page*

Inst.	**MIP**		**LNS**			**ALNS**			**RVNS**		
	Obj	LB	Obj	Gap obj	Gap LB	Obj	Gap obj	Gap LB	Obj	Gap obj	Gap LB
CS3-06	5486	5194	5647	2.85	8.02	5625.8	2.48	7.67	5504	0.32	5.63
CS3-07	5479	5213	5661	3.21	7.91	5620.2	2.50	7.24	5521.4	0.77	5.58
CS4-01	6734	6311.2	6895	2.34	8.47	6892.5	2.30	8.43	6792.4	0.86	7.08
CS4-02	6730	6293.9	6928	2.86	9.15	6913.7	2.66	8.96	6818.9	1.30	7.70
CS4-03	6706	6329.4	6935	3.30	8.73	6930.3	3.24	8.67	6845.4	2.04	7.54
CS4-04	10836	10228	11040	1.85	7.36	11002.7	1.51	7.04	10802.2	-0.31	5.32
G1-01	24942	9367.7	10019.8	-59.83	6.51	9692.1	-61.14	3.35	9715.4	-61.05	3.58
G1-02	10433	8819.2	9827.3	-5.81	10.25	9344.1	-10.44	5.61	9446.7	-9.45	6.64
G1-03	13026	11293	12284.7	-5.69	8.07	11801.6	-9.40	4.31	11953.6	-8.23	5.53
G1-04	9483	8910.9	9921.4	4.41	10.18	9651.1	1.73	7.66	21291.1	49.24	52.30
G1-05	4030134	11662	14735.9	-99.63	20.85	13987.6	-99.65	16.62	14019.9	-99.65	16.81
G1-06	10023395	17909	22763.8	-99.77	21.32	21637.3	-99.78	17.22	21983.2	-99.78	18.53
G1-07	112469	22510	24897.9	-77.86	9.58	23620.5	-79.00	4.70	24000.1	-78.66	6.21
G1-08	27044601	14267	16486.9	-99.94	13.46	215698.2	-99.20	27.15	315586.5	-98.83	35.35
G1-09	69096	21680	24263.8	-64.88	10.64	22981.9	-66.74	5.66	23547.8	-65.92	7.93
G1-10	94722	18878	21184.9	-77.63	10.88	20448.9	-78.41	7.68	20742.9	-78.10	8.99
G1-11	4029162	22788	27748	-99.31	17.87	25600.9	-99.36	10.99	26299	-99.35	13.35
G1-12	1086416	32921	41536.6	-96.18	20.74	38553.2	-96.45	14.59	40280.5	-96.29	18.26
G2-01	6775	6616.4	7783.8	12.95	14.99	6890.7	1.68	3.98	6957.6	2.62	4.90
G2-02	5060	5029	5588.7	9.46	10.01	5117.3	1.12	1.72	5149.1	1.73	2.33
G2-03	5859	5809.6	6085.9	3.72	4.54	5918.9	1.01	1.85	5903.5	0.75	1.59
G2-04	43520	11614	13059.9	-69.99	11.06	12612.5	-71.02	7.91	12750.4	-70.70	8.91
G2-05	4030410	13552	14922.8	-99.63	9.18	14380.1	-99.64	5.76	14403.4	-99.64	5.91
G2-06	1005370	4726.6	1505517.4	24.94	99.65	1605460.1	29.93	99.67	2305528.8	54.87	99.79
G2-07	7946	7761	8514.4	6.67	8.84	8120.8	2.15	4.43	8150.1	2.50	4.77
G2-08	55737	10016	11436	-79.48	12.40	10278.3	-81.56	2.55	10531.8	-81.10	4.89
G2-09	70253	22783	25674.4	-63.45	11.26	24530	-65.08	7.12	25120.2	-64.24	9.30
G2-10	2035580	17316	19765.4	-99.03	12.38	18679.6	-99.08	7.30	18936.6	-99.07	8.56
G2-11	58199	25320	28711.8	-50.67	11.81	27273.1	-53.14	7.16	27712.4	-52.38	8.63

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Table C.5 – *Continued from previous page*

Inst.	MIP		LNS			ALNS			RVNS		
	Obj	LB	Obj	Gap obj	Gap LB	Obj	Gap obj	Gap LB	Obj	Gap obj	Gap LB
G2-12	17198	16338	18767.3	8.36	12.94	17957.4	4.22	9.01	18018.5	4.54	9.32
G2-13	200660	32404	41059.2	-79.54	21.08	37584.4	-81.27	13.78	39940.1	-80.10	18.86
G3-01	2057388	11147	12837	-99.38	13.16	11729.5	-99.43	4.95	11712.1	-99.43	4.82
G3-02	1131389	12971	14558	-98.71	10.89	13487.3	-98.81	3.82	13441.5	-98.81	3.50
G3-03	150769	16976	19978.9	-86.75	15.02	17673.1	-88.28	3.94	18025.6	-88.04	5.82
G3-04	170329	22232	24013.4	-85.90	7.41	23358.1	-86.29	4.82	23598.1	-86.15	5.79
G3-05	169045011	19145	21716.3	-99.99	11.83	20793.4	-99.99	7.93	21091.8	-99.99	9.23
G3-06	104459	14761	16618.8	-84.09	11.18	15559	-85.11	5.13	15709.1	-84.96	6.03
G3-07	3105523	15263	17405.8	-99.44	12.31	16725.2	-99.46	8.74	16696.1	-99.46	8.58
G3-08	50033505	18539	3121466.2	-93.76	99.27	4832208.7	-90.34	99.51	2823728.2	-94.36	99.26
G3-09	5119796	24599	30984	-99.39	20.60	29710.9	-99.42	17.20	30180.9	-99.41	18.49
G3-10	226817	33831	36647	-83.84	7.68	35292.1	-84.44	4.14	35588.4	-84.31	4.94
G3-11	12166837	19119	22319.4	-99.82	14.33	22112.9	-99.82	11.27	20597.5	-99.83	7.17
G3-12	56116528	27074	7533017.2	-86.58	99.63	9136508.6	-83.72	99.67	9139087.9	-83.71	99.69
G3-13	263097	37717	42856.4	-83.71	11.98	41216.2	-84.33	8.49	41530.9	-84.21	9.18
G3-14	381597	53025	60176.9	-84.23	11.88	59182.6	-84.49	10.40	60908.8	-84.04	12.94
G3-15	262033	35477	44427.9	-83.04	20.14	41620.3	-84.12	14.76	42766.4	-83.68	17.04
G3-16	268692	49674	62963.8	-76.57	21.10	61173.6	-77.23	18.79	62964.3	-76.57	21.10
G4-01	8040160	9526	10577.9	-99.87	9.94	10230.2	-99.87	6.88	10150.7	-99.87	6.15
G4-02	1076210	14765	17136.1	-98.41	13.83	16403.5	-98.48	9.98	16392.7	-98.48	9.93
G4-03	82704	19782	23506.6	-71.58	15.84	22531.9	-72.76	12.20	22898.2	-72.31	13.60
G4-04	160553	22993	27935.1	-82.60	17.68	26466	-83.52	13.12	27216.1	-83.05	15.52
G4-05	6232840	33607	44003.6	-99.29	23.62	41671.9	-99.33	19.35	43515.3	-99.30	22.77
TH1-01	4331	4331	4368.6	0.86	0.86	4335.4	0.10	0.10	4341.9	0.25	0.25
TH1-02	6707	6635.6	7632.3	12.12	13.05	6856.7	2.18	3.22	6832.2	1.83	2.87
TH1-03	7231	7161.4	7893	8.38	9.26	7350.4	1.62	2.57	7398.3	2.25	3.19
TH1-04	7568	7568	8085.2	6.38	6.38	7634.8	0.87	0.87	7664.6	1.25	1.25
TH1-05	8487	8430.4	8919.3	4.84	5.48	8633.5	1.69	2.35	208695.9	21.84	22.36
TH1-06	8991	8991	9840.8	8.63	8.63	9214.1	2.41	2.41	409379.2	42.09	42.09

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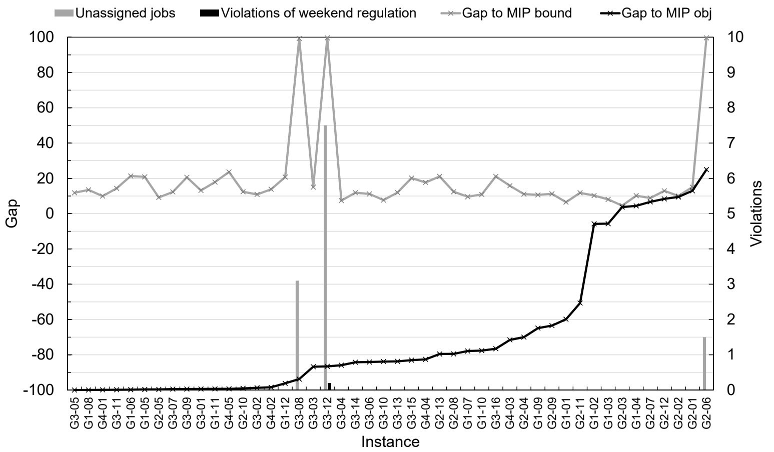
Table C.5 – *Continued from previous page*

Inst.	MIP		LNS			ALNS			RVNS		
	Obj	LB	Obj	Gap obj	Gap LB	Obj	Gap obj	Gap LB	Obj	Gap obj	Gap LB
TH1-07	8659	8282.3	9429.8	8.16	12.16	8930.4	3.03	7.25	9188.6	5.71	9.81
TH2-01	10443	10443	1010169	98.97	98.97	1009823.1	98.97	98.97	1009888.9	98.97	98.97
TH2-02	7544	7521.6	8535.6	11.61	11.87	7837.3	3.73	4.02	7975.8	5.40	5.68
TH2-03	9492	9492	11013.9	13.81	13.81	9665.8	1.79	1.79	9824.6	3.37	3.37
TH3-01	11087	11087	12156.2	8.79	8.79	11280.9	1.72	1.72	11246.5	1.42	1.42
TH3-02	10916	9909.8	411986	45.92	50.90	211172.9	22.08	29.26	1011094.5	60.38	64.03
TH3-03	13438	11762	14721.6	8.72	20.10	13861.1	3.05	15.14	814038.9	70.46	74.15
TH3-04	12862	12407	14401.8	10.69	13.85	13334.2	3.54	6.95	313569.1	33.33	35.69
TH3-05	10925	10240	12169.8	10.22	15.85	11403	4.18	10.19	11674.4	6.41	12.27
TH3-06	1011107	9934	2012890.2	49.77	99.51	2111870.3	51.41	99.52	2012374.9	49.76	99.51
TH3-07	13788	12476	14987.7	7.99	16.74	514442.9	51.00	55.66	314822.4	34.66	40.88
TH3-08	12842	11134	14070	8.70	20.84	113362.2	13.39	24.90	113606.4	15.02	26.32
TH4-01	12972	11580	14880.6	12.82	22.17	13639.7	4.86	15.07	13721.7	5.45	15.60
TH4-02	15626	14810	17677	11.60	16.21	116242.4	13.19	17.72	16451.5	5.00	9.96
TH4-03	14173	12931	16237.8	12.71	20.36	15053.8	5.84	14.09	14795.7	4.20	12.59

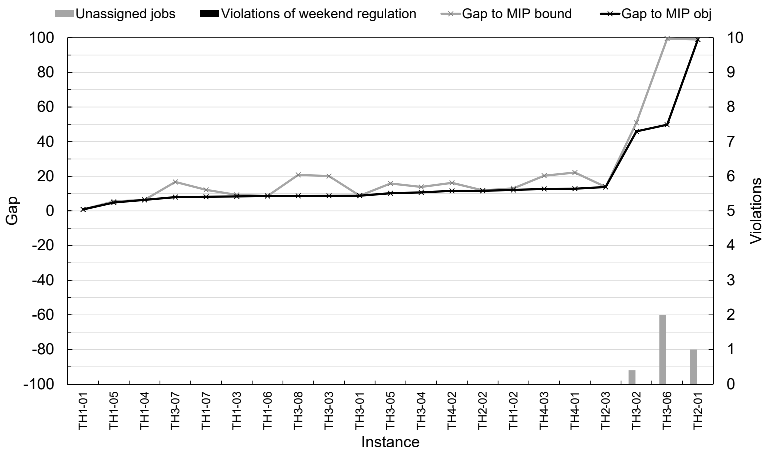
C.3. Additional figures

This section contains additional figures for the analysis of the results of the heuristics.

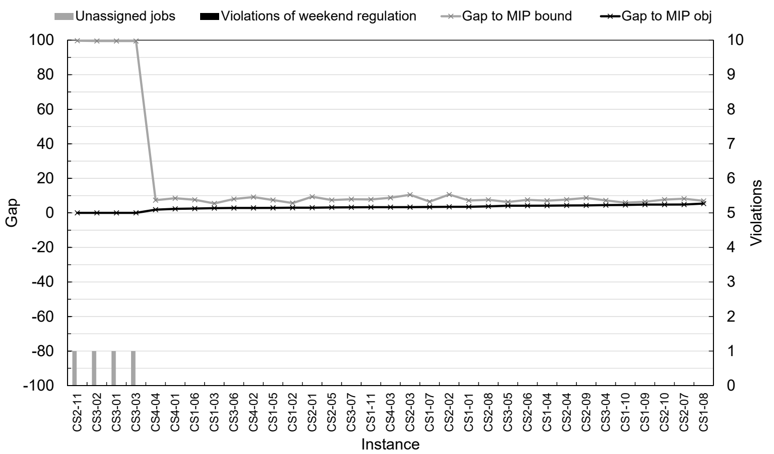
Figures C.1 and C.2 show the comparison of LNS and RVNS results to the MIP results based on the gaps to the MIP objective value and lower bound (values on the primary y-axis) and the violations of soft constraints (values on the secondary y-axis). For each dataset one subfigure is given.



(a) Generated instances

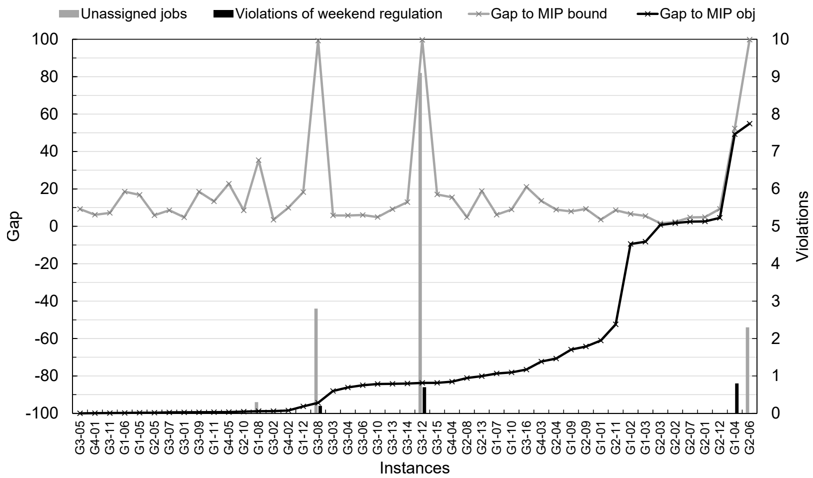


(b) TH instances

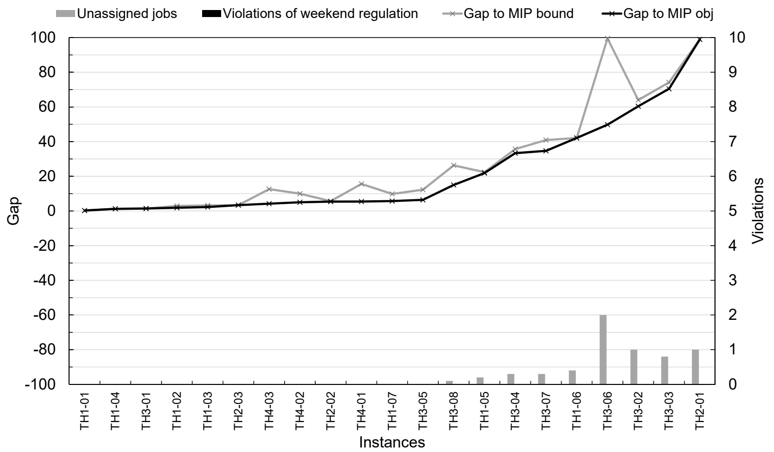


(c) CS instances

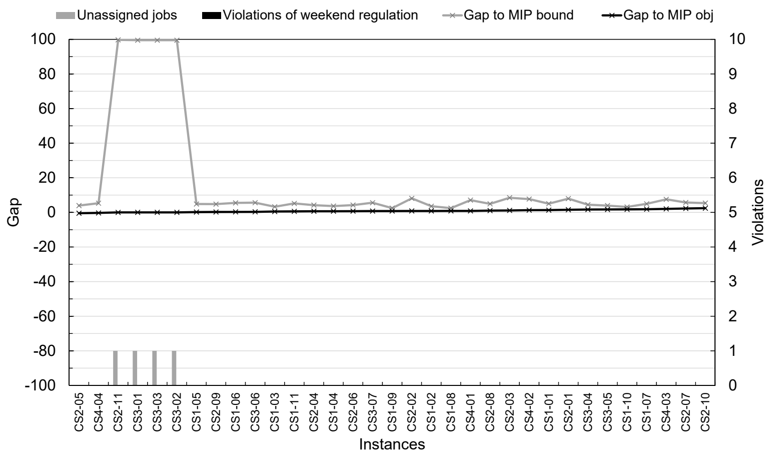
Figure C.1.: Comparison of LNS to MIP results



(a) Generated instances

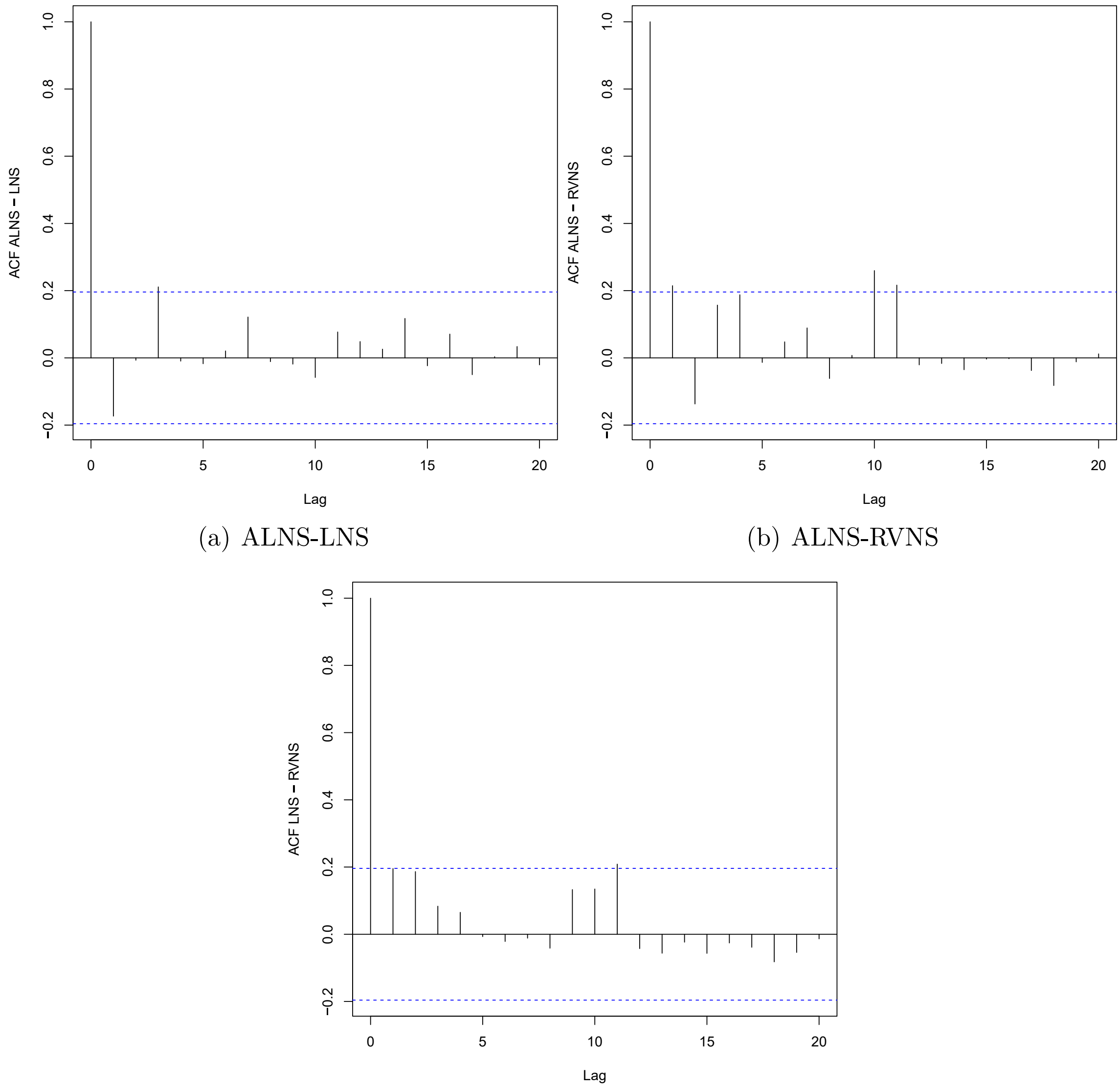


(b) TH instances



(c) CS instances

Figure C.2.: Comparison of RVNS to MIP results



(a) ALNS-LNS

(b) ALNS-RVNS

(c) LNS-RVNS

Figure C.3.: Auto correlation of heuristic runs

The autocorrelation for the pairwise differences of the heuristics results is depicted in Figure C.3. The values used for this analysis are the differences in the gaps to the MIP bound.

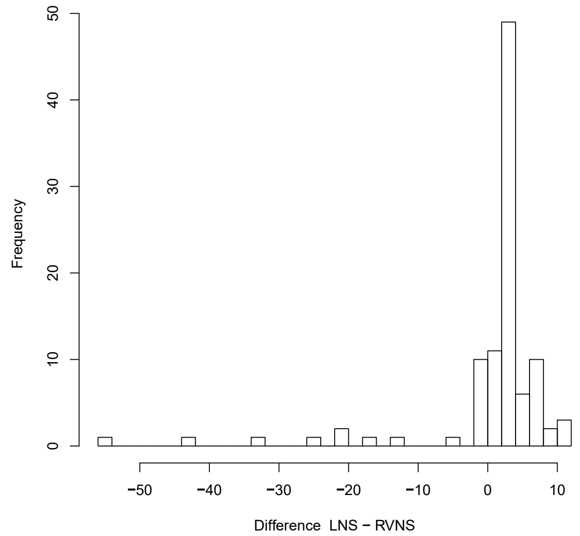
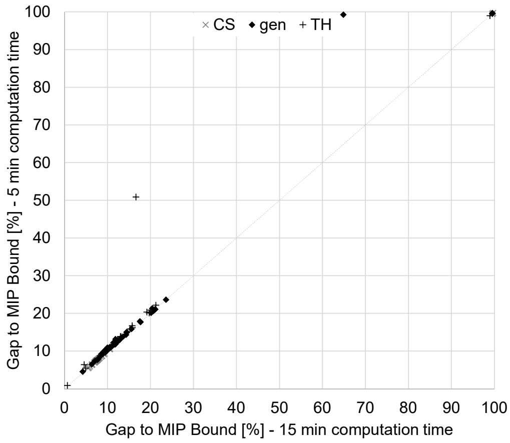
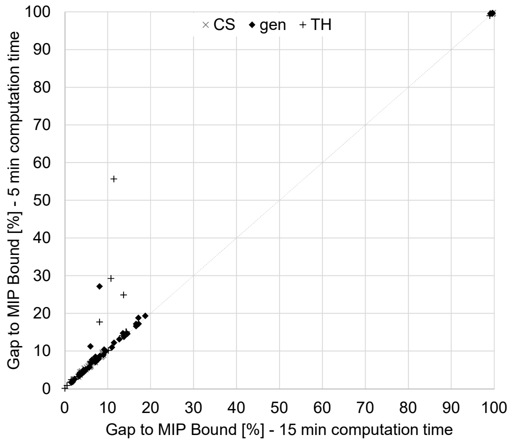


Figure C.4.: Empirical distribution of the difference between the gaps to the MIP bound of LNS-RVNS

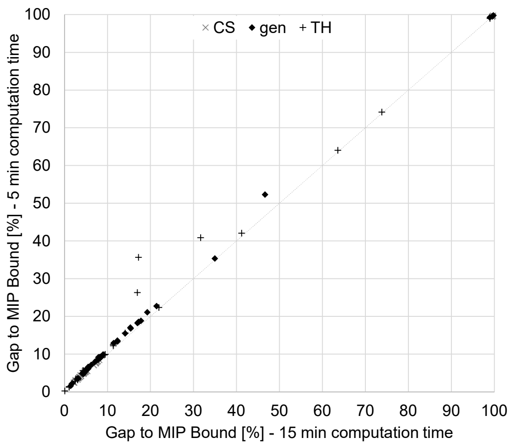
The empirical distribution of the difference between the gaps of the heuristics LNS and RVNS is shown in Figure C.4.



(a) LNS



(b) ALNS



(c) RVNS

Figure C.5.: Influence of computation time on heuristics based on test set data

The improvement of using 15 minutes compared to 5 minutes computation time are shown in Figure C.5 for each heuristic separately. The x-axis indicates the gap to the MIP best bound after 15 minutes and the y-axis the gap after 5 minutes.

Appendix D.

Normalization factors and additional formulas for continuity metrics

D.1. Normalization factors

The normalization for the continuity metrics presented in Section 7.2.1 are given by Equations (D.1) to (D.8). The worst cases are calculated based on the instance information not by values in the solutions.

The worst case for the *continuity of time* metrics is, if a job j is moved from the beginning of its time window a_i to the end b_i or vice versa, which is the maximum possible deviation for one job. To have an equally strong consideration of all jobs, the maximum deviation of all jobs per weeks multiplied by the number of jobs ($|\mathcal{J}_w^C|$) is used as normalization factor. If the worst case for each job individually would be used, the shift of one minute in short time windows would be weighted more than the same shift in a longer time windows. For $CoT^{NormMax}$ only the maximum is considered. The formulas for the normalization factors of metrics $CoT^{Sum}, CoT^{Max}, CoT^t$, and CoT^{Quad} are the following:

$$CoT_w^{NormSum} = \max_{j \in \mathcal{J}_w^C} \{|a_j - b_j|\} |\mathcal{J}_w^C| \tag{D.1}$$

$$CoT_w^{NormMax} = \max_{j \in \mathcal{J}_w^C} \{|a_j - b_j|\} \tag{D.2}$$

$$CoT_w^{NormT} = \max_{j \in \mathcal{J}_w^C} \{|a_j - b_j|\} |\mathcal{J}_w^C| \tag{D.3}$$

$$CoT_w^{NormQuad} = \max_{j \in \mathcal{J}_w^C} \{(a_j - b_j)^2\} |\mathcal{J}_w^C| \tag{D.4}$$

The worst case for the *continuity of care* metric based on nurse reassignments of clients is when a client had no nurses assigned and in the next week all nurses are assigned or vice versa. The normalization factor $CoC^{NormClient}$ (D.5) multiplies the number of nurses with the number clients to determine the value for each week.

$$CoC_w^{NormClient} = |\mathcal{N}_w||\mathcal{C}_w| \tag{D.5}$$

The normalization factor $CoC^{NormClient}$ (D.6) for the job-based continuity of care assumes a new assigned nurse for every job.

$$CoC_w^{NormJob} = |\mathcal{J}_w^C| \tag{D.6}$$

The normalization factors for the continuity of duty schedule metrics CoD^{Type} and CoD^{Time} are stated in Equations (D.7) and (D.8). The worst case for the number of shift type changes is a shift type change on each day of the week. This case is used in $CoD_w^{NormType}$ (D.7).

$$CoD_w^{NormType} = 7|\mathcal{N}_w| \tag{D.7}$$

The maximum time deviation for shift start and end times is a change from earliest possible time on the day to the latest possible time. The earliest and lastest times are determined based on the shift types and their time windows $[A_s, B_s]$. The maximum is multiplied by 14 for the seven days in the week and the two values (start and end).

$$CoD_w^{NormTime} = 14\big(\max_{s\in\mathcal{S}}\{B_s\} - \min_{s\in\mathcal{S}}\{A_s\}\big) \tag{D.8}$$

D.2. Definitions of continuity modes for remaining metrics

This section presents the formulas for the continuity metrics in different continuity modes.

The metric CoT^{Max} for the continuity modes *ToPrevious* and *ToReference* is given in Equations (D.9) and (D.10), respectively. In the former case, the maximum difference between the consecutive weeks is determined. In the latter case, the deviation to the given reference week is used as basis. The modes *Total* and *TotalRef* are not defined for CoT^{Max}.

$$\textit{ToPrevious/ToPrevRef.:} \qquad CoT^{Max} = \sum_{w\in\mathcal{W}} \max_{g\in\mathcal{G}_w} \big\{|z_{g,w} - z_{g,w-1}|\big\} \tag{D.9}$$

$$\textit{ToReference:} \qquad CoT^{Max} = \sum_{w\in\mathcal{W}} \max_{g\in\mathcal{G}_w} \big\{|z_{g,w} - z_{g,1}|\big\} \tag{D.10}$$

The metrics CoT^t and CoT^{Quad} are defined for the different continuity modes similar to CoT^{Sum} as presented in Section 7.2.3. In the modes *ToPrevious* and *ToPreviousPref* the deviation to the previous week is calculated whereas the difference to reference week is taken into account in mode *ToReference*. The deviations in each job group based on the entire planning period is used as basis for the modes *Total* and *TotalRef*. The respective Equations are (D.11) to (D.16).

$$\textit{ToPrevious/ToPrevRef.:} \qquad CoT^t = \sum_{g\in\mathcal{G}} \sum_{w\in\mathcal{W}_g} \max\{\theta, |z_{g,w} - z_{g,w-1}|\} \tag{D.11}$$

$$\textit{ToReference:} \qquad CoT^t = \sum_{g\in\mathcal{G}} \sum_{w\in\mathcal{W}_g} \max\{\theta, |z_{g,w} - z_{g,1}|\} \tag{D.12}$$

Total/TotalRef:
$$CoT^{t} = \sum_{g \in \mathcal{G}} \max\{\theta, \max_{w \in W_g}\{z_{g,w}\} - \min_{w \in W_g}\{z_{g,w}\}\} \tag{D.13}$$

ToPrevious/ToPrevRef.:
$$CoT^{Quad} = \sum_{g \in \mathcal{G}} \sum_{w \in \mathcal{W}_g} \max\{\theta, (z_{g,w} - z_{g,w-1})^2\} \tag{D.14}$$

ToReference:
$$CoT^{Quad} = \sum_{g \in \mathcal{G}} \sum_{w \in \mathcal{W}_g} \max\{\theta, (z_{g,w} - z_{g,1})^2\} \tag{D.15}$$

Total/TotalRef:
$$CoT^{Quad} = \sum_{g \in \mathcal{G}} \max\{\theta, (\max_{w \in W_g}\{z_{g,w}\} - \min_{w \in W_g}\{z_{g,w})^2\}\} \tag{D.16}$$

Only the continuity modes *ToPrevious*, *ToPreviousRef* and *ToReference* are defined for metric CoC^{Job} in Equations (D.18) and (D.20). The difference is the week considered as comparison in (D.17) and (D.19). In the former case the previous week is taken and in the latter case the reference week is used.

ToPrevious/ToPreviousRef:
$$\gamma_{g,w} = \begin{cases} 1, & \text{if } n_{g,w} \neq n_{g,w-1} \\ 0, & \text{otherwise} \end{cases} \tag{D.17}$$
$$CoC^{Job} = \sum_{g \in \mathcal{G}} \sum_{w \in \mathcal{W}_g} \gamma_{g,w} \tag{D.18}$$

ToReference:
$$\gamma'_{g,w} = \begin{cases} 1, & \text{if } n_{g,w} \neq n_{g,1} \\ 0, & \text{otherwise} \end{cases} \tag{D.19}$$
$$CoC^{Job} = \sum_{g \in \mathcal{G}} \sum_{w \in \mathcal{W}_g} \gamma'_{g,w} \tag{D.20}$$

For the continuity of duty schedules metric CoC^{Type} only the continuity modes *ToPrevious*, *ToPreviousRef* and *ToReference* are defined. Again, the difference is the week considered as comparison in (D.17) and (D.19). In the former case the day in the previous week $(d-7)$ is taken and in the latter case the day in the reference week $(d-7w)$ is used.

ToPrevious/ToPreviousRef:
$$\psi_{n,d} = \begin{cases} 1, & \text{if } s_{n,d} \neq s_{n,d-7} \\ 0, & \text{otherwise} \end{cases} \tag{D.21}$$
$$CoD^{Type} = \sum_{n \in \mathcal{N} \setminus \{\mathcal{N}^{\mathrm{R}}\}} \sum_{w \in \mathcal{W}_n} \sum_{d \in w} \psi_{n,d} \tag{D.22}$$

ToReference

$$\psi'_{n,d} = \begin{cases} 1, & \text{if } s_{n,d} \neq s_{n,d-7w} \\ 0, & \text{otherwise} \end{cases} \tag{D.23}$$

$$CoD^{Type} = \sum_{n \in \mathcal{N} \setminus \{\mathcal{N}^{\mathrm{R}}\}} \sum_{w \in \mathcal{W}_n} \sum_{d \in w} \psi'_{n,d} \tag{D.24}$$

(D.25)

The normalization factors presented in the previous section have to adapted if one of the modes *ToReference*, *Total* or *TotalRef* is used.

Appendix E.

Further results for the solution approach in a dynamic setting

E.1. Comparison of continuity metrics with weight 1.0

Additional boxplot diagrams for the continuity types time, care and duty schedules are given in Figures E.1, E.2 and E.3, respectively. The weight of the metrics is 1.0 which means that the tour length is neglected in the objective function.

The values in Figure E.1 are the maximum deviation of start times in each job group. Figures E.2(a) and E.2(b) show the number of nurses assigned to each client and the percentage of jobs in one job group with nurse change, respectively. Figures E.3(a) and E.3(b) show the number of shift type changes per nurse and the deviations in start and end times, respectively.

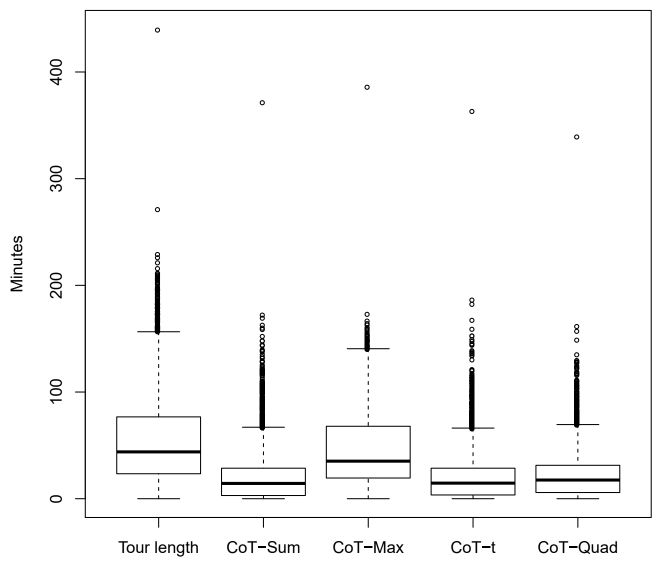
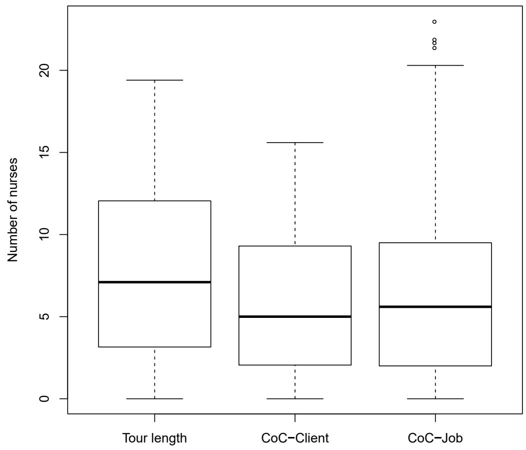
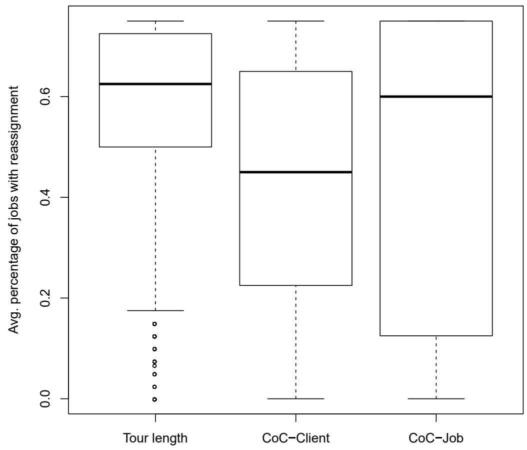


Figure E.1.: Boxplot diagram for continuity of time metrics with weight 1.0. Data points represent the maximum deviation in a job group

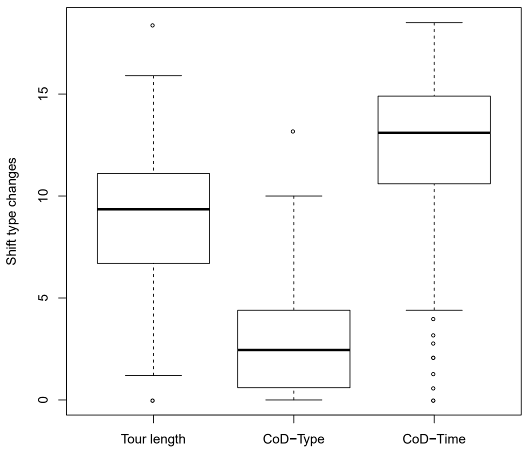


(a) Continuity of care per client. Data points represent the average number of nurses assigned to a client

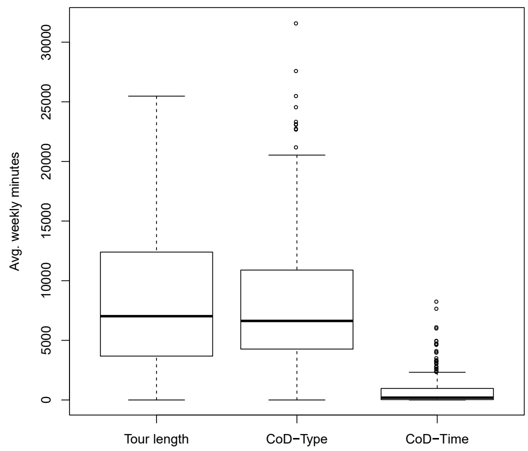


(b) Continuity of care per job group .Data points represent the average percentage of jobs per job group that have a reassignment

Figure E.2.: Boxplot diagrams for continuity of care metrics with weight 1.0



(a) Continuity of duty schedules based on shift types. Data points represent the average sum of shift type changes in the planning horizon

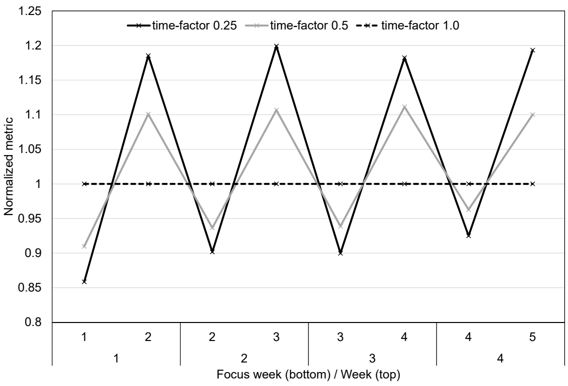


(b) Continuity of duty schedules based on time fluctuations. Data points represent the average sum of minutes that the start and ends deviate per week

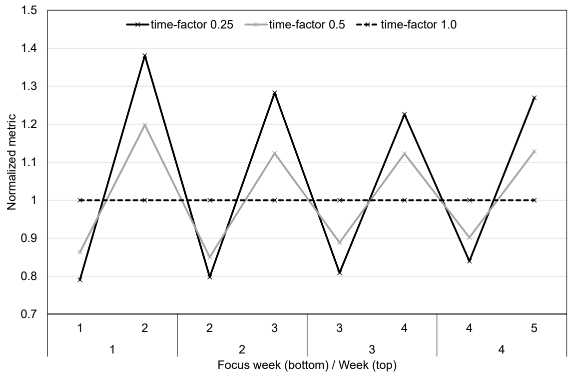
Figure E.3.: Boxplot diagrams for continuity of duty schedules with weight 1.0

E.2. Influence of time-dependent weighting in two week planning period

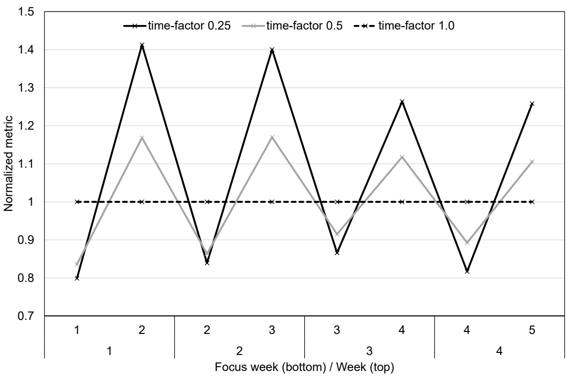
The results for the influence of time-dependent weighting in a two week planning period are given in Figure E.4 for each metric separately. The used time-factors are 1.0, 0.5 and 0.25. The values are normalized based on the value for a time-factor of 1.0. The x-axis indicates the focus weeks (bottom) and weeks in the planning period (top).



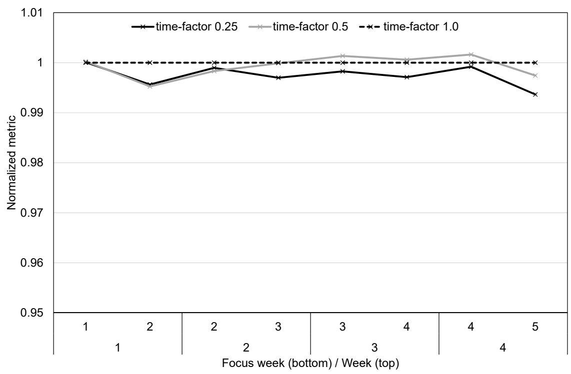
(a) Continuity of time



(b) Continuity of care



(c) Continuity of duty schedules



(d) Tour length

Figure E.4.: Influence of time-dependent weighting in a two week planning horizon

E.3. Analysis of planning period length for scenarios *increase* and *extreme*

The results for the different lengths of planning periods for the scenarios *increase* and *extreme* are given in Figures E.5 and E.6, respectively. The relative values for a planning period of length 4 weeks, 2 weeks and 1 week are compared to each other whereas the value with 1 week planning period is the reference value. The x-axis indicates the metric (bottom) and the planning week (top).

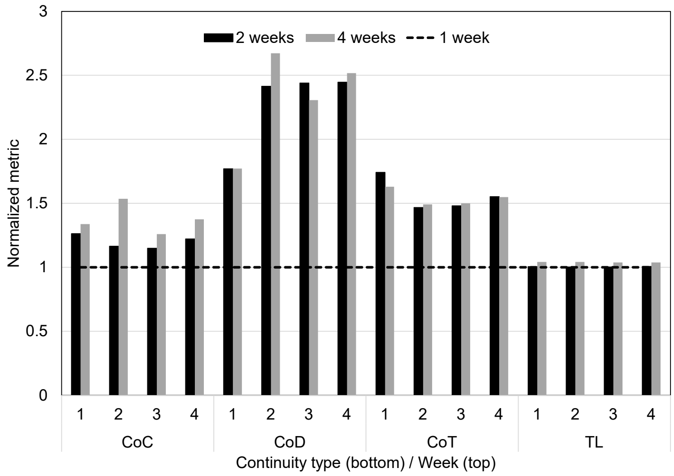


Figure E.5.: Comparison of planning period length in a rolling horizon setting for *increase* scenario

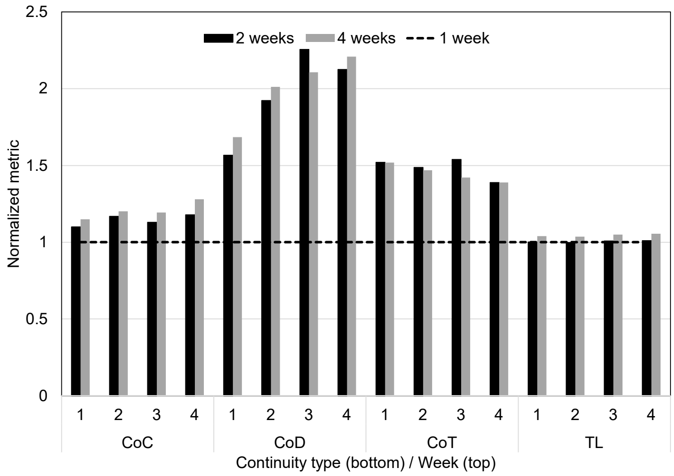


Figure E.6.: Comparison of planning period length in a rolling horizon setting for *extreme* scenario