Lobbying over Exhaustible-Resource Extraction*

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Abstract Consider a lobby group of exhaustible-resource suppliers, which bargains with the government over the extraction of an exhaustible resource and over contribution payments. We characterize the equilibrium extraction path and the development of contribution payments in time. The latter relates to the development of the conflict of interest between profit-maximization and welfare-maximization. Due to stock pollution damages, the government prefers a lower level of cumulative extraction than the lobby group in the long run. In the meantime, the resource suppliers' aim to maximize profits implies that equilibrium extraction may be too slow to maximize welfare, while flow-pollution damages imply that it may be too fast.

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1 Introduction

Once we accept the assumption that lobbyists can have an influence on policy, and if we additionally assume that there are no sufficient counterforces from other interest groups, this distortion is true almost by definition. The aim of this chapter is to show how this lobbying distortion develops in a dynamic model of exhaustible resources.

The literature in the tradition of the Grossman & Helpman (1994) common-agency interest-group model assumes that interest groups offer conditional bribes to the government to induce political distortions in their preferred direction. While such a depiction may be an acceptable caricature of lobbies' political influence for static problems, the political economy of exhaustible resources raises some specific questions due to its inherently dynamic nature. Firstly, how do contribution payments develop over time? This question arises because it might be more convincing to think of a repeated bargaining process instead of a one-shot take-it-or-leave-it offer in a dynamic model. Secondly, how does policy develop over time? And combining the questions, how do contribution valuation and bargaining power affect payments and distortions?

The current chapter characterizes resource extraction in an equilibrium influenced by lobbying. We assume that the government can choose the extraction path, but we also demonstrate how this extraction path can be implemented via resource taxes, while the suppliers get the revenues. As usual in the literature, the government's utility is a weighted sum of a utilitarian welfare function and utility from contribution payments. To focus on the relation between the government and the resource owners' lobby, we do not consider a common-agency setting with many competing lobbies, but assume that only one lobby group exists. Most interest-group models assume that lobby groups are first movers and offer contribution schedules that maximize their surplus given that the government is indifferent about accepting them. In our model, we instead assume that the government has some influence on the outcome as well, that is, there is a (Nash) bargaining, which determines policy and contribution payments. If the government has this more active role in shaping policy, the question arises how the threat of ending cooperation in the future shapes the bargaining outcome. We assume that the parties cannot commit to cooperation. Instead, they bargain in each period about the current values of their variables. We follow Petrosyan (1997) in assuming that the parties leave the bargaining table forever if negotiations fail once.

In our model, the government aims at maximizing the economy's surplus net of environmental externalities. By contrast, the lobby group's sole objective is the firms' intertemporal profit, i.e., it takes the price elasticity of resource demand into account. It turns out that whether extraction in the lobbying equilibrium occurs faster or slower than in the social optimum crucially depends on the magnitudes of these environmental damage and monopoly effects. Because extraction costs are increasing in cumulative extraction and marginal utility of resource consumption is finite, both welfare maximization and profit maximization lead to well-defined convergence levels of cumulative extraction. Although the lobby may want too fast or too slow extraction from the social planner's point of view as long as cumulative extraction is still small, the lobby's preferred total extraction is too high compared to welfare maximization due to, i.a., stock-pollution damages. We show how these considerations cause the contribution payments to develop over time.

In particular, we explicitly derive and discuss the lobbying equilibrium with linear-quadratic functions. Firstly, as long as total extraction has not exceeded the socially optimal level, contribution payments decline if the damage effect outweighs the monopoly effect and stay constant if these effects offset each other; if the government's preferred speed of extraction is greater than that of the lobby, contribution payments increase temporarily. Secondly, in the last case, contribution payments may turn negative for a while. Thirdly, one period before the socially optimal level is reached, the lobby's willingness to pay increases sharply. Lastly, when the socially optimal extraction becomes zero, contribution payments can increase temporarily, but converge to zero while extraction costs increase towards a prohibitively high level.

Our model of the resource market is fairly general. Given that we assume that there is no intertemporal behavior on the demand side, the resource might best be thought of as a fuel like coal or oil. In the model there is flow pollution, like for example soot or dust, and stock pollution, like for example carbon dioxide or permanent landscape changes. The political model is a pure lobbying model without elections. Thus, the best application may be smog and permanent landscape changes in developing or newly industrialized countries with weak institutions. Alternatively, the "government" may be thought of as an environmental regulation agency that has discretion over its policy field.

The main contributions of this chapter are as follows. Firstly, we analyze the distortive influence of a resource owners' lobby on resource extraction and demonstrate that this may lead to too fast or too slow extraction, depending on how much has already been extracted in the past. This result can easily be transferred to the welfare effects of monopolistic extraction. Secondly, we characterize the contribution payments' development and show how they relate to the conflict of interest between the government and the resource owners. Thirdly, we show how a non-negativity constraint on a choice variable shapes the result of dynamic bargaining; in the time period in which accumulated extraction becomes so high that the government would switch to zero extraction if no agreement were reached, the bargaining positions – and thus the contribution payments – change drastically. Moreover, there is no dynamic smoothing effect preventing this drastic change.

This chapter proceeds as follows. In the next section, we discuss the chapter's relation to the literature. Section 3 introduces the basic resource-economic model and derives the welfare-maximizing and the monopolistic profit-maximizing extraction path. Section 4 models lobbying over the extraction path and discusses the results and comparative dynamics. In Section 5, we demonstrate how the bargained extraction path can be implemented via resource taxes. Section 6 concludes.

2 Relation to the Literature

We apply the idea of distortive lobby influence to the extraction of exhaustible resources. This relates this chapter to different strands of literature, namely lobbying, resource extraction, and dynamic bargaining.

The idea how the interaction between the lobby and the government takes place and shapes policy follows the tradition of the Grossman & Helpman (1994, 2001) common-agency lobbying model in so far as the government in our model has a mixed motivation of welfare maximization and contributions, and firms pay for a favorable policy. This literature usually assumes that there are multiple lobby groups that offer competing contribution schedules to the government. In our context, having many lobby groups would not add much insight, so we focus on the influence of one lobby group – that of the resource owners. The bargaining process is summarized by an asymmetric Nash bargaining solution that determines how the surplus of the favorable policy is shared. This is suggested in Grossman & Helpman (2001, Section 7.5), as a generalization of the surplus sharing and, as a shortcut to the results of the usual lobbying-game structure, in Goldberg & Maggi (1999).

There are several applications of this framework to questions of environmental policy. Our model is most closely related to those that also analyze dynamic problems. Damania

& Fredriksson (2000) model a repeated game of collusion between lobbying firms. The first resource-dynamics model we know of that is similar to ours is Barbier et al. (2005). Boyce (2010) models lobbying in the context of renewable resources and common-pool resources. Different from both papers, we explicitly take different price elasticities of demand for the extracted resource into account. By contrast, Barbier et al. (2005) assume a small open economy with an exogenous price, and Boyce (2010) assumes that harvesters have a logarithmic utility function of their resource extraction. Additionally, we explicitly analyze the effects of flow and stock pollution caused by resource extraction.

In modeling resource extraction, we assume that extraction is not limited by the physical stock of the resource, but by the fact that extraction costs increase with cumulative past extraction (cf. Levhari & Liviatan 1977), which is both convenient to model and realistic. This implies that there are no Hotelling rents, but Ricardian rents due to increasing costs (Hartwick 1982). On top of these, however, there are monopoly rents, and increasing them at the expense of welfare is the objective of the lobby. There is a large literature analyzing how the governments of resource-importing countries try to reap the rents of foreign resource suppliers, either Hotelling rents (see, e.g., Bergstrom 1982, Keutiben 2014) or monopoly rents or both (see, e.g., Wirl 1994, Wirl 1995, Rubio & Escriche 2001, Daubanes 2008). In our model, resource suppliers are part of the same country as consumers so that a welfare-maximizing government has no particular interest in distributing rents away from them. However, monopoly rents distort the market outcome exactly because they are linked to monopolistic supply behavior, and this is what the government would like to avoid.

In our model, we see the typical effect that a resource monopolist chooses a slower extraction than competitive, unregulated suppliers in order to increase monopoly rents (cf. Krautkraemer 1998).[1] Compared to this benchmark, however, a welfare-maximizing government would also prefer slower extraction due to the second distortion which is relevant in the model context: environmental damage effects of resource extraction. It has long been known that both a monopoly's tendency to restrict supply and an unregulated industry's ignorance of environmental externalities have to be taken into account for welfare judgments so that a monopoly may be a second-best solution (see, in the context of static Pigouvian taxation, Buchanan 1969, Barnett 1980). Such a trade-off in the judgment of market power also exists in the dynamic context; if a monopolist prefers to restrict supply, this benefits the environ-

[1]Situations where a monopolist chooses a faster extraction are possible, but less common. For an overview of the literature on monopolistic resource supply, see the list of Fischer & Laxminarayan (2005).

ment (see Sweeney 1977). Thus, governmental welfare maximization does not always mean reducing the speed of extraction, even if there are environmental externalities. Nevertheless, the accumulation of stock pollution implies that welfare maximization requires reducing total extraction in the long run and thus a lower speed of extraction from some moment on (cf. Muzondo 1993). In our model, in every moment of time the conflict of interest between the government and the resource lobby is shaped by whether the welfare-maximizing extraction is higher or lower than what a monopolist would choose.

The last relevant strand of literature is that of dynamic cooperative games. Modeling the bargaining between the lobby and the government in an intertemporal context requires an assumption about the bargaining parties' outside options and commitment. We assume that bargaining takes place every period to determine lobby payments and extraction. If no agreement can be reached, the bargaining parties leave the table and choose uncooperative strategies for the rest of infinity, which means that no payments are made any more and the government enforces the welfare-maximizing amount of extraction. This modeling assumption may represent situations where after a bargaining failure, trust is destroyed. Additional to this interpretation, we opt for this assumption concept to ensure analytical solvability. It follows Petrosyan (1997) and is used by, e.g., Kaitala & Pohjola (1990), Fanokoa et al. (2011) and Jørgensen et al. (2005). For alternative approaches see Sorger (2006), or Boyce (2010), who applies the truthful Markov perfect equilibrium of Bergemann & Välimäki (2003); these solution concepts would in our model context require numerical solutions.

Before discussing the resulting equilibrium in detail, we introduce the economy with benchmarks for the lobbying model in the following section.

3 The Economy

3.1 Basics

Our aim is to derive the policy determined by a bargaining process between the government and a resource owners' lobby group. In this section, we prepare this analysis by introducing the model setting and characterizing two benchmarks for the lobbying model: welfare maximization and monopolistic profit maximization. These are the polar cases that span the bargaining range of our later political model. First we use general functions to describe the benchmarks, then we specify linear-quadratic functions to derive explicit solutions. Finally, we discuss the development of the conflict of interest between the welfare-maximizing government and the profit-maximizing monopolist.

We analyze the extraction of a non-renewable resource in a partial-equilibrium model.[2] The supply side is a sector of resource owners, who optimize intertemporally. The demand side is represented by a stationary demand function. Thus, the resource may best be thought of as an energy resource like coal, which is directly burnt by its buyers so that there are no demand-side stock effects, and whose share of the economy is small enough so that, for example, the interest rate can be taken as exogenous.

Let $q(t)$ denote resource extraction in period t and $z(t)$ cumulative extraction of all previous periods. Then the equation of motion of z is

$$z_+(t) \equiv z(t+1) = z(t) + q(t). \tag{1}$$

In the following, we drop t where no ambiguities arise. Gross consumer surplus in each period is $u \equiv u(q)$, and net consumer surplus is $u - pq$, where p is the market price of the resource. Consumers take the price as given, which implies

$$p \equiv p(q) = \tfrac{\partial u(q)}{\partial q} \tag{2}$$

in equilibrium. Extraction costs $c \equiv c(q,z)$ are increasing and convex in each argument, current and cumulative extraction. Cumulative extraction increases the marginal cost of current extraction, $\frac{\partial c(0,z)}{\partial z} = 0$ and $\frac{\partial^2 c(q,z)}{\partial q \partial z} > 0$. The economy's instantaneous utilitarian welfare w is gross consumer surplus minus extraction costs and environmental damages $x \equiv x(q,z)$, which is caused by current and cumulative resource consumption:

$$w \equiv w(q,z) = u(q) - c(q,z) - x(q,z). \tag{3}$$

$x(q,z)$ is assumed to be additively separable as well as increasing and convex in each argument, flow and stock pollution.

The agents in our model have an infinite planning horizon and a discount rate r, implying a discount factor $\beta \equiv 1/(1+r)$. In t the present value of the discounted welfare stream, in the following just called intertemporal welfare, thus is

$$W(t) = \sum_{s=0}^{\infty} \beta^s \cdot w(q(t+s), z(t+s)), \tag{4}$$

[2] Our model economy is a standard partial-equilibrium resource-economic setting with Ricardian (instead of Hotelling) rents, as for example in Hartwick (1982).

where $s \in \mathbb{N}$ is the summation index. Environmental damages $x(q, z)$ are external to resource owners, so that their flow profit π is

$$\pi \equiv \pi(p, q, z) = p \cdot q - c(q, z) \tag{5}$$

so that in t the present value of profits, in the following just called intertemporal profit, is

$$\Pi(t) = \sum_{s=0}^{\infty} \beta^s \cdot \pi(p(t+s), q(t+s), z(t+s)). \tag{6}$$

3.2 Benchmark Solution

In the following, we derive the welfare-maximizing and the profit-maximizing extraction paths and convergence levels of cumulative extraction. To do this, suppose that extraction from some moment on is determined by a state-dependent extraction function $q = q(z)$. Taking the equation of motion (1) into account, we can then write intertemporal welfare (4) as

$$W(z) = w(q(z), z) + \beta \cdot W(z + q(z)). \tag{7}$$

A social planner chooses q so as to maximize (7). Thus, the optimal $q(z)$ is given by the following Bellman equation:

$$W^{**}(z) = \max_q \left[u(q) - c(q, z) - x(q, z) + \beta \cdot W^{**}(z_+)\right], \tag{8}$$

where the double-asterisk denotes the planner's optimal solution. The first-order condition is

$$\tfrac{\partial u(q^{**})}{\partial q} - \tfrac{\partial c(q^{**},z)}{\partial q} - \tfrac{\partial x(q^{**},z)}{\partial q} + \beta \cdot \tfrac{\partial W^{**}(z_+^{**})}{\partial z} = 0. \tag{9}$$

Differentiating the Bellman equation yields the Envelope Condition:

$$\begin{aligned} \tfrac{\partial W^{**}(z)}{\partial z} &= -\tfrac{\partial c(q^{**},z)}{\partial z} - \tfrac{\partial x(q^{**},z)}{\partial z} + \beta \cdot \tfrac{\partial W^{**}(z_+^{**})}{\partial z} \\ &= -\tfrac{\partial c(q^{**},z)}{\partial z} - \tfrac{\partial x(q^{**},z)}{\partial z} - \tfrac{\partial u(q^{**})}{\partial q} + \tfrac{\partial c(q^{**},z)}{\partial q} + \tfrac{\partial x(q^{**},z)}{\partial q}. \end{aligned} \tag{10}$$

Shifting this in time and substituting into the first-order condition yields the planner's Euler equation, which is the Hotelling rule modified for stock-dependent cost effects as well as flow- and stock-pollution damages:

$$\tfrac{\partial u(q^{**})}{\partial q} - \tfrac{\partial c(q^{**},z)}{\partial q} - \tfrac{\partial x(q^{**},z)}{\partial q} = \beta \cdot \left[\tfrac{\partial c(q_+^{**},z_+^{**})}{\partial z} + \tfrac{\partial x(q_+^{**},z_+^{**})}{\partial z} + \tfrac{\partial u(q_+^{**})}{\partial q} - \tfrac{\partial c(q_+^{**},z_+^{**})}{\partial q} - \tfrac{\partial x(q_+^{**},z_+^{**})}{\partial q}\right]. \tag{11}$$

Thus, the current welfare created by marginal resource extraction, which is its consumer benefit net of extraction cost and flow externalities, has to equal the discounted welfare that could be gained from the resource if it were extracted a period later, plus the additional extraction cost and environmental damages due to the higher cumulative extraction.

Now suppose that a monopolist supplies the resource. Because the monopolist internalizes the price reaction (2), (5) can be written as

$$\pi(q,z) = p(q)\cdot q - c(q,z) \tag{12}$$

and with a state-dependent extraction function $q(z)$, we have, similar to (7):

$$\Pi(z) = \pi(q(z),z) + \beta\cdot\Pi(z+q(z)). \tag{13}$$

Letting a (single-)asterisk denote the monopolist's optimal solution, the Bellman equation is

$$\Pi^*(z) = \max_q \left[p(q)\cdot q - c(q,z) + \beta\cdot\Pi^*(z_+)\right]. \tag{14}$$

Following the same steps as in the welfare-maximizing case yields the following Euler equation:

$$p(q^*) + \tfrac{\partial p(q^*)}{\partial q}q^* - \tfrac{\partial c(q^*,z)}{\partial q} = \beta\cdot\left[\tfrac{\partial c(q_+^*,z_+^*)}{\partial z} + p(q_+^*) + \tfrac{\partial p(q_+^*)}{\partial q}q_+^* - \tfrac{\partial c(q_+^*,z_+^*)}{\partial q}\right]. \tag{15}$$

The interpretation is similar to that of the planner's Euler equation, except that the monopolist does not care for environmental damages, but for his influence on the price.

The welfare-maximizing extraction path (11) and the profit-maximizing extraction path (15) coincide if along the whole path it holds that

$$-\tfrac{\partial p(q^{**})}{\partial q}q^{**} - \tfrac{\partial x(q^{**},z)}{\partial q} = \beta\cdot\left[\tfrac{\partial x(q_+^{**},z_+)}{\partial z} - \tfrac{\partial x(q_+^{**},z_+)}{\partial q} - \tfrac{\partial p(q_+^*)}{\partial q}q_+^{**}\right]. \tag{16}$$

This would be fulfilled if there were no stock pollution and the effects of flow pollution and market power exactly canceled out.

Competitive, unregulated resource owners would neither internalize the environmental damages nor their influence on the price. Thus, their Euler equation can be derived from (15) by substituting the price derivatives by zero, or from (11) by dropping the environmental-damage derivatives. In what follows, we neglect the competitive, unregulated case and derive explicit solutions for the planner's and the monopolist's optimization problems only.

We assume that all functions are well-behaved in the sense that they result in extraction paths $q(t)$ that monotonically converge to $q = 0$ and a *convergence level* $\hat{z}$ of cumulative extraction for $t \to \infty$. For $\hat{z}$, $z_+ = z$ by $q(\hat{z}) = 0$. Substituting this into (11) and (15) and rearranging, we see that the welfare-maximizing and profit-maximizing convergence levels of cumulative extraction $\hat{z}_w$ and $\hat{z}_\pi$ are defined by

$$\tfrac{\partial w(0,\hat{z}_w)}{\partial q} = \tfrac{\partial u(0)}{\partial q} - \tfrac{\partial c(0,\hat{z}_w)}{\partial q} - \tfrac{\partial x(0,\hat{z}_w)}{\partial q} = \tfrac{\partial x(0,\hat{z}_w)}{\partial z} \Big/ r \,, \tag{17a}$$

$$\tfrac{\partial \pi(0,\hat{z}_\pi)}{\partial q} = p(0) - \tfrac{\partial c(0,\hat{z}_\pi)}{\partial q} = 0. \tag{17b}$$

Thus, extraction ceases when the net gain due to the first marginal extracted unit exactly matches the present value of its future effects due to the added cumulative extraction. Because this is only about a marginal unit, market power is not relevant anymore. The welfare effects include environmental damages so that the social planner's convergence level is definitely lower than that of the monopolist. For the monopolist's convergence level, stock-dependent cost effects are irrelevant because $\frac{\partial c(0,z)}{\partial z} = 0$. We assume $\frac{\partial u(0)}{\partial q}$ to be finite so that the convergence levels are well-defined; for instance, $\frac{\partial u(0)}{\partial q}$ can be the price of a backstop technology.

3.3 Benchmark Solution: Explicit Example

We now specify the functions to be linear-quadratic so that we can explicitly derive and discuss the solution. The demand price is a linear function of the quantity q. Marginal extraction costs in a given period are a linear function of cumulative extraction and of extraction within that period. Marginal flow-pollution damage is a linear function of extraction in the same period, and marginal stock-pollution damage is constant so that total stock-pollution damage is proportional to cumulative extraction. The assumed explicit forms of the functions are summarized in Table 1. Collecting terms, we have

$$w(q, z) = (b_w - \kappa_z z) \cdot q - \frac{a_w}{2} q^2 + \left(\frac{q}{r} - z \right) \cdot \chi_z, \tag{18a}$$

$$\pi(q, z) = (b_\pi - \kappa_z z) \cdot q - \frac{a_\pi}{2} q^2, \tag{18b}$$

where

$$b_w \equiv \rho_1 - \kappa_1 - \chi_1 - \frac{\chi_z}{r}, \tag{19a}$$

$$b_\pi \equiv \rho_1 - \kappa_1, \tag{19b}$$

$$a_w \equiv \kappa_2 + \rho_2 + \chi_2, \tag{19c}$$

$$a_\pi \equiv \kappa_2 + 2\rho_2. \tag{19d}$$

Table 1: Explicit functions.

Functions		Explicit forms
$u(q)$	$=$	$\rho_1 q - \frac{\rho_2}{2} q^2$
$p(q) = \frac{\partial u(q)}{\partial q}$	$=$	$\rho_1 - \rho_2 q$
$c(q, z)$	$=$	$\left(\kappa_z z + \kappa_1 + \frac{\kappa_2}{2} q\right) \cdot q$
$\frac{\partial c(q,z)}{\partial q}$	$=$	$\kappa_z z + \kappa_1 + \kappa_2 q$
$\frac{\partial c(q,z)}{\partial z}$	$=$	$\kappa_z q$
$x(q, z)$	$=$	$\chi_z z + \chi_1 q + \frac{\chi_2}{2} q^2$
$\frac{\partial x(q,z)}{\partial q}$	$=$	$\chi_1 + \chi_2 q$
$\frac{\partial x(q,z)}{\partial z}$	$=$	χ_z

The parameter index indicates the power of the variable that the parameter relates to.

If not stated otherwise, all coefficients and the summarized coefficients b_w, b_π, a_w, and a_π are assumed to be positive in the following. Thus, we also assume $b_\pi > b_w$ and ignore the case of $b_\pi = b_w$.

From (18a), the present net welfare gain of the first (marginal) unit of extraction is

$$\frac{\partial w(0, z)}{\partial q} = b_w - \kappa_z z + \frac{\chi_z}{r}. \tag{20}$$

The direct welfare loss of that unit due to stock pollution from the next period on is $\chi_z(1 + r)/r$, which discounted to t is χ_z/r. Thus, the direct intertemporal welfare effect of the first marginal unit of extraction is $b_w - \kappa_z z$. Likewise, the direct intertemporal effect of the first marginal unit of extraction on profit is $b_\pi - \kappa_z z$. The former is smaller than the latter because the social planner takes present flow-pollution damage (χ_1) and future discounted stock-pollution damage (χ_z/r) from extracting the first unit into account. For brevity, we refer to $b_w - \kappa_z z$ and $b_\pi - \kappa_z z$ as *first-unit welfare* and *first-unit profit* in the following, or *first-unit gains* if we mean both. For any z, it is worthwhile to extract at the margin if the first-unit gains are positive, that is, if $b_w - \kappa_z z > 0$ or $b_\pi - \kappa_z z > 0$, respectively. Stated the other way round, extraction would cease for $z = \hat{z}_w \equiv b_w/\kappa_z$ or $z = \hat{z}_\pi \equiv b_\pi/\kappa_z$, respectively. If cumulative extraction is at this convergence level, the gains from extracting cannot be high enough to cover the costs, which include the environmental damages in the planner's case.

a_w and a_π are the (absolute) slopes of the intratemporal marginal welfare function and the intratemporal marginal profit function, respectively. If $a_w > a_\pi$, then marginal welfare within a period decreases faster than marginal profit so that a social planner would have a stronger tendency to shift consumption into the future than a resource monopolist.

For stability of the resulting system of difference equations, we assume

$$a_i > \kappa_z \qquad \text{for } i = w, \pi. \tag{21}$$

Then we can derive the welfare-maximizing and the profit-maximizing extraction functions:

Proposition 3.1 (Explicit Example: Benchmark Extractions) *The welfare-maximizing and the profit-maximizing extraction functions are given by*

$$q^{**}(z) = \begin{cases} \psi_w \cdot (b_w - \kappa_z z) & \text{if } z \leq \hat{z}_w, \\ 0 & \text{if } z > \hat{z}_w, \end{cases} \tag{22a}$$

$$q^{*}(z) = \begin{cases} \psi_\pi \cdot (b_\pi - \kappa_z z) & \text{if } z \leq \hat{z}_\pi, \\ 0 & \text{if } z > \hat{z}_\pi, \end{cases} \tag{22b}$$

where

$$\psi_i \equiv \frac{2}{a_i + \sqrt{a_i^2 + \frac{4}{r}\kappa_z\,(a_i - \kappa_z)}} \leq \frac{1}{a_i} \qquad \text{for } i = w, \pi. \tag{23}$$

Proof. See Appendix A.1. □

Moreover, we can determine how the state variable z develops:

Proposition 3.2 (Explicit Example: Benchmark Cumulative Extractions) *Along both the welfare-maximizing and the profit-maximizing extraction paths, cumulative extraction develops as follows:*

$$z^{**}(t+s) = \begin{cases} \hat{z}_w - (1 - \psi_w \kappa_z)^s \cdot \left[\hat{z}_w - z(t)\right] & \text{if } z(t) \leq \hat{z}_w, \\ z(t) & \text{if } z(t) > \hat{z}_w, \end{cases} \tag{24a}$$

$$z^{*}(t+s) = \begin{cases} \hat{z}_\pi - (1 - \psi_\pi \kappa_z)^s \cdot \left[\hat{z}_\pi - z(t)\right] & \text{if } z(t) \leq \hat{z}_\pi, \\ z(t) & \text{if } z(t) > \hat{z}_\pi, \end{cases} \tag{24b}$$

where $0 < 1 - \psi_i \kappa_z \leq 1$ *for* $i = w, \pi$.

Proof. See Appendix A.1. □

Furthermore, we can state the maximized intertemporal welfare and profit:

Proposition 3.3 (Explicit Example: Benchmark Intertemporal Welfare and Profit) *The maximized intertemporal welfare and profit are given by*

$$W^{**}(z) = A_w^{**} \cdot q^{**}(z)^2 - \frac{\chi_z z}{1-\beta}, \tag{25a}$$

$$\Pi^*(z) = A_\pi^* \cdot q^*(z)^2 \geq 0, \tag{25b}$$

where

$$A_w^{**} \equiv \frac{\frac{1}{\psi_w} - \frac{a_w}{2}}{1-\beta\left(1-\psi_w\kappa_z\right)^2} > 0, \tag{26a}$$

$$A_\pi^* \equiv \frac{\frac{1}{\psi_\pi} - \frac{a_\pi}{2}}{1-\beta\left(1-\psi_\pi\kappa_z\right)^2} > 0. \tag{26b}$$

Proof. See Appendix A.1. □

Remark 3.1 *We can determine the lower bounds of A_w^{**} and A_π^* as follows. By the definition of ψ_w* (23)*:*

$$A_w^{**} > \frac{\frac{1}{2}a_w}{1-\beta\left(1-\psi_w\kappa_z\right)^2} = \frac{\frac{1}{2}\left(\kappa_2+\rho_2+\chi_2\right)}{1-\beta\left(1-\psi_w\kappa_z\right)^2} > 0. \tag{27}$$

The equality follows from substituting (19)*. $A_\pi^* > 0$ follows along the same lines.*

The extraction paths are mainly described by two characteristics. The first is the level of cumulative extraction that would cause extraction to cease. The second is the amount of extraction given any level of cumulative extraction.

In Proposition 3.2, we can see that the convergence levels are only reached asymptotically, given that the planning horizon starts with a z below $\hat{z}_w$ or $\hat{z}_\pi$, respectively. Thus, along an optimal extraction path from Proposition 3.1, the constraint never actually binds. Because the first extracted unit has positive flow externalities, $\chi_1 > 0$, and because there are stock externalities, $\chi_z > 0$, we have $b_\pi > b_w$, and the monopolist's convergence level of cumulative extraction is higher than that of the social planner: The resource owners still find it worthwhile to extract if $z = \hat{z}_w$ because they do not have to pay for the environmental damages.

Now consider the amount of extraction when z is below the convergence level. In Proposition 3.1, we can see that it is determined by, again, the first-unit gains, but also by the respective ψ_i. This term represents the decrease in marginal gains due to effects both within the current period and in the future. With infinite discounting ($r \to \infty$), so that the problem were static, we would have $\psi_i = 1/a_i$: Extraction would be limited by increasing marginal environmental damages and decreasing marginal consumer rents within the extraction period. While the monopolist ignores environmental damages, he would be more sensible to the consumer-rents effect because of the monopolistic price distortion. With a lower r, the same effects are at work, but the effect of today's extraction on tomorrow's marginal gains is also taken into account.

Due to the positive extraction, we always have $z_+ > z$ so that, by the functions in Proposition 3.1, extraction decreases in time. A smaller a_w, for instance, implies that ψ_w is larger so that extraction is increased. But given that the convergence level $\hat{z}_w$ is not changed, we can say that ψ_w (only) represents the speed of convergence: A larger ψ_w implies that extraction $q^{**}(z)$ is higher for a given z, but as z then increases, $q^{**}(z)$ also declines faster.

3.4 Differences in Preferred Extraction: Four Cases

In the following, we characterize the conflict of interest between the welfare-maximizing government and the profit-maximizing monopolist. To do this, suppose that from t on either the welfare-maximizing or the profit-maximizing extraction path is chosen. Which would lead to faster extraction? We have

$$\Delta_{q^*}(z) \equiv q^*(z) - q^{**}(z) = (\psi_\pi - \psi_w)(b_\pi - \kappa_z z) + \psi_w (b_\pi - b_w). \tag{28}$$

For the discussion of this difference it is useful to keep the following relation in mind:

Remark 3.2 *By* (23) *and* (19)*, we see that*

$$\psi_w \gtreqless \psi_\pi \qquad \Leftrightarrow \qquad a_w \lesseqgtr a_\pi \qquad \Leftrightarrow \qquad \chi_2 \lesseqgtr \rho_2. \tag{29}$$

For example, if ρ_2 is large, the demand price strongly reacts to extraction within any given period. This decreases both a_w and a_π and thus implies slower extraction. The effect on a_π – and thus on the profit-maximizing extraction – is stronger due to the monopolistic distortion associated with $q^*(z)$: A monopolist has a suboptimally strong tendency to shift extraction into the future to smooth out marginal revenues.[3] In the welfare-maximization

[3]This is related to the standard result that a monopoly implies slower extraction (Solow 1974). However, we are comparing a monopolist with a welfare-maximizer, not with competitive, unregulated firms.

Table 2: Benchmark cases.

Case	Relation	$\Delta_{q^*}(0)$	$\Delta'_{q^*}(z)$
1	$\psi_w < \psi_\pi$	> 0	< 0
2	$\psi_w = \psi_\pi$	> 0	$= 0$
3	$\psi_\pi \frac{b_\pi}{b_w} > \psi_w > \psi_\pi$	≥ 0	> 0
4	$\psi_w > \psi_\pi \frac{b_\pi}{b_w} > \psi_\pi$	< 0	> 0

problem, however, the increasing marginal flow-pollution damage also calls for smoothing out extraction: If χ_2 were large, the social planner would like to postpone extraction because this reduces the marginal flow-pollution damage. As seen in (29), this *marginal flow-pollution effect* may or may not outweigh the *market-power effect* so that a monopolist may extract too fast or too slow from the planner's point of view. The relations between the parameters lead to four distinguishable cases summarized in Table 2.

In Case 1, the marginal flow-pollution effect outweighs the market-power effect, $\chi_2 > \rho_2$, so that $\psi_w < \psi_\pi$. Because $b_\pi > b_w$, equation (28) is definitely positive: Due to all kinds of pollution, the social planner would want slower extraction than a monopolist.

Case 2 is defined by $\chi_2 = \rho_2$ so that $\psi_w = \psi_\pi$. Then the marginal flow-pollution effect and the market-power effect cancel out. We can see in (28) that the difference is completely driven by $b_\pi - b_w$. Because of this difference, the planner would still want slower extraction and a lower convergence level due to flow- and stock-pollution damages that *every* unit of extraction causes.

Now suppose that $\chi_2 < \rho_2$ so that $\psi_w > \psi_\pi$. Equation (28) then is ambiguous

$$\Delta_{q^*}(z) = \underbrace{(\psi_\pi - \psi_w)}_{<0} (b_\pi - \kappa_z z) + \psi_w \underbrace{(b_\pi - b_w)}_{\geq 0}. \tag{30}$$

We can definitely say, however, that the derivative of this difference with respect to z is positive. Thus, if the difference is positive for $z = 0$, then it will remain so as z increases. Substituting $z = 0$ and rearranging, we see that this holds for

$$\psi_\pi b_\pi - \psi_w b_w \geq 0. \tag{31}$$

(31) thus defines Case 3. Even though the market-power effect is stronger than the marginal flow-pollution effect, the additional pollution effects in b_w compensate for this. Therefore,

the social planner would still want less extraction than the monopolist for a given z. Moreover, the welfare-maximizing convergence level of cumulative extraction would be approached faster than the profit-maximizing convergence level so that for any z the desired additional extraction of the social planner shrinks faster than that of the monopolist.

Finally, in Case 4, we have $\chi_2 < \rho_2$ so that $\psi_w > \psi_\pi$, but (31) does not hold. Then the difference is negative for small z, but positive for large z. Substituting $\Delta_{q^*}(z) = 0$ in (28) and rearranging shows that there is a switching-level $z = \tilde{z}$, defined by

$$\tilde{z} = \hat{z}_\pi - \frac{\psi_w}{\psi_w - \psi_\pi}\left(\hat{z}_\pi - \hat{z}_w\right)\left(= \hat{z}_w - \frac{\psi_\pi}{\psi_w - \psi_\pi}\left(\hat{z}_\pi - \hat{z}_w\right)\right). \tag{32}$$

Consequently, up to $\tilde{z}$ the social planner would extract faster than a monopolist, but once $\tilde{z}$ is reached, this relation turns around; the reason is that the welfare-maximizing convergence level is still lower so that at some point the gains from extracting for the social planner decrease faster in z. Put another way, in total the monopolist would want to extract more than the social planner, but to smooth marginal revenue, the monopolist has a stronger incentive to stretch extraction over time.

4 Lobbying and Policy Determination

4.1 Political Agents

We assume that policy is set by a government that interacts with a resource-owner lobby group. More precisely, we assume that in each period the government and the lobby group bargain the resource suppliers' extraction quantity q and a contribution payment m that the lobby pays to the government. The bargained quantity may, for example, be enforced among the firms by announcing it as a maximum extraction for the period and implementing it via an allowance trading system or by allocating extraction quotas. We assume the individual resource suppliers to be so small that they individually neither internalize the environmental nor the marginal-revenue effects of supply (see Section 3.2), and competition policy prevents the lobby organization from serving as a cartel. Such competitive supply would lead to a faster extraction than either profit maximization or welfare maximization; the limit would thus be binding as long as it is somewhere between the two. Alternatively, we demonstrate how the lobby outcome can be implemented via resource taxes in Section 5.

In each period, the government has the following utility function:

$$g(q, m, z) = w(q, z) + \gamma m. \tag{33}$$

Thus, the government cares for welfare w, but it also derives utility γm from the lobby's payments. Let the present value of payments to the government discounted to period t, or intertemporal payments, be denoted by

$$M(t) = \sum_{s=0}^{\infty} \beta^s \cdot m(t+s). \tag{34}$$

Intertemporal utility of the government is the discounted sum of the utility stream:

$$G(t) = \sum_{s=0}^{\infty} \beta^s \cdot g(q(t+s), m(t+s), z(t+s)) = W(t) + \gamma M(t). \tag{35}$$

The (collective) utility function of resource owners is

$$l(q, m, z) = \pi(q, z) - \lambda m, \tag{36}$$

consisting of the sector's profits π and the lobby's cost of paying contributions λm. In π, the price reaction is taken into account – see (13). The marginal-cost parameter λ may, for example, reflect the coordination problems within the group. Intertemporal utility of the lobby group is

$$L(t) = \sum_{s=0}^{\infty} \beta^s \cdot l(q(t+s), m(t+s), z(t+s)) = \Pi(t) - \lambda M(t). \tag{37}$$

The assumptions that resource owners have overcome the collective action problem to form a lobby group and that the government's utility function (33) is additively separable between contribution utility and utilitarian welfare are usual in the interest-group literature (cf. Grossman & Helpman 1994, Grossman & Helpman 2001). A more important feature, which is also typical in the literature, is the assumption of constant marginal contribution utility, which simplifies the derivation of the time paths in the following.[4]

In each period, the government and the lobby bargain about a vector consisting of contribution payments and an extraction quantity for that period. To determine the bargaining result, it is crucial to define the parties' outside options. We assume that if no agreement is reached, cooperation breaks down forever. Then a non-cooperative solution is implemented for the rest of time: The government unilaterally decides the quantity and the lobby pays no contributions. The utility in this non-cooperative solution determines how much of the gains from cooperation each bargaining party can appropriate. The cooperative solution is chosen as the outcome of a Nash bargaining.

[4] See Klein et al. (2008) for the complications that can arise when current choices affect future marginal utility from the control variables in settings without commitment.

Both bargaining parties always have to be better off with the bargained policy than they would be with their outside options. Consequently, the bargained policy vector for the current period and the anticipation that the same kind of cooperation will take place in the future must always imply a higher intertemporal utility than the non-cooperative alternative; otherwise they would not agree. Therefore, this solution is time-consistent and does not require commitment (cf. Jørgensen & Zaccour 2001).

Commitment exists in one sense, however: The fact that the threat in the bargain is to play uncooperatively forever may be seen as a commitment not to cooperate (Sorger 2006). An alternative interpretation would be that the parties do not trust each other anymore. Given that party contribution payments in exchange for a favor are hardly enforceable in court, trust may be crucial.

Moreover, we assume that the government has an active role in the bargain, and its strength is represented by the respective parameter in the asymmetric Nash bargaining solution. A take-it-or-leave-it offer by the lobby, which is more typical in the literature, is a special case in this solution.

4.2 Nash Bargaining Solution

In the following, we formally define the Nash bargaining solution. The bargaining outcome is marked by $\star$ and the threat outcome by $\#$. We define the Nash bargaining solution as follows:[5]

Definition 4.1 (Nash Bargaining Solution) *The Nash bargaining solution of our lobbying game consists of two pairs of stationary Markovian strategies, $(q^\star, m^\star)$ and $(q^\#, m^\#)$, and two pairs of value functions, $(G^\star, L^\star)$ and $(G^\#, L^\#)$, such that the following is true. For all z it holds that*

$$z_+^\star = z + q^\star(z), \tag{38a}$$

$$z_+^\# = z + q^\#(z), \tag{38b}$$

[5] The form of the definition is borrowed from Sorger (2006); the assumption about the threat outcome, however, follows Petrosyan (1997), as pointed out in Section 2.

$$G^\star(z) = g(q^\star(z), m^\star(z), z) + \beta \cdot G^\star(z_+^\star), \tag{39a}$$

$$L^\star(z) = l(q^\star(z), m^\star(z), z) + \beta \cdot L^\star(z_+^\star), \tag{39b}$$

$$G^\#(z) = g(q^\#(z), m^\#(z), z) + \beta \cdot G^\#(z_+^\#), \tag{39c}$$

$$L^\#(z) = l(q^\#(z), m^\#(z), z) + \beta \cdot L^\#(z_+^\#), \tag{39d}$$

$$q^\#(z) \in \arg\max_q \left[g(q, m^\#(z), z) + \beta \cdot G^\#(z_+) | q \geq 0 \right], \tag{40a}$$

$$m^\#(z) \in \arg\max_m \left[l(q^\#(z), m, z) + \beta \cdot L^\#(z_+^\#) | m \geq 0 \right], \tag{40b}$$

$$\begin{aligned} N(q, m, z) \equiv{}& \eta \cdot \ln \left[g(q, m, z) + \beta \cdot G^\star(z_+) - G^\#(z) \right] \\ &+ (1 - \eta) \cdot \ln \left[l(q, m, z) + \beta \cdot L^\star(z_+) - L^\#(z) \right], \end{aligned} \tag{41}$$

$$\big(q^\star(z), m^\star(z)\big) \in \arg\max_{q,m} \left[N(q, m, z) | q \geq 0, m \geq 0 \right]. \tag{42}$$

(38a) and (38b) are just the definitions of the equation of motion (1) along the cooperative and non-cooperative path, respectively. Accordingly, the present-value equations (35) and (37) have a cooperative form, (39a) and (39b), and a non-cooperative form, (39c) and (39d). For the non-cooperative form, (40a) and (40b) define the choice variables. The objective function of the cooperative game is the (logarithm of the) Nash product (41), where η and $(1 - \eta)$ measure the bargaining powers of the government and the lobby group, respectively. The maximands are defined by (42). Thus, we assume that the Nash bargaining implements values of the choice variables so as to maximize the government's and the lobby's intertemporal utility under the (rational) assumption that cooperation is continued. This assumption is justified because the parties always have a higher intertemporal utility within the cooperative equilibrium than without it.

We start deriving the solution by discussing the non-cooperative solution. The government can enforce any desired quantity path if it wants to. If negotiations fail, the government would be best off by choosing the welfare-maximizing path described by (11): $q^\#(z) = q^{**}(z)$, $z_+^\# = z_+^{**}$. At the same time, the lobby would be best off paying no contributions, as these are costly in the present and have no intertemporal effect. The government's utility would be the maximized intertemporal welfare as defined in (8), and the lobby's utility would equal the intertemporal profit (12) for $q = q^{**}(z)$:

$$G^{\#}(z) = W^{**}(z), \tag{43a}$$

$$L^{\#}(z) = \Pi^{**}(z) = \pi(q^{**}(z), z) + \beta \cdot \Pi^{**}(z_+^{**}). \tag{43b}$$

Now consider the bargaining outcome. From (41) and (42), we have for $q^\star(z)$ (after rearranging):

$$\tfrac{\partial w(q^\star(z),z)}{\partial q} + \beta \cdot \tfrac{\partial G^\star(z_+^\star)}{\partial z} + \frac{1-\eta}{\eta}\frac{\Delta_{G^\star}(z)}{\Delta_{L^\star}(z)} \cdot \left[\tfrac{\partial \pi(q^\star(z),z)}{\partial q} + \beta \cdot \tfrac{\partial L^\star(z_+^\star)}{\partial z}\right] = 0, \tag{44}$$

where

$$\Delta_{G^\star}(z) \equiv G^\star(z) - W^{**}(z), \tag{45a}$$

$$\Delta_{L^\star}(z) \equiv L^\star(z) - \Pi^{**}(z) \tag{45b}$$

are the gains from cooperating for the government and the lobby, respectively. Thus, q is chosen so as to maximize a weighted sum of, on the one hand, current welfare and discounted government utility, and, on the other hand, current profit and discounted lobby utility. The weight depends on the bargaining power and the respective gains from cooperating. Likewise, the first-order condition for the contribution payment m is equivalent to

$$\tfrac{\partial g(q^\star(z),m^\star(z),z)}{\partial m} + \frac{1-\eta}{\eta}\frac{\Delta_{G^\star}(z)}{\Delta_{L^\star}(z)}\tfrac{\partial l(q^\star(z),m^\star(z),z)}{\partial m} = 0, \tag{46}$$

which is simpler than the condition for the extraction quantity (44) because m has no stock effect. By (33) and (36), utilities are linear in m so that the respective marginal utilities are constant. Substituting them in (46) and rearranging yields:

$$\frac{1-\eta}{\eta}\frac{\Delta_{G^\star}(z)}{\Delta_{L^\star}(z)} = \mu \equiv \frac{\gamma}{\lambda} \tag{47}$$

so that the lobby's weight, which is denoted by μ in the following, is constant and defined by the relative contribution valuation. Thus, due to the bargaining, $q = q^\star(z)$ is chosen so as to maximize the weighted sum:

$$V(t) = G(t) + \mu \cdot L(t) = W(t) + \mu \cdot \Pi(t) + \underbrace{(\gamma - \mu \cdot \lambda)}_{=0} \cdot M(t) = W(t) + \mu \cdot \Pi(t). \tag{48}$$

The q path is thus defined by the following Bellman equation:

$$V^\star(z) = \max_q \left[w(q, z) + \mu \cdot \pi(q, z) + \beta \cdot V^\star(z_+)\right], \tag{49}$$

which, along the lines of Section 3.2, yields the Euler equation:

$$\begin{aligned}
&p(q^\star) - \tfrac{\partial c(q^\star,z)}{\partial q} - \tfrac{\partial x(q^\star,z)}{\partial q} + \mu \cdot \left[p(q^\star) + \tfrac{\partial p(q^\star)}{\partial q} q^\star - \tfrac{\partial c(q^\star,z)}{\partial q}\right] \\
=&\beta \cdot \Big\{ \tfrac{\partial c(q_+^\star,z_+)}{\partial z} + \tfrac{\partial x(q_+^\star,z_+)}{\partial z} + p(q_+^\star) - \tfrac{\partial c(q_+^\star,z_+)}{\partial q} - \tfrac{\partial x(q_+^\star,z_+)}{\partial q} \\
&\quad + \mu \cdot \left[\tfrac{\partial c(q_+^\star,z_+)}{\partial z} + p(q_+^\star) + \tfrac{\partial p(q_+^\star)}{\partial q} q_+^\star - \tfrac{\partial c(q_+^\star,z_+)}{\partial q} \right] \Big\}.
\end{aligned} \tag{50}$$

The resource price p in the square brackets stems from the suppliers' marginal returns, while the price outside the square brackets just reflects the consumers' marginal surplus. Thus, (50) is a weighted sum of the planner's (11) and the monopolist's (15) Euler equation. The higher the government values the contributions, the more the lobby determines the path (and vice versa).

How do we obtain the contribution payment path? Take (47) and rearrange to get the present value of payments that must hold in equilibrium:

$$M^\star(z) = \frac{\eta}{\lambda} \cdot \left[\Pi^\star(z) - \Pi^{**}(z)\right] + \frac{1-\eta}{\gamma} \cdot \left[W^{**}(z) - W^\star(z)\right]. \tag{51}$$

Here, $\Pi^\star(z)$ and $W^\star(z)$ are known from (49) and $\Pi^{**}(z)$ and $W^{**}(z)$ are known from the welfare-maximizing extraction path. $M^\star(z)$ must be positive because the lobbying-equilibrium policy maximizes a weighted average of both bargaining parties' utility. The present value of the anticipated payments at least exactly compensates the government for the welfare losses and at most transfers all additional profits from the lobby to the government. The intertemporal relation (51) must hold every period, so the payments in any period have to fulfill

$$m^\star(z) = M^\star(z) - \beta \cdot M^\star(z_+^\star) \tag{52}$$

and can thus easily be calculated.[6] Moreover, substituting (51) and simplifying yields:

$$\begin{aligned}
m^\star(z) = &\frac{\eta}{\lambda} \cdot \left[\pi^\star(z) - \pi^{**}(z)\right] + \frac{1-\eta}{\gamma} \cdot \left[w^{**}(z) - w^\star(z)\right] \\
&+ \beta \cdot \left\{ \frac{\eta}{\lambda} \cdot \left[\Pi^{**}(z_+^\star) - \Pi^{**}(z_+^{**})\right] + \frac{1-\eta}{\gamma} \cdot \left[W^{**}(z_+^{**}) - W^{**}(z_+^\star)\right] \right\},
\end{aligned} \tag{53}$$

where $\pi^\star(z) \equiv \pi(q^\star(z), z)$, $\pi^{**}(z) \equiv \pi(q^{**}(z), z)$, $w^\star(z) \equiv w(q^\star(z), z)$, $w^{**}(z) \equiv w(q^{**}(z), z)$. If the bargaining problem were static, we would only have the first line, which would have to be positive for the same reason that makes (51) positive.

[6]Note that in a full-commitment situation, the lobby could just as well pay $M^\star(z(0))$ at the beginning of time, but without this commitment assumption, a payment that takes place every period is more plausible.

The dynamic version (53), however, also takes into account how the outside options change tomorrow due to cooperation today. These outside options are intertemporal welfare and profit along the welfare-maximizing extraction path. The government's part of (53) is always positive because it reflects the welfare loss from deviating one period from the welfare-maximizing extraction path. The lobby's part, however, can temporarily be negative; deviating one period from the government's preferred extraction path to the bargained one does not necessarily increase intertemporal profits. Nevertheless, the intertemporal payments (51) are always positive even if they are only determined by the lobby's willingness to pay ($\eta = 1$). In Section 4.4, we discuss the payments and their development with linear-quadratic functions in detail.

4.3 Nash Bargaining Solution: Explicit Example

For the explicit example, we build on the model from Section 3.3 and thus use the values and functions from Table 1. Thereby, the planner's explicit instantaneous welfare (18a) and the monopolist's explicit flow profit (18b) are complemented by utility from and cost of contributions, respectively. We thus have

$$g(q, z) = (b_w - \kappa_z z) \cdot q - \frac{a_w}{2} q^2 + \gamma m + \left(\frac{q}{r} - z\right) \cdot \chi_z, \tag{54a}$$

$$l(q, z) = (b_\pi - \kappa_z z) \cdot q - \frac{a_\pi}{2} q^2 - \lambda m. \tag{54b}$$

From the discussion in Section 4.2, we know that $q^{\#}(z) = q^{**}(z)$ and $m^{\#}(z) = 0$. Then the threat value functions are given as follows:

Proposition 4.1 (Explicit Example: Nash Bargaining Threat Values) *In the non-cooperative solution, the government's intertemporal utility is defined by $W^{\#}(z) = W^{**}(z)$ as given in Proposition 3.3. The lobby's intertemporal utility is given by*

$$L^{\#}(z) = \Pi^{**}(z) = A_\pi^{**} \cdot q^{**}(z)^2 - B_\pi^{**} \cdot q^{**}(z) \geq 0, \tag{55}$$

where

$$A_\pi^{**} \equiv \frac{\frac{1}{\psi_w} - \frac{a_\pi}{2}}{1 - \beta\left(1 - \psi_w \kappa_z\right)^2} > 0, \tag{56a}$$

$$B_\pi^{**} \equiv \frac{b_w - b_\pi}{1 - \beta\left(1 - \psi_w \kappa_z\right)} < 0. \tag{56b}$$

Proof. See Appendix A.2. □

Remark 4.1 *The inequality in* (56b) *follows from* $b_\pi > b_w$. *We can determine the lower bound of* A_π^{**} *along the lines of Remark 3.1. By the definition of* ψ_w (23)*:*

$$A_\pi^{**} > \frac{a_w - \frac{1}{2}a_\pi}{1-\beta\left(1-\psi_w\kappa_z\right)^2} = \frac{\frac{1}{2}\kappa_2 + \chi_2}{1-\beta\left(1-\psi_w\kappa_z\right)^2} > 0. \tag{57}$$

The equality follows from substituting (19).

We define

$$a \equiv \frac{a_w + \mu a_\pi}{1+\mu} > \kappa_z, \tag{58a}$$

$$b \equiv \frac{b_w + \mu b_\pi}{1+\mu} \geq b_w \tag{58b}$$

where $a > \kappa_z$ follows from (21) and

$$\psi \equiv \frac{2}{a + \sqrt{a^2 + \frac{4}{r}\kappa_z\left(a-\kappa_z\right)}} < \frac{1}{a}. \tag{59}$$

Then the policy functions are given as follows:

Proposition 4.2 (Explicit Example: Nash Bargaining Policies) *The equilibrium and the threat extraction functions as well as the equilibrium and the threat contribution payment functions are given by*

$$q^\star(z) = \begin{cases} \psi\cdot\left(b-\kappa_z z\right) & \text{if } z \leq \hat{z}, \\ 0 & \text{if } z > \hat{z}, \end{cases} \tag{60a}$$

$$q^\#(z) = q^{**}(z), \tag{60b}$$

$$\begin{aligned} m^\star(z) = {} & \frac{\eta}{\lambda}\cdot\left[\left(b_\pi - \kappa_z z\right)\Delta_{q^\star}(z) - \frac{a_\pi}{2}\Delta_{q^\star,2}(z)\right] - \frac{1-\eta}{\gamma}\cdot\left[\left(b_w-\kappa_z z\right)\Delta_{q^\star}(z) - \frac{a_w}{2}\Delta_{q^\star,2}(z)\right] \\ & + \beta\cdot\left\{\frac{\eta}{\lambda}\cdot\left[A_\pi^{**}\cdot\Delta_{q^\star,2}^{+**}(z) - B_\pi^{**}\cdot\Delta_{q^\star}^{+**}(z)\right] - \frac{1-\eta}{\gamma}\cdot A_w^{**}\cdot\Delta_{q^\star,2}^{+**}(z)\right\}, \end{aligned} \tag{60c}$$

$$m^\#(z) = 0, \tag{60d}$$

where (60b) *is given in Proposition 3.1 and where*

$$\Delta_{q^\star}(z) \equiv q^\star(z) - q^{**}(z), \tag{61a}$$

$$\Delta_{q^\star,2}(z) \equiv q^\star(z)^2 - q^{**}(z)^2, \tag{61b}$$

$$\Delta_{q^\star}^{+**}(z) \equiv q^{**}(z_+^\star) - q^{**}(z_+^{**}), \tag{61c}$$

$$\Delta_{q^\star,2}^{+**}(z) \equiv q^{**}(z_+^\star)^2 - q^{**}(z_+^{**})^2. \tag{61d}$$

Proof. See Appendix A.2. □

The state variable z develops as described in the following proposition:

Proposition 4.3 (Explicit Example: Nash Bargaining Equilibrium Cumulative Extraction) *Along the equilibrium extraction path, cumulative extraction develops as follows:*

$$z^\star(t+s) = \begin{cases} \hat{z} - (1-\psi\kappa_z)^s \left[\hat{z} - z(t)\right] & \text{if } z(t) \le \hat{z}, \\ z(t) & \text{if } z(t) > \hat{z}, \end{cases} \tag{62}$$

where $0 < 1 - \psi\kappa_z \le 1$.

Proof. See Appendix A.2. $\square$

Finally, the equilibrium value functions are given as follows:

Proposition 4.4 (Explicit Example: Nash Bargaining Equilibrium Values) *In the cooperative solution, intertemporal welfare and profit are given by*

$$W^\star(z) = A_w^\star \cdot q^\star(z)^2 - B_w^\star \cdot q^\star(z) - \frac{\chi_z z}{1-\beta}, \tag{63a}$$

$$\Pi^\star(z) = A_\pi^\star \cdot q^\star(z)^2 - B_\pi^\star \cdot q^\star(z), \tag{63b}$$

where

$$A_i^\star \equiv \frac{\frac{1}{\psi} - \frac{a_i}{2}}{1-\beta\left(1-\psi\kappa_z\right)^2} \qquad \text{for } i = w, \pi, \tag{64a}$$

$$B_i^\star \equiv \frac{b - b_i}{1-\beta\left(1-\psi\kappa_z\right)} \qquad \text{for } i = w, \pi. \tag{64b}$$

Intertemporal payments are given by

$$\begin{aligned} M^\star(z) \equiv \frac{\eta}{\sigma} \cdot \Big\{ A_\pi^\star \cdot q^\star(z)^2 - B_\pi^\star \cdot q^\star(z) - \left[A_\pi^{\star\star} \cdot q^{\star\star}(z)^2 - B_\pi^{\star\star} \cdot q^{\star\star}(z)\right] \Big\} \\ + \frac{1-\eta}{\zeta} \cdot \Big\{ A_w^{\star\star} \cdot q^{\star\star}(z)^2 - \left[A_w^\star \cdot q^\star(z)^2 - B_w^\star \cdot q^\star(z)\right] \Big\}. \end{aligned} \tag{65}$$

Intertemporal utility of the government and the lobby, $G^\star(z)$ *and* $L^\star(z)$*, are then given by the weighted sums* (35) *and* (37).

Proof. See Appendix A.2. $\square$

Remark 4.2 *Note that* $A_i^\star$ *and* $B_i^\star$ *differ from* $A_i^{\star\star}$ *and* $B_i^{\star\star}$ *in Proposition 4.1 because the former use the bargained* ψ *coefficient and the latter use the governmental* ψ_w *coefficient.*

$B^\star_w > 0$ and $B^\star_\pi < 0$ because $b_\pi > b > b_w$. We can determine the lower bounds of $A^\star_w$ and $A^\star_\pi$ along the lines of Remark 3.1. By the definition of ψ (59)*:*

$$A^\star_w > \frac{a - \frac{1}{2}a_w}{1-\beta\left(1-\psi\kappa_z\right)^2} = \frac{\frac{1}{2}\left(\kappa_2+\rho_2+\chi_2\right)+\frac{\mu}{1+\mu}\left(\rho_2-\chi_2\right)}{1-\beta\left(1-\psi\kappa_z\right)^2}, \tag{66a}$$

$$A^\star_\pi > \frac{a - \frac{1}{2}a_\pi}{1-\beta\left(1-\psi\kappa_z\right)^2} = \frac{\frac{1}{2}\kappa_2+\chi_2+\frac{\mu}{1+\mu}\left(\rho_2-\chi_2\right)}{1-\beta\left(1-\psi\kappa_z\right)^2} > 0. \tag{66b}$$

The equalities follow from substituting (19) *and* (58a)*.*

The form of the equilibrium extraction function (60a) is the same as that in the benchmark cases, but its concrete value for any given level of cumulative extraction z shows a distortive influence of the lobby because b and ψ are compromise values. By (59), a larger value of a implies a lower ψ and thus, by (60a), a lower extraction for a given amount of cumulative extraction z. a represents the weighted average of the decrease in marginal gains within a period. As discussed in Section 3.3, the individual slopes are driven by marginal cost and marginal consumer benefit, which has a stronger influence on a_π than on a_w, and a_w is also higher if marginal flow-pollution damage is higher. With the weight μ, a is thus pulled into the lobby's direction, which may increase or decrease extraction given z, depending on whether $a_w > a_\pi$. Likewise, $b - \kappa_z z$ is the weighted average of the direct intertemporal benefits that the bargaining parties assign to the first marginal unit of extraction. A larger value of b implies higher extraction given z and higher total extraction. b increases if either b_w or b_π increases; due to environmental damages, $b_w < b_\pi$ so that b also increases with μ. Along the equilibrium extraction path, z converges to $\hat{z} \equiv b/\kappa_z$.

Let us now consider the government's threat and the contribution payments. From Section 4.2 we know that the contribution payments at least compensate the government for two things: Firstly, for the instantaneous welfare loss and secondly, for the future welfare loss on the welfare-maximizing path due to the present choice of $q^\star(z)$ instead of $q^{**}(z)$. In both cases, the "loss" is defined in comparison to the non-cooperative solution, namely welfare-maximizing extraction, so we can write (60b): $q^{\#}(z) = q^{**}(z)$. However, comparing the trajectories of cumulative extraction in Propositions 3.2 and 4.3, we see that due to the lobbying distortion, z at some point of time exceeds the government's convergence level, $z > \hat{z}_w = b_w/\kappa_z$. Afterwards, the government can only threaten to switch to zero extraction forever.

This change must affect the equilibrium contribution payments. Firstly, as long as $z < \hat{z}_{\underline{w}} \equiv \hat{z}_w - q^\star(z)$, the welfare-maximizing extraction path starting today and that starting tomorrow would involve consumption smoothing. Thus, the change of tomorrow's extraction path due to the equilibrium extraction today affects the firms' and the government's future outside option. Secondly, for $\hat{z}_{\underline{w}} \leq z < \hat{z}_w$, switching to the welfare-maximizing path today by choosing $q^{**}(z)$ would still imply positive extraction forever, but choosing $q^{\star}(z)$ today means that this option will not be available anymore in the future; the government's future threat extraction will be zero. As this is relevant for at most one period, we do not analyze how the equilibrium contribution payments develop between $z = \hat{z}_{\underline{w}}$ and $z = \hat{z}_w$, but how this development changes around $z = \hat{z}_{\underline{w}}$ and $z = \hat{z}_w$ in the following. Finally, for $z \geq \hat{z}_w$, the government can only change to zero extraction, which would imply zero profits. Thus, the payments at least have to compensate the government for the welfare loss today and the direct welfare loss due to additional stock-pollution damages in the future, but the impact of $q^{\star}(z)$ on the welfare-maximizing extraction path has become irrelevant. Taken together, there are three distinguishable cases concerning $q^{**}(z^{\star})$ and $q^{**}(z_{+}^{\star})$ in (60c): $q^{**}(z^{\star}), q^{**}(z_{+}^{\star}) > 0$ for $z < \hat{z}_{\underline{w}}$, $q^{**}(z^{\star}) > 0$ and $q^{**}(z_{+}^{\star}) = 0$ for $\hat{z}_{\underline{w}} \leq z < \hat{z}_w$ and $q^{**}(z^{\star}), q^{**}(z_{+}^{\star}) = 0$ for $z \geq \hat{z}_w$.[7]

As discussed in Section 4.2, the lobby's threat is always paying no contributions at all, so we can write (60d): $m^{\#}(z) = 0$.

In the following section, we illustrate the development of extraction quantities and contribution payments over time. Thereby, we use specific parameter values that cover four relevant cases similar to those discussed in Section 3.4.

4.4 Illustration of the Lobbying Equilibrium

For the benchmark solution, we discussed the conflict of interest between a welfare-maximizing government and a monopolist and the development of this conflict in Section 3.4, and we characterized them by the marginal flow-pollution effect χ_2, the market-power effect ρ_2, and the respective first-unit gains b_w and b_π; their relations constitute four cases. In the following, we characterize the conflict of interest of the lobbying-equilibrium solution in a similar manner, using the difference between the equilibrium extraction quantity $q^{\star}(z)$ and the extraction that would be chosen if the government changed to the welfare-maximizing extraction path $q^{\#}(z) = q^{**}(z)$ (see Proposition 4.2). From Proposition 4.2 and similar to

[7]For the intertemporal payments (65), there are just two distinguishable cases because $q^{**}(z_{+}^{\star})$ does not appear.

Table 3: Lobbying-equilibrium cases.

		$z < \hat{z}_w$		$z \geq \hat{z}_w$		
Case	Relation	$\Delta_{q^\star}(0)$	$\Delta'_{q^\star}(z)$	$\Delta_{q^\star}(z)$	$\Delta'_{q^\star}(z)$	Implied by
I	$\psi_w < \psi$	> 0	< 0	> 0	< 0	Case 1
II	$\psi_w = \psi$	> 0	$= 0$	> 0	< 0	Case 2
III	$\psi \frac{b}{b_w} > \psi_w > \psi$	≥ 0	> 0	> 0	< 0	Case 3 and μ large
IV	$\psi_w > \psi \frac{b}{b_w} > \psi$	< 0	> 0	> 0	< 0	Case 3 and μ small or Case 4

(28), we have

$$\Delta_{q^\star}(z) \equiv q^\star(z) - q^{**}(z) = \begin{cases} (\psi - \psi_w)\,(b - \kappa_z z) + \psi_w\,(b - b_w) & \text{if } z \leq \hat{z}_w, \\ \psi\,(b - \kappa_z z) & \text{if } z > \hat{z}_w. \end{cases} \tag{67}$$

For the discussion of this difference, it is useful to keep an adapted version of Remark 3.2 in mind:

Remark 4.3 *By* (23), (59), (19), *and* (58), *we see that*

$$\psi_w \gtreqless \psi \gtreqless \psi_\pi \qquad \Leftrightarrow \qquad a_w \lesseqgtr a \lesseqgtr a_\pi \qquad \Leftrightarrow \qquad \chi_2 \lesseqgtr \rho_2. \tag{68}$$

The second line of (67) is positive: For $z > \hat{z}_w$, the welfare-maximizing extraction is zero, while the resource owners still want positive extraction. The first line leads to four distinguishable cases in which the equilibrium contribution payments develop in a qualitatively different manner, namely $\chi_2 > \rho_2 \Leftrightarrow \psi > \psi_w$ (Case I), $\chi_2 = \rho_2 \Leftrightarrow \psi = \psi_w$ (Case II), and for $\chi_2 < \rho_2$ first $\psi_w b/b_w > \psi_w > \psi$ (Case III) and finally $\psi_w > \psi b/b_w > \psi$ (Case IV). These are similar to Cases 1–4 of the benchmark relations laid out in Section 3.4, but not exactly equal. If we are in Case 1 or 2, then we are also in Case I or II, respectively, because $\psi_\pi \gtreqless \psi_w \Leftrightarrow \psi \gtreqless \psi_w$. For the other cases, this equivalence is not given. In all of them $\psi_\pi < \psi < \psi_w$; but while Case 4 always implies Case IV, Case 3 can also imply Case IV if the lobby's weight μ is small (see Lemma A.7 in Appendix A.3). The benchmark cases are summarized in Table 2 (on page 15) and the lobbying-equilibrium cases and their relations to the benchmark cases can be found in Table 3.

In the following, we describe the development of resource extraction and contribution payments in Cases I-IV. For illustration, we use diagrams for specific parameter values for

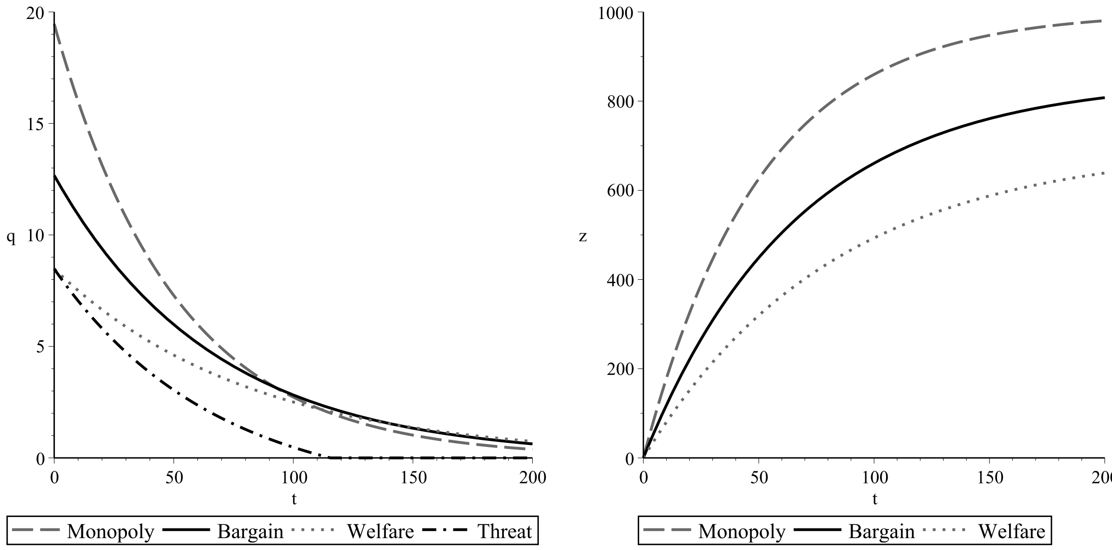


Figure 1: Extraction path and cumulative extraction path for $\chi_2 > \rho_2$ (Case I).

the functions in Table 1 (on page 11). The parameters used in the figures are $\beta = 15/16$, $\rho_1 = 100$, $\kappa_z = 1/10$, $\kappa_1 = 0$, $\kappa_2 = 0$, $\chi_z = 2$, and $\chi_1 = 0$ so that $b_\pi = 100$ and $b_w = 80$. At the beginning of time ($t = 0$), cumulative extraction is assumed to be zero, $z = 0$. We assume $\lambda = \gamma = 1$, so that the lobby's policy weight is $\mu = 1$ and $b = 90$, and the bargaining power is symmetric, $\eta = 1/2$. The relation of the remaining economic parameters, ρ_2 and χ_2, constitutes the four cases.

Case I: $\chi_2 > \rho_2$.

The first case is defined by $\chi_2 > \rho_2$ and thus $\psi_w < \psi < \psi_\pi$. This case is what most would intuitively expect, so we treat it in some detail. As discussed for the benchmarks, the first case implies that the marginal flow-pollution effect outweighs the market-power effect (cf. Section 3.4). Figure 1 shows the extraction paths (left-hand side figure) and cumulative extraction paths (right-hand side figure) for $\chi_2 = 5, \rho_2 = 2$. The dashed gray curve is the profit-maximizing path and the dotted gray curve is the welfare-maximizing path, each starting from $z(t = 0) = 0$. As discussed in Section 3.4, a monopolist would prefer a higher extraction than a welfare-maximizing planner for any given level of z and is also willing to reach a higher level of cumulative extraction. Using Propositions 3.1 and 3.2, we can write the q and z differences for the paths starting from $z(t = 0) = 0$:

$$z^*(t) - z^{**}(t) = \left[1 - (1 - \psi_\pi \kappa_z)^t\right] \cdot \hat{z}_\pi - \left[1 - (1 - \psi_w \kappa_z)^t\right] \cdot \hat{z}_w, \tag{69a}$$

$$q^*(t) - q^{**}(t) = (1 - \psi_\pi \kappa_z)^t \psi_\pi b_\pi - (1 - \psi_w \kappa_z)^t \psi_w b_w. \tag{69b}$$

By $\hat{z}_\pi = b_\pi/\kappa_z > \hat{z}_w = b_w/\kappa_z$, we can see that cumulative extraction along the monopolist's extraction path, $z^*(t)$, will always be higher than what a social planner would reach in the same time, $z^{**}(t)$; the monopolist's extraction will at some point be *below* the extraction of a social planner at the same time, exactly because of the higher accumulated extraction, which increases costs.

The equilibrium extraction path is a compromise between these extremes, shown as the black curve in Figure 1. Cumulative extraction converges towards $\hat{z} = b/\kappa_z$, which is a weighted average between the welfare-maximizing and profit-maximizing convergence levels $\hat{z}_\pi$ and $\hat{z}_w$.[8]

From the point of view of this lobbying equilibrium, $q^*(t)$ and $q^{**}(t)$ are only hypothetical reference paths once that $q^\star(z)$ has been chosen for a while. By contrast, the dash-dotted black curve represents government's threat extraction $q^\#(z^\star) = q^{**}(z^\star)$ in the corresponding period, given that z up to that period has been determined by the bargained policy. Each point along that curve represents extraction in the first period of deviation from the lobbying equilibrium to the welfare-maximizing path, so that each point is the beginning of an extraction path converging to $\hat{z}_w$. Each term of equation (67) is positive, so that $\Delta_{q^\star}(z) > 0$; the government would always switch to lower extraction. At the same time, the size of this change declines in z, both in the periods in which the non-negativity constraint is binding for the government and in those in which it is not; the derivative of equation (67) with respect to z is negative. This relation can also be seen in Figure 1 as the vertical difference between the solid black curve and the dash-dotted curve.

Figure 2 shows the contribution payment path in Case I. It is easiest to consider first the development once that the constraint has become binding, $z \geq \hat{z}_w$. By Proposition 4.2, $m^\star(z)$ then is

$$m^\star(z) = \frac{\eta}{\lambda} \cdot \underbrace{\left[(b_\pi - \kappa_z z)\, q^\star(z) - \frac{a_\pi}{2} q^\star(z)^2\right]}_{>0} - \frac{1-\eta}{\gamma} \cdot \underbrace{\left[(b_w - \kappa_z z)\, q^\star(z) - \frac{a_w}{2} q^\star(z)^2\right]}_{<0}. \quad (70)$$

In every period, the payments are at least as high as the welfare loss for that period due to the choice of $q^\star(z)$ instead of $q^{**}(z)$ – including the additional stock-pollution damages which are part of b_w – and at most as high as the resource owners' additional profit. Both terms are positive so that payments are definitely positive. Effects on tomorrow's outside options

[8]In the numerical example, $\mu = 1$ implies that both have equal weight and the equilibrium convergence level is halfway inbetween.

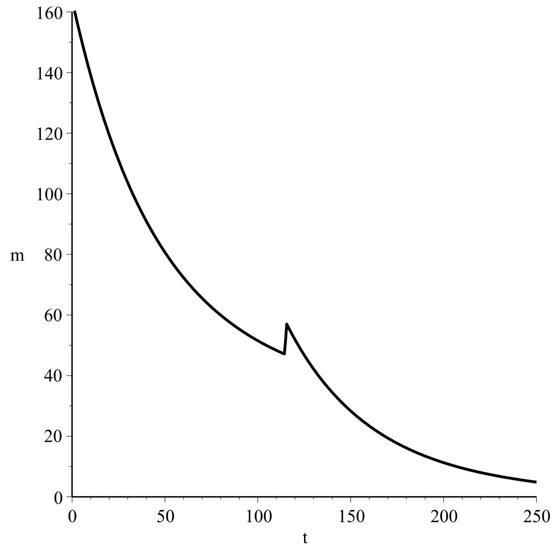


Figure 2: Contribution payment path for $\chi_2 > \rho_2$ (Case I).

as discussed in Section 4.2 are irrelevant because the government's future threat extraction will be zero, no matter how high z grows.

The development of payments can be understood by keeping in mind that z always increases, so differentiating (70) helps to understand the qualitative behavior between one period and another:

$$\frac{\partial m^\star(z)}{\partial z} = \frac{\eta}{\lambda} \cdot \Big[\underbrace{\big[b_\pi - \kappa_z z - a_\pi q^\star(z) \big]}_{>0} \underbrace{\tfrac{\partial q^\star(z)}{\partial z}}_{<0} \underbrace{- \kappa_z q^\star(z)}_{<0} \Big] - \frac{1-\eta}{\gamma} \cdot \Big[\underbrace{\big[b_w - \kappa_z z - a_w q^\star(z) \big]}_{<0} \underbrace{\tfrac{\partial q^\star(z)}{\partial z}}_{<0} \underbrace{- \kappa_z q^\star(z)}_{<0} \Big]. \quad (71)$$

Firstly, a higher z implies higher stock-dependent costs of extracting $q^\star(z)$. This directly changes the bargaining parties' gain from implementing $q^\star(z)$: It reduces the resource owners' equilibrium profits, so that they are less willing to pay for getting $q^\star(z)$ instead of $q^{**}(z)$, and it increases the welfare loss this would entail, so that the government would demand more. The former effect speaks in favor of declining, the latter in favor of increasing payments over time. Secondly, a higher z reduces the equilibrium extraction quantity, which reduces both profits and the welfare loss from cooperation. This indirect effect of z speaks in favor of declining payments. Thus, payments decline over time if the bargaining power of the government is high enough so that profits determine the compensation.[9] By contrast, if the lobby gets all the gains of cooperation ($\eta = 0$) so that the welfare loss determines the compensation, the effects work into opposing directions. In that case, payments are always

[9] We demonstrate in Proposition A.1 in Appendix A.3 that $\eta\,(1+\mu) \geq 1/2$ is sufficient.

declining if the indirect effect outweighs the direct one concerning the welfare loss.[10] Else, if the direct effect outweighs the indirect one and η is small, payments increase for $\hat{z}_w$ and z values not far above, but for some z reach a maximum and then converge towards zero because the indirect effect vanishes with the equilibrium extraction quantities: Additional marginal-cost increases are irrelevant if no extraction takes place. Finally, because a higher η both implies higher payments and makes it less likely that payments increase for $\hat{z}_w$, we can state that they can only increase in time if they are small in the first place.

Before the constraint starts binding, $z < \hat{z}_{\underline{w}}$, the basic forces determining the payments are similar, but more complicated. From Proposition 4.2, $m^\star(z)$ then is

$$m^\star(z) = \frac{\eta}{\lambda} \cdot \left[(b_\pi - \kappa_z z)\, \Delta_{q^\star}(z) - \frac{a_\pi}{2} \Delta_{q^\star,2}(z)\right] - \frac{1-\eta}{\gamma} \cdot \left[(b_w - \kappa_z z)\, \Delta_{q^\star}(z) - \frac{a_w}{2} \Delta_{q^\star,2}(z)\right] \\ + \beta \cdot \left\{ \frac{\eta}{\lambda} \cdot \left[A_\pi^{**} \cdot \Delta_{q^\star,2}^{+**}(z) - B_\pi^{**} \cdot \Delta_{q^\star}^{+**}(z)\right] - \frac{1-\eta}{\gamma} \cdot A_w^{**} \cdot \Delta_{q^\star,2}^{+**}(z) \right\}. \quad (72)$$

The first line contains the same effects as discussed before for the time when the constraint is binding. Now, however, implementing the lobbying equilibrium means choosing $q^\star(z)$ instead of some *positive* $q^{**}(z)$ so that the *difference* $\Delta_{q^\star}(z) \equiv q^\star(z) - q^{**}(z)$ determines the payments.[11] The second line reflects the implied deterioration of the respective outside option: If $q^\star(z)$ is chosen today instead of $q^{**}(z)$, how does the bargaining parties' position change tomorrow if welfare-maximizing policy is chosen from that period on? If only the welfare reduction is relevant ($\eta = 0$), payments are always positive because the government is compensated for the welfare loss that a deviation from the welfare-maximizing extraction path implies. For a higher governmental bargaining power, the effect on profits is more relevant so that things can be different. In Case I, $q^\star(z) > q^{**}(z) \Leftrightarrow \Delta_{q^\star}(z) > 0$ so that choosing $q^\star(z)$ implies higher z tomorrow, implying higher cost and reduced extraction: The square-bracketed term in the second line is negative, reducing payments. Nonetheless, total payments are always positive in Case I because the effect on current profit dominates (see Proposition A.2 in Appendix A.3).

As in the time when the constraint is binding, the direct effect of higher stock-dependent costs is ambiguous. To determine the indirect effect of a changing difference in preferred extraction, consider (67):

$$\frac{\partial \Delta_{q^\star}(z)}{\partial z} = -\left(\psi - \psi_w\right) \kappa_z < 0. \quad (73)$$

[10]We demonstrate in Proposition A.1 in Appendix A.3 that $a_w \geq 1/\psi$ is necessary and sufficient.

[11]For the payments after the constraint we had $\Delta_{q^\star}(z) = q^\star(z)$ by $q^{**}(z) = 0$.

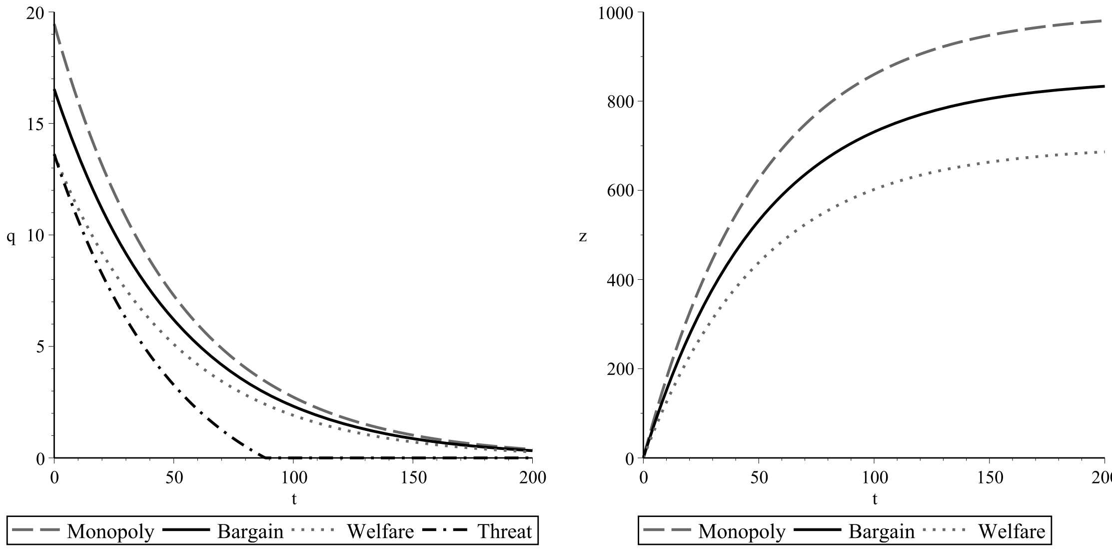


Figure 3: Extraction paths and cumulative extraction paths for $\chi_2 = \rho_2$ (Case II).

Thus, when the non-negativity constraint on the government's threat extraction is not yet binding, the difference in desired extraction is already decreasing.[12] This effect now also influences tomorrow's outside options. In Proposition A.2 in Appendix A.3, we demonstrate that the forces that speak for declining payments prevail.

In Figure 2, we can see that the development of payments before and after the constraint has been hit looks different and seems to be connected by a jump. This "jump", however, is constituted by two kink points of the contribution payment function or, equivalently, by two jumps in its derivative. If $\eta > 0$, the derivative jumps up for $z \to \hat{z}_{\underline{w}}$ and it jumps down for $z \to \hat{z}_w$; only if $\eta = 0$, there are no kink points (see Proposition A.4 in Appendix A.3). For $\eta = 0$, the contribution payments and their development are only depending on the welfare loss due to cooperation. The government's outside option only changes marginally when the constraint starts binding because extraction smoothing is hardly worth near the preferred convergence level. Thus, the course of the contribution payments does not change. For $\eta > 0$, the contribution payments and their development are also depending on the profit gain due to cooperation. When the constraint starts binding, the lobby's preferred extraction level is not yet reached. Thus, its outside option deteriorates when the government's threat does not involve extraction smoothing anymore so that the lobby's willingness to pay increases sharply, explaining the changing course of the contribution payments.

[12] Though less so than afterwards when we have $\Delta'_{q^\star}(z) = -\psi\kappa_z$.

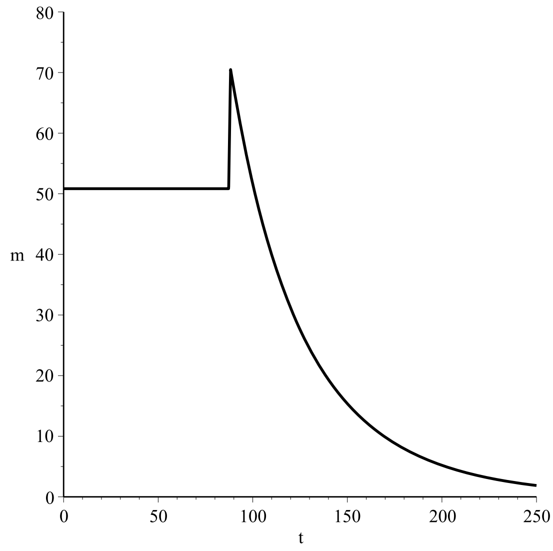


Figure 4: Contribution payment path for $\chi_2 = \rho_2$ (Case II).

Case II: $\chi_2 = \rho_2$.

The developments of extraction and cumulative extraction for the second case are depicted in Figure 3. This case is a knife-edge case: $\chi_2 = \rho_2$ (=2 in the figure) and thus $\psi_w = \psi = \psi_\pi$. The resulting conflict of interest is in line with that discussed for the benchmark relations (cf. Section 3.4), namely that the marginal flow-pollution effect is equal to the market-power effect. Thus, if there were no intertemporal effects, then a monopolist would choose the welfare-maximizing extraction quantity anyway. Accordingly, the difference between the welfare-maximizing extraction quantity and the bargained one is solely driven by the difference in first-unit gains or, equivalently, between the convergence levels, as long as the government and the resource owners want positive extraction. Each period, $q^\star(z)$ and $q^{**}(z)$ decrease by the same amount. Only when the non-negativity constraint starts binding for the government, this cannot go on; $q^{**}(z)$ then is and remains zero, while $q^\star(z)$ continues to shrink. (67) simplifies to

$$\Delta_{q^\star}(z) = q^\star(z) - q^{**}(z) = \begin{cases} \psi\,(b - b_w) & \text{if } z \leq \hat{z}_w, \\ \psi\,(b - \kappa_z z) & \text{if } z > \hat{z}_w. \end{cases} \tag{74}$$

Figure 4 shows the development of contribution payments. Payments remain at a positive, constant level as long as $q^{**}(z_+^\star) > 0$. When $q^{**}(z_+^\star) = 0$, payments sharply increase as in Case I. Once the non-negativity constraint starts binding for $q^{**}(z^\star)$, it is possible that payments decrease monotonously or first increase and then vanish in the long run (see discussion in Appendix A.3).

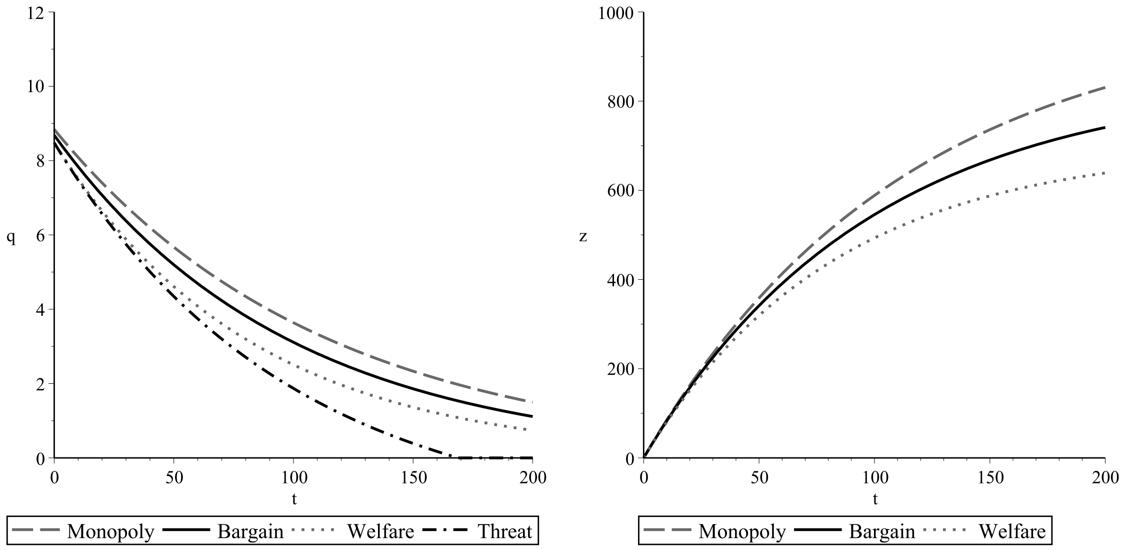


Figure 5: Extraction paths and cumulative extraction paths for $\chi_2 < \rho_2$ and $\psi_w b_w < \psi b$ (Case III).

Cases III and IV: $\boldsymbol{\chi_2 < \rho_2}$.

Now suppose that the market-power effect outweighs the marginal flow-pollution effect, $\chi_2 < \rho_2$ and thus $\psi_w > \psi > \psi_\pi$. For simplicity we focus in the discussion on cases where either $\psi_w b_w < \psi b$ *and* $\psi_w b_w < \psi_\pi b_\pi$ (Cases 3 and III) or $\psi_w b_w > \psi b$ *and* $\psi_w b_w > \psi_\pi b_\pi$ (Cases 4 and IV).

In Case III, the welfare-maximizing path would again imply lower extraction than the equilibrium extraction path, which in turn is below the profit-maximizing extraction. This can be seen in the same way as in the previous cases in Figure 5 for $\chi_2 = 2, \rho_2 = 5$. Comparing the equilibrium extraction $q^\star(z)$ with the welfare-maximizing threat extraction $q^{**}(z)$, we see an increasing divergence $\Delta_{q^\star}(z)$: The growth in z always implies reduced extraction, but the government wants to reduce extraction to a stronger extent than the resource owners. From (67):

$$\frac{\partial \Delta_{q^\star}(z)}{\partial z} = -\left(\psi - \psi_w\right)\kappa_z > 0. \tag{75}$$

Accordingly, and in contrast to Cases I and II, payments increase – see Figure 7a. Given that the bargaining parties can anticipate high payments in the future, they can also be negative and even declining for small values of z; in Appendix A.3, it is shown under which conditions this is the case. Once $q^{**}(z^\star_+) = 0$, the development of payments is similar to that in Cases I and II.

If, on the other hand, $\psi_w b_w > \psi b$, we have Case IV. In order to demonstrate this case, we

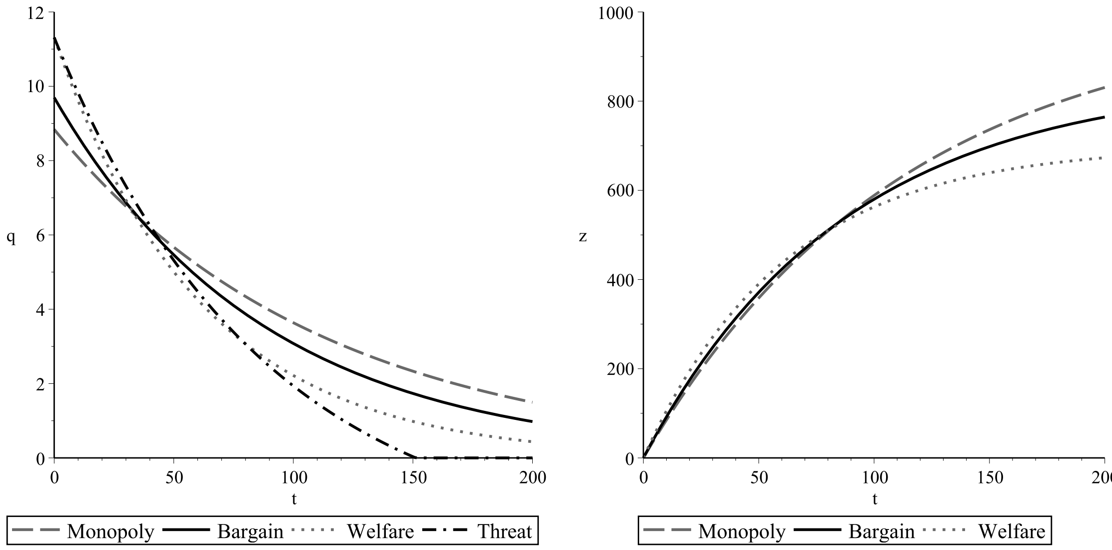
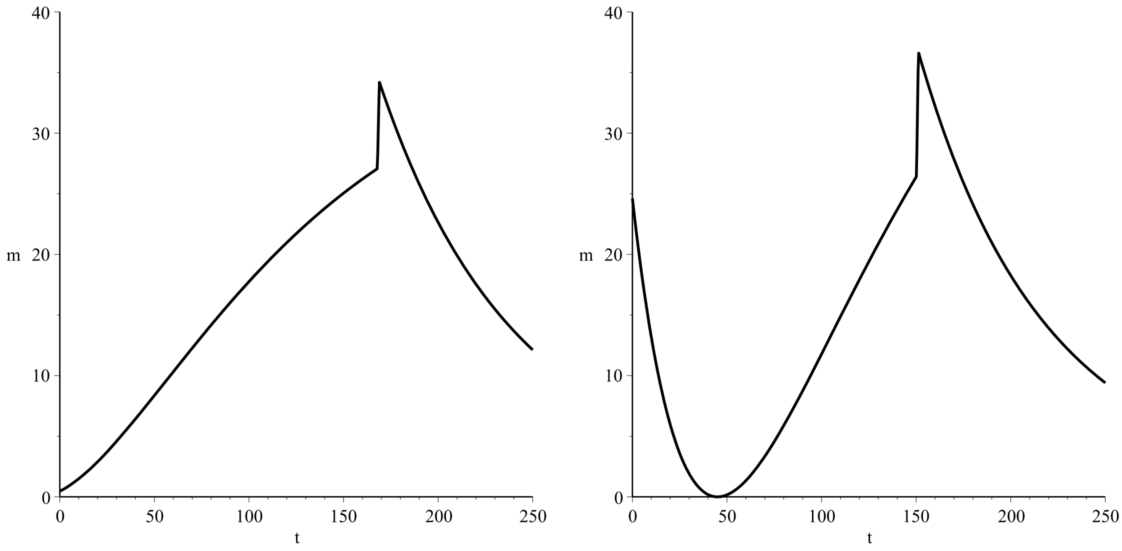


Figure 6: Extraction paths and cumulative extraction paths for $\chi_2 < \rho_2$ and $\psi_w b_w > \psi b$ (Case IV).

set $\chi_2 = 0$. The government's preferred extraction would exceed the bargained extraction for small z, in particular for $z = 0$, but as time goes by, the increase in z has (again) a stronger effect on the welfare-maximizing extraction than on the lobby's preferred extraction. Thus, it becomes lower than the bargained extraction for large z, in particular for $z = \hat{z}_w$. Therefore, the conflict of interest – the absolute value of $\Delta_{q^\star}(z)$ – first declines until the government's threat extraction is equal to the bargained extraction; afterwards, the two would diverge again as $\Delta_{q^\star}(z)$ increases, until the non-negativity constraint on the government's threat extraction becomes binding; see the left-hand side of Figure 6. Furthermore, on the government's preferred extraction path, the cumulative extraction would initially exceed that on the bargained extraction path, but fall short of it at some point after $\Delta_{q^\star}(z)$ becomes positive. Thus, the government would at first like to extract more than the resource owners, but in the long run, as extraction declines along any extraction path, the first-unit gains become more and more important. In the end, the lobby group's preferred convergence level is higher than that of the government; see right-hand side of Figure 6.

Figure 7b shows the development of contribution payments. The curve first slopes downwards. Payments are zero in the period when $\Delta_{q^\star}(z) = 0$ and afterwards, they turn negative as $\Delta_{q^\star}(z)$ becomes positive; this behavior is shown to be general for Case IV in Appendix A.3. After reaching a minimum, the curve slopes upwards and the payments become positive before $q^{**}(z^\star)$ hits the constraint. For z close to $\hat{z}_w$, the payment could also remain negative and even be declining. In Appendix A.3, it is shown under which conditions this is the case.



(a) $\psi_w b_w < \psi b$ (b) $\psi_w b_w > \psi b$

In Figure 7b, contribution payments are negative around $t = 45$.

Figure 7: Contribution payment paths for $\chi_2 < \rho_2$ (Cases III and IV).

When $q^{**}(z^{\star}_{+}) = 0$, payments again sharply increase and once the non-negativity constraint starts binding for $q^{**}(z^{\star})$, the development of payments is similar to that of the other cases.

Discussion.

Comparing the four extraction paths, we can see that the government's threat extraction is always below the bargained extraction in Cases I to III, while it initially exceeds the bargained extraction in Case IV. The period when the non-negativity constraint on the government's threat extraction becomes binding is delayed if either the slope of the marginal environmental damage function, χ_2, or the (absolute) slope of the marginal consumer-surplus function, ρ_2, increase (cf. Figure 3 with Figure 1 and Figure 5). If the slope of the marginal environmental damage function is increased, the government would prefer a lower extraction for a given amount of cumulative extraction; if the slope of the marginal consumer-surplus function increased, both bargaining parties would prefer a lower extraction for a given amount of cumulative extraction (see Proposition 3.1). Thus, if one of the slopes increased, the compromise path between welfare maximization and profit maximization would lead to a lower extraction for a given amount of cumulative extraction and the period when the constraint becomes binding would be delayed.

After the constraint has been hit, payments are never negative and become zero in the limit. One period before the constraint starts binding for $q^{**}(z^\star)$, the government's future threat extraction does not involve extraction smoothing anymore. Then the lobby's future outside option deteriorates so that payments increase more than before. In the time before the non-negativity constraint starts binding for $q^{**}(z^\star_+)$, payments are positive and declining if the government's preferred speed of extraction ψ_w is smaller than that of the lobby ψ_π. Then the difference between the equilibrium and the threat extraction $\Delta_{q^\star}(z)$ gradually declines. If both prefer the same speed of extraction $\psi_w = \psi_\pi$, payments are positive and constant. In this case, $\Delta_{q^\star}(z)$ does not change over time. Payments can temporarily be negative when the government prefers a higher speed of extraction than the lobby $\psi_w > \psi_\pi$. But as long as the threat exceeds the equilibrium extraction $\Delta_{q^\star}(z) < 0$, payments will be positive.

Finally, we concentrate on the two "political" parameter sets, namely the contribution valuations and the bargaining powers. The government's marginal-utility parameter γ and the lobby's marginal-cost parameter λ constitute the lobby's policy weight $\mu \equiv \gamma/\lambda$. If the government cares more for contribution payments or the lobby has a lower marginal cost of collecting them, the lobby has a higher weight in the equilibrium extraction choice. Then the convergence level $\hat{z}$ increases and the speed of extraction ψ shifts towards the lobby's preferred one. By contrast, we do not need the government's bargaining power η to determine the extraction path; it only influences the contribution payments. If the government has more bargaining power, the payments in the periods m usually increase. Only if the contribution payments turn negative for a while, which they possibly do in Case III and definitely in Case IV, they temporarily decline with the government's bargaining power. However, the present value of payments M always increases with the government's bargaining power.

5 Resource Taxes

The lobbying-equilibrium policy has been derived as a direct choice of extraction quantities. To generalize, we now also show how to establish the extraction path via resource taxes. Consider the behavior of resource suppliers that are so small that they take the price path as given; only through their lobby organization's influence on policy can they internalize the effect of supply on the price. Then along the lines of (15) the Euler equation of a resource supplier is

$$p(q^\circ) - \tau - \tfrac{\partial c(q^\circ, z)}{\partial q} = \beta \cdot \left[\tfrac{\partial c(q^\circ_+, z^\circ_+)}{\partial z} + p(q^\circ_+) - \tau_+ - \tfrac{\partial c(q^\circ_+, z_+)}{\partial q} \right], \tag{76}$$

where $^\circ$ denotes optimal extraction of a price-taking supplier and τ is the resource tax of the current period.

The tax path can be used to implement the extraction path bargained between the lobby and the government, (50). Comparing the two Euler equations, it must hold that

$$\tau^\star - \beta\cdot\tau^\star_+ = \frac{1}{1+\mu}\cdot\left\{\frac{\partial x(q^\star,z)}{\partial q} - \mu\cdot\frac{\partial p(q^\star)}{\partial q}q^\star - \beta\cdot\left[\frac{\partial x(q^\star_+,z_+)}{\partial z} - \frac{\partial x(q^\star_+,z_+)}{\partial q} + \mu\cdot\frac{\partial p(q^\star_+)}{\partial q}q^\star_+\right]\right\}. \tag{77}$$

Because the extraction path $q^\star(t+s)$ is known, it is straightforward to derive the tax path. For the explicit example, the tax path is given as follows:

Proposition 5.1 (Explicit Example: Tax Path) *The tax path $\tau^\star(t+s)$ that implements the extraction of $q^\star(t+s)$ by price-taking resource suppliers is defined by*

$$\tau^\star(t+s) = b_\pi - b + \left[a - (a_\pi - \rho_2)\right](1-\psi\kappa_z)^s\,\psi\left[b - \kappa_z z(t)\right]. \tag{78}$$

Equivalently, as a state-dependent policy rule we have

$$\tau^\star(z)b_\pi - b + \left[a - (a_\pi - \rho_2)\right]q^\star(z). \tag{79}$$

Proof. Using the explicit functions from Table 1 in (77) yields:

$$\tau^\star - \beta\cdot\tau^\star_+ = \frac{1}{1+\mu}\cdot\left\{\chi_1 + (\chi_2+\mu\rho_2)\,q^\star - \beta\cdot\left[\chi_z - \chi_1 - (\chi_2+\mu\rho_2)\,q^\star_+\right]\right\}. \tag{80}$$

We can substitute the bargaining-equilibrium extraction path from Propositions 4.2 and 4.3, which yields a difference equation for $\tau(t)$. Solving it and choosing a start value $\tau(0)$ that leads to a non-explosive path yields (78). □

The tax path consists of two parts. The first part, $b_\pi - b$, corrects for the different convergence levels due to the pollution effects. The resource taxes converge to this part in the long run, where they must just keep firms from extracting once that the lobbying-equilibrium convergence level of cumulative extraction, $z = \hat{z}$, has been reached. The second part is proportional to

$$a - (a_\pi - \rho_2) = \frac{\chi_2 + \mu\rho_2}{1+\mu}. \tag{81}$$

If the lobby's weight μ is very high, (81) goes to ρ_2 so that the resource suppliers are made to act like a monopolist. If μ goes to zero, (81) goes to χ_2 and we get a purely Pigouvian taxation.

Finally, note that implementing the lobbying equilibrium by resource taxation requires that the tax receipts are distributed to the resource suppliers as a lump-sum payment. While the time at which this happens is irrelevant in principle, in line of our lobbying model we would expect that in each period, the tax receipts of the respective period are paid back.

6 Conclusions

In this chapter, we derived resource extraction determined by the bargaining of a government and a lobby group. Equilibrium extraction is a compromise path between welfare maximization and profit maximization. The government would prefer the former and the lobby would prefer the latter if contribution payments and extraction were independent. Because marginal contribution utilities are constant, equilibrium extraction does not depend on the Nash bargaining powers: The weight of the lobby's influence on the equilibrium path increases in the government's preference for contribution payments and decreases in the lobby's cost of collecting them. Depending on flow-pollution damages and the price elasticity of resource demand, this implies that extraction is either too fast or too slow, compared to welfare maximization. Total extraction is too high due to first-unit flow-pollution damages and stock-pollution damages.

Along all equilibrium paths, extraction converges to zero as marginal costs increase with cumulative extraction. Thus, the conflict of interest between welfare maximization and profit maximization vanishes in the long run and so do the contribution payments after the welfare-maximizing convergence level is reached. Prior to that, contribution payments decline if the flow-pollution damage dominates, stay constant if the effects offset each other, and at least temporarily increase if the monopoly effect dominates. Partially, the development of contribution payments coincides with the change in the difference of the preferred extraction quantities. If the monopoly effect dominates, contribution payments may turn negative for a while. Then the weighted intertemporal profit loss from cooperating one period is higher than the corresponding welfare loss. In the period before the government's preferred convergence level is reached, the lobby's willingness to pay increases sharply. This is because the government's future threat extraction does not involve extraction smoothing anymore so that the lobby's future outside option deteriorates.

Finally, we demonstrate how the bargained extraction path can be implemented via resource taxes. These consist of a constant part, correcting for the difference in preferred convergence levels, and a part that is linear in equilibrium extraction quantities, correcting for the flow-pollution damage and the monopoly effect.

We believe the political-economy analysis of resource extraction to be a promising field of research, given that in this policy area many people seem to be convinced of the resource owners' distortive influence. In particular, an interesting research topic would be the political determination of backstop technologies' development, which would broaden the perspective on political distortions from resource consumption to investment.

A Appendix

A.1 Derivation of the Benchmark Solution

One way to derive a solution for the optimal (welfare-maximizing or profit-maximizing) extraction would be to use (18) in the Euler equations of Section 3.2. Together with the equation of motion of cumulative extraction (1) this constitutes a solvable system of difference equations. We instead guess the form of the extraction functions and verify it afterwards, as this is easier for our linear-quadratic system.

We guess that there exist constants $Y_{w,0}$, $Y_{w,1}$, $Y_{\pi,0}$, $Y_{\pi,1}$ such that the following state-dependent extraction functions exist

$$q^{**}(z) = \begin{cases} Y_{w,0} + Y_{w,1} z & \text{if } Y_{w,0} + Y_{w,1} z \geq 0, \\ 0 & \text{if } Y_{w,0} + Y_{w,1} z < 0, \end{cases} \tag{A.1a}$$

$$q^{*}(z) = \begin{cases} Y_{\pi,0} + Y_{\pi,1} z & \text{if } Y_{\pi,0} + Y_{\pi,1} z \geq 0, \\ 0 & \text{if } Y_{\pi,0} + Y_{\pi,1} z < 0. \end{cases} \tag{A.1b}$$

Thus, we expect the quadratic utility functions to lead to extraction functions that are linear in the state z as long as positive extraction is optimal. To solve for these coefficients, we first use them to state the value functions (8) and (14) in an explicit form.

Lemma A.1 (Benchmark Intertemporal Welfare and Profit) *Assume that*[13]

$$0 < 1 + Y_{i,1} \leq 1 \qquad \textit{for } i = w, \pi. \tag{A.2}$$

[13](A.2) is a stability condition; cf. Gandolfo (2009, Chapter 3).

Then

$$W^{**}(z)=\frac{b_w+\frac{Y_{w,0}}{Y_{w,1}}\kappa_z}{1-\beta\left(1+Y_{w,1}\right)}q^{**}(z)-\frac{\frac{a_w}{2}+\frac{1}{Y_{w,1}}\kappa_z}{1-\beta\left(1+Y_{w,1}\right)^2}q^{**}(z)^2-\frac{\chi_z z}{1-\beta}, \tag{A.3a}$$

$$\Pi^{*}(z)=\frac{b_\pi+\frac{Y_{\pi,0}}{Y_{\pi,1}}\kappa_z}{1-\beta\left(1+Y_{\pi,1}\right)}q^{*}(z)-\frac{\frac{a_\pi}{2}+\frac{1}{Y_{\pi,1}}\kappa_z}{1-\beta\left(1+Y_{\pi,1}\right)^2}q^{*}(z)^2, \tag{A.3b}$$

where $q^{**}(z)$ *and* $q^{*}(z)$ *are defined in* (A.1) *and* b_w *and* b_π *are defined in* (19).

Proof. For the proof, we ignore the case of (A.1) in which $q(z)$ is restricted to zero. The first reason is that due to the stability conditions, the case will not turn out to be relevant if z is not too high at the start. The second reason is that it can be easily seen that if the constraint becomes binding, we have

$$W^{**}(z)=-\frac{\chi_z z}{1-\beta}, \tag{A.4a}$$

$$\Pi^{*}(z)=0, \tag{A.4b}$$

which is obviously correct: $W^{**}(z)$ is then the present value of stock-pollution damage and $\Pi^{*}(z)$ is zero because zero production implies zero profits. Thus, let us now turn to the unconstrained case. Substituting (A.1a) into (1) yields:

$$z(t+1)=z(t)+Y_{w,0}+Y_{w,1}z(t). \tag{A.5}$$

Hence,

$$z(t+s)=z(t)+\sum_{\nu=0}^{s-1}q(t+\nu)=z(t)+sY_{w,0}+Y_{w,1}\sum_{\nu=0}^{s-1}z(t+\nu), \tag{A.6}$$

where $\nu\in\mathbb{N}$ is the summation index. After some substitutions and rearrangements, we get

$$z(t+s)=\left(1+Y_{w,1}\right)^s z(t)+Y_{w,0}\sum_{\nu=0}^{s-1}\left(1+Y_{w,1}\right)^\nu=\left(1+Y_{w,1}\right)^s\left[\frac{Y_{w,0}}{Y_{w,1}}+z(t)\right]-\frac{Y_{w,0}}{Y_{w,1}}, \tag{A.7}$$

where we have to assume (A.2) for stability. Substituting into (A.1a) yields:

$$q^{**}(t+s)=\left(1+Y_{w,1}\right)^s\left[Y_{w,0}+Y_{w,1}z(t)\right]. \tag{A.8}$$

From (18a), we then have (after some rearrangements)

$$\begin{aligned} w(t+s)=&\left(b_w+\frac{Y_{w,0}}{Y_{w,1}}\kappa_z\right)\left(1+Y_{w,1}\right)^s\left[Y_{w,0}+Y_{w,1}z(t)\right]\\ &-\left(\frac{a_w}{2}+\frac{1}{Y_{w,1}}\kappa_z\right)\left[\left(1+Y_{w,1}\right)^s\left[Y_{w,0}+Y_{w,1}z(t)\right]\right]^2\\ &+\left[\frac{Y_{w,0}}{Y_{w,1}}+\left(\frac{\beta}{1-\beta}-\frac{1}{Y_{w,1}}\right)\left(1+Y_{w,1}\right)^s\left[Y_{w,0}+Y_{w,1}z(t)\right]\right]\chi_z. \end{aligned} \tag{A.9}$$

Using this in (4) yields:

$$
\begin{aligned}
W^{**}(t) = &\left(b_w + \frac{Y_{w,0}}{Y_{w,1}}\kappa_z\right)\sum_{s=0}^{\infty}\beta^s\left(1+Y_{w,1}\right)^s\left[Y_{w,0}+Y_{w,1}z(t)\right] \\
&-\left(\frac{a_w}{2}+\frac{1}{Y_{w,1}}\kappa_z\right)\sum_{s=0}^{\infty}\beta^s\left[\left(1+Y_{w,1}\right)^s\left[Y_{w,0}+Y_{w,1}z(t)\right]\right]^2 \\
&+\left[\frac{Y_{w,0}}{Y_{w,1}}\sum_{s=0}^{\infty}\beta^s+\left(\frac{\beta}{1-\beta}-\frac{1}{Y_{w,1}}\right)\sum_{s=0}^{\infty}\beta^s\left(1+Y_{w,1}\right)^s\left[Y_{w,0}+Y_{w,1}z(t)\right]\right]\chi_z.
\end{aligned}
\tag{A.10}
$$

Evaluating the infinite sums yields (A.3a). (A.3b) can be obtained in the same manner using (A.1b) instead of (A.1a), (18b) instead of (18a), and substituting the results into (6). □

Using these value functions, we can explicitly derive the respective coefficients:

Lemma A.2 (Benchmark Extractions) *In the social planner's extraction function* (A.1a), *we have the following coefficients:*

$$Y_{w,0} = \psi_w \cdot b_w, \tag{A.11a}$$

$$Y_{w,1} = -\psi_w \cdot \kappa_z, \tag{A.11b}$$

implying $q^{**}(z) = \psi_w \cdot (b_w - \kappa_z z)$ *where* ψ_w *is defined as stated in* (23). *For the monopolist's extraction function* (A.1b), w *has to be replaced by* π *in* (A.11) *so that we get* $q^*(z) = \psi_\pi \cdot (b_\pi - \kappa_z z)$.

Proof. $q^{**}(z)$ must maximize (A.3a), which we can split into instantaneous welfare of the current period and discounted welfare of all periods afterwards

$$
\begin{aligned}
W^{**}(z) = &\left(b_w - \kappa_z z\right)q(z) - \frac{a_w}{2}q(z)^2 + \left[\frac{q(z)}{r} - z\right]\chi_z + \beta\left\{\frac{b_w + \frac{Y_{w,0}}{Y_{w,1}}\kappa_z}{1-\beta\left(1+Y_{w,1}\right)}\left(Y_{w,0}+Y_{w,1}z_+\right)\right. \\
&\left.-\frac{\frac{a_w}{2}+\frac{1}{Y_{w,1}}\kappa_z}{1-\beta\left(1+Y_{w,1}\right)^2}\left(Y_{w,0}+Y_{w,1}z_+\right)^2 - \frac{\chi_z z_+}{1-\beta}\right\}.
\end{aligned}
\tag{A.12}
$$

Substituting the equation of motion (1), differentiating with respect to $q(z)$, and substituting (A.1a), we get the following first-order condition:

$$
\begin{aligned}
&b_w - \kappa_z z - a_w\left(Y_{w,0}+Y_{w,1}z\right) + \beta\left\{\frac{Y_{w,1}b_w + Y_{w,0}\kappa_z}{1-\beta\left(1+Y_{w,1}\right)}\right. \\
&\left.-\frac{Y_{w,1}a_w + 2\kappa_z}{1-\beta\left(1+Y_{w,1}\right)^2}\left[Y_{w,0}+Y_{w,1}\left[Y_{w,0}+\left(1+Y_{w,1}\right)z\right]\right]\right\} = 0.
\end{aligned}
\tag{A.13}
$$

We can state (A.13) for $z_+ = z + q^{**}(z)$ and again substitute (A.1a). This generates two equations in two unknowns. These contain quadratic terms, but taking (A.2) into account, we can select the solution (A.11). $q^*(z)$ is derived in the same way. □

Substituting the coefficients from Lemma A.2 into (A.7), we can derive (24a) and, in the same way, (24b). Substituting them into (A.2) yields the inequalities in Proposition 3.2. We can further explicate the parameter restrictions as follows:

Lemma A.3 (Parameter Restrictions) (A.2) *implies*

$$2\kappa_z < a_i + \sqrt{a_i^2 + \frac{4}{r}\kappa_z\,(a_i - \kappa_z)} \qquad \textit{for } i = w, \pi, \tag{A.14}$$

which can only be fulfilled if $a_i > \kappa_z$ *holds, as stated in* (21).

Proof. (A.2) and (A.11) imply two inequalities, $0 < 1 - \psi_i \cdot \kappa_z$ and $1 - \psi_i \cdot \kappa_z \leq 1$. Substituting (23) into the first inequality yields (A.14), for which $a_i > \kappa_z$ is necessary. This implies that ψ_i is positive; see (23). Then the second inequality also holds. □

The inequality in (23) follows from Lemma A.3. Finally, we can replace the unknown coefficients in the value functions of Lemma A.1 and simplify, which yields the value functions in Proposition 3.3.

A.2 Derivation of the Nash Bargaining Solution

From the discussion in Section 4.2, we know that the present value of contribution payments can be derived with the threat value functions and the equilibrium values of intertemporal welfare and profit. Thus, we first derive the threat and equilibrium extraction paths and then determine the contribution payment path. From the discussion in Section 4.2, we also know that the threat extraction function must be the same as in the welfare-maximizing case. Furthermore, we guess that the equilibrium extraction function has the same linear form as in the benchmark cases. Using the coefficients from Lemma A.2, we thus have

$$q^\star(z) = X_{g,0} + X_{g,1} z, \tag{A.15a}$$

$$q^\#(z) = q^{**}(z) = \begin{cases} \psi_w \cdot (b_w - \kappa_z z) & \text{if } b_w - \kappa_z z \geq 0, \\ 0 & \text{if } b_w - \kappa_z z < 0. \end{cases} \tag{A.15b}$$

Strictly speaking, we would need to explicate the non-negativity constraint for $q^\star(z)$; but by the same logic as in the benchmark cases, the convergence level is only reached asymptotically so that the constraint never becomes binding. This is different for the threat extraction quantity, however. At some point, the lobbying extraction may lead to a total extraction above the welfare-maximizing convergence level. Then the non-negativity constraints are still irrelevant for the equilibrium path, but they bind for the government's threat.

The government's threat value function must be equal to the welfare-maximizing one from Proposition 3.3. The lobby's threat value function also results from the welfare-maximizing extraction path; it has the same form as (A.3b), but the π coefficients have to be replaced by the w ones. Using the coefficients from Lemma A.2, we thus have the present-value functions of Proposition 4.1.

We can now determine the present values of welfare and profits in equilibrium.

Lemma A.4 (Equilibrium Intertemporal Welfare and Profit) *Assume that*

$$0 < 1 + X_{g,1} \leq 1. \tag{A.16}$$

Then

$$W^\star(z) = \frac{b_w + \frac{X_{g,0}}{X_{g,1}}\kappa_z}{1-\beta\left(1+X_{g,1}\right)} q^\star(z) - \frac{\frac{a_w}{2} + \frac{1}{X_{g,1}}\kappa_z}{1-\beta\left(1+X_{g,1}\right)^2} q^\star(z)^2 - \frac{\chi_z z}{1-\beta}, \tag{A.17a}$$

$$\Pi^\star(z) = \frac{b_\pi + \frac{X_{g,0}}{X_{g,1}}\kappa_z}{1-\beta\left(1+X_{g,1}\right)} q^\star(z) - \frac{\frac{a_\pi}{2} + \frac{1}{X_{g,1}}\kappa_z}{1-\beta\left(1+X_{g,1}\right)^2} q^\star(z)^2, \tag{A.17b}$$

where $q^\star(z)$ is defined in (A.15a)*.*

Proof. The proof follows along the lines of the proof of Lemma A.1. □

The next step is to determine the equilibrium extraction path because we know by the discussion in Section 4.2 that this path maximizes (48) (and is therefore relatively easily characterizable):

Lemma A.5 (Equilibrium Extraction) *In the equilibrium extraction function* (A.15a)*, we have the following coefficients:*

$$X_{g,0} = \psi \cdot b, \tag{A.18a}$$

$$X_{g,1} = -\psi \cdot \kappa_z \tag{A.18b}$$

so that $q^\star(z) = \psi \cdot (b - \kappa_z z)$ where ψ and b are defined as stated in (59) *and* (58b)*, respectively.*

Proof. From (49) and (18), $q^\star(z)$ must maximize

$$V(z) = (b_w - \kappa_z z)\, q(z) - \frac{a_w}{2} q(z)^2 + \left[\frac{q(z)}{r} - z\right] \chi_z + \mu \left[(b_\pi - \kappa_z z)\, q(z) - \frac{a_\pi}{2} q(z)^2\right]$$
$$+ \beta \left[W^\star(z_+) + \mu \Pi^\star(z_+)\right], \tag{A.19}$$

where μ is defined by (47). By Lemma A.4:

$$W^\star(z_+) = \frac{b_w + \frac{X_{g,0}}{X_{g,1}} \kappa_z}{1 - \beta\left(1 + X_{g,1}\right)} \left(X_{g,0} + X_{g,1} z_+\right) - \frac{\frac{a_w}{2} + \frac{1}{X_{g,1}} \kappa_z}{1 - \beta\left(1 + X_{g,1}\right)^2} \left(X_{g,0} + X_{g,1} z_+\right)^2 - \frac{\chi_z z_+}{1 - \beta}, \tag{A.20a}$$

$$\Pi^\star(z_+) = \frac{b_\pi + \frac{X_{g,0}}{X_{g,1}} \kappa_z}{1 - \beta\left(1 + X_{g,1}\right)} \left(X_{g,0} + X_{g,1} z_+\right) - \frac{\frac{a_\pi}{2} + \frac{1}{X_{g,1}} \kappa_z}{1 - \beta\left(1 + X_{g,1}\right)^2} \left(X_{g,0} + X_{g,1} z_+\right)^2. \tag{A.20b}$$

The coefficients in (A.18) are then derived along the lines of the proof of Lemma A.2. □

Substituting the coefficients from Lemma A.5 into (A.7), we can derive (62). Substituting (A.18b) into (A.16) yields the inequality in Proposition 4.3. We can now also state the contribution payments that will be paid along the equilibrium path:

Lemma A.6 (Equilibrium Contribution Payments) *The equilibrium contribution payment function is as stated in equation* (60c) *of Proposition 4.2.*

Proof. Substituting the coefficients from Lemma A.5 into the equilibrium intertemporal welfare and profit (from Lemma A.4) and the threat value function (from Proposition 4.1) into (51) yields the present value of contribution payments in equilibrium, which we can state for z and $z_+^\star$. Using the equilibrium equation of motion (38a) together with the extraction functions from (60), we can plug the two equations into (52) and derive the equilibrium contribution payments. □

The inequality in (59) follows from Lemma A.3. Finally, we can simplify the equilibrium intertemporal welfare and profit and the present value of contribution payments in equilibrium derived in the proof of Lemma A.6, which yields the equilibrium value functions in Proposition 4.4.

A.3 Development of the Contribution Payments

For the benchmark cases, we distinguished four cases in Section 3.3, constituted by the relation between ψ_w, ψ_π, b_w, and b_π (Cases 1 to 4). We have similar cases for the relation between ψ_w, ψ, b_w, and b that we use for the discussion in Section 4.4 (Cases I to IV). In this Appendix, we first show how the sets of cases relate. Afterwards, we formally characterize the development of payments.

Lemma A.7 (Parameter Relations) *The benchmark cases and the lobbying-equilibrium cases are summarized in Table 2 (on page 15) and Table 3 (on page 27), respectively. In Cases 1 and 2, we have $\psi_w < \psi_\pi$ and $\psi_w = \psi_\pi$, respectively. Then, Cases I and II, $\psi_w < \psi$ and $\psi_w = \psi$, respectively, hold because $\psi_\pi \gtreqless \psi_w \Leftrightarrow \psi \gtreqless \psi_w$. In Cases 3 and 4, we have $\psi_w > \psi_\pi$ and in Cases III and IV, we have $\psi_w > \psi$. Case 3 is defined by $\psi_w < \psi_\pi b_\pi / b_w$. Then Case III, $\psi_w < \psi b / b_w$, holds if $\psi_w < \tilde{\psi}_\pi b_\pi / b_w$ where*

$$\tilde{\psi}_\pi \equiv \frac{2}{a_\pi + \frac{1+\mu}{\mu}\sqrt{a^2 + \frac{4}{r}\kappa_z (a - \kappa_z)} - \frac{1}{\mu}\sqrt{a_w^2 + \frac{4}{r}\kappa_z (a_w - \kappa_z)}} < \psi_\pi. \tag{A.21}$$

Else, Case IV, $\psi_w > \psi b / b_w$, holds, which it also does if $\psi_w > \psi_\pi b_\pi / b_w$ (Case 4).

Proof. From (23), (58), and (59), we have

$$\psi_w b_w \gtreqless \psi b \quad \Leftrightarrow \quad \psi_w b_w \gtreqless \tilde{\psi}_\pi b_\pi. \tag{A.22}$$

The derivative of $\tilde{\psi}_\pi$ with respect to μ is greater than or equal to zero:

$$\frac{\partial \tilde{\psi}_\pi}{\partial \mu} = \frac{1}{2}\left(\frac{\tilde{\psi}_\pi}{\mu}\right)^2 \left[\frac{a a_w + \frac{4}{r}\kappa_z \left(\frac{a+a_w}{2} - \kappa_z\right)}{\sqrt{a^2 + \frac{4}{r}\kappa_z (a - \kappa_z)}} - \sqrt{a_w^2 + \frac{4}{r}\kappa_z (a_w - \kappa_z)}\right] \geq 0. \tag{A.23}$$

From (A.23) and $0 < \mu < \infty$, we have

$$\lim_{\mu \to 0} \tilde{\psi}_\pi < \tilde{\psi}_\pi < \lim_{\mu \to \infty} \tilde{\psi}_\pi \quad \Leftrightarrow \quad \frac{2}{a_\pi + \frac{a_w a_\pi + \frac{4}{r}\kappa_z \left(\frac{a_w + a_\pi}{2} - \kappa_z\right)}{\sqrt{a_w^2 + \frac{4}{r}\kappa_z (a_w - \kappa_z)}}} < \tilde{\psi}_\pi < \psi_\pi. \tag{A.24}$$

By $\psi b = \tilde{\psi}_\pi b_\pi$ and $\psi_\pi > \tilde{\psi}_\pi$, $\psi_w > \psi_\pi b_\pi / b_w$ implies $\psi_w > \psi b / b_w$. By contrast, $\psi_w < \psi_\pi b_\pi / b_w$ does not necessarily imply $\psi_w < \psi b / b_w$. □

In Proposition 4.2, we have the formula for the contribution payments. Substituting from Proposition 4.1, taking into account that the non-negativity constraint on the government's threat extraction constitutes three distinguishable cases (see discussion at the end of Section 4.3), and simplifying yields:

$$m^\star(z) = \frac{1}{\gamma} \cdot \begin{cases} \Theta_1 \cdot (b - b_w)\left[q^\star(z) - q^{**}(z)\right] + \Theta_2 \cdot \left[q^\star(z) - q^{**}(z)\right]^2 & \text{if } z < \hat{z}_{\underline{w}}, \\ (b - b_w)\, q^\star(z) + \Theta_3 \cdot q^\star(z)^2 - \Theta_4 \cdot (b - b_w)\, q^{**}(z) - \Theta_5 \cdot q^{**}(z)^2 & \text{if } \hat{z}_{\underline{w}} \leq z < \hat{z}_w, \\ (b - b_w)\, q^\star(z) + \Theta_3 \cdot q^\star(z)^2 & \text{if } z \geq \hat{z}_w, \end{cases} \tag{A.25}$$

where

$$\Theta_1 \equiv \eta\,(1+\mu)\left[\frac{1-\beta}{1-\beta\,(1-\psi_w\kappa_z)}+\frac{\psi\,(a-a_w)}{2\left(\frac{1}{\psi}-\frac{a}{2}\right)(\psi-\psi_w)}\right] \qquad \gtreqless 0 \quad \Leftrightarrow \quad \psi \gtreqless \psi_w, \tag{A.26a}$$

$$\Theta_2 \equiv \frac{1-\beta}{1-\beta\,(1-\psi_w\kappa_z)^2}\left[\frac{1}{\psi_w}-\frac{a_w}{2}+\eta\,(1+\mu)\left(\frac{1}{\psi}-\frac{a}{2}\right)\right] \qquad > 0, \tag{A.26b}$$

$$\Theta_3 \equiv \frac{a_w}{2}-\frac{1}{\psi}+\eta\,(1+\mu)\left(\frac{1}{\psi}-\frac{a}{2}\right) \qquad > -\frac{1}{\psi}, \tag{A.26c}$$

$$\Theta_4 \equiv \frac{\eta\,(1+\mu)}{1-\beta\,(1-\psi_w\kappa_z)} \qquad > 0, \tag{A.26d}$$

$$\Theta_5 \equiv \frac{1}{1-\beta\,(1-\psi_w\kappa_z)^2}\left[\frac{a_w}{2}-\frac{1}{\psi_w}+\eta\,(1+\mu)\left(\frac{1}{\psi_w}-\frac{a}{2}\right)\right] \qquad > -\frac{1}{\psi_w}. \tag{A.26e}$$

Even though these coefficients are not intuitively interpretable, they allow a comfortable characterization of the development of payments. In the following, we discuss the payments and their development for different parameter relations. We start with the time after $\hat{z}_w$ has been reached so that $q^{**}(z) \geq 0$ and $q^{**}(z_+^\star) \geq 0$ are binding for the welfare-maximizing path because it is clear from Section 4.4 that this is easiest to analyze.

Proposition A.1 (Contribution Payments for $\boldsymbol{z \geq \hat{z}_w}$) *For $\hat{z} > z \geq \hat{z}_w$, contribution payments are positive. For $z \to \hat{z}$, they asymptotically converge towards zero. This convergence is monotone if*

$$\eta\,(1+\mu) \geq \frac{1}{2}\left(1-\frac{a_w-\frac{a}{2}}{\frac{1}{\psi}-\frac{a}{2}}\right). \tag{A.27}$$

Else, they increase for small z and decline for large z, in particular for $z \geq (\hat{z}+\hat{z}_w)/2$. The relation (A.27) *always holds if $\eta(1+\mu) \geq 1/2$. Else, if $\eta(1+\mu) < 1/2$, the left-hand side of the relation* (A.27) *must be the larger, the smaller a_w (or the larger a_π) for the convergence of the contribution payments to be monotone. Note that the relation* (A.27) *is independent of b_w and b_π.*

Proof. Using $\Theta_3 > -1/\psi$:

$$m^\star(z) > \frac{1}{\gamma}\left[(b-b_w)\,q^\star(z)-\frac{1}{\psi}q^\star(z)^2\right] = \frac{1}{\gamma}\underbrace{(\kappa_z z-b_w)}_{>0}q^\star(z) > 0, \tag{A.28}$$

except asymptotically where $q^\star(z)=0$ so that $m^\star(z)=0$. Furthermore,

$$\tfrac{\partial m^\star(z)}{\partial z} < \frac{1}{\gamma}\left[(b-b_w)-\frac{2}{\psi}q^\star(z)\right](-\psi\kappa_z) = \frac{1}{\gamma}\underbrace{(2\kappa_z z-b-b_w)}_{\gtreqless 0 \,\Leftrightarrow\, z \gtreqless (\hat{z}+\hat{z}_w)/2}\underbrace{(-\psi\kappa_z)}_{<0} \tag{A.29}$$

so that contribution payments at the latest decline when $z \geq (\hat{z} + \hat{z}_w)/2$. They always decline if they decline for small z, in particular for $z = \hat{z}_w$. Differentiating the first line of (A.25), substituting $z = \hat{z}_w$ and rearranging yields (A.27). Along the lines of the remark after Proposition 4.1, the fraction in (A.27) is positive so that $\eta(1+\mu) \geq 1/2$ is sufficient for the (weak) inequality in (A.27) to hold. Furthermore, the fraction in (A.27) is increasing in a_w and declining in a_π. □

We continue with the time before $\hat{z}_{\underline{w}}$ has been reached so that $q^{**}(z) \geq 0$ and $q^{**}(z_+^\star) \geq 0$ are not binding for the welfare-maximizing path. Here, we have to distinguish between the four lobbying-equilibrium cases. We start with Cases I and II in which $\psi_w \leq \psi$.

Proposition A.2 (Contribution Payments for $\boldsymbol{z < \hat{z}_{\underline{w}}}$ and $\boldsymbol{\psi \geq \psi_w}$) *Suppose that $\psi > \psi_w$ (Case I) or $\psi = \psi_w$ (Case II) and that $0 \leq z < \hat{z}_{\underline{w}}$. Then contribution payments are positive. They are declining in Case I and constant in Case II.*

Proof. In both cases, $q^\star(z) - q^{**}(z) > 0$; see (67). Additionally, $\psi > \psi_w$ implies $\Theta_1 > 0$ for Case I and $\psi = \psi_w$ implies $\Theta_1 = 0$ for Case II; see (A.26a). Thus, all (remaining) parts of (A.25) are positive. For the development of payments, we have

$$\tfrac{\partial m^\star(z)}{\partial z} = \frac{1}{\gamma}\left[\Theta_1 \cdot (b - b_w) + \Theta_2 \cdot 2\Delta_{q^\star}(z)\right](\psi_w - \psi)\,\kappa_z. \tag{A.30}$$

For Case I, every part of the square-bracketed term is positive and the round-bracketed difference with which it is multiplied is negative so that the whole derivative is negative and contribution payments decline. For Case II, the round-bracketed difference is zero so that contribution payments remain constant. □

Now we discuss Cases III and IV in which $\psi_w > \psi$.

Proposition A.3 (Contribution Payments for $\boldsymbol{z < \hat{z}_{\underline{w}}}$ and $\boldsymbol{\psi < \psi_w}$) *Define*

$$z_1 = \hat{z}_{\underline{w}} - \psi\frac{1-\psi_w\kappa_z}{\psi_w - \psi}\left(\hat{z} - \hat{z}_{\underline{w}}\right) \qquad = \frac{\psi_w b_w - \psi b}{(\psi_w - \psi)\,\kappa_z}, \tag{A.31a}$$

$$\tilde{z}_1 = z_1 - \frac{\Theta_1}{2\Theta_2}\frac{1-\psi\kappa_z}{\psi_w - \psi}\left(\hat{z} - \hat{z}_{\underline{w}}\right) \qquad = z_1 - \frac{\Theta_1}{2\Theta_2}\frac{b - b_w}{(\psi_w - \psi)\,\kappa_z}, \tag{A.31b}$$

$$z_2 = \tilde{z}_1 - \frac{\Theta_1}{2\Theta_2}\frac{1-\psi\kappa_z}{\psi_w - \psi}\left(\hat{z} - \hat{z}_{\underline{w}}\right) \qquad = \tilde{z}_1 - \frac{\Theta_1}{2\Theta_2}\frac{b - b_w}{(\psi_w - \psi)\,\kappa_z}, \tag{A.31c}$$

where $z_1 < \tilde{z}_1 < z_2$ by $\Theta_1 < 0$. If $z \leq z_1$ or $z \geq z_2$, contribution payments are positive. If $z_1 < z < z_2$, they are negative. If $z < \tilde{z}_1$, contribution payments are declining. If $z \geq \tilde{z}_1$, they are increasing.

Suppose that $\psi_w b_w \leq \psi b$ (Case III) and that $0 \leq z < \hat{z}_{\underline{w}}$. Then $z_1 \leq 0$. If $z_2 \leq 0$, contribution payments are globally positive and increasing. If $z_2 > 0$, they are negative for $0 \leq z < z_2$ and positive for $z \geq z_2$, and if $z_2 \geq \hat{z}_{\underline{w}}$, they are globally negative. Furthermore, if $\tilde{z}_1 > 0$, contribution payments are declining for $0 \leq z < \tilde{z}_1$ and increasing for $z \geq \tilde{z}_1$, and if $\tilde{z}_1 \geq \hat{z}_{\underline{w}}$, they are globally declining.

Now suppose that $\psi_w b_w > \psi b$ (Case IV) and that $0 \leq z < \hat{z}_{\underline{w}}$. Then $0 < z_1 < \hat{z}_w$ and contribution payments are positive and declining for $0 \leq z \leq z_1$. If $z_2 < \hat{z}_{\underline{w}}$, they are negative and declining for $z_1 < z < \tilde{z}_1$, negative and increasing for $\tilde{z}_1 \leq z < z_2$, and positive and increasing for $z_2 \leq z < \hat{z}_{\underline{w}}$. If $z_2 \geq \hat{z}_{\underline{w}}$, contribution payments remain negative for $z_1 < z < \hat{z}_{\underline{w}}$ and if $\tilde{z}_1 > \hat{z}_{\underline{w}}$, they remain declining for $z_1 < z < \hat{z}_{\underline{w}}$.

Proof. In both cases, $\psi < \psi_w$ implies $\Theta_1 < 0$; see (A.26a). In Case III, $\psi_w b_w \leq \psi b$ so that $z_1 \leq 0$ and in Case IV, $\psi_w b_w > \psi b$ so that $0 < z_1 < \hat{z}_w$; see (A.31a). We can write $m^\star(z)$ and $\frac{\partial m^\star(z)}{\partial z}$ as functions of z:

$$m^\star(z) = \frac{1}{\gamma} \cdot \Theta_2 \cdot (\psi\kappa_z - \psi_w\kappa_z)^2 \cdot (z_1 - z) \cdot (z_2 - z)\,, \tag{A.32a}$$

$$\frac{\partial m^\star(z)}{\partial z} = \frac{1}{\gamma} \cdot \Theta_2 \cdot (\psi\kappa_z - \psi_w\kappa_z)^2 \cdot (\tilde{z}_1 - z) \cdot (-2)\,. \tag{A.32b}$$

Those levels of cumulative extraction for which $m^\star(z)$ and $\frac{\partial m^\star(z)}{\partial z}$ change their signs directly follow from (A.32). □

Finally, we consider what happens in the period after $\hat{z}_{\underline{w}}$ (or before $\hat{z}_w$) has been reached. In this period, the current threat extraction $q^{**}(z)$ is positive but the future threat extraction $q^{**}(z_+^\star)$ is zero and its non-negativity constraint is binding. As this is relevant for at most one period, we do not analyze how the contribution payments develop between $z = \hat{z}_{\underline{w}}$ and $z = \hat{z}_w$, but how this development changes around $z = \hat{z}_{\underline{w}}$ and $z = \hat{z}_w$.

Proposition A.4 (Contribution Payments for $\hat{z}_{\underline{w}} \leq z < \hat{z}_w$) *Suppose that $\eta = 0$. Then $\frac{\partial m^\star(z)}{\partial z}$ is continuous for $z \to \hat{z}_{\underline{w}}$ and $z \to \hat{z}_w$. Now suppose that $\eta > 0$. Then $\frac{\partial m^\star(z)}{\partial z}$ is discontinuous for $z \to \hat{z}_{\underline{w}}$ and $z \to \hat{z}_w$. For $z \to \hat{z}_{\underline{w}}$, its right-hand limit is greater than its left-hand limit and for $z \to \hat{z}_w$, its right-hand limit is smaller than its left-hand limit.*

Proof. Subtracting the derivative of the first line in (A.25) from the derivative of the second line in (A.25) for $z \to \hat{z}_{\underline{w}}$ yields:

$$\begin{aligned}\lim_{z \to \hat{z}_{\underline{w}}+} \frac{\partial m^\star(z)}{\partial z} - \lim_{z \to \hat{z}_{\underline{w}}-} \frac{\partial m^\star(z)}{\partial z} &= \frac{\eta\,(1+\mu)\,\psi_w\kappa_z\,(b - b_w)}{1 - \beta\,(1 - \psi_w\kappa_z)} \\ &\cdot \frac{\beta\psi^2}{2\,(1 - \psi\kappa_z)} \left[(a - 2\kappa_z)\sqrt{a^2 + \frac{4}{r}\kappa_z\,(a - \kappa_z) + \kappa_z^2} + \left(a - \kappa_z + \frac{2}{r}\kappa_z\right)(a - \kappa_z)\right].\end{aligned} \tag{A.33}$$

The square-bracketed term is positive if $a \geq 2\kappa_z$. Else, if $a < 2\kappa_z$, the square-bracketed term is also positive because then

$$
\begin{aligned}
& \kappa_z^2 + \left(a - \kappa_z + \frac{2}{r}\kappa_z\right)(a - \kappa_z) > (2\kappa_z - a)\sqrt{a^2 + \frac{4}{r}\kappa_z (a - \kappa_z)} \\
\Leftrightarrow \quad & \left[\kappa_z^2 + \left(a - \kappa_z + \frac{2}{r}\kappa_z\right)(a - \kappa_z)\right]^2 > \left[(2\kappa_z - a)\sqrt{a^2 + \frac{4}{r}\kappa_z (a - \kappa_z)}\right]^2 \\
\Leftrightarrow \quad & \frac{4\kappa_z^2 (a - \kappa_z)^2}{(1 - \beta)^2} > 0.
\end{aligned} \tag{A.34}
$$

Thus, (A.33) is greater than (equal to) zero if η is greater than (equal to) zero. Subtracting the derivative of the second line in (A.25) from the derivative of the third line in (A.25) for $z \to \hat{z}_w$ yields:

$$
\lim_{z \to \hat{z}_w +} \tfrac{\partial m^\star(z)}{\partial z} - \lim_{z \to \hat{z}_w -} \tfrac{\partial m^\star(z)}{\partial z} = -\frac{\eta (1 + \mu) \psi_w \kappa_z (b - b_w)}{1 - \beta (1 - \psi_w \kappa_z)} < 0. \tag{A.35}
$$

Thus, (A.35) is smaller than (equal to) zero if η is greater than (equal to) zero. □

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