DISSERTATION

Robust planning in scheduled passenger traffic with applied stochastic programming and integrated risk management

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Schriftliche Arbeit zur Erlangung des akademischen Grades
- Doctor rerum politicarum (Dr. rer. pol.) -
im Fach Wirtschaftsinformatik

eingereicht an der
Fakultät für Wirtschaftswissenschaften der
Universität Paderborn

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Paderborn im Februar 2013

Acknowlegdement

This thesis is the result of the research I worked on as a member of the Decision Support & Operations Research Lab and of the International Graduate School Dynamic Intelligent Systems at the University of Paderborn. Completing this dissertation would not have been possible without the support of many people.

First of all I would like to thank my supervisor Prof. Dr. Leena Suhl for giving me the opportunity to join her research group and to write this thesis. Her constant guidance, her openness and her foresight were essential for the completion of this thesis. I also thank the International Graduate School Dynamical Intelligent Systems for providing a great doctoral program and all the support during my research project. Furthermore, I thank Prof. Dr. Natalia Kliewer for writing the second report on my thesis and Prof. Alper Atamtürk for his great collaboration during my time at the University of California, Berkeley.

I really enjoyed working at the DS&OR Lab. The close cooperation with so many fruitful discussions created an excellent working atmosphere where also fun was not missing. In particular, I would like to thank Stefan Kramkowski and Lin Xie for their collaboration and inspiration and my colleagues Jörg Wiese, Franz Wesselmann, Kostja Siefen and Florian Stapel for supporting and encouraging me.

Finally, I wish to thank my friends and family, especially my parents, for believing in me and supporting me. Special thanks to Franziska for her encouragement, her understanding and for showing me that there is more in life besides a thesis.

Paderborn, Februar 2013 Marc Naumann

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1 Introduction

The planning process in scheduled passenger traffic is a difficult task. Due to its complexity, airlines and public transport companies decompose their schedule planning into several planning phases. The resulting smaller problems of the strategic and operational planning phases can nowadays be solved with optimization tools for realistic instances.

Nonetheless many of the decisions during the schedule planning process have to be made under uncertainty. For example, airlines have to plan their routes and frequencies although they cannot foresee the fuel costs, which have become the largest part of their costs. Not only the costs but also their income is uncertain: The demand for a flight is not known until the last booking is completed. Therefore the aircraft type that is planned for a flight also has to be fixed under uncertainty. Furthermore, disruptions due to bad weather conditions or traffic density are a problem. An aircraft or a bus will be late and cannot start a consecutive flight or trip on time. Buffer times can be added, but as disruptions are unforeseeable, when, where and how long should they be? Furthermore the illness absences of personnel are an uncertainty in crew planning. Too much reserves cause high personnel costs but on the other hand the company must be able to replace every ill person on every day to maintain their service.

Against this background, this thesis aims to show that the strategic and operational planning process in scheduled passenger traffic can be improved by implementing an integrated risk management strategy with operational and contractual risk management into the mathematical optimization models of the planning phases.

Thereby applied stochastic optimization models for airline scheduling and scheduling in public transport are re-developed or developed from scratch. The goal is to show the practical advantages for robust planning with realistic case studies.

Chapter 2 begins with an introduction into the field of scheduled passenger traffic. It shows the scheduling process at airlines and in public transport as well as the uncertainties during the schedule planning process before an overview on the state-of-the-art in literature is given.

In Chapter 3 the techniques that are used in this thesis are introduced. Risk management and measurement are described and deterministic optimization models are defined briefly. Then stochastic optimization is explained in detail: It begins with an example, then stochastic optimization models are classified, solution methods are explained, and it ends with a discussion of methods for optimization under uncertainty.

Chapter 4, 5, 6 and 7 describe the new models and solution approaches developed in this thesis, which treat the open research questions discussed in Chapter 2. Furthermore they show the results and evaluate the models with realistic case studies. Chapter 4 describes an approach for robust airline schedule design under fuel price and demand uncertainty, Chapter 5 shows a model for robust re-fleeting under demand and fuel price uncertainty at airlines. Chapter 6 comes up with a new model and solution approach for robust vehicle scheduling in public bus transport. Furthermore, a model-extension that considers and counteracts weather uncertainty is proposed. In Chapter 7 a model for rota scheduling in public transport under uncertain illness absence rates is presented.

Finally, in Chapter 8 a conclusion is drawn and an outlook for further research is given.

2 Problem description

This chapter provides an introduction to scheduled passenger traffic. It first introduces the planning phases during the scheduling process, then explains the uncertainties during this process and ends with a literature review where the state-of-the-art and open research questions are discussed. Parts of this chapter have already been published.[1]

2.1 The planning process in scheduled passenger traffic

First, the schedule planning process of airlines is introduced. Figure 1 shows the process with its planning phases.

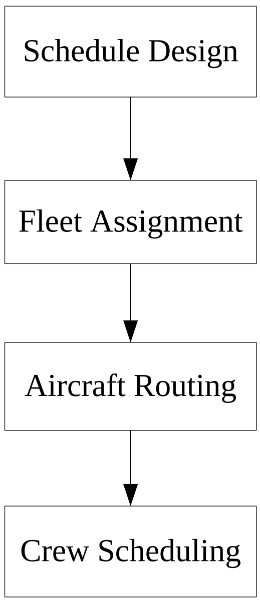

Figure 1 Airline schedule planning

The first planning phase is called *schedule design*. The airline decides which flights are flown with which frequency and flights are scheduled. This step is usually done manually and based on traffic forecasts, seasonal demand variations as well as tactical and strategic initiatives.

[1] See [NSK11], [NaSu12], [XNS12] and [NSF12]

The second step is the *fleet assignment*. In this step the aircraft type for each flight is determined. It is important that the capacity of the aircraft type matches the demand of the flight. Choosing a too large type wastes fuel and causes higher cost while a too small type can only serve a part of the demand so that the revenue decreases.

Next, the *aircraft routing* is planned. In this step, the set of flights which are flown with each airplane in a line is determined. Thereby the satisfaction of maintenance requirements is considered.

Finally, the *crew scheduling* is planned. It assigns the crews to the flights, so that the crew costs are minimized.

All these planning phases have numerous constraints and interdependent decisions. For example, the crew costs for a flight depend on the other flights that are assigned to the crew and on the ground-time between the flights. Because of the huge complexity and the interdependencies between the planning phases, even experienced planners are hardly able to find good solutions. Therefore airlines use optimization techniques. But even with optimization techniques, it is not possible to solve real-world problems considering all planning phases. Because of this fact, the whole process in decomposed and optimization models for schedule design, fleet assignment, aircraft routing and crew scheduling are solved sequentially, which nonetheless is still a challenging task.[2]

The planning process in public bus transport is very similar. Figure 2 shows the planning phases in public bus transport.

[2] See [BaCo04] and [GoTa98]

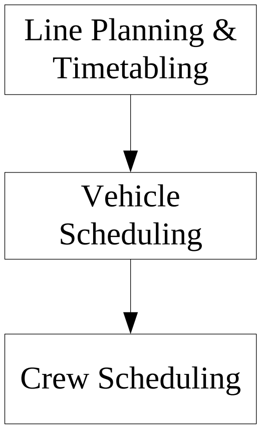

Figure 2 Schedule planning in public bus transport

At first the *line planning* and the *timetabling* is planned. Thereby the served lines are chosen and their frequencies and departure times are determined. Especially the estimated demand is used as planning-data. These planning phases are usually planned by a local authority (city or county) and followed by a tendering, in which a company can gain a license for one line or a set of lines.

The vehicle types, which transport the passengers on the given lines, and the routes of the vehicles are determined in the *vehicle scheduling*. Objectives can be cost minimization, minimization of used vehicles and/or the adherence to standards like a maximum number of line changes per circulation.

During the *crew scheduling*, first anonymous day duties are created and then assigned to the specific drivers. This second step is also called *crew rostering*. Collective agreements such as minimum times for breaks or maximum working time per week are important constraints in this planning phase.[3]

Like in Barnhart and Cohn[4] the term crew scheduling is used for all subproblems including the crew rostering in this thesis. The crew rostering itself can be divided into the *rota scheduling* and the *duty sequencing*. Shift types are assigned during the

[3] See [Kli05] p. 8ff and [HFW04]

[4] [BaCo04]

rota scheduling, in the duty sequencing the duties are assigned matching the shift type.[5]

For a global optimal solution all planning phases would have to be solved in one integrated model. But, like in airline schedule planning, this leads to too high complexity, so that the planning process is decomposed into smaller subproblems which can then be solved for each planning phase sequentially.

The scheduling processes for airlines and for public bus transport companies are very similar. For example, most constraints like flow conservation and covering, are equal. A small difference between vehicle scheduling in public bus transport and aircraft routing is the possibility of deadheads: A bus is allowed to drive without passengers while an aircraft is not, because this would be too expensive. As the processes are very similar, they are both considered in this thesis. From a methodological point of view, it makes sense to consider both planning processes together.

There are greater differences that separate airline scheduling and scheduling in public bus transport from rail-based traffic. For example the occupation of tracks has to be considered during planning in rail-based traffic, whereas busses and aircraft do not occupy their road or flight route exclusively.

Furthermore, the fleet assignment and aircraft routing respectively vehicle scheduling cannot be easily adapted to rail-based traffic, because railroad companies do not have a fixed fleet with a specific number of different types: Trains consist of a different number of cars which necessitates additional planning for balancing them.

Another strategic question that also differentiates planning in rail-based traffic from planning at airlines and in public bus transport is infrastructure planning. In contrast to the airspace between airports and the streets between bus stations, the connection between railway-stations – the network of railroads – often has to be constructed. Because of these greater differences, rail-based traffic is not considered in this thesis.

[5] See [EKS00]

2.2 Uncertainties in long-term planning phases

During their planning process the companies have to make many decisions under uncertainty. This chapter begins with the relevant uncertainties during the long-term planning phases (schedule design and fleet assignment or respectively line planning and timetabling) in scheduled passenger traffic.

2.2.1 Fuel price uncertainty

Fuel prices have become the largest part of the expenditures of airlines.

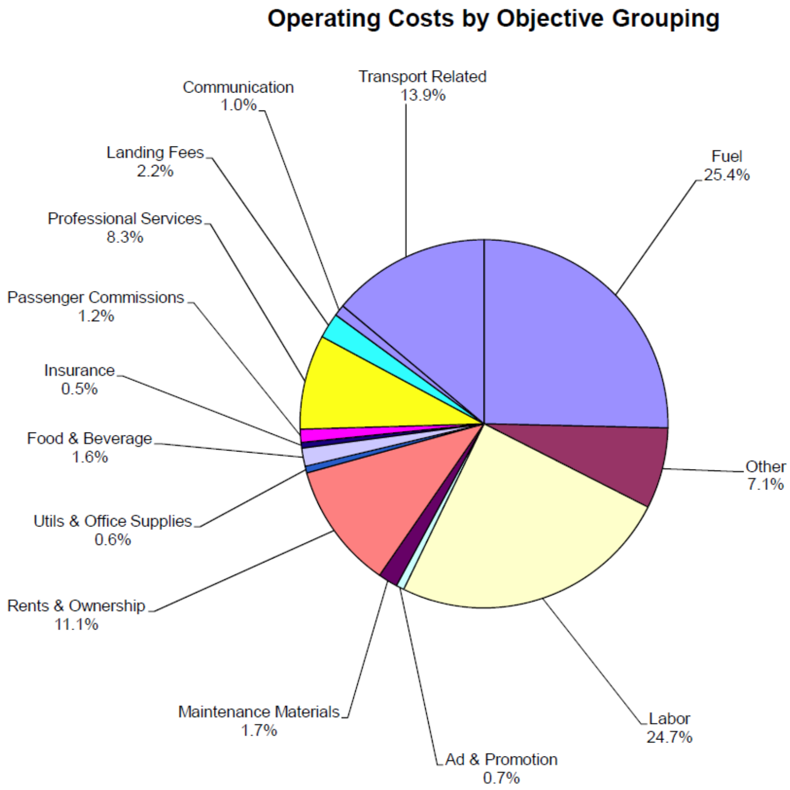

Figure 3 Cost structure in the airline industry

Figure 3[6] shows the cost structure of North American airlines in the third quarter of 2010. It can be seen that fuel costs are larger than labor costs, which have been the dominating part so far.

[6] Source: [ATA11]

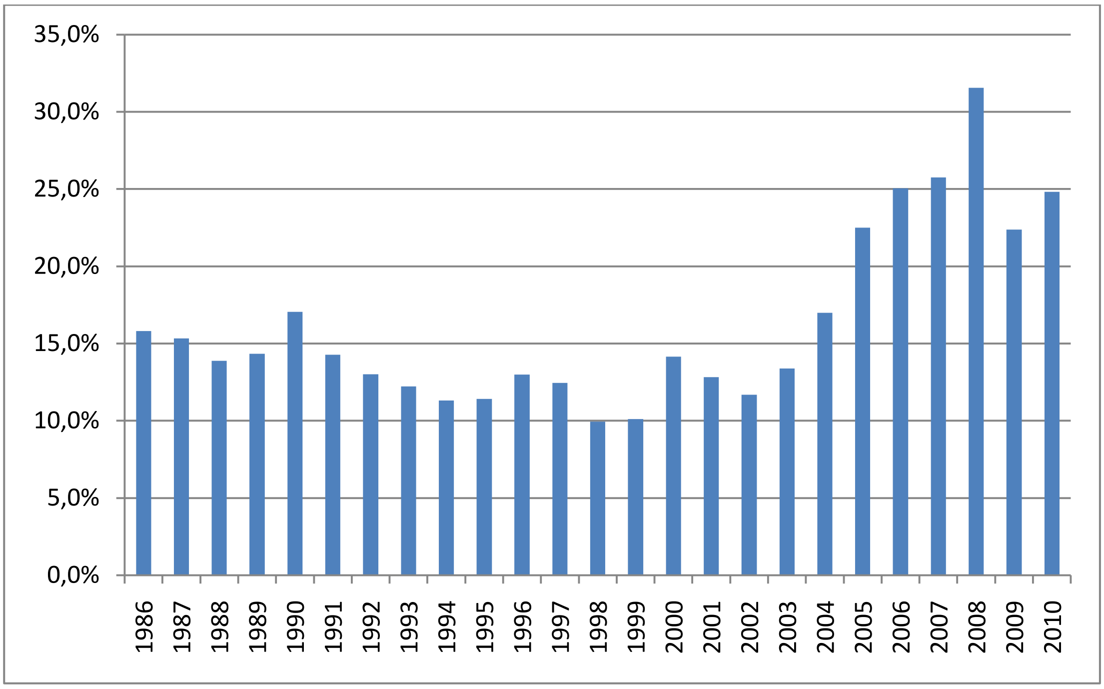

Figure 4 Share of jet fuel costs 1986-2010

Figure 4[7] shows the share of jet fuel costs of the total operational costs of airlines. In the 1990s this value always was about 10-15%, but it has grown to about 25% in the last years. In 2008 it even grew above 30%.

7 Created with data from [ATA11b]

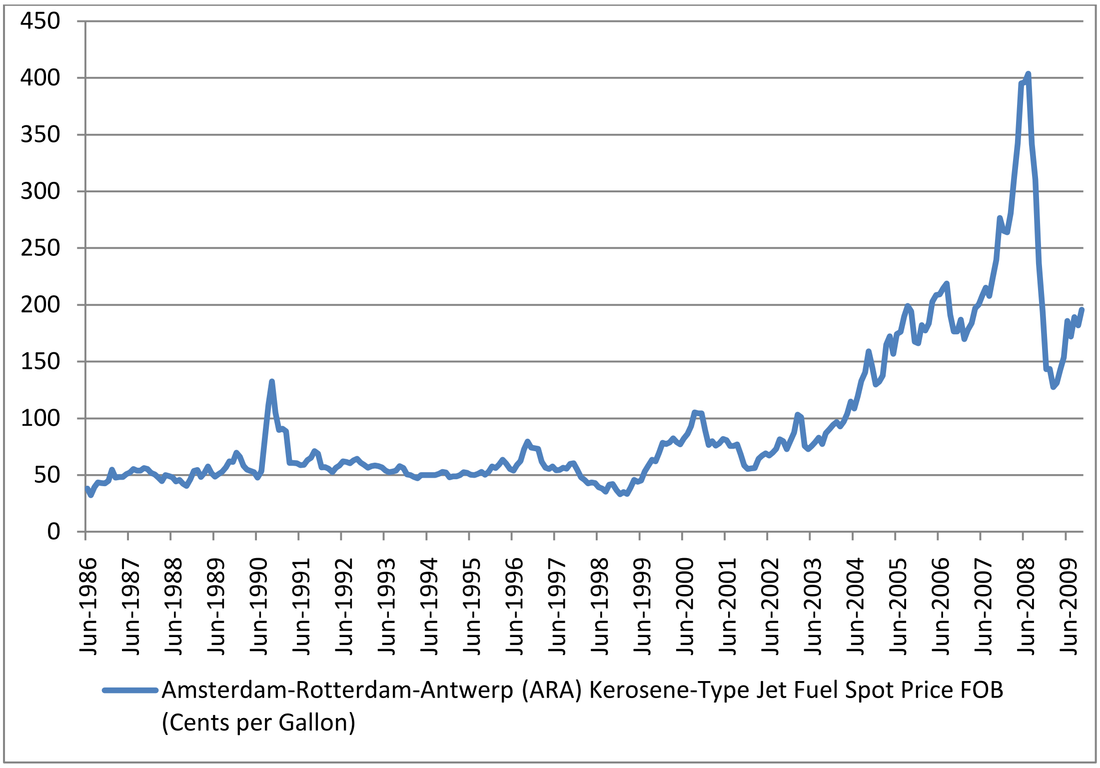

Figure 5 Fuel price development

Figure 5[8] shows the fuel price development from 1986 to 2009. The fuel price has quite high variations. It can double or halve within a few months, especially high variations in the last years can be seen. Therefore, airlines have to cope with high fluctuations for the major and further growing part of their expenditures.

This uncertainty especially influences the early planning phases of airlines, such as schedule design and fleet assignment which are planned several months or even years in advance.

For public transport companies the fuel price uncertainty has less impact than for the airline industry. There are several reasons why:

- The line planning and timetabling is done by the local authority. Therefore the company can only execute the plan and cannot change it because of variations in the fuel price.
- There is no separate planning phase for fleet assignment. Instead it is integrated into the vehicle scheduling. Furthermore, as the lines are given, the

[8] created with data from [EIA]

flexibility of changing compatible vehicle types on a line is more limited, because lines often require special vehicle types (low-floor busses for lines that stop at hospitals, etc.).

- The third reason is the economic impact of fuel price changes. Leuthardt[9] shows that the fuel costs for a standard bus in a city in 1997 are only 4.7% of the total operational costs for busses. The price for diesel in Germany in 2012 is 2.3 times higher than in 1997[10]. Therefore the fuel price share is not higher than the relative small share of fuel costs at airlines in the 1990s.

Figure 6[11] shows the yearly operating costs for a city bus in 1997.

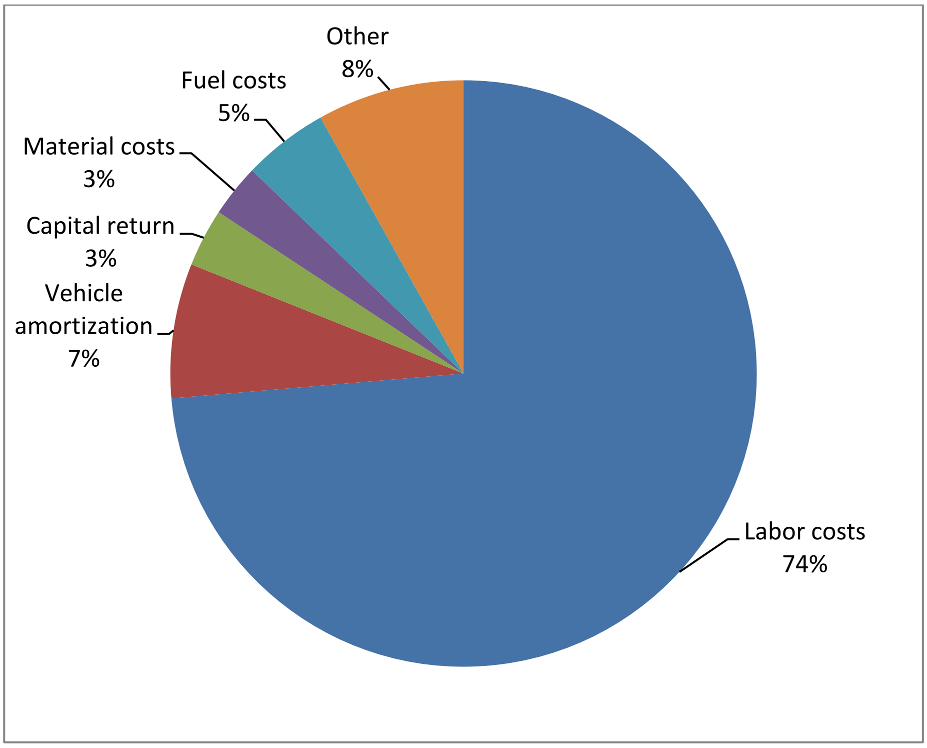

Figure 6 Bus operating costs

Figure 7[12] shows the diesel price development in Germany from 1990 to 2012.

9 [Leu98]

10 See [Sta12]

11 Created with data from [Leu98]

12 Created with data from [Sta12]

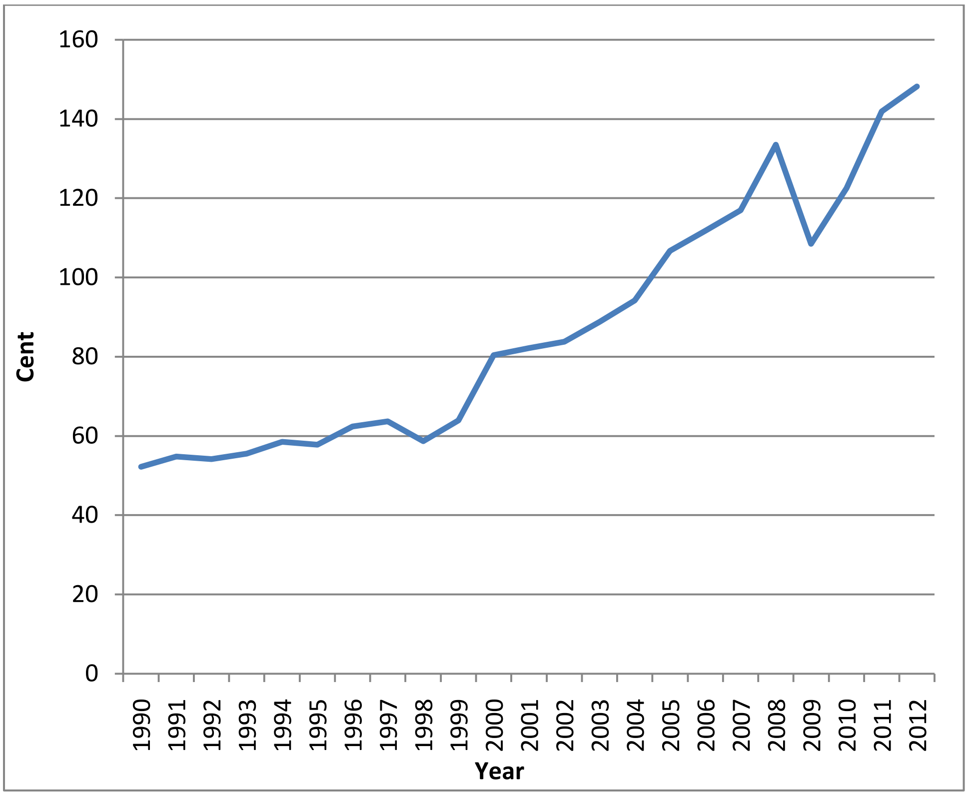

Figure 7 Diesel prices in Germany

2.2.2 Demand uncertainty

The demand uncertainty is also an important issue for airlines. Again, the planning phases schedule design and fleet assignment are affected, because they have to be planned before all passengers have booked their flights.

Figure 8 shows the yearly growth of global passenger traffic from 1951 to 2007[13]. Cento[14] argues that because the product of airlines is one of the most perishable, they have implemented techniques to counteract demand uncertainty: For short-term demand fluctuations the yield management is an efficient method, but to counteract long-term demand shifts the strategic network planning has to be adjusted.

[13] Data from [ATA10]

[14] [Cen09]

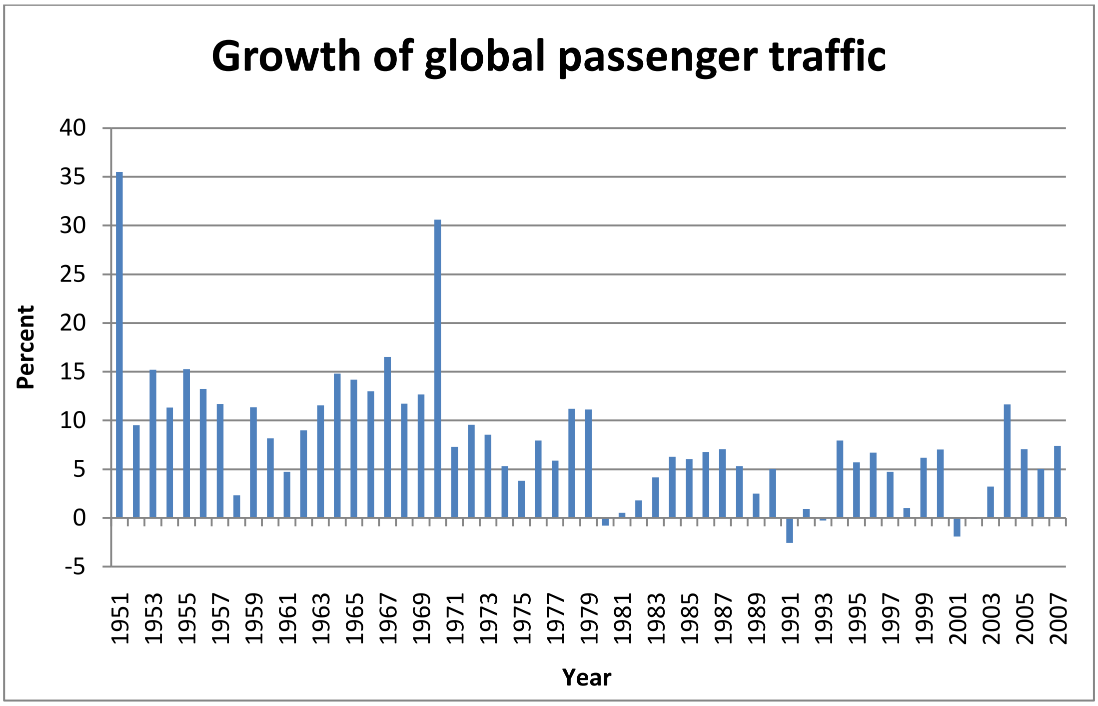

Figure 8 Growth of global passenger traffic

Compared to the data in Figure 8, the variations in the particular regions are even higher: The International Air Transport Association reports that the growth in passenger demand in March 2009 varies between 4.7% for the Middle Eastern carriers and -15.6% for the African carriers. Furthermore, the average load factor decreased because capacity was not adjusted as much as demand fell.[15]

As the schedule design determines the offered flights and their frequencies, which are often decisions for years, considering demand forecasts is very important. Furthermore, because offered airline seats are one of the most perishable services, the fleet assignment has to determine the right aircraft type under uncertainty to meet the final demand.

For public transport companies, the demand uncertainty is also not as important as for airlines. The reasons are again the line planning and timetabling by the local authority as well as the integration of fleet assignment into the vehicle scheduling. Therefore, lines, frequencies and many type assignments are already determined.

[15] Data see [IATAb]

Furthermore, public transport companies can better react on demand changes than airlines because there is standing room in busses.

2.2.3 Related work

Transportation is a sector where operations research is widely used; especially in the airline industry. In this chapter, an overview on the literature that treats the uncertainties shown before is given.

Yu[16] presents a wide variety of operations research applications in the airline industry. Gopalan and Talluri[17] give an overview on problems and mathematical models in airline schedule planning.

For the schedule design as the first planning phase in airline schedule planning, Etschmeier and Mathaisel[18] present an overview on early literature dealing with schedule construction and schedule evaluation. Cadarso and Marín[19] show a passenger oriented approach for robust airline schedule design. Lederer and Nambimadom[20] show that different network configurations such as hub and spoke or direct networks can be optimal in different situations. In Chapter 2 of their paper, Wen and Hsu[21] review the literature on airline flight frequency programming models. These models can also include several fleet types.

Demand uncertainty has also been considered in strategic airline planning. For example Barla and Constantatos[22] provide reasons why hub and spoke networks provide more flexibility to counteract uncertain demand. Barla[23] examines the effects of strategic interactions on an airline network under demand uncertainty with a duopoly

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16 [Yu98]

17 [GoTa98]

18 [EtMa85]

19 [CaMa10]

20 [LeNa98]

21 [WeHs06]

22 [BarCon99]

23 [Bar99]

game. Hsu and Wen[24] apply Gray Theory to the airline network design problem and consider demand uncertainty. The same authors[25] present a paper that evaluates the airline network design in response to demand fluctuations. Thereby they review the literature that considers demand uncertainty. Yan et al.[26] present an airline scheduling model that considers stochastic demands.

Sherali et al.[27] present a survey of models, concepts and algorithms for the fleet assignment problem. They also consider fleet assignment models that integrate schedule design decisions. An integrated model for fleet assignment and schedule design that considers flight leg selection is presented by Lohatepanont and Barnhart[28]. They also give a short overview on integrated models for schedule design and fleet assignment. Soumis et al.[29] present an integrated model that considers passenger satisfaction and the interaction between passenger and aircraft routing. Sherali et al.[30] present an integrated model for schedule design and fleet assignment considering itinerary-based demands for multiple fare classes.

Cobbs and Wolf[31] describe hedging strategies for airlines and perform an industry survey. They find out that the airline industry is not very much hedged at the time of their survey, although this would be a competitive advantage. Also Carter et al.[32] find out that hedging is positively related to the firm value of airlines. Triantis[33] presents general reasons for an integrated risk management strategy. He concludes that an integrated risk management strategy that considers contractual and operational risk management is superior to non-integrated risk management. To the best knowledge

[24] [HsWe00]

[25] [HsWe02]

[26] [YTF08]

[27] [SBZ06]

[28] [LoBa04]

[29] [SFR80]

[30] [SBH10]

[31] [CoWo04]

[32] [CRS06]

[33] [Tri05]

of the author, up to now financial hedging has not been integrated with operational decisions of airline schedule planning: Usually these decisions are made by different departments of the company and one department depends on the decision of the other department. Financial hedging instruments are well described by Hull.[34]

List et al.[35] present a stochastic model for fleet planning under uncertainty and consider partial moments as a measure for robustness. Fabian[36] shows how the Conditional Value at Risk can be integrated into linear optimization models.

To the knowledge of the author, a robust strategic planning model for the airline industry that considers schedule design and financial hedging under jet fuel price and demand uncertainty has not been developed yet.

The fleet assignment is one of the planning phases where operations research has more often been applied to than to schedule design. An overview on fleet assignment models is presented by Sherali et al.[37]. They also review the literature on re-fleeting approaches.

The idea of re-fleeting under demand uncertainty was introduced by Berge and Hopperstad[38]. They proposed significant cost improvements, if aircraft assignment is done closer to departure using more precise demand forecasts.

Further approaches for re-fleeting are presented by Jarrah et al.[39] and Talluri[40], who presents an approach for swapping aircraft. Newer re-fleeting publications are the papers from Sherali et al.[41] and Warburg et al.[42], including a case study of a European airline, for example.

[34] [Hull03]

[35] [LWN+03]

[36] [Fab08]

[37] [SBZ06]

[38] [BeHo93]

[39] [JGN00]

[40] [Ta96]

[41] [SBZ05]

[42] [WH+08]

A two-stage stochastic re-fleeting model is presented by Zhu[43] and Sherali and Zhu[44]. It considers demand uncertainty, but does not integrate fuel price uncertainty and financial hedging instruments. Their models are a basis for the development of the optimization model for re-fleeting in this thesis that considers fuel price uncertainty, demand uncertainty and risk management with financial hedging. To the best knowledge of the author, such a model has not been developed yet.

As shown before, the long-term planning phases in public bus transport - the timetabling and line planning - are planned by the local authority. Therefore they are from the companies' point of view not as interesting as the long-term planning phases at airlines. Public transport companies have not the degrees of freedom in planning the lines and timetables and are also not hit by such a significant demand and fuel price uncertainty like airlines.

2.3 Uncertainties in short-term planning phases

During the short term planning phases, aircraft routing / vehicle scheduling and crew scheduling, unforeseeable events like disruptions and illness absences cause uncertainty. These uncertainties are discussed in this section.

2.3.1 Disruptions because of weather and traffic

For the vehicle and crew scheduling in public bus transport, disruptions are a serious problem. The schedules are planned several weeks before the day of operations. Delays and disruptions cannot be avoided and not be well predicted, but are often not considered in the schedule planning. As schedules have become more cost-efficient because of the increased use of optimization techniques in the last years, they contain less buffer times that can absorb delays. Therefore disruptions lead to increased operational costs and contractual penalty costs, which have to be paid to the local authority.[45] Huisman et al.[46] present a function for calculating the penalty costs depend-

[43] [Zhu06] p. 61ff

[44] [ShZh08]

[45] See [KKM09]

[46] [HFW04]

ing on the delay length. It is a quadratic function to penalize large delays overproportionally and a delay length of about 1/2 hour costs as much as the fixed costs for a bus for one day. Therefore disruptions because of traffic delays (rush hour, road closures, etc.) and due to bad weather conditions (icy roads, road closures due to storm consequences) are important for the vehicle and crew scheduling problem in public bus transport.

Also for the aircraft routing and crew scheduling problem at airlines unavoidable disruptions during the execution of the plan are a problem. In contrast to most public transport companies, airlines usually have an operations control to dynamically counteract disruptions, but nevertheless disruption-tolerant planning of schedules is important as operations control can only counteract already existing disruptions and decrease their impacts. Eurocontrol[47] lists the reasons for delayed flights with a delay of 5 minutes or more. The main reasons for delays are traffic and weather: The air traffic control capacity and the airport capacity caused 37.54% and 7.45% of the delays in 2010, weather caused 20.86%.

2.3.2 Uncertainty because of illness

The crew scheduling has to be planned under the uncertainty that persons are absent due to illness. Crew schedules are planned several weeks before the execution of the schedule, so the number of ill persons cannot be anticipated.

During the rota scheduling in public bus transport, a set of given shift types is allocated to the drivers. The drivers can be classified into several groups depending on qualifications and work preferences. The rota scheduling problem considers specific work regulations as well as legal restrictions.

As the costs for personnel are 74% of the total bus operating costs and therefore the dominating part (see Figure 6), the allocation of reserve and attendance personnel has to be planned intelligently so that enough personnel is always available at reasonable costs.

[47] [EUR11]

2.3.3 Related work

The aircraft routing problem is often also called aircraft assignment, maintenance routing or aircraft rotation problem.

Clarke et al.[48] show their mathematical formulation of the aircraft rotation problem and discuss the similarities of the asymmetric traveling salesman problem and the aircraft rotation problem. They solve their formulation with real data with lagrangian relaxation and subgradient optimization. Desaulniers et al.[49] present a set-partitioning type model and a multi-commodity network flow model for the daily aircraft routing and scheduling problem.

A recent overview on the aircraft routing problem is given by Dück[50]. He also reviews the literature on robust scheduling for aircraft routing and crew scheduling.

Because of the large enterprise size of airlines and the therefore high economical impacts of disruptions due to weather or traffic, airlines, in contrast to most public transport companies, continuously monitor their flights to dynamically react on disruptions with aircraft changes, for example. Therefore schedule recovery is usual. Clausen et al.[51] present a recent overview on disruption management in the airline industry.

The vehicle scheduling problem in public bus transport is closely related to the aircraft routing problem. As differences, the aircraft routing does not integrate the assignment of the typeclass and does not consider the possibility of deadheads but usually plans for a longer horizon. In this field many optimization models have been developed, so that some models, like the model of Lan et al.[52], could be adjusted to the vehicle scheduling problem in public bus transport.

48 [CJNZ97]

49 [DDD+97]

50 [DÜ10] p. 32ff

51 [CLLR10]

52 [LCB06]

To the knowledge of the author, there is no literature on stochastic optimization models for robust vehicle scheduling in public bus transport. Bunte and Kliewer[53] give an overview on general vehicle scheduling models in public transport. Huisman et al.[54] solve the dynamic vehicle scheduling problem and use scenarios for travel times to consider disruptions and robustness. Dessouky et al.[55] present a summary of distribution functions for delays in public transport used in former studies.

An overview on airline crew scheduling is given by Gopalakrishnan and Johnson[56], they also review the solution approaches and discuss the robustness.

Schaefer et al.[57] consider disruptions during the planning of crew schedules and show that their approach is superior to deterministic optimization. Therefore they use approximate expected costs instead of planned costs to derive the schedule.

Dück[58] presents a recent overview on airline crew scheduling and also reviews the literature on robust crew scheduling for airlines. Furthermore see Clausen et al.[59] for an overview on crew recovery in the airline industry.

A general review on staff scheduling and rostering is given by Ernst et al.[60]. They review the existing models and solution techniques for personnel scheduling. This process is usually decomposed into several separate planning phases and applied to a wide area of applications such as scheduling in transportation systems, health care systems or emergency systems.

Emden-Weinert et al.[61] introduce the usual decomposition scheme in public bus transport. To consider the uncertainty because of illness absences, the rota scheduling

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53 [BuKl09]

54 [HFW04]

55 [DHNM99]

56 [GoJo05]

57 [SJKN05]

58 [Dü10] p. 34ff

59 [CLLR10]

60 [EJKS04]

61 [EKS00]

is the important planning phase. It assigns the shifts and free days to the drivers that are grouped by their qualification and/or their preferences (preferred shift types and free days).

Emden-Weinert et al.[62] modeled the rota scheduling problem as an integer linear program and compared this with a metaheuristic approach for small input data. Sodhi and Norris[63] developed a network-based model for the rota scheduling problem at London Underground and were able to solve data instances limited to 150 drivers and 1000 shifts in a reasonable time. Lau[64] showed that the rota scheduling problem is NP-hard.

Reserve shifts are used to cover the absences of drivers. The number of reserve shifts is planned as a fixed number[65] or a certain percentage of the total shifts. More detailed information such as historical or weekday-depended sickness absence rates have not been considered.

Moreover, to the best of the author’s knowledge, optional reserve shifts in addition to the present reserve shifts that can, but must not be exercised by the company, have not been considered in an optimization model for public transport, yet.

The literature on the used optimization techniques and on risk management will be shown in the detailed introductions in Chapter 3.

2.4 Open research questions and goals of the thesis

After introducing the schedule planning process with its uncertainties and discussing the existing literature, this section summarizes the open research questions that were discovered. The goal is to find new approaches for them: They are treated in later chapters of this thesis. Table 1 shows a summary of the open research questions and the work that has to be done as well as the corresponding chapters where the open questions will be treated.

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[62] [EKS00]

[63] [SoNo04]

[64] [Lau96]

[65] See [SoNo04]

Planning phase	Goal	
	Airline industry	Public transport
Schedule design / linep-lanning and timetabling	Develop a strategic model for robust schedule de-sign that considers sto-chastic fuel prices and fuel hedging (Chapter 4)	
Fleet assignment	Develop a re-fleeting model for robust two-stage fleet assignment considering fuel price uncertainty and fuel hedging (See Chapter 5)	
Aircraft routing / vehicle scheduling		Develop a stochastic pro-gramming approach for robust vehicle scheduling (See Chapter 6)
Crew scheduling		Develop a stochastic model for rota scheduling that considers optional reserve shifts (See Chap-ter 7)

Table 1 Open research questions

After an introduction of the methods that will be used in this thesis in Chapter 3, Chapter 4 presents a new model for robust airline schedule design under fuel price and demand uncertainty. In Chapter 5, a new model for airline re-fleeting under fuel price and demand uncertainty will be shown. Both models also consider financial instruments to enable hedging against jet fuel price variations. These two models extend the existing literature by considering fuel price uncertainty and integrating financial hedging instruments and operational decisions simultaneously in one optimization model.

Chapter 6 presents a new stochastic optimization model for robust vehicle scheduling in public bus transport. Furthermore, a model extension with weather derivatives is proposed and different solution methods are evaluated. The last application is introduced in Chapter 7: It is a stochastic model for robust rota scheduling in public bus transport. As stochastic optimization models have not been applied to this field of application, these two chapters provide a significant extension to the existing literature.

3 Principles of risk management and optimization under uncertainty

The central difficulty in a planning process is the uncertainty of relevant planning data. The system that has to be planned is usually dynamic and therefore changes in the surrounding environment as well as endogenous changes happen. The environmental changes can usually not be well predicted and also the impacts of endogenous changes are not always completely known.[66]

In this chapter, the methods used in this thesis are introduced. These are techniques for risk management and optimization under uncertainty. The chapter starts with an introduction in risk management and risk measures, before deterministic optimization models, which are a basis for stochastic optimization, are briefly defined. Then stochastic optimization is described in detail. The chapter ends with a discussion of methods for optimization under uncertainty and explains why stochastic programming is the most suitable method for this thesis.

3.1 Risk management

This chapter introduces risk management. The general risks that companies have to face are listed, it is explained why risk management is beneficial, and the operational and contractual methods to manage risks are shown. At last, the measurement of risk and the term robustness are introduced and defined.

For several years risk management has been widely used in companies: A survey published in the year 2000 shows that 90% of all companies in the German stock index DAX-100 use financial hedging instruments for risk management. Moreover, German companies have been obliged by law to install a risk management system in addition to the internal revision in 1998: A law (called KonTraG) obliges companies to manage risks that can lead to insolvency or illiquidity.[67]

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[66] See [Sch01] p. 89f

[67] See [Schä03]

3.1.1 Types of business risks

Companies face a wide variety of different risks. These business risks can be categorized into five dimensions: technological risks, economic risks, financial risks, performance risks and legal/regulatory risks.

Technological risks usually come up in research and development as well as the operational stages of a company. Especially companies in the high-tech-sector or pharmaceutical industry have to cope with technological risks to maintain their competitiveness.

Economic risks mainly arise due to changes in macroeconomic conditions. For example the production costs can vary because the costs for material and labor vary and the revenues can vary because the demand varies. Furthermore the economic risks are determined by the competitive environment of the company.

Most companies also have to cope with *financial risks* although their business is not focused in the financial industry. For example, nearly every company faces currency risks: Even if the company does not face translation or transaction risks, it loses competitiveness compared to other companies in depreciating currency areas.

Furthermore, companies more often face *performance risks* because they more and more enter into long-term contracts. The risk is that the contractual partner does not perform all contractual obligations satisfactory. For example a loss in quality from a supplier could have impacts on the companies' performance. Complete contracts that include all possible events are nearly impossible to create and taking legal action often has uncertain results.

At last, companies face *legal and regulatory uncertainty*: The companies have to stick to laws and regulations that can change over time. These can be changes in taxation for example and also expropriation as result of an extreme political change in a foreign country where a project is done, for example.

Table 2 shows the different risks and their categories.[68]

[68] See [Tri05]

Risk category	Risk
Technological	Research and development outcome risk
	Production breakdown
	Implementing new technology
	Defective products
	Force majeure risks
Economic	Material and labor costs
	Output price risk
	Product demand uncertainty
	Market share risk
Financial	Interest rate risk
	Currency rate risk
	Commodity price risk
	Security holdings risk
Performance	Subcontractor performance
	Judicial risk
	Credit risk of contract counterparties
Legal and regulatory	Tax law changes
	Environmental regulation changes
	Political regime switches or insurrection
	Expropriation

Table 2 Business risk categorization

3.1.2 Reasons for risk management

Obviously, companies face a lot of different risks. And financial theory states that taking risks will be compensated. Therefore taking more risks should lead to higher success. The question is now: Why should companies then manage risk?

Some of the reasons are:

- The probability of a bankruptcy as well as reorganization costs can both be decreased.

- Difficulties of attracting customers can be avoided and contracts with better terms can be entered into.
- When a firm approaches insolvency, often value-decreasing projects whose upside is covered by shareholders and whose downside is borne by bondholders are made. This could be avoided.
- With lower risk exposure, leverage effects and debt tax shields can be more utilized.
- Expensive external financing or underinvesting due to financial constraints can be avoided.
- The costs of performance-based compensation can be minimized for risk-averse employees.[69]

3.1.3 Methods for risk management

One method to manage risks are *contractual* mechanisms. When a contract is designed to share risks, it is designed with the following two goals. The first goal is that the company that takes the risk can better bear the risk because of diversification or a higher risk bearing capacity. For example insurance companies diversify their risks over a wide set of costumers. The second goal is that the company can better manage or control it, especially if the contract contains a risk premium. For example, the construction company of a large infrastructure project could best mitigate the risks of the project like technical risks or cost overrun risks.

There are many types of possible contracts to use for risk management: Financial derivatives, other forward- or option-like contracts related to delivery of real goods and insurance contracts can be used to exploit differences in risk-bearing capacity. Subcontractor performance contracts and employee compensation contracts are useful to transfer the risk to the party that is best able to control it. Finally the maturity structure of debt, convertible bonds, joint ventures and warranties can be used when controlling risk and risk-bearing should be considered in the contract simultaneously.[70]

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[69] See [Tri05]

[70] See [Tri05]

In this thesis, only a brief introduction of financial hedging instruments is given, for a detailed introduction, see Hull[71]. He argues that derivatives have become more and more important in the last years. Today, futures, forward-contracts, options and swaps are regularly traded. Their value depends on other underlying variables, for example traded assets or currencies.

The easiest hedging instruments are *forward-contracts* and *futures*. These are contracts to buy or sell a certain asset to a certain price at a certain time. Forward-contracts are usually traded in the over-the-counter market whereas futures are standardized and usually traded on an exchange. For example the future price of a certain amount of gold for December could be quoted as $300 in September. This is the price for which traders could buy or sell gold for delivery in December. The contract specifies the amount, the price and - in case of a commodity - also the product quality and delivery location. Contracts are usually available for several delivery periods in the future.

Options are a different type of derivatives. They give the owner the right to do something but, in contrast to forward-contracts and futures, they need not to be exercised. They also have a price, whereas it costs nothing to enter into a forward-contract or future.[72]

Another method for risk management besides using contracts is the usage of *real options*. A real option that minimizes risk exposure could be using the same currency area for production and sales, for example. A company could also try to expose itself to different risks such that they altogether decrease the companies' risk exposure reversely. There are many more real options for risk management shown by Triantis[73].

Triantis[74] comes to the conclusion that companies have to integrate a broad risk management strategy which integrates financial hedging and real options to conquer risk and maximize firm value. But as shown in Chapter 2.4, there are still open research

[71] [Hull03]

[72] See [Hull03]

[73] See [Tri05]

[74] See [Tri05]

questions in integrating real options and contractual risk management in the field of scheduled passenger traffic.

3.1.4 Robustness and risk measures

The word *robustness* is a broadly defined term. Therefore, this section is used to define this word for this thesis.

Scholl[75] generally defines the robustness of a plan as the property that the realization of the plan for (nearly) every situation that can occur in future leads to good or acceptable results regarding to the planned goals. Moreover he defines the term robustness in detail with its related terms and the criteria for robustness.

In this thesis, the term robustness is used as achievement of security or prevention of risk under uncertainty. Therefore, risk measures are used to measure robustness: A plan or a solution is more robust, when the risk measure used has a better value.

Risk measures can be the following.

The *worst scenario*: It is a very simple intuitive risk measure that can easily be implemented and measured.

The *variance*: The variance and the volatility, which is derived from the variance, are the risk measures that are used most often. But as risk is perceived as a negative deviation from a certain value and the variance also increases with positive variations, which are perceived as chances and not as risk, the variance can only be used for symmetric distributions as a suitable risk measure.[76]

The *Value at Risk (VaR)* is an established risk measure, especially since the "Basel II"-agreement. It measures the value that is not undercut by a certain probability α, for example 1%, 5% or 10%. It only considers downside variations and thus matches the perception of risk better than the variance. A drawback of the VaR is that it is

[75] See [Sch01] p. 93ff

[76] See [SZ08]

only a threshold value. It denotes the threshold that is not undercut by a certain probability, but does not consider the values that fall below this threshold.[77]

The *Conditional Value at Risk (CVaR)* considers the values that fall below this threshold. It denotes the expected value of the scenarios that fall below the Value at Risk. The CVaR is also known as mean excess loss, mean shortfall or tail VaR.[78]

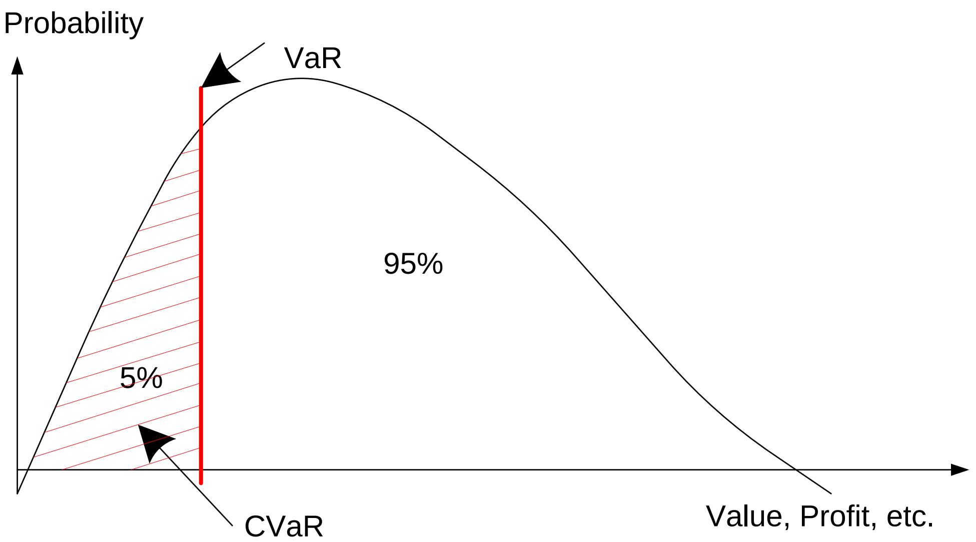

Figure 9 Value at Risk and Conditional Value at Risk

Figure 9 illustrates the Value at Risk and the Conditional Value at Risk. In some definitions, the robustness is often not measured by a general risk measure like those described above - for example the Conditional Value at Risk with an α of 5% for the profit of the next year - but by a more problem-specific measure. For example the sum of the expected delays of all service trips of a bus company for one day or the expected costs that the delays cause. For measuring robustness in this thesis, problem specific measures are translated into monetary values which can then be considered to calculate a general risk measure that is used to measure the robustness. To summarize: A solution is defined as more robust, if it has a better risk measure.

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[77] See [SZ08]

[78] See [RoUr00]

3.2 Deterministic Optimization Models

This chapter briefly introduces and defines linear programming and (mixed) integer programming. For readers who are not familiar with these techniques, there are detailed introductions in literature: Chvátal[79] describes linear programming in detail, while Wolsey[80] gives a wide introduction into (mixed) integer programming.

3.2.1 Linear Programming

The real numbers $c_1, c_2, c_3, \dots, c_n$ are used to define the function f of the real variables $x_1, x_2, x_3, \dots, x_n$:

$$f(x_1, x_2, x_3, \dots, x_n) = c_1x_1 + c_2x_2 + \cdots + c_nx_n = \sum_{j=1}^{n} c_j x_j \qquad (1.1)$$

This function is called a *linear function*. If b is a real number then the equation

$$f(x_1, x_2, x_3, \dots, x_n) = b \qquad (1.2)$$

is a *linear equation* and

$$f(x_1, x_2, x_3, \dots, x_n) \geq b \text{ and } f(x_1, x_2, x_3, \dots, x_n) \leq b \qquad (1.3)$$

are called *linear inequalities*. From now on, linear equations and linear inequalities are also called *linear constraints*. A *linear programming problem* or *linear program (LP)* consists of one *objective function* which is a linear function and is maximized or minimized subject to a number of linear constraints. If there are m constraints and n variables the linear program writes:

$$maximize \sum_{j=1}^{n} c_j x_j \qquad (2.1)$$

$$s.t. \sum_{j=1}^{n} a_{ij} x_j \leq b_i \quad \forall i = 1,2, \dots, m \qquad (2.2)$$

$$x_j \geq 0 \quad \forall j = 1,2, \dots, n \qquad (2.3)$$

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[79] [Chv83]

[80] [Wol98]

The numbers $x_1, x_2, x_3, \ldots, x_n$ that do not violate any constraint of the LP are called *feasible solution*. A feasible solution that maximizes (or minimizes) the objective function is an *optimal solution*. The corresponding value of the objective function is then the *optimal value* of the problem.

There are LPs with an optimal value, but an LP can also be infeasible or unbounded. A LP is *infeasible*, when it has no feasible solution and if for every feasible solution of an LP there is another feasible solution of the same LP that is better in terms of the objective function, the LP is *unbounded*.[81]

3.2.2 (Mixed) Integer Programming

If A is a $m \times n$-matrix, c a n-dimensional row-vector, b a m-dimensional column-vector and x a n-dimensional column-vector of variables,

$$maximize\ cx \qquad (3.1)$$

$$s.t.\ Ax \leq b \qquad (3.2)$$

$$x \geq 0 \qquad (3.3)$$

is a linear optimization model. If some but not all variables must have integer values, the problem

$$maximize\ cx + hy \qquad (4.1)$$

$$s.t.\ Ax + Gy \leq b \qquad (4.2)$$

$$x \geq 0, y \geq 0 \text{ and integer} \qquad (4.3)$$

is a *mixed integer program (MIP)*. (G is a m×p-matrix, h is a p-dimensional row-vector and y is a p-dimensional column-vector of integer variables).

[81] See [Chv83] p. 5ff

If all variables of the linear optimization model are integer variables, the problem

$$maximize\ cx \qquad (5.1)$$

$$s.t.\ Ax \leq b \qquad (5.2)$$

$$x \geq 0 \text{ and integer} \qquad (5.3)$$

is an *integer program (IP)*. If all integer variables of this problem can only have 0-1 values, the program

$$maximize\ cx \qquad (6.1)$$

$$s.t.\ Ax \leq b \qquad (6.2)$$

$$x \in \{0,1\} \qquad (6.3)$$

is called *binary integer program*.[82]

3.3 Stochastic Optimization Models

This section will introduce stochastic programming, which is the used technique for the models developed in thesis. It begins with a practical and easy understandable example before further terms and classifications of stochastic programming are presented.

3.3.1 An illustrative example

To show how stochastic programming can be used for optimization under uncertainty, consider the following problem of a transport agency.[83]

A transport agency offers a transport service, where customers can order transportation services with different types of vehicles. These orders have to be made at least one day before, so that the transport agency accepts, confirms and therefore knows its demand one day before and can then order the vehicles with their drivers. Due to

[82] See [Wol98] p. 3

[83] This example was inspired by the example from Birge and Louveaux [BiLo97] p. 4ff

contractual regulations, the transport agency has to order them for the whole day. For service quality, the agency guarantees transportation for confirmed orders.

There are three types of vehicles: Taxis have a transport capacity of 3, are offered for \$30 per order and cost the agency \$100 per day. A van has a capacity of 10, is offered for \$75 per order and costs \$200 per day. The largest vehicle is a bus with a capacity of 30: It is offered for \$200 per order and it costs the agency \$400 per day. All vehicles serve 10 orders per day on an average.

The transport agency can also hire taxis at short notice for the same day, but then the taxis have to be hired to a higher price. Larger vehicles cannot be hired on the day of operation, but taxis can be used to replace them. A bus can be replaced by 10 taxis and a van can be replaced by 4 taxis, if not enough vans or busses are available to transport the confirmed orders. However, the customers only pay the price for their ordered vehicle. Peak demand times are no problem for the agency as the vehicles are used to transport passengers and goods: The agency knows from former orders that it is possible to adjust the transport times for goods to have an equal workload during the day.

	Costs (\$) per day used	Costs per day (\$) if hired at short notice	Revenue (\$) per order	Average orders per day	Number of taxis to replace
Taxi	100	250	30	10	1
Van	200	-	75	10	4
Bus	400	-	200	10	10

Table 3 Parameters for the transport agency problem

Let us now consider the following orders for the next day: 125 taxi transport orders, 57 van transport orders and 34 bus transport orders.

It is now possible to set up an optimization model for this problem. The decision variables are:

x_1 Provided taxis the for next day

x_2 Provided vans the for next day

x_3 Provided busses the for next day

y_1 Taxis hired at short notice due to shortage and used for taxi orders

y_2 Vans replaced by taxis hired at short notice

y_3 Buses replaced by taxis hired at short notice

The deterministic optimization problem is:

$$\max 125 \cdot 30 + 57 \cdot 75 + 34 \cdot 200 - 100x_1 - 200x_2 - 400x_3 - 250y_1 - 4 \cdot 250y_2 - 10 \cdot 250y_3 \qquad (7.1)$$

$$s.t. 10x_1 + 10y_1 \geq 125 \qquad (7.2)$$

$$10x_2 + 10y_2 \geq 57 \qquad (7.3)$$

$$10x_3 + 10y_3 \geq 34 \qquad (7.4)$$

$$x_1, x_2, x_3, y_1, y_2, y_3 \geq 0 \text{ and integer} \qquad (7.5)$$

The optimal solution of the model is:

	Taxis	Vans	Buses
Vehicles ordered	13	6	4
Replaced by taxis hired at short notice	0	0	0
Costs ($) for ordered vehicles	1300	1200	1600
Costs ($) for hiring at short notice	0	0	0
Demand	125	57	34
Revenue ($)	3750	4275	6800
Profit ($)	10725		

Table 4 Optimal deterministic solution for the transport agency problem

In this solution, all demand is satisfied by provided vehicles. As the replacement by taxis hired at short notice is more expensive and therefore dominated, this option is not used in the optimal solution. Finding this solution is trivial and could be done without the help of mathematical optimization.

The transport agency is not satisfied with this solution, as it has a major drawback: It only considers the average number of orders per day that can be carried out by the vehicles. But due to the traffic situation on the day, the vehicles can carry out more or less orders per day: On bad days with heavy traffic, a bus can only carry out 6 orders, taxis and vans can handle 8 orders, as they can drive on smaller streets. On a good day, all vehicles can handle up to 12 orders. As a very basic representation of this uncertainty, three scenarios that consider the worst, average and best values for all vehicles are created. Table 5 shows the three possible scenarios.

Customer orders per day	Bad scenario	Average scenario	Good scenario
Taxi	8	10	12
Van	8	10	12
Bus	6	10	12

Table 5 Scenarios for the transport agency problem

The optimal solution for the average day was already calculated. Let us now set up an optimization model for the bad and for the good scenario and calculate the optimal solutions. The model for the bad scenario is:

$$\max 125 \cdot 30 + 57 \cdot 75 + 34 \cdot 200 - 100x_1 - 200x_2 - 400x_3 - 250y_1 - 4 \cdot 250y_2 - 10 \cdot 250y_3 \qquad (8.1)$$

$$s.t.\, 8x_1 + 8y_1 \geq 125 \qquad (8.2)$$

$$8x_2 + 8y_2 \geq 57 \qquad (8.3)$$

$$6x_3 + 8y_3 \geq 34 \qquad (8.4)$$

$$x_1, x_2, x_3, y_1, y_2, y_3 \geq 0 \text{ and integer} \qquad (8.5)$$

For the good scenario, it is:

$$\max 125 \cdot 30 + 57 \cdot 75 + 34 \cdot 200 - 100x_1 - 200x_2 - 400x_3 - 250y_1 - 4 \cdot 250y_2 - 10 \cdot 250y_3 \qquad (9.1)$$

$$s.t.\, 12x_1 + 12y_1 \geq 125 \qquad (9.2)$$

$$12x_2 + 12y_2 \geq 57 \qquad (9.3)$$

$$12x_3 + 12y_3 \geq 34 \qquad (9.4)$$

$$x_1, x_2, x_3, y_1, y_2, y_3 \geq 0 \text{ and integer} \qquad (9.5)$$

The optimal solutions are added to Table 4 so that Table 6 summarizes the different optimal solutions for the three scenarios:

Bad/average/good	Taxis	Vans	Buses
Vehicles ordered	16/13/11	8/6/5	6/4/3
Replaced by taxis hired at short notice	0/0/0	0/0/0	0/0/0
Costs ($) for ordered vehicles	1600/1300/1100	1600/1200/1000	2400/1600/1200
Costs ($) for hiring at short notice	0/0/0	0/0/0	0/0/0
Demand	125	57	34
Revenue ($)	3750	4275	6800
Profit ($)	9225/10725/11525		

Table 6 Optimal deterministic solutions for different scenarios

It can be seen that the optimal number of provided vehicles significantly differs depending on the scenario. The optimal number of buses is between 3 and 6, for vans between 5 and 8 and the optimal number of taxis is between 11 and 16.

As traffic forecasts for the next day are not available and the transport agency has to decide how much vehicles should be ordered today, it realizes that it has to make a decision under uncertainty which will not be perfect in every scenario. The agency could now maximize its expected profit under uncertainty and therefore implement the three scenarios in one optimization model.

The decisions that have to be made before the uncertainty is revealed are called *stage-1 decisions*. Their corresponding variables, in this example the variables x_1, x_2 and x_3, that denote the number of vehicles ordered the day before, are called *stage-1 variables* or *first-stage variables*.

The other variables are called *stage-2 variables* or *second-stage variables*. Their values are chosen depending on the scenario, when the values of the uncertain parameters are known. In this example, the taxis hired at short notice can be chosen when the agency knows if the day is a good, an average or a bad day. Because these variables can have different optimal values in the three scenarios, they are indexed by a scenario index. The y_i variables are replaced by y_{is} $(i = 1,2,3 \;\; s = 1,2,3)$. y_{32} then indicates the number of busses replaced by taxis hired at short notice because of bus shortage in scenario 2. If every scenario has the probability of $1/3$, the corresponding stochastic optimization model is:

$$\begin{aligned} \max 125 \cdot 30 + 57 \cdot 75 + 34 \cdot 200 - 100x_1 - 200x_2 - 400x_3 \\ -\frac{1}{3}(250y_{11} + 4 \cdot 250y_{21} + 10 \cdot 250y_{31}) \\ -\frac{1}{3}(250y_{12} + 4 \cdot 250y_{22} + 10 \cdot 250y_{32}) \\ -\frac{1}{3}(250y_{13} + 4 \cdot 250y_{23} + 10 \cdot 250y_{33}) \end{aligned} \tag{10.1}$$

$$s.t. 8x_1 + 8y_{11} \geq 125 \qquad (10.2)$$

$$8x_2 + 8y_{21} \geq 57 \qquad (10.3)$$

$$6x_3 + 8y_{31} \geq 34 \qquad (10.4)$$

$$10x_1 + 10y_{12} \geq 125 \qquad (10.5)$$

$$10x_2 + 10y_{22} \geq 57 \qquad (10.6)$$

$$10x_3 + 10y_{32} \geq 34 \qquad (10.7)$$

$$12x_1 + 12y_{13} \geq 125 \qquad (10.8)$$

$$12x_2 + 12y_{23} \geq 57 \qquad (10.9)$$

$$12x_3 + 12y_{33} \geq 34 \qquad (10.10)$$

$$x_1, x_2, x_3, y_{11}, y_{21}, y_{31}, y_{12}, y_{22}, y_{32}, y_{13}, y_{23}, y_{33} \geq 0 \text{ and integer} \qquad (10.11)$$

This stochastic model is presented in the *extensive form* because the stage-2 variables for every scenario are described explicitly.[84]

It can be seen that the stochastic optimization model is larger than the deterministic optimization model: It contains three blocks of restrictions, one for each scenario, compared to one in the deterministic model. When there is more than one stochasti-

[84] For definitions of terms see [BiLo97] p. 4ff

cally independent parameter, stochastic optimization models can become very large. Assume that there is a model with two random parameters with 15 and 18 realizations, which in real problems might not be much. Then the model would have $15 \cdot 18 = 270$ blocks of constraints. But the block structure of stochastic programs can be utilized by special solution algorithms, which will be described later in Chapter 3.3.4.1.[85]

Let us now go back to the example and look at the optimal solution of the stochastic model. It is shown in Table 7.

Bad/average/good	Taxis	Vans	Buses
Vehicles ordered	13	8	6
Replaced by taxis hired at short notice	3/0/0	0/0/0	0/0/0
Costs ($) for ordered vehicles	1300	1600	2400
Costs ($) for hiring at short notice	750/0/0	0/0/0	0/0/0
Demand	125	57	34
Revenue ($)	3750	4275	6800
Expected profit ($)	9275		

Table 7 Stochastic solution for the transport agency problem

The interpretation of the optimal solution is that the amount of vans and buses should be as high as necessary to cover the bad scenario without replacing them by taxis hired at short notice. In contrast to that, taxis should be hired at short notice in the worst scenarios to carry out all taxi orders.

[85] See [KW94] p. 11ff

It is obvious, that this solution is never optimal for every scenario: In the average and good scenario more busses and vans than necessary are ordered; in the bad scenario, taxis are hired at short notice, although hiring at short notice is very expensive. But as the stochastic model balances and/or hedges against the various scenarios, this solution is the best solution under uncertainty. As the agency must decide under uncertainty here and now, the solution of this stochastic model is also called the *here-and-now* solution.

If the agency knew that the next day will be a bad day, the one after the next an average day and the following a good day, it could use the optimal deterministic solutions for these days. This would lead to a profit of $9225, $10725 and $11525 with a mean profit of $10492. This solution is called the *wait-and-see* solution. But the agency would only realize a mean profit of $9275 using the *here-and-now* solution. The difference of $1217 cannot be gained because the agency does not have perfect information about the future. This loss of profit because of uncertainty is called the *expected value of perfect information (EVPI).*

If the agency does not want to use a stochastic model to optimize under uncertainty, it could use expected orders per day and use the optimal deterministic solution for the expected orders for every day. This solution is called the *expected value solution* and would lead to a mean profit of $8141 in the three scenarios. This solution is worse than the solution of the stochastic model ($9275) because the knowledge about the distribution of the future outcomes is not used. The difference between the expected value solution and the solution of the here-and-now model, which denotes the gain from solving the stochastic model, is called the *value of the stochastic solution (VSS).*[86]

3.3.2 Types of stochastic models

This chapter shows the various types of stochastic models and explains them. It first starts with recourse models.

In *recourse models*, the possible violation of constraints in the second stage, after the actual scenario is known, can be corrected by compensating measures. The compen-

[86] See also [BiLo97] p. 4ff

sation is calculated separately for each scenario and included in the objective function.

In the example above, the transport agency can order taxis at short notice to react on the scenario. Depending on the stochastic parameter “customer orders per day”, the agency can order additional taxis and use them as replacement for ordered buses, vans and taxis. These second-stage variables that compensate the violation of constraints are also called *recourse variables*.

- We talk about *fixed recourse*, when the necessary compensating activities and their extend is known and fixed. In other cases, the compensation is uncertain: Even if the violation of the constraint is known at the planning date, the compensating measures are uncertain and cannot be determined then.
- *Complete recourse* means that there is a compensating measure for every conceivable plan, so that the solution in the second stage will always be valid.
- *Relatively complete recourse*, in contrast, means if not always but for any solution that is valid in the first stage, a compensating measure can be implemented, so that a valid solution in the second stage always exists. To do this, however, the solution sets of the first and second stage have to be known, what is often a problem in practice.
- A special case of complete recourse is *simple recourse*. This means that every violation of a constraint is compensated by exactly one compensating measure, and not by a combination of several measures.
- In all other cases, it is not possible to compensate every possible constraint violation. This is called *incomplete recourse*.

Another approach are *chance-constrained models*. These models do not strictly require that every constrained is not violated, but they do this with a certain probability.

- *Simultaneous* chance-constrained models require that all constraints are not violated simultaneously with a certain probability. The second stage of the problem is not considered explicitly, although there might be a violation with the converse probability. Therefore this model does not consider the consequences of inadmissibilities in the realization of the plan.

- *Separate* chance-constrained models require each constraint to be not violated with a certain probability for each constraint.

Fat-solution models allow only such decisions that are valid for all scenarios. The consideration of the second stage is therefore unnecessary. The solution space is correspondingly small and fat-solution models give usually relative "expensive" decision support.

Deterministic replacement-value models: In these models, all uncertain parameters are replaced by a deterministic value. That means that the model is solved for only one scenario. Often the expected value (*expected-value model*), the worst value (*worst-case model*) or the expected value with certain surcharges or deductions is used.[87]

3.3.3 Stages of stochastic models

Stochastic programs can further be classified as *two-stage* or *multistage*. The example of the transport agency was formulated as a two-stage stochastic program. The transport agency has first to decide how many vehicles are ordered and then, after the uncertainty is revealed, it can order additional taxis at short notice. Because there is only one time when an uncertainty is revealed and there are decisions made before and after this uncertainty, this problem is called a *two-stage stochastic program*.

In some cases it might be useful to make decisions periodically in certain time intervals. For example, a producing company could decide every year to either expand or reduce their production capacity, depending on the development of the demand of the last year. In this case, a decision is made every year depending on the scenario of the last year. This program is called a *multistage stochastic program*. In multistage stochastic programs, every stage has its own scenarios. The next figure shows a scenario tree for a two-stage and for a multistage stochastic program with eight scenarios. The multistage program has two scenarios in every stage. Note that, although this is often the case, not every year or every period has to be one stage in the stochastic program: If every second year/period new decisions for the next two years/periods are made, then the program has twice as many years/periods as stages.

[87] See [Sch01] p. 73ff, [KW94] p. 9 and [BiLo97] p. 92

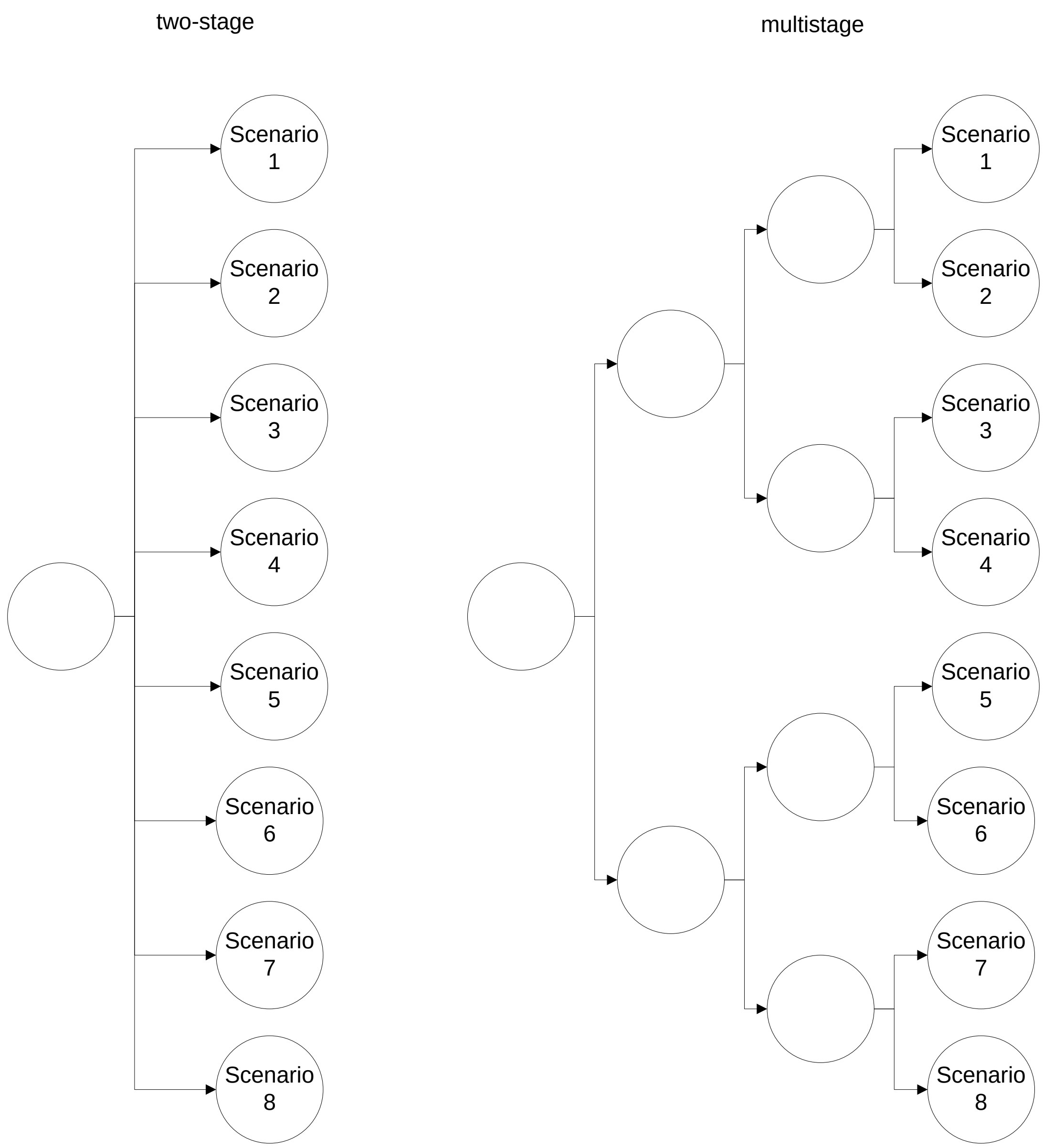

Figure 10 Two-stage vs. multistage scenario tree

With every scenario and its unique path through the scenario tree, a recourse model analogue to the two-stage recourse models described before can be created. The uncertain parameters, which change their values during the stages, are additionally indexed by a stage-index. In every stage greater than one a scenario has occurred and compensating measures for decisions taken in earlier stages can be done as well as new decisions for this stage have to be made. The different stages are usually connected by constraints, e.g. storage-balance-constraints.

If the decision variables are also indexed by stage and scenario, additional constraints have to be added, so that for every scenario which has identical stochastic data up to a certain stage the same decisions are made. These constraints are called *non-anticipativity-constraints*.[88] Omitting these constraints changes the model from a here-and-now model to a wait-and-see model.

3.3.4 Solving Stochastic Optimization Models

3.3.4.1 Benders‘ Decomposition

A very efficient solution method for stochastic programs is Benders’ Decomposition. Consider the linear optimization model:

$$\min cx + fy \qquad (11.1)$$

$$s.t.\ Ax = b \qquad (11.2)$$

$$Bx + Dy = d \qquad (11.3)$$

$$x, y \geq 0 \qquad (11.4)$$

In this model, the x-variables can be defined as stage-1 decisions and the y-variables as stage-2 decisions. Now a more complex format associated with two-stage stochastic programming is introduced:

[88] See [Sch01] p. 80ff

$$\min c^T x + \alpha_1 f_1^T y_1 + \alpha_2 f_2^T y_2 + \dots + \alpha_K f_K^T y_K \qquad (12.1)$$

$$s.t.\ Ax = b \qquad (12.2)$$

$$B_1 x + D_1 y_1 = d_1 \qquad (12.3)$$

$$B_2 x + D_2 y_2 = d_2 \qquad (12.4)$$

$$\dots \qquad \dots \qquad \dots$$

$$B_K x + D_K y_K = d_K \qquad (12.5)$$

$$x, y_1, \dots, y_K \geq 0 \qquad (12.6)$$

This format is called *block-ladder*, because every scenario represents one block of constraints. Figure 11 from Freund[89] shows this structure.

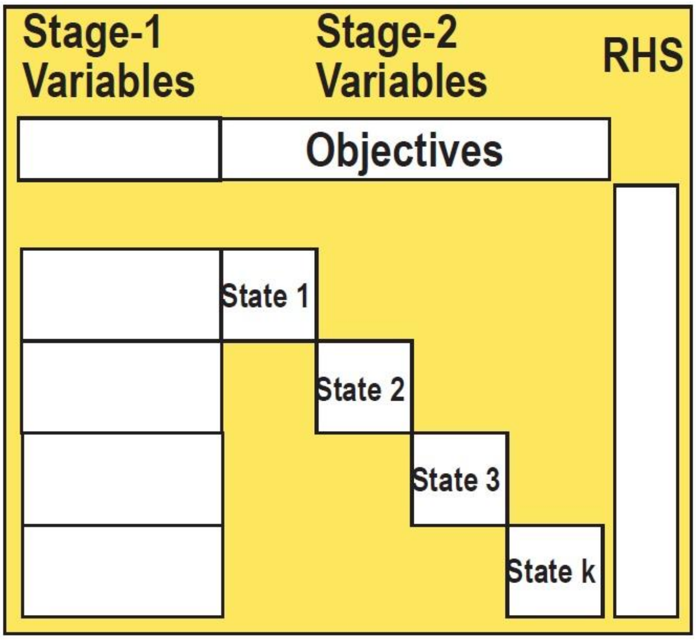

Figure 11 Block-ladder structure

This model can be reformulated to:

89 [Fre04] p. 3

$$\min cx + \sum_{i=1}^{K} a_i\, z_i(x) \qquad (13.1)$$

$$s.t.\ \ Ax = b \qquad (13.2)$$

$$x \geq 0 \qquad (13.3)$$

$$\text{with } P2_i\text{: } z_i(x) = \min f_i y_i \qquad (13.4)$$

$$s.t.\text{: } D_i y_i = d_i - B_i x \qquad (13.5)$$

$$y_i \geq 0 \qquad (13.6)$$

The problem P2 represents the second-stage decisions, when the first-stage decisions are already made.

The dual problem of P2 is:

$$D2_i\text{: } z_i(x) = \ max_{p_i} p_i^T (d_i - B_i x) \qquad (14.1)$$

$$s.t.\ \ D_i^T p_i \leq f_i \qquad (14.2)$$

The set of feasible solutions of $D2_i$ is: $D_2^i = \{p_i | D_i^T p_i \leq f_i\}$ and the extreme points and extreme rays can be enumerated: $p_i^1, \dots, p_i^{I_i}$ are the extreme points of D_2^i and $r_i^1, \dots, r_i^{I_i}$ are the extreme rays of D_2^i. Solving $D2_i$ then either returns an extreme ray $\left(r_i^j\right)^T (d_i - B_i x) > 0$ (if $D2_i$ is unbounded) or an extreme point and the optimal objective with: $z_i(x) = (p_i^{j2})^T (d_i - B_i x) = \max_{k=1,\dots,I_i} \left(p_i^k\right)^T (d_i - B_i x)$

$D2_i$ can therefore be written as:

$$D2_i\text{: } z_i(x) = min_{z_i} z_i \qquad (15.1)$$

$$s.t.\ \left(p_i^{j2}\right)^T (d_i - B_i x)\ \leq z_i \qquad \forall j2 = 1, \dots, I_i \qquad (15.2)$$

$$\left(r_i^j\right)^T (d_i - B_i x) \leq 0 \qquad \forall j = 1, \dots, J_i \qquad (15.3)$$

By inserting this formulation into the original problem, the y-variables are removed and the *Full-Master-Problem* with new constraints is obtained:

$$min_{x,z_1,\dots,z_K} cx + \sum_{i=1}^{K} a_i\, z_i \qquad (16.1)$$

$$s.t.\ Ax = b \qquad (16.2)$$

$$x \geq 0 \qquad (16.3)$$

$$\left(p_i^{j2}\right)^T (d_i - B_i x)\ \leq z_i \qquad \forall j2 = 1, \dots, I_i \quad \forall i = 1, \dots, K \qquad (16.4)$$

$$\left(r_i^j\right)^T (d_i - B_i x) \leq 0 \qquad \forall j = 1, \dots, J_i \quad \forall i = 1, \dots, K \qquad (16.5)$$

The Benders' Decomposition Algorithm begins with solving the Full-Master-Problem without the constraints, which are derived from the stage-2 variables. This model is called the *Restricted-Master-Problem*. They are left out, because many of them might be inactive because of their high number.

The optimal objective value of the Restricted-Master-Problem is a dual bound for the optimal objective value of the Full-Master-Problem. This value is saved. The optimal solution is saved in $\overline{\mathrm{x}}, \overline{\mathrm{z}_1}, \dots, \overline{\mathrm{z}_\mathrm{K}}$.

To test if the obtained solution is the optimal solution of the Full-Master-Problem, it has to be checked that all constraints that have not been considered are not violated. Therefore K subproblems are solved, with the first-stage solution $\overline{\mathrm{x}}, \overline{\mathrm{z}_1}, \dots, \overline{\mathrm{z}_\mathrm{K}}$.

If a subproblem is unbounded, an extreme ray is generated and the constraint $\left(\mathrm{r}_\mathrm{i}^\mathrm{j}\right)^\mathrm{T} (\mathrm{d}_\mathrm{i} - \mathrm{B}_\mathrm{i}\mathrm{x}) \leq 0$ is added to the restricted master problem.

If a subproblem has an optimal solution, the algorithm returns an optimal extreme point $\overline{\mathrm{p}_\mathrm{i}} = \mathrm{p}_\mathrm{i}^{\mathrm{j2}}$ for some j2 and the optimal solution $\overline{\mathrm{y}_\mathrm{i}}$. If $\left(\mathrm{p}_\mathrm{i}^{\mathrm{j2}}\right)^\mathrm{T} (\mathrm{d}_\mathrm{i} - \mathrm{B}_\mathrm{i}\overline{\mathrm{x}}) > \overline{\mathrm{z}_\mathrm{i}}$ the constraint $\left(\mathrm{p}_\mathrm{i}^{\mathrm{j2}}\right)^\mathrm{T} (\mathrm{d}_\mathrm{i} - \mathrm{B}_\mathrm{i}\overline{\mathrm{x}}) \leq \overline{\mathrm{z}_\mathrm{i}}$ is added to the Restricted-Master-Problem.

If every subproblem has a finite optimal objective function, the solution $\overline{\mathrm{x}}, \overline{\mathrm{y}_1}, \dots, \overline{\mathrm{y}_\mathrm{K}}$ is also valid for the original problem. If this solution is better than the primal bound, this solution is saved as the best solution and the primal bound is updated with its objective value.

Next, it is checked if the inequality $\left(\mathrm{p}_\mathrm{i}^{\mathrm{j2}}\right)^\mathrm{T} (\mathrm{d}_\mathrm{i} - \mathrm{B}_\mathrm{i}\overline{\mathrm{x}}) \leq \overline{\mathrm{z}_\mathrm{i}}$ holds for every subproblem. In this case the algorithm can be terminated, because the solution found is optimal for the whole problem. The algorithm also terminates if the difference between the dual and primal bound is smaller than a pre-defined tolerance, otherwise it starts a new iteration with solving the restricted master problem with the new constraints.[90]

3.3.4.2 The deterministic equivalent

Let us come back to the extensive form of the stochastic optimization model of the example from Chapter 3.3.1. In the extensive form, every stage-2 variable is additionally indexed by its scenario and the variables, objective-terms and constraints for every scenario were added to the model.

If the model has more than two stages, the scenario tree can be represented by indexing all decision variables by stage and scenario and adding additional constraints (non-anticipativity-constraints.[91]), so that for every scenario which has identical stochastic data up to a certain stage, the same decisions are made.

A stochastic optimization model in its extensive form is also called the *deterministic equivalent* of a stochastic optimization model. The deterministic equivalent is a LP or a MIP and can therefore be solved with a standard LP/MIP-solver. A formal description of the ways how the deterministic equivalent can be derived for different types of stochastic programs can be found in Kall and Wallace[92].

Like the deterministic equivalent of the transport agency problem, deterministic equivalents are usually large models, because they explicitly contain every constraint although many of them might be inactive.

Solving a deterministic equivalent with a standard LP/MIP-solver does not leave out inactive constraints, what is done in the Benders' Decomposition Algorithm. Therefore using deterministic equivalents seems to be inferior. But solving stochastic programs as deterministic equivalent also has practical advantages:

- Deterministic equivalents can easily be (re-)formulated and bookkeeping variables that calculate values over several scenarios can be easily integrated.

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[90] See [Fre04]

[91] See [Sch01] p. 80ff

[92] See [KW94] p. 25ff

- Because LPs and MIPs are widely used, modeling software and highly specialized implementations of solution algorithms are available.
- The inactive constraints of the model can often be eliminated by advanced preprocessing techniques of LP/MIP-solvers.

Therefore for practical applications it is an appropriate alternative to solve a stochastic program as deterministic equivalent.

3.3.5 Selected risk measures in stochastic optimization models

This section describes how to integrate the worst scenario and the Conditional Value at Risk into a stochastic optimization model.

The worst scenario, as a very basic risk measure, can often be integrated into optimization models very easily. It is especially easy to integrate it if you make use of bookkeeping variables like $profit_s$ or $cost_s$ that already contain the objective function values for each scenario. Then, in case of maximizing profits, maximizing or constraining $worstscen$ is possible by integrating the following constraint.

$$worstscen \leq profit_s \quad \forall s \in Scenarioset \qquad (17.1)$$

In case of minimizing costs the following inequality enables constraining or minimizing the worst scenario

$$worstscen \geq cost_s \quad \forall s \in Scenarioset \qquad (17.2)$$

Note that the variable $worstscen$ only contains the value of the worst scenario, if it is optimized in the objective function, otherwise it may contain worse values than the real value of the worst scenario.

The Conditional Value at Risk, as described in Chapter 3.1.4, is a measure for risk that matches the perception of risk very well. The CVaR can be integrated into optimization models and formulated with linear variables and constraints. Because of this nice fact integrating the CVaR into existing optimization models hardly increases their computational complexity. Therefore the dual formulation of Fabian[93] was

[93] See [Fab08]

slightly adjusted and can be integrated into a stochastic optimization model with the scenarioset S with the following additional variables and constraints:

Variables:

$cvar$	Bookkeeping variable which represents the Conditional Value at Risk
y_0	Dual variable
$y_s\ \forall s \in S$	Dual variables for every scenario

Constraints:

$$cvar = -\frac{1}{\alpha}\left(\alpha \cdot y_0 + \sum_{s \in S} y_s \cdot \frac{1}{|S|}\right) \qquad (18.1)$$

$$y_0 + y_s \geq -profit_s \qquad \forall s \in S \qquad (18.2)$$

$$y_s \geq 0 \qquad \forall s \in S \qquad (18.3)$$

The parameter $\alpha \in [0..1]$ represents the percentage of the scenarios with profits that fall below the corresponding Value at Risk. The bookkeeping variables $profit_s$ contain the profit in scenario s; in case of cost minimization $-profit_s$ should be replaced by a variable like $cost_s$.

The CVaR can be limited in constraints as well as be optimized in the objective function. Note that in cases where the bookkeeping variable $cvar$ is not optimized in the objective function it might also contain meaningless values in the solution. If the variable is limited in a constraint, but not optimized, (and the limit does not influence the optimal solution) it might have a value between the limit and the real CVaR. A 'feasible' solution for the variable $cvar$ is then worse or equal to the real CVaR. In this case the real CVaR should be calculated manually after the optimization.

Because the CVaR matches the perception of risk very well and adds only very little complexity to optimization models, it will be used as risk measure in this thesis.

3.3.6 Discussion of methods for optimization under uncertainty

This section now discusses other methods for optimization under uncertainty besides stochastic programming. These approaches can be divided into approaches that represent uncertainty indirectly and approaches that consider uncertainty directly.

A motivation to use a model with an indirect representation of uncertainty is that these models can usually be modeled and solved very easily. For example existing standard software for deterministic optimization can often be used.

One method is the use of deterministic replacement values. Thereby every stochastic value is replaced by a deterministic value. These can be the expected value, the most probable value or another estimated value. For risk averse planning, a safety margin can be added to the values. The most risk adverse solution will be obtained, if the worst-case values for all uncertain parameters are used. This will probably lead to a very expensive solution.

The second approach is sensitivity analysis. Sensitivity analysis is an ex-post analysis, because it analyzes the effects of changing one or several parameters to the objective function and/or to the solution structure. But especially for optimization models with integer variables, the sensitivity analysis can only be used in a very limited way. This is because an integer-solution can completely change if one parameter is changed to another value. Therefore, many optimization runs with different parameter combinations are often necessary to find valid results with sensitivity analysis.

The sensitivity analysis can be combined with simulation and statistic analysis. Thereby parameter combinations are randomly created and then the impacts of the parameter changes are evaluated with statistical measures. Because the results of the sensitivity analysis always depend on the parameter constellation, it cannot be seen as a method for optimization under uncertainty itself.

As a third approach, a risk profile can be created. Every possible scenario is optimized independently in an optimization model. After that it is evaluated with which probability a certain objective value is achieved. But in this approach all scenarios are optimized independently and therefore different decisions for every scenario may be made, but in reality only one decision can be made. Therefore a wait-and-see decision is evaluated, although the planner is in a here-and-now situation, which is a drawback of this approach.[94]

In contrast to indirect representation of uncertainties, direct representation explicitly uses the given information – scenarios and probabilities. Models that represent uncertainty explicitly and use the given information are stochastic optimization models.[95] The different types are described in Chapter 3.3.2.

In this thesis, stochastic optimization models are used. One reason is that it is a method that verifiably finds the optimal solution for the given data. This enables exact evaluations and complete comparability of test runs with different parameters or different model variations where certain variables are enabled or disabled. Thus, an additional inexactness caused by a heuristic solution procedure is avoided by using stochastic optimization models.

Furthermore, risk measures, such as the Conditional Value at Risk, can be integrated into stochastic optimization models and can be constrained to an exact value. That enables an exact evaluation of solutions with different risk levels.

Although stochastic optimization models are computationally harder to solve, practical instances of real-world problems can nowadays be solved. Therefore the applicability of stochastic optimization models is given and they are used because of the advantages shown above.

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[94] [Sch01] p. 186ff

[95] [Sch01] p. 196

4 Airline schedule design under fuel price and demand uncertainty

This chapter introduces a new stochastic programming approach for robust airline schedule design under fuel price and demand uncertainty.[96]

4.1 Motivation and goals

As shown in Chapter 2.2.1, today the two largest parts of the expenditures of airlines are the costs for labor and fuel.[97] The percentage for jet fuel expenditures has increased in the last years. They have grown from approx. one tenth to one third in only ten years.[98] As jet fuel price fluctuations are also high[99], airlines face a growing uncertainty for their costs. It therefore becomes more important for airlines to think about minimizing fuel costs and counteracting fuel price uncertainty.

As the schedule design significantly determines the fuel consumption of an airline, we aimed to develop a model that supports strategic decisions about this planning phase under fuel price uncertainty. We determine the optimal offered flights between a given set of airports with their frequency. To counteract jet fuel price uncertainty, we consider financial hedging instruments. As demand is highly uncertain at the time when the schedule is planned and aircraft seats are one of the most perishable services, we also introduce stochastic demands.

In this chapter, we focus on the schedule design phase and develop a strategic planning model for the airline industry under jet fuel price and demand uncertainty. Similarly to Lederer and Nambimadom[100] we mean by schedule the frequency of service between two airports. We therefore determine if and how often a flight between two airports should be flown with a certain aircraft type and how much fuel should be hedged as a decision under uncertainty. The optimal passenger flow in each scenario

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[96] Main parts of this chapter have been published in [NaSu12]

[97] [ATA11]

[98] [DB08]

[99] See [EIA]

[100] [LeNa98]

enables the evaluation. The flight times are not determined as this model aims to support decisions on a strategic level and focuses on uncertainty. To measure the robustness of the solutions, we integrate and restrict the Conditional Value at Risk (CVaR) as risk measure.

Although the flight schedule of an airline depends on other factors as well, the study will give some insight into the impacts of fuel price uncertainty on strategic airline schedule planning. We only consider the most important aspects of schedule design, because we aim to focus on uncertainties. Thereby we can integrate fuel price and demand scenarios and keep the model smaller and solvable.

4.2 Problem Description

Minimizing jet fuel consumption is possible with the use of larger aircraft. Larger aircraft usually have less jet fuel consumption per passenger.[101] The airbus A380 is the first long-haul aircraft that consumes less than 3 liters per passenger per 100 km. But using larger aircraft is only beneficial when there are enough passengers. Therefore it might be necessary to route passengers through hubs and to merge flights. For example if there are several flights from Europe to North America, the passengers from all the European locations could first be flown to London and one flight with a larger aircraft from London to North America could save jet fuel. On the other hand, this means less comfort for the European passengers who do not start in London, because they have to change the plane. Some possible passengers might then choose another airline that offers a non-stop-flight from their hometown. The airline could also offer a discount as compensation for the discomfort. This tradeoff between reducing passenger comfort, which might decrease revenues, and reducing jet fuel consumption will be considered in this study.

It is also possible to pass the higher jet fuel costs to the passengers via fuel surcharges. To consider that, the model could be solved with other demand and price data that could be calculated from a revenue management framework.

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[101] [AF08]

To counteract the high jet fuel price fluctuations financial hedging instruments can be used. With financial hedging the price for future purchases can be fixed. If an airline wants to hedge against higher jet fuel prices, it can sign a contract that fixes the price for jet fuel for a certain amount for a certain time. Then higher fuel prices do not have negative effects on the airline, but the airline is also not able to benefit from lower jet fuel prices anymore. Financial hedging instruments can be used to minimize fluctuations and are therefore an effective method for risk management.[102]

Cobbs and Wolf[103] argue that futures or forward-contracts for jet fuel are often not available, but show dynamic hedging strategies to hedge the jet fuel price using derivatives with other underlying assets like crude oil or heating oil, whose prices highly correlate with the jet fuel price. With an industry survey they also show that hedging was at the end of 2003 not very common at the majority of airlines. Their research results indicate that hedging creates market value and that the consideration of financial hedging instruments therefore could create a competitive advantage for an airline. Another study by Carter et al.[104] concludes that hedging is positively related to airlines' firm value.

In general, a good risk management strategy can be beneficial for companies. For a detailed description see Chapter 3.1.

4.3 Model

4.3.1 Model description

This section presents the developed mathematical optimization model that determines the optimal flights offered with their frequency and the optimal passenger flows for a given network of airports. The passengers can be directly transported to their destination on a non-stop flight or they can be indirectly transported via one or two airports, where they change the aircraft. When passengers do not fly non-stop, a discount on the price of the flight is given to compensate the discomfort. For a two-

[102] See chapter 3.1.3

[103] [CoWo04]

[104] [CRS06]

stop flight the discount is given two times. Passenger spill and recapture is not considered. Furthermore different aircraft types with their capacities and their fuel consumption are assigned to the flights to anticipate future fleet assignment.

As jet fuel costs become the major part of an airline's expenses and jet fuel prices have high fluctuations, this model explicitly considers the uncertainty of jet fuel prices with a scenarioset for each jet fuel price. The demand scenarios are also considered in a scenarioset and every demand scenario is combined with every fuel price scenario. The model is a two-stage stochastic program with $|\text{no. of fuel scenarios}| \times |\text{no. of demand scenarios}|$ scenarios.

To counteract the jet fuel price uncertainty, this model considers financial hedging instruments. With forward-contracts/futures the purchases of jet fuel can be hedged. This model assumes that there are futures for jet fuel, which may not exist, but as Cobbs and Wolf[105] argue, airlines can use futures on commodities whose prices highly correlate with jet fuel prices. The hedging can include a risk premium and the amount of the jet fuel bought that can be hedged is arbitrary from 0% to 100%. Reverse hedging or hedging more than 100% is not allowed.

The jet fuel price scenarios and the prices for the financial instruments are adjusted to each other, so that there is no riskless arbitrage strategy. As risk measure in this model we use the Conditional Value at Risk. It can be formulated as LP with the dual formulation of Fabian[106]. This formulation has been slightly adjusted and integrated.

Altogether this model is a strategic optimization model for airline schedule design under fuel price and demand uncertainty which considers risk/robustness measurement and financial instruments as countermeasures to uncertainty.

105 [CoWo04]

106 [Fab08]

4.3.2 Complete model

Sets:

A	Set of airports
T	Set of aircraft types
FS	Scenarioset for jet fuel prices
DS	Scenarioset for demands

Parameters:

$dist_{i,j}$	Distance from airport i to airport j in km
$p_{i,j}$	Sell-price for a flight from i to j
$d_{i,j,ds}$	Flight demand from airport i to airport j (stochastic parameter)
pd	Price discount given for every aircraft change
x^{min}	Percentage of expected demand for every possible flight connection that has to be satisfied. Can ensure a certain service level.
m_t	Passenger capacity of aircraft type t
$cons_t$	Jet fuel consumption of aircraft type t in liters per km
cpm_t	Operational cost per km of aircraft type t (without jet fuel costs)
$rmax_t$	Maximum range of aircraft type t
$rmin_t$	Minimum range of aircraft type t
ub_t	Maximum number of flights with aircraft type t
f_pr	Forwarded jet fuel price
f_margin	Margin for forwards in percent
pr_{fs}	Jet fuel price per liter in fuel-scenario fs (stochastic parameter)
$prob_{fs,ds}$	Probability for the combination of demand scenario ds and jet fuel price scenario fs
α	Probability value for the CVaR

Stage-1 variables:

$y_{i,j,t;\forall i,j\epsilon A:i<j}$	Number of flights per day from i to j and j to i with aircraft type t (nonnegative integer variable)
buy_stoch	Bought amount of fuel in liters to the stochastic price (nonnegative continuous variable)
buy_hedge	Bought amount of fuel in liters to the hedged price (nonnegative continuous variable)

Stage-2 variables:

$x0_{i,j,ds:i\neq j}$	Passenger flow - directly transported from i to j in demand-scenario ds (nonnegative continuous variable)
$x1_{i,k,j,ds:i\neq j\neq k}$	Passenger flow - transported passengers from i to j over k with aircraft change on airport k in demand scenario ds (nonnegative continuous variable)
$x2_{i,k,l,j,ds:i\neq j\neq k\neq l}$	Passenger flow - transported passengers from i to j over k and l with aircraft change on airport k and airport l in demand-scenario ds (nonnegative continuous variable)

Bookkeeping variables:

$revenue_{ds}$	Revenue in demand scenario ds
$consumption$	Jet fuel consumption
$fuel\_costs_{fs}$	Jet fuel costs in fuel price scenario fs
$operationcosts$	Sum of operational costs (without jet fuel costs)
$profit_{fs,ds}$	Profit in demand scenario ds in jet fuel scenario fs
$cvar$	Conditional Value at Risk
$cvar\_y0$	Auxiliary variable for the dual CVaR-formulation
$cvar\_y_{fs,ds}$	Nonnegative auxiliary variables for the dual CVaR-formulation

Objective Function:

$$\max \sum_{fs\epsilon FScen,ds\epsilon DScen} prob_{fs,ds} \cdot profit_{fs,ds} \qquad (19.1)$$

Constraints:

$$x0_{i,j,ds} + \sum_{k\in A:i\neq j\neq k} x1_{i,k,j,ds} + \sum_{k,l\in A:i\neq j\neq k\neq l} x2_{i,k,l,j,ds} \leq d_{i,j,ds} \quad \forall i,j\epsilon A: i \neq j, ds\epsilon DS \qquad (19.2)$$

$$x^{min} \cdot \sum_{ds\in DS}\sum_{fs\in FS} prob_{fs,ds} \cdot d_{i,j,ds} \leq \sum_{ds\in DS}\sum_{fs\in FS} prob_{fs,ds} \cdot \left(x0_{i,j,ds} + \sum_{k\in A:i\neq j\neq k} \left(x1_{i,k,j,ds} + \sum_{l\in A:i\neq j\neq k\neq l} x2_{i,k,l,j,ds} \right) \right) \quad \forall i,j\epsilon A: i \neq j \qquad (19.3)$$

$$x0_{i,j,ds} + \sum_{k\in A:i\neq j\neq k} \left(x1_{i,j,k,ds} + x1_{k,i,j,ds} \right) + \sum_{k,l\in A:i\neq j\neq k\neq l} \left(x2_{i,j,k,l,ds} + x2_{k,i,j,l,ds} + x2_{k,l,i,j,ds} \right) \leq \sum_{t\in T} m_t \cdot y_{ijt} \quad \forall i,j\epsilon A: i < j, ds\epsilon DS \qquad (19.4)$$

$$x0_{i,j,ds} + \sum_{k\in A:i\neq j\neq k} \left(x1_{i,j,k,ds} + x1_{k,i,j,ds} \right) + \sum_{k,l\in A:i\neq j\neq k\neq l} \left(x2_{i,j,k,l,ds} + x2_{k,i,j,l,ds} + x2_{k,l,i,j,ds} \right) \leq \sum_{t\in T} m_t \cdot y_{jit} \quad \forall i,j\epsilon A: i > j, ds\epsilon DS \qquad (19.5)$$

$$y_{ijt} = 0 \quad \forall i,j\epsilon A: i < j, t\epsilon T, if\ dist_{i,j} > rmax_t\ or\ dist_{i,j} < rmin_t \qquad (19.6)$$

$$\sum_{i,j\in A:i<j} 2 \cdot y_{ijt} \leq ub_t \quad \forall t\epsilon T \qquad (19.7)$$

$$revenue_{ds} = \sum_{i,j\in A:i\neq j} p_{i,j} \cdot \left(x0_{i,j,ds} + (1-\mathrm{pd}) \cdot \sum_{k\in A::i\neq j\neq k} x1_{i,k,j,ds} + (1-2\cdot \mathrm{pd}) \sum_{k,l\in A:i\neq j\neq k\neq l} x2_{i,k,l,j,ds} \right) \quad \forall ds\epsilon DS \qquad (19.8)$$

$$consumption = \sum_{i,j\in A:i<j,t\in T} 2\cdot y_{ijt} \cdot dist_{ij} \cdot cons_t \qquad (19.9)$$

$$operationcosts = \sum_{i,j\in A:i<j,t\in T} 2\cdot y_{ijt} \cdot dist_{ij} \cdot cpm_t \qquad (19.10)$$

$$buy\_stoch + buy\_hedge = consumption \qquad (19.11)$$

$$fuel\_costs_{fs} = buy\_stoch \cdot pr_{fs} + buy\_hedge \cdot f\_pr \cdot \left(1 + \frac{f\_margin}{100}\right) \quad \forall fs\epsilon FS \qquad (19.12)$$

$$profit_{fs,ds} = revenue_{ds} - fuel\_costs_{fs} - operationcosts \quad \forall fs\epsilon FS, ds\epsilon DS \qquad (19.13)$$

$$cvar = \frac{-1}{\alpha}\left(\alpha \cdot cvar\_y0 + \sum_{ds\in DS,fs\in FS} cvar\_y_{fs,ds} \cdot prob_{fs,ds}\right) \qquad (19.14)$$

$$cvar\_y0 + cvar\_y_{fs,ds} \geq -profit_{fs,ds} \quad \forall fs\epsilon FS, ds\epsilon DS \qquad (19.15)$$

The objective function maximizes the expected profit, but the Conditional Value at Risk can also be maximized. The constraint (19.2) ensures that the passenger flow variables do not exceed the demand; minimum demand satisfaction is forced by (19.3). This should ensure a connection (with 0, 1 or 2 aircraft changes) between every pair of airports in the network, if there is a demand between these airports. The inequalities (19.4) and (19.5) implement aircraft capacity, (19.6) assures that the maximum and minimum distance of the aircraft types is not exceeded. The constraint (19.7) ensures the maximum number of flights with an aircraft type while (19.8) assigns the revenue. The constraint (19.9) calculates the consumed jet fuel and (19.10) calculates the additional operational costs. The equality (19.11) sets the variables for hedged and non-hedged fuel purchases. The fuel costs and the profit for every scenario are calculated in (19.12) and (19.13). Finally (19.14) and (19.15) integrate the Conditional Value at Risk into the optimization model.

The stage-2 variables are only indexed by the demand scenario and not by the fuel price scenario. This is possible, because when the flights are planned by the stage-1 variables the fuel-price does not have any impact on the transported passengers – as much as possible are transported in every demand scenario. Thereby we only need one tenth of the stage-2 variables, when we use ten fuel price scenarios, and can decrease computational complexity. The connection of the fuel price scenarios and the demand scenarios is done by the bookkeeping variables.

4.3.3 An illustrative application

The data for this model is a small case study that was developed with a European airline. We consider two countries with six airports in each country; the countries are on different continents.

The considered aircraft types are a small one for domestic and medium-haul connections and a larger long-haul aircraft for intercontinental distances. The usage of aircraft types is constrained by a minimum range to avoid high consumption because of too short flights with large aircraft and by their maximum range. The flight distance between the airports is always the shortest line between the airports.

To calculate the demand scenarios, we use the expected demand for the flights, which is symmetric for each pair of airports, and create a random value for each scenario from a normal distribution with $\mu =$ expected demand and $\sigma = \text{expected demand}/6$ for each flight. These values are multiplied with factors from 77% to 122% depending on the five demand scenarios to create scenarios with different lower and higher total demands. Finally, negative values are set to 0. The value used for pd is 0.1; the value for α is also 0.1 – therefore the worst 10% of all scenarios are considered in the CVaR.

The jet fuel in November 2009 costs 0.55 \$ per liter.[107] The model takes ten scenarios for the future jet fuel price into account. The spread of the fuel prices in the scenarios is quite large in order to examine the effects of the jet fuel price development.

Scenario	1	2	3	4	5	6	7	8	9	10
Jet fuel price \$/l	0.06	0.11	0.17	0.28	0.41	0.55	0.69	0.83	1.05	1.38
Probability	10%	10%	10%	10%	10%	10%	10%	10%	10%	10%

Table 8 Jet fuel price scenarios

The fair forward rate for the scenarios is then calculated and an adjustable margin for the forward-contracts is added.

The parameter x^{min} is set to 0.5, which means that 50% of the expected demand for each connection has to be satisfied. The prices of the flights depend on the combination of the countries of the origin and destination airport.

Note that in this example a new network is constructed. With this model, it would also be possible to refine an existing network, which is commonly done in practice. Then the y-variables of non-modifiable flights should be fixed to the desired frequency of the connection between the two airports. This will also decrease the computational complexity of the model.

4.4 Results

4.4.1 Models for every jet fuel price scenario

Our results begin with a study of the impacts of different fuel prices on optimal offered flights. We would like to show how they are determined by the development of the jet fuel price. Therefore we optimize one model for each jet fuel price scenario

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[107] See [IATA]

without allowing financial instruments. The model for each jet fuel price scenario is also a stochastic model but the only uncertainty is the demand uncertainty. Then we look if the demand is satisfied and how many flights are carried out with each aircraft type. Figure 12 shows how many passengers are non-stop transported to their destination, how many are transported with an aircraft change and how many are not transported because their transportation would be unprofitable. (Note that the jet fuel price for scenario 1 is the lowest and for 10 the highest)

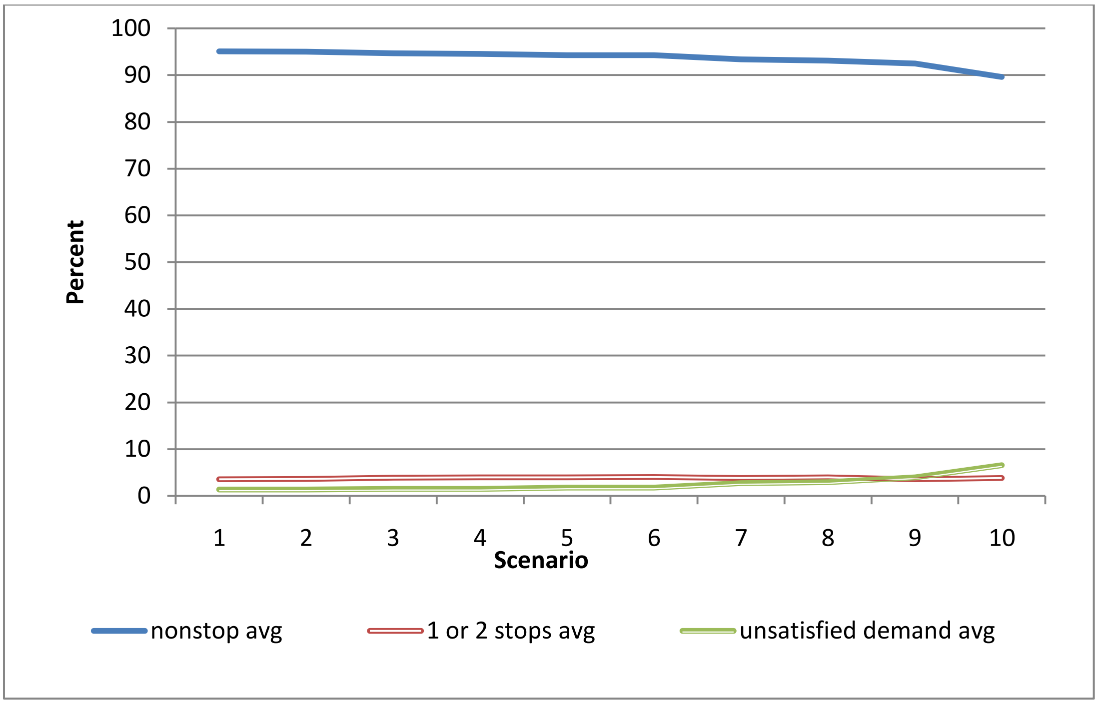

Figure 12 Transported demand

Figure 12 shows that if the jet fuel price rises, transporting fewer passengers becomes profitable and therefore more demand is not satisfied. The unsatisfied demand from scenario 1 to 5 is below 2%, but grows to nearly 6.5% in scenario 10. The percentage of non-stop transported passengers also decreases from 95% in scenario 1 to 89.5% in scenario 10, while the percentage of transported passengers with aircraft change is always between 3.5% and 4%. This shows that the amount of flights with aircraft changes of all flights slightly grows with higher jet fuel prices, although those passengers are less profitable because of the discount that is given for aircraft changes and the additional fuel and operational costs that they cause because of the indirect route. Note that the percentages are the average percentages over the five demand scenarios for each jet fuel price scenario.

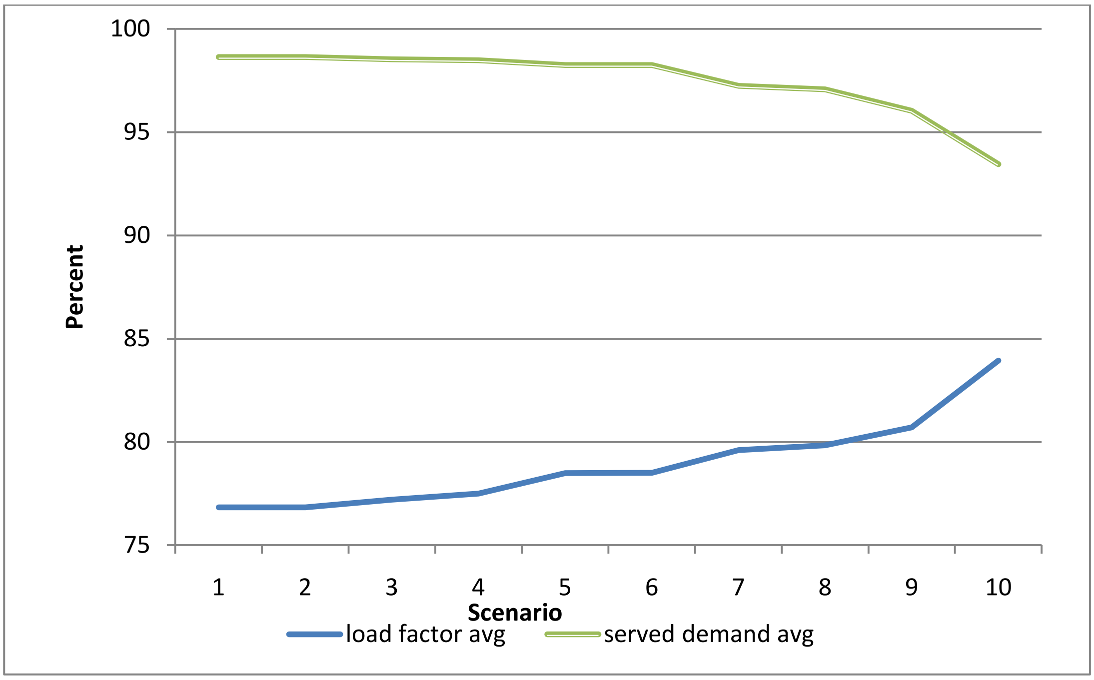

Figure 13 Load factor

Figure 13 shows the average load factor of all flights depending on the different jet fuel prices. From scenario 1 to 4 values between 76% and 78% are optimal. The optimal load factor grows to 84% in scenario 10, where the demand satisfaction decreases from >98% in the scenarios 1 to 6 to 93% in the highest fuel price scenario. The increase of the load factor has a minor impact on the demand satisfaction in scenario 1 to 6. The impact becomes higher from scenario 7 on where the load factor grows to values of 80% and higher. The increase of the load factor now decreases the satisfied demand more significantly.

This may be explained by the demand uncertainty: When the average seat usage of all demand scenarios grows to higher levels than 80%, we cannot transport all passengers in the higher demand scenarios. This additional demand in the higher demand scenarios is not satisfied, which causes the significant decrease in demand satisfaction. Connections also become unprofitable so that some flights are not offered anymore which decreases the demand satisfaction, too.

Financial hedging instruments are not considered in the models for every jet fuel price scenario, because as there is only one fuel price in each model and the profit is maximized for each scenario, the financial instruments could not change the risk.

Note that the results for this section, where one model for every fuel price scenario is created, might not be implemented without changes in practice. For example, the flight prices could be increased in the high fuel price scenarios so that more profitable flights could be offered, the demand satisfaction could increase, and lower load factors could be optimal because the aircraft does not need to have every seat occupied to be profitable. This section only shows the significant impacts of different fuel prices on offering the optimal flights by creating one solution for every fuel price scenario. In reality, one decision for offering flights has to be made here and now for all scenarios under fuel price uncertainty. This underlines the importance of considering fuel price uncertainty in the optimization model, what is done in the further results of this chapter with our proposed model.

4.4.2 Models considering both uncertainties and robustness

In this section, we present the calculations for the stochastic model that considers all jet fuel price and demand scenarios. The decision which flights should be offered and flown has to be done under uncertainty. Also the amount of jet fuel that should be hedged is a stage-1 decision. The stage-2 decisions are the passenger flows. To measure robustness, we restrict the Conditional Value at Risk at different risk levels.

4.4.2.1 Non-integrated hedging approaches

At first, we take a look at the risk/profit-distribution of the model without financial instruments. Therefore we first maximize the expected profit, then maximize the Conditional Value at Risk, and afterwards again the expected profit under the constraint of different risk levels. We first do this without financial instruments and obtain the pareto-frontline of optimal solutions shown in Figure 14.

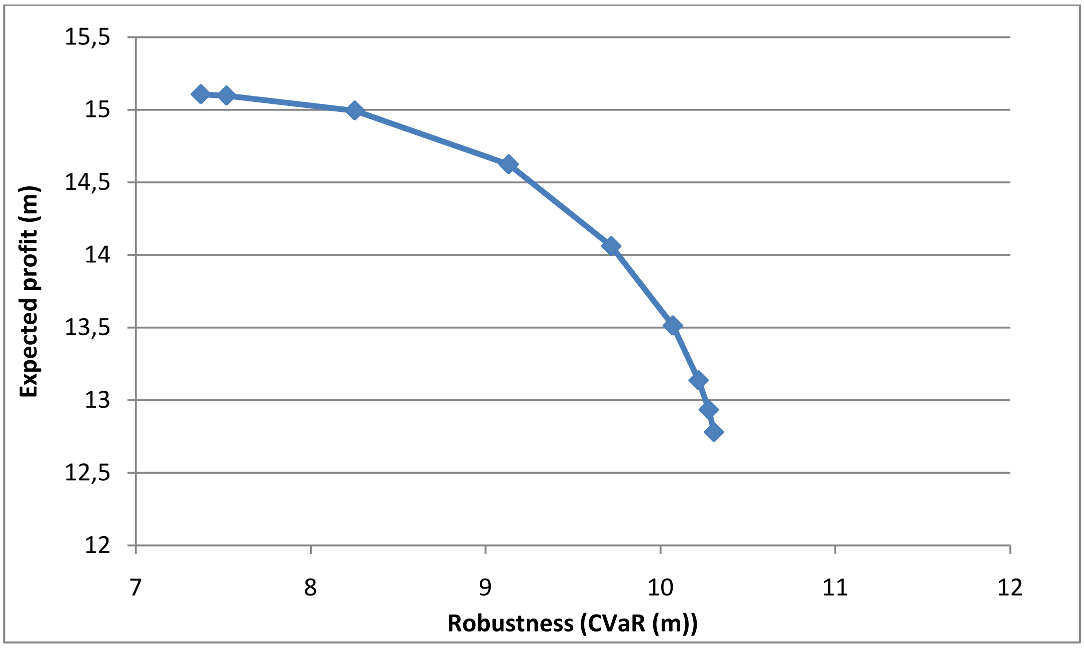

Figure 14 Profit/risk-profile without hedging

In Table 9, we compare the solutions with the highest expected profit (upper-left solution) and the lowest risk (bottom-right solution).

	Highest expected profit	Lowest risk
Expected profit	15.106m	12.780m
CVaR	7.372m	10.306m
Profit of the worst scenario	5.522m	9.296m
Profit the best scenario	22.450m	14.659m
Scenarios with profit < -10m	8	1
Flights medium-haul aircraft	502	330
Flights long-haul aircraft	26	16
Consumed jet fuel (l)	4.342m	2.715m

Table 9 Results highest expected profit - lowest risk

We can see that the most robust solution with the lowest risk has a Conditional Value at Risk of 10.306m instead of 7.372m. Also the profit in the worst scenario has increased from 5.522m to 9.296m, and the spread between the best and the worst scenario is significantly lower. On the other hand, the expected profit has also decreased from 15.106m to 12.780m. The risk is reduced by offering fewer flights and therefore consuming less jet fuel – the consumption is reduced by about one third.

These significant changes show that gaining maximum robustness needs severe operational changes, which might not be desired in practice. One medium solution with CVaR 9.132m, expected profit 14.623m and 436 and 20 flights with the aircraft types might be the most robust solution that is still practical. Therefore other methods for gaining robustness are necessary.

An approach could be to hedge the bought jet fuel. We expect that the minimum risk exposure is obtained by hedging 100% of the purchases. We then obtain an additional pareto-frontline shown in Figure 15.

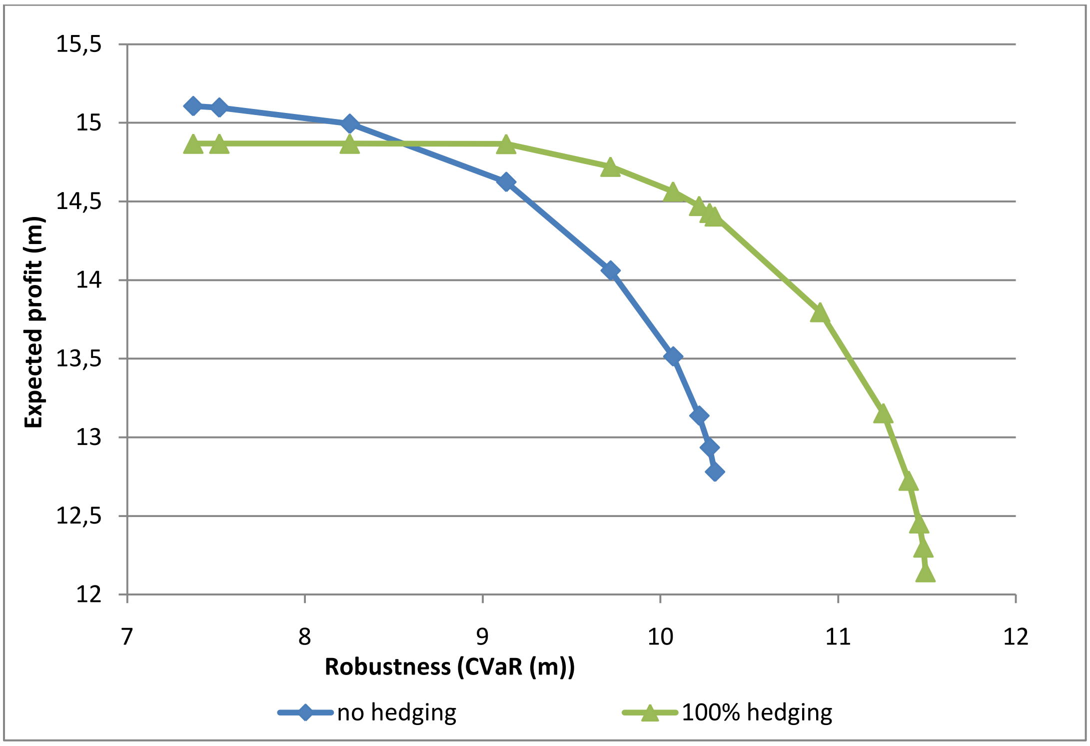

Figure 15 Profit/risk-profile with 0% and 100% hedging

If the fuel purchases are completely hedged, the best Conditional Value at Risk can be increased significantly from 10.306m to 11.492m. Hedging therefore can significantly increase robustness. But we also find out that the expected profit is lower than the maximum expected profit without hedging jet fuel. This is because the costs for the hedging premium lower the expected profit. Hedging all jet fuel purchases is better than hedging no fuel purchases when the minimum accepted CVaR is higher than ~8.5m (where the two lines cross each other).

We also see that the expected profit does not grow if we allow lower CVaR values than 9m. Up to this CVaR, it is not necessary to change any flight or any passenger flow to gain less risk. The risk for the solutions with a CVaR lower than 9m is completely covered by financial hedging. But could we gain more profit by hedging less jet fuel? Probably yes, because the costs for the hedging premium then would also decrease. But which percentage of the bought jet fuel should then be hedged? And if we first create a flight schedule and then determine the amount of hedged jet fuel or vice versa, do we disregard interactions? These questions lead to the integrated consideration of financial hedging instruments in the next section.

4.4.2.2 Integrated hedging approach

This section shows the results for the additional integration of financial hedging instruments into the optimization model. Now the amount of hedged fuel purchases is determined simultaneously with the other decisions in the model. This adds a degree of freedom and leads to the new pareto-frontline shown in Figure 16.

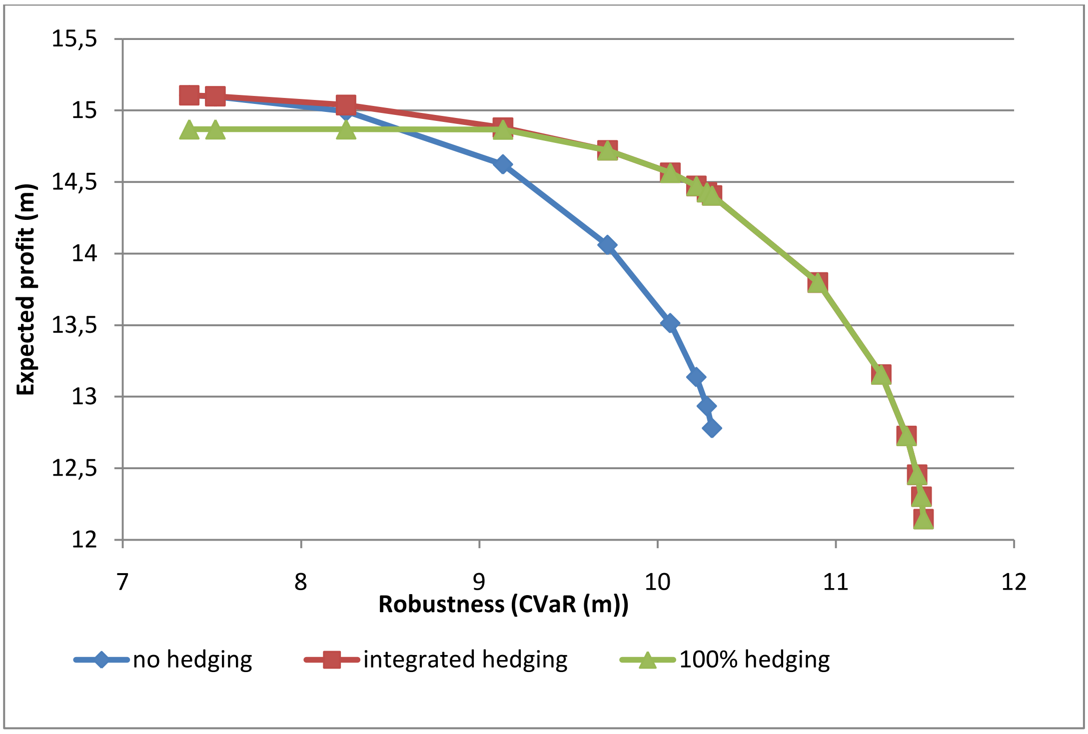

Figure 16 Integrated profit/risk-profile

The first obvious result is that the integrated model has the best solution at all risk levels. This is because it can save the hedging premium in high-risk solutions and is also able to determine the right amount of hedging in robust solutions in a way that not too much hedging (and thereby paying more hedging premium) lowers the expected profit. It therefore determines the best combination of hedging and operational changes simultaneously to gain a certain risk level with the best possible profit.

In the case where the risk is minimized, we calculate the same solution in the integrated model and in the model where 100% of the fuel purchases are hedged; in the case where the profit is maximized with no other risk-constraints, the model without financial instruments and the integrated model create the same solution. In between, when a risk limit is specified, the solutions of the integrated model dominate the non-integrated approaches and always find the global optimum.

As nowadays fuel hedging is usually planned independently from operational planning in airlines' financial departments, and operational planning departments only take the percentage of hedged fuel into account, this can lead to worse solutions than the global optimum.

Furthermore, we look at a more detailed comparison of the different solutions with integrated hedging and without hedging. Table 10 shows the expected profit, the number of flights with the different aircraft types and the amount of fuel hedged of the different solutions.

CVaR limit	Expected_profit (no hedging)	Expected_profit (integrated hedging)	Flights by medium aircraft (no hedging)	Flights by large aircraft (no hedging)	Flights by medium aircraft (integrated hedging)	Flights by large aircraft (integrated hedging)	Percentage of hedged fuel (integrated hedging)
7.37m	15.11m	15.11m	502	26	502	26	0
7.52m	15.10m	15.10m	492	26	494	26	0.3
8.25m	14.99m	15.04m	474	22	490	24	22.1
9.13m	14.62m	14.88m	436	20	488	24	89.0
9.72m	14.06m	14.72m	394	18	458	22	99.8
10.07m	13.51m	14.56m	350	18	450	20	99.6
10.22m	13.14m	14.47m	340	16	442	20	100
10.28m	12.93m	14.43m	334	16	436	20	99.8
10.31m	12.78m	14.41m	330	16	432	20	100
		13.80m			388	18	100
		13.16m			352	16	100
		12.73m			326	16	100
		12.30m			308	16	100

Table 10 Detailed results with and without hedging

We can see that a significant decrease of flights in the solutions with integrated hedging begins at a higher robustness level than in the solutions without hedging. The maximum robustness without hedging (CVaR= 10.31m) can be gained by using hedging and a practically reasonable decrease of flights and profit. Very large changes in the number of flights offered are usually not desired, because usually only a small share of the aircraft used is chartered and only a low number of aircraft is separated out during one flight plan period.

We also spot that fuel hedging sometimes slightly decreases (from 99.8%to 99.6% or from 100% to 99.8%) although the CVaR is limited to higher values. These can be explained by interactions between financial and operational planning that the integrated model can utilize.

Because financial instruments can be integrated with a LP-based formulation, the computational complexity of the integrated model is not significantly increased. Therefore, we recommend using an integrated approach.

4.4.3 Evaluation of the stochastic model

To evaluate the model developed, we calculate two well known ratios for stochastic optimization models: The expected value of perfect information (EVPI) and the value of the stochastic solution (VSS). The EVPI is the difference between the here-and-now solution and the wait-and-see solution of the stochastic model and therefore denotes the price that should be paid at maximum to purchase perfect information about the future. The VSS is the difference between the EEV-Solution and the here-and-now solution. To gain the EEV-solution, all uncertain parameters are set to their expected value, the optimal solution of a deterministic model with these parameters is calculated and evaluated for every scenario. The expected value of these solutions is the EEV. The VSS therefore denotes the advantage of solving a stochastic model instead of a deterministic model.

For this model maximizing the expected profit with both uncertainties and no hedging the EVPI is 1.195m. If hedging is allowed the EVPI grows to 1.773m. This is because hedging can gain additional profit, if the airline knew the future price of the jet fuel and can then hedge it only in the scenarios where the future price is higher than the fair hedge rate. This value is therefore hypothetical.

The VSS is 0.578m in both cases. It does not change because there are no wait-and-see decisions like in the models that calculate the EVPI. This value underlines the benefit of using stochastic optimization models for this application.

4.5 Conclusion

First we used models for every jet fuel price scenario to examine the impacts of different jet fuel prices. We showed that different jet fuel prices have impacts on optimal schedule planning: Higher jet fuel prices make more flights become unprofitable. Passengers will have to accept more aircraft changes when jet fuel prices increase. Also less empty seats are optimal at higher fuel prices. That showed the need for considering jet fuel price uncertainty in optimization models for airline schedule design.

Therefore we developed and used models that consider jet fuel price and demand uncertainty. We examined the robustness of the solutions by creating profit/risk-profiles for a model without hedging, for a model with hedging 100% purchases and for an integrated model. It was shown that the integrated model produces better solutions because it allows interactions between financial instruments and the operational decisions of the basic model and that financial instruments can significantly increase the robustness when risk is restricted.

Thus, we developed a new model for decision support in strategic airline schedule planning under fuel price uncertainty that considers operational and contractual risk management simultaneously.

5 Re-Fleeting under fuel price and demand uncertainty

This chapter introduces a new stochastic programming approach for robust re-fleeting under fuel price and demand uncertainty.[108]

5.1 Introduction

In the last years, the costs for jet fuel have increased, so that they have become the largest part of airlines' expenditures and are nowadays higher than crew costs that previously were the dominating part. Moreover, fuel prices have very high fluctuations, so that it becomes more important for airlines to counteract fuel price uncertainty.

As the fleet assignment allocates the different aircraft types to the airlines' flights, it highly determines the fuel consumption of an airline. Furthermore, the capacity of an aircraft has to fit to the demand. A too large aircraft with empty seats wastes fuel, while a too small aircraft cannot serve the demand, so that possible revenue is not generated.[109] As the fleet assignment has to be done at a time when the final demand is still unknown, we also add demand uncertainty to the model.

We present a two-stage stochastic program for fleet assignment to better cope with fuel price and demand uncertainty. We also integrate financial hedging instruments to enable improvements of the solutions' risk measures. The Conditional Value at Risk, as one of the risk measures that fit very well to the perception of risk, is integrated.

5.2 Problem description

The planning phase on which this chapter focuses on is called fleet assignment. It is usually carried out after the airline schedule design phase. An aircraft type for each offered flight is determined. Also, flow conservation and the number of available

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[108] Main parts of this chapter have been published in [NSF12]

[109] See [GoTa98]

aircraft have to be considered. Flight costs and expected revenue are important data for this planning phase.

The solution of the fleet assignment is used as a basis for the aircraft maintenance routing. In this step the flow of every single aircraft through the flight network is planned, so that every flight is covered and maintenance constraints are met before the crew scheduling assigns the crews to the flights.[110]

The fleet assignment is planned several months before the flight, since the crew scheduling depends on the fleet assignment and has to be planned 8-10 weeks prior to departure because of union regulations. But crews are usually able to fly several aircraft types within one family. Therefore changes within the type-family closer to the flight date without affecting the crew schedule are possible, and thus only the type-family for each flight has to be fixed before the crew scheduling is planned.[111]

This flexibility can be used to counteract uncertainty: More precise information for fuel prices and demand can be utilized, so that for the final type assignment a specific reaction in each scenario is possible. This approach, that first determines the type-family and later the specific type depending on more precise information of the uncertain parameters, is called re-fleeting.

Note that when re-fleeting is used, the usual airline schedule planning process, as described in chapter 2.1, has to be modified: Instead of the whole fleet assignment only the assignment of the type-family is done after the schedule design. The crew scheduling is planned directly after that. The final type assignment is done as late as possible to have more precise information on uncertain parameters. Finally, the aircraft routing is planned. Figure 17 shows the changes in airline schedule planning when re-fleeting is used.

[110] See [GoTa98] and [BaCo04]

[111] See [Zhu06] p. 62

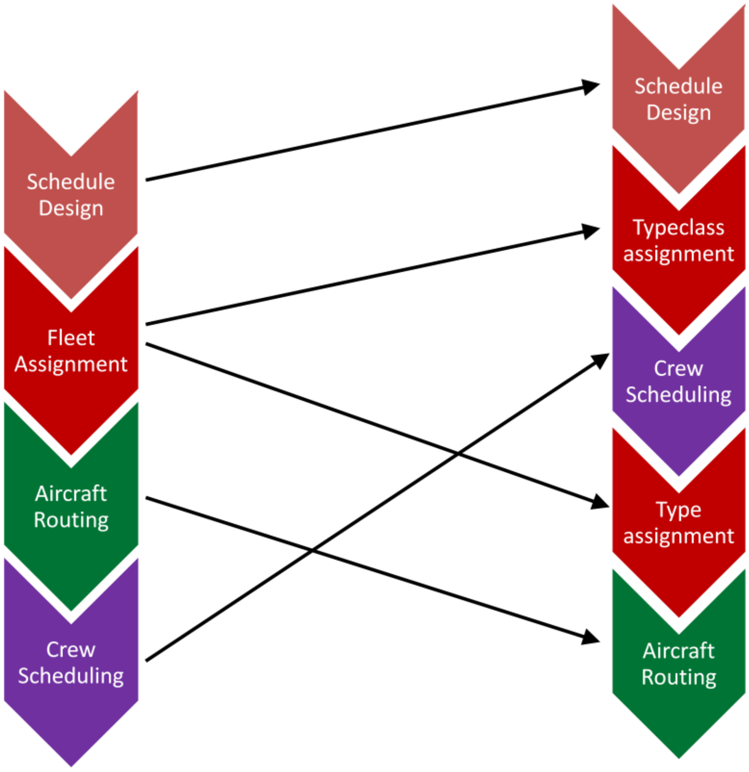

Figure 17 Process changes because of re-fleeting

Because of the advantages of an integrated risk management strategy[112], we again integrate financial hedging instruments into the optimization model to hedge against jet fuel price uncertainty. This enables interactions of financial hedging and operational flexibility. To the best knowledge of the author, there is no such model for re-fleeting that integrates financial hedging instruments and considers fuel price and demand uncertainty.

5.3 Model

This section describes our proposed two-stage stochastic program for re-fleeting. We first describe the underlying network structure and then show the mathematical formulation.

[112] See [Tri05]

5.3.1 Model description

Like Zhu[113] we use a time-space network as underlying network structure for our model. Figure 18 shows an exemplary network for one aircraft type.

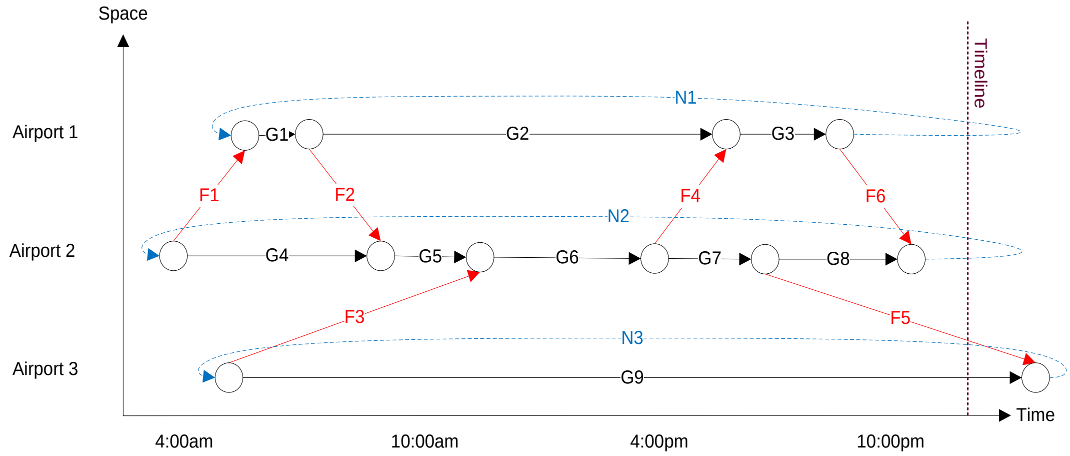

Figure 18 Time-space network for re-fleeting

The arcs (F1, …, F6) are flight arcs, the arcs (G1, …, G9) are the ground arcs and the dashed arcs (N1, N2 and N3) are the overnight arcs, which are a subtype of the ground arcs.

Such a network layer is constructed for every aircraft type. During this process we already consider the compatibility of aircraft type and airport as well as the maximum range of the specific types, so that incompatible flight arcs can be left out and layers therefore may look very different for domestic and long-haul aircraft types, for example.

For each flight, a flight arc is added to all compatible network layers. Every flight arc has one starting node and one ending node. Every node is then connected via a ground arc with the next node at the same airport. The last node in the planning horizon of each airport is connected with the first node of the airport via an overnight arc.

Finally, all arcs that pass forward in time through a certain timeline are marked. The sum of the flows of the marked arcs denotes the number of used aircraft of the corresponding aircraft type. This is done to consider the maximum available aircraft of the specific type later in the optimization model. In the example network layer above,

[113] [Zhu06] p. 61ff

these arcs are N1, N2, F5 and G9. N3 is not marked, because F5 and G9 already pass the timeline and N3 starts later.

For our calculations, we use the flight network and the fleet data of a large European airline. The fuel price and demand scenarios are considered in a scenarioset and combined with each other, so that the total number of scenarios is the number of fuel price scenarios multiplied by the number of demand scenarios. The rates for the financial hedging instruments considered in this model are adjusted to the jet fuel price scenarios, so that a riskless arbitrage strategy is impossible. Reverse hedging is not allowed and we furthermore integrate a margin for the financial instruments to consider costs like transaction costs or margins for contracting parties.

5.3.2 A two-stage stochastic re-fleeting model

Our proposed mathematical optimization model is a two-stage stochastic program and is formulated in the following way:

Sets:

K	Set of typeclasses
T_k	Set of types in class k
T	Set of all types
F	Set of flights
S	Scenarioset
P	Set of paths
$P(f)$	Set of paths that use flight f
Tl_t	Set of arcs passing forward in time through a counting time-line in network layer t
G_t	Set of ground arcs in network layer t
N_t	Set of nodes in network layer t

Parameters:

c_{ft}	Costs for flight f with type t, without jet fuel costs
pr_p	Selling price for path p
$prob_s$	Probability for scenario s

ub_t	Maximum available aircraft of type t
m_t	Capacity of aircraft type t
d_{ps}	Demand on path p in scenario s
bf_{fn}	+1 if flight f ends at node n, -1 if it begins there
bg_{gn}	+1 if ground arc g ends at node n, -1 if it begins there
α	Probability value for the CVaR
fp_s	Fuel price in scenario s
fph	Hedged fuel price
$cons_t$	Jet fuel consumption of aircraft type t in liters per km
$dist_f$	Distance of flight f in km
mar	Margin for hedging in percent

Variables:

Stage 1:

y_{fk}	Flight f is flown by typeclass k ($\epsilon\{0; 1\}$)
buy_h	Bought fuel in liters to the hedged fuel price (≥ 0)

Stage 2:

x_{fts}	Flight f is flown by type t in scenario s ($\epsilon\{0; 1\}$)
xg_{gs}	Flow on ground arc g in scenario s ($\geq 0, \in \mathbb{Z}$)
$pass_{ps}$	Passenger flow on path p in scenario s (≥ 0)
buy_s_s	Bought fuel to the non-hedged fuel price in scenario s

Bookkeeping variables:

$profit_s$	Profit in scenario s
$cvar$	Conditional Value at Risk
$cy0$	Auxiliary variable for the dual CVaR-formulation
cy_s	Auxiliary variables for the dual CVaR-formulation
$fuel_s$	Fuel costs in scenario s

Objective:

$$Max\colon \sum_{s \epsilon S} prob_s \cdot profit_s \;/\; Max\colon cvar \qquad (20.1)$$

Constraints:

$$\sum_{k \in K} y_{fk} = 1 \qquad \forall f \in F \qquad (20.2)$$

$$\sum_{t \in T_k} x_{fts} = y_{fk} \qquad \forall f \in F, k \in K, s \in S \qquad (20.3)$$

$$\sum_{f \in F} bf_{fn} \cdot x_{fts} + \sum_{g \in G_t} bg_{gn} \cdot xg_{gs} = 0 \qquad \forall n \in N_t, t \in T, s \in S \qquad (20.4)$$

$$\sum_{f \in Tl_t} x_{fts} + \sum_{g \in Tl_t} xg_{gs} \leq ub_t \qquad \forall t \in T, s \in S \qquad (20.5)$$

$$\sum_{p \in P(f)} pass_{ps} \leq \sum_{t \in T} m_t \cdot x_{fts} \qquad \forall f \in F, s \in S \qquad (20.6)$$

$$pass_{ps} \leq d_{ps} \qquad \forall p \in P, s \in S \qquad (20.7)$$

$$buy\_s_s + buy\_h \geq \sum_{f \in F} \sum_{t \in T} x_{fts} \cdot dist_f \cdot cons_t \qquad \forall s \epsilon S \qquad (20.8)$$

$$fuel_s = buy\_s_s \cdot fp_s + buy\_h \cdot fph \cdot \left(1 + \frac{mar}{100}\right) \qquad \forall s \epsilon S \qquad (20.9)$$

$$profit_s = \sum_{p \in P} pr_p \cdot pass_{ps} - \sum_{k \in K} \sum_{t \in T_k} \sum_{f \in F} c_{ft} \cdot x_{fts} - fuel_s \qquad \forall s \in S \qquad (20.10)$$

$$cvar = \frac{-1}{\alpha}\left(\alpha \cdot cy0 + \sum_{s \in S} cy_s \cdot prob_s\right) \qquad (20.11)$$

$$cy0 + cy_s \geq -profit_s \qquad \forall s \epsilon S \qquad (20.12)$$

The objective function can maximize the expected profit or the Conditional Value at Risk. Constraint (20.2) assigns the typeclass to each flight in stage 1. The second constraint assigns the type to the flights in stage 2 regarding the typeclass assigned in stage 1. The equation (20.4) ensures the flow-conservation at each node in the network while the inequality (20.5) implements the maximum number of aircraft of each type. The maximum aircraft capacity and the transported passengers are considered in (20.6) and (20.7). The fuel purchases and the fuel costs are modeled by the constraints (20.8) and (20.9), while (20.10) calculates the profit for the scenarios. Finally, the constraints (20.11) and (20.12) implement the Conditional Value at Risk.

5.4 Results

5.4.1 Implications of fuel prices on fleet assignment – a study with deterministic optimization

In this section, we study the impacts of different fuel prices on fleet assignment. For every fuel price scenario, we add different demand scenarios. Every scenario is optimized independently in its own optimization model. The proposed stochastic refleeting model then becomes a deterministic fleet assignment model because only one scenario is considered.

We study four fuel price scenarios and four demand scenarios resulting in a total number of 16 optimization models. The models 1-4, 5-8, 9-12 and 13-16 have the same fuel price, but other demand scenarios. The models 1-4 have the lowest fuel price and 13-16 the highest. As solution we have a plan where every flight has been assigned exactly one aircraft type.

The considered aircraft types are shown in Table 11.

Medium-haul types	Seats	Range	Long-haul types	Seats	Range
Boeing 737-500	111	2520	Airbus A330	221	10000
Boeing 737-300	127	2590	Airbus A340-300	221	12700
Airbus A319	132	3470	Airbus A340-600	306	12200
Airbus A320	156	3470	Boeing 747	352	12500
Airbus A321	190	4100	Airbus A380	526	12000

Table 11 Aircraft types

The fuel efficiency, measured as consumption per seat-kilometer, for the medium-haul types increases with their seat capacity. The order from minimum to maximum efficiency therefore is: Boeing 737-500, Boeing 737-300, Airbus A319, Airbus A320, and Airbus A321. The long-haul type efficiency ordered from minimum to maximum is: Airbus A340-300, Boeing 747, Airbus A330, Airbus A340-600, and Airbus A380. Thus, the Boeing 747 as a large aircraft is relatively inefficient. Also the A340-300 consumes more fuel than the A330, which has the same seat capacity.

First, we show the number of flights that are carried out by the different medium-haul aircraft types depending on the scenario.

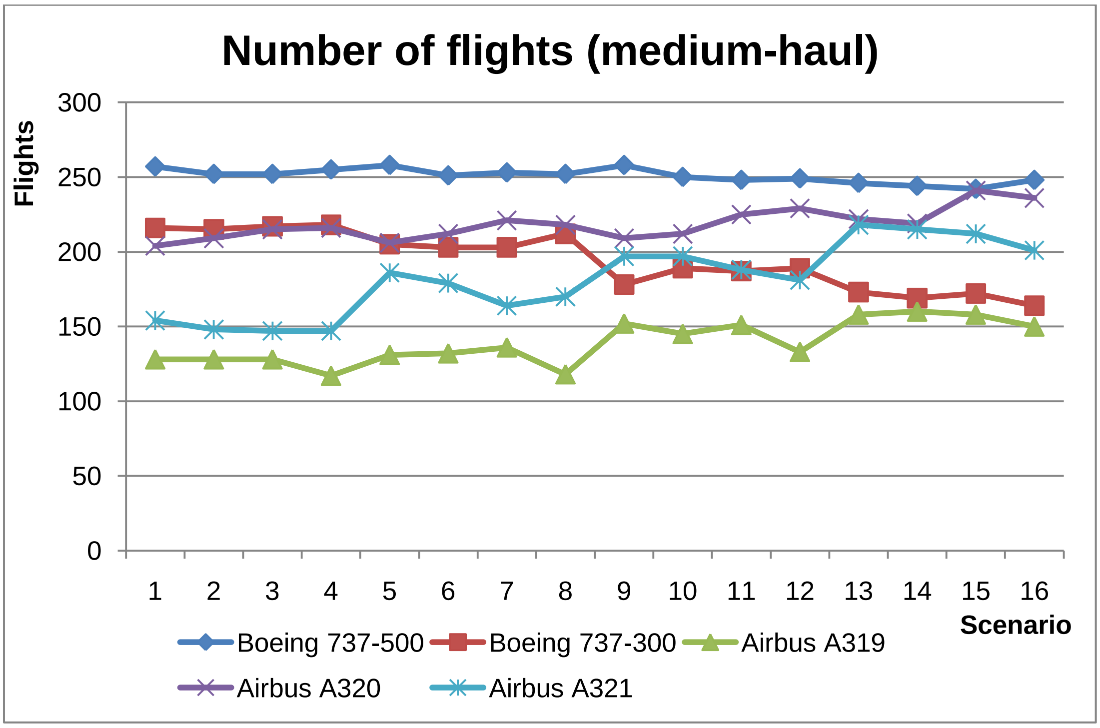

Figure 19 Flights with medium-haul types

It can be seen that the smaller less efficient Boeing types (B737-300 and B737-500) carry out less flights in higher fuel price scenarios, while the more efficient larger types fly on more flights. Especially the number of flights with the most efficient type, the Airbus A321, significantly increases in higher fuel price scenarios.

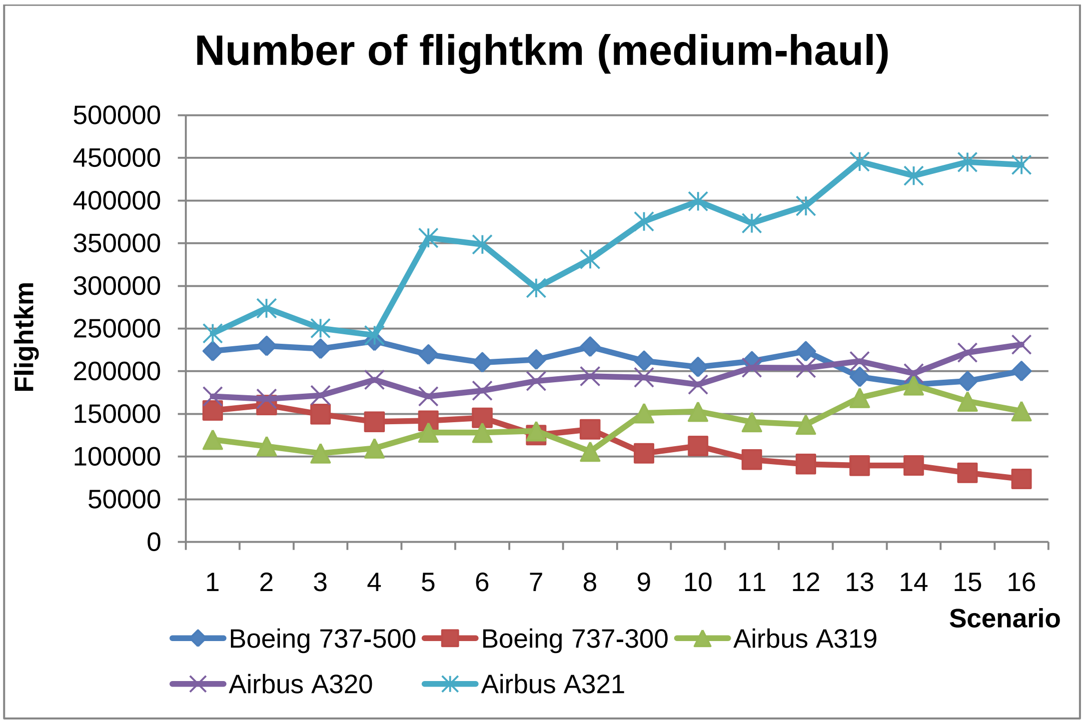

Figure 20 Flightkilometers with medium-haul types

When we analyze the flightkilometers[114] carried out by each type the results are intensified. Thus, more efficient larger types, especially the A321, do not only carry out more flights, they are also used for larger distances. This further saves fuel. The A319, as the median in efficiency and seat capacity, can probably increase flightkilometers because it can replace the less efficient B737-300, which has a very similar seat capacity, very well when fuel prices increase.

[114] total kilometers flown with a type

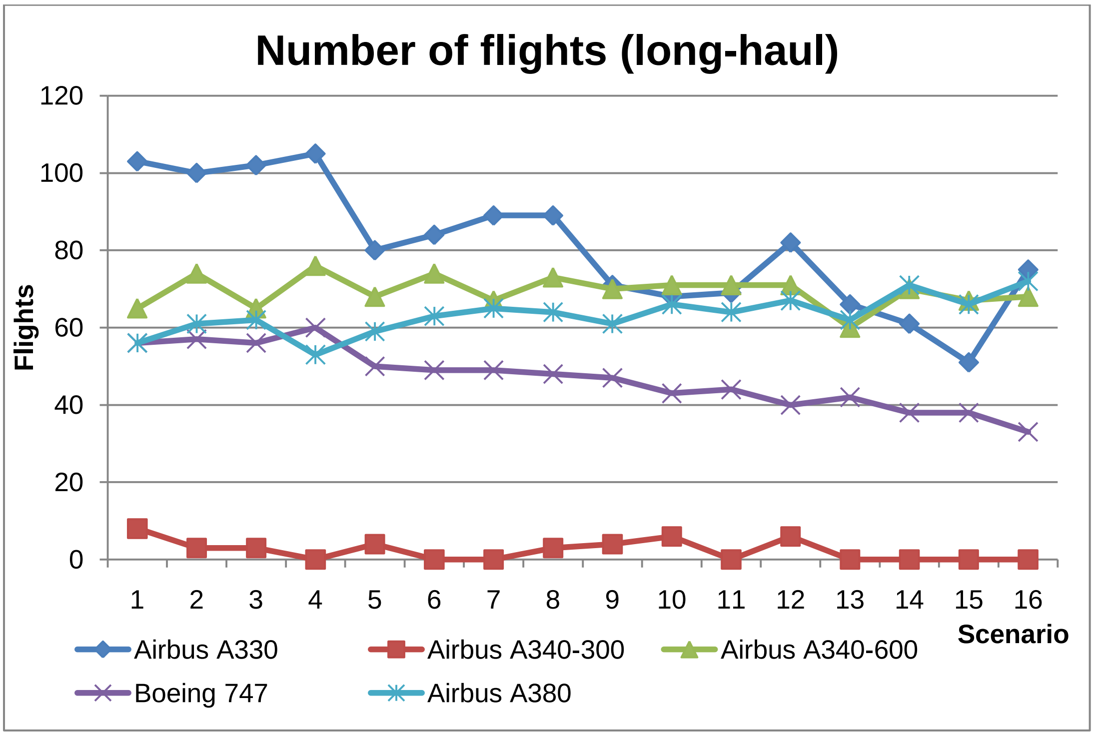

Figure 21 Flights with long-haul types

By examining the number of flights of the different long-haul types, it can be seen that the less efficient A340-300 is used on very few flights. In the four scenarios with the highest fuel price it does not even carry out one single flight. It can be perfectly replaced by the more efficient A330 with the same seat capacity. Also the less efficient Boeing 747 is used for fewer flights in higher fuel price scenarios. The A380, as a very efficient type, is used on more flights, and the usage of the A340-600 does not change in higher fuel price scenarios. But why does the A330, the median of the long-haul types, carry out fewer flights in higher fuel price scenarios, although it is a very good substitute for the less efficient A340-300? This will be explained later.

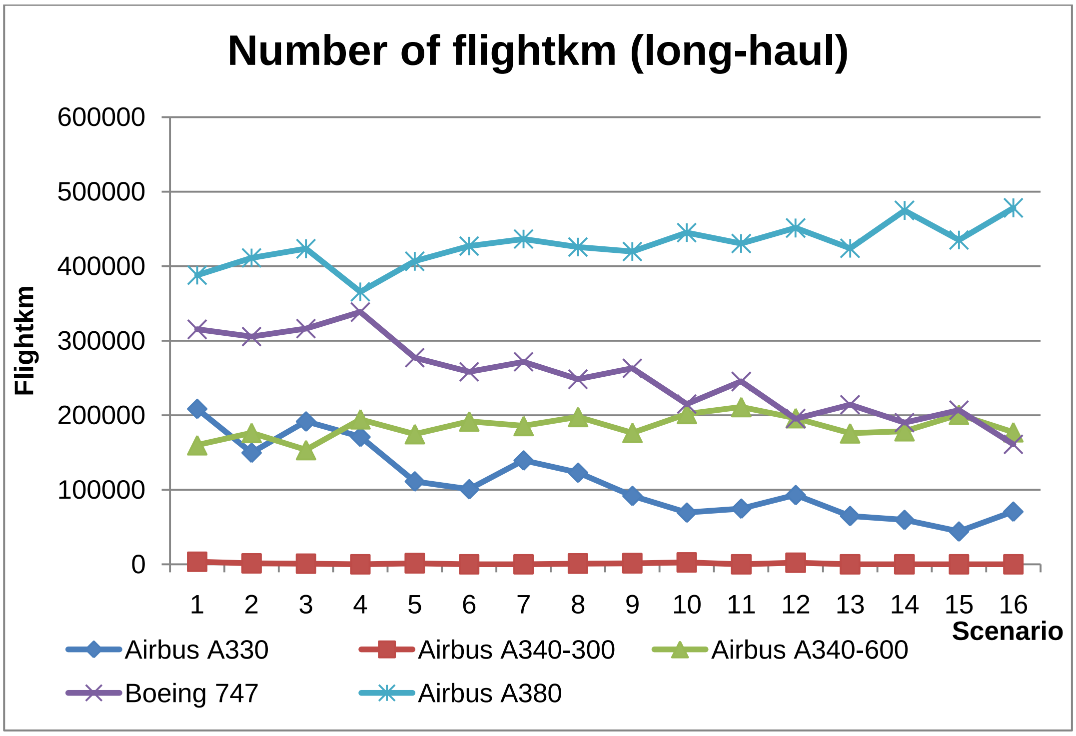

Figure 22 Flightkilometers with long-haul types

The detailed view of the flightkilometers of the different long-haul types confirms the preliminary results for the B747. It also shows that the A340-300 flies very few kilometers.

Furthermore, it is remarkable that the A380 dominates the other types, although it is used on fewer flights than other types (especially in the scenarios with lower fuel prices). Because of the high fuel efficiency of the A380, it is used for more flights in scenarios with high fuel prices and for large distances, such as the A321 in the results for the medium-haul types. The same tendency can be seen for the A340-600: The flightkilometers slightly increase with increasing fuel prices.

While the A330 carries out the highest number of flights in most scenarios, it flies only very few kilometers. Thus, it must be used for shorter distances. We now show the average distance flown with each type depending on the scenario.

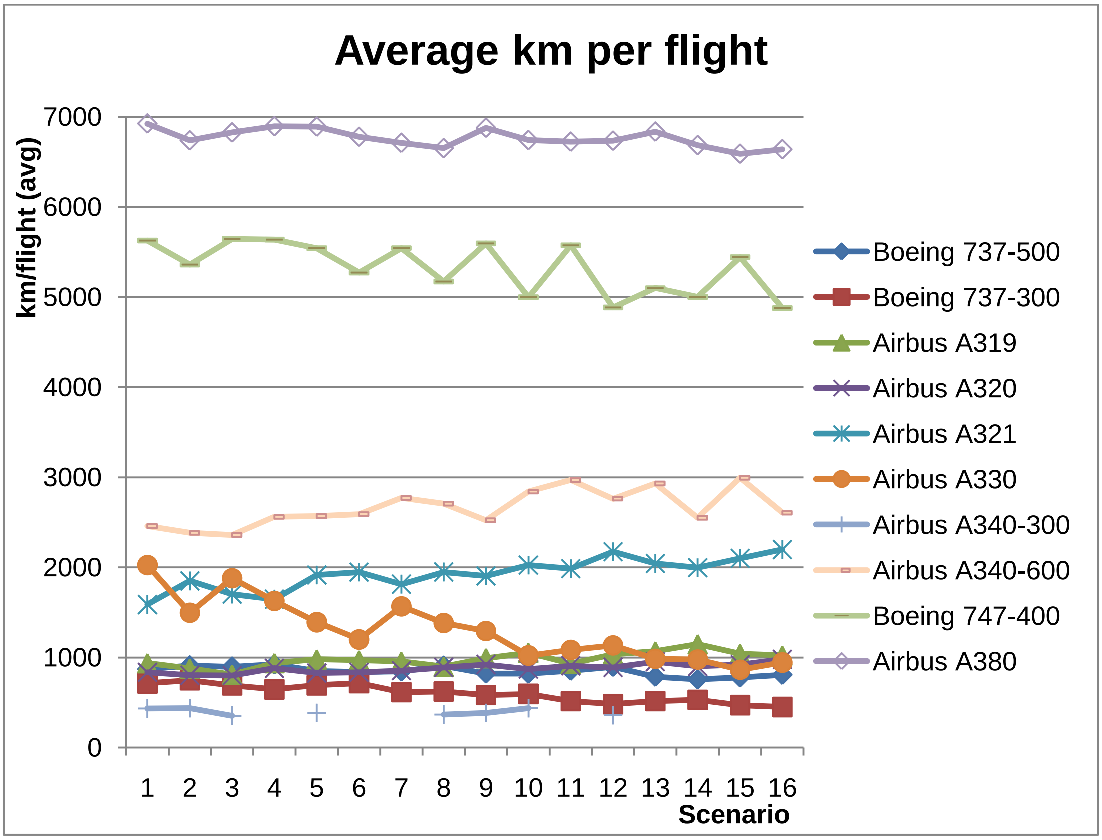

Figure 23 Average kilometer per flight

To complete the basic study, we can now explain the significant decrease in flights and flightkilometers of the A330. In higher fuel price scenarios, it seems to be replaced by the A321, which is more efficient. The A321 - compared to other medium-haul types - has a very high average distance, especially in higher fuel price scenarios, while the A330 has a low average distance for a long-haul type decreasing with growing fuel prices. In general, the more efficient types are used to fly larger distances in higher fuel price scenarios. We also notice a very small decrease in load factors in the four highest fuel price scenarios (0%-0.5%). This indicates that for profit-maximization fuel efficiency becomes more important than load factor maximization in the highest fuel price scenarios.

The result that the optimal fleet assignment highly depends on the fuel price and the demand scenario again shows the importance of our proposed re-fleeting model. Besides uncertain demands, fuel prices can change quickly while the crew assignment has to be fixed several weeks before the flight date. A re-fleeting model that enables short-term crew-independent changes of aircraft types can counteract these uncertainties more dynamically.

5.4.2 Stochastic optimization and risk measures

We now compare our proposed stochastic optimization model with deterministic optimization. We calculate the expectation of the expected value problem (EEV-solution). Therefore, we first optimize the expected value problem (EV-solution) which is a model where all stochastic parameters are set to their expected values. This solution is evaluated for every scenario. The weighted average of these objective functions denotes the EEV solution. This value is compared to the solution of the stochastic programming model in terms of expected profit and risk (measured with the risk measure CVaR). With stochastic optimization, we are able to create a pareto-optimal solution set in terms of expected profit and CVaR. Fuel hedging will be considered later. The different stochastic programming solutions are calculated by restricting the CVaR and optimizing expected profit. Note that we only use a part of the network for further calculations because of the increased computational complexity and the large number of optimization runs that have to be completed.

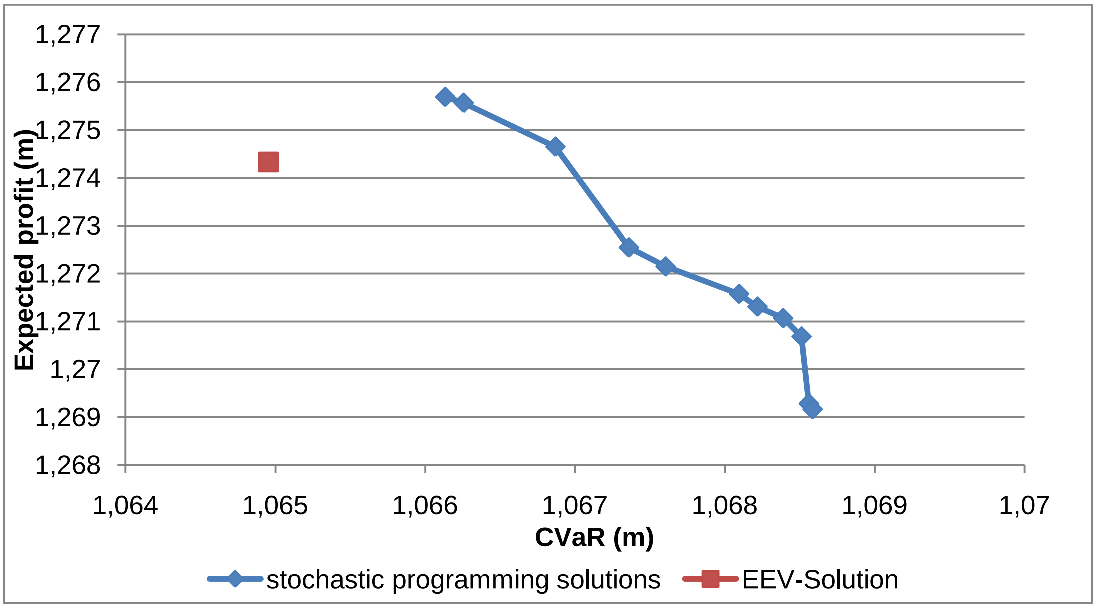

Figure 24 Comparison of stochastic and deterministic solutions

Because the stochastic optimization model has more flexibility – it can change aircraft types within their typeclass depending on fuel price and demand development – the stochastic programming solutions dominate the EEV-solution. With stochastic optimization, we are also able to restrict the CVaR to specific risk levels while this is not possible in deterministic optimization models.

5.4.3 Fuel hedging

To further decrease risk, we now compare the pareto-optimal set of stochastic solutions shown in Figure 24 with other solution sets where the fuel purchases are partly and completely hedged.

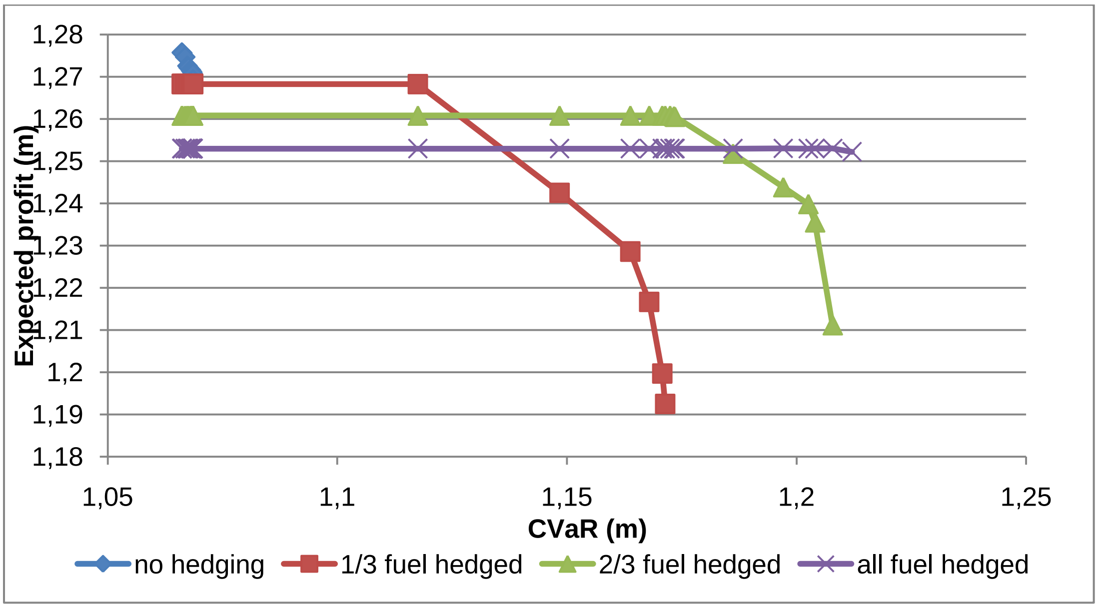

Figure 25 Solutions sets for different levels of fuel hedging

It can be seen that hedging fuel purchases can improve the CVaR very much. But on the other hand the maximum expected profit is decreased because of the hedging premium.

Up to now, the percentage of hedged fuel is usually planned by airlines' financial department, which decides independently from operational planning. When operational planning wants to restrict the risk measure to a certain level, it takes the fuel hedging into account and creates the fleet assignment depending on this data, which leads to the solutions presented in Figure 25.

But the different lines that represent the solution sets show that for different risk levels different levels of fuel hedging are better. But what is the right combination of hedging and changing the fleet assignment to obtain a certain risk level with the maximum expected profit?

To obtain the optimal combination, an integrated approach is necessary. Figure 26 shows the solutions of the integrated hedging decision compared to the preliminary results.

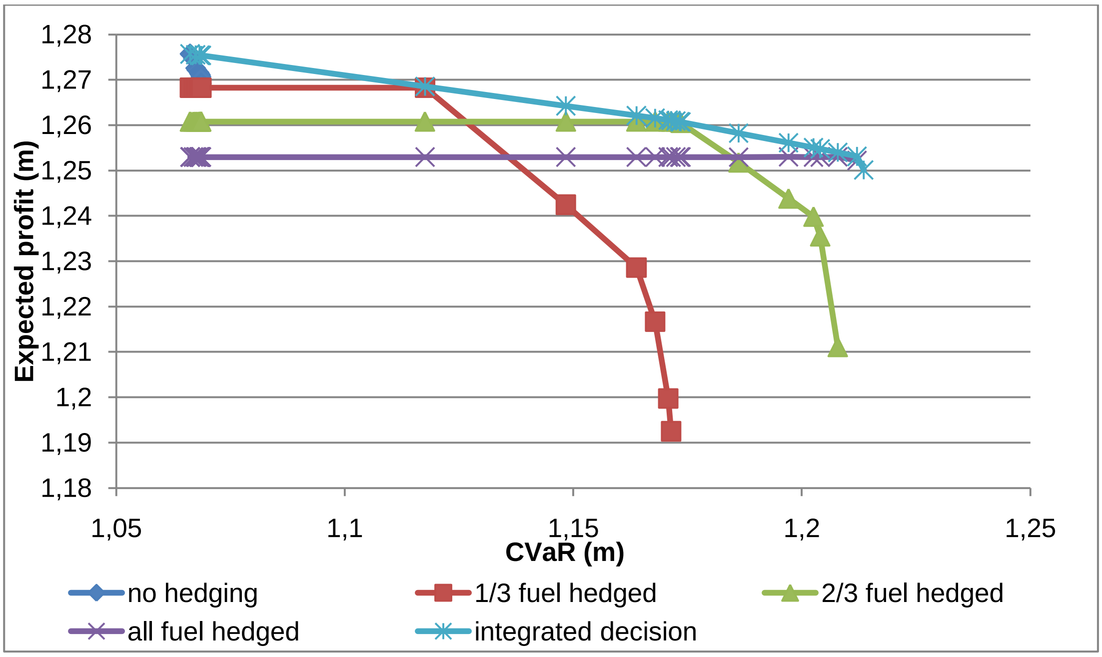

Figure 26 Integrated hedging decision

The result is that the solution set of the integrated approach dominates all the previous solution sets. The numerical results indicate that the pareto-optimal solution sets of the non-integrated hedging strategies are nearly tangent to the solution set, where the hedging decision is integrated. This shows that the right combination of hedging and operational planning has to be met if planning is not integrated.

But as the integration of financial hedging is possible with a LP-formulation and, therefore, the computational complexity is not notably increased, we suggest using this integrated approach for financial and operational risk management in re-fleeting.

5.5 Conclusion

A new two-stage stochastic optimization model for re-fleeting under demand and fuel price uncertainty that considers financial hedging was presented. We began with a study of the impacts of different fuel price scenarios on fleet assignment. The study reveals that higher fuel prices cause significant changes in optimal fleet assignment and therefore underlines the need for our proposed model.

Furthermore, the interdependency of risk management with financial hedging and fleet assignment was examined. It was shown that the usual non-integrated approach of hedging and fleet assignment can produce only one combination of risk and expected profit that can compete with the integrated planning approach. As the integration does not lead to an increased computational complexity, financial hedging and fleet assignment should be integrated to gain the optimal expected profit at different risk levels.

6 Robust vehicle scheduling in public bus transport

This chapter introduces a new stochastic programming approach for robust vehicle scheduling in public bus transport.[115]

6.1 Introduction

The vehicle schedules in public bus transport are traditionally planned several weeks before their execution. The buses are assigned to the given timetabled trips, so that every trip is covered by one bus. Thereby the trip has to be carried out by an allowed vehicle type for this trip. Furthermore, the vehicles have to start and end at their depot at the beginning and at the end of the planning horizon. The objective is usually to minimize costs. The costs are fixed cost per vehicle as well as variable cost per driven distance and time spent outside the depot. We call all these costs planned costs.

On the day of operations, the real driving times might vary. Disruptions cause delays and increase the operational costs. Moreover, they cause contractual penalty fees.

If there is waiting time between two service trips, delays can be absorbed. But as more buffer time in schedules causes more planned costs, cost-optimal schedules tend to contain few buffers. Thus, they are especially sensitive to disruptions. Therefore possible disruptions should be considered during the planning process to minimize the sum of planned costs and expected disruption costs. Figure 27 exemplarily illustrates the characteristics of different vehicle schedules. [116]

[115] The results of this chapter have partly been published in [NSK11]

[116] See [KKM09]

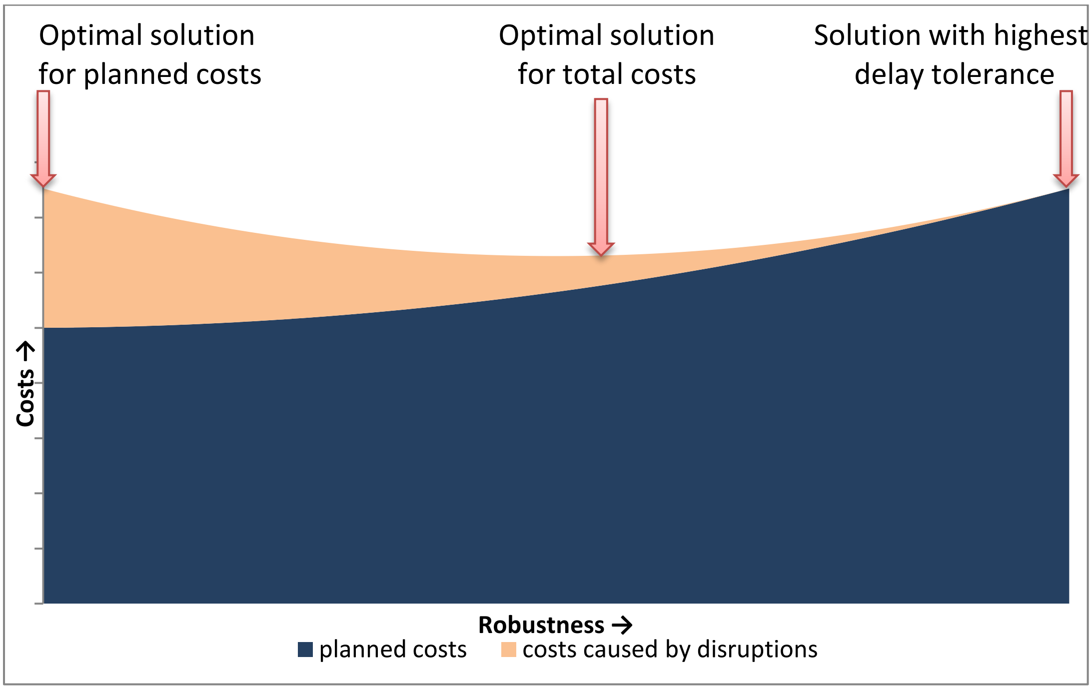

Figure 27 Planned costs and disruption costs

The solution shown on the left is cost-optimal for planned costs, but – as there are few buffers – disruptions cause high costs. On the other hand the solution at the right contains lots of buffers – every delay can be absorbed – but it is very expensive in planned costs. The solution in the middle has the lowest sum of planned and expected disruption costs.

6.2 Delay tolerance and robustness

Before describing how we aim to increase the delay tolerance and the robustness of vehicle schedules, we define some terms that will be used later.

A *primary delay* is a delay that is directly caused by a disruption, for example if a road is blocked, if there is a traffic jam, because of snow, etc. As disruptions occur, primary delays cannot be avoided. Primary delays cause a late arrival of a service trip that has started punctually.

If a delayed service trip causes a delayed start of a following service trip, we call this delay a *secondary delay*. Secondary delays occur because of dependencies of consecutive service trips. They can be prevented by inserting buffer time.

Most public transport companies, especially smaller companies, do not have an operations control center so that they cannot dynamically react on disruptions with re-planning. Therefore, if there is not enough buffer time to absorb the delayed arrival of a service trip, the subsequent service trip will start later. This effect is called *delay propagation*.

We call a vehicle schedule more *delay tolerant*, if it is able to absorb secondary delays better than a reference schedule. A schedule that has more buffer time is usually more expensive, because of additional operational costs (e.g. for waiting time outside the depot), but causes less penalty costs. Penalty costs are incurred for every secondary delay, depending on the length of the delay.

Our aim is to optimize the expected sum of planned costs plus additional operational costs because of disruptions plus penalty costs. Thus, we aim to create a schedule that has the lowest expected costs considering all delay scenarios.

We call a schedule more *robust*, if the total costs in the particular scenarios have a better risk measure. Precisely, we use the Conditional Value at Risk as a measure of robustness of the vehicle schedule.

6.3 Network Models

6.3.1 Delays in network models

Dessouky et al.[117] present a summary of distribution functions used in former studies. They define lateness as a deviation from scheduled arrival time, which fits our definition of a delay. The distribution function used for lateness in former studies was the exponential distribution. Because real delay scenarios for our timetable were not available, we also use the exponential distribution and extend it with a factor depending on the daytime of the service trip to consider the impacts of rush hours.

We use this modified distribution function to generate several delay scenarios. The scenarios are generated in a way that there are some scenarios with a low probability for a delay and a low delay length and some with a high probability and a high delay

[117] [DHNM99]

length and in between many other scenarios. Scenarios with a low delay length and a high probability for delays and scenarios with a low probability for a delay but a high length are not generated. Thus, we have scenarios with correlating delay length and delay probability and therefore some good and some bad scenarios and many in between. The reason for generating scenarios in this way is that we want to cover very bad days like days in winter with bad weather conditions or days with road closures downtown and on the other hand days where only few disruptions occur. Figure 28 exemplarily shows how the delay scenarios are generated.

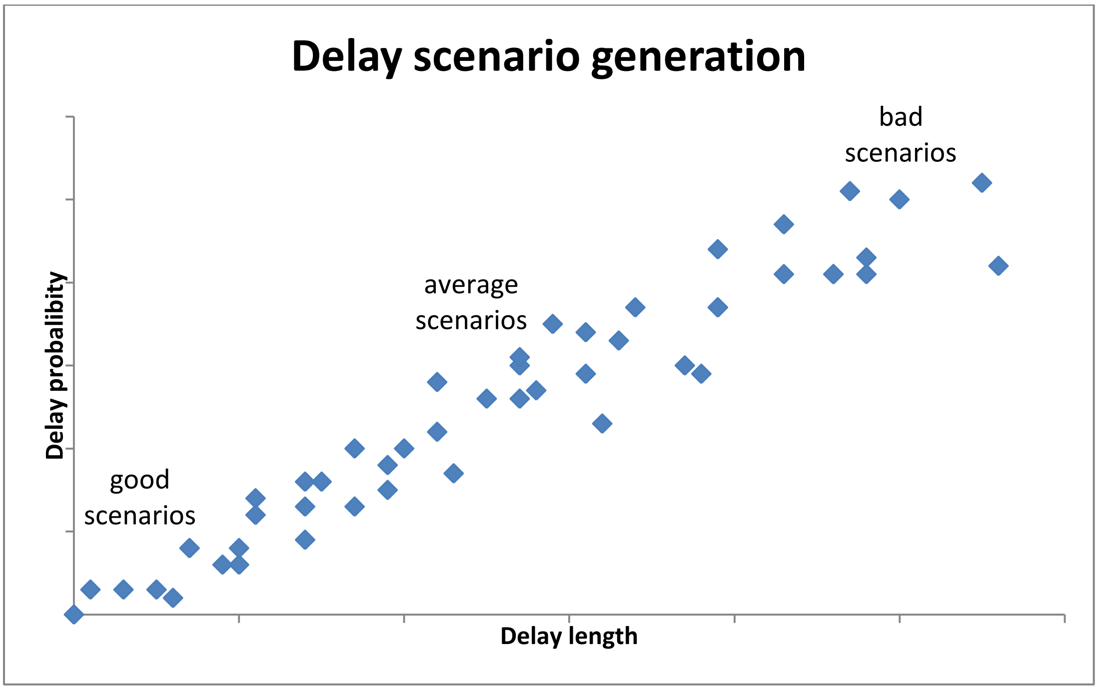

Figure 28 Generation of delay scenarios

Although we tried to generate the scenarios in a realistic way, the best method would be using real delay data of past days. This would regard the characteristics of the particular road network and also would be the most convenient way in practice.

6.3.2 Problems of modeling delays in a TSN

In our definition, a service trip is primarily delayed, if it has started on time and arrives not punctually at its ending station. We consider penalty cost, if a primarily delayed service trip causes a non-punctual start of a following service trip. To implement this, we add penalty costs to the connection of the delayed service trip with its following service trips, if the buffer time is too small to absorb the delay. This modeling leads to problems using a standard time-space network.

For modeling we use a time-space network (TSN) like in Kliewer et al.[118], but we have to use more deadhead and waiting arcs, because of the penalty costs. Let us consider the part of a time-space network at a certain bus station shown in Figure 29.

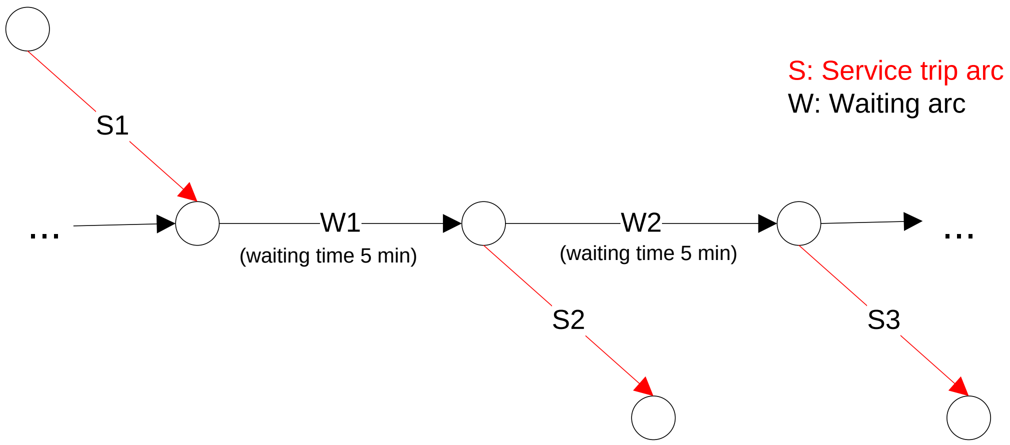

Figure 29 Penalty costs in a TSN 1

The service trip S1 ends at the bus station and the service trips S2 and S3 start 5 minutes and 10 minutes later at the same bus station. Now we assume that service trip S1 is delayed and arrives at the bus station with a delay of 8 minutes. As a consequence, S2 cannot start punctually when the same bus is used for it, because there is only a planned waiting time of 5 minutes. Therefore penalty costs are added to the waiting arc W1.

A more delay tolerant plan could now decide to use the same bus for S1 and S3 and use another bus for S2 to avoid the unpunctual start of S2. S3 could start punctually because of the buffer time of 10 minutes.

The problem of a TSN is now that if the same bus is used for S3 and S1, the arcs W1 and W2 would be used to connect these service trips. Using W1 and W2 would absorb the delay of 8 minutes, but the penalty costs added to W1 are still added to the

[118] See [KMS06] for a description of time-space networks in public transport

solution's costs. Therefore we need an additional waiting arc connecting the end-node of S1 with the start-node of S3:

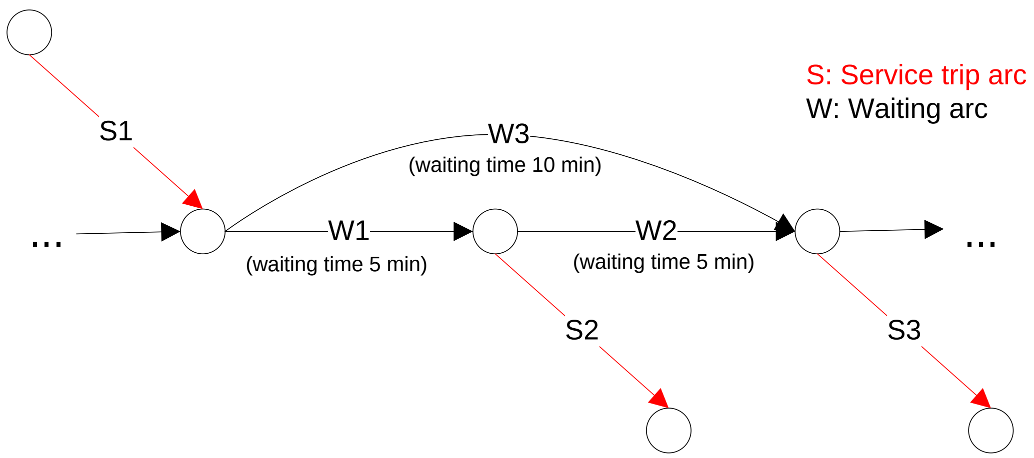

Figure 30 Penalty costs in a TSN 2

For the planned costs of the waiting arcs, the equality

$$planned\ cost(W1) + planned\ cost(W2) = planned\ cost(W3)$$

is satisfied, but

$$planned\ cost(W1) + penalty\ (W1) + planned\ cost(W2) + penalty(W2) \\ \neq planned\ cost(W3) + penalty(W3)$$

is not satisfied. Because of this non-additivity of the penalty costs, the transitivity of waiting arcs in a TSN cannot be utilized anymore. Therefore, we have to use a TSN with all connecting arcs to consider penalty costs in our model. This resulting network model also fits the definition of a connection-based network (CBN), but, in contrast to many implementations of CBNs, nodes still denote only one point in time and service trips are still modeled as arcs.

6.3.3 Implementation of delays in a network with all connecting arcs

This section describes the integration of delays in such a network. As argued before, we want to penalize the arcs that connect two service trips, if they do not have enough buffer time to absorb the delay. There are five possible types of arcs between two service trip arcs:

- A waiting arc connects them, if the service trips end and start at the same bus station at different times. (W1 in Figure 31)
- If the two service trips end and start at the same time and station, a waiting arc with the time 0 is utilized to connect them. (W2)
- An additional waiting arc is used to model a deadhead to the depot, waiting time in the depot, and a deadhead back to the bus station, if there is enough time to do this. (DH1)
- A deadhead arc is used, if the two service trips end and start at two different bus stations and there is enough time for the connection. (DH2)
- An additional deadhead arc is used again to model a deadhead to the depot, waiting time in the depot and a deadhead to the bus station where the next service trip starts, if there is enough time to do this. (DH3)

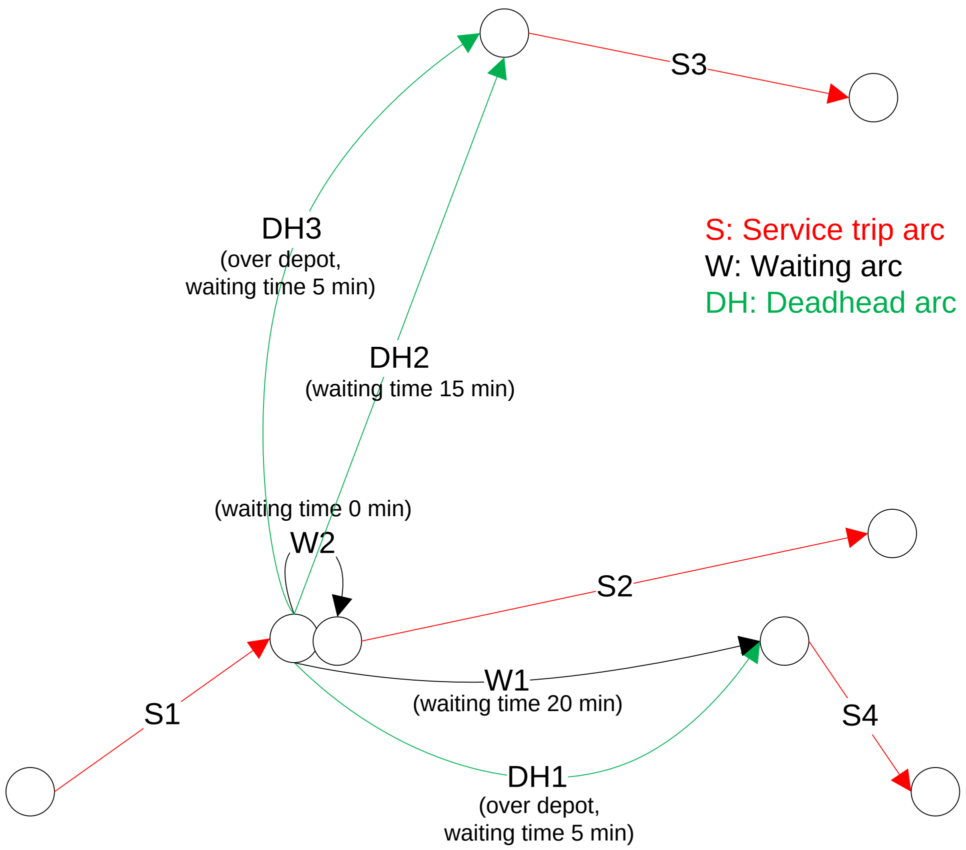

Figure 31 Penalty costs in a network with all connecting arcs

Now, it is possible to penalize every connection independently. For example, if S1 arrives with a delay of 8 minutes, the arcs W2, DH1 and DH3 would be penalized, because there is not enough buffer time to absorb the delay. If planning did not consider delays, DH1 instead of W1 and DH3 instead of DH2 could be chosen because of lower costs. But including delays and penalty costs could lead to a solution that uses W1 instead of DH1 and DH2 instead of DH3 because the larger waiting times of W1 and DH2 can better absorb delays and therefore cause less penalty costs. Therefore we cannot exclude any arc because no arc can be dominated by another arc.

Like Huisman et al.[119], we consider delays on service trips; delays on depot-trips and deadheads are not considered.

6.4 Mathematical Optimization Model

This chapter now shows the proposed mathematical optimization model for the robust vehicle scheduling problem. Therefore a basic formulation of a vehicle scheduling problem is taken and adjusted, so that stochastic delays and penalty costs are considered. The resulting model is a stochastic optimization model. The delays are represented in a scenarioset.

6.4.1 Basic optimization model

We use the following deterministic model formulation for the vehicle scheduling problem as a basis for our further development. For an overview on vehicle scheduling problems see Bunte and Kliewer[120].

Sets:

NL	Set of network layers
F	Set of service trips
ES_f	Set of all service trip arcs representing service trip f (one arc is generated for every compartible combination of depot and vehicle type)
V_{nl}	Set of nodes in network layer nl
E_{nl}	Set of arcs in network layer nl

[119] [HFW04]

[120] [BuKl09]

Parameters:

c_e	Cost of arc e
va_e	Beginning-node of arc e
ve_e	Ending-node of arc e
l_e	Lower bound of arc e (usually 0)
u_e	Upper bound of arc e (usually 1 for service trip arcs)

Variables:

x_e	Flow of arc e

Objective function:

$$\min \sum_{nl \in NL} \sum_{e \in E_{nl}} x_e \cdot c_e \qquad (21.1)$$

Flow-conservation constraints:

$$\sum_{i \in E_{nl} | va_i = v} x_i - \sum_{j \in E_{nl} | ve_j = v} x_j = 0 \qquad \forall v \in V_{nl}, nl \in NL \qquad (21.2)$$

Cover constraints:

$$\sum_{e \in ES_f} x_e = 1 \qquad \forall f \in F \qquad (21.3)$$

Integrality constraints:

$$x_e \in Z \qquad \forall e \in E_{nl}, nl \in NL \qquad (21.4)$$

Bounds:

$$l_e \leq x_e \leq u_e \qquad \forall e \in E_{nl}, nl \in NL \qquad (21.5)$$

6.4.2 Integration of penalty costs

As argued before, delays cause additional penalty costs. Like Huisman et al.[121], we use a quadratic function for the penalty costs to penalize larger delays overproportionally. The costs of one delay of α seconds should be as high as the fixed costs of one vehicle for one day. Therefore the penalty costs for arc e, which is one of the five arc types between two service trips described above, in scenario s are:

$$penalty_{s,e} = y_{s,e}^2 \cdot \frac{c_{nl}^{fix}}{\alpha^2} \qquad (22.1)$$

The variable $y_{s,e}$ is the delayed starting time of the service trip following on connection arc e in scenario s or 0 if e is not used. The parameter c_{nl}^{fix} denotes the fixed costs for the usage of one additional bus for one day of the bus type in network layer nl. The objective function now is:

$$\min \sum_{nl \in NL} \sum_{e \in E_{nl}} c_e \cdot x_e + \frac{1}{|S|} \sum_{nl \in NL} \sum_{e \in E_{nl}} \sum_{s \in S} \left(y_{s,e}^2 \cdot \frac{c_{nl}^{fix}}{\alpha^2} \right) \qquad (22.2)$$

The set S is the scenarioset. The penalty costs for service trips only have to be considered in the objective function, if the corresponding service trip arc has a flow greater than 0. Therefore the following constraints are added.

$$y_{s,e} \geq 0 \qquad \forall s \in S, e \in E_{nl}, nl \in NL \qquad (22.3)$$

$$y_{s,e} = d_{s,e} \cdot x_e \qquad \forall s \in S, e \in E_{nl}, nl \in NL \qquad (22.4)$$

Now, if $d_{s,e}$ is the delayed starting time of a service trip following on connection arc e in scenario s, $y_{s,e}$ represents the delayed starting time for a service trip arc e in scenario s if this service trip arc has a flow greater than 0 and can therefore be added to the objective function as described above.

Unfortunately this model is now a quadratic optimization model and therefore computationally harder to solve than a linear optimization model. But as the flow on each arc in a network with all connecting arcs, except on the circulation arc, is 0 or 1 and penalty costs do not need to not be added to the circulation arc, this model can be

[121] [HFW04]

reformulated to a linear model. We plug the equation $y_{s,e} = d_{s,e} \cdot x_e$ in the objective function and obtain:

$$\min \sum_{nl \in NL} \sum_{e \in E_{nl}} c_e \cdot x_e + \frac{1}{|S|} \sum_{nl \in NL} \sum_{e \in E_{nl}} \sum_{s \in S} \left((d_{s,e} \cdot x_e)_{s,e}^2 \cdot \frac{c_{nl}^{fix}}{\alpha^2} \right) \qquad (22.5)$$

$$= \min \sum_{nl \in NL} \sum_{e \in E_{nl}} c_e \cdot x_e + \frac{1}{|S|} \sum_{nl \in NL} \sum_{e \in E_{nl}} \sum_{s \in S} \left(x_e^2 \cdot \frac{d_{s,e}^2 \cdot c_{nl}^{fix}}{\alpha^2} \right) \qquad (22.6)$$

$$\xRightarrow{x_e \in \{0,1\}} \min \sum_{nl \in NL} \sum_{e \in E_{nl}} c_e \cdot x_e + \frac{1}{|S|} \sum_{nl \in NL} \sum_{e \in E_{nl}} \sum_{s \in S} \left(x_e \cdot \frac{d_{s,e}^2 \cdot c_{nl}^{fix}}{\alpha^2} \right) \qquad (22.7)$$

$$= \min \sum_{nl \in NL} \sum_{e \in E_{nl}} x_e \cdot \left(c_e + \frac{1}{|S|} \sum_{s \in S} \frac{d_{s,e}^2 \cdot c_{nl}^{fix}}{\alpha^2} \right) \qquad (22.8)$$

6.4.3 Complete model

This chapter now shows the stochastic optimization model after its reformulation of the previous section. It is a stochastic model for the multi depot vehicle scheduling problem with multiple vehicle types.

Objective function:

$$\min \sum_{nl \in NL} \sum_{e \in E_{nl}} x_e \cdot \left(c_e + \frac{1}{|S|} \sum_{s \in S} \frac{d_{s,e}^2 \cdot c_{nl}^{fix}}{\alpha^2} \right) \qquad (23.1)$$

Flow-conservation constraints:

$$\sum_{i \in E_{nl} | va_i = v} x_i - \sum_{j \in E_{nl} | ve_j = v} x_j = 0 \qquad \forall v \in V_{nl}, nl \in NL \qquad (23.2)$$

Cover constraints:

$$\sum_{e \in ES_f} x_e = 1 \qquad \forall f \in F \qquad (23.3)$$

Integrality constraints:

$$x_e \in Z \qquad \forall e \in E_{nl}, nl \in NL \qquad (23.4)$$

Bounds:

$$l_e \leq x_e \leq u_e \qquad \forall e \in E_{nl}, nl \in NL \qquad (23.5)$$

For the calculations in the next section the objective function can either be the expected costs or the Conditional Value at Risk. The model for the calculations, where the objective is the Conditional Value at Risk, is:

Objective function:

$$\min \frac{1}{\alpha_{cvar}} \left(\alpha_{cvar} \cdot y_0 + \sum_{s \in S} y_s \cdot \frac{1}{|S|} \right) \qquad (24.1)$$

Flow-conservation constraints:

$$\sum_{i \in E_{nl} | va_i = v} x_i - \sum_{j \in E_{nl} | ve_j = v} x_j = 0 \qquad \forall v \in V_{nl}, nl \in NL \qquad (24.2)$$

Cover constraints:

$$\sum_{e \in ES_f} x_e = 1 \qquad \forall f \in F \qquad (24.3)$$

Integrality constraints:

$$x_e \in Z \qquad \forall e \in E_{nl}, nl \in NL \qquad (24.4)$$

Bounds:

$$l_e \leq x_e \leq u_e \qquad \forall e \in E_{nl}, nl \in NL \qquad (24.5)$$

Costs in scenarios:

$$cost_s = \sum_{nl \in NL} \sum_{e \in E_{nl}} x_e \cdot \left(c_e + \frac{d_{s,e}^2 \cdot c_{nl}^{fix}}{\alpha^2} \right) \qquad \forall s \in S \qquad (24.6)$$

Implementation of the Conditional Value at Risk:

$$y_0 + y_s \geq cost_s \qquad \forall s \in S \qquad (24.7)$$

$$y_s \geq 0 \qquad \forall s \in S \qquad (24.8)$$

Additional variables are: $cost_s$, y_s and y_0; α_{cvar} is an additional parameter. For a description of them and of the Conditional Value at Risk as well as for its integration into optimization models see Chapter 3.1.4 and 3.3.5.

6.5 Results

In this section, we show the results for the vehicle schedules calculated with our stochastic programming approach. We compare them with the cost-optimal vehicle schedule and a simple approach that adds fixed buffer times between service trips. If a service trip is delayed, these buffer times are used to absorb (at least a part of) the delay. The instances are real timetables from small- and medium-sized German cities.

6.5.1 Tradeoff of planned-costs and penalty-costs

At first, we show the planned and penalty costs for different solutions. We expect that there are solutions with low planned costs, but high penalty costs as well as with low penalty costs, but high planned costs. The value for α is 1800, so that a delay of 30 minutes is as expensive as using one additional bus. Figure 32 and Figure 33 show the planned costs and the penalty costs for two different timetables.

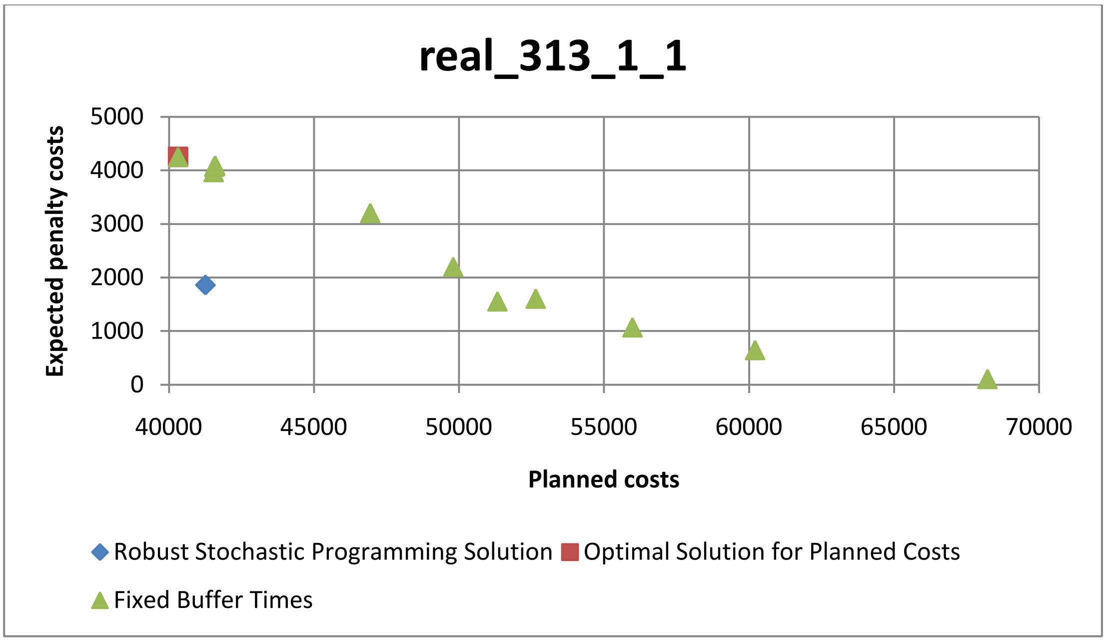

Figure 32 Planned and penalty costs for real_313_1_1

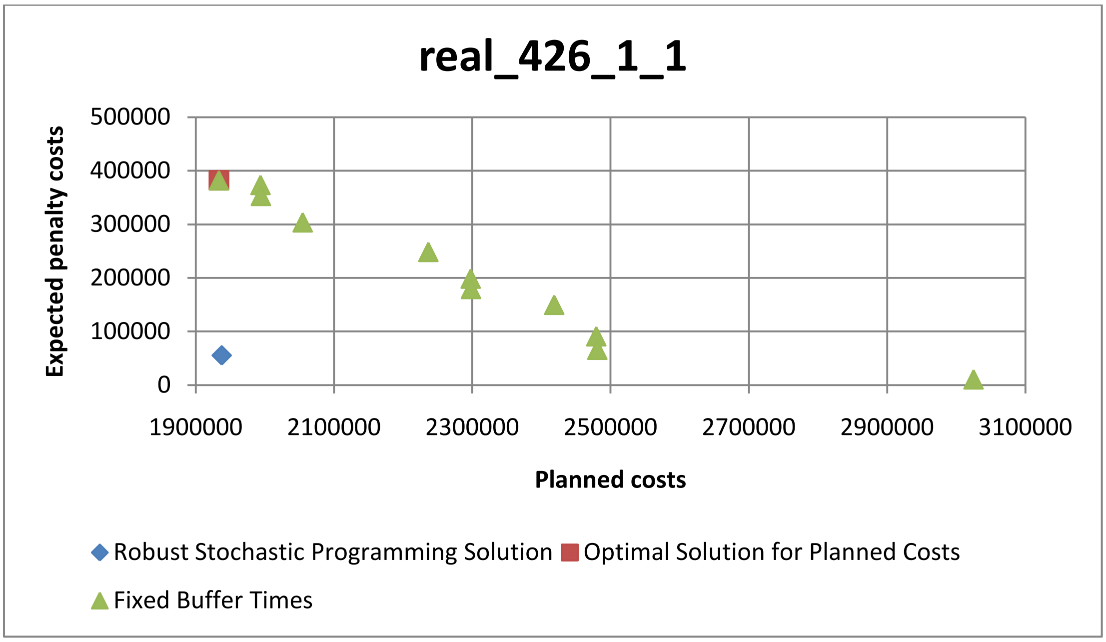

Figure 33 Planned and penalty costs for real_426_1_1

The figures show that the optimal solution for planned costs has the highest penalty costs in every instance. It can also be seen, that the additional fixed buffer times cause a decrease in penalty costs, but they significantly increase planned costs. The reason is that the buffer times are added after every service trip, so that it is very likely that more busses have to be used. This happens for all buffer durations. The solutions for fixed buffer times are calculated with buffer durations of 15, 30, 60, 120, 180, 240, 300, 420, 600 and 1200 seconds. These durations are added after every service trip and then the planned costs are minimized.

The solution calculated with stochastic programming reduces the penalty costs to very small values, while it only causes a slight increase in planned costs. The next tables show a more detailed view of the solutions found for these timetables.

Solution Approach	Total Costs	Planned Costs	Penalty Costs	Used Vehicles
Minimize planned costs	44563	**40311**	4252	**30**
Stochastic programming	**43115**	41256	1859	**30**
Fixed Buffer Time 15s	45508	41537	3971	31
Fixed Buffer Time 30s	45627	41555	4072	31
Fixed Buffer Time 60s	45683	41591	4092	31
Fixed Buffer Time 120s	50142	46935	3207	35
Fixed Buffer Time 180s	51999	49798	2201	37
Fixed Buffer Time 240s	52880	51322	1558	38
Fixed Buffer Time 300s	54249	52641	1608	39
Fixed Buffer Time 420s	57051	55978	1073	41
Fixed Buffer Time 600s	60855	60206	649	44
Fixed Buffer Time 1200s	68329	68221	**108**	49

Table 12 Solution comparison for real_313_1_1

Solution Approach	Total Costs	Planned Costs	Penalty Costs	Used Vehicles
Minimize planned costs	2315517	**1933416**	382100	**32**
Stochastic programming	**1992585**	1937304	55282	**32**
Fixed Buffer Time 15s	2367236	1993618	373619	33
Fixed Buffer Time 30s	2347397	1994178	353219	33
Fixed Buffer Time 60s	2358511	2054656	303856	34
Fixed Buffer Time 120s	2485118	2236416	248701	37
Fixed Buffer Time 180s	2496536	2297684	198852	38
Fixed Buffer Time 240s	2477632	2298080	179552	38
Fixed Buffer Time 300s	2568495	2418512	149982	40
Fixed Buffer Time 420s	2570616	2479332	91284	41
Fixed Buffer Time 600s	2547004	2480862	66142	41
Fixed Buffer Time 1200s	3036038	3025127	**10911**	50

Table 13 Solution comparison for real_426_1_1

The tables proof that the stochastic programming solutions are always best in total costs. This could have been expected, because stochastic programming always finds the optimal solution for the given data. The more interesting result is that the traditional approach that adds fixed buffer times between service trips finds solutions that are much worse than the stochastic programming solutions. They can decrease penalty costs to a minimum, but this increases the planned costs so much, that the total costs and the number of used vehicles result in very high values. The solutions produced with the simple approach, that adds buffer times after the service trips, therefore cannot compete with the stochastic programming solutions and are practically not usable.

6.5.2 Tradeoff of total costs and robustness

We now introduce the Conditional Value at Risk and calculate a set of pareto-optimal solutions. One extreme solution of this set has the minimum expected costs and one has the best Conditional Value at Risk. The intermediate solutions are calculated by restricting the Conditional Value at Risk and optimizing expected costs. Figure 34 shows the pareto-optimal set of solutions for two different timetables and the improvement in terms of total costs (planned costs plus penalty costs) and Conditional Value at Risk.

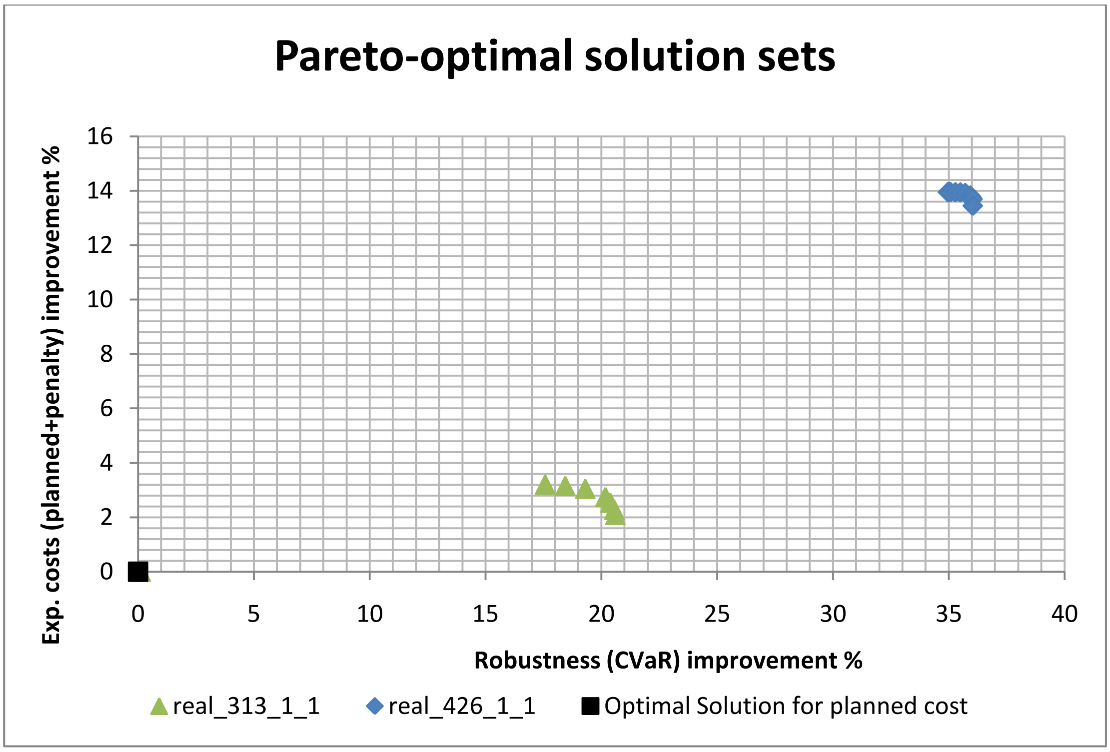

Figure 34 Pareto-optimal solutions calculated with stochastic programming

The solutions with the lowest expected costs are at the upper left and with the best CVaR at the bottom right of the solution sets. There are no solutions with a better CVaR or lower costs for these instances, so that these sets of solutions are pareto-optimal for these timetables. The set of pareto-optimal solutions generally show a convex behavior.

If we compare the solutions found by stochastic programming with the solutions with fixed buffer times in terms of total costs and CVaR, we see again that they are inferior. Figure 35 and Figure 36 show this comparison for the same timetables.

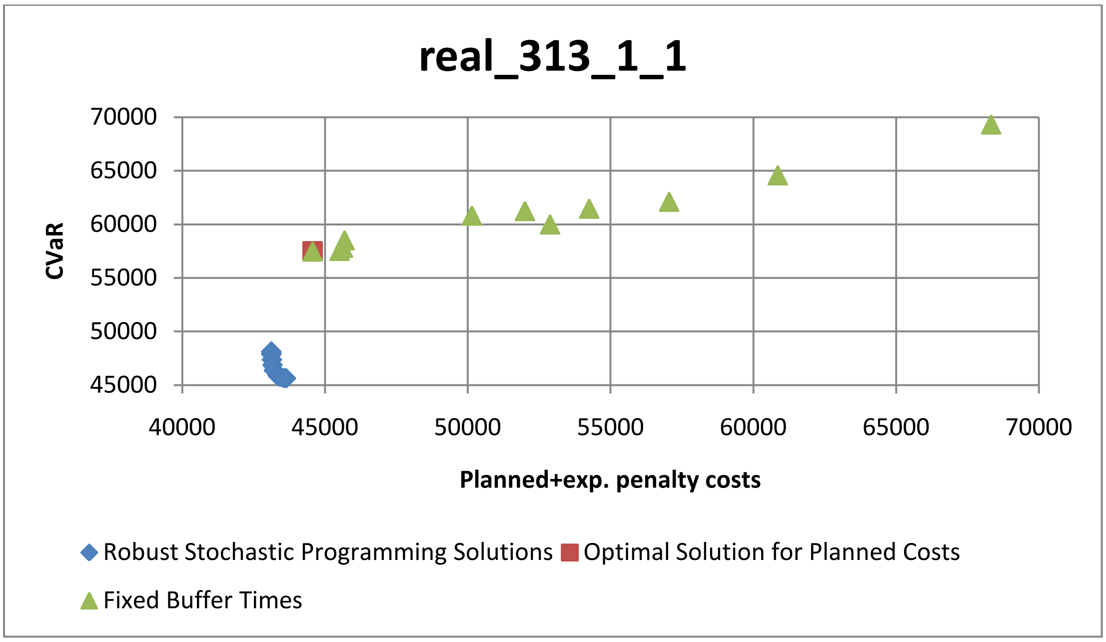

Figure 35 Total Costs and CVaR for real_313_1_1

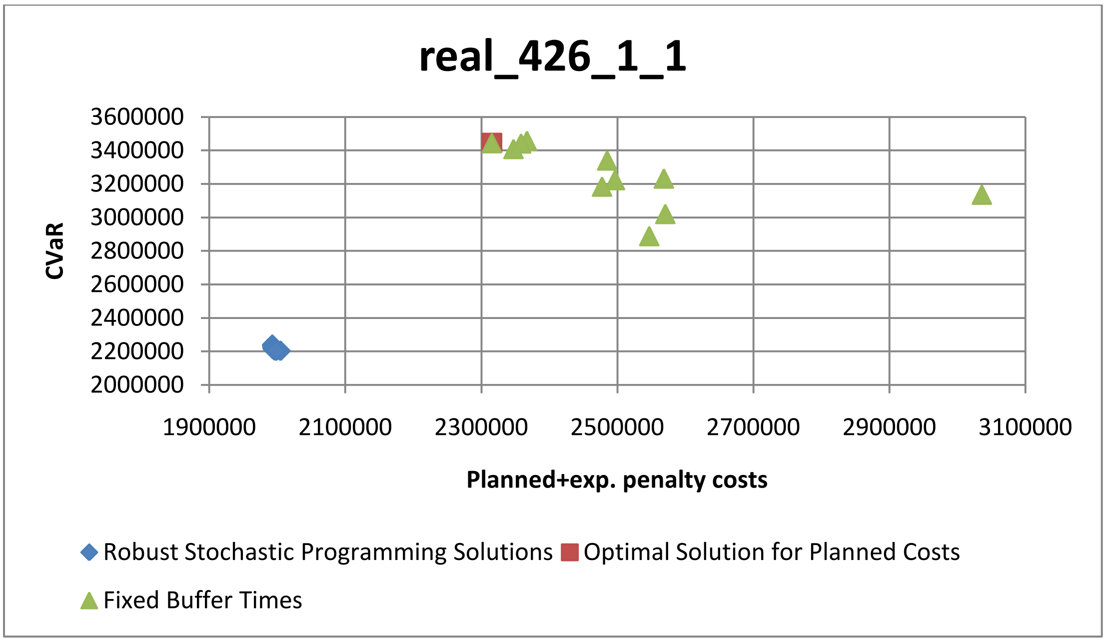

Figure 36 Total Costs and CVaR for real_426_1_1

Figure 36 shows that the CVaR can be improved by using fixed buffer times, but costs are always increased. In the instance “real_313_1_1” the CVaR cannot be improved by using fixed buffer times (see Figure 35), so that these solutions are dominated by the cost-optimal solution in terms of total costs and CVaR.

6.5.3 Introducing delay propagation

In our preliminary results, we compared the optimal solutions calculated with stochastic programming, those with fixed buffer times and the cost-optimal solution. The penalty costs were calculated when a disruption causes a delayed start of a following service trip. Further delay propagation was not considered. Under these assumptions, stochastic programming finds the optimal solution.

Now, we consider entire delay propagation and again compare our solutions calculated with stochastic programming with the solutions that add buffer times between service trips. We do this with a simulation software to examine if our stochastic programming approach that is heuristic, because it does not consider entire delay propagation during optimization, also leads to better results when entire delay propagation is considered. This is the real-world situation.

Because entire delay propagation is not considered in the stochastic model, the impacts of disruptions and the resulting costs caused by disruptions are underestimated. To compensate this, we change the parameter α in the optimization model to other values. That means that we change the function of the penalty costs. If we choose a lower value for α, a smaller delay will cause penalty costs of the amount of the fixed costs for the usage of one bus for one day. We therefore overestimate the penalty costs of a delayed start of a service trip to compensate the underestimation because of the lack of entire delay propagation in the optimization model.

After calculating the vehicle schedules with different values of α, we use a simulation software to evaluate the vehicle schedules with the starting value of α, which is 1800 seconds. Figure 37, Figure 38, Figure 39 and Figure 40 show the simulated results for the different approaches for four timetables.

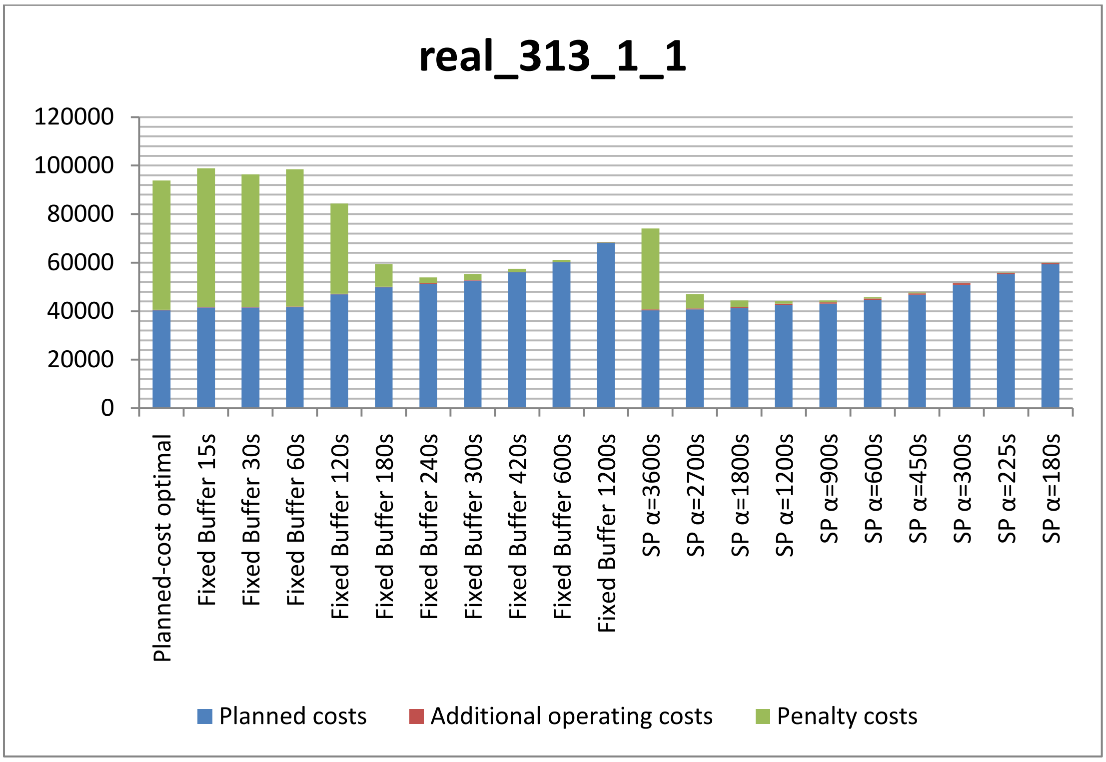

Figure 37 Costs of vehicle schedules for real_313_1_1

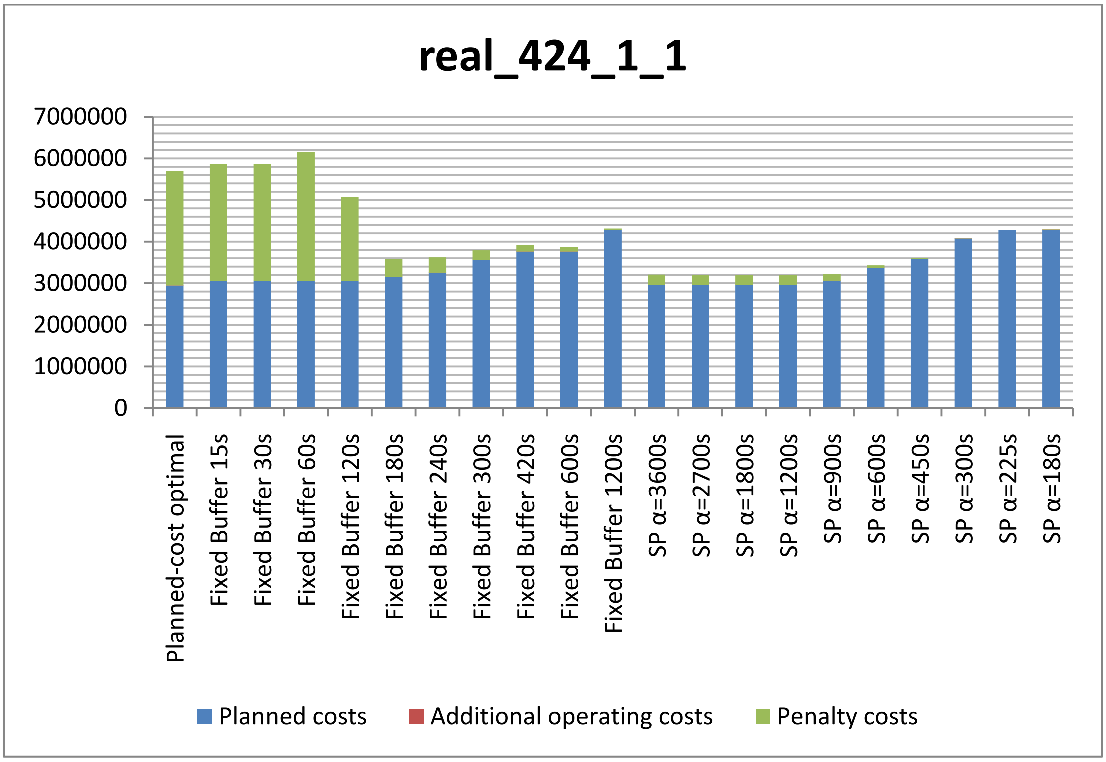

Figure 38 Costs of vehicle schedules for real_424_1_1

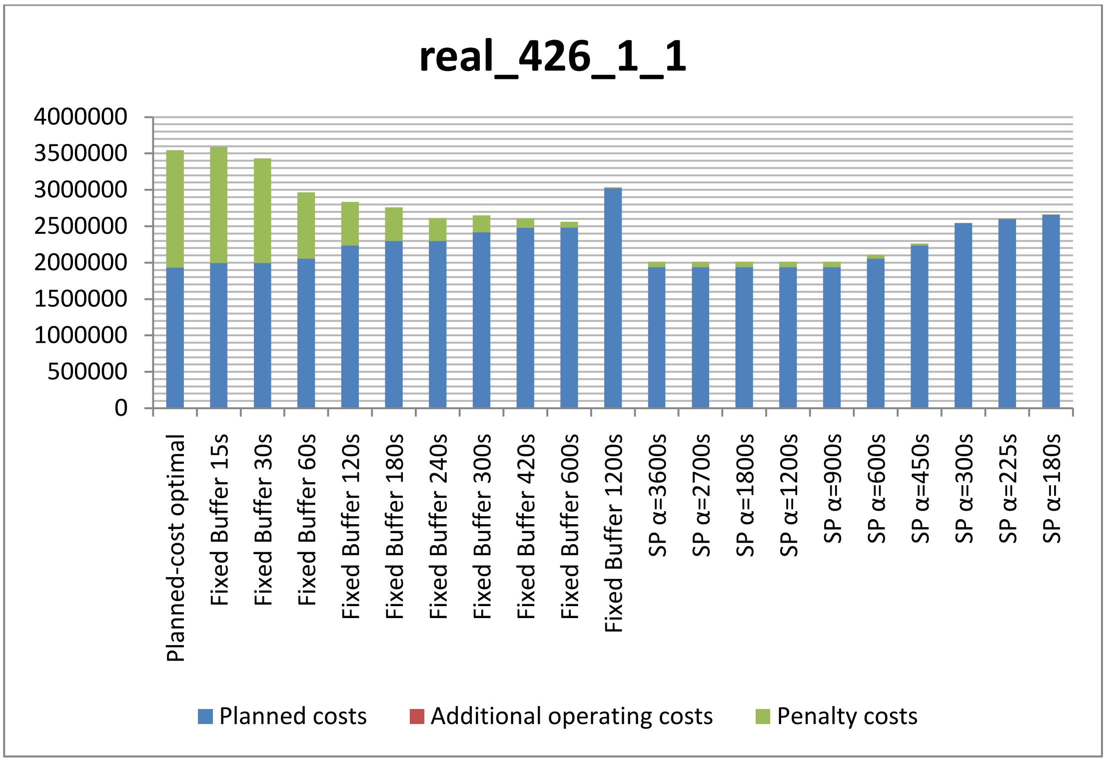

Figure 39 Costs of vehicle schedules for real_426_1_1

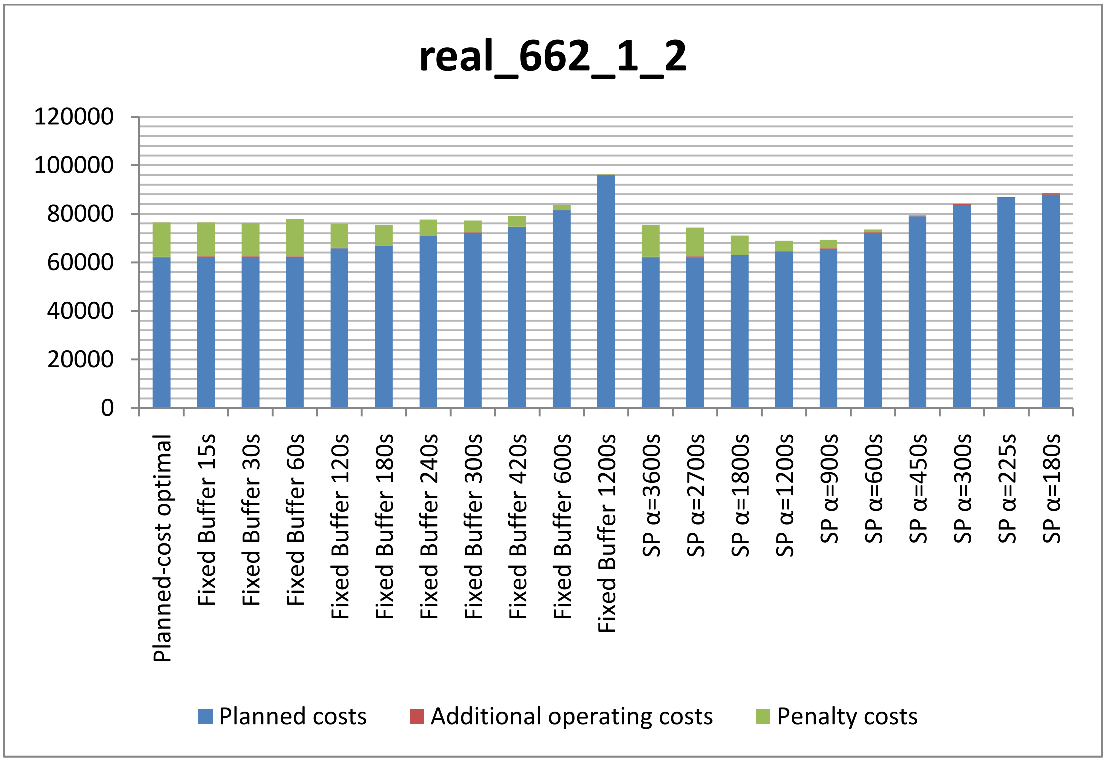

Figure 40 Costs of vehicle schedules for real_662_1_2

The first observation is that the penalty costs for the planned-cost optimal solution are significantly higher when we consider entire delay propagation. Because of this fact adding fixed buffer times between service trips can now decrease total costs. This was not possible in our preliminary results. Once more it can be seen, that adding fixed buffer times between service trips cannot compete with stochastic programming. A fixed buffer time of 240s can reduce total costs to 74.1% on average (compared to the planned cost-optimal solution) whereas stochastic programming with a value of 1200 for α can reduce them to 62.5% on average.

How should we now choose the value for α? The results indicate that the values 1800, 1200 and 900 produce good results for all instances. 1800 is the value that is also used in the simulation to calculate the penalty. Figure 41 shows the average changes in total costs of the four vehicle schedules above mentioned depending on the method used to create the vehicle schedule.

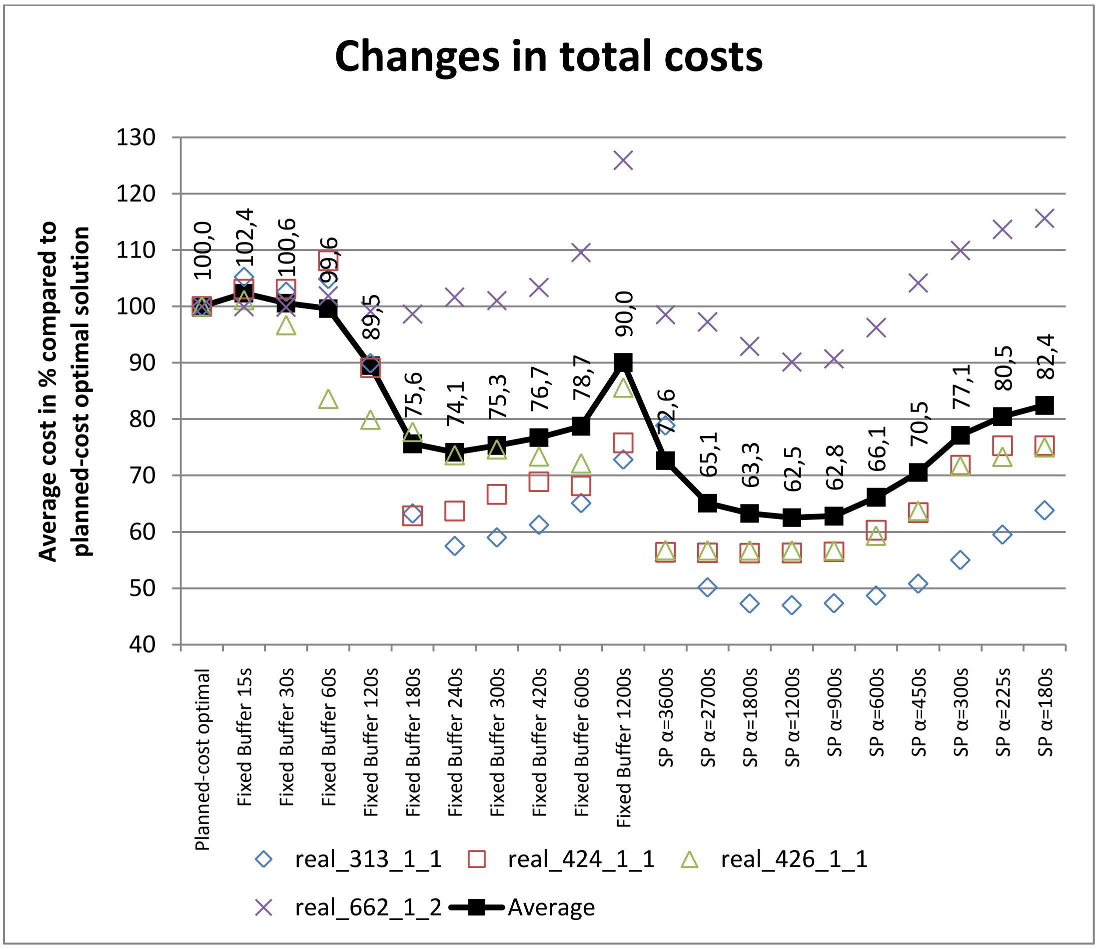

Figure 41 Changes in total cost

The result that an α of 1800 produces good results shows that our stochastic programming approach is applicable even though it does not consider entire delay propagation. The best results can be obtained with a value of 1200 for α. Thus, a small overestimation of the delay costs in the optimization is the best choice to compensate the additional delay costs because of entire delay propagation.

6.5.4 Evaluation with other scenariosets

As the real instances used to calculate the optimal solutions contain a large number of service trips that can be delayed, we cannot calculate every possible combination of delayed service trips and use them as a scenario. Therefore a certain number of scenarios, in our case 100, were included into the optimization model and the model was optimized with this data. It is now necessary to show, that our approach does not only produce good solutions because the solutions fit to the specific scenarios used for optimization.

This is important because the delay scenario of a day during the execution of a vehicle schedule will probably not be the same as it was on a former day. Disruptions will occur on other service trips, so that former days can be used as scenarios for optimization, but they have to lead to a good vehicle schedule for other days with the same distribution for the disruptions. Therefore, we use another scenarioset with 300 scenarios and evaluate the solutions, which were calculated with the first set, with the other scenarioset. Figure 42 shows the results.

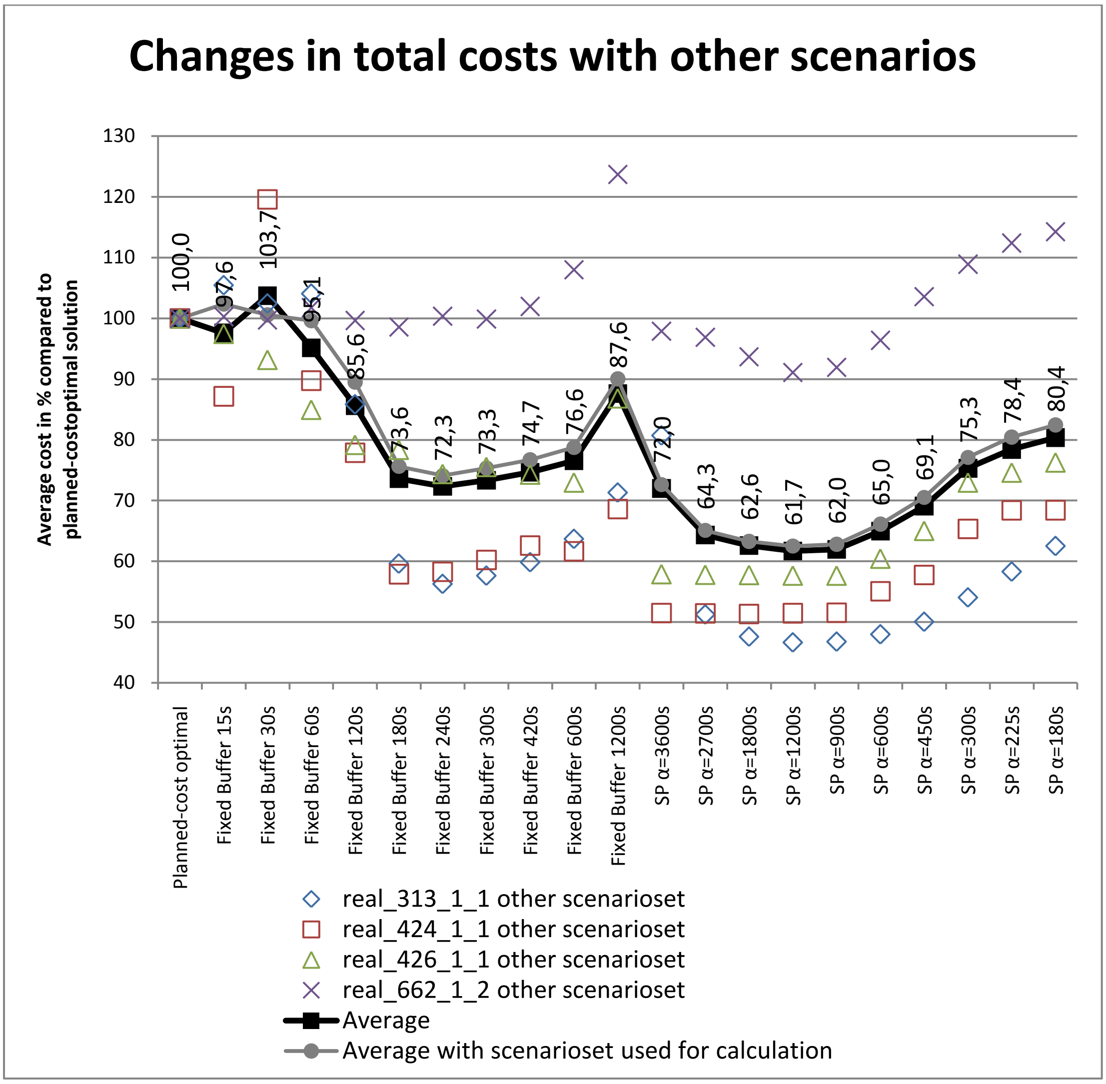

Figure 42 Evaluation with different scenariosets

The other scenarioset does not lead to different conclusions. The stochastic programming solutions are still the best solutions and 1200s is the best value for α. Again 1800s, 1200s and 900s are appropriate values for α. Thus, the developed approach does not depend on a specific scenarioset.

By using real delay data of past days, the characteristics of the timetable and road network are considered, so that using real past data is a valid method for finding good results for future delay scenarios as long as these characteristics do not change.

If the road network or other substantial factors change, new delay scenarios are necessary. The generation of these or the modification of past scenarios and the examination if the approach developed still leads to such good results when the scenarios used for optimization and simulation have different characteristics or distributions is not analyzed in this thesis. Such a sensitivity analysis of the whole method with different data records and their implications is a subject for further research.

6.5.5 Delay propagation and risk measures

We now again introduce the Conditional Value at Risk to examine, if the tradeoff between robustness and costs is still apparent when entire delay propagation is considered. For this examination, we use the same set of scenarios in the simulation and in the optimization and a value of 1200s for α during optimization and 1800s in the simulation.

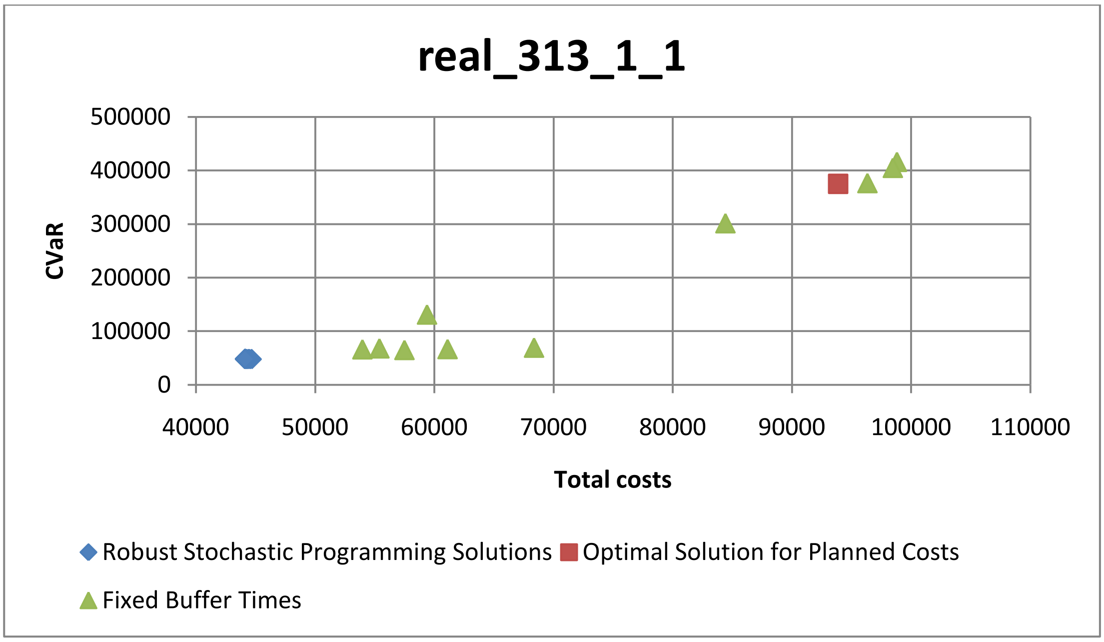

Figure 43 Costs and CVaR - entire delay propagation real_313_1_1

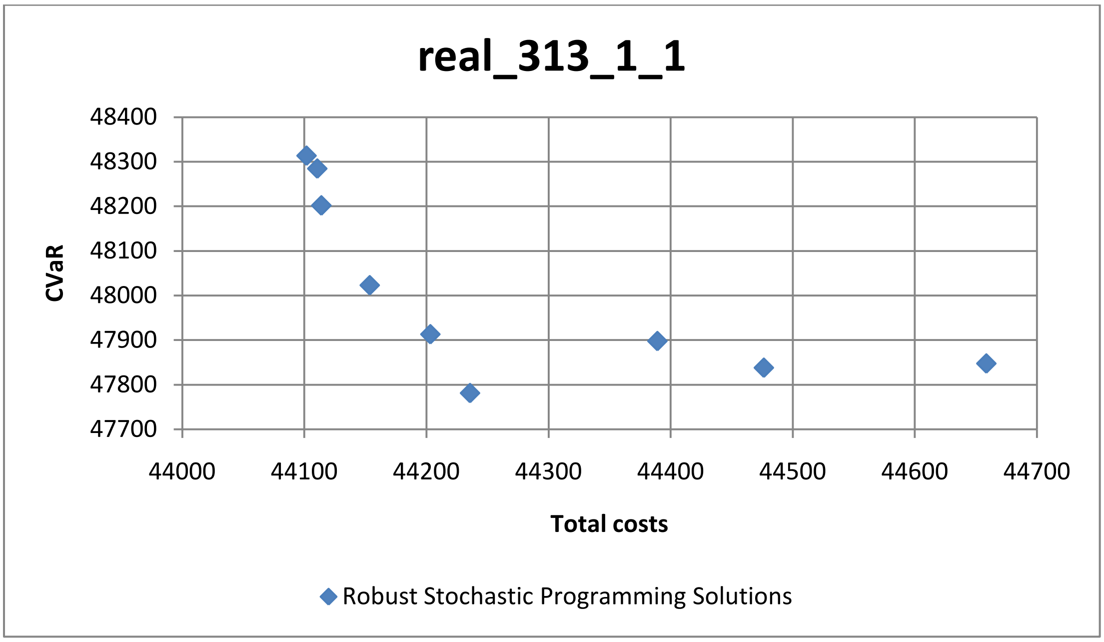

Figure 44 Costs and CVaR - entire delay propagation real_313_1_1 (2)

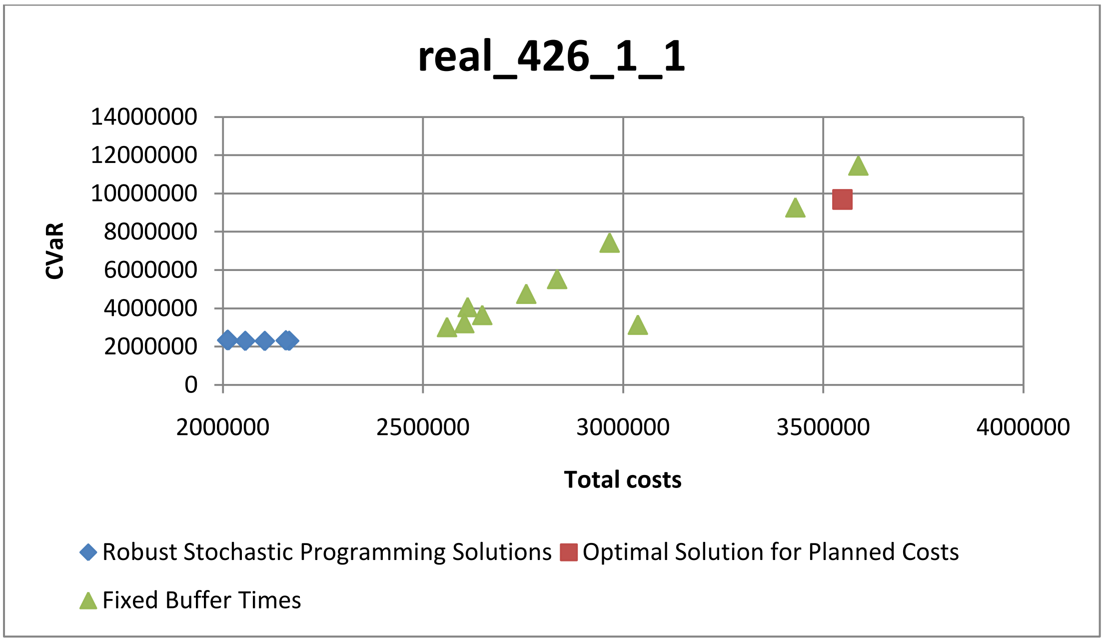

Figure 45 Costs and CVaR - entire delay propagation real_426_1_1

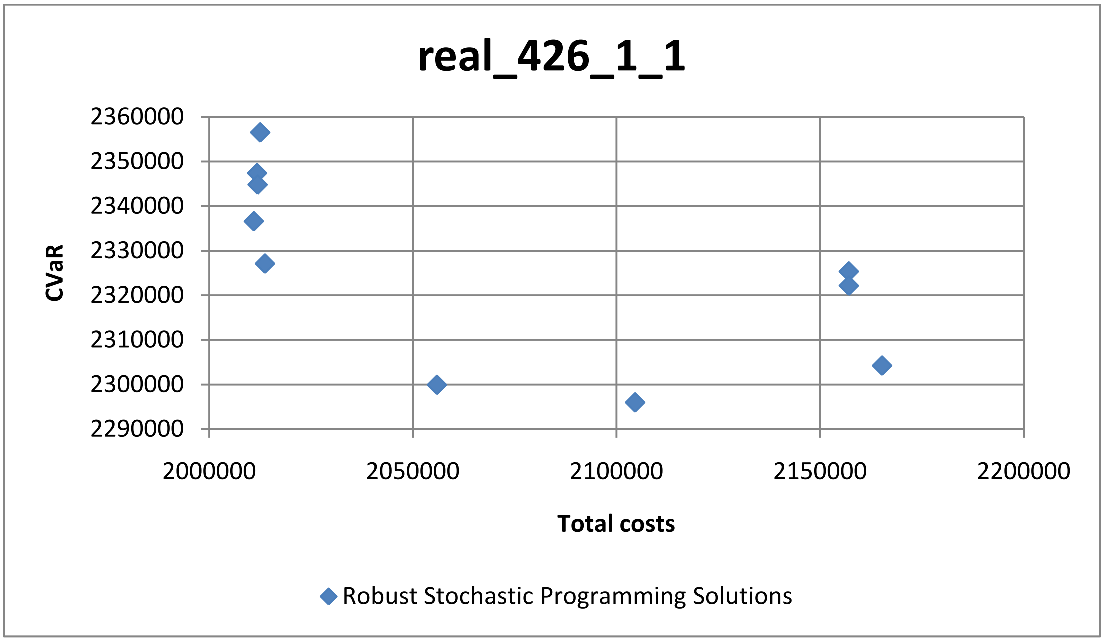

Figure 46 Costs and CVaR - entire delay propagation real_426_1_1 (2)

It comes out, that the set of pareto-optimal solutions and the tradeoff between optimizing costs and CVaR still exists. Nevertheless, the solutions should be evaluated via simulation to find good solutions and to sort out the few dominated outliers.

6.5.6 Other scenarioset, delay propagation and risk measures

Furthermore, we evaluate these solutions in the simulation tool with another scenarioset. Again, α is 1200s in the optimization and 1800s in the simulation.

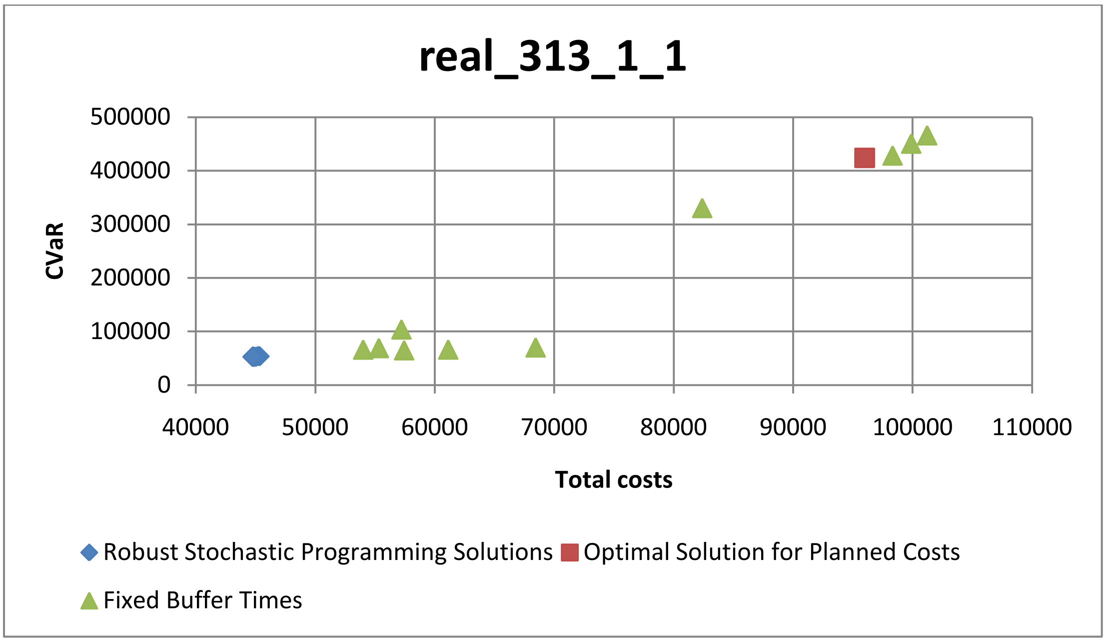

Figure 47 Optimizing risk measures with other scenarios real_313_1_1

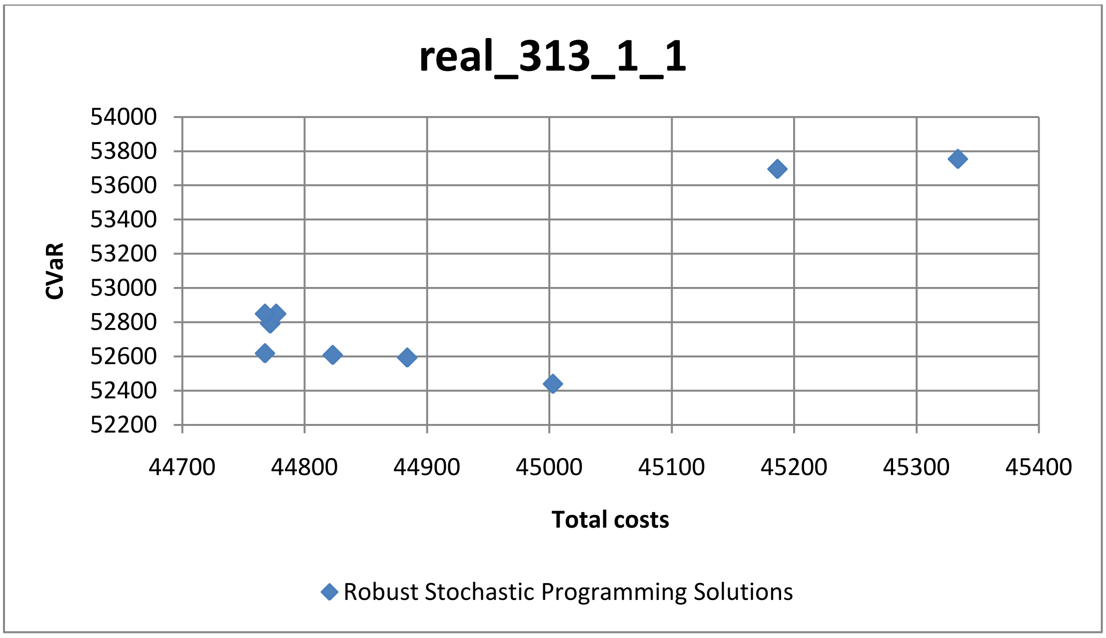

Figure 48 Optimizing risk measures with other scenarios real_313_1_1 (2)

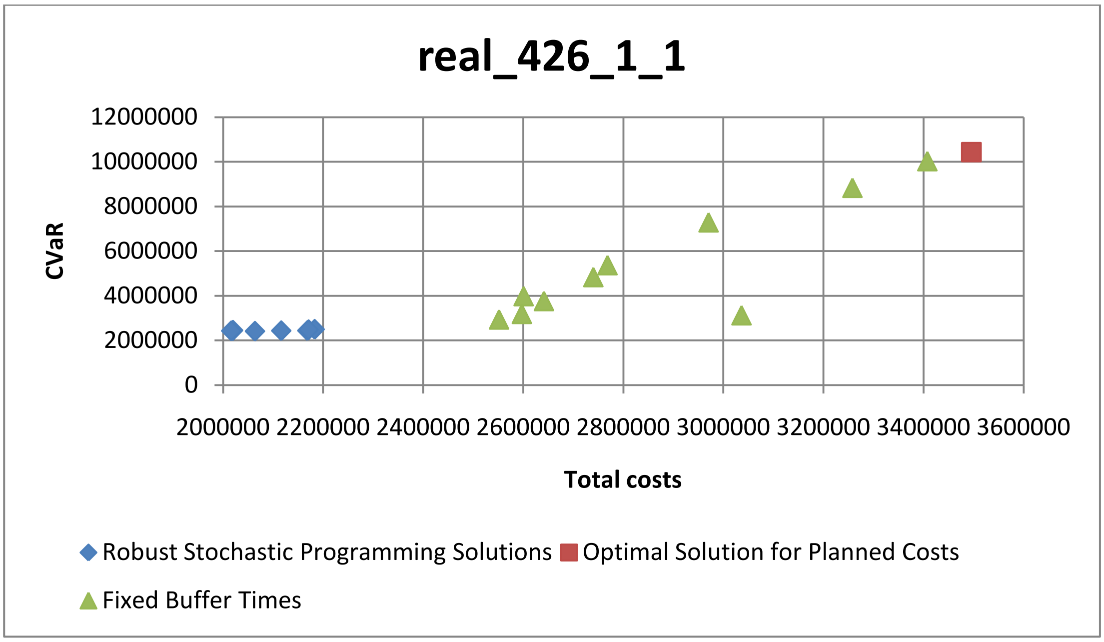

Figure 49 Optimizing risk measures with other scenarios real_426_1_1

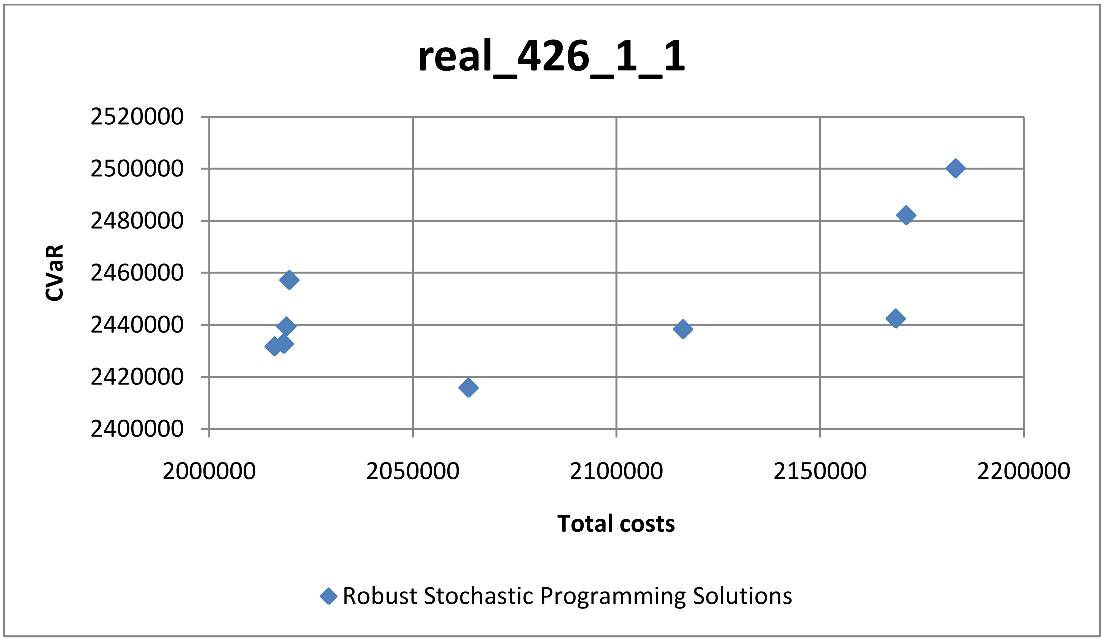

Figure 50 Optimizing risk measures with other scenarios real_426_1_1 (2)

Apparently the solutions are only partly usable. Figure 48 and Figure 50 show a set of pareto-optimal solutions with a tradeoff between optimizing costs and CVaR, but many solutions are dominated. The data shows that the solutions where the CVaR was optimized often have a worse CVaR and also higher costs.

Why does that happen? The definition of the CVaR says that it is the expected value of the x%, in our case 5%, worst scenarios. Therefore, by optimizing the CVaR we create a plan that optimizes the 5% of the worst scenarios. But when the scenarios change, the increase in planned costs cannot be rewarded with additional robustness, because the 5% of the worst scenarios have changed. The new 5% worst scenarios may have disruptions on other service trips.

Because of this fact, an optimization of the risk measure CVaR only makes sense, if at least the 5% worst scenarios are known or can be well approximated. For example, if it is known that certain service trips are always late when there is frost, snow or rain, etc. It is also applicable, when the more robust solutions lead to a higher number of vehicles; then the solution with the lowest expected total costs from the solutions with the higher number of vehicles should be chosen. Otherwise, the solution with the lowest expected costs is best in costs as well as in robustness. But the solutions created with stochastic programming still outperform the optimal solution for planned costs and the solutions with fixed buffer times as shown in Figure 47 and Figure 49.

6.6 Model extension with weather-derivatives

As an extension of this work in robust vehicle scheduling in public bus transport, an approach that integrates weather derivatives into the optimization model to further increase robustness is presented in this chapter. Although some research about the reasons of disruptions has to be done, this model extension shows that using weather derivatives is a promising approach in this area.

6.6.1 Model adaption

As a basis for integrating weather derivatives, we use the formulation of the stochastic optimization model of Chapter 6.4.3 and limit the Conditional Value at Risk to the parameter $CVaRlimit$:

Objective function:

$$\min \frac{1}{|S|} \cdot \sum_{s \epsilon S} cost_s \qquad (25.1)$$

Flow-conservation constraints:

$$\sum_{i \in E_{nl} | va_i = v} x_i - \sum_{j \in E_{nl} | ve_j = v} x_j = 0 \qquad \forall v \in V_{nl}, nl \in NL \qquad (25.2)$$

Cover constraints:

$$\sum_{e \in ES_f} x_e = 1 \qquad \forall f \in F \qquad (25.3)$$

Integrality constraints:

$$x_e \in Z \qquad \forall e \in E_{nl}, nl \in NL \qquad (25.4)$$

Bounds:

$$l_e \leq x_e \leq u_e \qquad \forall e \in E_{nl}, nl \in NL \qquad (25.5)$$

Costs in scenarios:

$$cost_s = \sum_{nl \in NL} \sum_{e \in E_{nl}} x_e \cdot \left(c_e + \frac{d_{s,e}^2 \cdot c_{nl}^{fix}}{\alpha^2} \right) \qquad \forall s \in S \qquad (25.6)$$

Conditional Value at Risk:

$$\frac{1}{\alpha_{cvar}} \left(\alpha_{cvar} \cdot y_0 + \sum_{s \in S} y_s \cdot \frac{1}{|S|} \right) \leq CVaRlimit \qquad (25.7)$$

$$y_0 + y_s \geq cost_s \qquad \forall s \in S \qquad (25.8)$$

$$y_s \geq 0 \qquad \forall s \in S \qquad (25.9)$$

Hull[122] describes how prices for weather derivatives can be calculated: The correct price of a derivative is the expected payoff discounted by the risk-free interest rate. We therefore use historical weather data of a German city and integrate two weather derivatives. For the sake of simplicity, we assume zero interest rate. Furthermore, a margin for the derivatives can be defined. Therefore, we add the following variables and parameters into the optimization model:

Variables:

d^{wind}	Amount of money spent for *windy* day derivatives (stage-1 variable)
d^{ice}	Amount of money spent for *icy* day derivatives (stage-1 variable)
ho_s	Payoffs from derivatives in scenario s (stage-2 variable)

Parameters:

p^{wind}	Probability that a day is a *windy* day (highest wind speeds ≥ 9bft)
p^{ice}	Probability that a day is an *icy* day (precipitation > 1mm and average daily temperature below 0˚C)
$margin$	Margin for weather derivatives
i_s^{wind}	1 if scenario s is a windy day, 0 if not
i_s^{ice}	1 if scenario s is an icy day, 0 if not

[122] [Hull03] p. 678

The model with weather derivatives is:

Objective function:

$$\min \frac{1}{|S|} \cdot \sum_{s \epsilon S} (cost_s - ho_s) + d^{wind} + d^{ice} \qquad (26.1)$$

Flow-conservation constraints:

$$\sum_{i \in E_{nl} | va_i = v} x_i - \sum_{j \in E_{nl} | ve_j = v} x_j = 0 \qquad \forall v \in V_{nl}, nl \in NL \qquad (26.2)$$

Cover constraints:

$$\sum_{e \in ES_f} x_e = 1 \qquad \forall f \in F \qquad (26.3)$$

Integrality constraints:

$$x_e \in Z \qquad \forall e \in E_{nl}, nl \in NL \qquad (26.4)$$

Bounds:

$$l_e \leq x_e \leq u_e \qquad \forall e \in E_{nl}, nl \in NL \qquad (26.5)$$

Costs in scenarios:

$$cost_s = \sum_{nl \in NL} \sum_{e \in E_{nl}} x_e \cdot \left(c_e + \frac{d_{s,e}^2 \cdot c_{nl}^{fix}}{\alpha^2} \right) \qquad \forall s \in S \qquad (26.6)$$

Conditional Value at Risk implementation:

$$\frac{1}{\alpha_{cvar}} \left(\alpha_{cvar} \cdot y_0 + \sum_{s \in S} y_s \cdot \frac{1}{|S|} \right) \leq CVaRlimit \qquad (26.7)$$

$$y_0 + y_s \geq cost_s + d^{wind} + d^{ice} - ho_s \qquad \forall s \in S \qquad (26.8)$$

Outcomes from hedging:

$$ho_s = (1 - margin) \cdot \left(\frac{d^{wind} \cdot i_s^{wind}}{p^{wind}} + \frac{d^{ice} \cdot i_s^{ice}}{p^{ice}} \right) \qquad \forall s \in S \qquad (26.9)$$

$$y_s \geq 0 \qquad \forall s \in S \qquad (26.10)$$

$$d^{wind}, d^{ice} \geq 0 \qquad (26.11)$$

We do not exactly know how severe weather conditions affect the traffic in public bus transport because there is no available data. Further data collection is necessary to find out in which way snow, freezing rain, storm and/or other weather phenomenons affect the occurrence of delays in vehicle schedules. We therefore assume that the traffic is influenced on days with these weather conditions:

- Strong winds with breaking branches from trees (peak wind velocity $\geq$ 75 km/h $\triangleq \geq 9bft$)
- Snowfall or freezing rain (average daily temperature below 0˚C and precipitation > 1mm)

We call these scenarios *windy* and *icy* days. We implement these conditions by indicating delay scenarios from the worst 10% of all scenarios, therefore days with a large delay length and a high delay probability, as windy and/or icy days.

The data used to calculate the scenarios is the historical data of the weather-station Berlin-Tempelhof from 1/1/1991 until 31/12/2009[123]. In this period a windy day happened in average about eight times per year and an icy day about four times per year. Weather derivatives are integrated independently for windy and icy days.

6.6.2 Computational Results

For our results, we again use the real schedule of a German city with 313 service trips. Even, if we assume a margin of 5% for the weather derivatives, we obtain results that are very promising.

[123] [DWD]

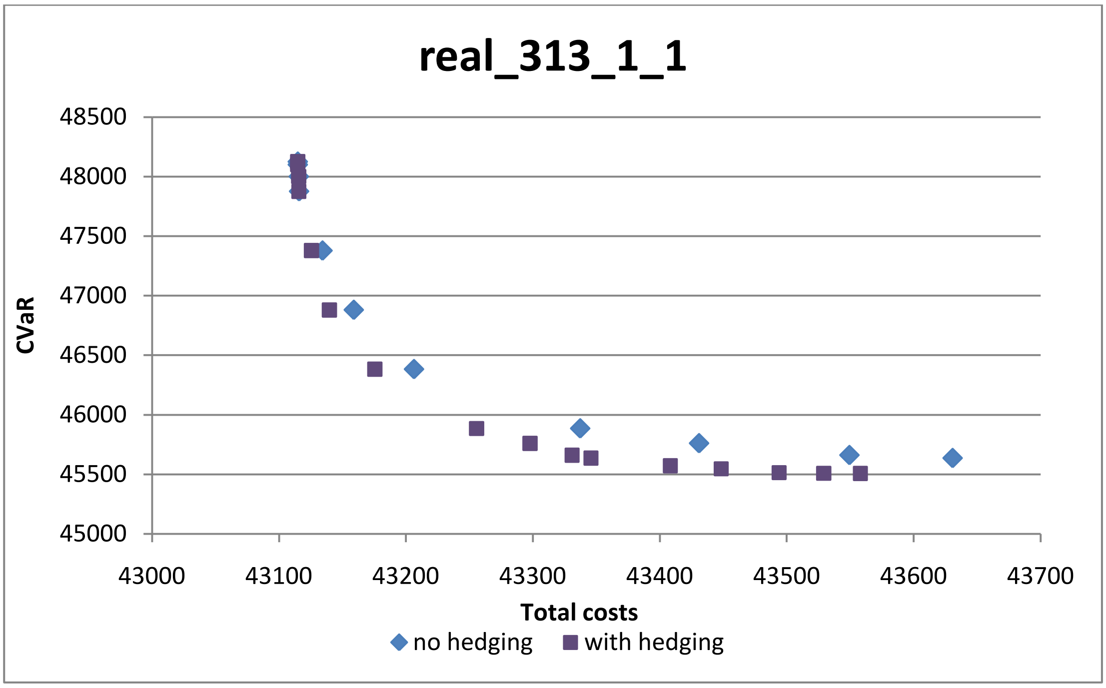

Figure 51 Weather derivatives for real_313_1_1

Figure 51 shows the pareto-optimal solution sets with hedging and without hedging. We can see that the minimum total costs cannot be decreased with weather derivatives, but at levels where the CVaR is restricted to better values the use of financial hedging instruments can gain the same level of robustness with lower costs. Furthermore, the maximum robustness level could be slightly increased with weather derivatives, as the best CVaR decreases from 45636 to 45508.

The results in this chapter were not simulated, so that delay propagation is not considered. Since we have shown that the vehicle schedules are applicable under the presence of delay propagation in Chapter 6.5, we do not do this again for the results of this model expansion with weather derivatives.

Let us remark that for robustness considerations it is important that some of the worst scenarios, which are considered to calculate the Conditional Value at Risk, are bad because of weather. Otherwise weather derivatives would not be able to change the Conditional Value at Risk and the robustness would not be increased. This assumption is also important for transferring the results to reality, where entire delay propagation has to be considered, as argued in Chapter 6.5.6. On the other hand, when changing the robustness measure to another measure, for example to the variance, weather derivatives could also decrease the variance if icy or windy days do not lead to such bad scenarios that they are considered in the CVaR. As detailed data for weather impacts on public bus transport is not available, data collection and future research in this area is necessary.

To conclude we can say that the use of weather derivatives enables further cost savings at the same robustness level if the disruption scenarios caused by weather have impacts on the robustness measure.

6.7 Considering entire delay propagation with a column generation approach

As a further extension of this work, a specialized solution algorithm is presented. It enables a consideration of entire delay propagation during optimization. The aim is to find the optimal solution that might not necessarily be found with the prior (heuristic) solution approach.

6.7.1 Introduction

In the former model, delay propagation was only considered from one service trip to the next following service trip, but not entirely over several service trips. Implementing entire delay propagation was not possible because of the modeling, so that an overestimation during the optimization was used to compensate the lack of considering entire delay propagation.

To consider entire delay propagation a mathematical model that uses paths instead of trips as variables is created. Then for each path through the network the delays in each scenario and the resulting penalty costs can be calculated at the beginning of every service trip, so that entire delay propagation can be considered in the optimization model.

Enumerating all paths from the first node on the day to the last node on the day would lead to an astronomically large number of variables. Because the resulting model would even for very small instances be unsolvable, a specialized solution method is necessary.

As solution method, column generation is used. Column generation is a method that begins with a small subset of variables that contain at least one feasible solution and adds variables to the master problem until the optimal solution is found. These variables are derived in the pricing that uses the dual solution of the master problem.

Solution approaches with column generation algorithms have been used in the context of public transport by Borndörfer et al. [124] for the line planning problem, by Desrochers and Soumis[125] for the crew scheduling problem and by Ribeiro and Soumis[126] for the vehicle scheduling problem, for example. Steinzen et al.[127] present an integrated model for vehicle and crew scheduling. Column generation will not be described in detail in this thesis. For an introduction see Wolsey[128].

6.7.2 Master Problem

The Master problem is:

Sets:

P	Set of paths in the network layers (from and to first arc in morning, including circulation arc as last arc)

Parameters:

$\delta_{p,f}$	1 if path p serves service trip f, 0 otherwise
$cost_p$	Expected costs of path p including operational costs and penalty costs

Variables:

z_p	Flow on path p (binary variable)

124 [BGP07]

125 [DeSo89]

126 [RiSo94]

127 [SGSK10]

128 [Wol98] p. 185ff

Objective function:

$$\min \sum_{p \in P} z_p \cdot cost_p \qquad (27.1)$$

Constraints:

$$\sum_{p \in P} z_p \cdot \delta_{p,f} = 1 \qquad \forall f \in F \qquad (27.2)$$

$$z_p \epsilon \{0,1\} \qquad \forall p \in P \qquad (27.3)$$

As starting variables, the optimal solutions for total expected costs for different values of α were used. The values were 600s, 900s, 1200s, 1800s and 2700s.[129]

6.7.3 Pricing

In the pricing, new paths are created and added to the master problem. The network is updated with the dual values of the solution of the master problem. The reduced costs for each service trip arc are:

$$\begin{aligned} reducedCosts_{\,trip} &= Expected\ Costs_{trip}\ (depending\ on\ earlier\ trips\ on\ the\ path) \\ &- dual\ value\ of\ corresponding\ row\ in\ master_{trip} \qquad (28.1) \end{aligned}$$

The expected costs are the operational costs plus the expected penalty costs of all delay scenarios. The penalty costs of a service trip in a delay scenario depend on the path on which the service trip is served, as the penalty is a function of the time the service trip has been delayed (which depends on the propagated delays of earlier service trips).

Therefore a simple version of Dijkstra's Algorithm[130] cannot be used, as it cannot consider different arc costs of the same arc depending on the path on which the arc was arrived. A modified version that updates and saves a certain set of parameters on each node was used. The parameters saved on each node were the propagated delays

[129] See chapter 6.5.3

[130] See [CERS01] p. 595ff for a description of Dijkstra's Algorithm

for each scenario and the predecessor node on the path on which the node was arrived.

As this modified version of Dijkstra's Algorithm only adds one variable in each pricing-iteration, another algorithm called Bellman-Ford[131] that adds several variables with different path-lengths was also implemented. This algorithm was selected because it finds short and long paths and therefore very different paths.

6.7.4 Results

The solutions found with the prior approach could not be improved by the column generation algorithm within reasonable time: The objective value did not improve within 12 hours for several schedules. As the penalty costs of the solutions found with the prior approach are only a very small percentage of total costs, the optimal solution might have been found by the prior approach. That shows that the stochastic programming approach produces very good solutions that are suitable in practice, when real delay scenarios of past days are used and there are no substantial changes in the road network or timetable.

6.8 Conclusion and outlook

We have shown that stochastic programming for the vehicle scheduling problem with disruptions leads to superior solutions compared to other approaches. If delay-propagation is not considered, stochastic programming finds the optimal solution for the given data, which outperforms a simple approach that adds fixed buffer times to the schedule. We created a set of pareto-optimal solutions in terms of maximum robustness (measured as CVaR) and minimum total costs.

When entire delay propagation is considered, the solutions of stochastic programming are still superior compared to a simple approach that adds fixed buffer times, although this simple approach can decrease total costs significantly compared to the cost-optimal solutions. A small overestimation of the delay costs in the developed stochastic optimization model is appropriate to compensate the heuristic delay prop-

[131] See [CERS01] p. 588ff for a description of the Bellman-Ford Algorithm

agation in the model. Using another scenarioset for evaluation has confirmed the applicability of the stochastic programming approach. Furthermore, we have seen that optimizing a risk measure is still applicable, when entire delay propagation is considered, but it highly depends on the nature of the specific scenarios considered in the risk measure.

Moreover, an extension of the model with weather derivatives was proposed and it was shown that the costs for gaining the same robustness level with stochastic programming could further be decreased if weather leads to disruption scenarios that are considered in the risk measure.

The integration of this model with a column generation approach could not improve the solutions found. That shows that although the solution approach is heuristic because of not considering entire delay propagation, the solutions found are of very high quality.

Finally, it was shown that the scenarios in the optimization model and the consideration of a quadratic penalty function do not add significant complexity to the optimization model. This is done with a reformulation of the optimization model and the calculation of the penalty costs in the network model. This necessitates a network with all connecting arcs, which is computationally more complex than the traditionally used time-space network in this application.

Despite the increased computational complexity, real instances can be solved in reasonable time. For our calculations we used instances of small- and medium-sized German cities with few network layers and with several hundred service trips. Instances of larger cities or large metropolitan areas can up to now not be solved with our approach. When more computational power, more memory and more efficient solution algorithms for MIPs will be available, this model will also be solvable for timetables of larger cities.

7 Rota scheduling in public transport under uncertainty

This chapter introduces a new stochastic programming approach for robust rota scheduling in public transport. [132]

7.1 Motivation

In public bus transport, the reserve shifts currently are evenly planned for all drivers without considering more detailed information such as historical or weekday-depended sickness absence rates. In case that the absence rate exceeds the available reserve personnel, usually additional drivers are called manually. This causes discontent for the drivers as well as a continuous organizational effort for the bus company. In order to reduce the discrepancy between a planned roster and the actual one, we present a new stochastic optimization model for rota scheduling in public bus transport. In addition to the present reserve shifts, optional reserve shifts are introduced.

7.2 Problem description

The rota scheduling problem assigns drivers to a set of crew duties. Each duty has its corresponding shift type (for example early shift). For each group of drivers a cyclic-based rota is developed in the rota scheduling problem. The given data is the set of shifts and free days as well as reserve shifts; management considerations, labor laws and the preferences of drivers have to be considered. All drivers in one group have the same qualification and work preferences.

Each column in the rota represents one weekday. The rows represent the weeks. A rota has as many rows as drivers in the group. Each driver begins in one row (the first driver in the first row, the second in the second row etc.). After every driver has accomplished the whole rota, they again begin at their starting point.

As every driver in a group has to accomplish the whole rota and thus has to do the same work, such a cyclic-based rota is very fair and therefore widely adopted. The time period for that a rota is used, is mostly a longer period.

[132] Parts of this chapter have been published in [XNS12]

The reserve shifts are nowadays usually planned as a fixed number or percentage. As absences due to illness are normally not foreseeable, it can happen that there are more drivers absent than reserve personnel is available. In this case, usually additional drivers are called manually at short notice. This inaccurate planning of reserve shifts has several disadvantages:

- It is an additional constant task to manage the reserves for every day for the company.
- It causes more costs for the reserves than necessary.
- It causes discontent for the drivers.

A new stochastic optimization model is therefore formulated for the rota scheduling problem and is compared to deterministic optimization. The scenarios of the stochastic program are created on a basis of historical and weekday-dependent sickness absence rates. In addition to the present reserve shifts, optional reserve shifts are introduced. This integration of contractual risk management offers better working-conditions for the reserve drivers and enables a specific reaction on the sickness absence rate of the day by exercising the optional reserves. Optional reserves can, but do not have to be exercised.

Thus, instead of either planning a large number of costly reserves or continuously calling drivers at short notice, the best combination of traditional present reserves for covering normal absence rates and optional reserves for handling exceptional high absence rates can be used.

7.3 Model

7.3.1 Case study

The case study used for our calculations was freely invented, but has the structure of real data used for the rota scheduling problem. We use 10 different shift types, 6 shift types with different starting and ending times during the day, 2 shift types in the night and 2 split shift types. A total number of 220 drivers are grouped into 3 rota groups that represent the desired shift types depending on the weekday.

We add 50 different scenarios to the stochastic optimization model. For calculating them, we use the historical sickness absence rate of Germany. The average value for the sickness absence rate of the years 2000-2009 is 3.64%[133]. We multiply this data with a factor depending on the weekday to consider that in a bus company historically the absence rate was higher or lower on different weekdays. These weekday-dependent numbers are used as mean values for a Normal-distribution from which we select a random number (random values <0 are set to 0). One week with different sickness absence rates for the weekdays is one scenario.

The costs for assigning undesired shift types, reserve shifts and optional reserve shifts as well as for exercising optional reserve shifts are different on weekdays and weekends. Nonetheless, the costs for the reserve types satisfy the following inequality on every weekday:

$$\begin{aligned} &cost(planned\ optional\ reserve) \leq cost(present\ reserve) \\ &\qquad \leq cost(exercised\ optional\ reserve) \qquad (29.1) \end{aligned}$$

The percentage of the worst scenarios considered in the CVaR is 10%.

7.3.2 A two-stage stochastic optimization model

The stochastic optimization model in its extensive form is presented in this chapter. The sets and parameters are:

ST	Set of different shift types
G	Set of rota-groups / driver groups
$Scen$	Scenarioset
a_g	Number of weeks for group g
$s_{d,t}$	Number of shifts of type d to assign on weekday t (sum over all weeks)
$abs_{t,s}$	Number of absent drivers on weekday t in scenario s

133 Calculated with data from the German Federal Ministry of Labour and Social Affairs [BMAS10]

u_d	(Average) working time in minutes (duration) of shift type d
b_d	Starting time of shift type d in minutes
e_d	Ending time of shift type d in minutes
ur	(Average) working time (duration) of a reserve shift
br	Starting time of a reserve shift
er	Ending time of a reserve shift
aw	Maximum total work time per week per driver
$c_{d,g,t}$	Penalty cost if shift type d is not a desired shift type of group g on weekday t, 0 otherwise
$cr_{g,t}$	Cost for one present reserve shift in group g on weekday t
$co_{g,t}$	Cost for one optional reserve shift in group g on weekday t (without exercising)
$coe_{g,t}$	Additional cost for exercising the optional reserve shift in group g on weekday t
cu	Penalty cost for an unassigned shift and for an outage due to reserve shortage
rd	Minimum rest period between two consecutive shifts
rw	Minimum rest period per week
sf_g	Minimum number of single free days for group g
$dfmin_g$	Minimum number of double free days for group g
$dfmax_g$	Maximum number of double free days for group g
$wdmin_g$	Minimum number of working days for group g
$wdmax_g$	Maximum number of working days for group g
fd_g	Maximum number of free days for group g
α	Probability value for the CVaR

$day(w, t, i)$ Function that returns the tupel (week w', day t') that represents the 'date' i days after day t in week w

Bookkeeping variables and auxiliary variables:

$f2_{g,w,t}$ Equal to 1, if a two-day free period begins on day t in week w in group g

$f1_{g,w,t}$ Equal to 1, if a one-day free period begins on day t in week w in group g (not including two-day free periods)

$cost_s$ Costs in scenario s

$cvar$ Conditional Value at Risk

$x0$ Auxiliary variable needed to integrate the CVaR

x_s Auxiliary variables needed to integrate the CVaR

Decision Variables:

Stage 1:

$y_{d,g,w,t}$ Equal to 1, if shift type d is assigned to weekday t in week w in group g, 0 otherwise

$yf_{g,w,t}$ Equal to 1, if weekday t in week w in group g is a free day, 0 otherwise

$yr_{g,w,t}$ Equal to 1, if weekday t in week w in group g is a present reserve shift day, 0 otherwise

$yo_{g,w,t}$ Equal to 1, if weekday t in week w in group g is an optional reserve shift day, 0 otherwise

Stage 2:

$yoe_{g,w,t,s}$ Equal to 1, if the optional reserve on weekday t in week w in group g is used in scenario s

$yun_{t,s}$ Number of outages on day t in scenario s due to lack of available reserve

The mathematic model is described as follows. The objective function minimizes the expected costs or the Conditional Value at Risk

$$\min \frac{1}{|Scen|} * \sum_{s \in Scen} cost_s \qquad (30.1)$$

or

$$\min cvar \qquad (30.2)$$

s.t.

Only one/exactly one shift type/free day/reserve type is assigned to a day:

$$yf_{g,w,t} + yr_{g,w,t} + yo_{g,w,t} + \sum_{d \in ST} y_{d,g,w,t} = 1 \quad \forall g \in G, t = 1, \dots, 7, w = 1 \dots a_g \qquad (30.3)$$

No overassignment of shift types:

$$\sum_{g \in G} \sum_{w \in \{1..a_g\}} y_{d,g,w,t} \le s_{d,t} \quad \forall d \in ST, t = 1, \dots, 7 \qquad (30.4)$$

Minimum rest periods between each two sequencing duties within one group:

$$1440 - \sum_{d \in ST} y_{d,g,day\,(w,t,-1)} * e_d - yr_{g,day\,(w,t,-1)} * er + \sum_{d \in ST} y_{d,g,w,t} * b_d + yr_{g,w,t} * br + 1440 * \left(yf_{g,day\,(w,t,-1)} + yo_{g,day\,(w,t,-1)} + yf_{g,w,t} + yo_{g,w,t} \right) \ge rd \quad \forall g \in G, t = 1, \dots, 7, w = 1 \dots a_g \qquad (30.5)$$

Upper bound of the working time for each driver per week:

$$\sum_{t \in \{1..7\}} \left(ur * yr_{g,w,t} + ur * yo_{g,w,t} + \sum_{d \in ST} u_d * y_{d,g,w,t} \right) \le aw \quad w = 1, \dots, \alpha_g, \forall g \in G \qquad (30.6)$$

Minimum weekly rest period:

$$\sum_{t\in\{1..7\}}\left(ur * yr_{g,w,t} + ur * yo_{g,w,t} + \sum_{d\in ST} u_d * y_{d,g,w,t}\right) \leq 7 * 1440 - rw$$

$$w = 1, \dots, \alpha_g, \forall g \in G \qquad (30.7)$$

Maximum number of possible working days:

$$\sum_{w\in\{1..a_g\}}\sum_{t\in\{1..7\}}\left(yr_{g,w,t} + yo_{g,w,t} + \sum_{d\in ST} y_{d,g,w,t}\right) \leq wdmax_g \ \forall g \in G \qquad (30.8)$$

Minimum number of working days:

$$\sum_{w\in\{1..a_g\}}\sum_{t\in\{1..7\}}\left(yr_{g,w,t} + \sum_{d\in ST} y_{d,g,w,t}\right) \geq wdmin_g \ \forall g \in G \qquad (30.9)$$

Maximum number of possible free days:

$$\sum_{w\in\{1..a_g\}}\sum_{t\in\{1..7\}} yf_{g,w,t} + yo_{g,w,t} \leq fd_g \ \forall g \in G \qquad (30.10)$$

Single free days:

$$\left(1 - yf_{g,w,t}\right) + yf_{g,day\,(w,t,1)} + yf_{g,day\,(w,t,-1)} + f1_{g,w,t} \geq 1 \quad \forall g \in G, t = 1, \dots, 7, w = 1 \dots a_g \qquad (30.11)$$

$$\left(1 - f1_{g,w,t}\right) + yf_{g,w,t} \geq 1 \quad \forall g \in G, t = 1, \dots, 7, w = 1 \dots a_g \qquad (30.12)$$

$$\left(1 - f1_{g,w,t}\right) + (1 - yf_{g,day\,(w,t,1)}) \geq 1 \quad \forall g \in G, t = 1, \dots, 7, w = 1 \dots a_g \qquad (30.13)$$

$$\left(1 - f1_{g,w,t}\right) + (1 - yf_{g,day\,(w,t,-1)}) \geq 1 \quad \forall g \in G, t = 1, \dots, 7, w = 1 \dots a_g \qquad (30.14)$$

Double free days:

$$\left(1-yf_{g,w,t}\right)+\left(1-yf_{g,day\,(w,t,1)}\right)+f2_{g,w,t}\geq 1 \quad \forall g\in G, t=1,\dots,7, w = 1\dots a_g \qquad (30.15)$$

$$\left(1-f2_{g,w,t}\right)+yf_{g,w,t}\geq 1 \quad \forall g\in G, t=1,\dots,7, w=1\dots a_g \qquad (30.16)$$

$$\left(1-f2_{g,w,t}\right)+yf_{g,day\,(w,t,1)}\geq 1 \quad \forall g\in G, t=1,\dots,7, w=1\dots a_g \qquad (30.17)$$

Minimum number of single free days:

$$\sum_{t\in\{1..7\}}\sum_{w\in\{1..a_g\}} f1_{g,w,t}\geq sf_g \ \forall g\in G \qquad (30.18)$$

Minimum number of double free days:

$$\sum_{t\in\{1..7\}}\sum_{w\in\{1..a_g\}} f2_{g,w,t}\ \geq dfmin_g \ \forall g\in G \qquad (30.19)$$

Maximum number of double free days:

$$\sum_{t\in\{1..7\}}\sum_{w\in\{1..a_g\}} f2_{g,w,t}\ \leq dfmax_g \ \forall g\in G \qquad (30.20)$$

At least one double free in two weeks:

$$\sum_{j=1}^{14} f2_{g,day\,(w,0,j)}\ \geq 1 \ \ w=0,\dots,a_g, \forall g\in G \qquad (30.21)$$

Reserve usage:

$$yun_{t,s}+\sum_{g\in G}\sum_{w\in\{1..a_g\}} yoe_{g,w,t,s}+yr_{g,w,t}\ \geq abs_{t,s}\ \forall\ t=1,\dots,7, s\in Scen \quad (30.22)$$

Only use available options:

$$yoe_{g,w,t,s}\leq yo_{g,w,t}\ \forall g\in G, t=1,\dots,7, w=1\dots a_g, s\in Scen \qquad (30.23)$$

Total costs in scenarios:

$$cost_s = \sum_{d\in ST} \sum_{t\in\{1..7\}} \left(s_{d,t} - \sum_{g\in G} \sum_{w\in\{1..a_g\}} y_{d,g,w,t} \right) * cu$$
$$+ \sum_{g\in G} \sum_{t\in\{1..7\}} \sum_{w\in\{1..a_g\}} \sum_{d\in ST} y_{d,g,w,t} * c_{d,g,t}$$
$$+ \sum_{g\in G} \sum_{t\in\{1..7\}} \sum_{w\in\{1..a_g\}} yr_{g,w,t} * cr_{g,t}$$
$$+ \sum_{g\in G} \sum_{t\in\{1..7\}} \sum_{w\in\{1..a_g\}} yo_{g,w,t} * co_{g,t}$$
$$+ \sum_{g\in G} \sum_{t\in\{1..7\}} \sum_{w\in\{1..a_g\}} yoe_{g,w,t,s} * coe_{g,t} + \sum_{t\in\{1..7\}} yun_{t,s} * cu \; \forall s \in Scen \qquad (30.24)$$

CVaR-Integration:

$$x0 + x_s \geq cost_s \; \forall s \in Scen \qquad (30.25)$$

$$cvar = \frac{1}{\alpha}\left(\alpha \cdot x0 + \sum_{s\in Scen} x_s \cdot \frac{1}{|Scen|} \right) \qquad (30.26)$$

Binary variables, integer variables and bounds for variables:

$$y_{d,g,w,t} \in \{0,1\} \; \forall g \in G, t = 1, \dots, 7, w = 1 \dots a_g, d \in ST \qquad (30.27)$$

$$f2_{g,w,t} \in \{0,1\} \; \forall g \in G, t = 1, \dots, 7, w = 1 \dots a_g \qquad (30.28)$$

$$f1_{g,w,t} \in \{0,1\} \; \forall g \in G, t = 1, \dots, 7, w = 1 \dots a_g \qquad (30.29)$$

$$yf_{g,w,t} \in \{0,1\} \; \forall g \in G, t = 1, \dots, 7, w = 1 \dots a_g \qquad (30.30)$$

$$yr_{g,w,t} \in \{0,1\} \; \forall g \in G, t = 1, \dots, 7, w = 1 \dots a_g \qquad (30.31)$$

$$yo_{g,w,t} \in \{0,1\} \; \forall g \in G, t = 1, \dots, 7, w = 1 \dots a_g \qquad (30.32)$$

$$yoe_{g,w,t,s} \in \{0,1\} \; \forall g \in G, t = 1, \dots, 7, w = 1 \dots a_g, \forall s \in Scen \qquad (30.33)$$

$$yun_{t,s} \in \mathbb{N}_0 \; \forall t = 1, \dots, 7, \forall s \in Scen \qquad (30.34)$$

$$x_s \geq 0 \; \forall s \in Scen \qquad (30.35)$$

The constraint (30.3) ensures that for every driver on every day in every week either exactly one shift or reserve type is assigned or the day is a free day. The inequality (30.4) prevents that more shifts than needed are assigned, while (30.5) implements the minimum rest period between two consecutive shifts. With the constraints (30.6) and (30.7) the maximum working time per week and the minimum rest period per week are considered. The inequalities (30.8) and (30.9) bound the maximum and minimum number of working days per rota before (30.10) limits the maximum number of possible free days in the rota. The set of constraints (30.11), (30.12), (30.13) and (30.14) implement the correct calculation of single free days; the set (30.15), (30.16) and (30.17) do the same for double free days. These constraint groups have been derived by applying propositional logic. Single free days are only counted if the day before and the day after are not free while a double free is also counted if the days before and after are free days. Therefore a period with three free days is counted as two double frees, a period with four consecutive free days as three double frees etc. The total single frees and double frees per rota are constrained with the inequalities (30.18), (30.19) and (30.20), while (30.21) requires at least one double free every two weeks. The constraints (30.5)-(30.21) implement the legal and firm-specific work regulations and may be different depending on the country and company. The constraint (30.22) assigns the reserve usage and the outages due to reserve shortage for each scenario. The inequality (30.23) ensures that only planned optional reserve shifts are used. The equality (30.24) calculates the total costs for each scenario before (30.25) and (30.26) finally implement the Conditional Value at Risk.

7.4 Results

7.4.1 Advantages of the stochastic model

We begin with the evaluation of the stochastic optimization model by introducing the expectation of the expected value problem (EEV-solution). For this, we solve a model with expected values to obtain the EV-solution and evaluate this solution with all scenarios. The weighted average is the EEV-solution.

We first do not consider optional reserve shifts (from now on also called options) and minimize costs. Table 14 shows the results.

	Exp. Costs	CVaR	Worst Scen.	Best Scen.
EEV (no options)	66800	122200	142200	12200

Table 14 EEV Solution without options

The EV-solution has costs of only 12200 after optimization, but when the solution obtained is evaluated with the full scenarioset this solutions has expected costs of 66800. That shows that using expected values and deterministic optimization leads to a very bad approximation of real costs.

We now compare the EEV-solution with the stochastic solution (here-and-now solution). Table 15 shows the results.

	Exp. Costs	CVaR	Worst Scen.	Best Scen.
EEV (no options)	66800	122200	142200	12200
Stochastic solution (no options)	19200	21000	29000	19000

Table 15 EEV and here-and-now comparison

The results show that stochastic optimization can reduce the expected costs to 28.7%, the VSS is 47600. Also the robustness in terms of CVaR and worst scenario of the solution is greatly increased. Deterministic optimization with average values can therefore not compete with stochastic optimization.

We now allow optional reserve shifts and again compare the results. The results are shown in Table 16.

	Exp. Costs	CVaR	Worst Scen.	Best Scen.
Stochastic solution (no options)	19200	21000	29000	19000
Stochastic solution (with options)	14891	16525	16825	12625

Table 16 Stochastic solutions with and without options

Adding optional reserve shifts to the model can again significantly decrease expected costs and increase robustness simultaneously. The costs are decreased by 22.4% and the CVaR is decreased by 21.3%.

Up to now we only have optimized the expected costs. To check if a further improvement in robustness is possible, we minimize the CVaR and show all pareto-optimal solutions with and without options in Figure 52.

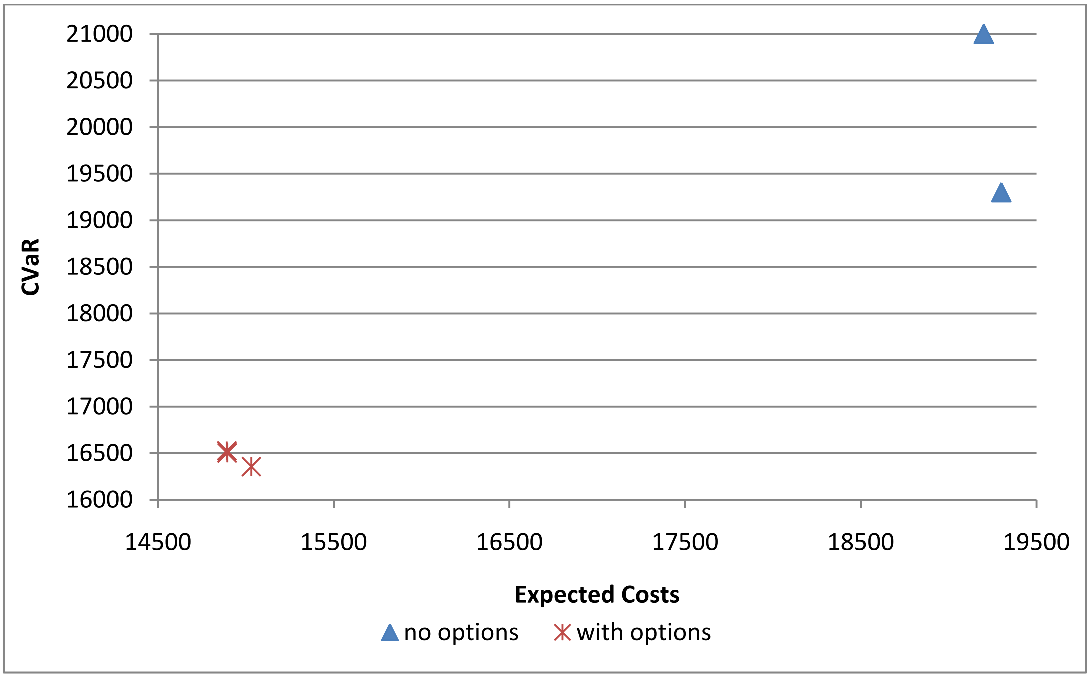

Figure 52 Pareto-optimal solution sets

We see that with and without options a further improvement in robustness is possible when costs are slightly increased. Without options a significant decrease of 1700 is possible with a cost increase of only 100. With options the CVaR can be decreased by 170 with a cost increase of 140, or may also be slightly increased by 22.75 with a cost increase of only 2.

As a result we can conclude that using optional reserve shifts leads to better solutions in terms of expected costs and robustness simultaneously. The solutions can be slightly adjusted by restricting CVaR to levels between the cost-optimal solution and the maximum CVaR.

At last, we compare the here-and-now solutions with the wait-and-see solutions with and without optional reserve shifts and calculate the EVPI in order to examine the impact of uncertainty on the objective value. Table 17 shows the results.

	No options	With options
Here-and-now	19200	14891
Wait-and-see	12154	12154
EVPI	7046	2737

Table 17 EVPI with and without options

The expected costs of both wait-and-see solutions have the same value because using optional reserve shifts is more expensive than any other shift type. In the wait-and-see models we can make an independent decision for every scenario, because we assume that we can anticipate the scenario because of having perfect information. Therefore, optional reserve shifts are not used under this unrealistic assumption. The wait-and-see models are only useful to assess the value of perfect information, if it was available.

We also can see that the EVPI is much lower in the model with options. That shows that optional reserve shifts are an efficient method to counteract uncertainty and lead to better solution quality.

7.4.2 Reserve usage

We now take a closer look at the reserve usage of the stochastic solutions without options. We compare the solution with the best Conditional Value at Risk and the cost-optimal solution.

Objective	Expected costs	CVaR
Options allowed	no	no
Expected costs	19200	19300
Present reserves	63	64
Outages due to reserve shortage	1	0

Table 18 Reserve usage comparison without options

The cost-optimal solution uses 63 present reserves while the solution with the best CVaR has 64 present reserves. With this additional reserve, the costs of one or several of the expensive scenarios are improved, so that the CVaR decreases. The cost-optimal solution causes one outage although that causes penalty costs that are between 33.33 times and 50 times higher than one reserve (depending on the weekday).

We now add the solutions with options to the table to analyze them.

Objective	Exp. costs	CVaR	Exp. costs	CVaR
Options allowed	no	no	yes	yes
Expected costs	19200	19300	14891	15031
Present reserves	63	64	22	28
Options	-	-	42	36
Exercised Options (avg. per scenario)	-	-	11.62	7.68
Outages due to reserve shortage	1	0	0	0

Table 19 Reserve usage comparison with and without options

The results show that present reserves are significantly less used when options are available. Because options enable a specific reaction on the scenario, present reserves are not needed for days where further absences are unlikely: These can be covered by options, what is altogether cheaper. This explains the decrease of the expected costs.

Furthermore, the outage disappears. Concluding, it is inefficient to have many outages: Only one outage in one solution is optimal; all other absences are covered by reserve shifts.

The sum of present reserves and average exercised options, which denotes the number of average used reserves, also decreases significantly. This is possible because of the additional flexibility of optional reserve shifts. Figure 53 shows a comparison.

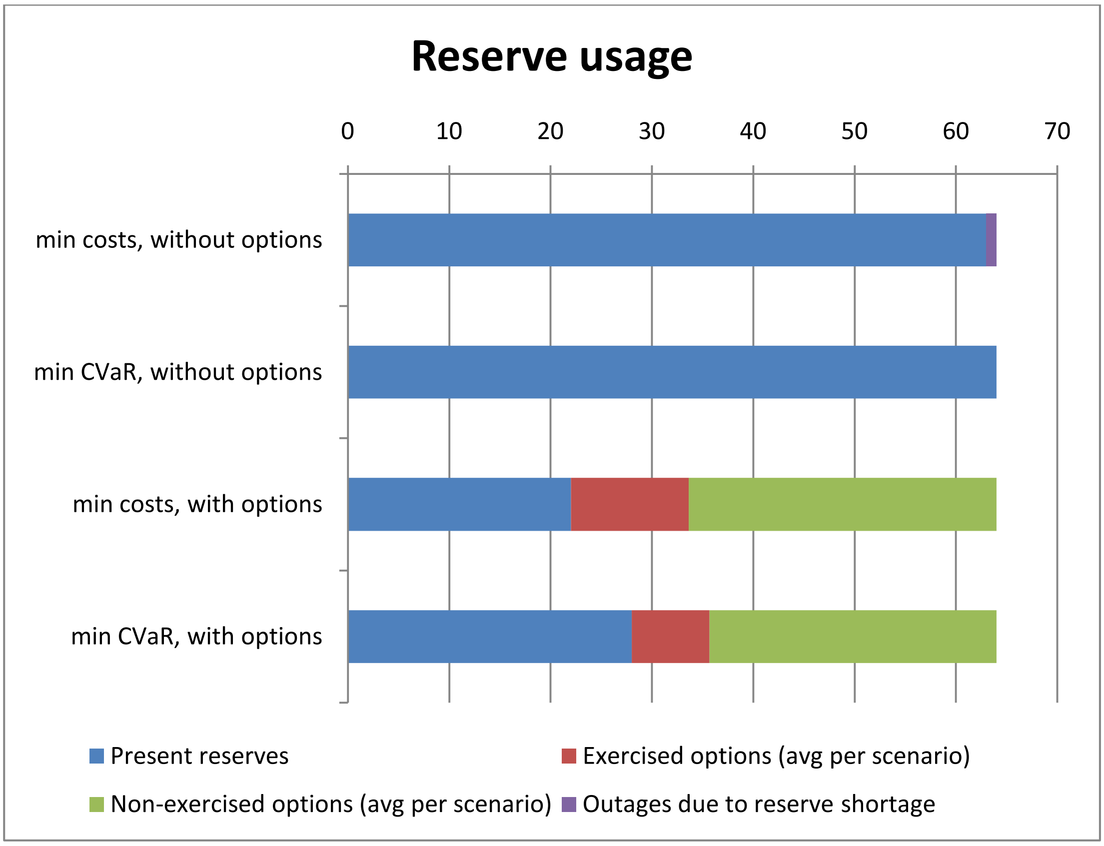

Figure 53 Reserve usage comparison

We see that the number of non-exercised options, which are relatively cheap, is higher than the number of exercised options, which are relatively expensive. This again shows the decrease of expected costs.

The solution with options where the CVaR is optimized uses more present reserves compared to the cost-optimal solution with options, because then in more scenarios the cheaper present reserves can be used so that less options need to be exercised in costly scenarios what results in a solution where the values of the costs of the scenarios are less wide spread. Therefore the CVaR is better. On the other hand, due to the usage of more present reserves, which are not needed in all scenarios, the expected costs of this solution increase.

7.4.3 Evaluation with another scenarioset

In our preliminary results, we used the same scenarioset for optimization and evaluation. But as we can only use a limited number of scenarios during the optimization and therefore not include all integer combinations of different sickness absence rates for all weekdays and future scenarios might be different to those used during optimization, we have to evaluate the solutions with another scenarioset to validate the solutions found with the stochastic optimization model.

To do this, we solve the stochastic model with the scenarioset for optimization, fix all stage-1 variables to the values of the optimal solution and then optimize the model again with the other scenarioset for evaluation. Now the only unfixed decision variables are the stage-2 variables that are the recourse decisions for the scenarios. Thus, only the planned optional reserves can be used to adjust the solution to the scenarioset used for evaluation.

These solutions are now compared with the EEV-solution, which is also evaluated with the other scenarioset. Figure 54 shows the expected costs and the CVaR of the solutions found; it compares these values and shows their changes due to the evaluation with the other scenarioset.

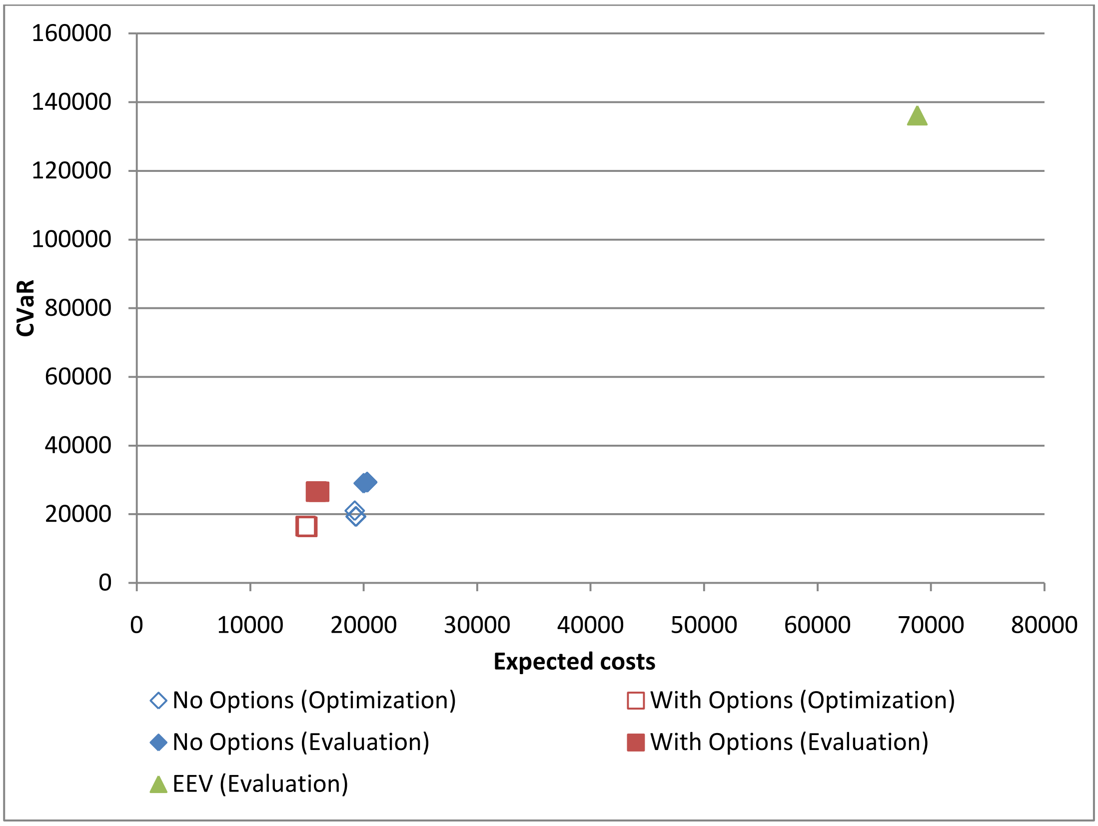

Figure 54 Solutions evaluated with other scenarioset

We can see that although the costs of the stochastic solutions with options increase after simulating them with the other scenarioset, they are by far better than the deterministic EEV-solution. Even the stochastic model without using options is significantly better than a deterministic solution. This is because the EEV-solution does not use any information about the scenario distribution. A further drawback that results from this fact is that the deterministic solution does not use options although it may do: Using options is more expensive than using present reserves to cover absences and as there is only one scenario in the deterministic model, it only uses present reserves.

If we now take a closer look at the costs and the CVaR of the solutions before and after the simulation, we spot that after the simulation some solutions are dominated. Table 20 and Table 21 show these values for the solutions; dominated solutions are italic.

Costs before simulation	CVaR before simulation	Costs after simulation	CVaR after simulation
19200	21000	20000	29000
19300	19300	*20300*	*29300*

Table 20 Solutions without options before and after simulation

Regarding the solutions without options, the solution with a better CVaR after the optimization has higher costs and a worse CVaR after simulating it with the other scenarioset. Therefore it is dominated.

Costs before simulation	CVaR before simulation	Costs after simulation	CVaR after simulation
14891	16525	15793	26565
14893	16475	*15835*	*26635*
15031	16355	16055	26555

Table 21 Solutions with options before and after simulation

The same can be observed at the second solution in Table 21, where options are allowed. This is because by restricting or optimizing the CVaR, additional costs are used to change the worst scenarios of the scenarioset. By changing the scenarioset, other scenarios are considered in the CVaR, so that the additional costs may not improve the worst scenarios in the other scenarioset. Nonetheless, the last solution of Table 21 is just as good as to be not dominated. Therefore, some common properties in the scenariosets seem to exist, that enable slightly better CVaRs after evaluation with another scenarioset.

7.5 Conclusion

A new stochastic optimization model for rota scheduling in public transport with optional reserve shifts was developed. The stochastic model outperforms the deterministic optimization model significantly. The expected costs could be decreased to 29% by using stochastic optimization.

It is possible to solve stochastic models with data-instances of real size. The integration of optional reserve shifts was also possible without a significant increase of solution times. This integration as a method of contractual risk management could further increase solution quality and robustness simultaneously.

Finally, the solutions were validated with simulation: The stochastic solutions with and without options will find good solutions independently from the scenarioset, as costs only slightly increase after simulation. Deterministic solutions led to excessive high costs and can therefore not compete with the stochastic solutions. This new approach developed is therefore better as it produces cheaper solutions than deterministic optimization. Furthermore, it reduces the daily managerial burden of bus companies as plans fit better to the real situation and additional reserve drivers do not have to be called at short notice anymore.

8 Conclusion and outlook

The aim of this thesis was to create better models for robust planning under uncertainty in scheduled passenger traffic by implementing an integrated risk management strategy into the models of the planning process. First, the planning process with its uncertainties was introduced, the relevant literature was reviewed and open research questions were discussed in Chapter 2.

The foundations for this thesis were introduced in Chapter 3. The terms risk and robustness were defined, risk management was explained and the advantages of an integrated risk management strategy were discussed. Also the methods for optimization under uncertainty were reviewed and stochastic programming, as the most suitable method for this work, was introduced in detail.

The open research questions were treated in the Chapters 4 - 7. For each developed model realistic case studies were used as data for the optimization. In Chapter 4 a new model for robust airline schedule design under fuel price and demand uncertainty was developed. The impacts of different fuel price scenarios on optimal schedule design were shown. Furthermore the integration of financial hedging instruments into the operational planning enabled best profits at every robustness level.

A new model for robust airline re-fleeting under fuel price and demand uncertainty was introduced in Chapter 5. The fuel price scenarios also had a significant impact on optimal fleet assignment. Again, only the integrated planning of financial hedging instruments and operational planning could guarantee the best profit for all risk levels.

Chapter 6 showed a new approach for robust vehicle scheduling in public bus transport. A new stochastic programming model was developed. Although this model is only heuristic, the simulation and integration of a column generation approach could prove that it produces good results for robust planning under real conditions. Furthermore, a promising extension that integrated weather derivatives was proposed, but real data on the impact of weather on delays in public bus transport is necessary to score the effectiveness of this extension.

Chapter 7 finally presented a model for rota scheduling under uncertain illness absence rates in public transport. The results show that an explicit consideration of historical illness absence rates in a stochastic optimization model leads to better results than deterministic optimization. The integration of contractual risk management via optional reserve shifts could again create superior solutions in terms of robustness and costs.

Therefore all open research questions found were treated. In every case, a stochastic optimization model with an integrated risk management strategy was used to show that robust planning could be improved. Every time a set of pareto-optimal solutions in terms of profit or costs and robustness, including solutions from the most robust solution to the solution with optimal costs/profit, was created. The maximum robustness could be increased by integrating contractual risk management in all four models.

All models were implemented in C# and solved as deterministic equivalent with Cplex 11[134]. As they offer precise evaluation, stochastic programming models with an integrated risk management strategy were a suitable method to enable better risk-aware planning in scheduled passenger traffic. Although computational power still prevents solving large-scale instances to optimality, it was possible to solve real small and medium-sized instances of the stochastic optimization models developed. This shows their applicability. Thus, stochastic optimization was an appropriate method to show the tradeoffs between robustness and profit-optimality.

In future, when more computationally powerful computers and better software for solving stochastic optimization models might be available, also large instances may be solvable. It may also be possible to create more specialized solution methods for the models developed in this thesis in order to solve larger instances.

Another topic for future research is the scenario generation and the sensitivity analysis of the whole models developed, if the distributions for the uncertain parameters are unknown. Like for most stochastic programming applications, it was assumed that at least the distributions for the uncertain parameters are known or that real scenarios exist and can be used for optimization, as argued in chapter 6. If this is not the case, the models developed could be optimized with one distribution and evaluated

[134] Product of IBM ILOG [IBM]

with several other distributions to score if they also produce good results under these circumstances.

Furthermore it may be possible to integrate several planning phases into one stochastic optimization model to take advantage of potential interactions between planning phases in future. This could, for example, be done for airline schedule design and airline re-fleeting to take advantage of interactions between network planning and fleet assignment under uncertainty. In addition, several booking classes like economy, business or first class could be added to these models.

The integration of several planning phases is already done with deterministic optimization models in this field of application and often shows good results, but the development of applicable approaches for large real problem instances is still a task for future research because of the highly increased computational complexity.

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