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## **Die Statik im Stahlbetonbau**

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Tabellen für die Stütz- und Schnittkräfte des einfachen Balkenträgers und  
des Freitragers

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Tabelle 6. Balken auf zwei Stützen.

$l$ : Stützweite,  $\xi = x/l$ ,  $\xi' = x'/l$ ;  $A, B$ : Stützkräfte;  $Q_m$ : Querkraft;  $M_m$ : Biegemoment im Querschnitt  $m$ ;  $x_0$ : Querschnitt mit dem größten Biegemoment  $M_{\max}$ .

Jede Abbildung zeigt der Reihe nach die Art der Belastung und die Zustandslinien für die Querkraft  $Q_m$  und das Biegemoment  $M_m$ .

	$A = \frac{Pb}{l} \quad B = \frac{Pa}{l} \quad Q_1 = +\frac{Pb}{l} \quad Q_2 = -\frac{Pa}{l}$ $M_1 = Pb\xi \quad M_2 = Pa\xi'$ $x_0 = a \quad M_{\max} = \frac{Pab}{l}$
	$A = \frac{pl}{2} \quad B = \frac{pl}{2} \quad Q = \frac{pl}{2}(1 - 2\xi)$ $M = \frac{pl^2}{2}\omega_R$ $x_0 = \frac{l}{2} \quad M_{\max} = \frac{pl^2}{8} = 0,125pl^2$
	$A = \frac{pc}{2l}(2n+c) \quad B = \frac{pc}{2l}(2m+c)$ $Q_1 = A \quad Q_2 = A - p(x-m) \quad Q_3 = -B$ $M_I = Am \quad M_{II} = Bn \quad M_1 = Ax$ $M_2 = M_I \frac{x}{m} - p \frac{(x-m)^2}{2} \quad M_3 = Bx'$ $x_0 = m + \frac{c}{2l}(2n+c) = m + \frac{A}{p} \quad M_{\max} = M_I + \frac{A^2}{2p}$
	$A = \frac{pc}{2l}(2n+c) \quad B = \frac{pc^2}{2l}$ $Q_1 = A - px \quad Q_2 = -B$ $M_I = Bn \quad M_1 = M_I \frac{x}{c} + \frac{pc^2}{2}\omega_R^{(c)}$ $M_2 = M_I \frac{x'}{n}$ $x_0 = \frac{A}{p} \quad M_{\max} = \frac{A^2}{2p}$
	$A = p_1c_1 - R \quad B = p_2c_2 + R \quad R = \frac{p_1c_1^2 - p_2c_2^2}{2l}$ $Q_1 = A - p_1x \quad Q_2 = -R \quad Q_3 = -B + p_2x'$ $M_I = \frac{p_1c_1^2}{2} - Rc_1 \quad M_{II} = \frac{p_2c_2^2}{2} + Rc_2$ $M_1 = M_I \frac{x}{c_1} + \frac{p_1c_1^2}{2}\omega_R^{(c_1)}; \quad M_2 = \frac{p_1c_1^2}{2} - Rx; \quad M_3 = M_{II} \frac{x'}{c_2} + \frac{p_2c_2^2}{2}\omega_R^{(c_2)}$ $\frac{p_1}{p_2} > \frac{c_2^2}{c_1^2}: \quad x_0 = \frac{A}{p_1} \quad M_{\max} = \frac{A^2}{2p_1}$ $\frac{p_1}{p_2} < \frac{c_2^2}{c_1^2}: \quad x'_0 = \frac{B}{p_2} \quad M'_{\max} = \frac{B^2}{2p_2}$

\*  $\omega_R^{(c)}$  und  $\omega_D^{(c)}$  nach Tab. 22 für das Intervall  $0 \leq \frac{x}{c} \leq 1$ .



Tabelle 6 (Fortsetzung)

	$A = B = pc \quad Q_1 = A - px$ $M_I = \frac{pc^2}{2} \quad M_1 = M_I \frac{x}{c} + \frac{pc^2}{2} \omega_R^{(c)} \quad M_2 = M_I$ $c \leq x_0 \leq c + m \quad M_{\max} = M_I$
	$A = B = pc \quad Q_1 = A \quad Q_2 = A - p(x - n)$ $M_I = pcn \quad M_{II} = \frac{pc^2}{2} \left( 1 + 2 \frac{n}{c} \right)$ $M_1 = pcx \quad M_2 = \frac{pc^2}{2} \left( \frac{n+x}{c} + \omega_R^{(c)} \right) \quad M_3 = M_{II}$ $n + c \leq x_0 \leq n + c + m \quad M_{\max} = M_{II}$
	$A = \frac{pl}{6} \quad B = \frac{pl}{3} \quad Q = -\frac{pl}{6} \omega_M$ $M = \frac{pl^2}{6} \omega_D$ $x_0 = \frac{l}{\sqrt{3}} = 0,5774l \quad M_{\max} = \frac{pl^2}{9\sqrt{3}} = 0,06415 pl^2$
	$A = \frac{2p_1 + p_2}{6} l \quad B = \frac{p_1 + 2p_2}{6} l$ $Q = A - \frac{p_1 + p_2}{2} l \xi - \frac{p_1 - p_2}{2} l \omega_R$ $M = \frac{l^2}{2} p_1 \omega_R - (p_1 - p_2) \frac{l^2}{6} \omega_D = \frac{p_1 l^2}{6} \left( \omega_D' + \frac{p_2}{p_1} \omega_D \right)$ $x_0 = \frac{1 - \nu}{1 - \mu} l \quad \nu = \sqrt{\frac{p_1^2 + p_1 p_2 + p_2^2}{3 p_1^2}} \quad \mu = \frac{p_2}{p_1}$ $M_{\max} = \frac{p_1 l^2}{6} \cdot \frac{2\nu^3 - \mu(1 + \mu)}{(1 - \mu)^2}$
	$A = \frac{pc}{6} \left( 3 - 2 \frac{c}{l} \right) \quad B = \frac{pc^2}{3l}$ $Q_1 = A - \frac{pc}{2} \frac{x^2}{c^2} \quad Q_2 = -\frac{pc^2}{3l}$ $M_I = \frac{pc^2}{3l} n \quad M_1 = M_I \frac{x}{c} + \frac{pc^2}{6} \omega_D^{(c)} \quad M_2 = M_I \frac{x'}{n}$ $x_0 = c \sqrt{\frac{A}{pc/2}} = c \sqrt{1 - \frac{2c}{3l}} \quad M_{\max} = \frac{2}{3} A x_0 = \frac{p x_0^3}{3c}$



Tabelle 6 (Fortsetzung).

	$A = \frac{pc^2}{6l} \quad B = \frac{pc}{6} \left( 3 - \frac{c}{l} \right)$ $Q_1 = \frac{pc^2}{6l} \quad Q_2 = A - \frac{pc}{2} \left( \frac{x-m}{c} \right)^2$ $M_I = \frac{pc^2}{6l} m \quad M_1 = M_I \frac{x}{m} \quad M_2 = M_I \frac{x'}{c} + \frac{pc^2}{6} \omega_D^{(c)}$ $x_0 = m + c \sqrt{\frac{c}{3l}} \quad M_{\max} = \frac{pc^2}{6l} \left( m + \frac{2}{3} c \sqrt{\frac{c}{3l}} \right) = A \left( x_0 - \frac{c}{3} \sqrt{\frac{c}{3l}} \right)$
	$A = \frac{pc}{2} \frac{b}{l} \quad B = \frac{pc}{2} \frac{a}{l}$ $Q_1 = \frac{pc}{2} \frac{b}{l} \quad Q_2 = A - \frac{p(x-m)}{2} + \frac{pc}{2} \omega_D^{(c)} \quad Q_3 = -\frac{pc}{2} \frac{a}{l}$ $M_I = \frac{pc}{2} \frac{b}{l} m \quad M_{II} = \frac{pc}{2} \frac{a}{l} n$ $M_1 = M_I \frac{x}{m} \quad M_2 = A x - \frac{pc^2}{6} \left( \frac{x-m}{c} \right)^3 \quad M_3 = M_{II} \frac{x'}{n}$ $x_0 = m + c \sqrt{\frac{A}{pc/2}} \quad M_{\max} = \frac{A}{3} (2x_0 + m)$
	$A = \frac{p(l+c_2)}{6} \quad B = \frac{p(l+c_1)}{6}$ $Q_I = p \frac{c_2 - c_1}{3} \quad Q_1 = A - \frac{pc_1}{2} \frac{x^2}{c_1^2} \quad Q_2 = -B + \frac{pc_2}{2} \frac{x'^2}{c_2^2}$ $M_I = p \frac{c_1 c_2}{3} \quad M_1 = \frac{pc_2}{3} x + p \frac{c_1^2}{6} \omega_D^{(c_1)} \quad M_2 = \frac{pc_1}{3} x' + \frac{pc_2^2}{6} \omega_D^{(c_2)}$ $c_1 > c_2: \quad x_0 = c_1 \sqrt{\frac{A}{pc_1/2}} = \sqrt{\frac{c_1(l+c_2)}{3}} \quad M_{\max} = \frac{2}{3} A x_0$ $c_1 < c_2: \quad x'_0 = c_2 \sqrt{\frac{B}{pc_2/2}} = \sqrt{\frac{c_2(l+c_1)}{3}} \quad M'_{\max} = \frac{2}{3} B x'_0$
	$A = \frac{pl}{2} - \frac{\Delta M}{l} \quad B = \frac{pl}{2} + \frac{\Delta M}{l} \quad Q = \frac{pl}{2} \left( 1 - 2 \frac{x}{l} \right) - \frac{\Delta M}{l}$ $M = \frac{pl^2}{2} \omega_R - \Delta M \frac{x}{l} - M_a \quad M_0 = \frac{pl^2}{8}$ $x_0 = \frac{l}{2} - \frac{\Delta M}{pl} \quad M = 0 \quad \text{für } x = x_0 \pm \sqrt{x_0^2 - \frac{2M_a}{p}}$ $M_{\max} = \frac{px_0^2}{2} - M_a = \frac{pl^2}{8} - \frac{M_a + M_b}{2} + \frac{\Delta M^2}{2pl^2}$
	$A = \frac{pl}{3} \quad B = \frac{pl}{3} \quad Q = \frac{pl}{3} (1 - 6\xi^2 + 4\xi^3)$ $M = \frac{pl^2}{3} (\xi - 2\xi^3 + \xi^4) = \frac{pl^2}{3} \omega''_p$ $x_0 = \frac{l}{2} \quad M_{\max} = \frac{5}{48} pl^2 = 0.1042 pl^2$



Tabelle 6 (Fortsetzung).

	$A = \frac{pl}{12} \quad B = \frac{pl}{4} \quad Q = \frac{pl}{12}(1 - 4\xi^3)$ $M = \frac{pl^2}{12}\omega_P \quad \omega_P = \xi - \xi^4$ $x_0 = \frac{l}{2}\sqrt[3]{2} = 0,6300l \quad M_{\max} = \frac{pl}{16}x_0 = 0,03935pl^2$
	$A = B = \frac{p(a+c)}{2} \quad Q_I = \frac{pc}{2}$ $Q_1 = A - \frac{pa}{2}\frac{x^2}{a^2} \quad Q_2 = Q_1 - p(x-a)$ $M_I = \frac{pl^2}{6}\frac{a}{l}\left(3 - 4\frac{a}{l}\right) \quad M_{II} = \frac{pl^2}{24}\left(3 - 4\frac{a^2}{l^2}\right)$ $M_1 = Ax - \frac{pa^2}{6}\left(\frac{x}{a}\right)^3 \quad M_2 = M_1 + \frac{pc^2}{2}\omega_R^{(c)}$ $x_0 = \frac{l}{2} \quad M_{\max} = M_{II}$
	$A = B = \frac{pl}{4} \quad Q_I = \frac{pl}{6} \quad Q_{II} = \frac{pl}{12} \quad Q_1 = \frac{pl}{4}(1 - 12\xi^2)$ $Q_2 = \frac{pl}{12}[1 + 4(1 - 3\xi)^2] \quad Q_3 = \frac{pl}{12}[1 - 4(1 - 3\xi)^2]$ $M_I = \frac{pl^2}{27} \quad M_{II} = \frac{pl^2}{18} \quad M_{III} = \frac{7}{108}pl^2$ $M_1 = \frac{pl^2}{4}\xi(1 - 4\xi^2) \quad M_2 = \frac{pl^2}{108}[-1 + 9\xi(5 - 12\omega_R)]$ $M_3 = \frac{pl^2}{108}[7 - 27\xi(1 - 4\omega_R)] \quad x_0 = l/2 \quad M_{\max} = M_{III}$
	$A = B = \frac{pc}{2} \quad Q_1 = \frac{pc}{2} \quad Q_2 = A - \frac{pc}{2}\left(\frac{x-n}{c}\right)^2$ $M_I = \frac{pcn}{2} \quad M_{II} = \frac{pc^2}{12}\left(3\frac{l}{c} - 2\right)$ $M_1 = \frac{pcx}{2} \quad M_2 = Ax - \frac{pc^2}{6}\left(\frac{x-n}{c}\right)^3$ $x_0 = \frac{l}{2} \quad M_{\max} = M_{II}$
	$A = B = \frac{pl}{4} \quad Q_I = \frac{pl}{8}$ $Q_1 = \frac{pl}{4}(1 - 8\xi^2) \quad Q_2 = \frac{pl}{2}(1 - 2\xi)^2$ $M_I = \frac{5}{96}pl^2 \quad M_{II} = \frac{pl^2}{16} \quad M_1 = \frac{pl^2}{2}(1 - 8\xi^2)\xi$ $M_2 = \frac{pl^2}{48}[3 - 4(1 - 2\xi)^2] \quad x_0 = \frac{l}{2} \quad M_{\max} = M_{II}$



Tabelle 6 (Fortsetzung).

	$A = B = \frac{pl}{4} \quad Q_1 = \frac{pl}{8}$ $Q_1 = \frac{pl}{8} [1 + (1 - 4\xi)^2] \quad Q_2 = \frac{pl}{8} [1 - (1 - 4\xi)^2]$ $M_I = \frac{pl^2}{24} \quad M_{II} = \frac{pl^2}{16}$ $M_{\frac{1}{2}} = \frac{pl^2}{96} [4 - 3(1 - 4\xi) \mp (1 - 4\xi)^3]$ $x_0 = \frac{l}{2} \quad M_{\max} = M_{II}$
	$R = \frac{p_1 c_1^2 - p_2 c_2^2}{6l} = -Q_I \quad A = \frac{p_1 c_1}{2} - R \quad B = \frac{p_2 c_2}{2} + R$ $Q_1 = A - \frac{p_1 c_1}{2} \left( \frac{x}{c_1} + \omega_R^{(c_1)} \right) \quad Q_2 = -B + \frac{p_2 c_2}{2} \left( \frac{x'}{c_2} + \omega_R^{(c_2)} \right)$ $M_I = \frac{c_1 c_2}{6l} (p_1 c_1 + p_2 c_2); \quad M_I = M_I \frac{x}{c_1} + \frac{p_1 c_1^2}{6} \omega_D^{(c_1)}; \quad M_2 = M_I \frac{x'}{c_2} + \frac{p_2 c_2^2}{6} \omega_D^{(c_2)}$ $R > 0: \quad x_0 = c_1 - \sqrt{\frac{2Rc_1}{p_1}} \quad M_{\max} = M_I + \frac{2}{3} R \sqrt{\frac{2Rc_1}{p_1}}$ $R < 0: \quad x'_0 = c_2 - \sqrt{\frac{-2Rc_2}{p_2}} \quad M'_{\max} = M_I - \frac{2}{3} R \sqrt{\frac{-2Rc_2}{p_2}}$
	$A = B = \frac{pl}{4} \quad Q_1 = \frac{pl}{4} (1 - 2\xi)^2$ $M_I = \frac{pl^2}{24} \quad M_{II} = \frac{pl^2}{24} [1 - (1 - 2\xi)^3]$ $x_0 = \frac{l}{2} \quad M_{\max} = M_I$
	$A = B = \frac{pc}{2} \quad Q_1 = \frac{pc}{2} \left( \frac{c-x}{c} \right)^2$ $M_I = \frac{pc^2}{6} \quad M_{II} = \frac{pc^2}{6} \left( 1 - \left( \frac{c-x}{c} \right)^3 \right)$ $c \leq x_0 \leq c + m \quad M_{\max} = M_I$
	$A = -\frac{M_a}{l} \quad B = +\frac{M_a}{l} \quad Q = -\frac{M_a}{l}$ $M = +M_a \xi'$ $x_0 = 0 \quad M_{\max} = +M_a$



Tabelle 6 (Fortsetzung).

	$A = -\frac{M_a + M_b}{l} \quad B = +\frac{M_a + M_b}{l} \quad Q = -\frac{M_a + M_b}{l}$ $M = M_a \xi' - M_b \xi$ $M_a > M_b: \quad x_0 = 0 \quad M_{\max} = +M_a$ $M_a < M_b: \quad x_0 = l \quad M'_{\max} = -M_b$
	$A = -\frac{M_c - M_d}{l} \quad B = +\frac{M_c - M_d}{l} \quad Q = -\frac{M_c - M_d}{l}$ $M_1 = -(M_c - M_d) \xi; \quad M_2 = +M_c \xi' + M_d \xi; \quad M_3 = +(M_c - M_d) \xi'$ $x_0 = m \text{ für } M_c > M_d$ $M_{\max} = -(M_c - M_d) \frac{m}{l} \quad \text{oder} \quad M_{\max} = M_c - (M_c - M_d) \frac{m}{l}$

Tabelle 7. Freitragler.

l: Länge,  $\xi = x/l$ ; C: Stützkraft;  $M_c$ : Einspannmoment;  $Q_m$ : Querkraft;  $M_m$ : Biegemoment im Querschnitt m;  $x_0$ : Querschnitt mit dem größten Biegemoment  $M_{\max}$ .

	$C = \sum_1^n P_k \quad M_c = + \sum_1^n P_k b_k$ $Q_m = - \sum_1^{m-1} P_k = Q_{m-1} - P_{m-1}$ $M_m = - \sum_1^{m-1} P_k (b_k - x'_m) = M_{m-1} + Q_m c_m$ $x_0 = l \quad M_{\max} = -M_c$
	$C = pl \quad M_c = \frac{pl^2}{2} \quad Q = -px$ $M = -\frac{px^2}{2}$ $x_0 = l \quad M_{\max} = -\frac{pl^2}{2}$
	$C = pc \quad M_c = \frac{pc}{2}(l+n) \quad Q_1 = -px \quad Q_2 = -pc$ $M_1 = -\frac{px^2}{2} \quad M_I = -\frac{pc^2}{2} \quad M_2 = -\frac{pc^2}{2} \left(\frac{2x}{c} - 1\right)$ $x_0 = l \quad M_{\max} = -\frac{pc}{2}(l+n)$



Tabelle 7 (Fortsetzung).

	$C = \frac{p l}{2} \quad M_c = \frac{p l^2}{6} \quad Q = -\frac{p l}{2} \xi^2$ $M = -\frac{p l^2}{6} \xi^3$ $x_0 = l \quad M_{\max} = -\frac{p l^2}{6}$
	$C = \frac{p l}{2} \quad M_c = \frac{p l^2}{3} \quad Q = -\frac{p l}{2} (2 \xi - \xi^2)$ $M = -\frac{p l^2}{6} (3 \xi^2 - \xi^3)$ $x_0 = l \quad M_{\max} = -\frac{p l^2}{3}$
	$C = (p_1 + p_2) \frac{l}{2} \quad M_c = (2 p_1 + p_2) \frac{l^2}{6}$ $Q = -\frac{l}{2} [2 p_1 \xi + (p_2 - p_1) \xi^2]$ $M = -\frac{l^2}{6} [3 p_1 \xi^2 + (p_2 - p_1) \xi^3]$ $x_0 = l \quad M_{\max} = -(2 p_1 + p_2) \frac{l^2}{6}$
	$C = \frac{p c}{2} \quad M_c = \frac{p c}{6} (l + 2 n)$ $Q_1 = -\frac{p}{2 c} x^2 \quad Q_2 = -\frac{p c}{2}$ $M_1 = -\frac{p}{6 c} x^3 \quad M_I = -\frac{p c^2}{6} \quad M_2 = +\frac{p c^2}{6} \left(2 - 3 \frac{x}{c}\right)$ $x_0 = l \quad M_{\max} = -\frac{p c}{6} (l + 2 n)$
	$C = \frac{p c}{2} \quad M_c = \frac{p c}{6} (2 l + n)$ $Q_1 = -\frac{p c}{2} \left(2 \frac{x}{c} - \frac{x^2}{c^2}\right) \quad Q_2 = -\frac{p c}{2}$ $M_1 = -\frac{p c^2}{6} \left(3 \frac{x^2}{c^2} - \frac{x^3}{c^3}\right)$ $M_I = -\frac{p c^2}{3} \quad M_2 = -\frac{p c^2}{6} \left(3 \frac{x}{c} - 1\right)$ $x_0 = l \quad M_{\max} = -\frac{p c}{6} (2 l + n)$



Tabelle 7 (Fortsetzung).

	$C = \frac{pl}{2} \quad M_c = \frac{pl^2}{4}$ $Q_1 = -\frac{pl}{4} \quad Q_1 = -pl\xi^2; \quad Q_2 = -\frac{pl}{4}(2\xi + \omega_R^{(c)})$ $M_1 = -\frac{pl^2}{3}\xi^3 \quad M_1 = -\frac{pl^2}{24} \quad M_2 = -\frac{pl^2}{24}(10\xi - 4 - \omega_D^{(c)})$ $x_0 = l \quad M_{\max} = -\frac{pl^2}{4}$
	$C = \frac{pl}{3} \quad M_c = \frac{pl^2}{12} \quad Q = -\frac{pl}{3}\xi^3$ $M = -\frac{pl^2}{12}\xi^4$ $x_0 = l \quad M_{\max} = -\frac{pl^2}{12}$
	$C = \frac{2}{3}pl \quad M_c = \frac{pl^2}{4} \quad Q = -\frac{pl}{3}(3\xi^2 - \xi^3)$ $M = -\frac{pl^2}{12}(4\xi^3 - \xi^4)$ $x_0 = l \quad M_{\max} = -\frac{pl^2}{4}$
	$C = \frac{2}{3}pl \quad M_c = \frac{pl^2}{3} \quad Q = -\frac{2}{3}pl(3\xi^2 - 2\xi^3)$ $M = -\frac{pl^2}{3}(2\xi^3 - \xi^4)$ $x_0 = l \quad M_{\max} = -\frac{pl^2}{3}$
	$C = 0 \quad M_c = M_d + M_e \quad Q = 0$ $M_1 = -M_d \quad M_2 = -(M_d + M_e)$ $x_0 = (l - n) \text{ bis } l \quad M_{\max} = -(M_d + M_e)$