Die Statik im Stahlbetonbau

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Tabellen für die Formänderungen und Schnittkräfte symmetrisch belasteter Kreis- und Kreisringplatten ist, entstehen nach H. Marcus die beiden simultanen Differentialgleichungen zweiter Ordnung

$$\frac{d^2 M}{d r^2} = -\left[p - \frac{1}{r^2}\int_0^r p\, r\, d r\right], \qquad \frac{d^2 w}{d r^2} = -\left[\frac{M}{N} - \frac{1}{r^2}\int_0^r \frac{M}{N}\, r\, d r\right], \tag{958}$$

die wiederum eine Analogie zu den Differentialgleichungen der Seilkurve und der Biegelinie des biegungssteifen Stabes bilden und sich zur Berechnung des Spannungs- und Formänderungszustandes der Kreisplatte ebenfalls eignen.

Tabelle 63. Formänderungen und Schnittkräfte symmetrisch belasteter Kreis- und Kreisringplatten.

$$\varrho = \frac{r}{a}, \qquad \beta = \frac{b}{a}, \qquad N = \frac{E h^3}{12(1-\mu^2)}, \qquad w' = \frac{d w}{d r}.$$

$$\Phi_0 = 1 - \varrho^4, \quad \Phi_1 = 1 - \varrho^2, \quad \Phi_2 = \varrho^2 \ln \varrho, \quad \Phi_3 = \ln \varrho, \quad \Phi_4 = \frac{1}{\varrho^2} - 1.$$

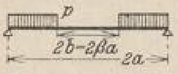
Die Funktionen Φ_0 bis Φ_4 sind in Tabelle 64 enthalten.

$$w = \frac{p a^4}{64 N (1+\mu)}\,[2(3+\mu)\Phi_1 - (1+\mu)\Phi_0],$$

$$M_r = \frac{p a^2}{16}(3+\mu)\Phi_1; \qquad M_t = \frac{p a^2}{16}[2(1-\mu) + (1+3\mu)\Phi_1], \qquad Q_r = -\frac{p a}{2}\varrho.$$

$\varrho = 0$: $\quad w = \frac{p a^4}{64 N}\,\frac{5+\mu}{1+\mu}; \qquad M_r = M_t = \frac{p a^2}{16}(3+\mu).$

$\varrho = 1$: $\quad w' = -\frac{p a^3}{8 N (1+\mu)}; \qquad M_t = \frac{p a^2}{8}(1-\mu); \qquad Q_r = -\frac{p a}{2}.$



$$\varkappa_1 = [(5+\mu) - (7+3\mu)\beta^2](1-\beta^2) - 4(1+\mu)\beta^4 \ln \beta,$$
$$\varkappa_2 = [(3+\mu) - (1-\mu)\beta^2](1-\beta^2) + 4(1+\mu)\beta^2 \ln \beta.$$

$\varrho \leqq \beta$: $\quad w = \frac{p a^4}{64 N (1+\mu)}[\varkappa_1 - 2\varkappa_2 + 2\varkappa_2 \Phi_1], \qquad M_r = M_t = \frac{p a^2}{16}\varkappa_2, \qquad Q_r = 0.$

$\varrho \geqq \beta$: $\quad w = \frac{p a^4}{64 N (1+\mu)}\{2[(3+\mu)(1-2\beta^2) + (1-\mu)\beta^4]\Phi_1 - (1+\mu)\Phi_0 - 4(1+\mu)\beta^4\Phi_3 - 8(1+\mu)\beta^2\Phi_2\},$

$$M_r = \frac{p a^2}{16}[(3+\mu)\Phi_1 - (1-\mu)\beta^4\Phi_4 + 4(1+\mu)\beta^2\Phi_3], \qquad Q_r = -\frac{p a}{2}\left(\varrho - \frac{\beta^2}{\varrho}\right),$$

$$M_t = \frac{p a^2}{16}[(1+3\mu)\Phi_1 + (1-\mu)\beta^4\Phi_4 + 4(1+\mu)\beta^2\Phi_3 + 2(1-\mu)(1-\beta^2)^2].$$

$\varrho = 0$: $\quad w = \frac{p a^4}{64 N (1+\mu)}\varkappa_1.$

$\varrho = 1$: $\quad w' = -\frac{p a^3}{8 N (1+\mu)}(1-\beta^2)^2, \quad M_t = \frac{p a^2}{8}(1-\mu)(1-\beta^2)^2, \quad Q_r = -\frac{p a}{2}(1-\beta^2).$

$$\varkappa_1 = 4 - (1-\mu)\beta^2, \qquad \varkappa_2 = [\varkappa_1 - 4(1+\mu)\ln\beta]\,\beta^2,$$
$$\varkappa_3 = 4(3+\mu) - (7+3\mu)\beta^2 + 4(1+\mu)\beta^2 \ln \beta.$$

$\varrho \leqq \beta$: $\quad w = \frac{p a^4}{64 N}\left\{1 + [4 - 5\beta^2 + 4(2+\beta^2)\ln\beta]\beta^2 + 2\frac{\varkappa_2}{1+\mu}\Phi_1 - \Phi_0\right\}, \qquad Q_r = -\frac{p a}{2}\varrho.$

$$M_r = \frac{p a^2}{16}[\varkappa_2 - (3+\mu) + (3+\mu)\Phi_1], \qquad M_t = \frac{p a^2}{16}[\varkappa_2 - (1+3\mu) + (1+3\mu)\Phi_1].$$

$\varrho \geqq \beta$: $\quad w = \frac{p\,a^4}{64\,N} \cdot 2\,\beta^2 \left[\frac{2\,(3+\mu) - (1-\mu)\,\beta^2}{1+\mu}\,\Phi_1 + 4\,\Phi_2 + 2\,\beta^2\,\Phi_3 \right].$

$$M_r = \frac{p\,a^2}{16}\,[(1-\mu)\,\beta^4\,\Phi_4 - 4\,(1+\mu)\,\beta^2\,\Phi_3], \qquad Q_r = -\frac{p\,b}{2}\,\frac{\beta}{\varrho}.$$

$$M_t = \frac{p\,a^2}{16}\,[-(1-\mu)\,\beta^4\,\Phi_4 - 4\,(1+\mu)\,\beta^2\,\Phi_3 + 2\,(1-\mu)\,\beta^2\,(2-\beta^2)].$$

$\varrho = 0$: $\quad w = \frac{p\,a^2\,b^2}{64\,N\,(1+\mu)}\,\varkappa_3, \qquad M_r = M_t = \frac{p\,a^2}{16}\,\varkappa_2.$

$\varrho = \beta$: $\quad M_r = \frac{p\,a^2}{16}\,[\varkappa_2 - (3+\mu)\,\beta^2], \qquad M_t = \frac{p\,a^2}{16}\,[\varkappa_2 - (1+3\,\mu)\,\beta^2]. \qquad Q_r = -\frac{p\,b}{2},$

$\varrho = 1$: $\quad w' = -\frac{p\,a\,b^2}{8\,N\,(1+\mu)}\,(2-\beta^2), \qquad M_t = \frac{p\,b^2}{8}\,(1-\mu)\,(2-\beta^2). \qquad Q_r = -\frac{p\,b}{2}\,\beta.$

$$\varkappa_1 = (3+\mu)\,(1-\beta^2) + 2\,(1+\mu)\,\beta^2 \ln\beta.$$
$$\varkappa_2 = (1-\mu)\,(1-\beta^2) - 2\,(1+\mu) \ln\beta.$$

$\varrho \leqq \beta$: $\quad w = \frac{P\,a^2\,b}{8\,N\,(1+\mu)}\,[(\varkappa_1 - \varkappa_2) + \varkappa_2\,\Phi_1], \qquad M_r = M_t = \frac{P\,b}{4}\,\varkappa_2, \qquad Q_r = 0.$

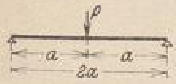
$\varrho \geqq \beta$: $\quad w = \frac{P\,a^2\,b}{8\,N\,(1+\mu)}\,\{[(3+\mu) - (1-\mu)\,\beta^2]\,\Phi_1 + 2\,(1+\mu)\,\beta^2\,\Phi_3 + 2\,(1+\mu)\,\Phi_2\},$

$$M_r = \frac{P\,b}{4}\,[(1-\mu)\,\beta^2\,\Phi_4 - 2\,(1+\mu)\,\Phi_3]. \qquad Q_r = -P\,\frac{\beta}{\varrho}.$$

$$M_t = \frac{P\,b}{4}\,[-(1-\mu)\,\beta^2\,\Phi_4 - 2\,(1+\mu)\,\Phi_3 + 2\,(1-\mu)\,(1-\beta^2)].$$

$\varrho = 0$: $\quad w = \frac{P\,a^2\,b}{8\,N\,(1+\mu)}\,\varkappa_1.$

$\varrho = 1$: $\quad w' = -\frac{P\,a\,b}{2\,N\,(1+\mu)}\,(1-\beta^2), \qquad M_t = \frac{P\,b}{2}\,(1-\mu)\,(1-\beta^2). \qquad Q_r = -P\,\beta.$

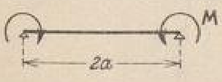


$$w = \frac{P\,a^2}{16\,\pi\,N}\left[\frac{3+\mu}{1+\mu}\,\Phi_1 + 2\,\Phi_2\right].$$

$$M_r = -\frac{P}{4\,\pi}\,(1+\mu)\,\Phi_3. \qquad M_t = \frac{P}{4\,\pi}\,[(1-\mu) - (1+\mu)\,\Phi_3], \qquad Q_r = -\frac{P}{2\,\pi\,a\,\varrho},$$

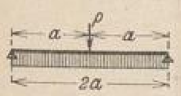
$\varrho = 0$: $\quad w = \frac{P\,a^2}{16\,\pi\,N}\,\frac{3+\mu}{1+\mu}.$

$\varrho = 1$: $\quad w' = -\frac{P\,a}{4\,\pi\,N\,(1+\mu)}, \qquad M_t = \frac{P}{4\,\pi}\,(1-\mu). \qquad Q_r = -\frac{P}{2\,\pi\,a}.$



$$w = \frac{\mathsf{M}\,a^2}{2\,N\,(1+\mu)}\,\Phi_1, \qquad M_r = M_t = \mathsf{M}, \qquad Q_r = 0.$$

$\varrho = 1$: $\quad w' = -\frac{\mathsf{M}\,a}{N\,(1+\mu)}.$



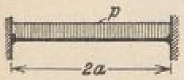
$$w = \frac{P\,a^2}{64\,\pi\,N}\left(2\,\frac{3+\mu}{1+\mu}\,\Phi_1 + \Phi_0 + 8\,\Phi_2\right).$$

$$M_r = -\frac{P}{16\,\pi}\,[(3+\mu)\quad\Phi_1 + 4\,(1+\mu)\,\Phi_3],$$

$$M_t = -\frac{P}{16\,\pi}\,[(1+3\,\mu)\,\Phi_1 + 4\,(1+\mu)\,\Phi_3 - 2\,(1-\mu)]. \qquad Q_r = -\frac{P}{2\,\pi\,a}\left(\frac{1}{\varrho} - \varrho\right)$$

$\varrho = 0$: $$w = \frac{P a^2}{64 \pi N} \frac{7 + 3\mu}{1 + \mu}.$$

$\varrho = 1$: $$w' = -\frac{P a}{8 \pi N (1 + \mu)}, \qquad M_t = \frac{P}{8 \pi} (1 - \mu).$$

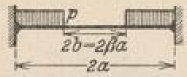


$$w = \frac{p a^4}{64 N} (2 \Phi_1 - \Phi_0). \qquad M_r = \frac{p a^2}{16} [(3 + \mu) \Phi_1 - 2],$$

$$M_t = \frac{p a^2}{16} [(1 + 3\mu) \Phi_1 - 2\mu], \qquad Q_r = -\frac{p a}{2} \varrho.$$

$\varrho = 0$: $$w = \frac{p a^4}{64 N}, \qquad M_r = M_t = \frac{p a^2}{16} (1 + \mu),$$

$\varrho = 1$: $$M_t = \mu M_r = -\frac{p a^2}{8} \mu, \qquad Q_r = -\frac{p a}{2}.$$



$$\varkappa_1 = 1 - 4\beta^2 + \beta^4 (3 - 4 \ln \beta),$$
$$\varkappa_2 = 1 - \beta^2 (\beta^2 - 4 \ln \beta).$$

$\varrho \leq \beta$: $$w = \frac{p a^4}{64 N} [(\varkappa_1 - 2 \varkappa_2) + 2 \varkappa_2 \Phi_1], \quad M_r = M_t = \frac{p a^2}{16} (1 + \mu) \varkappa_2, \quad Q_r = 0.$$

$\varrho \geq \beta$: $$w = \frac{p a^4}{64 N} [2 (1 - 2\beta^2 - \beta^4) \Phi_1 - \Phi_0 - 4 \beta^4 \Phi_3 - 8 \beta^2 \Phi_2],$$

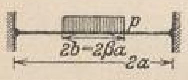
$$M_r = \frac{p a^2}{16} [-2 (1 - \beta^2)^2 + (3 + \mu) \Phi_1 - (1 - \mu) \beta^4 \Phi_4 + 4 (1 + \mu) \beta^2 \Phi_3],$$

$$M_t = \frac{p a^2}{16} [-2 \mu (1 - \beta^2)^2 + (1 + 3\mu) \Phi_1 + (1 - \mu) \beta^4 \Phi_4 + 4 (1 + \mu) \beta^2 \Phi_3],$$

$$Q_r = -\frac{p a}{2} \left(\varrho - \frac{\beta^2}{\varrho} \right).$$

$\varrho = 0$: $$w = \frac{p a^4}{64 N} \varkappa_1.$$

$\varrho = 1$: $$M_t = \mu M_r = -\frac{p a^2}{8} \mu (1 - \beta^2)^2, \qquad Q_r = -\frac{p a}{2} (1 - \beta^2).$$



$$\varkappa_1 = \beta^2 [4 - \beta^2 (3 - 4 \ln \beta)],$$
$$\varkappa_2 = \beta^2 (\beta^2 - 4 \ln \beta).$$

$\varrho \leq \beta$: $$w = \frac{p a^4}{64 N} [(\varkappa_1 - 2 \varkappa_2 + 1) + 2 \varkappa_2 \Phi_1 - \Phi_0],$$

$$M_r = \frac{p a^2}{16} \{[(1 + \mu) \varkappa_2 - (3 + \mu)] + (3 + \mu) \Phi_1\},$$

$$M_t = \frac{p a^2}{16} \{[(1 + \mu) \varkappa_2 - (1 + 3\mu)] + (1 + 3\mu) \Phi_1\}.$$

$\varrho \geq \beta$: $$w = \frac{p a^2 b^2}{32 N} [(2 + \beta^2) \Phi_1 + 2 \beta^2 \Phi_3 + 4 \Phi_2],$$

$$M_r = \frac{p b^2}{16} [-2 (2 - \beta^2) + (1 - \mu) \beta^2 \Phi_4 - 4 (1 + \mu) \Phi_3], \qquad Q_r = -\frac{p b}{2} \frac{\beta}{\varrho},$$

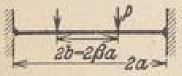
$$M_t = \frac{p b^2}{16} [-2 \mu (2 - \beta^2) - (1 - \mu) \beta^2 \Phi_4 - 4 (1 + \mu) \Phi_3].$$

$\varrho = 0$: $$w = \frac{p\,a^4}{64\,N}\varkappa_1, \qquad M_r = M_t = \frac{p\,a^2}{16}(1+\mu)\,\varkappa_2 .$$

$\varrho = \beta$: $$M_r = \frac{p\,a^2}{16}[(1+\mu)\,\varkappa_2 - (3+\mu)\,\beta^2], \qquad M_t = \frac{p\,a^2}{16}[(1+\mu)\,\varkappa_2 - (1+3\,\mu)\,\beta^2];$$

$$Q_r = -\frac{p\,b}{2}.$$

$\varrho = 1$: $$M_t = \mu\,M_r = -\frac{p\,b^2}{8}\mu\,(2-\beta^2), \qquad Q_r = -\frac{p\,b}{2}\beta .$$



$$\varkappa_1 = 1 - \beta^2\,(1 - 2\ln\beta),$$
$$\varkappa_2 = \beta^2 - 1 - 2\ln\beta .$$

$\varrho \leqq \beta$: $$w = \frac{P\,a^2\,b}{8\,N}[(\varkappa_1 - \varkappa_2) + \varkappa_2\,\Phi_1], \qquad M_r = M_t = \frac{P\,b}{4}(1+\mu)\,\varkappa_2, \qquad Q_r = 0 .$$

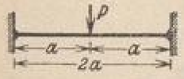
$\varrho \geqq \beta$: $$w = \frac{P\,a^2\,b}{8\,N}[(1+\beta^2)\,\Phi_1 + 2\,\beta^2\,\Phi_3 + 2\,\Phi_2],$$

$$M_r = -\frac{P\,b}{4}[2\,(1-\beta^2) - (1-\mu)\,\beta^2\,\Phi_4 + 2\,(1+\mu)\,\Phi_3],$$

$$M_t = -\frac{P\,b}{4}[2\,\mu\,(1-\beta^2) + (1-\mu)\,\beta^2\,\Phi_4 + 2\,(1+\mu)\,\Phi_3], \qquad Q_r = -P\frac{\beta}{\varrho}.$$

$\varrho = 0$: $$w = \frac{P\,a^2\,b}{8\,N}\varkappa_1 .$$

$\varrho = 1$: $$M_t = \mu\,M_r = -\frac{P\,b}{2}\mu\,(1-\beta^2), \qquad Q_r = -P\,\beta .$$

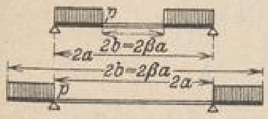


$$w = \frac{P\,a^2}{16\,\pi\,N}(\Phi_1 + 2\,\Phi_2).$$

$$M_r = -\frac{P}{4\,\pi}[1 + (1+\mu)\,\Phi_3], \qquad M_t = -\frac{P}{4\,\pi}[\mu + (1+\mu)\,\Phi_3], \qquad Q_r = -\frac{P}{2\,\pi\,a\,\varrho}.$$

$\varrho = 0$: $$w = \frac{P\,a^2}{16\,\pi\,N}.$$

$\varrho = 1$: $$M_t = \mu\,M_r = -\frac{P}{4\,\pi}\mu, \qquad Q_r = -\frac{P}{2\,\pi\,a}.$$



$$\varkappa_1 = (3+\mu) + 4\,(1+\mu)\frac{\beta^2}{1-\beta^2}\ln\beta,$$
$$\varkappa_2 = (3+\mu) - 4\,(1+\mu)\frac{\beta^2}{1-\beta^2}\ln\beta,$$

$$w = \frac{p\,a^4}{64\,N}\left\{\frac{2}{1+\mu}[(3+\mu) - \beta^2\,\varkappa_2]\,\Phi_1 - \Phi_0 - \frac{4}{1-\mu}\beta^2\,\varkappa_1\,\Phi_3 - 8\,\beta^2\,\Phi_2\right\},$$

$$M_r = \frac{p\,a^2}{16}[(3+\mu)\,\Phi_1 - \beta^2\,\varkappa_1\,\Phi_4 + 4\,(1+\mu)\,\beta^2\,\Phi_3], \qquad Q_r = -\frac{p\,a}{2}\left(\varrho - \frac{\beta^2}{\varrho}\right),$$

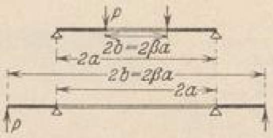
$$M_t = \frac{p\,a^2}{16}\{(1+3\,\mu)\,\Phi_1 + \beta^2\,\varkappa_1\,\Phi_4 + 4\,(1+\mu)\,\beta^2\,\Phi_3 + 2\,(1-\mu) - 2\,\beta^2\,[2\,(1-\mu) - \varkappa_1]\}.$$

$\varrho = \beta$: $$w = \frac{p\,a^4}{64\,N}\left\{[(5+\mu) - (7+3\,\mu)\,\beta^2]\frac{1-\beta^2}{1+\mu} - \frac{4}{1-\mu}\beta^2\,\varkappa_1\ln\beta\right\},$$

$$w' = -\frac{p\,a^2\,b}{8\,N\,(1+\mu)}\left(\frac{\varkappa_1}{1-\mu} - \beta^2\right), \qquad M_t = \frac{p\,a^2}{8}[\varkappa_1 - (1-\mu)\,\beta^2].$$

$\varrho = 1:\qquad w' = -\frac{p\,a^3}{8\,N\,(1+\mu)}\left[1 - \beta^2\left(2 - \frac{\varkappa_1}{1-\mu}\right)\right],$

$$M_t = \frac{p\,a^2}{8}\{(1-\mu) - \beta^2\,[2\,(1-\mu) - \varkappa_1]\},\qquad Q_r = -\frac{p\,a}{2}(1-\beta^2).$$



$$\varkappa = \frac{\beta^2}{1-\beta^2}\ln\beta\,.$$

$$w = \frac{P\,a^2\,b}{8\,N}\left[\left(\frac{3+\mu}{1+\mu} - 2\,\varkappa\right)\Phi_1 + 4\,\frac{1+\mu}{1-\mu}\,\varkappa\,\Phi_3 + 2\,\Phi_2\right].$$

$$M_r = -\frac{P\,b}{2}(1+\mu)\,(-\varkappa\,\Phi_4 + \Phi_3);\qquad Q_r = -P\,\frac{\beta}{\varrho},$$

$$M_t = -\frac{P\,b}{2}(1+\mu)\left[\varkappa\,\Phi_4 + \Phi_3 + \left(2\,\varkappa - \frac{1-\mu}{1+\mu}\right)\right].$$

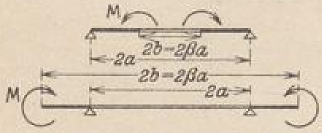
$\varrho = \beta:\qquad w = \frac{P\,a^2\,b}{8\,N}\left[\frac{3+\mu}{1+\mu}(1-\beta^2) + 4\,\frac{1+\mu}{1-\mu}\,\varkappa\ln\beta\right],$

$$w' = -\frac{P\,a^2}{2\,N\,(1+\mu)}\left(\beta^2 - 2\,\varkappa\,\frac{1+\mu}{1-\mu}\right),$$

$$M_t = -\frac{P\,b}{2}(1+\mu)\left(2\,\frac{\varkappa}{\beta^2} - \frac{1-\mu}{1+\mu}\right);\qquad Q_r = -P.$$

$\varrho = 1:\qquad w' = -\frac{P\,a\,b}{2\,N\,(1+\mu)}\left(1 - 2\,\varkappa\,\frac{1+\mu}{1-\mu}\right);\qquad Q_r = -P\,\beta,$

$$M_t = -\frac{P\,b}{2}(1+\mu)\left(2\,\varkappa - \frac{1-\mu}{1+\mu}\right).$$



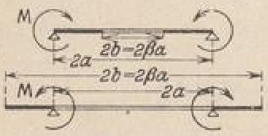
$$w = -\frac{\mathsf{M}\,b^2}{2\,N\,(1+\mu)}\,\frac{1}{1-\beta^2}\left(\Phi_1 - 2\,\frac{1+\mu}{1-\mu}\,\Phi_3\right),$$

$$M_r = \mathsf{M}\,\frac{\beta^2}{1-\beta^2}\,\Phi_4;\qquad M_t = -\mathsf{M}\,\frac{\beta^2}{1-\beta^2}(\Phi_4 + 2),\qquad Q_r = 0\,.$$

$\varrho = \beta:\qquad w = -\frac{\mathsf{M}\,b^2}{2\,N\,(1+\mu)}\left(1 - 2\,\frac{1+\mu}{1-\mu}\,\frac{\ln\beta}{1-\beta^2}\right),$

$$w' = \frac{\mathsf{M}\,b}{N\,(1+\mu)}\,\frac{1}{1-\beta^2}\left(\beta^2 + \frac{1+\mu}{1-\mu}\right);\qquad M_t = -\mathsf{M}\,\frac{1+\beta^2}{1-\beta^2}\,.$$

$\varrho = 1:\qquad w' = 2\,\frac{\mathsf{M}\,b}{N\,(1-\mu^2)}\,\frac{\beta}{1-\beta^2};\qquad M_t = -2\,\mathsf{M}\,\frac{\beta^2}{1-\beta^2}\,.$



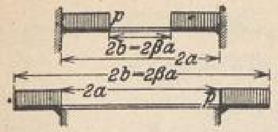
$$w = \frac{\mathsf{M}\,a^2}{2\,N\,(1+\mu)\,(1-\beta^2)}\left(\Phi_1 - 2\,\frac{1+\mu}{1-\mu}\,\beta^2\,\Phi_3\right).$$

$$M_r = \mathsf{M}\left(1 - \frac{\beta^2}{1-\beta^2}\,\Phi_4\right);\qquad M_t = \mathsf{M}\left(\frac{1+\beta^2}{1-\beta^2} + \frac{\beta^2}{1-\beta^2}\,\Phi_4\right).\qquad Q_r = 0$$

$\varrho = \beta:\qquad w = \frac{\mathsf{M}\,a^2}{2\,N\,(1+\mu)}\left(1 - 2\,\frac{1+\mu}{1-\mu}\,\frac{\beta^2}{1-\beta^2}\ln\beta\right),$

$$w' = -\frac{\mathsf{M}\,b}{N\,(1-\mu^2)}\,\frac{2}{1-\beta^2};\qquad M_t = \mathsf{M}\,\frac{2}{1-\beta^2}\,.$$

$\varrho = 1:\qquad w' = -\frac{\mathsf{M}\,a}{N\,(1+\mu)\,(1-\beta^2)}\left(1 + \frac{1+\mu}{1-\mu}\,\beta^2\right),\qquad M_t = \mathsf{M}\,\frac{1+\beta^2}{1-\beta^2}\,.$



$$\varkappa_1 = (1+\mu) + (1-\mu)\,\beta^2, \qquad \psi_1 = 4\,(1+\mu)\,\beta^2 \ln \beta,$$

$$\varkappa_2 = (1-\mu) + (1+\mu)\,\beta^2, \qquad \psi = \frac{\varkappa_1 + \psi_1}{\varkappa_2}\,\beta^2.$$

$$w = \frac{p\,a^4}{64\,N}\left[2\,(1 - 2\,\beta^2 - \psi)\,\Phi_1 - \Phi_0 - 4\,\psi\,\Phi_3 - 8\,\beta^2\,\Phi_2\right].$$

$$M_r = -\frac{p\,a^2}{16}\left[2\,(1 - 2\,\beta^2 + \psi) - (3+\mu)\,\Phi_1 + (1-\mu)\,\psi\,\Phi_4 - 4\,(1+\mu)\,\beta^2\,\Phi_3\right].$$

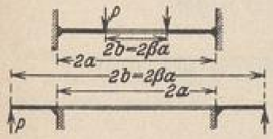
$$M_t = -\frac{p\,a^2}{16}\left[2\,\mu\,(1 - 2\,\beta^2 + \psi) - (1+3\,\mu)\,\Phi_1 - (1-\mu)\,\psi\,\Phi_4 - 4\,(1+\mu)\,\beta^2\,\Phi_3\right].$$

$$Q_r = -\frac{p\,a}{2}\left(\varrho - \frac{\beta^2}{\varrho}\right).$$

$\varrho = \beta$: $$w = \frac{p\,a^4}{64\,N}\left[(1-\beta^2)^2 - 2\,(1-\beta^2)\,(\psi + 2\,\beta^2) - 4\,(\psi + 2\,\beta^4)\ln\beta\right],$$

$$w' = -\frac{p\,a^3}{8\,N\,(1+\mu)}\,\frac{\psi - \beta^4}{\beta}; \qquad M_t = \frac{p\,a^2}{8}\,\frac{1-\mu^2}{\varkappa_2}\,(1 - \beta^4 + 4\,\beta^2 \ln\beta).$$

$\varrho = 1$: $$M_t = \mu\,M_r = -\frac{p\,a^2}{8}\,\mu\,(1 - 2\,\beta^2 + \psi); \qquad Q_r = -\frac{p\,a}{2}\,(1-\beta^2).$$



$$\varkappa = (1-\mu) + (1+\mu)\,\beta^2;$$

$$\psi = \left[1 + (1+\mu)\ln\beta\right]\frac{\beta^2}{\varkappa}.$$

$$w = \frac{P\,a^2\,b}{8\,N}\left[(1 + 2\,\psi)\,\Phi_1 + 4\,\psi\,\Phi_3 + 2\,\Phi_2\right].$$

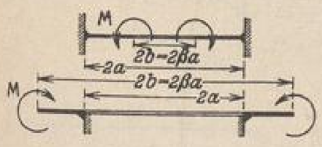
$$M_r = -\frac{P\,b}{2}\left[(1 - 2\,\psi) - (1-\mu)\,\psi\,\Phi_4 + (1+\mu)\,\Phi_3\right]. \qquad Q_r = -P\,\frac{\beta}{\varrho}.$$

$$M_t = -\frac{P\,b}{2}\left[\mu\,(1 - 2\,\psi) + (1-\mu)\,\psi\,\Phi_4 + (1+\mu)\,\Phi_3\right].$$

$\varrho = \beta$: $$w = \frac{P\,a^2\,b}{8\,N}\left[(1 + 2\,\psi)\,(1-\beta^2) + 2\,(\beta^2 + 2\,\psi)\ln\beta\right],$$

$$w' = \frac{P\,b^2}{2\,N\,\varkappa}\,(1 - \beta^2 + 2\ln\beta), \qquad M_t = -\frac{P\,b}{2}\,\frac{1-\mu^2}{\varkappa}\,(1 - \beta^2 + 2\ln\beta).$$

$\varrho = 1$: $$M_t = \mu\,M_r = -\frac{P\,b}{2}\,\mu\,(1 - 2\,\psi),$$



$$\varkappa = (1-\mu) + (1+\mu)\,\beta^2,$$

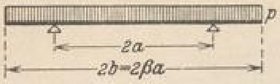
$$w = \frac{\mathsf{M}\,b^2}{2\,N\,\varkappa}\left[\Phi_1 + 2\,\Phi_3\right]; \qquad Q_r = 0,$$

$$M_r = \frac{\mathsf{M}\,\beta^2}{\varkappa}\left[2 + (1-\mu)\,\Phi_4\right]; \quad M_t = \frac{\mathsf{M}\,\beta^2}{\varkappa}\left[2\,\mu - (1-\mu)\,\Phi_4\right].$$

$\varrho = \beta$: $$w = \frac{\mathsf{M}\,b^2}{2\,N\,\varkappa}\,(1 - \beta^2 + 2\ln\beta); \qquad w' = \frac{\mathsf{M}\,b}{N\,\varkappa}\,(1-\beta^2).$$

$$M_t = -\frac{\mathsf{M}}{\varkappa}\left[(1-\mu) - (1+\mu)\,\beta^2\right].$$

$\varrho = 1$: $$M_t = \mu\,M_r = \frac{2\,\mathsf{M}\,\beta^2}{\varkappa}\,\mu.$$



$$\varkappa_1 = 2(1-\mu) + (1+3\mu)\beta^2 - 4(1+\mu)\beta^2 \ln\beta,$$
$$\varkappa_2 = 2(1-\mu) - (3+\mu)\beta^2 - 4(1+\mu)\beta^2 \ln\beta.$$

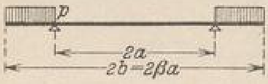
$\varrho \leqq 1$:
$$w = \frac{p a^4}{64 N}\left(\frac{2\varkappa_1}{1+\mu}\Phi_1 - \Phi_0\right),$$
$$M_r = \frac{p a^2}{16}[\varkappa_1 - (3+\mu) + (3+\mu)\Phi_1],$$
$$M_t = \frac{p a^2}{16}[\varkappa_1 - (1+3\mu) + (1+3\mu)\Phi_1], \qquad Q_r = -\frac{p a}{2}\varrho.$$

$\varrho \geqq 1$:
$$w = \frac{p a^4}{64 N}\left(\frac{2\varkappa_2}{1+\mu}\Phi_1 - \Phi_0 - 8\beta^2\Phi_3 - 8\beta^2\Phi_2\right), \qquad Q_r = \frac{p a}{2}\left(\frac{\beta^2}{\varrho} - \varrho\right).$$
$$M_r = \frac{p a^2}{16}[\varkappa_1 - (3+\mu) + (3+\mu)\Phi_1 - 2(1-\mu)\beta^2\Phi_4 + 4(1+\mu)\beta^2\Phi_3],$$
$$M_t = \frac{p a^2}{16}[\varkappa_1 - (1+3\mu) + (1+3\mu)\Phi_1 + 2(1-\mu)\beta^2\Phi_4 + 4(1+\mu)\beta^2\Phi_3].$$

$\varrho = 0$:
$$w = \frac{p a^4}{64 N}\left(\frac{2\varkappa_1}{1+\mu} - 1\right), \qquad M_r = M_t = \frac{p a^2}{16}\varkappa_1.$$

$\varrho = 1$:
$$w' = -\frac{p a^3}{16 N}\left(\frac{\varkappa_1}{1+\mu} - 1\right), \qquad Q_{ri} = -\frac{p a}{2}, \qquad Q_{ra} = \frac{p a}{2}(\beta^2 - 1),$$
$$M_r = \frac{p a^2}{16}[\varkappa_1 - (3+\mu)], \qquad M_t = \frac{p a^2}{16}[\varkappa_1 - (1+3\mu)].$$

$\varrho = \beta$:
$$w = -\frac{p a^4}{64 N(1+\mu)}\{[(3-5\mu) - (7+3\mu)\beta^2](\beta^2 - 1) + 16(1+\mu)\beta^2\ln\beta\},$$
$$w' = -\frac{p a^2 b}{8 N(1+\mu)}(2-\beta^2), \qquad M_t = \frac{p a^2}{8}(1-\mu)(2-\beta^2).$$



$$\varkappa_1 = \frac{1}{\beta^2}[(1-\mu) + 4\mu\beta^2 - (1+3\mu)\beta^4 + 4(1+\mu)\beta^4\ln\beta],$$
$$\varkappa_2 = \frac{1}{\beta^2}[(1-\mu)(1-2\beta^2) + (3+\mu)\beta^4 + 4(1+\mu)\beta^4\ln\beta].$$

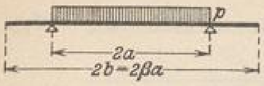
$\varrho \leqq 1$:
$$w = -\frac{p a^4}{32 N(1+\mu)}\varkappa_1\Phi_1, \qquad M_r = M_t = -\frac{p a^2}{16}\varkappa_1, \qquad Q_r = 0.$$

$\varrho \geqq 1$:
$$w = -\frac{p a^4}{64 N(1+\mu)}[2\varkappa_2\Phi_1 + (1+\mu)\Phi_0 + 4(1+\mu)(2\beta^2 - 1)\Phi_3 + 8(1+\mu)\beta^2\Phi_2],$$
$$M_r = -\frac{p a^2}{16}[\varkappa_1 - (3+\mu)\Phi_1 + (1-\mu)(2\beta^2 - 1)\Phi_4 - 4(1+\mu)\beta^2\Phi_3],$$
$$M_t = -\frac{p a^2}{16}[\varkappa_1 - (1+3\mu)\Phi_1 - (1-\mu)(2\beta^2 - 1)\Phi_4 - 4(1+\mu)\beta^2\Phi_3].$$
$$Q_r = \frac{p a}{2}\left(\frac{\beta^2}{\varrho} - \varrho\right).$$

$\varrho = 0$:
$$w = -\frac{p a^4}{32 N(1+\mu)}\varkappa_1, \quad M_r = M_t = -\frac{p a^2}{16}\varkappa_1.$$

$\varrho = 1$:
$$w' = \frac{p a^3}{16 N(1+\mu)}\varkappa_1, \quad M_r = M_t = -\frac{p a^2}{16}\varkappa_1, \quad Q_{ri} = 0, \quad Q_{ra} = \frac{p a}{2}(\beta^2 - 1).$$

$\varrho = \beta$:
$$w = \frac{p a^4}{64 N(1+\mu)} \cdot \left\{[2(1-\mu) - (3-5\mu)\beta^2 + (7+3\mu)\beta^4]\frac{\beta^2 - 1}{\beta^2} - 4(1+\mu)(4\beta^2 - 1)\ln\beta\right\},$$
$$w' = \frac{p a^3}{8 N(1+\mu)}\frac{(\beta^2 - 1)^2}{\beta}, \qquad M_t = -\frac{p a^2}{8}(1-\mu)\frac{(\beta^2 - 1)^2}{\beta^2}.$$



$$\varkappa = \frac{1-\mu}{\beta^2} + 2\,(1+\mu)\,.$$

$\varrho \leqq 1$: $$w = \frac{p\,a^4}{64\,N}\left(2\,\frac{\varkappa}{1+\mu}\,\Phi_1 - \Phi_0\right), \qquad M_r = \frac{p\,a^2}{16}\,[\varkappa - (3+\mu) + (3+\mu)\,\Phi_1]\,,$$

$$M_t = \frac{p\,a^2}{16}\,[\varkappa - (1+3\,\mu) + (1+3\,\mu)\,\Phi_1]\,, \qquad Q_r = -\frac{p\,a}{2}\,\varrho\,.$$

$\varrho \geqq 1$: $$w = \frac{p\,a^4}{32\,N}\left[\frac{1-\mu}{1+\mu}\,\frac{1}{\beta^2}\,\Phi_1 - 2\,\Phi_3\right], \qquad Q_r = 0\,.$$

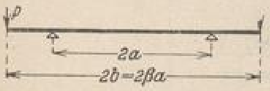
$$M_r = -\frac{p\,a^2}{16}\,(1-\mu)\left(\frac{\beta^2-1}{\beta^2} + \Phi_4\right), \qquad M_t = -\frac{p\,a^2}{16}\,(1-\mu)\left(-\frac{\beta^2+1}{\beta^2} - \Phi_4\right).$$

$\varrho = 0$: $$w = \frac{p\,a^4}{64\,N}\left(2\,\frac{\varkappa}{1+\mu} - 1\right), \qquad M_r = M_t = \frac{p\,a^2}{16}\,\varkappa\,.$$

$\varrho = 1$: $$w' = -\frac{p\,a^3}{16\,N}\left(\frac{\varkappa}{1+\mu} - 1\right), \qquad M_r = -\frac{p\,a^2}{16}\,(1-\mu)\,\frac{\beta^2-1}{\beta^2}\,,$$

$$M_t = \frac{p\,a^2}{16}\,(1-\mu)\,\frac{\beta^2+1}{\beta^2}\,, \qquad Q_r = -\frac{p\,a}{2}\,.$$

$\varrho = \beta$: $$w = -\frac{p\,a^4}{32\,N}\left[\frac{1-\mu}{1+\mu}\,\frac{\beta^2-1}{\beta^2} + 2\ln\beta\right], \qquad w' = -\frac{p\,a^3}{8\,N\,(1+\mu)\,\beta}\,, \qquad M_t = \frac{p\,a^2}{8}\,\frac{1-\mu}{\beta^2}\,.$$



$$\varkappa = (1-\mu)\left(\beta - \frac{1}{\beta}\right) + 2\,(1+\mu)\,\beta\ln\beta\,.$$

$\varrho \leqq 1$: $$w = -\frac{P\,a^3}{8\,N}\,\frac{\varkappa}{1+\mu}\,\Phi_1\,, \qquad M_r = M_t = -\frac{P\,a}{4}\,\varkappa\,, \qquad Q_r = 0\,.$$

$\varrho \geqq 1$: $$w = \frac{P\,a^3}{8\,N}\left\{-\left[\frac{\varkappa}{1+\mu} + 2\,\beta\right]\Phi_1 - 2\,\beta\,\Phi_3 - 2\,\beta\,\Phi_2\right\},$$

$$M_r = -\frac{P\,a}{4}\,[\varkappa + (1-\mu)\,\beta\,\Phi_4 - 2\,(1+\mu)\,\beta\,\Phi_3]\,, \qquad Q_r = +P\,\frac{\beta}{\varrho}\,.$$

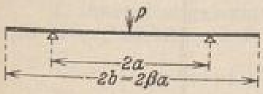
$$M_t = -\frac{P\,a}{4}\,[\varkappa - (1-\mu)\,\beta\,\Phi_4 - 2\,(1+\mu)\,\beta\,\Phi_3]\,.$$

$\varrho = 0$: $$w = -\frac{P\,a^3}{8\,N\,(1+\mu)}\,\varkappa\,.$$

$\varrho = 1$: $$w' = \frac{P\,a^2}{4\,N\,(1+\mu)}\,\varkappa\,; \qquad M_r = M_t = -\frac{P\,a}{4}\,\varkappa\,.$$

$\varrho = \beta$: $$w = \frac{P\,a^3}{8\,N\,(1+\mu)}\left\{[(1-\mu) + (3+\mu)\,\beta^2]\left(\beta - \frac{1}{\beta}\right) - 2\,\varkappa\right\},$$

$$w' = \frac{P\,a^2}{2\,N\,(1+\mu)}\,(\beta^2-1)\,; \qquad M_t = \frac{P\,a}{2\,\beta}\,(1-\mu)\,(1-\beta^2)\,.$$



$$\varkappa = 2\,(1+\mu)\,\beta^2\,.$$

$\varrho \leqq 1$: $$w = \frac{P\,a^2}{8\,\pi\,N}\left[\left(\frac{1-\mu}{\varkappa} + 1\right)\Phi_1 + \Phi_2\right].$$

$$M_r = -\frac{P}{8\,\pi\,\beta^2}\,[(1-\mu)\,(\beta^2-1) + \varkappa\,\Phi_3]\,.$$

$$M_t = -\frac{P}{8\,\pi\,\beta^2}\,[-(1-\mu)\,(\beta^2+1) + \varkappa\,\Phi_3]\,; \qquad Q_r = -\frac{P}{2\,\pi\,a\,\varrho}\,.$$

$\varrho \geqq 1$: $$w = \frac{P\,a^2}{8\,\pi\,N}\left(\frac{1-\mu}{\varkappa}\,\Phi_1 - \Phi_3\right), \qquad M_r = -\frac{P}{8\,\pi\,\beta^2}\,(1-\mu)\,[(\beta^2-1) + \beta^2\,\Phi_4],$$

$$M_t = -\frac{P}{8\,\pi\,\beta^2}\,(1-\mu)\,[-(\beta^2+1) - \beta^2\,\Phi_4]; \qquad Q_r = 0.$$

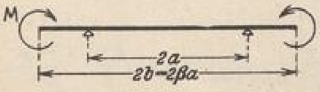
$\varrho = 0$: $$w = \frac{P\,a^2}{8\,\pi\,N}\left(\frac{1-\mu}{\varkappa} + 1\right).$$

$\varrho = 1$: $$w' = -\frac{P\,a}{8\,\pi\,N}\left(2\,\frac{1-\mu}{\varkappa} + 1\right), \qquad M_r = -\frac{P}{8\,\pi\,\beta^2}\,(1-\mu)\,(\beta^2-1);$$

$$M_t = \frac{P}{8\,\pi\,\beta^2}\,(1-\mu)\,(\beta^2+1).$$

$\varrho = \beta$: $$w = -\frac{P\,a^2}{8\,\pi\,N}\left[\frac{1-\mu}{\varkappa}\,(\beta^2-1) + \ln\beta\right]; \qquad w' = -\frac{P\,a}{4\,\pi\,N\,(1+\mu)\,\beta},$$

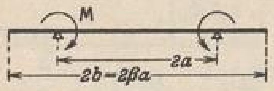
$$M_t = \frac{P}{4\,\pi\,\beta^2}\,(1-\mu).$$



$$w = \frac{\mathsf{M}\,a^2}{2\,N\,(1+\mu)}\,\Phi_1; \qquad M_r = M_t = \mathsf{M}; \qquad Q_r = 0.$$

$\varrho = 0$: $$w = \frac{\mathsf{M}\,a^2}{2\,N\,(1+\mu)};$$ $\varrho = 1$: $$w' = -\frac{\mathsf{M}\,a}{N\,(1+\mu)}.$$

$\varrho = \beta$: $$w = -\frac{\mathsf{M}\,a^2}{2\,N\,(1+\mu)}\,(\beta^2-1); \qquad w' = -\frac{\mathsf{M}\,b}{N\,(1+\mu)}.$$



$$\psi = \frac{1-\mu}{\beta^2}; \qquad \varkappa = (1+\mu) + \psi.$$

$\varrho \leqq 1$: $$w = \frac{\mathsf{M}\,a^2}{4\,N}\,\frac{\varkappa}{1+\mu}\,\Phi_1, \qquad M_r = M_t = \frac{\mathsf{M}}{2}\,\varkappa, \qquad Q_r = 0.$$

$\varrho \geqq 1$: $$w = \frac{\mathsf{M}\,a^2}{4\,N}\left(\frac{\psi}{1+\mu}\,\Phi_1 - 2\,\Phi_3\right); \qquad Q_r = 0,$$

$$M_r = \frac{\mathsf{M}}{2}\,(1-\mu)\left[\left(\frac{1}{\beta^2} - 1\right) - \Phi_4\right], \qquad M_t = \frac{\mathsf{M}}{2}\,(1-\mu)\left[\left(\frac{1}{\beta^2} + 1\right) + \Phi_4\right].$$

$\varrho = 0$: $$w = \frac{\mathsf{M}\,a^2}{4\,N}\,\frac{\varkappa}{1+\mu}.$$

$\varrho = 1$: $$w' = -\frac{\mathsf{M}\,a}{2\,N}\left(1 + \frac{\psi}{1+\mu}\right),$$

$$M_{ri} = \frac{\mathsf{M}}{2}\,\varkappa; \qquad M_{ra} = -\frac{\mathsf{M}}{2}\,(2-\varkappa),$$

$$M_{ti} = \frac{\mathsf{M}}{2}\,\varkappa; \qquad M_{ta} = \frac{\mathsf{M}}{2}\,\psi\,(\beta^2+1).$$

$\varrho = \beta$: $$w = -\frac{\mathsf{M}\,a^2}{4\,N}\left[\frac{\psi}{1+\mu}\,(\beta^2-1) + 2\ln\beta\right].$$

$$w' = -\frac{\mathsf{M}\,a}{N\,(1+\mu)\,\beta}, \qquad M_t = \mathsf{M}\,\psi.$$