Formelsammlung und Repetitorium der Mathematik

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I. Goniometrie.

Ebene Trigonometrie.

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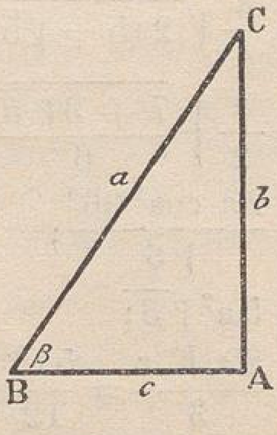
§ 56. Funktionen einfacher Winkel.

1. Erklärung der Funktionen.

a) Am rechtwinkligen Dreieck (a Hypotenuse, b und c Katheten.)

$$\begin{cases} \sin\beta = \frac{b}{a}; \quad \operatorname{tg}\beta = \frac{b}{c}; \quad \operatorname{cosec}\beta = \frac{a}{b}; \\ \cos\beta = \frac{c}{a}; \quad \operatorname{ctg}\beta = \frac{c}{b}; \quad \sec\beta = \frac{a}{c}. \end{cases}$$



b) Am Koordinatensystem (r Fahrstrahl, x und y Abscisse und Ordinate.)

$$\begin{cases} \sin\alpha = \frac{y}{r}; \quad \operatorname{tg}\alpha = \frac{y}{x}; \\ \cos\alpha = \frac{x}{r}; \quad \operatorname{ctg}\alpha = \frac{x}{y}. \end{cases}$$

Der sin hat das Vorzeichen der Ordinate, der cos das der Abscisse, tg und ctg haben gleiches Vorzeichen.

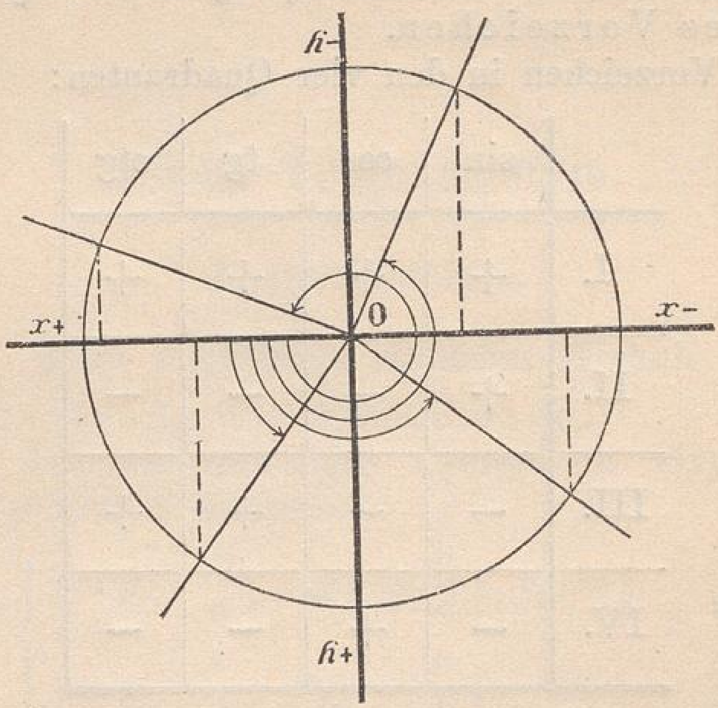
c) Vorzeichen in den vier Quadranten:

	sin	cos	tg	ctg
I.	+	+	+	+
II.	+	—	—	—
III.	—	—	+	+
IV.	—	+	—	—

2.

	$-\alpha$	$R-\alpha$	$R+\alpha$	$2R-\alpha$	$2R+\alpha$
sin	$-\sin\alpha$	$\cos\alpha$	$\cos\alpha$	$\sin\alpha$	$-\sin\alpha$
cos	$\cos\alpha$	$\sin\alpha$	$-\sin\alpha$	$-\cos\alpha$	$-\cos\alpha$
tg	$-\mathrm{tg}\,\alpha$	$\mathrm{ctg}\,\alpha$	$-\mathrm{ctg}\,\alpha$	$-\mathrm{tg}\,\alpha$	$+\mathrm{tg}\,\alpha$
ctg	$-\mathrm{ctg}\,\alpha$	$\mathrm{tg}\,\alpha$	$-\mathrm{tg}\,\alpha$	$-\mathrm{ctg}\,\alpha$	$+\mathrm{ctg}\,\alpha$

	$3R-\alpha$	$3R+\alpha$	$4nR\pm\alpha$		
sin	$-\cos\alpha$	$-\cos\alpha$	$\sin(\pm\alpha)$		
cos	$-\sin\alpha$	$+\sin\alpha$	$\cos(\pm\alpha)$		
tg	$+\mathrm{ctg}\,\alpha$	$-\mathrm{ctg}\,\alpha$	$\mathrm{tg}(\pm\alpha)$		
ctg	$+\mathrm{tg}\,\alpha$	$-\mathrm{tg}\,\alpha$	$\mathrm{ctg}(\pm\alpha)$		



3. Grenzwerte und besondere Werte:

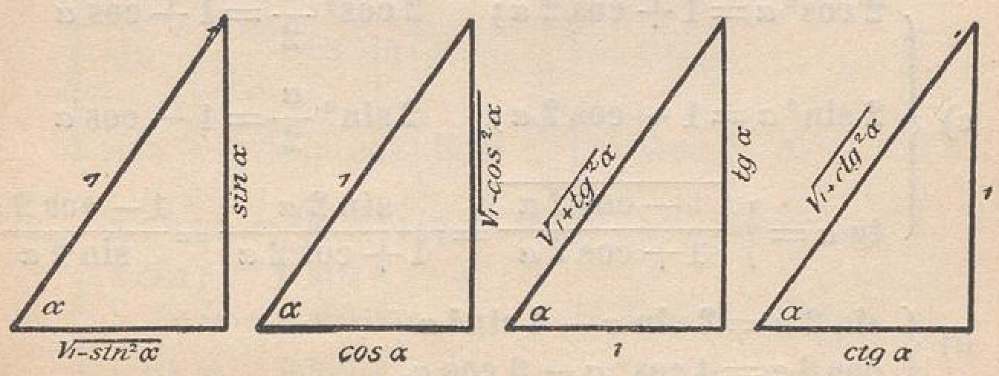
	0° 360°	90°	180°	270°	45°	30°	60°
sin	0	1	0	— 1	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}$	$\frac{1}{2}\sqrt{3}$
cos	1	0	— 1	0	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{3}$	$\frac{1}{2}$
tg	0	∞	0	— ∞	1	$\frac{1}{3}\sqrt{3}$	$\sqrt{3}$
ctg	∞	0	— ∞	0	1	$\sqrt{3}$	$\frac{1}{3}\sqrt{3}$

4. Zusammenhang der Funktionen:

$$\sin^2\alpha + \cos^2\alpha = 1$$

$$\left\{\begin{aligned} \operatorname{tg}\alpha &= \frac{\sin\alpha}{\cos\alpha} = \frac{1}{\operatorname{ctg}\alpha} \\ \operatorname{ctg}\alpha &= \frac{\cos\alpha}{\sin\alpha} = \frac{1}{\operatorname{tg}\alpha} \\ \operatorname{tg}\alpha \cdot \operatorname{ctg}\alpha &= 1 \\ 1 + \operatorname{tg}^2\alpha &= \frac{1}{\cos^2\alpha} \\ 1 + \operatorname{ctg}^2\alpha &= \frac{1}{\sin^2\alpha}. \end{aligned}\right.$$

	$\sin\alpha$	$\cos\alpha$	$\operatorname{tg}\alpha$	$\operatorname{ctg}\alpha$
$\sin\alpha =$		$\sqrt{1-\cos^2\alpha}$	$\frac{\operatorname{tg}\alpha}{\sqrt{1+\operatorname{tg}^2\alpha}}$	$\frac{1}{\sqrt{1+\operatorname{ctg}^2\alpha}}$
$\cos\alpha =$	$\sqrt{1-\sin^2\alpha}$		$\frac{1}{\sqrt{1+\operatorname{tg}^2\alpha}}$	$\frac{\operatorname{ctg}\alpha}{\sqrt{1+\operatorname{ctg}^2\alpha}}$
$\operatorname{tg}\alpha =$	$\frac{\sin\alpha}{\sqrt{1-\sin^2\alpha}}$	$\frac{\sqrt{1-\cos^2\alpha}}{\cos\alpha}$		$\frac{1}{\operatorname{ctg}\alpha}$
$\operatorname{ctg}\alpha =$	$\frac{\sqrt{1-\sin^2\alpha}}{\sin\alpha}$	$\frac{\cos\alpha}{\sqrt{1-\cos^2\alpha}}$	$\frac{1}{\operatorname{tg}\alpha}$	



§ 57. Funktionen zusammengesetzter Winkel.

1\. a)

$$\begin{cases} \sin(\alpha \pm \beta) = \sin\alpha \cos\beta \pm \cos\alpha \sin\beta \\ \cos(\alpha \pm \beta) = \cos\alpha \cos\beta \mp \sin\alpha \sin\beta \\ \operatorname{tg}(\alpha \pm \beta) = \dfrac{\operatorname{tg}\alpha \pm \operatorname{tg}\beta}{1 \mp \operatorname{tg}\alpha \operatorname{tg}\beta} \\ \operatorname{ctg}(\alpha \pm \beta) = \dfrac{\operatorname{ctg}\alpha \operatorname{ctg}\beta \mp 1}{\operatorname{ctg}\beta \pm \operatorname{ctg}\alpha} \end{cases}$$

b)

$$\begin{cases} \sin 2\alpha = 2\sin\alpha\cos\alpha; & \sin\alpha = 2\sin\dfrac{\alpha}{2}\cos\dfrac{\alpha}{2} \\ \cos 2\alpha = \cos^2\alpha - \sin^2\alpha; & \cos\alpha = \cos^2\dfrac{\alpha}{2} - \sin^2\dfrac{\alpha}{2} \\ \operatorname{tg} 2\alpha = \dfrac{2\operatorname{tg}\alpha}{1-\operatorname{tg}^2\alpha}; & \operatorname{tg}\alpha = \dfrac{2\operatorname{tg}\dfrac{\alpha}{2}}{1-\operatorname{tg}^2\dfrac{\alpha}{2}} \\ \operatorname{ctg} 2\alpha = \dfrac{\operatorname{ctg}^2\alpha - 1}{2\operatorname{ctg}\alpha}; & \operatorname{ctg}\alpha = \dfrac{\operatorname{ctg}^2\dfrac{\alpha}{2} - 1}{2\operatorname{ctg}\dfrac{\alpha}{2}} \end{cases}$$

c)

$$\begin{cases} 2\cos^2\alpha = 1 + \cos 2\alpha; & 2\cos^2\dfrac{\alpha}{2} = 1 + \cos\alpha \\ 2\sin^2\alpha = 1 - \cos 2\alpha; & 2\sin^2\dfrac{\alpha}{2} = 1 - \cos\alpha \\ \operatorname{tg}\alpha = \sqrt{\dfrac{1-\cos 2\alpha}{1+\cos 2\alpha}} = \dfrac{\sin 2\alpha}{1+\cos 2\alpha} = \dfrac{1-\cos 2\alpha}{\sin 2\alpha} \end{cases}$$

d)

$$\begin{cases} \sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha \\ \cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha. \end{cases}$$

2. Umformung von Summen und Differenzen.

$$\text{a)}\begin{cases} \sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} \\ \sin\alpha - \sin\beta = 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2} \\ \cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} \\ \cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2} \end{cases}$$

$$\text{b)}\begin{cases} \cos\alpha + \sin\alpha = \sqrt{2}\,.\,\sin(45^0+\alpha) \\ \cos\alpha - \sin\alpha = \sqrt{2}\,.\,\cos(45^0+\alpha) \\ \frac{1+\operatorname{tg}\alpha}{1-\operatorname{tg}\alpha} = \operatorname{tg}(45+\alpha) \\ \frac{\operatorname{ctg}\alpha+1}{\operatorname{ctg}\alpha-1} = \operatorname{ctg}(45^0-\alpha) \end{cases}$$

$$\text{c)}\begin{cases} \operatorname{ctg}\alpha + \operatorname{tg}\alpha = \frac{2}{\sin 2\alpha} \\ \operatorname{ctg}\alpha - \operatorname{tg}\alpha = 2\operatorname{ctg}2\alpha. \end{cases}$$

II. Das Dreieck etc.

§ 58. Formeln über das schiefwinklige Dreieck.

$$1.\begin{cases} \alpha+\beta+\gamma = 2\,\mathrm{R};\ \frac{\alpha}{2}+\frac{\beta}{2}+\frac{\gamma}{2} = \mathrm{R} \\ \sin(\beta+\gamma) = \sin(2\,\mathrm{R}-\alpha) = \sin\alpha \\ \cos(\beta+\gamma) = \cos(2\,\mathrm{R}-\alpha) = -\cos\alpha \\ \sin\frac{\beta+\gamma}{2} = \sin\left(\mathrm{R}-\frac{\alpha}{2}\right) = \cos\frac{\alpha}{2} \\ \cos\frac{\beta+\gamma}{2} = \cos\left(\mathrm{R}-\frac{\alpha}{2}\right) = \sin\frac{\alpha}{2}. \end{cases}$$

2. $\mathbf{a : b : c = \sin\alpha : \sin\beta : \sin\gamma}$. (Sinussatz.)

$$\begin{cases} a\sin\beta = b\sin\alpha = h'' \\ b\sin\gamma = c\sin\beta = h \qquad \text{(Höhenformel.)} \\ c\sin\alpha = a\sin\gamma = h'. \end{cases}$$