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BIBLIOTHEK  
PADERBORN

# Formelsammlung und Repetitorium der Mathematik

**Bürklen, O. Th.**

**Leipzig, 1896**

II. Das Dreieck etc.

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2. Umformung von Summen und Differenzen.

$$a) \left\{ \begin{array}{l} \sin \alpha + \sin \beta = 2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2} \\ \sin \alpha - \sin \beta = 2 \cos \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2} \\ \cos \alpha + \cos \beta = 2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2} \\ \cos \alpha - \cos \beta = -2 \sin \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2} \end{array} \right.$$

$$b) \left\{ \begin{array}{l} \cos \alpha + \sin \alpha = \sqrt{2} \cdot \sin (45^\circ + \alpha) \\ \cos \alpha - \sin \alpha = \sqrt{2} \cdot \cos (45^\circ + \alpha) \\ \frac{1 + \operatorname{tg} \alpha}{1 - \operatorname{tg} \alpha} = \operatorname{tg} (45^\circ + \alpha) \\ \frac{\operatorname{ctg} \alpha + 1}{\operatorname{ctg} \alpha - 1} = \operatorname{ctg} (45^\circ - \alpha) \end{array} \right.$$

$$c) \left\{ \begin{array}{l} \operatorname{ctg} \alpha + \operatorname{tg} \alpha = \frac{2}{\sin 2\alpha} \\ \operatorname{ctg} \alpha - \operatorname{tg} \alpha = 2 \operatorname{ctg} 2\alpha. \end{array} \right.$$

II. Das Dreieck etc.

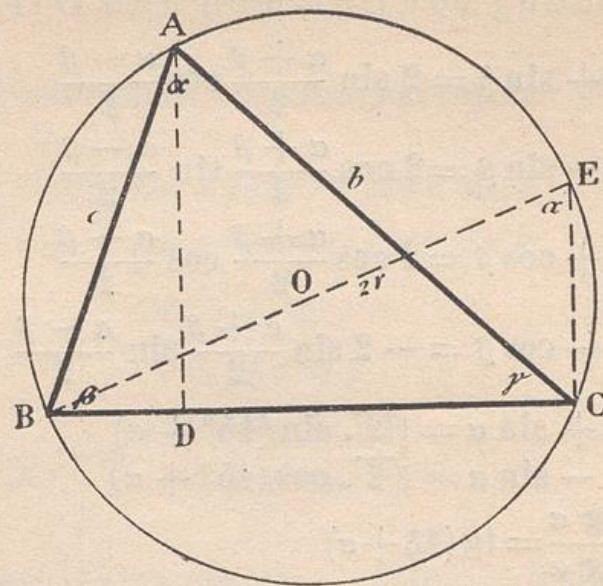
§ 58. Formeln über das schiefwinklige Dreieck.

$$1. \left\{ \begin{array}{l} \alpha + \beta + \gamma = 2R; \quad \frac{\alpha}{2} + \frac{\beta}{2} + \frac{\gamma}{2} = R \\ \sin (\beta + \gamma) = \sin (2R - \alpha) = \sin \alpha \\ \cos (\beta + \gamma) = \cos (2R - \alpha) = -\cos \alpha \\ \sin \frac{\beta + \gamma}{2} = \sin \left( R - \frac{\alpha}{2} \right) = \cos \frac{\alpha}{2} \\ \cos \frac{\beta + \gamma}{2} = \cos \left( R - \frac{\alpha}{2} \right) = \sin \frac{\alpha}{2}. \end{array} \right.$$

2.  $a : b : c = \sin \alpha : \sin \beta : \sin \gamma.$  (Sinussatz.)

$$\left\{ \begin{array}{l} a \sin \beta = b \sin \alpha = h'' \\ b \sin \gamma = c \sin \beta = h \quad (\text{Höhenformel.}) \\ c \sin \alpha = a \sin \gamma = h'. \end{array} \right.$$





$$\left\{ \begin{array}{l} \frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma} = 2r \\ a = 2r \sin \alpha \\ b = 2r \sin \beta \\ c = 2r \sin \gamma. \end{array} \right. \quad (\text{Sehnenformel.})$$

$$3. \left\{ \begin{array}{l} a = b \cos \gamma + c \cos \beta \\ b = c \cos \alpha + a \cos \gamma \\ c = a \cos \beta + b \cos \alpha. \end{array} \right. \quad (\text{Projektionssatz.})$$

$$4. \left\{ \begin{array}{l} \frac{b+c}{b-c} = \frac{\operatorname{tg} \frac{\beta+\gamma}{2}}{\operatorname{tg} \frac{\beta-\gamma}{2}} \\ \frac{c+a}{c-a} = \frac{\operatorname{tg} \frac{\gamma+\alpha}{2}}{\operatorname{tg} \frac{\gamma-\alpha}{2}} \\ \frac{a+b}{a-b} = \frac{\operatorname{tg} \frac{\alpha+\beta}{2}}{\operatorname{tg} \frac{\alpha-\beta}{2}} \end{array} \right. \quad (\text{Neper'sche Gleichgn.})$$



$$5. \left\{ \begin{array}{l} (b+c) \sin \frac{\alpha}{2} = a \cos \frac{\beta-\gamma}{2} \\ (c+a) \sin \frac{\beta}{2} = b \cos \frac{\gamma-\alpha}{2} \\ (a+b) \sin \frac{\gamma}{2} = c \cos \frac{\alpha-\beta}{2} \\ (b-c) \cos \frac{\alpha}{2} = a \sin \frac{\beta-\gamma}{2} \\ (c-a) \cos \frac{\beta}{2} = b \sin \frac{\gamma-\alpha}{2} \\ (a-b) \cos \frac{\gamma}{2} = c \sin \frac{\alpha-\beta}{2} \end{array} \right. \quad (\text{Mollweide'sche Gleichgn.})$$

6. Pythagoräischer Lehrsatz für das schiefwinklige Dreieck.

$$1. \left\{ \begin{array}{l} a^2 = b^2 + c^2 - 2bc \cdot \cos \alpha \\ b^2 = c^2 + a^2 - 2ca \cdot \cos \beta \\ c^2 = a^2 + b^2 - 2ab \cdot \cos \gamma. \end{array} \right.$$

Folgerungen:

$$2. a^2 = \left\{ \begin{array}{l} (b+c)^2 - 4bc \cos^2 \frac{\alpha}{2} \\ (b-c)^2 + 4bc \sin^2 \frac{\alpha}{2}. \end{array} \right.$$

$$\begin{array}{l|l} a+b+c=2s & a-b+c=2(s-b) \\ -a+b+c=2(s-a) & a+b-c=2(s-c). \end{array}$$

$$3. \left\{ \begin{array}{l} \sin \frac{\alpha}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}; \quad \cos \frac{\alpha}{2} = \sqrt{\frac{s(s-a)}{bc}}; \\ \sin \frac{\beta}{2} = \sqrt{\frac{(s-c)(s-a)}{ca}}; \quad \cos \frac{\beta}{2} = \sqrt{\frac{s(s-b)}{ca}}; \\ \sin \frac{\gamma}{2} = \sqrt{\frac{(s-a)(s-b)}{ab}}; \quad \cos \frac{\gamma}{2} = \sqrt{\frac{s(s-c)}{ab}}. \end{array} \right.$$



$$4. \begin{cases} \sin \alpha = \frac{2}{bc} \sqrt{s(s-a)(s-b)(s-c)} \\ \sin \beta = \frac{2}{ca} \sqrt{s(s-a)(s-b)(s-c)} \\ \sin \gamma = \frac{2}{ab} \sqrt{s(s-a)(s-b)(s-c)}. \end{cases}$$

$$5. \begin{cases} \operatorname{tg} \frac{\alpha}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}} = \frac{\varrho}{s-a} \\ \operatorname{tg} \frac{\beta}{2} = \sqrt{\frac{(s-c)(s-a)}{s(s-b)}} = \frac{\varrho}{s-b} \\ \operatorname{tg} \frac{\gamma}{2} = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}} = \frac{\varrho}{s-c}. \end{cases}$$

## 7. Inhalt, In- und Ankreishalbmesser.

$$1. \quad 2J = ab \sin \gamma = bc \sin \alpha = ca \sin \beta.$$

$$2. \quad J = \begin{cases} 2r^2 \sin \alpha \sin \beta \sin \gamma \\ \frac{abc}{4r}. \end{cases}$$

$$3. \quad J = \sqrt{s(s-a)(s-b)(s-c)} \\ = \varrho \cdot s = \varrho_1 (s-a) = \varrho_2 (s-b) = \varrho_3 (s-c).$$

$$4. \quad \varrho \cdot \varrho_1 \cdot \varrho_2 \cdot \varrho_3 = J^2.$$

$$5. \quad \begin{cases} \varrho = (s-a) \operatorname{tg} \frac{\alpha}{2} = (s-b) \operatorname{tg} \frac{\beta}{2} = (s-c) \operatorname{tg} \frac{\gamma}{2} \\ \varrho_1 = s \operatorname{tg} \frac{\alpha}{2}; \quad \varrho_2 = s \operatorname{tg} \frac{\beta}{2}; \quad \varrho_3 = s \operatorname{tg} \frac{\gamma}{2}. \end{cases}$$

$$6. \quad \begin{cases} \varrho = 4r \sin \frac{\alpha}{2} \sin \frac{\beta}{2} \sin \frac{\gamma}{2} \\ s = 4r \cos \frac{\alpha}{2} \cos \frac{\beta}{2} \cos \frac{\gamma}{2}. \end{cases}$$



## § 59. Berechnungen.

## I. Das rechtwinklige Dreieck,

a Hypotenuse.

1. Gegeben  $a, \beta$ .

$$b = a \sin \beta; \quad c = a \cos \beta.$$

2. Gegeben  $b, \beta$ .

$$a = \frac{b}{\sin \beta}, \quad c = b \operatorname{ctg} \beta.$$

3. Gegeben  $a, b$ .

$$\sin \beta = \frac{b}{a}, \quad c = a \cos \beta = b \operatorname{ctg} \beta; \quad c = \sqrt{a^2 - b^2}.$$

4. Gegeben  $b, c$ .

$$\operatorname{tg} \beta = \frac{b}{c}; \quad a = \frac{b}{\sin \beta} = \frac{c}{\cos \beta}; \quad a = \sqrt{b^2 + c^2}$$

$$2J = bc = ab \cos \beta = ab \sin \gamma = b^2 \operatorname{tg} \gamma.$$

## II. Das gleichschenklige Dreieck.

1. Gegeben  $b, \beta$ .

$$a = 2b \cos \beta; \quad h = b \sin \beta.$$

2. Gegeben  $a, \alpha$ .

$$b = \frac{a}{2 \cos \beta}; \quad h = \frac{a}{2} \operatorname{tg} \beta.$$

3. Gegeben  $a$  und  $b$ .

$$\cos \beta = \frac{a}{2b}; \quad h = b \sin \beta = \frac{a}{2} \operatorname{tg} \beta = \sqrt{b^2 - \frac{a^2}{4}}$$

$$2J = b^2 \sin \alpha = \frac{a^2}{4} \operatorname{tg} \beta.$$

## III. Das regelmässige Vieleck.

1. Gegeben  $a$ .

$$r = \frac{a}{2} : \sin \frac{180^\circ}{n}$$



$$\rho = \frac{a}{2} \operatorname{ctg} \frac{180^\circ}{n}$$

$$J = \frac{na^2}{4} \operatorname{ctg} \frac{180^\circ}{n}.$$

2. Gegeben  $r$ .

$$a = 2r \sin \frac{180^\circ}{n}$$

$$\rho = r \cos \frac{180^\circ}{n}$$

$$J = \frac{nr^2}{2} \sin \frac{360^\circ}{n}.$$

3. Gegeben  $\rho$ .

$$r = \rho : \cos \frac{180^\circ}{n}$$

$$a = 2\rho \operatorname{tg} \frac{180^\circ}{n}$$

$$J = n\rho^2 \operatorname{tg} \frac{180^\circ}{n}.$$

#### IV. Segment.

$$\text{Sektor} = \frac{r^2 \pi \alpha^\circ}{360^\circ} = \frac{r^2}{2} \operatorname{arc} \alpha.$$

$$\Delta = \frac{r^2}{2} \sin \alpha$$

$$\text{Segment} = \frac{r^2}{2} \left( \frac{\pi \alpha^\circ}{180^\circ} - \sin \alpha \right) = \frac{r^2}{2} (\operatorname{arc} \alpha - \sin \alpha).$$

#### V. Das schiefwinklige Dreieck.

1. Gegeben  $a, \beta, \gamma$ .

$$\alpha = 180^\circ - (\beta + \gamma); \quad b = \frac{a \sin \beta}{\sin \alpha}; \quad c = \frac{a \sin \gamma}{\sin \alpha}.$$



2. Gegeben,  $b, c, \alpha$ .

$$\frac{\beta + \gamma}{2} = R - \frac{\alpha}{2}$$

$$\operatorname{tg} \frac{\beta - \gamma}{2} = \frac{b - c}{b + c} \operatorname{tg} \frac{\beta + \gamma}{2}$$

$$\begin{cases} \beta = \frac{\beta + \gamma}{2} + \frac{\beta - \gamma}{2} \\ \gamma = \frac{\beta + \gamma}{2} - \frac{\beta - \gamma}{2} \end{cases}$$

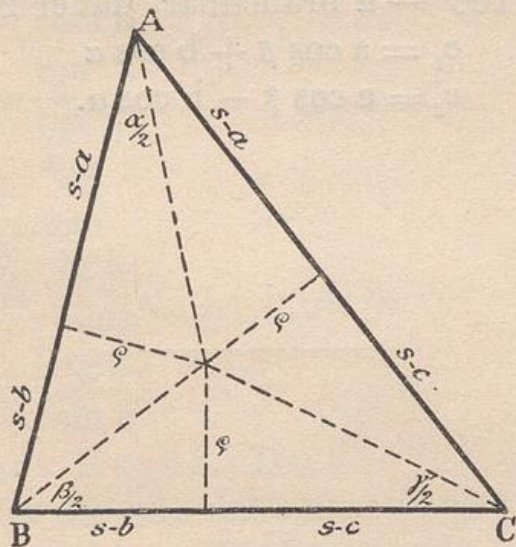
$$a = \frac{b \sin \alpha}{\sin \beta}$$

$$(\text{=} \sqrt{b^2 + c^2 - 2bc \cdot \cos \alpha})$$

oder:

$$\operatorname{tg} \beta = \frac{b \sin \alpha}{c - b \cos \alpha}$$

$$a = \frac{b \sin \alpha}{\sin \beta} = \frac{c - b \cos \alpha}{\cos \beta}$$

3. Gegeben  $a, b, c$ .

$$1. J = \sqrt{s(s-a)(s-b)(s-c)}.$$

$$2. \rho = \frac{J}{s}. \quad (2s = a + b + c).$$

$$3. \operatorname{tg} \frac{\alpha}{2} = \frac{\rho}{s-a}; \quad \operatorname{tg} \frac{\beta}{2} = \frac{\rho}{s-b}; \quad \operatorname{tg} \frac{\gamma}{2} = \frac{\rho}{s-c}.$$

$$\text{Proben: } 1. (s-a) + (s-b) + (s-c) = s.$$



$$2. \quad s \cdot \operatorname{tg} \frac{\alpha}{2} \cdot \operatorname{tg} \frac{\beta}{2} \cdot \operatorname{tg} \frac{\gamma}{2} = e.$$

$$3. \quad \frac{\alpha}{2} + \frac{\beta}{2} + \frac{\gamma}{2} = 90^\circ.$$

4. Gegeben  $a, b, \beta$ .

a)  $b > a$ .

$$1. \quad \sin \alpha = \frac{a \sin \beta}{b}; \quad \alpha < 90^\circ$$

$$2. \quad \gamma = 180^\circ - (\alpha + \beta).$$

$$3. \quad c = \frac{b \sin \gamma}{\sin \beta} = \frac{a \sin \gamma}{\sin \alpha}.$$

$$b) \quad b < a; \quad \sin \alpha = \frac{a \sin \beta}{b};$$

hiebei  $\alpha$  und  $180^\circ - \alpha$  brauchbar, daher 2 Werte für  $c$ :

$$c_1 = a \cos \beta + b \cos \alpha$$

$$c_2 = a \cos \beta - b \cos \alpha.$$

