Formelsammlung und Repetitorium der Mathematik

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§ 13. Wurzeln.

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7. $\frac{a^m - b^m}{a - b} = a^{m-1} + a^{m-2}b + a^{m-3}b^2 + \ldots + ab^{m-2} + b^{m-1}$.

8. $\left.\frac{a^m - b^m}{a - b}\right|_{a=b} = \frac{0}{0} = m\,a^{m-1}$.

II. Negative Exponenten.

Erklärung: $a^{-m} = \frac{1}{a^m}$.

1. $\begin{cases} a^m \cdot a^{-r} = a^{m-r} \\ a^{-m} \cdot a^r = a^{-m+r} \\ a^{-m} \cdot a^{-r} = a^{-m-r} = a^{-(m+r)}. \end{cases}$

2. $\begin{cases} a^m : a^{-r} = a^{m+r} \\ a^{-m} : a^r = a^{-m-r} \\ a^{-m} : a^{-r} = a^{-m+r}. \end{cases}$

3. $(ab)^{-m} = a^{-m}b^{-m}$.

4. $\left(\frac{a}{b}\right)^{-m} = \frac{a^{-m}}{b^{-m}} = \left(\frac{b}{a}\right)^m$.

5. $\begin{cases} (a^m)^{-r} = a^{-mr} \\ (a^{-m})^r = a^{-mr} \\ (a^{-m})^{-r} = a^{mr}. \end{cases}$

§ 13. Wurzeln.

Erklärung: Wenn $x^n = a$, dann $x = \sqrt[n]{a}$, also

$$\left(\sqrt[n]{a}\right)^n = a, \quad \sqrt[n]{a^n} = a$$

Benennungen: Bei $\sqrt[n]{a}$ heisst a Radikand, n Wurzelexponent;

$$\sqrt[2]{a} = \sqrt{a}$$

I. Wurzelformeln:

1. $\sqrt[n]{1} = 1$; $\sqrt[n]{0} = 0$; $\sqrt[2n+1]{1} = 1$; $\sqrt[2n+1]{-1} = -1$.

2. $\sqrt[1]{a} = a$; $\sqrt[2n+1]{a^{2n+1}} = a$, $\sqrt[2n+1]{-a^{2n+1}} = -a$;
$\sqrt[2n]{(-a)^{2n}} = -a$.

3. $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$.

4. $\sqrt[n]{a} : \sqrt[n]{b} = \sqrt[n]{a : b} = \sqrt[n]{\frac{a}{b}}$.

5. $\sqrt[n]{a^r} = \left(\sqrt[n]{a}\right)^r$.

6. $\sqrt[n]{a^r} = \sqrt[nx]{a^{rx}}$; $\sqrt[-n]{a^r} = \sqrt[n]{a^{-r}}$.

7. $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} = \sqrt[n]{\sqrt[m]{a}}$.

8. $a\sqrt[n]{b} = \sqrt[n]{a^n b}$.

II. Rationalmachen des Nenners:

1. $\frac{z}{\sqrt{a}} = \frac{z\sqrt{a}}{a}$; $\frac{z}{\sqrt[n]{a}} = \frac{z\sqrt[n]{a^{n-1}}}{a}$.

2. $\frac{z}{\sqrt{a}+\sqrt{b}} = \frac{z(\sqrt{a}-\sqrt{b})}{a-b}$;

$$\frac{z}{\sqrt[3]{a} \pm \sqrt[3]{b}} = \frac{z\left(\sqrt[3]{a^2} \mp \sqrt[3]{ab} + \sqrt[3]{b^2}\right)}{a \pm b}$$

3. $\frac{z}{\sqrt{a}+\sqrt{b}+\sqrt{c}} = \frac{z(\sqrt{a}+\sqrt{b}-\sqrt{c})}{(\sqrt{a}+\sqrt{b})^2 - c}$

$$= \frac{z(\sqrt{a}+\sqrt{b}-\sqrt{c})(a+b-c-2\sqrt{ab})}{(a+b-c)^2 - 4ab};$$

besonderer Fall $a + b = c$.

4. $$\frac{z}{\sqrt{a+\sqrt{b}}} = \frac{z\sqrt{a+\sqrt{b}}}{a+\sqrt{b}} = \frac{z\sqrt{a+\sqrt{b}}(a-\sqrt{b})}{a^2-b}$$

$$= \frac{z\sqrt{(a^2-b)(a-\sqrt{b})}}{a^2-b}.$$

III. Zerlegung einer Quadrat-Wurzel:

$$\sqrt{a \pm \sqrt{b}} = \sqrt{\frac{a+r}{2}} \pm \sqrt{\frac{a-r}{2}}, \text{ wobei } r = \sqrt{a^2-b}.$$

IV. Beispiele für Quadrat- und Kubikwurzelausziehung:

1. $\sqrt{12|53|16} = 354$

```
  9
 ---
 6|353
   325
  -----
 70|2816
    2816
   -----
   ====
```

2. $\sqrt[3]{44|361|864} = 354$

```
    27
    ---
  27|17361
    |135
      225
       125
     ------
3675|1486864
    |14700
       1680
         64
   ----------
   =======
```

3. $\sqrt{\left(25x^6 - 30x^5 + 79x^4 - 57x^3 + 58x^2 - 21x + \frac{9}{4}\right)} = 5x^3 - 3x^2 + 7x - \frac{3}{2}$

$$\pm 25x^6$$
$$10x^3 \,|\, -30x^5 + 79x^4$$
$$\mp 30x^5 \pm 9x^4$$
$$10x^3 - 6x^2 \,|\, +70x^4$$
$$\pm 70x^4 \mp 42x^3 \pm 49x^2$$
$$10x^3 - 6x^2 + 14x \,|\, -15x^3 + 9x^2$$
$$\mp 15x^3 \pm 9x^2 \mp 21x \pm \frac{9}{4}$$
$$= \quad = \quad = \quad =$$

4. $\sqrt[3]{(125x^9 - 225x^8 + 660x^7 - 657x^6 + 924x^5 - 441x^4 + 343x^3)} = 5x^3 - 3x^2 + 7x$

$$\pm 125x^9$$
$$75x^6 \,|\, -225x^8$$
$$\mp 225x^8 \pm 135x^7 \mp 27x^6$$
$$75x^6 - 90x^5 + 27x^4 \,|\, +525x^7 - 630x^6$$
$$\pm 525x^7 \mp 630x^6 \pm 189x^5$$
$$\pm 735x^5 \mp 441x^4 \pm 343x^3$$
$$= \quad = \quad = \quad = \quad =$$

§ 14. Potenzen mit gebrochenen Exponenten.

Erklärung: $a^{\frac{r}{n}} = \sqrt[n]{a^r}$; $a^{\frac{-r}{n}} = a^{\frac{-r}{n}} = a^{\frac{r}{-n}} = \frac{1}{a^{\frac{r}{n}}} = \sqrt[n]{a^{-r}}$

$$= \sqrt[-n]{a^r} = \frac{1}{\sqrt[n]{a^r}}$$

1. $1^{\frac{r}{n}} = 1$; $0^{\frac{r}{n}} = 0$.
2. $a^{\frac{r}{n}} \cdot a^{\frac{p}{q}} = a^{\frac{r}{n} + \frac{p}{q}}$.
3. $a^{\frac{r}{n}} : a^{\frac{p}{q}} = a^{\frac{r}{n} - \frac{p}{q}}$.