Formelsammlung und Repetitorium der Mathematik

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§ 57. Funktionen zusammengesetzter Winkel.

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1. a)
$$\begin{cases} \sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta \\ \cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta \\ \operatorname{tg}(\alpha \pm \beta) = \dfrac{\operatorname{tg}\alpha \pm \operatorname{tg}\beta}{1 \mp \operatorname{tg}\alpha\operatorname{tg}\beta} \\ \operatorname{ctg}(\alpha \pm \beta) = \dfrac{\operatorname{ctg}\alpha\operatorname{ctg}\beta \mp 1}{\operatorname{ctg}\beta \pm \operatorname{ctg}\alpha} \end{cases}$$

b)
$$\begin{cases} \sin 2\alpha = 2\sin\alpha\cos\alpha; & \sin\alpha = 2\sin\dfrac{\alpha}{2}\cos\dfrac{\alpha}{2} \\ \cos 2\alpha = \cos^2\alpha - \sin^2\alpha; & \cos\alpha = \cos^2\dfrac{\alpha}{2} - \sin^2\dfrac{\alpha}{2} \\ \operatorname{tg} 2\alpha = \dfrac{2\operatorname{tg}\alpha}{1 - \operatorname{tg}^2\alpha}; & \operatorname{tg}\alpha = \dfrac{2\operatorname{tg}\dfrac{\alpha}{2}}{1 - \operatorname{tg}^2\dfrac{\alpha}{2}} \\ \operatorname{ctg} 2\alpha = \dfrac{\operatorname{ctg}^2\alpha - 1}{2\operatorname{ctg}\alpha}; & \operatorname{ctg}\alpha = \dfrac{\operatorname{ctg}^2\dfrac{\alpha}{2} - 1}{2\operatorname{ctg}\dfrac{\alpha}{2}} \end{cases}$$

c)
$$\begin{cases} 2\cos^2\alpha = 1 + \cos 2\alpha; & 2\cos^2\dfrac{\alpha}{2} = 1 + \cos\alpha \\ 2\sin^2\alpha = 1 - \cos 2\alpha; & 2\sin^2\dfrac{\alpha}{2} = 1 - \cos\alpha \\ \operatorname{tg}\alpha = \sqrt{\dfrac{1 - \cos 2\alpha}{1 + \cos 2\alpha}} = \dfrac{\sin 2\alpha}{1 + \cos 2\alpha} = \dfrac{1 - \cos 2\alpha}{\sin 2\alpha} \end{cases}$$

d)
$$\begin{cases} \sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha \\ \cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha. \end{cases}$$

2. Umformung von Summen und Differenzen.

a)
$$\begin{cases} \sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} \\ \sin\alpha - \sin\beta = 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2} \\ \cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} \\ \cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2} \end{cases}$$

b)
$$\begin{cases} \cos\alpha + \sin\alpha = \sqrt{2}\,.\,\sin(45^\circ + \alpha) \\ \cos\alpha - \sin\alpha = \sqrt{2}\,.\,\cos(45^\circ + \alpha) \\ \frac{1+\operatorname{tg}\alpha}{1-\operatorname{tg}\alpha} = \operatorname{tg}(45+\alpha) \\ \frac{\operatorname{ctg}\alpha + 1}{\operatorname{ctg}\alpha - 1} = \operatorname{ctg}(45^\circ - \alpha) \end{cases}$$

c)
$$\begin{cases} \operatorname{ctg}\alpha + \operatorname{tg}\alpha = \frac{2}{\sin 2\alpha} \\ \operatorname{ctg}\alpha - \operatorname{tg}\alpha = 2\operatorname{ctg} 2\alpha. \end{cases}$$

II. Das Dreieck etc.

§ 58. Formeln über das schiefwinklige Dreieck.

1.
$$\begin{cases} \alpha + \beta + \gamma = 2\,\mathrm{R};\ \frac{\alpha}{2} + \frac{\beta}{2} + \frac{\gamma}{2} = \mathrm{R} \\ \sin(\beta + \gamma) = \sin(2\,\mathrm{R} - \alpha) = \sin\alpha \\ \cos(\beta + \gamma) = \cos(2\,\mathrm{R} - \alpha) = -\cos\alpha \\ \sin\frac{\beta+\gamma}{2} = \sin\left(\mathrm{R} - \frac{\alpha}{2}\right) = \cos\frac{\alpha}{2} \\ \cos\frac{\beta+\gamma}{2} = \cos\left(\mathrm{R} - \frac{\alpha}{2}\right) = \sin\frac{\alpha}{2}. \end{cases}$$

2. $\mathrm{a} : \mathrm{b} : \mathrm{c} = \sin\alpha : \sin\beta : \sin\gamma.$ (Sinussatz.)

$$\begin{cases} \mathrm{a}\sin\beta = \mathrm{b}\sin\alpha = \mathrm{h}'' \\ \mathrm{b}\sin\gamma = \mathrm{c}\sin\beta = \mathrm{h} \\ \mathrm{c}\sin\alpha = \mathrm{a}\sin\gamma = \mathrm{h}'. \end{cases}$$ (Höhenformel.)