Formelsammlung und Repetitorium der Mathematik

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§ 58. Formeln über das schiefwinklige Dreieck.

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2. Umformung von Summen und Differenzen.

a)
$$\begin{cases} \sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} \\ \sin\alpha - \sin\beta = 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2} \\ \cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} \\ \cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2} \end{cases}$$

b)
$$\begin{cases} \cos\alpha + \sin\alpha = \sqrt{2}\,.\sin(45^\circ+\alpha) \\ \cos\alpha - \sin\alpha = \sqrt{2}\,.\cos(45^\circ+\alpha) \\ \frac{1+\operatorname{tg}\alpha}{1-\operatorname{tg}\alpha} = \operatorname{tg}(45+\alpha) \\ \frac{\operatorname{ctg}\alpha+1}{\operatorname{ctg}\alpha-1} = \operatorname{ctg}(45^\circ-\alpha) \end{cases}$$

c)
$$\begin{cases} \operatorname{ctg}\alpha + \operatorname{tg}\alpha = \frac{2}{\sin 2\alpha} \\ \operatorname{ctg}\alpha - \operatorname{tg}\alpha = 2\operatorname{ctg}2\alpha. \end{cases}$$

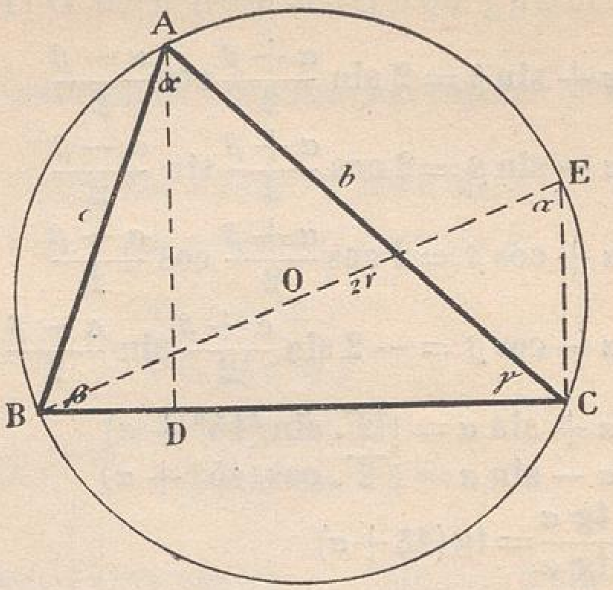
II. Das Dreieck etc.

§ 58. Formeln über das schiefwinklige Dreieck.

1.
$$\begin{cases} \alpha+\beta+\gamma = 2\,\mathrm{R};\quad \frac{\alpha}{2}+\frac{\beta}{2}+\frac{\gamma}{2} = \mathrm{R} \\ \sin(\beta+\gamma) = \sin(2\,\mathrm{R}-\alpha) = \sin\alpha \\ \cos(\beta+\gamma) = \cos(2\,\mathrm{R}-\alpha) = -\cos\alpha \\ \sin\frac{\beta+\gamma}{2} = \sin\left(\mathrm{R}-\frac{\alpha}{2}\right) = \cos\frac{\alpha}{2} \\ \cos\frac{\beta+\gamma}{2} = \cos\left(\mathrm{R}-\frac{\alpha}{2}\right) = \sin\frac{\alpha}{2}. \end{cases}$$

2. $\mathrm{a} : \mathrm{b} : \mathrm{c} = \sin\alpha : \sin\beta : \sin\gamma.$ (Sinussatz.)

$$\begin{cases} \mathrm{a}\sin\beta = \mathrm{b}\sin\alpha = \mathrm{h}'' \\ \mathrm{b}\sin\gamma = \mathrm{c}\sin\beta = \mathrm{h} \\ \mathrm{c}\sin\alpha = \mathrm{a}\sin\gamma = \mathrm{h}'. \end{cases}$$
(Höhenformel.)



$$\left\{\begin{array}{l} \dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma} = 2\,r \\ a = 2\,r \sin\alpha \\ b = 2\,r \sin\beta \\ c = 2\,r \sin\gamma. \end{array}\right.$$ (Sehnenformel.)

3. $$\left\{\begin{array}{l} a = b \cos\gamma + c \cos\beta \\ b = c \cos\alpha + a \cos\gamma \\ c = a \cos\beta + b \cos\alpha. \end{array}\right.$$ (Projektionssatz.)

4. $$\left\{\begin{array}{l} \dfrac{b+c}{b-c} = \dfrac{\operatorname{tg}\dfrac{\beta+\gamma}{2}}{\operatorname{tg}\dfrac{\beta-\gamma}{2}} \\[2ex] \dfrac{c+a}{c-a} = \dfrac{\operatorname{tg}\dfrac{\gamma+\alpha}{2}}{\operatorname{tg}\dfrac{\gamma-\alpha}{2}} \\[2ex] \dfrac{a+b}{a-b} = \dfrac{\operatorname{tg}\dfrac{\alpha+\beta}{2}}{\operatorname{tg}\dfrac{\alpha-\beta}{2}} \end{array}\right.$$ (Neper'sche Gleichgn.)

5. $$\begin{cases} (b+c)\sin\frac{\alpha}{2} = a\cos\frac{\beta-\gamma}{2} \\ (c+a)\sin\frac{\beta}{2} = b\cos\frac{\gamma-\alpha}{2} \\ (a+b)\sin\frac{\gamma}{2} = c\cos\frac{\alpha-\beta}{2} \\ (b-c)\cos\frac{\alpha}{2} = a\sin\frac{\beta-\gamma}{2} \\ (c-a)\cos\frac{\beta}{2} = b\sin\frac{\gamma-\alpha}{2} \\ (a-b)\cos\frac{\gamma}{2} = c\sin\frac{\alpha-\beta}{2} \end{cases}$$ (Mollweide'sche Gleichgn.)

6. Pythagoräischer Lehrsatz für das schiefwinklige Dreieck.

1. $$\begin{cases} \mathbf{a^2 = b^2 + c^2 - 2\,bc \cdot \cos\alpha} \\ \mathbf{b^2 = c^2 + a^2 - 2\,ca \cdot \cos\beta} \\ \mathbf{c^2 = a^2 + b^2 - 2\,ab \cdot \cos\gamma.} \end{cases}$$

Folgerungen:

2. $$a^2 = \begin{cases} (b+c)^2 - 4\,bc\cos^2\frac{\alpha}{2} \\ (b-c)^2 + 4\,bc\sin^2\frac{\alpha}{2}. \end{cases}$$

$$a+b+c = 2s \qquad a-b+c = 2(s-b)$$
$$-a+b+c = 2(s-a) \qquad a+b-c = 2(s-c).$$

3. $$\begin{cases} \sin\frac{\alpha}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}; & \cos\frac{\alpha}{2} = \sqrt{\frac{s(s-a)}{bc}}; \\ \sin\frac{\beta}{2} = \sqrt{\frac{(s-c)(s-a)}{ca}}; & \cos\frac{\beta}{2} = \sqrt{\frac{s(s-b)}{ca}}; \\ \sin\frac{\gamma}{2} = \sqrt{\frac{(s-a)(s-b)}{ab}}; & \cos\frac{\gamma}{2} = \sqrt{\frac{s(s-c)}{ab}}. \end{cases}$$

4. $$\begin{cases} \sin\alpha = \frac{2}{bc}\sqrt{s(s-a)(s-b)(s-c)} \\ \sin\beta = \frac{2}{ca}\sqrt{s(s-a)(s-b)(s-c)} \\ \sin\gamma = \frac{2}{ab}\sqrt{s(s-a)(s-b)(s-c)}. \end{cases}$$

5. $$\begin{cases} \operatorname{tg}\frac{\alpha}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}} = \frac{\varrho}{s-a} \\ \operatorname{tg}\frac{\beta}{2} = \sqrt{\frac{(s-c)(s-a)}{s(s-b)}} = \frac{\varrho}{s-b} \\ \operatorname{tg}\frac{\gamma}{2} = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}} = \frac{\varrho}{s-c}. \end{cases}$$

7. Inhalt, In- und Ankreishalbmesser.

1. $\mathbf{2\,J = ab\sin\gamma} = bc\sin\alpha = ca\sin\beta$.

2. $$J = \begin{cases} 2r^2\sin\alpha\sin\beta\sin\gamma \\ \frac{abc}{4r}. \end{cases}$$

3. $$J = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \varrho \cdot s = \varrho_1(s-a) = \varrho_2(s-b) = \varrho_3(s-c).$$

4. $\varrho \cdot \varrho_1 \cdot \varrho_2 \cdot \varrho_3 = J^2$.

5. $$\begin{cases} \varrho = (s-a)\operatorname{tg}\frac{\alpha}{2} = (s-b)\operatorname{tg}\frac{\beta}{2} = (s-c)\operatorname{tg}\frac{\gamma}{2} \\ \varrho_1 = s\operatorname{tg}\frac{\alpha}{2}; \quad \varrho_2 = s\operatorname{tg}\frac{\beta}{2}; \quad \varrho_3 = s\operatorname{tg}\frac{\gamma}{2}. \end{cases}$$

6. $$\begin{cases} \varrho = 4r\sin\frac{\alpha}{2}\sin\frac{\beta}{2}\sin\frac{\gamma}{2} \\ s = 4r\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\gamma}{2}. \end{cases}$$