Formelsammlung und Repetitorium der Mathematik

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§ 59. Berechnungen.

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I. Das rechtwinklige Dreieck,

a Hypotenuse.

1. Gegeben a, β.

$$b = a \sin \beta; \quad c = a \cos \beta.$$

2. Gegeben b, β.

$$a = \frac{b}{\sin \beta}, \quad c = b \operatorname{ctg} \beta.$$

3. Gegeben a, b.

$$\sin \beta = \frac{b}{a}, \; c = a \cos \beta = b \operatorname{ctg} \beta; \; c = \sqrt{a^2 - b^2}.$$

4. Gegeben b, c.

$$\operatorname{tg} \beta = \frac{b}{c}; \; a = \frac{b}{\sin \beta} = \frac{c}{\cos \beta}; \; a = \sqrt{b^2 + c^2}$$

$$2\,J = b\,c = a\,b \cos \beta = a\,b \sin \gamma = b^2 \operatorname{tg} \gamma.$$

II. Das gleichschenklige Dreieck.

1. Gegeben b, β.

$$a = 2\,b \cos \beta; \quad h = b \sin \beta.$$

2. Gegeben a, α.

$$b = \frac{a}{2 \cos \beta}; \; h = \frac{a}{2} \operatorname{tg} \beta.$$

3. Gegeben a und b.

$$\cos \beta = \frac{a}{2\,b}; \quad h = b \sin \beta = \frac{a}{2} \operatorname{tg} \beta = \sqrt{b^2 - \frac{a^2}{4}}$$

$$2\,J = b^2 \sin \alpha = \frac{a^2}{4} \operatorname{tg} \beta.$$

III. Das regelmässige Vieleck.

1. Gegeben a.

$$r = \frac{a}{2} : \sin \frac{180^0}{n}$$

$$\varrho = \frac{a}{2} \operatorname{ctg} \frac{180^0}{n}$$

$$J = \frac{n a^2}{4} \operatorname{ctg} \frac{180^0}{n}.$$

2. Gegeben r.

$$a = 2 r \sin \frac{180^0}{n}$$

$$\varrho = r \cos \frac{180^0}{n}$$

$$J = \frac{n r^2}{2} \sin \frac{360^0}{n}.$$

3. Gegeben ϱ.

$$r = \varrho : \cos \frac{180^0}{n}$$

$$a = 2 \varrho \operatorname{tg} \frac{180^0}{n}$$

$$J = n \varrho^2 \operatorname{tg} \frac{180^0}{n}.$$

IV. Segment.

$$\text{Sektor} = \frac{r^2 \pi \alpha^0}{360^0} = \frac{r^2}{2} \operatorname{arc} \alpha.$$

$$\Delta = \frac{r^2}{2} \sin \alpha$$

$$\text{Segment} = \frac{r^2}{2} \left(\frac{\pi \alpha^0}{180^0} - \sin \alpha \right) = \frac{r^2}{2} (\operatorname{arc} \alpha - \sin \alpha).$$

V. Das schiefwinklige Dreieck.

1. Gegeben a, β, γ.

$$\alpha = 180^0 - (\beta + \gamma); \quad b = \frac{a \sin \beta}{\sin \alpha}; \quad c = \frac{a \sin \gamma}{\sin \alpha}.$$

2. Gegeben, b, c, α.

$$\frac{\beta+\gamma}{2} = R - \frac{\alpha}{2}$$

$$\operatorname{tg}\frac{\beta-\gamma}{2} = \frac{b-c}{b+c}\operatorname{tg}\frac{\beta+\gamma}{2}$$

$$\begin{cases} \beta = \dfrac{\beta+\gamma}{2} + \dfrac{\beta-\gamma}{2} \\ \gamma = \dfrac{\beta+\gamma}{2} - \dfrac{\beta-\gamma}{2} \end{cases}$$

$$a = \frac{b \sin\alpha}{\sin\beta}$$

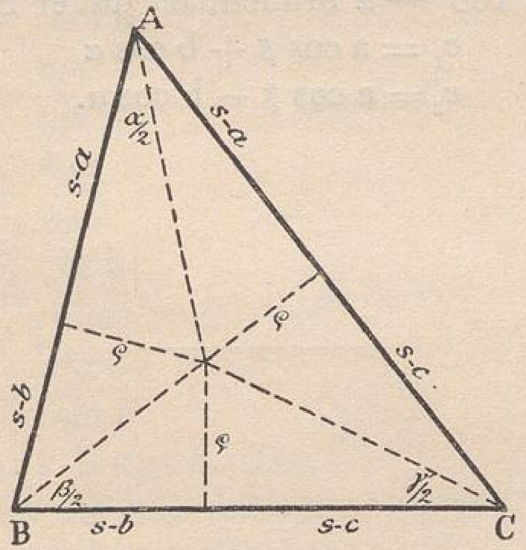
$$\left(= \sqrt{b^2 + c^2 - 2bc \, . \cos\alpha}\right)$$

oder:

$$\operatorname{tg}\beta = \frac{b \sin\alpha}{c - b\cos\alpha}$$

$$a = \frac{b\sin\alpha}{\sin\beta} = \frac{c - b\cos\alpha}{\cos\beta}.$$

3. Gegeben a, b, c.



1. $J = \sqrt{s(s-a)(s-b)(s-c)}.$

2. $\varrho = \frac{J}{s}. \qquad (2s = a + b + c).$

3. $\operatorname{tg}\frac{\alpha}{2} = \frac{\varrho}{s-a}; \quad \operatorname{tg}\frac{\beta}{2} = \frac{\varrho}{s-b}; \quad \operatorname{tg}\frac{\gamma}{2} = \frac{\varrho}{s-c}.$

Proben: 1. $(s-a) + (s-b) + (s-c) = s.$

$$2.\ s \cdot \operatorname{tg}\frac{\alpha}{2} \cdot \operatorname{tg}\frac{\beta}{2} \cdot \operatorname{tg}\frac{\gamma}{2} = \varrho.$$

$$3.\ \frac{\alpha}{2} + \frac{\beta}{2} + \frac{\gamma}{2} = 90^0.$$

4. Gegeben a, b, β.

a) **b > a.**

$$1.\ \sin\alpha = \frac{a \sin\beta}{b};\quad \alpha < 90^0$$

$$2.\ \gamma = 180^0 - (\alpha + \beta).$$

$$3.\ c = \frac{b \sin\gamma}{\sin\beta} = \frac{a \sin\gamma}{\sin\alpha}.$$

b) **b < a**; $\sin\alpha = \frac{a \sin\beta}{b}$;

hiebei α und $180^0 - \alpha$ brauchbar, daher 2 Werte für c:

$$c_1 = a \cos\beta + b \cos\alpha$$

$$c_2 = a \cos\beta - b \cos\alpha.$$