



UNIVERSITÄTS-
BIBLIOTHEK
PADERBORN

Algebra

Barth, Friedrich

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Aufgaben

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Nun weißt du, wie du Aggregate aus Bruchtermen berechnen kannst. Was machst du aber mit dem Term $3x + \frac{6x}{7x-2}$? Wir erinnern uns, dass man jede Zahl als Bruch mit dem Nenner 1 schreiben kann, z. B. $5 = \frac{5}{1}$. Genauso kann man jeden Nicht-Bruchterm als Bruchterm mit dem Nenner 1 schreiben. Machen wir's!

$$\begin{aligned} 3x + \frac{6x}{7x-2} &= \frac{3x}{1} + \frac{6x}{7x-2} = \frac{3x(7x-2) + 6x \cdot 1}{7x-2} = \\ &= \frac{21x^2 - 6x + 6x}{7x-2} = \frac{21x^2}{7x-2} \end{aligned}$$

Aufgaben

1. a) $\frac{a}{10} + \frac{2a}{5}$ b) $\frac{5p}{9} - \frac{3p}{4}$ c) $\frac{a}{5} - \frac{b}{3}$ d) $\frac{3p}{14} + \frac{11q}{21}$
2. a) $\frac{7}{a} + \frac{3}{b}$ b) $\frac{6}{a} - \frac{2}{3a}$ c) $\frac{11}{8m} + \frac{3}{4n}$ d) $\frac{-1}{7z} + \frac{27}{196z}$
3. a) $\frac{6n}{7m} - \frac{5}{4m}$ b) $\frac{11u}{13vw} - \frac{8u}{3vw}$ c) $\frac{7b}{a^2} + \frac{3}{b}$ d) $\frac{x}{y} - \frac{y}{x}$
4. a) $\frac{b^2+1}{3a^2} + \frac{a}{6b}$ b) $\frac{4a+b}{16a^2b} + \frac{a-3b}{12ab^2}$
 c) $\frac{7y-1}{x^2} - \frac{1}{4x^4}$ d) $\frac{2m+1}{4m^2n} - \frac{3n^2-m}{6mn^3}$
5. a) $1 + \frac{a}{b}$ b) $a - \frac{b}{a}$ c) $\frac{1}{x^2} - x$ d) $\frac{w^2}{u^2v} + v$
6. a) $\frac{a^2+b^2}{ab} + 2$ b) $\frac{c^2+d^2}{cd} - 2$ c) $\frac{(m+n)^2}{4mn} - 1$ d) $\frac{(x-y)^2}{4} + xy$
7. a) $\frac{(a+b)^2}{a-b} - (a-b)$ b) $\frac{(x-y)^2}{x+y} + (x+y)$ c) $\frac{(u+v)^2}{u-v} + (u+v)$
8. a) $\frac{a}{a+b} + \frac{a}{a-b}$ b) $\frac{3a}{4a-1} - \frac{2b+1}{3b}$ c) $\frac{3x-1}{2x^2-xy} - \frac{3y-2}{2xy-y^2}$
9. a) $\frac{a}{a-b} - \frac{b}{a+b}$ b) $\frac{x}{x+y} - \frac{y}{x-y}$
 c) $\frac{a+b}{a-b} - \frac{a-b}{a+b}$ d) $\frac{1}{x+y} + \frac{1}{x-y}$
10. a) $\frac{3a+5b}{2a+3b} - \frac{2a-3b}{3a-5b}$ b) $\frac{4u-7v}{6u-v} + \frac{6u+v}{4u+7v}$

$$11. \text{ a) } \frac{4a-5b}{6a+7b} - \frac{5b+4a}{6a-7b}$$

$$\text{b) } \frac{2a(a-b)}{a^2-2ab+b^2} - \frac{a+b}{a-b}$$

$$12. \text{ a) } \frac{2a}{9} - \frac{b}{9} + \frac{4c}{9}$$

$$\text{b) } \frac{-7m}{23} + \frac{3n}{-23} - \frac{8m-n}{23}$$

$$\text{c) } \frac{5p}{18} + \frac{-3q}{45} + \frac{4}{21}$$

$$\text{d) } \frac{a-b}{16} - \frac{2a+b}{36} + \frac{-a+b}{-24}$$

$$13. \text{ a) } \frac{1}{a} - \frac{1}{b} + \frac{a+b}{ab}$$

$$\text{b) } \frac{m-2}{2m} + \frac{1}{3mn} + \frac{3n+2}{-6n}$$

$$\text{c) } \frac{a^2+3b}{a^2b} + \frac{1-3a}{3ab} - \frac{5b-3}{5b^2}$$

$$\text{d) } \frac{3x-1}{7x^3y} - \frac{x^2+y}{2x^2y^2} + \frac{y^2-1}{3y^4}$$

$$14. \text{ a) } \frac{4x-3y}{a+b} - \frac{6x-5y}{2a+2b} + \frac{2y-3x}{3a+3b}$$

$$\text{b) } \frac{3z-1}{15a-9} + \frac{1-4z}{20a-12} + \frac{-3}{21-35a}$$

$$\bullet 15. \text{ a) } \frac{4p-7q}{7m-4n} + \frac{p+5q}{-7m-4n} - \frac{3p+q}{7m+4n}$$

$$\text{b) } \frac{3a-b}{x-2y} - \frac{4a+b}{2y-x} + \frac{2a-b}{2x-y}$$

$$\bullet 16. \text{ a) } \frac{1}{4p-7q} + \frac{49q}{21pq-12p^2} - \frac{8p+3q}{12pq-21q^2}$$

$$\text{b) } \frac{3}{32yz^2+28y^2} - \frac{4}{-24z^3-21yz} - \frac{y^2+z^2}{8y^2z^3+7y^3z}$$

$$17. \text{ a) } \frac{4a+1}{4a+7} - \frac{7}{16a^2-49}$$

$$\text{b) } \frac{x}{45x^2-5} + \frac{3x+1}{3x-1}$$

$$18. \text{ a) } \frac{x+b}{a^2-b^2} - \frac{x+b}{a^2-ab}$$

$$\bullet \text{ b) } \frac{5m+2n}{125m^2-20n^2} - \frac{4m-5n}{16m^2-20mn}$$

$$19. \text{ a) } \frac{3}{24x-4} - \frac{6x-1,25}{36x^2-12x+1}$$

$$\text{b) } \frac{z+4}{8+40z+50z^2} + \frac{1}{-22-55z}$$

$$20. \text{ a) } \frac{-13x}{y^2-26yz+169z^2} + \frac{13x}{169z^2-13yz}$$

$$\text{b) } \frac{3a-b}{243a^2-3b^2} + \frac{b-3a}{27ab-3b^2}$$

$$21. \text{ a) } \frac{5s+7}{t^2-4t+4} - \frac{6s-7}{4-t^2}$$

$$\text{b) } \frac{5r+12s}{4r^2-64s^2} - \frac{r-4s}{-r^2-8rs-16s^2}$$

$$22. \text{ a) } \frac{x}{2x-6} - \frac{x^2}{x^2-4x+3}$$

$$\text{b) } \frac{y}{2y^2+4y-6} + \frac{1}{y-1}$$

$$23. \text{ a) } \frac{2a^2-a+21}{12a^2-27} + \frac{2a-7}{6a-9} - 1$$

$$\text{b) } \frac{m^2}{m^2-4n^2} - \frac{7m+4n}{10m+20n} + \frac{1}{5}$$

$$24. \text{ a) } 5 - \frac{5x+9}{x+2} + \frac{8}{x^4-16}$$

$$\text{b) } \frac{16y^4+60y+9}{16y^4-81} + \frac{5}{4y^2+9} - 1$$

$$25. \text{ a) } \frac{a^2}{16a^2 + 40ab + 25b^2} - \frac{31a^2 - 50b^2}{16a^2 - 25b^2} + 2$$

$$\text{ b) } \frac{a^2 + ab + b^2}{a^2} + \frac{a^2 - 2ab - b^2}{ab} + \frac{a^2 - ab + b^2}{b^2} - 2$$

$$26. \text{ a) } \frac{1}{a} - \frac{2}{b} - \frac{5a - 12b}{a^2 - 4ab + 4b^2}$$

$$\text{ b) } \frac{1}{2x} + \frac{2x - 3y}{4x^2 + 12xy + 9y^2} - \frac{1}{3y}$$

$$27. \text{ a) } \frac{m-1}{m^2} - \frac{4}{6m-4} + \frac{15m}{4-9m^2}$$

$$\text{ b) } \frac{u+5v}{uv} - \frac{10u}{u^2 - 25v^2} + \frac{5}{u-5v}$$

$$28. \text{ a) } \frac{8x^2 - 0,5a}{2x^2 - ax} - \frac{7x + 0,5a - 1}{2x - a} + \frac{x-1}{2x}$$

$$\text{ b) } \frac{6m^2 + 9}{7m^3 - 63m} + \frac{m}{9 - m^2} + \frac{1}{7m}$$

$$29. \text{ a) } \frac{1}{2-b} + \frac{1}{2+b} - \frac{b^2}{4-b^2}$$

$$\text{ b) } \frac{x-a}{5x+3} - \frac{x+a}{5x-3} + \frac{(4a+6)x}{25x^2-9}$$

$$30. \text{ a) } \frac{a+b}{a-b} - \frac{a-b}{a+b} + \frac{a^2+b^2}{a^2-b^2}$$

$$\text{ b) } \frac{2a-3b}{2a+3b} - \frac{2a+3b}{2a-3b} + \frac{8a^2+18b^2}{4a^2-9b^2}$$

$$\text{ c) } \frac{3p+4q}{3p-4q} + \frac{3p-4q}{3p+4q} - \frac{48pq}{9p^2-16q^2}$$

$$\text{ d) } \frac{2(4a-b)}{4a+b} - \frac{3(4a+b)}{4a-b} + \frac{32a^2+32ab+2b^2}{16a^2-b^2}$$

$$\text{ e) } \frac{5(2x-3y)}{4x-5y} - \frac{4(2x+3y)}{4x+5y} - \frac{4x^2-25xy}{16x^2-25y^2}$$

$$\text{ f) } \frac{-3a+4}{4a-32} - \frac{2a-1}{6a+48} - \frac{64-64a-26a^2}{24a^2-1536}$$

$$31. \text{ a) } \frac{2(3m+4n)}{6m+9n} - \frac{2,5m-n}{10m-15n} - \frac{3m^2-1,5mn-7n^2}{4m^2-9n^2}$$

$$\text{ b) } \frac{a}{(a+b)^2} - \frac{b}{(a-b)^2} + \frac{2b}{a^2-b^2} - \frac{1}{a+b} + \frac{a^3-3a^2b+3ab^2+3b^3}{(a^2-b^2)^2}$$

$$\text{ c) } \frac{a}{a+b-c} - \frac{b}{a-b+c} + \frac{2ab-c(a+b)}{a^2-(b-c)^2}$$

$$32. \text{ a) } \frac{6x+1}{4x-6} + \frac{8x-3}{6x+9} - \frac{20x^2-4x+9}{8x^2-18}$$

$$\text{ b) } \frac{4-5y}{4+5y} + \frac{4+5y}{5y-4} - \frac{12-145y}{32-50y^2}$$

$$\bullet 33. \text{ a) } \frac{12a+5b}{0,9a+0,6b} + \frac{9a+4b}{0,6a+0,4b} - \frac{2(27a^2+33ab-4b^2)}{2,7a^2-1,2b^2}$$

$$\text{ b) } \frac{3a^2+6ab}{15a-20b} - \frac{ab}{3a-4b} - \frac{5ab-4b^2}{6a-8b} - \frac{5a-14b}{30}$$

$$34. \text{ a) } \frac{3a-b}{3x-6y} + \frac{4a-b}{-4x+8y} + \frac{b(x+y)}{2x^2-8y^2}$$

$$\text{ b) } \frac{3z}{6u+12v} - \frac{y+2z}{4u+8v} + \frac{y}{(2u+4v)^2}$$

$$35. \text{ a) } \frac{22x^2-27x-49}{735-15x^2} - \frac{-2x+4}{3x-21} + \frac{4x-1}{5x+35}$$

$$\text{ b) } \frac{2a-3b}{4a^2+2ab} - \frac{6a+4b}{6ab+3b^2} + \frac{5a+8b}{6ab} - \frac{8a-b}{8a^2+4ab}$$

$$36. \text{ a) } \frac{a^4+1}{a^4-1} + \frac{a^2+1}{a^2-1} - \frac{a+1}{a-1} \quad \text{ b) } \frac{x^2}{1-x^4} + \frac{x}{1+x^2} + \frac{1}{1+x}$$

$$\bullet 37. \text{ a) } \frac{3ab}{9a^2-b^2} + \frac{a+b}{2b-6a} + \frac{3a^2-4ab-b^2}{18a^2-2b^2}$$

$$\text{ b) } \frac{3a-5b}{6a^2-9ab} - \frac{2a+7b}{4ab+6b^2} - \frac{5ab-15b^2}{12a^3-27ab^2} + \frac{4a^2-23b^2}{8a^2b-18b^3}$$

$$\bullet 38. \text{ a) } \frac{-2x+3}{3x+9} - \frac{11x^2+51x-54}{6x^2-54} - \frac{11x-12}{x^2-9}$$

$$\text{ b) } \frac{7x+2y}{6xy-2y^2} - \frac{3x-4y}{9x^2+3xy} - \frac{6x^2-4y^2}{27x^3-3xy^2} - \frac{6x^2+7y^2}{9x^2y-y^3}$$

$$39. \text{ a) } \frac{2}{x+1} - \frac{x-3}{x^2-1} - \frac{x}{x^2-2x+1} \quad \text{ b) } \frac{1}{3y-2z} - \frac{24y}{36y^2-16z^2} + \frac{2z}{(3y+2z)^2}$$

$$\bullet 40. \text{ a) } \frac{1}{3a-4b} + \frac{2a-b}{-9a^2+24ab-16b^2} - \frac{a-3b}{9a^2+24ab+16b^2}$$

$$\text{ b) } \frac{4a^2}{6ab+9b^2} - \frac{2a}{3b} + \frac{3b}{2a} - \frac{9b^2}{4a^2+6ab}$$

$$\bullet 41. \text{ a) } \frac{m}{2n} + \frac{m^2}{m^2-2mn+n^2} - \frac{m(m^2+n^2)}{2m^2n-4mn^2+2n^3}$$

$$\text{ b) } \frac{4a^2}{10ab-25b^2} + \frac{25b^2}{4a^2-10ab} - \frac{2a}{5b} + \frac{5b}{2a}$$

$$\bullet 42. \frac{x-y}{3ax-2by+3ay-2bx} + \frac{1}{3a-2b} - \frac{x}{x^2-y^2}$$

$$\bullet 43. \frac{ax-a}{2ax+2x-a-1} + \frac{2ax^2+ax}{4ax^2+4x^2-a-1} - 1$$

$$\bullet 44. \frac{2a-3b}{6ab} + \frac{3}{4a-6b} - \frac{2b}{6a^2-9ab} - \frac{3a}{8ab-12b^2} + \frac{5a^2+6ab-19b^2}{24a^2b-36ab^2}$$

$$45. \frac{2}{x^2-1} - \frac{3}{x^2+x-2} + \frac{1}{x^2+3x+2}$$

$$46. \frac{1}{x^2-6x+9} + \frac{1}{x^2+8x+16} - \frac{2}{x^2+x-12}$$

$$\bullet 47. \frac{x^2+y^2}{2xy} - \frac{x}{x+y} - \frac{y}{x-y} + \frac{y^4-x^4+4x^3y}{2(x^3y-xy^3)}$$

$$\bullet 48. \frac{p}{p-q} - \frac{q^2}{p^2+pq+q^2} - \frac{p^2q}{p^3-q^3} \quad \bullet 49. \frac{1}{x+y} + \frac{x-y}{x^2-xy+y^2} - \frac{x(x-y)}{x^3+y^3}$$

$$\bullet 50. \frac{5}{4x^2-4x+1} - \frac{2}{4x^2+4x+1} + \frac{4x^2-28x-7}{16x^4-8x^2+1}$$

$$\bullet 51. \frac{a+b}{a^2-2ab+b^2} + \frac{a-b}{a^2+2ab+b^2} - \frac{2a-b}{a^2-b^2} - \frac{4(ab^2-b^3)}{(a^2-b^2)^2}$$

$$\bullet 52. \frac{x^2}{4x^2-4xy+y^2} - \frac{y^2}{4x^2+4xy+y^2} - \frac{x^2+y^2}{4x^2-y^2} + \frac{xy(6xy-5y^2)}{(4x^2-y^2)^2}$$

$$\bullet 53. \frac{7a^2-4b^2}{a^2+6ab+9b^2} - \frac{5a^2-9ab}{a^2-6ab+9b^2} - \frac{2a^2-3ab+4b^2}{a^2-9b^2} + \frac{462ab^3-82a^2b^2}{a^4-18a^2b^2+81b^4}$$

$$\bullet 54. \frac{2a-3b}{a^2+2ab+b^2} - \frac{2a+3b}{a^2-2ab+b^2} + \frac{10a^3+10a^2b}{a^4-2a^2b^2+b^4} - \frac{6b}{a^2-b^2}$$

$$\bullet 55. \frac{2x-1}{4x^2+4x+1} - \frac{2x+1}{4x^2-4x+1} + \frac{4x^2+7}{16x^4-8x^2+1}$$

$$\bullet 56. \frac{a}{a^2+3ab+2b^2} + \frac{b}{a^2+4ab+3b^2} - \frac{a+b}{a^2+5ab+6b^2}$$

$$\bullet 57. \text{ a) } \frac{1}{(a-b)(a-c)} + \frac{1}{(b-a)(b-c)} + \frac{1}{(c-a)(c-b)}$$

$$\text{ b) } \frac{a}{(a-b)(a-c)} + \frac{b}{(b-a)(b-c)} + \frac{c}{(c-a)(c-b)}$$

$$\text{ c) } \frac{a^2}{(a-b)(a-c)} + \frac{b^2}{(b-a)(b-c)} + \frac{c^2}{(c-a)(c-b)}$$

$$\text{ d) } \frac{a^3}{(a-b)(a-c)} + \frac{b^3}{(b-a)(b-c)} + \frac{c^3}{(c-a)(c-b)}$$