New Implementations in R for Parametric and Semiparametric Modelling of Univariate and Spatial Short- and Long-Memory Time Series

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Zusammenfassung

Wir berücksichtigen lokal polynomiale Regression und lokal gewichtete Regression als Technik zur Schätzung diverser Merkmale in univariaten und räumlichen Zeitreihen und zur Konstruktion neuer semiparametrischer Zeitreihenmodelle, die zur Erstellung von Punkt- und Intervallprognosen genutzt werden können. Wann immer es sinnvoll ist, werden existierende Algorithmen zur Schätzung des Bandbreitenparameters genutzt oder neue Algorithmen hergeleitet, um den nichtparametrischen Schätzschritt zu automatisieren unter der Annahme autokorrelierter Fehler. Ferner werden Algorithmen zur Implementierung von (bedingten) Quasi-Maximum-Likelihood-Schätzern für eine weite Klasse von exponentiellen GARCH-Modellen mit kurz- oder langfristigen Abhängigkeitsstrukturen, optional mit simultaner Modellierung des bedingten Erwartungswerts, vorgestellt.

Zuerst implementieren wir einen vollständig nichtparametrischen Algorithmus zur Bandbreitenauswahl für lokal polynomiale Regression im Kontext univariater, trendstationärer Zeitreihen mit Fehlern mit kurzfristiger Abhängigkeitsstruktur und nutzen ihn, um ein semiparametrisches Zeitreihenmodell mit ARMA-Fehlern zu erstellen, das benutzt werden kann, um Zeitreihen zu analysieren, und anhand dessen Prognosen erstellt werden können. Prognoseintervalle werden anhand der Annahme normalverteilter Innovationen oder anhand eines einfachen Bootstraps generiert. Der Ansatz wird auf einfache semiparametrische Modelle für Finanzdaten erweitert. Es wird ein grafischer Test durch asymptotische Eigenschaften lokal polynomialer Regression hergeleitet, anhand dessen die Linearität einer Trendfunktion und die Stationarität von Log-Renditen getestet werden können. Gleichzeitig wird das neue R-Paket `smoots` vorgestellt, in dem all diese Ansätze zur direkten Anwendung in R einprogrammiert sind.

Zweitens verbessern wir einen bestehenden Algorithmus zur Bandbreitenauswahl in lokal gewichteter Regression, so dass dieser die Komponentenzerlegung saisonaler, univariater Zeitreihen mit Fehlern mit kurzfristiger Abhängigkeitsstruktur erlaubt anstelle unabhängiger Fehler. Der Algorithmus funktioniert vollständig nichtparametrisch und kann auf saisonale Zeitreihen mit allen möglichen saisonalen Perioden angewandt werden, sofern die saisonale Periode eine nicht-negative ganze Zahl ist, z.B. auf tägliche Daten mit einem wöchentlichen saisonalen Muster. Ferner wird ein saisonales semiparametrisches Modell mit ARMA-Fehlern vorgestellt, das, in Kombination mit dem neuen Zerlegungsansatz, zur Prognose saisonaler Zeitreihen genutzt werden kann. Prognoseintervalle werden hierbei erneut entweder unter der Normalitätsannahme oder anhand eines Bootstraps berechnet. Durch die Anwendung

auf reale Zeitreihendaten zeigen wir, dass anhand der neuen Methode qualitativ hochwertige Saisonbereinigungen durchgeführt werden können. Gleichzeitig produziert die Methode glattere Trends als herkömmliche Zerlegungsmethoden, da dem Trend ausschließlich Schwingungen kleiner Frequenzen zugeordnet werden, so dass Schwingungen mittlerer und höherer Frequenzen, und damit zyklisches Verhalten, erfolgreich von dem Trend entkoppelt und dem Fehlerterm zugeschrieben werden. Eine Simulationsstudie validiert die Genauigkeit des Bandbreitenalgorithmus und die Konsistenz der automatisierten Komponentenschätzer auf Basis unserer Methode. Eine weitere Simulationsstudie zeigt die verbesserte Saisonschätzung durch unseren Ansatz im Vergleich zu anderen Zerlegungsmethoden anhand des Mean-Squared-Error-Kriteriums, wenn die Anzahl der Beobachtungen ausreichend hoch ist und die Annahmen des Modells nur in geringem Maße verletzt werden. Bei stärkeren Verletzungen, wie bei einer stochastischen Saisonkomponente, funktioniert unsere Methode unter hohen Beobachtungszahlen zumindest vergleichbar gut wie die besten Zerlegungsmethoden. Die neuen Zerlegungs- und Prognosemethoden sind in dem neuen R-Paket `deseats` abrufbar, welches ebenfalls im Detail präsentiert wird.

Drittens stellen wir ein saisonales, semiparametrisches und räumliches Modell mit zufälligen Kurven innerhalb eines Tages und mit Fehlern mit langfristiger Abhängigkeitsstruktur vor, das genutzt werden kann, um räumliche Zeitreihen mit zweidimensionalem Saisonalitätsmuster zu analysieren, im Detail mit einem Muster innerhalb eines Tages und mit einem anderen Muster von Tag zu Tag. Zu diesem Zweck stellen wir zwei neue Methoden vor, um die saisonale Fläche zu schätzen, wobei jeweils nur die Daten eines Jahres getrennt herangezogen werden, um die jahresweisen Saisonflächen mit bivariater lokal polynomialer Regression mit manuell ausgewählten Bandbreiten zu schätzen. Ein Ansatz nutzt die so geschätzten, jahresweisen Saisonflächen direkt; der andere Ansatz bildet eine mittlere Saisonfläche aus allen geschätzten Flächen, die dann auf die Daten angewandt wird. Nach der Entfernung der zweidimensionalen Saisonalität aus räumlichen Temperaturzeitreihen können wir nicht-stationäres Verhalten in den Daten innerhalb der separaten Tagesresiduen beobachten in Form von zufälligen Kurven an jedem Beobachtungstag. Daher schätzen und entfernen wir diese Kurven in einem weiteren Schritt mit automatisierter, univariater lokal polynomialer Regression und passen im Anschluss SFARIMA-Modelle an die Gesamtresiduen der nichtparametrischen Schätzschritte an. Nachdem die Saisonalitätsfläche und die zufälligen Tageskurven geschätzt und entfernt wurden, sind die angepassten SFARIMA-Modelle stationär und zeigen statistisch signifikante langfristige Abhängigkeitsstrukturen in der Zeitdimension von Tag zu Tag. Eine Sammlung von internen R-Kodierungen wurde erstellt, um die Arbeit mit räumlichen Zeitreihen zu vereinfachen, insbesondere um die neuen Methoden anzuwenden.

Viertens stellen wir die erweiterte Modellfamilie des Exponential GARCH, inklusi-

ve Modelle mit langfristigen Abhängigkeitsstrukturen, und einige ihrer Eigenschaften vor. Wir erweitern sie zu Modellen mit sich langsam verändernder unbedingter Varianz und zu Modellen mit gleichzeitiger Modellierung des bedingten Erwartungswertes. Außerdem leiten wir die (bedingten) Quasi-Maximum-Likelihood-Schätzer für die Parameter der parametrischen Modelle her, auch unter der neu eingeführten Average-Laplace-Verteilung und ihrer schiefen Variante. Prognosemethoden werden vorgestellt und Backtesting-Ansätze werden zusammengefasst. Anhand verschiedener realer Anwendungsbeispiele durch das neue R-Paket `fEGarch` wird die Nützlichkeit der neuen Modellfamilie aufgezeigt für Trainingsdaten und zum Zwecke von Backtesting. Die Anwendung von `fEGarch` im Kontext von multivariaten GARCH-Modellen und von GARCH abweichenden Volatilitätsmodellen wird dargestellt und die Fähigkeiten des `fEGarch`-Pakets werden mit denen verwandter statistischer Software verglichen. Eine abschließende Simulationsstudie zeigt, dass die (bedingten) Maximum-Likelihood-Schätzer für die neue Modellfamilie, mit wenigen Ausnahmen, konsistent und asymptotisch normalverteilt sind.

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Abstract

We consider local polynomial and locally weighted regression techniques to estimate a variety of features in univariate and spatial time series and to construct new semiparametric models that may be used for producing point and interval forecasts. Where possible, existing bandwidth selection algorithms are employed or new algorithms are created to automate the nonparametric estimation step under the assumption of autocorrelated errors. Moreover, we introduce algorithms for estimating a broad class of exponential GARCH models via (conditional) QMLE, optionally with simultaneous modelling of the conditional mean.

Firstly, we implement a fully nonparametric bandwidth selection algorithm for local polynomial regression in the context of univariate trend-stationary time series with short-memory errors and use it to construct a semiparametric time series model with ARMA errors that can be used to analyze and forecast time series. Forecasting intervals are obtained either under the assumption of normal innovations or following a simple bootstrap procedure. The approach is extended to simple semiparametric models for financial data. We introduce a graphical test to check the linearity of a trend and the stationarity of log-return series from asymptotic properties of local polynomial estimators. Simultaneously, the newly created `R` package `smoots` is introduced, which makes all of these methods available in `R`.

Secondly, we improve an existing bandwidth selection algorithm for locally weighted regression so that it allows for the decomposition of seasonal univariate time series with short-memory errors instead of independent errors. The algorithm works fully nonparametrically and can be applied to seasonal time series with any seasonal period that is a positive integer, for example to daily data with a weekly seasonal pattern. Furthermore, a seasonal semiparametric model with ARMA errors is introduced that, in combination with the new decomposition approach, can be used for forecasting seasonal time series, where forecasting intervals can yet again be constructed following the normality assumption or a bootstrap. Using real-world examples, we show that our new method can be used to produce high-quality seasonal adjustments. In addition, our method produces smoother trends than common decomposition methods, because only low-frequency movements in the series are attributed to the trend. Simultaneously, medium-frequency and high-freuqency movements, i.e. cyclical behavior, are detached successfully from the trend and attributed to the errors. A simulation study validates the correctness of the bandwidth selection algorithm and shows the consistency of the component estimates according to our method. Another simulation study shows the improved decomposition quality of our approach compared to various other decomposition methods according to the mean-squared error criterion, whenever the number of observations is sufficiently large and model assumptions are not violated too strongly. Under stronger violations, such as a stochastic seasonal component, our method works at least similarly to the best-performing decomposition methods. The new decomposition and forecasting methods are combined into a new R package `deseats`, which is also described in detail.

Thirdly, we introduce a seasonal, semiparametric and spatial model with long-memory errors and random intraday curves that can be used to analyze spatial time series with a two-dimensional seasonal pattern, i.e. one in the intraday and one in the interday dimension. For this purpose, two new methods for estimating the seasonal surface are proposed, where each year worth of data is firstly treated separately using bivariate local linear regression with manually selected bandwidths. One approach keeps the various year-wise seasonal surfaces, the other obtains an average seasonal surface over all years and applies it to all the data. After removing the two-dimensional seasonal surfaces from spatial temperature time series, we discover nonstationary behavior in the intraday dimension in the form of random curves on each separate day. We estimate and remove these curves in a further step via automated univariate local polynomial regression and fit SFARIMA models to the overall model residuals. After the seasonal surface and the intraday curves have been removed, the fitted SFARIMA models are stationary and show statistically significant long memory in the interday dimension. A set of new internal R codes was created to simplify the treatment of spatial time series, in particular in order to apply the new approaches.

Fourthly, we introduce the exponential GARCH model family, including long-memory models, and some of its properties. We extend it to semiparametric models under changing unconditional variance and to models with simultaneous fitting of the conditional mean and derive (conditional) quasi-maximum-likelihood estimators of the parametric model parameters, also under the new conditional average Laplace distribution and its skewed variant. Forecasting procedures are introduced and backtesting procedures are summarized. Through various real-world application examples via the new R package `fEGarch`, the suitability of the exponential GARCH family models is illustrated both for the training data and for backtesting purposes. The implementation of `fEGarch` in the context of multivariate GARCH and non-GARCH volatility models is presented and the capabilities of `fEGarch` are compared to those of related statistical software. A concluding simulation study shows that the (conditional) quasi-maximum-likelihood estimation for the exponential GARCH family provides consistent and asymptotically normal estimators with only a few exceptions.

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List of acronyms and abbreviations

This is an alphabetical list of the most frequent acronyms and abbreviations in this thesis. It is by no means a complete list covering all instances in this document; however, in each chapter acronyms and abbreviations are generally introduced separately, whenever they are first introduced within a chapter.

ACD autoregressive conditional duration

AIC Akaike information criterion

ALD average Laplace distribution

AR autoregressive

ARCH autoregressive conditional heteroskedasticity

ARMA autoregressive moving-average

ARIMA integrated ARMA

BCBS Basel Committee on Banking Supervision

BIC Bayesian information criterion

BIS Bank for International Settlements

CIE Center for International Economics, Paderborn University

CGF cumulant-generating function

DCS double-conditional smoothing

DeSeaTS deseasonalize time series

EGF exponential GARCH family

ES expected shortfall

FARIMA fractionally integrated ARMA

GARCH generalized ARCH

GCV generalized cross-validation

GDP gross domestic product

GED generalized error distribution

IDC International Data Corporation

iid / i.i.d. independent and identically distributed

IPI iterative plug-in

MA moving average

MGF moment-generating function

MLE maximum-likelihood estimation / estimator

QMLE quasi-MLE

SARIMA seasonal ARIMA

Semi-ARMA semiparametric ARMA

S-Semi-ARMA seasonal Semi-ARMA

UK United Kingdom

US / USA United States of America

VaR value-at-risk

Chapter 1

Introduction

"The goal is to turn data into information, and information into insight."
— Carly Fiorina, 2004, former CEO of Hewlett-Packard

With the rise of affordable and computationally powerful computers in recent decades and with the increasing potential to collect data with modern electronic devices and websites, the words of Fiorina (2004, para. 11) are seemingly gaining more and more importance as time progresses. Humanity's capacity to store and share information is long known to increase at a rapid speed (Hilbert and López, 2011; Chen et al., 2014), and latest figures estimate the worldwide total data flow in 2025 at around 173.4 zettabytes ($= 173.4 \times 10^{21}$ bytes), while this number is expected to triple by 2029 (IDC, 2025). In conjunction with the growing amount of data, the demand for professional data analysts, as well as data analysis in general, is experiencing an escalating surge. For example, the demand for data professionals on the labor market of the European Union is expected to increase annualy by at least 3.5% until the year 2030, with high-growth projections estimating up to 6.1% of annual growth (La Croce et al., 2025).

Data itself, however, and tasks in data analysis present themselves in various shapes and bring forward numerous challenges. To overcome these challenges, the science of *statistics* emerged. Statistics, originally from the Latin word *status*, which means as much as *state* or *condition*, "is the science of collecting, describing, and interpreting data" (Härdle et al., 2015, p. 1), and is hence the foundation of all traditional and modern data analysis. Mathematical statistics makes use of insights from probability theory to introduce new models to analyze data in all possible forms. However, since new statistical models are being invented at all times around the globe, they are at least as multitudinous and heterogeneous as the types of data they are introduced for.

A central piece in this thesis is the field of *time series analysis*, which is a special subarea of probability theory and statistics. While traditional statistics often deals with cross-sectional data, where all observations are considered to be made for different subjects at the same time, time series analysis shifts the focus to the analysis of a single quantity that is observed multiple times over the course of time. This change in perspective is especially important in settings, where the development of a system throughout time and its future behavior are of particular interest. Due to the fact that challenges related to time series analysis arise in every professional environment, the importance of this field of study cannot be overstated. Commonly related tasks are, for example, among others: companies or entire economies devising plans as well as controlling and optimizing internal future production processes (Box et al., 2008, p. 2); banks understanding future development of market risks, so that they can be in alignment with official regulations (BCBS, 2016; BCBS, 1996a; BCBS, 1996b; BCBS, 2019); insurances needing to forecast the amount of funds to set aside in order to be able to cover future claims by insurants (see for example Chapter 2 in Wüthrich and Merz, 2008). In all of these settings, both time and uncertainty play key factors, as reliable forecasts about the future are required.

Generally, the main objectives of time series analysis can be summarized to be (1) forecasting, (2) understanding the underlying mechanism of an observed time series, (3) seasonal adjustment of observed time series, and (4) hypothesis testing, among others (Brockwell and Davis, 2016, p. 5). In this thesis, each of these objectives (1) through (4) will play a prominent role in conjunction with various newly introduced time series models. Each of the new models presented in this thesis can be thought of as belonging to a specific type of model from a selection of time series model classes, namely (i) one-dimensional models in the mean, (ii) one-dimensional models in the variance and (iii) two-dimensional models in the mean. The number of dimensions refers here to the number of time dimensions considered simultaneously, where models from (iii) are also often referred to as spatial time series models (in the mean). In spite of these classifications, a strict demarcation between them is not always possible and thus not advisable: for example, as will be seen within this thesis, a model from (i) can in certain situations be applied for the purpose implied by class (ii). Another special case is in the form of one-dimensional dual models that simultaneously model the mean and the variance of a time series.

Apart from the theoretical exploration of time series analysis, the rise of statistical open-source software, for example of R (R Core Team, 2026), led to the availability of the latest time series methods in practice shortly after their initial publication. At the time of creation of this chapter, the Comprehensive R Archive Network (CRAN) and hence R feature approximately 23,000 add-on packages (CRAN, 2026) to conduct both traditional and the latest statistical analyses and more, with the number of packages growing regularly. A notable share of this software is dedicated explicitly to time series analysis: the official overview of relevant packages on CRAN for time series analysis lists 414 non-archived entries (Hyndman and Killick, 2026), while the overview of relevant R packages for empirical finance mentions 34 packages in its time series section (Eddelbuettel, 2026), some of which overlap between the two lists. These numbers highlight on one hand the importance of time series analysis in practice; on the other hand, they show that there is still an evergrowing need for new practically relevant contributions to the area of time series analysis. In order for new methods to have an impact on the scientific community and on practitioners, the simultaneous publication of statistical software alongside the introduction of theoretical papers is advisable, so that a large group of persons can check and use the new statistical ideas effectively.

In the remainder of this chapter, brief overviews of the aforementioned model classes are given including short historical contexts and empirical motivations for the improvement on current methods. In Section 1.1, one-dimensional models in the mean will be discussed, including models with trend or with both trend and seasonality as the nonstationary elements, whereas in Section 1.2 models in the variance will be reviewed. Section 1.3 provides information about spatial models in the mean. Ultimately, Section 1.4 will give an overview of the contents and chapters as well as of the detailed contributions of this thesis.

1.1 One-dimensional models in the mean

Modelling the mean of an observed time series has a long tradition. The idea of an autoregressive (AR) model, where the current value of a stationary time series is regressed on its past values, is largely accredited to Yule (1927), who studied sunspot data. Likewise, the foundations for moving-average processes, sums of past and current independent random shocks, were first introduced by Slutsky (1937), who also highlighted the inherently cyclic behavior such processes can produce. The theorem by Wold (1938) stated that every weakly stationary time series can be stated in terms of the sum of two time series, where one is an infinite-order MA process and the other is weakly stationary and specified linearly through its past values. It set the groundwork for the autoregressive moving-average (ARMA) and integrated ARMA (ARIMA) models (Whittle, 1953; see also the first edition of Box et al., 2008, from 1970). Wiener (1949) introduced concepts such as linear prediction of time series, while Box and Pierce (1970) and Ljung and Box (1978) introduced model checking ideas for ARIMA models. The introduction of fractional differencing led to the fractionally differenced ARMA (FARIMA) process (Granger and Joyeux, 1980; Hosking, 1981; Baillie, 1996), which allowed for long-range dependence in the elements of a time series. Estimation of the fractional differencing parameter in such long-memory time series via a log-periodogram approach was introduced by Geweke and Porter-Hudak (1983) and several refinements of it were suggested since then, for example by Hurvich and Deo (1999), McCloskey and Perron (2013) and Smith (2005), among others. A comprehensive overview on long-memory processes is given by Beran et al. (2013). Related works are those on Markov-switching ARMA models, i.e. models that allow for regime changes (Hamilton, 1989; Francq and Zakoïan, 2001; Stelzer, 2009).

When discussing time series models in the mean, the idea of time series decomposition needs to be explored. As pointed out by Dokumentov and Hyndman (2022), one of the earliest known works on time series decomposition is that of Poynting (1884), where trends and seasonal fluctuations in observed time series were removed using averaging techniques. Other authors followed; however, it was Persons (1919) who firstly stated assumptions on the various components. In particular, he accentuated the trend component T_t, the seasonal component S_t, the cyclical component C_t and an error component R_t (Persons, 1919; Dagum, 2010). In contemporary works (see for example Feng, 1999, Cleveland et al., 1990, and Hodrick and Prescott, 1997), the cyclical fluctuations are often either attributed directly to the trend, leading to a single trend-cycle component, or to the error term, thus often implying the exclusion of an explicitly stated component C_t in time series component models.

From early on, the usage of local regression approaches was popular in the identification of the trend component (Macaulay, 1931), whereas seasonal fluctuations were estimated through averages over the detrended values of the same month or quarter. It was then the electronic census program introduced by Shiskin (1957) that firstly considered moving averages for a first trend and subsequent seasonal component estimation and then implemented a refinement step for both components based on weighted moving averages; a framework that was kept, yet still improved on, until the latest seasonal adjustment program version *X-13ARIMA-SEATS* of the US census (Findley et al., 1998; U.S. Census Bureau, 2025). The two sub-methods in the program are X13-ARIMA (X13 for short) and TRAMO-SEATS. A locally weighted regression idea with fixed-bandwidth polynomial and trigonometric polynomial regressors that laid the foundation for the official German seasonal adjustment procedure was developed by Heiler (1970) and then later improved by Speth (2004); various data-driven improvements were suggested by Heiler and Feng (2000), Heiler and Feng (2004) and Feng (2013a). Nonetheless, these methods assumed independent errors throughout.

Under consideration of data consisting only of a mean function and independent errors, Nadaraya (1964) and Watson (1964) popularized kernel regression as a generalized version of the (weighted) moving average. Statistical properties of a huge class of nonparametric smoothers, including local linear regression, were discussed by Stone (1977). Locally weighted regression was discussed by Cleveland (1979) with local polynomial regression as a special case (Fan and Gijbels, 1996), which solved the boundary problem of kernel methods (Hastie and Loader, 1993). In the context of time series analysis, the major drawback of the existing local regression ideas was that they were usually studied under independent and identically distributed (iid) errors, whereas time series typically admit autocorrelated errors. This gave rise to further studies of the local polynomial estimator under short- and long-memory errors as well as first bandwidth selection algorithms under this setup (see for example Herrmann et al., 1992, Fan and Gijbels, 1996, Masry and Fan, 1997, Beran and Feng, 2002a, Beran and Feng, 2002b, and Beran and Feng, 2002c); however, those algorithms were restrictive in the sense that they, during the bandwidth estimation, already assumed a fixed model type for the errors, namely iid errors or ARMA or respectively FARIMA models.

Stochastic trends and seasonal fluctuations instead of deterministic ones were for example considered through ARIMA and seasonal ARIMA (SARIMA) models (see for example Box et al., 2008); a decomposition idea for official statistics following SARIMA models was introduced by Gómez and Maravall Herrero (1996) in Spain and is a recently added option in the X-13ARIMA-SEATS program (U.S. Census Bureau, 2025).

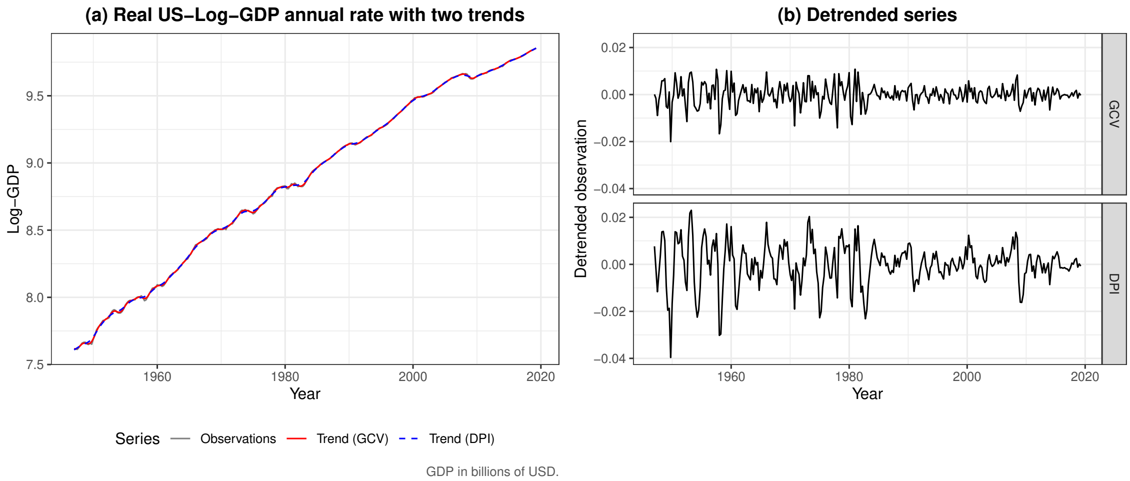


Figure 1.1: Quarterly real US-Log-GDP together with two automated local linear trends with bandwidth selection by GCV by `PLRModels` and by DPI by `KernSmooth` in (a) and the corresponding detrended series in (b).

In Figure 1.1(a), the quarterly and already seasonally adjusted log-real GDP of the US is depicted from the first quarter of 1947 to the second quarter of 2019. The data was downloaded from the databank of the Federal Reserve Bank of St. Louis. We have used two popular methods to fit a local linear trend to the series in a data-driven way: firstly, the bandwidth is selected by means of generalized cross-validation (GCV, Craven and Wahba, 1978) through the R package `PLRModels` (Aneiros-Pérez and López-Cheda, 2023); secondly, the direct plug-in (DPI) estimator of the bandwidth by Ruppert et al. (1995) is applied via the R package `KernSmooth` (Wand, 2024). As is observable, the estimated trend following GCV (red) follows the observations very closely, which implies that most of the autocorrelation in the time series errors is actually attributed to the estimated trend. This is a clear sign of undersmoothing. The accordingly detrended values at the top in Figure 1.1(b) show little variation and no clear pattern that would hint at positive or negative autocorrelation.

The estimated local linear trend with the bandwidth being selected by the DPI method (blue) seems slightly smoother in comparison to the GCV approach, but it overall still follows smaller movements of the observed series. The residuals illustrated at the bottom of Figure 1.1(b) show more variation and positive autocorrelation than for GVC; nonetheless, the results are still not satisfactory, as most of the variation is instead attributed to the trend. Aside from the inadequate trend estimates, none of the aforementioned approaches allows for the automated estimation of the first or second derivative of the trend, which could be of crucial interest in areas such as macroeconomics in order to assess the latest trend development in economic indicators properly. This motivates the development of a suitable and fully nonparametric trend estimation algorithm via automated local polynomial regression under (short-memory) time series errors as well as an analogous algorithm for data-driven estimation of the trend's derivatives. The trend estimation algorithm should be extended, so that the detrended values are modelled through further parametric procedures, i.e. resulting in a semiparametric model that can be used for forecasting.

Since seasonal adjustment is a paramount task of all statistical offices in the world, a mostly data-driven and unified adjustment framework is desirable to ensure seasonal adjustments being comparable between nations. The more settings need to be adjusted manually by the user, the more seasonal adjustments become incomparable. There are several popular packages on CRAN for the decomposition of seasonal time series, some of them being more and some being less automated. Most notably, the R function `stl()` within the basic installation of R implements seasonal decomposition of time series by Loess (STL, Cleveland et al., 1990), with `mstl()` of the R package `forecast` (Hyndman and Khandakar, 2008) being an automated STL implementation, and the official X-13ARIMA-SEATS program is available in various R packages, for example in `RJDemetra` (Quartier-la-Tente et al., 2024). While X-13ARIMA-SEATS does by default work in an automated way, it allows for many individual adjustments in the procedure. However, using those methods' default settings usually results in trends that also contain a lot of cyclical fluctuations and thus short-term movements.

As an example, Figure 1.2 shows the decomposition of the monthly civilian labor force time series of the US from January 1948 to December 2019. The data was once again downloaded from the website of the Federal Reserve Bank of St. Louis and the decomposition was done using X13 with its default settings. In Subfigure 1.2(a), we can clearly observe that the estimated trend by X13 is in fact the estimated

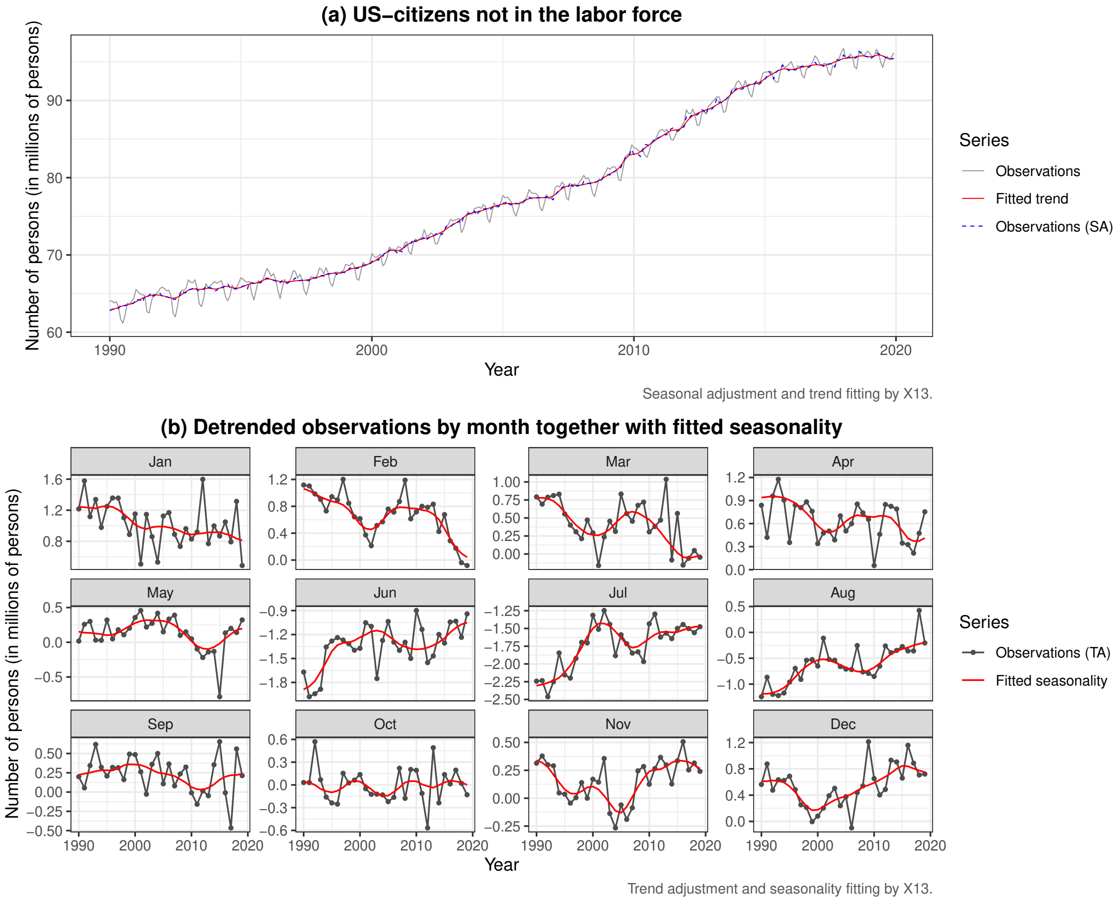


Figure 1.2: Automated X13-decomposition for the US-citizens not in the labor force. (a) shows the original observations together with the seasonally adjusted (SA) observations and the fitted trend, whereas (b) shows the trend adjusted (TA) observations by month together with the seasonality estimates.

trend-cycle, as it shows many short-term details. At the same time, the seasonal components seem to be estimated roughly suitably according to Subfigure 1.2(b), maybe also sometimes with a slight tendency to be undersmoothed like in February. While the consideration of a trend-cycle component is not in contrast to the X13 method's main goal, i.e. a suitable seasonal adjustment, such a model can hardly be used for forecasting or identifying pure low-frequency trend movements, the latter of which is of particular interest to understand the true long-term course of a series. Consequently, a method is desirable that fulfills multiple purposes at once at a high level of quality: decomposition optimized for the decomposition into various components, including a pure slowly changing trend, suitable seasonal adjustment, and the capibility to produce reliable forecasts. This motivates the introduction of a new seasonal decomposition algorithm under short-memory errors that works in a fully data-driven and nonparametric way. It should be able to attribute only low-fequency movements to the trend, seasonal-frequency movements to the seasonal component, and movements of all other frequencies to the error component, so that during seasonal adjustment all information aside from seasonal movements is kept and medium- and high-frequency fluctuations in the errors may still be modelled parametrically. Additionally, the new algorithm should be extended into a new semiparametric model that also enables forecasting procedures.

1.2 One-dimensional models in the variance

Originally stated by Mandelbrot (1963) and Fama (1965) and at later time points extended by other authors, observed (log-)return series $r_t = \ln(P_t) - \ln(P_{t-1})$, with P_t as closing prices at time $t \in \mathbb{N}_+$, usually exhibit certain characteristics, including (a) approximate stationarity of observed return series (at least for small observation periods), (b) returns following approximately a weak white noise process, i.e. the returns have either no or barely any autocorrelation directly, (c) notable autocorrelation in the squared returns (or in some other transform of the returns), (d) heavy tails in the empirical unconditional return distributions, (e) volatility clustering, i.e. large $|r_t|$ often following a large $|r_{t-1}|$ and small $|r_t|$ often following a small $|r_{t-1}|$, (f) asymmetry or leverage effects, i.e. negative returns commonly causing a larger increase in volatility than a positive return of same magnitude (see also Francq and Zakoïan, 2019, p. 7, or Daníelsson, 2011, p. 9).

To model and forecast volatility, in particular on the stock market, a variety of approaches have been proposed in the literature. Commonly known are unweighted and weighted moving average methods, stochastic volatility models, options-based volatility forecasts and more (Ghysels et al., 1996; Poon and Granger, 2003; J.P. Morgan, 1996). Through the introduction of the autoregressive conditional heteroskedasticity (ARCH, Engle, 1982) and the generalized ARCH (GARCH, Bollerslev, 1986) models, modelling of the variation in a time series was popularized immensely, because these models allowed to capture a selection of the aforementioned characteristics of return series. The basic idea is simple: describe the theoretical (log-)return process by

$$r_t \mid \mathcal{F}_{t-1} \sim \mathcal{D}(0, \sigma_t^2), \quad t \in \mathbb{Z}, \tag{1.1}$$

where $\mathcal{F}_t$ is the set of past information up until and including time point t and $\mathcal{D}(0, \sigma_t^2)$ implies some continuous (conditional) distribution with mean zero and variance σ_t^2. $\sigma_t^2 = \mathrm{var}\,(r_t \mid \mathcal{F}_{t-1}) > 0$ is also known as the conditional variance in r_t and is the key component being modelled in ARCH and GARCH models. Notwithstanding the initial success of GARCH models, an almost immeasurable amount of new members has been added to the class of GARCH-type models ever since the publication of the original ARCH and GARCH papers by Engle (1982) and Bollerslev (1986), for example in form of the popular exponential GARCH (EGARCH, Nelson, 1991), the asymmetric power

ARCH (APARCH, Ding et al., 1993), the threshold GARCH Zakoian (TGARCH, 1994), the Glosten-Jagannathan-Runkle GARCH (GJR-GARCH, Glosten et al., 1993), or the Log-GARCH (logarithmic GARCH, Pantula, 1986; Geweke, 1986; Milhøj, 1987), usually with the objective to capture more characteristics of observed returns. Each of such models uses Equation (1.1) in combination with a different model equation on σ_t^2 directly or on a transformation of it.

With the introduction of the fractionally integrated GARCH (FIGARCH), Baillie et al. (1996) paved the way for long-memory GARCH-type models, to which later, among others, also the fractionally integrated EGARCH (FIEGARCH, Bollerslev and Mikkelsen, 1996), the fractionally integrated APARCH (FIAPARCH, Tse, 1998) and the fractionally integrated Log-GARCH (FILog-GARCH, Feng et al., 2020a) were added. While a variety of short-memory GARCH-type models is applicable through the popular R packages `fGarch` (Wuertz et al., 2024) and `rugarch` (Galanos, 2024), the options for long-memory models are quite limited within the R-universe. `rugarch` only allows access to the FIGARCH. While there is no direct quasi-maximum-likelihood-estimator (QMLE) with common conditional distribution settings for a FILog-GARCH, it may be fitted based on the FARIMA-representation of a FILog-GARCH (Feng et al., 2020a), for example using `fracdiff` (Maechler, 2024). Commonly, for fitting FIEGARCH and FIAPARCH models, one has to leave the R-cosmos and use licensed software like `G@RCH` of *OxMetrics* (Laurent and Peters, 2002).

In Figure 1.3 we can observe the daily log-returns of the Standard and Poor's 500 index (S&P 500) from the start of the year 2006 to January 04, 2021, and some of its characteristics. The data was collected from Yahoo Finance. While Figure 1.3(a) displays the returns and highlights high-volatility periods in more color, Figure 1.3(b) shows the heavy-tailedness of the returns in comparison to a normal distribution through density estimates of the returns and an overlayed normal density with corresponding (sample) mean and (sample) variance. The sample ACFs of the simple and squared log-returns in Figures 1.3(c) and (d) underline the lack of autocorrelation in the simple log-returns (aside from maybe a significant autocorrelation small in magnitude at lag 1 for this particular example), whereas there is a strong autocorrelation in the squared returns. In fact, the ACF decays slowly for increasing lag, indicating the presence of long memory or of a changing scale function in the volatility.

The capabilities of current long-memory GARCH options in R are shown in Figure 1.4, where the in-sample fitted conditional standard deviations for a standard FIGARCH$(1, d, 1)$ and for a FILog-GARCH$(1, d, 1)$ - both with conditional skew-t distribution - are displayed in Subfigures 1.4(a) and (c). The FIGARCH was fitted using `rugarch`, the FILog-GARCH using `fracdiff`. In Subfigures 1.4(b) and (d), the corresponding one-step forecasts of the value-at-risk (VaR) and the expected shortfall

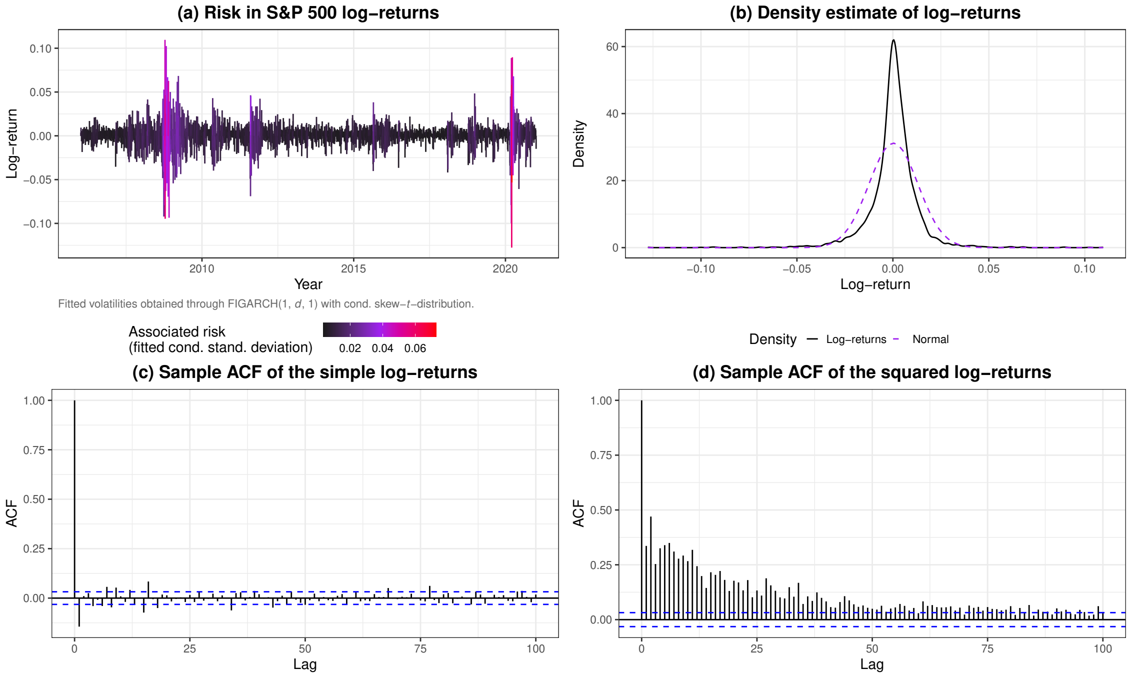


Figure 1.3: An example for heteroskedasticity in stock market returns: (a) shows the daily S&P 500 log-returns, (b) the density estimate of the log-returns together with an overlayed density of a variable following a same-mean and same-variance normal distribution, (c) the sample ACF of the simple log-returns, and (d) the sample ACF of the squared log-returns.

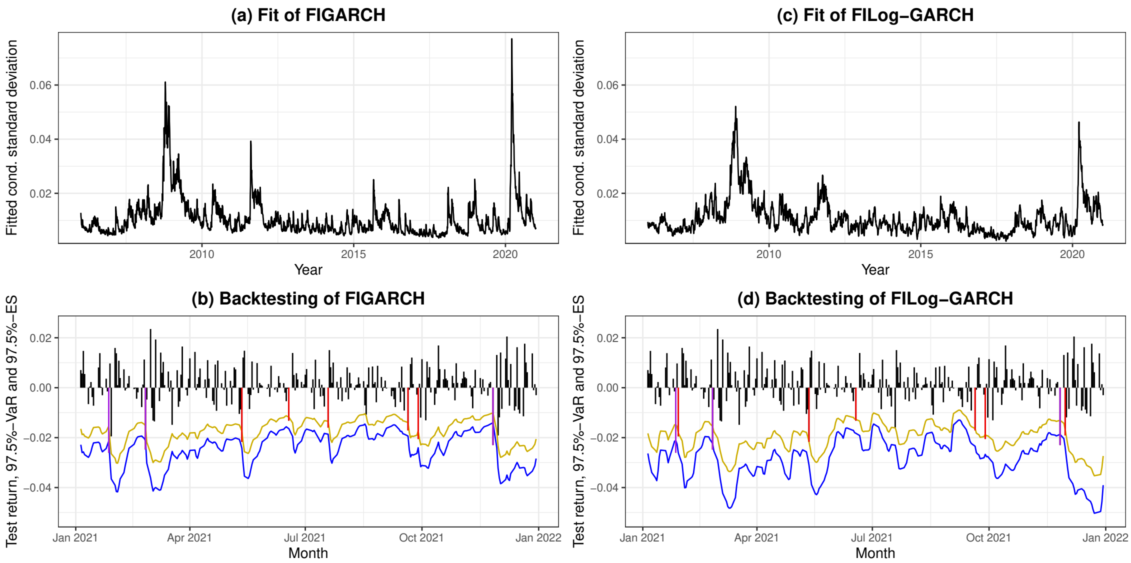


Figure 1.4: The fitted in-sample conditional standard deviations following a FIGARCH$(1, d, 1)$ and FILog-GARCH$(1, d, 1)$ with conditional skew-t distribution by `rugarch` and `fracdiff` and the corresponding out-of-sample one-step forecasted 97.5%- VaR and ES with with test returns and highlighted violations.

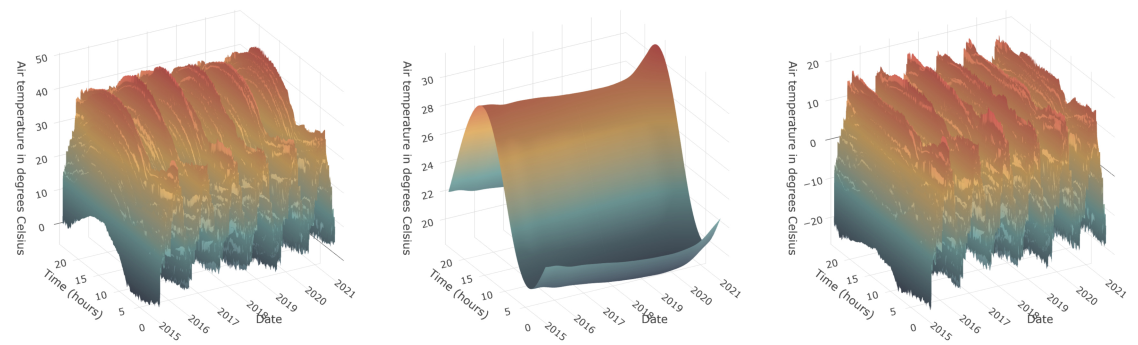
(ES) are included at the 97.5%-level alongside S&P 500 test returns. It becomes apparent that the FIGARCH and the FILog-GARCH do not allow for the inclusion of a leverage effect, as risk measures obtained by these models react equally strongly to both positive and negative shocks (of same magnitude). This fact is supported by the application of a sign bias test (Engle and Ng, 1993) to the in-sample residuals following the FIGARCH and the FILog-GARCH, because the tests do reject some of the null hypotheses on the absence of further sign effects in the residuals. Thus, to explain the return series the models can be improved. This lack of capturing asymmetry is also visible in the forecasted risk measure series. While the models would pass so-called traffic light tests for VaR and ES (see also Costanzino and Curran, 2018), we can observe a strong decrease in the risk measure forecasts also for positive returns, for example in March 2021 or in December 2021, because the return values large in magnitude are mostly positive compared to negative returns smaller in magnitude appearing at those times. Since VaR and ES influence the amount of capital reserves to hold for financial institutions, it is desirable to not have to hold unnecessarily large capital reserves during times with mostly large positive returns. Another visible aspect is that the fitted conditional volatility following a FILog-GARCH does not increase as strongly for larger shocks in magnitude compared to the FIGARCH.

This small selection of basic long-memory GARCH-type models in R implies a necessity for more advanced long-memory GARCH models already available in other statistical software. Furthermore, the FILog-GARCH has a major disadvantage: in its definition, the log-transformation is applied to the squared returns. While in theory, the probability that a return becomes zero is zero due to the continuous underlying conditional distribution, an observed return may in some cases be zero. This issue may be even more severe for high-frequency data. In practice, it is therefore of interest to introduce a stationary long-memory exponential GARCH-type model that does not suffer from the zero-returns and a reliable method to estimate it from given data. On a further note, it is well-known in the literature that the standard FIGARCH can never be weakly stationary (Baillie et al., 1996), which again drives the necessity for further applicable, weakly stationary long-memory GARCH models.

A further hindrance of existing R packages is their limitation when it comes to VaR and ES computation and the backtesting capabilities of these quantities. `rugarch` as the most sophisticated GARCH estimation package on CRAN does not allow for traffic light tests (Costanzino and Curran, 2018) in VaR and ES (Galanos, 2024); furthermore, it can only compute the VaR and ES for the implemented models without capabilities to extend such computations for more generalized models. From the viewpoint of practitioners, a piece of software is desirable that combines the entire model fitting, forecasting and backtesting pipeline in volatility modelling.

1.3 Spatial models in the mean

Random processes do not only occur in one (time) dimension like in the classical time series context. A spatial viewpoint has been established among various areas such as agriculture, epidemiology and ecology, see Goovaerts (1999), Lawson (2013) and Wiegand and Moloney (2013) as examples. Fundamental research on spatial random fields, however, was developed for example by Whittle (1954), Tjøstheim (1978) and Guyon (1982). Properties and estimators of spatial unilateral ARMA models were investigated by Basu and Reinsel (1993), Huang and Anh (1992), Bustos et al. (2009) and Drapatz (2016). A review on spatial ARMA models is provided in Bustos et al. (2009). A spatial FARIMA was introduced by Beran et al. (2009b) as well as a residual sum of squares estimator for the model and its asymptotic properties. Central limit theorems for spatial long-memory processes were studied for example by Lahiri and Robinson (2016). A closely related spatial ARFIMA model was introduced by Otto and Sibbertsen (2023). Surface smoothing in spatial time series consisting of a mean surface component and a stationary non-deterministic spatial error process was researched by Schäfer (2021) and Schäfer (2022). An automated bandwidth selection procedure for spatial local polynomial regression was introduced for this purpose and released under the R package name `DCSmooth` (Schäfer, 2021) to CRAN, enabling R-users to apply data-driven double-conditional local polynomial smoothing. Double-conditional smoothing, originally under bivariate kernel regression (Härdle, 1990; Ruppert and Wand, 1994), was introduced as a fast smoothing algorithm for spatial data by Feng (2013b); however, Feng (2013b) proposed it in the context of the estimation of volatility surfaces in spatial high-frequency log-returns.



(a) Observations (b) Smoothed surface (c) Residuals

Figure 1.5: Imputed five-minute air temperature data at Yuma station (2015-2020) displayed as a spatial time series. The smoothed surface and the residuals are obtained using the data-driven algorithm in the `DCSmooth` package under spatial long-memory errors, leading to implausible results.

As an example, in Figure 1.5(a), the five-minute air temperatures (in degrees Celsius) at Yuma station are displayed for each day from January 01, 2025, to

December 31, 2020. The data was downloaded from the website of the National Oceanic and Atmospheric Administration (NOAA) and imputed (more information in Chapter 5). In Figures 1.5(b) and (c), the estimated mean surface and the corresponding residuals are presented. The surface was obtained calling the `DCSmooth` package in R with option `"farima_RSS"` to account for potential long memory in the residuals in the bandwidth identification. 0.5437 and 0.2687 were selected as bandwidths by the procedure. The surface is clearly oversmoothed, because it does not account for any of the seasonal variation in the day-to-day dimension. This is also observable from the residuals that still include the seasonal interday pattern. Thus, the automated smoothing idea by Schäfer (2022) is not suitable to decompose such data and other approaches must be introduced.

1.4 Overview of contents and contributions

The main contribution of this thesis is the creation and publication of the R packages `smoots` (Feng et al., 2023b), `deseats` (Feng and Schulz, 2026) and `fEGarch` (Schulz et al., 2026b) for (almost all of) the methods presented and developed in this thesis: `smoots` makes the methods of Chapters 2 and 3 applicable, `deseats` those of Chapter 4, and `fEGarch` those of Chapter 6. Their package logos are displayed in Figure 1.6.

Figure 1.6: The logos of the three R packages `smoots`, `deseats` and `fEGarch` contributed through this thesis.

Every person can download and install the packages freely from CRAN as an extension to the free statistical software R. Furthermore, the packages were carefully crafted and designed to allow for the employment of the methods with simple application pipelines and without the need for deeper theoretical knowledge on the methods behind the packages. Internal R codes were produced to handle spatial data and apply the methods of Chapter 5. Accompanying to the R packages written for this thesis, which in of themselves include thousands of lines of R and C++ code, a respective software manual was written and published alongside each of the packages on CRAN, amounting to a total of 256 additional manual pages written as a side product of this thesis. However, to save space and costs in the printed version of this thesis, it was decided to omit the manual pages from the appendix of this thesis and instead the readers are referred to the published manuals on the corresponding CRAN webpages of the R packages presented here.

The structure of this thesis and further theoretical contributions are described in the following. In Chapter 2, the R package `smoots` (Feng et al., 2023b) is introduced that allows for automated semiparametric modelling of trend-stationary time series with short-memory errors. It makes use of trend estimation (and also of estimation of a trend's derivatives) via local polynomial regression (Fan and Gijbels, 1996), where the crucial bandwidth parameter is estimated in a data-driven and fully nonparametric way following the algorithms by Feng et al. (2020b). After the trend estimation, an ARMA model is fitted to the detrended series to finalize the entire semiparametric ARMA (Semi-ARMA) model. The methods are summarized briefly, the functionalities of the package are described and empirical examples in the context of macroeconomic series for mean modelling are provided with direct application code. Furthermore, semiparametric Log-GARCH and Log-ACD models are fitted to log-financial data using the same Semi-ARMA model with data-driven local polynomial trend. We show empirically that the model under consideration of the presented algorithm produces reasonable results for time series from various areas of research and stabilizes otherwise non-stationary data.

Chapter 3 extends on the `smoots` framework: with a later version of the package, new functionalities were introduced like a graphical test for the linearity of a trend that can also be adapted to check if an observed return series has a constant unconditional variance. The tests are derived from asymptotic theory of local polynomial regression. Furthermore, forecasting capabilities for the main model of Chapter 2, i.e. for the Semi-ARMA model, are implemented. Point forecasts are derived for Semi-ARMA models and corresponding forecasting intervals are introduced both under the assumption of normal innovations as well as following a nonparametric bootstrap procedure, where the uncertainty in the forecasts due to the trend portion is omitted. Chapter 3 hence firstly gives an introduction into the underlying theory and then provides various application examples of the established methods in Chapter 3, including a comparison between the point forecasting performance of `smoots` and other established time series methods. We show that many time series contain trends that are more complex than a simple linear function and that daily log-return series observed over a longer time period can indeed contain a time-varying scale function that renders them non-stationary. Furthermore, we show that the Semi-ARMA can produce more precise point forecasts at least for some observed time series in comparison to other established methods.

In Chapter 4, the R package `deseats` (Feng and Schulz, 2026) is presented. It can be used to decompose a seasonal time series with short-memory errors using automated locally weighted regression, where the bandwidth is estimated using an improved version of the algorithm by Feng (2013a) that in its original version only allowed for independent errors. The improved algorithm not only allows for short-memory errors, but also estimates the bandwidth in a fully nonparametric way, i.e. without assuming a specific short-memory time series model during the bandwidth estimation. The derivation of the asymptotically optimal bandwidth under short-memory errors is sketched, the algorithm for the decomposition is described and an entire semiparametric model, called an S-Semi-ARMA (seasonal Semi-ARMA) model, where after the decomposition an ARMA model is fitted to the residuals, is introduced together with corresponding forecasting methods. The forecasting intervals are derived under normal innovations and under a bootstrap, where the uncertainty in the forecasted trend and the forecasted seasonality is omitted. The chapter continues with example decompositions and the forecasting of real-world seasonal time series from various fields, where also example codes are provided directly. As benchmarks, seasonal adjustments by STL (Speth, 2004), X13 (Findley et al., 1998; U.S. Census Bureau, 2025) and TRAMO-SEATS (Gómez and Maravall Herrero, 1996) are also shown. A first simulation study then both highlights the validity of the bandwidth selection algorithm and the consistency of our corresponding data-driven component estimators. A second simulation study compares the quality of seasonal component estimation among various established methods under a huge variety of different settings following the mean squared error criterion. Our method can successfully compete with other approaches even under strong model assumption violations, and it is usually the best at identifying the seasonal component, if our method's model assumptions are violated only mildly, given that the number of observations is sufficiently large.

Methods for decomposing a spatial time series with a two-dimensional seasonal pattern (but without trend) using spatial locally weighted regression are introduced in Chapter 5. As will the emipirical analysis within that chapter show, spatial temperature time series may have random intraday curves, which render the remaining observations after the removal of the seasonal surface non-stationary. An additional procedure is proposed, where the random curves are estimated and removed from the residuals of the previous surface estimation. The model estimation is finalized by fitting SFARIMA models to the residuals, hence building a full semiparametric SFARIMA (Semi-SFARIMA) model with random intraday curves. No R package was published for this chapter; however, a collection of internal R codes was produced to simplify dealing with spatial time series and fit the aforementioned semiparametric spatial model with random intraday curves. Furthermore, the codes also allow for obtaining asymptotically valid standard errors of the SFARIMA coefficients in accordance with Beran et al. (2009b). The analysis shows that the spatial temperature data under consideration contains significant long memory in the interday dimension, while there is no notable long memory in the intraday dimension.

Chapter 6 introduces the R package `fEGarch`, which can be used to fit parametric and semiparametric GARCH-type models as well as dual extensions of parametric GARCH-type models with simultaneous modelling in the mean to observed return series and other time series. The estimation method is (conditional) quasi-maximum-likelihood estimation for parametric models or parametric model parts; nonparametric model parts are estimated through automated local polynomial regression. Furthermore, `fEGarch` implements multistep forecasts, rolling one-step forecasting over a test period and common backtesting approaches for GARCH-type models. The chapter introduces the general exponential GARCH family (EGF) that can be estimated via `fEGarch` and summarizes selected backtesting methods for VaR and ES. An overview of the package's functions is also provided. Extensive example applications are given with codes to show the fitting of various models available in the package and to illustrate the forecasting and backtesting capabilities of the package and the suitability of the models in practice. Applications of the backtesting capabilities under multivariate GARCH models and other volatility models are presented and comparisons to related statistical software are drawn, showing that `fEGarch` is a free alternative that offers more model and backtesting options than other statistical software in one software pipeline. The chapter closes with a simulation study that illustrates the consistency and asymptotic normality of the vast majority of the EGF estimators in `fEGarch`. Mathematically inclined readers are referred to the chapter's appendix, where some of the EGF's theoretical properties are derived, the available conditional distributions in `fEGarch` are presented, estimation methods as implemented in `fEGarch` are described, and forecasting approaches for the available models in `fEGarch` are introduced.

Chapter 7 summarizes main findings of this thesis and briefly outlines potential for further research.

Since this thesis is a cumulative thesis, i.e. the chapters were written individually as independent articles, the relevant mathematical notation is introduced separately and independently within each chapter and should not be transferred to other chapters within this thesis. Moreover, certain concepts may be repeated in multiple chapters and, if necessary, minor adjustments are made in comparison to the original articles, such as corrections of typing errors found after the articles' initial publications. Indications of co-authorships and of where each article is published are given as brief notes at the beginning of each main chapter. The personal amount of contribution to each of the articles is one divided by the corresponding number of authors.

Chapter 7

Conclusion

In this thesis, various new parametric and semiparametric time series models with short- and long-range dependence were introduced in the one-dimensional and two-dimensional time context. Alongside those models, estimation and forecasting procedures were established. All methods have been arranged into code scripts for the free statistical software R; in fact, almost all of the software has been published in form of the R packages `smoots`, `deseats` and `fEGarch` on CRAN, making them applicable for free for everyone with a laptop and a working internet connection. The packages were designed to make them easily and intuitively employable, also for persons without a deeper understanding of the theoretical topics, and further measures, such as the translation into pre-compiled C++ code and instances of parallel programming, were taken to improve on the computational efficiency as much as possible.

In Chapter 2, data-driven algorithms for the bandwidth selection in local polynomial regression for (one-dimensional) trend-stationary time series with short-memory errors were summarized. The algorithms allow for an automated selection of the globally optimal bandwidth in the sense of the asymptotic mean integrated squared error. For this purpose, an iterative plug-in approach is used. The result is an automated trend estimation as well as an automated estimation of a trend's derivatives. At the same time, the R package `smoots` for implementing these methods was introduced and applied to different example series from varying research areas, including a Log-GDP series and temperature data. The estimated trends and derivatives look plausible and align with the courses of the observed time series. The accordingly detrended series appear to be roughly stationary; however, potential long memory effects that might be prevalent in the data are not considered through the method. Fitting ARMA models to the detrended data highlighted the stabilizing effect of the automated detrending procedure on the observations. In a further step, the trend estimation algorithm was also applied to fit Semi-Log-GARCH and Semi-Log-ACD models to observed stock market log-returns and trading volumes, respectively. The application showed that such series can sometimes also exhibit signs of non-stationarity, which can be accounted for through the presented R package.

In Chapter 3, the capabilities of the `smoots` package were extended. A graphical test for checking the linearity of a trend was introduced that makes use of of an adjusted bandwidth that leads to asymptotically unbiased local polynomial trend (and derivative) estimates. This and the asymptotic normality of the (unbiased) estimators were derived from established asymptotics in local polynomial regression. The same testing procedure can, furthermore, also be applied to check, if the unconditional scale in an observed return series is constant. Additionally, methods for producing point and interval forecasts in accordance with a Semi-ARMA, that can be implemented using `smoots`, were presented. The forecasting intervals can be produced under the assumptions of normal innovations or following a bootstrap, where the uncertainty due to the trend is omitted. A backtesting procedure was summarized that compares out-of-sample point and interval forecasts of a Semi-ARMA with actual observations. To compare the accuracy of the point forecasts, the criteria RMSSE and MASE were considered, whereas a simple comparison between the number of observations inside the forecasting intervals gives an idea of the overall suitability of the model for forecasting. All approaches were applied to real-world examples, where it was shown that trends in time series are often clearly different from a simple linear trend and that stock series like returns and trading volumes can be non-stationary over a long period of time due to slowly changing scale functions or trends. Moreover, we illustrated the forecasting capabilities of the Semi-ARMA (with data-driven local linear trend) for a few example series and compared the one-step rolling forecasting performance over a hold-out period of length five to other established methods. In that application, the Semi-ARMA was demonstrated to produce the best point forecasts according to RMSSE and MASE in most cases and overall the most stable forecasting performance due to the lowest average RMSSE and MASE for the analyzed example series. Forecasting the trend in Semi-ARMA models, however, relies on the (linear) extrapolation of the trend, which in many cases works well in practice at least for short forecasting horizons. The robustness of this idea needs to be investigated more thoroughly, in particular with respect to forecasts for time points further into the future and for multi-step forecasting. In addition, since `smoots` works only under the assumption of short-memory errors, all ideas in Chapters 2 and 3 should be extended to cases with long-memory errors, i.e. to a fully nonparametric bandwidth selection for local polynomial regression under long-memory errors, to point and interval forecasts for a long-memory Semi-ARMA, and to trend testing under long-memory.

In Chapter 4, a data-driven algorithm called DeSeaTS for the bandwidth selection in locally weighted regression for the decomposition of seasonal time series with short-memory errors was introduced. The theoretical foundation for the bandwidth estimator is the asymptotically optimal bandwidth according to the asymptotic mean integrated squared error. The procedure can be applied to seasonal time series with any positive-integer seasonal period. A forecasting procedure was implemented that results in point and interval forecasts, where the interval forecasts can either be obtained theoretically following normal innovations or using a bootstrap. Once again, the uncertainty in the forecasts of the nonparametric model components is omitted. A new R package called `deseats` was presented that makes the chapter's methods easily applicable. The package was used to show with respect to observed time series with various seasonal periods that DeSeaTS provides high-quality seasonal adjustment, while it is simultaneously capable of distinguishing between a slowly moving trend and short-memory (cyclical) errors. Seasonal adjustments by STL, X13 and TRAMO-SEATS were provided as benchmarks. In addition, the application of the package for forecasting was presented. An initial simulation study showed that the bandwidth selection via DeSeaTS and the corresponding automated component estimators are all consistent. A second simulation study compared the seasonality estimation quality of various methods and showed that DeSeaTS usually outperforms other established methods (according to the mean squared error), when the seasonal component is slowly changing but still deterministic regardless of independent or autocorrelated errors and regardless of deterministic or stochastic trends, if the number of observations is sufficiently large. If the seasonal component itself, however, is stochastic, DeSeaTS is rarely the best method even for large numbers of observations, but its performance is often only marginally worse than that of the best method under such a setting. At the same time, the strong decrease in average mean squared error with a doubling of the numbers of observations, stronger than for other methods, indicates that DeSeaTS may perform best at identifying the seasonal component, even if the seasonality is stochastic, if the number of observations was even clearly larger than the ones tested in this article. Ultimately, DeSeaTS is valuable in research, but it may also provide some usefulness in official statistics, for example by enriching existing algorithms by a main decomposition step via DeSeaTS. Such research, however, needs to be conducted elsewhere. In addition, there is still potential for further research on the method itself. Most importantly, the DeSeaTS algorithm may misbehave, if an observed series contains jumps and structural breaks. Such particular violations of model assumptions were not yet discussed within this thesis. Therefore, the algorithm should potentially be made more robust for such cases. As a further aspect, DeSeaTS estimates one bandwidth for a simultaneous trend and seasonal component estimation. A more realistic approach would be to have separate estimation steps for those components and to implement two individually estimated bandwidths for this purpose; however, deriving the asymptotically optimal bandwidths under such a setting proves to be challenging and is beyond the scope of this thesis. A further extension of DeSeaTS should cover the automated locally weighted regression decomposition of seasonal time series under long-memory errors, including also the introduction of corresponding forecasting techniques. Moreover, the forecasting quality of the existing seasonal Semi-ARMA should be investigated more thoroughly for different forecasting horizons and a more suitable forecasting idea than an empirical linear extrapolation should be developed under local cubic trends.

In Chapter 5, we introduced a model and corresponding estimation method for spatial time series with two dimensional seasonality and spatial long-memory errors. Double conditional smoothing was employed with manually selected bandwidths to estimate the seasonal surface in spatial temperature time series. Our empirical findings after the surface removal motivated the nonparametric estimation of random intraday curves via automated local polynomial regression. The presented approach succeeded in obtaining stationary residuals for the analyzed example data that showed clear and statistically significant signs of long memory in the interday dimension according to the SFARIMA fitted to residuals. Further research should focus on improving the automated bandwidth selection for local polynomial double conditional smoothing in the context of the seasonal surface estimation. Additionally, forecasting ideas for the proposed spatial model should be researched as well as model extensions with both a trend surface and a seasonal surface, in particular for extended observation periods.

The R package `fEGarch` was presented in Chapter 6, which allows for the estimation of a huge variety of parametric and semiparametric short-memory and long-memory GARCH-type models. First and foremost, the new exponential GARCH family models were introduced and common backtesting strategies for volatility models according to official regulations were summarized. In the application section, a guide was provided on how to apply the `fEGarch` package for volatility forecasting and backtesting, most notably with respect to the EGF and its extensions. It was discussed, how to apply the package for backtesting multivariate GARCH models and non-GARCH volatility models, and the strengths of `fEGarch` were highlighted in comparison to its competitors `rugarch`, `ufRisk` and *OxMetrics*, as it is the only software that implements an entire estimation, forecasting and backtesting pipeline, most notably for a vast selection of long-memory GARCH-type models. A simulation study showed the consistency and asymptotic normality of the (conditional) QMLE for the EGF models in almost all evaluated cases. The appendix provided detailed theoretical information on the employed (conditional) QML estimation technique for the EGF, on VaR and ES forecasting methods for the EGF, and on properties of the available conditional distributions in `fEGarch`. Furthermore, some theoretical properties were derived for the EGF. Further research should extend the EGF by models with trend in the mean and simultaneous scale in the variance in order to allow for even more flexible modelling of observed time series. Such an idea could potentially be implemented in a step-wise procedure, where each model element could be estimated individually in a mostly data-driven way. Another aspect is the consideration of further conditional distributions for the EGF. A practically important future addition to the `fEGarch` package is the introduction of pipelines for direct multivariate GARCH implementation and backtesting.

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