



Scaling detection and characterization methods of photon-number-resolving superconducting detectors

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Summary

Light is usually measured as a continuous optical power in everyday applications, but fundamentally it consists of individual photons. This thesis investigates how photon counting can be extended from the single-photon regime toward much larger photon numbers, thereby reducing the gap to classical optical power meters.

Three approaches are developed and combined. First, quantum detector tomography, a method for fully characterizing complex photon-counting detectors, is scaled using efficient high-performance-computing approaches. Second, photon-number resolution with superconducting nanowire single-photon detectors is investigated by analyzing electrical traces, developing an arrival-time-based model, and quantifying the influence of jitter, optical pulse duration, and photon-number assignment. Third, intrinsic photon-number resolution is combined with spatial and temporal multiplexing. The result is a high-dynamic-range photon-counting detector capable of measuring thousands of photons per pulse.

This work shows that photon-number-sensitive measurements can be extended toward regimes that were previously accessible mainly through classical optical power measurements. Further, these methods are important for future photonic quantum technologies and detector systems with increasing numbers of optical modes, which require a quantum-mechanically consistent description.

Zusammenfassung

Licht wird in alltäglichen Anwendungen meist als optische Leistung gemessen, besteht jedoch grundsätzlich aus einzelnen Photonen. Diese Arbeit untersucht, wie die Photonenzählung vom Einzelphotonenbereich auf größere Photonenzahlen ausgeweitet werden kann, um so die Lücke zu klassischen optischen Leistungsmessern zu verringern.

Es werden drei Ansätze entwickelt und kombiniert. Erstens wird die Quantendetektortomographie, eine Methode zur vollständigen Charakterisierung komplexer Detektoren, mithilfe effizienter Hochleistungsrechneransätze skaliert. Zweitens wird die Photonenzahlauflösung bei supraleitenden Nanodraht-Einzelphotonen-Detektoren untersucht, indem elektrische Signalkurven analysiert, ein auf Ankunftszeiten basierendes Modell entwickelt und der Einfluss von Jitter, optischer Pulsdauer und Photonenzahlzuordnung quantifiziert wird. Drittens wird die intrinsische Photonenzahlauflösung mit räumlichem und zeitlichem Multiplexing kombiniert. Das Ergebnis ist ein Detektor mit hohem Dynamikbereich, der Tausende Photonen pro Puls messen kann.

Diese Arbeit zeigt, dass photonenzahlauflösende Messungen auf Bereiche ausgeweitet werden können, die bisher hauptsächlich durch klassische optische Leistungsmessungen zugänglich waren. Darüber hinaus sind diese Methoden wichtig für zukünftige photonische Quantentechnologien und Detektorsysteme mit einer steigenden Anzahl optischer Moden, die eine quantenmechanisch konsistente Beschreibung erfordern.

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List of abbreviations

AC	Alternating Current
AWG	Arbitrary Wave-form Generator
BS	Beam Splitter
CPU	Central Processing Unit
DC	Direct Current
EMG	Exponentially-Modified Gaussian
EOM	Electro-Optic Modulator
FPGA	Field Programmable Gate Array
FWHM	Full Width at Half Maximum
GPU	Graphics Processing Unit
HDR	High-Dynamic-Range
HWP	Half-Wave Plate
HPC	High-Performance Computing
LUT	Lookup Table
MKID	Microwave Kinetic Inductance Detector
MPI	Message Passing Interface
PBS	Polarizing Beam Splitter
PCA	Principal Component Analysis
PM	Polarization-Maintaining
PNR	Photon-Number Resolution
QDT	Quantum Detector Tomography
QWP	Quarter-Wave Plate
SPDC	Spontaneous Parametric Down-Conversion
SNSPD	Superconducting Nanowire Single-Photon Detector
SMSPD	Superconducting Microstrip Single-Photon Detector
SPAD	Single-Photon Avalanche Photodiode
STaND	Superconducting Tapered Nanowire Detector
STJ	Superconducting Tunnel Junction
TBP	Time-Bandwidth Product
TES	Transition Edge Sensor
TMD	Time-Multiplexed Detector
VOA	Variable Optical Attenuator

Author publications and contributions

Publications

First-author and relevant publications

The following publications (sorted by date of publication) form the main work presented in this thesis. Authors marked with [†] contributed equally to the work.

Quantum detector tomography of a high dynamic-range superconducting nanowire single-photon detector

Timon Schapeler, Jan Philipp Höpker, and Tim J. Bartley

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TS designed the experiment, developed the measurement procedure and performed the experiment with input from JPH. TS performed the data analysis and implemented the reconstruction. TS wrote and reviewed the manuscript with input from all authors.

Information extraction in photon-counting experiments

Timon Schapeler and Tim J. Bartley

Physical Review A **106**, 013701 (2022)

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TS designed the experiment, developed the measurement procedure, performed the experiment as well as the data analysis. TS wrote and reviewed the manuscript with input from TB.

Electrical trace analysis of superconducting nanowire photon-number-resolving detectors

Timon Schapeler, Niklas Lamberty, Thomas Hummel, Fabian Schlue, Michael Stefszky, Benjamin Brecht, Christine Silberhorn, and Tim J. Bartley

Physical Review Applied **22**, 014024 (2024)

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TS and TB conceived the idea. TS designed the experiment and measurement procedure with input from NL. TS and NL performed the experiment with input from TH and FS. NL and TS performed the data analysis. TS wrote and reviewed the manuscript with input from all authors.

Scalable quantum detector tomography by high-performance computing

Timon Schapeler[†], Robert Schade[†], Michael Lass, Christian Plessl, and Tim J. Bartley

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TS and TB conceived the idea. TS designed the experiment, developed the measurement

procedure, performed the measurements and analyzed the measurement data. TS and ML performed pretesting for the large-scale reconstruction. RS programmed the solver, performed all benchmarking and calculated the Wigner functions. TS performed the POVM verification. TS, RS and TB wrote and reviewed the manuscript with input from all authors.

Optimizing photon-number resolution with superconducting nanowire multi-photon detectors

Timon Schapeler, Fabian Schlue, Michael Stefszky, Benjamin Brecht, Christine Silberhorn, and Tim J. Bartley

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Jitter in photon-number-resolved detection by superconducting nanowires

Mariia Sidorova, **Timon Schapeler**, Alexej D. Semenov, Fabian Schlue, Michael Stefszky, Benjamin Brecht, Christine Silberhorn, and Tim J. Bartley

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MS conceived the idea with input from TS and AS. TS designed and performed the experiment with input from FS. The theoretical model was developed by MS with input from TS and AS. TS tested the model against the experimental data and analyzed the measurement data. TS created the visualizations. MS, TS, and TB wrote and reviewed the manuscript with input from all authors.

Practical considerations for assignment of photon numbers with SNSPDs

Timon Schapeler, Isabell Mischke, Fabian Schlue, Michael Stefszky, Benjamin Brecht, Christine Silberhorn and Tim J. Bartley

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Macroscopic photon counting beating the Poisson noise limit

Timon Schapeler, Fabian Schlue, Isabell Mischke, Michael Stefszky, Benjamin Brecht, Christine Silberhorn and Tim J. Bartley

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Co-author contributions

The following publications (sorted by date of publication) are additional work I have contributed to during my PhD, that are not directly relevant for this thesis. Two additional publications are not included, as they have been published prior to my PhD. Authors marked with [†] contributed equally to the work.

Degenerate photons from a cryogenic spontaneous parametric down-conversion source

Nina Amelie Lange, **Timon Schapeler**, Jan Philipp Höpker, Maximilian Protte, and Tim J. Bartley

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Low-noise balanced homodyne detection with superconducting nanowire single-photon detectors

Maximilian Protte[†], **Timon Schapeler**[†], Jan Sperling, and Tim J. Bartley

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Harnessing temporal dispersion for integrated pump filtering in spontaneous heralded single-photon generation processes

Julian Brockmeier, **Timon Schapeler**, Nina Amelie Lange, Jan Philipp Höpker, Harald Herrmann, Christine Silberhorn, and Tim J. Bartley

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Benchmarking Gaussian and non-Gaussian input states with a hybrid sampling platform

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Learning to detect optical nonclassicality

Martina Jung, Suchitra Krishnaswamy, **Timon Schapeler**, Annabelle Bohrdt, Tim J. Bartley, Jan Sperling, and Martin Gärtner

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Conference Contributions

The following conference contributions (sorted by date) are all first-author contributions during my PhD.

Quantum detector tomography of high-dimensional multiplexed superconducting detectors

Timon Schapeler, Jan Philipp Höpker, and Tim J. Bartley
Conference on Lasers and Electro-Optics, OSA Technical Digest
(Optica Publishing Group, 2021), paper FW2P.5, Oral presentation



Using outcome purity to compare multiplexed detectors

Timon Schapeler, Jan Philipp Höpker, and Tim J. Bartley
Quantum Information and Measurement VI, Technical Digest Series
(Optica Publishing Group, 2021), paper Th2A.3, Oral presentation



Information Extraction in Photon Counting Experiments

Timon Schapeler and Tim J. Bartley
DPG-Frühjahrstagung (2022), Oral presentation

Information Extraction in Photon Counting Experiments

Timon Schapeler and Tim J. Bartley
Conference on Lasers and Electro-Optics, Technical Digest Series
(Optica Publishing Group, 2022), paper FF3K.6, Oral presentation



Quantum characterization of superconducting detectors

Timon Schapeler and Tim J. Bartley
DPG-Frühjahrstagung (2023), Oral presentation

Approaching Mega-Scale Quantum Detector Tomography using High-Performance Computing

Timon Schapeler, Michael Lass, Christian Plessl and Tim J. Bartley
Optica Quantum 2.0 Conference and Exhibition, Technical Digest Series
(Optica Publishing Group, 2023), paper QW4C.6, Oral presentation



Mega-Scale Quantum Detector Tomography using High-Performance Computing

Timon Schapeler, Michael Lass, Robert Schade, Christian Plessl and Tim J. Bartley
European Quantum Technologies Conference (2023), Poster presentation

Photon-number resolution with SNSPDs via statistical analysis of electrical traces

Timon Schapeler, Niklas Lamberty, Thomas Hummel, Fabian Schlue, Michael Stefszky, Benjamin Brecht, Christine Silberhorn, and Tim J. Bartley
Quantum 2.0 Conference and Exhibition, Technical Digest Series
(Optica Publishing Group, 2024), paper QM2A.5, Oral presentation



Scalable Quantum Detector Tomography using High-Performance Computing

Timon Schapeler, Fabian Schlue, Michael Stefszky, Benjamin Brecht, Christine Silberhorn, and Tim J. Bartley

Single Photon Workshop (2024), Poster presentation

Massively Multiplexing Superconducting Nanowire Photon-Number-Resolving Detectors

Timon Schapeler, Michael Lass, Robert Schade, Christian Plessl and Tim J. Bartley

Single Photon Workshop (2024), Poster presentation

Optimizing photon-number resolution with superconducting nanowire multi-photon detectors

Timon Schapeler, Fabian Schlue, Michael Stefszky, Benjamin Brecht, Christine Silberhorn, and Tim J. Bartley

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Invited oral presentation

**Macroscopic Photon-Number Resolution with Sub-Poisson-Noise Precision**

Timon Schapeler, Fabian Schlue, Isabell Mischke, Michael Stefszky, Benjamin Brecht, Christine Silberhorn, and Tim J. Bartley

Single Photon Workshop (2026), Oral presentation

Light is ubiquitous and therefore one of the most accessible physical quantities in everyday life. We feel optical radiation as warmth on our skin and use our eyes to image the world around us. Beyond human perception, light is essential for many current technologies that enable us to take photos, communicate over long distances by transmitting information through optical fibers, generate electricity and measure distances with interferometry and time-of-flight techniques. Light can be described as discrete energy packets [1, 2], known as photons. However, in most macroscopic applications, we use continuous quantities such as intensity or optical power and wavelength to describe light. This description works well, when many photons contribute to the measured signal. However, it hides the quantized nature of light due to noise in the detection process. When the noise level of a detector is sufficiently low, measurement of the optical intensity reveals individual photons.

Classically, optical power meters are used to measure a continuous optical power, by linearly mapping an incident photon flux on a photodiode into a measurable macroscopic electrical current [3]. In the quantum regime, photon-counting detectors are used to detect individual photons. However, there exists an intermediate mesoscopic regime of optical power, where we can neither reliably measure the power nor count the number of individual photons. In this attowatt-to-picowatt regime, the optical power drops below the noise floor of commercial optical power meters, which renders the photocurrent generated by single photons unmeasurable. Conversely, photon-counting detectors do not nearly have the necessary dynamic range and often generate nonlinear responses based on internal amplification processes to overcome the noise floor, as in avalanche photodiodes.

Imagine, however, you can count the number of incident photons in an optical state corresponding to the sensitivity limit of standard commercial power meters. This limit is around 100 pW, which corresponds to approximately one billion telecom-wavelength photons distributed over one second. This power equates one photon per pulse at a repetition rate of 1 GHz. For experimentally more common rates, ranging from hundreds of kilohertz to hundreds of megahertz, this necessitates counting up to thousands of photons per pulse. This raises the central questions that motivate this thesis. Which detector technology can bridge the gap between single-photon sensitivity and large optical powers? How to build a detector capable of large-scale photon counting? And how can such a detector be characterized?

Bridging the gap between microscopic single-photon measurements and macroscopic power meter readings is a very fascinating task in physics, as it links the classical everyday world with the quantum world. This orders-of-magnitude blind spot between these two vastly different regimes is especially important for optical power metrology. Enabling a high-dynamic-range (HDR) photon-counting detector can establish traceability down to the single-photon regime. A common metrology framework allows to calibrate and compare future photonic quantum technologies. These have demonstrated growth toward the currently inaccessible regime, for example in the form of large-scale detector arrays [4–9], and photonic quantum systems [10–13]. This provides another motivation for large-scale photon counting. As photonic processors, used for quantum computing, grow in their number of modes, the measurement system needs to scale accordingly.

Here, photon-number-resolved (PNR) detection for each mode is essential to enable photonic quantum applications [14] to probe the true photon statistics of quantum states.

Performing accurate measurements is a fundamental requirement for photonic quantum technologies. As photonic quantum states [15], experiments [10–13] and detectors [4–9] become larger and more complex, the characterization methods need to scale up accordingly to allow a thorough understanding of measurement results and detector responses. If this understanding is not guaranteed, interpretation of data becomes unreliable. Therefore, a consistent and quantitative characterization of optical systems, specifically detectors, is essential for quantum state characterization [16–20], quantum sensing [21], quantum communication [22, 23], photonic quantum computing [24, 25] and optical quantum metrology [26]. As access to photon-number information becomes increasingly important, photon-counting, i.e., photon-number-resolving detectors will become indispensable to future quantum photonic experiments.

With the development of HDR photon-counting detectors it is important to answer the question of how to characterize such detectors in a way that is consistent and valid over the entire range of photon numbers to which they are sensitive. A follow-up question is whether a single efficiency value and dark-count rate are sufficient to adequately describe such a detector. Addressing the first question requires an experimental and computational testbed to investigate the scaling behavior of common characterization techniques. Regarding the second question, an adequate description entails knowledge about the probabilities that certain detector outcomes occur in response to given incident photon numbers.

Quantum detector tomography (QDT) provides a rigorous framework for this task [27–32]. In detector tomography, the response of a measurement device is probed with a tomographically complete set of known input states, typically the readily accessible coherent states produced by lasers. This allows to reconstruct the measurement process without assuming a detailed physical model of the detector. The result is a positive operator-valued measure (POVM) [33], whose elements completely describe the detector [34]. This provides precisely the information required to address the question posed above. Therefore, this makes QDT a powerful tool that provides a model-independent, quantum-mechanically consistent characterization of parameters such as efficiency, dark-count probability, saturation effects, cross-talk between detector elements in an array or potential nonlinear responses [31, 35, 36].

For phase-insensitive detectors, the POVM elements are diagonal in the photon-number basis and are commonly represented in a single matrix, which contains the conditional probabilities of obtaining a given detector outcome for a given incident photon number [30, 34]. Even in this case, the size of the reconstruction task grows rapidly. If the detector has N outcomes and is sensitive to a Hilbert space with dimension M (i.e., a maximum photon number of $M - 1$), the reconstructed POVM matrix contains $M \cdot N$ elements. For phase-sensitive detectors, this scaling becomes even more demanding, as the number of elements scales with $M^2 \cdot N$ [34]. Quantum detector tomography has previously been demonstrated for detectors with comparatively few outcomes and modest Hilbert space dimensions [30, 31, 35, 37–44]. Standard reconstruction methods, such as semi-definite programming and maximum-likelihood estimation can be applied successfully in this regime. However, as the availability of large-scale detectors [4–9] and photonic systems [10–13] grows, the matrices involved in tomography become too large for straightforward reconstruction on conventional hardware, making their characterization increasingly challenging. Therefore, one aim of this thesis is to *enable the characterization of high-dynamic-range photon-counting detectors through a scalable QDT approach*.

This characterization problem is closely connected to the choice of detector technology. To count photons over a range approaching classical optical power measurements, a detector must combine properties that are usually found in different devices: single-photon sensitivity, access to photon-number information, large dynamic range, high efficiency, and scalability. Existing detector technologies typically optimize only part of this parameter space [34, 45]. Detectors that are sensitive to single photons are either nonlinear threshold detectors that probe the presence or absence of photons [34], or they saturate at tens of photons [46–48]. Conversely, detectors that measure large optical powers lose their sensitivity to single photons due to thermal noise, which generates a dark current that completely masks the current generated by single photons. Therefore, the task of counting large numbers of photons is nontrivial and the choice of detector technology to achieve the goal of HDR photon-counting must be assessed carefully.

Detector technologies

Ordinary photodiodes are excellent for classical power measurements, as they operate at room temperature and enable linear power measurements with a high dynamic range. However, they lose their sensitivity around the low-picowatt regime, preventing them to count individual photons. This is due to their inherent noise floor, which originates from thermally generated dark current and electronic noise [49]. At the other end of the detection spectrum, photomultiplier tubes are an established technology that offers single-photon sensitivity through the photoelectric effect [34]. However, these devices suffer from low efficiency (of 2% at 1550 nm [34]) and limited scalability due to bulky packaging of vacuum tubes [50]. A widely used single-photon detector, the single-photon avalanche photodiode (SPAD) is a device that offers practical and compact near room-temperature operation. Although they have demonstrated limited PNR through advanced signal processing, they suffer from low detection efficiency at 1550 nm [51], which is an important wavelength in the telecom range used in low-loss fiber networks. Most commonly, SPADs are operated as threshold detectors [34] (also referred to as Geiger mode or click detectors). In this operation detectors can only distinguish between the absence or presence of light, specifically zero or at least one photon. These limitations notwithstanding, due to their compactness, SPADs can be readily multiplexed, i.e., combined into large arrays. The concept of multiplexing can regain photon-number information by distributing light onto multiple detectors separated in space or time [7, 52–58] and thus lead to quasi-PNR. Only in the limit of infinitely many equally illuminated detectors can actual photon-number resolution be achieved; generally the number of detectors needs to be orders of magnitude larger than the number of photons [59]. In addition to finite numbers of detectors and the resulting nonzero probability that multiple photons end up in the same detection bin, experimental imperfections such as nonuniform splitting and loss limit the practically achievable resolution [59, 60]. Relying solely on multiplexing large numbers of threshold detectors to emulate PNR introduces a severe scalability bottleneck. As the number of accessible modes in experiments increases [13], the number of necessary detectors would scale prohibitively into the millions. Therefore, access to photon-number information per detector is essential, to minimize the probability of multiple photons being falsely registered as one by binary threshold detection [61].

The class of superconducting detectors is highly promising for PNR applications. Some of these detectors uniquely offer energy-resolving capabilities, which can be directly related to the number of absorbed photons under monochromatic illumination. The pioneering devices in superconductor-based single-photon detection are superconducting tunnel junctions (STJs), where photons absorbed by the superconducting material break Cooper pairs and create quasiparticles [62, 63]. These

tunnel across a thin junction, where the resulting current is proportional to the photon energy. Similarly in microwave kinetic-inductance detectors (MKIDs) the generated quasiparticles change the kinetic inductance of the superconductor, which is measured with a microwave resonator circuit [34]. Another prominent example for true photon-number resolution are transition edge sensors (TESs) [46–48, 64]. These devices are highly efficient and highly sensitive energy-resolving bolometers, which measure the deposited energy from impinging photons. The absorber material of the sensor is held at the superconducting transition temperature while being voltage biased. When a photon is absorbed, its energy increases the temperature of the material, which results in a small change of the electrical resistance. An array of superconducting quantum interference device (SQUID) amplifier circuits measures the current through the sensor [63]. A single TES can resolve on the order of tens of photons [6, 15, 47], and operation in the saturated regime can extend photon-number sensitivity by exploiting recovery-time information and elaborate data analysis [65, 66]. Similarly to TESs, the operation of STJs and MKIDs comes at the cost of requiring millikelvin temperatures, reachable with adiabatic demagnetization refrigerators or dilution refrigerators [67]. In addition to complex readout schemes, slow recovery times and large timing jitter [63], these energy-resolving detectors are limited in their applicability in experiments requiring high repetition rates or precise timing [68]. These limitations motivate alternative detector technologies that combine photon-number resolution with high speed, low noise, and experimental accessibility.

This is where superconducting nanowire single-photon detectors (SNSPDs) play an important role. Since the first demonstration of an SNSPD by Gol’tsman et al. in 2001 [69], their development offers a promising route to address the challenges, faced by other superconducting detectors. Over the years numerous review papers have been published on SNSPDs [50, 63, 67, 70–75], thoroughly discussing the applications, scaling challenges, and contemporary research trends of these devices. Originally regarded as threshold detectors, they have already become a leading detector technology for quantum optics due to their near-unity detection efficiencies (especially for the telecom range [76, 77], with active research for mid-infrared photons [75]), low dark-count rates (excellent signal-to-noise ratios) [78], picosecond timing jitter [79], fast reset times [80], and the possibility of constructing detector arrays [81–83], thereby enabling GHz detection rates [58, 84, 85] and imaging [4, 5, 8, 86].

An SNSPD consists of a superconducting material patterned into a wire, typically approximately 100 nm in width and < 10 nm in thickness. Very short wires achieve record temporal resolution [79]. However, generally a meandering pattern is fabricated to increase the fill factor and thus the active area of the device for polarization insensitivity [87] and near-unity detection efficiency, when combined with a dielectric layered stack and optimized fiber coupling [76]. Recent work investigates wires with up to 100 μm width [88–90], these show polarization insensitivity, mid-infrared sensitivity and reduction in dark counts. Generally, the wire is cryogenically cooled below the critical temperature of the superconducting material, and current-biased below its switching current. In this state the wire is single-photon sensitive. In contrast to the other superconducting detectors, the cryogenic operation of SNSPDs is less complex. Commercial two-stage Gifford-McMahon closed-cycle cryostats can be used, which continuously operate at temperatures as low as 2 K or reach 0.8 K in combination with a helium sorption fridge [67].

The fundamental detection mechanism of SNSPDs is still under active research; in literature there are two main descriptions based on the hotspot model [69, 94–96] and the vortex model [94, 97–100]. A schematic of the detection mechanism is shown in Fig. 1.1(a). The hotspot model provides an intuitive understanding, in which the absorption of a photon by the superconducting wire has enough energy to disrupt the superconducting state and create quasiparticles. This creates a

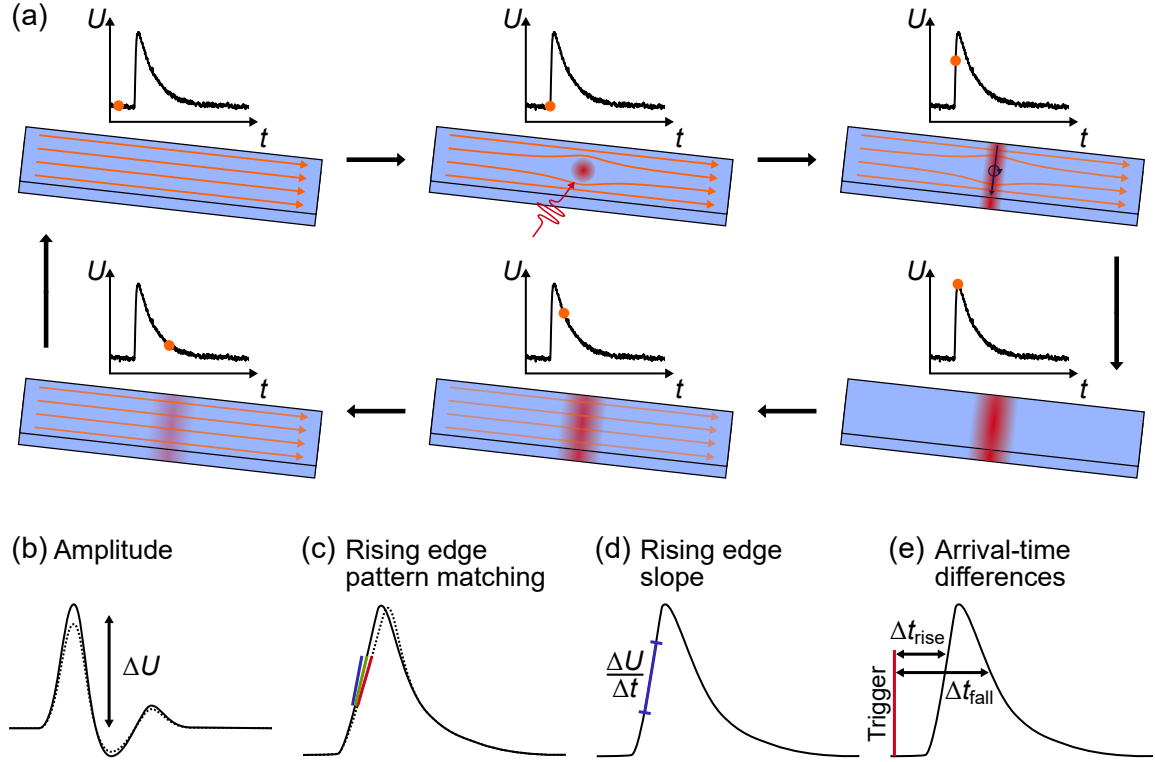


Figure 1.1: (a) Schematic representation of the detection mechanism of SNSPDs (starting top left in a clockwise direction). The wire is in a superconducting state. An absorbed photon creates a hotspot with normal resistance, which is avoided by the bias current. Increased current density and magnetic vortex entry assist the hotspot growth, until it covers the entire wire width. The wire resistance increases into the $k\Omega$ on a picosecond timescale, resulting in a measurable voltage pulse. The wire regains superconductivity via thermal dissipation. The insets show a typical output trace, where the orange dot indicates the voltage at the specific detection stages [91]. Adapted with permission from Ref. [92]. (b)-(e) Schematic representation of different techniques to retrieve photon-number information from the electrical output signals of SNSPDs. (b) The amplitude of the signal is sensitive to the number of photons, either through an inductor-resistor differentiating circuit or an impedance-matching taper. (c) Different colored lines represent reference waveforms of a subset of the rising edge. (d) A linear fit is made to the rising edge slope. (e) Arrival-time differences from a trigger signal to the rising (and falling) edge are used (either with an oscilloscope or a time-to-digital converter). Adapted with permission from Ref. [93].

localized resistive hotspot, where the superconductivity is broken. While Joule heating assists the hotspot growth, the current in the wire avoids the hotspot, which eventually exceeds the critical current density of the wire. When the superconducting state is disrupted along the full wire width, the bias current is shunted by an external circuit. The wire resistance increases into the $k\Omega$ on a time scale of picoseconds, generating a measurable voltage pulse. As the wire cools via thermal dissipation, the resistive state subsides, allowing the system to fully regain its superconducting state [70]. Literature shows that the hotspot model cannot describe the full detection mechanism, as it leads to inconsistencies when applied to experimental results [94].

The second model uses magnetic vortex dynamics to explain the detection mechanism with two possible scenarios [72]. In one scenario, an absorbed photon creates a vortex-antivortex pair,

which is separated by the Lorentz force arising from the bias current in the wire. When the vortex-antivortex pair breaks, the wire transitions into the resistive state and a measurable voltage pulse is generated. In another scenario, the energy of the absorbed photon leads to local heating of the wire, which reduces the energy barrier for magnetic vortex entry, ultimately disrupting the superconducting state [63, 72]. The preceding description provides only a brief overview of potential models for the detection mechanism of SNSPDs. A detailed discussion can be found in Ref. [94]. Regardless of which of these models describes the detection process, the question arises as to what happens when multiple photons are absorbed by the detector, and whether this can be resolved.

PNR SNSPD methods

The first demonstration of multi-photon detection using an SNSPD was initially demonstrated in 2007 by Cahall et al [101], which further complemented the already impressive figures of merit of SNSPDs. Although, the detection mechanism of SNSPDs is not fully understood, we know that the physical origin of PNR in SNSPDs is fundamentally different from that of energy-resolving detectors; an SNSPD does not directly measure the total deposited energy.¹ Instead, in their work, Cahall et al. [101] observed a time- and photon-number dependent hotspot resistance of the nanowire. Impinging photons can create multiple resistive hotspots; assuming no hotspots overlap, the total resistance of the wire scales with the number of created hotspots. In the initial demonstration, the voltage signal of the SNSPD was amplified and the rise time (rising edge slope or slew rate) of the signal was converted to an amplitude with an analog inductor-resistor differentiating circuit [101]. Faster rise times lead to larger signal amplitudes, which were attributed to higher photon numbers (see Fig. 1.1(b)). The resulting histogram of the peak height of the differentiated waveform was described by a sum of Gaussian distributions, which were attributed to photon-number contributions of the underlying Poissonian statistics (in their case up to four photons). They identified the main limitations of PNR to be variations in the hotspot resistance and the signal-to-noise ratio. Additionally, they note that multi-photon events cannot be detected unless the temporal optical pulse width is shorter than the stagnation time of the hotspot [101] (also referred to as the hotspot relaxation time, i.e., the time until superconductivity is locally recovered [102]).

Follow-up work by Nicolich et al. [91] presented a generalized electro-thermal model for the turn-on dynamics of SNSPDs based on Ref. [103]. The model connected the hotspot growth with features of the rising edge of the electrical readout pulse and showed a $1/\sqrt{n}$ scaling of the rising-edge rise time with photon number. This scaling shows a potential limitation of PNR with SNSPDs, as the differentiation between rise times becomes less pronounced for larger photon numbers. Additionally, the signal cannot rise arbitrarily fast, presenting an upper limit to the number of resolvable photons.

Zhu et al. [104] presented another approach to PNR with SNSPDs; they called their device a superconducting tapered nanowire detector (STaND). Their detector was combined with an integrated impedance-matching taper, which made the amplitude of the output signal of the SNSPD (see

¹The following four paragraphs build upon work originally published in Timon Schapeler, Fabian Schlue, Michael Stefszky, Benjamin Brecht, Christine Silberhorn, and Tim J. Bartley, Optimizing photon-number resolution with superconducting nanowire multi-photon detectors, *Proc. SPIE* **13448**, Advanced Photon Counting Techniques XIX, 134480C (2025), <https://doi.org/10.1117/12.3054905>. The original text has been modified to fit the narrative of this thesis.

Fig. 1.1(b)) sensitive to the number of hotspots induced by absorbed photons; this was already predicted in 2007 by Bell et al. [105]. The photon-number contributions in the pulse-height histograms were much more pronounced compared to Ref. [101], leading to less overlap and thus better distinguishability between photon numbers up to four. Here it was again noted that impinging photons need to overlap in time (up to tens of picoseconds), as for larger optical pulse widths they observed an underestimation of the counting statistics.

Instead of using the photon-number information encoded in the amplitude of the output signal, Endo et al. [39] used an oscilloscope to record reference waveforms corresponding to different photon numbers, which could then be used to distinguish different events by waveform pattern matching (see Fig. 1.1(c)). The authors used a subset of the rising edge, which provided more resilience against noise. This was especially interesting for higher photon numbers, where the overlap between waveforms increased. This technique was able to improve PNR, distinguishing up to five photons at a wavelength of approximately 770 nm. A similar approach was presented by Sempere-Llagostera et al. [106], where the authors use a commercial SNSPD and fit a linear function to part of the rising edge of recorded oscilloscope traces (see Fig. 1.1(d)); the extracted slopes were photon-number dependent. The disadvantage of these techniques is that new reference waveforms need to be acquired when the operating parameters of the SNSPD are modified, especially for a change in bias current, as this changes the signal amplitude and slew rate. Additionally, recording waveforms generates larger amounts of data, potentially limiting real-time applications.

An alternative promising method to extract photon-number information used arrival-time differences from single- and multi-photon events relative to a trigger signal (see Fig. 1.1(e)). This method, demonstrated by Davis et al. [107], used a limited number of data points of the detector's output signal and thus enabled real-time readout, and filtering of multi-photon events. Sauer et al. [108] refined the retrieval of photon-number information based on arrival-time differences to a trigger signal, by utilizing both the rising and falling edge of SNSPD traces (see Fig. 1.1(e)). Two-dimensional arrival-time histograms (differences of a trigger signal to the rising edge and falling edge on either axis) revealed that a simple projection onto the rising edge does not retrieve the full photon-number information. Instead, using the additional information from the falling edge, the two-dimensional histogram could be projected on an angle, which optimized the separation of photon-number events.

We have seen that the initial demonstration of PNR SNSPDs [101] has since been refined in various works using different techniques to access photon-number information [39, 61, 91, 104, 106–116]. The central question is therefore where this information is located in the SNSPD electrical output trace and how it can be extracted most efficiently. A common theme of many PNR techniques is the utilization of timing information from the rising edge of the SNSPD output signal, as this encodes the dynamics of hotspot formation from multiple photons. Modeling the photon-number dependence of the SNSPD traces, especially the photon-number-dependent timing jitter, is crucial for future detector engineering. A physical understanding makes it possible to assess how parameters such as optical pulse duration, detector material, device geometry, and readout bandwidth influence the achievable photon-number resolution [111, 113, 114], and can provide guidance for application-specific detector engineering. The desired PNR performance depends on the intended application, and therefore there may not be a single detector design that is optimal for all tasks. Detectors for heralding single photons from spontaneous parametric down-conversion sources should be optimized to achieve perfect distinguishability between single-photon events and multi-photon events [106], whereas photon-counting applications benefit from resolving larger numbers of photons. This motivates the second aim of this thesis, which is to *understand*

intrinsic PNR with SNSPDs. This entails the systematic investigation of electrical traces to identify photon-number-dependent features, determining how the relevant information can be extracted using as few data points as possible, modeling the photon-number-dependent traces of SNSPDs and thoroughly investigating the assignment of photon numbers using SNSPDs.

Now that we have established SNSPDs as a promising detector technology, because of their impressive figures of merit in combination with intrinsic PNR, the next question is how to develop them into a HDR photon-counting detector system. As mentioned above, SNSPDs can already be constructed into large arrays with up to 400000 pixels [8]. These imaging arrays typically use elaborate readout schemes, such as the thermally-coupled row-column readout [117]. These schemes reduce the number of readout lines, which is an important aspect for scalability, because of the limited heat load compatible with cryogenic operation of current refrigeration systems [117]. However, this comes with the disadvantage of ambiguous detection patterns; this means that if a certain combination of pixels registered a photon, detection events from other pixels are masked, i.e., not every pixel contributes to the read-out signal. In the best case, a photon-counting detector would require access to all pixels of a multiplexed array as well as intrinsic PNR for every pixel, to not hide any detection events, due to threshold detection or limitations in the readout scheme. Therefore, besides the development of intrinsic PNR from SNSPDs, multiplexing has an important scaling motivation for HDR photon counting. Detector multiplexing [52–54] and intrinsic PNR are very much complementary, as a demonstration by Kong et al. [61] has shown. This can increase the dynamic range by distributing photons over many multiplexing bins, while retaining photon-number information of each bin. A HDR photon-counting detector can bridge the gap between single-photon-level measurements and bright light. The final goal of this thesis is therefore to *bring these ingredients together and investigate scalable photon counting far beyond the conventional single-photon regime.*

Thesis Overview

This is a cumulative thesis, which is based on a series of my publications addressing the questions and aims identified in the introduction. Each chapter begins with a short motivation of the chapter-specific aims. This is followed by short sections for each publication, in which I summarize the relevant state of the art, the central result of the work, and its contribution to the aims of this thesis. The corresponding publications are then reprinted in full.

Chapter 2 addresses the scalability of quantum detector tomography. A high-dynamic-range time-multiplexed SNSPD is used to demonstrate high-dimensional detector tomography and to show that reconstructed POVMs are reliable and can be used to infer the mean photon number of bright optical states in Section 2.1. Afterwards, in Section 2.2, the computational bottleneck of large-scale reconstructions is overcome by an high-performance-computing-based reconstruction method capable of handling experimentally relevant POVMs in Hilbert spaces containing millions of photons. Section 2.3 discusses how reconstructed POVMs can be used to quantify the information that can be extracted about the photon number by various multiplexing architectures.

The rest of the thesis follows the progression from intrinsic PNR of an individual SNSPD to scalable photon counting. Chapter 3 investigates photon-number resolution with SNSPDs. The chapter follows the natural progression from locating where the photon-number information is encoded on the electrical output signal of an SNSPD in Section 3.1 and then continues with modeling the photon-number-dependent jitter in SNSPDs in Section 3.2, serving as a physical basis for the understanding of arrival-time-based PNR. The chapter finishes with a thorough investigation and

characterization quantifying the intrinsic PNR capability of SNSPDs, including photon-number misidentification and POVM reconstruction in Section 3.3. The chapter finishes with Section 3.4, a continuation of the intrinsic PNR SNSPD methods, completing the discussion about possible ways to retrieve photon-number information using intrinsic PNR SNSPDs.

The penultimate Chapter 4 unites the two main themes of the thesis. Intrinsic photon-number resolution is combined with large-scale temporal and spatial multiplexing to realize scalable photon counting to thousands of photons. In this architecture, multiplexing extends the dynamic range, while intrinsic PNR increases the information obtained from each detection bin. The detector is characterized using the scalable tomography method developed in Chapter 2. Therefore, the motivation of this thesis is to turn SNSPDs from highly efficient threshold detectors into scalable photon-number-resolving measurement devices for quantum optics and photonic quantum technologies and to enable a scalable tool to characterize these large-scale detectors. Finally, Chapter 5 concludes the thesis with an overview of the main results and discusses future directions for scalable photon-number-resolving detection and quantum detector tomography.

Scalable quantum detector tomography for high-dynamic-range photon counting

2

Detectors form the interface between quantum optical experiments and the recorded measurement outcomes. Therefore, it is essential that these devices are characterized thoroughly and consistently. For threshold detectors, figures of merit such as efficiency and dark-count probability provide a useful description about the device performance. However, these quantities do not fully characterize devices with multiple outcomes, specifically detectors with photon-number sensitivity. For these devices it is important to find the probabilities of certain detector outcomes in response to different input states. This reveals how the detector responds to one, two or three photons and whether there is significant overlap between measurement outcomes. This information is essential when the detector is used to infer photon-number statistics, reconstruct unknown optical states or to compare multiplexing architectures.

Quantum detector tomography (QDT) is a tool that achieves this characterization, as it reconstructs the positive operator-valued measures (POVMs) of the measurement device, providing a quantum-mechanically consistent description [30, 31]. To find the POVMs, described by the set of operators $\{\pi_n\}$, measurements with a set of known input probe states need to be performed. Typically a set of coherent states is used, which can be easily produced with a laser and expanded in the photon-number basis [34]. Many repetitions of the experiment probe the probability of the possible detection outcomes occurring. It is important that the entire dynamic range of the device is spanned with these input probe states to yield a complete description.

For phase-insensitive detectors, which applies to all detectors in this thesis, it is possible to simplify QDT substantially. These devices only register the number of incident photons, and no phase information. Therefore, all off-diagonal elements of the POVMs can be assumed to be zero, and the detector POVM can be reduced to a matrix Π containing the diagonal elements of the set of POVMs. Given the input states are known, the measurement outcomes of the device can be used to infer the POVMs of the device. This is done using the Born rule, which can be cast as a matrix equation $P = F\Pi$ [34]. These matrices contain the photon-number statistics of the D probe states $F_{D \times M}$, the measurement outcomes of the device $P_{D \times N}$, as well as the unknown POVMs $\Pi_{M \times N}$. To describe a physical device, the POVMs need to be normalized positive semi-definite operators, which is ensured by constraints during the reconstruction process.

Quantum detector tomography is established for small-scale phase-insensitive detectors with few outcomes N and small Hilbert space dimensions M . For these devices, the matrices involved in the reconstruction have only on the order of 10^3 elements. However, as detectors become increasingly large, these matrices will have $> 10^8$ elements, which corresponds to gigabytes of memory requirements for storage alone. Therefore, QDT itself must become scalable to remain useful for future quantum photonic technologies. This motivates the use of high-dynamic-range (HDR) detectors as a testbed for developing and validating reconstruction methods capable of

characterizing large detector systems. HDR detectors provide an ideal platform for this purpose, as their sensitivity to optical fields ranging from single photons to bright light [118] allows them to span a large Hilbert space in the photon-number basis. Characterizing such detectors naturally probes the potential limits and scaling behavior of QDT, especially in the context of memory requirements, runtime, and numerical stability.

The aim of this chapter is to *enable the characterization of high-dynamic-range photon-counting detectors through a scalable QDT approach*. This extends the applicability of QDT from small photon-counting devices to detectors with large numbers of outcomes and Hilbert spaces containing hundreds, thousands or even millions of photons. To achieve this goal, the chapter first establishes the experimental testbed for large-scale detector characterization, a HDR time-multiplexed detector, and then investigates whether QDT is generally scalable (see Section 2.1 or Publication 2.1). Afterwards, the chapter continues with the development of a high-performance-computing-based reconstruction method, to enable large-scale detector characterization (see Section 2.2 or Publication 2.2). Finally, the chapter presents how POVMs can be used to give insights into the photon-number-resolving (PNR) capability of detectors and compare different multiplexing architectures (see Section 2.3 or Publication 2.3).

2.1 Establishing the experimental testbed

Previously, QDT has been used for detectors with comparatively few outcomes $N \lesssim 10$ and Hilbert space dimensions $M \lesssim 100$ [30, 31, 35, 37–44]. This raises the questions whether QDT is generally scalable to characterize large-scale detectors. Publication 2.1 demonstrates the utilization of a HDR time-multiplexed SNSPD and a numerical reconstruction approach based on a convex optimization solver. This extends QDT to Hilbert space dimensions $M > 5000$, while maintaining numerical stability. This is validated by an extrapolation of the POVMs and inferring the mean photon number of 70000 of a bright coherent state [119]. In the context of this thesis, this establishes the experimental testbed for scalable QDT and identifies a computational bottleneck addressed later in Section 2.2.

Publication 2.1 is reprinted from Timon Schapeler, Jan Philipp Höpker, and Tim J. Bartley, Quantum detector tomography of a high dynamic-range superconducting nanowire single-photon detector, *Superconductor Science and Technology* **34**, 064002 (2021), <https://doi.org/10.1088/1361-6668/abee9a>. This article is licensed under a Creative Commons Attribution 4.0 (CC BY 4.0) license: <https://creativecommons.org/licenses/by/4.0/>. No changes were made.

2.2 Reaching scalability of large-scale detector characterization

High-performance computing (HPC) is well established in modern computational science and is routinely used for problems whose runtime or memory requirements exceed the capabilities of conventional computers. It therefore provides a natural route to address the computational bottleneck observed in Publication 2.1 and enable a scalable characterization method for large-scale detectors. In this approach, the reconstruction algorithm must be designed such that the benefits of parallelization can be used without losing the constraints required for a valid quantum-mechanical description of the detector. This motivates reconstruction methods optimized for HPC hardware and for the memory requirements of large-scale POVM reconstructions that can reach

up to gigabytes or even terabytes [120]. The implementation of such an HPC-based reconstruction method is discussed in Publication 2.2. Within this thesis, this completes the development from high-dimensional detector tomography to scalable detector characterization and provides a tomographic framework needed for HDR photon-counting detectors.

Publication 2.2 is reprinted from Timon Schapeler, Robert Schade, Michael Lass, Christian Plessl, and Tim J. Bartley, Scalable quantum detector tomography by high-performance computing, *Quantum Science and Technology* **10**, 015018 (2024), <https://doi.org/10.1088/2058-9565/ad8511>. This article is licensed under a Creative Commons Attribution 4.0 (CC BY 4.0) license: <https://creativecommons.org/licenses/by/4.0/>. No changes were made.

2.3 Interpreting positive operator-valued measures

As mentioned in the introduction, for phase-insensitive detectors (i.e., most single-photon-detecting schemes) the POVMs can be represented as a matrix containing the conditional probabilities of detector outcome n' occurring given n incident photons. These probabilities form the basis for common figures of merit such as efficiency, dark-count rate and cross-talk between outcomes [35, 121]. Which additional information can POVMs provide, for example about the photon-number-resolving capability of a detector? Publication 2.3 uses reconstructed POVMs to compare multiplexing architectures through outcome purity [121], which is a measure of how many orthogonal input states contribute to a given detector outcome. The concepts of purity and information extraction show that the HDR time-multiplexed detector introduced as the experimental testbed in Section 2.1 can probe large Hilbert spaces, however, it lacks outcome purity and thus extracts less photon-number information [122]. This indicates the need for detectors that can resolve the number of photons, which motivates Chapter 3.

Publication 2.3 is reprinted from Timon Schapeler and Tim J. Bartley, Information extraction in photon-counting experiments, *Physical Review A* **106**, 013701 (2022), <https://doi.org/10.1103/PhysRevA.106.013701>. This article is licensed under a Creative Commons Attribution 4.0 International (CC BY 4.0) license: <https://creativecommons.org/licenses/by/4.0/>. No changes were made.

Quantum detector tomography of a high dynamic-range superconducting nanowire single-photon detector

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Abstract

We demonstrate and verify quantum detector tomography of a superconducting nanowire single-photon detector (SNSPD) in a multiplexing scheme which permits measurement of up to 71 000 photons per input pulse. We reconstruct the positive operator valued measure (POVM) of this device in the low photon-number regime, and use the extracted parameters to show the POVMs spanning the whole dynamic range of the device. We verify this by finding the mean photon number of a bright state. Our work shows that a reliable quantum description of large-scale SNSPD devices is possible, and should be applicable to other multiplexing configurations.

Keywords: SNSPD, multiplexed detection, tomography

(Some figures may appear in colour only in the online journal)

1. Introduction

Since their invention at the turn of the century [1], superconducting nanowire single-photon detectors (SNSPDs) have become ubiquitous in quantum optics experiments, due to their low noise, high speed and high efficiency [2–6]. Although recent work has shown that some photon number resolution can be recovered directly from the output of the detector by modifying the bias circuitry [7–10], in standard operation, however, these detectors do not resolve the number of photons incident. To overcome this, SNSPDs can be multiplexed in arrays, either spatial [11–24] or temporal [25–27], such that the number of pixels in the array that fire gives some information about the number of photons that were incident.

The exact nature of the response of the device to different numbers of incident photons can be formally reconstructed using quantum detector tomography [28]. This technique provides a fully quantum mechanical description of the operation of the device, without relying on any underlying physical model of its detection mechanism. This allows one to state the probability that a specific number of photons were incident, given a particular detector outcome, known as a positive operator valued measure (POVM). Quantum detector tomography has been carried out for single-pixel SNSPD devices [29, 30], as well as temporally- [25] and spatially- [31] multiplexed SNSPDs.

At its heart, quantum detector tomography relies on a matrix inversion algorithm. The size of the matrices depends on the number of independent outcomes of the detector, as well as the dimension of the Hilbert space in which the quantum states reside. As the size of a multiplexed detector grows, not only does it exhibit more outcomes, but the Hilbert space to which it is sensitive also increases. This soon leads to a situation where the matrix inversion becomes computationally very expensive. Therefore, one of the challenges with quantum detector tomography is to develop techniques to handle large

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detectors, which are becoming increasingly experimentally available [18, 19, 24].

In this paper, we demonstrate tomographic reconstruction of a multiplexed detector with a Hilbert space up to 10^5 . Building on [31], we carry out the full reconstruction of up to 11 outcomes of a multiplexed detector. This result is used to verify a model of how the POVMs behave at much higher input photon numbers, which mitigates the effect of limited computational power when reconstructing the large matrices which result from such high-dimensional reconstructions. To do so, we make use of the logarithmic time-multiplexed detector architecture, recently demonstrated with a dynamic range over 120 dB [26]. We then demonstrate the utility of this technique by recovering the mean photon number of a bright coherent state.

This paper is organized as follows. We begin with a brief description of the detector under test and its applicability to recovering photon number information from bright optical states based on ensemble measurements. We then discuss detector tomography and the challenges associated with scaling to large numbers of outcomes and Hilbert space dimensions, before showing how extrapolating POVMs can be used to overcome this challenge. This is followed by the experimental methods and results, and then a verification of this approach using a bright coherent state.

2. Achieving high dynamic range with a superconducting single-photon detector

The detector upon which we will perform tomography is based on the multiplexing architecture presented in [26], and shown in figure 1. High dynamic range is achieved by splitting an input pulse into a series of output pulses with decaying intensity. The rate of intensity reduction is precisely controlled by the beam splitter and the losses inside the loop. However, these apply identically to each pulse, therefore the intensity reduction follows an exponential decay precisely [26]. The train of output pulses is incident on a single SNSPD, which is sensitive to each pulse if they are separated by a time greater than the recovery time of the detector, in our case the separation is $\tau = 156$ ns. In this picture, the SNSPD measures the presence or absence of photons within a time-bin. We used a standard commercial SNSPD, with nominal efficiency of 85%, jitter < 100 ps and a recovery time < 100 ns.

For the purposes of detector tomography, we define outcomes n of this detector as the number of time-bins occupied. The POVM matrix elements $\theta_i^{(n)}$ corresponding to these outcomes are thus given by the probability $p(n|i)$ of a particular outcome n , given a pulse containing i photons.

3. Experimental methods and results

Detector tomography is performed by subjecting a detector under test to a series of input states spanning the Hilbert space of the detector. In the case of our detector architecture, the detector cannot be saturated, since it is possible in principle for infinite time-bins to be occupied. In practice of course this

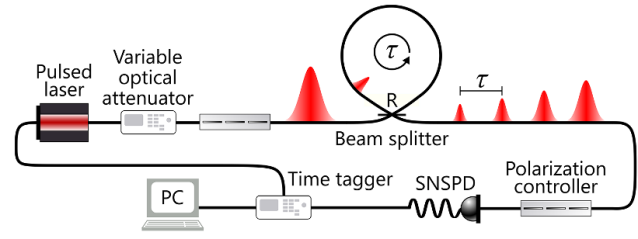


Figure 1. Schematic depiction of the experimental setup and multiplexing architecture. The multiplexing is based on a beam splitter with adaptable out-coupling R , loop-efficiency η_{loop} and loop-length τ . A 1556 nm pulsed laser generates coherent states at a repetition rate of 15 kHz, which can be attenuated by a computer-controlled variable optical attenuator. The first polarization controller sets the polarization for the beam splitter loop, as the components are polarization dependent. The same applies to the SNSPD, therefore, we use a second polarization controller in front of the detector. The electronic response of the SNSPD is measured by a time-tagger.

is not possible, since pulses would begin to pile up, and the detector may latch. For this reason, we truncated the device after $N_{\text{bin}} = 10$ time-bins. Nevertheless, the size of the Hilbert space is still very large, of the order $> 10^3$ photons.

In the experiment we subjected the detector under test (figure 1) to a total of $D = 71$ coherent states with quadratically increasing mean photon numbers $|\alpha_d|^2 \approx d^2$, with $d \in [0, 70]$. The coherent states are generated by a 1556 nm laser ($g^{(2)}(0) = 1.00006(17)$), which emits 9 ps pulses at a repetition rate of 15 kHz. The input states can be expressed in a finite matrix $\mathbf{F}$ containing Poisson distributions, which is truncated at a maximum dimension of $M = 5328$ in the photon number basis to include probability amplitudes at six standard deviation greater than the largest coherent state. For more details on the preparation and characterization of the input states see [31].

The train of pulses, created by the beam splitter, is detected by an SNSPD (figure 1), whose output pulses are measured by a time-tagger in a histogram containing 1.5×10^5 bins with a bin-width of 10 ps per bin. We collect data in an ensemble measurement with a measurement time of 30 s per coherent state. In post processing the raw time-tagger bins are integrated to find the total number of counts in each of the ten detector time-bins with a width of 2 ns. This allows to filter out dark counts or reflections and leads to a dark-count probability of 3×10^{-8} per 2 ns time bin, which is negligibly small.

3.1. Defining outcomes

The quotient of the number of counts per detector time-bin and the total number of generated laser pulses in the measurement time, describes the probabilities p_j of a specific bin $j \in [1, 10]$ firing. However, as mentioned in section 2, we consider an outcome n of this device as the number of time-bins occupied, which leads to $N = 11$ possible outcomes. Therefore, the probabilities of specific bins firing need to be transformed into probabilities p'_n of $n \in [0, 10]$ time-bins being occupied. This can be done using the Poisson binomial distribution, which

describes a discrete probability distribution of a sum of independent Bernoulli trials, where, in general, the success probability of each trial is not identical. The probability of having n time-bins occupied out of a total of N_{bin} can be written as the sum [32]

$$f(n) = \sum_{A \in F_n} \prod_{j \in A} p_j \prod_{k \in A^c} (1 - p_k), \quad (1)$$

where F_n is the set of all subsets of n bins that can be selected from $\{1, 2, \dots, N_{\text{bin}}\}$ and A^c is the complement of A . As an example, the calculation of the probability of one time-bin being occupied p'_1 , uses the set $F_1 = \{\{1\}, \{2\}, \dots, \{10\}\}$. For large numbers of time-bins, this expression is expensive to calculate, as it scales with the binomial coefficient $\binom{N_{\text{bin}}}{k}$. However, a closed-form expression for the Poisson binomial distribution is given by [33]

$$f(n)|_d = \frac{1}{N_{\text{bin}} + 1} \sum_{l=0}^{N_{\text{bin}}} C^{-ln} \prod_{j=1}^{N_{\text{bin}}} (1 + [C^l - 1] p_j|_d), \quad (2)$$

where

$$C = \exp\left(\frac{2i\pi}{N_{\text{bin}} + 1}\right), \quad (3)$$

and $p_j|_d$ are the probabilities of a particular bin j firing, given a coherent input state d . With this the probabilities of n time-bins being occupied are $p'_n|_d = f(n)|_d$. Note that if all bins have an equal probability, i.e. $p_j = p \forall j$, then this distribution reduces to the binomial distribution. Finally, the outcome matrix $\mathbf{P}$ can be expressed as $P_{d,n} = p'_n|_d$.

3.2. Matrix inversion

Given the input state matrix $\mathbf{F}$ and outcome matrix $\mathbf{P}$, a matrix $\mathbf{\Pi}$ corresponding to the set of POVMs $\{\pi_n\}$ can be found by solving the optimization problem [28]

$$\min \{ \|\mathbf{P} - \mathbf{F}\mathbf{\Pi}\|_2 + g(\mathbf{\Pi}) \}, \quad (4)$$

where $\|\cdot\|_2$ indicates the Frobenius norm [34] and the function

$$g(\mathbf{\Pi}) = \epsilon \sum_{i,n} \left(\theta_i^{(n)} - \theta_{i+1}^{(n)} \right)^2, \quad (5)$$

scaled by a factor ϵ , ensures that the resulting distributions are smooth [35]. For more details on the reconstruction process and importance of the correct choice of the smoothing parameter ϵ can be found in [31]. The reconstructed POVM elements for all eleven outcomes are shown in figure 2. The errors are calculated based on assuming 5% uncertainty in the amplitudes of the coherent states, which stems from the uncertainty in the calibration procedure.

Using the analysis from [31] we can use the reconstructed POVMs to directly find figures of merit of the device. From this we find an overall device efficiency $\eta = 0.44 \pm 0.03$,

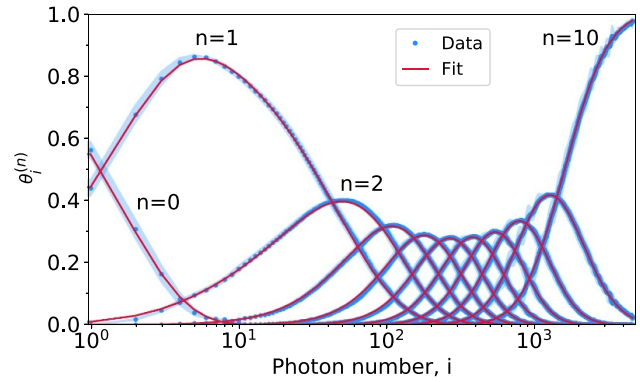


Figure 2. Diagonal elements of the experimentally reconstructed POVM operators in the photon-number basis for all eleven outcomes (blue dots). With increasing detector outcomes n , the distributions shift to higher photon numbers i . The error region is based on assuming 5% uncertainty in the amplitudes of the coherent states. The best fit (red line) using equation (2) (compare section 3.3).

dark-count probability $p_{\text{dark}} = (0.0 + 4.1) \times 10^{-7}$ and cross-talk probability $p_{\text{xtalk}} = (4.6 \pm 2.8) \times 10^{-6}$.

The computational effort for the reconstruction depends on the size of the POVM matrix $\mathbf{\Pi}$, i.e. the total number of outcomes N and the Hilbert space dimension M . Our current tomographic reconstruction method was able to reconstruct the POVM elements of the ten time-bin detector, however, due to limited computational power (3.6 GHz CPU with six cores and 16 GB RAM) this seems to be the limit of this implementation. Analysis of our code indicates that it is the smoothing function which consumes the most computation time and memory, and optimizing this routine would be the starting point for improving scalability.

3.3. Extrapolation

From the experimental results, it is clear that our computational power limits us to reconstructing up to eleven outcomes reliably. However, to reconstruct higher-order POVMs, a different approach is required. The crux of this approach is to use parameters calculated in the low outcome regime and extrapolate this to higher order. This deviates a little in spirit from detector tomography, since it requires some modelling of the device. However, this technique can be used to test different models of the device, since it provides a bridge between the top-down (model-free) and bottom-up (first-principles) approaches to characterizing a detector.

The first assumption about our device is that each bin has a binary result: either ‘occupied’ or ‘not occupied.’ Outcomes of the detector correspond to the total number of occupied time-bins. As mentioned above, this can be considered as the result of a series of independent Bernoulli trials, where, in general, each trial can have a different success probability. The general form of the probability of n out of N_{bin} time-bins being occupied, given an input Fock state $|i\rangle$, is given by equation (2).

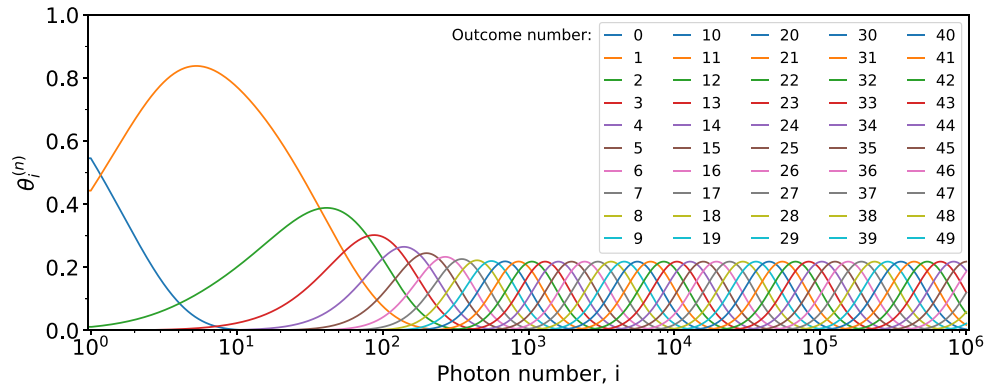


Figure 3. Extrapolated POVMs using equations (2) and (6), shown up to $N = 50$ outcomes, corresponding to a Hilbert space dimension of 10^6 .

Using this formalism, an alternative approach to reconstructing the operation of the detector is to find the probabilities $p_j(i)$ for each bin to be occupied, given the input Fock states $|i\rangle$ for $i \in [0, M]$, where M is the Hilbert space dimension of the experimentally measured coherent input states. Note, however, this assumes that each bin is independent—this approach cannot account for conditional probabilities based on other bins firing, thus it cannot resolve effects like cross-talk.

In fact, for the loop detector considered in this paper, the probabilities $p_j(i)$ can be estimated by modelling the detector. From [26] and neglecting dark counts (of the order 10^{-8} per time-bin), these probabilities are

$$p_j(i) = \begin{cases} 1 - (1 - R\eta_{\text{det}})^i & j = 1 \\ 1 - \left[1 - \frac{(1-R)^2\eta_{\text{det}}}{R} (R\eta_{\text{loop}})^{j-1}\right]^i & j \geq 2 \end{cases}, \quad (6)$$

which have been adjusted to include the detection efficiency η_{det} . For the specific number of time-bins N_{bin} of the multiplexed detector, equation (2) depends on three parameters: the detection efficiency η_{det} , the out-coupling R and the loop-efficiency η_{loop} . The optimal set of parameters can be found by fitting the Poisson binomial approximation, for all outcomes $n \in [0, 10]$ simultaneously, to the experimental data. The fit is based on a minimization of the Frobenius norm between the experimental, and fit matrix and is shown in figure 2 together with the experimentally reconstructed POVMs. The fit revealed the parameters $\eta_{\text{det}} = 0.4912 \pm (9 \times 10^{-4})$, $R = 0.89613 \pm (8 \times 10^{-5})$ and $\eta_{\text{loop}} = 0.9064 \pm (2 \times 10^{-4})$.

Using these parameters and combining equations (2) and (6), we can then extrapolate the POVMs out to higher order. In figure 3 we show POVMs up to 50 outcomes, corresponding to a Hilbert space dimension of 10^6 .

3.4. Verification using extrapolated POVMs

To show that the reconstructed POVMs of this device from the low photon-number and low outcome regime can be used to predict the behavior of the device for higher photon-numbers

and outcomes, we use the model from equation (2) to extrapolate up to $N = 120$ outcomes, which will be expressed in the matrix Π^{ext} . We verify this by finding the mean photon number of a bright state.

In this experiment we subject the detector to one coherent input state generated from the same laser as in section 3 with a repetition rate of 50 kHz. The higher repetition rate enables a measurement of the mean photon number per pulse with a power meter. The train of pulses is again detected by the SNSPD (figure 1) and the output pulses are measured by a time-tagger in a histogram. We allow a total of $N_{\text{bin}} = 119$ time-bins, by choosing the histogram to consist of 1.87×10^5 bins with a bin width of 100 ps. The measurement time is 300 s. By integrating over the raw bins we are able to find the probabilities of a specific bin $j \in [1, 119]$ firing, which will be transformed into the probabilities p'_n of $n \in [0, 119]$ time-bins being occupied, using equation (2). These probabilities then constitute the outcome matrix $\mathbf{P}$.

Subsequently, we found the mean photon number of the bright state, by finding the coherent state which best fit the measurement data $\mathbf{P}$ and extrapolated POVMs Π^{ext} by minimizing the expression

$$\|\mathbf{P} - \mathbf{F}^{\text{fit}}\Pi^{\text{ext}}\|_2. \quad (7)$$

The resulting mean photon number of $\bar{n}^{\text{fit}} = 71\,000 \pm 3000$ per pulse agrees with a separate measurement with a power meter of (440 ± 22) pW, which results in a mean photon number of $\bar{n}^{\text{pm}} = 69\,000 \pm 4000$ per pulse.

4. Conclusion

Characterizing the response of single-photon detectors is an important task in quantum optics. Configuring these detectors to be sensitive to bright states will become increasingly important as ever brighter photonic states become available [36]. Furthermore, having a self-consistent characterization of these detectors is important wherever they are utilized. We show that a tomographic reconstruction is possible even if

the dynamic range of such a detector is extremely large. This is enabled by configuring a single-photon detector, in this case an SNSPD, to respond logarithmically to the number of photons. Nevertheless, the resulting photon number can be determined with very high accuracy. While we have demonstrated this method for a time-multiplexed system, we expect a similar approach to be applicable to spatially multiplexed arrays of superconducting detectors [18–20, 22, 24].

Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.

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Scalable quantum detector tomography by high-performance computing

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Abstract

At large scales, quantum systems may become advantageous over their classical counterparts at performing certain tasks. Developing tools to analyze these systems at the relevant scales, in a manner consistent with quantum mechanics, is therefore critical to benchmarking performance and characterizing their operation. While classical computational approaches cannot perform like-for-like computations of quantum systems beyond a certain scale, classical high-performance computing (HPC) may nevertheless be useful for precisely these characterization and certification tasks. By developing open-source customized algorithms using HPC, we perform quantum tomography on a megascale quantum photonic detector covering a Hilbert space of 10^6 . This requires finding 10^8 elements of the matrix corresponding to the positive operator valued measure, the quantum description of the detector, and is achieved in minutes of computation time. Moreover, by exploiting the structure of the problem, we achieve highly efficient parallel scaling, paving the way for quantum objects up to a system size of 10^{12} elements to be reconstructed using this method. In general, this shows that a consistent quantum mechanical description of quantum phenomena is applicable at everyday scales. More concretely, this enables the reconstruction of large-scale quantum sources, processes and detectors used in computation and sampling tasks, which may be necessary to prove their nonclassical character or quantum computational advantage.

1. Introduction

Photonic quantum computing paradigms are built around large scale generation, manipulation and measurement of quantum light. At sufficient scale, the computations these devices perform cannot be verified by conventional computing. Pertinent examples are quantum simulators [1] and Boson sampling [2], where in the latter beyond a certain system size, the process of sampling from the output distribution of a nonclassical input state in a photonic circuit implementing a random unitary matrix is a task which is computationally easier for a photonic quantum processor. Nevertheless, the same photonic processor, under illumination from ‘classical’ light, is computationally ‘easy’ to compute using classical approaches. Since this classical light is tomographically complete, one can use techniques such as quantum tomography to characterize the device and verify its underlying quantum mechanical structure, without performing the full Boson sampling task [3]. However, just because the computational complexity class suggests that the problem is ‘easy’ for a classical computer, still the question arises what the practical limits of this approach are. To that end, high-performance computing (HPC) is a very well established field which has great potential to assist in these quantum tomography tasks, provided the benefits of parallelization can be reconciled with the constraints imposed by the quantum mechanical objects to be reconstructed.

Quantum detector tomography [4–9] is a well-established technique for providing a consistent quantum mechanical characterization of the measurement process. This approach to characterizing experiments is particularly attractive, since it provides a model-free method to connect the underlying quantum mechanics of systems to the measurement results we observe. The aim of a tomography experiment is to reconstruct the set of positive operator valued measures (POVMs) $\{\pi_n\}$ by mapping the detector response to a tomographically complete set of input states, i.e., the set of input states that span the full outcome space of the detector. The size of the problem is governed by the dimensionality of the Hilbert space M occupied by the set of input states in the photon-number (Fock) basis and the number of detector outcomes N . In order to be tomographically complete, the Hilbert space spanned by the input states is necessarily at least as large as the outcome space, i.e. $M \gtrsim N$. In general, the size of the set of POVMs $\{\pi_n\}$ is then $M^2 \cdot N$; for non-phase-sensitive detectors this reduces to $M \cdot N$, as the POVM for a particular outcome n is then given by $\pi_n = \sum_{k=0}^{M-1} \theta_k^{(n)} |k\rangle\langle k|$. Nevertheless, the challenge is thus to devise techniques to reconstruct POVMs covering ever larger system sizes, to enable state of the art quantum optics experiments [3, 10].

Up to now, almost all detector tomography experiments have described detectors with few outcomes ($N \lesssim 10$) covering a relatively small Hilbert space ($M \lesssim 100$) [7, 8, 11–17], where approaches such as semi-definite programming and maximum likelihood estimation could be readily applied with standard computing hardware. Alternatively, data pattern tomography can be used to characterize a relevant subset of the detector's outcome space [18]. More recently, numerical approaches using convex optimization solvers have been pursued, to interrogate larger system sizes [19]. This approach has been applied to HPC hardware, as investigated by Liu *et al* [20] using simulated data. Their results suggested an upper limit of system size $M \cdot N$ of the order 10^5 , based on available computational resources. In the case of phase sensitive detectors, the size of the matrix required to map the Hilbert space dimension M becomes M^2 , significantly increasing the computational resource requirements. In this context, the largest tomographic reconstruction to date has been performed on a phase-sensitive photon counter, requiring the reconstruction of $1.8 \cdot 10^6$ elements [21].

In this work, we demonstrate the feasibility of a large-scale reconstruction algorithm by performing experimental detector tomography of a high dynamic range time-multiplexed detector up to $N = 50$ occupied outcomes covering a Hilbert space of $M = 1.2 \cdot 10^6$ photons, surpassing the limit suggested by Liu *et al* [20] as well as the size of the experimental reconstruction by Zhang *et al* [21], by two orders of magnitude. To do so, we developed a convex minimization solver optimized for operation on HPC hardware. We apply this solver to the simulated data from Liu *et al* (340 outcomes, Hilbert space up to $3 \cdot 10^4$ photons), and show two orders of magnitude saving in runtime, and four orders of magnitude saving in memory usage. Extending the simulated data further, we demonstrate reconstructions of system sizes $> 10^{12}$ (e.g. 10^6 outcomes, 10^6 photons). Among many other applications, this allows for a quantum characterization of state-of-the-art single-photon sensitive detector arrays [10, 22] and Boson sampling machines [3], in minutes of computation time.

2. Tomographic approach

The principle of operation of detector tomography is depicted in figure 1. Given a tomographically complete set of input states (beige), many measurements are performed, resulting in a set of outcome probabilities (green) related to the outcomes via the set of POVMs (red). Quantum mechanically, measurement outcomes $p_{\rho,n}$ on each state ρ via the POVM π_n are governed by the Born rule $p_{\rho,n} = \text{Tr}[\rho\pi_n]$. Taking all measurements together, one can recast the Born rule as the matrix equation

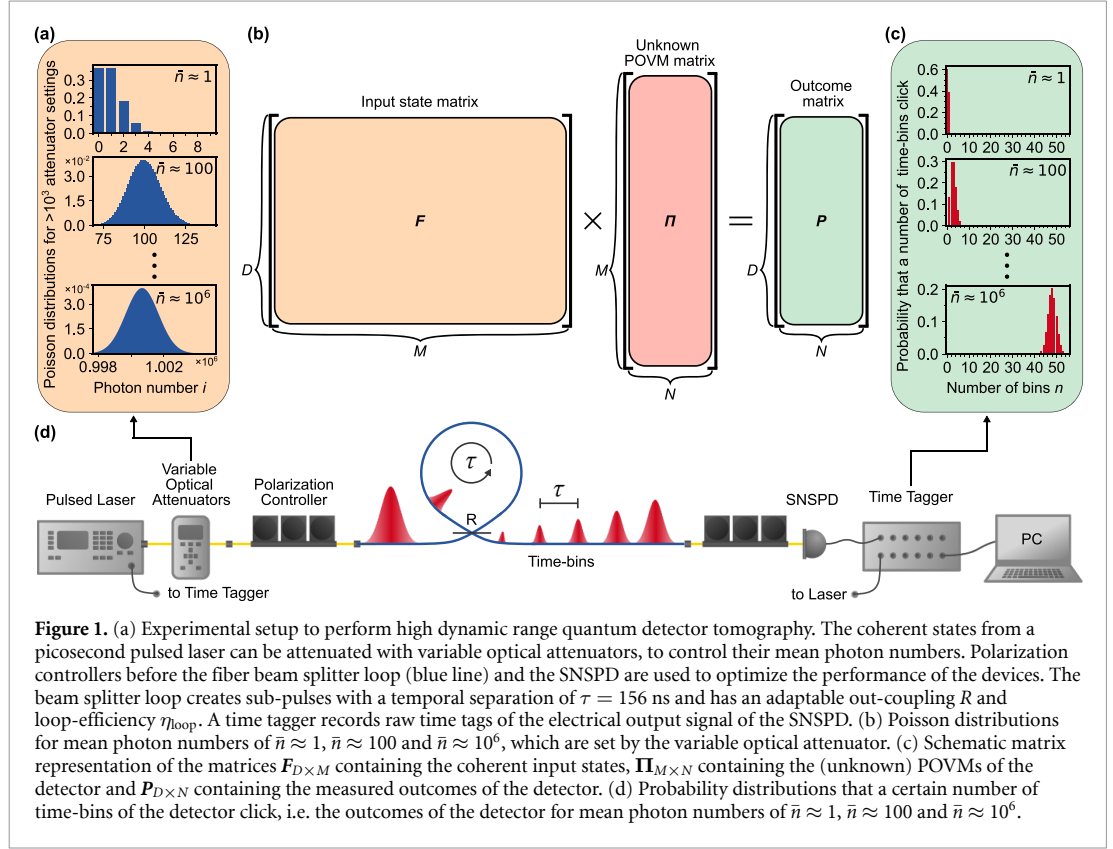
$$P = F\Pi \quad (1)$$

where the matrix of outcome probabilities $P_{D \times N}$ (green) are related to the set of input states $F_{D \times M}$ (beige) via the set of POVMs $\Pi_{M \times N}$ (red). The experimental setup used to generate and measure the set of input states is shown in figure 1, with experimental details given in section 2.2.

In principle, one can invert equation (1) to recover the unknown matrix of POVMs Π . In practice, however, the direct inversion of equation (1) often yields nonphysical results, i.e. POVMs which are not positive semi-definite and/or unit trace. This is due to the presence of unavoidable noise in the experimental outcomes. To overcome this, one can recast equation (1) into the constrained minimization problem

$$\min ||P - F\Pi||_2, \quad (2a)$$

$$\text{subject to } \pi_n \succeq 0, \sum_{n=0}^{N-1} \pi_n = I, \quad (2b)$$



which searches for the set of parameters in the POVM matrix Π which, when multiplied by the set of input states F , most closely match the measured data P . The two-norm, $\|A\|_2 = \sqrt{\sum_{ij} A_{ij}^2}$, is used and $A \succeq 0$ denotes the positive semi-definiteness constraint of the matrix A .

The solution of the minimization problem equation (2a) with the constraints equation (2b) for large POVM matrices is a numerically challenging convex optimization problem. Furthermore, the structure of the problem and the constraints render parallelization nontrivial, since an efficient communication scheme between nodes is required. The solution's computational effort and memory usage mainly depend on the number of free variables, i.e., the size of the matrix of POVMs Π . The construction of Π from π_n , and therefore the matrix dimensions, depend on whether or not the detector is sensitive to coherence present in the input states. In what follows, we consider phase insensitive detectors, the POVMs π_n of which can be expressed as diagonal matrices, with elements $(\pi_n)_{k,l}$ nonzero for $k=l$. For each π_n , these nonzero elements comprise each row of the matrix of POVMs Π , i.e., $\Pi_{i,n} = (\pi_n)_{k=i,l=i}$. In this case, the size of the matrix and the number of entries in the matrix Π is thus $M \cdot N$, where $M-1$ denotes the maximal photon number in the Hilbert space with dimension M , and N is the number of detector outcomes. Furthermore, in this case the constraints on π_n are such that the elements of Π are non-negative. The constrained minimization problem in the conventional form is thus

$$\min_{\Pi \in \mathbb{R}^{M \times N}} \|\mathbf{P} - \mathbf{F}\Pi\|_2^2 \quad (3a)$$

$$\Pi_{i,n} \geq 0 \quad \forall i \in \{0, \dots, M-1\}, \quad n \in \{0, \dots, N-1\} \quad (3b)$$

$$\sum_{n=0}^{N-1} \Pi_{i,n} = 1 \quad \forall i \in \{0, \dots, M-1\} \quad (3c)$$

for a given $\mathbf{P} \in \mathbb{R}^{D \times N}$ and given $\mathbf{F} \in \mathbb{R}^{D \times M}$. D denotes the number of probe states. The memory needed for storing Π , F and P is roughly

$$\text{mem}_{\text{storage}} = 8 \text{ byte} \cdot (MN + DM + DN). \quad (4)$$

The recent work by Liu *et al* [20] has shown the practical solution to this problem for the order of 10^5 free variables using the general minimization solver MOSEK [23] via CVXPY [24, 25]. The detector setup in their work requires M and D for a given number of outcomes N to be approximately

$$M_{\text{Liu}} \approx 6.6 \cdot N^{1.06} \quad (5)$$

$$D_{\text{Liu}} \approx 4.0 \cdot N^{1.14}. \quad (6)$$

Thus, the memory usage in the situation of Liu *et al.* for storing Π , F and P is

$$\text{mem}_{\text{storage}} \lesssim 2.6 \cdot 10^{-7} \cdot N^{2.2} \text{ GiB}. \quad (7)$$

Using our HPC hardware [26], we investigate the practical main memory usage (see appendix A for hardware and memory usage benchmark) of the solution approach of Liu *et al* [20] for the standard detector tomography (SDT) problem and the simplified modified detector tomography (MDT) problem. Fitting the measured memory usage to the model $\text{mem} = aN^b$ results in

$$\text{mem}_{\text{Liu,SDT}} \approx 1.8 \cdot 10^{-5} \cdot N^{2.95} \text{ GiB} \quad (8)$$

$$\text{mem}_{\text{Liu,MDT}} \approx 2.0 \cdot 10^{-5} \cdot N^{2.93} \text{ GiB}. \quad (9)$$

This differs slightly from the scaling approximation given in [20]; on the one hand, we track the memory usage of the solver continually, and on the other, we use a wider range of total outcomes $N = 11, 21, \dots, 201$.

Notwithstanding these slight differences, the prefactors in the memory usage scaling of the solver of Liu *et al* equations (8) and (9), and the memory needed for the storage of matrices, equation (7), are different by two orders of magnitude and the solver exhibits a scaling in N that is significantly higher than for storage. This strongly suggests that a suitable numerical solver with a much lower memory footprint can be constructed to solve this problem more efficiently and scalable.

The goal of our approach is, on the one hand, a memory usage that is only a few times larger than the memory required for storing Π , F and P and, on the other hand, the possibility not only to use multiple CPU-cores of a single compute node, but to scale to very large problems by using the distributed memory of many compute nodes. For this purpose, we propose a two-stage variant of the two-metric projected Newton approach [27–29] for the problem in equation (3). The algorithm is described in detail in appendix B.

2.1. Detector under test

We apply our solver to an experimental tomographic dataset obtained from a high dynamic range optical detector, with sensitivity from the single-photon level up to bright light [30]. The high dynamic range is achieved using a multiplexing scheme in which an incoming optical pulse is split into sub-pulses of exponentially decreasing pulse energies [31], incident on a superconducting nanowire single-photon detector (as shown schematically in figure 1(d)). The number of outcomes of the detector is governed by the number of sub-pulses which result in a measurement event. Since this detector has no intrinsic phase sensitivity, the POVMs are fully described by matrices diagonal in the number basis, given by $\pi_n = \sum_{k=0}^{M-1} \theta_k^{(n)} |k\rangle\langle k|$. The aim of the reconstruction is thus to find all $\theta_k^{(n)}$ for the different detector outcomes n , up to a maximum photon number $M - 1$, corresponding to a Hilbert space of size M .

To carry out the reconstruction, we need a set of input states which span the Hilbert space to which the detector is sensitive. For this task, it is convenient to use the set of coherent states $|\alpha\rangle = \sum_{i=0}^{\infty} e^{-\frac{|\alpha|^2}{2}} \frac{\alpha^i}{\sqrt{i!}} |i\rangle$. These states provide a photon number distribution governed by Poissonian statistics defined by their mean photon number $|\alpha|^2$. We require a set of D states of different mean photon number $|\alpha_d|^2$, such that the elements of the input state matrix may be written as

$$F_{d,i} = \langle i|\alpha_d\rangle\langle\alpha_d|i\rangle = \frac{|\alpha_d|^{2i}}{i!} e^{-|\alpha_d|^2} \quad (10)$$

for all photon numbers $i \in [0, M - 1]$. We limit ourselves to the diagonal elements of the density matrix $|\alpha\rangle\langle\alpha|$ since our detector is insensitive to phase.

While in principle a single mean photon number is sufficient to cover any Hilbert space, since the coefficients are nonzero for all photon numbers i , in practice one requires sufficient statistics for all photon numbers for a reliable reconstruction. We choose mean photon numbers to scale quadratically, i.e. $|\alpha_d|^2 \approx d^2$.

Previously, we have conducted detector tomography of 11 outcomes of this device up to a Hilbert space of size $M \approx 5 \times 10^3$ [19], limited by standard computational methods using CVXPY [24, 25]. Nevertheless, the logarithmic response of the detector means that the Hilbert space to which it is sensitive goes far beyond this limit. Constructing the analytical POVMs, enables us to test the accuracy of the computational reconstruction up to arbitrary size, which is essential to ensure correct reconstruction with our solver.

2.2. Experimental methods

The experimental setup is shown in figure 1(d). We use a picosecond pulsed laser with a wavelength of 1556 nm and a repetition rate of 25 kHz to generate the coherent probe states. We can control the mean photon number per pulse by two variable optical attenuators that are placed after the laser. In order to ensure proper operation of the beam splitter loop (which is made from polarization maintaining components) the polarization is controlled by a manual fiber polarization controller. Subsequently, the light pulses are coupled into the beam splitter loop with adaptable out-coupling R , loop-efficiency η_{loop} and temporal loop length τ . The light is partially coupled out of the loop creating sub-pulses, which are regarded as the time-bins of the time-multiplexed detector. These sub-pulses have a separation of $\tau = 156$ ns, which needs to be larger than the dead time of the SNSPD. Before the detector another polarization controller is used to optimize the polarization dependent detection efficiency of the SNSPD. We consider the beam splitter loop to be part of the overall time-multiplexed detector system, which we henceforth refer to as the detector. Given the repetition rate of the laser and the bin separation τ a maximum of 256 time-bins are allowed. We use a total of $D = 1076$ different coherent probe states, whose mean photon numbers scale quadratically, in order to efficiently span the Hilbert space (with dimension $M \approx 1.2 \cdot 10^6$) of the detector. We note that the experiment was limited by the pulse energy of the laser and not the detector itself, as in principle it is not possible to saturate this type of multiplexed detector. We record raw time tags with a time tagger for $5 \cdot 10^5$ trials of every input state.

The coherent probe state matrix $\mathbf{F}$ is constructed by expanding the coherent states $d \in [0, D-1]$ in the photon-number basis according to equation (10) up to the Hilbert space dimension $i \in [0, M-1]$.

The outcome matrix $\mathbf{P}$ of the time-multiplexed detector is populated by counting the number of occupied time-bins per trial in a 5 ns coincidence window and dividing by the total number of trials. This leads to the probabilities $P_{d,n} = p_n|_d$ for different outcomes n and input states d . We can truncate the number of time-bins at 150 (resulting in $N = 151$ possible outcomes of the time-multiplexed detector), as subsequent time-bins are dominated by dark noise only.

It is possible to derive the POVMs of this high dynamic range detector analytically, as the detector design is such that a relatively simple model of the device can be constructed [30]. In order to calculate the analytical POVMs, we first need to find the experimental parameters of the device. This can be done by fitting the bin-click probabilities, i.e. the probabilities that a certain bin j fires for a given mean photon number. For a coherent state input to the detector, the bin-click probabilities are described by

$$p_j^{\text{coh}}(d) = \begin{cases} 1 - \exp[-R\eta_{\text{det}}|\alpha_d|^2] & j = 1 \\ 1 - \exp[-(1-R)^2 R^{-1} (R\eta_{\text{loop}})^{j-1} |\alpha_d|^2 \eta_{\text{det}}] & j \geq 2 \end{cases}, \quad (11)$$

which we adapted from [30] to include the detector efficiency η_{det} . We additionally neglect the dark-count probability, which is in the order of 5×10^{-8} per time-bin in a 5 ns coincidence window. We find the experimental parameters $R = 0.91644(9)$, $\eta_{\text{loop}} = 0.90524(8)$ and $\eta_{\text{det}} = 0.528(1)$ by fitting equation (11) simultaneously for all coherent input states $|\alpha_d|^2$ with $d \in [0, D-1]$.

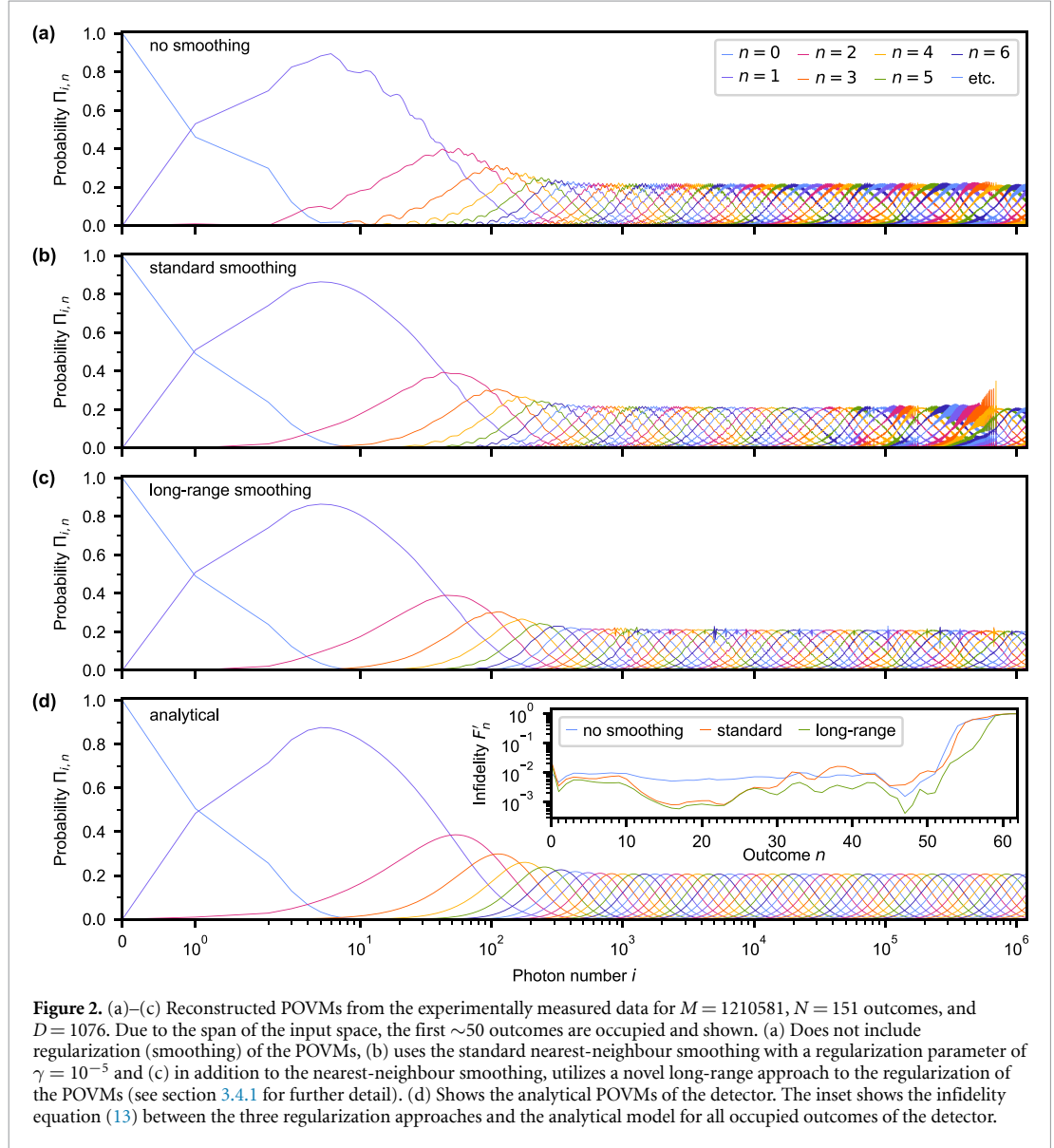
Given the experimental parameters of the high dynamic range detector, we can then calculate the bin-click probabilities for photon-number (Fock) state inputs

$$p_j^{\text{Fock}}(i) = \begin{cases} 1 - (1 - R\eta_{\text{det}})^i & j = 1 \\ 1 - [1 - (1-R)^2 R^{-1} (R\eta_{\text{loop}})^{j-1} \eta_{\text{det}}]^i & j \geq 2 \end{cases}, \quad (12)$$

which is again adapted from [30] to include the detector efficiency η_{det} . With these bin-click probabilities, it is possible to use a closed-form expression for the Poisson binomial distribution [32] to calculate the POVMs of the detector (see [19] for more detail).

2.3. POVM reconstruction

We investigate the proposed reconstruction method with the experimental measurement data, i.e. with a Hilbert space cutoff of $M = 1210581$, $N = 151$ outcomes, of which the first ~ 50 outcomes are tomographically covered by the $D = 1076$ input states. Figure 2 shows three different tomographic reconstructions, each of which take different approaches to regularization (see section 3.4.1 for details on the regularization routine). In figure 2(a) no regularization is used, resulting in somewhat noisy response, particularly at low outcome numbers. In figure 2(b), a regularization factor of $\gamma = 10^{-5}$ is used, limited to smoothing between nearest-neighbour photon numbers, as discussed in [8]. This successfully smooths the high-frequency noise at low photon numbers, but is insufficient (and indeed increases noise) at high photon numbers.



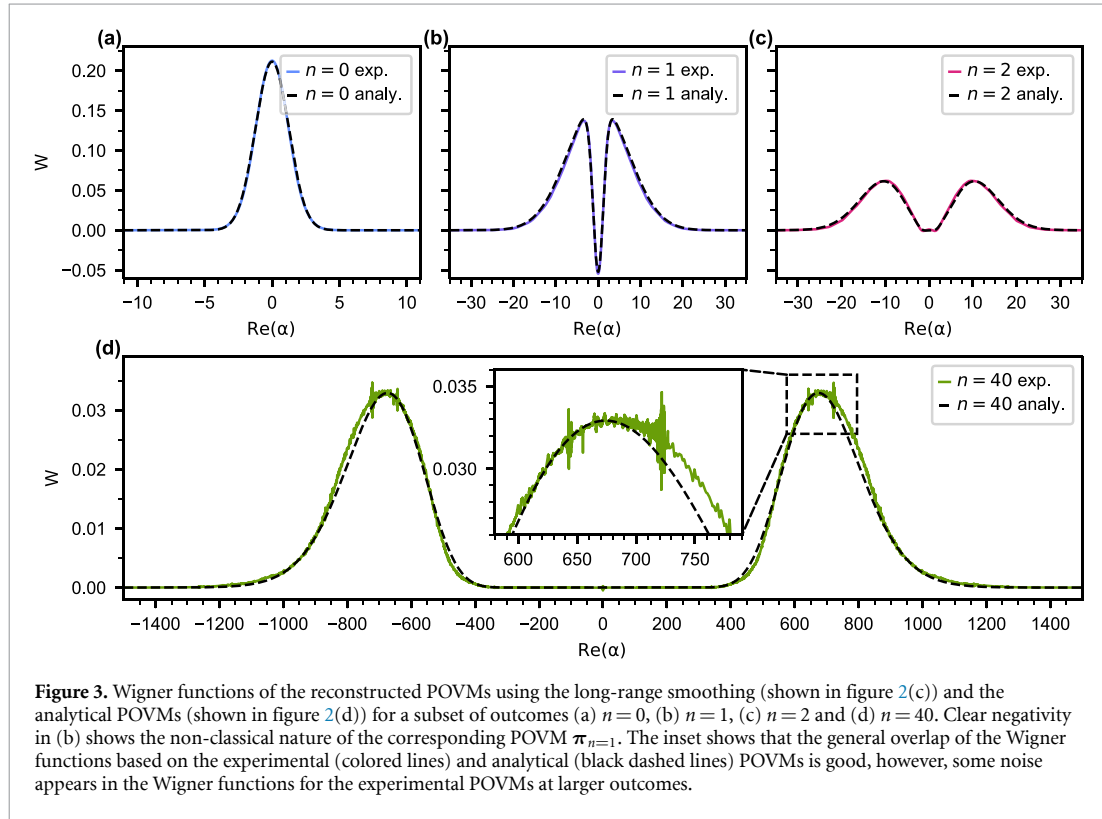
To mitigate this, we introduce a further step to the regularization which encourages smoother, more physical results at higher photon numbers (see section 3.4.1 for more detail). The result of this is shown in figure 2(c).

To evaluate the solver in general, and each approach to regularization in particular, we compare each reconstruction to the analytic model of the POVMs of the detector, shown in figure 2(d). In the inset of figure 2(d), we plot the infidelity (1-fidelity)

$$F'_n = 1 - F_n = 1 - \frac{\text{Tr} \left(\left(\sqrt{\pi_n} \pi_{n,\text{theo}} \sqrt{\pi_n} \right)^{1/2} \right)^2}{\text{Tr}(\pi_n) \text{Tr}(\pi_{n,\text{theo}})} \quad (13)$$

between the reconstructions and the analytic model. Each case shows excellent agreement between the reconstructed and analytical POVMs, with fidelities exceeding 99% for all occupied outcomes in all three cases. For the extended regularization, this fidelity increases to an average of 99.69% for all occupied outcomes up to $N = 50$. This demonstrates the unparalleled accuracy of the reconstruction at unprecedented Hilbert space size.

From the POVMs of the detector we can reconstruct the Wigner functions corresponding to different detector outcomes, as shown in figure 3. These Wigner functions give additional insight about the detector. Negativity at the origin in figure 3(b), shows the non-classical nature of the corresponding operator $\pi_{n=1}$. Our approach for calculating the Wigner functions (see appendix C) is stable even up to a Hilbert space



dimension of $M = 10^6$, as can be seen by the smooth nature of the Wigner functions of the analytical POVMs in figure 3(black dashed lines). Consequently, the noise in the Wigner function of the experimental POVM $\pi_{n=40}$ can be explained by noise in the reconstructed POVM (which is visible in figure 2(c)). Importantly, we see that the general overlap between Wigner functions of experimental and analytical POVMs is high, especially for small outcomes.

2.4. POVM verification with thermal statistics

Given the reconstructed POVMs, it is possible to apply them to reconstruct the statistics of unknown quantum states of light. With equation (1) it is possible to calculate the outcome matrix $\mathbf{P}$ by multiplying the input state matrix $\mathbf{F}$ with the POVMs $\mathbf{\Pi}$. For thermal states a specific row of $\mathbf{F}$ is given by

$$F_{\text{th}}(i) = \frac{1}{\bar{n}_{\text{th}} + 1} \left(\frac{\bar{n}_{\text{th}}}{\bar{n}_{\text{th}} + 1} \right)^i \quad (14)$$

in the photon-number basis up to the Hilbert space dimension $i \in [0, M - 1]$. With the input state matrix $\mathbf{F}_{\text{th}}$ and the previously reconstructed POVMs $\mathbf{\Pi}$ we can calculate the outcome matrix $\mathbf{P}_{\text{th,POVM}}$, i.e. the click probabilities and thus predict how the detector would behave under illumination with bright single-mode thermal light.

Additionally, it is possible to calculate the click probabilities of the detector for thermal light $p_{\text{click}}^{\text{th,GS}}(\bar{n}_{\text{th}})$ using the Glauber-Sudarshan P representation for thermal states as well as the click probabilities of the detector for coherent light $p_{\text{click}}^{\text{coh}}(z)$ using [33]

$$p_{\text{click}}^{\text{th,GS}}(\bar{n}_{\text{th}}) = \int_0^\infty \frac{e^{-\frac{z}{\bar{n}_{\text{th}}}}}{\bar{n}_{\text{th}}} p_{\text{click}}^{\text{coh}}(z) dz, \quad (15)$$

where $\bar{n}_{\text{th}}$ and z are mean photon numbers of the thermal and coherent states respectively.

The click probabilities for coherent states, i.e. probabilities of n bins being occupied $p_{\text{click}}^{\text{coh}}(z)$ are calculated from the bin-click probabilities for a coherent state input z (see equation (11)), using the closed-form expression for the Poisson binomial distribution [32] (see [19] for more detail). This calculation is similar to the calculation of the analytical POVMs (explained in section 2.2), for which Fock state inputs were used. Equation (15) entails an integration over the mean photon number of the coherent state z . As it is

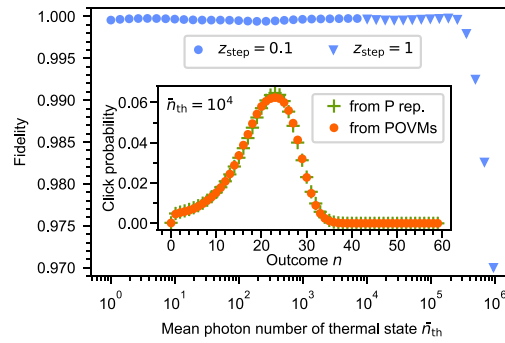


Figure 4. Fidelity of the reconstructed click probabilities for thermal states as a function of the mean photon number of the thermal state $\bar{n}_{th}$. Different symbols indicate different step sizes for the summation. The inset shows the click probabilities of the detector as a function of detector outcomes n for both calculation methods for a mean photon number of $\bar{n}_{th} = 10^4$. The green plus symbols correspond to the calculation using the Glauber-Sudarshan P representation, i.e. equation (15) and the orange dots correspond to the calculation using the previously reconstructed POVMs, as shown in figure 2(c).

not straightforward to find the analytical expression for $p_{click}^{coh}(z)$ to perform the integration, we approximate the integration with a summation up to an upper limit of z_{max} with finite step sizes z_{step} .

Calculating the fidelity between the two methods for different upper limits of the summation z_{max} and step sizes z_{step} leads to slightly different results. Using $z_{step} = 1$ leads to decreased fidelities for small $\bar{n}_{th}$, due to less accuracy in the numerical approximation. This effect subsides for larger $\bar{n}_{th}$. Increasing the upper limit z_{max} allows to calculate $p_{click}^{th,GS}$ for larger $\bar{n}_{th}$, while maintaining numerical accuracy. Therefore, we use an adaptable algorithm to calculate $p_{click}^{th,GS}$ within a reasonable amount of time; for $\bar{n}_{th} < 10^4$ we use $z_{step} = 0.1$ and for $\bar{n}_{th} < 2 \cdot 10^6$ we use $z_{step} = 1$.

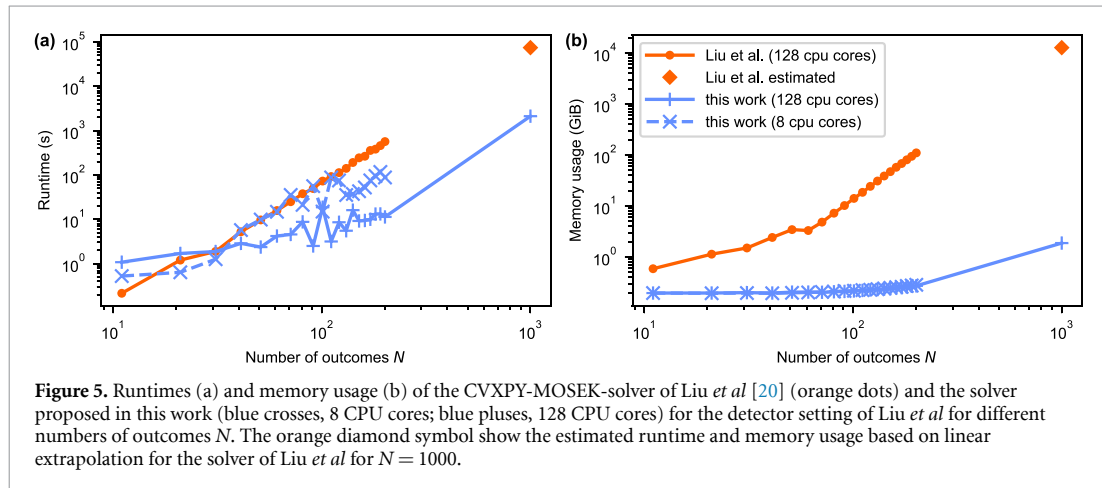
Using the fidelity definition as given in equation (13), we can calculate the fidelity between the two methods for calculating the click probabilities (using the POVMs and using equation (15)). The resulting fidelities as a function of the mean photon number of the thermal input state are shown in figure 4. The fidelity remains above 99.9% for all relevant $\bar{n}_{th}$, which verifies that the reconstructed POVMs are accurate. Given the high fidelity, the click probabilities for a thermal state for both calculation methods are almost identical, as shown in the inset of figure 4 for a mean photon number of $\bar{n}_{th} = 10^4$. The decrease in fidelity for $\bar{n}_{th} > 2 \cdot 10^5$ is due to a decrease in the sum of the photon-number contributions of the input states, i.e. a loss of normalization. This is attributed to the limited Hilbert space dimension M of the reconstructed POVMs of roughly $1.2 \cdot 10^6$. Thermal states with larger mean photon numbers have significant photon-number contributions beyond the Hilbert space of the POVM, which necessarily leads to errors in the calculation of the click probabilities using the POVMs.

2.5. Application to other detector geometries

To compare with the current state of the art, we evaluate the proposed solution method and implementation for the detector geometry of Liu *et al* [20]. They consider a spatially multiplexed detector in the form of an on-chip network of beam splitters, whose output modes terminate in an SNSPD. This splits the input light equally onto many SNSPDs, thus gaining quasi-photon-number resolution. Their setting is characterized by the ratio of the maximal photon number in the Hilbert space and the number of outcomes, i.e. M/N being of the order of only ten. Specifically, the relation in their case is given in equation (5). The inputs for the minimization problem, i.e. the matrices $\mathbf{F}$ and $\mathbf{P}$ were generated with the implementation of Liu *et al* [34]. In our analysis of their data, we use a single compute node with two AMD EPYC 7763 64-core CPUs. Further software and hardware details can be found in appendix A. The runtime and memory usage of the solver of Liu *et al* [20] and the proposed method are shown in figure 5 for different N . While the CVXPY-MOSEK-based solver of Liu *et al* has a lower runtime for small cases, i.e. $N \lesssim 30$, the solver proposed and implemented in this work is advantageous from intermediate sizes onwards. Performance of our proposed solver can be improved for small problems by using fewer CPU cores and, thus, threads which reduces the threading overhead. For $N = 1000$, which corresponds to $N \cdot M = 1.04 \cdot 10^7$ free variables in $\mathbf{\Pi}$, the estimated runtime and memory usage for the CVXPY-MOSEK-based solver can be estimated to ≈ 75000 seconds and ≈ 12700 GiB while the proposed solver requires 1450 seconds and 1.6 GiB.

2.6. Scalability for very large-scale problems

The limiting factor for the problem size is the main memory available on the compute nodes. We choose to evaluate the so-called ‘weak-scaling’ scenario, in which the portion of the problem that one node works on is



chosen to be constant. In this case, the maximum problem size depends on the available memory per compute node, the number of nodes, and their communication (message passing interface (MPI) ranks; see section 3.3 for details). Using our HPC hardware, the scalability of the approach up to $4.1 \cdot 10^{12}$ free parameters has been demonstrated to be feasible.

The question naturally arises as to how large a system can be reasonably reconstructed using this method. This cannot be answered in general since the iterative numerical solution depends on the number of iterations required to converge, which in turn depends on the specific problem to be solved as well as the experimental input data. Thus, the scalability of the proposed solver and implementation is evaluated based on the scalability of the required performance-relevant operations (e.g. evaluation of the objective value, gradient, Hessian-matrix products, and others, as listed in table 1) instead of the full reconstruction.

Nevertheless, one can estimate the scaling of the solver based on heuristics. Empirically for our two-stage iterative approach (see appendix B), the first stage requires about 10-15 Newton iterations, and the second stage requires about 30-200 Newton steps till convergence. Usually, between 20 and 50 conjugate-gradient iterations are needed per Newton iteration. Thus, the runtimes measured for individual operations (presented in figure 8) for $N \cdot M$ up to $3 \cdot 10^{12}$ can be multiplied by lower and upper estimates of iterations to give lower and upper estimates for the runtime of the reconstruction.

Given these results, in figure 6 we show estimated lower and upper limits of the runtime as applied to our detector geometry, up to a Hilbert space of $M \approx 3 \cdot 10^{12}$, number of outcomes $N \approx 150$ and requiring $D = \sqrt{M}$ input states to span the space. This is presented assuming that the number of iterations required for convergence remains similar to the experimentally realized case and that the number of compute nodes required to evaluate each system size is chosen according to the weak-scaling assumption (see section 3.4). The runtime for the reconstruction for the experimental setup of this work using one compute node of the Noctua 2 cluster [26] is also shown for comparison. As verification for the estimation in the large-scale limit, the measured runtime for the reconstruction for simulated POVMs with $N = 151$ and $M = 2.73 \cdot 10^{10}$ using 990 compute nodes taking 66 minutes is also shown. This large-scale case amounts to $M \cdot N = 4.1 \cdot 10^{12}$ free variables in the POVMs where 33 TB of storage is required for the resulting POVMs.

3. High-performance computing approach

3.1. Algorithmic approach

Instead of relying on general minimization solvers such as CVXPY [24, 25] that are designed for handling arbitrary problems from large classes of minimization problems, we propose a tailored algorithm that directly utilizes properties of the given problem.

We have evaluated several different approaches that only require the objective function and constraints as well as their derivatives but no matrix factorizations or decompositions. The goal of this restriction was to set up an algorithm that is well suited to be parallelized not only for using multiple CPU cores in one compute node but multiple CPU-nodes of a large HPC-cluster in parallel. Thus, we have evaluated the augmented Lagrangian approach [35, 36], different variants of active-space methods as well as projection approaches [37].

The minimization problem at hand is characterized by a large number of equality (M) and inequality constraints ($N \cdot M$) and a relatively low computational complexity of the objective function due to the

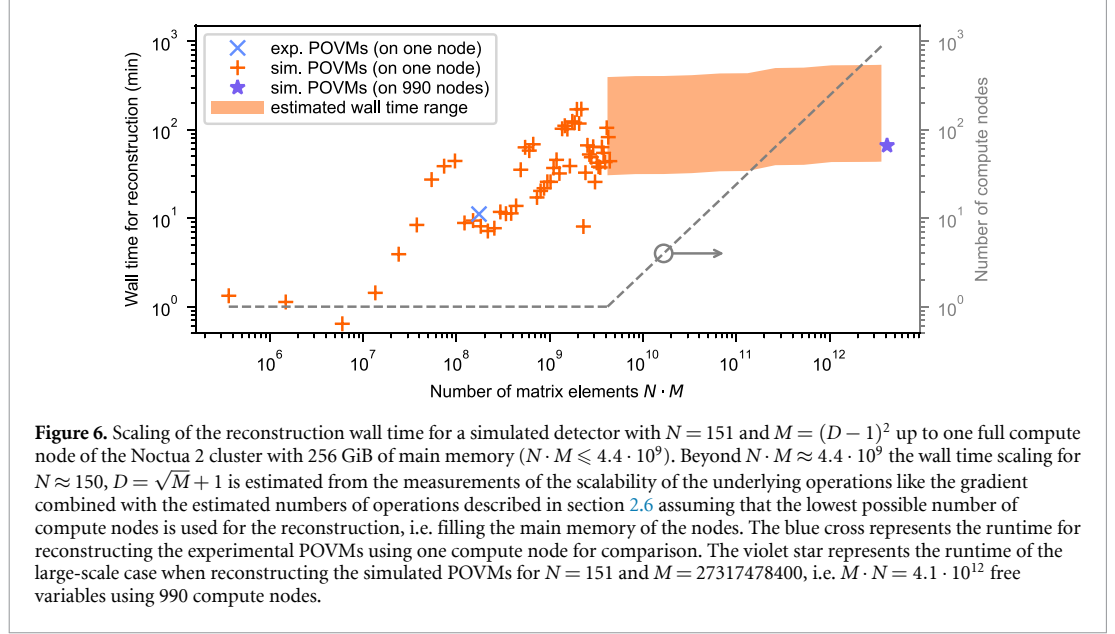


Table 1. The performance-relevant operations required for the approach described in section 3.1. $\mathbf{c}, \mathbf{d} \in \mathbb{R}^{M \times N}$, $\alpha \in \mathbb{R}$.

operation	equation	optimal parallelism
objective value	$O(\mathbf{\Pi}) = \ \mathbf{P} - \mathbf{F}\mathbf{\Pi}\ _2^2$	columns of $\mathbf{\Pi}$ with $D \times N$ -reduction
gradient	$\partial_{\mathbf{\Pi}} O(\mathbf{\Pi})$	columns of $\mathbf{\Pi}$ with $D \times N$ -reduction
Hessian products	$\mathbf{H}^{(k)} \mathbf{d}$	columns of $\mathbf{\Pi}$ with $D \times N$ -reduction
diagonal of Hessian	$H_{i,j;i,j}^{(k)} = \frac{\partial^2 O(\mathbf{\Pi}^{(k)})}{\partial \Pi_{i,j} \partial \Pi_{i,j}}$	columns of $\mathbf{\Pi}$ with $D \times N$ -reduction
simplex-projection	$\mathcal{P}_{SM}[\mathbf{d}]$	rows of $\mathbf{d}$
element-wise operations	e.g. $\mathbf{d} + \mathbf{c}, \mathbf{c} \cdot \mathbf{d}$	rows and/or columns of $\mathbf{d}$
scalar multiplications	$\alpha \mathbf{d}$	elements of $\mathbf{d}$
2-norm	$\sqrt{\sum_{i,n} d_{i,n}^2}$	rows/columns of $d_{i,j}$ with global reduction
row-maxima	$m(i) = \arg \max_{n \in \{0, \dots, N-1\}} d_{i,n}$	rows of $\mathbf{d}$
row-sums	$s(i) = \arg \sum_{n=0}^{N-1} d_{i,n}$	rows of $\mathbf{d}$

sparsity of $\mathbf{F}$. These characteristics directly impact the suitability of the classes of minimization algorithms: For example, the large number of equality constraints cause a large number of additional augmentation and penalty terms in augmented Lagrangian-based approaches.

Our implemented approach [38] is based on the two-metric projected Newton method [27]. We developed a two-stage extension which showed significantly faster convergence. Both stages use a diagonally preconditioned conjugate gradient for the approximate solution of the linear system in Newton's method. As line search a backtracking approach with Armijo-like conditions [37] is used. The essential difference between the first stage and the second stage is that in the first stage a projection $\mathcal{P}_{SM}$ onto the M -dimensional unit simplex is used [39] whereas the second stage projects only onto the non-negativity constraints. Full details and algorithms are given in appendix B. In the following section we present a few important aspects of the implementation.

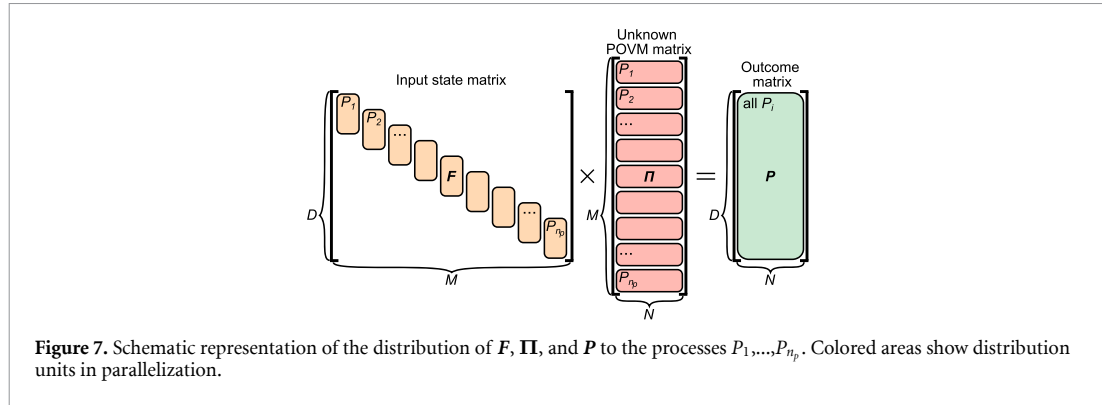
3.2. Implementation

3.2.1. Required operations

The performance-relevant operations required for the solution approach described in section 3.1 are listed in table 1 together with their possible parallelism scope. The parallelism scope refers to the level or dimension along which the POVMs $\mathbf{\Pi}$ could be distributed to different processes or compute nodes without requiring communication or, if not possible, the additional required reduction is specified.

3.2.2. Data distribution and parallelization

While all operations involving the objective function $O(\mathbf{\Pi}) = \|\mathbf{P} - \mathbf{F}\mathbf{\Pi}\|_2^2$ have the columns of $\mathbf{\Pi}$ as a natural parallelism level due to the product $\mathbf{F}\mathbf{\Pi}$, all other required operations have at least the rows of $\mathbf{\Pi}$ as a natural parallelism level. Thankfully, for large M , the matrix $\mathbf{F}$ is sparse and banded, making it possible to



also efficiently parallelize the operations that include the objective functions with respect to the rows of Π . Thus, Π is distributed in blocks of rows to different processes. The matrices P and $P - F\Pi$ are replicated on every compute node. Only the relevant non-zero blocks of F are stored on the corresponding processes. The distribution schema is schematically shown in figure 7. Thus, the computation of $F\Pi$ is performed in two steps: The first step is the process-local computation of the contributions to the auxiliary matrix $O = F\Pi$ with the rows of Π and the blocks of F that are present on the process. Secondly, the locally calculated contributions to O must be communicated to other processes. Due to the sparse banded structure of F for large M and because only parts of O are required on every process to compute the objective function, gradient, and Hessian products, no all-to-all communication is required for this step. Instead, the communication pattern can be handled efficiently with a butterfly graph. With the described distribution of blocks of rows of Π , the proposed reconstruction can scale to very large problem sizes because the entire main memory available in a large HPC cluster can be used to compute a reconstruction.

We have implemented the distributed memory parallelization with the MPI [40]. Additionally, we have parallelized the process-local operations with OpenMP [41] wherever possible. An emphasis was placed on avoiding false sharing of cache lines between the threads and an optimal usage of the available memory bandwidth.

3.3. Memory usage

The limiting factor for the size of the reconstruction in terms of the problem parameters N , M , and D that can be performed is the sum of the main memory available on the compute nodes.

The maximal possible Hilbert space dimension $M_{\max}$ possible for a reconstruction with our approach for a given number of outcomes N , number of probe states D , number of MPI-ranks n_{ranks} per compute node and number of compute nodes n_{nodes} and main memory per node mem_{node} can be estimated with

$$M_{\max} \approx \frac{n_{\text{nodes}}}{6} \left(\frac{\text{mem}_{\text{node}}}{N \cdot 8 \text{ byte}} - 2DN_{\text{ranks}} \right). \quad (16)$$

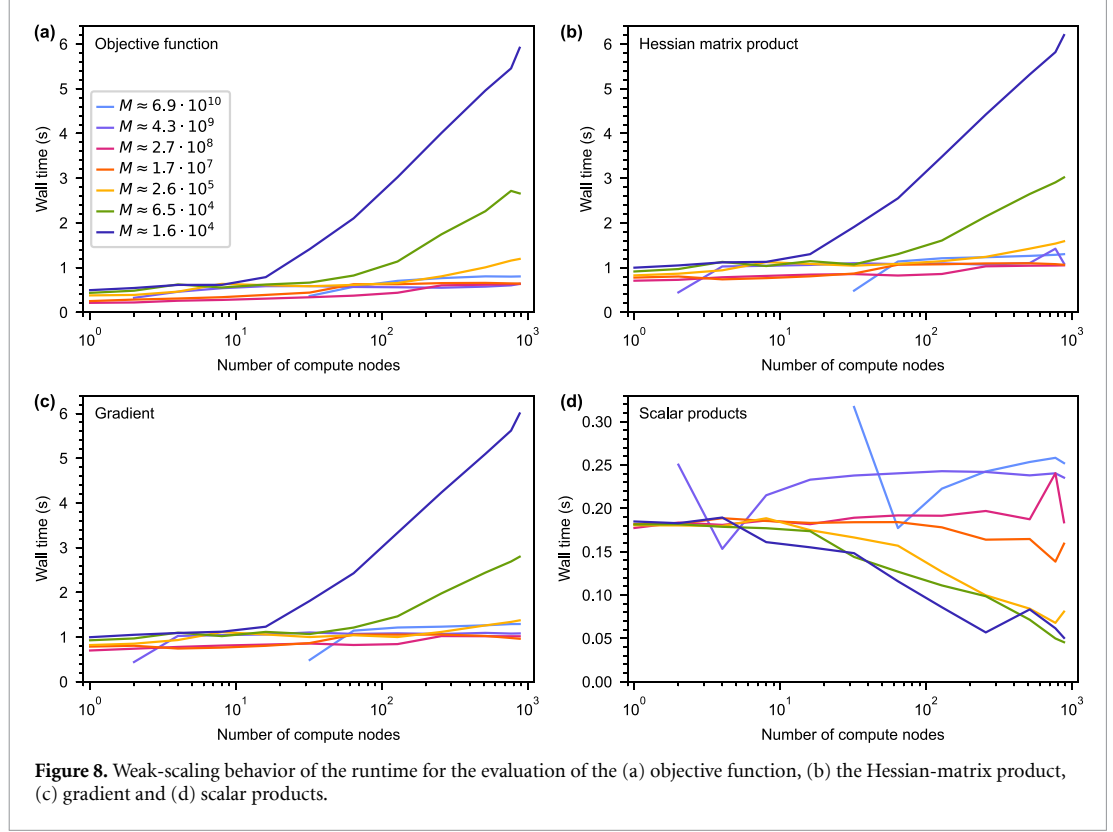
3.4. Scalability considerations

Due to the dependence of the number of iterations during the minimization on the input data, general statements on overall time-to-solution are limited. Nevertheless, the underlying operations can be analyzed in detail. We have chosen the quadratic scaling of probe states, i.e. $M = (D - 1)^2$ corresponding to the experimental situation presented in this work. The matrices P and the POVMs Π were chosen randomly and densely for the scalability experiments.

To analyze the scalability for very large reconstructions we chose a situation where the main memory of the available compute nodes is almost completely used and, thus, the limiting factor. Thus, the situation is related to the so-called weak-scaling case in parallelism where the portion of the problem that one compute node works on is kept constant when the number of compute nodes is increased.

For the compute nodes of Noctua 2 [26] with a main memory size of 256 GiB we have used $\text{mem}_{\text{node}} = 200 \text{ GB}$ to also account for buffer sizes of MPI transfers, OpenMP stack usage and other additional memory usages that are not covered in the approximation of equation (16). Scalability was investigated for 8 MPI ranks per node, i.e. one per NUMA domain, and 16 threads per MPI rank.

The scaling behavior of underlying operations is shown in figure 8 for the evaluation of the objective function (a), the Hessian-matrix product (b), gradient (c) and scalar products (d), which all require global communication between the compute nodes. Runtime results for the gradient are nearly indistinguishable from the Hessian-matrix product. Element-wise operations, scalar multiplications, and projections are



trivially parallel with the proposed parallelization scheme. For example, a conjugate-gradient iteration requires one Hessian product, several element-wise operations, and several scalar multiplications.

For larger photon number cutoffs $M \gtrsim 10^5$, the required runtime for the operations scales very favorably with the number of compute nodes. In contrast, for smaller M , the runtime increases visibly with the number of compute nodes. The underlying reason is that due to the choice of the weak-scaling scenario, in order to fill the main memory of the compute nodes in the case of small M , the number of outcomes N has to be very high as determined from equation (16).

3.4.1. Regularization and smoothing

In addition to the least-squares objective function in equation (2a) our implementation also supports the next-neighbor regularization term [7, 8]

$$g(\Pi) = \gamma \sum_{n=0}^{N-1} \sum_{i=0}^{M-2} (\Pi_{i,n} - \Pi_{i+1,n})^2 \quad (17)$$

with the regularization parameter $\gamma \geq 0$.

While long-range generalizations of the regularization term are, in principle, easily possible, they introduce an ambiguity due to the choice of the averaging and the range-dependence of the weighting. The minimization problem for this work's experimental situation exhibits a flat valley around the minimum. Thus, we propose a different way of encouraging a more physical, i.e. less noisy reconstruction by first performing a reconstruction starting with some initial Π . We use $\Pi_{i,n}^{(0)} = 1/N$ as a starting point. The reconstruction problem is solved with the proposed two-stage algorithm and the result is smoothed by replacing each $\Pi_{i,n}$ with its long-range average

$$\tilde{\Pi}_{i,n} = \frac{1}{2N_{\text{smooth}}(i) + 1} \sum_{j=i-N_{\text{smooth}}(i)}^{i+N_{\text{smooth}}(i)} \Pi_{j,n}, \quad (18)$$

where Π is the result of the minimization and the smoothing distance N_{smooth} depends on the photon number i so that the logarithmic photon-number scale in this experiment is accommodated. The smoothed POVMs $\tilde{\Pi}$ then serve as the starting point for a second minimization run where only the second stage is used.

The exclusion of the lowest ~ 100 photon numbers from the smoothing step improves the results because it prevents a severe disruption of the qualitative structure at low photon numbers. We have found that a smoothing distance of $N_{\text{smooth}} = i/50$ yields a significant improvement over the POVMs without a smoothing step as shown in figure 2(c).

3.4.2. Possible improvements

While our implementation assumes no *a priori* sparsity pattern of $\mathbf{\Pi}$, $\mathbf{P}$, and $\mathbf{F}$, a sparsity pattern forms in $\mathbf{\Pi}$ during the minimization. However, our current implementation performs all operations, like the gradients or Hessian-matrix-products, in the full $N \cdot M$ space, which reduces the code complexity but is likely wasteful in terms of memory bandwidth and floating-point operations. Also, due to the experimental setting with $M \gg N$, the parallelization and handling of matrices, especially $\mathbf{F}$, are optimized for this situation.

3.5. Extension to phase-sensitive detectors

The algorithm proposed in the previous section is tailored for the detector tomography reconstruction problem of a phase-insensitive detector given in equation (3). For a phase-sensitive detector [8, 42, 43], the minimization has the form

$$\min_{\pi_n \in \mathbb{C}^{M \times M}, n \in \{0, \dots, N-1\}} \|\mathbf{P} - \mathbf{F}\mathbf{\Pi}\|_2^2 \quad (19a)$$

$$\pi_n = \pi_n^\dagger \quad \forall n \in \{0, \dots, N-1\} \quad (19b)$$

$$\pi_n \succeq 0 \quad \forall n \in \{0, \dots, N-1\} \quad (19c)$$

$$\sum_{n=0}^{N-1} \pi_n = \mathbf{1}, \quad (19d)$$

with $\mathbf{\Pi} = (\pi_0, \dots, \pi_{N-1})$. The two-metric projected Newton approach can be generalized to this situation by replacing the projection on non-negativity constraints with a projection of a hermitian matrix on its nearest positive semi-definite matrix. Such a projection can, in the conceptually most straight-forward way, be calculated as

$$\mathcal{P}_{\text{sd}}[\mathbf{A}] = \mathbf{U}^\dagger \begin{pmatrix} \max(0, \lambda_1) & & \\ & \max(0, \lambda_2) & \\ & & \dots \end{pmatrix} \mathbf{U}, \quad (20)$$

where the unitary matrix $\mathbf{U}$ contains the eigenvectors of $\mathbf{A}$ as columns and λ_i are the eigenvalues of $\mathbf{A}$. As this projection problem also arises in other areas, more efficient methods have been developed, especially for structured matrices [44, 45].

4. Conclusion

Large-scale detectors for quantum light are becoming increasingly prevalent in quantum photonic technologies [10, 46, 47]. The ability to accurately characterize these devices, in a consistent quantum mechanical framework is central to using them effectively. This approach has been used to debug photonic quantum computing platforms [3], and could be applied to large-scale single-photon sensitive imaging systems [10, 22]. Developing computational tools to handle these large data sets, whilst preserving their quantum mechanical structure, is a vital task in the future proliferation of quantum photonic technologies.

We demonstrate the feasibility of applying HPC to characterize a quantum detector that covers a Hilbert space of dimension 10^8 , based on experimental data. The accuracy of this reconstruction is verified by comparison with an analytic model of the device, showing excellent agreement with fidelities above 99% per outcome. Moreover, we are able to plot Wigner functions of these very large quantum objects by exploiting arbitrary-precision floating-point numbers, and this freely available code is generally applicable for diagonal density matrices [38].

The solver itself, which is also freely available [38], is limited only by the number of compute nodes available. Using simulated data, we could reconstruct POVMs up to a system size of 10^{12} in 66 minutes. The memory required to represent and store the POVMs of a detector of this scale is 33 TB (2.4 TB compressed). This demonstrates the scale of photonic systems which can benefit from this approach.

At present, the solver is designed for use with phase-insensitive detectors. The degree and type of regularization can be adapted to the specific detector type. Extensions of the solver to cover phase sensitivity

are also discussed in section 3.5. Furthermore, the solver can be modified to solve other problems of the form of equation (3), provided the matrices fulfill the requirements (e.g. the banded structure of the F -matrix) as outlined in section 3.

The main bottleneck of the solver is the available memory bandwidth. Thus, it might be worthwhile to consider and explore approximate computing techniques like reduced floating-point or fixed-point representations of numbers to further increase scale. In the other direction, using hardware with higher memory bandwidths like GPUs or FPGAs is a promising route.

Data availability statement

The data that support the findings of this study are openly available at the following URL/DOI: <https://doi.org/10.5281/zenodo.10803758>, <https://doi.org/10.5281/zenodo.10810148>.

Code availability

The reconstruction code ‘Parallel Quantum Detector Tomography Solver’ (pqdts [38]) and the code for calculating Wigner functions is openly available on GitHub via <https://github.com/pc2/pqdts> and can be cited via Zenodo <https://doi.org/10.5281/zenodo.10853650>.

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Author contributions

T S and T B conceived the idea. T S designed the experiment, performed the measurements and analyzed the measurement data. T S and M L performed pretesting for the large-scale reconstruction. R S programmed the solver, performed all benchmarking and calculated the Wigner functions. T S performed the POVM verification. T S, R S and T B wrote the manuscript with inputs from all authors. C P and T B supervised the project. T S, R S and T B revised the manuscript.

Conflict of interest

The authors declare no competing interests.

Appendix A. Software and hardware details

All numerical results have been obtained using compute nodes of the Noctua 2 HPC cluster [26]. Each node hosts two AMD EPYC 7763 CPUs with 64 CPU cores each and 256 GiB of DDR4-3200 main memory in an 8-channel-per-cpu configuration, giving a usable memory bandwidth of 370 GB/s in the STREAM Triad benchmark. The CPUs are configured with 4 NUMA domains per CPU, SMT is disabled, and turbo-mode is enabled. The following software versions were used: Python 3.11.5, CVXPY 1.4.2, and MOSEK 10.1.24 for the solver of Liu *et al* and GCC 13.2.0 with OpenMPI 4.1.4 for the proposed solver.

Main memory usage refers to the maximal resident set size of the solver process as measured by the Linux kernel as the high-water mark (HWM).

Appendix B. Two-metric projected Newton method

B.1. Bertsekas two-metric projected Newton method

The two-metric projected Newton method for a general constrained minimization problem

$$\min_{\vec{x} \in \Omega} f(\vec{x}), \quad (\text{B1})$$

where Ω is the space in which the constraints for $\vec{x}$ are fulfilled, is an iterative approach derived from the well-known Newton method for unconstrained minimization [37]. For the unconstrained problem

$$\min_{\vec{x} \in \mathbb{R}^m} f(\vec{x}), \quad (\text{B2})$$

the unconstrained Newton method approximates the objective function $f(\vec{x})$ in the k -th step by a quadratic model around the current iterate $\vec{x}^{(k)}$ as

$$Q^{(k)}(\vec{x}) \approx f(\vec{x}^{(k)}) + (\vec{x} - \vec{x}^{(k)})^T \nabla f(\vec{x}^{(k)}) + \frac{1}{2} (\vec{x} - \vec{x}^{(k)})^T \mathbf{H}^{(k)} (\vec{x} - \vec{x}^{(k)}), \quad (\text{B3})$$

where $\mathbf{H}^{(k)}$ is the Hessian matrix in the k -th step, i.e. $H_{ij}^{(k)} = \frac{\partial^2}{\partial x_i \partial x_j} f(\vec{x}^{(k)})$. The next iterate $\vec{x}^{(k+1)}$ is then the solution of the minimization problem:

$$\vec{x}^{(k+1)} = \min_{\vec{x} \in \mathbb{R}^m} Q^{(k)}(\vec{x}). \quad (\text{B4})$$

The quadratic unconstrained minimization problem in equation (B4) can be rewritten as the well-known linear system for the minimization step $\vec{p}^{(k)} = \vec{x}^{(k+1)} - \vec{x}^{(k)}$:

$$\mathbf{H}^{(k)} \vec{p}^{(k)} = -\nabla f(\vec{x}^{(k)}), \quad (\text{B5})$$

which can be solved with various numerical methods. Usually, this schema is augmented by a line-search procedure where the next iterate $\vec{x}^{(k+1)}$ is determined from the step vector $\vec{p}^{(k)}$ by the solution of the line-search problem

$$\alpha^{(k)} = \arg \min_{\alpha \in \mathbb{R}} f(\vec{x}^{(k)} + \alpha \vec{p}^{(k)}) \quad (\text{B6})$$

$$\vec{x}^{(k+1)} = \vec{x}^{(k)} + \alpha^{(k)} \vec{p}^{(k)}. \quad (\text{B7})$$

While this additional line search procedure only adds a typically negligible computational cost, the approach is much more numerically reliable. For an objective function with positive definite Hessian, the unconstrained Newton method converges quadratically fast near the solution [37].

Analogous to the translation of the gradient descent method for unconstrained problems to the gradient projection method for constrained problems, the Newton method for unconstrained problems, equations (B3)–(B7) can be translated to problems with constraints [37]. For this purpose, the unconstrained quadratic approximation in equation (B3) is written as a constrained problem that determines the next iterate $\vec{x}^{(k+1)}$ as

$$\vec{x}^{(k+1)} = \arg \min_{\vec{x} \in \Omega} Q^{(k)}(\vec{x}) \quad (\text{B8})$$

the crucial difference is that the minimization, in contrast to the unconstrained case in equation (B4), is restricted to vectors $\vec{x}$ that fulfill the constraints. Consequently, the step direction $\vec{p}^{(k)}$ cannot be obtained by the solution of the linear system equation (B5), and instead, the constrained problem of equation (B8) is typically significantly harder to solve. In the case of bound-constrained problems, i.e. $\Omega = \{\vec{x} \in \mathbb{R}^m : l \leq x_i \leq u \forall i\}$, one possibility to circumvent this issue is the idea of two-metric projected Newton approaches [27]. Equation (B8) is rewritten as a projection

$$\vec{x}^{(k+1)} = \arg \min_{\vec{x} \in \mathbb{R}^m : l \leq x_i \leq u} \frac{1}{2} \left\| \vec{x} - \left(\vec{x}^{(k)} - (\mathbf{H}^{(k)})^{-1} \nabla f(\vec{x}^{(k)}) \right) \right\|_{\mathbf{H}^{(k)}}^2, \quad (\text{B9})$$

where the norm with respect to the metric defined by the positive-definite matrix $\mathbf{A}$ is defined as $\|\vec{y}\|_{\mathbf{A}} = \sqrt{\vec{y}^T \mathbf{A} \vec{y}}$. Instead of solving this projection with respect to the metric induced by the Hessian matrix $\mathbf{H}$, the two-metric approach solves the projection problem

$$\vec{x}^{(k+1)} \approx \arg \min_{\vec{x} \in \mathbb{R}^m : l \leq x_i \leq u} \frac{1}{2} \left\| \vec{x} - \left(\vec{x}^{(k)} - (\mathbf{D}^{(k)})^{-1} \nabla f(\vec{x}^{(k)}) \right) \right\|_{\mathbf{I}}^2 \quad (\text{B10})$$

with respect to the unit metric, but the inverse of the Hessian matrix is replaced by the inverse of a suitable positive definite matrix $\mathbf{D}$. Practically, the projection with respect to the unit metric in equation (B10) has the solution

$$\vec{x}^{(k+1)} \approx \mathcal{P}_{u,l} \left[\vec{x}^{(k)} - \left(\mathbf{D}^{(k)} \right)^{-1} \nabla f \left(\vec{x}^{(k)} \right) \right], \quad (\text{B11})$$

where $\mathcal{P}_{u,l}(\vec{x})$ is the projection of the vector $\vec{x}$ onto the constraint surface, i.e.

$$(\mathcal{P}_{u,l}(\vec{x}))_i = \begin{cases} l, & \text{if } x_i < l \\ u, & \text{if } x_i > u \\ x_i, & \text{otherwise} \end{cases}. \quad (\text{B12})$$

A line-search method is added to this approach as

$$\alpha^{(k)} = \arg \min_{\alpha \in \mathbb{R}} \mathcal{P}_{u,l} \left[\vec{x}^{(k)} - \alpha \left(\mathbf{D}^{(k)} \right)^{-1} \nabla f \left(\vec{x}^{(k)} \right) \right] \quad (\text{B13})$$

$$\vec{x}^{(k+1)} = \mathcal{P}_{u,l} \left[\vec{x}^{(k)} - \alpha^{(k)} \left(\mathbf{D}^{(k)} \right)^{-1} \nabla f \left(\vec{x}^{(k)} \right) \right] \quad (\text{B14})$$

Bertsekas [27] has shown that for a problem with non-negativity constraints,

$$\min_{\vec{x} \in \mathbb{R}^m : x_i \geq 0 \ \forall i} f(\vec{x}) \quad (\text{B15})$$

a choice of the matrix $\mathbf{D}^{(k)}$ as

$$\left(\mathbf{D}^{(k)} \right)_{ij} = \begin{cases} 0, & \text{if } i \neq j \text{ and either } i \in I \text{ or } j \in I \\ H_{ij}^{(k)}, & \text{otherwise} \end{cases} \quad (\text{B16})$$

with the index set I defined as

$$I = \left\{ i : 0 \leq x_i \leq \epsilon, \frac{\partial f(\vec{x})}{\partial x_i} > 0 \right\} \quad (\text{B17})$$

and a small ϵ , this approach leads to a globally convergent algorithm with a superlinear convergence rate under mild conditions.

For minimization problems over a probability simplex,

$$\min_{\vec{x} \in \mathbb{R}^m : x_i \geq 0 \ \forall i, \sum_{i=1}^m x_i = 1} f(\vec{x}), \quad (\text{B18})$$

Bertsekas [27] has proposed an approach that transforms the problem so that it can be handled with the two-metric projected Newton approach defined in equations (B14)–(B17). The transformation can be generalized straightforwardly to the tomography reconstruction problem in equation (3), which has M sum-constraints. The approach only requires evaluations of the objective function, the gradient, and the approximate solution of the linear system

$$\mathbf{D}^{(k)} \vec{p}^{(k)} = -\nabla f(\vec{x}^{(k)}) \quad (\text{B19})$$

for $\vec{p}^{(k)}$, which can be implemented as an iterative preconditioned conjugate-gradient (CG) method where only matrix-product operations between $\mathbf{D}^{(k)}$ and vectors as well as vector-vector operations are required. Thus, the memory usage footprint is only a few times larger than the memory usage for the storage of the $\mathbf{P}$, $\mathbf{F}$, and $\mathbf{\Pi}$ given in equation (7). The complete two-metric truncated Newton algorithm for the detector tomography problem equation (3) is given in algorithm 1.

Algorithm 1. Two-metric projected truncated Newton algorithm.

```

 $\Pi_{i,n}^{(0)} \leftarrow \frac{1}{N} \forall i, n$  ▷ Initialization that fulfills the constraints
choose  $0 < \beta < 1$ 
for  $k \leftarrow 0, 1, 2, \dots$  do ▷ projected Newton iteration
  for  $i \leftarrow 0, 1, 2, \dots, M$  do ▷ transformation step
     $m(i) \leftarrow \arg \max_{n \in \{0, \dots, N-1\}} \Pi_{i,n}^{(k)}$  ▷ determine per-row maxima
     $\tilde{\Pi}_{i,n}^{(k)} \leftarrow \Pi_{i,n}^{(k)} \forall n \neq m(i)$ 
     $\tilde{\Pi}_{i,m(i)}^{(k)} \leftarrow 1 - \sum_{j \neq m(i)} \Pi_{i,j}^{(k)}$  ▷ implicitly account for sum-constraint
  end for
   $\tilde{\mathbf{G}}^{(k)} \leftarrow -\partial_{\tilde{\Pi}^{(k)}} \|\mathbf{P} - \mathbf{F}\tilde{\Pi}^{(k)}\|_2^2$  ▷ gradient with respect to  $\tilde{\Pi}$ 
  solve  $\mathbf{D}^{(k)} \mathbf{P}^{(k)} = \tilde{\mathbf{G}}^{(k)}$  ▷ with diagonally-preconditioned CG
  for  $l \leftarrow 0, 1, 2, \dots$  ▷ line search
     $\alpha^{(k)} \leftarrow \beta^l$ 
     $\Pi_{i,n}^{(k+1)} \leftarrow \mathcal{P}_{0,\infty}[\tilde{\Pi}^{(k)} + \alpha^{(k)} \mathbf{P}^{(k)}]_{i,n} \forall i, n \neq m(i)$  ▷ update  $\Pi$ 
     $\Pi_{i,m(i)}^{(k+1)} \leftarrow 1 - \sum_{j \neq m(i)} \Pi_{i,j}^{(k+1)}$ 
    if  $\Pi_{i,m(i)}^{(k+1)} \geq 0 \forall i$  and  $\Pi^{(k+1)}$  fulfills Armijo-conditions [37] then
      exit loop
    end if
  end for
if converged then
  exit loop
end if
end for

```

B.2. Two-stage extension to the two-metric projected Newton approach

While the algorithm described in the previous section has a favorable memory usage characteristic and can be parallelized efficiently, we have found that due to a large number of sum constraints, equation (3c), the method converges slowly if it is not already rather close to the solution. The underlying reason is that in the line search in algorithm 1 the elements of $\Pi^{(k+1)}$ corresponding to the row-wise maxima are obtained implicitly from the sum-constraint as

$$\Pi_{i,m(i)}^{(k+1)} = 1 - \sum_{j \neq m(i)} \Pi_{i,j}^{(k+1)}, \quad (\text{B20})$$

where $m(i)$ is the column-index of the maximum in the row i of the matrix $\Pi^{(k)}$. If $\Pi^{(k)}$ is not close to the solution of the minimization problem, there is likely a row in $\Pi^{(k)}$ for which $\Pi_{i,m(i)}^{(k)} \ll 1$ and which, thus, forces the step length $\alpha^{(k)}$ to be small to fulfill the non-negativity constraint for $\Pi_{i,m(i)}^{(k+1)}$. Thus, in the next section, we propose a two-stage approach that avoids this convergence slowdown. In the first stage, we use a modified variant of the two-metric projected Newton method of Bertsekas by replacing the transformation step for the sum constraints with a projection onto the probability simplex that also enforces the sum constraints. The projection $\mathcal{P}_S[\vec{x}]$ of a vector $\vec{x} \in \mathbb{R}^m$ onto the probability simplex $S = \{\vec{y} \in \mathbb{R}^m | y_i \geq 0, \sum_i y_i = 1\}$ is defined as the point closest to $\vec{x}$ that is on S , i.e.

$$\mathcal{P}_S[\vec{x}] = \arg \min_{\vec{y} \in S} \|\vec{x} - \vec{y}\|_2. \quad (\text{B21})$$

In practice, we use the algorithm proposed by Condat [39] that has a best-case complexity of $\mathcal{O}(m)$, a worst-case complexity of $\mathcal{O}(m^2)$, and additional memory requirement $\mathcal{O}(1)$. For the purpose of the detector tomography reconstruction problem in equation (3), an M -dimensional generalization $\mathcal{P}_{S^M}$ of projection on a probability simplex can be defined as

$$\mathcal{P}_{S^M}(\Pi) = \begin{pmatrix} \mathcal{P}_S[(\Pi_{0,0}, \dots, \Pi_{0,N-1})] \\ \vdots \\ \mathcal{P}_S[(\Pi_{M-1,0}, \dots, \Pi_{M-1,N-1})] \end{pmatrix} \quad (\text{B22})$$

that projects the POVM Π on the constraints. The resulting algorithm is shown in algorithm 2. However, due to the presence of the M -dimensional projection $\mathcal{P}_{S^M}$, we have not yet been able to formally show global convergence for this modified variant and, thus, only use it as a first stage to accelerate convergence towards

Algorithm 2 Two-metric projected truncated Newton algorithm.

```

 $\Pi_{i,n}^{(0)} \leftarrow \frac{1}{N} \forall i, n$  ▷ Initialization that fulfills the constraints
choose  $0 < \beta < 1$ 
for  $k \leftarrow 0, 1, 2, \dots$  do ▷ projected Newton iteration
   $\mathbf{G}^{(k)} \leftarrow -\partial_{\Pi^{(k)}} \|\mathbf{P} - \mathbf{F}\Pi^{(k)}\|_2^2$  ▷ gradient with respect to  $\Pi$ 
  solve  $\mathbf{D}^{(k)} \mathbf{P}^{(k)} = \mathbf{G}^{(k)}$  ▷ with diagonally-preconditioned CG
  for  $l \leftarrow 0, 1, 2, \dots$  do ▷ line search
     $\alpha^{(k)} \leftarrow \beta^l$ 
     $\Pi^{(k+1)} \leftarrow \mathcal{P}_{\mathcal{S}^{M+1}} [\Pi^{(k)} + \alpha^{(k)} \mathbf{P}^{(k)}]$  ▷  $M + 1$ -dimensional projection
    if  $\Pi^{(k+1)}$  fulfills Armijo-conditions then
      exit loop
    end if
  end for
  if converged then
    exit loop
  end if
end for

```

Algorithm 3 Two-stage, two-metric projected truncated Newton algorithm.

```

 $\Pi_{i,n}^{(0)} \leftarrow \frac{1}{N} \forall i, n$  ▷ Initialization that fulfills the constraints
choose  $0 < \beta < 1$ 
minimize  $\Pi$  with algorithm 2 starting from  $\Pi^{(0)}$  ▷ Stage 1
minimize  $\Pi$  with algorithm 1 starting from the result of the previous stage ▷ Stage 2

```

the solution. Once sufficiently close to the solution, we switch to the second stage with the proven globally convergent two-metric projected Newton approach in algorithm 1. Practical results that demonstrate the efficiency of this approach are shown in figure 9. The overall two-stage, two-metric projected Newton approach is shown in algorithm 3. Thus, the two-stage algorithm is globally convergent and improves on the convergence issue of algorithm 1. The only required expensive operations are evaluating the objective function, gradient, and products of the modified Hessian $\mathbf{D}^{(k)}$ with vectors for the conjugate-gradient procedure. The constraints are enforced to numerical precision in both stages. The memory usage of the algorithm is given as the sum of the memory needed to store the matrices $\mathbf{F}$, $\mathbf{P}$, $\Pi^{(k)}$ plus the auxiliary matrices $\mathbf{G}^{(k)}$, $\mathbf{O}^{(k)} = \mathbf{F}\Pi^{(k)}$, $\Pi^{(k+1)}$, the index array I , and auxiliary matrices for the diagonally-preconditioned CG iteration. The overall memory requirement is

$$\text{mem}_{2\text{metric}} = (2ND + 6NM + MD + \mathcal{O}(M) + \mathcal{O}(N)) \cdot 8 \text{ byte.} \quad (\text{B23})$$

For large M , a sparse storage of $\mathbf{F}$ instead of a dense representation is used to drastically reduce the term MD in the memory estimation. For comparison, in the case of the detector geometry of Liu *et al* i.e. the dependence of M and D given by equations (5)–(6), and sufficiently large N , the memory estimate given in equation (B23) results in

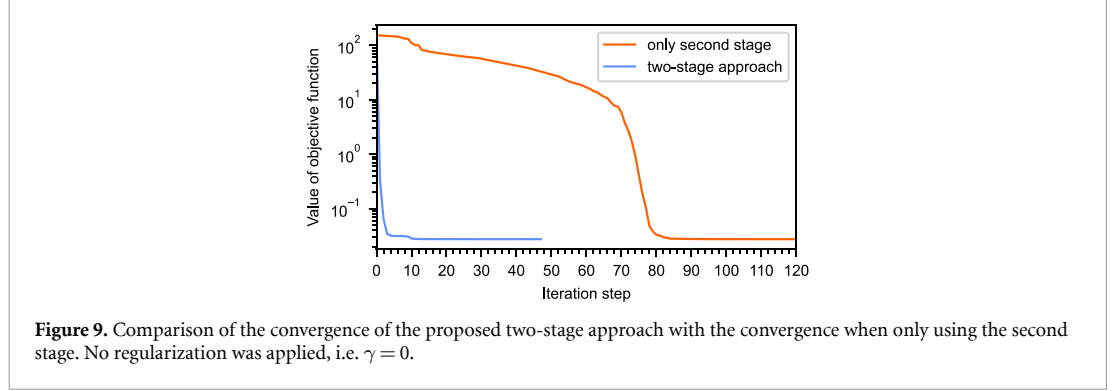
$$\text{mem}_{2\text{metric,Liu}} \lesssim 6.8 \cdot 10^{-7} \cdot N^{2.2} \text{ GiB.} \quad (\text{B24})$$

The convergence criterion for the line searches in both stages are the Armijo-like conditions where the smallest $m \in \mathbb{N}_0$ that fulfills

$$f\left(\mathcal{P}\left[\Pi^{(k)} + \beta^m \mathbf{P}^{(k)}\right]\right) \leq f\left(\Pi^{(k)}\right) + c\beta^m \frac{\partial}{\partial \alpha} f\left(\mathcal{P}\left[\Pi^{(k)} + \alpha \mathbf{P}^{(k)}\right]\right) \Big|_{\alpha=0} \quad (\text{B25})$$

with the objective function f , $\beta = \frac{3}{4}$, and $c = \frac{1}{10}$ is used for the step length $\alpha^{(k)} = \beta^m$ [27]. For the second stage also the non-negativity conditions, $\Pi_{i,n} \geq 0$, have to be satisfied explicitly which is fulfilled implicitly for the first stage. For the convergence criterion for the transition from stage 1 to stage 2, the criterion

$$|\partial_{\alpha} f\left(\mathcal{P}_{\mathcal{S}^M}\left[\Pi^{(k)} + \alpha \mathbf{P}^{(k)}\right]\right)| \leq 10^{-4} \quad (\text{B26})$$



is chosen. The convergence criterion for the second stage and, thus, for the overall algorithm is derived from the KKT-conditions [37] of the minimization problem and defined as

$$\sqrt{\frac{1}{NM} \sum_{n=0}^{N-1} \sum_{i=0}^{M-1} (\Pi_{i,n} \cdot (\partial_{\Pi_{i,n}} f(\Pi) + \lambda_i))^2} \leq \epsilon_{\text{KKT}}, \quad (\text{B27})$$

where $\lambda_i = \max(0, -\min_n \partial_{\Pi_{i,n}} f(\Pi))$ is the estimate for the Lagrange multipliers of the sum-constraints.

B.3. Convergence of the two-stage approach

We investigate the convergence behavior of the proposed method with the experimental measurement data, i.e. with a Hilbert space cutoff of $M = 1210581$, $N = 151$ outcomes, and $D = 1076$ probe states.

The effectiveness of the proposed two-stage approach can be seen by comparing the convergence speed when only using the second stage, i.e. Bertseka's two-metric projected Newton method in contrast to the two-stage approach depicted in figure 9. While the convergence close to the minimum is very similar by construction, replacing the first iterations with the modified variant drastically accelerates convergence when far away from the minimum.

Appendix C. Computation of Wigner functions

The computation of Wigner functions for high photon numbers is numerically challenging due to the high absolute values occurring during the calculation. For the high photon numbers in this work, the intermediate values can easily exceed the maximal values that are representable with conventional double-precision floating-point numbers. Thus, we have set up a variant [38] by generalizing the implementation of the Wigner function available in QuantumOptics.jl [48] to use arbitrary-precision floating-point numbers via the BigFloat data type in Julia which uses the GNU MPFR library [49].

Computations with arbitrary-precision floating-point numbers required for the computation of the Wigner function are much more expensive than double-precision floating-point numbers because no direct hardware implementation is available. We have optimized the implementation of the Wigner function for phase-insensitive detectors, i.e. diagonal density matrices, and have implemented a trivial parallelization via MPI to scale the computation of the Wigner function to many CPU cores/compute nodes [38].

The arbitrary-precision floating-point numbers in MPFR are represented as the product of a sign $s \in \{-1, 1\}$, a 64-bit exponent e , and an arbitrary-sized fixed-point mantissa m as

$$s \cdot 2^e \cdot m. \quad (\text{C1})$$

The important ingredient is the very large range $\approx 10^{262} \approx 10^{10^{18}}$ of numbers that can be represented compared to the range of 64-bit floating-point numbers with a range of $\approx 10^{307}$. The bit size of the mantissa can be varied. For the Wigner functions in this work, mantissa sizes of about 60-70 bits are sufficient in the sense that larger mantissa sizes yield binary-identical Wigner functions when the final result is converted to double-precision floating-point numbers.

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Information extraction in photon-counting experiments

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We demonstrate a comparison of different multiplexing architectures based on quantum detector tomography. Using the purity of their measurement outcomes, we gain insight into the photon-number-resolving ability of the devices. Further, we calculate the information each measurement outcome can extract from a Hilbert space with given dimension. Our work confirms that more multiplexing outcomes enable higher photon-number-resolving ability; however, the splitting between those outcomes must be optimized as well.

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I. INTRODUCTION

The ability to count photons is crucial for quantum optical experiments and technologies, such as quantum metrology [1], quantum information [2], and single-photon imaging [3]. Many types of single-photon detectors have been demonstrated, all with certain advantages and disadvantages. The quality of these detectors can be quantified by many figures of merit, such as efficiency, dark counts, or timing resolution [4]. However, the ability of a detector to resolve the number of photons is a crucial quantifier in photon-counting experiments.

Resolving the photon number can be fully achieved using, for example, superconducting transition-edge sensors (TESs) [5]. For monochromatic light, a TES has full photon-number-resolving ability, which means that a specific photon number maps to a specific output signal. Some information about photon number can be obtained using click detectors such as superconducting nanowire single-photon detectors (SNSPDs) or single-photon avalanche diodes in combination with multiplexing schemes. However, multiplexing can only achieve quasi-photon-number resolution, as some information is lost due to the nonzero probability of multiple photons causing the same outcome [6].

Many different multiplexing architectures have been shown. Some of these schemes rely on equally splitting the incoming light onto the detectors, such as the conventional spatial [7] and temporal [8,9] multiplexing trees or detector arrays [10]. Others make use of logarithmic multiplexing such as the time-loop detectors [11] or integrated in-line detector arrays [12].

Multiplexed detectors can be parameterized by figures of merit such as efficiency, dark counts, and crosstalk [4,13]. Although these quantities provide intuition of the quality of the detectors, they do not quantify how these figures of merit combine to determine the utility of the devices for certain

tasks. The photon-number-resolving ability of the device is one important example which is not directly described by these figures of merit.

To overcome this limitation, van Enk introduced the concept of measurement outcome purity [14], which can be directly linked to the ability of a detector architecture to resolve photon number. A related question arises: How much information can be extracted by a measurement outcome? In this paper we experimentally investigate measurement outcome purity and information extraction for a variety of different detectors to quantify and compare their photon-number-resolving ability.

II. MEASUREMENT OUTCOME PURITY

The specific detector architectures we consider are shown in Figs. 1(a)–1(d). In order to compare the different devices in a common framework, we use quantum detector tomography [16], which yields a quantum mechanical description of a detector under test (DUT) in terms of its so-called positive-operator-valued measures (POVMs). The set of POVMs $\{\pi_n\}$ fully describes the detector with different outcomes n . The operators are non-negative $\pi_n > 0$ with $\sum_n \pi_n = 1$.

An important property of a general detection scheme is that repeated measurements do not necessarily yield the same outcome, i.e., a measurement may not necessarily project onto pure states. van Enk [14] showed that it is thus possible to define a measurement outcome purity of the POVM corresponding to outcome n , which is analogous to the purity of a quantum state, as

$$\text{Pur}(\pi_n) = \frac{\text{Tr}[(\pi_n)^2]}{[\text{Tr}(\pi_n)]^2}. \quad (1)$$

The purity is upper bounded by unity, as a perfect measurement would be a one-to-one mapping of one input state to one outcome. The lower bound is given by the Hilbert space dimension M as $\frac{1}{M} \leq \text{Pur}(\pi_n) \leq 1$. A nonpure POVM means that multiple orthogonal input states contribute to the same outcome with significant probabilities [14]. It is possible to estimate the number of orthogonal input states which contribute to outcome n by the inverse of the purity $\text{Pur}(\pi_n)^{-1}$. This directly follows from the lower bound of the purity, where the

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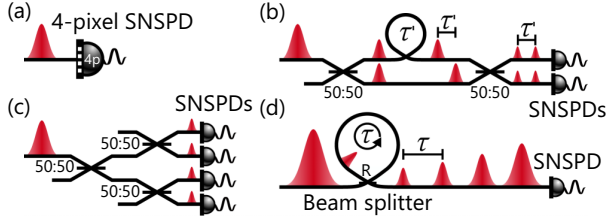


FIG. 1. Schematic representations of (a) a commercially available four-pixel (2x2) SNSPD array, (b) a four-bin time-multiplexed detector (eight bins can be achieved with an additional fiber loop of length $2\tau'$ and beam splitter), (c) a four-bin spatially multiplexed detector, and (d) a ten-bin time-multiplexed loop detector with out-coupling R , bin separation τ , and loop efficiency η_{loop} , recently shown in [15].

minimal purity coincides with a contribution from every input state in the Hilbert space [17].

The investigated multiplexing schemes [shown in Figs. 1(a)–1(d)] have different dynamic ranges, which means they are sensitive to different numbers of photons. This translates to different Hilbert space dimensions M in the POVM description. Above a certain photon number, the POVMs of a detector will not change. This can directly be seen in the outcome statistics, as with increasing mean photon numbers, the detector will only respond with the largest outcome. Therefore, the POVM corresponding to that outcome will be saturated (occur with unit probability), while the other POVMs converge to zero probability. This can be seen in Fig. 2, which shows the POVM elements for the four-bin time-multiplexed detector (TMD) as an example.

In order to compare the detector architectures, we use a fixed Hilbert space dimension of $M = 5000$ without loss of generality, while assuming that the outcome statistics will not change after saturation of the largest outcome. We use Eq. (1) to calculate the purities of the detectors, which are shown in Fig. 3. It can be seen that the detectors with four bins have comparable purities. This is expected, as the choice in the

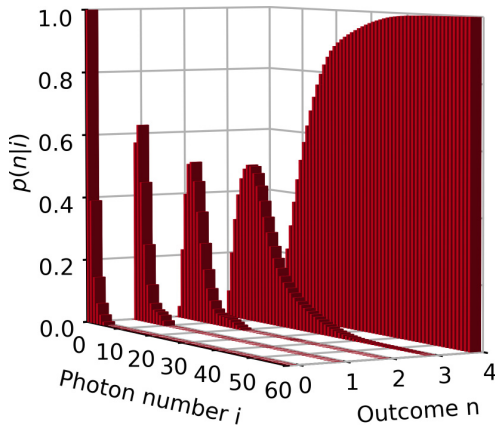


FIG. 2. Diagonal elements of the experimentally reconstructed POVM operators in the photon-number basis for the four-bin time-multiplexed detector, as an example to give a sense of the narrowness of outcomes and show saturation of the largest outcome.

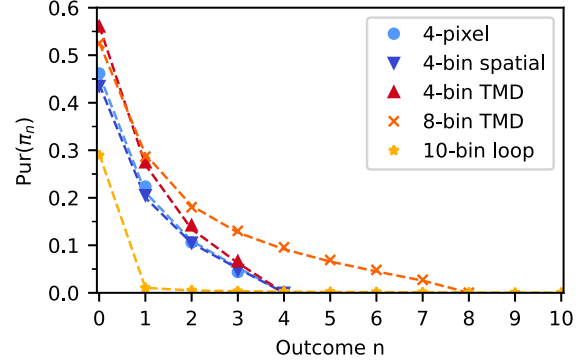


FIG. 3. Purities $\text{Pur}(\pi_n)$ of the five multiplexed detectors per outcome n . Different colors and markers correspond to experimental data of the different multiplexing schemes, as labeled in the legend. Dashed lines correspond to modeled data (see the text for details).

degree of freedom of multiplexing (in space or time) should not provide any significant advantages with regard to photon-number resolution. The small differences can be explained by slightly different performance metrics of the detectors (summarized in Table I), with the efficiency being the main contribution, as the purity suffers most for decreasing efficiencies [17]. Figure 3 also shows that the outcome purities of the eight-bin TMD are higher compared to the four-bin TMD. This makes sense, since fewer input states will contribute to each individual outcome, as more outcomes are available in the eight-bin TMD. Narrower POVM distributions imply higher purities.

We verified this behavior by implementing a model of multiplexed click detectors by Miatto *et al.* [18]. We modeled the equal splitting devices (four-pixel, four-bin, and eight-bin TMD and four-bin spatial) by solely their efficiencies (i.e., neglecting noise), which we extracted from the experimental data. The ten-bin loop detector model is based on previous work [19].

The measurement outcome purities closely agree with the experimental data, as can be seen by the dashed lines with corresponding color in Fig. 3. However, having more available outcomes does not directly imply higher outcome purities, as can be seen by the purities of the ten-bin time-multiplexed loop detector in Fig. 3. This is due to the different multiplexing scheme. The device is based on logarithmic multiplexing,

TABLE I. Figures of merit (efficiency η , dark-count probability p_{dark} , and crosstalk probability p_{stalk}) of the five different multiplexed detectors, obtained from the POVMs as in Ref. [21]. The errors are based on assuming 5% uncertainty in the amplitudes of the coherent states.

Detector	η (%)	p_{dark} (%)	p_{stalk} (%)
four-pixel	63 ± 4	$(5.9 \pm 1.6) \times 10^{-4}$	14 ± 1
four-bin spatial	61 ± 4	$(5.4 \pm 1.8) \times 10^{-4}$	3.6 ± 0.1
four-bin TMD	72 ± 4	$(1.6 \pm 0.1) \times 10^{-4}$	$< 2 \times 10^{-4}$
eight-bin TMD	69 ± 4	$< 10^{-5}$	$(4.3 \pm 1.1) \times 10^{-5}$
ten-bin loop	44 ± 3	$< 4.1 \times 10^{-5}$	$(4.6 \pm 2.8) \times 10^{-4}$

while the other architectures rely on equal splitting. The ten-bin loop detector responds logarithmically to the number of photons, which results in broad POVM distributions (many orthogonal input states contribute to a given outcome) and thus in poor purities.

III. INFORMATION EXTRACTION

The purity of a photon-counting detection outcome can be intuitively regarded as the photon-number-resolving ability of the device or the ability of the detector to distinguish between different photon-number states. A related concept is information extraction, namely, how much information about the photon number can be obtained given a specific outcome. Here van Enk [14] introduced the entropy

$$H^{(n)} = - \sum_i p(i|n) \log_2[p(i|n)], \quad (2)$$

which quantifies the information (in bits) about the photon number i that is still missing after having obtained the outcome n . This quantity uses the conditional probability $p(i|n)$, which describes the probability of having a photon number equal to i given a measurement outcome n .

We are able to obtain the probability $p(i|n)$, utilizing the POVMs of the detectors. A single element of the POVM π_n is the conditional probability $p(n|i)$ of n clicks occurring given i incident photons. Using Bayes' theorem

$$p(i|n) = \frac{p(n|i)p(i)}{p(n)} = \frac{p(n|i) \sum_{i,n} p(n|i)}{M \sum_i p(n|i)}, \quad (3)$$

it is possible to obtain the probability $p(i|n)$ directly from the POVMs. Here we assume a flat prior probability $p(i) = \frac{1}{M}$, meaning that all input states are equally likely.

From Eq. (2) we can calculate the information that can be extracted by a certain outcome n about the photon number i as

$$H_{\text{extr}}^{(n)} = H_{\text{total}}(M) - H^{(n)}, \quad (4)$$

where $H_{\text{total}}(M) = -\log_2(\frac{1}{M}) = \log_2(M)$ is the total information contained in the Hilbert space of dimension M . Using Eq. (4), we calculate the amount of information that can be extracted from the Hilbert space of size $M = 5000$. The total information available is calculated to $H_{\text{total}}(5000) = 12.3$ bits, which is shown by the black dashed line in Fig. 4. Having similar performance metrics and outcome purities, the detectors with four bins extract the same amount of information from the Hilbert space. Figure 4 also shows that the outcomes of the eight-bin TMD extract the most information. This directly follows from the argument that more available outcomes lead to narrower POVMs and make each outcome more specific. If a narrow outcome is observed, more knowledge about the photon number can be extracted, as fewer photon-number states (or orthogonal input states) contribute to this outcome. This also explains the sudden drop in the extracted information for the largest outcome of the detectors based on equal splitting. These detectors have a low dynamic range (roughly $M \approx 100$) compared to the maximum Hilbert space of size $M = 5000$. This means that obtaining the largest outcome reveals very little about the possible input photon number, due to the saturation of the outcome (compare Fig. 2).

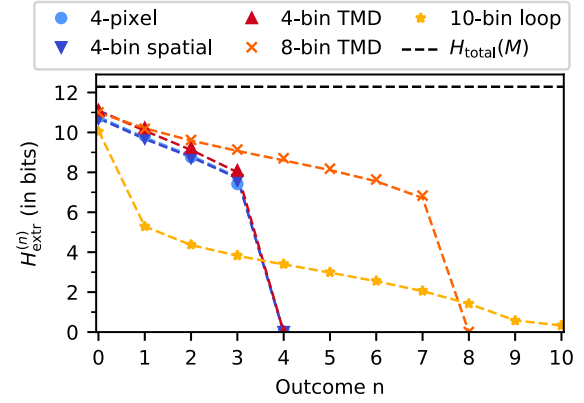


FIG. 4. Extracted information $H_{\text{extr}}^{(n)}$ in bits of the five multiplexed detectors per outcome n . The total available information in the Hilbert space of dimension $M = 5000$ is shown by the black dashed line. Different colors and markers correspond to experimental data of the different multiplexing schemes, as labeled in the legend. Dashed lines correspond to modeled data (see the text for details).

The outcomes of the ten-bin loop detector, based on logarithmic multiplexing, span the Hilbert space with broad POVM distributions. This leads to less information extracted per outcome. However, even the largest outcome is able to extract some information from the total available Hilbert space, even for large input states with hundreds to thousands of photons. From this, it is clear that solely increasing the number of multiplexing outcomes will not yield better photon-number resolution. Optimizing the photon-number-resolving ability of a device requires optimizing the splitting between multiplexed detectors [17].

IV. EXPERIMENTAL METHODS

Calculating the purity and information extraction relies on a tomographic reconstruction of each detector. We summarize the methods here; more detail can be found in Refs. [16,19–21].

In order to reconstruct the POVMs, the detector under test needs to be subjected to a set of input states

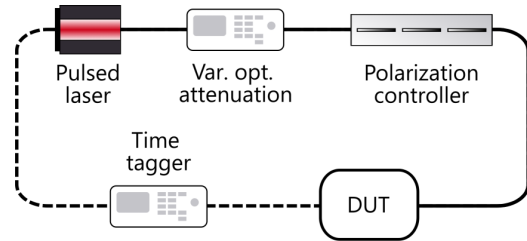


FIG. 5. Experimental setup for the detector tomography analysis. A 1556-nm pulsed laser produces coherent states, which can be attenuated by variable optical attenuators. The polarization can be controlled to optimize detection efficiencies for the polarization-dependent SNSPDs, which are connected next. A time tagger records the electrical responses from the detectors. Solid and dashed lines represent optical and electrical connections, respectively.

TABLE II. Total counts in thousands with corresponding uncertainties for the four-pixel detector.

$ \alpha ^2$	$n = 1$	$n = 2$	$n = 3$	$n = 4$
333(22)	4990.2(23)	4992.5(23)	4993.2(23)	4994.2(23)
290(19)	4997.7(23)	4991.2(23)	4993.1(23)	4992.2(23)
253(17)	4991.0(23)	4993.6(23)	4992.5(23)	4993.7(23)
221(15)	4991.2(23)	4992.8(23)	4991.6(23)	4993.1(23)
193(13)	4990.6(23)	4991.3(23)	4993.9(23)	4993.9(23)
168(11)	4999.7(23)	4992.1(23)	4992.6(23)	4991.7(23)
147(10)	4990.8(23)	4992.1(23)	4992.9(23)	4991.6(23)
123(8)	4989.7(23)	4994.0(23)	4991.6(23)	4992.9(23)
102(7)	4990.6(23)	4993.4(23)	4992.2(23)	4991.9(23)
82(6)	4990.8(23)	4993.2(23)	4992.3(23)	4991.4(23)
65(5)	4990.8(23)	4993.7(23)	4992.5(23)	4984.1(23)
50(4)	4990.5(23)	4993.3(23)	4993.3(23)	4954.9(23)
36.0(24)	4999.0(23)	4993.0(23)	4993.0(23)	4844.9(23)
25.1(17)	4988.0(23)	4992.5(23)	4981.1(23)	4508.6(22)
15.9(11)	4991.8(23)	4984.4(23)	4840.5(23)	3587.7(19)
9.2(6)	4979.7(23)	4832.5(22)	4037.9(21)	2014.8(15)
4.09(27)	4617.3(22)	3490.0(19)	1747.4(14)	445.4(7)
1.00(7)	2340.0(16)	845.2(10)	148.8(4)	17.46(14)
0	1.90(5)	0.150(13)	0(1)	0(1)

spanning the Hilbert space of the given device. Coherent states are an ideal choice, as they form a tomographically complete set [16]. Here we compare four different multiplexing schemes (five detectors) in total: a commercially available four-pixel SNSPD array, a four-bin (and eight-bin) TMD, a four-bin spatially multiplexed detector, and finally a ten-bin time-multiplexed loop detector, which relies on a logarithmic time-multiplexing architecture enabling a high dynamic range over 120 dB [15] [all shown in Figs. 1(a)–1(d)]. The experimental setup, shown schematically in Fig. 5, begins with a pulsed laser producing coherent states with a wavelength of 1556 nm. The mean photon number can be controlled by variable optical attenuators and the polarization can be set to optimize the detection efficiency of the polarization-dependent SNSPDs. Afterward, the different multiplexing

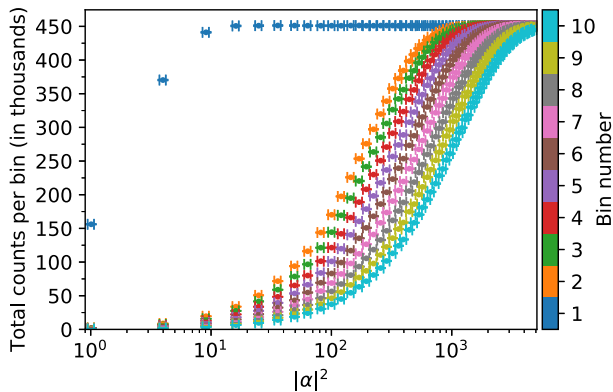


FIG. 6. Total counts per bin for the ten-bin time-multiplexed loop detector vs mean photon number of the coherent probe states with corresponding uncertainties. The uncertainty in the total counts is too small to resolve.

TABLE III. Total counts in thousands with corresponding uncertainties for the four-bin spatially multiplexed detector. Here D_1 – D_4 correspond to the four detectors.

$ \alpha ^2$	D_1	D_2	D_3	D_4
122(8)	4997.5(23)	4997.5(23)	4997.5(23)	4997.1(23)
102(7)	4997.8(23)	4997.8(23)	4997.8(23)	4997.4(23)
81(6)	4998.2(23)	4998.4(23)	4998.4(23)	4998.1(23)
65(5)	4996.6(23)	4998.1(23)	4998.4(23)	4997.9(23)
49(4)	4986.1(23)	4994.3(23)	4998.1(23)	4995.8(23)
35.9(23)	4931.0(23)	4968.7(23)	4993.8(23)	4978.8(23)
25.0(17)	4760.6(22)	4863.6(23)	4957.2(23)	4898.7(23)
16.6(11)	4345.7(21)	4551.4(22)	4793.5(22)	4631.0(22)
9.2(6)	3365.1(19)	3676.5(20)	4143.8(21)	3813.0(20)
4.08(27)	1923.0(14)	2194.5(15)	2676.4(17)	2326.7(16)
1.05(7)	590.3(8)	695.4(9)	900.7(10)	751.5(9)
0	0.101(11)	0.134(12)	0.348(19)	1.44(4)

schemes are connected and the outcome statistics are recorded by a time tagger in a few-nanosecond-wide coincidence window. The mean photon numbers of the coherent input states are chosen to scale quadratically to efficiently span the Hilbert space of the different detectors. Different numbers of input states are necessary, as the Hilbert space dimension depends on the detector under test.

The raw data (see the Appendix for total counts for all different detectors) are converted to the outcome statistic matrix $\mathbf{P}$, depending on the form of the acquired counts. This conversion is described in detail in Ref. [21] for the four-pixel detector, in Ref. [22] for the four-bin spatially multiplexed detector, and in Ref. [19] for the ten-bin loop detector. For the four- and eight-bin TMDs, the total counts (given in Tables IV and V, respectively) are divided by the total number of input pulses to obtain $\mathbf{P}$. After knowing the input states, typically represented by a matrix $\mathbf{F}$, it is possible to reconstruct the POVM matrix $\mathbf{\Pi}$ of the different multiplexed detectors. This is done using the Born rule, which allows one to formulate the matrix equation $\mathbf{P} = \mathbf{F}\mathbf{\Pi}$ and a subsequent matrix inversion routine [23–25].

TABLE IV. Total counts in thousands with corresponding uncertainties for the four-bin TMD.

$ \alpha ^2$	$n = 0$	$n = 1$	$n = 2$	$n = 3$	$n = 4$
49(4)	0(1)	0(1)	0(1)	0.046(7)	100.0(4)
40.1(26)	0(1)	0(1)	0(1)	0.210(15)	99.8(4)
31.3(21)	0(1)	0(1)	0.007(2)	1.17(4)	98.8(4)
23.9(16)	0(1)	0(1)	0.071(9)	4.44(7)	95.5(4)
17.9(12)	0(1)	0.019(5)	0.686(27)	13.01(12)	86.3(3)
12.2(8)	0.010(4)	0.359(19)	5.13(8)	30.32(18)	64.18(26)
8.0(6)	0.269(17)	3.62(7)	19.01(14)	42.25(21)	34.85(19)
4.45(29)	4.08(7)	20.17(15)	36.92(20)	29.98(18)	8.85(10)
1.99(13)	24.10(16)	41.00(21)	26.44(17)	7.58(9)	0.839(29)
0.49(4)	70.20(27)	25.90(17)	3.72(7)	0.190(14)	0.005(2)
0.050(4)	96.4(4)	3.51(6)	0.055(8)	0(1)	0(1)
0.0051(4)	99.6(4)	0.377(20)	0.002(1)	0(1)	0(1)
0.00052(4)	100.0(4)	0.042(7)	0(1)	0(1)	0(1)
0	100.0(4)	0(1)	0(1)	0(1)	0(1)

TABLE V. Total counts in thousands with corresponding uncertainties for the eight-bin TMD.

$ \alpha ^2$	$n = 0$	$n = 1$	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 6$	$n = 7$	$n = 8$
98(7)	0(1)	0(1)	0(1)	0(1)	0(1)	0(1)	0(1)	0.102(11)	99.9(4)
80(6)	0(1)	0(1)	0(1)	0(1)	0(1)	0(1)	0(1)	0.558(24)	99.4(4)
63(5)	0(1)	0(1)	0(1)	0(1)	0(1)	0.001(1)	0.033(6)	2.82(6)	97.1(4)
48(4)	0(1)	0(1)	0(1)	0(1)	0(1)	0.012(4)	0.459(22)	9.8(1)	89.7(3)
35.7(23)	0(1)	0(1)	0(1)	1(1)	0.013(4)	0.305(18)	3.66(7)	24.84(16)	71.18(27)
24.4(16)	0(1)	0.001(1)	0.005(2)	0.067(9)	0.686(27)	4.57(7)	17.81(14)	39.2(2)	37.7(2)
15.9(11)	0.002(1)	0.035(6)	0.304(18)	2.09(5)	8.2(1)	20.36(15)	31.23(18)	27.42(17)	10.320(11)
8.9(6)	0.198(15)	2.00(5)	8.3(1)	19.00(14)	26.89(17)	24.63(16)	13.83(12)	4.53(7)	0.610(25)
3.97(26)	6.39(8)	21.44(15)	30.26(18)	24.51(16)	12.41(12)	4.08(7)	0.807(29)	0.10(1)	0.003(1)
0.99(7)	50.80(23)	35.84(19)	11.18(11)	1.93(5)	0.230(16)	0.015(4)	0(1)	0(1)	0(1)
0.100(7)	93.3(4)	6.48(9)	0.202(15)	0.002(1)	0(1)	0(1)	0(1)	0(1)	0(1)
0.0102(7)	99.3(4)	0.692(27)	0.006(2)	0(1)	0(1)	0(1)	0(1)	0(1)	0(1)
0.00104(5)	99.9(4)	0.073(9)	0(1)	0(1)	0(1)	0(1)	0(1)	0(1)	0(1)
0	100.0(4)	0(1)	0(1)	0(1)	0(1)	0(1)	0(1)	0(1)	0(1)

V. CONCLUSION

We have shown that quantum detector tomography is the basis to enable a comparison of different multiplexing schemes. The outcome purity, which can be calculated directly from the POVM elements of given detection outcomes, indicates how pure a certain outcome is. Quantum detector tomography enables a mapping to photon-number states; thus the purity gives insight into the photon-number-resolving ability of the devices. Narrow POVM distributions are purer, in the sense that fewer photon-number states contribute to the given outcome. This directly translates into the amount of information the outcome can extract from the total available Hilbert space. More information can be extracted by narrow POVM distributions. Outcomes of multiplexing architectures based on equal splitting, such as standard spatial or temporal multiplexing, show significantly better purities and can extract larger amounts of information compared to the logarithmic multiplexing scheme of the ten-bin loop detector. Nevertheless, logarithmic multiplexing enables a high dynamic range, which gives information about large photon numbers, where the equal splitting schemes are already saturated. Our analysis further confirms that more multiplexing outcomes enable higher photon-number-resolving ability [6,26,27], but also that the splitting between those outcomes must be optimized.

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APPENDIX: MEASUREMENT DATA

The total counts of the five different multiplexing architectures are shown in Tables II–V and Fig. 6. The uncertainty of the mean photon number of the coherent input states is based on assuming 5% uncertainty for the calibration detector efficiency. The uncertainty of the total counts is calculated using Poisson statistics, i.e., the square root of the total counts. The uncertainty of the total counts is negligible in comparison to the uncertainty in the coherent state mean photon number.

For the four-pixel detector and the four-bin spatially multiplexed detector, data were recorded for 10 s at a repetition rate of 500 kHz, leading to a maximum of 5×10^6 total counts in Tables II and III. For the four-bin and eight-bin TMDs, data were recorded for 1 s at a repetition rate of 100 kHz, leading to maximum of 1×10^5 total counts in Tables IV and V. For the ten-bin time-multiplexed loop detector, data was recorded for 30 s at a repetition rate of 15 kHz, leading to a maximum of 4.5×10^5 total counts in Fig. 6.

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Photon-number resolution with SNSPDs

3

Many experiments in quantum optics as well as photonic quantum technologies need access to the photon-number statistics of an optical field. Photon-number-resolving (PNR) detectors are therefore essential for quantum optical state characterization [16] as well as quantum imaging and sensing [21]. PNR detection is a key enabling technology for quantum communication [22, 23], photonic quantum computing [24, 25], and quantum metrology [26]. It is also crucial for tasks such as heralding nonclassical states [17–20], optimizing single-photon sources based on spontaneous parametric down-conversion or spontaneous four-wave mixing [106, 107], and improving the performance of computational paradigms such as Gaussian boson sampling [11, 12, 123].

Despite this broad relevance, the practical meaning of photon-number resolution is not as simple as the term suggests. Conceptually, photon counting appears straightforward: the detector returns the exact number of incident photons. In practice, however, it is much more difficult to realize and quantify. Users of PNR detectors are interested in the discrete photon number, while typical measurement outcomes are continuous in nature and subject to noise (e.g., the voltage or timing information from electrical output traces). This has the important implication that different (especially neighboring) photon numbers have non-negligible overlap in their outcome distributions, leading to potential ambiguity in photon assignment. Hence, the at first straightforward task of photon counting raises the nontrivial question [93]: How many photons can the detector count and more importantly with what resolution?

This question is particularly relevant for SNSPDs. As established in Chapter 1, SNSPDs combine excellent figures of merit with the possibility of retrieving photon-number information from their electrical response. Fundamentally, an SNSPD is not an energy-resolving detector. Instead, its PNR capability can be understood by viewing the superconducting nanowire as a highly multiplexed system of effective hotspot detection regions [39, 114]. If several photons are absorbed within a sufficiently short time window and at different positions along the nanowire, multiple resistive hotspots can form. The resulting resistance, and therefore the electrical output signal, can then depend on the number and dynamics of the absorbed photons. This intrinsic form of multiplexing provides the physical basis for what has become known as intrinsic PNR with SNSPDs [109, 112, 113, 115].

The aim of this chapter is to *understand intrinsic PNR with SNSPDs*. This entails the systematic investigation of electrical traces to identify photon-number-dependent features, and determining how the relevant information can be extracted using as few data points as possible (see Section 3.1 or Publication 3.1). It further requires modeling the photon-number-dependent traces of SNSPDs, including underlying jitter contributions (see Section 3.2 or Publication 3.2) and a careful investigation of photon number assignment (see Section 3.3 or Publication 3.3). These steps will provide a better physical understanding of intrinsic PNR SNSPDs. Section 3.4 extends the state of the art of intrinsic PNR SNSPDs, which started in Chapter 1, to provide an overview of the methods that have been used to retrieve photon-number information from SNSPDs.

3.1 Locating photon-number information on SNSPD electrical traces

Prior to this work, many techniques for retrieving photon-number information from SNSPDs have been investigated, using different features of the electrical output trace [39, 61, 91, 101, 104, 106–116] (an overview is provided in Chapter 1 and Section 3.4). This motivates a systematic investigation of the complete electrical trace to identify where photon-number information is encoded. With sufficient measurement bandwidth, full waveform acquisition can access all information contained in the electrical trace of a detection event. Publication 3.1 applies principal component analysis (PCA) to a data set of electrical SNSPD traces to identify which features vary with the incident mean photon numbers. This work establishes the signal-analysis basis for PNR with SNSPDs.

Publication 3.1 is reprinted from Timon Schapeler, Niklas Lamberty, Thomas Hummel, Fabian Schlue, Michael Stefszky, Benjamin Brecht, Christine Silberhorn and Tim J. Bartley, Electrical trace analysis of superconducting nanowire photon-number-resolving detectors, *Physical Review Applied* **22**, 014024 (2024), <https://doi.org/10.1103/PhysRevApplied.22.014024>. This article is licensed under a Creative Commons Attribution 4.0 International (CC BY 4.0) license: <https://creativecommons.org/licenses/by/4.0/>. No changes were made.

3.2 Modeling photon-number-dependent jitter

Publication 3.1 motivates arrival-time-based methods with a constant-threshold time-to-digital converter (time tagger) as a particularly suitable way to retrieve photon-number information with a minimal number of data points. Arrival-time-based PNR relies on the separation of photon-number-dependent timing distributions, whose widths are determined by several jitter mechanisms. Publication 3.2 develops a physical model for photon-number-dependent jitter in SNSPDs and shows that arrival-time histograms can be described using a sum of exponentially-modified Gaussian distributions, over orders of magnitude in probability and a broad range of photon fluxes. This provides physical interpretation of the timing distributions used for photon-number assignment, informs future improvements in the PNR capability, and can enable detector optimization for specific applications [112, 114].

Publication 3.2 is reprinted from Mariia Sidorova, Timon Schapeler, Alexej D. Semenov, Fabian Schlue, Michael Stefszky, Benjamin Brecht, Christine Silberhorn and Tim J. Bartley, Jitter in photon-number-resolved detection by superconducting nanowires, *APL Photonics* **10**, 086113 (2025), <https://doi.org/10.1063/5.0273752>. This article is licensed under a Creative Commons Attribution-NonCommercial 4.0 International (CC BY-NC 4.0) license: <https://creativecommons.org/licenses/by-nc/4.0/>. No changes were made.

3.3 Characterizing photon-number assignment

A practical PNR detector must not only show photon-number-dependent signal features, but also provide reliable information by quantifying the uncertainty of photon-number assignment. The preceding sections have shown that this is particularly relevant for SNSPDs, where photon-number information is inferred from continuous and noisy measurement outcomes. Assigning certain

regions of the continuous measurement outcome distribution to specific photon numbers can lead to misidentification of photon numbers [106–108]; careful post-selection of data could be used to reduce misassignment, at the cost of discarding data due to invalid detection events [6, 111, 115]. Building on the model from the previous section (Publication 3.2), it is possible to quantify possible misidentification in arrival-time-based photon-number assignment. In addition, Publication 3.3 studies the influence of optical pulse duration on the ability to discriminate photon numbers. This work advances the chapter’s progression toward the aim of *understanding intrinsic PNR with SNSPDs*. A full understanding of SNSPD-based PNR will require continued efforts, including specialized measurements and analysis methods tailored to the intended application. Nevertheless, this work connects the physical understanding of PNR to a quantitative detector-characterization framework.

Publication 3.3 is reprinted from Timon Schapeler, Isabell Mischke, Fabian Schlue, Michael Stefszky, Benjamin Brecht, Christine Silberhorn and Tim J. Bartley, Practical considerations for assignment of photon numbers with SNSPDs, *APL Quantum* 3, 016102 (2026), <https://doi.org/10.1063/5.0304127>. This article is licensed under a Creative Commons Attribution 4.0 International (CC BY 4.0) license: <https://creativecommons.org/licenses/by/4.0/>. No changes were made. The supplementary material can be found under the following DOI: <https://doi.org/10.60893/figshare.aip.c.8197994>.

3.4 PNR SNSPD methods – continued

In addition to the methods for retrieving photon-number information discussed in Chapter 1, which had been demonstrated prior to this work, a number of other methods were demonstrated contemporaneously to the work presented in this thesis (Publication 3.1, Publication 3.2 and Publication 3.3). This section serves as a continuation of the state of the art of intrinsic PNR SNSPDs.¹

Using micron-wire detectors, Kong et al. [111] showed PNR up to ten photons. This improvement over previous detectors is achieved by a higher inductance of the wire, which increases the rise time of the traces. The increased rise time leads to better distinguishability of different photon-number events. The authors implemented a real-time readout of the rising edge time by splitting the electrical signal and measuring at a low and high voltage level using a time-to-digital converter (see Fig. 3.1(a)). Stretching out the rising edges has the disadvantage of an overall slower signal and thus an increased reset time of the detector. This shows the trade-off between improved PNR and maximum detector count rate for timing-based PNR readouts.

Specifically exploring the impact of kinetic inductance on PNR performance of their devices, Jaha et al. [113] used waveguide-integrated SNSPDs. They found that increasing the kinetic inductance of the detector improves PNR, as it increases the distance between photon-number peaks in arrival-time measurements relative to a trigger signal (see Fig. 1.1(e)). Again, this improvement of PNR comes at the cost of an increased reset time, leading to a reduced maximum count rate of the detector, which is consistent with findings from Kong et al. [111].

¹This section builds upon work originally published in Timon Schapeler, Fabian Schlue, Michael Stefszky, Benjamin Brecht, Christine Silberhorn, and Tim J. Bartley, Optimizing photon-number resolution with superconducting nanowire multi-photon detectors, *Proc. SPIE* 13448, Advanced Photon Counting Techniques XIX, 134480C (2025), <https://doi.org/10.1117/12.3054905>. The original text has been modified and expanded to include up-to-date references.

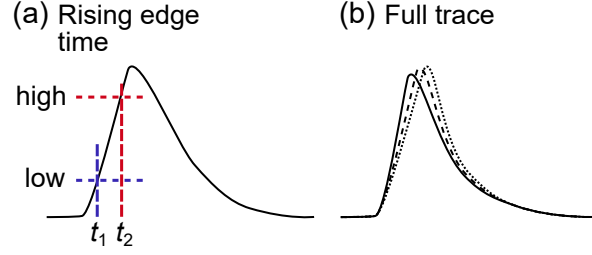


Figure 3.1: Schematic representation of the different techniques to retrieve photon-number information from the electrical output signals of SNSPDs. (a) The rising-edge time is measured using a time-to-digital converter at two threshold levels. (b) A set of electrical traces of the SNSPD is used for PCA. Adapted with permission from Ref. [93].

Many experiments operate at a wavelength of 1550 nm, with SNSPDs being optimized for this specific regime. With the aim to investigate a wavelength range important for, e.g., quantum dots and quantum memories, Los et al. [112] studied PNR with SNSPDs optimized for 850 nm to 950 nm. The authors utilized an oscilloscope to measure the distributions between rising edge of the SNSPD traces and a trigger signal (see Fig. 1.1(e)). Reducing the jitter of the SNSPD is crucial for improving PNR; as jitter reduces with increasing photon energy (shorter wavelength) [79, 124, 125], they achieved PNR up to seven photons at a wavelength of 1064 nm. This again shows that, in order to maximize the benefits of PNR SNSPDs, detectors need to be optimized for specific tasks.

Work from Kuijff et al. [116] builds upon work presented in Publication 3.1 and revealed that the first principal component of a principal component analysis (PCA) corresponds to the derivative of the mean response trace, which makes the result physically interpretable. They also show that this method (see Fig. 3.1(b)) can, in principle, be implemented on FPGA hardware, which is important for real-time and feedforward operation.

In contrast to timing-based PNR methods, impedance-matching tapers, originally used by Zhu et al. [104] and later by Colangelo et al. [109], have demonstrated their utility to retrieve photon-number information encoded in the amplitude of the electrical trace (see Fig. 1.1(b)). Recently, Kong et al. [61] utilized an impedance-matching taper in a 16-pixel detector array, designed to distinguish between all possible single- and two-photon detection events of their device. The authors leveraged signal-amplitude differences due to the impedance matching taper to distinguish between events where one or two photons impinge on the same pixel. This work showed that intrinsic PNR is complementary with multiplexing techniques and has great potential for improved PNR detection.

Lastly, Endo et al. [126] used the method of optical sampling to improve separation, and thus distinguishability, between the one- and two-photon components of a PNR SNSPD. The authors split a laser pulse into two and impinged one part into a dual-output Mach-Zehnder modulator. The other part can be used for the experiment and is finally detected by an SNSPD. The amplified output signal is then fed into the radio frequency (RF) input port of the modulator, where an RF delay line allows to optically sample the amplitude of the SNSPD signal with high temporal resolution. A balanced photodetector sampled the output of the modulator and gave rise to clearly distinguishable photon-number contributions in a voltage histogram.

All intrinsic PNR SNSPD methods covered in Chapter 1 as well as this chapter are consolidated in Table 3.1, where the methods are ordered chronologically based on the date of publication. Besides the detector type and a short description of the PNR technique, also the quoted PNR (maximum

resolvable photon number), the wavelength and the pulse duration of the laser pulses is given. The wavelength is important when comparing timing-based PNR methods, as shorter wavelengths lead to decreased jitter [79, 124, 125], which improves PNR. The optical pulse duration is also crucial for PNR, as longer optical pulses can mask photon-number information due to a large contribution to the overall jitter of the system (as discussed in Publication 3.2 and shown in Publication 3.3).

Besides the methods that rely on the intrinsic PNR capability of SNSPDs, alternative methods for retrieving photon-number information from SNSPDs should also be highlighted. Quasi-PNR can be achieved when multiple threshold detectors are configured in a spatial or temporal multiplexing architecture [7, 52–54, 56, 57]. These implementations rely on the probabilistic splitting of incident photons; however, with a nonzero probability that multiple photons impinge on the same threshold detector. Therefore, only partial photon-number information can be retrieved. Arrays of SNSPDs, where the active area of the device is split into multiple pixels, have been demonstrated with custom readout designs [58, 81, 127–133], yielding quasi-PNR and pixel-level readout. Increasing the number of pixels in the active area reduces the probability of two photons being absorbed by the same pixel, which improves the PNR capability of the device [81].

Table 3.1: Summary of the approaches for intrinsic PNR with SNSPDs. [†]The number of resolved photons as given by the authors. ^{*}The aim here was the discrimination of one and more than one photon. [‡]Here the optical pulse is carved from a continuous-wave laser source.

Source	Detector Type (Material)	PNR Technique	Quoted PNR up to [†]	Wave-length	Pulse Duration
Cahall et al. [101] (2017)	SNSPD (amorphous)	Amplitude of differentiated detection waveforms	four	1550 nm	80 ps [‡]
Nicolich et al. [91] (2019)	SNSPD (amorphous)	Electrothermal model for turn-on dynamics	-	1550 nm	80 ps [‡]
Zhu et al. [104] (2020)	STaND (NbN)	Amplitude of the output pulses	five	1550 nm	33 ps
Endo et al. [39] (2021)	SNSPD (NbTiN)	Rising edge waveform pattern matching	five	772 nm	~ 1 ps
Sempere et al. [106] (2022)	Commercial SNSPD	Fitting rising edge slope	two [*]	1550 nm	1 ps
Davis et al. [107] (2022)	SNSPD (NbN)	Rising edge arrival-time difference to trigger signal	two [*]	1550 nm	600 ps [‡]
Colangelo et al. [109] (2023)	Tapered SNSPD (NbN)	Amplitude of the output pulses	three	775 nm, 1550 nm	1 ps
Sauer et al. [108] (2023)	Commercial SNSPD (NbTiN)	Rising and falling edge arrival-time difference to trigger signal	five	1554 nm	~ 300 fs
Schapeler et al. [110] (2024)	Commercial SNSPD (NbTiN)	PCA of full electrical trace; Rising and falling edge arrival-time difference to trigger signal	five	1550 nm	~ 3 ps
Kong et al. [111] (2024)	SMSPD (NbN)	Rising edge time	ten	1064 nm	< 1 ps
Los et al. [112] (2024)	SNSPD (NbTiN)	Rising edge arrival-time difference to trigger signal	seven	1064 nm	2.3 ps
Jaha et al. [113] (2024)	Waveguide-integrated SNSPD (NbTiN)	Rising edge arrival-time difference to trigger signal	three	1550 nm	100 fs
Kong et al. [61] (2025)	Tapered SNSPD array (NbN)	Amplitude of the output pulses	two	1550 nm	6 ps
Endo et al. [126] (2025)	SNSPD (NbTiN)	Optical sampling method	two	1545 nm	6 ps
Sidorova et al. [114] (2025)	Commercial SNSPD (NbTiN)	Multi-photon jitter model	-	1550 nm	1 ps
Schapeler et al. [115] (2026)	Commercial SNSPD (NbTiN)	Rising edge arrival-time difference to trigger signal	three	1550 nm	1 ps
Kuijf et al. [116] (2026)	Commercial SNSPD	PCA of full electrical trace	two	1551 nm	23 ps

Electrical trace analysis of superconducting nanowire photon-number-resolving detectors

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We apply principal component analysis (PCA) to a set of electrical output signals from a commercially available superconducting nanowire single-photon detector (SNSPD) to investigate their photon-number-resolving capability. We find that the rising edge as well as the amplitude of the electrical signal have the most dependence on photon number. Accurately measuring the rising edge while simultaneously measuring the voltage of the pulse amplitude maximizes the photon-number resolution of SNSPDs. Using an optimal basis of principal components, we show unambiguous discrimination between one- and two-photon events, as well as partial resolution up to five photons. This expands the use case of SNSPDs to photon-counting experiments, without the need of detector multiplexing architectures.

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I. INTRODUCTION

The ability to count photons is desirable in a great many quantum optics experiments and applications, not least quantum computing paradigms [1], quantum communication [2,3], and multiphoton metrology [4]. While some energy-resolving detectors function at the single-photon level, they are often slow devices with limited timing resolution [5]. Instead, single-photon detectors, which detect the presence or absence of photons but cannot resolve the photon number, can offer quasi-photon-number resolution when configured in multiplexing architectures [6–12] or array readout design [13–18]. Within this class of detectors, superconducting nanowire single-photon detectors (SNSPDs) have become the gold standard due their unmatched signal-to-noise ratio [19] and high timing resolution [20].

While long regarded as a threshold detector, careful analysis of the detector signal has recently demonstrated that they also show limited photon-number information in

their electrical output signal. The first evidence of photon-number resolution using an SNSPD has been presented by Cahall *et al.* [21] and was attributed to a time- and photon-number-dependent hot-spot resistance of the nanowire. By investigating the different rise times from the electrical output signal of the detector, the authors show evidence up to four-photon detection events. These findings are consistent with a generalized electrothermal model [22] based on Ref. [23]. This model links hot-spot growth and the rising edge of the electrical output signal of the detector. Another approach to gain photon-number resolution from SNSPDs has been presented by Zhu *et al.* [24], where the amplitude of the detector output signal directly correlates with the photon number up to five photons. This has been achieved by combining a single superconducting nanowire with an impedance-matching taper, which makes the amplitude of the output signal sensitive to the number of hot spots induced by absorbed photons, as predicted by Bell *et al.* [25]. Further, in an attempt to be more resistant against noise, Endo *et al.* have used an oscilloscope to record reference wave forms corresponding to different photon numbers, which could then be used to distinguish different events by wave-form pattern matching. They have found that using a subset of the rising edge of the SNSPD output trace improves the photon-number discrimination for larger input states, compared to merely using a single point in time [26]. Alternative approaches for photon-number resolution involve analyzing the rising

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edge of the output signal. One method utilizes a linear fit to a section of the rising edge, with the photon number determined from the corresponding slope measured using an oscilloscope [27]. Another method measures the variation in the slew rate of the SNSPD output signal, employing a constant-threshold time tagger to determine the difference in arrival time of detection events relative to a trigger signal [28].

The most recent method to resolve photon number with a single SNSPD builds on this idea and measures the relative time difference between a trigger signal and the rising and falling edges of the electrical detector output signal, which enhances the discrimination of different photon numbers [29].

While investigating the full electrical output signal of a detector maximizes the amount of information from a detection event, this is not practical in experiments where photon-number information must be extracted in real time. The question naturally arises: which aspects of the electrical output provide most information about the photon number? To answer this question, in this paper we use a popular technique from multivariate statistics known as principal component analysis (PCA) [30] to deduce maximum information about the photon number from a minimum number of data points.

In general, PCA is a dimensionality-reduction technique that determines a set of optimal orthogonal basis functions (principal components) describing the maximum amount of information in a data set. That means that it transforms the data set, containing lots of variables, into a smaller data set, being less accurate but with reduced complexity. This is done by calculating the covariance matrix of the data set and finding the eigenvectors of the largest eigenvalues of the covariance matrix. The eigenvectors (also called principal components in PCA terms) correspond to the directions of maximum variance, i.e., information in the data set. PCA has already been applied to the electrical output signals from transition-edge sensors (TESs) [31], where it has been successfully used to reduce the complexity of the data set by focusing on the first principal components.

In this work, we make use of the full electrical output signal of the commercially available SNSPD, maximizing the amount of information from any detection event. From this, we will use PCA to investigate the extent to which SNSPDs can resolve the photon number and to identify which features of the electrical output signal are most relevant for maximizing the photon-number-resolving capability. Following that, we make use of the knowledge from the PCA and use a time tagger to verify our findings. We use a more application-oriented method of recording the relative time differences between a trigger signal and the rising and falling edges of the SNSPD electrical output signal [29].

As this method relies on a trigger signal to gain photon-number information, pulsed light is necessary. Previous

methods [21,24,27] do not rely on a trigger signal and could thus be used for continuous-wave light. However, to have significant multiphoton components in the short time window, where the photon-number-resolving detection mechanism works (up to tens of picoseconds [32]), the photon flux would cause the SNSPDs to latch. We investigate a single-pixel SNSPD, to show the photon-number-resolving capability of just one pixel. However, this method can also be applied to SNSPD arrays, where each pixel is read out individually [10,12], to further increase the photon-number resolution of these arrays.

II. EXPERIMENTAL SETUP

The experiment consists of recording the electrical output signal from a commercially available SNSPD with an oscilloscope. The experimental setup can be seen in Fig. 1. We use a < 200 -fs pulsed laser, as recent theoretical simulations suggest that longer delays between absorbed photons decrease resolvability [32]. After taking fiber dispersion into account, the temporal pulse width is approximately 3 ps at the detector. Since the SNSPD cannot be operated at the repetition rate of the pulsed laser of 80 MHz, we pick every 800th pulse to reduce the repetition rate to 100 kHz. This is done by dividing the synchronization output of the laser with a pulse divider, which is then used as a trigger for an arbitrary wave-form generator (AWG) to generate a 8.3-ns pulse for the pulse-picking electro-optic modulator (EOM). We use a power supply to set a dc offset voltage to the EOM, in order to optimize the extinction ratio of the pulse-picking system. The polarization controller before and the fiber polarizing beam splitter (PBS) after the EOM are used to optimize the polarization of the light toward the SNSPD. One port of the PBS is connected to a fiber attenuator (in order to attenuate the light close to the single-photon level) and two variable optical attenuators (VOA) control the precise mean photon number impinging on the detector. The second polarization controller is used to optimize the detection efficiency of the polarization-dependent SNSPD. The second port of the fiber PBS is connected to a fast photodiode (1.2-GHz bandwidth), which acts as a trigger for the oscilloscope. We use a 21-GHz-bandwidth oscilloscope with 128 GSa/s to record the electrical signal of the SNSPD. This gives us a sampling period of roughly 8 ps, while maintaining the full information of the electrical signal of the detector. The commercially available SNSPD has a timing jitter of 11.3 ps (FWHM), specified by the manufacturer (Single Quantum).

The oscilloscope is operated in a sequential acquisition mode, where 1000 electrical output traces are recorded at once and then streamed to a computer. One trace consists of 3×10^4 points, which corresponds to a length in time of approximately 234 ns. We scan through mean photon numbers from 0.5 to 5 in steps of 0.5 and record 1×10^5 traces

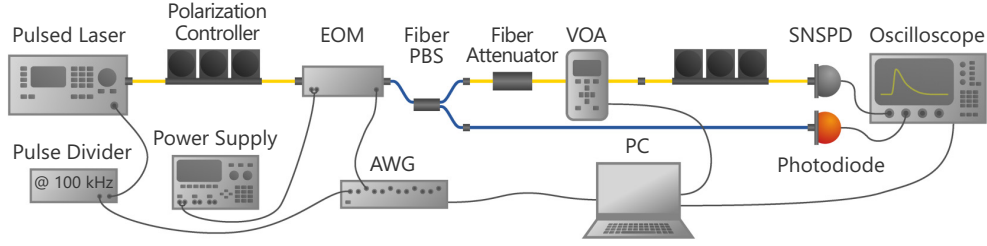


FIG. 1. We use a femtosecond-pulsed laser at 80 MHz, which is pulse-picked down to 100 kHz using an electro-optic modulator (EOM). The EOM receives a square-wave function from an arbitrary wave-form generator (AWG) and a dc offset voltage from a power supply. The polarization controller and the fiber polarizing beam splitter (PBS) are used to optimize the optical power incident on the SNSPD. A fiber attenuator and a variable optical attenuator are used to attenuate the laser light and set different mean photon numbers. The second polarization controller is used to optimize the signal on the SNSPD. The second port of the fiber PBS is connected to a fast photodiode, which is used as the trigger for the oscilloscope. The yellow and blue fibers represent single-mode and polarization-maintaining fiber, respectively.

per mean photon number. An exception is the mean photon number of 0.5, for which we record twice as many traces, since a lot of pulses contain zero photons. Every instance of zero impinging photons results in an empty trace, which contains no relevant information for the PCA. We randomize the order of the 1100 measurements, to avoid any bias toward photon number that might occur due to possible drifts during the measurement.

In a second measurement, we substitute the oscilloscope for a time tagger with a time resolution of < 1.9 ps. This measurement is used to confirm our findings of the subsequent PCA method, in order to maximize the photon-number-resolving capability of the commercial SNSPD using a time tagger.

The mean photon numbers as a function of attenuation are calibrated before the measurement. This is done by recording the count rate (r_{count}) of the SNSPD over the full operating range [from dark counts until saturation, where the count rate equals the repetition rate (r_{rep}) of the experiment] with a time tagger, by sweeping the attenuation using the VOAs. Assuming Poissonian statistics of the laser, the mean detected photon number per pulse is then given by

$$\bar{n} = -\ln \left(1 - \frac{r_{\text{count}}}{r_{\text{rep}}} \right). \quad (1)$$

III. RESULTS

The full data set of electrical output signals consists of 1.1×10^6 traces (also called samples) and 3×10^4 points per trace, i.e., time-dependent voltages (also called features). However, we only use 1.9×10^4 points, as this is enough to contain the complete SNSPD electrical trace. In a first step, we filter out traces from suboptimal pulse picking (i.e., traces with multiple peaks or traces at the wrong time delay with respect to the trigger signal) as well as empty traces (when no photon was detected, after

the trigger signal from the fast photodiode). This means that in the PCA we will not investigate the zero-photon (vacuum) component of the coherent state, as we want to investigate the photon-number-resolving capability of the detector. Since the discrimination of click and no-click events is trivial, we focus only on the click events (at least one photon).

As a next step, we fit the PCA model with a subset of 1000 samples from each mean photon number (using more samples did not change the fit). This yields the principal components f_1 and f_2 in Fig. 2, which describe the most information of the data set (we show covariance matrices in Fig. 6). The model is then successively applied to the full data set in order to find the weights corresponding to the principal components for each sample. From our analysis, we find that only the first two principal components show photon-number dependency. Plotting the weights of the first component against the weights of the second component in a two-dimensional histogram shows a clear

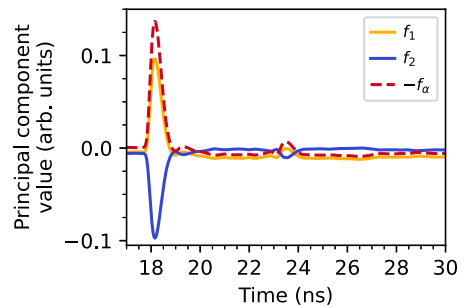


FIG. 2. First two principal components (basis functions), f_1 and f_2 , and the principal component that results from the optimal angle projection $-f_\alpha$ of the two-dimensional histogram in Fig. 3(a). Only the relevant time regime is shown; the basis functions are flat otherwise.

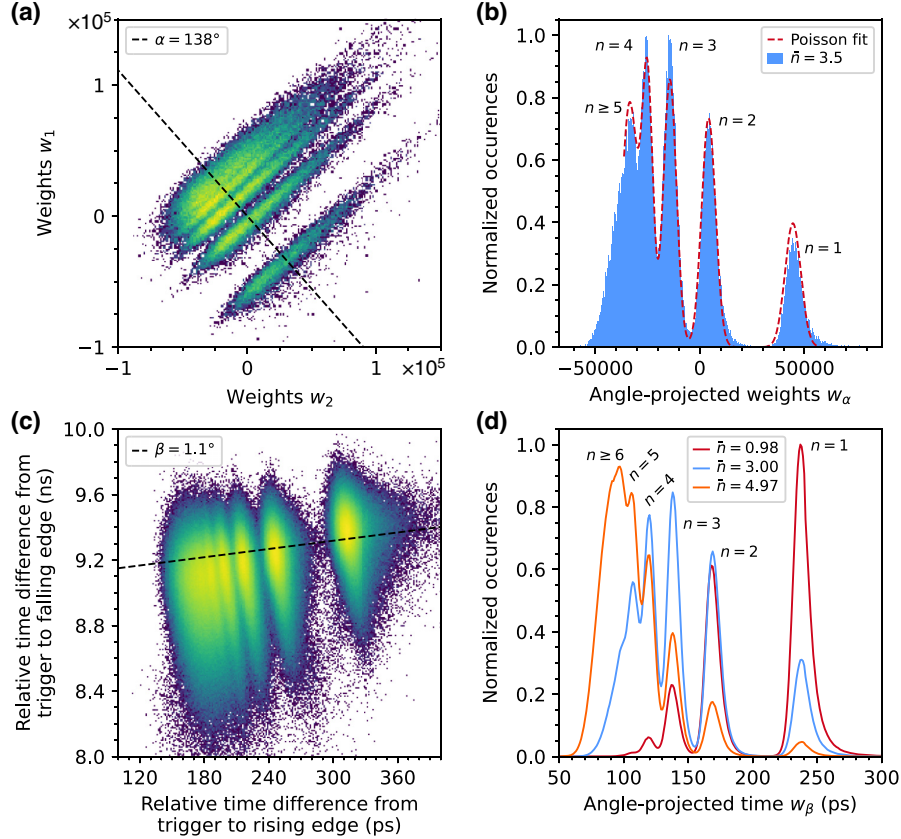


FIG. 3. (a) The two-dimensional histogram of the weights of the first two principal components from the PCA model for a mean photon number of $\bar{n} = 3.5$. These two components show a photon-number dependency, which can be seen by the clear differentiation between the ellipses. For smaller mean photon numbers, the occupation of the ellipses shifts to the lower right and for higher mean photon numbers to the upper left. (b) The histogram of the optimal projection at an angle of $\alpha = 138^\circ$ and a multi-Gaussian fit with an enforced Poisson distribution (red dashed line). Additional two-dimensional histograms and their optimal projection (for mean photon numbers of $\bar{n} = 1$, $\bar{n} = 3$, and $\bar{n} = 5$) are shown in Fig. 7. (c) The two-dimensional histogram of the relative time difference between the trigger signal and the rising and falling edge. The full data set is displayed, which is the sum of the individual measurements for the three mean photon numbers indicated in (d). (d) The optimal projection at an angle of $\beta = 1.1^\circ$ of the two-dimensional histogram of (c) for three photon numbers (as labeled in the legend).

differentiation between photon numbers [see Fig. 3(a), where the mean photon number is $\bar{n} = 3.5$].

Due to the angle of the ellipses, the components on their own will not reveal the optimal photon-number-resolving capability of the detector. However, projecting the histogram on an angle of $\alpha = 138^\circ$ [as indicated by the black dashed line in Fig. 3(a)], the maximal photon-number resolution can be unveiled. This projection is required as the principal components describe the maximal variance in the data set, which can be a mixture from various information sources. The projection distills the maximal photon-number resolution from the principal components. This leads to the histogram shown in Fig. 3(b) for a mean photon number of $\bar{n} = 3.5$, where the angle-projected weights are calculated as $w_\alpha = w_2 \sin(\alpha) + w_1 \cos(\alpha)$. After the

angled projection, only one basis function will describe the photon-number dependency. Interpreting this basis function reveals which features of the electrical output trace of an SNSPD need to be observed when it comes to photon-number resolution. We find that the relative time difference between the trigger signal and the rising edge (peak of $-f_\alpha$ around 18 ns in Fig. 2), as well as the peak amplitude of the electrical output signal from the SNSPD (part of $-f_\alpha$ after the peak), contain the most photon-number information.

In a recent study, the relative time difference between a trigger signal and the rising edge as well as the falling edge has been observed in a two-dimensional histogram using a time tagger [29]. In order to compare our oscilloscope measurement (utilizing PCA to show us the best method

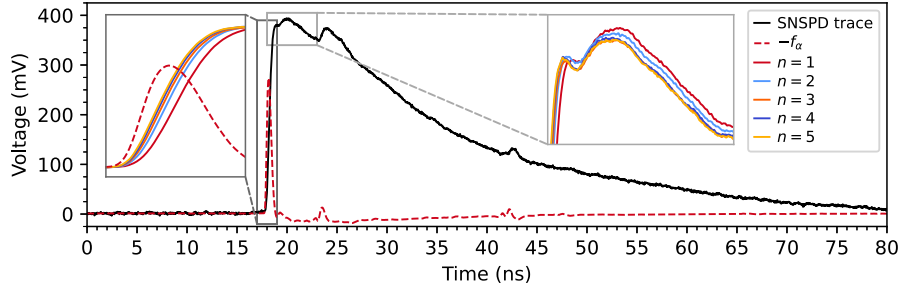


FIG. 4. The electrical output signal of the SNSPD, as well as the angled principal component $-f_\alpha$ (scaled by a factor of 2000 for better comparison). The left inset shows the rising edges of the traces for different photon numbers, where an earlier rising edge correlates with a higher photon number. The right inset shows the peaks of the traces, where a reduced height correlates with a higher photon number. The colors correspond to different photon numbers, as labeled in the legend. For reduced influence of electrical noise, an average over 20 traces is shown for both insets. Features around 24 ns and 43 ns are typically seen for SNSPDs from Single Quantum (see, e.g., Ref. [27]).

to retrieve photon-number information) with the method from Ref. [29], we have also recorded a two-dimensional histogram of the rising and falling edge using a time tagger with a time resolution of < 1.9 ps on the same commercial SNSPD. In the end, this needs less data analysis to show photon-number resolution and is thus more application oriented. We use the same experimental setup shown in Fig. 1, except with a time tagger that records the relative time differences from the trigger signal to the rising and falling edge of the trace. The photon-number resolution depends on the voltage threshold of the time tagger. We find that at half peak height (corresponding to 185 mV), the separation between photon numbers is best. Afterward, we plot the complete data set recorded with the time tagger in a two-dimensional histogram [see Fig. 3(c)], find the optimal projection angle ($\beta = 1.1^\circ$), and plot the distributions for different mean photon numbers [see Fig. 3(d)]. A comparison of the resulting distributions in Figs. 3(b) and 3(d) reveals a very similar performance.

In order to visualize why the two methods lead to similar results, we show an electrical output signal of the SNSPD in Fig. 4 and highlight the key features for photon-number resolution as identified by the PCA. In the insets of Fig. 4, we show an average over 20 SNSPD traces for different photon-number events as identified by the PCA. An increased photon number correlates with a reduced time difference between a trigger signal and the rising edge. This is consistent with previous studies where the rising edge has been observed in order to show photon-number resolution using an SNSPD [21,22,26–29].

In our measurements, an increased photon number correlates with a reduced peak amplitude. This is the opposite of earlier work [21], which has shown an increase in the peak amplitude with the photon number. The difference can be explained with the readout electronics. Reference [21] uses a high-bandwidth amplifier combined with a high-pass filter and therefore amplifies slower signals less

than fast signals. This means that traces corresponding to lower photon numbers are less amplified since they have slower rise times, resulting in a lower peak amplitude. Our readout electronics are a commercial device supplied by the manufacturer of the SNSPD. It is a safe assumption that the amplifier is optimized for signal to noise and thus has a bandwidth closely matching the SNSPD for one-photon detection events. Faster signals are then amplified less, resulting in lower peak amplitudes for multiphoton detection events.

While it is certainly challenging to make simultaneous precise time and voltage measurements, this can be circumvented by only focusing on the time measurement of rising and falling edges using a precise time tagger. The voltage measurement can be converted into a time measurement of the falling edge, since the threshold voltage is at a fixed value. A lower peak amplitude then means an earlier crossing of the threshold voltage. As mentioned above, the threshold of the time tagger needs to be chosen carefully in order to maximize the photon-number resolution. The left inset in Fig. 4 shows that the separation between different photon-number events appears largest at half peak height, which is consistent with the optimized threshold set for the time-tagger measurement.

A comparison of the optimal projection for the PCA in Fig. 3(b) and the time-tagger analysis in Fig. 3(d) shows very close agreement. Due to the increased time resolution of the time tagger, this method is suitable to circumvent the need to measure the peak amplitude of the electrical output signal to optimize the photon-number resolution. In order to characterize the photon-number resolution of the SNSPD, we make use of the confidence measure

$$C_n = \int_{-\infty}^{\infty} \frac{p(w|n)^2 p(n)}{p(w)} dw, \quad (2)$$

which has been proposed by Humphreys *et al.* [31]. This measure describes the probability that a certain detector response yields the correct photon number. Here, $p(n)$ is the probability of an n -photon input, which in our case is a Poissonian distribution. $p(w)$ is the overall probability distribution of the angle-projected weight w_α (angle-projected time w_β) of the two-dimensional histograms in Figs. 3(b) and 3(d). Finally, $p(w|n)$ is the probability $p(w)$ given an incident photon number n . This is derived by fitting Gaussian functions to the individual peaks in the histograms shown in Figs. 3(b) and 3(d). As an example, the distribution $p(w|n=1)$ corresponds to a Gaussian function fitted to the first peak from the right (corresponding to a photon number of $n=1$) to either histogram.

In Fig. 5, we show the confidence C_n for the two data sets (PCA and time-tagger method) for three mean photon numbers. Since this work is focused on the readout of the SNSPD, in Fig. 5 we show the confidence where we do not account for the detector efficiency of $\eta_{\text{det}} = (77 \pm 3)\%$ (nonfaded lines), which means that the confidence only takes into account the correct readout of the photon number absorbed by the SNSPD and not the actual incident photon number. In this way, only the two investigated methods are compared, without the effect of the limited detection efficiency. Taking the nonunity detector efficiency into account reduces the confidence measure, as can be seen by the faded lines in Fig. 5. This is the effect of photon loss, where, e.g., a two-photon event can be caused by a three-photon input, where one photon has been lost, or a four-photon input, where two photons have been lost. When comparing different detectors (and not readout methods), it makes sense to include the detector efficiency when calculating the confidence. However, this work compares readout methods on the same detector. More detail about the calculation of the confidence when including the detector efficiency can be found in Appendix C, where we also show probability distributions $p(w|n)$ and $p(n|w)$. Note that this analysis neglects the effects of dark counts, since these are negligible (10^{-8}) within the detection window. As can be seen in Fig. 3(b) for the PCA data set and in Fig. 3(d) for the time-tagger data set, the photon-number contributions overlap significantly for $n \geq 5$ and $n \geq 6$, respectively. Therefore, we only calculate the confidence up to $n=4$ (PCA) and $n=5$ (time tagger). In Fig. 5, we show that the confidence decreases with increasing photon number, which results from the fact that the spacing of the ellipses (corresponding to the photon number) in Figs. 3(a) and 3(c) decreases.

The confidence for the time-tagger measurement is always slightly higher, which can be explained by the higher timing resolution of < 1.9 ps compared to the timing resolution of the oscilloscope of roughly 8 ps. This means that focusing merely on the temporal measurement of the rising and falling edges with a time tagger can effectively substitute measuring the SNSPD

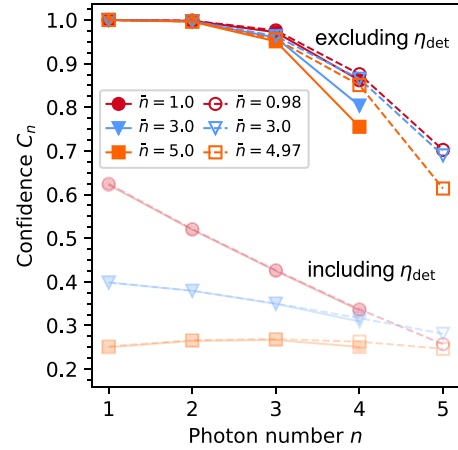


FIG. 5. The confidence C_n versus the photon number for the analysis of the oscilloscope data using PCA (solid symbols and solid lines) and the time-tagger data (hollow symbols and dashed lines) for three mean photon numbers. The nonfaded lines are confidence values excluding the detector efficiency η_{det} and the faded lines are including the detector efficiency. Probability distributions $p(w)$ for the displayed mean photon number are shown in Fig. 3(d) (time-tagger method) and in Fig. 7 (PCA method).

signal amplitude (proposed by the PCA). This makes a time tagger the ideal candidate to extract photon-number information from SNSPDs. Additionally, the width of the ellipses of the time-tagger measurement in Fig. 3(c) is limited by the system jitter of the experimental setup. Reducing the system jitter should therefore improve the photon-number discrimination, more noticeably for higher photon numbers.

IV. CONCLUSIONS

Using PCA (a tool from multivariate statistics) on a set of electrical output signals from an SNSPD allows us to investigate which features of the SNSPD output trace are most relevant for photon-number resolution. Our analysis indicates that the rising edge, as well as the amplitude of the electrical output signal, show the most photon-number information. Thus, simultaneously measuring the rising edge and the amplitude of the SNSPD output signal should maximize the photon-number resolution.

We additionally show that a more application-oriented approach of recording the relative time difference from a trigger signal to the rising and falling edge of the SNSPD output signal in a two-dimensional histogram (recently shown in Ref. [29]) can effectively substitute the voltage measurement by a temporal measurement of the falling edge of the output signal using a time tagger. We explain this by the much better timing resolution of < 1.9 ps of the time tagger compared to the timing resolution of the oscilloscope. We also characterize the photon-number

resolution by calculating a confidence measure [31] (which is the probability that a certain detector response yields the correct photon number) of the two methods presented in this work. We find that the confidence using the time-tagger method is always slightly higher and that the time tagger can in principle resolve higher photon numbers, due to the better timing resolution. This makes the time tagger an excellent tool to enable photon-number resolution with SNSPDs. This method relies on extremely precise timing resolution of the measurement signal, since it is sensitive to the effect of multiple localized breakdowns in superconductivity (“hot spots”) being induced by multiple incident photons. Factors that limit this resolution, primarily system jitter and optical pulse length, will reduce the photon-number-resolving capabilities. It may be possible to modify the detector geometry to increase the optical pulse duration for which this effect persists but this remains an open research question.

The data set of the raw SNSPD traces is freely available at Ref. [33].

ACKNOWLEDGMENTS

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APPENDIX A: COVARIANCE MATRIX

PCA is a dimensionality-reduction technique. To reduce the complexity of the data set, one first calculates the covariance matrix of the data set. Subsequently, one finds the eigenvectors (principal components in PCA terms) of the largest eigenvalues of the covariance matrix, which correspond to the directions of maximum variance, i.e., information in the data set. The covariance matrix is a square matrix that represents how each data point relates to every other data point. The covariance is a measure of how two data points tend to vary together.

We calculate the covariance matrix of a subset of the full data set, which contains 1000 samples (oscilloscope traces) from each mean photon number (in total, 10^4 samples). Only a small region of the $19\,000 \times 19\,000$ covariance matrix contains relevant entries, which is the region around the rising edge of the electrical output signal of the SNSPD. We show the relevant region of the covariance matrix in Fig. 6(a), together with the first principal component f_1 (white line). It can be seen in Fig. 6(a) that the largest covariance values occur at the rising edge of the electrical output signal of the SNSPD. This indicates that the rising edge causes the most variance (i.e., contains the most information) in the data set. The negative values along the horizontal cut at $y \approx 18.25$ ns (which corresponds to the rising edge) in Fig. 6(a) mean variation in the opposite direction, i.e., increasing values at the rising edge correlate with decreasing values at the falling edge. This is consistent with our findings that an increasing photon number correlates with an earlier rising

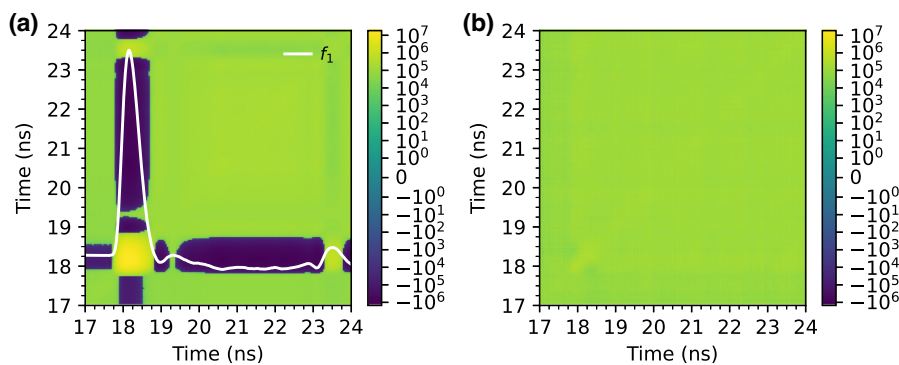


FIG. 6. (a) The covariance matrix of a subset of the full data set [1000 samples (oscilloscope traces) from each mean photon number]. Therefore, this subset contains all photon-number components. The largest (positive) values are around the rising edge of the electrical output signal of the SNSPD. The smallest (negative) values correspond to the falling edge of the signal. The white line (arbitrary scaling) corresponds to the first principal component f_1 (also shown in Fig. 2). f_1 essentially corresponds to the color-axis values of a horizontal cut of the matrix at $y \approx 18.25$ ns. (b) The covariance matrix of 50 samples corresponding to the one-photon component, as identified by the PCA.

edge and a reduced height (or, correspondingly, an earlier falling edge). This behavior is also represented by f_1 , which is the first principal component found by the PCA.

In Fig. 6(b), we show the covariance matrix of 50 samples (oscilloscope traces) corresponding to the one-photon component, as identified by the PCA. This means that we extract only those traces that the PCA has identified as a one-photon event ($n = 1$) and we calculate the covariance matrix based on 50 of these traces. In Fig. 6(b),

it can be seen that there are no relevant contributions to the covariance matrix, which means that all data points vary in the same way. This is expected, as we only use one-photon events, which should produce traces that look very similar and thus have little to no variation between them. When closely examining Fig. 6(b) around the point (18 ns, 18 ns) (which corresponds to the rising edge of the SNSPD traces), slightly larger covariances can be noted. This can be explained by slight variations of the rising edge of the SNSPD traces.

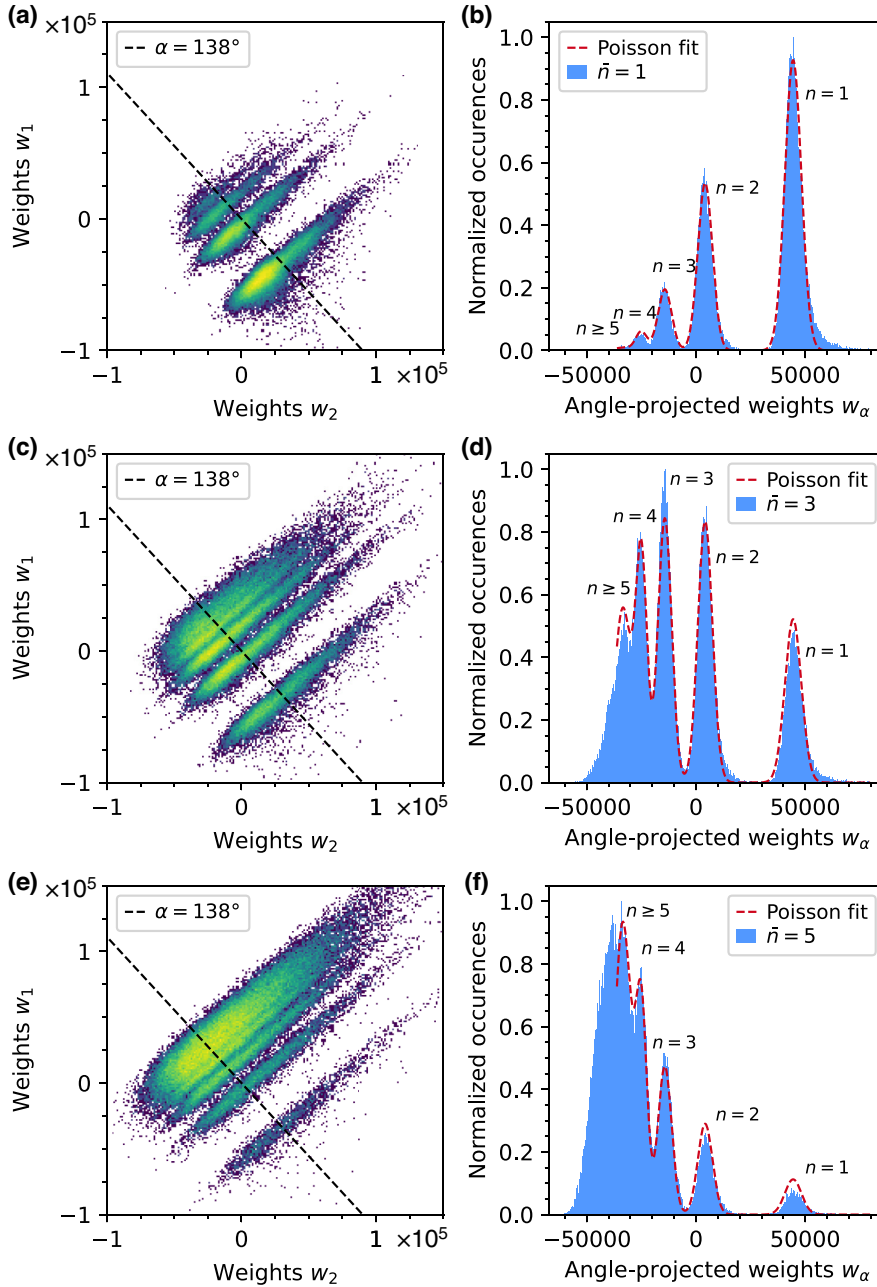


FIG. 7. (a),(c),(e) Two-dimensional histograms of the weights of the first two principal components from the PCA model for mean photon numbers of $\bar{n} = 1$, $\bar{n} = 3$, and $\bar{n} = 5$, respectively. (b),(d),(f) Histograms of the optimal projection at an angle of $\alpha = 138^\circ$ and a multi-Gaussian fit with an enforced Poisson distribution (red dashed line) according to (a), (c), and (e), respectively.

APPENDIX B: PRINCIPAL COMPONENT ANALYSIS

Here, we provide additional two-dimensional histograms from the PCA model, together with their optimal projection at an angle of $\alpha = 138^\circ$, for mean photon numbers of $\bar{n} = 1$, $\bar{n} = 3$ and $\bar{n} = 5$ (see Fig. 7). It can be seen that the occupation of the ellipses shifts toward the upper left of the two-dimensional histograms with increasing mean photon number. These mean photon numbers are used to calculate the confidence in Fig. 5.

APPENDIX C: CONFIDENCE CALCULATION

We characterize the photon-number resolution of the NSPSPD using the confidence measure introduced by Humphreys *et al.* [31]:

$$C_n = \int_{-\infty}^{\infty} p(w|n)p(n|w)dw = \int_{-\infty}^{\infty} \frac{p(w|n)^2 p(n)}{p(w)} dw. \quad (C1)$$

Using Bayes' theorem, the confidence can also be expressed using the probabilities $p(w|n)$ and $p(n|w)$, of observing a specific weight w given a photon number n and of observing a specific photon number n given a certain weight w , respectively. To take the limited efficiency of the detector η_{det} into account, the loss matrix

$$L_{n,n'} = \binom{n}{n'} \eta_{\text{det}}^{n'} (1 - \eta_{\text{det}})^{n-n'} \quad (C2)$$

can be used, which is an expression that describes the probability of retaining n' out of n photons including efficiency η_{det} .

In Fig. 8(a), we show $p(w|n)$ as a function of the angle-projected time w_β (for the time-tagger method) for different

photon numbers n . These are (multi-)Gaussian distributions with the mean and standard deviation determined by the underlying probability distribution $p(w)$ [which is the angle-projected histogram shown in Fig. 3(d)]. The amplitudes of the Gaussians are determined by the loss matrix in Eq. (C2), to account for the efficiency of the detector. This is necessary, as it is possible that, e.g., a two-photon event can be caused by a three-photon input, where one photon has been lost, or a four-photon input, where two photons have been lost. In Fig. 8(b), we show $p(n|w)$ for a mean photon number of $\bar{n} = 1$, which can be calculated with Bayes' theorem using $p(w|n)$ [shown in Fig. 8(a)], $p(n)$ (the Poisson distribution for a certain mean photon number $\bar{n}$), and $p(w)$.

To correctly calculate the confidence, the zero-photon contribution needs to be included as well. Neither the PCA nor the time-tagger method include zero-photon events. Therefore, we use the mean photon-number-dependent closed-form expression for the zero-photon contribution to the confidence,

$$C_{0,\bar{n}} = L_{n,0} \frac{L_{n,0} p(n, \bar{n}')}{p(0, \bar{n})}, \quad (C3)$$

which is then added separately as an additional term to Eq. (C1). Here, $L_{n,0}$ is the probability of losing all n out of n photons given by Eq. (C2), $p(0, \bar{n})$ is the probability of measuring $n = 0$ photons for a mean photon number of $\bar{n}$ (given by a Poisson distribution), and $p(n, \bar{n}')$ is the probability of actually having n photons for a mean photon number of $\bar{n}'$ corrected by the efficiency (also given by a Poisson distribution), i.e., $\bar{n}' = \bar{n}/\eta_{\text{det}}$.

Integrating over the product of the two probability distributions $p(w|n)$ and $p(n|w)$ [shown in Figs. 8(a) and 8(b), respectively] and adding the zero-photon contribution [given by Eq. (C3), when including the detector efficiency η_{det}] for a photon number n and a mean photon number $\bar{n}$

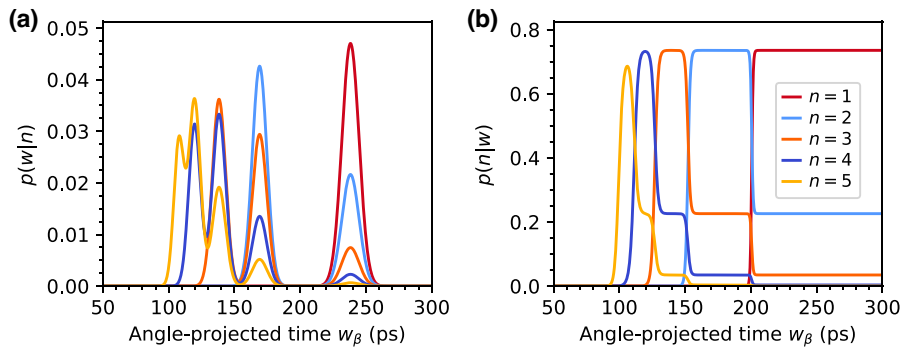


FIG. 8. (a) The probability $p(w)$ given an incident photon number n , adjusted for the efficiency of the detector η_{det} . (b) The probability $p(n)$ given a certain weight w_β for a mean photon number of $\bar{n} = 1$. These distributions depend on the mean photon number, as they are calculated from $p(w|n)$, $p(w)$ and $p(n)$ using Bayes' theorem.

gives the confidence $C_{n,\bar{n}}$. Confidence values for the PCA method and the time-tagger method are shown in Fig. 5 (including and excluding the detector efficiency η_{det}).

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Jitter in photon-number-resolved detection by superconducting nanowires

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ABSTRACT

By analyzing the physics of multi-photon absorption in superconducting nanowire single-photon detectors (SNSPDs), we identify physical components of jitter. From this, we formulate a quantitative physical model of the multi-photon detector response that combines the local detection mechanism and local fluctuations (hotspot formation and intrinsic jitter) with the thermoelectric dynamics of resistive domains. Our model provides an excellent description of the arrival-time histogram of a commercial SNSPD across several orders of magnitude, both in arrival-time probability and across mean photon number. This is achieved with just three fitting parameters: the scaling of the mean arrival time of voltage response pulses, as well as the Gaussian and exponential jitter components. Our findings have important implications for photon-number-resolving detector design, as well as applications requiring low jitter, such as light detection and ranging (LIDAR).

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I. INTRODUCTION

Photon-number measurements are a cornerstone of experimental quantum optics and fundamental to our understanding of the quantized nature of light. They are used to measure photon statistics in order to classify quantum optical states¹ and perform projective measurements, which herald the presence of particular nonclassical photonic states.^{2–5} Photon-number-resolved (PNR) measurements are also required for a broad range of applications in photonic quantum information processing, such as error identification in photonic quantum computing^{6,7} and quantum key distribution.⁸

To measure photon number, one can rely on devices with an intrinsic photon-energy resolution. One archetypal example is the superconducting transition-edge sensor.^{9,10} Nevertheless, these devices require significant millikelvin cryogenic overhead, and their

timing characteristics (on the nano- to microsecond level) limit the range of experiments and applications. A more experimentally accessible alternative has been to use spatially and/or temporally multiplexed threshold detectors,^{11–13} also called “on-off” or “click” detectors, such as single-photon avalanche photodiodes (SPADs) or superconducting nanowire single-photon detectors (SNSPDs). These detectors trigger on detecting at least one photon. By splitting an unknown number of photons into multiple modes, each of which is incident on its own detector, one can achieve quasi-photon-number resolution. This approaches perfect PNR when the number of detectors is much larger than the number of photons to be measured.^{14,15}

SNSPDs are now well-established as the detector of choice for many experiments and applications of quantum optics and photonics because of their high efficiency, low noise, and accurate timing characteristics. SNSPDs were initially regarded as

threshold detectors. However, Cahall *et al.*¹⁶ observed that some degree of photon-number information could be obtained by analyzing the electrical trace of the detector response, and since then a number of works have further refined this technique.^{17–24} All of these rely on photons with an optical pulse duration shorter than the jitter of the detectors, typically below 100 ps. This is, however, well within the range of quantum light sources generated by nonlinearities.²⁵

Schapeler *et al.*²² showed that almost all photon-number information is contained within the rising edge of the electrical trace of the detector and that this is most accurately measured with a time-to-digital converter conditioned on a threshold. Central to extracting photon-number information is therefore the ability to measure this arrival time accurately, which requires an understanding of the underlying photon-number-dependent jitter mechanisms. However, while models for SNSPD jitter are well accepted,^{26,27} the effects of photon number on jitter are far less well understood. Moreover, studying multi-photon absorption effects not only enhances our understanding of the underlying device physics of SNSPDs but also allows detectors to be designed to maximize this figure of merit, thereby unlocking further applications of this fundamental photonic technology.

In this paper, we present a physical model that accounts for the photon-number-dependent jitter of SNSPDs and apply it to experimental data obtained from a commercial device operated over a broad range of photon fluxes. We obtain excellent agreement between theory and experiment over several orders of magnitude in the arrival-time probability histogram. Based on this understanding, we suggest how SNSPDs can be designed to optimize their photon-number-resolving power.

II. SUPERCONDUCTING NANOWIRE DETECTOR PHYSICS AND JITTER MECHANISMS

Photon detection in superconducting nanowires involves sequential processes:²⁸ hotspot formation (vortex crossing in wide wires), resistive domain evolution, and electrical signal propagation. Each process introduces delays and variations that jointly determine the total jitter. The arrival-time histogram of voltage response pulses in single-photon detection with SNSPDs is well described with an exponentially modified Gaussian (EMG) distribution,²⁹

$$P_{\text{EMG}}(t, \sigma, \tau, \mu) = \frac{1}{2\tau} \exp\left(\frac{1}{2\tau} \left[\frac{\sigma^2}{\tau} - 2t + 2\mu \right]\right) \times \left[1 - \operatorname{erf}\left(\frac{\frac{\sigma^2}{\tau} - t + \mu}{\sigma\sqrt{2}}\right) \right], \quad (1)$$

where μ is the mean value and σ and τ are the Gaussian and exponential components, respectively.

For SNSPDs, the total measured jitter is typically determined as the histogram width, given by the standard deviation $\sigma_{\text{tot}} = \sqrt{\sigma^2 + \tau^2}$. The Gaussian jitter component can be decomposed into its constituent contributions

$$\sigma = \sqrt{\sigma_{\text{noise}}^2 + \sigma_{\text{inst}}^2 + \sigma_{\text{opt}}^2 + \sigma_{\text{geom}}^2 + \sigma_{\text{int}}^2}. \quad (2)$$

Among these, the first three components are due to external sources: noise in the electronics σ_{noise} , data acquisition instrumentation

(e.g., time-to-digital converter, trigger photodiode) σ_{inst} , and optical pulse duration σ_{opt} . Geometric jitter σ_{geom} arises from the temporal uncertainty of where photons are absorbed along the wire, i.e., it is determined by the SNSPD geometry. The Gaussian σ_{int} together with exponential τ components constitute the jitter intrinsic to the photon detection process; throughout the paper, we refer to it as the intrinsic jitter ($\sqrt{\sigma_{\text{int}}^2 + \tau^2}$).

Intrinsic jitter is determined by the physics of the hotspot formation process. It arises from thermal fluctuations affecting the detection energy barrier that dominate other jitter sources³⁰ like Fano fluctuations (phonon loss),³¹ non-uniformities in the superconducting energy gap and variations in the film thickness,³² local position-dependent threshold,²⁶ and vortex time-of-flight. In narrow nanowires, the vortex time-of-flight jitter is negligible (<1 ps).³³ Wider strips exhibit large vortex-induced jitter at low bias currents, but this diminishes at high currents, where vortex motion becomes fast and the position-dependent threshold flattens.³⁴ In this case, thermal fluctuations may again become the dominant jitter source.

After photon absorption, the hotspot region enters a metastable state with the free energy U defined by the photon energy and the actual local temperature, which itself is the subject of background thermal fluctuations in the absence of photons. Thermal fluctuations in the hotspot³⁰ (a Poisson stochastic process) may provide enough energy to escape the metastable state over the still remaining energy barrier ΔU (difference between the detection threshold and the actual energy). The escape rate (inverse of first hitting time³⁵), distributed exponentially, varies with ΔU . For large ΔU , escapes are rare, producing an exponential tail in the arrival-time histogram parameterized by τ , while for smaller ΔU (e.g., in high-energy photon detection), escapes occur frequently, and the histogram retains the Gaussian shape inherent to the background fluctuations. The jitter in this case is defined by events driving the hotspot to energies above the detection threshold.²⁶

The arrival-time histogram in multi-photon detection can be described using a sum of P_{EMG} [Eq. (1)] with different means μ corresponding to different photon numbers, each weighted by the probability for a given photon-number component in the underlying photon distribution P_n . For coherent light, this distribution is Poissonian with $P_n(\bar{n}) = \frac{\bar{n}^n e^{-\bar{n}}}{n!}$, resulting in

$$P_{\text{PNR}}(\bar{n}) = \sum_{n=1}^{\infty} P_n(\bar{n}) P_{\text{EMG}}(t, \sigma_n, \tau_n, \mu_n), \quad (3)$$

for simplicity, uniform illumination is assumed. The key challenge in generalizing this model for multi-photon absorption is therefore to account for photon-number dependence in each of the parameters in the P_{EMG} distribution, the jitter components, and μ_n . The photon-number dependence of μ_n has previously been studied by Nicolich *et al.*³⁶ Here, we refine this model and discuss additionally the photon-number dependence of σ and τ arising from multi-photon absorption.

To do this, we first consider the underlying physics of different multi-photon absorption scenarios.

A. Multi-photon absorption at the same site

When multiple photons n , each of energy $h\nu$, are absorbed simultaneously within the hotspot size, the deposited energy is

$E = n \cdot h\nu$ [see Fig. 1(a-i)]. Higher energy deposition leads to a shorter intrinsic delay, as this is proportional to $1/(E - E_0)$ due to the energy-dependent collapse time of the superconducting order parameter (E_0 is the photon energy corresponding to the detection threshold^{26,27,37}), depicted in Fig. 1(a-iii). Furthermore, for the same dispersion in the background energy fluctuations, intrinsic jitter decreases with n . Hence, for $E > E_0$, increasing n results in earlier domain growth and, therefore, earlier voltage response pulse arrivals μ (alongside reduced intrinsic jitter, akin to detecting a higher energy photon³⁸). Note, this does not affect the dynamics of the resistive domain (i.e., rise time t_{rise}) but only the delay μ and the intrinsic jitter [see Fig. 1(a-ii)]. Despite the fact that for $E < E_0$ the detection mechanism is different and is governed by fluctuations,³⁰ the qualitative result remains, i.e., intrinsic jitter decreases as n increases.

Given the small characteristic hotspot size ($L_T \approx 50$ nm in NbTiN³⁹ and $L_T \approx 220$ nm in WSi⁴⁰), the probability of multi-photon absorption at the same site is higher in detectors with smaller active areas, such as those integrated with waveguides⁴¹ or nano-detectors.⁴² If the area does not exceed the hotspot size, the intrinsic jitter monotonously decreases as both the number of photons and the mean photon number (light intensity) increase.

For detectors with areas exceeding the hotspot size, there are several independent absorption sites (bins) over which incident photons are distributed randomly. This results in a combination of measurement events in which multiple photons are absorbed at the same site or independently at different sites.

B. Simultaneous multi-photon absorption at independent sites

When non-overlapping (independent) resistive domains [see Fig. 1(b)] are created, they share the same transport current, which correlates their dynamics. As the number of domains increases, the wire resistance R grows faster, causing the current to divert more rapidly into the external circuit and producing a voltage response pulse with a shorter rising time $t_{\text{rise}} = L_k/R \propto 1/\sqrt{n}$ [see Fig. 1(b)].

This is because $R \propto ntu$, where u is the domain wall propagation rate, and the voltage rise $V(t) \propto (1 - e^{-t^2/nu})$ reaches $(1 - e^{-1})$ when $t \propto 1/\sqrt{n}$ (which holds for linear slopes near the 50% pulse amplitude). This results in photon-number dependent mean arrival times $\mu_n \propto 1/\sqrt{n}$, as shown by Nicolich *et al.*³⁶ The non-linear electrothermal model⁴³ refines this to $t_{\text{rise}} \propto n^{-\alpha}$, where $\alpha \in [0.3, 0.4]$ due to the current-dependence of the domain wall propagation rate, in agreement with experimental results.²³

C. Delayed multi-photon absorption at independent sites

When photons are absorbed at independent sites with a time delay, typically due to optical pulse durations σ_{opt} longer than the hotspot formation time (about the electron-phonon cooling time), the detection process becomes more complex. The first photon creates a hotspot and a growing resistive domain, causing the current to start diverting into the external circuit. The second photon, absorbed at a different site, encounters a reduced current. At smaller currents, the remaining energy barrier is larger, leading to even later emergence of the second domain, larger intrinsic jitter, and reduced detection efficiency (if not saturated). Under the reduced current, domain growth is slower, so that when the first domain has already disappeared, the second domain may still exist. This increases the voltage rise time compared to the case of simultaneous photon absorption, affecting not only intrinsic jitter but also the mean of a distribution. The resolvable delay between photons is determined by how long the current persists in the detector above the level at which the detection efficiency remains equal to the steady-state saturated value. This timescale is set either by the circuit time constant (in long, high-inductance, or shunted wires) or by the electron-phonon cooling time (in short, low-inductance wires).

D. Impact of resistive domain overlap

Overlapping domains compromise photon-number resolution, as two photons absorbed within the domain length are detected

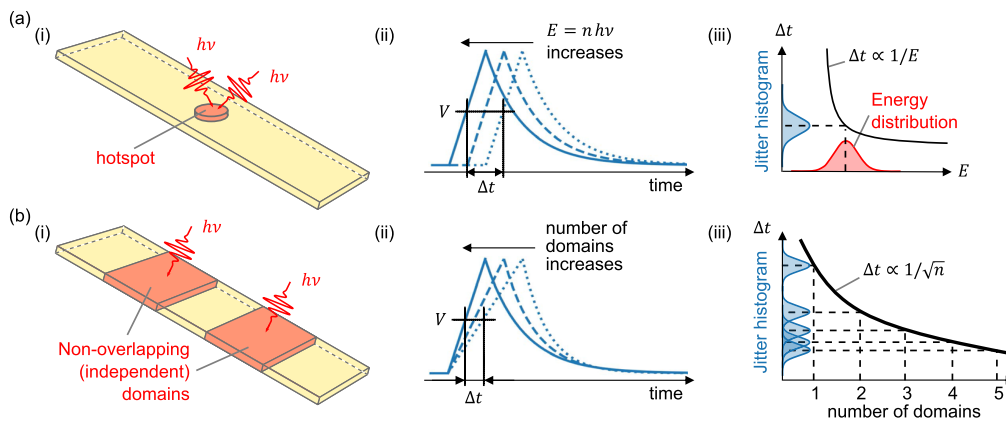


FIG. 1. Jitter in different scenarios of multi-photon detection in superconducting strips. (a) (i) Upon multi-photon absorption at the same site (within a hotspot), the deposited energy $E = n \cdot h\nu$ scales with the number of absorbed photons n , each of energy $h\nu$. (ii) Arrival time decreases as E increases. (iii) Higher energy reduces the intrinsic delay time and narrows the jitter distribution. (b) (i) When multiple photons induce n independent hotspots and each generates a non-overlapping resistive domain, (ii) the slope of the voltage response pulse becomes faster $\propto 1/\sqrt{n}$. (iii) The corresponding arrival-time histogram as a function of photon number.

as one. For a nanowire composed of M independent bins, each of the domain length, the probability that no bin contains more than one photon ($n \leq M$, photons are uniformly distributed) is $P(\text{no overlap}) = \prod_{i=0}^{n-1} \frac{M-i}{M}$. This arises because the first photon can occupy any of the M bins with probability $1/M$, the second photon avoids the already-occupied bin with probability $(M-1)/M$, and so on. Then the probability that at least one bin contains two photons is

$$P(\text{overlap}) = 1 - \prod_{i=0}^{n-1} \frac{M-i}{M} \approx 1 - \exp\left(-\frac{n(n-1)}{2M}\right), \quad (4)$$

the approximation is valid for $n \ll M$.

This is a special case of the inclusion–exclusion principle for the probability that k from n photons generate independent non-overlapped domains¹¹

$$P_M(k|n) = \frac{1}{M^n} \frac{M!}{k!(M-k)!} \sum_{j=0}^k (-1)^j \frac{k!}{j!(k-j)!} (k-j)^n, \quad (5)$$

for $k \leq n \leq M$. For $k = n$, i.e., no overlap at all, one gets $P_M(n|n) = M!/[M^n] = P(\text{no overlap})$.

The probability given by Eq. (5) can be added to the PNR model [under the sum in Eq. (3)] to correct for overlap (see discussion in Sec. IV A). Note, the overlap probability is never zero.

E. Multi-photon geometric jitter

Geometric jitter σ_{geom} arises from spatial variations in the propagation time of the electrical signal from the photon absorption position x to the readout electronics. If the nanowire can be treated as a transmission line (which may not hold under all circumstances⁴⁴), two electrical pulses propagate with velocity v toward either end of the nanowire with length l . In the differential readout scheme,^{45,46} the absolute arrival time difference at the readout is $\Delta t = |x/v - (l-x)/v| = 2x/v$. For uniform illumination and single-photon detection, the time difference $\Delta t/2 = x/v$ is also

uniformly distributed, resulting in position-dependent uncertainty, i.e., geometric jitter. The corresponding geometric jitter is $\sigma_{\text{geom},1} = l/(2\sqrt{3}v)$, which yields ≈ 5 ps per 100 μm of nanowire length (for typical $v = 6 \mu\text{m}/\text{ps}$ ^{45,46}).

For two-photon detection, the distribution of the time difference Δt follows a symmetric triangular distribution because at least one site is more likely to be closer to the nanowire ends [see orange bars in Fig. 2(a)]. The geometric jitter for this case, $\sigma_{\text{geom},2} = l/(2\sqrt{6}v)$, is reduced by a factor of $\sqrt{2}$ compared to single-photon detection. For three-photon detection, the time difference Δt is determined by the two outermost absorption sites. Since one site always falls between the other two, the variation of Δt is constrained, leading to a narrower, more peaked distribution than the triangular case for two sites [see Fig. 2(a)]. Geometric jitter therefore decreases as the photon-number state increases, following $\sigma_{\text{geom},n} \propto n^{-0.75}$ (with the exponent determined numerically by Monte-Carlo simulations).

The time-difference distribution was experimentally observed in Ref. 46 [see Fig. 3(a) therein] for different pulsed light intensities. It represents a convolution of distributions corresponding to single- and multi-photon events weighted by the Poisson probabilities of the photon number, although this interpretation was not recognized at the time.

Now consider a conventional readout scheme, where one end of the nanowire is shorted to the ground. At the absorption site x , one electrical pulse propagates directly to the readout end, and the other first reaches the shorted end and then, propagating through the ground, arrives at the readout end. The arrival time of the “center of mass” $\Delta t = |t_1 + t_2|/2 = (l/v + l/v_g)/2$ is independent of position and, therefore, does not cause geometric jitter unless the dispersion and losses differ between the nanowire and the ground plane.²⁹ Here, $v_g \approx 140 \mu\text{m}/\text{ps}$ is the propagation velocity through the ground plane; since $v_g \gg v$, the contribution of the last term l/v_g is negligible. Alternatively (depending on the device layout and circuit implementation), the second pulse can be reflected by the ground and propagate back toward the readout through the nanowire itself. In this scenario, the geometric jitter can be canceled at a specific voltage trigger.⁴⁷

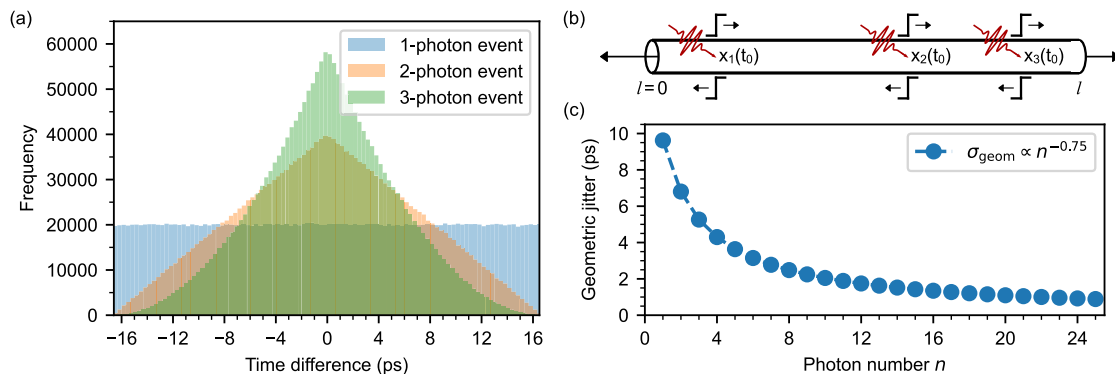


FIG. 2. (a) Distributions of time differences Δt for one-, two-, and three-photon events show flat, triangular, and peaked shapes, respectively. (b) Schematic showing the origin of geometric jitter. (c) Geometric jitter σ_{geom} as a function of photon number n .

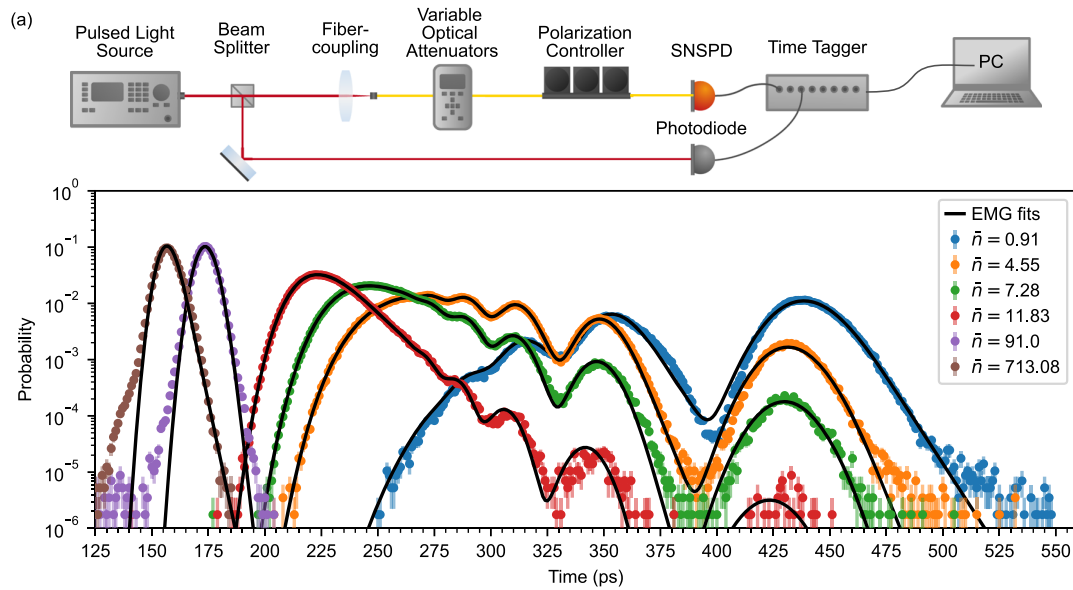


FIG. 3. (a) Experimental setup to measure photon-number resolution with SNSPDs based on the rising edge arrival times relative to a trigger signal. Refer to the main text for more details. (b) Arrival-time histograms for different mean photon numbers $\bar{n}$ specified in the legend (the error bars are based on Poisson errors of the counting statistics). The black lines correspond to the best fits based on a sum of exponentially modified Gaussian distributions [Eq. (3)].

III. COMPARISON TO EXPERIMENT

We characterize the PNR response of a commercial SNSPD (Single Quantum, 2023) made of a 1.4 mm-long, 70 nm-wide, meandered NbTiN nanowire with 140 nm pitch at an operating temperature of 2.3 K (the sheet resistance and transition temperature were not disclosed). The detector is operated at a bias current of 17.7 μ A with a switching current of 19.3 μ A as specified by the manufacturer; it achieves a system detection efficiency of $(91 \pm 3)\%$ with 19 ps jitter (FWHM, or 8 ps std; we assume that this was obtained at $\bar{n} \ll 1$ with a short pulsed laser). The single-ended readout chain, designed by the manufacturer (the circuit was not disclosed), includes cryogenic amplifiers with a 1 GHz bandwidth.

From the recorded electrical traces, we extract the following parameters: the detector dead time of ≈ 150 ns defined as the time at which the voltage signal drops below the noise level; the root-mean-square baseline noise $\sigma_{\text{elec}} = 4.9$ mV; the 20%–80% voltage rise time in response to a single photon $t_{\text{rise},1} = 0.68$ ns, and the corresponding slew rate $\gamma_{\text{slew},1} = dA/dt = 1.08$ mV ps^{-1} (which holds for linear slopes near 50% of the pulse amplitude A). Using these values, we estimate the noise jitter as $\sigma_{\text{noise}} = \sigma_{\text{elec}}/\gamma_{\text{slew}} = 4.5$ ps.

The experimental setup, Fig. 3(a), comprises a 1550 nm pulsed light source with 2.4 ps pulse duration (FWHM, $\sigma_{\text{opt}} = 1$ ps std) at a 9.5 kHz repetition rate, which was reduced from 80 MHz using a pulse picker. Two variable optical attenuators accurately set the mean photon number per optical pulse (with uncertainty $\pm 5\%$, see Ref. 22). Manual fiber polarization controllers are used to maximize the SNSPD count rate. The arrival times of SNSPD signals are recorded using a Time Tagger X (Swabian Instruments, < 2 ps std jitter) triggered by the fast photodiode (5 GHz bandwidth). The time

tagger threshold is set at half the peak amplitude of the SNSPD signal for optimal photon-number resolution.²² We acquired 122 datasets spanning mean photon numbers from 1 to ~ 68 000 photons per pulse, with 570 000 events recorded for each set. Figure 3(b) shows six representative experimental datasets for different mean photon numbers per optical pulse $\bar{n}$ (colored dots).

We describe the arrival-time histograms with our PNR model [Eq. (3)], making the following assumptions: (i) as an ansatz, we assume the intrinsic jitter components, σ_{int} and τ , do not depend on the photon number n , but we allow them to vary with the mean photon number $\bar{n}$ (see the beginning of the discussion in Sec. IV); (ii) we disregard time-walk artifacts,⁴⁸ arising from count rate dependent variations in the bias current, due to the long optical pulse period (105 μ s) compared to the SNSPD dead time (150 ns); (iii) we empirically allow the one-photon peak position μ_1 to vary with photon flux to account for unknown circuit effects (e.g., ringing, bandwidth); (iv) we neglect delayed photon absorption effects despite the optical pulse duration (2.4 ps FWHM) being comparable to the hotspot formation time in NbTiN (≈ 5 ps).

Under these assumptions, the photon-number-dependent model parameters are

$$\mu_n \rightarrow \mu_\infty + \frac{\mu_1 - \mu_\infty}{\sqrt{n}} \propto \frac{t_{\text{rise},1}}{\sqrt{n}}, \quad (6a)$$

$$\sigma_{\text{noise},n} \rightarrow \frac{\sigma_{\text{elec}}}{\gamma_{\text{slew},1}\sqrt{n}} \approx \frac{\sigma_{\text{elec}} t_{\text{rise},1}}{A\sqrt{n}}, \quad (6b)$$

$$\sigma_{\text{geom},n} \rightarrow \frac{\sigma_{\text{geom},1}}{n^{0.75}}, \quad (6c)$$

and the remaining parameters are instrumental jitter $\sigma_{\text{inst}} = 3$ ps (from time-tagger and trigger photodiode), optical jitter $\sigma_{\text{opt}} = 1$ ps, and intrinsic jitter τ and σ_{int} . In Eq. (6a), $\mu_{\infty} = 144$ ps is an arbitrary offset delay in our setup (the time difference between the trigger signal and the steepest rising edge). In Eq. (6c), the one-photon geometric jitter $\sigma_{\text{geom},1} = 9$ ps was pre-fitted and held constant across all datasets while the exponent 0.75 was determined numerically (see Sec. II E). The three primary fitting parameters are μ_1 , τ , and σ_{int} .

As shown in Fig. 3(b) (black curves), the model is in excellent agreement over several orders of magnitude of the arrival-time probability and across a wide range of $\bar{n}$ from 1 to 500 per optical pulse.

To cover a wider range of fluxes $\bar{n}$ from 0.001 to 1000 and validate the model robustness, we performed additional arrival-time measurements (see Fig. 5 in Appendix) with a laser diode with a 10 kHz repetition rate and a larger pulse duration of $\sigma_{\text{opt}} = 4$ ps (std). The datasets with both light sources are well described by the model.

Figure 4(a) (red and green colored symbols) shows how the total width of the arrival-time histogram depends on the photon flux (similar experimental data were obtained from a waveguide-integrated SNSPD⁴⁹). In the limit of large photon fluxes, the SNSPD jitter diminishes, which has been addressed in Ref. 37, attributed to the bolometric regime, i.e., when the total deposited photon energy turns a large section of the wire normal.

In Figs. 4(b) and 4(c), we show the best-fit values of the intrinsic jitter $\sqrt{\sigma_{\text{int}}^2 + \tau^2}$ and the one-photon peak position μ_1 obtained when describing the experimental histograms partly shown in Fig. 3(b) and in Fig. 5. As expected, in the few-photon range $\bar{n} \ll 1$, the intrinsic jitter stays broadly constant, while it decreases at high photon flux due to overlap and heating effects. The one-photon peak position μ_1 shows inconsistent behavior. For the data acquired with a

pulse-picked system, μ_1 shifts toward earlier times, likely due to residual unpicked pulses causing detector heating (this agrees with electrothermal simulations), while for diode laser measurements, μ_1 shifts toward later times and then decreases.

We took a closer look at this discrepancy by changing the diode laser repetition rate from 1 kHz to 1 MHz (see Fig. 5 in the Appendix); however, we leave these effects outside the scope of this work, as they do not impact our main conclusions regarding jitter mechanisms.

IV. DISCUSSION

While it is clear that our results agree with the PNR model very well, we made a number of assumptions that may not be applicable in all uses.

First, while circuit-related effects (finite bandwidth, over-/undershoot in current, etc.) remain uncharacterized in our setup, they likely influence the observed shifts in the one-photon peak position in Figs. 4(b) and 4(c).

Second, as expected, the total jitter at high photon fluxes $\bar{n} > 100$ drops below that at low fluxes $\bar{n} \ll 1$ [see Fig. 4(a)] as its components due to noise, geometry, and intrinsic physics diminish. However, the reduction in intrinsic jitter [Figs. 4(b) and 4(c)] appears at a much smaller $\bar{n} \approx 10$ than anticipated. We estimate that our nanowire comprises $\sim 10^3$ independent domain detection bins (from our simulations, the domain length scales as $2 \mu\text{m}/\sqrt{\bar{n}}$) and $\sim 10^4$ independent hotspot detection bins (with hotspot size $L_T \approx 50$ nm in NbTiN), suggesting that overlap effects should only become relevant at $\bar{n} > 10^2$. We attribute this premature reduction in jitter to the limited bandwidth of our readout electronics (BW = 1 GHz), which distorts rising edges of voltage pulses generated by photon numbers $n > 4$ ($t_{\text{rise}} = 0.35/\text{BW}$). This limited bandwidth may also shift the one-photon peak position μ_1 .

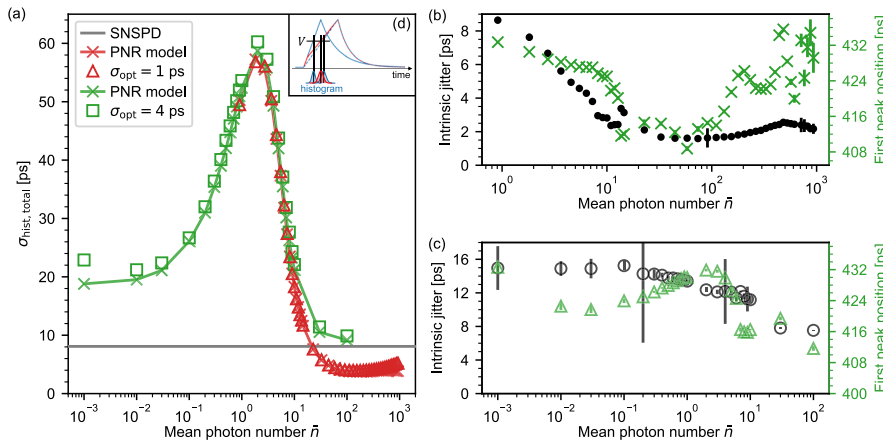


FIG. 4. (a) Total width of the arrival-time histogram $\sigma_{\text{hist,total}}$ as a function of mean photon number for datasets taken with two light sources with different optical jitter (as indicated in the legend). The model and experiment show close agreement. The gray line corresponds to the reference SNSPD jitter of 8 ps (std, as specified by the manufacturer). (b) Best-fit intrinsic jitter $\sqrt{\sigma_{\text{int}}^2 + \tau^2}$ (left, black axis) and the mean position μ_1 (right, green axis) of the one-photon histogram peak from Eq. (1), for the dataset with optical jitter of $\sigma_{\text{opt}} = 1$ ps. The error bars are based on the fitting uncertainty. (c) Best-fit parameters for the dataset with optical jitter of $\sigma_{\text{opt}} = 4$ ps. The arrival-time histograms and fits are shown in Fig. 5(b). The inset (d) shows a schematic of voltage response pulses from a single domain (blue dashed line), two independent domains (blue solid line), and initially two independent domains that then merge into each other (red solid line).

In scenarios of multi-photon absorption at independent sites, intrinsic jitter is influenced by the photon number due to the following: each absorbed photon initiates a delay in the emergence of a resistive domain, according to the local fluctuations, which are not correlated. The shortest delay determines the voltage pulse arrival time, similar to the effect of delayed multi-photon absorption described in Sec. II C. As the number of photons increases, the intrinsic jitter decreases for the same reason as for multi-photon geometric jitter in Sec. II E. Statistical analysis showed that for the studied detector this effect is small and can be neglected.

A. Limitations due to overlap

Although our primary analysis neglects domain overlap, Eq. (5) provides a first-order correction that accounts for it. This leads to a positive skew in the arrival-time distribution (underestimation of high photon-number events and overestimation of low photon-number events). Increasing the nanowire length reduces the overlap probability, while adding a series inductance or a shunt resistance has the opposite effect; it slows down the current dynamics and effectively increases the domain size, thereby increasing the overlap probability.

The situation is different when domains merge while growing, which is not described by Eq. (5). Electrothermal simulations⁵⁰ show that in such cases, the voltage response initially follows the two-domain dynamics but transitions to one-domain behavior as domains merge [see Fig. 4(d) for a schematic]. These events populate earlier times in the first peak, shifting its mean value to earlier times as $\bar{n}$ increases. This process is of a continuous nature, as the distance between two initially non-overlapping domains varies continuously. Therefore, the time at which the domains grow into each other and the voltage transitions to the one-domain dynamics is also continuous.

Hotspot overlap becomes significant only at higher fluxes $\bar{n}$ due to their smaller size compared to the domain. While hotspot overlap does not affect the rise time, it reduces the arrival time, populating earlier times in the histogram [see Fig. 1(a-ii)]. For SNSPDs operating below detection efficiency saturation, this can result in overestimation of high photon-number states.

B. Implications for detector engineering

First of all, this PNR method can only work for optical pulse durations σ_{opt} below characteristic timescales determined by the detector material and circuit. This limit is set either by hotspot formation time [typically about the electron-phonon cooling time, ≈ 5 ps in NbTiN,³⁹ ≈ 15 ps in WSi,³⁹ and $\approx 12 - 20$ ps in MoSi (unpublished)] or, in SNSPDs with saturated detection efficiency, by the time during which the current in the nanowire remains above the level with saturated efficiency. In devices with no saturation, PNR signatures may still be resolvable for longer optical pulses, though with reduced efficiency and increased jitter, either up to the resistive domain lifetime [≈ 50 ps⁵¹ in NbTiN, ≈ 630 ps in WSi,⁵² and $\approx 70 - 130$ ps in MoSi (unpublished)] or up to the time the current remains in the detector (Sec. II C).

Optimizing PNR efficiency, i.e., distinguishability of different photon numbers, requires maximizing the separation between the one-photon and the many-photon (steepest) voltage pulse ($\mu_1 - \mu_\infty$) while minimizing all the jitter contributions. From

Eq. (6a), maximizing ($\mu_1 - \mu_\infty$) means increasing the rise time $t_{\text{rise},1}$. One approach is increasing the wire length and, therefore, the kinetic inductance L_k , since $t_{\text{rise}} \propto L_k$, which also helps to suppress domain overlap, albeit at the cost of increased geometric jitter and increased detector's dead time $t_{\text{dead}} = t_{\text{fall}} \log(N/A)$, where N is the noise level and $t_{\text{fall}} = L_k/50 \Omega$ is the voltage falling time. These trade-offs must be carefully considered when optimizing the detector's design for PNR, as has been noted in Refs. 23 and 41.

Longer rise times relax the bandwidth requirements, which must satisfy $\text{BW} = 0.35\sqrt{\bar{n}}/t_{\text{rise},1}$ to avoid pulse distortion. However, the variance due to noise increases proportionally [Eq. (6b)]. Reducing the electrical noise by using low-noise cryogenic amplifiers has become a well-established technique in the community, therefore enhancing PNR (which has also been noted in Ref. 24). Furthermore, increasing the voltage pulse amplitude, which means maximizing the bias current, can also help. The combination of large bias currents and large area detectors is readily achieved by using micron-wide wires.⁵³⁻⁵⁵ With this approach, Kong *et al.*²³ achieved PNR up to ten photons, sacrificing the reset time.

It is also apparent from Eqs. (6b) and (6c) that noise and geometric jitter can be effectively removed when using a large number of photons per pulse due to the inverse proportionality on n . This has practical implications in laser ranging: higher photon fluxes improve resolution, as multi-photon events reduce the timing jitter (also the intrinsic jitter due to overlap effects) and sharpen the histogram peak.⁵⁶ This was observed in very early work on SNSPDs.⁵⁷

Finally, in real-world quantum photonics applications, single-photon sources typically used emit photons with durations ranging from 100 ps to a few nanoseconds, which SNSPDs must be able to handle for optimal PNR in such settings.

V. CONCLUSION

In this work, we have provided a quantitative model of photon-number dependent traces of SNSPDs, which includes both the arrival time and photon-number dependent jitter. Our model accurately predicts the arrival-time probability histogram across several orders of magnitude for a broad range of mean numbers of incident photons. Implementation of the model will enable the design of detectors with maximized PNR capability. While these results are certainly promising, there are additional effects that can be incorporated into future modeling, such as double-ended or differential read-out of the SNSPDs (eliminating geometric jitter) or the use of impedance matching tapers.⁵⁸

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The identification of any manufacturer or supplier in this work shall not be taken as an endorsement or recommendation of specific products provided by the said manufacturer.

AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts to disclose.

Author Contributions

Mariia Sidorova: Conceptualization (lead); Formal analysis (equal); Investigation (equal); Methodology (equal); Software (equal); Validation (supporting); Visualization (supporting); Writing – original draft (equal); Writing – review & editing (equal). **Timon Schapeler:** Conceptualization (supporting); Data curation (lead); Formal analysis (equal); Investigation (equal); Methodology (equal); Software (equal); Visualization (lead); Writing – original draft (equal); Writing – review & editing (equal). **Alexej D. Semenov:**

Conceptualization (supporting); Formal analysis (equal); Methodology (equal); Validation (lead); Writing – review & editing (equal). **Fabian Schlue:** Data curation (supporting); Investigation (equal); Writing – review & editing (equal). **Michael Stefszky:** Data curation (supporting); Funding acquisition (supporting); Project administration (equal); Supervision (equal); Writing – review & editing (equal). **Benjamin Brecht:** Data curation (supporting); Funding acquisition (supporting); Project administration (equal); Supervision (equal); Writing – review & editing (equal). **Christine Silberhorn:** Funding acquisition (equal); Project administration (equal); Resources (equal); Supervision (equal); Writing – review & editing (equal). **Tim J. Bartley:** Funding acquisition (equal); Project administration (equal); Resources (equal); Supervision (equal); Writing – original draft (equal); Writing – review & editing (equal).

DATA AVAILABILITY

The data that support the findings of this study are openly available in Zenodo at <https://doi.org/10.5281/zenodo.14888684>.

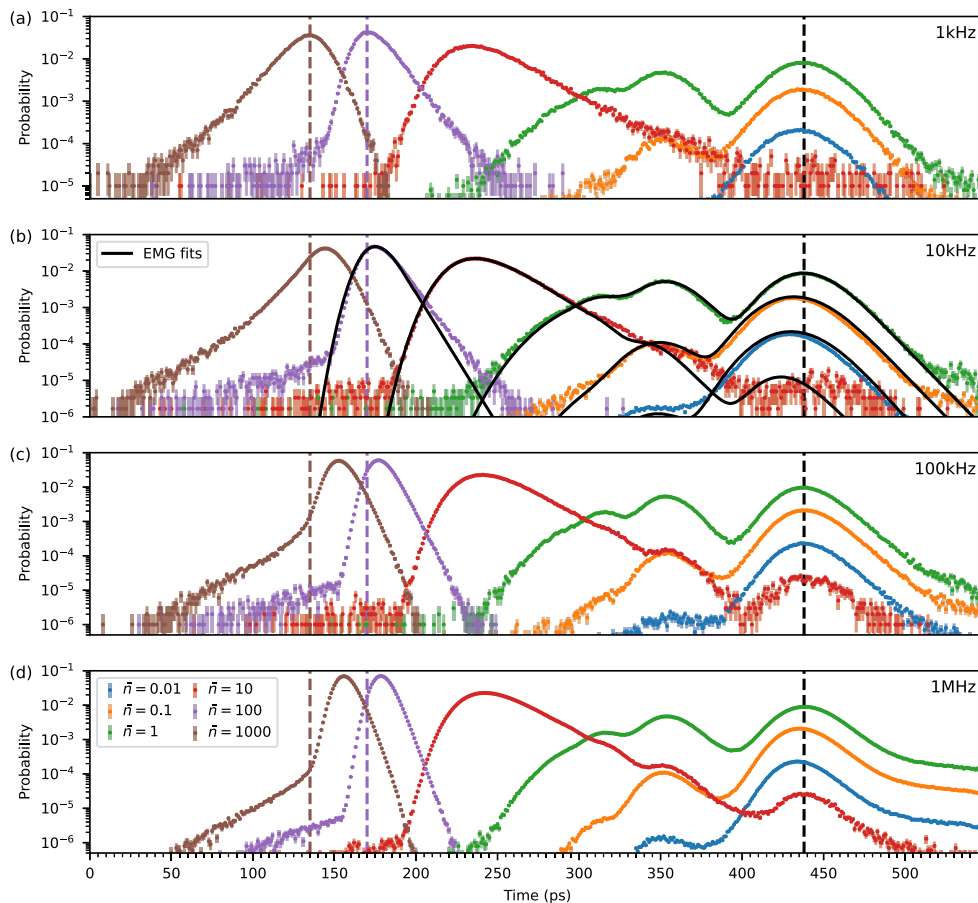


FIG. 5. Arrival-time probabilities measured at fixed laser repetition rates of (a) 1 kHz, (b) 10 kHz, (c) 100 kHz, and (d) 1 MHz. The shared legend shows the mean photon number per optical pulse (the error bars are based on Poisson errors of the counting statistics). The black lines in (b) for a 10 kHz repetition rate show the best fits with Eq. (3); the best fit parameters are plotted in Fig. 4(c).

APPENDIX A: ARRIVAL-TIME PROBABILITIES AT DIFFERENT PULSE REPETITION RATES

Here, we present arrival-time histograms measured with an alternative light source: a diode laser emitting at 1556 nm wavelength, with a pulse duration of 9.0 ps (FWHM, 3.8 ps std). We varied the laser pulse repetition rate from 1 kHz to 1 MHz and the mean photon number per pulse from 0.01 to 1000 (see Fig. 5). For reference, the detector dead time is ~ 150 ns (≈ 7 MHz).

Histograms obtained at 10 kHz were described with our PNR model; the best-fit parameters are shown in Fig. 4(c). The extracted intrinsic jitter is larger, which we attribute to the longer optical pulse duration exceeding the hotspot formation time in NbTiN (~ 5 ps). This means we are operating in the regime closer to the delayed photon absorption rather than the simultaneous photon absorption.

The data in Fig. 5 show two trends: (i) at large mean photon numbers ($\bar{n} = 1000$, brown points in Fig. 5), where the detector operates in the bolometric regime, a large part of the wire becomes normal, and the histogram shifts systematically toward longer arrival times as the repetition rate increases (e.g., 135 ps at 1 kHz vs 156 ps at 1 MHz); and (ii) the one-photon peak also slightly shifts to later arrival times (for $\bar{n}$ increasing from 0.01 to 1, the shift is 2.5 ps at 1 kHz and 5 ps at 1 MHz). We also note that the width of the one-photon histogram peak increases at lower repetition rates, likely due to Allan variance over long measurement times.

Since neither the specific circuit design nor the thermal coupling of the commercial detector is known, we leave the interpretation of these behaviors outside the scope of this work. However, we note that one may disregard the effect of heating because electro-thermal simulations show that for fixed bias current, increasing the bath temperature in contrast leads to earlier arrival times. One may also exclude current over- and undershooting because the laser repetition rate remains much below the detector's inverse dead time of 7 MHz.

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Practical considerations for assignment of photon numbers with SNSPDs

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ABSTRACT

Superconducting nanowire single-photon detectors (SNSPDs) can enable photon-number resolution (PNR) based on accurate measurements of the detector's response time to few-photon optical pulses. In this work, we investigate the impact of the optical pulse shape and duration on the accuracy of this method. We find that Gaussian temporal pulse shapes yield cleaner arrival-time histograms and, thus, more accurate PNR, compared to bandpass-filtered pulses of equal bandwidth. For low system jitter and an optical pulse duration comparable to the other jitter contributions, photon numbers can be discriminated in our system with a commercial SNSPD. At 60 ps optical pulse duration, photon-number discrimination is significantly reduced. Furthermore, we highlight the importance of using the correct arrival-time histogram model when analyzing photon-number assignment. Using exponentially modified Gaussian distributions, instead of the commonly used Gaussian distributions, we can more accurately determine photon-number misidentification probabilities. Finally, we reconstruct the positive operator-valued measures of the detector, revealing sharp features that indicate the intrinsic PNR capabilities.

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I. INTRODUCTION

Photon-number-resolving detectors are indispensable tools for advancing photonic quantum information science and technology, being at the heart of many experiments in fields including quantum communication,^{1,2} quantum computing,³ quantum imaging and sensing,⁴ and quantum metrology.⁵ These detectors are crucial components to characterize quantum states,⁶ optimize single-photon sources based on, e.g., spontaneous parametric down-conversion,^{7,8} and improve the efficiency of schemes such as Gaussian boson sampling by enabling more precise photon-number measurements than conventional single-photon detectors.^{9,10}

While transition-edge sensors (TESs)^{11,12} represent a state-of-the-art PNR technology, offering photon-energy resolution with high efficiencies, their operation requires millikelvin temperatures and complex readouts, and their intrinsically slow recovery time limits their experimental accessibility. Superconducting nanowire

single-photon detectors (SNSPDs) have rapidly emerged as the leading technology for single-photon detection owing to their excellent performance characteristics, including near-unity detection efficiency,^{13,14} picosecond timing jitter,¹⁵ fast reset times,¹⁶ and exceptionally low dark count rates.¹⁷ Historically, SNSPDs have been employed as threshold detectors, distinguishing only between the absence and presence of one or more photons. Multiplexing of conventional threshold detectors (in space or time) can be used to gain quasi-PNR.^{18–24} In this context, recent work has achieved high efficiency, high maximum count rates, low dark counts, and low crosstalk by multiplexing up to 64 SNSPDs into one active area.^{25–29} However, multiplexing can only emulate true PNR with sufficiently many threshold detectors.²⁴ As quantum systems grow, more detectors are needed, posing a significant scaling challenge. Therefore, true PNR or intrinsic PNR with sufficiently high resolution, from TESs or SNSPDs, respectively, is beneficial.

Groundbreaking research revealed that SNSPDs possess an inherent photon-number-resolving capability.³⁰ Since then, models for photon-number resolution have been constructed,^{31,32} and different PNR readout techniques have been investigated based on signal amplitude via impedance-matching tapers^{33–35} or signal timing characteristics of rising (and falling) edges.^{7,8,36–41}

Fundamentally, the SNSPD is not an energy-resolving detector. The PNR capability stems from the fact that the nanowire itself can be seen as a highly multiplexed device, consisting of many independent hotspot detection bins (as long as the number of impinging photons is much smaller than the number of independent detection bins).³² Thus, PNR arises from the formation of multiple resistive regions (hotspots) on the superconducting nanowire upon absorption of multiple photons. When multiple photons are absorbed within a sufficiently short time window, they create multiple distinct hotspots, and the total resistance of the nanowire becomes proportional to the number of absorbed photons. Signatures of PNR should still be resolvable up to the resistive domain lifetime,³² which is roughly 50 ps for NbTiN SNSPDs.⁴² In this regime, multiple hotspots provide an “intrinsic multiplexing” of the detector response, which may be resolved with sufficiently fast timing resolution of the detector rise time. This has become known as intrinsic PNR (distinguishing it from “true PNR” of energy-resolving detectors under monochromatic illumination).

In this work, we aim to advance the fundamental understanding of operating SNSPDs as PNR detectors, since they offer a promising pathway for numerous cutting-edge quantum applications that demand precise photon-number resolution with high speed and low noise. To do so, we investigate the intrinsic PNR capabilities of a commercial SNSPD based on a detailed analysis of arrival-time histograms derived from time differences between a trigger signal and the rising edge of the SNSPD’s electrical signal.^{32,37,38} We specifically examine the photon-number differentiation under varying temporal optical pulse widths, which significantly influences resolution. We compare a recently constructed model for arrival-time histograms based on exponentially modified Gaussian distributions³² with a simpler model based on Gaussian distributions, focusing on misidentification of photon numbers. Finally, to provide a complete and rigorous characterization of the detector performance, we reconstruct the positive-operator-valued measures (POVMs) of the PNR SNSPD.

II. EXPERIMENTAL SETUP

The experimental setup is shown in Fig. 1. We use a 1550 nm pulsed laser with a repetition rate of 80 MHz. An electro-optic modulator (EOM) is used as a pulse picker to achieve a repetition rate of 9.5 kHz of the laser pulses. The EOM is modulated by a square pulse from a digital delay generator, which is triggered by an electrical synchronization pulse from the pulsed laser. A quarter- and half-wave plate are used to rotate the polarization of the laser to horizontal for optimal pulse picking. Another set of wave plates is used after the EOM to optimize the extinction ratio of the pulse-picking, which was maximally 23 dB. The unpicked pulses are dumped at a beam dump after a polarizing beam splitter (PBS). The transmitted pulses are further split by a beam sampler, where the transmitted part is used to trigger a fast photodiode with a bandwidth of 5 GHz, and the reflected part is fiber-coupled. We use a programmable

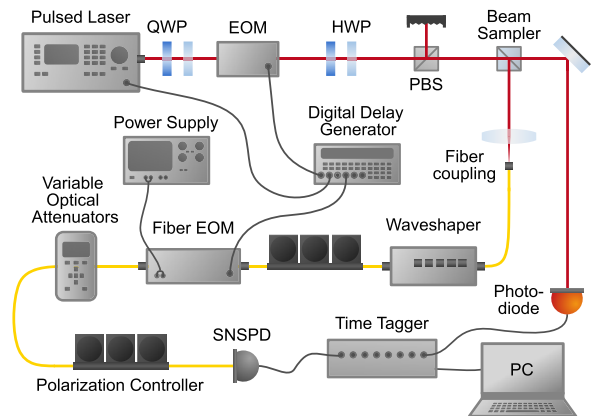


FIG. 1. Experimental setup used to investigate the influence of multiple experimental parameters of the input light pulses on the photon-number resolution of SNSPDs. The laser pulses from a pulsed laser pass through two electro-optic modulators for high extinction ratio pulse-picking, a programmable optical processor (wvshaper) to manipulate the temporal shape of the pulse and variable optical attenuators to set the mean photon number per pulse. For more details, refer to Sec. II. QWP: quarter-wave plate; HWP: half-wave plate; EOM: electro-optic modulator; PBS: polarizing beam splitter.

optical processor (WaveShaper 4000B from Coherent) to investigate different pulse shapes and pulse widths. Afterward, the light passes through a fiber polarization controller to optimize the polarization for another (fiber-coupled) EOM to enhance the extinction ratio of the pulse picking, achieving over 50 dB. Two variable optical attenuators (VOAs) are used to precisely set the mean photon number per pulse, and another set of fiber polarization controllers are used to optimize the efficiency of the polarization-dependent SNSPD. The commercial SNSPD (Single Quantum) has a jitter of 19 ps (FWHM) while biased at saturated internal detection efficiency (17.7 μ A, manufacturer-specified operating current). More information about the SNSPD’s figures of merit and nanowire parameters can be found in the [supplementary material](#). A time tagger (Time Tagger X from Swabian Instruments) is used to record the time stamp of the trigger signal from the fast photodiode, as well as the rising and falling edges of the electrical trace of the SNSPD. The threshold of the time tagger is set to half peak height, which was previously found to give optimal separation of photon-number events.³⁸ Calibration of the input states is performed following the procedure outlined in Refs. 38 and 43, which relates the mean photon number per pulse of the coherent states to the attenuation of the variable optical attenuators.

For the main dataset used for Sec. IV, we record a total of $D = 122$ different coherent input states (with an optical temporal pulse duration of 2.9 ps). We vary the mean photon number per pulse $\bar{n}$ linearly from 0 to 16 in steps of 1, then quadratically from 25 to 10 000, then with linear attenuation steps up to a maximum mean photon number of $\sim 68\,000$ (more information about the input state calibration procedure can be found in Ref. 44). Every input state is sampled 570 000 times; we record the timestamps of the trigger signal and rising and falling edge of the SNSPD trace.

III. PULSE-DURATION-DEPENDENT PNR

The intrinsic photon-number-resolving capability of an SNSPD depends on the total measured jitter in the experimental setup.³² One contribution to this jitter is the optical temporal pulse duration of the laser pulses, i.e., the uncertainty of the photon arrival time. We investigate the PNR capability of the SNSPD for varying optical pulse durations by shaping the pulse in the spectral domain (see Appendix A). The effect of different spectral filters on the PNR capability is described in Appendix B and visualized in Fig. 7; all following measurements use a Gaussian filter, as this leads to a cleaner differentiation of photon-number events. In Fig. 2, we show arrival-time histograms for different bandwidth settings, i.e., optical temporal pulse durations. The bandwidth of 2.66 nm (orange line) leads to the calculated optical pulse duration of 2.9 ps, which gives the cleanest photon-number differentiation. The light blue line in Fig. 2 shows that the photon-number differentiation starts to decrease for an optical pulse duration of 25 ps (a filter bandwidth of 0.14 nm). If one of the contributions to the system jitter begins to dominate (in this case, the optical temporal pulse duration), the underlying distributions for all photon-number contributions begin to broaden. This is clearly visible by the increased one-photon contribution (compare rightmost peaks in Fig. 2). For an optical pulse duration of 60 ps, the separation between the one- and two-photon contributions is significantly affected (see peaks at relative time differences of 310 and 180 ps of the red line in Fig. 2, respectively). The FWHM of the one-photon peaks in Fig. 2 is a representation of the total measured system jitter. This jitter is a convolution of multiple contributions, originating from noise in the electronics, the data acquisition instrumentation, the optical temporal pulse duration, geometric jitter in the nanowire, and intrinsic jitter from the detection process (more information can be found in the [supplementary material](#) and Ref. 32).

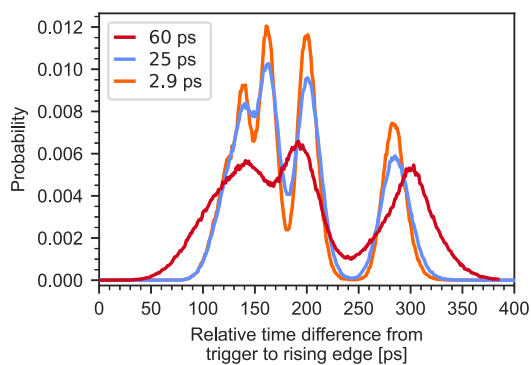


FIG. 2. Arrival-time histogram of different waveshaper bandwidth settings, for a mean photon number per pulse of $\bar{n} = 3$ and center wavelength of $\lambda = 1550$ nm. The temporal pulse widths of 60 ps (red), 25 ps (light blue), and 2.9 ps (orange) correspond to bandwidths of 0.01, 0.14, and 2.66 nm, respectively. The mean photon number for the bandwidth of 0.01 nm is 2.5, due to limited optical power. The mean photon numbers of the coherent input states are calibrated before every measurement.

IV. ANALYSIS

For every incident mean photon number on the detector, we analyze the data as arrival-time histograms between the trigger signal and the rising edge of the SNSPD trace. With these histograms, we investigate the quality of photon-number assignment. We note that, although we also recorded the falling edge of the SNSPD trace, we found that, for this dataset, no additional photon-number information could be extracted,⁴⁵ as shown in Refs. 37 and 38. Subsequently, we will describe the fitting procedure of the arrival-time histograms.

A. Exponentially modified Gaussian distributions

Arrival-time histograms from SNSPDs are well described by exponentially modified Gaussian (EMG) distributions.⁴⁶ Recently, the authors of Ref. 32 extended this into a model incorporating photon-number resolution based on arrival-time measurements, including the underlying jitter mechanism in superconducting nanowires.

Our analysis based on EMG distributions uses the first nine input states $\bar{n} = 1$ to $\bar{n} = 9$ to incorporate a slight shift in the peak positions of the underlying distributions when varying the mean photon number [further details can be found in Ref. 32; the slight shift can be seen in Fig. 8(a), colored lines]. This ensures that the analysis is independent of the mean photon number. We fit EMG distributions (based on the model from Ref. 32) to the histograms for different mean photon numbers and sum the individual fits. Using this model, we achieve high accuracy between data and fit, as can be seen in Fig. 3(a), where the exponential tail, which is present in the experimental data, is accounted for. By incorporating the slight shift in the one-photon peak position, we can fully characterize the quality of the photon-number assignment in a later step. This becomes important in experiments where photon numbers need to be distinguished on a shot-by-shot basis, but the mean photon number may vary.

Depending on the SNSPD material or design, the exponential contribution in the arrival-time measurements may differ. Alternatively, fitting models based on sums of Gaussian distributions have been used.^{40,41} We also apply a Gaussian model (which is explained in detail in Appendix C) to show the difference in accuracy between the models. Figure 3(b) shows that the fit (red dashed line) has reduced overlap with the data (black line). Plotting the underlying Gaussian distributions on a logarithmic scale also reveals further details about how the overlap decreased (colored lines). The EMG model [Fig. 3(a)] is accurate over four orders of magnitude, whereas the Gaussian model [Fig. 3(b)] is accurate for merely one order of magnitude.

B. Assignment quality

To quantify the assignment quality of the photon-number events, we extract fitting parameters of the underlying distributions (for both the EMG and the Gaussian distribution model). We then calculate the intersection points of all neighboring distributions, which are where we draw separation lines to distinguish the photon-number components into regions of arrival times (see vertical dashed lines in Fig. 3).

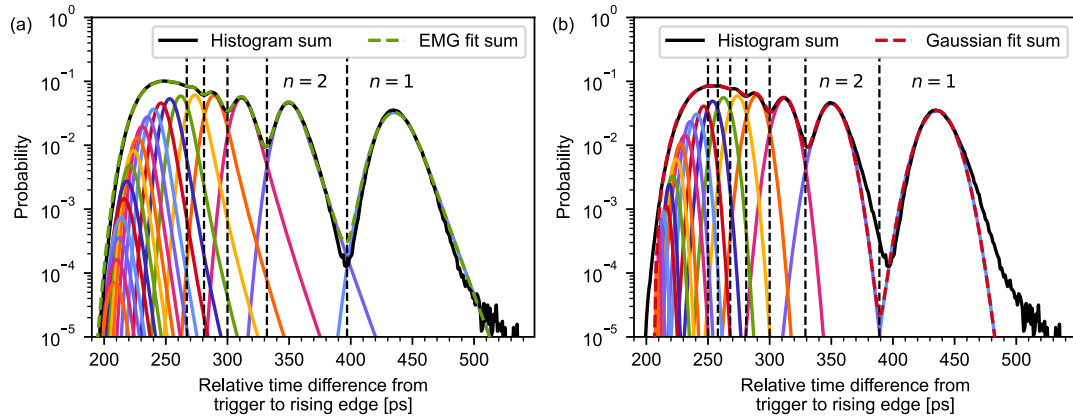


FIG. 3. (a) Sum of the arrival-time histograms (black line) with fits (green dashed line and the colored lines) based on EMG distributions (for an ensemble of arrival-time histograms for different mean photon numbers), according to the model from Ref. 32 with $\chi_{\text{EMG}}^2 = 0.02$. (b) Sum of the arrival-time histograms (black line) with a fit (red dashed line) based on a sum of Gaussian distributions weighted according to Poissonian statistics [Eq. (C1)] with $\chi_{\text{Gauss}}^2 = 0.06$. Vertical black dashed lines separate the photon-number regions, based on the intersection points between neighboring distributions. Individual underlying Gaussian distributions for photon-number contributions up to $n = 20$ are shown with different colored lines.

For each fitting method, we calculate the overlap of every distribution with every photon-number region (bounded by the upper and lower bounds), which results in an overlap matrix. The overlap matrix based on the EMG distributions is shown in Fig. 4(a), where the color axis is displayed on a logarithmic scale.

Subsequently, we use the type I and type II errors,⁴⁷ namely, the probabilities to misidentify (false positive) or miss (false negative)⁴⁸ an n -photon event to quantify the assignment of photon numbers. The probability of missing an n -photon event, which happens if the n th distribution is not fully contained within the n th photon-number region is given by

$$p_{\text{missing},n} = 1 - \int_{l_n}^{u_n} g_n, \quad (1)$$

where g_n is the n th distribution, and u_n and l_n are the upper and lower bounds of the n th photon-number region, respectively. The probability of misidentifying other photon numbers $m \neq n$ as n , which happens if other distributions $m \neq n$ overlap with the n th photon-number region is given by

$$p_{\text{misidentified},n} = \frac{\sum_{m \neq n} \int_{l_n}^{u_n} g_m}{\sum_i \int_{l_n}^{u_n} g_i}, \quad (2)$$

which is normalized to all contributions in the n th photon-number region. The probabilities for both fitting methods are shown in Fig. 4(b), where it can be seen that, due to the larger overlap of the underlying distributions, the probabilities of missing and misidentifying an n -photon event increase as a function of n .

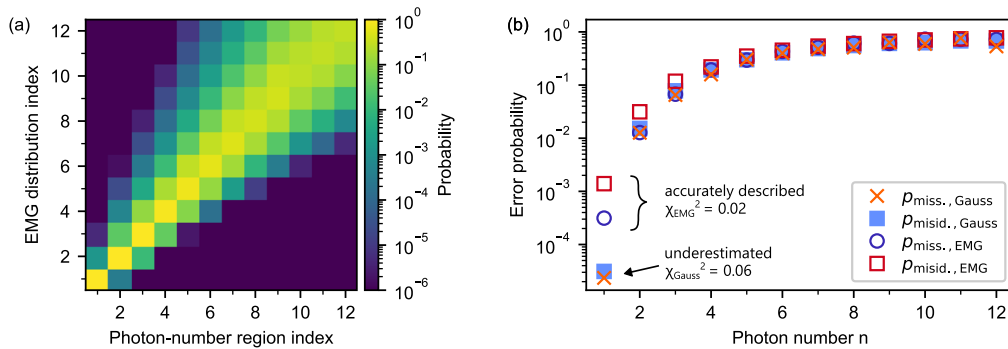


FIG. 4. (a) Overlap matrix of every EMG distribution with every photon-number region, bounded by the separation lines, i.e., intersection points of the underlying distributions [from Fig. 3(a)]. (b) Probability of missing $p_{\text{missing},n}$ and misidentifying $p_{\text{misidentified},n}$, an n -photon event as a function of photon number n for both fitting methods (based on Gaussian and EMG distributions). For the Gaussian model, the probabilities are underestimated due to inaccuracies in the fit of the exponential tail of the underlying distributions [see Fig. 3(b)].

TABLE I. Probabilities to miss or misidentify photon-number components based on the two analysis methods using either Gaussian distributions or EMG distributions.

	$P_{\text{misidentified}}$		P_{missing}	
	Gaussian (%)	EMG (%)	Gaussian (%)	EMG (%)
$n = 1$	0.0032	0.14	0.0024	0.032
$n = 2$	1.53	3.14	1.27	1.28

The error probabilities are generally larger for the fits based on the EMG distributions (i.e., the model from Ref. 32), due to capturing the exponential tail in the experimental data. The simplified fits based on Gaussian distributions lead to an underestimation of the error probabilities, which can be seen most clearly in Fig. 4(b) for a photon number of $n = 1$ (exact probabilities are listed in Table I). For unity efficiency of the detector, based on the Gaussian fits, one would assume to misidentify only 1 out of roughly 30 000 one-photon events (which would be a two-photon event), whereas for more accurate EMG fits, 1 out of roughly 700 one-photon events is misidentified. This shows that the exponential tail contributes significantly to misidentification, which further motivates making the separation between photon-number components as large as possible to reduce the misidentification probability.

Alternatively, it is possible to reduce the misidentification probability $P_{\text{misidentified},n}$ by choosing narrower photon-number regions around the respective peak centers. This technique has also been used to reduce assignment errors for photon counting using TESs.⁴⁹ The drawback of reducing the size of the region is that “loss” is introduced. When the photon-number regions do not cover the entire arrival-time axis [see Fig. 9(a)], some detection events lead to invalid assignments if they do not fall into any region. In Appendix D, we show that sacrificing 6% of one-photon events to loss can reduce the misidentification probability of one-photon events to 0.01%, i.e., 1 out of 10 000 one-photon events is misidentified.

Finally, we use the resolvability criterion defined in Ref. 50 to determine the maximal photon number the SNSPD can resolve, i.e., the photon number n for which the inequality

$$(\mu_n - \mu_{n+1}) \geq \text{FWHM}_n, \quad (3)$$

is still valid. Here, μ_n is the peak center of the n th peak in the arrival time histogram, and FWHM_n is the full-width-half-maximum of the n th photon-number contribution. According to this criterion, the SNSPD can resolve up to $n = 3$ photons (for both analysis methods), and every event contributing to earlier times in the arrival-time histogram may be regarded as a “four or more photon” event.

V. QUANTUM DETECTOR TOMOGRAPHY

Outcomes of the photon-number-resolving SNSPD are allocated based on their arrival times, with the separation lines defining different outcomes [see vertical dashed lines in Fig. 3(a)]. This is done by counting the occurrences in each photon-number region (i.e., creating a histogram with variable bin sizes) for every input state. The $n' = 0$ outcome is populated by subtracting the number of all $n' > 0$ outcomes from the total number of trials (i.e., 570 000). Subsequently, the entire matrix is normalized, i.e., divided by the total number of trials, resulting in the outcome matrix P with elements $P_{d,n'}$ [see Fig. 5(a)]. Here, d indexes the known coherent input states, which are also represented in a matrix F with elements $F_{d,n} = \frac{|\alpha_d|^{2n}}{n!} e^{-|\alpha_d|^2}$ for photon numbers $n \in [0, M-1]$ and $|\alpha|^2 = \bar{n}$ as the mean photon number of a coherent state. M is the Hilbert space dimension, which describes the maximum photon-number space where the detector was operated. We truncate the matrices P and F after $d = 40$ input states, as the detector is already saturated (only responds with the largest outcome, i.e., a “four or more photon” event); this can be seen by the yellow solid line in Fig. 5(a) reaching a probability of one.

Finally, to reconstruct the set of positive operator-valued measures (POVMs) $\{\pi_{n'}\}$, commonly represented in a matrix Π , which describes our (phase-insensitive) detector in a quantum mechanically consistent way, we make use of the Born rule in matrix representation, namely,

$$P = F\Pi. \quad (4)$$

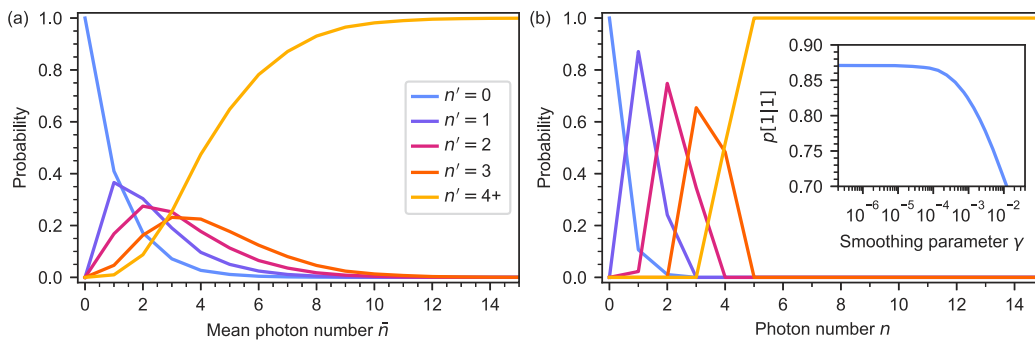


FIG. 5. (a) Outcome matrix P that results from applying the separation lines from Fig. 3(a) to the recorded data, by creating histograms with variable bin sizes for the possible detector outcomes. The largest outcome is $n' = 3$, given by the resolvability criterion Eq. (3). (b) Reconstructed POVM elements for a smoothing parameter of 10^{-6} . The inset shows the $\Pi_{1,1} = p(1|1)$ POVM element as a function of the smoothing parameter. Over-smoothing is manifested in a reduction of the $p(1|1)$ probability for a larger smoothing parameter.

For that, we solve the constrained minimization problem,^{43,51–53}

$$\min \left\{ \|\mathbf{P} - \mathbf{F}\mathbf{\Pi}\|_2 + \gamma \sum_{n,n'} (\Pi_{n,n'} - \Pi_{n+1,n'})^2 \right\}, \quad (5)$$

which is subject to the constraints $\pi_n \geq 0$ and $\sum_{n'=0}^{N-1} \pi_{n'} = \mathbf{I}$ and where γ is a smoothing parameter. The diagonal elements of the reconstructed POVMs for the different outcomes of the detector (corresponding to the columns of the matrix $\mathbf{\Pi}$) are shown in Fig. 5(b) for a smoothing parameter of $\gamma = 10^{-6}$. To find the optimal smoothing parameter, we sweep γ and investigate the matrix element $\Pi_{1,1} = p(n' = 1|n = 1)$, i.e., the conditional probability to observe the $n' = 1$ outcome, given a one-photon input in the photon-number basis [see the inset in Fig. 5(b)]. We choose a value that does not affect the $\Pi_{1,1}$ element to prevent over-smoothing.

The POVMs [based on a resolution limit of $n = 3$ from Eq. (3)] in Fig. 5(b) have sharp features. This is expected due to the intrinsic photon-number resolution and high efficiency of the SNSPD. For a photon-number resolving detector, the outcomes of a POVM reconstruction should directly map to photon numbers, i.e., the basis is the photon-number (Fock) basis, excluding the largest outcome of “ n or more photons.” When setting the resolvability limit of the PNR detector too high (i.e., there is too much overlap of the underlying photon-number contributions in the photon-number regions of the arrival-time histogram), the reconstructed POVMs cannot represent the photon-number basis anymore, at least not for larger outcomes. Thus, the outcomes need to be seen as labels, describing events that occur in a specific arrival-time window, rather than photon numbers.

Using the resolvability criterion [Eq. (3)] leads to the expected sharp features of the reconstructed POVMs [shown in Fig. 5(b) and Table II], as we limit the photon-number differentiation based on arrival times to reasonably low overlaps between neighboring photon-number contributions. To investigate the POVMs of the detector for a case where the overlap between the underlying photon-number contributions is large, we allow the detector to resolve up to $n = 6$ photons, i.e., the highest outcome is a “seven or more photon” event. Following the procedure of the previous POVM reconstruction, we find the POVMs shown in Fig. 6(a). Based on the individual EMG distributions shown in Fig. 3(a) and

the overlap matrix shown in Fig. 4(a), the larger outcomes $n' > 3$ are sharper than expected. As more photon numbers contribute to a photon-number region with increasing n , the POVMs should broaden (become smoother) for larger outcomes.

In an effort to visualize the expected behavior of the POVMs, we model the POVMs of the detector as $\mathbf{\Pi} = \mathbf{L}\mathbf{O}$ using the overlap matrix $\mathbf{O}$ from Fig. 4(a) and the loss matrix $\mathbf{L}$. The elements of $\mathbf{L}$ are given by⁵⁴

$$L_{n,m} = \binom{n}{m} \eta^m (1 - \eta)^{n-m}, \quad (6)$$

which describes the probabilities to retain m out of n photons with an efficiency $\eta = \eta_{\text{det}} = 0.91 \pm 0.03$. The overlap matrix $\mathbf{O}$ acts similar to the convolution matrix,^{55,56} which is used for multiplexed detectors. The simulated POVMs shown in Fig. 6(b) behave as expected, as larger outcomes become broader due to the increasing overlap from multiple EMG distributions with the photon-number regions.

To test the accuracy of the reconstructed POVMs $\mathbf{\Pi}_{\text{rec}}$ and the simulated POVMs $\mathbf{\Pi}_{\text{sim}}$, we can use Eq. (4) to calculate back the outcome matrices $\mathbf{P}_{\text{rec}}$ and $\mathbf{P}_{\text{sim}}$ and compare it to the measured outcome matrix $\mathbf{P}$ (based on our experimental data). For the comparison, we use a fidelity measure,^{52,53}

$$F = \frac{\left(\sum_i \sqrt{a_i a'_i} \right)^2}{\sum_i a_i \sum_i a'_i}, \quad (7)$$

to quantify how well two vectors (columns) of the outcome matrices match, where a and a' correspond to the same column in the experimentally measured outcome matrix $\mathbf{P}$ and the outcome matrix using the POVMs $\mathbf{P}_{\text{rec}}$ or based on the simulation $\mathbf{P}_{\text{sim}}$, respectively. Even though the POVMs in Fig. 6 are different, we find that both $\mathbf{P}$ matrices, based on the reconstructed and simulated POVMs, describe the experimental data well with average fidelities of 99.92% and 99.80% across the non-saturated outcomes, respectively.

To smooth larger outcomes, one might consider increasing the smoothing parameter γ . However, that does not work, as all outcomes are affected by the smoothing, thus also decreasing the sharpness of outcomes $n' \leq 3$. This can be seen by the dashed lines in Fig. 6(a), which shows the result of a reconstruction using $\gamma = 10^{-3}$,

TABLE II. Reconstructed POVM elements, i.e., conditional probabilities $\Pi_{n,n'} = p(n'|n)$ with a smoothing parameter of $\gamma = 10^{-6}$ [from Fig. 5(b)] up to a photon number of $n = 7$.

	$\Pi_{n,n'}$	Detector outcome n'				
		0	1	2	3	4+
Photon number n	0	0.999 96	0.000 04	0	0	0
	1	0.106 40	0.870 92	0.022 68	0	0
	2	0.010 81	0.241 09	0.748 10	0	0
	3	0	0	0.345 61	0.654 39	0
	4	0	0	0	0.483 57	0.516 43
	5	0	0	0	0	1.000 00
	6	0	0	0	0	1.000 00
	7	0	0	0	0	1.000 00

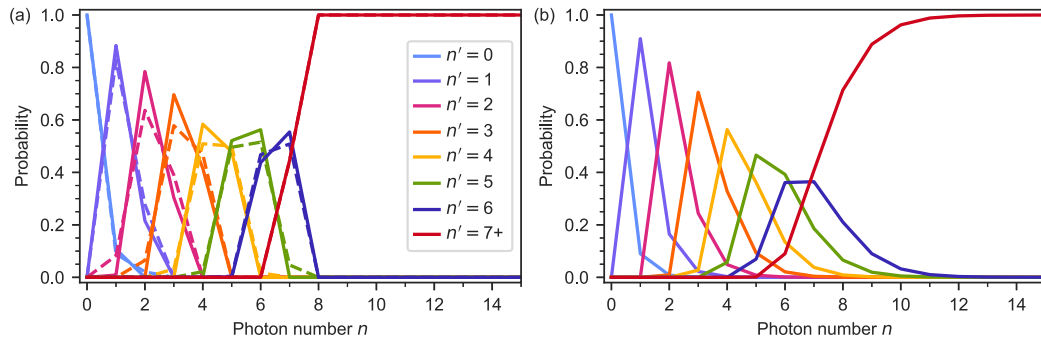


FIG. 6. (a) Reconstructed POVMs Π_{rec} when allowing the largest outcome to be $n' = 6$. The outcomes remain sharp even for larger n' . Solid lines show the reconstructed POVMs for $\gamma = 10^{-6}$ and dashed lines for $\gamma = 10^{-3}$. (b) Simulated POVMs Π_{sim} using the overlap matrix $\mathbf{O}$ in Fig. 4(a) and the loss matrix $\mathbf{L}$ [Eq. (6)]. The outcomes broaden as n' increases.

with an average fidelity of 99.89%. We find that the reconstruction with $\gamma = 10^{-6}$ yields the highest average fidelity, thus giving the most accurate representation of the experimental data.

VI. CONCLUSION

The assignment of photon numbers using SNSPDs depends on many experimental parameters. An important parameter is the jitter of the system, which influences the resolution of arrival-time measurements. Minimizing the jitter components of an experimental setup is crucial for optimized PNR.^{32,40} One contribution to jitter, i.e., the uncertainty in arrival time of the photons, is the optical pulse duration of the light source. Using a programmable optical pulse shaper, we investigate different pulse shapes and pulse durations by setting different filters and bandwidths, respectively. We find that a Gaussian spectral filter gives a much cleaner arrival-time histogram compared to a bandpass filter (see Fig. 7). We also show that the photon-number discrimination remains accurate for temporal pulse

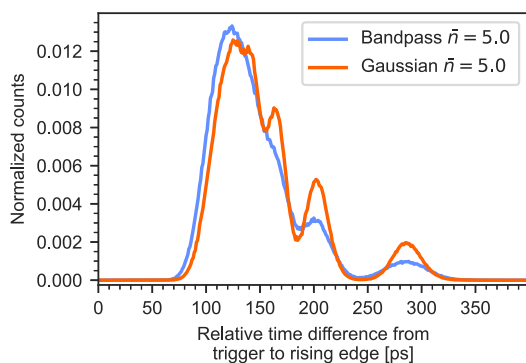


FIG. 7. Arrival-time histogram given two waveshaper filter types, for a mean photon number per pulse of $\tilde{n} = 5$, bandwidth of $\Delta\lambda = 0.14$ nm, and center wavelength of $\lambda = 1550$ nm.

durations in the same order of magnitude as other jitter contributions of the experiment. For 60 ps optical pulses, the discrimination is significantly reduced (see Fig. 2).

In addition, we show a comparison between two different models for photon-number measurements using SNSPDs, based on exponentially modified Gaussian distributions and Gaussian distributions. At first glance, an analysis of the arrival-time histograms using Gaussian distributions leads to accurate fits of the photon-number contributions [see Fig. 3(b)]. However, our analysis shows that the probability of misidentifying photon-number events is significantly underestimated due to the exponential tail in the arrival-time histograms (see Table I). Using a more accurate model for arrival-time histograms (based on a sum of EMG distributions³² for the SNSPD in this investigation) leads to excellent agreement between experimental data and fit, and, thus, to an accurate characterization of the misidentification probability of photon-number events (see Table I). Minimized misidentification probability is achieved when the photon-number contributions to the arrival-time histograms have a large separation in time, or alternatively narrowing acceptance regions in the arrival times for photon-number events (see Appendix D); this will enable high accuracy in heralding single-photons based on, e.g., spontaneous parametric down-conversion sources.

We follow our analysis with a reconstruction of the POVMs of the detector. The POVMs show sharp features, which are expected from photon-number resolving detectors, which ideally show a one-to-one mapping from input photon numbers n to detector outcomes n' . We allow the detector to resolve higher photon numbers (up to $n = 6$ instead of $n = 3$), at the cost of increased overlap of photon-number contributions in the arrival-time histograms. We find that the POVMs remain moderately sharp, even for larger outcomes. This does not fit the expected broadening based on simulated POVMs, due to contributions from multiple photon numbers, due to higher overlap of the underlying distributions [see Figs. 3(a) and 4(a)]. In this operation mode, the outcomes correspond to mixtures of photon numbers, which describe events that occur in a specific arrival-time window. Notwithstanding, the POVMs can still represent the data with high fidelity.

SUPPLEMENTARY MATERIAL

The [supplementary material](#) includes details about the SNSPD, such as the nanowire parameters, figures of merit, and an example electrical trace, as well as more details on measurement jitter contributions.

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AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts to disclose.

Author Contributions

Timon Schapeler: Conceptualization (equal); Data curation (lead); Formal analysis (lead); Investigation (lead); Methodology (lead); Software (lead); Visualization (lead); Writing – original draft (lead); Writing – review & editing (equal). **Isabell Mischke:** Formal analysis (supporting); Investigation (supporting). **Fabian Schlue:** Investigation (supporting); Writing – review & editing (equal). **Michael Stefszky:** Data curation (supporting); Funding acquisition (supporting); Project administration (equal); Supervision (equal); Writing – review & editing (equal). **Benjamin Brecht:** Funding acquisition (equal); Project administration (equal); Supervision (equal); Writing – review & editing (equal). **Christine Silberhorn:** Funding acquisition (equal); Project administration (equal); Resources (equal); Supervision (equal); Writing – review & editing (equal). **Tim J. Bartley:** Conceptualization (equal); Funding acquisition (equal); Project administration (equal); Resources (equal); Supervision (equal); Writing – original draft (supporting); Writing – review & editing (equal).

DATA AVAILABILITY

The data that support the findings of this study are openly available in Zenodo at <https://doi.org/10.5281/zenodo.14888684>.

APPENDIX A: OPTICAL PULSE SHAPING

The programmable optical processor (wavelength shaper) allows one to filter the original laser spectrum with different filter shapes and bandwidths. Spectral shaping of the optical pulse may also change the temporal shape of the optical pulse and for Fourier-transform-limited pulses, the temporal pulse duration can be calculated as

$$\Delta\tau = \text{TBP} \frac{\lambda^2}{c\Delta\lambda}. \quad (\text{A1})$$

Here, $\Delta\lambda$ is the filtered spectral bandwidth (set by the wavelength shaper), and TBP is the time-bandwidth product, which is 0.441 for Gaussian pulses and 0.315 for sech^2 pulses. We use a free-space femtosecond-pulsed laser (with a sech^2 pulse shape), which is coupled into fiber. Due to dispersion in the fiber, the optical pulse is not Fourier-transform-limited when arriving at the wavelength shaper. We use an autocorrelator to measure the optical pulse duration of 1.06 ps before the spectral shaping (after 5 m of fiber in total; 1 m fiber for fiber-coupling and 4 m of fiber inside the wavelength shaper before the shaping stage).

Given the limited optical power, we are not able to measure the pulse duration using an autocorrelator after spectral shaping. Therefore, we calculate the optical pulse duration based on the spectral parameters. Due to group-velocity dispersion, the output pulse length $\Delta\tau_{\text{out}}$ is given by⁵⁴

$$\Delta\tau_{\text{out}} = \Delta\tau_{\text{in}} \sqrt{1 + \left(\frac{2 \ln(2)}{\pi} \frac{\lambda^2}{c' \Delta\tau_{\text{in}}^2} D_{\lambda} L_{\text{fiber}} \right)^2}, \quad (\text{A2})$$

where $\Delta\tau_{\text{in}}$ is the FWHM pulse length of the input pulse, λ is the wavelength, $c' = \frac{c}{n}$ is the speed of light in fiber (with $n = 1.4682^{57}$), D_{λ} is the dispersion coefficient, and L_{fiber} is the length of fiber. The dispersion coefficient at 1550 nm for standard SMF28 fiber is given by $D_{\lambda} \leq 18 \frac{\text{ps}}{\text{nm km}} = 1.8 \times 10^{-5} \frac{\text{s}}{\text{m}^2}$.⁵⁷

We use Eq. (A2) to calculate the optical pulse length of the photons arriving at the cryostat, accounting for approximately $L_{\text{fiber}} = 32$ m of fiber-based group-velocity dispersion. Different wavelength shaper bandwidths lead to different optical temporal pulse durations of our probe pulses (see Table III). Based on Eq. (A1), we would calculate an optical temporal pulse duration of >300 ps for the bandwidth of $\Delta\lambda = 0.01$ nm. Given the experimental result, i.e., the arrival-time histogram from Fig. 2 (red line) and the bandwidth setting accuracy of the wavelength shaper of ± 0.04 nm, an optical temporal pulse duration of (60 ± 10) ps is more realistic. Potentially, there are also effects from the TBP not being purely described by the Gaussian filter, but rather by a convolution from the original sech^2 shape of the laser and the Gaussian filter. Bandwidths and corresponding optical temporal pulse durations are listed in Table III.

TABLE III. Calculated optical pulse durations according to different wavelength shaper bandwidth settings. The uncertainties are based on the bandwidth setting accuracy of the wavelength shaper (± 0.04 nm) and the TBP that might be a convolution from the original sech^2 shape (0.315) of the laser and the Gaussian filter (0.441).

Bandwidth (nm)	0.01	0.14	2.66
Pulse duration (ps)	60 ± 10	25 ± 10	2.9 ± 0.3

APPENDIX B: DEPENDENCE ON FILTER SHAPE

Figure 7 shows a comparison of the arrival-time histogram when setting a bandpass filter (light blue) and a Gaussian filter (orange) with a bandwidth of $\Delta\lambda = 0.14$ nm for the waveshaper. The Gaussian filter leads to a much cleaner photon-number differentiation, as the bandpass filter seems to wash out the photon-number events. Applying a bandpass or Gaussian filter in the spectral domain leads to a sinc-like or Gaussian temporal pulse shape, respectively. With this in mind, it explains that the Gaussian filter leads to a better performance, as the side lobes of the sinc temporal shape likely cause the pulse to be broader in time.

APPENDIX C: GAUSSIAN DISTRIBUTIONS

In Fig. 8(a), it can be seen that the arrival-time histograms for different incident mean photon numbers, specifically the one-photon peak position, slightly shift to earlier times with increasing mean photon number. To incorporate this slight shift, we sum the histograms for incident mean photon numbers from one to nine (in steps of one) and find the peak position of the first five photon-number components.

According to Nicolich *et al.*³¹ the rise time of the trace scales with $1/\sqrt{n}$; this matches our data, which can be seen by the linear dependence when plotting $1/\sqrt{n}$ over the peak position of the n th photon-number contribution [see Fig. 8(b)].

We then fit the histogram sum with a sum of Gaussian distributions,

$$f(a, \sigma_n, x_n) = a \cdot \sum_{n=1}^{20} \frac{1}{\sigma_n \sqrt{2\pi}} e^{-\frac{1}{2} \left(\frac{x - x_n}{\sigma_n} \right)^2} P'(n), \quad (C1)$$

with a as an overall scaling factor, σ_n as the standard deviations, and $x_n = \left(\frac{1}{\sqrt{n}} - b_{\text{lin}} \right) / m_{\text{lin}}$ as the peak position of the Gaussian distributions. Here, we use b_{lin} and m_{lin} from the linear fit in Fig. 8(b) as fitting parameters to enforce the $1/\sqrt{n}$ scaling. Each Gaussian distribution is scaled according to the sum of the Poisson distribution components from the mean photon numbers used in the histogram

sum, i.e., $P'(n) = \sum_{\tilde{n}=1}^9 \frac{(\tilde{n}\eta)^n}{n!} e^{-\tilde{n}\eta}$, where $\eta = 0.91$ is the efficiency of the detector.

APPENDIX D: MINIMIZING MISIDENTIFICATION

It is possible to come up with a minimization scheme that minimizes the misidentification probability $p_{\text{misidentified},n}$ while simultaneously minimizing the probability to miss events $p_{\text{missing},n}$; in other words, the overlap of the n th distribution with the n th region should be maximized, while the overlap of all other distributions with the n th region should be minimized simultaneously. Choosing the weights of the two tasks sensibly (i.e., deciding which case has higher priority for an experiment) will optimize the bounds for the photon-number regions. The probabilities to misidentify or miss events are shown in Table IV for the optimized photon-number regions shown in Fig. 9(a), where higher weights are set to penalize misidentification. We visualize the probabilities of missing and misidentification as a function of region width in Fig. 9(b). To create Fig. 9(b), we leave the right border of each photon-number region fixed [as shown in Fig. 9(a)] and shrink the region from the left border, starting from the intersection point of the distributions. Sacrificing roughly $p_{\text{missing},1} \approx 6\%$ of one-photon events will reduce the misidentification probability to $p_{\text{misidentified},1} = 0.01\%$ [see Fig. 9(b)] at a region width of 131 ps, i.e., only 1 out of roughly 10 000 one-photon events is actually a two-photon event.

Using the minimization scheme described above, the probabilities to miss events $p_{\text{missing},n}$ are not equal for all photon numbers

TABLE IV. Probabilities to miss or misidentify photon-number components based on EMG distributions and optimized photon-number regions from Fig. 9(a).

	$p_{\text{misidentified}} (\%)$	$p_{\text{missing}} (\%)$
$n = 1$	0.01	5.87
$n = 2$	0.60	24.30
$n = 3$	2.78	48.85

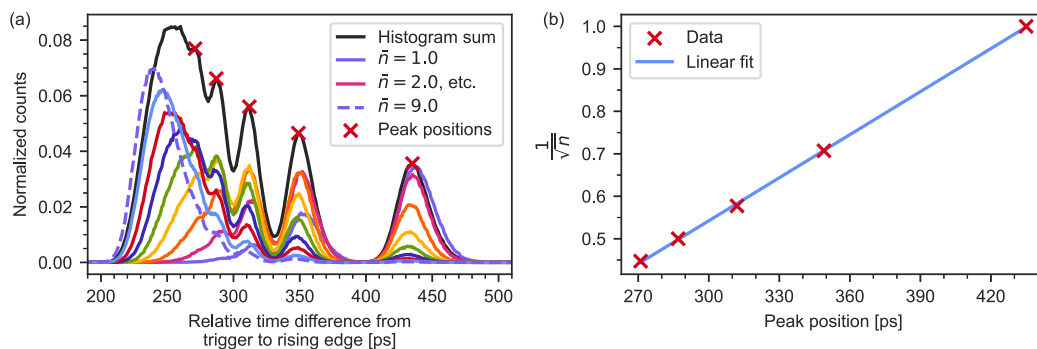


FIG. 8. (a) Arrival-time histograms for mean photon numbers from $\tilde{n} = 1$ to $\tilde{n} = 9$ (colored lines are scaled up by a factor of three, to improve visibility) as well as the sum of the histograms (black line). Peak positions of the first five photon-number contributions are marked by red crosses. (b) Predicted $1/\sqrt{n}$ scaling³¹ of the peak positions from (a) with a linear fit (blue line).

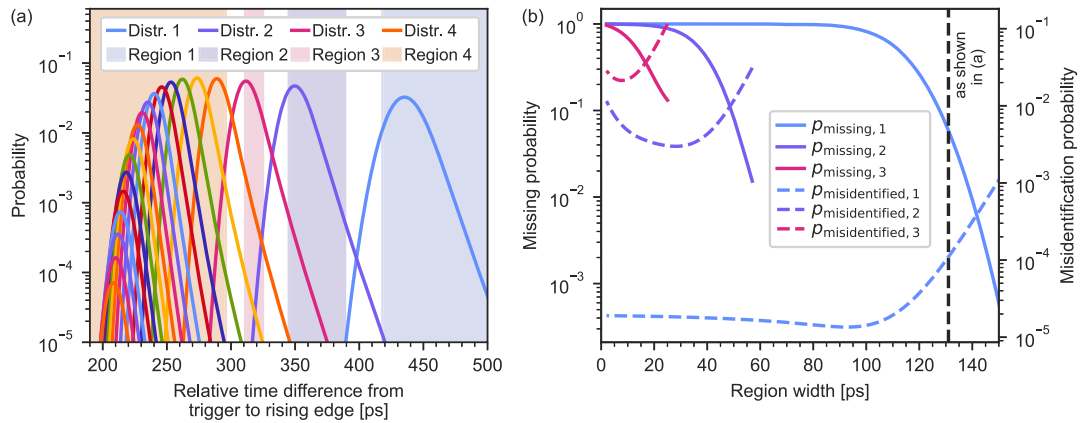


FIG. 9. (a) Arrival-time histogram fits based on EMG distributions [equivalent to Fig. 3(a)] with photon-number regions optimized to simultaneously reduce $p_{\text{misidentified},n}$ and $p_{\text{missing},n}$. The overlap between (EMG) distributions and regions (with the same color) is maximized, while the overlap of any region with all other (EMG) distributions is minimized. Higher weights are set to penalize misidentification, which shifts the left border of the photon-number regions to larger relative time differences. (b) Probabilities of missing (left axis, solid lines) and misidentification (right axis, dashed lines) as a function of region width for $n = 1$, $n = 2$, and $n = 3$. As an example, we leave the right border of the region fixed as shown in (a) and shrink the region from the left border, starting from the intersection point of the distributions. The actual optimization process is more complex, as the region can also shrink from the right edge simultaneously.

n (see Table IV). Without making assumptions about the photon-number distribution of an input state, one may not be able to faithfully reconstruct the distribution. To reconstruct the mean photon number of the distribution, the formula

$$\bar{n} = \sum_n \frac{N_n}{N_{\text{Tot}}} \frac{1}{\eta_n} n \quad (\text{D1})$$

can be used, which compensates for lost events due to invalid assignment outside of the photon-number region. Here, N_n is the number of events in the photon-number region n , N_{Total} is the total number of events in the arrival-time histogram, and $\eta_n = 1 - p_{\text{missing},n}$ is the efficiency of measuring photon number n (compensating for the unequal $p_{\text{missing},n}$).

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Scalable high-dynamic-range photon counting

4

In Chapter 3 we have investigated how SNSPDs can retrieve photon-number information, making them extremely valuable for quantum photonic technologies. However, a single SNSPD can only count a small number of photons (in most cases < 10), which motivates a combination of existing technologies to achieve HDR photon counting. Chapter 1 already mentioned large SNSPD arrays used for imaging [8] but simultaneously identified the readout scheme as a bottleneck for photon-counting applications. The problem is that not all pixels of the array can contribute to the detector output due to the commonly used row-column readout scheme [117], which leads to a loss of information. This is a common limitation of multiplexing architectures, whether they use the spatial [7, 52, 57] or temporal [53, 54, 56] degree of freedom. The probabilistic splitting of photons leads to a nonzero probability of multiple photons occupying the same threshold detection bin and thus being falsely counted as one.

When SNSPDs are multiplexed into a large system, the possibility to access all pixels individually needs to be maintained, which can be achieved with custom readout designs [58, 81, 127–133]. To achieve scalability, it is important to limit the number of readout lines from the cryogenic environment, due to finite heat load of current refrigeration systems. Ideally, the individual readout of array pixels or multiplexing bins retains potential photon-number-dependent features of the separate devices, which significantly increases the information about the incident photon number [61], and thus the fidelity of the photon-number assignment.

The aim of this chapter is to combine multiplexing with individual bin readout and intrinsic PNR, and thereby *bring these ingredients together and investigate scalable photon counting far beyond the conventional single-photon regime*. Publication 4.1 investigates how temporal and spatial multiplexing increase the dynamic range of the detector by distributing the incident light over many temporal and spatial detection bins, while arrival-time-based PNR SNSPDs allow each multiplexing bin to contribute photon-number information rather than only threshold information. In this way, the detector architecture is designed to count photons at substantially larger photon numbers than a single PNR SNSPD could access.

A central challenge is that arrival-time-based PNR produces continuous and noisy measurement outcomes. Assigning a photon number based on fixed arrival-time regions of a histogram necessarily discards part of the available information and can lead to photon-number misidentification, as identified in Publication 3.3. To avoid potential misassignment of photon numbers, a novel probabilistic treatment of the outcome distributions is developed. Here the PNR model from Publication 3.2 is used to construct a probabilistic description of the arrival-time histograms for each detection bin. A measured arrival time is mapped to a probability distribution, incorporating all possible photon-number contributions. Therefore, the utilized model corresponds to a POVM of each individual detection bin in the arrival-time basis. This provides a shot-by-shot estimate of the measured photon number with a corresponding uncertainty.

The detector consists of 1024 multiplexed detection bins and the PNR model incorporates photon-number contributions up to 15 photons per detection bin. Based on the assumption that the individual bins can be treated independently, this allows the detector to count thousands of photons and the resulting Hilbert space dimension is of the order 10^4 photons. As the detector remains photon-number resolving across the entire Hilbert space, the number of detection outcomes is also of the order 10^4 . The detector POVMs are sufficiently large that the reconstruction requires the scalable QDT method developed in Publication 2.2 for a more efficient use of computational resources; this allows to use a conventional computer for the reconstruction.

In this sense, the work presented in this chapter is the synthesis of this thesis. It combines the scalable characterization methods of Chapter 2, the physical and statistical understanding of intrinsic PNR developed in Chapter 3, and large-scale multiplexing into a single detector. Publication 4.1 shows that these ingredients can enable scalable photon counting over thousands of photons per pulse and to bridge the gap to macroscopic power measurements.

Macroscopic photon counting beating the Poisson noise limit

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Photon counting is a cornerstone of quantum optics. Here, we demonstrate precisely counting from 0 to over 9000 photons, beating the Poisson noise limit by at least 4.1 dB across this range. We achieve sub-single-photon precision up to 276 photons per pulse. To do so, we multiplex eight intrinsically photon-number-resolving superconducting nanowire single-photon detectors across 128 temporal modes. We use a model-informed characterization of each of the 1024 detection bins, for optimal precision. We perform quantum detector tomography to reconstruct the positive operator valued measures (POVMs) of the complete device, which consists of $1.38 \cdot 10^8$ matrix elements. At the repetition rate of our experiment of 80 kHz, we can precisely count photons corresponding to an optical power of approximately 71 pW, bridging the gap from single-photon measurements to high-sensitivity optical power meters. A photon-number-resolving detector of this size, and the tools used to analyze it, will become increasingly important to characterize large quantum states, as well as tasks in precision metrology and optical power standards.

I. INTRODUCTION

The quantization of the energy of a light wave is one of the most fundamental aspects of quantum mechanics. Nevertheless, building detectors which remain sensitive to the discrete nature of photons when the intensity of the light field is large is highly challenging. The most commonly used single-photon detectors (such as avalanche photodiodes, or superconducting nanowire single-photon detectors) typically distinguish only the presence or absence of photons, not their number [1]. Some photon-number information can be recovered by multiplexing these devices: splitting the incident light across many detector elements or pixels [2–5]. In the limit of equal splitting across infinite pixels, this recovers the photon number information precisely [6]. In practice, finite pixel numbers and nonuniform splitting between them mean that there is non-zero probability of multiple photons incident on the same pixel, causing significant loss and statistical noise [6]. Other types of detector, such as transition edge sensors (TESs), may resolve energy directly (corresponding to photon-number resolution if the incident light is monochromatic) but these devices become saturated beyond ~ 20 photons [7–9]. TES can also be operated in the saturated regime, where photon-number information up to millions of photons is contained in their recovery time [10]. Alternatively, detectors sensitive to large numbers of photons and which respond monotonically to incident power (such as photodiodes) are widespread; however, they exhibit significant electrical noise, which washes out any photon-number-resolving capability.

Bridging the gap between single-photon-level measurements and bright light has significant implications for

both fundamental studies of large-scale photonic quantum states, as well as practical implications for optical metrology. In this paper, we show that by combining intrinsic photon-number resolution and large-scale multiplexing, we not only bridge this gap, but do so whilst exceeding the precision of Poisson-noise-limited single-photon-level measurements.

To achieve this, we use the spatially- and temporally-multiplexed detector shown in Fig. 1(a). This device comprises 128 temporal modes incident on eight intrinsically photon-number-resolving superconducting nanowire single-photon detectors (PNR-SNSPDs), resulting in 1024 multiplexing bins. Each bin is rigorously characterized by mapping its arrival time relative to a trigger signal to the probability a certain number of photons were absorbed, following the analysis in Refs. [11, 12]. For a given arrival time measured by the time tagger, we thus have a distribution of photon-number probabilities corresponding to that arrival time. In each shot of the experiment, we obtain 1024 individual samples of the incident light, each of which corresponds to a calibrated photon number distribution. These samples are convolved to yield an overall distribution of photon numbers, and thus an estimate of the photon number absorbed by the device.

II. EXPERIMENTAL METHOD

Our experimental approach is shown in Fig. 1(a), which allows us to split an optical pulse into eight 128-bin pulse trains, resulting in a total of 1024 detection bins, each with intrinsic photon-number resolution. Optical pulses enter a 128-bin-per-output temporal multiplexing network (green box). This fiber network consists of balanced beam splitters and delay lines (made from polarization maintaining components). The upper arms of the beam splitters are connected to delay lines with lengths

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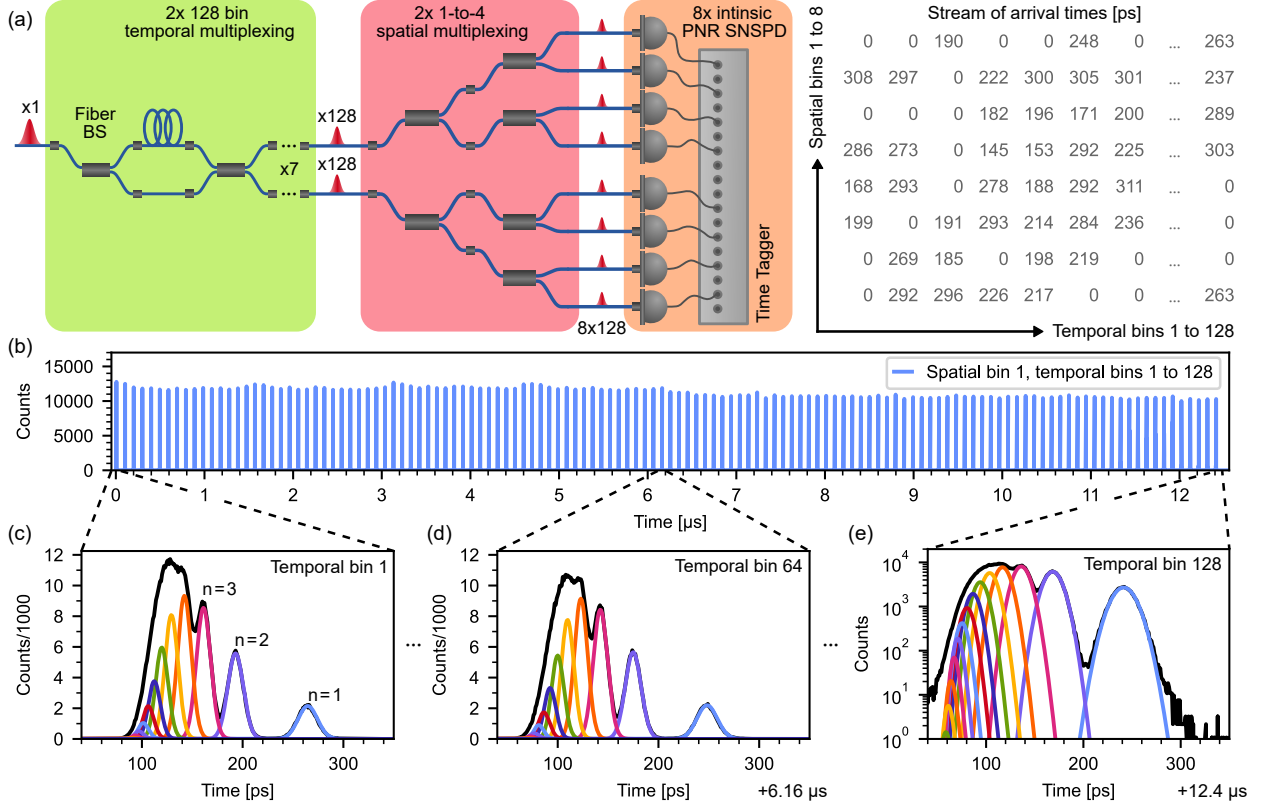


FIG. 1. (a) Combination of temporal multiplexing (green box), spatial multiplexing (red box) and intrinsic PNR SNSPDs (orange box). One input pulse is (temporally) split into two pulse trains with 128 sub pulses each, afterwards both pulse trains are (spatially) split onto four SNSPDs, which can resolve the photon number in each of the $8 \cdot 128 = 1024$ resulting sub pulses, i.e. bins. The output of each of the eight spatial bins is a stream of 128 arrival times measured by the time tagger, which is depicted in the upper right corner. (b) Arrival-time histogram of the entire pulse train, i.e., all 128 temporal bins for the first spatial bin. The 128 peaks correspond to the individual temporal bins, which are spaced approximately 100 ns apart for a total duration of ≈ 12.5 μ s. Zooming into the histogram yields the arrival-time histograms of the individual temporal bins shown in (c), (d) and (e) for bins 1, 64 and 128, respectively (with (e) shown on a logarithmic scale). Here, for an incident mean photon number $\bar{n} = 5$, the individual photon-number contributions for $n = 1, 2, 3, \dots$ can be seen. The colored lines correspond to exponentially modified Gaussian (EMG) fits based on the model from Sidorova et al. [11].

according to $2^x L$, where $x \in [0, 6]$ and $L = 20$ m, corresponding to a time delay of approximately 100 ns (see Ref. [13] for more detail on the temporal multiplexing setup). For every impinging optical pulse, the network creates a pulse train consisting of 128 pulses for both outputs (shown for one spatial bin in Fig. 1(b)). This means that the intensity of the impinging pulse is split evenly across all 256 sub-pulses.

Afterwards, the generated pulse trains enter a spatial multiplexing network (red box), which divides each input into four spatial bins using three balanced beam splitters. Every spatial bin is connected to a commercial photon-number-resolving SNSPD (Single Quantum). Finally, a time-to-digital converter (Time Tagger X, Swabian Instruments) records the raw time tags for later analysis (orange box).

We use PNR SNSPDs (based on arrival-time measurements), therefore, every bin can resolve the number of photons in the sub-pulses (shown in Fig. 1(c)-(e) for three temporal bins). In total, we measure 125 input states with mean photon numbers scaling quadratically from 0 to 15393 per pulse; we record $3.2 \cdot 10^6$ samples for each input state. The calibration of the coherent input states is performed following the procedure outlined in Refs. [14, 15], relating the incident mean photon number per pulse of the coherent states to the attenuation of the variable optical attenuators. More details about the full experimental setup is presented in the supplemental document (see Sec. 1 and Fig. 1 therein).

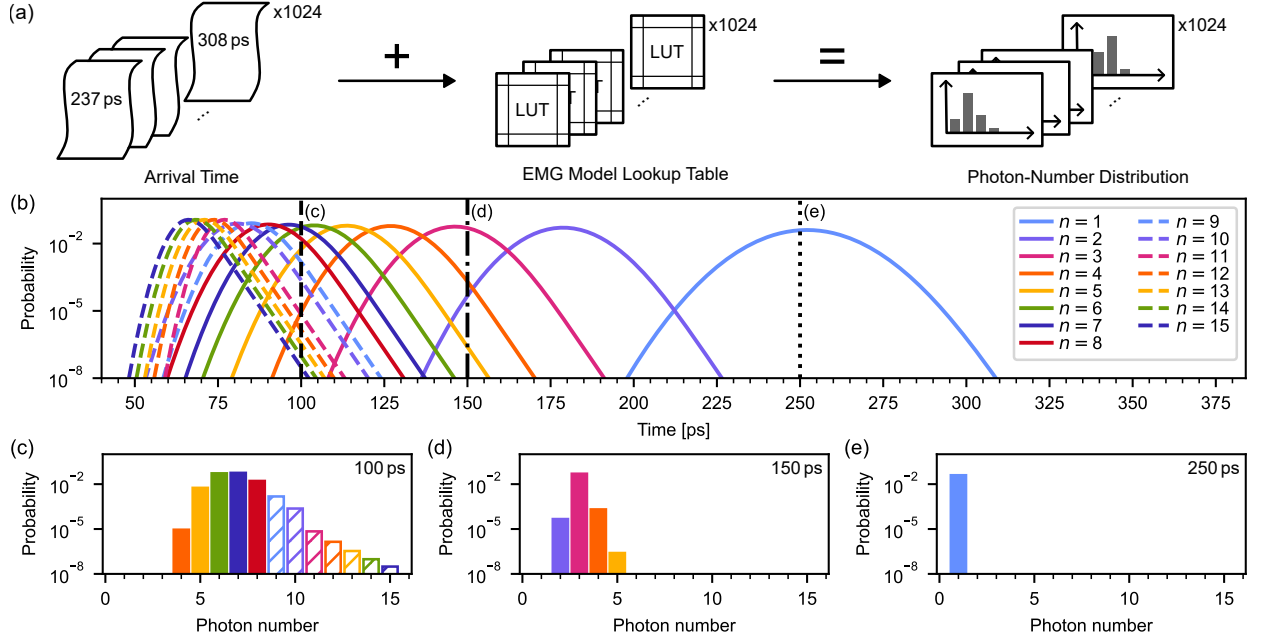


FIG. 2. (a) For every input state that impinges on the detection system, we record up to 1024 arrival times (corresponding to the $8 \cdot 128 = 1024$ spatial and temporal bins). For every bin, we have a lookup table (LUT), which is created through an EMG model describing the $n = 15$ underlying photon-number contributions (b). With this, every arrival time is then translated into a photon-number distribution (c)-(e). (b) The LUT is created by fitting EMG distributions to the arrival-time histograms (see Fig. 1(c)-(e)) for all 1024 bins and then assuming a flat prior distribution. As an example we show the LUT for spatial bin 1 and temporal bin 48. (c)-(e) The photon-number distributions are a result of a vertical cut at the arrival times highlighted in (b), i.e., 100 ps, 150 ps and 250 ps, respectively, obtaining the probability values of the underlying distributions and renormalization.

III. SHOT-TO-SHOT PHOTON-NUMBER DISTRIBUTIONS FROM CONSTITUENT DETECTORS

In general, the mapping of a phase insensitive detector outcome to photon number is described by the positive operator valued measure (POVM) of the detector [16]. Before investigating the POVMs of the device in its entirety, it is helpful to consider the measurement outcomes arising from the constituent detection bins. As we have previously shown [11], the rise-time of the electrical signal from SNSPDs depends on the number of hotspots generated within a characteristic time (typically $\lesssim 50$ ps). Therefore, assuming these hotspots are caused by multiple photons, the detector becomes photon-number resolving. Moreover, since these hotspots are small compared to the total nanowire area, the probability of overlapping hotspots or two photons being absorbed within the same hotspot is negligible [11]. Thus the detector is considered photon-number resolving, up to the point where the photon-number dependent rise times cannot be distinguished (typically $\lesssim 5$ photons [11, 12, 15, 17–23]).

A. Arrival time to photon number mapping

In the experiment, we measure an arrival time, from which we wish to infer the photon number. Recently, Sidorova et al. [11] presented an analytical model, based on exponentially-modified Gaussian (EMG) distributions, which maps the number of absorbed photons to arrival times, taking into account various sources of jitter and finite timing resolution. Therefore, no unique mapping between arrival time and photon number is possible. Instead, for a given measured arrival time, there exists a distribution of contributions from different numbers of photons. This mapping can be seen as POVMs of each individual detection bin on a restricted Hilbert space; the mapping is only valid when a detection event occurs, i.e., the vacuum event is not covered by this restricted space. Assuming saturated internal detection efficiency, the model describes the behavior of the detector upon photon absorption, and is ignorant of any loss. Vacuum events are registered as no response from the detector during a measurement time window. This is incorporated into the mapping by the distribution $p(n) = \delta_{n,0}$, i.e., the unit probability for the vacuum component and zero for all other photon numbers.

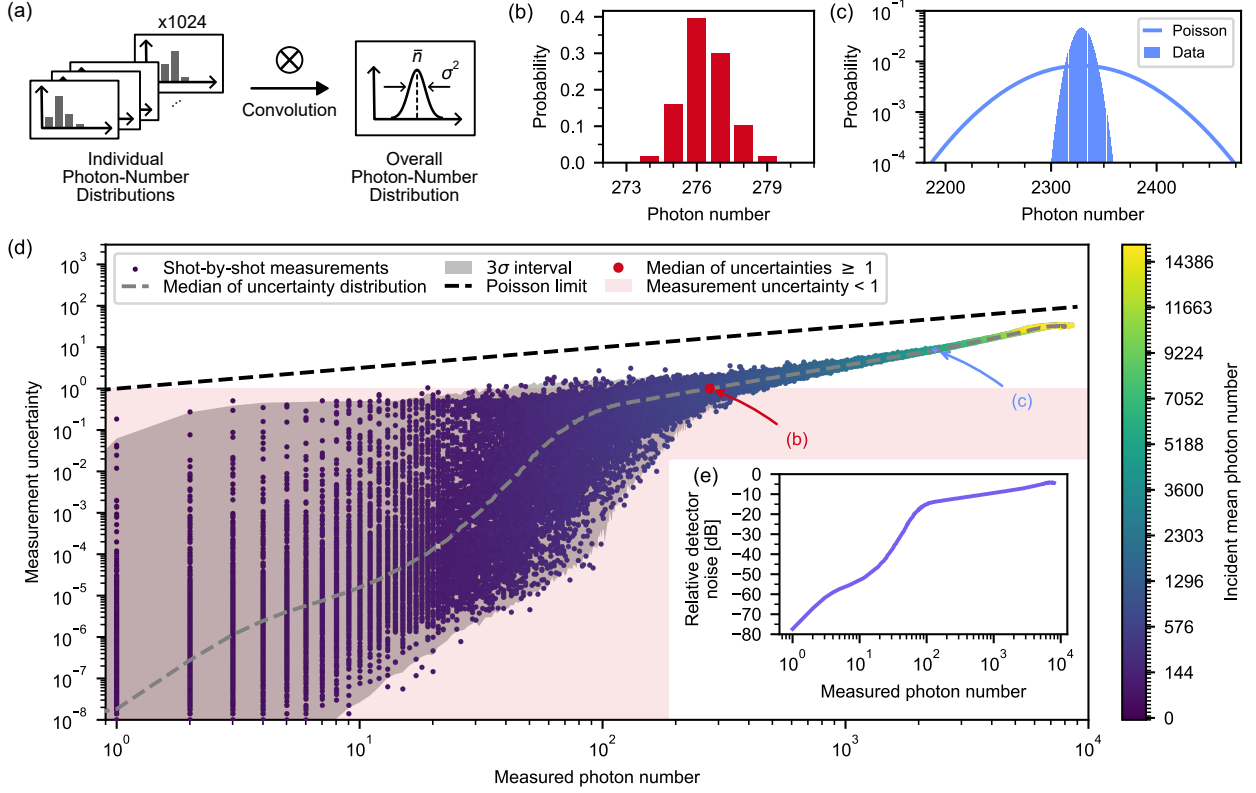


FIG. 3. (a) From the 1024 individual photon-number distributions generated with the LUT (as depicted in Fig. 2(c)-(e)) we can generate an overall photon-number distribution for every single input state that impinges on the detection system. This is done via a convolution. (b), (c) Photon-number distribution of a single shot for two different incident mean photon numbers (484 and 4904, respectively). These shots are marked in (d) with red and blue dots, respectively. (d) Scatter plot of the measurement uncertainty vs. the measured photon number on a shot-by-shot basis. For every incident mean photon number (corresponding to different colors of the scatter plot dots, see color bar) we plot 1000 samples. It can be seen that all data points are below the Poisson noise limit (black dashed line), which corresponds to $\sigma = \sqrt{n}$. For every measured photon number we find the distribution of measurement uncertainties and determine the median (gray dashed line) and 3σ interval (gray shaded area). The red shaded region indicates where the measurement uncertainty is below 1. The red dot indicates the measured photon number, where the median of the uncertainty distributions exceeds 1 for the first time, which happens for a measured photon number of 276. (e) Relative detector noise vs. measured photon number. The detector operates consistently below the Poisson noise limit, with a maximum relative detector noise of -4.1 dB at > 6800 photons.

Given a flat prior of input photon numbers (i.e., a uniform distribution, as not to assume any prior knowledge about the input photon statistics), one can use the Sidorova model to find this restricted POVM. In practice, this results in a lookup table (LUT), whereby each arrival time corresponds to a probability distribution of photon numbers (schematically shown in Fig. 2(a)). We truncate the model to include photon-number contributions (per bin) up to 15 (a lookup table for one detection bin is shown in Fig. 2(b)). This approach has the significant advantage that we incorporate and maintain the probabilistic contribution of all photon numbers throughout the analysis, which allows us to put error bars on each measurement shot-to-shot.

B. Photon-number distributions

For each shot of the experiment, we generate 1024 distributions $p_i(n)$ corresponding to the probability that a photon number n contributed to the outcome of each bin i (shown for one bin for three different arrival times in Fig. 2(c)-(e)). To generate the aggregated distribution $P(n)$, we convolve all 1024 distributions ($p_1 * p_2 * \dots * p_N$)(n) = $(\ast_{i=1}^N p_i)$ (n), which is schematically shown in Fig. 3(a). This convolution is computationally efficient by multiplying each distribution in Fourier space and then taking the inverse Fourier transform of the result to obtain the final distribution. The result comprises the probability of each photon number contributing to the final outcome. The mean of the resulting distribution is the photon number, and the standard deviation its error,

TABLE I. Comparison of recent works, showing the maximum photon number $n_{\max}$ with a ± 1 photon uncertainty ($\sigma \leq 1$). *Based on the LUTs for all bins, where the standard deviation exceeds one for the first time. †Values are calculated with Eq. 8 from Ref. [24], where the standard deviation of the click distribution is ≤ 1 . ‡Inferred from the photon-number error rates in the extended data given in Ref. [9].

Source	Detector type	Methodology	$n_{\max}$
This work	SNSPD	single-pixel	8*
Stasi et al. [25]	SNSPD	28-pixel interleaved	10†
Cheng et al. [5]	SNSPD	100-pixel waveguide-integrated	16†
This work	SNSPD	1024-bin multiplexing (no PNR)	47†
Levine et al. [26]	TES	Poisson-influenced K -means algorithm [27]	100
Eaton et al. [9]	TES	3-bin spatial multiplexing	$\lesssim 106^\ddagger$
This work	SNSPD	1024-bin multiplexing combined with intrinsic PNR	276

generated on a shot-by-shot basis.

An example of a measurement outcome following this analysis is shown in Figs. 3(b),(c). As can be seen, this corresponds to measuring a mean of 276 and 2329 photons, respectively. The width of the distribution is significantly narrower than the Poisson noise limit (i.e., shot noise following Poissonian statistics) indicated by the solid line in Fig. 3(c). This corresponds to a significant improvement in precision compared to the Poisson noise limit of over an order of magnitude (12.1 dB and 7.5 dB, respectively).

While the above analysis maintains complete information about all photon-number contributions to the final distribution, this may not be necessary in some measurement tasks. In an alternative analysis, we can consider each of the 1024 bins as sampling the initial incident photon-number distribution. One can calculate the means $\bar{n}_i$ and variances σ_i^2 of each of these samples, and, since they are independent and therefore additive, sum them over the total number of bins $B = 1024$ to yield the resulting mean $\bar{n} = \sum_i^B \bar{n}_i$ and variance $\sigma^2 = \sum_i^B \sigma_i^2$ or standard deviation $\sigma = \sqrt{\sigma^2}$. This mean and variance are identical to the mean and variance that can be calculated from the previously discussed method, i.e., the complete photon-number distribution.

C. Shot-by-shot measurement uncertainty vs. measured photon number

We apply the alternative (second) method to measure a series of coherent states, ranging in incident mean photon number from 0 to 15393. In Fig. 3(d) we plot 10^3 shot-by-shot data points (measurement uncertainty σ vs. measured photon number $\bar{n}$) for each incident mean photon number (the colors of the dots correspond to different incident mean photon numbers, as shown in the color bar).

As stated above, these can be calculated directly from the aggregated distribution, or equivalently the sums of the variances or means of the 1024 constituent samples. For a subset of our data set ($12.5 \cdot 10^6$ data points, i.e., 10^5 shots per incident mean photon number), we aggregate the distribution of measurement uncertainties for all

integer valued measured photon numbers (corresponding to the x axis in Fig. 3(d), from 1 to the largest measured photon number of approximately 9000). We indicate the two example shot-by-shot measurements from Figs. 3(b),(c) with dots and arrows in the same color as the respective plots. The median as well as the 3σ interval of these distributions are shown as the gray dashed line and the gray shaded area in Fig. 3(d). To display the uncertainty, we use the median, i.e., the midpoint of the measurement uncertainty distribution for a certain measured photon number.

For Poisson noise limited measurements, the measured mean and the variance are equal, hence the measurement uncertainty scales as the square root of the mean. This limit is indicated by the black dashed line (corresponding to $\sigma = \sqrt{\bar{n}}$). The measurement uncertainties are consistently below the Poisson noise limit for all measured photon numbers. In Fig. 3(e) we show the relative detector noise $N_{\text{dB}} = 10 \cdot \log_{10} \left(\frac{\sigma}{\sqrt{\bar{n}}} \right)$ in dB, which compares the measurement uncertainty to the Poisson noise limit. We find an average relative detector noise of -6.7 dB across the entire tested dynamic range of the detector, i.e., almost a factor of five below the Poisson noise limit. The maximum detector noise is -4.1 dB at > 6800 photons and the minimum is -77.3 dB at 1 photon. Note, at this point, we are limited by the dark-count probability of our detector, which is $< 3.7 \cdot 10^{-5}$ based on a combined dark-count rate of < 80 Hz of the eight SNSPD channels.

Measurement uncertainties below one, i.e., all data points in the red shaded region in Fig. 3(d), correspond to better than ± 1 photon precision in the measurements ($\sigma < 1$), which we achieve up to a measured photon number of 276. To give a sense of scale, we can calculate the number of click detectors one would need, to get a standard deviation of the click distribution of ≤ 1 . Using Eq. 8 from Ref. [24] this results in $3.7 \cdot 10^4$ click detectors, i.e., well over one order of magnitude more detectors than detection bins used in this work. Table I compares the maximum photon number $n_{\max}$ with a ± 1 photon uncertainty; it shows that our detector significantly outperforms other published work. This shows the power of combining multiplexing with intrinsic photon-number resolution, where intrinsically resolving the photon number is the crucial ingredient for the high preci-

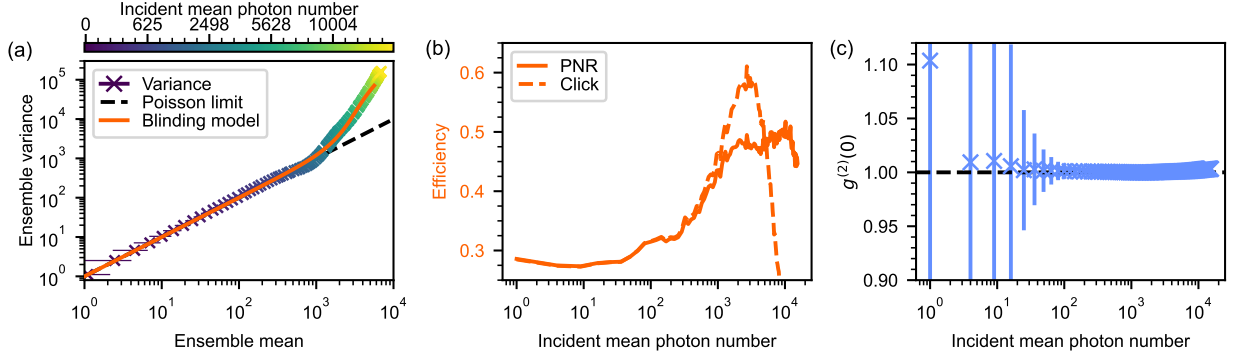


FIG. 4. (a) Ensemble variance vs. ensemble mean from 10^5 samples for every incident mean photon number, corresponding to the colored cross symbols. These lie on the Poisson noise limit (black dashed line) as we measure coherent light, which follows Poissonian statistics. Above a measured mean of approximately 1000 we see a deviation, which we can attribute to a probabilistic blinding of our detection bins. Our blinding model (orange line) accurately explains the deviation from the Poisson noise limit (see Sec III D in the supplemental document for more detail). (b) The efficiency of the device is the fraction of the measured mean photon number over the incident mean photon number (orange line). The dashed orange line corresponds to the efficiency when operating the detector in click-detection mode. (c) Second-order correlation function $g^{(2)}(0)$ as a function of incident mean photon number. The weighted average is $g^{(2)}(0) = 1.000270(7)$.

sion, enabled by the large separation of the one-photon and “more than one”-photon events in the arrival-time-based PNR measurements (compare Fig. 1(c)-(e)).

IV. PHOTON-NUMBER RESOLVED MEASUREMENTS OF COHERENT STATES

Here we discuss how ensemble data from the different coherent input states is used to further characterize the detector and our coherent light source. We investigate the variance and mean of ensemble data, estimate the efficiency of the detector, and calculate the second order correlation function using our photon-number resolved measurements.

A. Ensemble data

In Fig. 4(a) we show the ensemble variance over the ensemble mean of 10^5 samples for every incident mean photon number of the experiment (colored cross symbols, where the color corresponds to different incident mean photon numbers according to the color bar). We expect these to follow the Poisson noise limit, since they are sampling the underlying Poissonian photon statistics of the coherent input states. This is the case up to a measured mean photon number of approximately 1000 photons. Beyond this point, the variance of these distributions increase significantly. This is due to blinding effects of the detector bins, which leads to a zero-inflated distribution, with a mean photon number dependent probability of a “blinded” detector bin. More details are discussed in Sec. III D in the supplemental document, where we developed a model (orange line in Fig. 4(a)) to explain this behavior.

B. Estimating the efficiency

We calculate the efficiency of the device as the fraction of the measured mean photon number and the incident mean photon number, which is shown in Fig. 4(b) as a function of the incident mean photon number (orange line). It is apparent that the efficiency has a strong non-linear dependence on the incident power. We attribute this to the detector count rate dependent efficiency for AC-coupled amplification circuits [28–30]. Due to the temporal multiplexing architecture higher mean photon numbers lead to higher detector count rates, as more bins contain photons, that will lead to more detection events. More detail is given in Sec. III A in the supplemental document. Despite these effects, the detector can reach an efficiency of above 50% for any incident mean photon number, when setting the appropriate operational bias current. For this data set the efficiency of optimized for an incident mean photon number of approximately 5000 photons per pulse.

As a point of comparison, we can treat the detector as a “click” detector, and estimate the mean photon number based on the probability of measuring vacuum (“no click” events) in a 450 ps coincidence window for each bin. Since we assume the incident light to be Poissonian, the probability of vacuum is $P(0, \bar{n}) = e^{-\bar{n}\eta}$ and the efficiency can be expressed as $\eta = -\ln(P_{\text{no click}})^{\frac{1}{\bar{n}}}$. This is also shown in Fig. 4(b) as the orange dashed line. Here again, the nonuniform efficiency is apparent. Furthermore, while the two estimates of the mean photon number agree up to an incident mean photon number of approximately 500, they deviate significantly beyond this point. This is due to noise-induced “blinding” of some detector bins, which increases the probability of measuring a zero-event, when the detector is under high illumina-

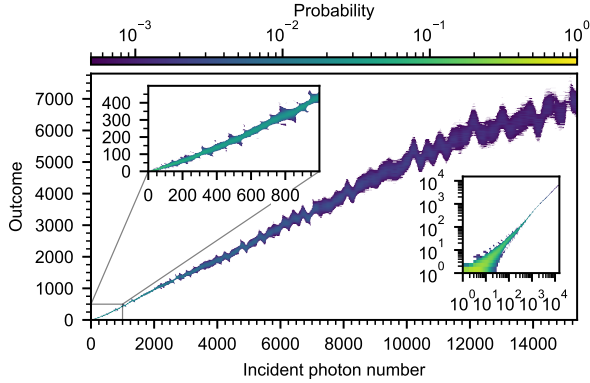


FIG. 5. Reconstructed POVM matrix of the entire detector shown as a heatmap of the conditional probabilities. The x axis corresponds to the incident mean photon number and the y axis to the detector outcome, i.e., the measured photon number. The color axis shows the probability of observing the conditional probability of a certain measured photon number given an incident photon number. The top left inset shows a zoom-in of the first 500 outcomes. The bottom right inset shows the content of the main plot on a logarithmic scale for both axes.

tion. This is discussed in more detail in Sec. III D in the supplemental document. For much higher incident mean photon numbers the efficiency

The accuracy of the detector, namely how well it can infer the input photon number from the measurement outcomes, depends on how well the losses can be characterized. However, its underlying precision, i.e., how precisely number of incident photons can be counted, depends on the variance of the measurements, which is independent of loss.

C. Second order autocorrelation

Given photon-number resolved measurements we can also calculate the second order correlation function from

$$g^{(2)}(0) = 1 + \frac{\sigma^2 - \bar{n}}{\bar{n}^2}, \quad (1)$$

using the ensemble variance and ensemble mean (i.e., the cross symbols from Fig. 4(a)). We show the $g^{(2)}(0)$ values for all incident mean photon numbers in Fig. 4(c). The weighted average calculates to $\bar{g}^{(2)}(0) = 1.000270(7)$. As expected, our light source produces coherent light across a broad range of mean photon numbers.

V. QUANTUM DETECTOR TOMOGRAPHY

The entire outcome space of the device is 1024^{450} , i.e., all combinations of each bin and each possible arrival time. This unfeasibly large number makes many approaches to detector tomography intractable, since the space cannot be sampled in reasonable time.

We can reduce our outcome space significantly by attributing photon numbers to each outcome, given an input, truncated at some reasonable incident photon number. This is possible due to the model-based LUTs (see Sec. III A), which map arrival times to photon-number distributions for all detection bins. Note that the assignment is limited by the fidelity of the model, which may lose accuracy under certain illumination regimes (see Sec. III C in the supplemental document). Specifically, we populate the outcome matrix $\mathbf{P}_{D \times N}$, by creating a histogram (with a total of N photon-number bins) of the measured photon numbers $n' \in [0, 9010]$. This is done for an ensemble of 10^5 samples for every of the $D = 125$ incident mean photon numbers (more detail about the outcome matrix can be found in Sec. II and Fig. 2 in the supplemental document). The input state matrix $\mathbf{F}_{D \times M}$ corresponds to Poisson distributions of the different incident coherent states truncated at a maximum photon number of 15393.

The POVMs are commonly displayed in a matrix $\mathbf{\Pi}$ consisting of the diagonal elements of the underlying set of POVMs $\{\pi\}$, which contains all relevant data for phase-insensitive detectors (which applies to our detector). Therefore, this matrix displays the conditional probabilities $p(n'|n)$ of measuring n' photons given n incident photons. The size of the POVM matrix $\mathbf{\Pi}_{M \times N}$ is thus $M \cdot N = 15394 \cdot 9011 \approx 1.38 \cdot 10^8$ elements. We reconstruct the POVM matrix using established methods, optimized for reconstructing large matrices [31], which are shown in Fig. 5. In general, it can be seen that the POVMs are narrow across the entire Hilbert space $M \in [0, 15393]$ and follow a slope of approximately 0.5, indicating a detection efficiency of approximately 50%. Some numerical artifacts are visible, especially at large incident photon numbers, which are amplified due to the logarithmic color bar scaling. The top left inset shows a zoom-in on the first 500 outcomes, displaying a linear dependency from input to output photon number, particularly in this regime. The bottom right inset shows the POVMs on logarithmic axes. Here, it can be seen that the detector is not yet saturating, which means that the dynamic range of the detector is not being fully utilized by the incident optical power. The most probable outcome for the largest input state is approximately 7000 photons. At the repetition rate of the experiment of 80 kHz, this corresponds to an optical power of over 71 pW. As the dynamic range of the device was not fully utilized, we could measure up to 156 pW of optical power per pulse, if all detection bins yield the maximal photon-number event of 15 (based on the current implementation of the EMG-model, i.e., LUTs).

VI. CONCLUSION

As the size of photonic quantum systems increases, building and analyzing sufficiently large detection systems will become increasingly important. Building up larger systems from well-characterized constituent components is key to this task. We applied this approach to intrinsically photon-number-resolving detectors. Convolution of the resulting outcome distributions of each detection bin allowed us to construct the response of the device as a whole. This, in turn, allowed us to perform quantum detector tomography on the complete device. We show sub-single-photon precision up to 276 photons, and measure up to 71 pW of optical power per pulse.

Our results show that this approach can maintain *precise* photon-number resolution when measuring thousands of photons. Nevertheless, our *accuracy* is limited by detector imperfections, particularly effects caused by temporary blinding of certain detector bins. Nevertheless, we show that the principle of combining detectors, each with intrinsic PNR, vastly outperforms multiplexed click detectors alone. Furthermore, we do so at a scale that is an order of magnitude larger than previous multiplexed single-photon counting systems [5, 9].

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DATA AVAILABILITY STATEMENT

The data that support the findings of this study are openly available in Zenodo at <https://doi.org/10.5281/zenodo.19678965>.

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Supplemental document: Macroscopic photon counting beating the Poisson noise limit

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This supplemental document includes further information about the complete experimental setup in Sec. I. We also give more details about the construction of the outcome matrix, which is used for the reconstruction of the positive operator valued measures (POVMs) of the detector (i.e., in the quantum detector tomography routine) in Sec. II. We discuss detector imperfections such as efficiency, time-walk effect, AC-coupling and our blinding model in Sec. III. Finally, we give further details about the arrival-time histogram fitting in Sec. IV.

I. EXPERIMENTAL SETUP

The full experimental setup is shown in Fig. 1, which is divided into three main parts: probe state preparation (blue box), temporal multiplexing (green box) and spatial multiplexing (red box). To justify the choice of the laser repetition rate of the experiment, we need to start with the SNSPDs that are used to detect the optical pulses. These SNSPDs have an efficiency recovery time of approximately 60 ns. Therefore, the temporal multiplexing network (green box in Fig. 1) is designed to generate a pulse train with a pulse separation of 100 ns, corresponding to a fiber length of $L = 20$ m (the average pulse separation is 97.31 ns). The pulse train consists of 128 sub-pulses, which means that the temporal delay between consecutive optical laser pulses needs to be at least 12.5 μ s to avoid temporal overlap. This means that the repetition rate of the laser must be < 80 kHz (the actual repetition rate of the laser was 79.665 kHz). This repetition rate aligns pulse trains from consecutive laser pulses right after each other with a small temporal delay to distinguish them more easily.

Blue box We use a 1550 nm pulsed laser with a repetition rate of 80 MHz. A free-space electro-optic modulator (EOM) is used as a pulse picker to achieve a repetition rate of 80 kHz. The EOM is modulated by a square-pulse from a digital delay generator, which is triggered by an electrical synchronization pulse from the pulsed laser. We use a quarter-wave plate (QWP) and half-wave plate (HWP) to rotate the polarization of the laser to horizontal for optimal pulse picking. Another set of wave plates are used after the EOM to optimize the extinction ratio of the pulse-picking to > 20 dB. The unpicked pulses are dumped at a beam dump, after a polarizing beam splitter (PBS). The transmitted pulses are further split by a beam sampler, where the transmitted part is used to trigger a fast photodiode with a bandwidth of 5 GHz, and the reflected part is fiber-coupled. We use a programmable optical processor (WaveShaper 4000B from Coherent) to spectrally

shape the optical pulses. A 0.28 nm bandwidth Gaussian spectral filter is programmed, which gives clean PNR signatures in arrival-time measurements [1] corresponding to an optical temporal pulse duration of (25 ± 5) ps. This value is calculated following Ref. [1, Appendix A] taking into account the optical pulse shaping and fiber dispersion. We broaden the optical pulse in time before coupling to the temporal multiplexing network (total length of approximately 2.54 km), as the initially ultra-short optical temporal pulse duration of 300 fs would suffer from high group-velocity dispersion. This would destroy any photon-number information in the arrival-time measurements using the PNR SNSPDs [1]. Before spectrally shaping the pulse (after 5m of fiber), the temporal pulse duration is 1.06 ps. The 0.28 nm Gaussian spectral filter broadens the temporal pulse duration to 11 ps, which then disperses in the 2.54 km of fiber to the final temporal pulse duration of (25 ± 5) ps.

After spectral shaping, the light passes through another (fiber-coupled) EOM to enhance the extinction ratio of the pulse picking. Both EOMs are aligned in time with the digital delay generator, in order to pick the same laser pulse. We optimize the input polarization to the EOM with a fiber polarization controller, the DC offset voltage with a power supply and the voltage pulse amplitude of the modulating pulse with the digital delay generator, achieving a total extinction ratio > 54 dB. Two variable optical attenuators (VOAs) are used to precisely set the mean photon number per pulse.

Green box Then the optical pulses enter the temporal multiplexing network, which generates a 128-bin pulse train per output arm from every impinging pulse. This fiber network consists of balanced beam splitters and delay lines. The upper arms of the beam splitters are connected to delay lines with lengths according to $2^x L$, where $x \in [0, 6]$ and $L = 20$ m, corresponding to a time delay of approximately 100 ns (see Ref. [2] for more detail on the temporal multiplexing setup). For every impinging optical pulse, the network creates a pulse train consisting of 128 pulses for both outputs. This means that the intensity of the impinging pulse is split evenly across all 256 sub-pulses. The components in the net-

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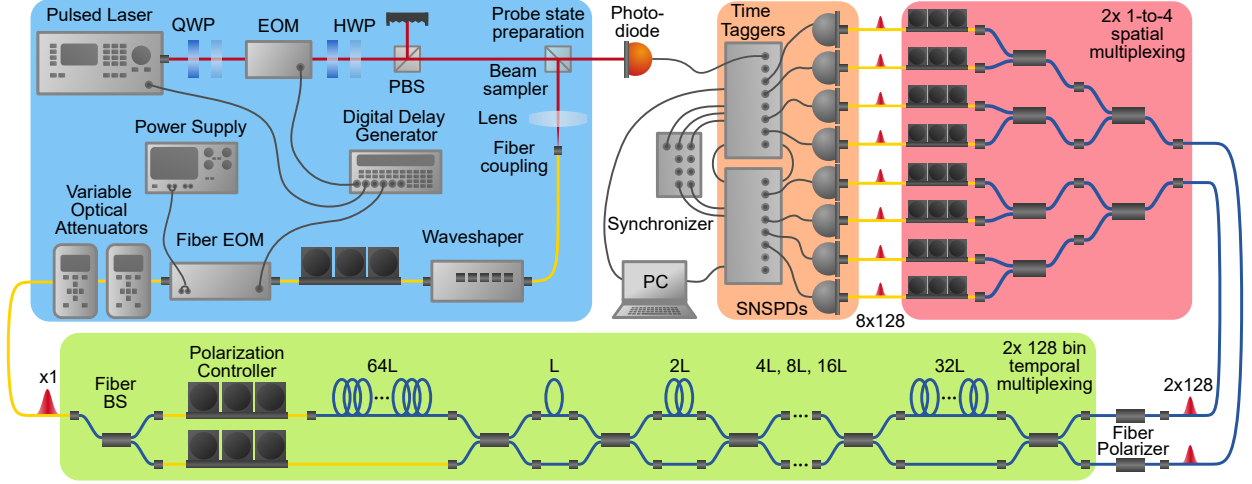


FIG. 1. The experimental setup can be divided into four main parts: probe state preparation (blue box), temporal multiplexing (green box), spatial multiplexing (red box) and PNR detection (orange box). A detailed explanation is given in Sec I.

work are polarization-maintaining; one polarization controller per arm is used to align the polarization to the slow axis of the optical fiber. Additionally, fiber polarizers are used to filter on the slow axis, as we observed polarization-mode dispersion between the slow axis and fast axis, if the polarization of the light was not perfectly aligned (see Sec. III D for more detail).

Red box Subsequently, both pulse trains enter a one-to-four spatial multiplexing network, consisting of three balanced beam splitters each. All eight output ports are connected to polarization controllers, to optimize the polarization-dependent efficiency of the commercial PNR SNSPDs (Single Quantum), which each receive the 128-bin pulse train. The SNSPDs have an efficiency $> 90\%$ and a jitter < 20 ps (FWHM).

Orange box Finally, we use two time taggers connected with a synchronizer (Time Tagger X and Synchronizer from Swabian Instruments) to record the trigger signal from the fast photodiode as well as the rising and falling edges of the electrical traces of the SNSPDs. Using two time taggers (connected with a Synchronizer) partially avoids overflows in the time tag streaming. The threshold of the time taggers is set to half peak height, giving optimal separation of photon-number events [3]. Calibration of the input states is performed following the procedure outlined in Refs. [3, 4], which relates the mean photon number per pulse of the coherent states with the attenuation of the variable optical attenuators.

In order to cover to full dynamic range of the detector, we measure 125 coherent input states with mean photon numbers scaling quadratically from 0 to 15393. The measurement time is 40 s, thus every input state is sampled $3.2 \cdot 10^6$ times.

II. OUTCOME MATRIX

As stated in the main document, the outcome matrix $P_{D \times M}$ is constructed by creating a histogram of the measured photon numbers for all incident mean photon numbers (for 10^5 samples). The means and variances of these outcome distributions are presented Fig. 4(a) in the main document. In Fig. 2 we show the outcomes in more detail. Specifically, in Fig. 2(a) we show that the outcomes for different incident mean photon numbers follow Poissonian statistics, as expected from our coherent light source (for clarity we show the red dashed lines for every second outcome that is shown). In Fig. 2(b) we visualize the region where the ensemble mean and variance from Fig. 4(a) in the main document begin to deviate from the Poisson noise limit. Here it can be seen that the outcomes begin to broaden, i.e., the variance of the distribution exceeds the mean, which is due to the blinding effects discussed in Sec. III D.

This outcome matrix is then used in the reconstruction of the POVMs, which is discussed in Sec. V and shown in Fig. 5 in the main document.

III. DETECTOR IMPERFECTIONS

It is possible to improve upon the results shown in the main document. In this section we discuss a number of factors which can be improved to further enhance the precision and accuracy of the detector.

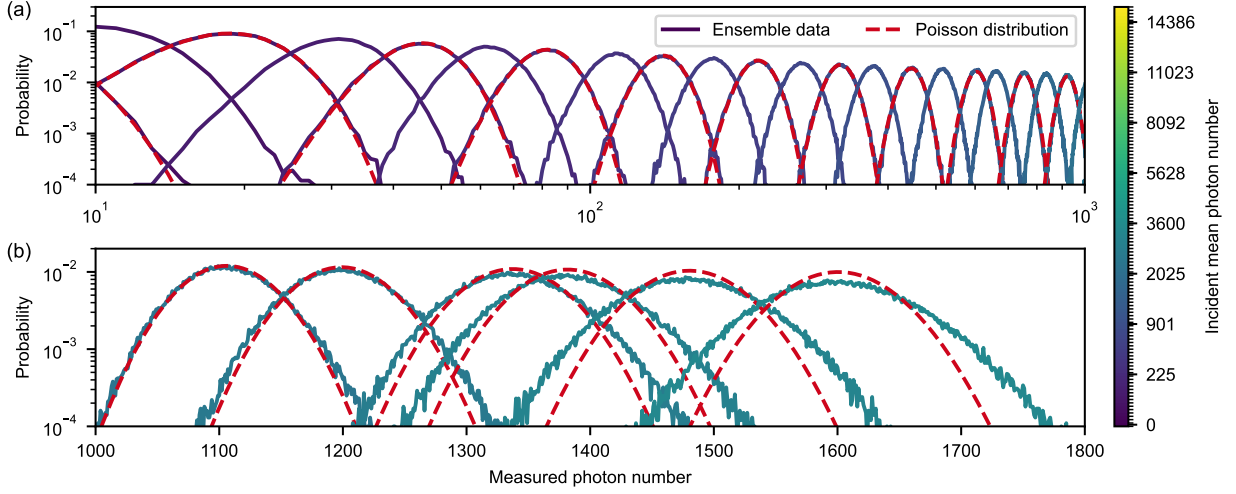


FIG. 2. (a) Outcome matrix $P_{D \times M}$ for every second coherent input state in the measured photon number region from 10 to 1000. We include Poisson distributions for every fourth coherent input state (for clarity), using the mean of the outcome distribution as the mean of the Poisson distribution, i.e., the distributions are not fitted. This shows, that the ensemble data of 10^5 samples follows Poissonian statistics from our coherent light source. (b) Outcome matrix for measured photon numbers from 1000 to 1800, to visualize the region where the ensemble mean and variance begin to deviate from the Poisson noise limit (compare Fig. 4(a) in the main document).

A. Efficiency

From Fig. 4(b) in the main document, it is clear that the efficiency of the detector is strongly dependent on the incident mean photon number. Here we will discuss what causes this dependency. The repetition rate of the laser is 80 kHz, within each repetition the 128 temporal bins (with a separation of 100 ns each) are located. Under high incident mean photon numbers, this leads to an effective repetition rate of 10 MHz (at the detector), as all temporal bins contain photons and thus lead to detection events.

At their normal operating bias current we observed latching of the SNSPDs for large mean photon numbers. To avoid latching, we lowered the operational bias current slightly under illumination with large mean photon numbers per pulse, which then remains unchanged for the entire data set. The detector efficiency is dependent on the detector count rate [5, 6], i.e., the incident mean photon number per pulse. This is due to the AC coupling (capacitive coupling), where current from a capacitor in the electronic circuit can flow back into the nanowire, which increases the effective bias current in the nanowire and thus the efficiency [5] (more details are discussed in Sec. III C). When the operational bias current is optimized for large mean photon numbers, it has the effect that for significantly reduced mean photon numbers, i.e., lower detector count rates, the efficiency also reduces (see Fig. 2(b) in the main document).

Importantly, we are in principle still able to achieve high efficiency of our detector for any mean photon num-

ber, by setting the appropriate operational bias current for the given incident mean photon number. For our data set, we optimized the efficiency for an incident mean photon number of approximately 5000 photons per pulse, as can be seen in Fig. 2(b) in the main document. If the efficiency should be maximized for small incident mean photon numbers, the device will begin to latch at larger mean photon number, i.e., reducing the dynamic range of the detector. Alternatively, the operational bias current could be adjusted depending on the measured detector count rate, to maintain high efficiency for all incident mean photon numbers. However, this only works when taking ensemble data, where the expected incident mean photon number does not vary over orders of magnitude.

B. Time-walk effect

Our model-based characterization of the detection bins relies on the fact that our model parameters remain constant between the calibration and data taking (under different incident mean photon numbers). In essence, the arrival-time histograms (e.g, shown in Fig. 1(c)-(e) in the main document) must be independent of input state parameters from shot-to-shot. In Ref. [7], Mueller et al. showed that SNSPDs can suffer from increased jitter at high detector count rates, regarded as “time-walk”. For SNSPDs at high detector count rates (when the pulse inter-arrival time is on the order of the reset time of the detector), current reset and amplifier effects may lead to distorted or especially smaller voltage pulses. This leads

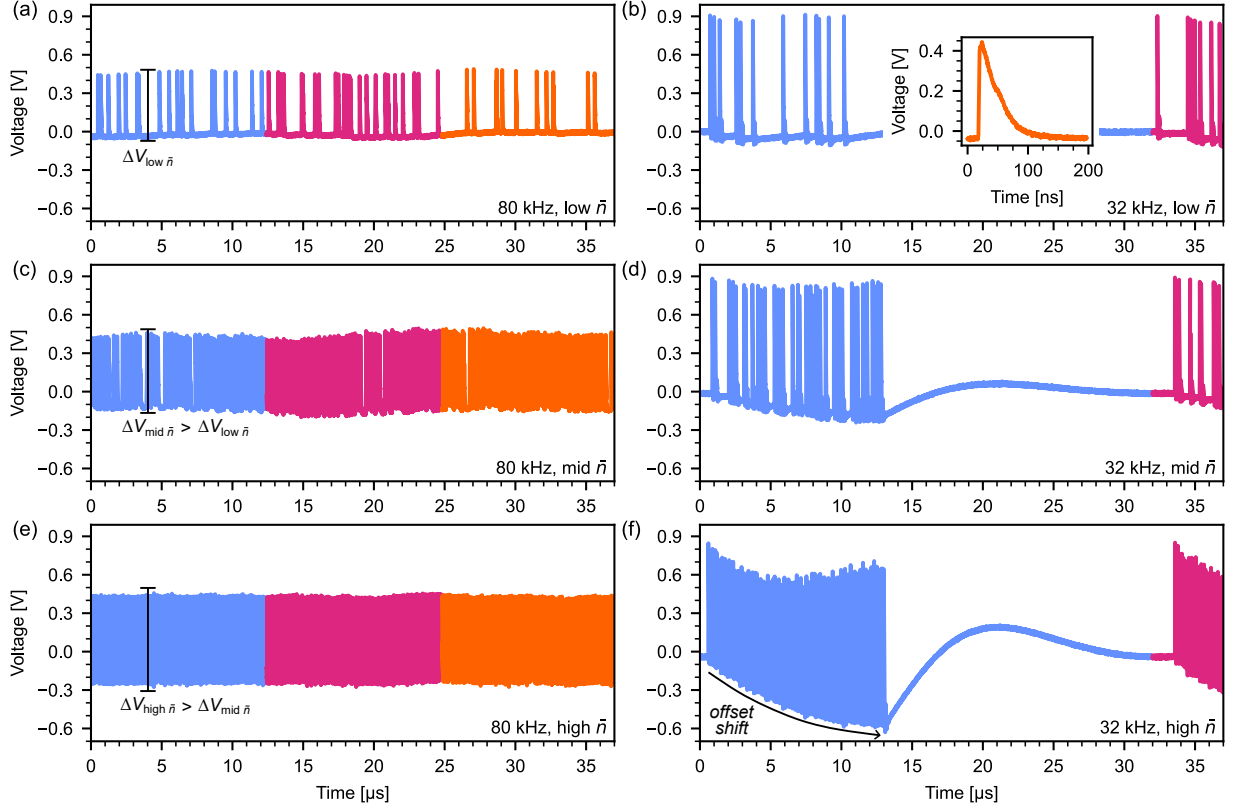


FIG. 3. (a), (c) and (e) Oscilloscope traces of the 128 bin pulse train for three different mean photon numbers (low, medium and high, respectively) for a repetition rate of the laser of 80 kHz. Subsequent pulse trains (here differentiated by different colors) are directly aligned after each other. (b), (d) and (f) Oscilloscope traces when the laser repetition rate is 32 kHz. Here, when subsequent pulse trains are separated in time, the “charging” effect is clearly visible. To avoid this we operate at 80 kHz, to be at a constant DC-offset. The spacing between subsequent traces is approximately 100 ns, i.e., just enough time for the SNSPD to reset after a detection event. The inset in (b) shows a single electrical output trace of an SNSPD.

to increased jitter for timing measurements using a fixed threshold time tagger [7].

In our experiment, a larger incident mean photon number corresponds to a higher detector count rate. However, due to the fiber delay lines in the temporal multiplexing architecture, the inter-arrival time of pulses in the temporal pulse train is predetermined to approximately 100 ns. This time is longer than the reset time of the SNSPDs. The time-walk effect may still be present when dark counts or stray photons (due to, e.g., imperfect pulse picking or reflections) are detected between the predetermined detection bins. This leads to potential inaccuracies of the LUTs, which do not take this effect into account directly. However, from our analysis we can still see clean arrival-time histograms for all 1024 bins and for all incident mean photon numbers. Therefore, we can conclude that although the detector might suffer from slightly increased jitter at higher detector count rates, i.e., larger mean photon numbers, the time-walk effect is not a primary problem.

C. AC-coupled amplification

At high detector count rates we observe a shift in the DC-offset of the voltage output (i.e., the voltage trace starting point of different temporal multiplexing detection bins). This is caused by the AC-coupled amplification circuit [5]. This effect can be clearly seen in Fig. 3(f), when the laser repetition rate is lower (32 kHz) to allow for discharging to happen after a 128-bin pulse train. To avoid a constant charging and discharging, which introduces significant jitter, worsening the PNR readout based on arrival-time measurements, we operate at 80 kHz which aligns subsequent 128-bin pulse trains directly after each other. This still causes the DC-offset of the voltage output to deviate from zero. With more detection events per time (i.e., a higher detector count rate) this DC-offset increases (towards negative voltages; compare the different DC-offsets in Fig. 3(a),(c),(e)).

We observed that the maximum of the voltage traces remains rather constant (see Fig. 3(a),(c),(e)). The SNSPD experiences additional biasing [5, 8], which arises

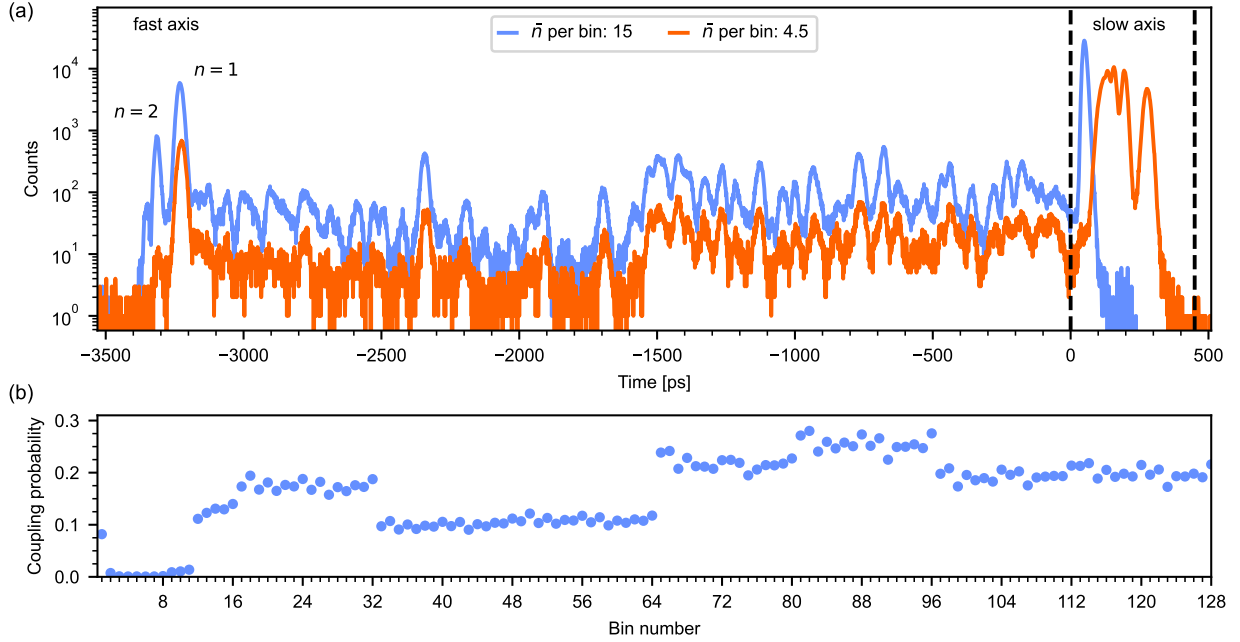


FIG. 4. (a) Arrival-time histogram of the last temporal bin (128) for spatial bin 1 for two different incident mean photon numbers per bin. The region of interest corresponds to the slow-axis photons from 0 ps to 450 ps (vertical dashed lines). The fast-axis photons arrive 3.5 ns earlier. The peaks between the two extremes correspond to photons that couple back and forth between the axes at the splice interfaces, due to slight angle mismatches of the polarization-maintaining fibers. (b) Coupling probability p_c for all temporal bins for spatial bin 1.

due to charges from a detection pulse that accumulate at a capacitor in the AC-coupled readout electronics of the SNSPD. This additional biasing, caused by bias current flowing back into the nanowire, leads to an increase in the effective voltage trace height at higher detector count rates (compare Fig. 3(a),(c),(e)). In our system, using a constant threshold time tagger, this means that we trigger on different relative trigger levels of the trace for different detector count rates, i.e., mean photon numbers. This is effectively the same as the time-walk effect [7] discussed in Sec. III B, which leads to a slight increase in jitter (a similar effect is also discussed in Ref. [9]). Under illumination with a constant mean photon number, this effect is not visible. However, for different incident mean photon numbers, this effect manifests in slight temporal shifts of the arrival-time histogram. Histograms for larger mean photon numbers are shifted slightly to larger arrival-time differences, as the effective relative trigger level is higher. We observe a shift of < 15 ps from the lowest to the highest mean photon number.

To mitigate this effect in the analysis, we considered characterizing the device using multiple coherent states with several different mean photon numbers per bin (e.g., one, four and seven), to provide an average arrival time histogram. This essentially adds jitter to each photon-number term, increasing the estimate of the contributions of different photon numbers. However, this lead to

inaccuracies and convergence problems in the EMG distribution fitting. Therefore, we selected an incident mean photon number per bin of five (to be approximately in the middle of the observed shift). Here the EMG distribution fits show excellent agreement with the data (compare Fig. 1(c)-(e) in the main document).

Importantly, this effect can also be overcome in hardware by using DC-coupled amplification.

D. Blinding effects

The multiplexing network consists of polarization maintaining fibers, to ensure the polarization is optimal for each detector, independent of the temporal bin. These fibers guide photons in the “slow” and the “fast” axis, which travel at different velocities, leading to polarization-mode dispersion. The total length of the network is approximately 2.54 km, during which the two polarization modes acquire a 3.5 ns temporal offset (see the time difference between the slow-axis photons and fast-axis photons in Fig. 4(a)). The beat length (a measure for the birefringence, i.e., the difference of the refractive indices for the slow and fast axes) of the fiber we used (Corning PM 15-U25D) is < 5 mm at 1550 nm. Therefore we expect to see a $\frac{1550 \text{ nm}}{1.46} \cdot \frac{1}{5 \text{ mm}} \cdot 2540 \text{ m} = 3.8$ ns temporal difference between the arrival times of the two

axes, which agrees with our observation.

Each temporal bin of the network samples this offset at different fiber lengths. The fibers and beam splitters of the network are fusion spliced with a polarization aligning fusion splicer. Nevertheless, we see back and forth coupling between the different polarization modes at the splices (see Fig. 4(a)) due to non-ideal alignment during splicing. However, importantly, if a photon couples from the slow axis to the fast axis once, it will always arrive before a photon that travels in the slow axis the entire network. If this photon is detected, the detector will be in its dead time during the expected arrival time of photons traveling in the slow axis, and is therefore blind to those photons.

We can express this as a general coupling probability p_c , which is determined by integrating all counts outside of the region of interest (from -3500 ps to 0 ps in Fig. 4), i.e., everything except the slow axis and then dividing by the total number of trigger events. This yields a lower bound for the coupling probability. From this we can calculate the probability of at least one of n photons coupling to the fast axis $p_{\text{fast}} = 1 - (1 - p_c)^n$. The probability of blinding the slow axis is then $p_{\text{blind}} = p_{\text{fast}} \eta_{\text{det}}$. Therefore, the zero-inflated photon-number distribution becomes

$$P(n, \bar{n}) = \begin{cases} e^{-\bar{n}'} + \sum_{n=1}^{\infty} \frac{(\bar{n}')^n}{n!} e^{-\bar{n}'} p_{\text{blind}} & , n = 0 \\ \frac{(\bar{n}')^n}{n!} e^{-\bar{n}'} (1 - p_{\text{blind}}) & , n \neq 0 \end{cases} \quad (1)$$

with $\bar{n}' = \bar{n} \eta_{\text{det}} (1 - p_c)$ as the mean photon number that remains in the slow axis.

Independent measurements of this effective coupling probability (see Fig. 4(b)) yields an average effective $p_c = 0.2$. Applying this model to the data in Fig. 4(a) in the main document shows excellent agreement across the measurement range of four orders of magnitude. While a full system description requires bin-specific coupling probabilities, we find that a single effective coupling probability is a valid first-order approximation. The limitation of blinding effects can be overcome with more accurate splicing between polarization-maintaining fibers, or fibers with minimal polarization mode dispersion. Additionally, it is possible to choose the fast axis for the investigation, as these photons always arrive first and are not affected by blinding. However, this comes at the cost of a higher susceptibility to bending loss [10].

IV. ARRIVAL-TIME HISTOGRAM FITTING

We follow the procedure from Ref. [1] to investigate the photon-number-resolving capability of all 1024 bins independently. First, we find the region of interest for every detection bin (i.e., the position in time of the pulse train, where the arrival-time histogram is located, as depicted in Fig. 1(b)-(e) in the main document). We then take the arrival-time histograms for a mean photon number of 5 per bin, to find the peak positions of the first three photon number contributions. The peak position scaling ($1/\sqrt{n}$) is then used as a fitting parameter to fit the exponentially-modified Gaussian distributions [11] to the arrival-time histogram. Based on the fitted distributions we can generate the lookup table for all 1024 bins (as shown in Fig. 2(b) in the main document).

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The central question of this thesis was whether photon counting can be extended beyond the conventional single-photon regime toward photon numbers that are normally approached with classical optical power measurements. Addressing this question required three ingredients, which defined the main aims of this thesis. First, high-dynamic-range (HDR) photon-counting detectors require a characterization framework that remains valid over large photon-number Hilbert spaces and large outcome spaces. Quantum detector tomography (QDT) provides such a framework, but its reconstruction methods must become scalable to be applicable to large detectors. Second, a suitable detector platform is needed that allows for large-scale photon counting. Here, superconducting nanowire single-photon detectors (SNSPDs) were investigated, as they combine excellent figures of merit with intrinsic photon-number resolution (PNR). This required a detailed understanding of intrinsic PNR with SNSPDs. And third, we aimed to investigate scalable photon counting and to build a HDR detector, by combining appropriate multiplexing architectures with intrinsic PNR SNSPDs.

Overall, this thesis shows that photon counting does not have to remain restricted to the single-photon regime. By combining scalable characterization, intrinsic PNR, and multiplexing, photon-number-sensitive measurements can be extended toward regimes that were previously accessible mainly through classical optical power measurements.

5.1 Scalable detector characterization based on QDT

The characterization of HDR photon-counting detectors through scalable QDT, not only allows a quantum-mechanically consistent description of the detector, it also provides a common characterization framework across orders of magnitude in the detectable number of photons per pulse, i.e., optical power. This is important to establish traceability in optical power metrology. In Chapter 2, I have established the experimental testbed that allows the investigation of arbitrarily large Hilbert spaces and detector outcomes. The logarithmic time-multiplexing architecture served as an ideal tool to access a high dynamic range. While Publication 2.1 showed the first successful application of QDT to a HDR detector, it also demonstrated a clear bottleneck in terms of memory requirements and runtime. This stems from limitations of conventional approaches of reconstructing detector POVMs, as the dimensions of the matrices involved in QDT increase. Addressing this bottleneck, we have utilized for the first time high-performance computing (HPC) to enable scalable QDT methods. We have reconstructed the POVMs of a detector with a Hilbert space dimension with over 10^6 photons and 10^2 detector outcomes, based on experimental data in Publication 2.2.

At this point, it might seem like we are already close to our desired aim of a HDR photon-counting detector, as this device can respond to over one million photons. However, it is missing the counting part of our aim. This becomes apparent when investigating the purity of the outcomes of this multiplexing architecture, as demonstrated in Publication 2.3. To achieve PNR, a detector must have a one-to-one mapping from input photon number to output photon number, i.e., the POVM matrix needs to be square and diagonal. If this is not the case, there will inherently be ambiguity

in the photon-number assignment, as multiple photons numbers contribute to the same detection outcome. That said, based on simulated data, we demonstrated the reconstruction of a POVM matrix with $4 \cdot 10^{12}$ elements. This corresponds to a PNR detector that can resolve up to two million photons. We were able to reach this scale partially due to the sparse and banded structure of the input state matrix, thanks to the photon statistics of the coherent probe states, which allowed for efficient parallelization on a cluster with many compute nodes. This means that we have successfully achieved the first aim of this thesis, which was to *enable the characterization of high-dynamic-range photon-counting detectors through a scalable QDT approach*. Notwithstanding this achievement, there is one caveat: to characterize a detector capable of counting millions of photons through its POVMs, we need over one hour of access to an entire supercomputer cluster with almost one thousand compute nodes. Additionally, the storage of all matrix elements, representing the conditional probabilities that describe the detector and give rise to figures of merit such as efficiency and dark-count probability, will require multiple terabytes of memory.

Future directions for scalable QDT

To increase the accessibility of this characterization technique, a promising route for future improvements is to explore options that reduce memory footprints or leverage higher memory bandwidth. The main bottleneck of the current implementation is the available memory bandwidth, i.e., the rate at which data can be transferred to the processor. Because the solver is efficient and quickly performs tasks, it stays idle while waiting for data to arrive. On the software level, improvements to mitigate this bottleneck can involve approximate computing techniques. These methods sacrifice a marginal amount of accuracy for computational efficiency, either by reducing the precision of floating-point numbers or by operating on fixed-point numbers. This way, the processor remains efficient and the memory requirements are reduced. On the hardware level, transitioning to alternative architectures is promising. GPUs can be utilized because they have significantly higher memory bandwidth compared to traditional CPUs due to their parallel architecture. Field-Programmable Gate Arrays (FPGAs) can also bypass memory bottlenecks by enabling highly customized, application-specific hardware designs that optimize data usage.

Alternatively, recent work uses gradient-descent methods to achieve scalable QDT [44, 134]. These methods can be more efficient in terms of time and memory complexity [44], as they are unconstrained optimization algorithms. This comes with the disadvantage of reduced accuracy due to the absence of physicality constraints. In contrast, this can be readily implemented in typical constrained convex optimization techniques. The gradient-descent method achieves memory efficiency by mini-batch sampling [135] of the stochastic algorithm, which allows for computation on random subsets of the involved matrices rather than loading the complete matrices [44].

Standard optimization solvers from open-source libraries are built to allow for a broad range of problems to be solved. Available knowledge about the problem is important, to increase the efficiency of the optimization process. This enables building customized solvers, for example, specific to the task of POVM reconstruction for phase-insensitive high-dynamic-range photon-number-resolving detectors. For efficient PNR detectors, the POVM matrix is very close to being square and diagonal (the number of outcomes N is ideally equal to the Hilbert space dimension M , providing the one-to-one mapping mentioned above). Adaptive methods could be used, in which the POVMs are first described on a coarse grid with an interpolation between the grid points; subsequent iterations can then refine the grid at locations where more variability is required. Additionally, prior knowledge about the detector can inform the solver about regions with nonzero elements, for example based on the outcome matrix. Another interesting alternative might be

to decompose the rows of the POVM matrix into a complete set of basis functions to describe the individual outcomes; these basis sets are commonly used to describe molecular systems in computational science [136]. This can greatly reduce the number of parameters to optimize.

5.2 Advances in intrinsic PNR with SNSPDs

The second aim was to *understand intrinsic PNR with SNSPDs*, as this detector technology presented itself as a promising candidate to achieve large-scale photon counting. The literature showed many approaches to retrieve the photon-number information from the electrical output trace of the SNSPD. Therefore, the first task was to identify the location on the trace where photon-number information is encoded, to conclusively find an appropriate readout method. We recorded an extensive data set of SNSPD traces for varying incident mean photon numbers, using a high-bandwidth and high-sample-rate oscilloscope, to gain the maximum information about the detection events. Principal Component Analysis (PCA) identified which features from the data set consisting of many samples vary the most. The analysis in Publication 3.1 showed that the photon-number dependence is concentrated primarily in the early electrical response, especially the rising edge and amplitude of the signal, making a time-to-digital converter an ideal and experimentally practical tool to extract photon-number information. If the bandwidth of the amplification stage is appropriately matched to the SNSPD output, it is possible that all relevant information is contained in the rising edge, as the amplitude of the trace for different photon-number events does not change.

The identification of a timing-based approach showed that timing jitter is a central figure of merit to understand PNR with SNSPDs. Arrival-time histograms contain photon-number-dependent peaks with finite width, which determines how well neighboring photon numbers can be distinguished. Our model from Publication 3.2 provided a quantitative description of the photon-number-dependent arrival times, based on a sum of exponentially-modified Gaussian distributions. The model incorporated different jitter components, some of which are due to internal sources governed by the geometry of the SNSPD as well as the intrinsic photon detection process. Typically, amorphous materials such as tungsten silicide (WSi) and molybdenum silicide (MoSi) showed larger jitter compared to polycrystalline materials such as niobium nitride (NbN) or niobium titanium nitride (NbTiN) [137]. Based on their disordered structure amorphous materials have limited electronic conductivity, resulting in lower carrier densities and lower critical temperatures, which increases jitter [137, 138]. SNSPDs based on niobium have demonstrated below 3 ps jitter [79] and are dominantly used for PNR investigations and applications [9, 39, 61, 104, 107–115, 126].

Besides the internal jitter sources, external contributions are due to electronic noise, measurement instrumentation and optical temporal pulse duration. If these components have large contributions to the overall jitter of the system, photon-number information will be washed out due to a large uncertainty in the arrival time of the detection signals. This shows the importance of early signal amplification, for example by cryogenic amplifiers. This reduces noise pickup in the coaxial cables from cryogenic temperature to room temperature, and improves the signal-to-noise ratio. Almost all studies in this thesis were conducted on commercially available devices with < 20 ps jitter that utilize cryogenic amplification, which shows the accessibility of PNR SNSPDs. As for the measurement instrumentation, fast photodiodes and time taggers with picosecond timing resolution are ideal, as this keeps their contributions to the overall system jitter small. Potentially the experimentally most crucial contributions to jitter is the optical temporal pulse duration of the light itself. Many laser systems produce ultrashort pulses. This is ideal for PNR, as photons

arrive almost simultaneously on the superconducting wire (see Publication 3.2). However, SNSPDs are mostly fiber-coupled devices, and ultrashort pulses suffer from group-velocity dispersion in optical fibers. This broadens the temporal pulse width and affects the PNR capability of the SNSPD due to delayed photon absorption.

Future directions for timing-based PNR

Enabling PNR for longer pulse durations is desirable for single-photon sources based on quantum dots due to pulse durations (lifetimes) of typically hundreds, but down to tens, of picoseconds [139, 140], which provides compatibility between detection hardware and photonic quantum processors [141]. The developed model from Publication 3.2 can provide the necessary insights into how detector design influences the timescales relevant for PNR. Arrival-time-based PNR requires the absorbed photons to contribute to the early detector response within a sufficiently short time window. This window is set by material- and circuit-dependent dynamics, including hotspot formation, electron–phonon cooling, current redistribution, and the resistive-domain lifetime. For devices with saturated detection efficiency, the limiting timescale can be close to the hotspot formation time, typically of the order of the electron–phonon cooling time (approximately tens of picoseconds), or up to the time the bias current remains in the nanowire [114]. For devices without saturated efficiency, PNR signatures may persist for longer optical pulses, but with reduced efficiency and increased jitter, potentially up to the resistive-domain lifetime (approximately hundreds of picoseconds) [114].

In Publication 3.3, I investigated how optical pulse duration affects the PNR capability of SNSPDs. Since the pulse duration adds uncertainty to the photon arrival time, longer pulses broaden the photon-number-dependent timing distributions and reduce their distinguishability [115]. Future work should systematically study arrival-time-based PNR for longer optical pulses, in particular beyond 100 ps, to determine whether photon-number information is washed out or whether partial PNR can be retained through the detector response to the earliest absorbed photons.

Intrinsic PNR with SNSPDs should not be understood as a binary figure of merit that a detector either does or does not have, but rather as a probabilistic detector capability that must be carefully characterized. We have shown that the electrical trace, originating from a detection event, contains photon-number information and that this can be modeled. However, the reliability of assigning photon numbers depends on the separation and overlap of the corresponding outcome distributions. In Publication 3.3, I have shown a characterization of photon-number assignment dependent on model choice, based on purely Gaussian distributions or EMG distributions, including exponential tails, which increase overlap between photon-number contributions. For arrival-time-based measurements, the EMG model provides the more accurate description. This is especially important for applications that rely on accurate photon-number assignment with minimized misidentification probability, i.e., a detected n photon event should only originate from an n photon input. Such applications include heralding tasks, for example based on spontaneous parametric down-conversion sources, to distinguish between single photons and multiple photons with maximum fidelity.

The arrival-time method measures the difference in time from a trigger signal to the rising edge at a constant threshold. This has the inherent benefit of a conditional measurement, which filters out dark counts. An alternative timing-based method measures the rising-edge time, based on a low and high voltage threshold, which works well even if no trigger signal is present. However, the disadvantage is that dark counts cannot be filtered out. Although, not presented in this thesis,

we studied both methods in detail as part of a Bachelor thesis that I have supervised [142]. We investigated optimal low and high threshold combinations (rising-time method), as well as an optimal thresholds (arrival-time method), by sweeping through the voltage thresholds with a time tagger. Based on these results and the results presented in this thesis, which originate from different SNSPDs, I can conclude that optimal thresholds are highly detector dependent. To maximize the photon-number information one should always carefully characterize the PNR capability prior to a measurement.

Similar to the two-dimensional arrival-time histogram presented in Publication 3.1 we have also investigated a combination of the two methods, i.e., recording the arrival-time differences from a trigger signal to the low and high thresholds. By projecting the two-dimensional histogram on an axis under an optimal angle, it might be possible to decrease photon-number misassignment. Although, there is an interplay between the exponential tail of the EMG distribution originating from the arrival-time measurements, the Gaussian distribution from the rising-time measurement and the separation of photon-number peaks that needs to be studied in further detail. Only then is it possible to assess whether the added complexity of the data acquisition and analysis is worth the potential improvement in PNR.

In Publication 3.2 we have seen that the arrival-time histogram continuously shifts to earlier times for increasing mean photon numbers, even up to hundreds or thousands of photons. Nevertheless, in this bolometric regime, no information about individual photons can be obtained, as the contributions from photon numbers fully overlap. It is not the rising edge that becomes steeper, it is the overall delay of the SNSPD trace. This means that the arrival-time distribution is now dependent on the mean photon number of the incident light pulses. This feature might still be useful, to estimate mean photon numbers per pulse. Further studies based on the data set produced for Publication 3.2 and Publication 3.3 can be conducted to investigate the uncertainty in the determination of the mean photon number based on width of the arrival-time distributions (see Fig. 4(a) in Publication 3.2). The bolometric regime is also important for laser ranging applications, due to a reduced timing jitter [114, 143].

Although a complete understanding of SNSPD-based PNR will require further specialized measurements and application-specific analysis methods, this work establishes a connection between the physical mechanisms underlying PNR and a quantitative detector-characterization framework. With this in mind, this thesis has made significant advances toward the goal of *understanding intrinsic PNR with SNSPDs*.

Comments on the phrase “intrinsic PNR”

The use of this phrase “intrinsic PNR” is inconsistent in the literature on PNR with SNSPDs. It can be misleading if it is understood as equivalent to true PNR. However, true PNR can only be achieved with energy-resolving detectors under illumination with monochromatic light, whereas SNSPDs do not measure the deposited photon energy. Instead, the nanowire itself can be viewed as a highly multiplexed detector consisting of many effective hotspot detection regions, as discussed in Publication 3.2. At the same time, this mechanism also differs from quasi-PNR, also known as pseudo-PNR, which is achievable with “extrinsic” multiplexing architectures. In extrinsic multiplexed detectors, the incident light is deliberately split into several spatial or temporal modes (multiplexing bins) using additional optics, such as beam splitters and fibers. In SNSPD-based intrinsic PNR, by contrast, the multiplexing resource is provided intrinsically by the nanowire itself. Therefore, a more explicit term for this mechanism could be intrinsic-multiplexing-based PNR.

This terminology emphasizes that the effect is neither true calorimetric PNR nor purely extrinsic multiplexing, but rather originating from the distributed detection dynamics of the nanowire.

5.3 Scalability of photon-number-resolving detectors

Motivated by an improvement over the investigation of photon-number assignment in Publication 3.3, which uses regions in the arrival-time histogram for assignment, we introduced a novel probabilistic treatment of the PNR distributions in Publication 4.1. This avoids discarding valuable information and potential misassignment. In this analysis method, photon-number distributions are generated for every arrival time recorded by the time tagger. These distributions originate from a prior characterization of the arrival-time histogram using the PNR model from Publication 3.2. Either the entire photon-number distribution can be used for further analysis of the data, or a mean and standard deviation can parameterize the distribution. The mean estimates the number of photons that were measured, while the standard deviation estimates the uncertainty of the assigned photon number. Crucially, this is possible shot-by-shot, no ensemble measurement is required.

This novel technique was an important contributions to the final aim of the thesis, as it allowed to use larger photon-number contributions from individual SNSPDs, and to directly estimate the uncertainty. Still, an individual SNSPD can only resolve a limited number of photons, as shown in Chapter 3. Therefore, a combination of detection architectures is required to achieve our goal of a HDR photon-counting detector. Multiplexing many SNSPDs allowed us to achieve the high dynamic-range, and reading out every SNSPD individually allowed us to retain the photon-number information of the underlying detection bins. This increases the amount of extracted information compared to other detectors, that either have many more multiplexing bins, but lack PNR and individual readout of detectors [8], or have better PNR, but lack in the number of multiplexing bins [6].

Publication 4.1 combined these ideas into a single detector system. Arrival-time measurements from PNR SNSPDs allowed to construct model-based lookup tables, mapping each arrival time to a photon number and a corresponding uncertainty. The large-scale temporal and spatial multiplexing architectures allowed to individually read out every detection bin, and together these could then be combined into a final detector output shot-by-shot of the experiment. We used a spatial-multiplexing architecture with eight SNSPDs, each of which received the 128 bin pulse train from the time-multiplexing architecture. The intrinsic PNR of each of the resulting 1024 detection bins, achieved a dynamic range in the order of 10^4 photons. Therefore, the POVMs that describe this detector have on the order of 10^8 matrix elements. Although the size of this task still only requires a single compute node, the improved efficiency of the parallel quantum detector tomography solver [144] developed in Publication 2.2 was necessary to characterize the detector, and allowed us to use a conventional computer for the reconstruction.

As mentioned above, the POVM matrix of an efficient PNR detector is close to being square and diagonal. The POVMs shown in Fig. 5 of Publication 4.1 are not diagonal, which is a result from the limited efficiency, in the range of 30% to 50%, of the detector. For low detector outcomes from small incident photon numbers, the POVMs are very narrow, indicating an accurate photon-number assignment. Larger outcomes broaden due to an increased uncertainty in the assignment, originating from less separation and thus larger overlap of higher photon-number components in the arrival-time histograms.

Future directions for scalable photon counting

This result presents a straightforward solution to improve the precision of the detector, even for large incident photon numbers. Distributing the same number of photons across more multiplexing bins, results in less photons per bin, which reduces the uncertainty in the photon-number estimate. Scaling the number of spatial bins to 16 or even 64 SNSPDs brings an improvement by a factor of two or four, respectively, in the resulting dynamic range. Although manufacturers offer solutions in this range, it requires significant commercial efforts, as the number of physical SNSPD channels needs to be increased. An alternative method is to increase the number of temporal bins, which comes with negligible costs but the disadvantage of a reduced maximal repetition rate of the experiment, as the temporal degree of freedom becomes a multiplexing resource. For future developments, I would advise to choose the former method of increasing the number of physical SNSPDs. This provides most flexibility with the available laser repetition rate. For example, our detector was limited to 80 kHz due to the temporal multiplexing, while many experiments operate in the megahertz regime.

Further development is also required in the readout of SNSPDs. Typical commercial SNSPDs use AC-coupled amplification, which was identified as a main limitation in our detector. For large incident photon numbers, many temporal multiplexing bins of the detection channels contain photons, which results in a high effective detector count rate of approximately 10 MHz. At this high rate, charges from each output pulse of the SNSPD accumulate at a capacitor in the AC-coupled readout electronics, resulting in a DC-offset change of the voltage traces. For measurements with a constant-threshold time tagger, this corresponds to a change in the relative trigger level and therefore an increase of the system jitter, which negatively affects the PNR capability. This is less of a problem for coherent states, as the number of photons closely vary around the mean, determined by the underlying Poissonian statistics. This means that the DC-offset stays constant after the capacitor has charged completely.

For broader photon statistics, especially thermal statistics, this problem is much more pronounced, to the extent that any photon-number information was completely masked. For thermal light, the photon-number variance is large. The most probable photon-number contribution, for any mean photon number of a thermal state, is zero photons, i.e., vacuum. This has the result that although in one shot of the experiment, all detection bins were occupied, the subsequent shot can have completely empty detection bins. If this happens, the capacitor of the readout electronics has enough time to discharge (in the approximately 12 μ s of one pulse train), which means it will charge again in a future shot of the experiment. This results in a fluctuating DC-offset, increased jitter, and finally a reduced PNR capability of the device. Appendix A.2 shows example voltage traces with DC-offset fluctuations recorded for a two-mode squeezed state generated by a spontaneous parametric down-conversion source and for a pseudo-thermal state.

The development of DC-amplifiers for commercial systems can overcome this issue. These amplifiers are already used by the Jet Propulsion Laboratory at NASA [145], explicitly to avoid the re-biasing associated with AC-coupled amplifiers operating at high count rates [146], discussed in the supplemental document for Publication 4.1. The ideal detector combines both improvements; a detector system that only relies on spatial multiplexing offers the capability of high detector count rates provided by SNSPDs, and DC-coupled amplification maintains constant jitter performance independent of the incident mean photon number, provided that the PNR capability is maintained.

Next to hardware developments, there are also further investigations on the data analysis that are interesting, specifically the dependence of the detector performance on model fidelity and detector stability. The probabilistic analysis relies on lookup tables that map arrival times to photon-number distributions. These lookup tables are only as accurate as the underlying calibration and model. Changes in the detector response between calibration and measurement, for example due to count-rate-dependent changes of the system jitter, which distort the arrival-time histograms and limit the accuracy of photon-number assignment. This distinction between precision and accuracy is especially important for metrological applications, where small systematic errors dominate for small statistical uncertainty.

Notwithstanding these future research opportunities, the final aim of the thesis to *investigate scalable photon counting far beyond the conventional single-photon regime*, was successfully achieved. Based on the 1024 multiplexing bins and photon-number contributions of maximally 15 per bin, the detector allows to count 15360 photons per pulse. At the repetition rate of approximately 80 kHz and for 1550 nm photons, this corresponds to an optical power of 156 pW. Given the limited, but still impressive, efficiency of 30% to 50%, due to many fiber connections and beam splitters as potential loss sources, the largest input coherent state produced detection events around 7000 measured photons. This corresponds to an optical power of 71 pW, which can be measured with an optical power meter. The thesis therefore demonstrates a route toward bridging microscopic single-photon measurements and macroscopic optical power measurements, while beating the Poisson noise limit across the entire dynamic range.

5.4 Outlook: frontiers of SNSPDs in the context of PNR & future research directions

The next step toward practical high-dynamic-range photon counting is the co-design of detector hardware, readout, and analysis. On the hardware side, SNSPDs should be engineered to maximize the separation of photon-number contributions. Here, large-kinetic-inductance detectors are a promising candidate, which provide increased separation with the trade-off of slower signals and thus reduced maximum count rate [111, 113]. Simultaneously, minimizing all jitter contributions and preserving fast reset times to allow for high repetition rates is crucial for optimal photon-number assignment. Particularly important is the operation with longer optical pulses, since this is essential for compatibility with many single-photon sources and photonic quantum processors. Photon-number resolution toward visible wavelengths has already been studied [112], which is relevant for quantum dot sources. Here, shorter wavelengths can lead to reduced jitter [79, 124, 125], which improves PNR. For longer wavelengths toward the mid-infrared, however, further investigations are required. Lower photon energies lead to increased delay times (and increased variations of the delay time, i.e., jitter) between photon absorption and the formation of the resistive domain [147]. The development of wide-wire detectors [88–90], which can be used for long-wavelength detection, opens up further research opportunities in the context of PNR. These detectors can be operated at milliampere bias currents, instead of microampere currents for typical SNSPDs. Increasing the bias current reduces jitter; however, fluctuations in the photon absorption site across the wire are a source of jitter [147], which can become significant for wide-wire designs. For the readout electronics of SNSPDs, DC-coupled cryogenic amplification should be pursued for commercially available detectors. These can alleviate the problems due to DC-offset drifts at high detector count rates, which were observed with AC-coupled amplification circuits.

On the system level, spatial multiplexing and integrated SNSPD arrays offer a path to larger dynamic range without the repetition-rate limitations of temporal multiplexing, provided that the readout scheme preserves photon-number information from individual bins [9, 61]. Simultaneously, spatially multiplexed PNR SNSPDs greatly enhance the fidelity of photon-number assignment, as this reduces the number of photons detected by each PNR SNSPD. At lower photon numbers, adjacent photon-number events are more clearly separated, enabling more accurate photon-number assignment.

On the analysis side, the probabilistic analysis method should be extended to include model uncertainty and potential detector drift due to the readout electronics, which will be important for applications in optical metrology. Our QDT solver already demonstrated scalability, however, it can be made more accessible by investigating approximate computing techniques, highly customized solvers, or hardware such as GPUs and FPGAs, which reduce memory requirements.

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A.1 Statement on the use of artificial intelligence

The following table outlines the generative AI tools and Large Language Models employed during the preparation of this thesis. These tools were used to assist with the writing process, improve linguistic clarity, and support data visualization, document formatting and literature review tasks. Crucially, all AI-generated suggestions, summaries, and code snippets were manually reviewed, verified against primary literature, and edited by the author to ensure scientific accuracy and personal stylistic consistency. All scientific core contents, data analysis, and interpretations remain the original, independent work of the author.

Table A.1: Overview of AI tools and their specific applications in the preparation of this thesis.

AI Tool	Scope of Application and Usage
Gemini 3 (Flash and Thinking)	L^AT_EX & Technical Support: Assisted with typesetting and troubleshooting, specifically debugging compiler errors. Python Plotting: Extending data plotting scripts via <code>matplotlib</code> to more complex figures, including the implementation of consistent styling, subplot alignment, and figure scaling.
DeepL Translator	Translation: Used to assist with the German translation of the summary.
NotebookLM	Literature Synthesis: Assisted with the comparison of various research papers, specifically for the state-of-the-art section. All findings were subsequently validated through a direct comparison with the primary literature.
OpenAI ChatGPT (GPT-5.5)	Linguistic Polishing: Assisted in refining technical descriptions, academic tone and reorganizing complex sentences to improve logical flow and readability.

A.2 Arrival-time measurements based on thermal statistics

Figure A.1 shows oscilloscope traces from one SNSPD channel from Publication 4.1 for approximately 20 subsequent laser pulses. This means that in total 20 of the 128 bin pulse trains are visible. As mentioned in Chapter 5, we have observed a significant offset drift of the SNSPD voltage traces under illumination with thermal statistics. Figure A.1(a) shows traces for a two-mode squeezed state from a spontaneous parametric down-conversion (SPDC) source and Fig. A.1(b) for a pseudo-thermal state based on a rotating ground glass, which was placed in the beam path of the laser. A clear drift is noticeable, which results from empty pulse trains from a vacuum input to the detector. In this time of approximately 12 μs , the capacitor in the readout electronics has enough time to discharge accumulated charges, which originated from the AC-coupled amplification at high detector count rates [146]. This leads to an overall instability, compared to coherent input states, due to the large photon-number variance of the thermal statistics. To verify that the drifts are not caused by the SPDC source, we tested pseudo-thermal states (with a $g^2(0) \approx 1.5$). This allowed us to first ensure functionality of the detector for coherent input states and then add the rotating ground glass, without changing the rest of the experimental setup. Another aim of the detector from Publication 4.1 was to measure photon-number correlations between the two modes of a two-mode squeezed state from a bright SPDC source. Unfortunately, due to large offset drifts and the resulting increase in system jitter, the photon-number information obtained from the arrival-time measurements of the PNR SNSPDs was highly unreliable or completely absent.

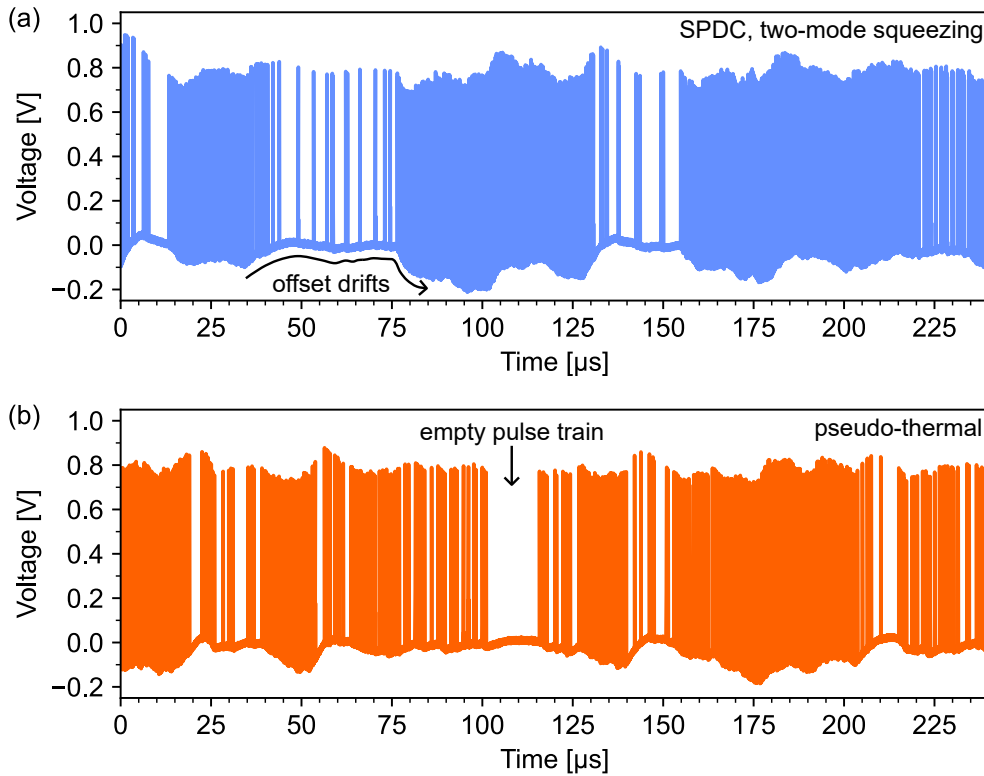


Figure A.1: Oscilloscope traces of the output signal of one spatial channel, for approximately 20 subsequent laser pulses, i.e., 20 of the 128 bin pulse trains. (a) For a two-mode squeezed state from an SPDC source. (b) For a pseudo-thermal state using a rotating ground glass, with a $g^2(0) \approx 1.5$.

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Erklärung

Ich versichere, dass ich die Arbeit ohne fremde Hilfe und ohne Benutzung anderer als der angegebenen Quellen angefertigt habe und dass die Arbeit in gleicher oder ähnlicher Form noch keiner anderen Prüfungsbehörde vorgelegen hat und von dieser als Teil einer Prüfungsleistung angenommen worden ist. Alle Ausführungen, die wörtlich oder sinngemäß übernommen worden sind, sind als solche gekennzeichnet.

Ort, Datum

Unterschrift