Dynamic Decision Making and the Perception of Risk for Low Probability Events: A Literature Review

Behnud Mir Djawadi ♣

University of Paderborn

Abstract

The present study reviews the literature about dynamic decision-making and judgment of low-probability, high-consequence events. The specific features of this situation under risk and uncertainty imply an anomaly: while the single probability of an event with high negative consequences may be small, being exposed to the same situation repeatedly over time, however, makes the one-time occurrence of this event highly probable. Evidence is presented which demonstrates that people violate the principles of rationality in dynamic settings and make their decisions in isolation instead of integrating all future consequences. Moreover, systematic biases and errors in belief formation lead to judgments which do not coincide with those obtained by probability theory and Bayesian updating. The fundamental proposition of this literature review is that policy-makers can benefit from an integrated view of psychological factors and economic (non-)rational choice behavior. A profound understanding of how people think and make decisions concerning repeated risks of low-probability events conceivably leads to effective policies and risk management strategies

JEL Classification: D03, D04, D81

Keywords: Decision Analysis, Dynamic Choice, Risk Perception, Cumulative Risk, Low-Probability Events, Repeated Gambles, Prospect Theory, Risk Aggregation, Risk Segregation, Literature Review

♣ Department of Management, University of Paderborn, Warburger Str. 100, D-33098 Paderborn, Germany. Email: behnud.djawadi@wiwi.uni-paderborn.de, phone: +49-(0)5251-60-2093, fax: +49-(0)5251-60-3550

1. Introduction

> *I once wanted to set up a campground with New Guineans under a dead tree. My companions were appalled. After all, the tree could fall over any time! Oh no, I said, the tree will stand there for another 40 years. But no, they refused to sleep under that tree. In that moment it became clear to me where this noise came from, that I heard every night in the jungle: nothing but falling trees. I worked it out it for myself: if the chances of a tree falling over are one to a thousand, then, provided one sleeps in the jungle every night, the risk of being killed by a falling tree is high. (Excerpt of an interview with Pulitzer-Prize Winner Jared Diamond about life and traditions of indigenous tribes, 2013)*

Similar to the anecdote illustrated in the preceding quotation, many risky situations can be found which seem to be subtle at first sight, yet in fact bear dreadful risks. Consider for example situations with which we are confronted in our daily lives: making many car trips during one year, living near a nuclear power plant or in regions known for natural hazards such as floods and earthquakes, using a vast amount of electronic home appliances, exercising extreme sports such as mountain-climbing, smoking cigarettes over a long period of time or not complying with medication for chronic diseases in therapeutic treatments. Consider further the risk investors face when they frequently invest in stocks with low risks of a total loss, or corrupt officials when they frequently take bribes. All the described situations under risk and uncertainty potentially generate an anomaly: while the single probability of an event with high negative consequences may be small, being exposed to the same situation repeatedly over time however makes the one-time occurrence of this event highly probable. As the negative consequences in form of an accident, a hazard or a medical relapse are either lethal or in form of imprisonment and severe financial damages all crucial for future life, the impact of judgment and choices about these low-probability, high-consequence events deserves special attention (Camerer and Kunreuther, 1989). This review describes how people make judgments and choices concerning these risks by using insights from different fields of studies: decision theory, behavioral economics and psychology. In particular, we specify normative theories how choices in a dynamic situation should be made, and present evidence of how actual choices often violate these theories. In face of this evidence, we present descriptive models to explain the deviation from normative theories. As many decisions are based on beliefs concerning the likelihood of uncertain events, the present study in addition reviews the psychological research on risk perception of hazardous events which occur repeatedly over time to understand the ways in which people think about and respond to risk and its cumulative nature. An analysis of this evidence is important for the following reasons:

people tend to ignore low-probability risks (Kunreuther et al., 2001; Weinstein, 1984) and are limited in their ability to judge probabilities for repeated and compound events (Bar-Hillel, 1973; Cohen and Hansel, 1957). A prime example of this judgment is shown in Slovic et al. (1978). The majority of people were not interested in wearing seat belts when they were informed that the probability of a car accident with fatal consequences was 0.00001 per single car trip. However, when the chance of an accident was framed over a 50 year lifetime of driving rather than the single chance of a car accident of a single trip, 78% of the interviewed people favored a law for wearing seat belts. Thus, the basic proposition of this literature review is that policy-makers can benefit from an integrated view of psychological factors and economic (non-)rational choice behavior so that a better understanding of how people think and make decisions about cumulative risk of extreme events conceivably leads to effective policies and risk management strategies. With these qualifications in mind, the concluding section discusses different themes which are seen as promising areas for future research.

The literature review is organized as follows: the next section provides a description of the economist's model of dynamic choice under risk and uncertainty. Subsequently, the folding back analysis to evaluate dynamic choices is demonstrated and the empirical evidence of non-expected utility models is discussed. Section 3 presents an application of dynamic decision-making with low-probability, high-consequence events in the field of financial economics. Investment decisions facing the repeated risk of a loss over time are viewed from both a normative perspective and from a descriptive perspective, the latter by the use of concepts rooted in behavioral economics. Section 4 describes the research about risk perception which is concerned with psychological and emotional factors that have been shown to affect the judgment of low-probability and compound events. Thereafter, different risk management strategies are highlighted which aim at increasing the capacity of dealing with risks in a rational manner. Section 5 provides suggestions for further research, while section 6 concludes.

2. Normative Theory: Dynamic Decision Making

Dynamic Expected Utility Theory

Conscious decision-making in an uncertain, complex and dynamic world involves choices being made in accordance to a set of criteria by which the outcomes and consequences of each alternative are evaluated (Howard, 1968; Shoemaker, 1993). Expected utility theory (EUT) as the cornerstone of most of the modern economic analyses of behavior in the presence of risk requires the decision-maker in this respect to choose the option which maximizes expected

utility (Green, 1987). Specifically, preferences of the individual over risky prospects or lotteries satisfy a set of logically sound principles of rational choice and ensure that a utility function which assigns values to more and less preferred lotteries has certain numerical properties. Based on these assumptions and characteristics, EUT states that the decision maker chooses between risky lotteries by comparing their expected utility values, i.e., the weighted sums obtained by adding the utility values of outcomes multiplied by their respective probabilities (Mongin, 1997). In its basic form, EUT specifies preferences for choices in static situations which require the individual to irrevocably make all his decisions before any risk is resolved (Machina, 1989). To extend the theory to dynamic situations with decisions that are sequentially made after the resolution of some uncertainty, certain axioms have to be derived from principles of rational dynamic choice: ordering and independence. The ordering axiom states that any function which numerically represents preferences over a sequence of choices must satisfy completeness, reflexiveness and transitivity. Independence requires preferences to obey separability, timing independence[1] and the reduction of compound lotteries (Cubitt, 1996).[2] Separability requires that dynamic choice satisfies the following condition: when lottery A is preferred over lottery B, then, given any sequence of choices made in the past, A should again be preferred over B at any point in the future. Timing independence means that planned behavior equals actual behavior even after reconsidering the plan before carrying out each action. The decision-maker will always stick to his original strategy even if he is given the opportunity to reconsider his choice (Volij, 1994). More specifically, situations in which the decision-maker has to commit himself to all choices before the uncertainty of the first choice is resolved (called normal form situations) are equivalent to situations in which the decision-maker subsequently makes one choice after the uncertainty of the preceding choice is resolved (called extensive form situation). Finally, the principle of the reduction of compound lotteries establishes equivalence between multi-stage and reduced simple gambles, obtained from the former by multiplying out the probabilities of the compound lotteries.

[1] Another term is dynamic consistency.

[2] Hammond (1988) regards ordering and independence as one joint principle which can be derived by the concept of consequentialism. Roughly, consequentialism means that when the next choice situation is reached all choices in the past should be ignored and the original preference function should only be applied to consequences of reachable outcomes in the future.

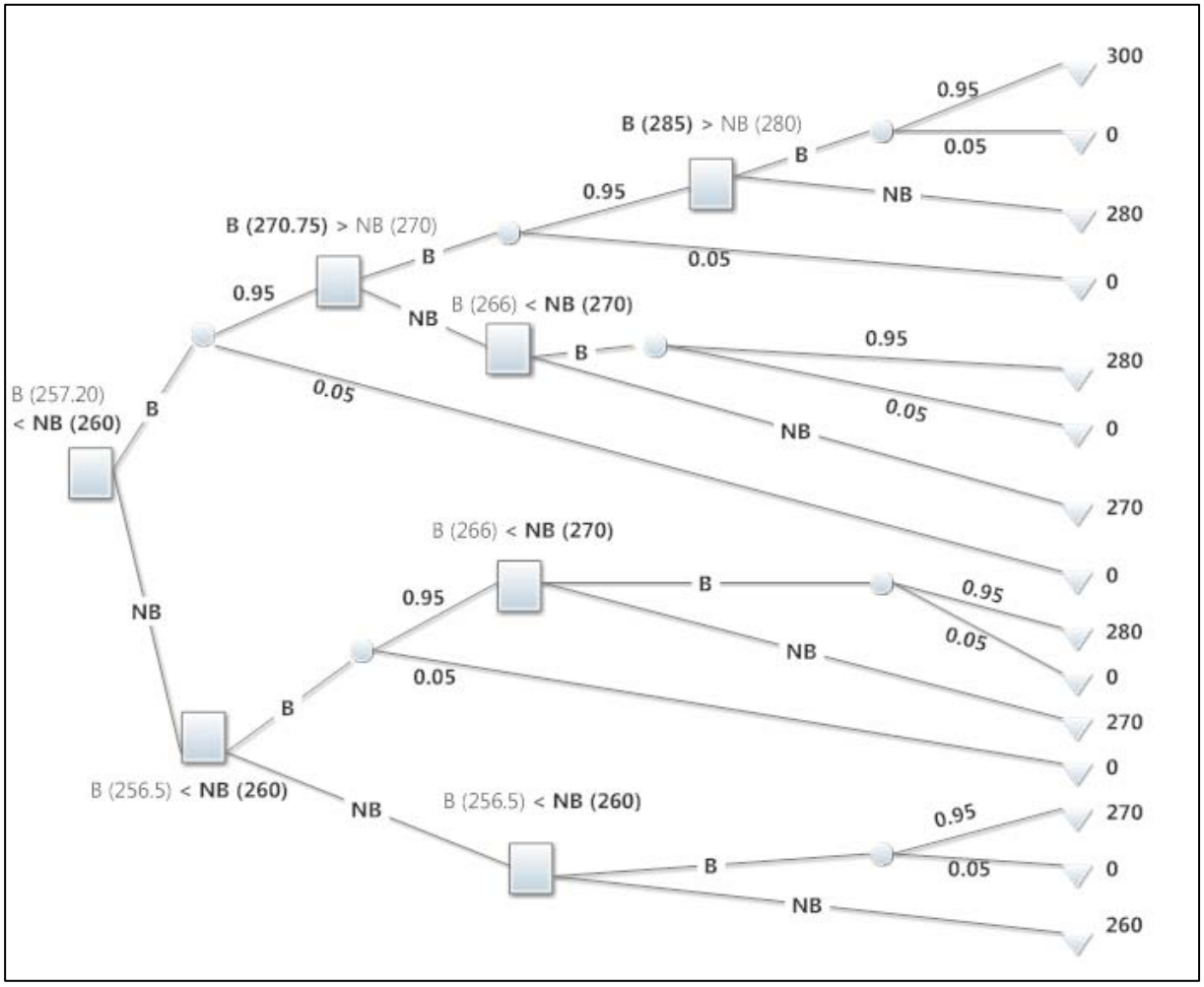
Folding Back Analysis for Evaluating Alternatives in a Decision Tree

If preferences over a sequence of choices satisfy the axioms of ordering and independence then repeated decisions can be evaluated using the folding back procedure[3] (Sarin and Wakker, 1994). Starting from the very end, each choice over lotteries with final outcomes is substituted by the certainty equivalent of that lottery which is mostly preferred by the decision-maker according to some defined criteria. These optimal certainty equivalents themselves become the final outcomes of the lotteries in the second last decision situation, so that the substitution procedure can be applied to choices of the second last decision situation as well. Recursively, this procedure is applied to all preceding decision situations until the first decision situation is reached. The optimal decision sequence can then be derived by choosing the lottery with the optimal underlying certainty equivalent in each decision situation. The procedure is best explained by using an example. Illustrated in Figure 1 is a decision tree which is a common graphical representation of dynamic choice problems in decision theory (Markowitz, 1959; Raiffa, 1968). Let us assume that this decision tree belongs to a public official who is faced with three consecutive decisions and that the decision process does not involve an economically relevant amount of time. Starting from the left end or the root of the tree he can make the decision, indicated by a square, to accept a bribe (B) or not accept a bribe (NB). Accepting a bribe leads him to a chance node (circle) which determines whether the public official will be caught in his deed or not with the associated probabilities.
In the decision tree, nature will move the upper or the lower branch with the displayed probabilities of respectively 0.95 and 0.05. When nature moves the upper branch, then our public official can again decide about accepting or not accepting a bribe, otherwise, he has been detected and reaches a terminal node with a final outcome of zero. In cases of not accepting a bribe there is no probability detection so that the public official moves straight to the next decision. At the terminal nodes to the far right of the tree, indicated by triangles, the final outcomes of each possible combination of the three decisions are displayed. Note that this example represents a situation with repeated decisions of low-probability, high-consequence events. The probability of being detected when taking bribes might be very small in each case. However, since getting caught terminates the career of a public official the economic consequences are very high (outcome of 0). Let us now assume that the criterion by which the public official evaluates a sequence of bribe choices is the maximization of

[3] Other terms are backward induction or dynamic programming.

expected value[4]. Folding back the decision tree requires starting from the terminal nodes and evaluating the lottery associated with the choice of B against the sure outcome of the NB choice.

Figure 1: Example of a decision tree representing the dynamic choice problem of a public official*



* Note: Time in a decision tree flows from left to right. Decisions are denoted by squares, chance nodes by circles. The branches emanating from each decision node represent all of the possible decisions under consideration at that point in time. Branches emanating from each chance node represent a set of mutually exclusive moves of nature. Every final branch, indicated by a triangle, has a numerical value (e.g. representing monetary value) associated with the combinations of decisions and events. Evaluation of the next decision based on the certainty equivalents of risky or sure outcomes in the subsequent choice situation is displayed above each decision node. The optimal decision at that node is in bold script.

The highest expected values as optimal certainty equivalents are then used as outcomes for the preceding decision situation (285 in the far upper branch, 270 in the second upper branch, 270 in the second lower branch, and 260 in the far lower branch), and the calculus for this situation is reapplied until the first decision node is reached. The result of the calculus and the evaluation of the alternatives are displayed above each decision node, respectively. In this

[4] Different criteria can be derived from the shape of the underlying utility function one has in mind which satisfies ordering and independence. The criterion of expected value assumes that the utility function is linear in shape.

example, the optimal decision sequence is always not accepting the bribe because this choice combination results in a higher overall outcome of 260 than accepting three times the bribe which only yields an expected value of 257 units.

Violations of Dynamic Expected Utility and Non-Expected Utility Models

Similar to the research of the basic form of EUT, there is a growing body of empirical evidence demonstrating that individuals' preferences over a sequence of choices violate at least one of the axioms of ordering and independence. For example, the majority of participants violate the principle of separability when they are given a version of the Allais Paradox (1953, 1979) as dynamic choice problem (Machina, 1989). Shafir and Tversky (1992) find similar results using a set of different choice problems such as Prisoner's dilemma, Newcomb's problem and Wason's selection task. In each choice problem, participants prefer a choice x over y per se yet reveal reverse preferences depending on choices in the past and whether or not it is known in what branch of the decision tree the choice of x and y has to be made. Kahneman and Tversky (1979) provide evidence against the principle of the reduction of compound lotteries. In their comparison between a one-stage and an equivalent two-stage compound lottery, participants tend to isolate the uncommon elements of the two gambles by cancelling common first-stage probabilities or common final-stage payoffs. This leads to preferences which are inconsistent with those observed when participants face the equivalent one-stage lottery. In the context of decision strategies for equity of public risks, Keeney and Winkler (1985) show circumstances under which the principle of timing independence is violated. In their setting, ex ante risk equity is not compatible with preferences obtained by von Neumann-Morgenstern utility functions (von Neumann and Morgenstern, 1944) so that the optimal strategies derived from folding back analysis depend on the presentation mode (normal form vs. extensive form representation) of the dynamic choice problem and are not necessarily equivalent. Similarly, LaValle and Wapman (1985) show in the generalized case that even the slightest deviation from von Neumann-Morgenstern utility functions causes a violation of timing independence and produces in normal form a different optimal decision sequence than in extensive form.

Cubitt and Sugden (2001) experimentally test the validity of several principles in dynamic choice problems of low-probability, high consequence events. Their experiment involves a special form of lotteries called accumulator gambles. An accumulator gamble is a multi-stage lottery in which the probability of winning money decreases with the number of rounds at a

constant rate. Each subject plays a number of compulsory rounds of the lottery followed by a number of voluntary rounds, stepwise one at a time. Each subject himself specifies how many voluntary rounds he wants to play. Once the subject loses one round, he drops out of the experiment and all his savings from previous rounds are taken away, so that losing a lottery only one time bears a huge economic damage for the subject. To test the different principles of dynamic rational choice the choice problems are designed in the following way: in the first choice problem the subject states the number of voluntary rounds beforehand and commits to this decision before he starts playing the first compulsory round (choice problem C); in the second choice problem the subject plays the compulsory and then the voluntary rounds one at a time (choice problem P), while in the third choice problem the subject skips the compulsory rounds and directly starts with the number of voluntary rounds as if he had won all the compulsory rounds (choice problem S). If the principle of separability holds, then the number of specified voluntary rounds in choice problems P and S should be the same. If the principle of timing independence holds, then the number of specified voluntary rounds in choice problems C and P should be the same, while the number of specified voluntary rounds in choice problems C and S should be the same if the principle of reduction of compound lotteries holds. Subjects faced exactly one choice problem and were able to decide up to 3 voluntary rounds. The results indicate that the principles of separability and timing independence hold while only the principle of reduction of compound lotteries is violated. However, as soon as only one principle is violated the whole independence axiom fails, so that folding back analysis is not an adequate means for evaluating decisions in dynamic choice problems.

Researchers have responded to this growing body of evidence and developed more general theories which relax some of the principles of the independence axiom. Sarin and Wakker (1998) introduce the concept of sequential consistency, which requires that the decision-maker commits to a family of (non-)expected models (e.g. the multiple priors family (Gilboa and Schmeidler, 1989), rank-dependent family (Quiggin, 1982), or the betweenness family (Fishburn, 1988)) and uses this family of models throughout. Conditions are shown in which folding back analysis is still applicable for the non-expected utility maximizer. Similarly, Nielsen and Jaffray (2006) present a decision model in which the decision-maker selects in any decision tree non-dominated strategies while allowing for non-expected utility preferences. Using an operational approach, their algorithm uses folding back by selecting a set of acceptable strategies from which the decision maker will choose according to a specified ranking. Machina (1989) argues that outcomes in different states of nature can be

complimentary with one another so that risks borne in the past may be relevant for the future. His backtracking procedure denies separability, but incorporates timing independence and the reduction of compound lotteries. Segal (1987, 1990) substitutes the principle of the reduction of compound lotteries by a weaker stochastic dominance axiom for multi-stage gambles. In this respect he integrates the concept of compound dominance with rank-dependant utility theory as weaker form of EUT. This approach denies the reduction of compound lotteries, while asserting separability and timing independence. Karni and Safra (1989, 1990) propose a model of behaviorally consistent choice in which choices are evaluated using generalized expected utility. More specifically, choices at each decision node are assumed to be made by separate "agents" who all restrict themselves to choices which correspond to a subgame perfect equilibrium of the remaining decision tree in extensive form. Hence, this approach violates timing independence, while adopting separability and the reduction of compound lotteries. Cubitt et al. (1998) report an experiment which tests these alternative theoretical models under monetary incentives. Similar to the study of accumulator gambles, the authors design five choice problems 1-5 and derive from observed choice behavior across these problems which principles are violated and thus which models account for the observed choices. Any theory which follows the non-separability strategy implies that choice behavior in the problems 2-5 are equivalent. Any theory that only asserts the violation of timing independence involves equivalent choice behavior in problems 1 and 2, and between 3 and 5, respectively. Finally, any theory that only denies the reduction of compound lottery implies equivalence in choice behavior between the problems 1 to 4. The results indicate that only the principle of timing independence is violated so that alternative models which accept timing independence while rejecting separability or the reduction of compound lotteries fail to describe observed choice behavior. Thus, the results favor the alternative model proposed by Karni and Safra and suggest that choice behavior between normal and equivalent extensive form situations differ. Subsequent experimental research reported by Busemeyer et al. (2000) and Johnson and Busemeyer (2001) has found more evidence of this specific violation in similar contexts. Hey and Pannaccione (2011) argue that reactions to these potential dynamic inconsistencies vary depending on which type of behavior subjects follow. Three types are considered: naïve, resolute and sophisticated behavior. The naïve decision-maker works through time always choosing the best decision as viewed from the present perspective, even though this may lead to actual choices which might differ from planned ones. The concept of resolute choice formalized by Machina (1989) and McClennen (1990) states that the decision-maker acts according to a plan judged best from an ex ante perspective, and intentionally acts

on that resolve when the plan imposes on him ex post to make a choice he does not prefer at that decision point. Lastly, the sophisticated decision-maker obeys the principle of timing independence and acts dynamically consistent. To discriminate between the different types that may exist, the authors implement an experimental design in which subjects are asked to take two sequential decisions concerning the allocation of a given sum of money. The resulting data show that the majority of subjects are resolute, a significant few are sophisticated and rather few are naïve. Similar findings have been reported in an earlier study by Hey and Lotito (2009) using a slightly different dynamic choice problem.

3. Descriptive Theory in Financial Economics

Repeated Gambles and Compound Risk

A prominent application of repeated decisions with low-probability, high-consequence events can be found in the field of financial economics. Essentially, this discipline explores how rational investors would apply decision theory to the problem of investment. The underlying assumption entails that investors care about losses at every step along their investment path. As such, investors face the repeated decision problem that the risk of a severe financial loss is the same in each time period of the investment horizon in which the portfolio of risky assets is held.

Since Markowitz (1952), there has been extensive research on how the factor "time" affects the distribution of risky outcomes and the probability of a loss, as well as on how investors should evaluate the combined risk of multiple choices. However, similar to dynamic decision-making in general, observed investment behavior is often inconsistent with the predictions of normative investment theories (e.g. Merton, 1969; Merton and Samuelson, 1974; Samuelson, 1969; Siegel, 1998). One controversial topic is the role of time diversification in portfolio selections which states that investors with a long time horizon should invest more heavily in risky assets, namely stocks, than investors with a short time horizon. This strategy follows the idea that the longer an investment is held, the less likely it is to produce an overall loss because losses will be, in the long run, cancelled out by gains in expected terms. Hence, to reduce the risk of a loss, the strategy suggests that the decision-maker should turn down a risky gamble or lottery when played once while accepting a repeated play of that gamble over several independent trials. Samuelson (1963) rejected this way of reasoning about repeated gambles as irrational. When asked about preferences of a gamble with positive expected value, a colleague of him (henceforth referred to as SC, as has been commonly the case in subsequent references in the literature) declined the offer of a chance to win $200 if the flip of

a coin yielded heads but lose $100 if the coin yielded tails. However, the colleague asserted that he was willing to accept a series of 100 such bets, claiming that in a hundred tosses of a coin the law of large numbers would make it virtually certain that he would come out ahead. Samuelson uses an induction argument to prove an inconsistency theorem which asserts that no one who wants to maximize expected utility can agree to a sequence of gambles if each of the single gambles is unacceptable at every asset position throughout the relevant range of outcomes (Aloysius, 2007; Tversky and Bar-Hillel, 1983). In this respect, Samuelson argues that multiple plays of a gamble compounds risk rather than reducing it and explains the acceptance of repeated gambles by a fallacious use of the law of large numbers. The intuition behind the law is that above and below average realizations of a random variable tend to cancel out each other, making it more certain that the expected value is realized in the long run (Klos et al., 2005). However, as Samuelson (1963) points out, playing a gamble repeatedly N-times does not lead to an overall outcome of the gamble's expected value multiplied by N. Instead, the probability to end up in a given small interval around the long-run expected value declines with the number of repeated plays (Klos et al., 2005). In this sense, a reduction of risk can only be achieved by subdividing the single gamble into smaller independent gambles or risks, rather than adding up more independent risks by repeating the play of the gamble. Similarly, Bodie (1995), Kritzman (1994) and Kritzman and Rich (2002) show that time actually increases risk. Other than the usual approach of using expected utility frameworks, Bodie uses theories from put and call options in a preference-free framework and measures the riskiness of stocks by the costs of insuring against earning less than the risk-free rate of interest. Derived from the put-call parity theorem, Bodie shows that the costs, and equivalently, the riskiness of stocks increase rather than decrease with the length of the investment horizon. Kritzman (1994) argues that, by time diversification, the growing improbability of a loss is offset by the increasing magnitude of a potential loss. Using different utility functions commonly proposed in the financial literature to express investors' preferences, Kritzman shows that expected utility of risky assets remains constant as a function of investment horizon. Kritzman and Rich (2002) analyze situations in which the one-time occurrence of a loss is already unacceptable and leads to the abrupt termination of the investment. The risk of such occurrences is frequently mismeasured as investors typically measure risk as the probability of a given loss or the amount that can be lost with a given probability only at the end of their investment horizons. The authors introduce two new ways of measuring risk, namely within-horizon losses and continuous value at risk, and

demonstrate through statistical analysis known as "first-passage time probability" that risk increases with time even if investors care only about the probability of a loss.

Decision-Making in the Short and Long Run: Single-Play vs. Multiple-Play Gambles

Samuelson's frequently quoted analysis initiated a stream of research and discussion which continues to present (Aloysius, 2007). The central debate focuses on whether SC's pattern of being risk-averse in the short run by rejecting the single play of a gamble, while at the same time being risk-seeking in the long run by accepting repeated plays of the gamble, can be viewed as a behavioral anomaly. Applied to situations of daily life, people who routinely avoid wearing seat belts should also wear no seatbelt on the next single car trip. People who found it acceptable to live in areas threatened by natural hazards for years should also spend the next day there. Vice versa, if already living one day in that area is not acceptable than also spending the whole life there should be rejected.

Tversky and Bar-Hillel (1983) develop an axiomatic proof to show that the different risk-taking behavior in the short and long run violates basic rationality conditions like dominance and transitivity. However, other researchers argue that the existing proofs and axiomatic analyses to refuse SC's behavior are based on an assumed condition rather than on a normative axiom of EUT. Hellwig (1995), Lippman and Mamer (1988), Nielsen (1985) and Ross (1999) formalize conditions on utility functions for the eventual acceptance of sequences of gambles. One principal condition is that the left tail of the utility function does not decline exponentially as wealth declines - a condition which is satisfied by many perfectly normal expected utility functions. This entails that the benefits of a sufficiently long sequence of gambles with positive expected value overwhelm the risk of incurring losses. Gollier (1996) and Peköz (2002) argue that the acceptance of a sequence of gambles depends on whether the decision-maker has an option to stop the repeated play at any time. Peköz's theoretical work shows that in the case of a stopping option, large sequences of gambles with positive expected value should always be accepted. He proposes that the optimal time to stop the repeated play is when the decision-maker has incurred losses. Gollier (1996) examines the impact of the number of options to replay the gamble on the decision to gamble today. Without any restrictions on the utility function, his theory shows that the willingness of the decision-maker to accept a number of repeated plays of a gamble increases with the number of options offered to him.

A variety of experiments explored different themes that bear on the difference between risk-taking behavior in the short and the long run. For example, several authors challenged the

proposition that the decision of a single play vs. repeated plays of a risky gamble should be treated alike. Luce (1996) points to the difficulty that classical theories do not provide any formula to derive utility of a sequence of gambles form the utility of playing the gamble once. In this respect, SC cannot be judged guilty of inconsistency. According to Lopes (1981; 1996), different criteria operate in short-run decisions as compared to long-run decisions (see also Shoemaker and Hershey, 1996). Whereas she accepts EUT and the notion of expected value in long-run situations, she questions their rationality and applicability to short-run circumstances and unique events. To support her conjectures, three experiments are conducted which demonstrate that preferences of gambles in the short run are not necessarily based on expected utility maximization. Wedell and Böckenholt (1994) show that subjects in the short run focus more on characteristics of single outcomes or their associated odds and probabilities rather than integrating probabilities and outcomes. Keren and Wagenaar (1987) provide further empirical support for the need to distinguish between risk-taking behavior in the short and the long run. Using the choice task by Kahneman and Tversky (1979), the authors observe preferences over single-play gambles which violate EUT. However in repeated conditions, subjects' preferences are congruent with EUT, so that results obtained in the short run cannot be generalized to situations in the long run. The role of expected value in single vs. multi-play decisions has been investigated in several experimental studies as well. Lichtenstein et al. (1969) find that subjects consider expected value of minor importance in the short run. These findings are replicated by Montgomery and Adelbratt (1982) who present evidence of a series of experiments which show that information about expected value has only marginal effects on subjects' choice behavior in a variety of situations except when repeated gambles are allowed. Especially when a sufficient number of gambles were allowed, all subjects were willing to base their choices on expected value, while perceiving this notion as too abstract for single-play circumstances. Joag et al. (1990) find similar results: in their simulated industrial purchasing task it becomes evident that decision-makers combine probabilities and outcomes consistent with the principle of expected value. In contrast, decisions in the short run are not observed following a multiplicative integration pattern. Experimental data provided by Wedell and Böckenholt (1994) and Li (2003) also suggests that in multiple play conditions, subjects' choice behavior can be best described by the principle of maximizing expected value, while in the short run, observed choices do not seem to follow any kind of expectation rule.

Evaluation Procedures of Repeated Gambles: Aggregation and Segregation of Risk

Besides normative approaches based on probability theory and axiomatic logic, there is a growing body of research which draws psychological considerations into theories of decision-making with respect to risk and uncertainty. Termed as behavioral economics (Camerer et al., 2003; Diamond and Vartiainen, 2007), this economic field seeks to increase the explanatory power of economics by providing it with more realistic psychological foundations (Camerer and Loewenstein, 2003). As illustrated above, with regard to standard finance theory as a descriptive enterprise, there is a great deal of evidence which suggests that a variety of behavioral factors appear to be highly consequential for economic outcomes. Consequently, the sub-discipline of behavioral economics known as behavioral finance (Thaler, 1993; 2005; Barberis and Thaler, 2003) catalogs behavioral anomalies, accounts for the conflict between standard benchmark models and the rich supply of contrary data and proposes alternative models based on cognitive, perceptual and social insights of human behavior toward a deeper understanding of decision-making in financial markets and institutions therein. In the following, we will present the behavioral concepts relevant for analyzing how investors think about investment decisions and act over long horizons, besides showing experimental evidence concerning which behavioral principles might cause the systematic departure of decision-making expected from normative theory.

One of the most prominent and influential alternatives of modeling choice under risk and uncertainty is prospect theory (Kahneman and Tversky, 1979) which is not a mathematical generalization of EUT but rather built on observed perceptual and psychological principles of human behavior. Prospect theory accounts for many of the violations of EUT and differs from EUT in five basic ways (see Camerer and Kunreuther, 1989): firstly, decision-makers are assumed to value gains and losses from a reference point rather than final wealth positions. Secondly, the value of a gain and likewise the disvalue of a loss both increase at a diminishing rate. Hence, the value function is concave in the domain of gains and convex in the domain of losses. Thirdly, the value function incorporates the concept of loss aversion whose property is that reductions in wealth, relative to the current reference point, are weighted much more heavily than increases in wealth of the same size. Roughly speaking, losses are weighted about twice as much as gains (Tversky and Kahneman, 1991; Thaler et al., 1997). Fourthly, decision-makers are assumed to edit risks before they make a choice by using procedures which simplify the risks and make them easier to understand. Lastly, decision-makers are assumed to weight probabilities non-linearly by either overestimating or ignoring low-

probability events while underweighting events of all other probabilities.
In the realm of financial decision-making, prospect theory is integrated in the descriptive model of mental accounting (Thaler, 1985; 1999) which is defined as the set of cognitive operations used by individuals and households to organize, evaluate, and keep track of financial activities. This concept has been used in order to explain a wide range of consumption, budgeting, spending and savings behavior (Heath, 1995; Levav and McGraw, 2009; Read et al., 1999; Thaler and Bernatzi, 2004). In the context of investment and asset allocation, the key mental accounting issues concern the aggregation and segregation of risk: an investor who frames decisions narrowly in an isolated manner (segregation of risk) will tend to make short-term choices rather than adopting long-term policies, a result which is only obtained if the investor frames decisions in a broader way by evaluating the overall outcome distribution of a series of repeated gambles (aggregation of risk). Rationality demands that, whenever decision-makers face two or more concurrent decisions, they consider the joint consequences of their choices rather than treat each decision as a separate event. However, as Kahneman and Lovallo (1993) observe, decision-makers are prone to treat problems as unique, neglecting both the statistics of the past and the multiple opportunities of the future. As a consequence, a decision-maker who acts on a case-by-case basis may devote too much attention to the specific features of an individual case with insufficient regard to the predictive validity of these features and to the relevant base rate of outcomes (Redelmeier and Tversky, 1992). The authors illustrate the costs of isolating rather than compounding consecutive decisions by the following set of three independent gambles: one gamble with a 0.5 chance to win $500, and a 0.5 chance to win $0. Two gambles, each with a 0.5 chance to win $250, and a 0.5 chance to win $0. Simple arithmetic yields the compound gamble: A 0.125 chance to win $1,000, a 0.25 chance to win $750, $500 and $250, and a 0.125 chance to win $0.
Assuming that the decision-maker has preferences according to a power utility function which captures the characteristics of risk aversion and risk proportionality commonly observed in previous experiments (Tversky and Kahneman, 1991; Stevens, 1975), then the sum of the computed certainty equivalents of the individual gambles are worth $300 to the decision maker, while the compound gamble yields a certainty equivalent of $433. Hence, the evaluation of segregating and aggregating the risk over joint decisions is of major importance, as it leads to a different basis for the final decision, for example what minimum selling price the decision-maker would be willing to accept if he owned this set of gambles. Read et al. (1999) refer to the way how two or more concurrent decisions are evaluated as choice bracketing and show that this concept is an important determinant of human behavior. Their

review on choice bracketing includes a wide range of decision contexts, revealing that decision-makers often fail to integrate the consequences of all the choices of a sequence and tend to assess the consequences of the choices in isolation. In most cases, this behavior leads to lower utility. Bernatzi and Thaler (1999) argue that SC's behavior is not the result of risk aversion in the short run but due to the segregated evaluation procedure of repeated gambles. As a specific example of segregating risk, they introduce the behavioral concept of myopic loss aversion: myopic loss aversion rests on loss aversion and mental accounting, and refers to the fact that decision-makers tend to be more sensitive towards losses and have the tendency to evaluate outcomes frequently. The effect of myopic loss aversion was most prominently used as an explanation for the equity premium puzzle in finance (Barberis and Huang, 2008; Mehra and Prescott, 1985; Bernatzi and Thaler, 1995; Gneezy and Potters, 1997). Applied to repeated plays of SC gambles, decision-makers with loss-averse utility functions, who evaluate each choice in a series of risky individual decisions myopically in isolation, prefer not to make choices that may lead to losses, even when the expected value of these choices is positive. However, evaluating a series of such gambles jointly yields positive expected utility even under loss aversion. Thus, myopic loss aversion amplifies risk aversion in standard utility models and leads to inconsistent preferences depending on whether decision-makers take a short-run or long-run perspective (Baz et al., 1999). Consistent with myopic loss aversion, Bernatzi and Thaler (1999) find that aversion to short-term losses can be eliminated by providing the subjects with the explicit distribution of potential outcomes. Many more subjects were willing to accept a gamble when it was described in terms of its distribution of outcomes than in terms of N repetitions of the single gamble. Hence, similar to Kahneman and Lovallo (1993), the way information is presented influences the choice of the decision-maker.

Langer and Weber (2001) use gambles of different gain/loss probabilities and magnitudes to investigate how a portfolio of risky choices should be presented to make it more attractive for the decision-maker. Assuming that the decision-maker has preferences according to a more general value function of prospect theory which expresses loss aversion and diminishing sensitivity, their formal analysis demonstrates that repeated plays of SC gambles and gambles with a high loss probability yield higher utility if evaluated in aggregation, so that presenting the gambles in aggregated rather than in segregated form should be more attractive to the decision-maker. The opposite is true for gambles with low-probability and high-negative consequence events. Here, segregating the consequences of the repeated choices leads to higher overall utility, so that the decision-maker should be more willing to accept the repeated

play of the gambles when they are presented in segregated rather than in aggregated form. Consistent with their theoretical findings, the authors provide experimental evidence that subjects are more willing to accept the repeated play of SC gambles and gambles with a high probability of a loss when the overall outcome distribution of the sequence is explicitly displayed. Subjects are less willing to accept the repeated play of the gamble with low-probability and high-negative consequences if presented in aggregated form.

Even if both evaluation strategies of aggregating and segregating risk theoretically lead to the same distribution of outcomes, decision-makers perceive the choice situation differently. Redelmeier and Tversky (1992) give evidence in the context of SC gambles. If subjects are shown the description of the single SC lottery to be repeated five times, their willingness to accept the lottery is lower than if the subjects are shown the distribution of final outcomes of the five-fold SC lottery. This framing effect violates the notion of description invariance which states that preferences over gambles do not depend on the way they are presented. In this respect, the observed behavior rejects the ability of the decision-maker to reduce compound gambles to single ones and thus violates the principle of the reduction of compound gambles (see for a discussion of the isolation effect: Camerer, 1995; Kahneman and Tversky, 1979). Similar results are obtained in the experiment by Keren (1991) which shows that gambles presented in aggregated form are more attractive to the decision-maker because the risk involved in the gamble becomes more transparent compared to the repeated mode presentation of the gamble.

4. Risk Perception of Low Probability Events

Many decisions under risk and uncertainty are based on beliefs concerning the likelihood of uncertain events. Traditionally, the standard principle to judge the risk of uncertain events originates from mathematical probability theory and Bayes's rule for updating probabilities in the face of new evidence. However, as will be shown, these concepts are an inadequate description of the way humans estimate probabilities and judge the risk of events, especially in situations with repeated low-probability, high-consequence events. Instead, cognitive biases retrieved from heuristic rules and errors in belief formation lead to judgments which do not coincide with those obtained by probability theory and Bayesian updating (Selten 1998; Simon, 1972). This problem becomes even more serious, as also experts' judgments appear to be prone to some of the biases commonly attributed to the general public (Fischhoff et al., 1982; Tversky and Kahneman, 1971). Hence, for developing effective policies and risk management strategies, it is of major importance to understand the bounds of human

reasoning with regards to repeated low-probability events people may encounter in life. Whereas technological experts and risk analysts build upon scientific work and quantitative data to estimate the occurrence of these events, the majority of lay people lacks this experience and rather relies on intuitive judgments, typically called risk perception. Studies of risk perception examine the judgments people make when they are asked to characterize and evaluate hazardous activities and technologies (Slovic, 1987). Based mostly on surveys and laboratory experiments, a selection of these studies are reviewed to gain insights on how individuals perceive the risk of hazardous events and on the factors which account for possible anomalies when judging the likelihood of repeated low-probability, high-consequence events. Thereafter, different risk management strategies are described which aim at increasing the capacity of dealing with hazardous risks in a rational manner.

(Mis-) Perception of Hazardous Events and Cumulative Risk as Result of Biases in Judgment

Slovic et al. (1980) ask lay people to "consider the risk of dying" for each of 30 presented hazards including nuclear power, motor vehicles, smoking, surgery, electric power and home appliances. Using the psychometric paradigm (Brown and Green, 1980), the authors correlate the respondents' mean ratings with risk characteristics across different hazards. Risks are perceived to be higher if the activity is perceived to be involuntary, catastrophic, not personally controllable, inequitable in the distribution of its risk and benefits, unfamiliar, and highly complex (see for additional summaries: Covello, 1983; Slovic et al., 1985). In contrast, hazards which are continuous and which many people are exposed to, like smoking, motor vehicles and home appliances, are perceived to be of little concern. However, risk experts estimate these events to be much more risky than perceived by the general public. One explanation of this deviation is that people judge the probability or frequency of an event by the ease with which relevant instances can be retrieved from memory (Lichtenstein et al., 1978). This biased judgment called availability heuristic (Tversky and Kahneman, 1973) is heavily observed in studies concerning judged frequency of lethal events. In studies by Lichtenstein et al. (1978) and Combs and Slovic (1979), it becomes evident that risk perceptions of events depend on the individual's last experience with the specific hazard and the coverage of the hazard's consequences in the news media. People tend to overestimate the likelihood if the specific event has recently been experienced personally or if the event is easier to imagine and distinct. While dramatic events such as accidents, cancer, homicide, or multiple-death catastrophes, which tend to be publicized disproportionately, are overestimated, frequencies of "quiet killers" such as diabetes, stroke and heart disease are

highly underestimated. However, these are the most occurring events in daily life. Field surveys by Burton and Kates (1964) and Burton et al. (1978) have also indicated that people misjudge the risk of low-probability events like natural hazards, such as natural floods, earthquakes, hurricanes and drought. Estimates become only accurate if the hazard's last major occurrence was experienced recently and its impact on one's livelihood is large in magnitude. Weinstein (1984) provides evidence in line with past studies (Weinstein 1980; 1982) that low-probability events are especially underestimated if subjects perceive the situation under risk to be personally controllable. This anomaly can be explained by an optimism bias which implies an attitude of "it can't happen to me". For example, Svenson (1981) finds that almost 90% of drivers felt they were better than average because of their past driving records. The fact that only half of them can be better than the median was simply disregarded. Optimism effects are also observed in later studies with respondents from Sweden and the U.S. (Svenson et al., 1985). In some cases, the optimism bias expresses itself in an illusion of control (Langer, 1975; Langer et al, 1975; Koehler et al., 1994; Charness and Gneezy, 2010). An illusion of control is defined as an expectancy of a personal success which is inappropriately higher than the objective probability would warrant, like expecting a "6" with a higher probability than 1/6 if the individual himself throws the die. Especially with regard to repeated events, the cognitive task of integrating multiple risks into a reasonable overall judgment seems to be a significant challenge for the individual decision-maker. In an early work by Cohen and Hansel (1957) studying children aged 10 years and older, subjects underestimate the value of probability of a multiple additive choice. In situations in which all multiple rounds have to be won to receive a prize, the overall winning probability is overestimated compared to the prescribed way of multiplying the single probabilities of winning each round. Similar behavioral patterns can be observed with adolescents. In an experimental study by Bar-Hillel (1973), subjects have to choose between a single gamble and a compound gamble made out of a series of single gambles. In the compound version, all the single gambles have to be won to receive the prize, so that the overall winning probability is obtained by simply multiplying the winning probability of each gamble in the series. Indicated by the observed preferences about the single and compound gambles, the majority of subjects overestimates the overall winning probability of the compound gamble and therefore prefers to play a series of gambles instead of the single gamble with higher expected value. It seems that subjects only look at the single probability of winning one gamble of the series and fail to integrate the probabilities of all future gambles. Similarly, Slovic (1969) examines whether the attractiveness of a gamble can be increased by dividing its structure

into a series of single gambles. He finds that the gamble in compound form is much more preferred to the equivalent one-shot gamble because in compound form the higher probability of winning one gamble in a series catches the attention of the decision-maker more than the overall probability of the one-shot gamble. Svenson (1984) asks subjects about the cumulative risk of dying over different periods of time. Subjects are presented a set of hypothetical persons, each characterized by different levels of risk for different periods of time. One person was characterized by risk exposures of 18 weeks at 2.3 deaths per thousand persons per year of exposure, the second person with 28 weeks at 2.5 deaths per thousand persons per year, and the third person with 6 weeks of 16 deaths per thousand persons per year. Svenson finds that most of the subjects did not properly include the exposure time in their calculus so that the cumulative risk was often misjudged. Slovic (2000) investigates how smokers perceive the cumulative risk of getting lung cancer as a consequence of long-term smoking. He finds that especially young people underestimate the risks of long-term smoking. Specifically, these people see no health risk from smoking the next cigarette or even from smoking regularly for the first few years which runs counter to the risk estimate of experienced smokers who have already smoked for a long period of time. Failures of integrating compound events are also observed if subjects are given concrete numbers about the single probabilities of each event. Abbink et al. (2002) and Djawadi and Fahr (2013) provide experimental evidence of misjudgments of compound events in the domain of corruption. Situations in which the public official frequently engages in illegal actions where small amounts of bribes are involved and the detection rate is rather low are examined. In their experimental bribery game, Abbink et al. (2002) state that subjects tend to significantly underestimate the overall probability of disqualification. Subjects are asked to give an estimation of the overall probability of disqualification for three different detection probabilities and three different time horizons. For almost each of these nine detection probability and time horizon combinations, subjects underestimate the overall probability of disqualification, in some cases severely by more than 300 percent. Djawadi and Fahr (2013) find similar results in their study about the impact of risk perception on corrupt behavior. Since getting caught terminates the career of a public official, what matters is the overall probability of getting caught once, which is much higher if the public official habitually engages in corruption. High rates of corruption can already be found in very early periods of the public official's career which strongly suggest a systematic underestimation of the overall probability of being detected once. Slovic et al. (1978) provide evidence in the case of seat belt usage. The majority of people were not interested in wearing seat belts when they were

informed that the probability of a car accident with fatal consequences was 0.00001 per single car trip. Hence, these subjects per se were not aware of the fact that being exposed to this single probability over a long period of time raises the one-time occurrence of a car accident to a much higher dimension. For example, if the single probability of a car accident was extrapolated to 50 years of driving with 800 car trips per year, the overall probability of never being involved in a car accident boils down to only 67%. When presented in this way, the majority of subjects changed their opinion and favored the use of seat belts. Related surveys and laboratory experiments on insurance behavior observe that people simply ignore low-probability risks. Slovic et al. (1977), Kunreuther et al. (1978) and Laury et al. (2009) observe that people only buy insurance if the risk of a fatality exceeds a special threshold, even when the insurance is subsidized. In a laboratory experiment on purchasing insurance, McClelland et al. (1993) find that many subjects bid zero for coverage implying that the low probability of a loss is viewed as completely negligible. Stone et al. (2004) observe that people are not willing to pay considerably more to reduce the risk of some adverse event if the likelihood is displayed in tiny numbers. Overestimation of compound events can also evolve as a result of a judgment bias called conjunction fallacy (Tversky and Kahneman, 1983). For instance, people often judge event probabilities by their plausibility. Adding details to the description of events make them appear to be more likely because such details add plausibility (Camerer and Kunreuther, 1989). In their famous "Linda" example, Tversky and Kahneman (1983) give their subjects a description of a hypothetical person called Linda and ask them whether the event that Linda is a bank teller or the compound event that Linda is a bank teller and active in the feminist movement is more probable. Because of the description of Linda, subjects perceived the compound event to be more probable, although, according to probability theory, the conjunction of two independent events can never be more probable than each of the single events alone.

Besides empirical evidence about cognitive errors that people make when they have to judge the likelihood of uncertain events, there is a growing body of research that views risk as feelings based on instincts and intuitive reactions to danger. While the role of emotions have been examined in decision theory and identified as an important determinant of human behavior and decision-making (Damasio, 1994; Loewenstein, 2000; Loewenstein and Lerner, 2003; Zajonc, 1980), recent developments in the evaluation and perception of risk started to recognize the importance of emotions for explaining anomalies in people's judgment about uncertain events (Loewenstein et al., 2001; Slovic et al., 2005; Slovic et al. 2007). Studies show that people judge a risk not only by how they think, but also by how they feel about it.

Slovic et al. (2007) use the term "affect heuristic" to characterize reliance on such feelings. Affect expresses good or bad feelings experienced with the situation or event and can been seen as more subtle feelings than visceral emotions, such as fear and anger. Finucane (2000) shows that information about benefits of a nuclear power plant changes people's perception of the risk of a nuclear meltdown and vice versa. Rottenstreich and Hsee (2001) demonstrate how feelings of affect lead to insensitivity of probabilities. If the potential outcome of a gamble is emotionally powerful for the individual, the gamble's attractiveness or unattractiveness is relatively insensitive to changes in probability as great as from 0.99 to 0.01. Loewenstein et al. (2001) develop the concept of "risk-as-feelings" and show that emotional responses to risky situations, including feelings such as fear and anger, often diverge from cognitive evaluations and have a different and sometimes greater impact on risk-taking behavior. For example, Loewenstein et al. (2001) observe that one's images and feelings towards winning a lottery are likely to be the same whether the chances of winning are 1 in 10 million or 1 in 10000. Responses to uncertain events appear to have an all-or-nothing characteristic that is quite sensitive to the possibility rather than the probability of strong negative or positive outcomes which imply that small probabilities carry too much weight. The authors argue that the relationship between probabilities and emotions can help to explain the paradox of playing a lottery while simultaneously buying insurance. Overweighting the small probabilities of winning the lottery might stem from a disproportionate pleasurable anticipation, while the fear of losses leads to an increased perceived likelihood of improbable adverse events which are desired to be reduced by buying insurance.

Risk Management Strategies

Until today, the vast empirical evidence about risk perception and judgment of low-probability, high-consequence events has been a fruitful source of knowledge for researchers to develop risk management strategies which account for psychological factors of human reasoning and which address some of the biases people fall prey to. Research has been especially centered on improving the communication of risk (Kuhnreuther, 2001; Morgan, 2002). Given that people do have marked difficulties interpreting low probabilities, different ways of presenting information about statistics were developed and empirically validated. As has been done in the work by Slovic et al. (1978), stretching the time frame, over which the probability of a low-probability event can occur, raises the awareness of risk: herein it is the overall risk of being involved in a heavy car accident with fatal consequences when not

wearing a seatbelt at some point of the lifetime. Similar evidence is obtained by Weinstein et al. (1996) in the context of earthquake protection. If a company considers to protect itself against a possible earthquake over the 25-year life of its plant, managers are more likely to be aware of the possible risk if they are told that the chances of an earthquake in 25 years is 1 to 5 instead of 1 to 100 in any given year. Stone et al. (1994) find that people are more willing to pay for risk reduction of an extreme adverse event if risks are presented in relative ratios rather than in absolute numbers. Hence, the awareness of risk increases if the individual is told that buying insurance will cut the risk of suffering a loss in half compared of saying that insurance will reduce the risk from 0.000006 probability to 0.000003 probability. Other studies have shown that response scales and numerical formats can affect the awareness of low-probability and compound events. Slovic et al. (2000) present case summaries of patients hospitalized with mental disorder to forensic psychologists and psychiatrists and ask them to judge the likelihood that the patient would harm someone within six months after discharge from the hospital. The authors find that respondents pay much more attention to this event if the risk is communicated by probability ratios of 10 in 1000 or 100 in 10000 instead of 1 in 100. Thus, multiplying the nominator and denominator by a constant factor and making the nominator more salient leads to a higher perceived likelihood of the event. Similar results are obtained in studies by Epstein (1994) and Yamagishi (1994a; 1994b; 1997). Numerous researchers have observed that judgment biases were reduced or eliminated when the likelihood of an event was assessed by frequencies rather than probabilities (Gigerenzer, 1994; 1996a; 1996b; Gigerenzer and Hoffrage, 1995; Hoffrage and Gigerenzer, 1998). Slovic et al. (2000) examine whether the risk assessment of low-probability events depends on the format displayed as probabilities such as 1% or as equivalent frequency of 1 in 100. As also observed by Koehler (2001) who studies the judgment of a DNA match in criminal trials, the authors find that risks which are presented as frequencies increase the perception of the event's likelihood. While this finding is consistent with the literature that thinking with frequencies tends to be different from probability judgment (Gigerenzer, 1991; Kahneman and Tversky, 1982), the question cannot be answered if this perception is more accurate than judgment from probability (Slovic et al., 2000). Several studies examined how visual displays can enhance the communication of risk (Bettman et al., 1986; Lipkus and Hollands, 1999). Examples of visual displays that have been used to communicate risk include the risk ladder, stick, human, and Chernoff faces, histograms, line graphs and pie charts (Tufte, 1990; Hollands and Spence, 1992; Kosslyn, 1989; Spence and Lewandowsky, 1991). Dots and related formats have been used to communicate low probabilities of different diseases. In an

experimental study, Kaplan et al. (1985) inform their subjects of one of three probabilities – 1/1000, 1/10000, 1/100000 - of having negative reaction to a flu vaccine. Half of their respondents were provided with a visual display of dots representing one of the three risks. For instance, those who reviewed a risk of 1/1000 saw one tenth of a page covered with 1000 dots. The authors find that subjects were more sensibilized towards risk and were more likely to get vaccinated when the information was displayed in visual form. Galesic et al. (2009) investigate whether icon arrays as substitutes for numerical values increase the accuracy of understanding medical risks. The authors present their participants three medical scenarios and communicate absolute and relative risk reductions by numbers and by graphical forms of icon arrays. It is shown that icon arrays increased the accuracy of understanding relative risk reductions, especially if participants suffer from low numeracy. Kunreuther et al. (2001) argue that insensitivities to low probabilities can be explained by the notion of evaluability introduced by Hsee and colleagues (Hsee, 1996; Hsee et al., 1999). According to the evaluability heuristic, if people have difficulties to evaluate how good/bad or high/low an attribute is, then a comparison with another related attribute which is known and meaningful to the individual will help to appreciate the value and characteristics of the attribute. Kunreuther et al. (2001) find that individuals are more able to evaluate low probabilities when they are given specific comparison information. In their study, subjects have to judge the safety of a chemical facility. It appears that no attention is paid to the low probabilities irrespective of displaying them in terms of probabilities or insurance premiums. However, when subjects are presented a scenario in which they can compare the fatality rates of several chemical plants and are additionally given risk information of other domains, such as fatality rates of car accidents, then the sensibility towards low probabilities increases and changes of low probabilities are more distinguished.

5. Avenues for Future Research

The existing literature about dynamic decision-making under uncertainty lacks an appropriate, incentive-compatible design to capture the characteristics of (i) repeated actions, (ii) high winning probabilities of single events (so that through repetition the probability of a loss significantly increases), and, (iii) an abrupt termination of the play associated with high losses. As presented in section 3, the standard lottery design involves a precommitted choice between a single gamble and a series of gambles. The stakes of the gambles are mostly of hypothetical nature, payments to subjects are in most cases fixed beforehand and not contingent on the individual choices, and/or, among the series of gambles, only one single

gamble is chosen at random and played according to the single probabilities (an incentive system known as random-lottery incentive system). However, this might not represent the situation one is interested in. Studies about violations of dynamic EUT indicate that precommitted choices differ from choices that are made one at a time after the resolution of some uncertainty. Another weakness is that feelings such as fear, thrill, joy and pleasure which appear to be an important determinant of risk-taking behavior are very likely not present, if all decisions have to be made beforehand, and thus, can never be studied. Even if people experience these feelings in the final stage of an experiment, they may fail to anticipate these feelings when making their decisions (Cubitt and Sugden, 2001).

Further, the random lottery incentive scheme might not elicit true preferences. Holt (1986) argues that the validity of this experimental design relies on the assumption that individual's preferences obey the principle of the reduction of compound lotteries. However, as shown in studies by Cubitt and Sugden (2001), Kahneman and Tversky (1979) and Starmer and Sugden (1991), observed behavior is not consistent with the reduction principle. Lastly, standard lottery designs cannot create feelings of losses as only one lottery is played so that subjects cannot generate any savings, before these may be taken away from them if they lose the lottery. Yet, the accumulator gamble developed by Cubitt and Sugden (2001) seems to be a promising alternative lottery design, as it addresses much of the weaknesses mentioned above. However, little is known about its empirical validity as the design has only been used to study violations of rationality principles.

As empirically shown, the experience of positive or negative affect may lead to changes in probability assessments which might help to explain the violation of timing independence reported in section 2. This divergence is probably even more pronounced if, prior to the next choice, far unexpected outcomes are realized which trigger even higher emotional states. However, evidence about the consequences for future behavior is missing.

As described in Section 3, people are observed to bracket consecutive choices in segregation rather than in aggregation. With exception of Langer and Weber (2001) who only asked about the attractiveness of gambles, there is no empirical study which investigates how repeated choices of low-probability, high-loss lotteries are actually evaluated. The strategy of segregating the risk, for example, would lead to severe underestimation of the overall risk. Pursuing this research would give valuable insights on whether myopic loss aversion is even more pronounced in these settings or whether other behavioral concepts affect the way individuals evaluate these consecutive choices.

It is not completely clear whether subjects lack the ability per se to employ folding back analysis for the evaluation of dynamic decisions or whether they have to be assisted towards this way of thinking. In most studies, only the outcomes of a lottery are presented to the subjects. However, the way how lotteries are presented appears to have an impact on behavior. Therefore, it would be interesting to examine whether choice behavior is more in line with the predictions of folding back, when subjects are given for example a graphical display of a decision tree. So far, it is not known how subjects will behave if they see the tree with many zeros on the branches, as in situations of low-probability, high-consequence events only some choice combinations lead to an overall positive outcome. This research can be extended by the use of different numerical formats such as probabilities or frequencies in combination with different ratio scales or graphical displays.

The risk management strategy of communicating the overall risk has been empirically shown to increase the awareness of the one-time occurrence of a low-probability event. Referring to the example illustrated in Figure 1, it might appear puzzling that a public official is worse off in expected terms if he accepts the bribes rather than doing nothing, despite a small detection rate of 5%. In fact, the overall probability that the public official will not be caught at least once when he accepts the bribes over three consecutive decisions is only 85.7%, and decreases rapidly to 59.8% over 10 decisions and below 30% over 25 decisions. If the risk is communicated in these terms and framed over a substantial number of future consecutive decisions then the awareness of the risk of getting caught only once should be incredibly raised. However, more research about the impact of communicating the overall risk on choice behavior is needed to give a satisfying answer to this hypothesis.

6. Conclusion

The present study reviews the literature about dynamic decision-making and judgment of low-probability, high-consequence events and offers insights gained from research of different disciplines such as decision theory, behavioral economics and psychology. Dynamic situations with low-probability, high-consequence events pose significant challenges for the individual decision-maker: low probabilities are difficult to weight and the link between choices today and consequences tomorrow is difficult to see. Economic decisions in dynamic situations are found to be not consistent with theories of dynamic choice under risk and uncertainty, or, applied to the field of financial economics, with classical portfolio theory. Instead, choices are made in isolation without integrating future consequences. In most cases, evaluating each choice one by one leads to an underestimation of the cumulative risk.

However, what matters is the cumulative risk, as the one-time occurrence of the low-probability event already creates severe damages from which one may never be able to recover in future life. The development of an appropriate experimental design is needed to empirically investigate the determinants of this choice behavior. Based on these findings, ways to evaluate compound events in a more rational manner are seen as a further promising field for future research.

People's judgment about the overall risk of low-probability, high-consequence events such as the meltdown of a nuclear plant, the one-time occurrence of natural hazards or other lethal events, often suffers from cognitive biases and errors in belief formation. Ignoring probabilities and heuristics of availability, plausibility or optimism systematically leads to a deviation from judgment based on probability theory.

Accounting for these shortcomings in human judgment and behavior is of major importance in developing effective policies and risk management strategies. For example, without these considerations, individuals will not use insurance to protect themselves against rare, large losses if their attention is only devoted to dealing with likely events. Lay investors will still end up with portfolios that are riskier than their time-independent risk tolerance would allow. Regarding possible institutions in the domain of corruption, increasing the detection rate will not be an effective governmental measure to fight corruption if public officials do not respond to changes of low detection probabilities of getting caught.

Strategies to cope with these biases in judgment involve different methods of communicating risk information. These methods include compounding probabilities over time, using frequencies instead of probabilities and using ratios instead of absolute numbers. Studies show that communicating the risk in terms of these new formats increases the awareness of the overall risk of the situation and mitigates the bias of ignoring low probabilities. Yet more promising research is needed to clarify the role of risk communication on risk-taking behavior under monetary incentive conditions, i.e. which communication format affects behavior the most or the least.

The notion of risk as feelings started to open a new and promising field of study. If we again refer to the introductory quotation by the anthropologists Jared Diamond, indigenous tribes base their risk assessment solely on instincts and experience rather than on analytics and logic. However, their risk perception of impending threats is not necessarily of inferior accurateness. Future research is needed to study the interplay between choices and risk as feelings, finding arguments for or against this notion as a further determinant of observed behavior.

As has been indicated throughout the present study, research on dynamic decision-making of low-probability, high-consequence events benefits from perspectives of decision theory and empirical investigation of behavioral economics and psychology. Without this integrative approach, knowledge of the determinants of judgment and choice behavior in these situations under risk and uncertainty will be incomplete.

References

Abbink, K. Irlenbusch, B., and Renner, E. (2002): An Experimental Bribery Game. Journal of Law, Economics and Organization, Vol. 18: 428-454.

Allais, M. (1953): Le comportement de l`homme rationel devant le risque, critique des postulates et axiomes de l`ecole Americaine. In: Econometrica, Vol. 21: 503-546.

Allais, M. (1979): The so-called Allais paradox and rational decisions under uncertainty. In: M. Allais and O. Hagen (eds.), The expected utility hypotheses and the Allais paradox. Dordrecht: Reidel.

Aloysius, J.A. (2007): Decision making in the short and long run: Repeated gambles and rationality. In: British Journal of Mathematical and Statistical Psychology, Vol. 60: 61-69.

Bar-Hillel, M. (1973): On the Subjective Probability of Compound Events. In: Organizational Behavior and Human Performance, Vol. 9: 396-406.

Barberis, N., and Huang, M. (2008): The Loss Aversion/Narrow Framing Approach to the Equity Premium Puzzle. In: R. Mehra, (ed.): Handbook of the Equity Risk Premium, Elsevier Science, North-Holland.

Barberis, N., and Thaler, R.H. (2003): A Survey of Behavioral Finance. In: G.M. Constantinides, M. Harris, and R.Stulz (eds.), Handbook of the Economics and Finance, North-Holland: Elsevier Science Ltd.

Baz, J., Briys, E., Bronnenberg, B.J., Cohen, M., Kast, R., Viala, P., Wathieu, L., Weber, M., and Wertenbroch, K. (1999): Risk Perception in the Short Run and in the Long Run. In: Marketing Letters, Vol. 10: 267-283.

Bernatzi, S., and Thaler, R.H. (1995): Myopic Loss Aversion and the Equity Premium Puzzle. In: The Quarterly Journal of Economics, Vol. 110: 73-92.

Bernatzi, S., and Thaler, R.H. (1999): Risk Aversion or Myopia? Choices in Repeated Gambles and Retirement Investments. In: Management Science, Vol. 45: 364-381.

Bettman, J.R., Payne, J.W., and Staelin, R. (1986): Cognitive Considerations in Designing Effective Labels for Presenting Risk Information. In: Journal of Public Policy & Marketing, Vol. 5: 1-28.

Bodie, Z. (1995): On the Risk of Stocks in the Long Run. In: Financial Analysts Journal, Vol. 51: 18-22.

Brown, R.A., and Green, C.H. (1980): Precepts of safety assessments. In: Journal of the Operational Research Society, Vol. 11: 563-571.

Burton, I., and Kates, R.W. (1964): The Perception of Natural Hazards in Resource Management. In: Natural Resources Journal , Vol. 3: 412-441.

Burton, I., Kates, R.W., and White, G.F. (1978): The environment as hazard. New York: Oxford University Press.

Busemeyer, J.R., Weg, E., Barkan, R., Li, X., and Ma, Z. (2000): Dynamic and consequential consistency of choices between paths of decision trees. In: Journal of Experimental Psychology: General, Vol. 129: 530-545.

Camerer, C.F. (1995): Individual Decision Making. In: J. H. Kagel and A.E. Roth (eds.), Handbook of Experimental Economics, Princeton: Princeton University Press.

Camerer, C.F., and Kunreuther, H. (1989): Decision Processes for Low Probability Events: Policy Implications. In: Journal of Policy Analysis and Management, Vol. 8: 565-592.

Camerer, C.F., and Loewenstein, G. (2003): Behavioral Economics: Past, Present, Future. In: C.F. Camerer, G. Loewenstein and M. Rabin (eds.), Advances in Behavioral Economics, Princeton: Russel Sage Foundations.

Camerer, C.F., Loewenstein, G., and Rabin, M. (eds.) (2003): Advances in Behavioral Economics, Princeton: Russel Sage Foundations.

Charness, G., and Gneezy, U. (2010): Portfolio Choice and Risk Attitudes: An Experiment. In: Economic Inquiry, Vol. 48: 133-146.

Cohen, J., and Hansel, C.E.M. (1957): The Nature of Decisions in Gambling - Equivalence of Single and Compound Subjective Probabilities. In: Acta Psychologica, Vol. 13: 357-370.

Combs, B., and Slovic, P. (1979): Newspaper Coverage of Causes of Death. In: Journalism & Mass Communication Quarterly, Vol. 56:837-849.

Covello, V.T. (1983): The Perception of Technological Risks: A Literature Review. In: Technological Forecasting and Social Change, Vol. 23: 285-297.

Cubitt, R.P. (1996): Rational Choice and Expected Utility Theory. In: Oxford Economic Journals, Vol. 48: 1-19.

Cubitt, R.P., and Sugden, R. (2001): Dynamic Decision-Making Under Uncertainty: An Experimental Investigation of Choices between Accumulator Gambles. In: The Journal of Risk and Uncertainty, Vol. 22: 103-128.

Cubitt, R.P., Starmer, C., and Sugden, R. (1998): Dynamic Choice and the Common Ration Effect: An Experimental Investigation. In: The Economic Journal, Vol. 108: 1362-1380.

Epstein, S. (1994): Integration of the cognitive and the psychodynamic unconscious. In: American Psychologist, Vol. 49: 709-724.

Finucane, M.L., Alhakami, A., Slovic, P., and Johnson, S.M. (2000): The affect heuristic in judgments of risks and benefits. In: Journal of Behavioral Decision Making, Vol. 13: 1-17.

Fishburn, P.C. (1988): Nonlinear Preference and Utility Theory. Baltimore: John Hopkins University Press.

Fischhoff, B., Slovic, P., and Lichtenstein, S. (1982): Lay Foibles and Expert Fables in Judgements about Risk. In: The American Statistician, Vol. 36: 240-255.

Damasio, A.R. (1994): Descartes´error: Emotion, reason, and the human brain. New York: Putnam.

Diamond, P., and Vartiainen, H. (eds.) (2007): Behavioral Economics and Its Applications, Princeton: Princeton University Press.

Djawadi, B.M. and Fahr, R. (2013): The Impact of Risk Perception and Risk Attitudes on Corrupt Behavior: Evidence from a Petty Corruption Experiment. IZA Discussion Paper No. 7383, Bonn, Germany.

Galesic, M., Garcia-Retamero, R., and Gigerenzer, G. (2009): Using Icon Arrays to Communicate Medial Risks: Overcoming Low Numeracy. In: Health Psychology, Vol. 28: 210-216.

Gigerenzer, G. (1991): How to make cognitive illusions disappear: Beyond “heuristics and biases.” In: W. Stroebe, and M. Hewstone (eds.), European Review of Social Psychology, Chichester: Wiley.

Gigerenzer, G. (1994): Why the distinction between single-event probabilities and frequencies is relevant for psychology and vice versa. In: G. Wright, and P. Ayton (eds.), Subjective probability, New York: Wiley.

Gigerenzer, G. (1996a): On narrow norms and vague heuristics: A rebuttal to Kahneman and Tversky. In: Psychological Review, Vol. 103: 592-596.

Gigerenzer, G. (1996b): The psychology of good judgment: Frequency formats and simple algorithms. In: Journal of Medical Decision Making, Vol. 16: 273-280.

Gigerenzer, G., and Hoffrage, U. (1995): How to improve Bayesian reasoning with instruction: Frequency formats. In: Psychological Review, Vol. 102: 684-704.

Gilboa, I., and Schmeidler, D. (1989): Maxmin Expected Utility Theory with a Non-Unique Prior. In: Journal of Mathematical Economics, Vol. 18: 141-153.

Gneezy, U., and Potters, J. (1997): An Experiment on Risk Taking and Evaluation Periods. In: The Quarterly Journal of Economics, Vol. 112: 631-645.

Gollier, C. (1996): Repeated Optional Gambles and Risk Aversion. In: Management Science, Vol. 42: 1525-1530.

Green, J. (1987): "Making Book Against Oneself," The Independence Axiom, and Nonlinear Utility Theory. In: The Quarterly Journal of Economics, Vol. 102: 785-796.

Hammond, P. J. (1988): Consequentialist foundations for expected utility. In: Theory and Decisions, Vol. 25: 25-78.

Heath, C. (1995): Escalation and De-Escalation of Commitment in Response to Sunk Costs: The Role of Budgeting in Mental Accounting. In: Organizational Behavior and Human Decision Processes, Vol. 62: 38-54.

Hellwig, M. F. (1995): The Assessment of Large Compounds of Independent Gambles. In: Journal of Economic Theory, Vol. 67: 299-326.

Hey, J.D., and Lotito, G. (2009): Naïve, resolute or sophisticated? A study of dynamic decision making. In: Journal of Risk and Uncertainty, Vol. 38: 1-25.

Hey, J.D., and Panaccione, L. (2011): Dynamic decision making: what do people do? In: Journal of Risk and Uncertainty, Vol. 42: 85-123.

Hollands, J.G., and Spence, I. (1992): Judgments of change and proportion in graphical perception. In: Human Factors, Vol. 34: 313-334.

Hoffrage, U., and Gigerenzer, G. (1998): Using natural frequencies to improve diagnostic influences. In: Academic Medicine, Vol. 73: 538-540.

Holt, C.A. (1986): Preference Reversals and the Independence Axiom. In: The American Economic Review, Vol. 76: 508-515.

Howard, R.A. (1968): The Foundations of Decision Analysis. In: IEEE Transactions on Systems Science and Cybernetics, Vol. SSC-4: 211-219.

Hsee, C.K. (1996): The Evaluability Hypothesis: An Explanation for Preference Reversals between Joint and Separate Evaluations of Alternatives. In: Organizational Behavior and Human Decision Processes, Vol. 67: 247-257.

Hsee, C.K., Loewenstein, G.F., Blount, S., and Bazerman, M.H. (1999): Preference reversals between joint and separate evaluations of options: A review and theoretical analysis. In: Psychological Bulletin, Vol. 125: 576-590.

Joag, S.G., Mowen, J.C., and Gentry, J.W. (1990): Risk Perception in a Simulated Industrial Purchasing Task: The Effects of Single versus Multi-Play Decisions. In: Journal of Behavioral Decision Making, Vol. 3: 91-108.

Johnson, J.G., and Busemeyer, J.R. (2001): Multiple-stage decision-making: the effect of planning horizon length on dynamic consistency. In: Theory and Decision, Vol. 51: 217-246.

Kahneman, D., and Lovallo, D. (1993): Timid Choices and Bold Forecasts: A Cognitive Perspective on Risk Taking. In: Management Science, Vol. 39: 17-31.

Kahneman, D., and Tversky, A. (1979): Prospect Theory: An Analysis of Decision under Risk. In: Econometrica, Vol. 47: 263-292.

Kahneman, D., and Tversky, A. (1982): Variants of Uncertainty. In: D. Kahneman, P. Slovic, and A. Tversky (eds.): Judgment under uncertainty: Heuristics and biases, Cambridge: Cambridge University Press.

Kaplan, R.M., Hammel, B., and Schimmel, L.E.: Patient information processing and decision to accept treatment. In: Journal of Social Behavior & Personality, Vol. 1: 113-120.

Karni, E., and Safra, Z. (1989): Ascending bid auctions with behaviorally consistent bidders. In: Annals of Operations Research, Vol. 19: 435-446.

Karni, E., and Safra, Z. (1990): Behaviorally Consistent Optimal Stopping Rules. In: Journal of Economic Theory, Vol. 51: 391-402.

Keeney, R.L., and Winkler, R.L. (1985): Evaluating Decision Strategies for Equity of Public Risks. In: Operations Research, Vol. 33: 955-970.

Keren, G. (1991): Additional Tests of Utility Theory under Unique and Repeated Conditions. In: Journal of Behavioral Decision Making, Vol. 4: 297-304.

Keren, G., and Wagenaar, W.A. (1987): Violation of Utility Theory in Unique and Repeated Gambles. In: Journal of Experimental Psychology, Vol. 13: 387-391.

Klos, A., Weber, E.U., and Weber, M. (2005): Investment Decisions and Time Horizon: Risk Perception and Risk Behavior in Repeated Gambles. In: Management Science, Vol. 51: 1777-1790.

Koehler, J.J. (2001): When are people persuaded by DNA match statistics? In: Law and Human Behavior, Vol. 25: 493-513.

Koehler, J.J., Gibbs, B.J., and Hogarth, R.M. (1994): Shattering the Illusion of Control: Multi-shot versus Single-shot Gambles. In: Journal of Behavioral Decision Making, Vol. 7: 183-191.

Kosslyn, S.M. (1989): Understanding charts and graphs. In: Applied Cognitive Psychology, Vol. 3: 185-226.

Kritzman, M. (1994): What Practitioners Need to Know about Time Diversification. In: Financial Analysts Journal, Vol. 50: 14-18.

Kritzman, M., and Rich, D. (2002): The Mismeasurement of Risk. In: Financial Analysts Journal, Vol. 58: 91-99.

Kunreuther, H. (2002): Risk Analysis and Risk Management in an Uncertain World. In: Risk Analysis, Vol. 22: 655-664.

Kunreuther, H., Ginsberg, R., Miller, L., Sagi, P., Slovic, P., Borkan, B., and Katz, N. (1978): Disaster Insurance Protection: Public Policy Lessons. New York: Wiley.

Kunreuther, H., Novemsky, N., and Kahneman, D. (2001): Making Low Probabilities Useful. In: The Journal of Risk and Uncertainty, Vol. 23: 103-120.

Langer, E.J. (1975): The Illusion of Control. In: Journal of Personality and Societal Psychology, Vol. 32: 311-328.

Langer, E.J., and Roth, J. (1975): Heads I Win, Tails It's Chance: The Illusion of Control as a Function of the Sequence of Outcomes in a Purely Chance Task. In: Journal of Personality and Societal Psychology, Vol. 32: 951-955.

Langer, T., and Weber, M. (2001): Prospect Theory, Mental Accounting, and Differences in Aggregated and Segregated Evaluation of Lottery Portfolios. In: Management Science, Vol. 47: 716-733.

Laury, S.K., McInnes, M.M., and Swarthout, J.T. (2009): Insurance decisions for low-probability losses. In: Journal of Risk and Uncertainty, Vol. 39: 17-44.

Lipkus, I.M., and Hollands, J.G. (1999): The Visual Communication of Risk. In: Journal of National Cancer institute Monographs, Vol. 25: 149-163.

LaValle, I.H., and Wapman, K.R. (1986): Rolling Back Trees Requires the Independence Axiom. In: Management Science, Vol. 32: 382-385.

Levav, J., and McGraw, A.P. (2009): Emotional Accounting: How Feelings About Money Influence Consumer Choice. In: Journal of Markting Research, Vol. 46: 66-80.

Li, S. (2003): The role of Expected Value illustrated in decision-making under risk: single-play vs. multiple-play. In: Journal of Risk Research, Vol. 6: 113-124.

Lichtenstein, S., Slovic, P., and Zink, D. (1969): Effect of instruction in expected value on optimality of gambling decisions‘. In: Journal of Experimental Psychology, Vol. 79: 46-55.

Lichtenstein, S., Slovic, P., Fischhoff, B., Layman, M., and Combs, B. (1978): Judged Frequency of Lethal Events. In: Journal of Experimental Psychology: Human Learning and Memory, Vol. 4: 551-578.

Lipkus, I.M., and Hollands, J.G. (1999): The Visual Communication of Risk. In: Journal of the National Cancer Institute Monographs, Vol. 25: 149-163.

Lippman, S.A., and Mamer, J.W. (1988): When many wrongs make a right. In: Probability in the Engineering and Information Sciences, Vol. 2: 115-127.

Loewenstein, G.F. (2000): Emotions in Economic Theory and Economic Behavior. In: American Economic Review, Vol. 90: 426-432.

Loewenstein, C.F. and Lerner, J.S. (2003): The Role of Affect in Decision Making. In: R.J. Davidson, K.R. Scherer, and H.H. Goldsmith (eds), Handbook of Affective Sciences, New York: Oxford University Press.

Loewenstein, G.F., Weber, E.U., Hsee, C.K., and Welch, E.S. (2001): Risk as feelings. In: Psychological Bulletin, Vol. 127: 267-286.

Lopes, L.L. (1981): Notes, Comments, and New Findings – Decision Making in the Short Run. In: Journal of Experimental Psychology, Vol. 7: 377-385.

Lopes, L.L. (1996): When Time Is of the Essence: Averaging, Aspiration, and the Short Run. In: Organizational Behavior and Human Decision Processes, Vol. 65: 179-189.

Luce, D. (1996): Commentary on aspects of Lola Lopes’ paper. In: Organizational Behavior and Human Decision Processes, Vol. 65: 190-193.

Machina, M.J. (1989): Dynamic Consistency and Non-Expected Utility Models of Choice Under Uncertainty. In: Journal of Economic Literature, Vol. 27: 1622-1668.

Markowitz, H. M. (1952): Portfolio selection. In: Journal of Finance, Vol. 7: 77-91.

Markowitz, H. M. (1959): Portfolio selection: Efficient diversification of investments. New Haven: Yale University Press.

McClelland, G., Schulze, W., and Coursey, D. (1993): Insurance for Low-Probability Hazards: A bimodal Response to Unlikely Events. In: Journal of Risk and Uncertainty, Vol. 7: 95-116.

McClennen, E.F. (1990): Rationality and dynamic choice: Foundational Explorations. Cambridge: Cambridge University Press.

Mehra, R., and Prescott, E.C. (1985): The Equity Premium Puzzle. In: Journal of Monetary Economics, Vol. 15: 145-161.

Merton, R. (1969): Lifetime Portfolio Selection: The Continuous-Time Case. In: Review of Economics and Statistics, Vol. 51: 247-257.

Merton, R., and Samuelson, P. (1974): Fallacy of the Log-Normal Approximation to Optimal Portfolio Decision-Making over many Periods. In: Journal of Financial Economics, Vol. 1: 67-94.

Mongin, P. (1997): Expected Utility Theory. In: J.Davis, W.Hands and U. Maki (eds.), Handbook of Economic Methodology, London: Edward Elgar.

Montgomery, H., and Adelbratt, T. (1982): Gambling Decisions and Information about Expected Value. In: Organizational Behavior and Human Performance, Vol. 29: 39-57.

Morgan, M.G., Fischhoff, B., Bostrom, A, and Atman, C.J. (2002): Risk communication: A mental models approach. New York: Cambridge University Press.

Nielsen, L.T. (1985): Attractive Compounds of Unattractive Investments and Gambles. In: Scandinavian Journal of Economics, Vol. 87: 463-473.

Nielsen, T.D., and Jaffray, J-Y. (2006): Dynamic decision making without expected utility: An operational approach. In: European Journal of Operational Research, Vol. 169: 226-246.

Peköz, E.A. (2002): Samuelson's Fallacy of Large Numbers and Optimal Stopping. In: The Journal of Risk and Insurance, Vol. 69: 1-7.

Quiggin, J. (1982): A theory of anticipated utility. In: Journal of Economic Behavior and Organization, Vol. 3: 323-343.

Raiffa, H. (1968): Decision Analysis. London: Addison-Wesley.

Read, D., Loewenstein, G., and Rabin, M. (1999): Choice Bracketing. In: Journal of Risk and Uncertainty, Vol. 19: 171-197.

Redelmeier, D.A., and Tversky, A. (1992): On the Framing of Multiple Prospects. In: Psychological Science, Vol. 3: 191-193.

Ross, S.A. (1999): Adding Risks: Samuelson's Fallacy of Large Numbers Revisited. In: Journal of Financial and Quantitative Analysis, Vol. 34: 323-339.

Rottenstreich, Y., and Hsee, C.K. (2001): Money, kisses and electric shocks: On the affective psychology of probability weighting. In: Psychological Science, Vol. 12: 158-190.

Samuelson, P. (1963): Risk and Uncertainty: A Fallacy of Large Numbers. In: Scientia, Vol. 98: 108-113.

Samuelson, P. (1969): Lifetime Portfolio Selection by Dynamic Stochastic Programming. In: Review of Economics and Statistics, Vol. 51: 239-246.

Sarin, R., and Wakker, P. (1994): Folding Back in Decision Tree Analysis. In: Management Science, Vol. 40: 625-628.

Sarin, R., and Wakker, P.P. (1998): Dynamic Choice and NonExpected Utility. In: Journal of Risk and Uncertainty, Vol. 17: 87-119.

Segal, U. (1987): The Ellsberg paradox and risk aversion: an anticipated utility approach. In: International Economic Review, Vol. 28: 175-202.

Segal, U. (1990): Two-Stage lotteries without the independence axiom. In: Econometrica, Vol. 58: 349-377.

Selten, R. (1998): Features of experimentally observed bounded rationality. In: European Economic Review, Vol. 42: 413-436.

Shafir, E., and Tversky, A. (1992): Thinking through Uncertainty: Nonconsequential Reasoning and Choice. In: Cognitive Psychology, Vol. 24: 449-474.

Shoemaker, P.J.H. (1993): Determinants of Risk-Taking: Behavioral and Economic Views. In: Journal of Risk and Uncertainty, Vol. 6: 49-73.

Shoemaker, P.J.H., and Hershey, J. (1996): Maximizing your chances of winning. In: Organizational Behavior and Human Decision Processes, Vol. 65: 194-200.

Siegel, J.J. (1998): Stocks for the Long Run: The Definitive Guide to Financial Market Returns and Long-Term Investment Strategies. New York: McGraw-Hill.

Simon, H.A. (1972): Theories of Bounded Rationality. In: C.B. McGuire and Roy Radner (eds.), Decision and Organization, North-Holland Publishing Company.

Slovic, P. (1969): Manipulating the Attractiveness of a Gamble without Changing its Expected Value. In: Journal of Experimental Psychology, Vol. 79: 139-145.

Slovic, P. (1987): Perception of Risk. In: Science, Vol. 236: 280-285.

Slovic, P. (2000): What Does it Mean to Know a Cumulative Risk? Adolescents' Perceptions of Short-term and Long-term Consequences of Smoking. In: Journal of Behavioral Decision Making, Vol. 13: 259-266.

Slovic, P., Fischhoff, B., and Lichtenstein, S. (1978a): Accident Probabilities and Seat Belt Usage: A Psychological Perspective. In: Accident Analysis and Prevention, Vol. 10: 281-285.

Slovic, P., Fischhoff, B., and Lichtenstein, S. (1985): Characterizing Perceived Risk. In: R.W. Kates, C.Hohenemse, and J.X. Kasperson (eds.), Perilous progress: Managing the hazards of technology, Boulder, CO: Westview.

Slovic, P., Fischhoff, B., and Lichtenstein, S. (1980): Facts and fears: understanding perceived risk. In: R. Schwing, and W.A. Albers, Jr. (eds.), Societal risk assessment: how safe is safe enough?, New York: Plenum.

Slovic, P., Fischhoff, B., and Lichtenstein, S., Corrigan, B., and Combs, B. (1977): Preference for Insurance Against Probable Small Losses: Insurance Implications. In: Journal of Risk and Insurance, Vol. 44: 237-258.

Slovic, P., Monahan, J., and MacGregor, D.G. (2000): Violence risk assessment and risk communication. The effects of using actual cases, providing instruction, and employing probability versus frequency formats. In: Law and Human Behavior, Vol. 24: 271-296.

Slovic, P., Finucane, M.L., Finucane, M.L., and MacGregor, D.G. (2007): The affect heuristic. In: European Journal of Operational Research, Vol. 177: 1333-1352.

Slovic, P., Peters, E., Finucane, M.L., and MacGregor, D.G. (2005): Affect, Risk, and Decision Making. In: Health Psychology, Vol. 24: 35-40.

Spence, I, and Lewandowsky, S. (1991): Displaying proportions and percentages. In: Applied Cognitive Psychology, Vol. 5: 61-77.

Starmer, C., and Sugden, R. (1991): Does the Random-Lottery Incentive System Elicit True Preferences? An Experimental Investigation. In: American Economic Review, Vol. 81: 971-978.

Stevens,S.S. (1975): Psychophysics. New York: John Wiley and Sons.

Stone, E., Yates, F., and Parker, A. (1994): Risk Communication: Absolute versus Relative Expression of Low-Probability Risks. In: Organizational Behavior and Human Decision Processes, Vol. 60: 387-408.

Svenson, O. (1981): Are We All Less Risky and More Skillful When Our Fellow Drivers Are? In: Acta Psychologica, Vol. 47: 143-148.

Svenson, O. (1984): Cognitive Processes in Judging Cumulative Risk over Different Periods of Time. In: Organizational Behavior and Human Performance, Vol. 33: 22-41.

Svenson, O., Fischhoff, B., and MacGregor, D. (1985): Perceived Driving Safety and Seatbelt Usage. In: Accident Analysis & Prevention, Vol. 17: 119-133.

Thaler, R.H. (1985): Mental Accouting and Consumer Choice. In: Marketing Science, Vol. 4: 199-214.

Thaler, R.H. (eds) (1993): Advances in Behavioral Finance. New York: Russell Sage Foundation.

Thaler, R.H. (1999): Mental Accouting matters. In: Journal of Behavioral Decision Making, Vol. 12: 183-206.

Thaler, R.H. (eds) (2005): Advances in Behavioral Finance Vol.II. New York: Russell Sage Foundation.

Thaler, R.H., and Bernatzi, S. (2004): Save More Tomorrow^{TM}: Using Behavioral Economics to Increase Employee Saving. In: Journal of Political Economy, Vol. 112: 164-187.

Thaler, R.H., Tversky, A., Kahneman, D., and Schwartz, A. (1997): The Effect of Myopia and Loss Aversion on Risk Taking: An Experimental Test. In: Quarterly Journal of Economics, Vol. 112: 647-661.

Tufte, E.R. (1990): Envisioning information. Cheshire: Graphics Press.

Tversky, A., and Bar-Hillel, M. (1983): Risk: The Long and the Short. In: Journal of Experimental Psychology, Vol. 9: 713-717.

Tversky, A., and Kahneman, D. (1971): Belief in the Law of Small Numbers. In: Psychological Bulletin, Vol. 76: 105-110.

Tversky, A., and Kahneman, D. (1973): Availability: A heuristic for judging frequency and probability. In: Cognitive Psychology, Vol. 5: 207-232.

Tversky, A., and Kahneman, D. (1983): Exceptional vs. Intuitive Reasoning: The Conjunction Fallacy in Probability Judgment. In: Psychological Review, Vol. 90: 293-315.

Tversky, A., and Kahneman, D. (1991): Loss Aversion in Riskless Choice: A Reference-Dependent Model. In: Quarterly Journal of Economics, Vol. 106: 1039-1061.

Volij, O. (1994): Dynamic consistency, consequentialism and reduction of compound lotteries. In: Economic Letters, Vol. 46: 121-129.

von Neumann, J., and Morgenstern, O. (1944): Theory of games and economic behavior. Princeton: Princeton Unversity Press.

Wedell, D.H., and Böckenholt, U. (1994): Contemplating single versus multiple encounters of a risky prospect. In: The American Journal of Psychology, Vol. 107: 499-518.

Weinstein, N.D. (1980): Unrealistic optimism about future life events. In: Journal of Personality and Social Psychology, Vol. 39: 806-820.

Weinstein, N.D. (1982): Unrealistic optimism about susceptibility to health problems. In: Journal of Behavioral Medicine, Vol. 5: 441-460.

Weinstein, N.D. (1984): Why It Won't Happen to Me: Perceptions of Risk Factors and Susceptibility. In: Health Psychology, Vol. 3: 431-457.

Weinstein, N., Kolb, K., and Goldstein, B. (1996): Using time intervals between expected events to communicate risk magnitudes. In: Risk Analysis, Vol. 16: 305-308.

Yamagishi, K. (1994a): Consistencies and biases in risk perception: I. Anchoring process and response-range effect. In: Perceptual and Motor Skills, Vol. 79: 651-656.

Yamagishi, K. (1994b): Consistencies and biases in risk perception: II. Individual subjects' performance. In: Perceptual and Motor Skills, Vol. 79: 659-663.

Yamagishi, K. (1997): When a 12.86% mortality is more dangerous than 24.14%: Implications for risk communication. In: Applied Cognitive Psychology, Vol. 11: 495-506.

Zajonc, R.B. (1980): Feeling and Thinking: Preferences Need No Inference. In: American Psychologist, Vol. 35: 151-175.