Essays on the Theory of Industrial Organization: Credence Goods, Vertical Relations, and Product Bundling

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Chapter 4

Bundling in a Distribution Channel with Retail Competition

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Abstract

We analyze the incentives for retail bundling and the welfare effects of retail bundling in a decentralized distribution channel with two retailers and two monopolistic manufacturers. One manufacturer exclusively sells his good to one retailer, whereas the other manufacturer sells his good to both retailers. Thus, one retailer is a monopolist for one product but competes with the other retailer in the second product market. The two-product retailer has the option to bundle his goods or to sell them separately. We find that bundling aggravates the double marginalization problem for the bundling retailer. Nevertheless, when the retailers compete in prices, bundling can be more profitable than separate selling for the retailer as bundling softens the retail competition and leads to an extension of his monopoly power. The ultimate outcome depends on the marginal costs of the manufacturers. Given retail quantity competition, however, bundling is in no case the retailer's best strategy. Furthermore, we show that profitable bundling is welfare harming because it reduces consumer and producer surplus in the equilibrium.

JEL classification: L11; L13; L41; L81; M31

Keywords: Double marginalization; Leverage theory; Oligopoly; Retail bundling

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4.1 Introduction

The practice of selling two or more distinct goods as a bundle is a strategy widely used by downstream firms. Many electronic retailers sell packages containing separate items, for example, packages consisting of a video game console and video games or of a personal computer and an operating system. Another example is a Subscription-Video-on-Demand (SVoD) streaming service like Netflix that offers subscriptions to its whole content at a monthly fee. Netflix plays a pure bundling strategy as it supplies its content solely as a bundle (Bhargava, 2012). In this paper, we study downstream retail bundling in a distribution channel. Our focus lies on the analysis of the incentives for retail bundling and of the consequences of retail bundling. We consider various aspects of retail bundling and bundling in general in our analysis. We next expose these aspects.

Downstream firms often bundle goods that are produced by powerful upstream firms. Many electronic retailers buy goods from large market players like Microsoft or Sony. Streaming services distribute movies and television shows produced by major film studios or powerful television production companies like Paramount Pictures or Warner Bros. Television, respectively. The presence of upstream market power in a distribution channel might lead to double marginalization (DM) and thus to inefficiencies in the channel. Such inefficiencies may harm upstream firms, downstream firms and consumers (Spengler, 1950). A downstream firm's decision to bundle might aggravate a DM problem which, in turn, could affect the downstream firm's incentives to bundle in the first place (see, for example, Bhargava, 2012).

Downstream firms usually include products in their bundles which are also supplied by their competitors. For instance, a SVoD streaming service often offers content like certain TV series or movies that are also available at other streaming services. By purchasing and distributing such products, a downstream firm enlarges its product range but might induce intrabrand competition. An upstream firm benefits from selling to several downstream firms as this leads to a larger output but it could induce interbrand competition (Dobson and Waterson, 1996). Streaming services sometimes include exclusive goods in their bundles which can be content that producers supply solely via one streaming service. A further example for such an exclusive vertical agreement is the *Amazon Exclusives* program. Producers involved in this program are not allowed to sell their products through any online marketplace other than Amazon. The reason for establishing this kind of exclusive relationship from a downstream firm's perspective may be to limit intrabrand competition, whereas an upstream firm's motivation to distribute through only one retailer may be to restrict interbrand competition (Dobson and Waterson, 1996; Moner-Colonques et al., 2004). In sum, the component goods of a downstream firm's bundle could be products for which a downstream firm is a monopolist or products for which the downstream firm competes in an oligopolistic market. A firm's bundling strategy might affect the intensity of oligopolistic competition and that this effect can have a pivotal influence on a firm's bundling decision (see e.g. Carbajo et al., 1990).

Moreover, bundling has raised anti-competitive concerns. It is widely regarded a type of price discrimination since it can reduce the heterogeneity in consumers' reservation prices (see e.g. Stigler, 1963; Adams and Yellen, 1976). In addition, bundling is a strategy that could be used by a multi-product firm with monopoly power in one market but facing competition in a second market to leverage its monopoly position of the first market into the second market. This means that the firm could potentially leverage its competitors out of the market and thereby create another monopoly. Analyzing such interrelations between bundling, monopoly power and competition is a typical feature of the *leverage theory* of bundling (see e.g. Carbajo et al., 1990; Whinston, 1990; Martin, 1999; Carlton and Waldman, 2002; Egli, 2007; Spector, 2007; Peitz, 2008; Mantovani, 2013; Chung et al., 2013; Vamosiu, 2018).

In order to study downstream bundling, we develop a theoretical model that fits our motivational example(s). We consider a distribution channel with two downstream retailers and two monopolistic upstream manufacturers. One manufacturer sells his good to both retailers. The other manufacturer sells his good only to one retailer due to an exclusivity agreement that makes the according retailer a monopolist in one product market. Both retailers supply the second product and thus compete in a duopoly for this product. The two-product retailer has the option to purely bundle the two goods or to solely supply them as separate products. One goal of our research is to investigate the incentives of the two-product retailer to bundle. We additionally ask how bundling by the retailer affects equilibrium market results such as other firms' strategies or profits. We also analyze the welfare effects of (profitable) retail bundling. A crucial element of our research is to examine the roles of retail competition and upstream market power regarding the bundling incentives and the consequences of retail bundling. We analyze our research goals under retail price and retail quantity competition.

The purpose of our study and the structure of our retail market relate to the leverage theory such as presentend in Carbajo et al. (1990) and Martin (1999). Both papers analyze the bundling incentives for a two-product firm and the welfare effects of bundling in a non-vertical industry, where the two-product firm is a monopolist for one good and competes with a second firm regarding the other good. Carbajo et al. (1990) assume an inelastic demand and find that given price competition in the duopoly, bundling is always more profitable than separate selling for the two-product firm. Under quantity competition, however, selling the products independently could be more beneficial for the two-product firm, depending on the marginal costs of production. Furthermore, Carbajo et al. (1990) highlight that bundling always reduces consumer surplus and has an ambiguous effect on the total welfare in both modes of competition. Martin (1999) regards quantity competition, a linear demand structure and differentiated goods. He illustrates that bundling may change or create substitution relationships between goods. In Martin's model, bundling always leads to an increase in profit for the bundling firm and a reduction in social welfare. We use the market set-up of Carbajo et al. (1990) and Martin (1999) and extend it to a vertical structure while also considering the linear demand structure of Martin (1999) in our framework.

The analysis of downstream retail bundling in a decentralized distribution channel is a relatively new research topic. The most prominent papers dealing with this topic are Rennhoff and Serfes (2009); Bhargava (2012); Chakravarty et al. (2013); Girju et al. (2013); Cao et al. (2015); Giri et al. (2017); Ma and Mallik (2017); Cao et al. (2019). Bhargava (2012) and Cao et al. (2015) are closely related to our work. Bhargava (2012) studies retail bundling in a market with a two-product downstream retailer and two monopolistic manufacturers. He shows that if goods are valued independently of each other, the manufacturers tend to overprice their goods under retail bundling. As a consequence, the profits of all firms are reduced by bundling in comparison to separate selling. Cao et al. (2015) evaluate retail bundling in a distribution channel with a two-product retailer, one monopolistic wholesale market and one perfectly competitive wholesale market. They demonstrate a pivotal role of the monopolistic manufacturer's marginal production cost regarding a retailer's bundling decision. When this cost is low, retail bundling worsens the double marginalization problem in the channel in most cases and thus reduces the channel profit. If the manufacturer's marginal cost is moderately high, bundling weakens the DM problem, resulting in bundling being profitable and raising the channel profit.

Bhargava (2012) and Cao et al. (2015) and all other above-mentioned papers focusing on retail bundling do not examine or even consider retail competition with the exception of Rennhoff and Serfes (2009). The consideration of retail competition, however, is a crucial element in our bundling analysis. This is the major contrast of our model to these papers but further differences in terms of modeling and assumptions can be found. Rennhoff and Serfes (2009) investigate downstream bundling in an upstream-downstream market with two downstream and two upstream firms, where they regard downstream and upstream competition. Both downstream firms sell two products which they purchase from the two upstream firms. Rennhoff and Serfes find that imposing a regulation that forces downstream firms to unbundle could benefit consumers when the firms play pure bundling in the unregulated equilibrium. Additionally, they show that the upstream firms can influence the bundling incentives of the downstream firms with their pricing strategies. The model of Rennhoff and Serfes is not connected to the leverage theory, which poses one main difference to our work, but their framework also differs significantly from ours in terms of the theoretical model. To sum up, we are, to the best of our knowledge, the first to evaluate downstream retail bundling in a distribution channel, in which the retail market and thus the purpose of the study relates to the leverage theory.

Our main results can be summarized as follows. Bundling always aggravates the double marginalization problem for the two-product retailer. Nevertheless, bundling might be more profitable for the retailer than selling the products separately when the retailers compete in prices. This is because bundling greatly reduces the intensity of retail competition and leads to an extension of market power for the bundling retailer under price competition. Yet, bundling is profitable only when the manufacturer's marginal cost of the good sold exclusively by the retailer is sufficiently high and the marginal cost of the good sold in the duopoly is sufficiently low. Under retail quantity competition, however, bundling is never profitable for the two-product retailer. Interestingly, this is the case even when bundling has similar positive effects on the two-product retailer's profit and market position as with price competition.

We find a negative influence of the presence of upstream market power on the profitablity of retail bundling. In both modes of retail competition, the retailer's bundling incentives are stronger in a centralized channel, where the full market power in the channel is on the retailers' site, than in the decentralized channel with upstream market power. Also, when we keep the decentralized structure and the two goods are manufactured by a multi-product monopolist, then the bundling incentives are qualitatively the same as in the centralized channel. Hence, it is a combination of vertical externalities and horizontal externalities between upstream producers that lowers the profitability of retail bundling in our framework, which is in line with Bhargava (2012).

We analyze the welfare effects only for the retail price competition setting as only there bundling is the retailer's equilibrium strategy. We observe that profitable bundling reduces consumer and producer surplus. The latter observation is rather surprising because in the bundling equilibrium, only the manufacturer distributing to both retailers is harmed by bundling. Consumer surplus falls because the prices of both retailers are raised by bundling. We consequently identify retail bundling as a welfare harming strategy.

The rest of the paper is organized as follows. Section 4.2 introduces the basic model. We analyze retail bundling under retail price competition in Section 4.3 and under retail quantity competition in Section 4.4. Section 4.5 concludes.

4.2 The Basic Model

We consider a distribution channel with two downstream retailers, R_A and R_B, two upstream manufacturers, M_1 and M_2, and a continuum of final consumers. Manufacturer M_1 produces good 1 at constant marginal cost $k_1 > 0$ and manufacturer M_2 produces good 2 at constant marginal cost $k_2 > 0$. We presume that good 1 and good 2 are independent in demand from the consumer perspective. The retailers supply the goods that they purchase from the producers without any product transformation as final goods since this is normally the case for retailers. Therefore, one unit of input equals one unit of a final good and the only costs the retailers have to bear are the wholesale prices for the

goods that they procure from the manufacturers. We assume that M_1 has an exclusivity contract with retailer R_A that says that M_1 is only allowed to sell his good 1 to R_A. This set-up makes R_A a monopolist for good 1 in the retail market. Manufacturer M_2 sells his product to both retailers and therefore we have a retail duopoly regarding good 2. Manufacturer M_1 charges a per-unit wholesale price of w_1 for good 1 and manufacturer M_2 a per-unit wholesale price of w_2 for good 2.
Retailer R_A as two-product firm can decide between two pricing strategies. He can either sell good 1 and good 2 separately or supply them solely as a bundle to the final consumers. First assume that R_A sells good 1 and good 2 **separately** as depicted in Figure 4.1.

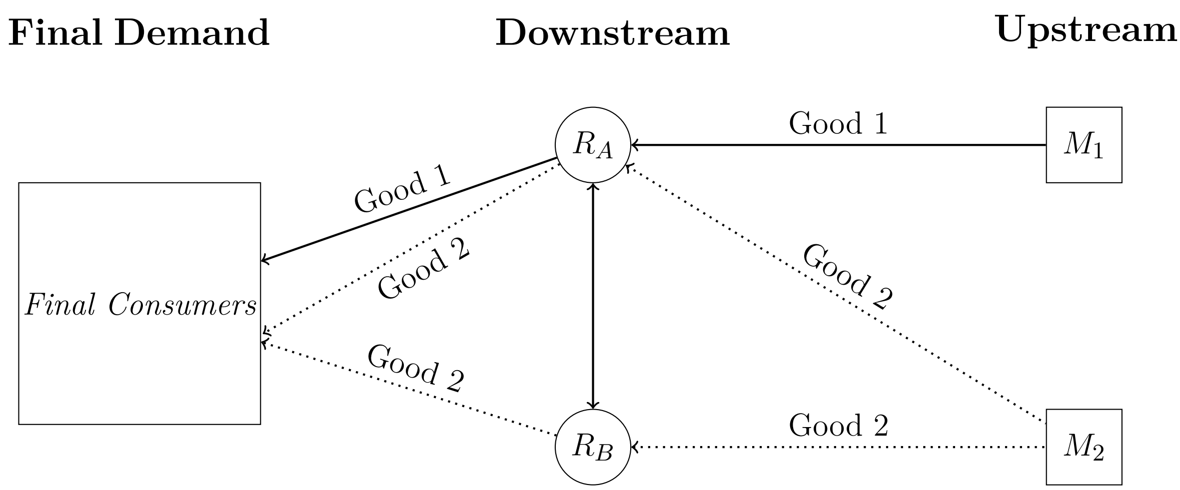

Figure 4.1: Market structure in case of *Separate Selling*

In the separate selling market, we have a homogeneous retail duopoly for good 2 and a retail monopoly for good 1. We use a Dixit (1979)-type utility function to represent the aggregate final consumers' preferences regarding good 1 and good 2 as well as other goods. The representative consumer's utility is given as

$$U(m, Q_1, Q_2) = m + a(Q_1 + Q_2) - 0.5(Q_1^2 + Q_2^2), \tag{4.1}$$

where m denotes the consumption of other goods and $Q_1(Q_2)$ the consumption of good 1 (2) for the representative consumer. The parameter $a > 0$ is the consumer's valuation for a good and can be interpreted as the product quality like in Häckner (2000). We assume the two goods to be of the same quality. Note that we also presume $a > k_1, k_2$ in order to ensure market transactions. The price of the composite good m is normalized to one. The retail price of good 1 (2) is given by p_1 (p_2).
Solving the representative consumer's optimization problem yields the inverse demand functions for the two stand-alone goods

$$p_1(Q_1) = a - Q_1, \tag{4.2}$$

$$p_2(Q_2) = a - Q_2, \tag{4.3}$$

where $Q_1 = q_{A1}$ is the quantity of good 1 supplied by retailer R_A. Furthermore, we have $Q_2 = q_{A2} + q_{B2}$, where q_{A2} and q_{B2} are the quantities of good 2 supplied by retailer R_A and R_B, respectively. Consequently, the demand functions for the two goods are

$$Q_1(p_1) = a - p_1, \tag{4.4}$$

$$Q_2(p_2) = a - p_2. \tag{4.5}$$

Under separate selling, retailer R_A and R_B maximize the profits π_A and π_B, respectively, which are given as

$$\pi_A = (p_1 - w_1)q_{A1} + (p_2 - w_2)q_{A2}, \tag{4.6}$$

$$\pi_B = (p_2 - w_2)q_{B2}. \tag{4.7}$$

Notice that the wholesale prices represent the retailers' marginal or unit costs.

Now suppose that retailer R_A **bundles** his products. When retailer R_A bundles, he combines one unit of good 1 with one unit of good 2 and offers the combination as bundle b_A. For notational purposes, we refer to the good supplied by retailer R_B in the bundling market as bundle b_B even though bundle b_B consists solely of one unit of good 2. The bundling market is displayed in Figure 4.2.

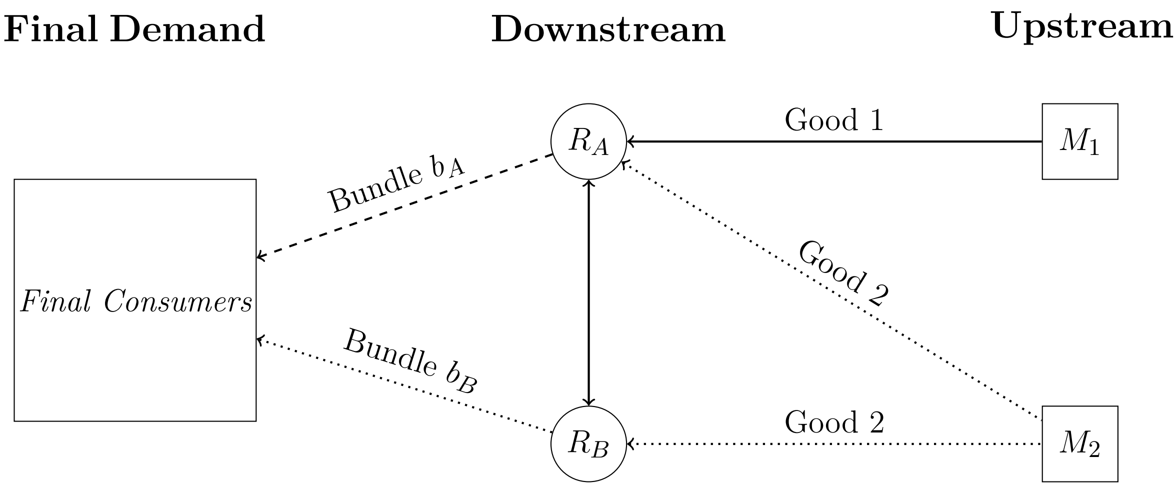

Figure 4.2: Market structure in case of *Bundling*

The number of bundles supplied by retailer R_A is given by Q_{b_A} and the amount of bundles supplied by retailer R_B is given by Q_{b_B}. Thus, the relations between the quantities under bundling are

$$Q_1 = Q_{b_A}, \tag{4.8}$$

$$Q_2 = Q_{b_A} + Q_{b_B}. \tag{4.9}$$

Following a method used by Martin (1999), we substitute the quantity relations (4.8) and (4.9) into the representative consumer's utility U. We receive a utility function V, which describes the consumers' preferences for the two bundles and other goods m:

$$V(m, Q_{b_A}, Q_{b_B}) = m + a(2Q_{b_A} + Q_{b_B}) - 0.5(2Q_{b_A}^2 + 2Q_{b_A}Q_{b_B} + Q_{b_B}^2). \tag{4.10}$$

The price of bundle b_A is denoted by p_{b_A} and the price of bundle b_B by p_{b_B}. Solving the representative consumer's optimization problem regarding the bundles yields the inverse demand functions

$$p_{b_A}(Q_{b_A}, Q_{b_B}) = 2a - 2Q_{b_A} - Q_{b_B}, \tag{4.11}$$

$$p_{b_B}(Q_{b_A}, Q_{b_B}) = a - Q_{b_A} - Q_{b_B}. \tag{4.12}$$

Note that $\frac{\partial p_{b_A}}{\partial Q_{b_B}} = -1 < 0$ and $\frac{\partial p_{b_B}}{\partial Q_{b_A}} = -1 < 0$. That is, in line with Martin (1999) the two bundles are (imperfect) demand substitutes even though the bundled products are independent in demand. The product differentiation between the bundles can be interpreted as differentiation in a vertical sense since good 1 adds some value to bundle b_A that bundle b_B does not provide (compare Egli, 2007). We further observe that bundling differentiates the products sold by the retailers: under separate selling, the goods sold in the retail duopoly are homogeneous, whereas under bundling, the products sold in the retail duopoly are differentiated since the bundles of the two retailers are imperfect substitutes.

For the bundles, we obtain the demand functions

$$Q_{b_A}(p_{b_A}, p_{b_B}) = a - p_{b_A} + p_{b_B}, \tag{4.13}$$

$$Q_{b_B}(p_{b_A}, p_{b_B}) = p_{b_A} - 2p_{b_B}. \tag{4.14}$$

When R_A bundles, then the profit of retailer $R_A(R_B)$ is given by $\Pi_A(\Pi_B)$:

$$\Pi_A = (p_{b_A} - w_1 - w_2)Q_{b_A}, \tag{4.15}$$

$$\Pi_B = (p_{b_B} - w_2)Q_{b_B}. \tag{4.16}$$

In the upstream market, the manufacturers M_1 and M_2 maximize the profits π_1 and π_2, respectively, under **separate selling** and **bundling**. Their profits are given by

$$\pi_1 = (w_1 - k_1)Q_1, \tag{4.17}$$

$$\pi_2 = (w_2 - k_2)Q_2. \tag{4.18}$$

The timing of the here considered game is as follows (compare Figure 4.3). At first, retailer R_A decides whether to bundle or to sell the two goods separately. Retailer R_A

bundles only when bundling is more profitable than separate selling. In the second stage, the manufacturers set their optimal wholesale prices. In the third stage, the retailers play their optimal prices or quantities, depending on the competition mode. In the last stage, retail sales are materialized.

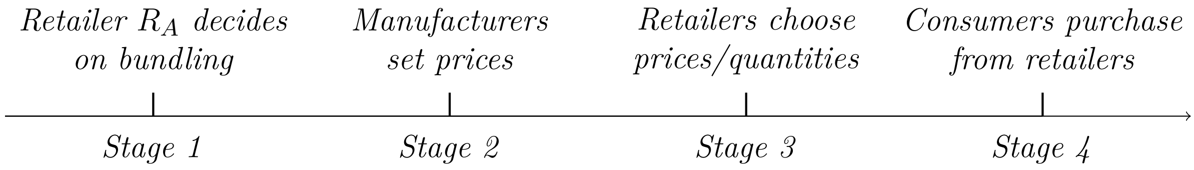

Figure 4.3: Timing of the game

In what follows, we solve the game for the subgame perfect Nash equilibrium in pure strategies applying backward induction, first for the retail price and then for the retail quantity competition setting. All first- and second-order conditions, proofs and further calculations can be found in Appendix 4.6. Let the superscripts S and BU denote the optimal solutions under separate selling and bundling, respectively, in the following.

4.3 Retail Price Competition

Suppose for now that the retailers compete in prices. For this setting, we first analyze the case where retailer R_A sells his products separately, and then the case where retailer R_A bundles. In a last step, we compare the two cases to derive the bundling incentives and the consequences of bundling. We further need to restrict the marginal cost of good 2, k_2, to values lower than $\frac{29a+36k_1}{65}$ in order to guarantee non-negative market results under price competition.[1] This means that we assume $k_2 < \frac{29a+36k_1}{65} < a$ and $k_1 < a$ for the price competition case.

4.3.1 Separate Selling

Assume that retailer R_A sells his products separately. Then, retailer R_A and retailer R_B maximize the profits:

$$\pi_A(p_1, p_{A2}, p_{B2}) = (p_1 - w_1)q_{A1}(p_1) + (p_{A2} - w_2)q_{A2}(p_{A2}, p_{B2}), \tag{4.19}$$

$$\pi_B(p_{A2}, p_{B2}) = (p_{B2} - w_2)q_{B2}(p_{A2}, p_{B2}), \tag{4.20}$$

where $p_{A2}(p_{B2})$ denotes the retail price of good 2 set by retailer $R_A(R_B)$. Maximizing the profits of the retailers with respect to prices yields

[1]See Lemma 4.2 further below.

$$p_1^S = \frac{a + w_1}{2}, \tag{4.21}$$

$$p_2^S = p_{A2}^S = p_{B2}^S = w_2. \tag{4.22}$$

In the market for good 1, R_A simply charges a monopoly price. In the second product market, both retailers play a standard Bertrand game since they sell homogeneous goods. Thus, they charge prices equal to the marginal cost or wholesale price of good 2. Thereby, the retailers gain zero profits in the duopoly.
The wholesale demand for good 1, Q_1, is derived from plugging $p_1^S = \frac{a+w_1}{2}$ into (4.4) and the wholesale demand for good 2, Q_2, by plugging $p_2^S = w_2$ into (4.5). We obtain

$$Q_1(w_1) = \frac{a - w_1}{2}, \tag{4.23}$$

$$Q_2(w_2) = a - w_2. \tag{4.24}$$

Considering the wholesale demands, the manufacturers M_1 and M_2 maximize their profits $\pi_1(w_1) = (w_1 - k_1)Q_1(w_1)$ and $\pi_2(w_2) = (w_2 - k_2)Q_2(w_2)$ by setting their profit-maximizing wholesale prices

$$w_1^S = \frac{a + k_1}{2}, \tag{4.25}$$

$$w_2^S = \frac{a + k_2}{2}. \tag{4.26}$$

Substituting the optimal wholesale prices into all market entities provides the market results for the case of separate selling, which are depicted in Lemma 4.1.

Lemma 4.1. *Market results under retail price competition and separate selling:*

p_1^S	$\frac{3a+k_1}{4}$	π_B^S	0
p_2^S	$\frac{a+k_2}{2}$	π_1^S	$\frac{(a-k_1)^2}{8}$
q_{A1}^S	$\frac{a-k_1}{4}$	π_2^S	$\frac{(a-k_2)^2}{4}$
q_{A2}^S	$\frac{a-k_2}{4}$	PS^S	$\frac{7a^2-6ak_1-8ak_2+3k_1^2+4k_2^2}{16}$
q_{B2}^S	$\frac{a-k_2}{4}$	CS^S	$\frac{(a-k_1)^2}{32} + \frac{(a-k_2)^2}{8}$
Q_2^S	$\frac{a-k_2}{2}$	W^S	$\frac{19a^2-14ak_1-24ak_2+7k_1^2+12k_2^2}{32}$
π_A^S	$\frac{(a-k_1)^2}{16}$		

Proof. See Appendix 4.6.1. □

In our framework, total welfare W is the sum of consumer surplus CS and producer surplus PS. The producer surplus is the sum of profits of all firms and consequently the channel profit. Note that retailer R_A's total equilibrium profit simply equals the monopoly profit he gains by selling good 1 due to the zero profits in the second retail market. Manufacturer M_2 benefits from the fierce competition in the retail duopoly because it leads to a high wholesale demand for good 2, whereas R_A demands only a (retail) monopoly quantity from manufacturer M_1.

4.3.2 Bundling

Assume now that retailer R_A bundles good 1 and good 2 and solely supplies the bundle. Since bundling differentiates the products of the retailers in the duopoly, the bundle prices are not driven down to wholesale prices. The bundle prices are determined by maximizing the following profits of the retailers:

$$\Pi_A(p_{b_A}, p_{b_B}) = (p_{b_A} - w_1 - w_2)Q_{b_A}(p_{b_A}, p_{b_B}), \tag{4.27}$$

$$\Pi_B(p_{b_A}, p_{b_B}) = (p_{b_B} - w_2)Q_{b_B}(p_{b_A}, p_{b_B}). \tag{4.28}$$

Solving the optimization problems of the retailers yields

$$p_{b_A}^{BU} = \frac{4a + 4w_1 + 6w_2}{7}, \tag{4.29}$$

$$p_{b_B}^{BU} = \frac{a + w_1 + 5w_2}{7}. \tag{4.30}$$

In order to obtain the wholesale demand functions, we insert $p_{b_A}^{BU} = \frac{4a+4w_1+6w_2}{7}$ and $p_{b_B}^{BU} = \frac{a+w_1+5w_2}{7}$ into (4.13) and (4.14). Considering (4.8) and (4.9), the wholesale demands read

$$Q_1(w_1, w_2) = \frac{4a - 3w_1 - w_2}{7}, \tag{4.31}$$

$$Q_2(w_1, w_2) = \frac{6a - w_1 - 5w_2}{7}. \tag{4.32}$$

We observe that $\frac{\partial Q_1}{\partial w_2} < 0$ and $\frac{\partial Q_2}{\partial w_1} < 0$. This means that good 1 and good 2 become complementary wholesale goods due to being bundled together.
Under bundling, the profits of the manufacturers are $\pi_1(w_1, w_2) = (w_1 - k_1)Q_1(w_1, w_2)$ and $\pi_2(w_1, w_2) = (w_2 - k_2)Q_2(w_1, w_2)$. They are maximized by setting

$$w_1^{BU} = \frac{34a + 30k_1 - 5k_2}{59}, \tag{4.33}$$

$$w_2^{BU} = \frac{32a - 3k_1 + 30k_2}{59}. \tag{4.34}$$

Both wholesale prices depend on k_1 and k_2 due to the complementary relationship between the goods. Wholesale price $w_1^{BU}(w_2^{BU})$ increases with $k_1(k_2)$ but decreases with $k_2(k_1)$ as $\frac{\partial w_1^{BU}}{\partial k_1}, \frac{\partial w_2^{BU}}{\partial k_2} > 0$ and $\frac{\partial w_1^{BU}}{\partial k_2}, \frac{\partial w_2^{BU}}{\partial k_1} < 0$.
The following Lemma 4.2 provides the market results under bundling given the equilibrium wholesale prices.

Lemma 4.2. *Market results under retail price competition and bundling:*

$p_{b_A}^{BU}$	$\frac{2(282a+51k_1+80k_2)}{413}$		Π_B^{BU}	$\frac{2(29a+36k_1-65k_2)^2}{170569}$
$p_{b_B}^{BU}$	$\frac{253a+15k_1+145k_2}{413}$		π_1^{BU}	$\frac{3(34a-29k_1-5k_2)^2}{24367}$
$Q_{b_A}^{BU}$	$\frac{3(34a-29k_1-5k_2)}{413}$		π_2^{BU}	$\frac{5(32a-3k_1-29k_2)^2}{24367}$
$Q_{b_B}^{BU}$	$\frac{2(29a+36k_1-65k_2)}{413}$		PS^{BU}	$\frac{72202a^2-61704ak_1-82700ak_2+28137k_1^2+38635k_2^2+5430k_1k_2}{170569}$
Q_2^{BU}	$\frac{5(32a-29k_2-3k_1)}{413}$		CS^{BU}	$\frac{18002a^2+3897k_1^2+3480k_1k_2+10625k_2^2-11274ak_1-24730ak_2}{170569}$
Π_A^{BU}	$\frac{9(34a-29k_1-5k_2)^2}{170569}$		W^{BU}	$\frac{6(15034a^2+5339k_1^2+1485k_1k_2+8210k_2^2-12163ak_1-17905ak_2)}{170569}$

Proof. See Appendix 4.6.1. □

Notice that the assumption $k_2 < \frac{29a+36k}{65}$ is necessary because $Q_{b_B} > 0$ holds only if $k_2 < \frac{29a+36k}{65}$. The product differentiation induced by bundling is the main reason why bundling might be profitable for R_A since it makes the retailers less competitive when they engage in price competition. By Lemma 4.2, we immediately see that retailer R_B gains a positive profit under bundling due to the reduction in the intensity of retail competition and thus benefits from bundling. We explain in the following in more detail when and how the two-product retailer R_A benefits from bundling.

4.3.3 Bundling Decision

We now analyze whether there is a bundling equilibrium and how bundling affects the market magnitudes in comparison to separate selling. A bundling equilibrium is an equilibrium where retailer R_A prefers bundling over separate selling.

The aim of R_A's bundling strategy is to raise his profit by (i) extending his monopoly power to the second product market and (ii) extracting more consumer surplus from consumers that buy good 1 absent bundling by making them pay a higher price for good 2. As a consequence of the latter reason, we adopt that the price of good 1, p_1^S, must be larger than the price of good 2, p_2^S, under separate selling for bundling to be considered by R_A.[2] When consumers with a high willingness to pay for good 1 can buy good 1 only tied with good 2, they might be willing to purchase the bundle even if that means paying a high bundle price and hence a relatively high price for good 2. However, the condition $p_1^S > p_2^S$ is fulfilled in any case under retail price competition because of the restriction $k_2 < \frac{29a+36k_1}{65}$.[3] Even though $p_1^S > p_2^S$ is always given, it is not guaranteed that it is R_A's best strategy to bundle. Nevertheless, we identify a region, where retailer R_A's bundling profit exceeds his separate selling profit.

Proposition 4.1. *Given retail price competition, $k_2 < \frac{29a+36k_1}{65}$ and $k_1 < a$, then there exists a unique bundling equilibrium with the bundle prices $p_{b_A}^{BU} = \frac{2(282a+51k_1+80k_2)}{413}$ and $p_{b_B}^{BU} = \frac{253a+15k_1+145k_2}{413}$.*

[2]This is in line with Carbajo et al. (1990).

[3]We have $p_1^S > p_2^S$ when $k_2 < \frac{a+k_1}{2}$, where $\frac{29a+36k_1}{65} < \frac{a+k_1}{2}$.

As the bundles are imperfect substitutes, retailer R_B charges a price above the wholesale price of good 2 under bundling in contrast to the separate selling market. Additionally, the price $p_{b_B}^{BU}$ he sets for his bundle b_B exceeds the retail price of good 2 under separate selling, p_2^S. This price raise induced by bundling, in turn, allows R_A to charge a very high price for bundle b_A, which clearly exceeds the sum of wholesale prices under bundling and the price of bundle b_B. This shows that bundling greatly softens the competition between the retailers in the duopoly.[4] In addition, R_A sets a price for bundle b_A that is larger than the sum of the prices of the stand-alone goods, i.e. $p_{b_A}^{BU} > p_1^S + p_2^S$. In conclusion, bundling allows R_A to extend his monopoly power from the market for good 1 to the second product market by bundling. The softened competition and the subsequent extension of market power positively affect R_A's profit and therefore bundling can be profitable here.
We find that the marginal costs of the manufacturers decisively influence whether bundling is finally profitable for R_A as summarized by the following theorem.

Theorem 4.1. *Given retail price competition,* $k_2 < \frac{29a+36k_1}{65}$, $k_1 < a$ *and*

(a) if $k_1 < \frac{a}{13}$, *then retailer* R_A *prefers not to bundle,*

(b) if $k_1 > \frac{a}{13}$ *and* $k_2 \in \left(\frac{13k_1-a}{12}, \frac{29a+36k_1}{65}\right)$, *then retailer* R_A *prefers not to bundle,*

(c) if $k_1 > \frac{a}{13}$ *and* $k_2 \in \left(0, \frac{13k_1-a}{12}\right)$, *then retailer* R_A *prefers to bundle.*

Proof. See Appendix 4.6.3. □

By Theorem 4.1, we observe in addition that the marginal cost of good 2, k_2, must be smaller than the marginal cost of good 1, k_1, for bundling to be profitable for retailer R_A since $\frac{13k_1-a}{12} < k_1$. The pivotal role of the marginal costs of the monopolistic manufacturers with respect to the retailer's bundling decision is consistent with Cao et al. (2015). We provide the intuition for Theorem 4.1 in the following.
In the separate selling case, only changes in k_1 and not in k_2 affect R_A's profit because (i) the two retailers set the equilibrium prices for good 2 equal to the wholesale price of good 2 and (ii) the wholesale price of good 1, w_1^S, only depends on k_1 as the two goods are independent in demand. A higher marginal cost of good 1 means a higher wholesale price of good 1 $\left(\frac{\partial w_1^S}{\partial k_1} > 0\right)$ and thus a lower separate selling profit for R_A $\left(\frac{\partial \pi_A^S}{\partial k_1} < 0\right)$. Therefore, when k_1 is lower than $\frac{a}{13}$, separate selling is more profitable than bundling for R_A. In contrast to the separate selling market, changes in k_2 affect R_A's profit under bundling because the bundle prices exceed the wholesale prices. More precisely, a decrease in k_2 reduces the wholesale price of good 2 (and also the sum of wholesale prices) and

[4] Following Carbajo et al. (1990) and Chung et al. (2013), less intense competition is defined by higher prices under price competition and by lower quantities under quantity competition.

therefore raises the bundling profit (note that $\frac{\partial \Pi_A^{BU}}{\partial k_2} < 0$). As a consequence, given a sufficiently high k_1 (i.e. $k_1 > \frac{a}{13}$), that makes independent selling less profitable, and a sufficiently low k_2 (i.e. $k_2 < \frac{13k_1 - a}{12}$), that makes bundling more profitable, bundling is retailer R_A's best strategy.

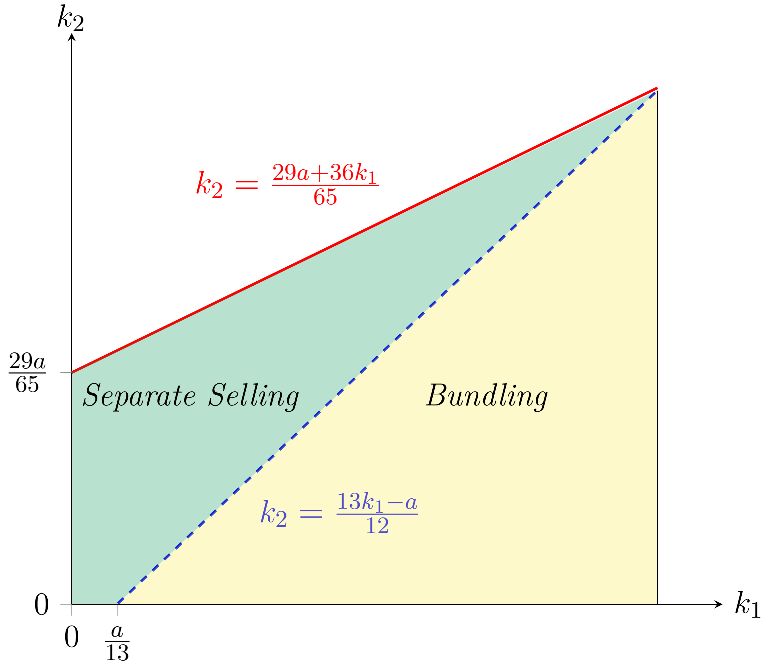

Figure 4.4: Bundling vs. Separate Selling

The profitable bundling region is captured by the yellow shaded area in Figure 4.4. Notice with a too large k_2 (i.e. $k_2 > \frac{13k_1 - a}{12}$), retailer R_A has no motivation to bundle and, consequently, decides to sell his products separately. This is the case even when $k_1 > \frac{a}{13}$ and despite the positive effects bundling has on the retailer's profit. The green shaded area in Figure 4.4 captures the region where R_A prefers separate selling over bundling. The area above the red line in the figure is excluded by our assumptions.

The observed reduction in the intensity of competition in the duopoly with price competition is in line with other articles of the leverage theory such as Carbajo et al. (1990), Egli (2007) or Chung et al. (2013). Yet, in contrast to Carbajo et al. (1990), in our model separate selling might be the multi-product firm's best strategy when facing price competition. This difference can be explained with the market structure. Carbajo et al. (1990) consider a non-vertical market structure, but in our model the price-setting behavior of the manufacturers negatively affects the bundling incentives and therefore bundling may be the inferior strategy for the two-product firm. This is illustrated in more detail in the following.

Proposition 4.2 summarizes how the manufacturers response to the bundling strategy by R_A, irrespective of bundling being profitable for the retailer.

Proposition 4.2. *Under retail price competition, bundling induces*

(a) manufacturer M_1 to raise the wholesale price for good 1,

(b) manufacturer M_2 to raise the wholesale price for good 2 if $k_1 < \frac{5}{6}a$ or if $k_1 > \frac{5}{6}a$ and $k_2 \in \left(-5a + 6k_1, \frac{29a+36k_1}{65}\right)$,

(c) M_2 to reduce the wholesale price for good 2 if $k_1 > \frac{5}{6}a$ and $k_2 \in (0, -5a + 6k_1)$,

(d) a greater sum of both wholesale prices.

Proof. See Appendix 4.6.4. □

Bundle b_A consists of the products of both manufacturers and hence an increase in the wholesale price of good 1 has only a partial influence on manufacturer M_1's own sales. In addition, good 1 and good 2 become complementary wholesale goods due to being bundled, which makes it hard for R_A to forego one good. These effects induce M_1 to raise his price in order to benefit from R_A's bundling strategy. Good 2 is also supplied by retailer R_B and not only in bundle b_A and therefore the rationale to charge a higher price is weakened for M_2. However, the impact of an increase in wholesale prices on the retailers' sales is weaker when the retailers set prices above wholesale prices compared to the situation where the retailers set prices equal to wholesale prices. Thus, the less intense competition due to bundling and the induced complementarity between the wholesale goods allow M_2 to raise his price. Nevertheless, manufacturer M_2 may lower his price when R_A bundles. This is the case given $k_1 > \frac{5}{6}a$ and $k_2 < -5a + 6k_1$. The reason for this is that under separate selling, M_2's price does not depend on k_1 and under bundling, M_2's price is reduced by an increase in k_1 and by a decrease in k_2. This means, in turn, that when k_1 is too small or k_2 too large, M_2 raises his price. Even when M_2 lowers his price due to bundling, the reduction of his wholesale price is lower than the increase in M_1's wholesale price. As a consequence, the sum of wholesale prices with bundling is always greater than with separate selling.

Another issue with respect to the firms' pricing behavior is that in the separate selling market, there is only double marginalization in the market for good 1. There is a bilateral monopoly regarding good 1 in this case, but we have standard Bertrand competition in the second product market leading to a retail price equal to the wholesale price. By contrast, bundling leads to double marginalization also in the second product market since both bundles and both wholesale goods under bundling are priced with a positive mark-up. Looking at the changes of wholesale prices and the changes in the market scenario caused by R_A's bundling strategy, we establish that bundling aggravates the double marginalization problem between R_A and the manufacturers.

As illustrated, R_A might prefer to bundle despite the worsened DM problem. If the wholesale price of good 2 is decreased, bundling is always more profitable than separate selling for R_A.[5] Clearly, the lower wholesale price positively influences the profitability of bundling. Yet, even when both wholesale prices are raised, bundling could still be the better option for R_A than separate selling.[6] In sum, the positive effects that bundling has on R_A's profit in the form of softening retail competition and extending market power can outweigh the negative impact that bundling has on R_A's profit in the form of aggravating the double marginalization problem. To further elaborate on this, consider the paper of Bhargava (2012). In contrast to our framework, in Bhargava's model separate selling is always the two-product retailer's best strategy because bundling aggravates the DM problem in the channel. Since Bhargava considers a distribution channel with only a single retailer that is a monopolist for both traded goods, no impact of bundling on retail competition can outweigh the negative effect of an aggravated DM problem. For that reason, bundling is never profitable in Bhargava's model, but it can be profitable in our framework due its impact on retail competition.

Nevertheless, the exacerbated DM problem has a pivotal (negative) influence on R_A's bundling incentives in our set-up. To see this, suppose that the retailers hold all the market power in the distribution channel such that the manufacturers have no bargaining power.[7] In this *centralized channel*, the retailers set retail as well as wholesale prices and, consequently, the manufacturers are simply price-takers. To keep their costs low, the retailers set wholesale prices equal to the marginal production costs of the two goods in both settings, no bundling and bundling, i.e. $w_1^S = w_1^{BU} = k_1$ and $w_2^S = w_2^{BU} = k_2$. Therefore, the double marginalization in the channel is eliminated. Comparing R_A's profits in this centralized channel under price competition, we find that it is always more profitable for R_A to bundle than to sell his goods separately, which is in contrast to the decentralized channel.

Notably, the double marginalization problem in the decentralized channel is only aggravated when the vertical externalities are combined with horizontal externalities upstream. This finding is consistent with Bhargava (2012). To illustrate this, assume that the two manufacturers in the decentralized channel merge such that both goods are produced by a single manufacturer. The two-product manufacturer charges the same wholesale prices in the bundling market and in the separate selling market, i.e. $w_1^S = w_1^{BU} = \frac{a+k_1}{2}$ and $w_2^S = w_2^{BU} = \frac{a+k_2}{2}$. That is, the monopolist internalizes the cross-price effects regarding the wholesale demands under bundling, where the two stand-alone goods are complementary wholesale products. This means that the extent of double marginalization in the bilateral monopoly for good 1 is not affected by bundling in this case. Nevertheless, bundling creates DM in the channel regarding good 2 but only to R_A's advantage be-

[5]This is the case because $k_1 > \frac{5}{6}a$ and $k_2 < -5a + 6k_1$ (lowered wholesale price for good 2) imply $k_1 > \frac{a}{13}$ and $k_2 < \frac{13k_1 - a}{12}$ (increased profit profit for R_A) since $\frac{5}{6}a > \frac{a}{13}$ and $-5a + 6k_1 < \frac{13k_1-a}{12}$.

[6]It might hold that $k_1 > \frac{5}{6}a > \frac{a}{13}$ and $-5a + 6k_1 < k_2 < \frac{13k_1-a}{12}$ or $\frac{a}{13} < k_1 < \frac{5}{6}a$ and $k_2 < \frac{13k_1-a}{12}$.

[7]Alternatively, the wholesale markets could be assumed to be perfectly competitive.

cause of the softened competition and no change in wholesale prices. Finally, bundling is again always more profitable than separate selling for R_A when there is a multi-product upstream monopoly in the decentralized channel. In conclusion, it is the mix of vertical externalities and horizontal externalities in the upstream market that mitigates R_A's bundling incentives and not the presence of powerful upstream firms alone.[8]

4.3.4 Consequences of Profitable Bundling

We next examine the impact of bundling on market entities when bundling is profitable for R_A and hence only in the region $k_1 > \frac{a}{13}$ and $k_2 < \frac{13k_1-a}{12}$.[9]

We first regard the downstream market. As already explained, retailer R_B benefits from R_A's bundling strategy as bundling generates a positive profit for R_B opposed to separate selling. Moreover, profitable bundling has interesting effects on the sales of both retailers. Profitable bundling raises R_A's quantity of good 1 despite the increase in wholesale price w_1 because the monopolistic good 1 is bundled with the more competitive good 2. By contrast, profitable bundling lowers R_A's sales of good 2 due to the softening in competition and a possible raise in the wholesale price w_2. We observe that if $k_1 > \frac{181}{288}a$ and $k_2 \in \left(0, \frac{-181a+288k_1}{107}\right)$, bundling actually increases R_B's equilibrium quantity of good 2.[10]

This is always the case when the wholesale price of good 2 is lowered by bundling but even for $\frac{5}{6}a > k_1 > \frac{181}{288}a$ such that w_2 is increased, R_B's quantity rises when the marginal cost of good 2 is sufficiently low. The explanation for the potential raise in R_B's quantity is that he sets a lower price for his bundle than retailer R_A. In conclusion, on the one hand R_A can raise his profit and strengthen his position in the retail duopoly by bundling, but on the other hand he might actually help to raise R_B's market share.

In the upstream market, bundling has an ambiguous influence on the profits of the manufacturers as summarized by

Proposition 4.3. *Under retail price competition, profitable bundling results in*

(a) an increase in manufacturer M_1's profit,

(b) a decrease in manufacturer M_2's profit.

Proof. See Appendix 4.6.5. □

As a consequence of the fierce competition in the retail market for good 2 under separate selling, manufacturer M_2 produces and supplies a large quantity of good 2. As the

[8]For the bundling incentives in the centralized channel and the multi-product manufacturer case see Appendix 4.6.1.

[9]'Profitable bundling' is sometimes abbreviated to 'bundling' in the rest of the section.

[10]The condition $\Delta q_{B2} = q^S_{B2} - Q^{BU}_{b_B} < 0$ is met for $k_2 < \frac{-181a+288k_1}{107}$. Note that $\frac{-181a+288k_1}{107} < \frac{13k_1-a}{12}$ and that $\frac{-181a+288k_1}{107} > 0$ when $k_1 > \frac{181}{288}a$. Further note that $\frac{181}{288}a > \frac{a}{13}$.

softening in competition lowers the wholesale demand for good 2, M_2 sells a lower quantity possibly even at a lower price in the bundling equilibrium. Hence, he gains a smaller profit under bundling than under separate selling. In contrast, M_1 sells a larger quantity at a higher price under bundling and thereby bundling raises M_1's profit.
Even though manufacturer M_1's profit and the profits of both retailers are raised by bundling, the producer surplus is reduced by it as our welfare analysis illustrates.

Proposition 4.4. *Under retail price competition, profitable bundling results in*

(a) a decrease in consumer surplus,

(b) a decrease in producer surplus,

(c) a decrease in total welfare.

Proof. See Appendix 4.6.6. □

This means that the loss in M_2's profit is larger than the total gain in profits of the other three firms. The total quantity of good 2 is lowered and retail prices as well as the sum of wholesale prices are raised by profitable bundling. Therefore, the consumer surplus is diminished too. Consequently, bundling always harms social welfare on all levels in the equilibrium. This result is (partly) in contrast to Carbajo et al. (1990). In their model, bundling might increase social welfare since it always raises the producer surplus under price competition as the only two firms in the market gain from the softening in competition. In our vertical market with four firms, however, one firm loses from bundling and the subsequent softening in competition which finally decreases producer surplus.

4.4 Retail Quantity Competition

Suppose now that the retailers engage in quantity competition. For this case, we must impose the condition $k_2 < \frac{a+3k_1}{4}$ to guarantee $p_1^S > p_2^S$ and non-negative market results.[11] Therefore, we assume $k_2 < \frac{a+3k_1}{4} < a$ and $k_1 < a$ here. The following analysis is analogous to the price competition case.

4.4.1 Separate Selling

When retailer R_A sells his products separately, the two retailers maximize the profits

$$\pi_A(q_{A1}, q_{A2}, q_{B2}) = (p_1(q_{A1}) - w_1)q_{A1} + (p_2(q_{A2}, q_{B2}) - w_2)q_{A2}, \tag{4.35}$$

$$\pi_B(q_{A2}, q_{B2}) = (p_2(q_{A2}, q_{B2}) - w_2)q_{B2}. \tag{4.36}$$

[11] See Lemma 4.4 and Proposition 4.5 further below.

The profit-maximizing quantities of the retailers with respect to good 1 and good 2 are

$$q^S_{A1} = \frac{a - w_1}{2}, \tag{4.37}$$

$$q^S_{A2} = q^S_{B2} = \frac{a - w_2}{3}. \tag{4.38}$$

The wholesale demand for good 1 is given by (4.37) since $Q_1 = q_{A1}$ and concerning good 2 by $Q_2 = q_{A2} + q_{B2}$. Finally, the wholesale demand functions are

$$Q_1(w_1) = \frac{a - w_1}{2}, \tag{4.39}$$

$$Q_2(w_2) = \frac{2\,(a - w_2)}{3}. \tag{4.40}$$

Solving the optimization problems of the manufacturers yields the equilibrium wholesale prices

$$w^S_1 = \frac{a + k_1}{2}, \tag{4.41}$$

$$w^S_2 = \frac{a + k_2}{2}, \tag{4.42}$$

which are equivalent to the according wholesale prices under separate selling and price competition.

The equilibrium outcomes with separate selling and quantity competition considering the optimal wholesale prices are displayed in Lemma 4.3.

Lemma 4.3. *Market results under retail quantity competition and separate selling:*

p^S_1	$\frac{3a+k_1}{4}$	π^S_B	$\frac{(a-k_2)^2}{36}$
p^S_2	$\frac{2a+k_2}{3}$	π^S_1	$\frac{(a-k_1)^2}{8}$
q^S_{A1}	$\frac{a-k_1}{4}$	π^S_2	$\frac{(a-k_2)^2}{6}$
q^S_{A2}	$\frac{a-k_2}{6}$	PS^S	$\frac{59a^2+27k_1^2+32k_2^2-54ak_1-64ak_2}{144}$
q^S_{B2}	$\frac{a-k_2}{6}$	CS^S	$\frac{25a^2+9k_1^2+16k_2^2-18ak_1-32ak_2}{288}$
Q^S_2	$\frac{a-k_2}{3}$	W^S	$\frac{143a^2+63k_1^2+80k_2^2-126ak_1-160ak_2}{288}$
π^S_A	$\frac{13a^2-8ak_2+9k_1^2+4k_2^2-18ak_1}{144}$		

Proof. See Appendix 4.6.2. □

We find that manufacturer M_2 earns a lower profit here than under price competition and separate selling as $\frac{(a-k_2)^2}{6} < \frac{(a-k_2)^2}{4}$. This is because retail quantity competition with homogeneous goods induces ceteris paribus a lower wholesale demand for good 2 than the more aggressive retail price competition.

4.4.2 Bundling

In the bundling market, the retailers maximize their profits with respect to the quantities of the bundles. Their bundling profits are given by

$$\Pi_A(Q_{b_A}, Q_{b_B}) = (p_{b_A}(Q_{b_A}, Q_{b_B}) - w_1 - w_2)Q_{b_A}, \tag{4.43}$$

$$\Pi_B(Q_{b_A}, Q_{b_B}) = (p_{b_B}(Q_{b_A}, Q_{b_B}) - w_2)Q_{b_B}. \tag{4.44}$$

The optimal bundle quantities of the retailers are

$$Q_{b_A}^{BU} = \frac{3a - 2w_1 - w_2}{7}, \tag{4.45}$$

$$Q_{b_B}^{BU} = \frac{2a - 3w_2 + w_1}{7}. \tag{4.46}$$

Again the relations (4.8) and (4.9) determine the wholesale demands. Consequently, the wholesale demands for good 1 and good 2, respectively, are

$$Q_1(w_1, w_2) = \frac{3a - w_2 - 2w_1}{7}, \tag{4.47}$$

$$Q_2(w_1, w_2) = \frac{5a - w_1 - 4w_2}{7}. \tag{4.48}$$

The two goods become complementary wholesale goods due to bundling like in the price competition case since $\frac{\partial Q_1}{\partial w_2} < 0$ and $\frac{\partial Q_2}{\partial w_1} < 0$.
Under bundling, we receive the equilibrium wholesale prices

$$w_1^{BU} = \frac{19a + 16k_1 - 4k_2}{31}, \tag{4.49}$$

$$w_2^{BU} = \frac{17a - 2k_1 + 16k_2}{31}. \tag{4.50}$$

As under price competition, we observe concerning the relationships between marginal costs and wholesale prices that $\frac{\partial w_1^{BU}}{\partial k_1}, \frac{\partial w_2^{BU}}{\partial k_2} > 0$ and $\frac{\partial w_1^{BU}}{\partial k_2}, \frac{\partial w_2^{BU}}{\partial k_1} < 0$.
The market results for bundling are summarized in the following Lemma 4.4.

Lemma 4.4. *Market results under retail quantity competition and bundling:*

$p_{b_A}^{BU}$	$\frac{2(164a+19k_1+34k_2)}{217}$	Π_B^{BU}	$\frac{4(15a+11k_1-26k_2)^2}{47089}$
$p_{b_B}^{BU}$	$\frac{149a+8k_1+60k_2}{217}$	π_1^{BU}	$\frac{2(19a-15k_1-4k_2)^2}{6727}$
$Q_{b_A}^{BU}$	$\frac{2(19a-15k_1-4k_2)}{217}$	π_2^{BU}	$\frac{2(17a-2k_1-15k_2)^2}{6727}$
$Q_{b_B}^{BU}$	$\frac{2(15a+11k_1-26k_2)}{217}$	PS^{BU}	$\frac{2(8467a^2+2773k_1^2+1016k_1k_2+4678k_2^2-6562ak_1-10372ak_2)}{47089}$
Q_2^{BU}	$\frac{4(17a-15k_2-2k_1)}{217}$	CS^{BU}	$\frac{2(1517a^2+241k_1^2+360k_1k_2+916k_2^2-842ak_1-2192ak_2)}{47089}$
Π_A^{BU}	$\frac{8(19a-15k_1-4k_2)^2}{47089}$	W^{BU}	$\frac{4(4992a^2+1507k_1^2+688k_1k_2+2797k_2^2-3702ak_1-6282ak_2)}{47089}$

Proof. See Appendix 4.6.2. □

Notice that $Q_{b_B} > 0$ is given if $k_2 < \frac{15a+11k_1}{26}$. We have $\frac{a+3k_1}{4} < \frac{15a+11k_1}{26}$ and thus $Q_{b_B} > 0$ is guaranteed. In contrast to the price competition case, we here have double marginalization under separate selling and bundling for each part of the supply chain and therefore cannot directly identify whether any retailer benefits from bundling.

4.4.3 Bundling Decision

We again regard that the retail price of good 1, p_1^S, is larger than the retail price of good 2, p_2^S, under separate selling as a necessary condition for bundling to be considered by R_A. The condition $p_1^S > p_2^S$ holds if $k_2 < \frac{a+3k_1}{4}$, which we imposed as assumption.[12] Even though $p_1^S > p_2^S$ is satisfied by assumption, we do not find a bundling equilibrium:

Proposition 4.5. *When the retailers engage in quantity competition, then retailer R_A prefers not to bundle in the equilibrium.*

Proof. See Appendix 4.6.7. □

While in the case of price competition, the product differentiation as a consequence of bundling can lead to bundling being R_A's best strategy, this is not the case when the retailers engage in quantity competition. For that reason, R_A opts for separate selling in the equilibrium.

The observation that bundling is never profitable for the multi-product firm given quantity competition is contrary to parts of the leverage theory, which only consider non-vertical industries and therefore no price-setting upstream firms, see e.g. Carbajo et al. (1990); Martin (1999); Chung et al. (2013). However, also under quantity competition, the price setting behavior of the manufacturers plays a pivotal role with respect to the retailer's bundling incentives in our framework, as depicted in the following.

The price setting reactions of the two manufacturers to retailer R_A's bundling strategy are summarized by

Proposition 4.6. *Under retail quantity competition, bundling induces*

(a) manufacturer M_1 to raise the wholesale price for good 1,

(b) manufacturer M_2 to raise the wholesale price for good 2 if $k_1 < \frac{3}{4}a$ or if $k_1 > \frac{3}{4}a$ and $k_2 \in \left(-3a + 4k_1, \frac{a+3k_1}{4}\right)$,

(c) M_2 to reduce the wholesale price for good 2 if $k_1 > \frac{3}{4}a$ and $k_2 \in (0, -3a + 4k_1)$,

(d) a greater sum of both wholesale prices.

Proof. See Appendix 4.6.8. □

[12] Our insights about the (non-)profitability of bundling do not depend on this assumption.

The manufacturers want to benefit from R_A's bundling strategy and have the same incentives to raise their prices as under retail price competition. Hence, M_1 raises his price due to bundling. Analogous to the price competition case, manufacturer M_2 lowers his price given a sufficiently large k_1 and a sufficiently small k_2. Finally, we find that bundling also leads in the quantity competition game to a heavier double marginalization problem between R_A and the manufacturers as the sum of the two wholesale prices is again greater than the sum without bundling here. The rise in the sum of wholesale prices makes R_A charge a very high bundle price (again higher than the sum of the retail prices of the two stand-alone goods), which results in this case in too little sales of bundle b_A and thus a lower profit for R_A under bundling than under separate selling.

To further illustrate the influence of double marginalization on the bundling decision, consider again first a centralized channel and then a two-product upstream monopoly. As under price competition, the wholesale prices equal the respective manufacturers' marginal costs, i.e. $w_1^S = w_1^{BU} = k_1$ and $w_2^S = w_2^{BU} = k_2$, in the centralized channel with quantity competition. We find that in the centralized case here, retailer R_A may prefer to bundle depending on the marginal costs of the manufacturers. This, however, implies that R_A could prefer separate selling too, as in the decentralized channel.[13] When we have a decentralized vertical industry where both goods are produced by a single manufacturer, we obtain $w_1^S = w_1^{BU} = \frac{a+k_1}{2}$ and $w_2^S = w_2^{BU} = \frac{a+k_2}{2}$. Then, R_A's bundling incentives are qualitatively the same as in the centralized channel. In sum, we demonstrate that also under quantity competition, the bundling incentives are weakened by the aggravation in the DM problem and that the problem is aggravated only if the vertical externalities are combined with horizontal externalities upstream.[14]

It is particularly interesting that there is no bundling equilibrium with quantity competition considering the impact of bundling on certain market outcomes. If we have $\frac{a+3k_1}{4} > k_2 > \frac{-11a+180k_1}{169}$, R_A's quantity of good 2 rises whereas R_B's quantity of good 2 falls due to bundling.[15] Ultimately, the total quantity of good 2 decreases because of bundling. This means that bundling could improve R_A's market position in the retail duopoly while weakening R_B's position. Nevertheless, bundling would reduce R_A's profit. The intuition behind this observation is that when R_A raises his quantity of good 2 while lowering R_B's quantity, both wholesale prices increase with bundling.[16] The increase in wholesale prices and in R_A's quantity of good 2 incurs high input costs for R_A which

[13]The two-product firm's bundling incentives given a centralized channel and quantity competition are consistent with Carbajo et al. (1990) for a non-vertical industry.

[14]For the bundling incentives in the centralized channel and the case with a multi-product manufacturer see Appendix 4.6.2.

[15]We obtain $\Delta q_{A2} = q_{A2}^S - Q_{b_A}^{BU} < 0$ for $k_2 > \frac{-11a+180k_1}{169}$ and $\Delta q_{B2} = q_{B2}^S - Q_{b_B}^{BU} > 0$ in case $k_2 > \frac{132k_1-37a}{95}$. It holds that $\frac{a+3k_1}{4} > \frac{-11a+180k_1}{169} > \frac{132k_1-37a}{95}$.

[16]If $k_2 > \frac{-11a+180k_1}{169}$, then $k_2 > -3a+4k_1$ (higher wholesale price of good 2 under bundling) is always fulfilled because of $\frac{-11a+180k_1}{169} > -3a+4k_1$.

negatively affect the bundling profit. In fact, bundling displays another advantage for R_A because it reduces the intensity of competition in the retail market for product 2 as it reduces the total quantity of good 2. Nonetheless, separate selling is in any case more profitable than bundling for R_A.

Summing up, opposed to the price competition setting, the positive effects that bundling has on R_A's profit under quantity competition are not sufficient to outweigh the negative impact bundling has on R_A's profit. As a consequence, there exist no bundling equilibrium given the retailers compete in quantities. Since there is no bundling equilibrium, we do not analyze any welfare consequences for this case.

4.5 Conclusion

In this paper, we theoretically examine the incentives for a retail bundling and the allocative effects of retail bundling in a decentralized distribution channel with powerful manufacturers. We consider a retail market that is connected to the leverage theory of bundling with a two-product retailer that is a monopolist in one product market but competes with another retailer in the second product market. We analyze the two-product retailer's bundling strategy under retail price and retail quantity competition.

We observe that bundling aggravates the double marginalization problem between the two-product retailer and the manufacturers in either mode of retail competition. This happens because of the combination of vertical externalities and horizontal externalities upstream, which we identify in line with Bhargava (2012) as a factor that weakens the incentives for retail bundling. However, the influence that bundling has on retail competition in our leverage theory framework can outweigh this negative impact of bundling on the retailer's bundling profit, but only when the retailers engage in price competition. Then, bundling greatly softens the retail competition and results in an extension of market power for the bundling retailer. It finally depends on the marginal costs of the manufacturers whether bundling is profitable under retail price competition. We therefore identify the marginal costs as pivotal factors concerning the rationale for retail bundling. As also separate selling might be preferred by the retailer, the negative effects of bundling can also outweigh the positive effects regarding the profitability of bundling. Interestingly, this is always the case when the retailers compete in quantities. Even though bundling reduces the intensity of retail competition and might extend the two-product retailer's market power under quantity competition too, the retailer always gains a higher profit with separate selling than with bundling in this case.

We further study how bundling influences social welfare when bundling is the equilibrium strategy and thus only for the scenario with retail price competition. We find that profitable bundling diminishes the consumer surplus since it raises the prices of both retailers. Furthermore, it reduces the producer surplus despite both retailers and the

manufacturer, that sells exclusively to the bundling retailer, benefiting from bundling. Consequently, retail bundling harms social welfare in the equilibrium on all levels.
In conclusion, our study derives the implication that bundling may not necessarily be the best strategy for retailers in digital markets such as streaming services or for retailers in more traditional industries such as electronic retailers. In some cases, a retailer might be better off to offer his products separately, especially when dealing with powerful manufacturers. However, our findings suggest too that bundling may a profitable strategy for retailers if they face particularly fierce competition. Our results additionally indicate that retail bundling should be evaluated carefully from a competition policy perspective since it could harm welfare efficiency and consumers due to high retail prices.
One natural extension of our model would be to incorporate mixed bundling as a potential strategy for the multi-product retailer. This could generate interesting insights about the optimal bundling strategy for retailers. Further room for future research leaves the consideration of downstream retail competition. Many competition related issues, such as product differentiation, collusion or variations in market size, could be implemented and investigated. This article serves as a starting point for research that combines aspects of retail bundling and of the leverage theory. In addition, this article enlarges the currently small literature on the interplay of retail competition and retail bundling.

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4.6 Appendix

4.6.1 Retail Price Competition

Equilibrium Conditions

Separate Selling

Retailer R_A's separate selling profit is strictly concave in p_1 because $\frac{\partial^2 \pi_A}{\partial p_1^2} = -2 < 0$. Thus, the second-order condition (SOC) for a (global) maximum is always fulfilled. The first-order condition (FOC) that determines R_A's optimal price for good 1 reads

$$\frac{\partial \pi_A}{\partial p_1} = a - 2p_1 + w_1 \stackrel{!}{=} 0. \tag{4.51}$$

Solving the FOC for p_1 generates the monopoly price.

Manufacturer M_1's profit and manufacturer M_2's profit are strictly concave in w_1 and w_2, respectively, as $\frac{d^2\pi_1}{dw_1^2} = -1 < 0$ and $\frac{d^2\pi_2}{dw_2^2} = -2 < 0$. The FOCs determining the equilibrium wholesale prices are

$$\frac{d\pi_1}{dw_1} = \frac{a - 2w_1 + k_1}{2} \stackrel{!}{=} 0, \tag{4.52}$$

$$\frac{d\pi_2}{dw_2} = a - 2w_2 + k_2 \stackrel{!}{=} 0. \tag{4.53}$$

Solving the FOCs for w_1 and w_2, respectively, leads to the optimal wholesale prices.

Bundling

Retailer R_A's and retailer R_B's bundling profits are strictly concave in p_{b_A} and p_{b_B}, respectively, since $\frac{\partial^2 \Pi_A^{BU}}{\partial p_{b_A}^2} = -2 < 0$ and $\frac{\partial^2 \Pi_B^{BU}}{\partial p_{b_B}^2} = -4 < 0$. The FOCs concerning the optimal bundle prices are

$$\frac{\partial \Pi_A}{\partial p_{b_A}} = a + p_{b_B} - 2p_{b_A} + w_2 + w_1 \stackrel{!}{=} 0, \tag{4.54}$$

$$\frac{\partial \Pi_B}{\partial p_{b_B}} = -4p_{b_B} + p_{b_A} + 2w_2 \stackrel{!}{=} 0. \tag{4.55}$$

From the FOCs we can derive the reaction functions of the retailers as

$$p_{b_A}(p_{b_B}) = \frac{a + p_{b_B} + w_2 + w_1}{2}, \tag{4.56}$$

$$p_{b_B}(p_{b_A}) = \frac{p_{b_A} + 2w_2}{4}. \tag{4.57}$$

The intersection of the two reaction functions generates the optimal bundle prices.

In the upstream market, we have $\frac{\partial^2 \pi_1}{\partial w_1^2} = -\frac{6}{7} < 0$ and $\frac{\partial^2 \pi_2}{\partial w_2^2} = -\frac{10}{7} < 0$. The FOCs of the manufacturers regarding the profit-maximizing wholesale prices are

$$\frac{\partial \pi_1}{\partial w_1} = \frac{4a - w_2 - 6w_1 + 3k_1}{7} \stackrel{!}{=} 0, \tag{4.58}$$

$$\frac{\partial \pi_2}{\partial w_2} = \frac{6a - 10w_2 - w_1 + 5k_2}{7} \stackrel{!}{=} 0. \tag{4.59}$$

We obtain the reaction functions by solving the FOCs for w_1 and w_2, respectively:

$$w_1(w_2) = \frac{4a - w_2 + 3k_1}{6}, \tag{4.60}$$

$$w_2(w_1) = \frac{6a - w_1 + 5k_2}{10}. \tag{4.61}$$

The fixed point of the reaction functions generates the equilibrium wholesale prices.

Multi-Product Upstream Monopoly

Separate Selling

Consider that both goods, 1 and 2, are produced by a single firm, denoted M_{12}. When retailer R_A sells his products separately, M_{12} earns the profit

$$\pi_{12}(w_1, w_2) = (w_1 - k_1)Q_1(w_1) + (w_2 - k_2)Q_2(w_2). \tag{4.62}$$

When retailer R_A plays a separate selling strategy, goods 1 and 2 are independent in demand. As a consequence, solving M_{12}'s optimization problem yields the same profit-maximizing wholesale prices as in the case with two independently operating manufacturers so that $w_1^S = \frac{a+k_1}{2}$ and $w_2^S = \frac{a+k_2}{2}$.

Bundling

When retailer R_A bundles, M_{12}'s profit is analogous to M_{12}'s profit under separate selling, where $\frac{\partial^2 \pi_{12}}{\partial w_1^2} = -\frac{6}{7} < 0$ and $\frac{\partial^2 \pi_{12}}{\partial w_2^2} = -\frac{10}{7} < 0$. The FOCs with respect to the optimal wholesale prices are here given by

$$\frac{\partial \pi_{12}}{\partial w_1} = \frac{4a + k_2 - 2w_2 + 3k_1 - 6w_1}{7} \stackrel{!}{=} 0, \tag{4.63}$$

$$\frac{\partial \pi_{12}}{\partial w_2} = \frac{6a + 5k_2 - 10w_2 + k_1 - 2w_1}{7} \stackrel{!}{=} 0. \tag{4.64}$$

Solving this equation system for w_1 and w_2 generates the optimal wholesale prices, w_1^{BU} and w_2^{BU}, which are equivalent to the according wholesale prices in the separate selling market.

Welfare Outcomes

Separate Selling

The consumer surplus in the market for good 1 is given as

$$CS_1 = \frac{a - p_1}{2} Q_1, \tag{4.65}$$

and the consumer surplus for good 2 as

$$CS_2 = \frac{a - p_2}{2} Q_2. \tag{4.66}$$

We obtain the consumer surplus of good 1 by substituting $p_1^S = \frac{3a+k_1}{4}$ and $Q_1^S = \frac{a-k_1}{4}$ into (4.65) and we obtain the consumer surplus of good 2 by substituting $p_2^S = \frac{a+k_2}{2}$ and $Q_2^S = q_{A2}^S + q_{B2}^S = \frac{a-k_2}{2}$ into (4.66). We get

$$CS_1^S = \frac{(a-k_1)^2}{32}, \tag{4.67}$$

$$CS_2^S = \frac{(a-k_2)^2}{8}. \tag{4.68}$$

Consequently, the total consumer surplus is

$$CS^S = CS_1^S + CS_2^S = \frac{(a-k_1)^2}{32} + \frac{(a-k_2)^2}{8}. \tag{4.69}$$

The producer surplus is the sum of profits of all firms in the channel. Here, the producer surplus is

$$PS^S = \frac{3k_1^2 + 4k_2^2 - 6ak_1 - 8ak_2 + 7a^2}{16}, \tag{4.70}$$

and total welfare is

$$W^S = PS^S + CS^S = \frac{19a^2 - 14ak_1 - 24ak_2 + 7k_1^2 + 12k_2^2}{32}. \tag{4.71}$$

Bundling

The consumer surplus in the market for bundle b_A is given by

$$CS_A = \frac{2a - p_{b_A}}{2} Q_{b_A}, \tag{4.72}$$

and in the market for bundle b_B by

$$CS_B = \frac{a - p_{b_B}}{2} Q_{b_B}. \tag{4.73}$$

We insert $p_{b_A}^{BU} = \frac{2(282a+51k_1+80k_2)}{413}$ and $Q_{b_A}^{BU} = \frac{3(34a-29k_1-5k_2)}{413}$ into (4.72) as well as $p_{b_B}^{BU} = \frac{253a+15k_1+145k_2}{413}$ and $Q_{b_B}^{BU} = \frac{2(29a+36k_1-65k_2)}{413}$ into (4.73). We receive

$$CS_A^{BU} = \frac{3(34a - 5k_2 - 29k_1)(131a - 80k_2 - 51k_1)}{170569}, \tag{4.74}$$

$$CS_B^{BU} = \frac{5(29a - 65k_2 + 36k_1)(32a - 29k_2 - 3k_1)}{170569}. \tag{4.75}$$

The total consumer surplus under bundling amounts to

$$\begin{aligned} CS^{BU} &= CS_A^{BU} + CS_B^{BU} \\ &= \frac{18002a^2 + 3897k_1^2 + 3480k_1k_2 + 10625k_2^2 - 11274ak_1 - 24730ak_2}{170569}. \end{aligned} \tag{4.76}$$

The producer surplus is given by

$$PS^{BU} = \frac{28137k_1^2 + 5430k_1k_2 + 38635k_2^2 - 61704ak_1 - 82700ak_2 + 72202a^2}{170569}. \tag{4.77}$$

The total welfare is then

$$\begin{aligned} W^{BU} &= CS^{BU} + PS^{BU} \\ &= \frac{6(15034a^2 + 5339k_1^2 + 1485k_1k_2 + 8210k_2^2 - 12163ak_1 - 17905ak_2)}{170569}. \end{aligned} \tag{4.78}$$

Further Calculations

- The price of bundle b_A is greater than the price of retailer R_B's bundle b_B when $\Delta p^{BU} = p_{b_A}^{BU} - p_{b_B}^{BU} = \frac{311a+15k_2+87k_1}{413} > 0$. This condition is obviously fulfilled.

- The price of bundle b_A is larger than the sum of retail prices under separate selling when $\Delta p_{b_A} = p_1^S + p_2^S - p_{b_A}^{BU} = \frac{-191a+186k_2+5k_1}{1652} < 0$. The condition $\Delta p_{b_A} < 0$ is satisfied for $191a > 5k_1 + 186k_2$, which is clearly given because of $a > k_1, k_2$.

- R_B's price is raised by bundling if $\Delta p_{b_B} = p_2^S - p_{b_B}^{BU} = \frac{3(-31a+41k_2-10k_1)}{826} < 0$. We have $\Delta p_{b_B} < 0$ for $k_2 < \frac{31a+10k_1}{41}$, which is always fulfilled because it holds that $k_2 < \frac{29a+36k_1}{65} < \frac{31a+10k_1}{41}$.

- Retailer R_A's quantity and thus the total quantity of good 1 is increased due to bundling when $\Delta q_{A1} = q_{A1}^S - Q_{b_A}^{BU} = \frac{5(a+12k_2-13k_1)}{1652} < 0$, which is fulfilled for $k_2 < \frac{13k_1-a}{12}$. The condition $k_2 < \frac{13k_1-a}{12}$ is obviously always given under profitable bundling. Retailer R_A's quantity of good 2 is decreased as a consequence of bundling in case we have $\Delta q_{A2} = q_{A2}^S - Q_{b_A}^{BU} = \frac{5a-352k_2+348k_1}{1652} > 0$. We have $\Delta q_{A2} > 0$ when $k_2 < \frac{5a+348k_1}{353}$, which is satisfied with certainty in the bundling equilibrium since $\frac{13k_1-a}{12} < \frac{5a+348k_1}{353}$.

- Comparing retailer R_B's quantities, we get $\Delta q_{B2} = q_{B2}^S - Q_{b_B}^{BU} = \frac{181a+107k_2-288k_1}{1652} < 0$ for $k_2 < \frac{-181a+288k_1}{107}$. Note that $\frac{-181a+288k_1}{107} > 0$ only for $k_1 > \frac{181}{288}a$. Further, note that $\frac{13k_1-a}{12} > \frac{-181a+288k_1}{107}$ and $\frac{181}{288}a > \frac{a}{13}$. Finally, when $k_1 > \frac{181}{288}a$ holds, profitable bundling increases R_B's quantity if $0 < k_2 < \frac{-181a+288k_1}{107}$ and decreases R_B's quantity if $\frac{13k_1-a}{12} > k_2 > \frac{-181a+288k_1}{107}$. In case $\frac{a}{13} < k_1 < \frac{181}{288}a$, profitable bundling decreases R_B's quantity with certainty since then $\frac{13k_1-a}{12} > k_2 > 0 > \frac{-181a+288k_1}{107}$ holds in the equilibrium.

- Bundling reduces the quantity of good 2 if $\Delta Q_2 = Q_2^S - Q_2^{BU} = \frac{3(31a-41k_2+10k_2)}{826} > 0$, which is fulfilled for $k_2 < \frac{31a+10k_1}{41}$. The condition $k_2 < \frac{31a+10k_1}{41}$ is always met under (profitable) bundling since $k_2 < \frac{13k_1-a}{12} < \frac{29a+36k_1}{65} < \frac{31a+10k_1}{41}$.

- In the *centralized channel*, we impose $k_2 < \frac{a+k_1}{2}$ to ensure $Q_{b_B}^{BU} = \frac{2(a-2k_2+k_1)}{7} > 0$. The restriction $k_2 < \frac{a+k_1}{2}$ also guarantees $p_1^S > p_2^S$. Regarding R_A's profit, we obtain $\Delta\pi_A = \pi_A^S - \Pi_A^{BU} = -\frac{(a-2k_2+k_1)(15a-2k_2-13k_1)}{196}$. Note that $\Delta\pi_A$ is quadratic and strictly concave in k_2 as $\frac{\partial^2 \Delta\pi_A}{\partial k_2^2} = -\frac{2}{49} < 0$. We obtain $\Delta\pi_A < 0$ when $k_2 < \frac{a+k_1}{2}$ or when $k_2 > \frac{15a-13k_1}{2}$. As $k_2 < \frac{a+k_1}{2}$ by assumption, we always have $\Delta\pi_A < 0$. In conclusion, bundling always increases retailer R_A's profit in the centralized channel.

- In case we have only a *multi-product manufacturer* producing both goods, we need to impose the assumption $k_2 < \frac{a+k_1}{2}$ to ensure $Q_{b_B}^{BU} = \frac{a-2k_2+k_1}{7} > 0$. As above, this assumption also guarantees $p_1^S > p_2^S$. Moreover, the profit difference for R_A's profits is $\Delta\pi_A = \pi_A^S - \Pi_A^{BU} = -\frac{(a-2k_2+k_1)(15a-2k_2-13k_1)}{784}$, where $\Delta\pi_A$ is quadratic and strictly concave in k_2 since $\frac{\partial^2 \Delta\pi_A}{\partial k_2^2} = -\frac{1}{98} < 0$. We obtain $\Delta\pi_A < 0$ for $k_2 < \frac{a+k_1}{2}$ or for $k_2 > \frac{15a-13k_1}{2}$. Since $k_2 < \frac{a+k_1}{2}$ is given by assumption, bundling is here also more profitable than separate selling for R_A in any case, like in the centralized channel.

4.6.2 Retail Quantity Competition

Equilibrium Conditions

Separate Selling

Retailer R_A's separate selling profit is strictly concave in q_{A1} and q_{A2} because we have $\frac{\partial^2 \pi_A}{\partial q_{A1}^2} = -2 < 0$ and $\frac{\partial^2 \pi_A}{\partial q_{A2}^2} = -2 < 0$. The FOC for R_A's optimal quantity of good 1 reads

$$\frac{\partial \pi_A}{\partial q_{A1}} = a - 2q_{A1} - w_1 \stackrel{!}{=} 0. \tag{4.79}$$

Solving the FOC for q_{A1} yields R_A's profit-maximizing quantity of good 1.

The FOC that generates R_A's optimal quantity of good 2 reads

$$\frac{\partial \pi_A}{\partial q_{A2}} = a - q_{B2} - 2q_{A2} - w_2 \stackrel{!}{=} 0. \tag{4.80}$$

Solving the FOC for q_{A2} gives us R_A's reaction function

$$q_{A2}(q_{B2}) = \frac{a - q_{B2} - w_2}{2}. \tag{4.81}$$

For retailer R_B's profit, $\frac{\partial^2 \pi_B}{\partial q_{B2}^2} = -2 < 0$ holds. The FOC determining retailer R_B's optimal quantity of good 2 is

$$\frac{\partial \pi_B}{\partial q_{B2}} = a - q_{B2} - 2q_{A2} - w_2 \stackrel{!}{=} 0, \tag{4.82}$$

and solving the FOC for q_{B2} leads to the reaction function

$$q_{B2}(q_{A2}) = \frac{a - q_{A2} - w_2}{2}. \tag{4.83}$$

The intersection of the two reaction functions (4.81) and (4.83) provides the profit-maximizing quantities of good 2 for both retailers.

In the upstream market, $\frac{d^2\pi_1}{dw_1^2} = -1 < 0$ and $\frac{d^2\pi_2}{dw_2^2} = -\frac{4}{3} < 0$ hold. The FOCs for the optimal wholesale price of good 1 and good 2, respectively, are

$$\frac{d\pi_1}{dw_1} = \frac{a + k_1 - 2w_1}{2} \stackrel{!}{=} 0, \tag{4.84}$$

$$\frac{d\pi_2}{dw_2} = \frac{2(a + k_2 - 2w_2)}{3} \stackrel{!}{=} 0. \tag{4.85}$$

Solving the FOCs for w_1 and w_2, respectively, generates the equilibrium wholesale prices.

Bundling

Retailer R_A's profit and retailer R_B's profit are strictly concave in Q_{b_A} and Q_{b_B}, respectively, because $\frac{\partial^2 \Pi_A}{\partial Q_{b_A}^2} = -4 < 0$ and $\frac{\partial^2 \Pi_B}{\partial Q_{b_B}^2} = -2 < 0$. The FOCs with respect to the optimal quantities of the bundles are

$$\frac{\partial \Pi_A}{\partial Q_{b_A}} = 2a - Q_{b_B} - 4Q_{b_A} - w_2 - w_1 \stackrel{!}{=} 0, \tag{4.86}$$

$$\frac{\partial \Pi_B}{\partial Q_{b_B}} = a - 2Q_{b_B} - Q_{b_A} - w_2 \stackrel{!}{=} 0. \tag{4.87}$$

We derive the corresponding reaction functions by solving the FOCs for Q_{b_A} and Q_{b_B}, respectively, and obtain

$$Q_{b_A}(Q_{b_B}) = \frac{2a - Q_{b_B} - w_2 - w_1}{4}, \tag{4.88}$$

$$Q_{b_B}(Q_{b_A}) = \frac{a - Q_{b_A} - w_2}{2}. \tag{4.89}$$

The intersection of the two reaction functions gives us the equilibrium quantities of the two bundles.

For the manufacturers, we have $\frac{\partial^2 \pi_1}{\partial w_1^2} = -\frac{4}{7} < 0$ and $\frac{\partial^2 \pi_2}{\partial w_2^2} = -\frac{8}{7} < 0$. The FOCs regarding the optimal wholesale prices of the manufacturers M_1 and M_2 are

$$\frac{\partial \pi_1}{\partial w_1} = \frac{3a + 2k_1 - w_2 - 4w_1}{7} \stackrel{!}{=} 0, \tag{4.90}$$

$$\frac{\partial \pi_2}{\partial w_2} = \frac{5a + 4k_2 - 8w_2 - w_1}{7} \stackrel{!}{=} 0. \tag{4.91}$$

We derive the following reaction functions of the manufacturers:

$$w_1(w_2) = \frac{3a - w_2 + 2k_1}{4}, \tag{4.92}$$

$$w_2(w_1) = \frac{5a - w_1 + 4k_2}{8}. \tag{4.93}$$

The intersection of the reaction functions determines the equilibrium wholesale prices.

Multi-Product Upstream Monopoly

Separate Selling

Again assume that solely firm M_{12} produces goods 1 and 2. Given quantity competition, manufacturer M_{12}'s optimization problems under separate selling and bundling are analogous to the according ones under retail price competition. For the separate selling market, it holds again that the two goods are independent in demand. Consequently, the equilibrium wholesale prices in the setting, where M_{12} is the only manufacturer, are the same as in the setting, where we have two independent manufacturers, i.e. $w_1^S = \frac{a+k_1}{2}$ and $w_2^S = \frac{a+k_2}{2}$.

Bundling

Consider that R_A bundles. Then, we have strict concavity because of $\frac{\partial^2 \pi_{12}}{\partial w_1^2} = -\frac{4}{7} < 0$ and $\frac{\partial^2 \pi_{12}}{\partial w_2^2} = -\frac{8}{7} < 0$. The FOCs for M_{12}'s equilibrium wholesale prices read

$$\frac{\partial \pi_{12}}{\partial w_1} = \frac{3a + k_2 - 2w_2 + 2k_1 - 4w_1}{7} \stackrel{!}{=} 0, \tag{4.94}$$

$$\frac{\partial \pi_{12}}{\partial w_2} = \frac{5a + 4k_2 - 8w_2 + k_1 - 2w_1}{7} \stackrel{!}{=} 0. \tag{4.95}$$

We receive the profit-maximizing wholesale prices by solving the equation system above for w_1 and w_2.

Welfare Outcomes

Separate Selling

We plug $p_1^S = \frac{3a+k_1}{4}$ and $Q_1^S = q_{A1}^S = \frac{a-k_1}{4}$ into (4.65) as well as $p_2^S = \frac{2a+k_2}{3}$ and $Q_2^S = q_{A2}^S + q_{B2}^S = \frac{a-k_2}{3}$ into (4.66). We obtain

$$CS_1^S = \frac{(a-k_1)^2}{32}, \tag{4.96}$$

$$CS_2^S = \frac{(a-k_2)^2}{18}. \tag{4.97}$$

The total consumer surplus then amounts to

$$CS^S = CS_1^S + CS_2^S = \frac{25a^2 + 9k_1^2 + 16k_2^2 - 18ak_1 - 32ak_2}{288}. \tag{4.98}$$

The producer surplus is

$$PS^S = \frac{59a^2 - 64ak_2 - 54ak_1 + 32k_2^2 + 27k_1^2}{144}. \tag{4.99}$$

The total welfare is

$$W^S = CS^S + PS^S = \frac{143a^2 + 63k_1^2 + 80k_2^2 - 126ak_1 - 160ak_2}{288}. \tag{4.100}$$

Bundling

We insert $p_{b_A}^{BU} = \frac{2(164a+19k_1+34k_2)}{217}$ and $Q_{b_A}^{BU} = \frac{2(19a-15k_1-4k_2)}{217}$ into (4.72) as well as $p_{b_B}^{BU} = \frac{149a+8k_1+60k_2}{217}$ and $Q_{b_B}^{BU} = \frac{2(15a+11k_1-26k_2)}{217}$ into (4.73). We get

$$CS_A^{BU} = \frac{2(19a - 4k_2 - 15k_1)(53a - 34k_2 - 19k_1)}{47089}, \tag{4.101}$$

$$CS_B^{BU} = \frac{4(15a - 26k_2 + 11k_1)(17a - 15k_2 - 2k_1)}{47089}. \tag{4.102}$$

The total consumer surplus amounts to

$$\begin{aligned} CS^{BU} &= CS_A^{BU} + CS_B^{BU} \\ &= \frac{2(1517a^2 + 241k_1^2 + 360k_1k_2 + 916k_2^2 - 842ak_1 - 2192ak_2)}{47089}. \end{aligned} \tag{4.103}$$

The producer surplus is

$$PS^{BU} = \frac{2(2773k_1^2 + 1016k_1k_2 + 4678k_2^2 - 6562ak_1 - 1372ak_2 + 8,467a^2)}{47089}, \tag{4.104}$$

and total welfare reads

$$
\begin{aligned}
W^{BU} &= CS^{BU} + PS^{BU} \\
&= \frac{4(4992a^2 + 1507k_1^2 + 688k_1k_2 + 2797k_2^2 - 3702ak_1 - 6282ak_2)}{47089}.
\end{aligned}
\tag{4.105}
$$

Further Calculations

- The sum of retail prices under separate selling is smaller than the price of bundle b_A if $\Delta p_{b_A} = p_1^S + p_2^S - p_{b_A}^{BU} = \frac{3(-19a+4k_2+15k_1)}{2604} < 0$, which is given if $19a > 4k_2 + 15k_1$. The condition $19a > 4k_2 + 15k_1$ is always fulfilled because of $a > k_1, k_2$.

- Retailer R_B's price is raised by bundling if $\Delta p_{b_B} = p_2^S - p_{b_B}^{BU} = \frac{-13a+37k_2-24k_1}{651} < 0$. We find that R_B's price is always greater under bundling than under separate selling. This is because $\Delta p_{b_B} < 0$ holds for $k_2 < \frac{13a+24k_1}{651}$, which is met with certainty because of $k_2 < \frac{a+3k_1}{4} < \frac{13a+24k_1}{37}$.

- The difference in R_A's quantity of good 2 is $\Delta q_{A2} = q_{A2}^S - Q_{b_A}^{BU} = \frac{-11a-169k_2+180k_1}{1302}$, where $\Delta q_{A2} < 0$ for $k_2 > \frac{180k_1-11a}{169}$. Note that $\frac{180k_1-11a}{169} < \frac{a+3k_1}{4}$ and that $\frac{180k_1-11a}{169} > 0$ when $k_1 > \frac{11}{180}a$. Thus, when $k_1 > \frac{11}{180}a$, bundling increases R_A's quantity of good 2 if $\frac{a+3k_1}{4} > k_2 > \frac{180k_1-11a}{169}$ and decreases it if $0 < k_2 < \frac{180k_1-11a}{169}$. Given $k_1 < \frac{11}{180}a$, we have $k_2 > 0 > \frac{180k_1-11a}{169}$ and then R_A's quantity of good 2 is increased due to bundling.

- Retailer R_B's quantity is decreased by bundling if $\Delta q_{B2} = q_{B2}^S - Q_{b_B}^{BU} > 0$, which is fulfilled when $k_2 > \frac{132k_1-37a}{95}$. We have $\frac{132k_1-37a}{95} < \frac{a+3k_1}{4}$ and $\frac{132k_1-37a}{95} > 0$ if $k_1 > \frac{37}{132}a$. Consequently, in case $k_1 > \frac{37}{132}a$, retailer R_B's quantity is reduced due to bundling for $\frac{a+3k_1}{4} > k_2 > \frac{132k_1-37a}{95}$ and it is increased for $0 < k_2 < \frac{132k_1-37a}{95}$. If $k_1 < \frac{37}{132}a$, bundling always reduces R_B's quantity as then $k_2 > 0 > \frac{132k_1-37a}{95}$.

- If $\Delta Q_2 = Q_2^S - Q_2^{BU} = \frac{13a-37k_2+24k_1}{651} > 0$, the quantity of good 2 is lowered by bundling. The condition $\Delta Q_2 > 0$ is given if $k_2 < \frac{13a+24k_1}{37}$, which is fulfilled with certainty since $k_2 < \frac{a+3k_1}{4} < \frac{13a+24k_1}{37}$.

- In the centralized channel, we need to assume $k_2 < \frac{2a+k_1}{3}$ in order to guarantee that $Q_{b_B}^{BU} = \frac{2a-3k_2+k_1}{7} > 0$ and $k_2 < \frac{a+3k_1}{4}$ to guarantee $p_1^S > p_2^S$. As $\frac{2a+k_1}{3} > \frac{a+3k_1}{4}$, it suffices to assume $k_2 < \frac{a+3k_1}{4}$. Also, $\Delta\pi_A = \pi_A^S - \Pi_A^{BU} = -\frac{(a+2k_2-3k_1)(11a-62k_2+51k_1)}{1764} < 0$ holds either for $k_2 \in \left(\frac{3k_1-a}{2}, \frac{11a+51k_1}{62}\right)$ if $k_1 > \frac{a}{3}$ or for $k_2 \in \left(0, \frac{11a+51k_1}{62}\right)$ if $k_1 < \frac{a}{3}$ since $\Delta\pi_A$ is quadratic and strictly convex in k_2 as $\frac{\partial^2 \Delta\pi_A}{\partial k_2^2} = \frac{62}{441} > 0$. When $\Delta\pi_A < 0$, retailer R_A's bundling profit exceeds his separate selling. Note that $\frac{11a+51k_1}{62} < \frac{a+3k_1}{4}$. Our results demonstrate that bundling may be profitable for R_A in the centralized case, depending on the marginal costs of production of the manufacturers.

- With a single manufacturer, we suppose $k_2 < \frac{2a+k_1}{3}$ to ensure $Q_{b_B}^{BU} = \frac{2a-3k_2+k_1}{14} > 0$ and $k_2 < \frac{a+3k_1}{4}$ to ensure $p_1^S > p_2^S$. Again, it suffices to assume $k_2 < \frac{a+3k_1}{4}$. We get $\Delta\pi_A = \pi_A^S - \Pi_A^{BU} = -\frac{11a^2-40ak_2+18ak_1-124k_2^2+288k_1k_2-441k_2+288k_1^2}{7056} < 0$ either for $k_2 \in \left(\frac{3k_1-a}{2}, \frac{11a+51k_1}{62}\right)$ if $k_1 > \frac{a}{3}$ or for $k_2 \in \left(0, \frac{11a+51k_1}{62}\right)$ if $k_1 < \frac{a}{3}$ because $\Delta\pi_A$ is quadratic and strictly convex in k_2 since $\frac{\partial^2\Delta\pi_A}{\partial k_2^2} = \frac{31}{882} > 0$. In conclusion, the bundling incentives here are analogous to the centralized channel.

4.6.3 Proof of Theorem 4.1

Retailer R_A's bundling profit exceeds his separate selling profit if and only if

$$\begin{aligned}\Delta\pi_A &= \pi_A^S - \Pi_A^{BU} \\ &= \frac{4105a^2 + a(-57170k_1 + 48960k_2) - 3600k_2^2 - 41760k_1k_2 + 49465k_1^2}{2729104} < 0.\end{aligned} \tag{4.106}$$

Notice that $\Delta\pi_A$ is quadratic and strictly concave in k_2 $\left(\frac{\partial^2\Delta\pi_A}{\partial k_2^2} = -\frac{450}{170569} < 0\right)$. Solving for k_2 yields that $\Delta\pi_A > 0$ for $k_2 \in \left(\frac{13k_1-a}{12}, \frac{821a-761k_1}{60}\right)$. We obtain $\Delta\pi_A < 0$ in case $k_2 < \frac{13k_1-a}{12}$ or $k_2 > \frac{821a-761k_1}{60}$. The marginal cost k_2 is bounded from below by 0 and from above by $\frac{29a+36k}{65}$, where $\frac{29a+36k}{65} < \frac{821a-761k_1}{60}$ and $\frac{13k_1-a}{12}$ may be positive or negative. We have $\frac{13k_1-a}{12} > 0$ if $k_1 > \frac{a}{13}$. Therefore, we can derive the bundling incentives as follows:

(a) If $k_1 < \frac{a}{13}$, then $\frac{13k_1-a}{12} < 0$ and, consequently, we have $k_2 > 0 > \frac{13k_1-a}{12}$. If $k_2 > \frac{13k_1-a}{12}$, then $\Delta\pi_A > 0$ holds, which means that R_A gains a higher profit by separate selling than by bundling. Thus, R_A prefers separate selling over bundling in case $k_1 < \frac{a}{13}$.

(b) If $k_1 > \frac{a}{13}$, then $\Delta\pi_A > 0$ holds for $k_2 \in \left(\frac{13k_1-a}{12}, \frac{29a+36k}{65}\right)$. Consequently, R_A prefers separate selling over bundling when $k_1 > \frac{a}{13}$ and $k_2 \in \left(\frac{13k_1-a}{12}, \frac{29a+36k}{65}\right)$.

(c) If $k_1 > \frac{a}{13}$, we obtain $\Delta\pi_A < 0$ for $k_2 \in \left(0, \frac{13k_1-a}{12}\right)$. When $\Delta\pi_A < 0$, then bundling raises R_A's profit in comparison to separate selling. In conclusion, bundling is R_A's equilibrium strategy if $k_1 > \frac{a}{13}$ and $k_2 \in \left(0, \frac{13k_1-a}{12}\right)$.

4.6.4 Proof of Proposition 4.2

Manufacturer M_1's wholesale price under bundling exceeds his wholesale price under separate selling, i.e. $w_1^{BU} > w_1^S$, if $k_2 < \frac{9a+k_1}{10}$. Since $\frac{29a+36k_1}{65} < \frac{9a+k_1}{10}$, we have $k_2 < \frac{29a+36k_1}{65} < \frac{9a+k_1}{10}$. Hence, $w_1^{BU} > w_1^S$ holds in any case.

Manufacturer M_2's wholesale price under bundling exceeds his wholesale price under separate selling, i.e. $w_2^{BU} > w_2^S$, if $k_2 > -5a + 6k_1$, where $-5a + 6k_1 < \frac{29a+36k_1}{65}$ and $-5a + 6k_1 > 0$ for $k_1 > \frac{5}{6}a$. In case $k_1 > \frac{5}{6}a$, we obtain $w_2^{BU} < w_2^S$ if $k_2 \in (0, -5a + 6k_1)$ and $w_2^{BU} > w_2^S$ if $k_2 \in \left(-5a + 6k_1, \frac{29a+36k_1}{65}\right)$. Furthermore, if $k_1 < \frac{5}{6}a$, we have $k_2 > 0 > -5a + 6k_1$ and hence $w_2^{BU} > w_2^S$ is given.

The sum of wholesale prices amounts to $w_1^S + w_2^S = \frac{2a+k_1+k_2}{2}$ under separate selling and to $w_1^{BU} + w_2^{BU} = \frac{66a+25k_2+27k_1}{59}$ under bundling. We have $\frac{2a+k_1+k_2}{2} < \frac{66a+25k_2+27k_1}{59}$ when $k_2 < \frac{14a-5k_1}{9}$, which is always fulfilled because of $k_2 < \frac{29a+36k_1}{65} < \frac{14a-5k_1}{9}$. Consequently, the sum of wholesale prices under bundling is always greater than the sum of wholesale prices under separate selling.

4.6.5 Proof of Proposition 4.3

We prove the cases according to the cases in the proposition:

(a) Manufacturer M_1's profit is increased by bundling if and only if

$$\begin{aligned}\Delta\pi_1 &= \pi_1^S - \pi_1^{BU} \qquad (4.107)\\ &= \frac{4183k_1^2 - 6960k_1k_2 - 600k_2^2 - 1406ak_1 + 8160ak_2 - 3377a^2}{194936} < 0,\end{aligned}$$

where $\Delta\pi_1$ is quadratic and strictly concave in k_2 $\left(\frac{\partial^2\Delta\pi_1}{\partial k_2^2} = -\frac{150}{24367} < 0\right)$. By solving for k_2, we derive that $\Delta\pi_1 > 0$ if $k_2 \in \left(\frac{25.636a+34.364k_1}{60}, \frac{790.364a-730.364k_1}{60}\right)$. Consequently, in case $k_2 < \frac{25.636a+34.364k_1}{60}$ or $k_2 > \frac{790.364a-730.364k_1}{60}$, we have $\Delta\pi_1 < 0$. It holds that $\frac{25.636a+34.364k_1}{60} > \frac{13k_1-a}{12}$. Thus, with profitable bundling, we have $k_2 < \frac{13k_1-a}{12} < \frac{25.636a+34.364k_1}{60}$ and therefore $\Delta\pi_1 < 0$ holds. In conclusion, M_1's profit is always increased by profitable bundling.

(b) Manufacturer M_2's profit is raised by bundling if and only if

$$\begin{aligned}\Delta\pi_2 &= \pi_2^S - \pi_2^{BU} \qquad (4.108)\\ &= \frac{-180k_1^2 - 3480k_1k_2 + 7547k_2^2 + 3840ak_1 - 11614ak_2 + 3887a^2}{97468} < 0.\end{aligned}$$

Notice that $\Delta\pi_2$ is quadratic and strictly convex in k_2 $\left(\frac{\partial^2\Delta\pi_2}{\partial k_2^2} = \frac{7547}{48734} > 0\right)$. Solving for k_2 yields that $\Delta\pi_2 < 0$ for $k_2 \in \left(\frac{3712.708a+3834.292k_1}{7547}, \frac{7901.292a-354.292k_1}{7547}\right)$. Furthermore, it yields that $\Delta\pi_2 > 0$ for $k_2 < \frac{3712.708a+3834.292k_1}{7547}$ or $k_2 > \frac{7901.292a-354.292k_1}{7547}$. We have $\frac{3712.708a+3834.292k_1}{7547} > \frac{13k_1-a}{12}$ and hence $k_2 < \frac{13k_1-a}{12} < \frac{3712.708a+3834.292k_1}{7547}$ in the bundling equilibrium. This means that $\Delta\pi_2 > 0$ is always satisfied under profitable bundling and therefore profitable bundling leads to a lower profit than separate selling for manufacturer M_2 in any case.

4.6.6 Proof of Proposition 4.4

We prove the cases according to the cases in the proposition:

(a) The consumer surplus is increased by bundling if and only if

$$\begin{aligned}\Delta CS &= CS^S - CS^{BU} \\ &= \frac{276781a^2 - 573192ak_2 + 19630ak_1 + 342276k_2^2 - 111360k_1k_2 + 45865k_1^2}{5458208} < 0,\end{aligned} \tag{4.109}$$

where ΔCS is quadratic and strictly convex in k_2 $\left(\frac{\partial^2 \Delta CS}{\partial k_2^2} = \frac{85569}{682276} > 0\right)$ with its vertex regarding k_2 at $V\left(\frac{23883a+4640k_1}{28523}\middle|\frac{6155(a-k_1)^2}{912736}\right)$. We clearly have $\Delta CS > 0$ because of $\frac{6155(a-k_1)^2}{912736} > 0$. Hence, the consumer surplus decreases with certainty when R_A bundles.

(b) The producer surplus is increased by bundling if and only if

$$\begin{aligned}\Delta PS &= PS^S - PS^{BU} \\ &= \frac{3(20505k_1^2 - 28960k_1k_2 + 21372k_2^2 - 12050ak_1 - 13784ak_2 + 12917a^2)}{2729104} < 0.\end{aligned} \tag{4.110}$$

Note that ΔPS is quadratic and strictly convex in k_2 $\left(\frac{\partial^2 \Delta PS}{\partial k_2^2} = \frac{16029}{341138} > 0\right)$. Regarding k_2, ΔPS has its vertex at $V\left(\frac{1723a+3620k_1}{5343}\middle|\frac{335(a-k_1)^2}{28496}\right)$. As $\frac{335(a-k_1)^2}{28496} > 0$ holds, we always have $\Delta PS > 0$. In conclusion, bundling always leads to a reduction in producer surplus.

(c) As the two previous cases show, producer and consumer surplus are always lowered by bundling and hence the total welfare is always lowered as well.

4.6.7 Proof of Proposition 4.5

Retailer R_A's bundling profit exceeds his separate selling profit under quantity competition if and only if

$$\begin{aligned}\Delta\pi_A &= \pi_A^S - \Pi_A^{BU} \\ &= \frac{196285a^2 - 201608ak_2 - 190962ak_1 + 169924k_2^2 - 138240k_1k_2}{6780816} \\ &+ \frac{423801k_2 - 259200k_1^2}{6780816} < 0. \qquad (4.111)\end{aligned}$$

Note that $\Delta\pi_A$ is quadratic and strictly convex in k_2 $\left(\frac{\partial^2 \Delta\pi_A}{\partial k_2^2} = \frac{42481}{847602} > 0\right)$. The vertex of $\Delta\pi_A$ with respect to k_2 is given as $V\left(\frac{25201a+17280k_1}{42481}\middle|\frac{13681(a-k_1)^2}{679696}\right)$. Notice that $\frac{13681(a-k_1)^2}{679696} > 0$. This means that we always have $\Delta\pi_A > 0$, which implies that R_A's separate selling profit exceeds his bundling profit in any case. As a consequence, R_A does not play a bundling strategy in the equilibrium under quantity competition.

4.6.8 Proof of Proposition 4.6

Manufacturer M_1's wholesale price under bundling is greater than his wholesale price under separate selling, i.e. $w_1^{BU} > w_1^S$, if $k_2 < \frac{7a+k_1}{8}$. The marginal cost k_2 is restricted from above by $\frac{a+3k_1}{4}$. We have $k_2 < \frac{a+3k_1}{4} < \frac{7a+k_1}{8}$ and, consequently, $w_1^{BU} > w_1^S$ always holds.

Manufacturer M_2's wholesale price in the bundling market exceeds his wholesale price in the separate selling market, i.e. $w_2^{BU} > w_2^S$, if $k_2 > -3a + 4k_1$. We have $-3a + 4k_1 > 0$ if $k_1 > \frac{3}{4}a$. For $k_1 > \frac{3}{4}a$ and $k_2 \in (0, -3a + 4k_1)$, the wholesale price of good 2 diminishes due to bundling. When $k_1 > \frac{3}{4}a$ and $k_2 \in \left(-3a + 4k_1, \frac{a+3k_1}{4}\right)$, however, the wholesale price increases. In case $k_1 < \frac{3}{4}a$, $k_2 > 0 > -3a + 4k_1$ holds, which implies that the wholesale price of good 2 rises as a consequence of bundling.

Under separate selling, the sum of wholesale prices amounts to $w_1^S + w_2^S = \frac{2a+k_2+k_1}{2}$, and to $w_1^{BU} + w_2^{BU} = \frac{2(18a+6k_2+7k_1)}{31}$ under bundling, where $\frac{2a+k_2+k_1}{2} < \frac{2(18a+6k_2+7k_1)}{31}$ is fulfilled if $k_2 < \frac{10a-3k_1}{7}$. The condition $k_2 < \frac{10a-3k_1}{7}$ is satisfied in any case because of $k_2 < \frac{a+3k_1}{4} < \frac{10a-3k_1}{7}$. Thereby, the sum of wholesale prices under bundling is always greater than the sum of wholesale prices under separate selling.

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