Essays on the Theory of Industrial Organization: Credence Goods, Vertical Relations, and Product Bundling

Der Fakultät für Wirtschaftswissenschaften der
Universität Paderborn

zur Erlangung des akademischen Grades
Doktor der Wirtschaftswissenschaften
- Doctor rerum politicarum -

vorgelegte Dissertation
von

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geboren am 28.10.1988
in Mönchengladbach

Oktober 2019

Chapter 6

Oligopolistic Upstream Competition with Differentiated Inputs

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Abstract

We consider a vertical supply chain that consists of a downstream final good producer and $n \geq 2$ upstream intermediate good producers. The final good producer transforms the n differentiated inputs into an output good via a CES production function and sells the composed good to the final consumers. We study the impact of upstream price and upstream quantity competition on the supply chain. We find that the intermediate good producers prefer price over quantity competition when the inputs are complements and vice versa when they are substitutes. However, the final good producer and the consumers prefer price over quantity competition for all degrees of input differentiation. We additionally observe that the welfare optimal solution materializes when a horizontally integrated upstream market merges vertically with the downstream producer.

JEL classification: L00; L13

Keywords: CES production function; Input competition; Product differentiation

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6.1 Introduction

The modern production process is often characterized by a supply chain of finite lengths in which different intermediate goods are successively combined to a final good. Consider the production of a car, for example. A car manufacturer sells the final good (the entire car) in the downstream market to the consumer. In order to build the car, however, the manufacturer acts as a consumer of intermediate goods in the upstream market. A car is made of complementary intermediate goods such as tires, seats, a body etc. Typically, such intermediate goods are supplied by different and competing suppliers. Another example consists of fruit juices or fruit smoothies, which are compositions of various different fruits that can be regarded substitutable inputs and may also be procured from competing suppliers. The present paper models the impact of such complementarity and substitutability of inputs on market outcomes. In our framework, the final good is a composition of inputs that are supplied by an oligopolistic upstream market. We would like to emphasize that inputs are generally not transformed on a one-to-one basis into outputs, but that the underlying production technology is captured by a constant elasticity of substitution (CES) production function (Arrow et al., 1961). We also take into account that there is often competition between input suppliers and consider cases without input competition or vertical separation as points of comparison.

Our model consists of three levels: a representative consumer buys a homogeneous good from a monopolistic final good producer in the downstream market. In order to produce the final good, the final good producer purchases heterogeneous intermediate goods from $n \geq 2$ upstream intermediate good producers. The n inputs are transformed into one unit of output via a CES production technology. We analyze price and quantity competition in the oligopolistic market for intermediate goods and then successively integrate the market from top to bottom. We will address the following research questions:

1. *How do the market results such as prices, quantities and profits vary under different modes of upstream competition (price or quantity) and with respect to integration (horizontal or vertical)?*

2. *What is the impact of intermediate good differentiation (different degrees of substitutability or complementarity) and the intensity of competition (number of upstream firms) on market results?*

3. *What are the welfare effects of the various market settings?*

There exists vast industrial organization literature that studies the impact of product differentiation on market outcomes. Product differentiation is often enclosed in the utility function of the consumer where one distinguishes between a linear-quadratic utility function that triggers a linear demand system (Dixit, 1979; Singh and Vives, 1984; Häckner,

2000) or a CES utility (Spence, 1976b; Dixit and Stiglitz, 1977). In their seminal work, Singh and Vives (1984) present a model that builds on the model of Dixit (1979) and which analyzes a duopoly with complementary and substitutable products. They find that prices under quantity competition are always higher than under price competition and that this has different effects on the profits of the firms, depending on the demand relations between products. If the products are substitutes, firms gain higher profits with quantity competition than with price competition. By contrast, if the products are complements, price competition results in higher profits for the firms than quantity competition. Häckner (2000) extends the Dixit (1979) model to a more general framework with an arbitrary number of firms and vertical product differentiation. Häckner's findings indicate that the results in Singh and Vives (1984) depend on the duopoly assumption. He illustrates that when there are more than two firms and the goods are complements, then the low quality firms may charge higher prices under price competition than under quantity competition. Additionally, the profits of the high-quality firms may be greater under price competition than under quantity competition for substitutable products if the quality differences between the firms are large.

Further seminal contributions to the literature on oligopolies with differentiated products were made by Spence (1976a,b).[1] He highlights that imperfect competition may lead to overprovision of (imperfect) substitutes and underprovision of complementary products. In case of substitutes, the existing firms suffer when more firms enter because they lose demand. Nevertheless, more firms would enter because prices are above marginal costs and because they do not take demand externalities among the products into account. This finally results in more products in the market than what is optimal. In case of complements, the firms supply too little at too high prices, which reduces the demand for other products as well as hampers entry (or induces exit) and output expansion of the existing firms. Spence hence states that one would expect complementary products to be supplied by multi-product firms. This is because they internalize cross-price externalities between their goods and want to guarantee the supply of each complement.

In the present paper, we do not consider product differentiation from the consumer, but from the *producer perspective*. While it is a typical feature of macroeconomic models to express intermediate goods differentiation via a CES production function[2] (e.g. Christiano et al., 2005), it has been barely addressed from a competition policy point of view. Noteworthy exceptions include Warren-Boulton (1974) and subsequent work of Mallela and Nahata (1980), Waterson (1982) and Abiru (1988) who build upon the same model. They consider two inputs that are combined to the final good via a CES function. Each input is supplied by separate markets. One input is supplied by a perfectly competitive

[1] Notice that in Spence (1976a) no utility function is given but the demand system is assumed to be linear.

[2] Differentiation of intermediate goods is usually enclosed in a variant of the CES function with a continuum of intermediate good producers in macroeconomic models.

upstream market, whereas the other input is either supplied by a monopolist (Warren-Boulton, 1974; Mallela and Nahata, 1980; Waterson, 1982) or by an oligopolistic upstream market in which the firms produce a homogeneous good (Abiru, 1988). They thus rather focus on the effects of vertical integration and do not consider the impact of product differentiation within the same upstream market. In our framework, however, there is only one upstream market and each upstream oligopolist supplies an input that is different from the inputs of the other firms. We therefore investigate how upstream competition with differentiated inputs affects the whole chain in terms of market outcomes.

Our article relates to other literature on vertically related markets (see, for instance, Spengler, 1950; Häckner, 2003; Inderst and Valletti, 2011; Matsushima and Mizuno, 2012; Reisinger and Schnitzer, 2012; Alipranti et al., 2014; Rozanova, 2015; Basak and Mukherjee, 2017; Reisinger and Tarantino, 2013, 2015, 2019). In a seminal paper, Spengler (1950) finds that in a vertical industry with an upstream monopoly and a downstream monopoly, inefficiencies arise due to double marginalization. A vertical merger between the two firms eliminates the vertical externalities in the form of double marginalization and therefore raise consumer surplus, the profits of the firms and overall welfare. The final price is lowered by vertical integration and the integrated firm's profit is greater than the sum of the profits of the vertically separated firms. Notice that in most theoretical models of vertically related markets, inputs are transformed into outputs on a one-to-one basis. Let us note some frameworks that allow for a more complex production technology. In Inderst and Valletti (2011), the output is a composition of a finite number of imperfectly substitutable inputs that are supplied by different upstream firms. The input-output transformation in their model does not need to be one-to-one, but in fixed proportions. Matsushima and Mizuno (2012) and Reisinger and Tarantino (2019) consider a Leontief production function with perfectly complementary inputs and thus a special case of the CES production function. By contrast, in the present paper we give a full characterization for the case in which inputs can range from perfect complements to perfect substitutes. Consequently, we generalize results for a class of vertical supply chain games, especially regarding upstream competition and its influence on a supply chain.

Our major results can be summarized as follows. We illustrate that the seminal results of Spence (1976a,b) and Singh and Vives (1984) regarding the relations between competition modes are robust for an upstream market with $n \geq 2$ firms in a vertical supply chain where the inputs are composed via a CES production technology. To be precise, we find that the intermediate good producers always set lower prices under price competition than under quantity competition. Additionally, we find that the upstream firms benefit more from price competition for complementary inputs and more from quantity competition for substitutable inputs. We further illustrate that when all intermediate good producers are merged to a single monopoly, intermediate good producers always benefit from the merger because the multi-product monopoly internalizes the cross-price effects between the differentiated input goods. Furthermore, we demonstrate that the upstream firms suffer from a rise in the number of upstream firms irrespective of the inputs being substitutes or complements. This is because the input prices are raised for complementary inputs when more firms enter to the disadvantage of all firms, as this effect reduces the demand for all firms. For substitutable inputs, the inputs prices are reduced when more firms enter due to more intense competition, which ultimately lowers the profits of all firms.

We additionally observe that the the downstream producer, the final consumers and a welfare maximizing social planner prefer upstream price competition over upstream quantity competition due to the lower input prices under price competition. Moreover, they benefit more from an upstream monopoly than from upstream competition when the inputs are complements and vice versa when they are substitutes. This is because the monopolist sets lower prices/higher quantities than the competing firms for complementary inputs but the reverse holds true for substitutable inputs. Consumer surplus, the downstream firm's profit, producer surplus and hence total welfare increase with an increase in n if the inputs are substitutes because of the reduction in input prices. We observe the opposite impact for complementary inputs because of the rise in input prices. We also demonstrate that vertical integration can improve social welfare and yield a (constrained) welfare efficient solution due to the elimination of vertical externalities. The positive impact of vertical integration on social welfare in our framework is consistent with the fundamental findings of Spengler (1950).

The remainder of the paper is organized as follows. In Section 6.2, we introduce the model and derive the demand functions. In Section 6.3, we analyze upstream price competition and upstream quantity competition. In Section 6.4, we examine horizontal integration as well as horizontal and vertical integration. In addition, we derive welfare maximizing solutions. In Section 6.5, we compare the market results. Section 6.6 concludes.

6.2 The Model

6.2.1 Basics

The market consists of $1 + 1 + n$ economic agents divided into three types, namely: one representative consumer C (who represents all identical final consumers in this market), one final good producer F and $n \in \{2, 3, \ldots\}$ *identical* intermediate good producers $\{I_i \mid i \in N = \{1, \ldots, n\}\}$. The production costs of the final good producer for composing the inputs and the production costs of the intermediate good producers are normalized to zero.

The consumer C buys $q \in \mathbb{R}_+$ quantities of a final good from the final good producer F. The final good producer charges a per unit price $p \in \mathbb{R}_+$ for the composed final output

good. In order to produce the output q, the final good producer buys n *differentiated* inputs $\mathbf{x} = (x_i)_{i \in N} \in \mathbb{R}^n_+$ from the intermediate good producers $\{I_1, \ldots, I_n\}$. Each intermediate good producer I_i charges a per unit price $t_i \in \mathbb{R}_+$.
Let $B \in \mathbb{R}_{++}$ denote the exogenous income of the consumer. He spends pq when buying the composed good. Producer F has an income of pq and spends $\mathbf{x} \cdot \mathbf{t}$ on the n inputs where analogously $\mathbf{t} = (t_i)_{i \in N}$. Each intermediate good producer I_i finally receives $x_i t_i$. See Figure 6.1 for an illustration of the market.

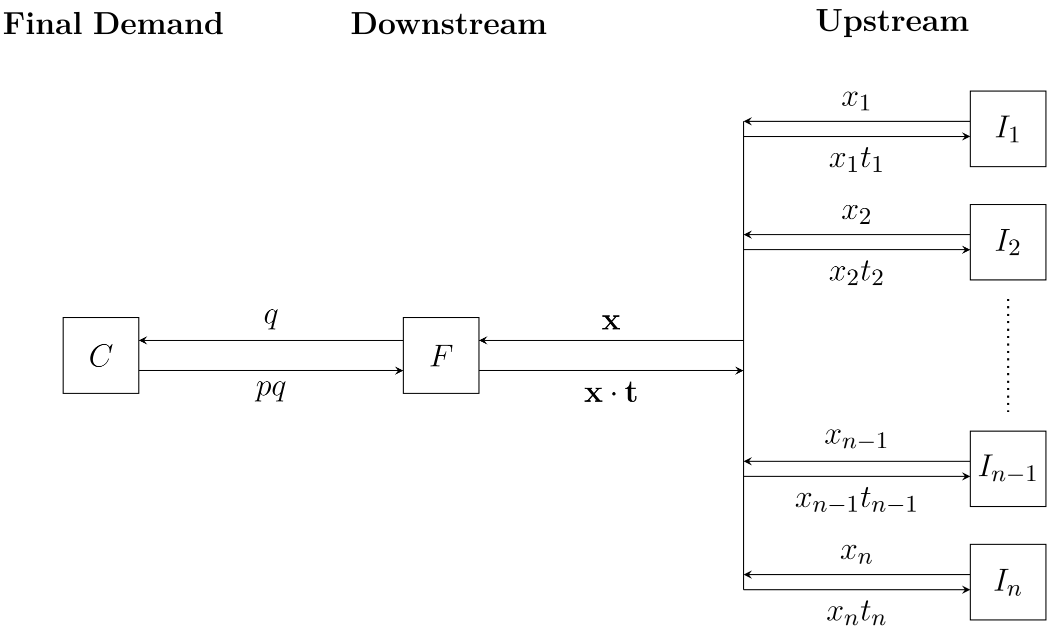


Figure 6.1: Schematic model

The game has three stages. In the first stage, the intermediate good producers either set prices or quantities. In the second stage, the final good producer purchases a basket of inputs to produce the profit-maximizing amount of the final good. In the third stage, the consumer demands the utility-maximizing amount of the final good. The game is solved backwards for a subgame perfect Nash equilibrium in pure strategies.

Consumer. The representative consumer derives an utility from consuming the quantity q of the composed good and the money $B - pq$ spent on some (numeraire) composite good. The price of the final good is given by p and the price of the composite good is normalized to unity. The consumer can spend his budget B on either of these two options. Let us assume that the preferences are captured by a linear-quadratic utility function $u : \mathbb{R}_+ \to \mathbb{R}_+$ of the following form:

$$u(q;p) = q - \frac{1}{2}q^2 + B - pq. \tag{6.1}$$

Maximizing (6.1) w.r.t. q yields the demand function $q(p) = \text{argmax}_{q \in \mathbb{R}_+} u(q)$.

Final good producer. The final good producer transforms the n differentiated inputs $\mathbf{x} \in \mathbb{R}_+^n$ into the output via a CES technology $q : \mathbb{R}_+^n \to \mathbb{R}_+$:

$$q(\mathbf{x};\sigma) = \left[\frac{1}{n}\sum_{i\in N} x_i^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \tag{6.2}$$

with $\sigma \in \mathbb{R}_{++} \setminus \{1\}$ being the constant elasticity of substitution (CES). One may note that the inputs are perfect complements for $\sigma \to 0$ and perfect substitutes for $\sigma \to \infty$. It is well known that for the limit cases the following production functions result:

$$q(\mathbf{x};\sigma) \to \begin{cases} \min_{i\in N}\{x_i\} & \text{for } \sigma \to 0, \\ \prod_{i\in N} x_i^{\frac{1}{n}} & \text{for } \sigma \to 1, \\ \frac{1}{n}\sum_{i\in N} x_i & \text{for } \sigma \to \infty. \end{cases} \tag{6.3}$$

By (6.3), we have a Leontief production function for $\sigma = 0$, a Cobb-Douglas production function for $\sigma = 1$ and a linear production function for $\sigma = \infty$. For the sake of brevity, we suppress the parameter σ as an explicit argument of q in the following.
Remember that the final good producer F has no production costs for composing the inputs but bears the costs for procuring the inputs from the intermediate good producers. We assume F to be sufficiently small such that it has no bargaining power towards the upstream firms. Hence, F does not negotiate with the upstream producers but considers the input prices to be given. The final good producer demands the input x_i from each intermediate good producer I_i and the intermediate good producers charge per unit prices t_i. The profit of F is finally given by

$$\Pi(\mathbf{x};\mathbf{t}) = q(\mathbf{x})p(q(\mathbf{x})) - \mathbf{x}\cdot\mathbf{t}, \tag{6.4}$$

where $p(q)$ is the inverse demand function. Maximizing $\Pi(\mathbf{x};\mathbf{t})$ with respect to each x_i yields the final good producer's input demand function $x_i(\mathbf{t})$ for each input.

Intermediate good producers. We analyze price and quantity competition between intermediate good producers. In the **price competition** mode, the strategic action of each intermediate good producer I_i is the price t_i. As the production costs of the upstream firms are normalized to zero, we obtain the following profit (objective) function for each intermediate good producer:

$$\omega_i(t_i,\mathbf{t}_{-i}) = x_i(t_i,\mathbf{t}_{-i})t_i \tag{6.5}$$

where $\mathbf{t}_{-i} = (t_j)_{j \in N \setminus \{i\}}$ are the actions of all intermediate good producers but I_i. Formally, the price competition game Γ^P is a triple $\langle N, \{\omega_i(t_i, \mathbf{t}_{-i})\}_{i \in N}, S_i \rangle$, where S_i is the strategy space for any $i \in N$.
Since the profits differ up only to a permutation $\rho : N \to N$ of the index $i \in N$, i.e.

$$\omega_i(t_1, \ldots, t_n) = \omega_{\rho(i)}(t_{\rho(1)}, \ldots, t_{\rho(n)}), \tag{6.6}$$

the game is symmetric. We thus consider symmetric Nash equilibria of the form $t_i = \bar{t}$ for all $i \in N$ and apply the symmetric opponents form approach (following Hefti, 2017). Therefore, we fix $i = 1$ as a representative firm. The profits then become

$$\omega(t, \bar{t}) = \omega_1(t, \bar{t}, \bar{t}, \ldots). \tag{6.7}$$

In the **quantity competition** mode, the strategic action of each intermediate good producer I_i is the quantity x_i. In order to define profits, we first invert the system of demand functions $x_i(\mathbf{t})$ into inverse demand functions $t_i(\mathbf{x})$. One then defines profits by

$$\pi_i(x_i, \mathbf{x}_{-i}) = x_i t_i(x_i, \mathbf{x}_{-i}). \tag{6.8}$$

Analogous to the price competition game, the quantity competition game Γ^Q is also a triple $\langle N, \{\pi_i(x_i, \mathbf{x}_{-i})\}_{i \in N}, S_i \rangle$. We again invoke the symmetric opponents form approach and consider firm $i = 1$ as a representative. The profits then become

$$\pi(x, \overline{x}) = \pi_1(x, \overline{x}, \overline{x}, \ldots). \tag{6.9}$$

In the next step, we derive the demand function of the final consumer regarding the output good and the demand functions of the final good producer regarding the inputs.

6.2.2 Demand Analysis

The maximizer of the consumer's utility (6.1) yields the familiar linear demand function:

$$q(p) = \max\{0, 1 - p\} = \operatorname*{argmax}_{q \in \mathbb{R}_+} u(q; p). \tag{6.10}$$

The inverse demand function is then readily given by $p(q) = \max\{0, 1 - q\}$. Considering interior solutions, the final good producer F maximizes its profit given by

$$\Pi(\mathbf{x}; \mathbf{t}) = q(\mathbf{x})(1 - q(\mathbf{x})) - \mathbf{x} \cdot \mathbf{t}. \tag{6.11}$$

The maximizer of F's profit is an n-tuple of input demand functions:

$$(x_1(\mathbf{t}), \ldots, x_n(\mathbf{t})) \in \underset{\mathbf{x}\in\mathbb{R}^n_+}{\operatorname{argmax}}\, \Pi(\mathbf{x};\mathbf{t}). \tag{6.12}$$

Maximizing the final good producer's profit ultimately yields

Proposition 6.1. *For $i \in N$, the input demand function is given by*

$$x_i(\mathbf{t}) = \frac{1}{2t_i^{\sigma} q(\mathbf{t}^{-\sigma})}\left(1 - \frac{n}{q(\mathbf{t}^{-\sigma})^{\frac{1}{\sigma}}}\right), \tag{6.13}$$

where $\mathbf{t}^{-\sigma} = (t_1^{-\sigma}, \ldots, t_n^{-\sigma})$.

Proof. See Appendix 6.7.1. □

We observe that the final good producer's input demand functions depend on the elasticity of substitution σ. In the rest of the paper, we further explore how the relationship between σ and the input demand affects the market results in the channel.

6.3 Upstream Competition Analysis

In this section, we analyze price and quantity competition between intermediate good producers. Given the demand for the final good and the demand for the intermediate goods, we can solve for the subgame perfect Nash equilibrium. Note that we consider a sequential game along the supply chain in which the intermediate good producers move first and the final good producer moves second, as well as an n-person simultaneous move game at the upstream tier.

6.3.1 Price Competition

We first assume that the intermediate good producers compete in prices. We find that there exists a unique interior price competition equilibrium for a sufficiently low elasticity of substitution σ as indicated by

Proposition 6.2. *Under upstream price competition, the symmetric equilibrium input prices are given as*

$$t^P(n,\sigma) = \begin{cases} \frac{\sigma+n(1-\sigma)}{n(1+\sigma+n(1-\sigma))} & \text{for } \sigma < \frac{n}{n-1}, \\ 0 & \text{for } \sigma \geq \frac{n}{n-1}. \end{cases}$$

Proof. See Appendix 6.7.2. □

For a sufficiently low σ such that $\sigma < \frac{n}{n-1} =: \overline{\sigma}$, there exists a unique interior solution because $\sigma < \overline{\sigma}$ ensures that the second-order condition for a (global) maximum is satisfied. We hence denote $\overline{\sigma}$ as the upper bound on the elasticity of substitution. We denote the price competition equilibrium profit of one intermediate good supplier by $\omega^P(n,\sigma) = \omega(t^P(n,\sigma), t^P(n,\sigma)) = \frac{\sigma+n(1-\sigma)}{2n(1+\sigma+n(1-\sigma))^2}$. As it turns out, the restriction $\sigma < \overline{\sigma}$ implies that the profits are positive so that $\omega^P(n,\sigma) > 0$. By contrast, the intermediate good suppliers make losses or zero profits if they set $t^P(n,\sigma) = \frac{\sigma+n(1-\sigma)}{n(1+\sigma+n(1-\sigma))}$ given $\sigma \geq \overline{\sigma}$. Hence, if $\sigma \geq \overline{\sigma}$, it is optimal for them to set zero prices such that they prevent losses and always gain zero profits instead.

6.3.2 Quantity Competition

Now we assume that each intermediate good producer I_i sets the quantity x_i as a strategic variable. We first inverse the input demand functions $x_i(\mathbf{t})$ of Proposition 6.1 into $t_i(\mathbf{x})$, leading to

Lemma 6.1. *For $i \in N$, the inverse input demand function is given by*

$$t_i(\mathbf{x}) = \frac{1-2q(\mathbf{x})}{n} \left[\frac{q(\mathbf{x})}{x_i} \right]^{\frac{1}{\sigma}}.$$

Proof. See Appendix 6.7.3. □

In order to guarantee a unique interior symmetric equilibrium and positive profits, the elasticity of substitution must be sufficiently large under upstream quantity competition:

Proposition 6.3. *Under upstream quantity competition, the symmetric equilibrium input quantities are given as*

$$x^Q(n,\sigma) = \begin{cases} \frac{1-(1-\sigma)n}{2(1+\sigma-n(1-\sigma))} & \text{for } \sigma > \frac{n-1}{n}, \\ 0 & \text{for } \sigma \leq \frac{n-1}{n}. \end{cases}$$

Proof. The proof follows the same steps as of Proposition 6.2. □

We therefore derive a lower bound $\sigma > \frac{n-1}{n} =: \underline{\sigma}$ on the elasticity of substitution. The lower bound ensures that the second-order condition for a (global) maximum holds in this case. The lower bound also guarantees that the input equilibrium profits under quantity competition, which are given by $\pi^Q(n,\sigma) = \pi^Q(x^Q(n,\sigma), x^Q(n,\sigma)) = \frac{\sigma(1-n(1-\sigma))}{2n(1+\sigma-n(1-\sigma))^2}$, are greater than zero such that $\pi^Q(n,\sigma) > 0$. However, the profits become negative (or zero) for $\sigma \leq \underline{\sigma}$. Hence, if $\sigma \leq \underline{\sigma}$, it is the best strategy of each input producer to produce nothing such that the profits always equal zero in the equilibrium.

Our main focus lies on interior solutions but we also take into account what happens at the boundaries of the elasticity of substitution σ. Consequently, we restrict our following

analysis to the values of $\sigma \in [\underline{\sigma}, \overline{\sigma}]$. We consider differentiation at the lower bound of σ as maximum complementarity and at the upper bound of σ as maximal substitutability within our set-up, but we are aware that we generally reach perfect complementarity (substitutability) for $\sigma \to 0$ ($\sigma \to \infty$).[3]
So far, we have only looked at market settings with oligopolistic upstream competition. However, as, for example, Spence (1976a,b) highlights, firms in imperfectly competitive markets do not necessarily take the horizontal demand externalities between them into account in their strategic decisions. The same holds true for vertical externalities that are present in channels where upstream and downstream firms are vertically separated. The disregard of externalities may be to the disadvantage of firms and consumers (Spengler, 1950; Spence, 1976a,b; Tirole, 1988). To analyze the impact of vertical and horizontal externalities on market results, we successively eliminate them in the next section.

6.4 Integration and Welfare Efficiency

We first consider a bilateral monopoly in which all intermediate good producers merge horizontally such that horizontal externalities are eliminated. In the next step, we let the upstream monopoly and the monopolistic final good producer merge vertically such that vertical externalities are eliminated. As a point of reference, we finally study the welfare efficient solution. When considering such cooperative solutions, the firms jointly maximize the sum of their profits over $\mathbf{x}$. We therefore regard optimization problems without strategic interaction. In order to compare the results for the integrated cases with the symmetric equilibria of the noncooperative games, we restrict the set of admissible strategies to $\{(x, \ldots, x) \in S_i^n\}$. Since the downstream and upstream profits are then given by $\Pi(x, \ldots, x) = x^2$ and $\pi(x, x) = \frac{(1-2x)x}{n}$, respectively, we consider one-dimensional problems. Note that the nonnegativity constraints on prices $p(q(x, \ldots, x)) = 1 - x \geq 0$ and $t_i(x, \ldots, x) = \frac{1-2x}{n} \geq 0$ further restrict the strategy space by $x \in [0, \frac{1}{2}]$.

6.4.1 Horizontal Integration

Presume that all intermediate good producers merge into a single firm such that the upstream market consists of a so-called *multi-input monopoly* M. The input monopolist maximizes the sum of upstream profits $US(x) = n\pi(x, x) = (1 - 2x)x$, which leads to

Proposition 6.4. *The multi-input monopolist sets as the optimal quantity per input*

$$x^M = \frac{1}{4}.$$

Proof. The result $\frac{1}{4} = \operatorname{argmax}_{x \in [0, \frac{1}{2}]} US(x)$ is immediate. □

[3]See Section 6.5.1 below regarding the complementarity of inputs in our framework.

The multi-input monopolist internalizes all demand externalities among the inputs. Consequently, the optimal quantity set by the upstream monopolist for each intermediate good is independent of the elasticity of substitution and hence constant. This also holds true for the next scenario.

6.4.2 Horizontal and Vertical Integration

Suppose now that the multi-input monopoly and the downstream producer merge vertically into a single firm so that there is only one firm, denoted V, in the channel. Then the integrated firm maximizes the producer surplus defined as the sum of profits of downstream and upstream firms:

$$PS(x) = \Pi(x, \ldots, x) + US(x) = (1 - x)x. \tag{6.14}$$

Solving the integrated firm V's optimization problem generates

Proposition 6.5. *In the horizontally and vertically integrated market, the optimal quantity for each input is given by*

$$x^V = \frac{1}{2}.$$

Proof. The result $\frac{1}{2} = \operatorname{argmax}_{x \in [0, \frac{1}{2}]} PS(x)$ is immediate. □

It follows from Proposition 6.5 that $t_i^V(x^V, \ldots, x^V) = 0$. Thus, vertical integration yields efficiency in the sense that the integrated firm's input prices equal the marginal costs of the inputs, which are normalized to zero. The intuition for this result is as follows. Even though we do not have typical double marginalization in our supply chain (in the original sense of Spengler, 1950), vertical externalities still arise when we consider a vertically separated structure. This is because the upstream firms set prices above their marginal costs, which negatively affects the downstream firm due to larger costs. When the downstream producer and the upstream monopoly merge, it is in the integrated firm's interest to keep the input costs as low as possible in order to internalize all vertical externalities. Therefore, firm V sets input prices equal to marginal costs in the equilibrium.

6.4.3 Welfare Efficient Solution

We now study welfare optimizing solutions. We concentrate on the constrained welfare optimum (Dixit and Stiglitz, 1977, Sec. 1. C.). This means that we rule out transfer payments and therefore do not allow for negative profits. Hence, the welfare efficient solution (denoted by the superscript E) is restricted to the strategy space $x \in [0, \frac{1}{2}]$. However, we also shortly discuss the unconstrained optimum (denoted by the superscript UE), which is the first best solution regarding social welfare.

Consider a benevolent planer who maximizes welfare, where welfare is defined as the sum of consumer surplus

$$CS(x) = \frac{1}{2}[1 - p(q(x, \ldots, x))]q(x, \ldots, x) = \frac{1}{2}x^2 \tag{6.15}$$

and producer surplus (6.14). When considering symmetric input quantities, the welfare coincides with the linear-quadratic subutility function of the consumer

$$W(x) = CS(x) + PS(x) = x - \frac{1}{2}x^2. \tag{6.16}$$

Maximizing welfare with respect to the input quantity yields

Proposition 6.6. *The welfare efficient quantity per input is given by*

$$x^E = x^V = \frac{1}{2}.$$

Proof. The result $\frac{1}{2} = \text{argmax}_{x \in [0, \frac{1}{2}]} W(x)$ is immediate. □

We find that the welfare efficient solution coincides with the vertically integrated market concerning market outcomes. Consequently, we find a positive impact of vertical integration on welfare, which is consistent with the fundamental findings of Spengler (1950). Interestingly, the efficient downstream price is positive as $p^E = \frac{1}{2} > 0$. This means that the constrained welfare efficient solution does not require that the final good price equals the final good producer's marginal costs, which are zero.
In the unconstrained case, one gets $x^{UE} = 1 = \text{argmax}_{x \in \mathbb{R}_+} W(x)$ such that the final good price p^{UE} equals zero. However, we would obtain negative input prices $t^{UE} < 0$ and thus we would need lump-sum transfers to the upstream firms. With lump-sum transfers, the upstream firms could cover losses and the first best solution could be sustained.

6.5 Comparison and Discussion of Market Results

We now compare the market outcomes of all settings. We especially discuss the impact of the elasticity of substitution, σ, and the number of upstream firms, n, on market outcomes and the relations between the different settings.[4] We take the number of upstream firms as an indicator for the intensity of upstream competition. We first focus on the profits and related market entities, and then analyze the welfare outcomes.

[4]Note that the comparison with the vertical integration (V) case is implied in the comparison with the (constrained) welfare efficient solution (E) because the settings coincide.

6.5.1 Profits

Let us first define the equilibrium input quantity under price competition as

$$x^P(n,\sigma) = \frac{1}{2}\left(1 - nt^P(n,\sigma)\right) = \frac{1}{2(1+\sigma+n(1-\sigma))}. \tag{6.17}$$

At $\sigma = 1$, the price (P) and quantity (Q) competition as well as monopoly (M) solution coincide so that $x^P(n,1) = x^Q(n,1) = x^M$ for all $n \in \mathbb{N}$. The reason is that the final good producer F regards the inputs as independent for $\sigma = 1$. However, F regards the inputs as complements for $\sigma < 1$ and as substitutes for $\sigma > 1$. We can show this by considering the cross-price derivative $\epsilon_{ij}(\mathbf{t})$ with respect to the input demand functions. For any different $i, j \in N$ define

$$\begin{aligned}\epsilon_{ij}(\mathbf{t}) = \frac{\partial x_i(\mathbf{t})}{\partial t_j} &= -\frac{1}{2t_i^\sigma q(\mathbf{t}^{-\sigma})^2}\frac{\partial q(\mathbf{t}^{-\sigma})}{\partial t_j}\left(1 - \frac{n}{q(\mathbf{t}^{-\sigma})^{\frac{1}{\sigma}}}\right)\\ &\quad + \frac{1}{2t_i^\sigma q(\mathbf{t}^{-\sigma})}\frac{n}{\sigma q(\mathbf{t}^{-\sigma})^{\frac{1}{\sigma}+1}}\frac{\partial q(\mathbf{t}^{-\sigma})}{\partial t_j}.\end{aligned} \tag{6.18}$$

In the symmetric equilibrium, this expression becomes

$$\bar{\epsilon}(t) = \epsilon_{ij}(t,\ldots,t) = -\frac{1}{2}\left(1 + \sigma\left(1 - \frac{1}{nt}\right)\right). \tag{6.19}$$

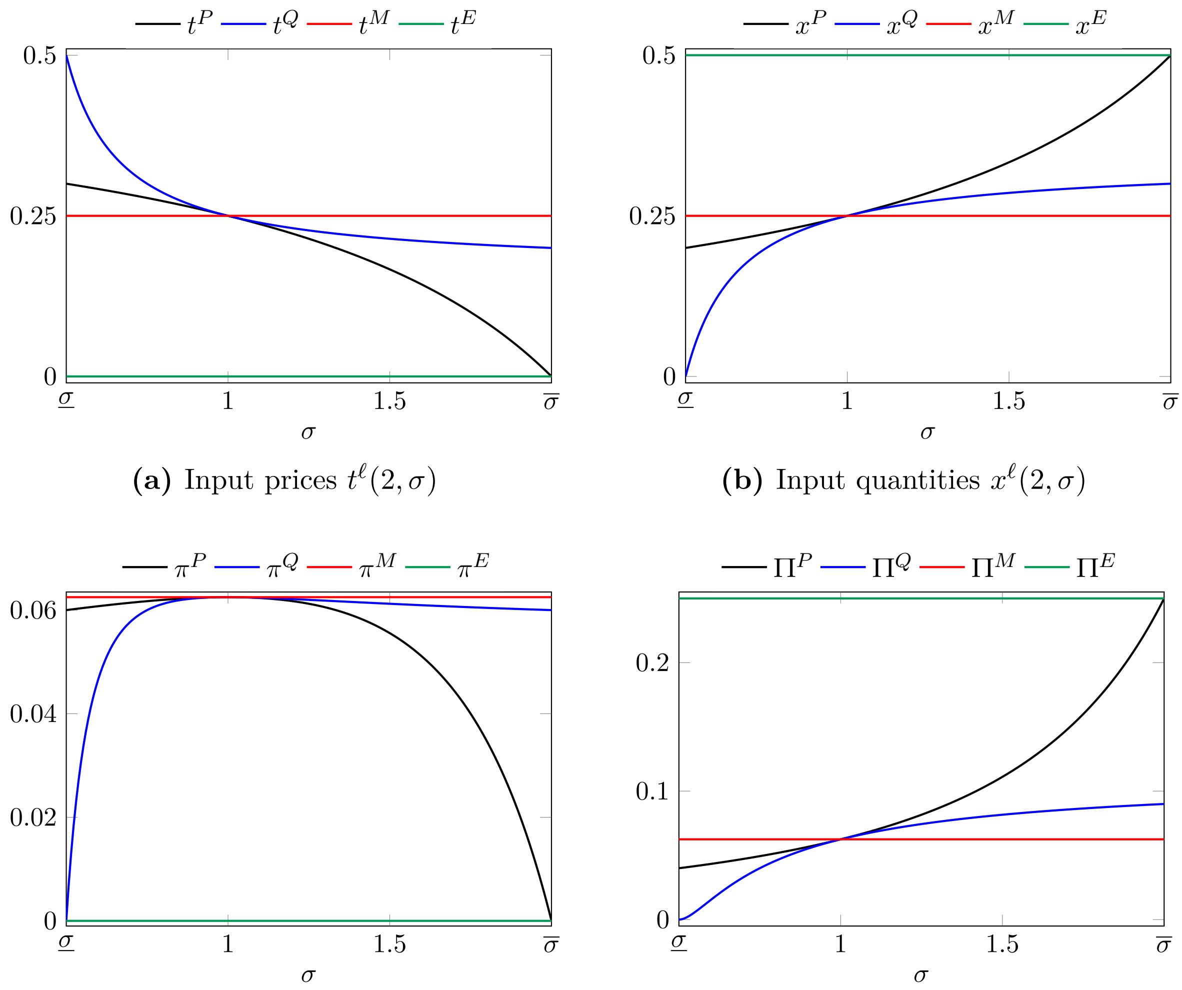
If we now consider $\bar{\epsilon}_{ij}(t)$ for $t \in \{t^P(n,\sigma), t^Q(n,\sigma)\}$, it turns out that for the two competition modes $\beta \in \{P, Q\}$ when considering interior solutions, the following holds:

$$\bar{\epsilon}(t^\beta(n,\sigma)) \begin{cases} < 0 & \text{for } \sigma < 1,\\ = 0 & \text{for } \sigma = 1,\\ > 0 & \text{for } \sigma > 1.\end{cases} \tag{6.20}$$

We next compare market outcomes, such as input prices and quantities, and then compare the profits for the various scenarios. Let us first note that the number of upstream firms n affects all results (including welfare outcomes) only level-wise, but not qualitatively. Considering the input quantities, for instance, it is straightforward to show the following holds for all n:

$$\begin{cases} x^Q(n,\sigma) < x^P(n,\sigma) < x^M < x^E & \text{for } \sigma < 1,\\ x^Q(n,\sigma) = x^P(n,\sigma) = x^M < x^E & \text{for } \sigma = 1,\\ x^M < x^Q(n,\sigma) < x^P(n,\sigma) < x^E & \text{for } \sigma > 1.\end{cases} \tag{6.21}$$

Let us thus fix $n = 2$. This leads to $\underline{\sigma} = 0.5$ and $\overline{\sigma} = 2$. We therefore plot the results for $\sigma \in [0.5, 2]$ in the following.[5] Figure 6.2 displays the input prices $t^\ell(2,\sigma)$, the input quantities $x^\ell(2,\sigma)$, the input profits $\pi^\ell(2,\sigma)$ and the output profit $\Pi^\ell(2,\sigma)$ for a varying elasticity of substitution $\sigma \in [0.5, 2]$ and for each mode of play $\ell \in \{P, Q, M, E\}$.[6]



(a) Input prices $t^\ell(2,\sigma)$

(b) Input quantities $x^\ell(2,\sigma)$

(c) Input profits $\pi^\ell(2,\sigma)$

(d) Output profit $\Pi^\ell(2,\sigma)$

Figure 6.2: Market results for $n = 2$ and $\sigma \in [\underline{\sigma}, \overline{\sigma}]$, where $\underline{\sigma} = 0.5$ and $\overline{\sigma} = 2$

For the two competition modes $\beta \in \{P, Q\}$, the input prices $t^\beta(2,\sigma)$ are monotonically decreasing in σ. If the inputs become less complementary or closer substitutes (i.e. σ increases), the market power of each input supplier diminishes, because the final good producer rather easily switches to another supplier. The prices under upstream quantity competition always exceed the prices under the more aggressive upstream price competition and prices in both competition types are above the efficient level. Consequently, we have $t^Q(2,\sigma) > t^P(2,\sigma) > t^E$ for all $\sigma \in [0.5, 2] \setminus \{1\}$. Furthermore, for $\sigma = 1$, the competition prices coincide with the multi-input monopoly price, because the in-

[5] In the verbal comparison, we sometimes only refer to the cases with substitutes or complements, since if $\sigma = 1$ the market results for the modes $\{P, Q, M\}$ always coincide.

[6] Note that the output price behaves according to the behavior of the input prices and that the output quantity equals any input quantity in the equilibrium, see Appendix 6.7.4.

puts are considered independent and thus each upstream firm has monopolistic market power, regardless of whether a firm chooses its price or its quantity as a strategic variable. Therefore, we have $t^Q(2,1) = t^P(2,1) = t^M$ (compare Figure 6.2a).

The input prices in the competition modes exceed the monopoly prices when the inputs are complements, i.e. $t^\beta(2,\sigma) > t^M(2,\sigma)$ for all $\sigma \in [0.5, 1)$, and vice versa when they are substitutes, i.e. $t^\beta(2,\sigma) < t^M(2,\sigma)$ for all $\sigma \in (1, 2]$. In case of complementary (substitutable) inputs, the upstream firms face negative (positive) cross-price effects concerning the demand for inputs (compare (6.20)). The monopoly internalizes the demand externalities in contrast to the competing firms. Hence, the monopolist sets lower prices than the competing firms in case of complementary inputs, but higher prices in case of substitutable inputs because it takes into account the impact of prices on the demand for all inputs. As the competition in the upstream market becomes fierce, prices are driven to the welfare efficient level under the hard price competition as $t^P(2,2) = t^E = 0$, but remain positive under quantity competition since $t^Q(2,2) > 0$ (see Figure 6.2a).

Quantities are inversely related to prices. That is, the higher the input price, the smaller the input quantity. One may note that the firms set prohibitively high prices in the case of quantity competition if the inputs are very complementary. This means that $x^Q(2, 0.5) = 0$ (compare Figure 6.2b). The reason for this is that quantity competition is more monopolistic than price competition and that the input suppliers overestimate their market power when the inputs reach maximum complementarity. They therefore charge very high input prices if $\sigma = \underline{\sigma}$, which results in a equilibrium demand of zero for each input. For the two cooperative modes $\{M, E\}$, the input prices as well as quantities are constant in σ, because the externalities of product differentiation are internalized as indicated above (compare Figures 6.2a and 6.2b).

The differences in prices or quantities between the modes of play leads to Proposition 6.7 with respect to the profits of the intermediate good suppliers.

Proposition 6.7.a. *The intermediate good producers always benefit from a full upstream market merger and always suffer from the welfare efficient solution.*

Proposition 6.7.b. *The intermediate good producers prefer price over quantity competition when the inputs are complements and vice versa when the inputs are substitutes.*

The profits of the intermediate good suppliers under competition are bounded from above by the monopoly profit and from below by the efficient profit so that $\pi^M > \pi^\beta(2,\sigma) > \pi^E$ for all $\sigma \in [0.5, 2] \setminus \{1\}$. The upper bound is basically the first best solution for the upstream firms, because they jointly maximize profits. The lower bound is driven by the fact that the efficient input prices equal marginal costs ($t^E = 0$) and therefore the profits of the intermediate good producers are also zero ($\pi^E = 0$). Moreover, the profits $\pi^\beta(2,\sigma)$ increase in σ for each competition mode if $\sigma < 1$ and decrease if $\sigma > 1$. This is because a reduction in one firm's input price (which follows from an increase in σ)

promotes the demand of the inputs of the other firms if the inputs are complements, due to the negative cross-price effects. The reverse holds true if the inputs are substitutable, due to the positive cross-price effects. The difference in demand effects is also the reason why the intermediate good producers gain higher profits under price competition than under quantity competition when the inputs are complements and vice versa when the inputs are substitutes. This means that $\pi^P(2,\sigma) > \pi^Q(2,\sigma)$ for all $\sigma \in [0.5, 1)$ and that $\pi^Q(2,\sigma) > \pi^P(2,\sigma)$ for all $\sigma \in (1, 2]$. Also, there is a payoff maximal degree of substitution at $\sigma = 1$, as then each firm has monopolistic power (see Figure 6.2c).
In sum, we observe that the relations and differences between quantity and price competition with respect to prices, quantities and profits for the upstream market in our model are in line with the seminal findings of Singh and Vives (1984). Consequently, we show that the findings of Singh and Vives for a non-vertical duopoly are robust for an upstream oligopoly in a vertical distribution channel with product differentiation via a downstream CES production function. In addition, our results are consistent with the seminal results of Spence (1976a,b), who highlights that firms in imperfect competition do not take their demand relationships into account, in contrast to multi-product firms. As our results illustrate, the multi-input monopolist is always better off than the competing input firms.
Let us now turn to the final good producer's profit. Comparing the profits for the different modes of play, we find the following:

Proposition 6.8.a. *The final good producer always benefits from the efficient solution.*

Proposition 6.8.b. *The final good producer prefers an upstream monopoly over upstream competition when the inputs are complements and vice versa when they are substitutes.*

Proposition 6.8.c. *The final good producer prefers upstream price competition over upstream quantity competition, regardless of the inputs being complements or substitutes.*

The profit of the final good producer is also inversely related to input prices as the input prices represent the final good producer's total costs. As a consequence, the producer's profit is bounded from above by the efficient profit because input prices are zero ($t^E = 0$) in the efficient market. Moreover, the final producer prefers an upstream monopoly over upstream competition for complementary inputs since $\Pi^M > \Pi^\beta(2,\sigma)$ for all $\sigma \in [0.5, 1)$ but the reverse holds true for substitutable inputs since $\Pi^M < \Pi^\beta(2,\sigma)$ for all $\sigma \in (1, 2]$. This divergence in profits follows from the divergence in input prices, depending on the input differentiation. The final good producer gains a higher profit under price competition than under quantity competition independent of the inputs being substitutes or complements such that $\Pi^P(2,\sigma) > \Pi^Q(2,\sigma)$ for all $\sigma \in [0.5, 2] \setminus \{1\}$ because of the lower input prices under price competition. Additionally, the profit is monotonically increasing in σ for each competition mode because the input prices monotonically decrease

in σ, where $\Pi^P(2,2) = \Pi^E$. This means that the maximum profit for F is reached under price competition as well given the input substitutability is at its maximum due to $t^P(2,2) = 0$ (compare Figure 6.2d).
Naturally, the differences between the modes of play, established so far, directly affect the differences in welfare outcomes, which are therefore for the most parts additionally in line with the fundamental literature results on differentiated oligopolies.[7] This is depicted in more detail in the next section.

6.5.2 Welfare

We now consider the comparative statics of the welfare measures upstream surplus US, producer surplus PS, consumer surplus CS and total welfare W with respect to a change in the number of upstream firms, n, and the degree of input differentiation, σ. Let us therefore reconsider any of the mentioned measures in terms of the input quantity x:

$$\begin{aligned} US(x) &= (1-2x)x, \\ PS(x) &= (1-x)x, \\ CS(x) &= \frac{1}{2}x^2, \\ W(x) &= x - \frac{1}{2}x^2. \end{aligned} \tag{6.22}$$

As $x^M = \frac{1}{4}$ and $x^E = \frac{1}{2}$ are constant, they do not change with respect to n or σ. Hence, the welfare measures for the cases $\{M, E\}$ are constant too. We are thus left with the welfare entities of the competition modes $\beta \in \{P, Q\}$. Remember the equilibrium quantities of the competition modes are

$$x^P(n,\sigma) = \frac{1}{2(1+\sigma+n(1-\sigma))}, \tag{6.23}$$

$$x^Q(n,\sigma) = \frac{1-(1-\sigma)n}{2(1+\sigma-n(1-\sigma))}. \tag{6.24}$$

For any competition setting, the equilibrium quantities are bounded from above as we observe that $x^\beta(n,\sigma) < \frac{1}{2}$ for $\underline{\sigma} < \sigma < \overline{\sigma}$ and $n > 1$. Additionally, they are further bounded for certain values of σ:

$$x^\beta(n,\sigma) \begin{cases} \in \left(0, \frac{1}{4}\right) & \text{for } \sigma < 1, \\ = \frac{1}{4} & \text{for } \sigma = 1, \\ \in \left(\frac{1}{4}, \frac{1}{2}\right) & \text{for } \sigma > 1. \end{cases} \tag{6.25}$$

[7] Except for the producer surplus since we have a vertical market where the channel profit constitutes the producer surplus and not simply the profits of the upstream oligopoly.

We do not need to distinguish between price and quantity competition, because for each mode β, the sign of the partial derivatives of $x^{\beta}(n, \sigma)$ with regard to n and σ is the same:

$$\operatorname{sgn}\left(\frac{\partial x^{\beta}(n,\sigma)}{\partial n}\right) = \begin{cases} -1 & \text{for } \sigma < 1, \\ 0 & \text{for } \sigma = 1, \\ 1 & \text{for } \sigma > 1, \end{cases} \tag{6.26}$$

$$\operatorname{sgn}\left(\frac{\partial x^{\beta}(n,\sigma)}{\partial \sigma}\right) = 1 \quad \forall n > 1. \tag{6.27}$$

If the intermediate goods are complements ($\sigma < 1$), then the input suppliers raise prices with an increase in firms and thus quantities fall. Anticipating that there is a higher number of complementary inputs in the market, which are needed for producing the output good, the firms tend to raise prices and hold back quantities. For substitutable inputs ($\sigma > 1$), the input producers reduce prices and therefore increase quantities with a rise in n. In this case, the entrants make the incumbent firms lose market shares and hence market power. This finally results in more intense competition and lower prices. For independent inputs ($\sigma = 0$), there is no influence of an increase in n on quantities, because any supplier is basically a monopolist as mentioned before. Note that our observed effects of entry on prices and quantities depending on the degree of product differentiation are in line with Spence (1976a,b). Also note that when the degree of substitutability among inputs, σ, rises, then the input suppliers always lower prices. Thereby, the quantities of the suppliers increase for any n if σ rises. This observation is a generalization of our results that are depicted by the Figures 6.2a and 6.2b.

Now we simply take the total derivative of any welfare measure of the two types of competition with respect to n and σ and check the sign. For the sake of clarity, we sometimes suppress the superscript β in the following. The functions $PS(x)$, $CS(x)$ and $W(x)$ are monotonically increasing for all $x \in [0, \frac{1}{2}]$ and they thus change in the same direction as x changes in n and σ. The sign of the total derivative of $PS(x)$ regarding σ, for example, is given by

$$\begin{aligned} \operatorname{sgn}\left(\frac{dPS^{\beta}(x(n,\sigma))}{d\sigma}\right) &= \operatorname{sgn}\left(\frac{dPS(x)}{dx}\frac{\partial x(n,\sigma)}{\partial \sigma}\right) \\ &= \operatorname{sgn}\left(\frac{dPS(x)}{dx}\right)\operatorname{sgn}\left(\frac{\partial x(n,\sigma)}{\partial \sigma}\right) \\ &= \operatorname{sgn}\left(1 - 2x^{\beta}\right) \cdot 1 \\ &= 1 \cdot 1 = 1. \end{aligned} \tag{6.28}$$

When considering the upstream surplus $US(x)$, however, the sign depends on x as well:

$$\operatorname{sgn}\left(\frac{dUS^{\beta}(x)}{dx}\right) = \operatorname{sgn}(1-4x^{\beta}) = \begin{cases} 1 & \text{for } \sigma < 1, \\ 0 & \text{for } \sigma = 1, \\ -1 & \text{for } \sigma > 1. \end{cases} \tag{6.29}$$

Taking this into account, we get case-sensitive results for the upstream surplus

$$\operatorname{sgn}\left(\frac{dUS^{\beta}(x(n,\sigma))}{d\sigma}\right) = \operatorname{sgn}\left(\frac{dUS(x)}{dx}\frac{\partial x(n,\sigma)}{\partial \sigma}\right) = \begin{cases} 1 & \text{for } \sigma < 1, \\ 0 & \text{for } \sigma = 1, \\ -1 & \text{for } \sigma > 1, \end{cases} \tag{6.30}$$

$$\operatorname{sgn}\left(\frac{dUS^{\beta}(x(n,\sigma))}{dn}\right) = \operatorname{sgn}\left(\frac{dUS(x)}{dx}\frac{\partial x(n,\sigma)}{\partial n}\right) = \begin{cases} -1 & \text{for } \sigma < 1, \\ 0 & \text{for } \sigma = 1, \\ -1 & \text{for } \sigma > 1. \end{cases} \tag{6.31}$$

The following Proposition 6.9 summarizes our findings regarding the impact of n and σ on welfare results.

Proposition 6.9. *An increase in the number of intermediate good suppliers n or in the elasticity of substitution σ affect upstream surplus, producer surplus, consumer surplus and social welfare in the competition modes $\beta \in \{P, Q\}$ as follows:*

	US^{β}	PS^{β}	CS^{β}	W^{β}
n	$-/0/-$	$-/0/+$	$-/0/+$	$-/0/+$
σ	$+/0/-$	$+$	$+$	$+$

Note: Backslash indicates the cases σ </=/>1.

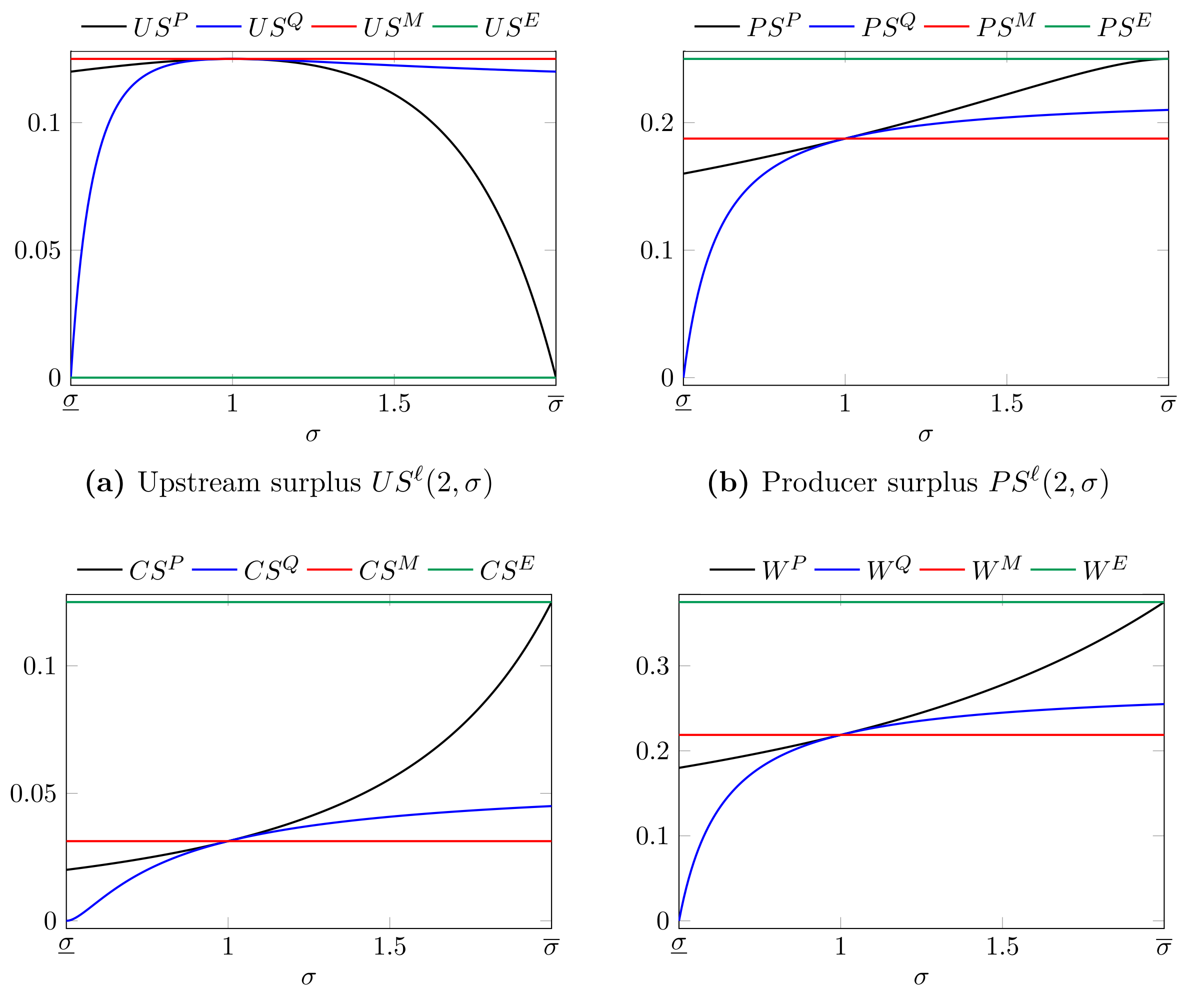
Let us focus for the intuition of the proposition only on the cases where the inputs are either complements ($\sigma < 1$) or substitutes ($\sigma > 1$). When the number of upstream firms increases, the payoffs of the input suppliers diminish irrespective of the characteristics of the inputs. If the inputs are complements, then an upstream firm's price increases (which it does with a rise in n) slower than its quantity decreases. This can again be reasoned with the negative cross-price externalities: a raise in the input price of one firm lowers the demand for this firm's input but also for the input of the other firms, due to the inputs being complements, which negatively affects the profits of all upstream firms. If the inputs are substitutes, however, an intermediate good producer's price diminishes (which it does with an increase in n) faster than its quantity rises. In this case, we have positive cross-price externalities. When one intermediate good supplier lowers its price, it raises the demand for its own input but reduces the demand for the inputs of the other firms. As the other upstream firms act in the same way, the competition becomes very

fierce and all firms lose from entry. This further shows that our findings are consistent with Spence (1976a,b) as we again observe that the upstream firms do not take their demand externalities into account in their strategic decisions.
For the producer surplus, consumer surplus and overall welfare, the analysis coincides. In any case, they gain from an increase in the number of upstream firms when the inputs are substitutes and they suffer from it when the inputs are complements. This is because the input prices rise with an increase in n if the inputs are complements and but decrease in n if they are substitutes. When the input quantities x^{β} increase (which they do when the input prices fall), then the downstream price $p = 1 - x^{\beta}$ falls. Hence, the consumer surplus increases (decreases) when the input prices decrease (increase). Additionally, the final good supplier benefits from lower input prices because of its lower costs and hence suffers from a rise in input prices. Therefore, the producer surplus increases in n for $\sigma > 1$ even though the upstream surplus always falls in n.
The influence of the degree of substitutability σ on the upstream surplus US^{β} is analogous to the influence of σ on the input profits (see Figure 6.2c) as the upstream surplus is simply the sum of the profits of all intermediate good producers. Furthermore, producer, consumer and thereby total welfare gain from a rise in the degree of substitutability since the input prices decrease in σ under upstream competition.
Last, we compare the welfare magnitudes for each mode of play $\ell \in \{P, Q, M, E\}$ for a varying degree of σ and $n = 2$ as depicted in Figure 6.3. Notice that as already indicated, the differences in the upstream surplus (compare Figure 6.3a) are simply analogous to the differences in the input profits (Figure 6.2c). The differences in producer surplus between the four modes coincide with the differences in the output profit (Figure 6.2d). Consequently, the producer surplus is always greater under price competition than under quantity competition, meaning that $PS^{P}(2,\sigma) > PS^{Q}(2,\sigma)$ for all $\sigma \in [0.5, 2] \setminus \{1\}$. As the upstream surplus under quantity competition exceeds the upstream surplus under price competition for substitutable inputs, the difference is outweighed by the difference in the output profit (compare Figure 6.3b).
Regarding consumer surplus and overall welfare, we find the following:

Proposition 6.10.a. *The final consumers and a benevolent planner always benefit from the efficient solution.*

Proposition 6.10.b. *The final consumers and a benevolent planner prefer upstream price competition over upstream quantity competition, independent of the inputs being complements or substitutes.*

Proposition 6.10.c. *The final consumers and a benevolent planner prefer upstream competition over an upstream monopoly when the inputs are substitutes and vice versa when they are complements.*



(a) Upstream surplus $US^{\ell}(2,\sigma)$

(b) Producer surplus $PS^{\ell}(2,\sigma)$

(c) Consumer surplus $CS^{\ell}(2,\sigma)$

(d) Welfare $W^{\ell}(2,\sigma)$

Figure 6.3: Welfare results for $n = 2$ and $\sigma \in [\underline{\sigma}, \overline{\sigma}]$, where $\underline{\sigma} = 0.5$ and $\overline{\sigma} = 2$

The consumer surplus (Figure 6.3c) is inversely related to the input prices and therefore inversely related to the output price. Obviously, social welfare (Figure 6.3d) and consumer surplus are bounded from above by the welfare efficient solution. Additionally, the final consumers benefit more from price competition than from quantity competition such that $CS^P(2,\sigma) > CS^Q(2,\sigma)$ for all $\sigma \in [0.5, 2] \setminus \{1\}$ because of the lower output price in the former. However, they are also better off with an upstream monopoly than with upstream competition, but only if the inputs are complements as $CS^M > CS^{\beta}(2,\sigma)$ for all $\sigma \in [0.5, 1)$. This is because the input prices and thus the output price are lower in the monopoly if the inputs are complements. By contrast, if the inputs are substitutes, the consumers prefer upstream competition over an upstream monopoly as $CS^{\beta}(2,\sigma) > CS^M$ for all $\sigma \in (1, 2]$, since then the input and output prices are smaller under upstream competition. The differences between the modes of play coincide for producer surplus and consumer surplus. Thereby, we can directly derive the differences between the modes of play regarding total welfare from them. Notice that the maximum welfare is realized under price competition in case we have maximal substitutability among inputs and thus very intense competition. This means that we obtain $W^P(2,2) = W^E$.

6.6 Conclusion

In this article, we study a two-tier vertical supply chain in which a monopolistic downstream final good producer purchases differentiated intermediate goods from $n \geq 2$ upstream producers and transforms them via a CES production function into a final good. We investigate the impact of the oligopolistic upstream competition with differentiated inputs on the market results of the vertical chain, where we analyze upstream price and upstream quantity competition. We also consider full horizontal integration upstream as well as horizontal and vertical integration to evaluate the impact of horizontal and vertical externalities on market outcomes. Additionally, we examine the welfare results of the different settings.

We find that the intermediate good producers always set higher prices prices under quantity competition than under price competition, independent of the inputs being substitutes or complements. As a consequence, they prefer price competition over quantity competition if the inputs are complements and vice versa if they are substitutes, due to different demand effects. Moreover, we illustrate that the intermediate good suppliers are better off when they merge to a multi-input monopoly than under upstream competition because of the elimination of horizontal externalities. Furthermore, we demonstrate that under upstream competition, a rise in the number of upstream firms n and hence in the intensity of competition diminishes the total upstream surplus in any case. When the input are complements, the intermediate good producers raise prices and lower quantities in case more firms enter, resulting in a reduction in demand and therefore a reduction in profits. By contrast, the upstream firms lower prices and raise quantities when more firms enter into the market if the inputs are substitutes. In this situation, the competition becomes very fierce with entry and hence the input profits fall.

We additionally show that the final good producer and the final consumers always prefer price competition over quantity competition among intermediate good producers, irrespective of the degree of substitutability between inputs because of lower input prices under price competition. The lower input prices result in a lower output price and in lower costs for the downstream producer and, consequently, they result in a greater consumer surplus and a greater profit for the downstream producer. Finally, producer surplus and total welfare are always greater under upstream price competition than under upstream quantity competition too. The final good producer, producer surplus, consumer surplus and total welfare benefit more from a multi-input upstream monopoly than from upstream competition in case the inputs are complements but vice versa when they are substitutes. This is because the monopoly sets lower prices than the competing firms for

complementary inputs but the reverse holds true for substitutable inputs. Furthermore, consumer surplus, producer surplus and total welfare decrease in n for complementary inputs due to the raised input prices but increase in n for substitutable inputs due to the reduced input input prices.
We further observe that an upstream horizontally integrated as well as vertically integrated supply chain yields a (constrained) welfare efficient solution. The reason for this is that the negative externalities of input differentiation and the negative externalities of vertical separation are internalized in this setting. The efficient solution also coincides with a corner equilibrium of the upstream price competition game in which the degree of substitutability is at its maximum within our model. In this case, input prices are driven down to marginal costs because of intense competition.
In conclusion, we establish that the seminal literature results of Spence (1976a,b) and Singh and Vives (1984) regarding the relations and market outcomes between different competition modes and market settings hold for the upstream market in our framework. This means that we highlight that their findings for differentiated oligopolies in non-vertical markets are robust for an upstream market in a more general (vertical) set-up with n upstream firms and input differentiation via a CES production function. We in addition illustrate that a vertical merger may be beneficial for social welfare, which is in line with the seminal findings of Spengler (1950).
An interesting extension of our model would be to incorporate oligopolistic downstream competition. Analyzing competition at every tier of a successive oligopoly with a general production function such as the CES function would complement our results and could provide further important managerial and economic implications.

Acknowledgments

We thank Angelika Endres, Claus-Jochen Haake, Burkhard Hehenkamp and Markus Reisinger for valuable comments. The paper was presented in the research seminar of the Frankfurt School of Finance and Management (Frankfurt, Germany, 2017), at the Paderborn University Faculty of Business Administration and Economics Workshop (Paderborn, Germany, 2017), at the SING14 (Bayreuth, Germany, 2018), the 29^{th} Jerusalem Summer School of Economics: Industrial Organization 2018 (Jerusalem, Israel, 2018) and at the 4^{th} Lancaster Game Theory Conference (Lancaster, United Kingdom, 2018).
This work was partially supported by the German Research Foundation (DFG) within the Collaborative Research Center "On-The-Fly Computing" (SFB 901) under the project number 160364472-SFB901/3.

6.7 Appendix

6.7.1 Proof of Proposition 6.1

The first-order condition (FOC) for determining the optimal demand of input $i \in N$ reads

$$\frac{\partial \Pi(\mathbf{x})}{\partial x_i} = \frac{\partial q(\mathbf{x})}{\partial x_i}(1 - 2q(\mathbf{x})) - t_i \overset{!}{=} 0. \tag{6.32}$$

The partial derivative of the production function is equal to

$$\frac{\partial q(\mathbf{x})}{\partial x_i} = \frac{1}{n}\left[\frac{q(\mathbf{x})}{x_i}\right]^{\frac{1}{\sigma}}. \tag{6.33}$$

For all $i \in N$, the FOC thus becomes

$$t_i x_i^{\frac{1}{\sigma}} = \frac{q(\mathbf{x})^{\frac{1}{\sigma}}}{n}(1 - 2q(\mathbf{x})). \tag{6.34}$$

Equating the left hand side for $i, j \in N$ yields

$$x_j = \left[\frac{t_i}{t_j}\right]^{\sigma} x_i. \tag{6.35}$$

For all $j \in N \setminus \{i\}$, we substitute (6.35) back into (6.32) and can determine x_i explicitly. With $\mathbf{t}^{-\sigma} = (t_1^{-\sigma}, \ldots, t_n^{-\sigma})$, the output then becomes

$$\begin{aligned}
q(\ldots, x_i t_i^{\sigma} t_{i-1}^{-\sigma}, x_i, x_i t_i^{\sigma} t_{i+1}^{-\sigma}, \ldots) &= \left[\frac{1}{n}\left(x_i^{\frac{\sigma-1}{\sigma}} + \sum_{j \in N\setminus\{i\}} \left[\frac{t_i}{t_j}\right]^{\sigma-1} x_i^{\frac{\sigma-1}{\sigma}}\right)\right]^{\frac{\sigma}{\sigma-1}} \\
&= x_i \left[\frac{1}{n}\left(\left[\frac{t_i}{t_i}\right]^{\sigma-1} + \sum_{j \in N\setminus\{i\}} \left[\frac{t_i}{t_j}\right]^{\sigma-1}\right)\right]^{\frac{\sigma}{\sigma-1}} \\
&= x_i t_i^{\sigma} \left[\frac{1}{n}\sum_{j \in N} t_j^{1-\sigma}\right]^{\frac{\sigma}{\sigma-1}} \\
&= x_i t_i^{\sigma} q(\mathbf{t}^{-\sigma}).
\end{aligned} \tag{6.36}$$

Equation (6.32) then simplifies to

$$0 = \frac{t_i q(\mathbf{t}^{-\sigma})^{\frac{1}{\sigma}}}{n}\left(1 - 2x_i t_i^{\sigma} q(\mathbf{t}^{-\sigma})\right) - t_i. \tag{6.37}$$

Eventually we solve (6.37) for $x_i(\mathbf{t})$:

$$x_i(\mathbf{t}) = \frac{1}{2t_i^\sigma q(\mathbf{t}^{-\sigma})}\left(1 - \frac{n}{q(\mathbf{t}^{-\sigma})^{\frac{1}{\sigma}}}\right). \tag{6.38}$$

6.7.2 Proof of Proposition 6.2

Let us first define $q(\mathbf{t}^{-\sigma})$ for $\mathbf{t} = (t, \bar{t}, \bar{t}, \ldots)$:

$$\overline{q}\left(t^{-\sigma}, \bar{t}^{-\sigma}\right) := q\left(t^{-\sigma}, \bar{t}^{-\sigma}, \bar{t}^{-\sigma}, \ldots\right) = \left[\frac{1}{n}\left(t^{1-\sigma} + (n-1)\bar{t}^{1-\sigma}\right)\right]^{\frac{\sigma}{\sigma-1}}. \tag{6.39}$$

The first-order condition for maximizing $\omega(t, \bar{t})$ reads

$$\begin{aligned}\frac{\partial\omega(t,\bar{t})}{\partial t} =& \frac{1}{2}\Bigg[\frac{(1-\sigma)t^{-\sigma}\overline{q}\left(t^{-\sigma},\bar{t}^{-\sigma}\right) - t^{1-\sigma}\frac{\partial\overline{q}\left(t^{-\sigma},\bar{t}^{-\sigma}\right)}{\partial t}}{\overline{q}\left(t^{-\sigma},\bar{t}^{-\sigma}\right)^2}\left(1 - \frac{n}{\overline{q}\left(t^{-\sigma},\bar{t}^{-\sigma}\right)^{\frac{1}{\sigma}}}\right)\\ &+ \frac{t^{1-\sigma}}{\overline{q}\left(t^{-\sigma},\bar{t}^{-\sigma}\right)}\frac{n}{\sigma\overline{q}\left(t^{-\sigma},\bar{t}^{-\sigma}\right)^{\frac{1}{\sigma}+1}}\frac{\partial\overline{q}\left(t^{-\sigma},\bar{t}^{-\sigma}\right)}{\partial t}\Bigg] \overset{!}{=} 0.\end{aligned} \tag{6.40}$$

The partial derivative in the term above can be simplified to

$$\frac{\partial\overline{q}\left(t^{-\sigma},\bar{t}^{-\sigma}\right)}{\partial t} = -\frac{\sigma}{n}\overline{q}\left(t^{-\sigma},\bar{t}^{-\sigma}\right)^{\frac{1}{\sigma}} t^{-\sigma}. \tag{6.41}$$

We consider a symmetric equilibrium of the form $t = \bar{t}$ such that $\overline{q}\left(t^{-\sigma},\bar{t}^{-\sigma}\right)$ simplifies to

$$\overline{q}\left(\bar{t}^{-\sigma},\bar{t}^{-\sigma}\right) = \bar{t}^{-\sigma}. \tag{6.42}$$

The first-order condition then becomes

$$\frac{\partial\omega(\bar{t},\bar{t})}{\partial t} = \frac{1}{2}\left[\left(1-\sigma+\frac{\sigma}{n}\right)(1-n\bar{t}) - \bar{t}\right] = 0. \tag{6.43}$$

Solving (6.43) for $\bar{t} = t^P(n,\sigma)$ yields a symmetric candidate

$$t^P(n,\sigma) = \frac{\sigma + n(1-\sigma)}{n(1+\sigma+n(1-\sigma))}. \tag{6.44}$$

A sufficient condition for the existence of a unique symmetric equilibrium is given in (Hefti, 2017, Theorem 1) and reads

$$\frac{\partial^2 \omega(\bar{t},\bar{t})}{\partial t^2} < 0. \tag{6.45}$$

Therefore, consider the second-order partial derivative evaluated at the symmetric equilibrium

$$\frac{\partial^2 \omega(t^P(n,\sigma), t^P(n,\sigma))}{\partial t^2} = \frac{(2n^2-3n+1)\sigma^2 + (-4n^2+3n+1)\sigma + 2n^2}{2n((n-1)\sigma - n)}. \tag{6.46}$$

Note that the enumerator is a parabola in σ. Since $2n^2 - 3n + 1 > 0$ for all $n > 1$, it is open to the top, and since the vertex $V\left(\frac{4n+1}{4n-2}, \frac{7n+1}{8n-4}\right) > (0,0)$ lies in the first quadrant, it is positive valued for all $(n,\sigma) \in \mathbb{N} \times \mathbb{R}_{++}$. The denominator is negative for $\sigma < \frac{n}{n-1}$. The second order condition is thus satisfied for all $\sigma < \frac{n}{n-1}$.

In addition, the equilibrium profits of the intermediate good suppliers are given by $\omega^P(n,\sigma) = \omega(t^P(n,\sigma), t^P(n,\sigma)) = \frac{\sigma+n(1-\sigma)}{2n(1+\sigma+n(1-\sigma))^2}$. We observe that $\sigma < \frac{n}{n-1}$ ensures that the profits are positive so that $\omega^P(n,\sigma) > 0$. If $\sigma \geq \frac{n}{n-1}$, the intermediate good suppliers would earn zero profits or make losses with $t^P(n,\sigma) = \frac{\sigma+n(1-\sigma)}{n(1+\sigma+n(1-\sigma))}$. Thus, they set $t^P = 0$ in the equilibrium if $\sigma \geq \frac{n}{n-1}$.

6.7.3 Proof of Lemma 6.1

We first solve (6.35) for t_j:

$$t_j = \left[\frac{x_i}{x_j}\right]^{\frac{1}{\sigma}} t_i. \tag{6.47}$$

Now we plug t_j for all $j \in N \setminus \{i\}$ into $q(\mathbf{t}^{-\sigma})$:

$$\begin{aligned} q\left(\ldots, t_i^{-\sigma} x_{i-1} x_i^{-1}, t_i^{-\sigma}, t_i^{-\sigma} x_{i+1} x_i^{-1}, \ldots\right) &= \left[\frac{1}{n}\left(t_i^{1-\sigma} + \sum_{j\in N\setminus\{i\}} \left[\frac{x_i}{x_j}\right]^{\frac{1-\sigma}{\sigma}} t_i^{1-\sigma}\right)\right]^{\frac{\sigma}{\sigma-1}} \\ &= \frac{q(\mathbf{x})}{x_i t_i^{\sigma}}. \end{aligned} \tag{6.48}$$

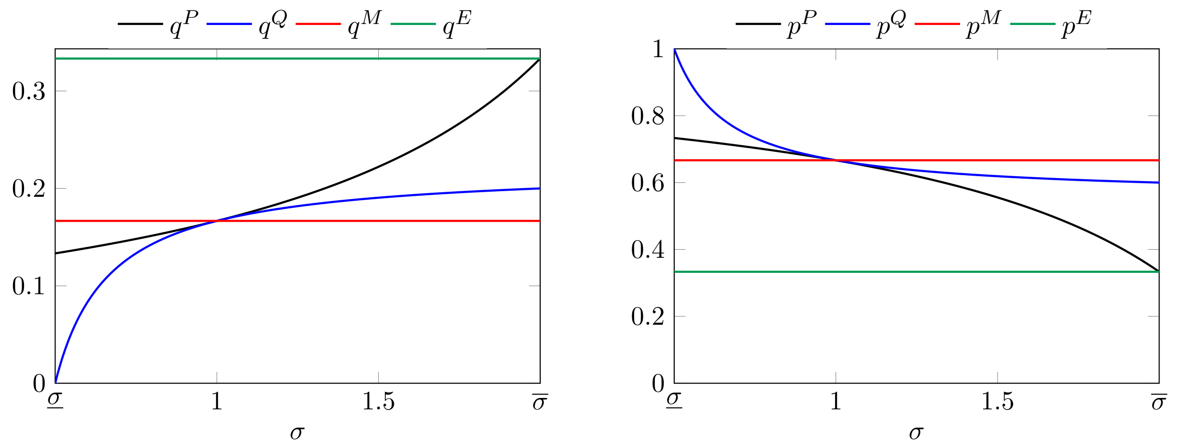
After substituting (6.48) for $q(\mathbf{t}^{-\sigma})$ into (6.13) one gets

$$x_i = \frac{1}{2t_i^{\sigma} \frac{q(\mathbf{x})}{x_i t_i^{\sigma}}} \left(1 - \frac{n}{\left[\frac{q(\mathbf{x})}{x_i t_i^{\sigma}}\right]^{\frac{1}{\sigma}}}\right) = \frac{x_i}{2q(\mathbf{x})}\left(1 - n t_i \left[\frac{x_i}{q(\mathbf{x})}\right]^{\frac{1}{\sigma}}\right). \tag{6.49}$$

Solving (6.49) for t_i eventually yields

$$t_i(\mathbf{x}) = \frac{1 - 2q(\mathbf{x})}{n} \left[\frac{q(\mathbf{x})}{x_i}\right]^{\frac{1}{\sigma}}. \tag{6.50}$$

6.7.4 Further Comparisons



(a) Output quantity $q^\ell(2, \sigma)$ **(b)** Output price $p^\ell(2, \sigma)$

Figure 6.4: Additional output outcomes for $n = 2$ and $\sigma \in [\underline{\sigma}, \overline{\sigma}]$

6.7.5 Detailed Market Results

t^P	$\frac{\sigma+n(1-\sigma)}{n(1+\sigma+n(1-\sigma))}$
x^P	$\frac{1}{2(1+\sigma+n(1-\sigma))}$
q^P	x^P
p^P	$\frac{2n\sigma-2\sigma-2n-1}{2(n\sigma-\sigma-n-1)}$
π^P	$\frac{\sigma+n(1-\sigma)}{2n(1+\sigma+n(1-\sigma))^2}$
Π^P	$\frac{1}{4(n\sigma-\sigma-n-1)^2}$
US^P	$\frac{n+\sigma-n\sigma}{2(1+n+\sigma-n\sigma)^2}$
PS^P	$\frac{1+2n+2\sigma-2n\sigma}{4(1+n+\sigma-n\sigma)^2}$
CS^P	$\frac{1}{8(n\sigma-\sigma-n-1)^2}$
W^P	$\frac{3+4n+4\sigma-4n\sigma}{8(1+n+\sigma-n\sigma)^2}$

Table 6.1: Price competition

t^Q	$\frac{\sigma}{n\,(1+\sigma-n(1-\sigma))}$
x^Q	$\frac{1-(1-\sigma)n}{2(1+\sigma-n(1-\sigma))}$
q^Q	x^Q
p^Q	$\frac{n\sigma+2\sigma-n+1}{2(n\sigma+\sigma-n+1)}$
π^Q	$\frac{\sigma(1-n(1-\sigma))}{2n(1+\sigma-n(1-\sigma))^2}$
Π^Q	$\frac{(n\sigma-n+1)^2}{4(n\sigma+\sigma-n+1)^2}$
US^Q	$\frac{\sigma(n\sigma-n+1)}{2(n\sigma+\sigma-n+1)^2}$
PS^Q	$\frac{(n\sigma-n+1)\,(n\sigma+2\sigma-n+1)}{4(n\sigma+\sigma-n+1)^2}$
CS^Q	$\frac{(n\sigma-n+1)^2}{8(n\sigma+\sigma-n+1)^2}$
W^Q	$\frac{(n\sigma-n+1)\,(3n\sigma+4\sigma-3n+3)}{8(n\sigma+\sigma-n+1)^2}$

Table 6.2: Quantity competition

t^M	$\frac{1}{2n}$
x^M	$\frac{1}{4}$
q^M	x^M
p^M	$\frac{3}{4}$
π^M	$\frac{1}{8n}$
Π^M	$\frac{1}{16}$
US^M	$\frac{1}{8}$
PS^M	$\frac{3}{16}$
CS^M	$\frac{1}{32}$
W^M	$\frac{7}{32}$

Table 6.3: Monopoly

t^E	0
x^E	$\frac{1}{2}$
q^E	x^E
p^E	$\frac{1}{2}$
π^E	0
Π^E	$\frac{1}{4}$
US^E	0
PS^E	$\frac{1}{4}$
CS^E	$\frac{1}{8}$
W^E	$\frac{3}{8}$

Table 6.4: Efficient solution

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