Gravity Models in Dynamic Economies

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Acknowledgment

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The optimal working environment and the interaction with other scientists, both inside and outside the faculty, have been significant parts of creating this thesis.

I owe my supervisor Prof. Dr. Bernard M. Gilroy a great debt of gratitude for his outstanding support. In 2017 we developed the idea to further extend and improve the existing gravity model literature. This initially very general idea gave rise to various approaches that are presented in this dissertation, a portfolio of different studies. The close ties between research and teaching at his professorship for macroeconomic theory and international economics have continuously given new impulses from which all members of the chair were able to benefit. Examining theoretical approaches collectively and creating a space for new ways of thinking were important aspects in working out innovative research questions. In this context I would also like to thank Dr. Christian Peitz and Dr. Volker Seiler for their inputs.

I dedicate this dissertation to my family, who have supported me extensively since I began my studies. Particularly noteworthy are my parents Martina and Volker Stoeckmann, who laid the foundation for my academic career in the early years. Thanks also go to my brother Max J. Stoeckmann for his support.

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Chapter 1

Introduction

In a global economy that is characterized by a variety of unpredictable shocks, rapidly advancing digitization, political disagreements and the overall international heterogeneity, it is difficult to capture all economic interactions in universally valid laws or theories. Only a few other sciences have to adapt to new circumstances so quickly. What can be accepted as state of the art among scientists today may be out of date tomorrow. That is the reason why the criticism of many non-economists, especially with regard to the failure of multiple models during the last crises, is partly appropriate.

While the general scientific discipline of economics can be roughly divided into three ways of thinking that build on each other, Classical, Austrian and Keynesian, macroeconomics in particular can be split into the period before and after Lucas (1976).

Until 1976, primarily straightforward models such as the IS-LM or the Mundell-Fleming model dominated economics. The Lucas criticism then lead further into the investigation of the foundation of such models. Teaching, however, is still largely based on these simple, but only in rare exceptional cases accurate, models. The required deeper analysis of these models in macroeconomic research is synonymous with a microeconomic foundation.

While current macroeconomic research deals primarily with Dynamic Stochastic General Equilibrium (DSGE) models based on Real Business Cycle (RBC) models, there is a little more variability to be found in the field of international economics. However, the basic microeconomic assumptions are largely identical. Households are utility maximizers and firms are portrayed as profit maximizers. Further theoretical cornerstones from publications by among others Dixit & Stiglitz (1977), Blanchard & Kiyotaki (1987), Blanchard & Quah (1989) or Romer (1990) can often be found in both DSGE models and models of international economics. Monopolistic competition, households' preferences for a variety in goods, is often used as a sufficient base for further analysis. They can be modeled, among others, by Constant Elasticity of Substitution (CES) utility functions introduced by Armington (1969).

CES functions are special cases of homothetic preferences, relationships represented by utility functions which are homogeneous of degree one. Jeffrey Bergstrand's (1985) model, which is one of the bases of this work, not only uses CES utility functions, but also Constant Elasticity of Transformation (CET) production functions based on Powell & Gruen's (1968) contribution to derive his general gravity approach. The choice for Bergstrand's (1985) *The gravity equation in international trade: some microeconomic foundations and empirical* *evidence* as a cornerstone of this work is based on the assumptions, the level of detail, the microeconomic foundation and the broad acceptance of the paper in the field of international economics.

In addition to Bergstrand's (1985, 1989) studies, many other important contributions have been added to the literature on gravity models over time. There are several meta-studies that adequately summarize this literature, such as Head & Mayer's (2014). Even if this dissertation is not reproducing any of these studies, there are important contributions to be mentioned.

Gravity models have become a *workhorse, toolkit and cookbook* for a variety of research questions (Head & Mayer 2014). Inspired by Newton's Law of Gravity, these models in international economics define interaction between two economies or regions by the product of their masses divided by their distance to each other. This definition allows flexibility for the whole range of different applications. In the literature, primarily the economic sizes of the respective countries represent the masses, which are positively correlated with trade flows. For special cases, variables such as the number of inhabitants, the per capita income or even stock market volumes representing these masses and therefore also are positively correlating with bilateral flows.

Gravity models are commonly used to estimate trade, especially for the effects of free trade agreements, a floating exchange rate, geographical peculiarities, cultural affinities or differences. These mentioned applications of the effects on trade flows are far from covering the whole bandwidth of possibilities. Listing all the options does not provide any added value to this thesis. While there is a number of theoretical foundations for the gravity model of trade, to which reference is be made in the following, one of which is even fully explained in Appendix A, there are no adequate justifications for other dependent variables that are described by gravity models, even though they follow similar patterns. Especially factor movements are also often explained by gravity models. These movements are divided into the two parts of production. Based on gravity, workers' mobility is represented by commuting flows on regional and national level (Flowerdew & Aitkin 1982) and by migration flows on international level (Vanderkamp 1977). Capital flows, in terms of Foreign Direct Investments (FDI) between states and trading places can be modeled as well (Brenton et al. 1999, Portes & Rey 2005).

The popularity of the model is based on the ongoing globalization and the increasing density of networks between states. While there is a variety of theoretical models describing the reason for trade more or less successfully, the gravity model takes the leading role in describing patterns for which country trades with whom. The advancing data availability allows a growing number of scientists to efficiently work with these kinds of models. Although the model was only used empirically for a long time, it has also gained theoretical relevance over the years. Many authors have committed themselves to the theoretical foundation with a range of different assumptions. Some of these have also been revised over time and would lead to biased results if applied.

In the 1960s, Tinbergen (1962) and Pöyhönen (1963) introduced the idea of explaining bilateral trade flows by a basic gravity model. Even if this relation could be validated empirically, their approaches lacked a sufficient theoretical foundation at that time. Among the multitude of different explanations, some of the most important were based on ideas of Linnemann (1966), Aitken (1973), Geraci & Prewo (1977), Prewo (1978), Abrams (1980), and Sapir (1981). They defined trade flows by a partial equilibrium of export supply and import demand resulting in a reduced form of the gravity model. Therefore country i's consumption of goods and services produced in country j was equal to the product of nation i's and j's Gross Domestic Product (GDP) divided by the world's GDP, later described by Baldwin & Taglioni (2006) as *potluck assumption*.

Anderson (1979), inspired by the *potluck* approaches, then sets new standards for the theoretical justification of the model. He provides a microfoundation that is still used by modern international economists for its simplicity. His theoretical approach based mainly on the assumption of countries producing unique goods that are imperfect international substitutes. Due to the success and the spread of the model, also in subsequent studies, there is also some criticism to be found. Deardorff (1984), for example, criticizes the theoretical *potluck* justification and demands a more accurate theoretical foundation in order to be able to continue working with the empirically successful model.

Bergstrand (1985) then provides a more detailed derivation of gravity. He combines the theory that was initially only designed to describe patterns of who trades with whom with classic trade theory to emphasize the reason for trade. Bergstrand (1985) links national factor endowment with bilateral trade. Where previous authors left too little space for individual adjustments, his model can be adapted to more specific research questions quite straightforward. He also implements price terms, which on the one hand lead to a detailed theoretical model but on the other hand makes the exact implementation in empirical studies more difficult. Bergstrand (1985) himself cannot depict these complex price terms due to data unavailability, but rather approximates them using price indices.

Inspired by Helpman & Krugman (1985), Bergstrand (1989, 1990) then links his existing model to more classical trade theory while sticking to price indices. In the following years, authors contributed with different approaches to the theoretical derivation of the gravity model. But by adding further assumptions, they were nowhere near as popular. These models can rather be seen as a backslide compared to Bergstrand's (1985, 1989, 1990).

About two and a half decades after Anderson's (1979) sole approach, Anderson & van Wincoop (2003) brought back gravity models to the stage of international economics literature, strongly oriented towards Anderson's (1979) initial assumptions. Their contribution is directed more towards the design of empirical studies, especially with respect to cross-section and panel data.

Current research in the field of gravity equations is primarily empirical, there is little that is new to the theoretical literature. This dissertation does not only apply the model empirically, but also corrects and expands Bergstrand's (1985) model to make it valid for more specific research question. Since the gravity model is rarely associated with other economic theories, this work also contributes to the integration of gravity into existing international economics concepts.

Even though the model is adapted and adjusted to a variety of individual research questions, there are primary determinants which are the same over all applications, the positive impact of the target market and the market of origin, as well as the negative impact of the distance between them. Their effects can be empirically validated for many different time periods and at almost all levels. To illustrate, Germany's foreign trade is briefly described in Figures 1.1, 1.2 and 1.3 with special regard to the primary determinants in a way how also Krugman et al. (2012) introduce the model. Data is collected from the Federal Statistical Office for Germany, Eurostat for Europe and the World Bank for the non-European states. In order not to have any external disturbances, specifically the Covid-19 pandemic, this introductory example uses data from the last year before the outbreak of the virus.

Germany exports goods valued EUR 1.32 trillion in 2018, ranking third behind China and the USA. In terms of exports, final products dominate German industries. This is due to two main factors. On the one hand, Germany is a low-resource country, on the other hand, Germany has a technological advantage over many trading partners and can thus export highly technical products.

In 2018, the German government achieved a trade surplus of EUR 227 billion. While reaching a lower surplus than in 2017, it remained the leader among all trading partners with the exception of China. While the largest bilateral trade surplus can be achieved with the US, the largest bilateral trade deficit is in the economic relationship with China. Although surpluses and deficits have their own research area and are highlighted in the media, there is no unique consequence. For this reason, the current account will not be discussed in the course of this thesis.

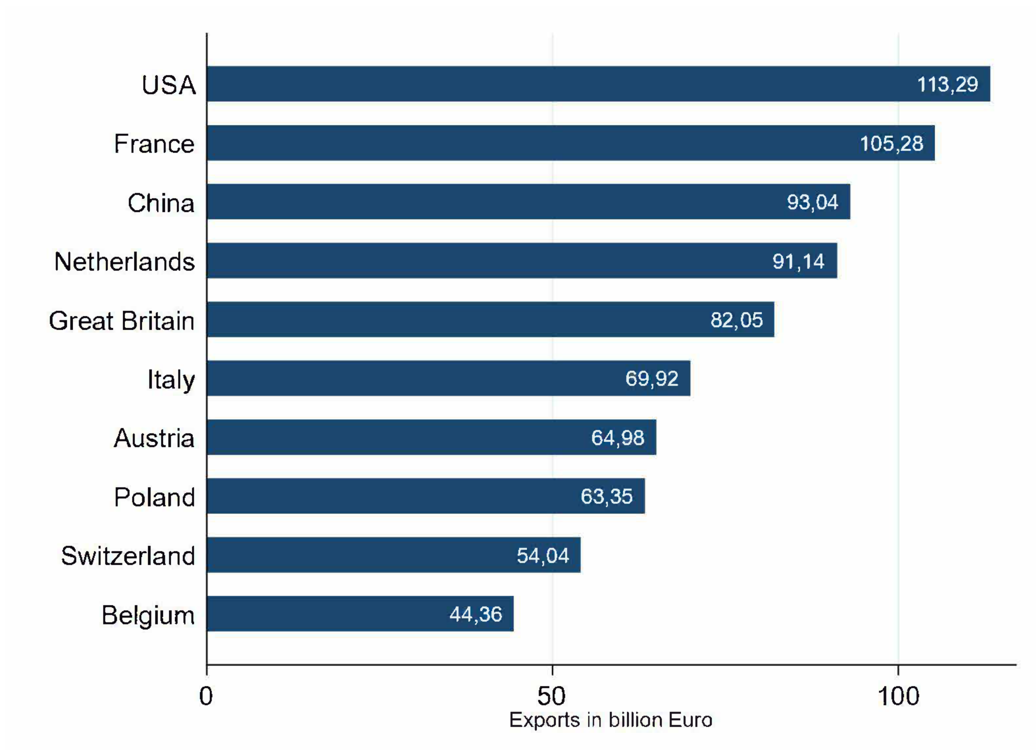

Figure 1.1: Germany's exports by partner 2018

Germany exports goods and services worth EUR 113.29 billion to the United States (US) in 2018, which accounted for almost ten percent of total exports. Trade relations with the US are therefore of great economic importance. Effects by possible protectionism could therefore be crucial. France, China and the Netherlands follow as further trading partners with regard to German exports. The United Kingdom and other neighbouring countries are also heavily dependent on German exports in 2018.

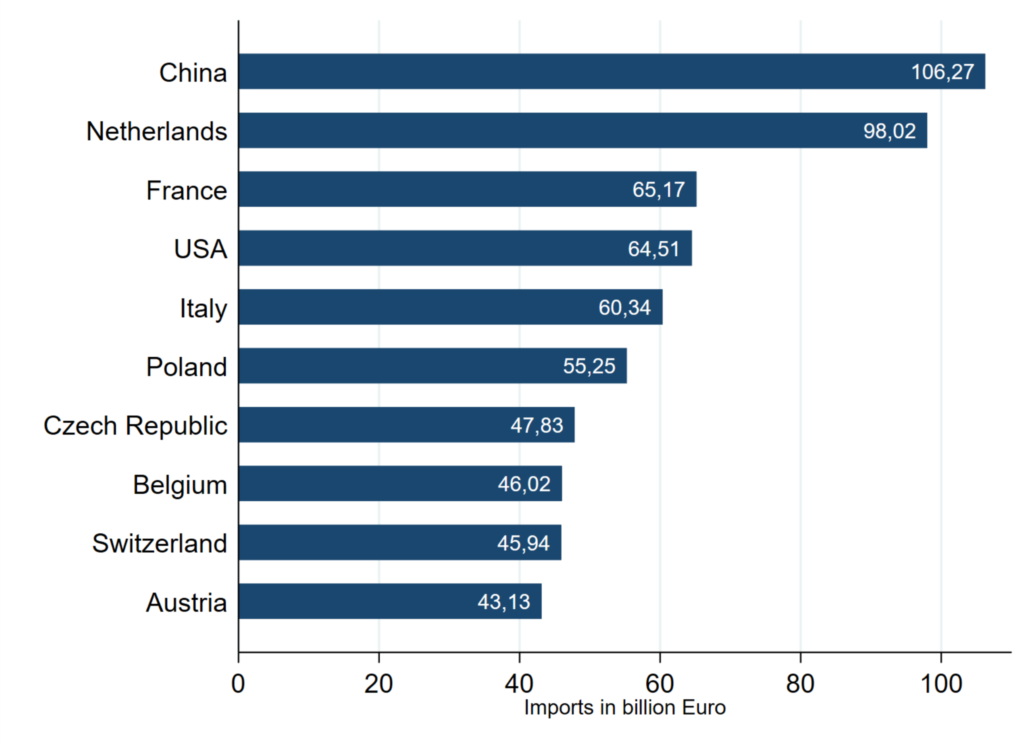

Figure 1.2: Germany's imports by partner 2018

Since Germany has the largest bilateral trade deficit with China, it is obvious that goods and services from this country amount to the largest volume in terms of all German imports of EUR 106.28 billion. While the US, as the largest consumer, dominates the ranking of German exports, they are only on the fourth place in terms of imports. The Netherlands and France, which are also heavily involved in exports, are second and third in imports. The states among Germany's ten largest trading partners are almost identical for exports and imports, only the Czech Republic and the United Kingdom can only be found in either Figure 1.1 or 1.2.

By taking a look at trade statistics at the industry level, similar rankings and identical trading partners for both exports and imports can be observed. This contradicts every classical trade theory resulting in specific branch specialization. In contrast, if states restricted themselves to the production of a specific good, they would be represented either as exporters or as importers, but never as both within an industry. In order to justify such observations, intra-industry trade theories are receiving wide acclaim from economists.

In addition to the US and China, all other trading partners among the ten largest in Germany, measured by exports and imports, are European countries. It is striking that, as predicted in the gravity model, only relatively large economies determine international trade patterns. Only developed nations, countries whose GDP is well above the average of the world's population are among the largest trading partners of Germany.

Nations that are as economically strong as the European partners in the top ten, but located in other parts of the world, are unattractive for both exports and imports, as the increased distance entails higher transport costs. For example, Italy and Brazil have an almost identical GDP of around EUR 2 trillion. In the ranking of German exports, however, Italy occupies 6th place, Brazil 29th. It is similar for imports. Italy is ranked 5th while Brazil is 31st. The same applies to Spain and Australia with a GDP of about 1.42 trillion. Spain ranks 12th place and Australia 28th on the list of exports. For imports, it is even clearer, Spain is in the 13th place and Australia in 46th.

In order to visualize the impact of the economic size on trade, only the nine neighboring states of Germany are considered in Figure 1.3. The distance and also other secondary factors, especially a common border are neglected since they would have been identical across all countries.

The countries to be considered are Denmark, Poland, the Czech Republic, Austria, Switzerland, France, Luxembourg, Belgium and the Netherlands. Of course, this introductory example is for illustrative purposes only and can not be generalized due to the small number of observations.

If data is available, the gravity model also allows for further subdivision to lower levels, such as those of the provinces. Thus differences within Germany between the regions could be identified. Trade with the Benelux (Belgium, Netherlands, Luxembourg) countries is higher in western Germany, trade with Poland and the Czech Republic in the eastern states and trade with Switzerland and Austria in southern Germany.

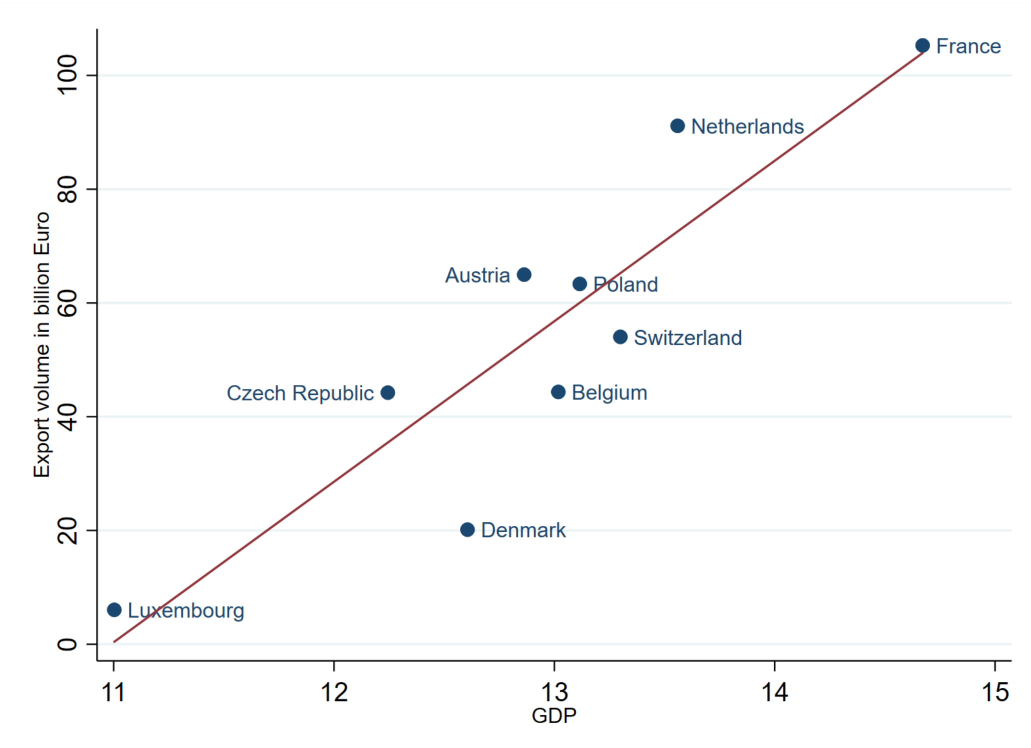

Figure 1.3: The size effect on bilateral trade

Figure 1.3 represents the positive effect of GDP on bilateral trade in 2018. Comparatively small states, measured by economic strength, such as Luxembourg, account for only a small part of Germany's export volume. The larger a country economically, the stronger the trade relations. The ordinary least squares (OLS) method further visualizes this positive relationship shown by the solid line even though there are other aspects as well, such as cultural affinities, which could define trade in more detail. The linearization requires a previous transformation of the influencing factor GDP by logarithms.

In empirical studies, the gravity model is primarily used to provide suitable control variables. By extracting the impact of market sizes, distance, and other secondary variables associated with the model, the effect of the factor under investigation can be studied separately. Regardless of the research question, the positive impact of the size of the economy and the negative influence of the distance in a bilateral relationship on arbitrary levels is always observable. Mainly economic and social science research questions are answered by applying gravity models. For example, it can be predicted how foreign trade will change if a country becomes a member in a trading union or what happens if the country leaves. The possibilities of extending the core of the model to a wide variety of research questions with further specifications are infinite.

The methodology used in Figure 1.3 is very similar to many previous studies, but is only intended to give an impression of the application possibilities. However, the following chapters really contribute to the literature of gravity models used in international economics.

Chapter two to six are self-contained studies that claim to be independent research work. In combination with other publications outside of this work they would also fulfill the requirements of a cumulative dissertation. The reason the chosen format is a monograph without all the studies included is that their topics are highly heterogeneous. In order to really make a significant contribution to the gravity literature, only topic-related studies are included. Even if all studies deal with the gravity model, there is a high degree of variability in its implementation. There are purely theoretical as well as empirical, methodical and hybrid forms.

Dynamics are crucial with respect to these research questions. Even if the model in its physical origin describes the gravitational force, the attraction of masses, and thus also predicts movements in space, the economic equivalent is rather static. However, this static is inadequate in the context of a rapidly changing international economy. A majority of the current global problems affect international trade, directly or indirectly. The worldwide Covid-19 pandemic, military conflicts, the US-China trade war, the exit of Great Britain from the EU, large migration flows, but also at first sight mundane events like the Suez Canal obstruction or climate change-depending failures of value chain parts. Since these dynamics inevitably have to be implemented, gravity models are expanded with innovative variables and traditional models are literally given an additional dimension in the following chapters.

At the time the dissertation is submitted, the individual studies are either published or in a publication process. Because they all deal with the topic of gravity models in a dynamic environment, it is possible that some explanations are repeated or that the same sources are quoted multiple times. Nevertheless, appendices are summarized at the end of this thesis.

Due to a detailed analysis of theoretical derivations of the gravity model, a fundamental error with regard to Bergstrand's frictionless case has been revealed. The second chapter therefore introduces the correction, which is published as *Stoeckmann, N. (2020). Recalculating Gravity: A Correction of Bergstrand's 1985 Frictionless Case. Econ Journal Watch, 17(2), 333.*

> Jeffrey Bergstrand's general gravity equation is used widely in studies of international trade. This note provides a correction for the special frictionless case of the gravity equation derived in Bergstrand (1985). In this particular case of perfect substitutability of goods in consumption and in production, zero tariffs, and zero transport costs, the model simplifies to $PX_{ij} = Y_i^{1/2} Y_j^{1/2}$, not $PX_{ij} = (1/2) \cdot Y_i^{1/2} Y_j^{1/2}$. The (1/2) should not be there. The correction has been confirmed by Professor Bergstrand. The special case has not been used much in empirical work, but a few theoretical models have used the special case and repeated the erroneous equation, so those papers too stand in need of correction. This note provides the derivations for the correction to the special frictionless case of Bergstrand's gravity equation.

The third chapter *Migration and bilateral trade: Demand-side effects in micro-founded models*, which is partly based on the results of the second, deals with the impact of migration on bilateral trade. Different stages of the paper's progress were presented at the *13th FIW-Research Conference "International Economics" - Vienna*, *Hagen Workshop on Global Economic Studies* and the *XXII Conference on International Economics - Murcia* and can therefore also be found in the respective conference reports.

> A variety of empirical studies already tried to show the impact of migration on both imports and exports. While the supply-side effect is sufficiently explained by Gould (1994) as an extension of Bergstrand's (1985) micro-founded gravity model, the demand-side effect still lacked a detailed theoretical justification and empirical separation. Therefore, Bergstrand's partial equilibrium is extended to also account for a transplanted home-bias, the preference of migrants for goods from their country of origin. By choosing a data set of trade flows from South and East Asian countries to OECD founding members between 1995 and 2014, a significant positive demand effect on trade and an even larger effect on traded household consumption in particular can be observed. Furthermore, non-linear effects depending on the amount of migrants are shown by quantile-based categories and are explained in a three-dimensional economies of scale model.

Chapter four is a critical thought experiment on the long-term effects of international trade on economic growth. This study is published as *Stoeckmann, N. (2022). Conditional Beta-Convergence by Gravity. Theoretical Economics Letters, 12(1), 98-110.*

> Globalization and the international interdependence of states have reached their climax at the beginning of the 21st century. At the same time, growing inequalities between and within countries are leaving some behind. While a variety of models have sufficiently explained national divergence, international divergence still remains subject of numerous studies. This work contributes to the set of possible explanations for worldwide disparities by combining the ideas of classical growth theories with the gravity model of trade. The circular relations between GDP, trade flows and TFP then explain long term differences in the development of states. Resulting path dependencies thus can be explained by an *International Innovation Spiral* that continuously leads developed economies towards potential higher outputs while existing alongside national peculiarities. In this way, the importance of trade unions and the openness to international markets can be theoretically further substantiated.

The fifth chapter *Unraveling the distance paradox: Alternative approach for the persisting effect in gravity models* tackles one of the recent problems of gravity model usage in international economics.

> Borchert & Yotov (2017), among others, have devoted themselves to solving the distance puzzle, the observation that despite advancing globalization, distance plays a persistent role in bilateral trade. This study analyzes more than just one determinant of the gravity model with respect to globalization. Therefore, both the fluctuation of binary variables' relevance and the relevance of the distance can be attributed towards individual decision making on the international level. This study also proposes an alternative empirical approach by measuring globalization effects through the traveled value per meter divided by the total trade per year in a panel data set concentrating on annual intra-OECD trade flows between 2000 and 2019.

Chapter six *Synchronized effects of interest rates on capital movements: How arbitrage models fail to sufficiently explain FDI* uses the gravity model as a basic construct for control variables in a Foreign Direct Investment context. This comes with an additional contribution on advantages and disadvantages of log-transformations. Intermediate results of this chapter were presented at the *The Economic Consequences of Trade, Finance and FDI at PKU - Beijing* and can therefore be found in the conference report.

> Negative interest rate policies to incentivize spending and investing money for long term growth is ubiquitous at times when positive economic shocks or stability is needed. While the policy affects the domestic liquidity, there is no guarantee that the intended investments remain within national borders. In conclusion, foreign countries may benefit from spillover effects through FDI as a transmission channel. Therefore, this paper focuses on determinants of bilateral FDI, especially on the individual impacts of domestic and foreign interest. The estimated effects of both the domestic and foreign interest rate are negative on outward FDI while the interest rate difference between domestic and foreign does not have a significant effect in a data set covering OECD countries between 2003 and 2013. Due to the lack of significance of the difference between the interest rates, capital arbitrage models can no longer adequately reflect reality. Because outward FDI positions can be negative, this study additionally provides three different transformation functions and their general effects on the data structure to solve log-linearization problems.

To conclude, chapter seven summarizes the greatest contributions, gives recommendations for working with gravity models in general and presents a specific exposé on a future study.

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Chapter 2

Recalculating gravity

Jeffrey Bergstrand (1985) derives a theoretical gravity equation for bilateral international trade flows focusing on the role of prices. The gravity equation has served as the basis for thousands of theoretical and empirical works in the field of international economics.

Bergstrand develops a general equilibrium model of world trade, which is based on utility and profit maximization in N countries endowed with one production factor each. He uses nested utility and production functions based on constant elasticity of substitution (CES) and constant elasticity of transformation (CET), respectively. The model generates N^2 partial equilibrium subsystems of 4 equations each with 4 endogenous variables and $3N$ constraints. This system of $4N^2 + 3N$ equations results in the general gravity model if the small market assumption - the neglectable impact of the market between country i and j on other markets - and the assumptions of identical preferences and technologies across countries hold.

$$PX_{ij} = \underbrace{Y_i^{\frac{\sigma-1}{\gamma+\sigma}} Y_j^{\frac{\gamma+1}{\gamma+\sigma}}}_{I} \underbrace{C_{ij}^{-\sigma\frac{\gamma+1}{\gamma+\sigma}} T_{ij}^{-\sigma\frac{\gamma+1}{\gamma+\sigma}} E_{ij}^{\sigma\frac{\gamma+1}{\gamma+\sigma}}}_{II}$$
$$\cdot \underbrace{\left(\sum_{k=1,k\neq i}^{N} P_{ik}^{1+\gamma}\right)^{-\frac{(\sigma-1)(\gamma-\eta)}{(1+\gamma)(\gamma+\sigma)}}}_{III} \underbrace{\left(\sum_{k=1,k\neq j}^{N} P_{kj}^{1-\sigma}\right)^{\frac{(\gamma+1)(\sigma-\mu)}{(1-\sigma)(\gamma+\sigma)}}}_{IV}$$
$$\cdot \underbrace{\left[\left(\sum_{k=1,k\neq i}^{N} P_{ik}^{1+\gamma}\right)^{\frac{1+\eta}{1+\gamma}} + P_{ii}^{1+\eta}\right]^{-\frac{\sigma-1}{\gamma+\sigma}}}_{V} \underbrace{\left[\left(\sum_{k=1,k\neq j}^{N} P_{kj}^{1-\sigma}\right)^{\frac{1-\mu}{1-\sigma}} + P_{jj}^{1-\mu}\right]^{-\frac{\gamma+1}{\gamma+\sigma}}}_{VI}$$

This equation is the same as equation (14) in Bergstrand's original paper (Bergstrand 1985), divided into VI parts. The value of bilateral trade is dependent on both countries' gross domestic products Y, the gross transport factor C, the tariff rate T, the exchange rate E, the f.o.b. price of i's good in k P_{ik}, the c.i.f. price of k in j P_{kj} and domestic prices and elasticities, where $\sigma(\mu)$ is the elasticity of substitution in consumption between imported goods (between imported and domestic goods) and $\gamma(\eta)$ is the elasticity of transformation in production between export markets (between foreign and domestic markets). This equation is often cited as Bergstrand's generalized gravity model. It is used to define sufficient control variables for empirical trade analysis and to provide theoretical justification for gravity.

To get a frictionless gravity model that excludes all price terms, further assumptions have to be made. Assuming perfect substitutability, perfect commodity arbitrage, zero tariffs, zero transport costs and normalizing exchange rate to unity implies that $C_{ij} = T_{ij} = E_{ij} = 1$,

$P_{ik} = P_{kj} = P_{ii} = P_{jj} = \bar{P}\ \forall\ i, j, k$, and $\sigma = \mu = \gamma = \eta = \infty$. Bergstrand's simplification resulted in his equation (15), $PX_{ij} = (1/2)Y_i^{1/2}Y_j^{1/2}$ - but recalculating the derivation, as shown in the following, actually leads to $PX_{ij} = Y_i^{1/2}Y_j^{1/2}$. The coefficient (1/2) in Bergstrand's equation (15) should have been omitted. This correcation has been confirmed by Professor Bergstrand as part of the valuable comments he provided on this paper.

$$I\colon\ Y_i^{\frac{\sigma-1}{\gamma+\sigma}} Y_j^{\frac{\gamma+1}{\gamma+\sigma}} = Y_i^{\frac{1}{2}} Y_j^{\frac{1}{2}}$$

$$II\colon\ = 1$$

$$III\colon\ \left(\sum_{k=1,k\neq i}^{N} \bar{P}^{1+\gamma}\right)^{-\frac{(\sigma-1)(\gamma-\eta)}{(1+\gamma)(\gamma+\sigma)}} = \left(\sum_{k=1,k\neq i}^{N} \bar{P}^{1+\gamma}\right)^{-\frac{(\sigma-1)(0)}{(1+\gamma)(\gamma+\sigma)}} = \left(\sum_{k=1,k\neq i}^{N} \bar{P}^{1+\gamma}\right)^{0} = 1$$

$$IV\colon\ \left(\sum_{k=1,k\neq j}^{N} \bar{P}^{1-\sigma}\right)^{\frac{(\gamma+1)(\sigma-\mu)}{(1-\sigma)(\gamma+\sigma)}} = \left(\sum_{k=1,k\neq j}^{N} \bar{P}^{1-\sigma}\right)^{\frac{(\gamma+1)(0)}{(1-\sigma)(\gamma+\sigma)}} = \left(\sum_{k=1,k\neq j}^{N} \bar{P}^{1-\sigma}\right)^{0} = 1$$

$$V\colon\ \left[\left(\sum_{k=1,k\neq i}^{N} \bar{P}^{1+\gamma}\right)^{\frac{1+\eta}{1+\gamma}} + \bar{P}^{1+\eta}\right]^{-\frac{\sigma-1}{\gamma+\sigma}} = \left\{\left[\left((N-1)\bar{P}^{1+\gamma}\right)^{\frac{1}{1+\gamma}}\right]^{1+\eta} + \bar{P}^{1+\eta}\right\}^{-\frac{\sigma-1}{\gamma+\sigma}}$$

$$= \left\{\left[(N-1)^{\frac{1}{1+\gamma}}\bar{P}\right]^{1+\eta} + \bar{P}^{1+\eta}\right\}^{-\frac{\sigma-1}{\gamma+\sigma}} = \left\{(N-1)^{\frac{1+\eta}{1+\gamma}}\bar{P}^{1+\eta} + \bar{P}^{1+\eta}\right\}^{-\frac{\sigma-1}{\gamma+\sigma}}$$

$$= \left\{N\bar{P}^{1+\eta}\right\}^{-\frac{\sigma-1}{\gamma+\sigma}} = \left\{N^{\frac{1}{1+\eta}}\bar{P}\right\}^{-\frac{(\sigma-1)(1+\eta)}{\gamma+\sigma}} = \bar{P}^{-\frac{(\frac{\sigma}{\sigma}-\frac{1}{\sigma})(\frac{1}{\sigma}+\frac{\eta}{\sigma})}{\frac{\gamma}{\sigma}+\frac{\sigma}{\sigma}}} = \bar{P}^{-\frac{1}{2}}$$

$$VI\colon\ \left[\left(\sum_{k=1,k\neq j}^{N} \bar{P}^{1-\sigma}\right)^{\frac{1-\mu}{1-\sigma}} + \bar{P}^{1-\mu}\right]^{-\frac{\gamma+1}{\gamma+\sigma}} = \left\{\left[\left((N-1)\bar{P}^{1-\sigma}\right)^{\frac{1}{1-\sigma}}\right]^{1-\mu} + \bar{P}^{1-\mu}\right\}^{-\frac{\gamma+1}{\gamma+\sigma}}$$

$$= \left\{\left[(N-1)^{\frac{1}{1-\sigma}}\bar{P}\right]^{1-\mu} + \bar{P}^{1-\mu}\right\}^{-\frac{\gamma+1}{\gamma+\sigma}} = \left\{(N-1)^{\frac{1-\mu}{1-\sigma}}\bar{P}^{1-\mu} + \bar{P}^{1-\mu}\right\}^{-\frac{\gamma+1}{\gamma+\sigma}}$$

$$= \left\{N\bar{P}^{1-\mu}\right\}^{-\frac{\gamma+1}{\gamma+\sigma}} = \left\{N^{\frac{1}{1-\mu}}\bar{P}\right\}^{-\frac{(\gamma+1)(1-\mu)}{\gamma+\sigma}} = \bar{P}^{-\frac{(\frac{\gamma}{\gamma}+\frac{1}{\gamma})(\frac{1}{\gamma}-\frac{\mu}{\gamma})}{\frac{\gamma}{\gamma}+\frac{\sigma}{\gamma}}} = \bar{P}^{\frac{1}{2}}$$

$$\bar{P}X_{ij} = Y_i^{\frac{1}{2}} Y_j^{\frac{1}{2}} \bar{P}^{\frac{1}{2}-\frac{1}{2}} = Y_i^{\frac{1}{2}} Y_j^{\frac{1}{2}}$$

While the empirical implications of this correction may not be serious, there is a formal theoretical relevance. Besides presenting the correct solution of the frictionless case, this note further contributes to the literature on gravity models in three ways.

Bergstrand (1985) introduces the theoretical section of his paper by stating that the gravity equation then in use, $PX_{ij} = \beta_0 Y_i^{\beta_1} Y_j^{\beta_2} D_{ij}^{\beta_3} A_{ij}^{\beta_4} u_{ij}$, lacks a sufficient theoretical foundation. He defines his goal to derive a gravity equation as similar as possible to this representation. The first equation he derives is from the general equilibrium model. With further constraints, he achieves his goal of creating a frictionless case closer to the widely used equation. With the correction of the constant in this special case, it is possible to get even closer to his goal. Not only the general equilibrium model is frequently used in empirical studies. Some authors also cite the frictionless case as a base for further theoretical studies (e.g. Földvári 2006 and Ramesh 2017). These works also need to be corrected.

The range of theoretical derivations of gravity models has grown significantly over time, with the most cited models next to Bergstrand being Anderson (1979), Anderson & van Wincoop (2003), Deardorff (1998) and Eaton & Kortum (2002). Meta-papers (such as Head & Mayer 2014), which have set themselves the task of classifying and comparing gravity approaches, occasionally criticize a lack of comparability. Corrections such as the one presented here contribute to the comparability.

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Chapter 3

Migration and bilateral trade

3.1 Introduction

Migration will play an increasingly important role in this globalized but heterogeneous world. Modern economic theories should therefore try to implement the resulting effects adequately. According to the United Nations, the number of global migrants reached an all-time high prior to the Covid-19 crisis and will not decrease afterwards. Their effects in the origin and destination countries strongly influence existing economic interactions. In times of deep specialization and diversified consumption, one of the most relevant economic interaction between two countries is bilateral trade. This study therefore shows the impacts of migration on bilateral trade, both theoretically and empirically.

Many studies have shown the effects of migration on trade empirically, only a few have tried to do it also theoretically. Their success has been unsatisfactory so far. Due to the fact that there is a variety of empirical evidence, this study first focuses on the theoretical implementation of both the demand and the supply side effect of migrants on bilateral trade with their country of origin. Traditional economic models based on factor endowment justify migration as labor compensation in the abundant country. Unfortunately, these models are not sufficient describing bilateral trade patterns. Besides the supply effects successfully shown by Gould (1994), where migrants extend bilateral trade relations in their country of destination with their country of origin due to decreasing market access costs, there also is a demand factor that has not yet been implemented theoretically in a Bergstrand-type (1985) gravity model. This demand-side effect is based on a greater preference of migrants for goods from their origin country, which consequently leads to an increase in trade inflows from that country.

In the theoretical section of this study, Bergstrand's (1985) micro-founded gravity model is extended. This extension specifically deals with the demand and supply effect of migrants on bilateral trade. The empirical analysis significantly demonstrates the demand effect separately by excluding the supply effect through the choice of data. There are two major reasons why the data consists of annual trade flows from East and South Asian countries to Organisation for Economic Co-operation and Development (OECD) founding members, a panel covering 396 country-pairs between 1995-2014. The quality and the continuity with which data is gathered in the OECD allows a detailed panel analysis. Furthermore, only a very small number or even none inhabitants of founding OECD members migrated to East and South

Asian countries in this period, which allows to exclude the supply effect. Established static estimation methods are then used resulting in a significant positive demand-side effect of migrants.

After introducing into the topic, this paper is divided into six further sections. Summarizing the results of the effect of migration on trade from previous studies in the second section provides several methods and control variables. The third section shows the derivation of the micro-founded model implementing migrants' effects on bilateral trade, which leads to the hypothesis. Section four then tests the separated demand-side effect empirically before section five embeds the amount-depending effect in a three-dimensional economies of scale model. Building on that, section six shows potential further application for that model before the seventh section concludes.

3.2 The effect of migration on trade

There have been numerous studies investigating the effect of migration on bilateral trade. Almost all of them show a positive correlation empirically, only a few try to model this relationship theoretically.

Gould (1994) is known as one of the first who investigated the impact of migration on bilateral trade between the United States (US) and 47 countries. These 47 trading partners are the origin country of a large number of migrants living in the US. The results in a data set covering the period 1970-1986 reveal the positive link between bilateral trade and migration. Gould attributes the effect to still existing links to the home country, through reduced language barriers or even through direct contacts. Moreover, migrants also have certain knowledge of the political and business environment of their country of origin. Hence, through these connections, transaction costs are reduced, which enhances exports to the origin country.

Head & Ries (1998) tested the correlation of migration and bilateral trade for Canada and 136 partners from 1980 to 1982 by also applying the gravity model. In general, they found that a ten percent increase in migration is associated with a one percent increase in exports and a three percent increase in imports. Furthermore, they differentiate migrants into three types: independent migrants, family migrants and refugees. Independent migrants tend to have the biggest impact on trade while refugees have the lowest. They also indicated that East Asian migrants have a larger effect compared to the other regions. Regarding the supply effect, this could be interpreted as the highest market information costs due to both the contrasting culture and the language, which is then decreased by their knowledge. On the demand side, their preference for East Asian goods could be higher than the preference of other migrant groups. This explicit demand effect is going to be tested in the empirical section of this study.

Girma & Yu (2002) conducted an empirical analysis of the United Kingdom's (UK) trade with 48 countries between 1981 and 1993. Their study focused on identifying the differences in the impacts of migration on trade flows. The authors divided migrants into two groups. The first consists of migrants coming from Commonwealth countries while the second covers non-Commonwealth migrants. Migration in the first group tend to have cultural and institutional factors in common with the UK whereas the second group does not. The results show that the UK's exports increased significantly by migrants from non-Commonwealth partners, whereas there is no effect on Commonwealth partners. In conclusion, migrants from different cultural and institutional background tend to facilitate total trade to a larger extent.

Rauch & Trindade (2002) show the effects of an ethnic Chinese network on international trade in a panel covering 63 countries in 1980 and 1990. The authors classify different effects of homogeneity and heterogeneity. According to their results, Chinese ethnic groups tend to have a greater impact on trade of heterogeneous products than on trade of homogeneous goods.

Their findings were then applied to support the hypothesis of Blanes (2005) for the case of Spain. Blanes similarly suggests that the greater impact of migration on trade of heterogeneous products in comparison to homogeneous products is due to the fact that it primarily occurs in intra-industry trade.

According to White (2007), there are two ways that migration affects bilateral trade. He differentiates between the demand effect of a transplanted home bias and a network effect with the impact on supply. While the network effect is sufficiently explained through the reduced market information costs by Gould (1994) before, the transplanted home bias refers to the fact that migrants prefer products from their country of origin which cannot be sufficiently substituted by the destination country's local market. The demand for these products would therefore promote the destination country to import goods from their country of origin.

In his paper, White (2007) examined the migration-trade link between the US and 73 trading partners in the period 1980 until 2001. On the one hand, Gould (1994) with a similar focus suggested that migration has positive effect on bilateral trade in general. On the other hand, White (2007) found that the increase in bilateral trade between the US and other countries is derived from migrants coming from low-income countries only.

Hatzigeorgiou (2010) investigated the impact of migration on trade flows of Sweden and 180 trading partners between 2002 and 2007. His argument of the positive impact of migrants on bilateral trade is also based on the network effect. The empirical results showed that migration has a significantly strong, positive and robust impact on Sweden's international trade. A ten percent increase in the stock of migration led to a six percent and nine percent increase in exports and imports, respectively. While many authors had to deal with endogeneity, Felbermayr & Jung (2009) find a robust, causal positive effect of migrants on trade using a regression-based F-test for strict exogeneity based on Wooldridge (2002).

Zhang (2020) takes up White's idea of the transplanted home bias and finds out that the home bias is heterogeneous among the different migrant groups and that ethnic taste bias explains a large part of bilateral trade relations.

In general, migration has positive effects on bilateral trade between the destination and origin country. Empirically, the import effect is larger than the export effect. Also the impact increases with larger cultural differences. Some studies indicate that the effect is also larger on heterogeneous goods in comparison to the effect on homogeneous goods. However, so far most studies have lacked a satisfactory theoretical justification implementing both the supply and the demand effect. In addition, it is difficult for all authors to differentiate these effects empirically from one another. It happens that they only want to show one of the effects, but do not simultaneously take the other into account and thus get biased results.

In order to fit the existing theoretical literature, this study extends Bergstrand's (1985) micro-founded gravity model to include migration in both directions.

3.3 Theoretical implementation

Bergstrand's (1985) micro-founded equilibrium model of world trade is modified to implement the positive effect of migration on both the supply and the demand side. Gould (1994) already introduced endogenous transaction costs that negatively correlate with the number of migrants due to easier market access. This study also adds demand-side effects of migrants. Utility and profit maximizing leads to a general equilibrium model between N countries endowed with one production factor each. Both a nested utility and production function based on constant elasticity of substitution (CES) and constant elasticity of transformation (CET) respectively are utilized. Equating bilateral import and export values generates N^2 partial equilibrium subsystems of 4 equations each with 4 endogenous variables and $3N$ constraints. This system of $4N^2 + 3N$ equations results in the general gravity model if the small market assumption, the neglectable impact of the market between country i and j, and the assumptions of identical preferences and technologies across countries hold.

Identical technologies across countries are assumed on the supply side. Therefore, the production factor labor L, consisting of both native and migrant workers, in country i can be represented equivalent to Bergstrand (1985) and Gould (1994) as

$$L_i = \left\{ \left[\left(\sum_{k=1,k\neq i}^{N} X_{ik}^{\phi} \right)^{\frac{1}{\phi}} \right]^{\delta} + X_{ii}^{\delta} \right\}^{\frac{1}{\delta}} \quad \forall\ i = 1, ..., N. \tag{3.1}$$

Referring to equation (3.1), the production factor can be allocated across industries according to the CET production surface, where X_{ik} is country i's good supplied to country k and X_{ii} is country i's good supplied to the domestic market. The exponent δ is equal to $(\eta+1)/\eta$, where η is the elasticity of transformation between any two goods produced in country i $(0 \leq \eta \leq \infty)$ and $\phi = (\gamma + 1)/\gamma$, where γ is the CET among exportable goods $(0 \leq \gamma \leq \infty)$. Maximizing profits $\pi_i = \sum_{k=1}^{N} P_{ik}X_{ik} - W_i L_i,\ \forall\ i = 1, ..., N$ under the restriction from equation (3.1) gives N^2 first order conditions (FOC) and generates $N(N-1)$ bilateral export volumes

$$\begin{aligned} X_{ij}^{S} =& Y_i P_{ij}^{*\gamma} \left[\left(\sum_{k=1,k\neq i}^{N} P_{ik}^{*1+\gamma} \right)^{\frac{1}{1+\gamma}} \right]^{-(\gamma-\eta)} \\ & \cdot \left\{ \left[\left(\sum_{k=1,k\neq i}^{N} P_{ik}^{*1+\gamma} \right)^{\frac{1}{1+\gamma}} \right]^{1+\eta} + P_{ii}^{*1+\eta} \right\}^{-1} \quad \forall\ i, j = 1, ..., N;\ i \neq j. \end{aligned} \tag{3.2}$$

The supply of country i to country j is therefore depending on the total income paid to labor $Y_i = W_i L_i$, where W_i is the wage, on the elasticities and on i's real prices P_{ik}^{*} received in different countries k as it is shown in equation (3.2). Real prices are further differentiated into $P_{ik}^{*} = P_{ik} C_{ik}^{-1} T_{ik}^{-1} Z_{ik}^{-1}$, where P_{ik} represents the nominal price received, $C_{ik}(\geq 1)$ the transport costs, which are positively correlating with distance, $T_{ik}(\geq 1)$ the ad valorem tariff rate on i's product in k plus one and $Z_{ik}(\geq 1)$ the costs of market access in i for k. This representation is equivalent to Gould's (1994). The differentiation of real prices and nominal prices plus other factors can be done on the supply side, like it is done here, or on the demand side, like Bergstrand (1985) does. It does not make a difference in the outcome, neither mathematically, nor theoretically.

For the demand side, identical utility functions across countries, depending on the quantity of demanded goods and their respective factors α and ζ, are assumed. While ζ is referring to the migrants M_{kj} that originated in that specific country k, α refers to any other cohort $M_j - M_{kj}$. To fit into the same utility function, both parts are weighted by dividing the number of migrants from that specific country respectively any other group by the total number of inhabitants M_j in country j. Since it is the goal to describe a transplanted home bias like White (2007) suggests, $\zeta > \alpha$ is assumed. In that way migrants are getting more utility from consuming goods from that country, where they are originated. For simplicity and consistency, native inhabitants in country j without any migration background are represented as M_{jj}.

$$U_j = \left\langle \left\{ \left[\sum_{k=1,k\neq j}^{N} \left(\frac{M_j - M_{kj}}{M_j} \alpha_{kj} X_{kj}^{\theta} + \frac{M_{kj}}{M_j} \zeta_{kj} X_{kj}^{\theta} \right) \right]^{\frac{1}{\theta}} \right\}^{\psi} + \frac{M_j - M_{jj}}{M_j} \alpha_{jj} X_{jj}^{\psi} + \frac{M_{jj}}{M_j} \zeta_{jj} X_{jj}^{\psi} \right\rangle^{\frac{1}{\psi}} \quad \forall\ j = 1, ..., N \tag{3.3}$$

Maximizing this utility function, equation (3.3), with respect to the budget constraint $Y_j = \sum_{k=1}^{N} P_{kj} X_{kj},\ \forall\ j = 1, ..., N$ yields the demand of country i's product in j

$$\begin{aligned} X_{ij}^{D} =& Y_j P_{ij}^{-\sigma} \left(\frac{M_j - M_{ij}}{M_j} \alpha_i + \frac{M_{ij}}{M_j} \zeta_i \right)^{\sigma} \\ & \cdot \left\{ \left[\sum_{k=1,k\neq j}^{N} P_{kj}^{1-\sigma} \left(\frac{M_j - M_{kj}}{M_j} \alpha_k + \frac{M_{kj}}{M_j} \zeta_k \right)^{\sigma} \right]^{\frac{1}{1-\sigma}} \right\}^{\sigma-\mu} \\ & \cdot \left\langle \left\{ \left[\sum_{k=1,k\neq j}^{N} P_{kj}^{1-\sigma} \left(\frac{M_j - M_{kj}}{M_j} \alpha_k + \frac{M_{kj}}{M_j} \zeta_k \right)^{\sigma} \right]^{\frac{1}{1-\sigma}} \right\}^{1-\mu} \right. \\ & \left. + P_{jj}^{1-\mu} \left(\frac{M_j - M_{jj}}{M_j} \alpha_j + \frac{M_{jj}}{M_j} \zeta_j \right)^{\mu} \right\rangle^{-1} \quad \forall\ i, j = 1, ..., N;\ i \neq j. \end{aligned} \tag{3.4}$$

If there is no differentiation in the preference of products, $\alpha = \zeta = 1$, equation (3.4) would converge to Bergstrand's (1985) representation of the demand, with the exception that he already distinguished between real and nominal prices on the supply side. Equating supply and demand, equation (3.2) and (3.4), leads to the partial equilibrium representation of bilateral trade flows.

Due to the small-market assumption, equation (3.5) displays the value of aggregate trade flows from country i to j depending on both national incomes Y_i and Y_j, the exogenous bilateral variables C_{ij}, T_{ij}, Z_{ij}, various prices and on different migrant groups in j. While the incomes affect bilateral trade positively, transport costs, tariff rates and costs for the market access affect it negatively. The impacts of prices cannot be determined a priori, they are depending on supply and demand elasticities. The focus, regarding this research, is explicitly on M_{ij}, the number of migrants in the importer originated in the exporter country. The market access or information costs are assumed to be dependent on the number of migrants and an unobserved factor z, therefore $Z_{ij} = f(z, M_{ji})$. The notation in the index follows consistently every other bilateral variable, therefore M_{ji} are migrants living in country i originated in j. The return of migrants on cost reduction is assumed to be positively diminishing $\partial Z_{ij}/\partial M_{ji} < 0, \partial^2 Z_{ij}/\partial M_{ji}^2 > 0$ (Gould 1994).

In addition to this supply effect, migrants in the importing country originated in the exporting country M_{ij} seem to lead to an overall increase in demand if $\zeta_{ij} > \alpha_{ij}$ holds.

$$\begin{aligned} PX_{ij} =& Y_i^{\frac{\sigma_j-1}{\gamma_i+\sigma_j}} Y_j^{\frac{\gamma_i+1}{\gamma_i+\sigma_j}} C_{ij}^{-\frac{(\gamma_i+1)\sigma_j}{\gamma_i+\sigma_j}} T_{ij}^{-\frac{(\gamma_i+1)\sigma_j}{\gamma_i+\sigma_j}} Z_{ij}^{-\frac{(\gamma_i+1)\sigma_j}{\gamma_i+\sigma_j}} \\ & \cdot \left(\frac{M_j - M_{ij}}{M_j}\alpha_{ij} + \frac{M_{ij}}{M_j}\zeta_{ij} \right)^{\frac{(\gamma_i+1)\sigma_j}{\gamma_i+\sigma_j}} \\ & \cdot \left[\left(\sum_{k=1,k\neq i}^{N} P_{ik}^{1+\gamma_i} \right)^{\frac{1}{1+\gamma_i}} \right]^{-\frac{(\sigma_j-1)(\gamma_i-\eta_i)}{(1+\gamma_i)(\gamma_i+\sigma_j)}} \\ & \cdot \left\{ \left[\sum_{k=1,k\neq j}^{N} P_{kj}^{1-\sigma_j} \left(\frac{M_j - M_{kj}}{M_j}\alpha_{kj} + \frac{M_{kj}}{M_j}\zeta_{kj} \right)^{\sigma_j} \right]^{\frac{1}{1-\sigma_j}} \right\}^{\frac{(\gamma_i+1)(\sigma_j-\mu_j)}{(1-\sigma_j)(\gamma_i+\sigma_j)}} \\ & \cdot \left\{ \left[\left(\sum_{k=1,k\neq i}^{N} P_{ik}^{1+\gamma_i} \right)^{\frac{1}{1+\gamma_i}} \right]^{1+\eta_i} + P_{ii}^{1+\eta_i} \right\}^{-\frac{(\sigma_j-1)}{\gamma_i+\sigma_j}} \\ & \cdot \left\langle \left\{ \left[\sum_{k=1,k\neq j}^{N} P_{kj}^{1-\sigma_j} \left(\frac{M_j - M_{kj}}{M_j}\alpha_{kj} + \frac{M_{kj}}{M_j}\zeta_{kj} \right)^{\sigma_j} \right]^{\frac{1}{1-\sigma_j}} \right\}^{1-\mu_j} \right. \\ & \left. + P_{jj}^{1-\mu_j} \left(\frac{M_j - M_{jj}}{M_j}\alpha_{jj} + \frac{M_{jj}}{M_j}\zeta_{jj} \right)^{\mu_j} \right\rangle^{-\frac{(\gamma_i+1)}{\gamma_i+\sigma_j}} \end{aligned} \tag{3.5}$$

3.4 Empirical analysis

The most obvious and precise way to analyze demand-side effects of migration households would be observing their direct individual demand behavior. Due to many reasons, such data is not existing and too complex to collect over a variety of countries and migrants. However, with the implementation of sufficient control variables and the reasonable choice of countries, a panel data can have a similar validation in defining the demand-side effect of migrants on bilateral trade.

Supply effects of migrants based on existing networks pushing the export flow to the country they are originated in, are already shown in several case studies. This study tries to state the demand effect as general as possible. Therefore 396 country pairs over the period 1995-2014 are observed in a panel. While the receiving country of both migrant and trade flows are exclusively OECD founding members, countries of origin are located in South and East Asia.

The explicit choice for the coverage in this data set is based on multiple reasons.
OECD countries usually have a higher data availability and quality in comparison to other countries. Furthermore, OECD countries are likely to become a destination for migration since they are more developed on average. East and South Asian countries as the origin of both migration and trade flows are the perfect choice for this analysis, because of both the results of Head & Ries (1998) and Girma & Yu (2002), the easier to observe effect in the Asian population, and the neglectable small stock of OECD migrants in the exporter country that could lead to biased results considering the push effect on bilateral trade supply. For many years and country-pairs this number of migrants in the panel is even zero, so there cannot even be a supply-side effect. Conversely, if there is an effect of migrants, it is clearly attributable to demand. Southeast, Central Asia and Western Asia are excluded since these countries are too small to be relevant, both on trade and on a migration stock perspective. They even have got too many real zeros in their observations which would lead to a transformation bias, if implemented. However, in some countries the data is simply not available. Only the founding members of the OECD were selected because several Asian and Oceanic countries have joined the organization over time, including South Korea, Japan, Australia and New Zealand, which would lead to a strong local bias if implemented in the data set. The detailed selection of origin and destination countries can be found in the appendix B.1 and B.2.
The last twenty years available, 1995-2014, have been chosen to be relevant to actual policy makers and due to the reached peak of globalization until now.
The theoretical framework in the third section is, with some adjustments, a good basis for the empirical analysis. Taking the partial derivative of equation (3.5) with respect to the number of migrants M_{ij} from that specific country i leads to the hypothesis, that M_{ij} increases bilateral trade volume PX_{ij} if $\zeta_{ij} > \alpha_{ij}$ and $\sigma_j > 1$ hold. Because $0 < \sigma_j < \infty$, this additional assumption of σ_j being greater than one is another limitation, but not too unlikely and widely assumed in the literature.

Hypothesis 1 *If all other factors remain constant and the number of migrants in the importing country j originated in the exporting country i increases, trade flows from i to j increase.*

To test Hypothesis 1 (H1), the derivative with respect to the number of migrants of equation (3.5) is transformed into a log-linear regression model

$$\begin{aligned} tra_{ijt} =& \beta_0 + \beta_1 mig_{ijt} + \beta_2 gdp_{it} + \beta_3 gdp_{jt} + \beta_4 aco_{ijt} + \beta_5 pop_{it} + \beta_6 pop_{jt} \\ &+ \beta_7 EUV_{it} + \beta_8 IUV_{jt} + \beta_9 DEF_{it} + \beta_{10} DEF_{jt} + \beta_{11} FTA_{ijt} + \varepsilon_{ijt}, \end{aligned} \tag{3.6}$$

where tra_{ijt} is the endogenous flow of goods and services in natural logarithms (ln) from country i to country j at any given time point t. In general, all variables in logarithms are represented in lower case letters, while absolute values are represented in capital letters. The exogenous variable of interest mig_{ijt} is the stock of migrants in country j originated in country i in every time point t. Both Y from the equilibrium, respectively the derivative, are represented by Gross Domestic Products gdp_{it} and gdp_{jt}. C_{ij} and aco_{ijt} both stand for the trade costs between i and j in t. It is important to note that distance normally lacks a time level. Since panel data analyses and especially fixed effect (FE) models are the most appropriate models for gravity approaches, the advantages of which will be discussed later in this work, every variable has to fit into those two dimensions of country-pairs and time points. FE models exclude time-invariant variables, therefore, if distance is explicitly implemented, the choice of the regression will erase a detailed analysis of its impact anyways. That is why, this paper suggests to adjust the proxy for costs C_{ij} by multiplying the time-invariant distance between the two biggest cities of each country-pair by a time-varying factor, an average freight index. If the explicit impact of distance is not important, FE estimations usually cover country-pair specifications. Variables pop_{it} and pop_{jt} representing the population size of each country. While the derivative suggest that only pop_{jt} due to M_j is relevant, Y_i and Y_j could also be represented by per capita income and therefore set the gdp in relation to inhabitants. Other authors also suggest the relevance of the population size due to the more exact representation of supply possibilities and a demand market. For that reason, they are both implemented in the estimation. Besides these logarithmic variables, the regressions also feature some indexes and a binary variable in absolute values. Following Bergstrand (1985), price indexes, export and import unit values approximate given prices in equation (3.5). The export unit value EUV_{it}, therefore, reflects $(\sum P_{ik}^{1+\gamma})^{1/(1+\gamma)}$ across all i on the one hand. On the other hand, the import unit value IUV_{jt} approximates $(\sum P_{kj}^{1-\sigma})^{1/(1-\sigma)}$. Likewise, $(\sum P_{ik}^{1+\gamma})^{(1+\eta)/(1+\gamma)} + P_{ii}^{1+\eta}$ and $(\sum P_{kj}^{1-\sigma})^{(1-\mu)/(1-\sigma)} + P_{ii}^{1-\mu}$ are represented by the GDP deflators DEF_{it} and DEF_{jt} for each country in any given time point. T_{ij} in the derivative is representing tariffs. These rates are approximated by the binary dummy variable FTA_{ijt} representing the common membership in a free trade agreement, precisely in the World Trade Organization, at any given time point. Z_{ij}, the market access costs, can be ignored in the empirical section for two reasons. It is assumed to be dependent on z, which is not observable, and on M_{ji} like Gould (1994) assumes. The dataset is compiled with the purpose that this M_{ji}, the number of migrants originated in the importing country living in the exporting country is zero or neglectable small.

The variable mig_{ijt} is represented by the stock of foreign population by country of birth. Other definitions for the migration stock, for instance stock of migrants by nationality goes along with a bias. Countries deal with the citizenship of migrants from different countries differently. The stock of foreign born labor by country of birth would also have been an alternative. However, this choice would not reflect the demand effect of an entire family and would consequently overestimate the impact. Considering only the working population would often neglect the actual transplanted home bias of a whole family. That is why, the stock of foreign population by country of birth is the best choice for this research design. The OECD data base provides a consistency in this annual data without the necessity for interpolation. Gould (1994) suggests the implementation of the number of migrants from the US in the home country, but does not do it due to data unavailability. In contrast, the focus of this study is on that demand effect and the supply effect is, due to the absence of migrants in the exporting country originated in the importing country, excluded. He is right that in a bilateral case, where migrants of the partner country living in the other, it is not possible to differentiate between the export and the import effect. Ignoring this fact and stating that the export effect dominates the import effect, without having data available, seems naive.

Often adjacency is also added to the set of control variables like McCallum (1995) successfully did. Due to the fact that this data set only considers country-pairs which do not share a border, such a binary dummy variable is not necessary. Further country-pair specific variables like a common official language or a colonial relationship are neglected, since the chosen estimator, particularly the fixed effects (FE) estimator, controls for these bilateral peculiarities. The detailed description of the data set can be found in the appendix B.3.

The most appropriate empirical tool is the panel data analysis. For the analysis of bilateral trade, this kind of method has been recommended by several researchers such as Egger (2002) and Baier & Bergstrand (2007). Ghosh & Yamarik (2004) even highlight these panel methods compared to cross-section analyses and show the specific advantages.

Basically, the application of panel data estimations allows to control for unobserved cross-section heterogeneity. It adds more information, variability, lowers collinearity, increases degrees of freedom and leads to an overall enlarged efficiency.

This paper uses panel data of 396 country-pairs in the period from 1995-2014 resulting in 1829 and 1964 observations in each estimation of total trade. In the literature, static and dynamic estimations of bilateral trade can be found with equal frequency. There is a fundamental difference between static and dynamic models. Dynamic models take time effects into account, which are represented by lagged values of the dependent variable. They assume that changes in the dependent variable rely on their own value in the past. In the short term, dependencies can be shown effectively by dynamic models and static models are almost always misspecified, because of the serial correlation of the within-group errors.

However, static methods are efficient for the investigation of long-term effects. The dynamic effects will not be ignored, but will be explained by the independent variables in the static estimation method. Moreover, time effects will be controlled by adding time fixed effects to the models. A Poisson pseudo maximum likelihood (PPML) regression reduces the informative value and is not necessary at first since the data set does not contain real zeroes as it is often observed in foreign direct investments data sets.

That is why in total, there are seven different estimations. The pooled Ordinary Least Squares (POLS) regression is first applied to give an idea of what the impacts of individual variables might be. However, the POLS regression leads to inconsistent estimation results, where at least one of the assumptions is usually violated due to omitted variable bias. Thus, the group of estimation is extended by random effects (RE) and fixed effects (FE) models.

The advantage of RE models is, that the error term can contain unobserved effects. RE models follow the assumption that these unobserved effects are independent of explanatory variables. On the contrary, FE models allow the relation between unobserved effects and the explanatory variable. Therefore, FE models are more robust because they allow to estimate direct effects in case that time-invariant omitted variables can be correlated with the explanatory variables. However, FE models cannot include time-invariant variables because a differentiation between observed and unobserved effects is not possible. That is the reason, so many static panel estimations analyzing the gravity model must suppress the effect of distance. Here, the time-invariant geographical distance is multiplied by the time-varying average freight costs. For all models except the POLS model, it is differentiated between the variables from the derivative of equation (3.5) represented by the estimations (1), (3), (5) and (7) in Table 3.1, and the friction-less gravity model plus the stock of migrants. Bergstrand (1985) reduces his general gravity model to the friction-less case consisting of $PX_{ij} = 0.5 \cdot Y_i^{1/2} Y_j^{1/2}$, which, if calculated correctly, is even purer and resulting in $PX_{ij} = Y_i^{1/2} Y_j^{1/2}$ (Stoeckmann 2020). Therefore, bilateral trade would be only defined by the sizes of the respective economies and the migration flows as stated in (2), (4) and (6) in Table 3.1.

Table 3.1: Estimated effects on bilateral trade

	(1) POLS	(2) RE	(3) RE	(4) FE	(5) FE	(6) FE	(7) FE
mig_{ijt}	0.097***	0.210***	0.088**	0.115**	0.128**	0.115**	0.095*
	(0.0000)	(0.0000)	(0.0037)	(0.0012)	(0.0020)	(0.0013)	(0.0268)
gdp_{it}	0.717***	0.485***	0.606***	0.372***	0.337***	0.481***	0.404***
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
gdp_{jt}	0.160***	0.051*	0.092	0.053	0.159*	0.177*	0.195*
	(0.0001)	(0.0231)	(0.0858)	(0.1430)	(0.0130)	(0.0158)	(0.0246)
aco_{ijt}	0.063***		0.033**		−0.004		0.000
	(0.0000)		(0.0045)		(0.7251)		(.)
pop_{it}	0.010		0.130***		−0.972**		−0.949**
	(0.5447)		(0.0008)		(0.0037)		(0.0087)
pop_{jt}	0.184***		0.270***		−0.308		0.547
	(0.0000)		(0.0000)		(0.4564)		(0.3085)
EUV_{it}	−0.133***		−0.021*		0.048***		0.054***
	(0.0000)		(0.0277)		(0.0000)		(0.0000)
IUV_{jt}	−0.034**		−0.049***		−0.005		0.062**
	(0.0011)		(0.0000)		(0.7141)		(0.0056)
DEF_{it}	0.040***		−0.008		−0.016**		−0.013*
	(0.0000)		(0.1340)		(0.0043)		(0.0296)
DEF_{jt}	−0.002		0.007		0.005		0.004
	(0.8498)		(0.1773)		(0.3093)		(0.5039)
FTA_{ijt}	−0.003		−0.001		−0.000		0.001
	(0.7533)		(0.9145)		(0.9602)		(0.8569)
Time FE	×	×	×	×	×	✓	✓
N	1829	1964	1829	1964	1829	1964	1829

Standardized beta coefficients; p-values in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

The individual impacts of variables from equation (3.6) on trade flows are shown in estimations (1) – (7). The sizes of the respective economies, gdp_{it} and gdp_{jt}, reflect the expectations of the literature and the theoretical derivation perfectly. The export market in particular has a consistently high significance in its positive impact on bilateral trade. No clear significance can be demonstrated for the distance, respectively the average transport costs aco_{ijt}. Estimation (7) cannot output a value for the impact of aco_{ijt}. Controlling for time fixed effects in addition to country-pair fixed effects results in ignoring a variable that has been multiplied from these. For estimations (1), (3) and (5), the aco_{ijt} has no consistent impact or significance. This is due to the fact that the data set was chosen to especially work out the impact of migration on trade. The regional selection is, therefore, crucial. Founding members of the OECD are mostly concentrated in Western Europe, capital cities of East and South Asian countries are also geographically concentrated, so that there are no major differences in the distances. In addition, important trade partners of the OECD, such as China, Japan and South Korea, are relatively further to the east than smaller, less significant economies in terms of trade flows. The population sizes pop_{it} and pop_{jt} are implemented for two reasons. On the one hand, pop is sometimes used as another proxy associated with the economic power in the literature, while on the other hand it represents the whole population of a country, which is related to the migrants as M_j in equation (3.5) predicts. Because of these opposing effects, there is no uniform result for this variable. The export unit value EUV and the import unit value IUV suggest that they can play a significant role. However, there is no clear direction proven overall estimations either.

In addition to the economic size of the exporter, only one variable stands out across all models, the stock of migrants in country j, whose origin is country i. mig_{ijt} is significant positive in the POLS model, as well as in the RE models and in the FE models with both a time fixed effect added and without a time FE factor. The impact varies around 0.1, which states that an increase in the migrant stock by ten percent leads to an increase in total trade flows by one percent.

The Hausman test suggests the estimation with fixed effects. If time-fixed effects are also taken into account, estimate (7) can be considered representative for this study. This estimation is also the one, where most of the variables reach the significance level and correspond to the sign predicted in the literature and derivation.

Adopting, among others, Felbermayr & Jung (2009) based on Wooldridge (2002), the country-pair time-fixed model (7) does not have a crucial endogeneity problem. A regression based F-test fails to reject the Null that the effect of migrants on trade is strictly exogenous.
From these facts it follows that the hypothesis H1, that migrants in the importer country increase bilateral trade with their country of origin due to a transplanted home-bias, cannot be rejected.
A larger and even more significant positive effect of migration can be observed when the endogenous bilateral total trade is substituted with bilateral trade in household consumption. It should be noted that the number of observations is significantly lower due to the availability, but is still in an acceptable range. Assuming the same conventions, model (7), a country-pair fixed effect model with time fixed effects, is the most representative. Accordingly, an increase in the number of migrants by ten percent leads to an increase in traded household consumption from the country of origin by 3.85 percent, which is almost four times the effect. The two GDP in logarithms also fulfill their assumed theoretical impacts.

Table 3.2: Estimated effects on traded household consumption

	(1) POLS	(2) RE	(3) RE	(4) FE	(5) FE	(6) FE	(7) FE
mig_{ijt}	0.204***	0.480***	0.329***	0.424***	0.641***	0.342***	0.385***
	(0.0002)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
gdp_{it}	0.100***	0.191***	0.258***	0.246***	0.335***	0.464***	0.450***
	(0.0008)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
gdp_{jt}	0.679***	0.263***	0.388***	0.144**	0.696***	0.662***	0.657***
	(0.0000)	(0.0000)	(0.0000)	(0.0049)	(0.0000)	(0.0000)	(0.0000)
aco_{ijt}	0.074**		0.073***		0.045***		0.000
	(0.0015)		(0.0000)		(0.0002)		(.)
pop_{it}	0.249***		0.072		−1.124		−1.165
	(0.0000)		(0.2582)		(0.1463)		(0.2393)
pop_{jt}	−0.061		0.120		−5.084***		−1.380
	(0.5631)		(0.2803)		(0.0000)		(0.0505)
EUV_{it}	0.006		0.005		0.022		0.031
	(0.8514)		(0.7379)		(0.2044)		(0.1374)
IUV_{jt}	−0.010		−0.071***		−0.088***		0.040
	(0.7318)		(0.0000)		(0.0000)		(0.1071)
DEF_{it}	−0.040		0.017		0.013		0.019
	(0.2260)		(0.1450)		(0.2038)		(0.0946)
DEF_{jt}	0.009		0.005		0.010		0.005
	(0.6630)		(0.4864)		(0.0946)		(0.4515)
FTA_{ijt}	−0.038		0.009		0.026***		0.026***
	(0.0533)		(0.1988)		(0.0002)		(0.0001)
Time FE	×	×	×	×	×	✓	✓
N	548	601	548	601	548	601	548

Standardized beta coefficients; p-values in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Even if the estimator is not optimal, the results are also robust for a PPML model (Table B.4, Table B.5). Theory and several studies suggest including importer-time and exporter-time fixed effects, as well as time fixed effects to control for multilateral resistances (Anderson & van Wincoop 2004). There is no additional value by adding time fixed effects separately as it is the same result as in column (3) and (4) in the respective Table B.4 and B.5. There is another difference for PPML in representing the dependent variables by absolute values. While the results for total trade are similar to the results of the fixed effect models, no clear significance can be ascertained for traded household consumption including FE.

3.5 Specification

This paper generalizes ζ over all k and j. However, the preference for products from the country of origin can be variable. So migrants from different countries can have different transplanted home biases, which affect trade flows individually. Furthermore, it is conceivable that migrants have a non-linear effect on bilateral trade in terms of supply and demand based on the mixed results especially with respect to the different effect on traded household consumption depending on the estimation method.

Gould (1994) suggests that the number of migrants in the exporting country originated in the importing has diminishing returns on the market access costs. That seems reasonable, but the omission of migrants from the exporting country in the importing country shows an essential weakness of his empirical analysis. Another aspect is the advancing globalization, which facilitates entry into foreign markets through the spread of the English language and through many trade agreements that are already controlled for. In contrast to the supply effect, the demand effect might have economies of scale. So if more and more migrants prefer products from their country of origin, it will be cheaper for merchants to offer them in large quantities. Nevertheless, the higher demand can also induce local companies to produce the goods, making imports unnecessary.

Since σ_j has already been limited in the FOC, it would not be more informative if further variables would be limited as well. Thus, the second order condition (SOC) of equation (3.5) with respect to the number of migrants cannot clearly define the effect, so that another theoretical approach has to test the behavior of migrants' demand effects.

In the literature both possible outcomes are mentioned. On the one hand, a higher demand through an increasing number of migrants tend to reduce costs for exporting producers while on the other hand it is assumed that a higher number of migrants demanding goods from their country of origin encourages local merchants or new market participants to offer these specific goods, which would automatically make imports redundant.

These opposing effects are not yet compatible with existing theories. However, the economies of scale in a trade environment model initially by Marshall's (1879) industrial clusters offers a sufficient basis to implement both effects. Local firms take over the production of the goods demanded at a point where domestic demand increased to a certain level. Vernon (1966) with the product life cycle and Helpman (2011) with the proximity-concentration trade-off provide a similar approach, but focus on foreign direct investment instead.

To reveal the non-linear demand effect of migration on bilateral trade, a one good - two country model is considered. Both states i and j own a representative firm facing a cost function C. They differ in that j can produce only for the domestic market, while i initially produces for both the domestic and foreign market. For this reason, i has to exclusively pay additional transport costs, tariffs and taxes τ for goods that are exported to j.

$$\begin{aligned} C_i =& \bar{F} + \bar{c}q_i + (\bar{c} + \tau)\, q_j \\ C_j =& \bar{F} + \bar{c}q_j \end{aligned} \tag{3.7}$$

Both countries' firms face identical fix costs $\bar{F}$ and variable costs $\bar{c}$. In equation (3.7), q_i and q_j represent the quantities of a homogeneous good supplied to the respective markets i and j. Assuming firms are forced down to their average cost, the average cost curve represents the price offered for any given quantity. The average cost curve is therefore equivalent to the supply curve as it is also presented in the theory of external economies and trade.

Country j imports from i as long as the offered price is below the local possibilities, so as long $AC_i < AC_j$. The average cost curves (AC) are the cost functions 3.7 divided by the quantity produced, therefore $AC_i = \bar{F}(q_i + q_j)^{-1} + \bar{c} + \tau q_j (q_i + q_j)^{-1}$ and $AC_j = \bar{F}q_j^{-1} + \bar{c}$. Equating them results in

$$q_j = \sqrt{\frac{\bar{F}q_i}{\tau}}. \tag{3.8}$$

As long as this condition 3.8 is true, the two average cost functions intersect and prices are equal between importing and producing locally. That is why, when $q_j \geq (\bar{F}q_i\tau^{-1})^{0.5}$, there is no need to still import from i since the local prices are the same or even lower in comparison to the imported. Because q_j is depended on fix costs, the collective variable τ and the quantity produced in i itself, a third dimension is needed to show the intersection graphically.

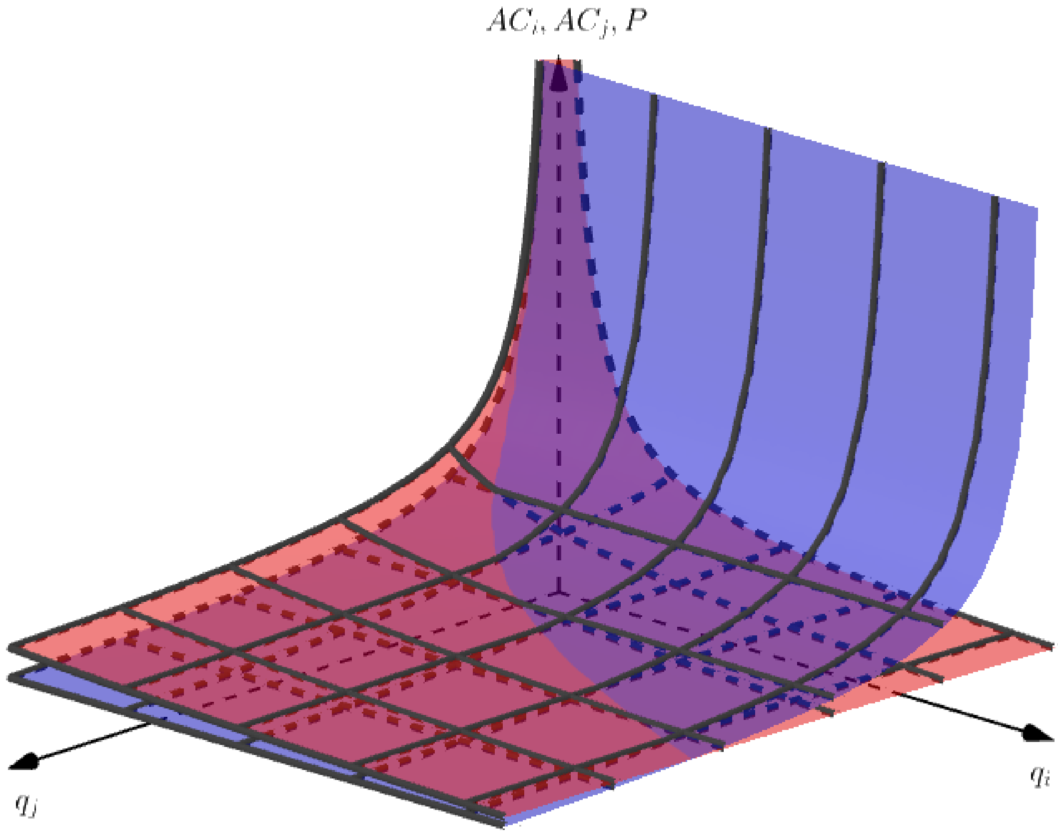

Figure 3.1: Side view of average cost surfaces

Figure 3.1 shows the intersection of both surfaces AC_i and AC_j from a side view. The red surface represents AC_i, while the blue surface represents AC_j. Because AC_j is not depended on q_i, it shows the same pattern over all q_i. AC_i is different. The supply of country i is depending on both axes q_i and q_j. Since the average costs are equal to the price, costumers always prefer the underlying surface. Going alongside the q_j-axis with given $q_i > 0$, shows an intersection of both surfaces, where AC_i is preferred until a specific q_j. After that intersection, AC_j is preferred. This point for a given q_i is defined by equation (3.8). If q_j goes to infinity AC_i converges to $\bar{c} + \tau$ and AC_j converges to $\bar{c}$. This can be observed on the q_j-axis in Figure 3.1. The distance between the axis itself and the blue surface is $\bar{c}$, while the distance between the axis and the red surface is $\bar{c} + \tau$, therefore $lim_{q_j \to \infty} AC_i - AC_j = \tau$ is displayed.

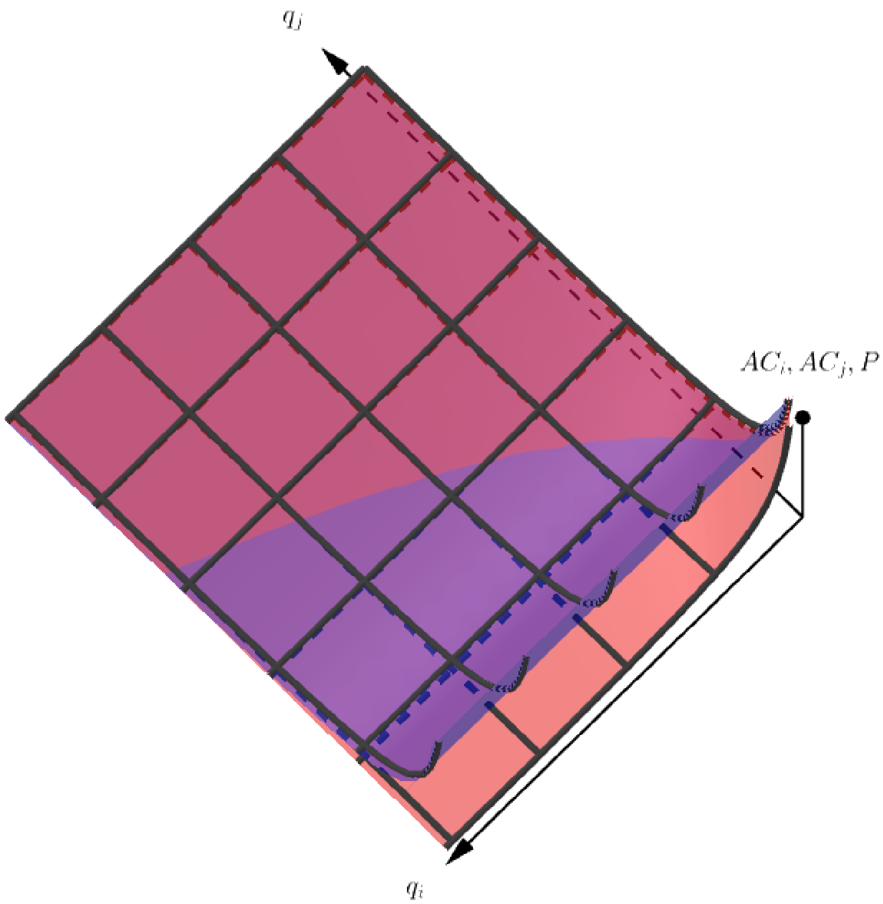

Figure 3.2: Top view of average cost surfaces

Figure 3.2 shows the same model as in Figure 3.1, but from a top view. From this two-dimensional view, the exponential dependency of q_j from the two constants and q_i is visible even better. The line that separates the two colors from each other is equation (3.8).

The efficiency of producing locally is therefore depended on three factors. The larger the quantity produced in i, the later producing in j is favorable. Also the larger the fix costs $\bar{F}$ and the lower the transport costs, tariffs and taxes τ, the longer importing is more efficient than self-producing.

If this relation is now related back to the demand effect of migrants on bilateral trade, imports increase with an increasing number of migrants in the first place. This should be followed by a sharp decrease if the critical quantity in demand is reached.

Due to the fact that firms need time to react, a dynamic transition is to be expected rather than an abrupt one, which could possibly also end in a negative demand-side effect of migrants on imports. Furthermore, it should be noted that economies consist of many industries in which the constants and q_i are different. Since the effect is measured on all imports, a parabolic dependence is to be expected.

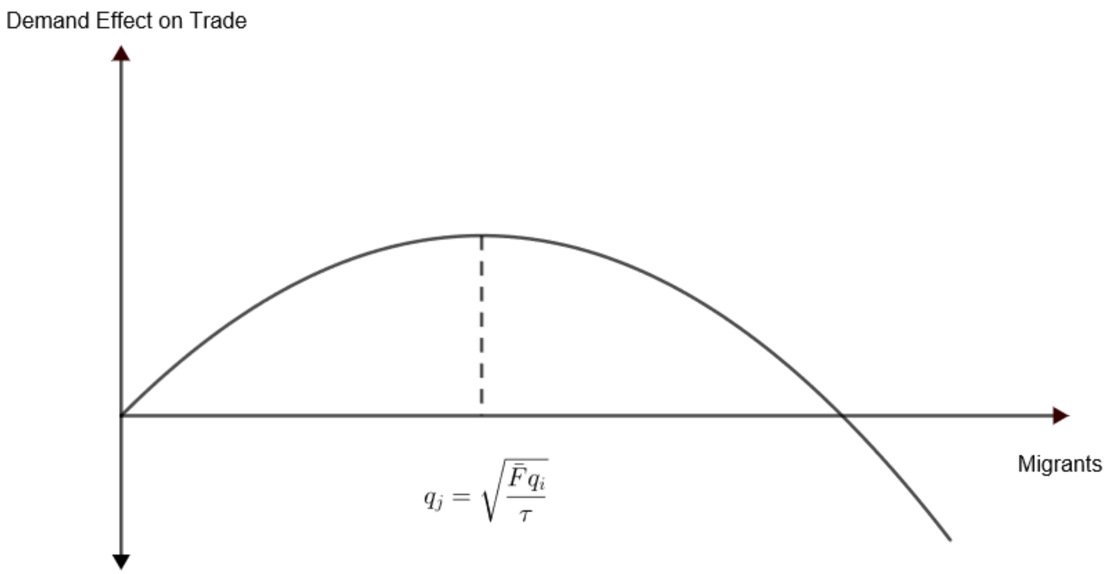

Figure 3.3: Reversing demand effect of migrants on trade

Figure 3.3 represents the expected non-linear demand effect of migrants on bilateral trade over all industries. The increase at the beginning can be justified by the economies of scale, while the turning point and the decrease of the impact on trade can be explained by the shift towards local production as explained in Figure 3.1 and 3.2. If the number of migrants increases to that extent that the parabolic function intersects the x-axis, it is even assumed that there will be a negative impact on demand. The dashed line at the maximum represents the average industry, where $q_j = (\bar{F} q_i \tau^{-1})^{0.5}$ is the turning point. If this effect can also be observed empirically, the idea of the effect reversal would be validated.

Hypothesis 2 *If all other factors remain constant and the data set is divided with respect to given quantiles of migrants, the effect of migration on trade flows from i to j first increases, reaches a maximum and then decreases after a turning point depending on an increasing amount of migrants in the importing country originated in the exporting country.*

To test Hypothesis 2 (H2), the same data set and method is used as in the empirical section.

The migration variable is split into quantiles, in particular in deciles. Results are robust for a division into other quantiles, but fewer quantiles lead to less clear results due to the lack of diversification and with more quantiles the number of observations per category decreases. Running the same regression as in equation (3.6), but with the number of migrants categorized, results in the beta coefficients represented in Figure 3.4.

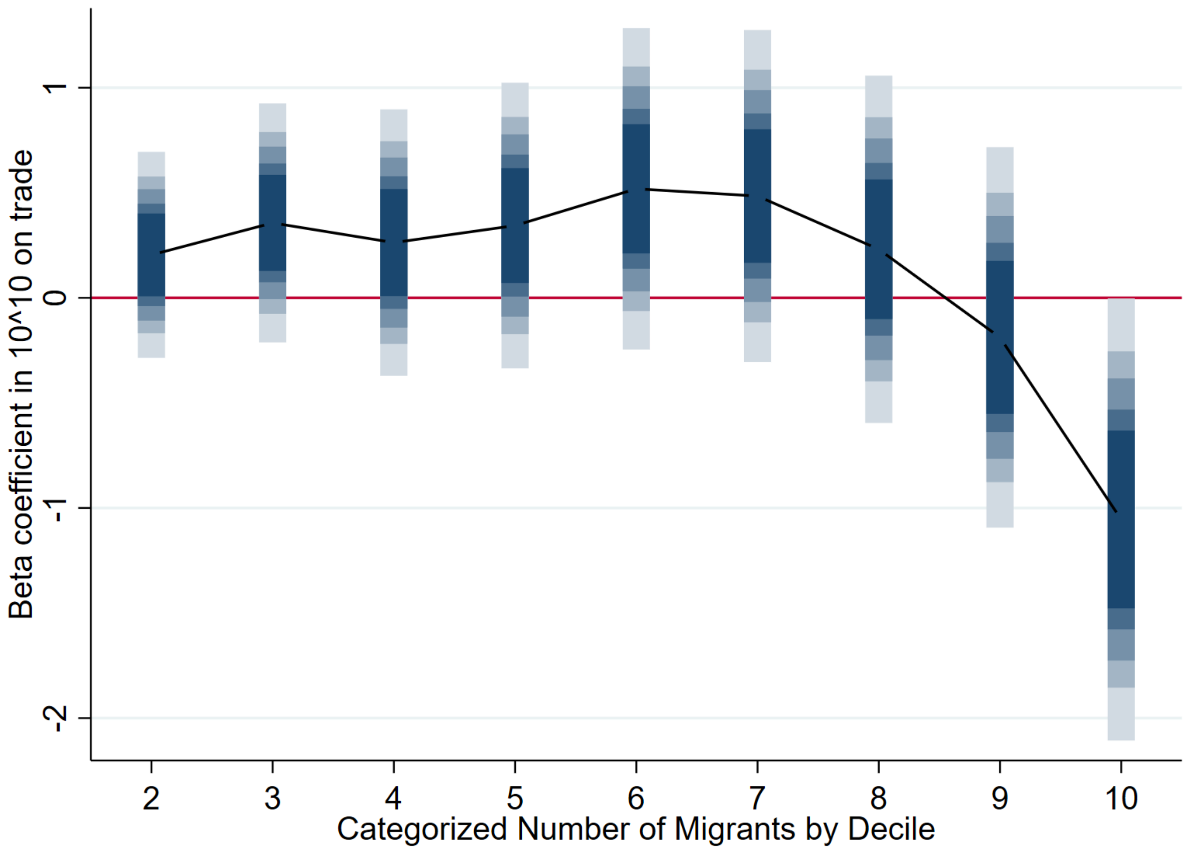

Figure 3.4: Effects of migration-based categories on trade

Although not all categories fulfill the highest significance level (the differently shaded areas represent a 99, 95, 90, 80 and 70 percent significance level respectively), Figure 3.4 allows some conclusions to be drawn. The first category by deciles is used as a reference group. In the following categories, there is a positive effect up to the eighth decile in the means before the ninth and tenth go into negative. A behavior similar to that predicted in Figure 3.3 can therefore be observed empirically. After reaching the peak around category six, the impact of migrants on bilateral trade turns, which could be justified with the incentive for national producers to enter the market due to the higher local demand. The peak corresponds to the point $q_j = (\bar{F} q_i \tau^{-1})^{0.5}$ where some producers in the host country begin to produce goods as perfect substitutes. The overall positive impact in the empirical analysis in the fourth section confirms the rather late turning point from a positive to a negative effect. The first category contains almost exclusively real zero observations with respect to bilateral migration. In conclusion, also the effect is not different from zero in this specific category. The ratios of the following categories in relation to the first represented in Figures 3.4 and 3.5 therefore also show the absolute effect.

Even if the empirical setup is not perfect, the result suggests that also H2 cannot be rejected and that there is a positive effect on trade by migrants' demand in the first place followed by a decline until a significant negative impact is reached at the tenth decile.

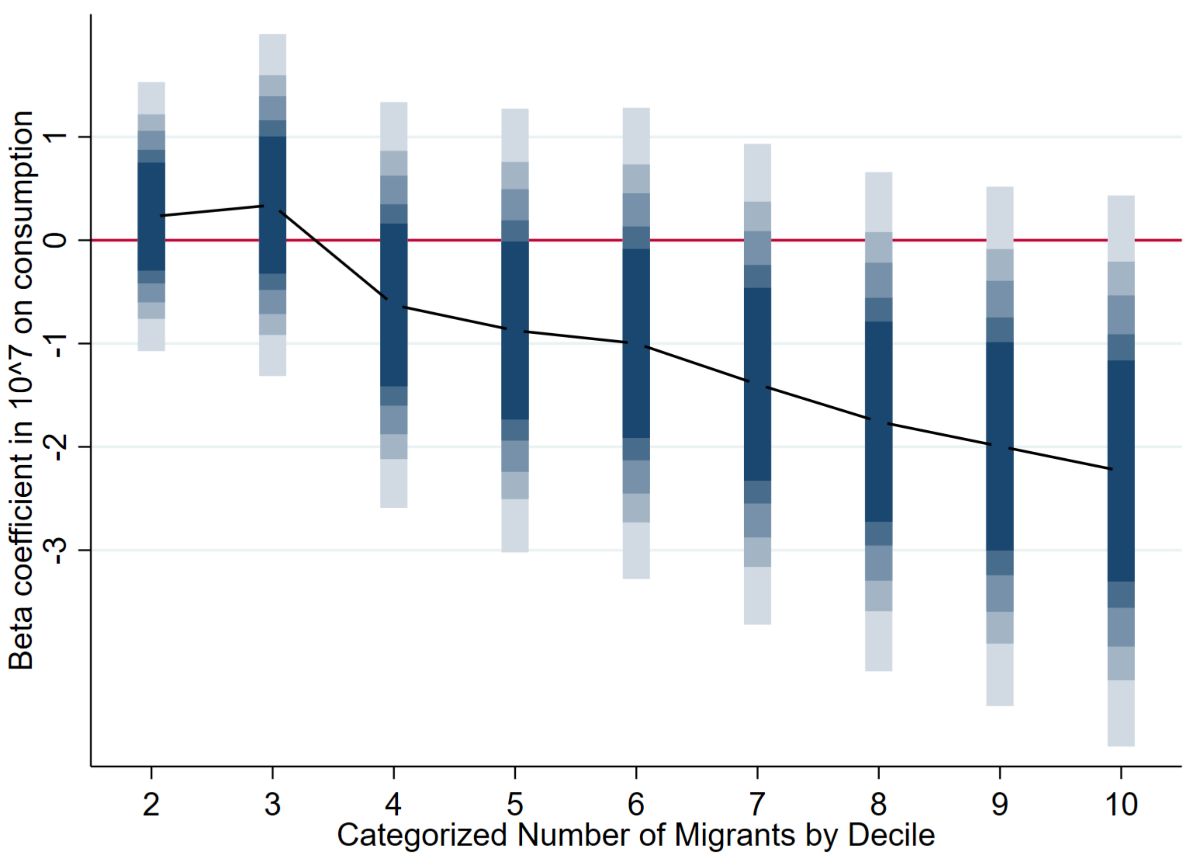

Figure 3.5: Effects of migration-based categories on traded household consumption

Focusing on the traded household consumption alternatively, like it is done in the empirical analysis, effects show a similar behavior. The impact on traded household consumption first increases, but drops between the third and the fourth decentile representing a negative effect. In conclusion, the effect can be observed much earlier than the data set with the entire trade suggests, because imported household consumption is much more preference-sensitive and only represents a fraction of total trade. The high significance of the ninth and tenth decentile together with the mean values justify the mixed results, especially the observed in B.5.

3.6 Further notes

Since the decision to import migrants' former local goods comes at a cost τ, there is a decrease in welfare after migration. This change is the negative of Figure 3.3, which is based on Figure 3.1 and 3.2. The loss of welfare is compensated for the moment that local production begins. The idea of the shift from imports to local production was initially designed to represent the impact of migrants on demand. But this concept can be basically transformed to several other cases.

It is applicable to all trade decisions on international level. It is enough to differentiate between two uneven consumer groups or markets in country i and j. Innovative or trending products that are initially offered in country i follow exactly the scheme from equation (3.8). Disregarding any legal protection, companies in the importer country have to wait for a critical amount of demand before it becomes profitable to produce locally. Here, in addition to the quantity produced in the exporting country q_i, the two constants $\bar{F}$ and τ play an essential role in determining the critical quantity. For established industries with high fixed cost and low transport cost, it is much longer profitable to export, synonymous it is reasonable for the demanding country to import much longer to delay own production. The opposite effect can be expected for established industries or specific goods that have low fixed costs with high transport costs. These branches reach the critical quantity relatively fast. Established industries in this context refers to an already existing demand q_i, which, if increases, pushes out the critical point of the shift to local production.

If this demand q_i does not yet exist, then q_j from equation (3.8) is also zero and it is straightforward to produce locally from the start. This model can also be transformed to the classical external economies and trade model, in which a solution for more efficient countries with insufficient demand is to withdraw from world trade for a short time in order to generate sufficient output with local demand. This kind of extreme protectionism can be represented by a τ which converges to infinity. If then protectionism cannot undercut the world market price, it must be maintained.

Even within multinational enterprises, this intersection of the areas AC_i and AC_j supports decision making. It is applicable to decide between exporting and foreign direct investment, especially greenfield investment. The latter should be taken into account when $q_j \geq (\bar{F} q_i \tau^{-1})^{0.5}$.

3.7 Conclusion

In summary, both a theoretical extension of the micro-founded gravity model by Bergstrand (1985) and the various estimation methods of the empirical section suggest that migration M_{ij} has a significant positive impact on bilateral trade flows tra_{ijt} and traded household consumption in particular. The most important estimation method (7) controls not only for all variables that are derived from equation (3.5), but also for country-pair-specific and time-specific fixed effects. In this data set, with the trade flows of South and East Asian countries to OECD founding members between 1995-2014, an increase in South and East Asian migrants in the OECD countries by ten percent leads to an increase in total trade from Asia in the OECD by one percent and to an increase in traded household consumption by 3.85 percent. PPML supports the results for total trade.

In addition to the theoretical implementation of the demand effect, the selection of the data set is essential. Countries were selected so that a supply push effect for exports can be nearly ruled out due to the absence of OECD migrants in the respective Asian countries. This selection of data also explains the insignificance of other variables, as shown in Table 3.1 and 3.2.

The market entry effect caused by migrants in the exporting country is theoretically also taken into account. However, it is also conceivable that transfer costs for exporters can be minimized if there are migrants from the exporting country in the importing country. This effect is estimated to be very low compared to the demand effect and would be difficult to clearly separate. The empirical observations in the regression with decentiles support the thesis of the higher demand effect through the larger observable effect with respect to traded household consumption.

For more general statements, it would now be obvious to extend the data set to include other countries and, above all, the variable of migrants in the exporting country. As many studies show, this would also lead to a higher significance for the other variables derived from theory. However, this is not as straightforward as it seems. For many of the countries in question, such data does not exist or is only available in five or ten year steps.

So the goal for an optimal data set would be to include all bilateral trade flows between countries that also provide the necessary data. As several studies showed, the solution of a cross-country analysis would be a step backwards. A more extensive data set would also allow the research question to be expanded with respect to heterogeneous demand effects among migration groups. In this study, the effect of heterogeneity in the migrant groups can only exist among different South and East Asian countries and, according to some authors like Head & Ries (1998), is not as great. Another important point is the consideration of differences in production and income. Migrants transfer their direct purchasing power to the host country first, but in some cases they indirectly transfer income back to the country of origin.

In order to be able to further specify the positive effect, migrants' non-linear demand effects have been embedded in a three-dimensional economies of scale model. The explanatory approach that migration initially leads to an increase in imports from their home country before the number of migrants increases to a specific point and thus companies are incentivized to produce goods locally also withstands further robustness checks. The general welfare that results experiences a decrease.

It cannot be completely ruled out that this effect does not depend on the skill level of migrants. For further studies, it is therefore advisable to check these results also for robustness towards migrant groups from the same country, but with different skills. Such a data set is currently not adequately available.

This economies of scale model could also be used for a variety of additional research questions in the field of multinational enterprises.

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Chapter 4

Conditional beta-convergence by gravity

4.1 Introduction

Even if globalization has resulted in countries being more closely connected than ever before, there are some that have benefited little or not at all. While there are many explanations for national inequalities and their implications, the literature on theoretical justifications for international divergences is still growing.

In the past, there have been a variety of models that predicted beta-convergence where developing countries would catch up through the more efficient use of capital. When this could not be empirically observed for most of the countries considered, the failure to catch up was justified by either conditional beta-convergence or a path dependence. However, path dependence has been like a black box to the growth literature while conditional beta-convergence just generally states that the catch-up in growth is related to additional factors mostly leading to club-convergence within a homogeneous group of states.

Since both approaches are rather imprecise, this study aims to put such pathways or conditional-beta convergences into the proper context. Therefore, three of the most important indicators of macroeconomics - Gross Domestic Product (GDP), Total Factor Productivity (TFP) and multilateral trade - are combined. The relationship between trade and GDP is already described by the gravity model. Looking at this model's implications in detail reveals the reciprocal relation between production and trade. A larger GDP comes with larger markets and production possibilities, which increase trade, while at the same time exports and imports increase GDP in the long run. In order to define this sequence of effects more clearly, TFP is introduced as a mediator, which increases the more it is traded due to spill-over and competition effects. The result is a spiraling path on which countries continuously reach higher output, technologies and total trade.

This *International Innovation Spiral* does not rule out any national effects, especially not the higher efficiency in the use of capital. They exist side by side, whereby the capital effect can lead countries to rise to a group in which they can benefit from the innovation spiral more efficiently. The combination of this capital efficiency with the openness of the markets then leads to long-term, self-sustaining growth. If this cannot be guaranteed in all countries, it could mean that the gap between rich and poor states will most certainly widen. High income countries (HIC) would dominate world markets and become an even smaller closed club.

There are already some options to integrate low income countries (LIC) globally, so that they can establish themselves on world markets. This study predicts which of them will be successful in the long run and also explains a part of China's disproportionate growth in recent years. The strong discrepancy to the other, especially to the remaining BRICS (Brazil, Russia, India, China, South Africa) states can thus be partially justified and does not have to continue to be regarded as an anomaly of economic models.

For this reason, the individual relationships between GDP growth, total trade and TFP are discussed based on already existing economic theories and recent literature in section two. Then the model and its implications are derived by piecing together the puzzle resulting in the *International Innovation Spiral* in the third section before section four concludes.

4.2 Growth, trade and TFP

Former United Nations-secretary Kofi Annan once stated that "open markets offer the only realistic hope of pulling billions of people in developing countries out of abject poverty, while sustaining prosperity in the industrialized world" (United Nations 2000). What was true in 2000 is now even more relevant in a world that is connecting and changing even faster. Unfortunately, decoupling does not only exist between east and west, but also between LIC and HIC. Even if globalization has mostly positive effects for many, it causes some to fall behind.

The economic growth literature on national inequalities provides several explanations, such as Kuznets' curve (1955). He was one of the first relating technological progress to wealth. His idea was that innovations first increase the level of inequality and while more people gain access to these new technologies, inequality decreases. It is analogical to the heterogeneity among states' economic growth rates. Countries have access to new technologies A at different points in time. That is why economists are aware, not just since the Solow model (1956, 1957), that such technological differences can also lead to income divergence at the international level. After Solow himself and countless other authors tested and improved the model intensively over several years, the era of endogenous growth models came during the 1980s with approaches by Romer (1986) or Lucas (1988). Further subsequent models concentrated even more intensively on the endogenization of technological progress primarily taking closed economies into account and explaining the innovation growth of countries with the voluntary redistribution of workers from manufacturing to the Research and Development (R&D) sector.

While these models explain a significant part of growth, most of them neglect advantages of an open economy. Trade and general global interaction are crucial and should therefore be taken as additional key factors in determining growth. Besides spill-over effects, innovations are mainly driven by substantial bigger markets.

The gravity model of trade already successfully links larger markets to more international connectivity. It indicates the intensity with which states interact by quantifying bilateral trade with the help of exogenous macroeconomic impact factors and geographical peculiarities. The fundamental determinants are equivalent to Newton's gravity, where the force of attraction between two bodies depends on the relation of their masses and the distance between them. Among others, Anderson (1979), Bergstrand (1985) and then Anderson & van Wincoop (2003) provide solid theoretical support for the gravity model of trade. Currently, this model dominates most of the trade-related literature. Although gravity is economically well founded and combines two essential variables of the economy: GDP and trade, there are no substantial studies that link the implications of the model itself to other macroeconomic theories. The rationale for the relationship between these two variables is rather straightforward. Larger economies provide more resources and can therefore produce goods and services at a higher quantity. At the same time, larger economies are also associated with a higher potential for imports. Similar to Newton's model, the distance between the two economies considered is inversely proportional to the bilateral trade. Greater distances correspond to higher transport costs, which have an impact on individual trade decision-making processes.

A large number of follow-up studies and meta-papers in the field of international trade literature define Anderson's (1979) approach as the first theoretical foundation of the gravity model. Anderson (1979) derives international goods movements from country i to country j in an ideal world without restrictions as X_{ij}, which is proportional to the product of the two sizes of respective economies Y_i and Y_j. He assumes that states have a differentiated output Y, preferences among the countries are homothetic and identical, prices are identical in all countries and that trade is in a multilateral equilibrium. The latter implies the market-clearing price. The assumptions lead to the fact that the state j demands the goods of i in proportion to its output. Thus $X_{ij_{And}} = b_i Y_j$, where b_i represents the portion of good i demanded by every importer in j. The market-clearing equilibrium price indicates that bilateral trade can be represented as $X_{ij_{And}} = Y_i Y_j / Y_w$, where Y_w reflects the global output (Anderson 1979). However, Anderson (1979) is not able to sufficiently implement transport costs. He assumes the convention that all free trade prices are identical.

Unlike Anderson (1979), Bergstrand (1985) includes prices and price indices in his derivation of the gravity model on both the theoretical and the empirical level. He assumes a constant-elasticity of substitution (CES) utility function based on Armington (1969) to show that products from different markets are imperfect substitutes. His derivation of the model is more complex and differentiated than Anderson's (1979), which on the one hand allows more detailed conclusions about the influencing factors of bilateral trade, but on the other hand limits the exact implementation in empirical estimations. There is also a simplified version of Bergstrand's (1985) model without any restrictions, the friction-less case. Trade flows between country i and country j are then also only dependent on the outputs of the respective countries and can be represented by $X_{ij_{Ber}} = Y_i^{1/2} Y_j^{1/2}$ (Bergstrand 1985, Stoeckmann 2020). Also with regard to further derivations of the gravity model such as Anderson & van Wincoop's (2003), it can be stated that bilateral trade flows between country i and j T_{ij} can be defined as a general function of the two economic sizes $T_{ij} = f(Y_i, Y_j)$ and further country-pair specific peculiarities based on the chosen model. The total trade of a country in time point t is therefore highly dependent on its own output

$$T = f(Y). \tag{4.1}$$

Larger economies thus have a higher trade volume than smaller. While the own output indisputably has a positive impact on trade, the impact of others' GDP is ambivalent. On the one hand both Anderson (1979) and Bergstrand (1985) multiply the numerator by the trading partner's GDP, while on the other hand Anderson (1979) implements the world's output Y_w as a denominator. For now, only the simplified form, as in equation (4.1), is considered. In the further analysis of this study, the positive impact of trading partners will also be discussed. Larger trading volumes and, above all, more intensive networking at the international level inevitably lead to a growth in innovation through more competition and spill-over effects. Even the smallest inefficiencies in markets lead to distortions and misallocations, which in turn lead to an obstacle to TFP growth (Restuccia & Rogerson 2008). While Ades & Glaeser (1999) or Frankel & Romer (1999) find a causal effect of trade on TFP, Rodrik (2000), Rodriguez & Rodrik (2000) and Irwin & Terviö (2002) argue that trade is not a direct determinant of TFP if institutional strength and geographical peculiarities are included. Alcalá & Ciccone (2004) find a significant and robust causal effect of trade on productivity across countries by using real openness as a measure of trade. Helpman (1988) even finds economies of scale in growth. According to him and previous studies (Bhagwati 1978, Feder 1983, Romer 1986) GDP growth goes hand in hand with export growth. In the majority of cases, however, there is no unique causality found.

Grossman & Helpman (1990) define trade as the engine of growth in one of their studies. First, the comparative advantage, according to Ricardo, determines in which direction states specialize and then Marshallian economies of scale in the respective specialized industry are assumed. They refer to Romer (1990), who defined innovation growth as non-diminishing economies of scale that ensure long term growth. There are also more recent studies that contribute to the relationship between innovation and trade including Eaton & Kortum (2002), who use a dynamic Ricardian model to assign innovation to lower trade barriers and more intense international competition. They also propose to distinguish between technologies that are accessible to everyone and technologies that only industrialized countries can use. Hsieh & Klenow (2010) summarize the results of previous studies and conclude that TFP has a strong impact on growth. They assume this influence to be at 50-70%. They even suggest that TFP growth attracts more capital and labor. The authors estimate this indirect accumulation effect to be bigger than the actual effect on output. Kucheryavyy et al. (2016) add Marshallian externalities to the Eaton & Kortum (2002) framework to show heterogeneous degrees of specialization between industries.

In summary of the entire recent trade-TFP literature it can be stated that there are two dominant reasons why trade has a positive causal impact on innovation growth. On the one hand, larger international markets are associated with more competition for both domestic companies through imports and for exporting companies on the world market. On the other hand, there are spill-over effects for open economies, both cross-border intra-industry and national cross-industry. Countries that pursue an outward development policy therefore grow faster than isolated states.

Furthermore, there are some model theoretical assumptions that are successful over a variety of approaches. In most cases the basic structure of the chosen model is a Ricardian-Marshallian type. States first specialize on the basis of their comparative advantage and can then expect economies of scale in their chosen industries. These advantages are then defined via branch-specific TFP. In addition, a distinction between different types of availability of technologies is made. Therefore, it is useful to differentiate between exclusive technologies and technologies that are available to every state. Sometimes, the attraction of further production factors by a higher TFP is theoretically implemented by the choice of a more specific production function. The general relationship between technological progress or innovation A_i and trade in country i can thus be represented as

$$A = g\left(c, T\right). \tag{4.2}$$

The more a country is involved in international trade, the higher the innovation, either from the urge to withstand international markets, to expand its own position, or through network-related spill-over effects. Technological progress in country i in time point t is therefore a function g of the commonly available technology in that period c_t and the country's respective total trade in the previous period $T_{i_{t-1}}$. This delay of one period is included in order to be able to better fit actual R&D processes.

The last of the three relationships is the most straightforward. As previously stated, Hsieh & Klenow (2010) estimate the individual effect of TFP on growth as 50-70%. The Solow residual represents the general efficiency of the respective country. It indicates what part of the growth in production cannot be traced back to an increase in the use of the production factors capital K and labor L. Thus, GDP in country i Y_i is generally defined by

$$Y = h\left(AL, K\right), \tag{4.3}$$

a function h of total factor productivity A_{i_t}, the capital stock K_{i_t} and the available labor force L_{i_t} in that respective country i in time point t.

Many growth models, especially Solow based, initially predicted that developing countries will catch up in per capita income (PCI) through a more efficient use of capital. These convergences, beta-convergences, mean that economies with initially lower PCI tend to grow faster than countries with comparably higher PCI which in conclusion leads to a convergence in steady-states in the long run. However, convergences like these only take place within homogeneous groups of similar countries like within the Organisation for Economic Co-Operation and Development (OECD). The most obvious, but less popular explanation is based on the availability of technologies, which is higher in the OECD than in other countries due to the closer community of states. Other states do not seem to have access to same technologies. For this reason, there are multiple steady states within the international community. Against the background of advancing globalization, however, it is surprising that such a conditional beta-convergence can be observed. Despite perfect networking through the internet, technologies are not used uniformly worldwide. So where does this divergence come from? Some authors argue with path dependency. The idea of such paths dominated the literature on technological change for the last decades. However, this path dependency is only vaguely described and poorly justified. Some social studies argue that geographic conditions and resources paved a way to a successful economy. Since there are many countries that are among the leading industrial nations, which are relatively poor in commodities or had to face a complete shutdown of the economy during and after World War II, these arguments about peculiarities initiating a successful path seem rather uninspired.

A synthesis of equations (4.1), (4.2) and (4.3) appears to be more promising. The resulting model allows to observe path dependencies, but does not need them as an underlying reason. Therefore, this study initially proposes an *International Innovation Spiral* that justifies the heterogeneous developments of countries quite intuitively.

4.3 International innovation spiral

Combining equations (4.1), (4.2) and (4.3) indicates that GDP has a positive impact on trade, trade strengthens technological progress and thus more output is generated. Although there are additional impact factors, the GDP of country i in time point t Y_{i_t} can thus be defined as a nested function of the previous period's GDP

$$Y_{i_t} = h\left\{g\left[f\left(Y_{i_{t-1}}\right)\right]\right\}. \tag{4.4}$$

Consequently, an increase in the current GDP will also lead to more GDP in the future which is then reflected on individual path dependencies. But instead of having linear paths as it is often assumed in the literature, these relationships in equation (4.4) can be sketched as in

Figure 4.1 by an *International Innovation Spiral* (IIS). In contrast to the previous literature's black box with respect to path dependencies, the two variables trade T and technological progress A serve as mediators that complete a circular movement towards higher outputs Y.

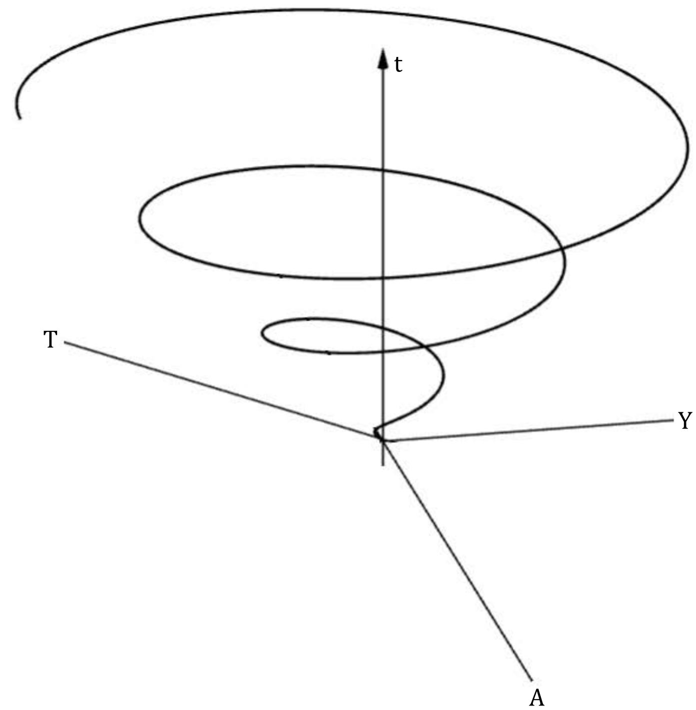

Figure 4.1: International innovation spiral

The IIS visualizes the previously derived dependencies in an intuitive manner. However, additional impact factors like the population growth or the national-specific efficiency in the use of capital, which classic models require, are not yet considered.

In this purist version as in Figure 4.1, different starting points would lead to a growing difference in the output of two countries if all other factors remain constant over time. The reason for this is that unequal outputs lead to changes in market shares, which in turn increase the technological advantage over the international markets for one country and decrease it for the other. Even if the IIS is not further specified, it can also be used for models in a Ricardian-Marshallian environment which then would justify growing scale effects for countries in specific comparatively advantageous industries. Formally, all variables of the equations (4.1), (4.2) and (4.3) and of the resulting Figure 4.1 would then each have an index for the specific industry considered. Consequently, there would be coexisting spirals with national inter-industry effects between. Technological progress would then be allowed to spill from one IIS to another by extending equation (4.2).

Two conjunctions of this circular movement through time are crucial for economies. On the one hand, it might be the case that the economic size Y of some country is not sufficient to profit from the transmission of technological growth on international markets. On the other hand, it may be that larger markets do not stimulate more trade due to a lack in openness of the economy itself.

The size of the economy and the trade volume should not be seen in absolute terms, but rather be put in relation to other economies if it is about technological progress in international markets through competition. If a country is relatively small, it can lose touch with international competitors and thus can only work with generally available technologies c. It would then find itself in a *Technology Trade Trap* in which the country would need a higher technological standard in order to survive in international markets, but at the same time also needs larger markets in order to develop technological progress from competition. Breaking out from this trap is difficult. Externally reintegrating those countries into the markets should be one straightforward solution to boost economic growth in even the poorest countries.

Contrary, already larger economies are expected to develop a growing output over a long period of time, from which a steadily growing technological advantage emerges. If this is combined with the opposite effects for the smaller, more isolated economies, a steadily growing international divergence can be deduced. Slight differences in the initial output can therefore lead to larger ones in the long run. In conclusion, the path dependency that is propagated in many studies can thus partly be observed, but should not be seen as a reason, rather as a symptom.

The longer the period considered, the more low output countries would be left behind until a point where they are completely irrelevant to international markets and only rely on the commonly available technologies. If it is additionally assumed, as many previous studies have done, that a higher TFP would attract more factors of production, economies could shrink not only in relevance, but also in absolute terms resulting in labor and capital outflows. An early intervention is therefore crucial and should be the goal to allow LIC to continue to benefit from technological progress through international markets. Both protectionism on the side of the LIC and exclusion in trade organizations on the side of HIC are therefore hindrances for stable long term growth.

Intra-HIC trade is robust and will continue to be used as a catalyst for innovation. Global technological progress depends, by definition, on how large the markets are, therefore, how many countries are included. From a development policy point of view, a stronger integration of LIC into world trade makes more sense than leaving states behind and then providing them with external help. From the LIC point of view, this innovation spiral can be seen as an incentive to integrate oneself even more into world trade, or to allow to be integrated via trade unions or free trade agreements.

With the ongoing Covid-19 pandemic, countries all over the globe experience a range of negative effects. Developed, more networked countries are initially more affected than developing countries and are experiencing declines in their GDP, which in absolute terms are greater than those of developing or emerging countries. Although HIC experience serious cuts for life, it can also be seen as an opportunity for LIC to make catching up on world markets easier, because some paths have been slightly reset. The goal should be to support LIC in the fight against the pandemic, so that on the one hand the humanitarian impact is kept within limits and on the other hand the economic cut is comparatively smaller and a K-recovery can be averted. Should such aid fail or should the pandemic become even more intense in these countries, it becomes increasingly unlikely that those countries will ever catch up gaining technological progress through international markets.

It can also be discussed to what extent such an integration of LIC should look like from the HIC viewpoint. It is difficult to justify why subsidies for imports from LIC should effectively improve technological growth in the exporting country. With respect to equation (4.2), the impact of trade on technological progress would then be lowered or even erased by noncompetitive prices.

The BRICS countries in particular have been the focus of convergence in growth-related studies since the beginning of the new millennium. It has long been assumed that these countries will catch up with leading industrial nations through technology spillovers and the more efficient use of capital. However, it can be stated that so far only China has been able to break away from this group and connects economically with the industrialized west.

Emerging economies, by definition within the LIC-HIC scale, can take away important implications from the IIS. China appears to have taken significant steps to embark on a successful path while the other members of the BRICS still struggle to establish their positions on the world market. It would be unsatisfactory to label China, as in many studies, a special case that cannot be captured with current models. Even though China can consider itself to be profiting sufficiently from the international exchange as in the IIS, there were initial national impact factors that got the country on track. The Chinese government took substantial steps to even start this process. Sticking to classical growth models, such as Solow's, the more efficient use of capital and an increased savings rate played a significant role with respect to the Chinese growth miracle allowing them to jump onto the next technology growth level where the club of industrialized nations is located. Also the over-proportional population size contributed indirectly as it is part of the general output function (4.3). These national factors do not contradict the idea of the IIS, this example rather shows the difference between national and international effects, which can coexist and even substitute each other in the short run. In the long run, however, national peculiarities are fading out while the countries developing through a spiraling growth.

The model even allows for more specifications. Equation (4.1) only generally defines trade in relation to an unspecific output while gravity models of both Anderson (1979) and Bergstrand (1985) suggest a more precise implementation. To measure bilateral trade flows, they also take GDP of trading partners into account. Converting this into the set-up presented could result in $T_i = f\left(Y_i \cdot \sum Y_{in}\right)$ with $n = 1, ..., N$, where N is i's number of trading partners. This definition intensifies the potential problems arising from the trap in which developing and emerging countries can find themselves in. If closed clubs would become literal, the divergence would even increase further.

The idea of the IIS resulting in individual levels of technological and economic growth can be applied to the whole bandwidth of growth theories. Wherever technological progress is integrated as an exogenous variable, the IIS can coexist with national effects. In the classic Solow model thus a country can find a connection to international markets through a higher savings rate or a higher capital-labor ratio in the short-run. In technology-endogenous models, such as Romer's (1990), on the other hand, processes can be justified even better using the spiral. The reallocation of a share of the workforce is therefore not voluntary, but serves to maintain or to increase competitiveness on international markets.

4.4 Conclusion

Combining GDP Y, trade T and technological progress A derives an *International Innovation Spiral*, which can explain a significant part of conditional beta-convergence between countries. Assumptions from existing models were adopted for the relationships between the individual variables. Total trade T is proportional to the output Y based on the gravity model, technological progress A is proportional to the total trade and the output is in turn proportional to the technological progress. Countries that are in close contact with the international community, above all with the industrialized nations, move over the time axis in ever larger cycles across the three variables while shaking off other nations on global markets. If the divergence between HIC and LIC is too large, LIC can no longer benefit from international markets and thus become irrelevant for trade.

What would further intensify this effect would be a more specific production function that makes the input factor accumulation dependent on the technological progress. Because the aggregation of production factors can be based on input allocation efficiency, it would also be justified why developing countries experience exponential capital outflows and outflows of labor, a brain drain through labor migration. Since, mostly outside classic economic theory, state funding can be essential in supporting technological progress, more available capital also leads to extra kick-off financing for innovations.

Another, more ambivalent, implication of the IIS is the impact of population on economic growth. Due to a higher population growth, per capita income naturally decreases in the short term. In the long run, however, an increase in the population could also lead to a higher per capita income since labor is also part of the output function. Through the IIS, the connection of an economy to the international community of states can then be achieved through larger markets, which in the following periods leads to greater technological and thus resulting in higher economic growth rates in comparison to population growth. China is one of the countries that, in addition to a higher savings rate and other individual political measures, was able to benefit from this factor and is therefore of great relevance on international markets due to its economic and population size.

Population growth consequences and also some further suggestions might be controversial. The extent to which national effects can be weighed against international is not clearly defined.

Also the relationship between the generally available technological progress and the progress resulting from international competition has not yet been specified. Due to the generalization of this work, all these ambiguities could, nonetheless, be implemented in specific models.

The spiral theoretically implies a potential dark future for the world economy. Because national peculiarities are becoming increasingly insignificant and industrial nations benefit exponentially more from international markets, a phenomenon corresponding to Newton's gravity model could be predicted. Only a few economies will stand out from the rest and absorb market shares and resources from others. If a long time horizon is considered, it can even be assumed that more and more of this closed club of the remaining nations will fall off the international market and that their remains will be absorbed by the dominant economy like in a literal black hole.

This dystopian scenario underlines the importance of trade unions from a theoretical perspective. To appear as an economic unit, for example as the European Single Market does, increases participation and thus technological progress due to the simple economic size. So, from this point of view, Britain's exit was bad for both parties. In many ways the European Union appears as a unit in which technologies are transferred transparently within, similar to China or the USA. The global dominance of these three economic powers will continue to grow in the future due to their size, but also due to the second important factor of the gravity model - the distance. Because these political and economic powers are geographically distributed across the globe, clusters and thus geographically dependent beta-convergences will continue to form.

In addition to the importance of the trade unions, further political recommendations for action can be derived. The aim should be to guarantee more competition without monopolies, to promote privatization in as many markets as possible and to support developing countries in their short-term national efforts to participate from international markets without being subsidized.

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Chapter 5

Unraveling the distance paradox

5.1 Introduction

Since Tinbergen (1962) and Pöyhönen (1963) initiated modeling bilateral trade relations by gravity models, which were later also successfully embedded in theory by Anderson (1979) and Bergstrand (1985), the model, which has its origins in physics, has steadily gained in importance in the field of international economics. While different application possibilities have been found, the primary variables have not changed for half a century.

In this model, trade flows are proportional to the size of the economies under consideration and inversely proportional to the distance between them. While the economic variables describe potential production factors and purchase markets, greater distances are associated with additional transfer cost. Gravity models can be used in many research areas and over any period of time. Larger periods of time show that the impacts of some variables vary. Among others, Disdier & Head (2008) showed that distance effects from 1870 till 1950 become weaker, but then stay relatively constant and even become stronger afterwards. Against the background of individual progress and general globalization, this is initially counterintuitive. Nevertheless, this phenomenon can be explained by different approaches, such as Borchert & Yotov's (2017). In their empirical study, they put the globalization effects in the context of national progress in order to be able to show a reduction in the distance elasticity. Although they make a major contribution in this field of research, their study leaves some parts of the paradox unexplained.

This paper presents a micro-based explanation for the distance residual alongside an alternative empirical approach. In this context, annual bilateral trade flows of current OECD members are analyzed in order to have crisis-persistent results and to cover the so-far peak of the globalization and digitization era. In addition, also secondary variables of the gravity model are tested for their varying influence on trade flows over time.

The main findings are the increasing elasticity of borders and the decreasing elasticity of a common language on trade flows over time (although both have a positive impact on trade flows). In addition, no other variable appears to fluctuate more in elasticity than regional trade agreements (RTA). Instead of applying Borchert & Yotov's (2017) quite static empirical strategy, this study proposes an alternative approach, calculating the traveled value per meter divided by the total trade per year as a measurement for globalization effects in international trade.

Section two contains literature contributing to the explanation of the varying effect of distance on trade. Individual decision-making processes then develop to an aggregated utility function which is able to explain a delayed development towards even more global trade. Thereby, individual decision makers face a trade-off between a reduction in cost and uncertainties towards alternatives. These relationships, manifested in utility functions are then applied empirically in the third section before section four concludes.

5.2 The effect of distance on trade

The relationship based on Newton's findings in physics has been criticized since the first publications on trade-related gravity models in the 1960s. It was firstly about the lack of theoretical foundation, then about the particular implications of this model. Anderson (1979) and Bergstrand (1985) made contributions early on to eradicate the first criticism by giving the model a theoretical foundation each. One major criticism with respect to the implications is related to the distance paradox, the persistent importance of distance in bilateral trade, which does not seem to decrease even with advancing globalization.

Frankel et al. (1997) were one of the first to discover the non-declining effect of the distance coefficient over time. Coe et al. (2002) have been the first to define this observation as the *missing globalization puzzle*. There is a variety of different subsequent studies attempting to resolve the distance *puzzle* or *paradox*.

Brun et al. (2005) highlight that the estimated coefficient of distance within gravity models is generally increasing over time. But instead of applying the traditional gravity model, rather applying an augmented barrier to trade, this initially counter-intuitive effect is reversed and a decrease in elasticity can be observed. However, their estimation reveals that the impact varies between high income (HIC) and low income countries (LIC). Berthelon & Freund's (2008) results, however, reveal a ten percent increase in the distance's elasticity since 1985. They figure out that especially homogeneous, large and high tariff goods are more distance-sensitive than others. Márquez-Ramos et al. (2007) link the phenomenon of the missing globalization puzzle piece to the empirical estimation method of other studies. They argue that there is no adequate explanation for non-linear estimation methods. If a linear estimator is applied, the results are as expected and there is indeed no paradox. Like Brun et al. (2005), the authors also assume that there are differences in distance elasticity between developing and industrialized economies. The decline in elasticity is also obvious when looking at the WTO World Trade Report (Bernan 2008). A clear decrease in costs, especially air transport costs, of 92 percent between 1955 and 2004 can be observed.

The fact that it is not about the direct transport cost, but about the persistence of the elasticity of the distance variable in gravity models and the fact that the economic community almost uniformly advocates non-linear estimation methods with fixed effects mean that the previous contributions are rather irrelevant. It was Borchert & Yotov (2017) that made the first substantial progress in this topic. They contribute a lot to the gravity literature with their study "Distance, globalization, and international trade". Their main goal was to clearly show a decreasing effect of distance on bilateral trade over time. Furthermore, they can see differences between heterogeneous countries in benefiting from this decline. They attribute these observations to the differences in the goods that dominate the respective trade balances of individual countries.

Borchert & Yotov (2017) argue that the failure to reflect decreasing trading costs in the standard gravity equation is mainly due to the choice of control variable and a general proper context. The basic gravity model is only able to determine relative trading costs. Since globalization has an equal effect on the international level, the associated effects influence international variables equally. Estimates with gravity models therefore become imprecise and general trends cannot be shown with the classic choice of impact factors. The two authors have therefore worked on an alternative to still be able to show globalization in trade data. Based on Yotov (2012) and other scientific contributions, Borchert & Yotov (2017) show alternative indicators but also an alternative empirical design in general.

Yotov (2012) already assumes that international indicators must be put in the context of national ones. Thus, in order to make globalization measurable, international integration must be measured in relation to national, respectively domestic markets. The effects of globalization could thus be determined from the relation between the actual change in the elasticity of distance on bilateral trade to changes in national transport costs. Consequently, it is empirically observable that international trade costs show a higher decline compared to national trade costs. Borchert & Yotov (2017) expand the results of Yotov (2012) in that estimates of distance elasticity in relation to trade at the country level are allowed to vary through the implementation of country-specific dummies for domestic trade. As a result, the international heterogeneity is better represented and country-specific peculiarities are absorbed. With this methodology, the authors are able to better identify the beneficiaries of globalization. By dividing the dataset into the three categories of low income countries (LIC), middle income countries (MIC) and high income countries (HIC), the two authors find that MIC have the highest decrease in distance coefficients within estimates of gravity models and so benefit most from globalization.

The u-shaped relation between the countries' per capita income and the beneficiaries of globalization via the decreasing distance effect can be interpreted in many ways. One possible interpretation, which Borchert & Yotov (2017) also name, is the different composition of export goods. They define the value-to-weight ratio to be an indicator for relatively fricton-less goods with respect to distance. With one further contribution they also check the results for the air-to-rail ratio as a proxy for export good composition, and find a high correlation with profits from globalization recognizable in the reduced distance elasticity.

There appear to be different distance sensitivities per industry and as economies are shaped by dominant branches, there are industries that benefit and branches that are unaffected by the globalization effects at distance. Beyond the interpretations of the two authors, it could be assumed that LIC play a subordinate role in world trade and that they are also the main supplier of food. Since these are perishable goods, transporting them is still difficult and associated with some frictions. The opposite is the case for HIC, which are high-tech goods exporting, that are either very difficult to transport or digital. This means that the globalization effect in transport also plays a subordinate role here. Only MIC, which export goods with few requirements, like raw materials, experience a large decline in the distance coefficient.

Besides distance and economic sizes, other determinants which define bilateral flows vary among applications. Also the impact of globalization plays a different role in each of the research fields. The secondary variable literature in gravity models is mostly dominated by four binaries: a common border, a common language, bilateral trade agreements and a colonial relationship. All these factors increase the trading potential. Of course, there are other influences that determine trade between two economies, but these are not found so consistently in empirical studies as the ones mentioned. There are also differences within the impacts of these secondary variables with regard to the specific research question. A common language for example is not as essential for export goods as a common border, whereas for international service trade, it is the other way around (Chang 2014).

The phenomenon of the missing globalization puzzle can be partially explained by the study by Borchert & Yotov (2017). Industries that produce goods with little requirements for transportation seem to benefit from globalization. However, a significant part of the distance paradox remains unexplained. Although they claim to control for home bias effects, these effects remain significant factors influencing bilateral trade at the global level. For illustration, trade in digital goods, that have no physical weight, can be modeled well using the primary variables of the gravity model, where the distance is still inversely proportional to the trading volume.

Explanatory approaches for this remaining part of the relevance of distance, especially in the case of weightless goods, are mostly culturally based. The assumption is that the secondary variables and potential country-specific fixed effects do not adequately cover cultural differences and thus a part is still reflected over the bilateral distance. The distance elasticity observed, therefore, consists of the physical part, which is based in the transport costs, and a cultural part, for which it cannot be sufficiently control for.

Blum & Goldfarb (2006) precisely examine this cultural distance. They analyze consumer behavior for purely digital products by looking at US internet users and their website visits. They confirm the hypothesis that there is a generally negative association between distance and website visits, even after controlling for language and infrastructural variables. McCallum (1995) in goods trading and French & Poterba (1991) in financial product trading also find an explicit home bias. Obstfeld & Rogoff (2000) include these findings in their list of the ”Six Major Puzzles in International Macroeconomics”. Loungani et al.(2002) show correlations between physical and informational distance.

The relationship between physical and cultural distance is also the subject of numerous recent studies (Nes et al. 2007, Chaiyabut 2013). Hofstede's national cultural dimensions (1984, 2011) usually play a central role in the quantification of these (Hancıoğlu et al. 2014, Kristjánsdóttir et al. 2017). In general, uncertainties based on cultural differences can be interpreted as increased transaction costs for trade (Tadesse et al. 2017).

The residual that remains when the transportation cost factor is subtracted from the physical distance is defined differently across a variety of studies. Whether it is called informational distance, cultural distance or psychological distance is irrelevant to its meaning. Therefore, in the following of this study, the all-encompassing term *distance residual* is chosen. The overall implication is an insecurity about the foreign. Implementing this risk in traditional gravity models then explains the part of distance elasticity that cannot be attributed to transport costs depending on weight, difficulty or time.

Even though gravity models work on a macro-level of international trade, the actual trade decisions are made by individual decision makers. In the following, this process is therefore converted into a utility function. This is not very specific, but could be straightforwardly implied in the explicit utility functions from Bergstrand (1985) or Anderson & van Wincoop (2003).

Assuming that individuals, who are actively involved in the decision about trading partners, are risk-averse, the expected utility $E(U)$ of such global interaction can be obtained by a combination of expected return $E(r)$ and an uncertainty factor $V(r)$ in a μ/σ - principle. To make it more convenient, γ represents the degree of risk aversion. Indexes G and L indicate global and local trade respectively, which are going to be compared.

$$E(U_G) = E(r_G) - \gamma V(r_G) \lesseqqgtr E(U_L) = E(r_L) - \gamma V(r_L) \tag{5.1}$$

$E(r)$ is the difference of expected sales $E(s)$ and expected cost $E(c)$. For simplicity, $E(s)$ is assumed to be equal across all indexes and with adjustment to inflation also over the whole time span. In conclusion, expected sales can be subtracted on both sides of the equation. Matching cost and variances on the respective sides solves for a break-even point. For convenience expected values are denoted as μ, variances as σ^2

$$\mu(c_L) - \mu(c_G) \lesseqqgtr \gamma \left(\sigma_G^2 - \sigma_L^2\right). \tag{5.2}$$

In this context, only when the difference between $\mu(c_L)$ and $\mu(c_G)$ is greater than the difference between $\gamma\sigma_G^2$ and $\gamma\sigma_L^2$, exports are restructured to rather be global than local. Rearranged, if the quotient

$$\frac{\mu(c_L) - \mu(c_G)}{\gamma(\sigma_G^2 - \sigma_L^2)} \lesseqqgtr 1 \tag{5.3}$$

is equal to one, the decision-maker is indifferent between trading globally or locally. Smaller than one equals a decision towards local trade, larger than one indicates global trade. In order to aggregate these individual decisions to a macroeconomic level, it is assumed that γ is variable across all decision-makers, but with a mean of one.

Since it is the goal to present an alternative approach for the empirical validation of Borchert & Yotov's (2017) findings, this study suggests a different measurement. By multiplying the bilateral trade volume in USD with the distance to the respective trading partner, it is possible to get a value per meter traveled measurement (VPM). This is averaged over all bilateral relations to get one value per year. In order to control for the general development on international level, it is then divided by the total trade of the respective year. By eliminating the growth trend in the sum of exports through the denominator, this function thus distinguishes local from global trade trends. In this context, if trade relations grow equally in relation to all different distances, this ratio would remain constant. If trading relationships with partners with higher distance increase at an above-average rate, the ratio would also increase. It decreases if local trade increases above average.

5.3 Alternative empirical approach

The data considered contains 38 OECD members and their intra-OECD bilateral annual trade flows between 1960 and 2019. Besides the primary variables for GDP in logarithms (logs) gdp in each time point and the log distance between dis, there are further secondary variables in the estimations following. To control for external effects, log population sizes pop, binaries for a common border BOR, a common language LAN, a colonial relationship COL and a bilateral regional trade agreement RTA are added. Furthermore, it is controlled for religious proximity REL, per capita income PCI and memberships in the General Agreement of Tariffs and Trade GAT, in the World Trade Organization WTO and in the European Union EUU (in order to adhere to the convention of having three-letter variables, GATT and the EU are atypically abbreviated). To make it more convenient, variables with lower case letters represent log-transformations, variables in upper case letter represent the absolute value observed.

In order to analyze the elasticity fluctuation of specific variables within this data set, both a

Pooled Ordinary Least Squares (POLS) and a Poisson Pseudo-Maximum Likelihood (PPML) estimation is performed. The results for the entire data set can be found in Tables C.1, C.2 and C.3. A majority of the variables fulfill the expected influences of the gravity model. The dependent variable varies in its sources in Table C.1 and C.2, even though it always represents the bilateral trade flow. Columns one and two come from Comtrade, first reported by origin and second by destination, third and fourth columns come from BACI, with the third representing total trade and the fourth representing manufactured goods only. Columns five and six represent the dependent variable from the International Monetary Fund (IMF) also reported from both the origin and destination country. These different sources are implemented to show that results do not vary significantly. Accordingly, for the following analyses, such as in Table C.3 and further, only bilateral trade reported through Comtrade by the origin country is considered (Column (1) in Tables C.1 and C.2) since it contains the most observations.

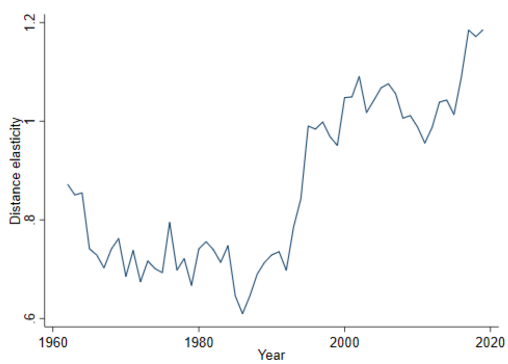

Figure 5.1: Distance elasticity (POLS)

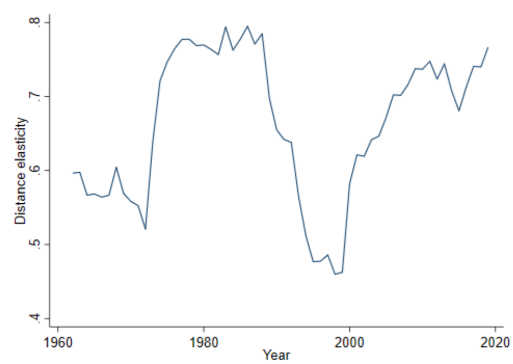

Figure 5.2: Distance elasticity (PPML)

The initial data is converted into cross-section data sets per year again regressed with a POLS and PPML estimator separately since it is the goal to uncover developments of variables' individual influences over time. Results for the development of the distance (Figure 5.1 and 5.2) reveal a high fluctuating elasticity. Even though the two look different, it can be stated that the absolute elasticity of the distance variable increases almost steadily from the 1990s until 2019. The underlying estimations are identical to those in Tables C.2 (column 1) and C.3 (column 3). Same applies for Figure 5.3 and 5.4, where the development of the four binaries is presented.

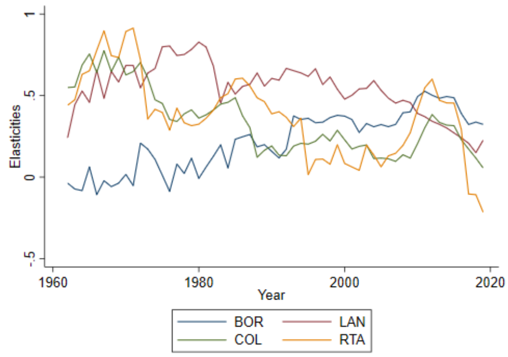

Figure 5.3: Binary elasticity (POLS)

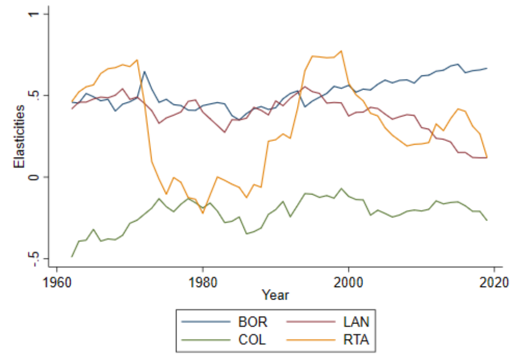

Figure 5.4: Binary elasticity (PPML)

Especially with the PPML estimator, which currently dominates the empirical literature, a clear trend of the variables for a common border, a common official language and for a regional trade agreement is observable. The elasticity of a common border increases while the elasticities of both a common language and for a regional trade agreement decline from year 2000. Combining these results with the increase in the absolute distance elasticity from 2000 to 2019 could justify an ongoing trend towards a local bias in international trade, where trading with neighbors gets more appealing than trading globally.

Just taking these results as given not considering Borchert & Yotov's (2017) findings, supports the thesis that the insecurity part of the individual utility function seems to outweigh a cost minimization effect through globalization in the world trade. It would be questionable whether, in addition to the minimized transfer costs based on the spreading business language English, transport costs through the international linkage also decrease. Effects of globalization would then only be observable, when a cost minimizing effect is strong enough. If it would not be the case, the trend would continue going on towards a more and more local trade, which is based on rising uncertainty and a lack of trust in the foreign.

In order to verify or falsify these conclusions, the alternative empirical approach is proposed, where the value per meter traveled divided by the total trade per year is considered. The advantage of this new approach is that it is not depending on specific estimation techniques, whether this is a continuous observation period or not, nor on the choice of additional national variables.

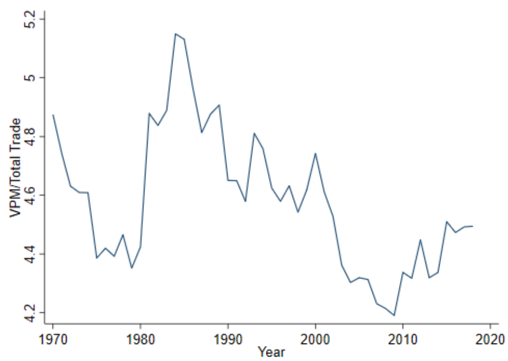

Figure 5.5: Trade Trends 1970-2019

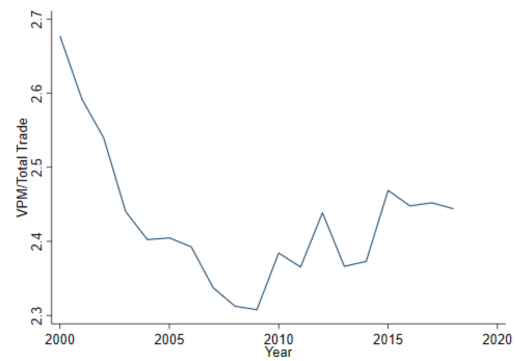

Figure 5.6: Trade Trends 2000-2019

The ratios on the vertical axes over the periods considered allow a more detailed insight then both the previous estimations and Borchert & Yotov's (2017) results. Although Figure 5.5 also covers the period from 2000 to 2019, there are differences to Figure 5.6 in this period. For this alternative empirical approach, having a perfectly balanced panel is of the utmost importance. If this is not checked, a bilateral trade relationship which is only implemented later in the data set, could lead to biased results. For this reason, Figure 5.5 represents a smaller data set than Figure 5.6. A valid differentiation between LIC, MIC and HIC is not possible due to data unavailability for this frequency. Even though the values are different, the trends starting in the year 2000 are identical. These trends open up new insights into global trade. It seems like there is a turning point after the Global Financial Crisis (GFC) where the ratio once more starts to increase making trading globally again more attractive. Comparing the results to those of Borchert & Yotov (2017) and Figures 5.1, 5.2, 5.3 and 5.4 raises questions.

How is it possible to combine all these results? Is the ratio of VPM and total trade also an indicator of global distrust predicting a part of the GFC? Only considering the period 2000-2019, the elasticity of the distance variable and other binaries suggest that there is a steady trend towards a local bias in international trade, the alternative empirical approach supports this up to the GFC, not beyond. Borchert & Yotov (2017) find a decreasing distance elasticity from 1986 to 2006 analyzing with a 10-year frequency. This can also be confirmed by looking at Figure 5.2, but the frequency and time period is crucial for their analysis, not allowing for detailed insights, especially the increase from 2000 until the GFC.

After the Global Financial Crisis, the distance elasticity decreases in the short-term, which can be observed in Figures 5.1 and 5.2. The alternative empirical approach also supports this by an increasing ratio after 2008. Overall, it is to be expected that the distance elasticity and the VPM/total trade ratio are negatively correlated, which is what they do over the period 2000-2019.

5.4 Conclusion

With the $\mu - \sigma^2$ decision criterion, utilities of individuals can be modeled with respect to the decision between trading globally and locally. Referring to this, insecurities about other countries might have been increased until the Global Financial Crisis, because declines in transport and transfer cost are observable while the binary elasticity for a common border increase and the elasticities for a regional trade agreement and a common language decrease in recent years. This will become even more relevant as telemigration, digital goods and knowledge pipelines through foreign direct investments get more popular and further reduce transport cost.

One explanation for the increasing elasticity of the distance variable from 2000 would be that goods that are comparatively easy to transport can be manufactured in the destination country itself and that the portion of traded high-tech goods that are also difficult to transport increase. Intra-industry trade (IIT) is also a possible explanation. Many authors assume that homogeneous goods are more likely to become more distance sensitive as compared with differentiated goods. Since Ricardian economics only exist in a very weak form at the international level, this IIT could lead to increasingly more important distance elasticity and thus transport costs in decision-making processes for potential imports.

In contrast to the study by Borchert & Yotov (2017), there is no distinction between LIC, MIC and HIC. Therefore, it is not possible to uncover individual globalization effects. It could be that such effects are simply not existing in this relatively homogeneous group of OECD countries or that some countries benefits are dominated by the majority's disadvantage. Further follow-up studies should therefore use the alternative empirical approach of the ratio of VPM and total trade in more heterogeneous data sets.

This ratio not only helps answering the research question, but also seems to be a good indicator of global distrust, since the minimum was reached during the GFC.

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Chapter 6

Synchronized effects of interest rates

6.1 Introduction

With lower interest rates, central banks incentivize both private households and firms to borrow and therefore spend more money. While the effects for households can be described by a primarily national increase in consumption, the resulting investments by companies, especially Multinational Enterprises (MNE), are much more international. This paper focuses on how interest rate policies affect Foreign Direct Investments (FDI).

FDI literature is still dominated by factors which are deducted from Dunning's eclectic paradigm of ownership, location and internalization (OLI) (Buckley et al. 2015, Luo & Tung 2007, George et al. 2005). That is why there is a focus on the foreign country's specific factors, while the domestic's are mostly neglected (Bénassy-Quéré et al. 2007). This study contributes to the empirical analysis of national factors, especially the domestic interest rate. In these estimates based on a gravity model, outward FDI stocks are taken as dependent variables. These reflect economic activity abroad more precisely and have econometric advantages over FDI flows. One of the primary factors in gravity models, the economics size of a country, is a significant determinant of FDI in previous studies (Markusen 2002, Busse et al. 2010). Markusen's (2002) knowledge-capital model suggests that foreign's economic size, almost always in terms of Gross Domestic Product (GDP), and horizontal MNE are positively correlated caused by their higher sales potential. Respectively, vertical MNE are positively affected by domestic GDP. The other primary factor, distance, represents transport costs while concerning bilateral trade. Authors like Egger & Winner (2006), Wei (2000) and Egger & Pfaffermayr (2004) show that distance also hinders FDI. The previous research outcome on the effect of interest rates is mixed. While exemplary Chakrabarti (2001) and Arbatli (2011) find positive significance, Onyeiwu & Shrestha (2004) find negative significance of interest rates on FDI.

In a data set of FDI between Organisation of Economic Co-operation and Development (OECD) members in the years 2003 to 2013, a non-neglectable share of negative and real zero FDI positions occur. They result from higher liabilities to the partner country in comparison to their income. Therefore, the standard procedure for gravity models, calculating with logarithms (logs), is not as effortless as it is in many estimations for bilateral trade, where negative values do not exist. Three different functions for transformed logs of FDI are

computed and advantages and disadvantages for each of them are discussed in addition.
After introducing into the topic, this paper is divided into four further sections starting with the basic framework containing previous studies about the relations between interest rates, FDI and the gravity model. Based on this, three hypotheses arise. The third section describes the estimation methods used in this report. Section five shows the empirical results, before section six concludes.

6.2 Interest rates, FDI and gravity

Effects of interest rate policy, the gravity model and reasons for FDI between the domestic country i and the foreign j form the basic framework for this study. Interest rate policy is investigated with special regard to effects on FDI and inflation. The gravity model provides a structure of control variables for the econometric strategy. Theoretical foundations of FDI lead to hypotheses which are tested in the empirical analysis.
The correlation of inflation INF and interest rates INT is one of the cornerstones of active interest rate policy. The traditional economic argumentation is that as INT decreases, the economy faces a positive shock where individuals are willing to borrow more money. On the one hand firms are able to finance investments more easily, on the other hand private households consume more and cause an inflation through an increase in money supply. In contrast, an increasing INT has opposite effects. For country i, this relation can be formally expressed as $\partial\ INF_i/\partial\ INT_i < 0$, the partial derivative of inflation in i with respect to i's interest rate.
Because of this inverse correlation between INF and INT, central banks set interest rates to manipulate economies' inflation rates in order to primarily maintain stability. The goal is not to directly target private households; instead, banks and financial institutions bind their intermediate lending and borrowing rates to a central given INT and pass the changes to companies. In most countries or monetary unions INT is represented by government bond returns.
A decreasing interest rate in country i comes with an increasing investment stock I. Formally, it can be expressed as $\partial\ I/\partial\ INT_i < 0$, the partial derivative of investments with respect to the interest rate of country i. Concerning this representation, I is explicitly not indicated for

a specific country as it is for the derivative of the inflation.

Central banks focus on domestic markets. While a lower interest rate primarily affects households nationally, investments are affected both nationally and internationally. Companies are therefore not forced to invest in domestic markets, but can also use the new liquidity to strengthen their investments abroad. As a consequence, foreign countries can get a positive economic shock through FDI by benefiting from a lowered domestic interest rate. It is conceivable that the inflation effect by household consumption abroad will also have international consequences. However, these are assumed to be significantly smaller than those on investments.

Dunning's OLI framework approaches MNE activity well in theory. Most FDI are justified with market access (Brainard 1993, Markusen & Venables 1999) defined as horizontal investments. Studies focusing on vertical FDI within the internalization process (Aizenman & Marion 2004) are comparatively rare. The effect on economic growth resulting from FDI depends on whether FDI is either substituting or complimenting domestic facilities (De Mello 1999). Since the FDI data set under consideration covers both types of investments, greenfield and mergers and acquisitions (M&A), there can be no distinction between different forms and their specific effects (Gilroy & Lukas 2006). It is therefore not possible to interpret the results of this study with respect to individual effects on different types of FDI. Nevertheless, Brakman et al.(2007) among others show a dominance of M&A in the past years of 78 percent compared with 22 percent greenfield investments. The literature, however, varies from substituting effects of greenfield and M&A to complementary effects (Qiu & Wang 2011).

There is a variety of mainly empirical literature defining determinants of FDI from both the perspective of the receiving country as well as the perspective of the investing country. GDP is the most common influencing positive factor, either denoted as an absolute value or as a growth rate (Chakrabarti 2001, Hansen & Rand 2006, Alfaro et al. 2004). GDP growth on the one hand results in a growing potential market for investors. On the other hand a higher GDP is related to a greater availability of resources and therefore causes an increase in FDI flows out of the country. To attract FDI, countries need to make sure that they are open to investment inflows. Accordingly, the second important factor is the level of economic openness (Chakrabarti 2001, Sekkat & Veganzones-Varoudakis 2007). Openness can be defined via trade openness, the relation of the sum of exports and imports to GDP or via specific openness indicators (Quazi 2007). Indicators can also be used for political rights (Busse & Hefeker 2007, Guerin & Manzocchi 2009) since stability also supports investment decisions. Due to its cost minimizing effect regarding the connection to other parts of the product chain or to markets themselves, infrastructure is a significant push factor in the literature as well (Mollick et al. 2006). In addition, exchange rates (Froot & Stein 1991), inflation rates (Munir & Mansur 2009) and interest rates (Caballero et al. 2008) influence inward and outward FDI.

These variables can also be linked to the gravity model which currently dominates the literature for bilateral trade. It can also be used to find suitable control variables in the FDI context. Countries with a high bilateral trade volume also share a lot of FDI and investors tend to search for opportunities in neighboring countries with similar characteristics, governmental systems and institutionally proximity.

Expectations for the second primary variable in the gravity model are more controversial. Long distances are associated with higher transportation costs, therefore the negative impact in gravity models for bilateral trade is straightforward. When it comes to FDI, however, the direct physical distance is not of much importance but correlates with information costs which could be assigned to institutional, psychological or cultural distance. This is consistent with Dunning's OLI of favoring specific market characteristics as FDI determinants (Brenton & Di Mauro 1999). Nevertheless, there are contrasting studies that assume a positive correlation between distance and FDI based on the investors' preferences to gain control through direct investments instead of choosing alternatives like establishing trade partnerships. Associated information asymmetries would then be minimized by intentions to manage locally.

The overall literature on FDI determination by gravity is large. Eaton & Tamura (1994) were one of the first using the model to determine international investment flows. They find that bilateral trade and FDI share common determinants in population size, income, the level of education and further peculiarities. Brenton et al. (1999) extend their approach by estimating gravity variables' effects on FDI between Central and Eastern European Countries (CEEC) and members of the European Union (EU). Building on the acceptance of the gravity variables for determining FDI flows, Wei (2000) then uses the model to determine suitable control variables when estimating the effect of corruption on investment inflows. Portes & Rey (2005) apply the basic gravity model in a further improved empirical setting and claim that the explanation power with regard to FDI is at least as high as it is for bilateral trade. In order to also have a theoretical foundation, Bergstrand & Egger (2013) create a general equilibrium model. The implementation of theoretical and empirical settings has since become more and more detailed. Among others, Keller & Yeaple (2009, 2013) implemented a gravity-based foreign affiliate sales model in such a way that conclusions about trade costs from observable trade flows were possible. There are even prominent recent contributions to the literature like Petri (2012), Chang (2014), Qian & Sandoval-Hernandez (2016) or Mistura & Roulet (2019). Besides these well established primary factors: GDP of the respective country and the distance DIS between i and j, the literature also adds individual secondary factors like the economic index of freedom EIF, similarities in religious majorities REL and several binary variables as dummies for a common border BOR, language LAN, colonial relationship COL, currency CUR and the membership in the European Union EUU. In some gravity models GDP, as a measurement for size, is extended by the respective population size or even by per capita income. The results, however, do not change significantly when gravity is only implemented to provide control variables, even with respect to this research question.

Based on the literature on interest rates, FDI and the gravity model, three hypotheses arise.

Hypothesis 3 *The impact of interest rates on inflation is national while it is international on investments.*

Hypothesis 3 (H3) summarizes the general assumption about the difference in impacts of the interest rates. It is mainly based on the greater influence of firms on international markets in comparison to the households' impact.

Economic theory predicts different directions of interest rates' impact on FDI. The prevailing academic opinion is that capital flows to the higher return yielding economy. Therefore, the difference between interest rates would determine outward FDI. If the interest rate in the foreign county INT_j is greater than the domestic one INT_i, then the difference $INT_i - INT_j$ is smaller than zero and pushes FDI from country i to j. MNE are able to borrow money in their home country relatively cheap and use arbitrage effects when investing abroad. MacDougall (1960) and later Kemp (1962) are one of the first economists implementing these differences of interest rates on international level theoretically. They assume a two-country model with free capital movement, where capital flows from a capital abundant country to a country where it is comparatively scarce. The difference in interest rates is based on the marginal productivity, movements of capital therefore increase the total welfare when factors equalize. This concept of factor price equalization, often called capital arbitrage approach, however, fails to explain current bilateral movements of capital, especially between similar countries. In such cases, it is observable that FDI flows from country i to country j with simultaneous FDI flows from j to i in the same period. Some studies suggest that if the effects of interest rates are isolated, so for a case where it is possible to control for all other effects determining MNE activities, capital arbitrage can actually be observed. Because this whole theory is based on vague assumptions and can not be satisfactorily validated in current empirical research, McDougall's and Kemp's theoretical work are split up into two separated hypotheses.

Hypothesis 4 *The domestic interest rate is negatively correlated with outward FDI.*

Hypothesis 4 (H4) shares the same justification for outward FDI as the first part of the capital arbitrage approach. MNE increase their foreign activities, respectively FDI, if the domestic interest rate decreases. Money is cheaper, so MNE follow central banks' intentions to increase their investment. H4 does not stand in contrast to capital arbitrage theories, only neglects effects of foreign interest rates on FDI.

Hypothesis 5 *The foreign interest rate is negatively correlated with outward FDI.*

Since $\partial\, I/\partial\, INT < 0$ defines the impact of interest rates on investments, it also includes the effect of the domestic interest on inward FDI. Lower interest rates in a partner country's stable economic environment therefore may support investment decisions. In summary, investments can be financed at home, abroad or in both countries via mixed financing. If hypothesis 5 (H5) cannot be rejected, then capital arbitrage models fail to explain bilateral FDI activities since not the interest differential, but both interest rates separately affect FDI activities.

6.3 Econometric approach

The control variables and empirical methods from the literature are used to test these hypotheses. GDP, DIS, EIF, REL, BOR, LAN, COL, CUR, EUU can be found as potential impact factors over a majority of the literature. Therefore, the initial estimation includes all of them logarithm (log)-linearized to reshape the skewed data set. Log transformations are theoretically based in the derivation of the gravity model itself. Due to the non-normal transformation of negative and real zero outward FDI stocks, there also will be an initial estimation in absolute terms to avoid bias based on the choice of the transformation function. Because the orthogonality assumption does not hold for this panel, an Ordinary Least Square (OLS) estimator is inconsistent. The error term would include country-pair specific effects correlating with the independent variable. Estimating gravity models in scientific literature started with cross-sectional data, but nowadays is performed by using panel data only. A panel model's best feature is its ability to control for specific effects, either random (RE) or fixed effects (FE). The choice will be supported by the Hausman test. If the FE estimator is preferred, then some limitations are relevant. FE estimations restrain variations across individuals. These unobserved effects can be correlated with the explanatory variable. The second limitation concerns time invariant variables. Factors which do not change over time are eliminated by a FE model. In the case of this study, this is going to relevant for DIS_{ij}, BOR_{ij}, LAN_{ij} and COL_{ij} indicated by the absence of a time index t.

The Hausman-Taylor (HT) estimation (Hausman & Taylor 1981) overcomes limitations mentioned for the RE and FE case. It allows explanatory variables to be included as instruments.

$$Y_{it} = \beta_0 + X_{1it}\beta_1 + X_{2it}\beta_2 + Z_{1i}\delta_1 + Z_{2i}\delta_2 + \mu_i + \epsilon_{it} \tag{6.1}$$

The HT model in equation (6.1) assumes that explanatory variables are independent, uncorrelated with ϵ_{it} but can be correlated with μ_i. To be more precise, these independent variables are time varying and uncorrelated X_{1it}, time varying and correlated with μ_i X_{2it}, time invariant and uncorrelated Z_{1i} and time invariant and correlated with μ_i Z_{2i}. As it is done in FE models, the estimator for X_{2i} is instrumented by the deviation from panel level means. Z_{2i}, in contrast, is instrumented by individual averages of independent variables X_{1it}. HT estimators therefore allow the usage of gravity models which are highly dependent on time-invariant variables.

Before deciding between RE, FE and HT it should be justified why a static panel is preferred over a dynamic. A generalized method of moments (GMM) estimation can implement time-invariant regressors in an instrumental variable (IV) setting, not in its difference (Arellano & Bond 1991) form, but in its system (Arellano & Bover 1995, Blundell & Bond 1998) form and supports dynamics in the panel. Dynamics are included by an autoregressive (AR) term which is a lagged variable of the outcome. However, the usage of system GMM leads to an exponential increase of the number of instruments with an increasing number of time periods. Thus, by using a static non-IV estimator it is possible to avoid a potential weak-instrument bias. Mixed modeling, extending static models with the lagged dependent variable as an explanatory, leads also to serious bias as Nickell (1981) and further econometric research state. Even though FDI and trade share similar push and pull factors, strategies determining FDI can be more complex (Gilroy & Lukas 2006). FDI flows are not nearly as stable as trade flows are. A closer look at the data reveals that they also change signs from period to period. To make it more convenient to investigate the interest rates' effects in a panel, the outward FDI stock is taken as the dependent variable. Although FDI occur much rarer, they have a extensively higher volume than individual trade flows. MNE plan their investments in independent projects. FDI in this data set contains both greenfield investments and M&A. Both often represent one-time cash flows. Therefore, there is a need for further control variables beyond the given in the literature.

6.4 Empirical results

The dataset is based on 31 OECD countries and their FDI relations with 36 partner countries inside and outside of the OECD betwwen 2003 and 2013. Since data is not available for some bilateral trade relations, only 977 country pairs are observable. Considering the completeness and balance, the number of observations is 7571 for most of the regressions.

The observations of outward FDI stock FDI_{ijt} range from a minimum of -30237 up to 645098 and measures the total level of direct investment from the country of origin i in j at the end of each year t. It is retrieved directly from the OECD database, but downscaled by 10^6. In addition to economic and econometric factors, the outward FDI stock is also preferable over the bilateral flows because of its greater availability.

A standard log transformation for FDI_{ijt} is not possible since a large portion of the observations is non-positive and could not be considered further. Thus, to consider the maximum amount of observations, three different transformation functions are applied. The specific functions and their advantages and disadvantages are discussed at the end of this section in detail.

Interest rates are included as short term interest rates INT_{S_t} as well as long term interest rates INT_{L_t} for both respective countries. INT_{S_t} is defined by the rate at which short-term borrowings between financial institutions are offered, or by the rate at which short-term government papers are placed in the markets. In particular, three-month market rates, often money market rate or treasury bill rate, are annualized for this panel. In contrast, INT_{L_t} refers to the government bonds emitted for a maturity of ten years. These interest rates are also annualized to fit the panel data. Population sizes POP_{it}, POP_{jt} and economic sizes in terms of GDP_{it}, GDP_{jt} in current United States Dollars (USD) are in absolute values. These observations are taken from World Development Indicators, World Bank. While these variables fluctuate over time, geographical distance DIS_{ij} between i and j in kilometers as well as binary variables for a common border BOR_{ij}, same official or primary language LAN_{ij} and a colonial relationship COL_{ij} do not fluctuate over time. Therefore, they do not have to be indexed by t. DIS_{ij} is gathered by CEPII and is population weighted. Binaries are taken from Head et al. (2010). Religious proximity REL_{ijt} by Disdier & Mayer (2007) is in an interval between zero and one. It is calculated by the sum of products of shares of same religious beliefs. CUR_{ijt}, a binary variable for common currencies is based on the CEPII data set updated by De Sousa (2012). The last variable for typical gravity models usually is a free trade agreement (FTA), a regional trade agreement (RTA) or a preferential trade agreement (PTA), here EUU_t indicates the membership in the European Union (in order to adhere to the convention of having three-letter variables, the European Union is atypically abbreviated). COS_t and TIM_t are representing entry costs and entry time to set up a business in the respective country. Entry costs are measured in percentage of per capita income (PCI), entry time is measured in days until the business is allowed to run. Further variables are the averaged annual exchange rates of the respective country's currency against the USD EXC_t, the annual inflation based on consumer price index (CPI) with year 2015 as the reference at 100 points INF_t and purchasing power parity PPP_t. All of these are taken from the OECD database. The Heritage Foundation provides a variety of equally important factors to measure the Economic Index of Freedom. These factors range from zero, completely unable to confirm, to 100, confirm completely. It is possible to observe the index itself, but also the individual factors separately. That is why the variables for both countries i and j for property rights

PRO_t, government integrity GOI_t, tax burden TAX_t, government spending GOS_t, business freedom BUS_t, monetary freedom MON_t, trade freedom TRF_t, investment freedom INV_t and financial freedom FIN_t are considered. Additionally, the natural logarithms (log) is taken for variables that are initially in absolute values. To make it more convenient, variables with lower case letters represent log-transformations, variables in upper case letter represent the absolute values observed.

Standard log transformation for the outward FDI stock FDI_{ijt} variable is not as straightforward as it is for exclusively positive values. Thus, three different transformations for such specific cases are introduced to keep the loss of data as low as possible. Each of them comes with advantages and disadvantages that will be compared in the following.

$$y_1 = log\left[x + \sqrt{(x^2 + 1)}\right] \tag{6.2}$$

In equation (6.2) and the others, x represents the outward FDI stock to be transformed for every country-pair at every point in time. Squaring x and adding one ensures positiveness in the root regardless of what value of x is assumed. Negative absolute values are also negative after transformation since the log is taken from an interval between greater than zero but smaller or equal to one. Although this inverse hyperbolic sine function is common knowledge in mathematics, Busse & Hefeker's (2007) application is widely accepted as a major contribution in the field of empirical investigations of FDI.

$$y_2 = log\left[x - min(x) + 1\right] \tag{6.3}$$

The second transformation, equation (6.3), shifts the observations interval into the positive. By subtracting the minimum of all observations in the sample and adding one, there will be always transformed data starting at zero no matter whether the minimum of x is negative, positive or zero. When adding or subtracting a constant from all observations of a data set, the mean will change by the same amount as the constant while the standard deviation remains unchanged.

$$y_3 = log\left[\frac{x - min(x)}{max(x) - min(x)} \cdot \lambda\right] \tag{6.4}$$

Equation (6.4) is the third transformation from absolute values to logs. Again, the minimum of observations x is subtracted from the observation itself and then scaled on the range of observations with $max(x) - min(x)$ in the denominator. When taking logs, the size of the interval is highly relevant due to the logarithm compression ability, therefore, it can be generally multiplied by λ. Setting $\lambda = 1$ results in negative transformed values with the maximum of zero. Unlike (6.2) and (6.3), transforming with (6.4) deletes one observation, because the numerator gets zero for the minimum of observations and log transforming is not possible.

The usage of transformation functions in an empirical setting should be handled with caution. The choice between (6.2), (6.3) and (6.4), but also further transformation options changes data structure essentially. For this rather straightforward application example, these functions are therefore compared with the help of discrete generated random numbers r for four different cases in Figure D.1. Just to give an indication about the different outcomes, each of these hypothetical distributions contains 10^5 generated observations. The first column in Figure D.1 represents the actual generated observations distributed in 10^2 equidistant bins. The second column reflects the transformation from equation (6.2), the third and the fourth from (6.3) and (6.4) respectively. The interval of values is chosen to be close to equal between minus and plus 10^2 with a zero mean. The first row of random discrete numbers are Gaussian distributed, second is close to equally distributed, third is Laplace distributed and fourth follows a beta distribution with very heavy tails. It can be observed that transformation functions affect data structure differently, therefore, the results of these estimations, but also estimations beyond this study, are highly dependent on the choice between (6.2), (6.3) and (6.4).

Busse & Hefeker's (2007) (second column of Figure D.1) is a valid transformation for specific data structures and empirical research questions especially if a two-peak distribution is in need, but should be handled with caution in other cases. Since it is applied in numerous subsequent studies, advantages, disadvantages and the overall usefulness should be checked for each individual case separately. For further analysis and research depending preference orders of different transformations, studies regarding the correct situational choice follow.

Table 6.1: Cross correlation

	$INT_{S_{it}}$	$INT_{S_{jt}}$	$INT_{L_{it}}$	$INT_{L_{jt}}$	EXC_{it}	EXC_{jt}	PPP_{it}	PPP_{jt}	INF_{it}	INF_{jt}
$INT_{S_{it}}$	1.0000									
$INT_{S_{jt}}$	0.2753	1.0000								
$INT_{L_{it}}$	0.4816	0.0533	1.0000							
$INT_{L_{jt}}$	0.0572	0.6108		1.0000						
EXC_{it}	0.1634		0.1130		1.0000					
EXC_{jt}		0.2280		0.3784		1.0000				
PPP_{it}	0.1836		0.1137		0.9934		1.0000			
PPP_{jt}		0.2062		0.3068		0.9594		1.0000		
INF_{it}	-0.6262	-0.1872	-0.2605		-0.1515		-0.1557		1.0000	
INF_{jt}	-0.1588	-0.6203		-0.4249		-0.1805		-0.1397	0.4223	1.0000

Table 6.1 represents the correlation matrix of international equilibria. Besides the two variables for interest rates INT_S and INT_L, exchange rates EXC, purchasing power parity PPP and inflation rates INF are listed to check for cross correlations and parity conditions. Their correlations are displayed for a statistical significance on a one-percent level. This Table is of high relevance for the choice between the two different interest rates and the control variables that have to be included in the following estimations. $INT_{S_{it}}$ correlates positively with $INT_{S_{jt}}$. This finding is problematic with regard to separated effects of interest rate policies on FDI, because foreign interest rate policy also potentially affects domestic and vice versa. In contrast $INT_{L_{it}}$ and $INT_{L_{jt}}$ do not correlate which indicates that they are the better choice to investigate the separated effect of interest rate policy on FDI.

The other variables are implemented to check also for the quality of the data. Purchasing power parity is represented by the high correlation of EXC_t and its respective INF_t and of course PPP_t itself, the Fisher effect by the correlation of INT_{L_t} and INF_t and the international Fisher effect by INT_{L_t} and EXC_t. Assuming interest rates INT_{L_t} to be exogenous while EXC_t, PPP_t and INF_t are following, so are endogenous, it is justifiable to only include INT_{L_t} for the empirical analysis. If short term interest rates INT_{S_t} are excluded from the data and therefore from Table 6.1, the extant variables behave like theories predict and therefore seem appropriate for further analysis. A spill-over effect from country i to j regarding INT_t and inflation INF_t cannot be observed concerning long term interest rates as it is raised in H3.

An introductory Pooled Ordinary Least Squares (POLS) regression in absolute values is applied to check for a broad overview of significant variables. As in the gravity model assumed,

GDP_{it} and GDP_{jt} are significantly positive, DIS_{ij} has a significant negative effect on absolute outward FDI stock in this OECD panel. Long term interest rates of the domestic country $INT_{L_{it}}$ are also negative significant on a one-percent level. Consequently, interest rates tend to play a major role in determining FDI.

Not only has POP_t a non-expected negative effect for both domestic and foreign country, it is also not significant. Concerning the country-pair specific effects, binary variables CUR_{ijt}, BOR_{ij}, LAN_{ij} and COL_{ij} are highly positive significant while REL_{ijt} can not reach the significance level set. It is noticeable that remaining country specific variables for TIM_t, COS_t, PRO_t, GOI_t, TAX_t, GOS_t, BUS_t, MON_t, TRF_t, INV_t and FIN_t are mostly significant for destination countries, rather less for origin countries. The high correlation of $INT_{L_{it}}$ and FDI_{ijt} tends to support H4.

This introductory POLS regression serves as the base for further, more specific analyses implementing a variety of models. From the initial estimate both POP_t, REL_{ijt} and previously mentioned remaining country specific variables for domestic thus become no longer necessary, because it is both theoretical justifiable and empirical non significant. With respect to the panel structure, a Generalized Least Squares (GLS) regression with random effects (RE) and fixed effects (FE) as well as a Hausman Taylor regression (HT) is applied. The estimates can be observed in Table D.1. It is divided into six different estimation with three models using either long-term interest rates $INT_{L_{it}}$ and $INT_{L_{jt}}$ separately or their difference $INT_{L_{i-jt}}$. It is observable that separated interest rates are negatively significant over all models while their difference is not. Both primary gravity variables GDP_t and DIS_{ij} are highly significant and fulfill their assumed impact. The group of binaries shows mixed results over the distinct estimations, therefore no general statement can be made. Surprisingly striking is the negative significance of the domestic country's membership in the European Union EUU_{it} while the foreign country's EUU_{jt} is not significant in any of these regression models. TIM_{jt}, days to set up a business in country j, seem to be positive significant with respect to their impact on outward FDI while property rights PRO_{jt} hinder FDI significantly in five of six estimations with absolute variables. The index for business freedom BUS_{jt} has a highly significant positive impact on FDI while on the contrary, financial freedom FIN_{jt} has a highly significant negative impact.

Since both theoretical and empirical literature recommend using log-log models in gravity estimations, Table 6.2 also contains six different regression models, but with most variables in logarithms. Two transformation functions of outward FDI stock coming from (6.3) and (6.4) representing the depended variables in RE, FE and HT estimations. Variables for indexes and binaries are not in logs, distance, GDP and interest rates are.

Table 6.2: Estimated effects on bilateral FDI (GLS log-log)

	RE (1)	RE (2)	FE (3)	FE (4)	HT (5)	HT (6)
$int_{L_{it}}$	-0.031^{**}	-0.039^{***}	-0.030^{*}	-0.040^{***}	-0.031^{**}	-0.039^{***}
	(0.0069)	(0.0000)	(0.0173)	(0.0000)	(0.0085)	(0.0000)
$int_{L_{jt}}$	-0.020	-0.035^{***}	-0.034^{*}	-0.034^{***}	-0.028^{*}	-0.036^{***}
	(0.1136)	(0.0000)	(0.0211)	(0.0000)	(0.0331)	(0.0000)
gdp_{it}	0.321^{***}	0.302^{***}	0.099	0.140^{***}	0.304^{***}	0.252^{***}
	(0.0000)	(0.0000)	(0.3333)	(0.0000)	(0.0000)	(0.0000)
gdp_{jt}	0.195^{***}	0.181^{***}	0.178^{*}	0.218^{***}	0.184^{***}	0.185^{***}
	(0.0000)	(0.0000)	(0.0375)	(0.0000)	(0.0000)	(0.0000)
dis_{ij}	-0.187^{***}	-0.202^{***}	0.000	0.000	-0.187^{**}	-0.217^{***}
	(0.0000)	(0.0000)	(.)	(.)	(0.0021)	(0.0000)
COL_{ij}	0.057^{**}	0.070^{**}	0.000	0.000	0.063^{*}	0.080^{*}
	(0.0067)	(0.0049)	(.)	(.)	(0.0313)	(0.0200)
BOR_{ij}	0.051^{*}	0.100^{***}	0.000	0.000	0.041	0.080
	(0.0290)	(0.0003)	(.)	(.)	(0.2799)	(0.0553)
LAN_{ij}	0.134^{***}	0.179^{***}	0.000	0.000	0.149^{***}	0.201^{***}
	(0.0000)	(0.0000)	(.)	(.)	(0.0000)	(0.0000)
CUR_{ijt}	0.017	-0.012	-0.014	-0.017^{*}	0.004	-0.016
	(0.3174)	(0.1408)	(0.6017)	(0.0483)	(0.8482)	(0.0541)
EUU_{it}	-0.078^{***}	-0.090^{***}	-0.059	-0.078^{***}	-0.079^{***}	-0.088^{***}
	(0.0000)	(0.0000)	(0.0527)	(0.0000)	(0.0004)	(0.0000)
EUU_{jt}	-0.034	-0.045^{***}	-0.028	-0.040^{***}	-0.039	-0.044^{***}
	(0.1227)	(0.0000)	(0.4627)	(0.0007)	(0.1625)	(0.0001)
TIM_{jt}	0.006	-0.008	-0.005	-0.013^{**}	0.002	-0.011^{*}
	(0.6253)	(0.0852)	(0.7328)	(0.0080)	(0.8671)	(0.0196)
COS_{jt}	-0.005	0.001	-0.006	-0.000	-0.002	-0.000
	(0.7905)	(0.9234)	(0.8217)	(0.9582)	(0.9015)	(0.9852)
PRO_{jt}	0.025	-0.097^{***}	-0.058	-0.133^{***}	0.016	-0.112^{***}
	(0.4721)	(0.0000)	(0.2052)	(0.0000)	(0.6597)	(0.0000)
GOI_{jt}	0.006	0.038^{**}	-0.077	0.005	-0.009	0.026
	(0.8615)	(0.0078)	(0.1270)	(0.7630)	(0.7975)	(0.0739)
TAX_{jt}	0.012	-0.044^{***}	0.001	-0.035^{**}	-0.003	-0.042^{***}
	(0.5689)	(0.0000)	(0.9765)	(0.0010)	(0.8937)	(0.0000)
GOS_{jt}	0.019	-0.015^{*}	-0.016	-0.018^{*}	0.005	-0.016^{*}
	(0.3185)	(0.0352)	(0.4936)	(0.0134)	(0.8159)	(0.0190)
BUS_{jt}	0.024	0.057^{***}	0.044^{*}	0.065^{***}	0.027	0.059^{***}
	(0.1053)	(0.0000)	(0.0191)	(0.0000)	(0.0776)	(0.0000)
MON_{jt}	-0.008	-0.011^{*}	-0.009	-0.018^{***}	-0.005	-0.014^{**}
	(0.5266)	(0.0118)	(0.5532)	(0.0001)	(0.7179)	(0.0012)
TRF_{jt}	-0.001	0.021^{***}	0.018	0.024^{***}	0.004	0.022^{***}
	(0.9305)	(0.0001)	(0.3315)	(0.0000)	(0.7815)	(0.0000)
INV_{jt}	0.068^{***}	0.020^{**}	0.027	0.013^{*}	0.053^{**}	0.017^{**}
	(0.0001)	(0.0011)	(0.1811)	(0.0361)	(0.0026)	(0.0068)
FIN_{jt}	0.019	-0.016^{**}	-0.024	-0.026^{***}	0.003	-0.021^{***}
	(0.2272)	(0.0047)	(0.2021)	(0.0000)	(0.8434)	(0.0003)
N	7571	7570	7571	7570	7571	7570

Standardized beta coefficients; p-values in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Since transformations with equation (6.4) lose the minimum, estimations have one observation less than estimations with (6.3). Both the domestic and the foreign interest rates are negatively significant on log-transformed FDI with the exception of the RE model for transformed FDI by (6.3). gdp_{it}, gdp_{jt} and also dis_{ij} again represent gravity models perfectly in almost all estimations. Membership in the European Union EUU_t for domestic and foreign is negatively significant in almost all cases. General statements about foreign-specific indexes are not possible since their coefficients vary among estimations. If interest rates are considered in absolute values in these estimations in Table 6.2, significance levels and signs of their coefficients do not change.

6.5 Conclusion

The choice between transformation functions highly affects outcome as Figure D.1 shows. For further research depending the preference order between them, studies regarding the correct situational choice of different transformation functions follow.

H3 cannot be rejected with the model chosen. It should be mentioned that only the second part, the international effect of interest rates, is tested sufficiently. Nevertheless, it was not the main goal of this study anyways to also check the correlation of interest rates and inflation. But the correlation matrix is a good indicator that this hypothesis should hold.

The more detailed estimations suggest that both interest rates affect investments separately. The difference in interest rates does not determine FDI as observed in Table D.1. Since H4 and H5 cannot be rejected, capital arbitrage approaches are not able to explain observed behavior within this specific data set and the chosen estimation methods. Both interest rates of countries i and j negatively affect outward FDI of country i in j separately. It is straightforward that this is the reason why difference in interest rates cannot be a driving factor for FDI. The theoretically assumed positive impact of interest rates in country j cannot be observed. It is rather a complementing effect by for example mixed financing or the interest rate itself is not a determinant at all and is just an indicator for the overall economic state in respective countries. However, following the observed signs, if the interest rate decreases, more money flows into the country. If the interest rate increases, there is less incoming FDI or even a withdrawal.

The gravity model confirms its relevance and validity as a provider for control variables over all models with its highly significant impact factors GDP and DIS. Binary variables, except a common border in the log-log regression with the Hausman-Taylor estimator, are positively significant and therefore increase FDI relations between considered countries. Noticeable, TIM_{jt}, the time for setting up a business in j, is increasing outward FDI in a variety of estimations. Property rights PRO and governmental spending GOS tend to hinder FDI while business freedom BUS, trade openness TRF and investment freedom INV strengthen relationships regarding FDI. These positive impacts are easy to interpret since MNE need to overcome fewer barriers to invest. Mentioned negative impacts can be explained by the search of MNE for investment opportunities in lesser developed countries. Assuming that the low numbers of some indexes correlate with cost effects for investors abroad, links observations to Dunning's location advantages. Therefore, the separation of the Economic Freedom Index in each of its parts is justified by their different signs of impact factors.

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Chapter 7

Limitations and prospect

This chapter includes a summary of the main contributions, a critical examination of the general model's limitations and an outlook on future research questions including an explicit exposé. Since the individual chapters of this dissertation are independent scientific contributions, the associated detailed conclusions of the research questions are already implemented in the respective last sections of each previous chapter.

In order to have a good basis for theoretical extensions, the friction-less case in Bergstrand's (1985) derivation of the gravity model was corrected in the second chapter, accurately it results in $\bar{P}X_{ij} = \sqrt{Y_i Y_j}$ instead of resulting in $\bar{P}X_{ij} = (1/2) \cdot \sqrt{Y_i Y_j}$.

Bergstrand's (1985) partial equilibrium model is then extended to also account for a transplanted home-bias in addition to the already implemented supply side effect by Gould (1994) in chapter three. The detailed derivation in Appendix A made it possible to derive the hypothesis on the positive effect of migrants in the importing country originated in the exporting country. Empirical results show the significant effect on total trade and an even greater effect on traded household consumption. In addition to this initial empirical analysis, migrants' non-linear demand effects were then included in a three-dimensional economies of scale model to set up a further hypothesis in which the demand effect for migrants in their country of origin first increases and then decreases as soon as local companies can also satisfy the demand. This reversing effect can also be demonstrated for total trade and traded household consumption in a quantile-based categorical regression.

Chapter four includes a theoretical approach to explain beta-convergences in growth. Therefore, GDP, total trade and technological progress were combined to derive an international innovation spiral. Besides various implications, the international divergence between countries' economic growth rates can thus be explained by changes in the respective shares on world markets through the gravity model. A simulation of these results has not yet been carried out perfectly, but is under development.

The fifth chapter shows that even though a fraction of the distance paradox could be explained by Borchert & Yotov (2017), there is still a part attributed towards information or cultural differences between economies. These differences are defined as the distance residual. By breaking global trade down to the level of individual decision-makers, it turns out that uncertainties about trading partners can outweigh potential cost benefits. The alternative empirical approach presented is in line with the distance elasticity's development over time and could be used as an indicator for global distrust.

Even though there is no theoretical foundation for gravity relations with respect to FDI flows, they follow similar patters. Therefore, chapter six utilizes this similarity to implement sufficient control variables to empirically check for the impact of interest rates on FDI. The empirical results suggest that capital arbitrage models are no longer able to explain FDI flows since the difference between the exporter's and the importer's interest rate is not significant while the individual interest rates are negatively proportional to FDI. In addition, there is a short introduction about the differences of log-transformation functions for cases, where some observations are non-positive. A further methodical paper could be based on these approaches.

A majority of this work is based on previous studies. However, the depths of the individual chapters and the heterogeneity of the applications stand out in comparison to other dissertations or meta-papers in this area. In addition to the individual research questions, this work also contributes to the optimization of the methodology when working with gravity models. Theoretical studies should be based on the publications by Anderson (1979), Bergstrand (1985) or Anderson & van Wincoop (2003). Here, Bergstrand (1985) offers the greatest level of detail, but at the same time this is accompanied by a more difficult empirical implementation. His approach can be manipulated straightforwardly to the models of Anderson & van Wincoop (2003) or Baier et al. (2018) (see Bergstrand, J. H., Cray, S. R., & Gervais, A. (2021). Increasing Marginal Costs, Firm Heterogeneity, and the Gains from "Deep" International Trade Agreements. Appendix B.3.). Embedding the model in other macroeconomic concepts, such as in a Dynamic Stochastic General Equilibrium (DSGE) model, should be done using the friction-less case in order to keep the complexity of an open economy within limits. The guidelines for empirical work are somewhat narrower. Two estimation methods differ from the others in their quality, but also in the quantity of their use. The fixed effect estimator and the PPML estimator dominate the empirical literature on gravity models even though this dissertations also proposes the HT estimator. In the case of explicit causality problems, a two-stage least square model can also be used. As time goes on, the availability of data also improves. This makes it possible to review old empirical studies and to partially improve them.

Dynamics are of great importance for the global system and are present from chapter three to six. Even if some economists speak of a *slowbilization*, there are more and more movements of goods, services, production factors and much more on an international level. So it is not just a possibility to increase the amount of control variables for international flows because of the higher data availability, but an obligation.

There is a range of recent meta-papers, just to mention four of them: Head & Mayer (2014), Yotov et al. (2016), Kabir et al. (2017) and Baier et al. (2018). These studies and others, including the chapters of this dissertation, summarize the advantages and disadvantages of the gravity model extensively.

The most obvious advantage is that the model is very intuitive in its primary variables. By adding secondary variables depending on the research question, gravity can be used for any international economics field. In empirical studies, there is a high predictive power over almost all periods and levels of observation.

Despite the wide range of possible applications and the solid theoretical justification, there are also some downsides of the model. The implementation of the models in empirical studies is getting better and better. It is therefore self-evident that outdated papers are repeatedly criticized in terms of their methodology over the course of time. However, these weak points will not be addressed in the following, but rather the general disadvantages will be revealed. The model is difficult to reconcile with other standard models in international economics. Many studies only analyze the highest of all levels of the economy, the macroeconomic, and go little or not into the industrial levels. A solid extension of the traditional gravity model with firm heterogeneity would be a clear added value. With regard to the primary variables, only those of the trading partners involved can be analyzed, even though trade costs of a third party can also affect bilateral trade. Regardless of the research question and methodology, the gravity model has high demands on data sets. Many case studies can therefore not be carried out due to poor data quality in the respective segment. The empirical methodology also gains ever higher standards, which is why studies are quickly outdated.

A further research question in the area of the gravity model is presented in the following exposé in order to give an outlook for a potential next project.

CSFTA

Bilateral trade agreements will become increasingly important in the future. They are assumed to be trade-creating and are in line with World Trade Organization (WTO) rules. In very heterogeneous economies, doubters about the benefits of such agreements are always voiced loudly. It is therefore good to examine the effect of such an agreement in more detail, also to predict outcomes of forthcoming contracts.

In July 2013, the China-Switzerland free trade agreements (CSFTA) was signed and became legally effective a year later (For an overview about the implications see e.g. *Lanteigne, M. (2019). The China–Switzerland Free Trade Agreement and Economic Identity-Building. Journal of Contemporary China, 28(118), 614-629.*). This bilateral agreement is of great importance as it could serve as a model for a potential trade deal between China and the EU. Switzerland is the second European country and the third member of the OECD to negotiate an FTA with China.

After joining the WTO in 2001, China's foreign trade policy is being watched with great interest by all sides. Some critics see China's involvement as a threat to the sovereignty of the countries involved. Also not all FTA are assumed to have a positive impact on both economies considered (Baier et al. 2019). In order to create a good basis for discussion, this study aims to empirically analyze the actual effects of the CSFTA.

There is no significant increase when looking at the bilateral trade statistic. It could therefore be assumed that this is an FTA without any direct effect. The explicit research question is therefore whether CSFTA has a positive impact on Swiss exports to China, although no direct effect can be observed.

Gravity models can address this issue. The general effect in large data sets is positive. The implementation is usually done via PPML or FE estimates of the effect of a dummy or binary variable for the FTA in particular. However, endogeneity problems because of the biased selection of agreement partner can arise with these estimates, which is why the results have to be validated using other approaches. An efficient but less prominent approach is the use of a synthetic control method (SCM). SCM can efficiently deal with the problem that FTA are not exogenous random variables. This methodology based on Abadie & Gardeazabal (2003) and later extended by Abadie et al. (2010) is therefore appropriate to identify the causal impact of CSFTA on trade.

In the SCM, a treated unit is compared with a synthetic control unit in order to draw conclusions about the effect of a political intervention based on the starting point of the treatment. A synthetic unit is a weighted average or linear combination of units that have not experienced this policy intervention. The weighting used to create the synthetic unit is chosen so that it is as similar as possible to the actual treated unit before the intervention.

The literature on estimating the effect of an FTA on bilateral trade using an SCM is still underdeveloped. However, examples are , but are not limited to Hannan (2017), Ritzel & Kohler (2017) and Barlow et al. (2017).

Regarding the analysis of the effect of CSFTA on Swiss exports to China, the donor pool, which includes the units from which the synthetic Swiss exports are constructed, consists of China-targeted exports from European countries that do not have an FTA with China. In addition to the variable of interest, there are other predictor variables, such as in a regression. In this case, the gravity model is not only used as a reference value for the effect, but also as a provider for these variables. For the exporters, the GDP, the population size, the distance, the entry cost, a dummy for the membership in the General Agreement of Tariffs and Trade, a common legal system and the exchange rate are used. An implementation for variables for the importing country makes no sense since the synthetic exports only consist of exports to China.

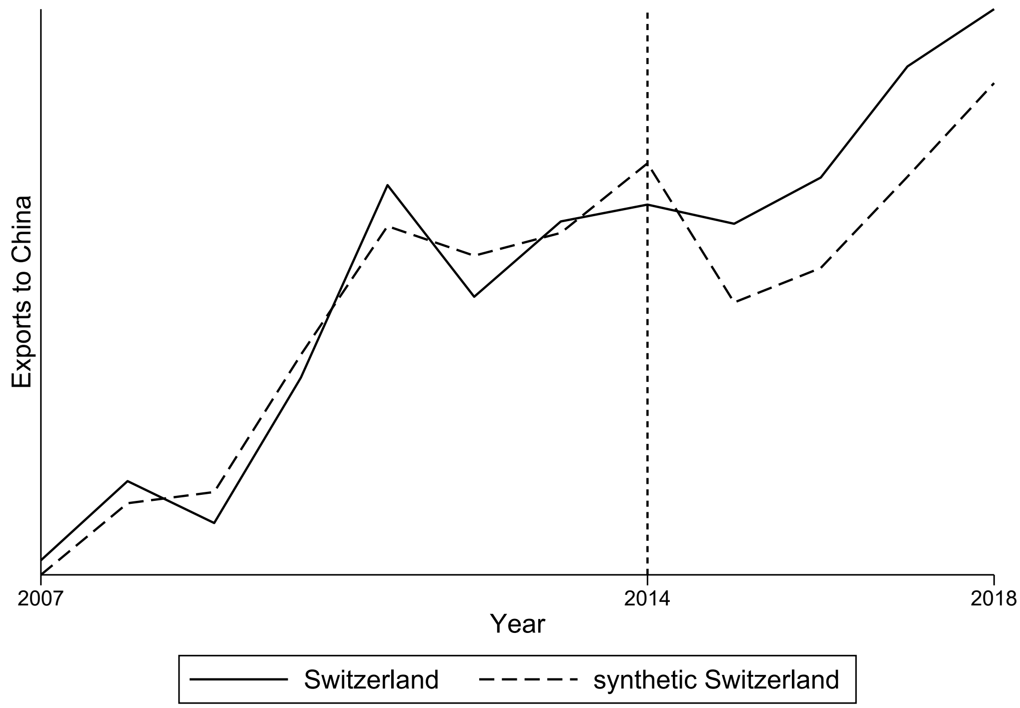

Figure 7.1: Synthetic control method: FTA effect on Swiss exports to China

Figure 7.1 shows both the actual Swiss exports to China and the synthetic exports to China. Now it is also clear why the effect was not directly observable with the introduction of the CSFTA 2014. Actual exports remain constant, although all other countries in the donor pool recorded a decrease, which can be seen by the declining dashed line. The donor countries consist only of exports from Austria, Germany, Luxembourg, Portugal, Slovakia and Slovenia. The algorithmic choice for these donors would even allow further conclusions to be drawn about similarities with these states. Although the synthetic control unit already seems to have a good pre-treatment fit, there is a discrepancy visible in the last few years before the treatment.

Ben-Michael et al. (2021) extend the existing SCM for cases where the pre-treatment fit is not optimal. Their augmented synthetic control method (ASCM) uses an outcome model to estimate the bias that occurs by non-optimal fits and corrects the SCM estimate.

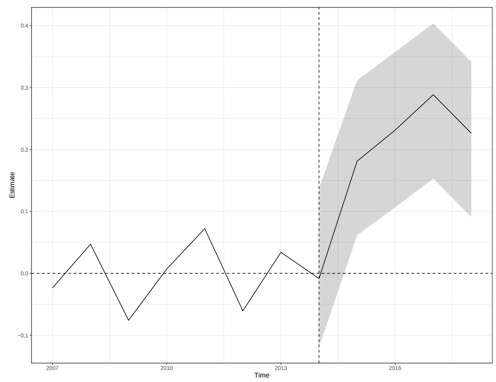

Figure 7.2: Augmented SCM (FE)

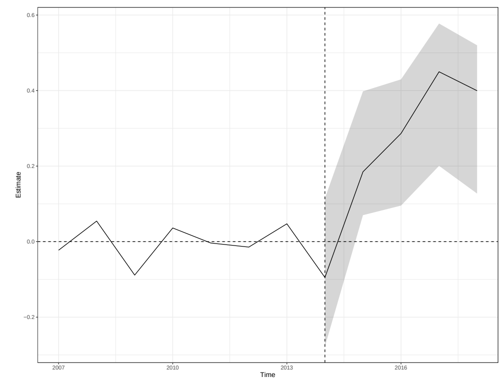

Figure 7.3: Augmented SCM (residual)

Two of the results can be seen in Figure 7.2 and 7.3. While the x-axis indicates time, the y-axis shows the discrepancy between the synthetic and the actual treated unit within an ASCM. The fit between these two is even better and the effect after the CSFTA was implemented in 2014 is recognizable. The significance boundaries are computed by a jackknife+ resampling technique.

In order to turn this expose into a reliable study, the implementation of the methodology must be improved in some places. The jackknife method for instance is vulnerable to criticism. The choice of predictor variables is not optimal and should be expanded. The restriction to only export flows from Switzerland or synthetic Switzerland to China also limits the empirical section. An analysis of the flows in the other direction, in which imports from China to Switzerland are analyzed, would also be conceivable. Here it would make sense for the donor pool to consist of imports from other Asian countries to optimize the fit.

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Appendices

Appendix A

Derivation

A.1 Supply

$$L_i = \left\{ \left[\left(\sum_{k=1,k\neq i}^{N} X_{ik}^{\phi} \right)^{\frac{1}{\phi}} \right]^{\delta} + X_{ii}^{\delta} \right\}^{\frac{1}{\delta}} \quad \forall\ i = 1, ..., N$$

- L_i: production factor labor of both migrant and native population in i
- Labor is allocated across industries for every country i according to a constant elasticity of transformation (CET)
- Labor can be transformed into producing different foreign goods at a constant elasticity
- Labor cannot be transformed from producing foreign goods to domestic goods at the same constant elasticity
- X_{ik}: country i's good supplied to country k
- X_{ii}: Country i's good supplied to domestic market
- $\delta = (\eta + 1)/\eta$, where η is the CET between any two goods in country i $(0 \leq \eta \leq \infty)$
- $\phi = (\gamma + 1)/\gamma$ where γ is the CET among exportable goods $(0 \leq \gamma \leq \infty)$

Maximizing profits

$$\pi_i = \sum_{k=1}^{N} P_{ik} X_{ik} - W_i L_i \ \forall\ i = 1, ..., N$$

gives N^2 first order conditions and generates $N(N-1)$ bilateral export volumes

$$X_{ij}^{S} = \left\{ Y_i P_{ij}^{\gamma} \left[\left(\sum_{k=1,k\neq i}^{N} P_{ik}^{1+\gamma} \right)^{\frac{1}{1+\gamma}} \right]^{-(\gamma-\eta)} \right\} \cdot \left\{ \left[\left(\sum_{k=1,k\neq i}^{N} P_{ik}^{1+\gamma} \right)^{\frac{1}{1+\gamma}} \right]^{1+\eta} + P_{ii}^{1+\eta} \right\}^{-1} \quad \forall\ i, j = 1, ..., N$$

- P_{ik}: the price received for selling i's product in country k
- Y_i: Total income paid to labor $Y_i = W_i L_i$ where W_i is the wage

First part of step by step calculation of supply X_{ij}^S

Simplified with a 3-country case: *i, 1, 2*

Assume country i has profit function

$$\pi_i = P_{i1}X_{i1} + P_{i2}X_{i2} + P_{ii}X_{ii} - W_iL_i \tag{A.1}$$

Assume joint product transformation surface

$$L_i = \left\{ \left[\left(X_{i1}^{\phi} + X_{i2}^{\phi} \right)^{\frac{1}{\phi}} \right]^{\delta} + X_{ii}^{\delta} \right\}^{\frac{1}{\delta}} \tag{A.2}$$

Maximize Lagrange with respect to X and λ

$$\begin{aligned} \pi_i =& P_{i1}X_{i1} + P_{i2}X_{i2} + P_{ii}X_{ii} - W_iL_i \\ & - \lambda \left\{ L_i - \left[\left(X_{i1}^{\phi} + X_{i2}^{\phi} \right)^{\frac{\delta}{\phi}} + X_{ii}^{\delta} \right]^{\frac{1}{\delta}} \right\} \end{aligned} \tag{A.3}$$

$$\frac{\partial \pi_i}{\partial X_{i1}} = P_{i1} + \lambda \frac{1}{\delta} \left[\left(X_{i1}^{\phi} + X_{i2}^{\phi} \right)^{\frac{\delta}{\phi}} + X_{ii}^{\delta} \right]^{\frac{1-\delta}{\delta}} \frac{\delta}{\phi} \left(X_{i1}^{\phi} + X_{i2}^{\phi} \right)^{\frac{\delta-\phi}{\phi}} \phi X_{i1}^{\phi-1} = 0 \tag{A.4}$$

$$\frac{\partial \pi_i}{\partial X_{i2}} = P_{i2} + \lambda \frac{1}{\delta} \left[\left(X_{i1}^{\phi} + X_{i2}^{\phi} \right)^{\frac{\delta}{\phi}} + X_{ii}^{\delta} \right]^{\frac{1-\delta}{\delta}} \frac{\delta}{\phi} \left(X_{i1}^{\phi} + X_{i2}^{\phi} \right)^{\frac{\delta-\phi}{\phi}} \phi X_{i2}^{\phi-1} = 0 \tag{A.5}$$

$$\frac{\partial \pi_i}{\partial X_{ii}} = P_{ii} + \lambda \frac{1}{\delta} \left[\left(X_{i1}^{\phi} + X_{i2}^{\phi} \right)^{\frac{\delta}{\phi}} + X_{ii}^{\delta} \right]^{\frac{1-\delta}{\delta}} \delta X_{ii}^{\delta-1} = 0 \tag{A.6}$$

$$\frac{\partial \pi_i}{\partial \lambda} = L_i - \left[\left(X_{i1}^{\phi} + X_{i2}^{\phi} \right)^{\frac{\delta}{\phi}} + X_{ii}^{\delta} \right]^{\frac{1}{\delta}} = 0 \tag{A.7}$$

Rewrite $(A.4) - (A.6)$, δ and ϕ as multiplicators cancel out

$$\frac{X_{i1}^{\phi-1}}{P_{i1}} = -\lambda^{-1} \left[\left(X_{i1}^{\phi} + X_{i2}^{\phi} \right)^{\frac{\delta}{\phi}} + X_{ii}^{\delta} \right]^{\frac{\delta-1}{\delta}} \left(X_{i1}^{\phi} + X_{i2}^{\phi} \right)^{\frac{\phi-\delta}{\phi}} \tag{A.8}$$

$$\frac{X_{i2}^{\phi-1}}{P_{i2}} = -\lambda^{-1} \left[\left(X_{i1}^{\phi} + X_{i2}^{\phi} \right)^{\frac{\delta}{\phi}} + X_{ii}^{\delta} \right]^{\frac{\delta-1}{\delta}} \left(X_{i1}^{\phi} + X_{i2}^{\phi} \right)^{\frac{\phi-\delta}{\phi}} \tag{A.9}$$

$$\frac{X_{ii}^{\delta-1}}{P_{ii}} = -\lambda^{-1} \left[\left(X_{i1}^{\phi} + X_{i2}^{\phi} \right)^{\frac{\delta}{\phi}} + X_{ii}^{\delta} \right]^{\frac{\delta-1}{\delta}} \tag{A.10}$$

Summarize $(A.8) - (A.10)$

$$\begin{aligned} & -\lambda^{-1} \left[\left(X_{i1}^{\phi} + X_{i2}^{\phi} \right)^{\frac{\delta}{\phi}} + X_{ii}^{\delta} \right]^{\frac{\delta-1}{\delta}} \\ =& \frac{X_{i1}^{\phi-1}}{\left(X_{i1}^{\phi} + X_{i2}^{\phi} \right)^{\frac{\phi-\delta}{\phi}} P_{i1}} = \frac{X_{i2}^{\phi-1}}{\left(X_{i1}^{\phi} + X_{i2}^{\phi} \right)^{\frac{\phi-\delta}{\phi}} P_{i2}} = \frac{X_{ii}^{\delta-1}}{P_{ii}} \end{aligned}$$

$$\frac{X_{i1}^{\phi-1}}{P_{i1}} = \frac{X_{i2}^{\phi-1}}{P_{i2}} = \frac{\left(X_{i1}^{\phi} + X_{i2}^{\phi} \right)^{\frac{\phi-\delta}{\phi}} X_{ii}^{\delta-1}}{P_{ii}} \tag{A.11}$$

Since $\delta > \phi > 1 \Rightarrow \gamma > \eta > 0$

$$\gamma = \frac{1}{\phi - 1}\ ,\ -\gamma = \frac{1}{1-\phi}\ ,\ \gamma + 1 = \frac{\phi}{\phi - 1}\ ,\ \phi = \frac{1+\gamma}{\gamma}$$

$$\eta = \frac{1}{\delta - 1}\ ,\ -\eta = \frac{1}{1-\delta}\ ,\ \eta + 1 = \frac{\delta}{\delta - 1}\ ,\ \delta = \frac{1+\eta}{\eta}$$

From $(A.11)$, rewrite

$$P_{i1}X_{i2}^{\phi-1} = P_{i2}X_{i1}^{\phi-1}$$

$$X_{i2} = X_{i1}P_{i1}^{\frac{1}{1-\phi}}P_{i2}^{\frac{1}{\phi-1}} \tag{A.12}$$

$$X_{ii} = X_{i1}^{\frac{\phi-1}{\delta-1}}P_{i1}^{\frac{1}{1-\delta}}P_{ii}^{\frac{1}{\delta-1}}\left(X_{i1}^{\phi}X_{i2}^{\phi}\right)^{\frac{\delta-\phi}{\phi(\delta-1)}} \tag{A.13}$$

$$L_i = \left[\left(X_{i1}^{\phi} + X_{i2}^{\phi}\right)^{\frac{\delta}{\phi}} + X_{ii}^{\delta}\right]^{\frac{1}{\delta}}$$

$$L_i^{\delta} = \left(X_{i1}^{\phi} + X_{i2}^{\phi}\right)^{\frac{\delta}{\phi}} + X_{ii}^{\delta}$$

Substitute X_{i2} (A.12) and X_{ii} (A.13)

$$L_i^{\delta} = \left[X_{i1}^{\phi} + \left(X_{i1}P_{i1}^{\frac{1}{1-\phi}}P_{i2}^{\frac{1}{\phi-1}}\right)^{\phi}\right]^{\frac{\delta}{\phi}} + \left[X_{i1}^{\frac{\phi-1}{\delta-1}}P_{i1}^{\frac{1}{1-\delta}}P_{ii}^{\frac{1}{\delta-1}}\left(X_{i1}^{\phi} + X_{i2}^{\phi}\right)^{\frac{\delta-\phi}{\phi(\delta-1)}}\right]^{\delta}$$

$$L_i^{\delta} = \left(X_{i1}^{\phi} + X_{i1}^{\phi}P_{i1}^{\frac{\phi}{1-\phi}}P_{i2}^{\frac{\phi}{\phi-1}}\right)^{\frac{\delta}{\phi}} + X_{i1}^{\frac{\delta\phi-\delta}{\delta-1}}P_{i1}^{\frac{\delta}{1-\delta}}P_{ii}^{\frac{\delta}{\delta-1}}\left(X_{i1}^{\phi} + X_{i2}^{\phi}\right)^{\frac{\delta^2-\delta\phi}{\phi(\delta-1)}}$$

Substitute exponents and X_{i2} (A.12)

$$L_i^{\delta} = \left(X_{i1}^{\phi} + X_{i1}^{\phi}P_{i1}^{-(\gamma+1)}P_{i2}^{\gamma+1}\right)^{\frac{\delta}{\phi}} + X_{i1}^{\frac{\delta\phi-\delta}{\delta-1}}P_{i1}^{-(\eta+1)}P_{ii}^{\eta+1}\left[X_{i1}^{\phi} + \left(X_{i1}P_{i1}^{\frac{1}{1-\phi}}P_{i2}^{\frac{1}{\phi-1}}\right)^{\phi}\right]^{\frac{\delta^2-\delta\phi}{\phi(\delta-1)}}$$

$$L_i^{\delta} = X_{i1}^{\delta}\left(1 + P_{i1}^{-(\gamma+1)}P_{i2}^{\gamma+1}\right)^{\frac{\delta}{\phi}} + X_{i1}^{\frac{\delta\phi-\delta}{\delta-1}}P_{i1}^{-(\eta+1)}P_{ii}^{\eta+1}\left[X_{i1}^{\phi}\left(1 + P_{i1}^{\frac{\phi}{1-\phi}}P_{i2}^{\frac{\phi}{\phi-1}}\right)\right]^{\frac{\delta^2-\delta\phi}{\phi(\delta-1)}}$$

Rewrite $1 = P_{i1}^{-(\gamma+1)} P_{i1}^{\gamma+1}$ and substitute exponents

$$L_i^\delta = X_{i1}^\delta \left(P_{i1}^{-(\gamma+1)} P_{i1}^{\gamma+1} + P_{i1}^{-(\gamma+1)} P_{i2}^{\gamma+1}\right)^{\frac{\delta}{\phi}}$$

$$+ X_{i1}^{\frac{\delta\phi-\delta}{\delta-1}} P_{i1}^{-(\eta+1)} P_{ii}^{\eta+1} \left[X_{i1}^\phi \left(P_{i1}^{-(\gamma+1)} P_{i1}^{\gamma+1} + P_{i1}^{-(\gamma+1)} P_{i2}^{\gamma+1}\right)\right]^{\frac{\delta^2-\delta\phi}{\phi(\delta-1)}}$$

Separate $P_{i1}^{\frac{-(\gamma+1)\delta}{\phi}}, P_{i1}^{-(\gamma+1)}$, rewrite $X_{i1}^{\frac{\delta\phi-\delta}{\delta-1}} + (X_{i1}^\phi)^{\frac{\delta^2-\delta\phi}{\phi(\delta-1)}} = X_{i1}^\delta$

and substitute exponents

$$L_i^\delta = X_{i1}^\delta P_{i1}^{\frac{-(\gamma+1)\delta}{\phi}} \left(P_{i1}^{\gamma+1} P_{i2}^{\gamma+1}\right)^{\frac{\delta}{\phi}}$$

$$+ X_{i1}^\delta P_{i1}^{\frac{-\delta}{\delta-1}} P_{ii}^{\frac{\delta}{\delta-1}} P_{i1}^{\frac{-\phi(\delta^2-\delta\phi)}{(\phi-1)(\phi(\delta-1))}} \left(P_{i1}^{1+\gamma} + P_{i2}^{1+\gamma}\right)^{\frac{\delta^2-\delta\phi}{\phi(\delta-1)}}$$

Rewrite $P_{i1}^{\frac{-(\gamma+1)\delta}{\phi}} = P_{i1}^{-\delta\gamma}$ and $\Omega^{\frac{\delta}{\phi}} = \Omega^{\frac{\delta\gamma}{1+\gamma}}$

Rewrite $P_{i1}^{\frac{-\delta}{\delta-1}} P_{i1}^{\frac{-\phi(\delta^2-\delta\phi)}{(\phi-1)(\phi(\delta-1))}} = P_{i1}^{\frac{-\delta}{\phi-1}} = P_{i1}^{-\delta\gamma}$

Rewrite $P_{ii}^{\frac{\delta}{\delta-1}} = P_{ii}^{\delta\eta}$

$$L_i^\delta = X_{i1}^\delta P_{i1}^{-\delta\gamma} \left(P_{i1}^{\gamma+1} P_{i2}^{\gamma+1}\right)^{\frac{\delta\gamma}{1+\gamma}} + X_{i1}^\delta P_{i1}^{-\delta\gamma} P_{ii}^{\delta\eta} \left(P_{i1}^{1+\gamma} P_{i2}^{1+\gamma}\right)^{\frac{\delta(\delta-\phi)}{\phi(\delta-1)}}$$

Separate $X_{i1}^\delta P_{i1}^{-\delta\gamma}$

$$L_i^\delta = X_{i1}^\delta P_{i1}^{-\delta\gamma} \left[\left(P_{i1}^{\gamma+1} P_{i2}^{\gamma+1}\right)^{\frac{\delta\gamma}{1+\gamma}} + P_{ii}^{\delta\eta} \left(P_{i1}^{1+\gamma} P_{i2}^{1+\gamma}\right)^{\frac{\delta(\delta-\phi)}{\phi(\delta-1)}}\right]$$

Rewrite $\Omega^{\frac{\delta(\delta-\phi)}{\phi(\delta-1)}} = (\Omega^{\frac{1}{1+\gamma}})^{\delta(\gamma-\eta)}$

$$L_i^\delta = X_{i1}^\delta P_{i1}^{-\delta\gamma} \left\{\left[\left(P_{i1}^{\gamma+1} P_{i2}^{\gamma+1}\right)^{\frac{1}{1+\gamma}}\right]^{\delta\gamma} + P_{ii}^{\delta\eta} \left[\left(P_{i1}^{1+\gamma} P_{i2}^{1+\gamma}\right)^{\frac{1}{1+\gamma}}\right]^{\delta(\gamma-\eta)}\right\}$$

Expand $\Omega^{\delta\gamma}$ in $\Omega^{\delta\eta}$ and $\Omega^{\delta\gamma-\delta\eta}$

$$L_i^\delta = X_{i1}^\delta P_{i1}^{-\delta\gamma} \left\{\left[\left(P_{i1}^{\gamma+1} P_{i2}^{\gamma+1}\right)^{\frac{1}{1+\gamma}}\right]^{\delta\eta} \left[\left(P_{i1}^{\gamma+1} P_{i2}^{\gamma+1}\right)^{\frac{1}{1+\gamma}}\right]^{\delta(\gamma-\eta)}\right.$$

$$\left. + P_{ii}^{\delta\eta} \left[\left(P_{i1}^{\gamma+1} P_{i2}^{\gamma+1}\right)^{\frac{1}{1+\gamma}}\right]^{\delta(\gamma-\eta)}\right\}$$

Separate $\left[\left(P_{i1}^{\gamma+1} P_{i2}^{\gamma+1}\right)^{\frac{1}{1+\gamma}}\right]^{\delta(\gamma-\eta)}$ and $\Omega^{\delta\eta} = \Omega^{1+\eta}$

$$L_i^\delta = X_{i1}^\delta P_{i1}^{-\delta\gamma} \left[\left(P_{i1}^{\gamma+1} P_{i2}^{\gamma+1}\right)^{\frac{1}{1+\gamma}}\right]^{\delta(\gamma-\eta)} \left\{\left[\left(P_{i1}^{\gamma+1} P_{i2}^{\gamma+1}\right)^{\frac{1}{1+\gamma}}\right]^{1+\eta} + P_{ii}^{1+\eta}\right\}$$

Divide by everything except X_{i1}^δ and L_i^δ

$$X_{i1}^\delta = L_i^\delta P_{i1}^{\delta\gamma} \left[\left(P_{i1}^{\gamma+1} P_{i2}^{\gamma+1}\right)^{\frac{1}{1+\gamma}}\right]^{-\delta(\gamma-\eta)} \left\{\left[\left(P_{i1}^{\gamma+1} P_{i2}^{\gamma+1}\right)^{\frac{1}{1+\gamma}}\right]^{1+\eta} + P_{ii}^{1+\eta}\right\}^{-1}$$

Generalize with indixes for elasticities

Take it to the power of $\frac{1}{\delta}$, then $\Omega^{\frac{-1}{\delta}} = \Omega^{\frac{-\eta_i}{1+\eta_i}}$

$$X_{i1}^S = L_i P_{i1}^{\gamma_i} \left[\left(P_{i1}^{\gamma_i+1} P_{i2}^{\gamma_i+1} \right)^{\frac{1}{1+\gamma_i}} \right]^{-(\gamma_i - \eta_i)} \left\{ \left[\left(P_{i1}^{\gamma_i+1} P_{i2}^{\gamma_i+1} \right)^{\frac{1}{1+\gamma_i}} \right]^{1+\eta_i} + P_{ii}^{1+\eta_i} \right\}^{\frac{-\eta_i}{1+\eta_i}}$$

$$X_{i2}^S = L_i P_{i2}^{\gamma_i} \left[\left(P_{i1}^{\gamma_i+1} P_{i2}^{\gamma_i+1} \right)^{\frac{1}{1+\gamma_i}} \right]^{-(\gamma_i - \eta_i)} \left\{ \left[\left(P_{i1}^{\gamma_i+1} P_{i2}^{\gamma_i+1} \right)^{\frac{1}{1+\gamma_i}} \right]^{1+\eta_i} + P_{ii}^{1+\eta_i} \right\}^{\frac{-\eta_i}{1+\eta_i}}$$

$$X_{ij}^S = L_i P_{ij}^{\gamma_i} \left[\left(\sum_{k=1, k\neq i}^{N} P_{ik}^{\gamma_i+1} \right)^{\frac{1}{1+\gamma_i}} \right]^{-(\gamma_i - \eta_i)} \cdot \left\{ \left[\left(\sum_{k=1, k\neq i}^{N} P_{ik}^{\gamma_i+1} \right)^{\frac{1}{1+\gamma_i}} \right]^{1+\eta_i} + P_{ii}^{1+\eta_i} \right\}^{\frac{-\eta_i}{1+\eta_i}} \tag{A.14}$$

Goods which stay in the country (A.13) are repesented differently

Rewrite $X_{i1}^{\frac{\phi-1}{\delta-1}}$ as $X_{i1}^{\frac{\eta}{\gamma}}$, $P_{i1}^{\frac{1}{1-\delta}}$ as $P_{i1}^{-\eta}$ and $P_{ii}^{\frac{1}{\delta-1}}$ as P_{ii}^{η}

$$X_{ii}^S = \left(X_{i1}^S \right)^{\frac{\eta}{\gamma}} P_{i1}^{-\eta} P_{ii}^{\eta} \left[\left(X_{i1}^S \right)^{\phi} + \left(X_{i2}^S \right)^{\phi} \right]^{\frac{\delta-\phi}{\phi(\delta-1)}}$$

Plug in the definitions for X_{i1}^S and X_{i2}^S and $\Omega^{\frac{\delta-\phi}{\phi(\delta-1)}} = \Omega^{\frac{\gamma-\eta}{\phi\gamma}}$

$$\begin{aligned} X_{ii}^S = & \left\langle L_i P_{i1}^{\gamma} \left[\left(P_{i1}^{\gamma+1} P_{i2}^{\gamma+1} \right)^{\frac{1}{1+\gamma}} \right]^{-(\gamma-\eta)} \left\{ \left[\left(P_{i1}^{\gamma+1} P_{i2}^{\gamma+1} \right)^{\frac{1}{1+\gamma}} \right]^{1+\eta} + P_{ii}^{1+\eta} \right\}^{\frac{-\eta}{1+\eta}} \right\rangle^{\frac{\eta}{\gamma}} P_{i1}^{-\eta} P_{ii}^{\eta} \\ & \cdot \left\langle L_i^{\phi} P_{i1}^{\phi\gamma} \left\{ \left[P_{i1}^{\gamma+1} P_{i2}^{\gamma+1} \right]^{\frac{1}{1+\gamma}} \right\}^{-\phi(\gamma-\eta)} \left\{ \left[\left(P_{i1}^{\gamma+1} P_{i2}^{\gamma+1} \right)^{\frac{1}{1+\gamma}} \right]^{1+\eta} + P_{ii}^{1+\eta} \right\}^{\frac{-\phi\eta}{1+\eta}} \right. \\ & \left. + L_i^{\phi} P_{i2}^{\phi\gamma} \left[\left(P_{i1}^{\gamma+1} P_{i2}^{\gamma+1} \right)^{\frac{1}{1+\gamma}} \right]^{-\phi(\gamma-\eta)} \left\{ \left[\left(P_{i1}^{\gamma+1} P_{i2}^{\gamma+1} \right)^{\frac{1}{1+\gamma}} \right]^{1+\eta} + P_{ii}^{1+\eta} \right\}^{\frac{-\phi\eta}{1+\eta}} \right\rangle^{\frac{\gamma-\eta}{\phi\gamma}} \end{aligned}$$

1. line Rewrite $P_{i1}^{-\eta} P_{ii}^{\eta}$ and solve $\frac{\eta}{\gamma}$

2./3. line Solve for $\frac{\gamma-\eta}{\phi\gamma}$ and separate $\left(P_{i1}^{\phi\gamma} + P_{i2}^{\phi\gamma} \right)^{\frac{\gamma-\eta}{\phi\gamma}}$

$$\begin{aligned} X_{ii}^S = & L_i^{\frac{\eta}{\gamma}} P_{i1}^{\eta-\eta} P_{ii}^{\eta} \left[\left(P_{i1}^{1+\gamma} + P_{i2}^{1+\gamma} \right)^{\frac{1}{1+\gamma}} \right]^{\frac{-\eta(\gamma-\eta)}{\gamma}} \left\{ \left[\left(P_{i1}^{\gamma+1} P_{i2}^{\gamma+1} \right)^{\frac{1}{1+\gamma}} \right]^{1+\eta} + P_{ii}^{1+\eta} \right\}^{\frac{-\eta^2}{\gamma(1+\eta)}} \\ & \cdot L_i^{\frac{\gamma-\eta}{\gamma}} \left[\left(P_{i1}^{1+\gamma} + P_{i2}^{1+\gamma} \right)^{\frac{1}{1+\gamma}} \right]^{\frac{-(\gamma-\eta)^2}{\gamma}} \left\{ \left[\left(P_{i1}^{\gamma+1} P_{i2}^{\gamma+1} \right)^{\frac{1}{1+\gamma}} \right]^{1+\eta} + P_{ii}^{1+\eta} \right\}^{\frac{-\eta(\gamma-\eta)}{(1+\eta)\gamma}} \\ & \cdot \left(P_{i1}^{\phi\gamma} + P_{i2}^{\phi\gamma} \right)^{\frac{\gamma-\eta}{\phi\gamma}} \end{aligned}$$

Rewrite $L_i^{\frac{\eta}{\gamma}} \cdot L_i^{\frac{\gamma-\eta}{\gamma}} = L_i$ and $P_{i1}^{\eta-\eta} = 1$

Calculate $\Omega^{\frac{-\eta(\gamma-\eta)}{\gamma}} \cdot \Omega^{\frac{-(\gamma-\eta)^2}{\gamma}} = \Omega^{-(\gamma-\eta)}$, also $\Omega^{\frac{-\eta^2}{\gamma(1+\eta)}} \cdot \Omega^{\frac{-\eta(\gamma-\eta)}{(1+\eta)\gamma}} = \Omega^{\frac{-\eta}{1+\eta}}$

Rewrite $\left(P_{i1}^{\phi\gamma} + P_{i2}^{\phi\gamma}\right)^{\frac{\gamma-\eta}{\phi\gamma}} = \left(P_{i1}^{1+\gamma} + P_{i2}^{1+\gamma}\right)^{\frac{\gamma-\eta}{1+\gamma}}$

$$X_{ii}^S = L_i P_{ii}^{\eta} \left[\left(P_{i1}^{1+\gamma} + P_{i2}^{1+\gamma}\right)^{\frac{1}{1+\gamma}}\right]^{-(\gamma_i-\eta)} \left\{\left[\left(P_{i1}^{\gamma+1} P_{i2}^{\gamma+1}\right)^{\frac{1}{1+\gamma}}\right]^{1+\eta} + P_{ii}^{1+\eta}\right\}^{\frac{-\eta}{1+\eta}}$$
$$\cdot \left(P_{i1}^{1+\gamma} + P_{i2}^{1+\gamma}\right)^{\frac{\gamma-\eta}{1+\gamma}}$$

Caclulate $\left(\Omega^{\frac{1}{1+\gamma}}\right)^{-(\gamma-\eta)} \cdot \Omega^{\frac{\gamma-\eta}{1+\gamma}} = \Omega^0 = 1$

$$X_{ii}^S = L_i P_{ii}^{\eta_i} \left\{\left[\left(P_{i1}^{\gamma+1} P_{i2}^{\gamma+1}\right)^{\frac{1}{1+\gamma}}\right]^{1+\eta} + P_{ii}^{1+\eta}\right\}^{\frac{-\eta}{1+\eta}}$$

Second part of step by step calculation of supply X_{ij}^S

Simplified with a 3-country case: *i, 1, 2*

$$\text{Profit } \pi_i = P_{i1}X_{i1} + P_{i2}X_{i2} + P_{ii}X_{ii} - W_iL_i \tag{A.15}$$

$$\text{Labor } L_i = \left\{\left[\left(X_{i1}^{\phi} + X_{i2}^{\phi}\right)^{\frac{1}{\phi}}\right]^{\delta} + X_{ii}^{\delta}\right\}^{\frac{1}{\delta}} \tag{A.16}$$

Wihout using Lagrange, substitute

$$\pi_i = P_{i1}X_{i1} + P_{i2}X_{i2} + P_{ii}X_{ii} - W_i\left\{\left[\left(X_{i1}^{\phi} + X_{i2}^{\phi}\right)^{\frac{1}{\phi}}\right]^{\delta} + X_{ii}^{\delta}\right\}^{\frac{1}{\delta}} \tag{A.17}$$

Maximize with respect to goods supplied by i to all three countries

$$\frac{\partial \pi_i}{\partial X_{i1}} = P_{i1} - W_i\frac{1}{\delta}\left[\left(X_{i1}^{\phi} + X_{i2}^{\phi}\right)^{\frac{\delta}{\phi}} + X_{ii}^{\delta}\right]^{\frac{1-\delta}{\delta}}\frac{\delta}{\phi}\left(X_{i1}^{\phi} + X_{i2}^{\phi}\right)^{\frac{\delta-\phi}{\phi}}\phi X_{i1}^{\phi-1} = 0 \tag{A.18}$$

$$\frac{\partial \pi_i}{\partial X_{i2}} = P_{i2} - W_i\frac{1}{\delta}\left[\left(X_{i1}^{\phi} + X_{i2}^{\phi}\right)^{\frac{\delta}{\phi}} + X_{ii}^{\delta}\right]^{\frac{1-\delta}{\delta}}\frac{\delta}{\phi}\left(X_{i1}^{\phi} + X_{i2}^{\phi}\right)^{\frac{\delta-\phi}{\phi}}\phi X_{i2}^{\phi-1} = 0 \tag{A.19}$$

$$\frac{\partial \pi_i}{\partial X_{ii}} = P_{ii} - W_i\frac{1}{\delta}\left[\left(X_{i1}^{\phi} + X_{i2}^{\phi}\right)^{\frac{\delta}{\phi}} + X_{ii}^{\delta}\right]^{\frac{1-\delta}{\delta}}\delta X_{ii}^{\delta-1} = 0 \tag{A.20}$$

Rewrite $(A.18)$ in detail

$$0 = P_{i1} - W_i\frac{1}{\delta}\left[\left(X_{i1}^{\phi} + X_{i2}^{\phi}\right)^{\frac{\delta}{\phi}} + X_{ii}^{\delta}\right]^{\frac{1-\delta}{\delta}}\frac{\delta}{\phi}\left(X_{i1}^{\phi} + X_{i2}^{\phi}\right)^{\frac{\delta-\phi}{\phi}}\phi X_{i1}^{\phi-1}$$

$$P_{i1} = W_i\left[\left(X_{i1}^{\phi} + X_{i2}^{\phi}\right)^{\frac{\delta}{\phi}} + X_{ii}^{\delta}\right]^{\frac{1-\delta}{\delta}}\left(X_{i1}^{\phi} + X_{i2}^{\phi}\right)^{\frac{\delta-\phi}{\phi}}X_{i1}^{\phi-1}$$

$$W_i = P_{i1}\left[\left(X_{i1}^{\phi} + X_{i2}^{\phi}\right)^{\frac{\delta}{\phi}} + X_{ii}^{\delta}\right]^{\frac{\delta-1}{\delta}}\left(X_{i1}^{\phi} + X_{i2}^{\phi}\right)^{\frac{\phi-\delta}{\phi}}X_{i1}^{1-\phi} \tag{A.21}$$

Rewrite $(A.19)$ and $(A.20)$

$$W_i = P_{i2}\left[\left(X_{i1}^{\phi} + X_{i2}^{\phi}\right)^{\frac{\delta}{\phi}} + X_{ii}^{\delta}\right]^{\frac{\delta-1}{\delta}}\left(X_{i1}^{\phi} + X_{i2}^{\phi}\right)^{\frac{\phi-\delta}{\phi}}X_{i2}^{1-\phi} \tag{A.22}$$

$$W_i = P_{ii}\left[\left(X_{i1}^{\phi} + X_{i2}^{\phi}\right)^{\frac{\delta}{\phi}} + X_{ii}^{\delta}\right]^{\frac{\delta-1}{\delta}}X_{ii}^{1-\delta} \tag{A.23}$$

Equate $(A.21) - (A.23)$

$$\begin{aligned}
&P_{i1}\left[\left(X_{i1}^{\phi} + X_{i2}^{\phi}\right)^{\frac{\delta}{\phi}} + X_{ii}^{\delta}\right]^{\frac{\delta-1}{\delta}}\left(X_{i1}^{\phi} + X_{i2}^{\phi}\right)^{\frac{\phi-\delta}{\phi}}X_{i1}^{1-\phi} \\
=&P_{i2}\left[\left(X_{i1}^{\phi} + X_{i2}^{\phi}\right)^{\frac{\delta}{\phi}} + X_{ii}^{\delta}\right]^{\frac{\delta-1}{\delta}}\left(X_{i1}^{\phi} + X_{i2}^{\phi}\right)^{\frac{\phi-\delta}{\phi}}X_{i2}^{1-\phi} \\
=&P_{ii}\left[\left(X_{i1}^{\phi} + X_{i2}^{\phi}\right)^{\frac{\delta}{\phi}} + X_{ii}^{\delta}\right]^{\frac{\delta-1}{\delta}}X_{ii}^{1-\delta}
\end{aligned}$$

Divide by $[\Omega]^{\frac{\delta-1}{\delta}}$

$$P_{i1}\left(X_{i1}^{\phi}+X_{i2}^{\phi}\right)^{\frac{\phi-\delta}{\phi}}X_{i1}^{1-\phi}=P_{i2}\left(X_{i1}^{\phi}+X_{i2}^{\phi}\right)^{\frac{\phi-\delta}{\phi}}X_{i2}^{1-\phi}=P_{ii}X_{ii}^{1-\delta}$$

$$P_{i1}X_{i1}^{1-\phi}=P_{i2}X_{i2}^{1-\phi}=\frac{P_{ii}X_{ii}^{1-\delta}}{\left(X_{i1}^{\phi}+X_{i2}^{\phi}\right)^{\frac{\phi-\delta}{\phi}}}$$

Take it to the power of -1

$$\frac{X_{i1}^{\phi-1}}{P_{i1}}=\frac{X_{i2}^{\phi-1}}{P_{i2}}=\frac{\left(X_{i1}^{\phi}+X_{i2}^{\phi}\right)^{\frac{\phi-\delta}{\phi}}X_{ii}^{\delta-1}}{P_{ii}}$$

As this is equal to $(A.11)$ of the first part, use equations from there

Solve for W_i from $(A.23)$ with first part definitions for X_{i1}, X_{i2} and X_{ii}

Multiply by ϕ in the exponent for X_{i1} and X_{i2} and by δ or $1-\delta$ for X_{ii}

$$\begin{aligned}W_i=&P_{ii}\left[\!\!\left[\left\langle L_i^{\phi}P_{i1}^{\phi\gamma}\left[\left(P_{i1}^{1+\gamma}+P_{i2}^{1+\gamma}\right)^{\frac{1}{1+\gamma}}\right]^{-(\gamma-\eta)\phi}\left\{\left[\left(P_{i1}^{1+\gamma}+P_{i2}^{1+\gamma}\right)^{\frac{1}{1+\gamma}}\right]^{1+\eta}+P_{ii}^{1+\eta}\right\}^{\frac{-\eta\phi}{1+\eta}}\right.\right.\right.\\&\left.+\,L_i^{\phi}P_{i2}^{\phi\gamma}\left[\left(P_{i1}^{1+\gamma}+P_{i2}^{1+\gamma}\right)^{\frac{1}{1+\gamma}}\right]^{-(\gamma-\eta)\phi}\left\{\left[\left(P_{i1}^{1+\gamma}+P_{i2}^{1+\gamma}\right)^{\frac{1}{1+\gamma}}\right]^{1+\eta}+P_{ii}^{1+\eta}\right\}^{\frac{-\eta\phi}{1+\eta}}\right\rangle^{\frac{\delta}{\phi}}\\&\left.\left.+\left\{L_i^{\delta}P_{ii}^{\delta\eta}\left[\left(P_{i1}^{1+\gamma}+P_{i2}^{1+\gamma}\right)^{\frac{1}{1+\gamma}}\right]^{1+\eta}+P_{ii}^{1+\eta}\right\}^{\frac{-\eta\delta}{1+\eta}}\right]\!\!\right]^{\frac{\delta-1}{\delta}}\\&\cdot L_i^{1-\delta}P_{ii}^{(1-\delta)\eta}\left\{\left[\left(P_{i1}^{1+\gamma}+P_{i2}^{1+\gamma}\right)^{\frac{1}{1+\gamma}}\right]^{1+\eta}+P_{ii}^{1+\eta}\right\}^{\frac{-\eta(1-\delta)}{1+\eta}}\end{aligned}$$

Solve $\frac{\delta}{\phi}$, $L^{\delta\frac{\delta-1}{\delta}}=L^{\delta-1}$, extend $(P_{i1}+P_{i2})^{\delta\gamma}$ to $\left(P_{i1}^{1+\gamma}+P_{i2}^{1+\gamma}\right)^{\frac{\delta\gamma}{1+\gamma}}$

$$\begin{aligned}W_i=&P_{ii}L_i^{\delta-1}\left\langle\left[\left(P_{i1}^{1+\gamma}+P_{i2}^{1+\gamma}\right)^{\frac{1}{1+\gamma}}\right]^{-\delta(\gamma-\eta)}\left\{\left[\left(P_{i1}^{1+\gamma}+P_{i2}^{1+\gamma}\right)^{\frac{1}{1+\gamma}}\right]^{1+\eta}+P_{ii}^{1+\eta}\right\}^{\frac{-\eta\delta}{1+\eta}}\right.\\&\left.\cdot\left(P_{i1}^{1+\gamma}+P_{i2}^{1+\gamma}\right)^{\frac{\delta\gamma}{1+\gamma}}+P_{ii}^{\delta\eta}\left\{\left[\left(P_{i1}^{1+\gamma}+P_{i2}^{1+\gamma}\right)^{\frac{1}{1+\gamma}}\right]^{1+\eta}+P_{ii}^{1+\eta}\right\}^{\frac{-\eta\delta}{1+\eta}}\right\rangle^{\frac{\delta-1}{\delta}}\\&\cdot L_i^{1-\delta}P_{ii}^{(1-\delta)\eta}\left\{\left[\left(P_{i1}^{1+\gamma}+P_{i2}^{1+\gamma}\right)^{\frac{1}{1+\gamma}}\right]^{1+\eta}+P_{ii}^{1+\eta}\right\}^{\frac{-\eta(1-\delta)}{1+\eta}}\end{aligned}$$

Rewrite $P_{ii}^{1} \cdot P_{ii}^{(1-\delta)\eta} = P_{ii}^{1} \cdot P_{ii}^{\frac{1-\delta}{\delta-1}} = P_{ii}^{1-1} = 1$

Rewrite $L_i^{(\delta-1)} \cdot L_i^{(1-\delta)} = L_i^{(\delta-1)+(1-\delta)} = 1$

Rewrite $\left(P_{i1}^{1+\gamma} + P_{i2}^{1+\gamma}\right)^{\frac{\delta\gamma}{1+\gamma}} = \left[\left(P_{i1}^{1+\gamma} + P_{i2}^{1+\gamma}\right)^{(1+\gamma)}\right]^{\delta\gamma}$

$$W_i = \left\langle \left[\left(P_{i1}^{1+\gamma} + P_{i2}^{1+\gamma}\right)^{\frac{1}{1+\gamma}}\right]^{-\delta(\gamma-\eta)} \left\{\left[\left(P_{i1}^{1+\gamma} + P_{i2}^{1+\gamma}\right)^{\frac{1}{1+\gamma}}\right]^{1+\eta} + P_{ii}^{1+\eta}\right\}^{\frac{-\eta\delta}{1+\eta}} \right.$$

$$\left. \cdot \left[\left(P_{i1}^{1+\gamma} + P_{i2}^{1+\gamma}\right)^{\frac{1}{1+\gamma}}\right]^{\delta\gamma} + P_{ii}^{\delta\eta} \left\{\left[\left(P_{i1}^{1+\gamma} + P_{i2}^{1+\gamma}\right)^{\frac{1}{1+\gamma}}\right]^{1+\eta} + P_{ii}^{1+\eta}\right\}^{\frac{-\eta\delta}{1+\eta}} \right\rangle^{\frac{\delta-1}{\delta}}$$

$$\cdot \left\{\left[\left(P_{i1}^{1+\gamma} + P_{i2}^{1+\gamma}\right)^{\frac{1}{1+\gamma}}\right]^{1+\eta} + P_{ii}^{1+\eta}\right\}^{\frac{-\eta(1-\delta)}{1+\eta}}$$

Summarize both $\left(P_{i1}^{1+\gamma} + P_{i2}^{1+\gamma}\right)^{\frac{1}{1+\gamma}}$

$$W_i = \left\langle \left[\left(P_{i1}^{1+\gamma} + P_{i2}^{1+\gamma}\right)^{\frac{1}{1+\gamma}}\right]^{-\delta(\gamma-\eta)+\delta\gamma} \left\{\left[\left(P_{i1}^{1+\gamma} + P_{i2}^{1+\gamma}\right)^{\frac{1}{1+\gamma}}\right]^{1+\eta} + P_{ii}^{1+\eta}\right\}^{\frac{-\eta\delta}{1+\eta}} \right.$$

$$\left. + P_{ii}^{\delta\eta} \left\{\left[\left(P_{i1}^{1+\gamma} + P_{i2}^{1+\gamma}\right)^{\frac{1}{1+\gamma}}\right]^{1+\eta} + P_{ii}^{1+\eta}\right\}^{\frac{-\eta\delta}{1+\eta}} \right\rangle^{\frac{\delta-1}{\delta}}$$

$$\cdot \left\{\left[\left(P_{i1}^{1+\gamma} + P_{i2}^{1+\gamma}\right)^{\frac{1}{1+\gamma}}\right]^{1+\eta} + P_{ii}^{1+\eta}\right\}^{\frac{-\eta(1-\delta)}{1+\eta}}$$

Rewrite $\left(\Omega^{-\eta\delta/(1+\eta)}\right)^{(\delta-1)/\delta} = \Omega^{\eta(1-\delta)/(1+\eta)}$

Separate $\left\{\left[\left(P_{i1}^{1+\gamma} + P_{i2}^{1+\gamma}\right)^{\frac{1}{1+\gamma}}\right]^{1+\eta} + P_{ii}^{1+\eta}\right\}^{\frac{\eta(1-\delta)}{1+\eta} - \frac{\eta(1-\delta)}{1+\eta}}$

$$W_i = \left\{\left[\left(P_{i1}^{1+\gamma} + P_{i2}^{1+\gamma}\right)^{\frac{1}{1+\gamma}}\right]^{1+\eta} + P_{ii}^{1+\eta}\right\}^{\frac{\eta(1-\delta)}{1+\eta} - \frac{\eta(1-\delta)}{1+\eta}}$$

$$\cdot \left\{\left[\left(P_{i1}^{1+\gamma} + P_{i2}^{1+\gamma}\right)^{\frac{1}{1+\gamma}}\right]^{-\delta(\gamma-\eta)+\delta\gamma} + P_{ii}^{\delta\eta}\right\}^{\frac{\delta-1}{\delta}}$$

Rewrite $\Omega^{\frac{\eta(1-\delta)}{1+\eta} - \frac{\eta(1-\delta)}{1+\eta}} = 1$

$$W_i = \left\{\left[\left(P_{i1}^{1+\gamma} + P_{i2}^{1+\gamma}\right)^{\frac{1}{1+\gamma}}\right]^{-\delta(\gamma-\eta)+\delta\gamma} + P_{ii}^{\delta\eta}\right\}^{\frac{\delta-1}{\delta}}$$

Rewrite $\left(P_{ii}^{\delta\eta}\right)^{\frac{\delta-1}{\delta}} = P_{ii}^{\eta(\delta-1)} = P_{ii}^{\frac{\delta-1}{\delta-1}}$, extend $\left(P_{ii}^{1+\eta}\right)^{1/(1+\eta)}$

Rewrite simultaneously for $P_{i1} + P_{i2}$

$$W_i = \left\{\left[\left(P_{i1}^{1+\gamma} + P_{i2}^{1+\gamma}\right)^{\frac{1}{1+\gamma}}\right]^{1+\eta} + P_{ii}^{1+\eta}\right\}^{\frac{1}{1+\eta}}$$

Generalize

$$W_i = \left\{ \left[\left(\sum_{k=1,k\neq i}^{N} P_{ik}^{1+\gamma} \right)^{\frac{1}{1+\gamma}} \right]^{1+\eta} + P_{ii}^{1+\eta} \right\}^{\frac{1}{1+\eta}}$$

Substitute W_i and $\frac{Y_i}{W_i}$ for L_i in X_{ij} from $(A.14)$

$$X_{ij}^S = \frac{Y_i}{W_i} P_{ij}^{\gamma} \left[\left(\sum_{k=1,k\neq i}^{N} P_{ik}^{1+\gamma} \right)^{\frac{1}{1+\gamma}} \right]^{-(\gamma-\eta)} \left\{ \left[\left(\sum_{k=1,k\neq i}^{N} P_{ik}^{1+\gamma} \right)^{\frac{1}{1+\gamma}} \right]^{1+\eta} + P_{ii}^{1+\eta} \right\}^{\frac{-\eta}{1+\eta}}$$

Rewrite $\left(\Omega^{\frac{1}{1+\eta}}\right)^{-1} \cdot \Omega^{\frac{-\eta}{1+\eta}} = \Omega^{\frac{-(1+\eta)}{1+\eta}} = \Omega^{-1}$

$$X_{ij}^S = Y_i P_{ij}^{\gamma} \left[\left(\sum_{k=1,k\neq i}^{N} P_{ik}^{1+\gamma} \right)^{\frac{1}{1+\gamma}} \right]^{-(\gamma-\eta)} \left\{ \left[\left(\sum_{k=1,k\neq i}^{N} P_{ik}^{1+\gamma} \right)^{\frac{1}{1+\gamma}} \right]^{1+\eta} + P_{ii}^{1+\eta} \right\}^{-1} \tag{A.24}$$

Substitute W_i and $\frac{Y_i}{W_i}$ for L_i in X_{ii}^S

$$X_{ii}^S = Y_i P_{ii}^{\eta} \left\{ \left[\left(\sum_{k=1,k\neq i}^{N} P_{ik}^{1+\gamma} \right)^{\frac{1}{1+\gamma}} \right]^{1+\eta} + P_{ii}^{1+\eta} \right\}^{-1}$$

Both equations can be extended by individual indexes i on each elasticity

A.2 Demand

Assume identical utility functions across countries depending on the quantity of demanded goods and their respective factors, where the demand factor $\zeta > \alpha$.

$$U_j = \left\langle \left\{ \left[\sum_{k=1,k\neq j}^{N} \left(\frac{1-M_{kj}}{M_j} \alpha_{kj} X_{kj}^{\theta} + \frac{M_{kj}}{M_j} \zeta_{kj} X_{kj}^{\theta} \right) \right]^{\frac{1}{\theta}} \right\}^{\psi} + \frac{1-M_{jj}}{M_j} \alpha_{jj} X_{jj}^{\psi} + \frac{M_{jj}}{M_j} \zeta_{jj} X_{jj}^{\psi} \right\rangle^{\frac{1}{\psi}} \quad \forall\ j = 1, ..., N$$

Maximizing utility with respect to the budget constraint

$$Y_j = \sum_{k=1}^{N} \tilde{P}_{kj} X_{kj}, \ \forall\ j = 1, ..., N$$

yields

$$\begin{aligned} X_{ij}^{D} =& Y_j \tilde{P}_{ij}^{-\sigma} \left(\frac{1-M_{ij}}{M_j} \alpha_i + \frac{M_{ij}}{M_j} \zeta_i \right)^{\sigma} \\ & \cdot \left\{ \left[\sum_{k=1,k\neq j}^{N} \tilde{P}_{kj}^{1-\sigma} \left(\frac{1-M_{kj}}{M_j} \alpha_k + \frac{M_{kj}}{M_j} \zeta_k \right)^{\sigma} \right]^{\frac{1}{1-\sigma}} \right\}^{\sigma-\mu} \\ & \cdot \left\langle \left\{ \left[\sum_{k=1,k\neq j}^{N} \tilde{P}_{kj}^{1-\sigma} \left(\frac{1-M_{kj}}{M_j} \alpha_k + \frac{M_{kj}}{M_j} \zeta_k \right)^{\sigma} \right]^{\frac{1}{1-\sigma}} \right\}^{1-\mu} \right. \\ & \left. + P_{jj}^{1-\mu} \left(\frac{1-M_{jj}}{M_j} \alpha_j + \frac{M_{jj}}{M_j} \zeta_j \right)^{\mu} \right\rangle^{-1} \quad \forall\ j = 1, ..., N \end{aligned}$$

- $\tilde{P}_{kj} = P_{kj} C_{kj} T_{kj} Z_{kj}$
- C_{kj}: transport cost factor assumed to be a correlating with distance, $C_{kj} \geq 1$
- T_{kj}: j's tariff on product from k, $T_{ik} = 1 +$ ad valorem tariff rate ≥ 1
- Z_{kj}: costs associated with gaining foreign market information about country j in k, $Z_{kj} \geq 1$ (if Z is increasing, the utility that the consumer gets decreases, because of the higher price)
- $Z_{kj} = f(z, M_{jk})$, where M_{jk} is the number of migrants in country k originated in country j and where z is a proxy for other factors influencing costs for gaining foreign market access

Step by step calculation of demand X_{ij}^D

Simplified with a 4-country case: *1, 2, 3, j*

Use Lagrange to maximize U_j with respect to the constraint Y_j

$$U_j = \Bigg\{ \Bigg[\Bigg(\frac{1-M_{1j}}{M_j}\alpha_{1j}X_{1j}^{\theta} + \frac{M_{1j}}{M_j}\zeta_{1j}X_{1j}^{\theta} + \frac{1-M_{2j}}{M_j}\alpha_{2j}X_{2j}^{\theta} + \frac{M_{2j}}{M_j}\zeta_{2j}X_{2j}^{\theta}$$
$$+ \frac{1-M_{3j}}{M_j}\alpha_{3j}X_{3j}^{\theta} + \frac{M_{3j}}{M_j}\zeta_{3j}X_{3j}^{\theta}\Bigg)^{\frac{1}{\theta}}\Bigg]^{\psi} + \frac{1-M_{jj}}{M_j}\alpha_{jj}X_{jj}^{\psi} + \frac{M_{jj}}{M_j}\zeta_{jj}X_{jj}^{\psi}\Bigg\}^{\frac{1}{\psi}}$$
$$-\lambda\left(Y_j - P_{1j}X_{1j} - P_{2j}X_{2j} - P_{3j}X_{3j} - P_{jj}X_{jj}\right)$$

Rewrite

$$U_j = \Bigg\{ \Bigg[\underbrace{\left(\frac{1-M_{1j}}{M_j}\alpha_{1j} + \frac{M_{1j}}{M_j}\zeta_{1j}\right)}_{\omega_1} X_{1j}^{\theta} + \underbrace{\left(\frac{1-M_{2j}}{M_j}\alpha_{2j} + \frac{M_{2j}}{M_j}\zeta_{2j}\right)}_{\omega_2} X_{2j}^{\theta}$$
$$+ \underbrace{\left(\frac{1-M_{3j}}{M_j}\alpha_{3j} + \frac{M_{3j}}{M_j}\zeta_{3j}\right)}_{\omega_3} X_{3j}^{\theta}\Bigg]^{\frac{\psi}{\theta}} + \underbrace{\left(\frac{1-M_{jj}}{M_j}\alpha_{jj} + \frac{M_{jj}}{M_j}\zeta_{jj}\right)}_{\omega_j} X_{jj}^{\psi}\Bigg\}^{\frac{1}{\psi}}$$
$$-\lambda(Y_j - P_{1j}X_{1j} - P_{2j}X_{2j} - P_{3j}X_{3j} - P_{jj}X_{jj}$$

$$\frac{\partial U_j}{\partial X_{1j}} = \frac{1}{\psi}\left[\left(\omega_1X_{1j}^{\theta} + \omega_2X_{2j}^{\theta} + \omega_3X_{3j}^{\theta}\right)^{\frac{\psi}{\theta}} + \omega_jX_{jj}^{\psi}\right]^{\frac{1-\psi}{\psi}}$$
$$\cdot\frac{\psi}{\theta}\left(\omega_1X_{1j}^{\theta} + \omega_2X_{2j}^{\theta} + \omega_3X_{3j}^{\theta}\right)^{\frac{\psi-\theta}{\theta}}\theta\omega_1X_{1j}^{\theta-1} + \lambda P_{1j} = 0 \tag{A.25}$$

$$\frac{\partial U_j}{\partial X_{2j}} = \frac{1}{\psi}\left[\left(\omega_1X_{1j}^{\theta} + \omega_2X_{2j}^{\theta} + \omega_3X_{3j}^{\theta}\right)^{\frac{\psi}{\theta}} + \omega_jX_{jj}^{\psi}\right]^{\frac{1-\psi}{\psi}}$$
$$\cdot\frac{\psi}{\theta}\left(\omega_1X_{1j}^{\theta} + \omega_2X_{2j}^{\theta} + \omega_3X_{3j}^{\theta}\right)^{\frac{\psi-\theta}{\theta}}\theta\omega_2X_{2j}^{\theta-1} + \lambda P_{2j} = 0 \tag{A.26}$$

$$\frac{\partial U_j}{\partial X_{3j}} = \frac{1}{\psi}\left[\left(\omega_1X_{1j}^{\theta} + \omega_2X_{2j}^{\theta} + \omega_3X_{3j}^{\theta}\right)^{\frac{\psi}{\theta}} + \omega_jX_{jj}^{\psi}\right]^{\frac{1-\psi}{\psi}}$$
$$\cdot\frac{\psi}{\theta}\left[\omega_1X_{1j}^{\theta} + \omega_2X_{2j}^{\theta} + \omega_3X_{3j}^{\theta}\right]^{\frac{\psi-\theta}{\theta}}\theta\omega_3X_{3j}^{\theta-1} + \lambda P_{3j} = 0 \tag{A.27}$$

$$\frac{\partial U_j}{\partial X_{jj}} = \frac{1}{\psi}\left[\left(\omega_1X_{1j}^{\theta} + \omega_2X_{2j}^{\theta} + \omega_3X_{3j}^{\theta}\right)^{\frac{\psi}{\theta}} + \omega_jX_{jj}^{\psi}\right]^{\frac{1-\psi}{\psi}}\psi\ \omega_jX_{jj}^{\psi-1} + \lambda P_{jj} = 0 \tag{A.28}$$

$$\frac{\partial U_j}{\partial \lambda} = Y_j - P_{1j}X_{1j} - P_{2j}X_{2j} - P_{3j}X_{3j} - P_{jj}X_{jj} = 0 \tag{A.29}$$

Rewrite $(A.25)$ in detail

$$
\begin{aligned}
-\lambda P_{1j} =& \left[\left(\omega_1 X_{1j}^{\theta} + \omega_2 X_{2j}^{\theta} + \omega_3 X_{3j}^{\theta}\right)^{\frac{\psi}{\theta}} + \omega_j X_{jj}^{\psi}\right]^{\frac{1-\psi}{\psi}} \\
& \cdot \omega_1 \left(\omega_1 X_{1j}^{\theta} + \omega_2 X_{2j}^{\theta} + \omega_3 X_{3j}^{\theta}\right)^{\frac{\psi-\theta}{\theta}} \omega_1 X_{1j}^{\theta-1} \\
\frac{\omega_1 X_{1j}^{\theta-1}}{P_{1j}} =& -\lambda \left[\left(\omega_1 X_{1j}^{\theta} + \omega_2 X_{2j}^{\theta} + \omega_3 X_{3j}^{\theta}\right)^{\frac{\psi}{\theta}} + \omega_j X_{jj}^{\psi}\right]^{\frac{\psi-1}{\psi}} \\
& \cdot \left(\omega_1 X_{1j}^{\theta} + \omega_2 X_{2j}^{\theta} + \omega_3 X_{3j}^{\theta}\right)^{\frac{\theta-\psi}{\theta}} \qquad (A.30)
\end{aligned}
$$

Rewrite $(A.26) - (A.28)$

$$
\begin{aligned}
\frac{\omega_2 X_{2j}^{\theta-1}}{P_{2j}} =& -\lambda \left[\left(\omega_1 X_{1j}^{\theta} + \omega_2 X_{2j}^{\theta} + \omega_3 X_{3j}^{\theta}\right)^{\frac{\psi}{\theta}} + \omega_j X_{jj}^{\psi}\right]^{\frac{\psi-1}{\psi}} \\
& \cdot \left(\omega_1 X_{1j}^{\theta} + \omega_2 X_{2j}^{\theta} + \omega_3 X_{3j}^{\theta}\right)^{\frac{\theta-\psi}{\theta}} \qquad (A.31) \\
\frac{\omega_3 X_{3j}^{\theta-1}}{P_{3j}} =& -\lambda \left[\left(\omega_1 X_{1j}^{\theta} + \omega_2 X_{2j}^{\theta} + \omega_3 X_{3j}^{\theta}\right)^{\frac{\psi}{\theta}} + \omega_j X_{jj}^{\psi}\right]^{\frac{\psi-1}{\psi}} \\
& \cdot \left(\omega_1 X_{1j}^{\theta} + \omega_2 X_{2j}^{\theta} + \omega_3 X_{3j}^{\theta}\right)^{\frac{\theta-\psi}{\theta}} \qquad (A.32) \\
\frac{\omega_j X_{jj}^{\psi-1}}{P_{jj}} =& -\lambda \left[\left(\omega_1 X_{1j}^{\theta} + \omega_2 X_{2j}^{\theta} + \omega_3 X_{3j}^{\theta}\right)^{\frac{\psi}{\theta}} + \omega_j X_{jj}^{\psi}\right]^{\frac{\psi-1}{\psi}} \qquad (A.33)
\end{aligned}
$$

Summarize $(A.30) - (A.33)$

$$
\frac{\omega_1 X_{1j}^{\theta-1}}{P_{1j}} = \frac{\omega_2 X_{2j}^{\theta-1}}{P_{2j}} = \frac{\omega_3 X_{3j}^{\theta-1}}{P_{3j}} = \frac{\left(\omega_1 X_{1j}^{\theta} + \omega_2 X_{2j}^{\theta} + \omega_3 X_{3j}^{\theta}\right)^{\frac{\theta-\psi}{\theta}} \omega_j X_{jj}^{\psi-1}}{P_{jj}} \qquad (A.34)
$$

Rewrite $(A.34)$

$$
\begin{aligned}
X_{2j}^{\theta-1} =& \omega_1 \omega_2^{-1} X_{1j}^{\theta-1} P_{1j}^{-1} P_{2j} \\
X_{2j} =& \omega_1^{\frac{1}{\theta-1}} \omega_2^{\frac{1}{1-\theta}} X_{1j} P_{1j}^{\frac{1}{1-\theta}} P_{2j}^{\frac{1}{\theta-1}} \qquad (A.35) \\
X_{3j} =& \omega_1^{\frac{1}{\theta-1}} \omega_3^{\frac{1}{1-\theta}} X_{1j} P_{1j}^{\frac{1}{1-\theta}} P_{3j}^{\frac{1}{\theta-1}} \qquad (A.36) \\
X_{jj} =& \left(\omega_1 X_{1j}^{\theta} + \omega_2 X_{2j}^{\theta} + \omega_3 X_{3j}^{\theta}\right)^{\frac{\psi-\theta}{\theta(\psi-1)}} \omega_1^{\frac{1}{\psi-1}} \omega_j^{\frac{1}{1-\psi}} X_{1j}^{\frac{\theta-1}{\psi-1}} P_{1j}^{\frac{1}{1-\psi}} P_{jj}^{\frac{1}{\psi-1}} \qquad (A.37)
\end{aligned}
$$

Since $-\infty < \psi < \theta < 1 \Rightarrow 0 < \mu < \sigma < \infty$

$$\sigma = \frac{1}{1-\theta}, -\sigma = \frac{1}{\theta-1}, \mu = \frac{1}{1-\psi}, -\mu = \frac{1}{\psi-1}, \theta = \frac{\sigma-1}{\sigma}$$

$$Y_j = P_{1j}X_{1j} + P_{2j}X_{2j} + P_{3j}X_{3j} + P_{jj}X_{jj}$$

Plug $(A.35)-(A.37)$ into budget constraint

Substitute exponents, $\Omega^{\frac{\psi-\theta}{\theta(\psi-1)}} = \Omega^{\frac{\psi-\theta}{\theta}\cdot(-\mu)} = \Omega^{(\frac{\psi}{\theta}-1)\cdot(-\mu)}$

$$\begin{aligned} Y_j =& P_{1j}X_{1j} + P_{2j}\left(\omega_1^{\frac{1}{\theta-1}}\omega_2^{\frac{1}{1-\theta}}X_{1j}P_{1j}^{\frac{1}{1-\theta}}P_{2j}^{\frac{1}{\theta-1}}\right) + P_{3j}\left(\omega_1^{\frac{1}{\theta-1}}\omega_3^{\frac{1}{1-\theta}}X_{1j}P_{1j}^{\frac{1}{1-\theta}}P_{3j}^{\frac{1}{\theta-1}}\right) \\ &+ P_{jj}\left[\left(\omega_1X_{1j}^{\theta} + \omega_2X_{2j}^{\theta} + \omega_3X_{3j}^{\theta}\right)^{\frac{\psi-\theta}{\theta(\psi-1)}}\omega_1^{\frac{1}{\psi-1}}\omega_j^{\frac{1}{1-\psi}}X_{1j}^{\frac{\theta-1}{\psi-1}}P_{1j}^{\frac{1}{1-\psi}}P_{jj}^{\frac{1}{\psi-1}}\right] \end{aligned}$$

Separate X_{1j}, extend P_{1j} to $P_{1j}^{\sigma+1-\sigma}$

$$\begin{aligned} Y_j =& X_{1j}\left(P_{1j}^{\sigma+1-\sigma} + P_{2j}\omega_1^{\frac{1}{\theta-1}}\omega_2^{\frac{1}{1-\theta}}P_{1j}^{\frac{1}{1-\theta}}P_{2j}^{\frac{1}{\theta-1}} + P_{3j}\omega_1^{\frac{1}{\theta-1}}\omega_3^{\frac{1}{1-\theta}}P_{1j}^{\frac{1}{1-\theta}}P_{3j}^{\frac{1}{\theta-1}}\right) \\ &+ P_{jj}\left(\omega_1X_{1j}^{\theta} + \omega_2X_{2j}^{\theta} + \omega_3X_{3j}^{\theta}\right)^{\frac{\psi-\theta}{\theta(\psi-1)}}\omega_1^{\frac{1}{\psi-1}}\omega_j^{\frac{1}{1-\psi}}X_{1j}^{\frac{\theta-1}{\psi-1}}P_{1j}^{\frac{1}{1-\psi}}P_{jj}^{\frac{1}{\psi-1}} \end{aligned}$$

Rewrite exponents and $X_{1j}^{\frac{\theta-1}{\psi-1}} = X_{1j}^{\frac{\mu}{\sigma}}$

Rewrite $\Omega^{\frac{\psi-\theta}{\theta(\psi-1)}} = \Omega^{(\frac{\psi}{\theta}-1)\cdot(-\mu)} = \Omega^{\frac{\mu(\theta-\psi)}{\theta}}$

$$\begin{aligned} Y_j =& X_{1j}\left(P_{1j}^{\sigma+1-\sigma} + P_{2j}\omega_1^{-\sigma}\omega_2^{\sigma}P_{1j}^{\sigma}P_{2j}^{-\sigma} + P_{3j}\omega_1^{-\sigma}\omega_3^{\sigma}P_{1j}^{\sigma}P_{3j}^{-\sigma}\right) \\ &+ P_{jj}\left(\omega_1X_{1j}^{\theta} + \omega_2X_{2j}^{\theta} + \omega_3X_{3j}^{\theta}\right)^{\frac{\mu(\theta-\psi)}{\theta}}\omega_1^{-\mu}\omega_j^{\mu}X_{1j}^{\frac{\mu}{\sigma}}P_{1j}^{\mu}P_{jj}^{-\mu} \end{aligned}$$

Summarize exponents

$$\begin{aligned} Y_j =& X_{1j}\left(P_{1j}^{\sigma+1-\sigma} + P_{1j}^{\sigma}P_{2j}^{1-\sigma}\omega_1^{-\sigma}\omega_2^{\sigma} + P_{1j}^{\sigma}P_{3j}^{1-\sigma}\omega_1^{-\sigma}\omega_3^{\sigma}\right) \\ &+ P_{1j}^{\mu}P_{jj}^{1-\mu}\omega_1^{-\mu}\omega_j^{\mu}X_{1j}^{\frac{\mu}{\sigma}}\left(\omega_1X_{1j}^{\theta} + \omega_2X_{2j}^{\theta} + \omega_3X_{3j}^{\theta}\right)^{\frac{\mu(\theta-\psi)}{\theta}} \end{aligned}$$

Substitute X_{2j} and X_{3j} with $(A.35)$ and $(A.36)$

$$
\begin{aligned}
Y_j =& X_{1j}\left(P_{1j}^{\sigma+1-\sigma} + P_{1j}^{\sigma}P_{2j}^{1-\sigma}\omega_1^{-\sigma}\omega_2^{\sigma} + P_{1j}^{\sigma}P_{3j}^{1-\sigma}\omega_1^{-\sigma}\omega_3^{\sigma}\right)\\
&+P_{1j}^{\mu}P_{jj}^{1-\mu}\omega_1^{-\mu}\omega_j^{\mu}X_{1j}^{\frac{\mu}{\sigma}}\left[\omega_1 X_{1j}^{\theta} + \omega_2\left(\omega_1^{\frac{1}{\theta-1}}\omega_2^{\frac{1}{1-\theta}}X_{1j}P_{1j}^{\frac{1}{1-\theta}}P_{2j}^{\frac{1}{\theta-1}}\right)^{\theta}\right.\\
&\left.+\,\omega_3\left(\omega_1^{\frac{1}{\theta-1}}\omega_3^{\frac{1}{1-\theta}}X_{1j}P_{1j}^{\frac{1}{1-\theta}}P_{3j}^{\frac{1}{\theta-1}}\right)^{\theta}\right]^{\frac{\mu(\theta-\psi)}{\theta}}
\end{aligned}
$$

Separate P_{1j}^{σ} and $X_{1j}^{\mu(\theta-\psi)}$ and rewrite $\Omega^{\frac{\mu(\theta-\psi)}{\theta}} = \Omega^{\frac{\sigma-\mu}{\sigma-1}}$

$$
\begin{aligned}
Y_j =& X_{1j}P_{1j}^{\sigma}\left(P_{1j}^{1-\sigma} + P_{2j}^{1-\sigma}\omega_1^{-\sigma}\omega_2^{\sigma} + P_{3j}^{1-\sigma}\omega_1^{-\sigma}\omega_3^{\sigma}\right)\\
&+P_{1j}^{\mu}P_{jj}^{1-\mu}\omega_1^{-\mu}\omega_j^{\mu}X_{1j}^{\frac{\mu}{\sigma}}X_{1j}^{\mu(\theta-\psi)}\\
&\cdot\left(\omega_1 + \omega_2\omega_1^{\frac{\theta}{\theta-1}}\omega_2^{\frac{\theta}{1-\theta}}P_{1j}^{\frac{\theta}{1-\theta}}P_{2j}^{\frac{\theta}{\theta-1}} + \omega_3\omega_1^{\frac{\theta}{\theta-1}}\omega_3^{\frac{\theta}{1-\theta}}P_{1j}^{\frac{\theta}{1-\theta}}P_{3j}^{\frac{\theta}{\theta-1}}\right)^{\frac{\sigma-\mu}{\sigma-1}}
\end{aligned}
$$

Rewrite exponents

$$
\begin{aligned}
Y_j =& X_{1j}P_{1j}^{\sigma}\left(P_{1j}^{1-\sigma} + P_{2j}^{1-\sigma}\omega_1^{-\sigma}\omega_2^{\sigma} + P_{3j}^{1-\sigma}\omega_1^{-\sigma}\omega_3^{\sigma}\right)\\
&+P_{1j}^{\mu}P_{jj}^{1-\mu}\omega_1^{-\mu}\omega_j^{\mu}X_{1j}^{\frac{\mu}{\sigma}}X_{1j}^{\mu(\theta-\psi)}\\
&\cdot\left(\omega_1 + \omega_2\omega_1^{1-\sigma}\omega_2^{\sigma-1}P_{1j}^{\sigma-1}P_{2j}^{1-\sigma} + \omega_3\omega_1^{1-\sigma}\omega_3^{\sigma-1}P_{1j}^{\sigma-1}P_{3j}^{1-\sigma}\right)^{\frac{\sigma-\mu}{\sigma-1}}
\end{aligned}
$$

Rewrite $X_{1j}^{\frac{\mu}{\sigma}}X_{1j}^{\mu(\theta-\psi)} = X_{1j}$, extend 1 to $\omega_1^{-\sigma}\omega_1^{\sigma}$ and 1 to $\omega_1^{1-\sigma}\omega_1^{\sigma-1}P_{1j}^{\sigma-1}P_{1j}^{1-\sigma}$

$$
\begin{aligned}
Y_j =& X_{1j}P_{1j}^{\sigma}\left(P_{1j}^{1-\sigma}\omega_1^{-\sigma}\omega_1^{\sigma} + P_{2j}^{1-\sigma}\omega_1^{-\sigma}\omega_2^{\sigma} + P_{3j}^{1-\sigma}\omega_1^{-\sigma}\omega_3^{\sigma}\right) + P_{1j}^{\mu}P_{jj}^{1-\mu}\omega_1^{-\mu}\omega_j^{\mu}X_{1j}\\
&\cdot\left(\omega_1\omega_1^{1-\sigma}\omega_1^{\sigma-1}P_{1j}^{\sigma-1}P_{1j}^{1-\sigma} + \omega_2\omega_1^{1-\sigma}\omega_2^{\sigma-1}P_{1j}^{\sigma-1}P_{2j}^{1-\sigma} + \omega_3\omega_1^{1-\sigma}\omega_3^{\sigma-1}P_{1j}^{\sigma-1}P_{3j}^{1-\sigma}\right)^{\frac{\sigma-\mu}{\sigma-1}}
\end{aligned}
$$

Separate X_{1j} and $\omega_1^{-\sigma}$ (1st line), separate $P_{1j}^{\mu}(P_{1j}^{\sigma-1})^{\frac{\sigma-\mu}{\sigma-1}} = P_{1j}^{\sigma}$

Rewrite $\omega_1^{-\mu}\left(\omega_1^{1-\sigma}\right)^{\frac{\sigma-\mu}{\sigma-1}} = \omega_1^{-\sigma}$ (2nd line)

$$
\begin{aligned}
Y_j =& X_{1j}\left[P_{1j}^{\sigma}\omega_1^{-\sigma}\left(P_{1j}^{1-\sigma}\omega_1^{\sigma} + P_{2j}^{1-\sigma}\omega_2^{\sigma} + P_{3j}^{1-\sigma}\omega_3^{\sigma}\right)\right.\\
&\left.+\,P_{jj}^{1-\mu}\omega_1^{-\sigma}\omega_j^{\mu}P_{1j}^{\sigma}\left(\omega_1\omega_1^{\sigma-1}P_{1j}^{1-\sigma} + \omega_2\omega_2^{\sigma-1}P_{2j}^{1-\sigma} + \omega_3\omega_3^{\sigma-1}P_{3j}^{1-\sigma}\right)^{\frac{\sigma-\mu}{\sigma-1}}\right]
\end{aligned}
$$

Summarize

$$Y_j = X_{1j}\left[P_{1j}^{\sigma}\omega_1^{-\sigma}\left(\sum_{k=1}^{3}P_{kj}^{1-\sigma}\omega_k^{\sigma}\right) + P_{jj}^{1-\mu}\omega_j^{\mu}P_{1j}^{\sigma}\omega_1^{-\sigma}\left(\sum_{k=1}^{3}P_{kj}^{1-\sigma}\omega_k\right)^{\frac{\sigma-\mu}{\sigma-1}}\right]$$

Divide by $[\Omega]$

$$X_{1j} = Y_j\left[P_{1j}^{\sigma}\omega_1^{-\sigma}\left(\sum_{k=1}^{3}P_{kj}^{1-\sigma}\omega_k^{\sigma}\right) + P_{jj}^{1-\mu}\omega_j^{\mu}P_{1j}^{\sigma}\omega_1^{-\sigma}\left(\sum_{k=1}^{3}P_{kj}^{1-\sigma}\omega_k\right)^{\frac{\sigma-\mu}{\sigma-1}}\right]^{-1}$$

Separate $P_{1j}^{-\sigma}\omega_1^{\sigma}$

$$X_{1j} = Y_jP_{1j}^{-\sigma}\omega_1^{\sigma}\left[\left(\sum_{k=1}^{3}P_{kj}^{1-\sigma}\omega_k^{\sigma}\right) + P_{jj}^{1-\mu}\omega_j^{\mu}\left(\sum_{k=1}^{3}P_{kj}^{1-\sigma}\omega_k\right)^{\frac{\sigma-\mu}{\sigma-1}}\right]^{-1}$$

Rewrite

$$X_{1j} = Y_jP_{1j}^{-\sigma}\omega_1^{\sigma}\left(\sum_{k=1}^{3}P_{kj}^{1-\sigma}\omega_k^{\sigma}\right)^{-\frac{\sigma-\mu}{\sigma-1}}\left[\left(\sum_{k=1}^{3}P_{kj}^{1-\sigma}\omega_k^{\sigma}\right)^{1-\frac{\sigma-\mu}{\sigma-1}} + P_{jj}^{1-\mu}\omega_j^{\mu}\right]^{-1}$$

Rewrite and generalize

$$X_{ij}^{D} = Y_jP_{ij}^{-\sigma}\omega_i^{\sigma}\left[\left(\sum_{k=1,k\neq j}^{N}P_{kj}^{1-\sigma}\omega_k^{\sigma}\right)^{\frac{1}{1-\sigma}}\right]^{\sigma-\mu}\left\{\left[\left(\sum_{k=1,k\neq j}^{N}P_{kj}^{1-\sigma}\omega_k^{\sigma}\right)^{\frac{1}{1-\sigma}}\right]^{1-\mu} + P_{jj}^{1-\mu}\omega_j^{\mu}\right\}^{-1}$$

The same as Bergstrand's 1985 representation if ω converges to 1

$$X_{ij}^{D} = Y_jP_{ij}^{-\sigma}\left(\frac{1-M_{ij}}{M_j}\alpha_{ij} + \frac{M_{ij}}{M_j}\zeta_{ij}\right)^{\sigma}\left\{\left[\sum_{k=1,k\neq j}^{N}P_{kj}^{1-\sigma}\left(\frac{1-M_{kj}}{M_j}\alpha_{kj} + \frac{M_{kj}}{M_j}\zeta_{kj}\right)^{\sigma}\right]^{\frac{1}{1-\sigma}}\right\}^{\sigma-\mu}$$
$$\cdot\left\langle\left\{\left[\sum_{k=1,k\neq j}^{N}P_{kj}^{1-\sigma}\left(\frac{1-M_{kj}}{M_j}\alpha_{kj} + \frac{M_{kj}}{M_j}\zeta_{kj}\right)^{\sigma}\right]^{\frac{1}{1-\sigma}}\right\}^{1-\mu}\right.$$
$$\left. + P_{jj}^{1-\mu}\left(\frac{1-M_{jj}}{M_j}\alpha_{jj} + \frac{M_{jj}}{M_j}\zeta_{jj}\right)^{\mu}\right\rangle^{-1} \quad \text{(A.38)}$$

Rewrite $(A.37)$

$$X_{jj} = \left(\omega_1 X_{1j}^{\theta} + \omega_2 X_{2j}^{\theta} + \omega_3 X_{3j}^{\theta}\right)^{\frac{\psi-\theta}{\theta(\psi-1)}} \omega_1^{\frac{1}{\psi-1}} \omega_j^{\frac{1}{1-\psi}} X_{1j}^{\frac{\theta-1}{\psi-1}} P_{1j}^{\frac{1}{1-\psi}} P_{jj}^{\frac{1}{\psi-1}}$$

Substitute X_{2j} with (11) and X_{3j} with (12)

$$X_{jj} = \left[\omega_1 X_{1j}^{\theta} + \omega_2 \left(\omega_1^{\frac{1}{\theta-1}} \omega_2^{\frac{1}{1-\theta}} X_{1j} P_{1j}^{\frac{1}{1-\theta}} P_{2j}^{\frac{1}{\theta-1}}\right)^{\theta} + \omega_3 \left(\omega_1^{\frac{1}{\theta-1}} \omega_3^{\frac{1}{1-\theta}} X_{1j} P_{1j}^{\frac{1}{1-\theta}} P_{3j}^{\frac{1}{\theta-1}}\right)^{\theta}\right]^{\frac{\psi-\theta}{\theta(\psi-1)}}$$
$$\cdot\, \omega_1^{\frac{1}{\psi-1}} \omega_j^{\frac{1}{1-\psi}} X_{1j}^{\frac{\theta-1}{\psi-1}} P_{1j}^{\frac{1}{1-\psi}} P_{jj}^{\frac{1}{\psi-1}}$$

Rewrite and separate $(X_{1j}^{\theta})^{\frac{\psi-\theta}{\theta(\psi-1)}} X_{1j}^{\frac{\theta-1}{\psi-1}} = X_{1j}$

$$X_{jj} = X_{1j} \left(\omega_1 + \omega_2 \omega_1^{\frac{\theta}{\theta-1}} \omega_2^{\frac{\theta}{1-\theta}} P_{1j}^{\frac{\theta}{1-\theta}} P_{2j}^{\frac{\theta}{\theta-1}} + \omega_3 \omega_1^{\frac{\theta}{\theta-1}} \omega_3^{\frac{\theta}{1-\theta}} P_{1j}^{\frac{\theta}{1-\theta}} P_{3j}^{\frac{\theta}{\theta-1}}\right)^{\frac{\psi-\theta}{\theta(\psi-1)}} \omega_1^{\frac{1}{\psi-1}} \omega_j^{\frac{1}{1-\psi}} P_{1j}^{\frac{1}{1-\psi}} P_{jj}^{\frac{1}{\psi-1}}$$

Substitute exponents, extend 1 to $\omega_1^{1-\sigma} \omega_1^{-(1-\sigma)} P_{1j}^{-(1-\sigma)} P_{1j}^{1-\sigma}$

$$X_{jj} = X_{1j} \left(\omega_1 \omega_1^{1-\sigma} \omega_1^{-(1-\sigma)} P_{1j}^{-(1-\sigma)} P_{1j}^{1-\sigma} + \omega_2 \omega_1^{1-\sigma} \omega_2^{-(1-\sigma)} P_{1j}^{-(1-\sigma)} P_{2j}^{1-\sigma}\right.$$
$$\left. + \omega_3 \omega_1^{1-\sigma} \omega_3^{-(1-\sigma)} P_{1j}^{-(1-\sigma)} P_{3j}^{1-\sigma}\right)^{\frac{\psi-\theta}{\theta(\psi-1)}} \omega_1^{\frac{1}{\psi-1}} \omega_j^{\frac{1}{1-\psi}} P_{1j}^{\frac{1}{1-\psi}} P_{jj}^{\frac{1}{\psi-1}}$$

Separate $\omega_1^{1-\sigma} P_{1j}^{-(1-\sigma)}$, summarize exponents

$$X_{jj} = X_{1j} \omega_1^{\frac{(1-\sigma)(\psi-\theta)}{\theta(\psi-1)}} P_{1j}^{\frac{-(1-\sigma)(\psi-\theta)}{\theta(\psi-1)}} \left(\omega_1^{\sigma} P_{1j}^{1-\sigma} + \omega_2^{\sigma} P_{2j}^{1-\sigma} + \omega_3^{\sigma} P_{3j}^{1-\sigma}\right)^{\frac{\psi-\theta}{\theta(\psi-1)}} \omega_1^{\frac{1}{\psi-1}} \omega_j^{\frac{1}{1-\psi}} P_{1j}^{\frac{1}{1-\psi}} P_{jj}^{\frac{1}{\psi-1}}$$

Substitute X_{1j} with $(A.38)$ or one line above

$$X_{jj} = Y_j P_{1j}^{-\sigma} \omega_1^{\sigma} \left[\left(\sum_{k=1,k\neq j}^{3} P_{kj}^{1-\sigma} \omega_k^{\sigma}\right)^{\frac{1}{1-\sigma}}\right]^{\sigma-\mu} \left\{\left[\left(\sum_{k=1,k\neq j}^{3} P_{kj}^{1-\sigma} \omega_k^{\sigma}\right)^{\frac{1}{1-\sigma}}\right]^{1-\mu} + P_{jj}^{1-\mu} \omega_j^{\mu}\right\}^{-1}$$
$$\cdot \omega_1^{\frac{(1-\sigma)(\psi-\theta)}{\theta(\psi-1)}} P_{1j}^{\frac{-(1-\sigma)(\psi-\theta)}{\theta(\psi-1)}} \left(\omega_1^{\sigma} P_{1j}^{1-\sigma} + \omega_2^{\sigma} P_{2j}^{1-\sigma} + \omega_3^{\sigma} P_{3j}^{1-\sigma}\right)^{\frac{\psi-\theta}{\theta(\psi-1)}} \omega_1^{\frac{1}{\psi-1}} \omega_j^{\frac{1}{1-\psi}} P_{1j}^{\frac{1}{1-\psi}} P_{jj}^{\frac{1}{\psi-1}}$$

Summarize exponents of P_{1j} and ω_1, summarize $(\Omega^{\frac{1}{1-\sigma}})^{\sigma-\mu} \Omega^{\frac{\psi-\theta}{\theta(\psi-1)}} = \Omega^{\frac{\sigma-\mu}{1-\sigma} + \frac{\psi-\theta}{\theta(\psi-1)}}$

$$X_{jj} = Y_j P_{1j}^{\mu-\sigma+\frac{(1-\sigma)(\theta-\psi)}{\theta(\psi-1)}} \omega_1^{\sigma-\mu+\frac{(1-\sigma)(\psi-\theta)}{\theta(\psi-1)}} \left(\sum_{k=1,k\neq j}^{3} P_{kj}^{1-\sigma} \omega_k^{\sigma}\right)^{\frac{\sigma-\mu}{1-\sigma} + \frac{\psi-\theta}{\theta(\psi-1)}}$$
$$\cdot \left\{\left[\left(\sum_{k=1,k\neq j}^{3} P_{kj}^{1-\sigma} \omega_k^{\sigma}\right)^{\frac{1}{1-\sigma}}\right]^{1-\mu} + P_{jj}^{1-\mu} \omega_j^{\mu}\right\}^{-1} \omega_j^{\mu} P_{jj}^{-\mu}$$

$$
\begin{aligned}
&\text{Rewrite } P_{1j}^{\mu-\sigma+\frac{(1-\sigma)(\theta-\psi)}{\theta(\psi-1)}} = P_{1j}^{\frac{\mu-\sigma\mu-\sigma+\sigma^2+\sigma-\mu-\sigma^2+\sigma\mu}{1-\sigma}} = P_{1j}^{0} = 1 \\
&\text{Rewrite } \omega_1^{\sigma-\mu+\frac{(1-\sigma)(\psi-\theta)}{\theta(\psi-1)}} = \omega_1^0 = 1 \text{ and } \Omega^{\frac{\sigma-\mu}{1-\sigma}+\frac{\psi-\theta}{\theta(\psi-1)}} = \Omega^{\frac{\sigma-\mu}{1-\sigma}+\frac{\mu-\sigma}{1-\sigma}} = \Omega^0 \\
X_{jj} =& Y_j \left\{ \left[\left(\sum_{k=1,k\neq j}^{3} P_{kj}^{1-\sigma}\omega_k^{\sigma} \right)^{\frac{1}{1-\sigma}} \right]^{1-\mu} + P_{jj}^{1-\mu}\omega_j^{\mu} \right\}^{-1} \omega_j^{\mu} P_{jj}^{-\mu} \\
&\text{Generalize}
\end{aligned}
$$

$$
X_{jj}^{D} = Y_j P_{jj}^{-\mu}\omega_j^{\mu} \left[\left(\sum_{k=1,k\neq j}^{N} P_{kj}^{1-\sigma}\omega_k^{\sigma} \right)^{\frac{1-\mu}{1-\sigma}} + P_{jj}^{1-\mu}\omega_j^{\mu} \right]^{-1}
$$

The same as Bergstrand's 1985 representation if ω converges to 1

$$
\begin{aligned}
X_{jj}^{D} =& Y_j P_{jj}^{-\mu} \left(\frac{1-M_{jj}}{M_j}\alpha_{jj} + \frac{M_{jj}}{M_j}\zeta_{jj} \right)^{\mu} \\
&\cdot \left\{ \left[\sum_{k=1,k\neq j}^{N} P_{kj}^{1-\sigma} \left(\frac{1-M_{kj}}{M_j}\alpha_{kj} + \frac{M_{kj}}{M_j}\zeta_{kj} \right)^{\sigma} \right]^{\frac{1-\mu}{1-\sigma}} \right. \\
&\left. + P_{jj}^{1-\mu} \left(\frac{1-M_{jj}}{M_j}\alpha_{jj} + \frac{M_{jj}}{M_j}\zeta_{jj} \right)^{\mu} \right\}^{-1} \qquad \text{(A.39)}
\end{aligned}
$$

A.3 Equilibrium

Step by step calculation for the equilibrium X_{ij}

Supply $(A.24)$

$$X_{ij}^S = Y_i P_{ij}^{\gamma} \left[\left(\sum_{k=1,k\neq i}^{N} P_{ik}^{1+\gamma} \right)^{\frac{1}{1+\gamma}} \right]^{-(\gamma-\eta)} \left\{ \left[\left(\sum_{k=1,k\neq i}^{N} P_{ik}^{1+\gamma} \right)^{\frac{1}{1+\gamma}} \right]^{1+\eta} + P_{ii}^{1+\eta} \right\}^{-1}$$

Demand $(A.38)$

$$X_{ij}^D = Y_j P_{ij}^{-\sigma} \omega_i^{\sigma} \left[\left(\sum_{k=1,k\neq j}^{N} P_{kj}^{1-\sigma} \omega_k^{\sigma} \right)^{\frac{1}{1-\sigma}} \right]^{\sigma-\mu} \left\{ \left[\left(\sum_{k=1,k\neq j}^{N} P_{kj}^{1-\sigma} \omega_k^{\sigma} \right)^{\frac{1}{1-\sigma}} \right]^{1-\mu} + P_{jj}^{1-\mu} \omega_j^{\mu} \right\}^{-1}$$

Equate

$$X_{ij} = X_{ij}^S = X_{ij}^D$$

It does not make a difference whether C, T and Z are substituted in supply or demand

$$
\begin{aligned}
&Y_i P_{ij}^{\gamma}\left[\left(\sum_{k=1,k\neq i}^{N} P_{ik}^{1+\gamma}\right)^{\frac{1}{1+\gamma}}\right]^{-(\gamma-\eta)}\left\{\left[\left(\sum_{k=1,k\neq i}^{N} P_{ik}^{1+\gamma}\right)^{\frac{1}{1+\gamma}}\right]^{1+\eta}+P_{ii}^{1+\eta}\right\}^{-1}\\
=&Y_j P_{ij}^{-\sigma}\omega_i^{\sigma}\left[\left(\sum_{k=1,k\neq j}^{N} P_{kj}^{1-\sigma}\omega_k^{\sigma}\right)^{\frac{1}{1-\sigma}}\right]^{\sigma-\mu}\left\{\left[\left(\sum_{k=1,k\neq j}^{N} P_{kj}^{1-\sigma}\omega_k^{\sigma}\right)^{\frac{1}{1-\sigma}}\right]^{1-\mu}+P_{jj}^{1-\mu}\omega_j^{\mu}\right\}^{-1}
\end{aligned}
$$

Set $P_{ij}^{D}=P_{ij}C_{ij}T_{ij}Z_{ij}$

$$
\begin{aligned}
&Y_i P_{ij}^{\gamma}\left[\left(\sum_{k=1,k\neq i}^{N} P_{ik}^{1+\gamma}\right)^{\frac{1}{1+\gamma}}\right]^{-(\gamma-\eta)}\left\{\left[\left(\sum_{k=1,k\neq i}^{N} P_{ik}^{1+\gamma}\right)^{\frac{1}{1+\gamma}}\right]^{1+\eta}+P_{ii}^{1+\eta}\right\}^{-1}\\
=&Y_j\left(P_{ij}C_{ij}T_{ij}Z_{ij}\right)^{-\sigma}\omega_i^{\sigma}\\
&\cdot\left[\left(\sum_{k=1,k\neq j}^{N} P_{kj}^{1-\sigma}\omega_k^{\sigma}\right)^{\frac{1}{1-\sigma}}\right]^{\sigma-\mu}\left\{\left[\left(\sum_{k=1,k\neq j}^{N} P_{kj}^{1-\sigma}\omega_k^{\sigma}\right)^{\frac{1}{1-\sigma}}\right]^{1-\mu}+P_{jj}^{1-\mu}\omega_j^{\mu}\right\}^{-1}
\end{aligned}
$$

Rewrite

$$
\begin{aligned}
P_{ij}^{\gamma+\sigma}=&Y_i^{-1}Y_jC_{ij}^{-\sigma}T_{ij}^{-\sigma}Z_{ij}^{-\sigma}\omega_i^{\sigma}\left[\left(\sum_{k=1,k\neq i}^{N} P_{ik}^{1+\gamma}\right)^{\frac{1}{1+\gamma}}\right]^{\gamma-\eta}\left[\left(\sum_{k=1,k\neq j}^{N} P_{kj}^{1-\sigma}\omega_k^{\sigma}\right)^{\frac{1}{1-\sigma}}\right]^{\sigma-\mu}\\
&\cdot\left\{\left[\left(\sum_{k=1,k\neq i}^{N} P_{ik}^{1+\gamma}\right)^{\frac{1}{1+\gamma}}\right]^{1+\eta}+P_{ii}^{1+\eta}\right\}\left\{\left[\left(\sum_{k=1,k\neq j}^{N} P_{kj}^{1-\sigma}\omega_k^{\sigma}\right)^{\frac{1}{1-\sigma}}\right]^{1-\mu}+P_{jj}^{1-\mu}\omega_j^{\mu}\right\}^{-1}
\end{aligned}
$$

Take it to the power of $\frac{1}{\gamma+\sigma}$

$$
\begin{aligned}
P_{ij}=&Y_i^{\frac{-1}{\gamma+\sigma}}Y_j^{\frac{1}{\gamma+\sigma}}C_{ij}^{\frac{-\sigma}{\gamma+\sigma}}T_{ij}^{\frac{-\sigma}{\gamma+\sigma}}Z_{ij}^{\frac{-\sigma}{\gamma+\sigma}}\omega_i^{\frac{\sigma}{\gamma+\sigma}}\\
&\cdot\left[\left(\sum_{k=1,k\neq i}^{N} P_{ik}^{1+\gamma}\right)^{\frac{1}{1+\gamma}}\right]^{\frac{\gamma-\eta}{\gamma+\sigma}}\left[\left(\sum_{k=1,k\neq j}^{N} P_{kj}^{1-\sigma}\omega_k^{\sigma}\right)^{\frac{1}{1-\sigma}}\right]^{\frac{\sigma-\mu}{\gamma+\sigma}}\\
&\cdot\left\{\left[\left(\sum_{k=1,k\neq i}^{N} P_{ik}^{1+\gamma}\right)^{\frac{1}{1+\gamma}}\right]^{1+\eta}+P_{ii}^{1+\eta}\right\}^{\frac{1}{\gamma+\sigma}}\\
&\cdot\left\{\left[\left(\sum_{k=1,k\neq j}^{N} P_{kj}^{1-\sigma}\omega_k^{\sigma}\right)^{\frac{1}{1-\sigma}}\right]^{1-\mu}+P_{jj}^{1-\mu}\omega_j^{\mu}\right\}^{\frac{-1}{\gamma+\sigma}}
\end{aligned}
\tag{A.40}
$$

Substitute $(A.40)$ into $(A.24)$

$$X_{ij} = Y_i \left\langle Y_i^{\frac{-1}{\gamma+\sigma}} Y_j^{\frac{1}{\gamma+\sigma}} C_{ij}^{\frac{-\sigma}{\gamma+\sigma}} T_{ij}^{\frac{-\sigma}{\gamma+\sigma}} Z_{ij}^{\frac{-\sigma}{\gamma+\sigma}} \omega_i^{\frac{\sigma}{\gamma+\sigma}} \left[\left(\sum_{k=1,k\neq i}^{N} P_{ik}^{1+\gamma}\right)^{\frac{1}{1+\gamma}}\right]^{\frac{\gamma-\eta}{\gamma+\sigma}} \left[\left(\sum_{k=1,k\neq j}^{N} P_{kj}^{1-\sigma}\omega_k^{\sigma}\right)^{\frac{1}{1-\sigma}}\right]^{\frac{\sigma-\mu}{\gamma+\sigma}} \right.$$
$$\left. \cdot \left\{\left[\left(\sum_{k=1,k\neq i}^{N} P_{ik}^{1+\gamma}\right)^{\frac{1}{1+\gamma}}\right]^{1+\eta} + P_{ii}^{1+\eta}\right\}^{\frac{1}{\gamma+\sigma}} \left\{\left[\left(\sum_{k=1,k\neq j}^{N} P_{kj}^{1-\sigma}\omega_k^{\sigma}\right)^{\frac{1}{1-\sigma}}\right]^{1-\mu} + P_{jj}^{1-\mu}\omega_j^{\mu}\right\}^{\frac{-1}{\gamma+\sigma}} \right\rangle^{\gamma}$$
$$\cdot \left[\left(\sum_{k=1,k\neq i}^{N} P_{ik}^{1+\gamma}\right)^{\frac{1}{1+\gamma}}\right]^{-(\gamma-\eta)} \left\{\left[\left(\sum_{k=1,k\neq i}^{N} P_{ik}^{1+\gamma}\right)^{\frac{1}{1+\gamma}}\right]^{1+\eta} + P_{ii}^{1+\eta}\right\}^{-1}$$

Rewrite

$$X_{ij} = Y_i Y_i^{\frac{-\gamma}{\gamma+\sigma}} Y_j^{\frac{\gamma}{\gamma+\sigma}} C_{ij}^{-\frac{\gamma\sigma}{\gamma+\sigma}} T_{ij}^{-\frac{\gamma\sigma}{\gamma+\sigma}} Z_{ij}^{-\frac{\gamma\sigma}{\gamma+\sigma}} \omega_i^{\frac{\gamma\sigma}{\gamma+\sigma}}$$
$$\cdot \left[\left(\sum_{k=1,k\neq i}^{N} P_{ik}^{1+\gamma}\right)^{\frac{1}{1+\gamma}}\right]^{\frac{\gamma(\gamma-\eta)}{\gamma+\sigma}} \left[\left(\sum_{k=1,k\neq j}^{N} P_{kj}^{1-\sigma}\omega_k^{\sigma}\right)^{\frac{1}{1-\sigma}}\right]^{\frac{\gamma(\sigma-\mu)}{\gamma+\sigma}}$$
$$\cdot \left\{\left[\left(\sum_{k=1,k\neq i}^{N} P_{ik}^{1+\gamma}\right)^{\frac{1}{1+\gamma}}\right]^{1+\eta} + P_{ii}^{1+\eta}\right\}^{\frac{\gamma}{\gamma+\sigma}} \left\{\left[\left(\sum_{k=1,k\neq j}^{N} P_{kj}^{1-\sigma}\omega_k^{\sigma}\right)^{\frac{1}{1-\sigma}}\right]^{1-\mu} + P_{jj}^{1-\mu}\omega_j^{\mu}\right\}^{\frac{-\gamma}{\gamma+\sigma}}$$
$$\cdot \left[\left(\sum_{k=1,k\neq i}^{N} P_{ik}^{1+\gamma}\right)^{\frac{1}{1+\gamma}}\right]^{-(\gamma-\eta)} \left\{\left[\left(\sum_{k=1,k\neq i}^{N} P_{ik}^{1+\gamma}\right)^{\frac{1}{1+\gamma}}\right]^{1+\eta} + P_{ii}^{1+\eta}\right\}^{-1}$$

Summarize $\Omega^{\frac{\gamma(\gamma-\eta)}{\gamma+\sigma}} \cdot \Omega^{-(\gamma-\eta)} = \Omega^{\frac{-\sigma(\gamma-\eta)}{\gamma+\sigma}}$ and $\Omega^{\frac{\gamma}{\gamma+\sigma}} \cdot \Omega^{-1} = \Omega^{\frac{-\sigma}{\gamma+\sigma}}$

$$X_{ij} = Y_i Y_i^{\frac{-\gamma}{\gamma+\sigma}} Y_j^{\frac{\gamma}{\gamma+\sigma}} C_{ij}^{-\frac{\gamma\sigma}{\gamma+\sigma}} T_{ij}^{-\frac{\gamma\sigma}{\gamma+\sigma}} Z_{ij}^{-\frac{\gamma\sigma}{\gamma+\sigma}} \omega_i^{\frac{\gamma\sigma}{\gamma+\sigma}}$$
$$\cdot \left[\left(\sum_{k=1,k\neq i}^{N} P_{ik}^{1+\gamma}\right)^{\frac{1}{1+\gamma}}\right]^{\frac{-\sigma(\gamma-\eta)}{\gamma+\sigma}} \left[\left(\sum_{k=1,k\neq j}^{N} P_{kj}^{1-\sigma}\omega_k^{\sigma}\right)^{\frac{1}{1-\sigma}}\right]^{\frac{\gamma(\sigma-\mu)}{\gamma+\sigma}}$$
$$\cdot \left\{\left[\left(\sum_{k=1,k\neq i}^{N} P_{ik}^{1+\gamma}\right)^{\frac{1}{1+\gamma}}\right]^{1+\eta} + P_{ii}^{1+\eta}\right\}^{\frac{-\sigma}{\gamma+\sigma}}$$
$$\cdot \left\{\left[\left(\sum_{k=1,k\neq j}^{N} P_{kj}^{1-\sigma}\omega_k^{\sigma}\right)^{\frac{1}{1-\sigma}}\right]^{1-\mu} + P_{jj}^{1-\mu}\omega_j^{\mu}\right\}^{\frac{-\gamma}{\gamma+\sigma}} \tag{A.41}$$

Multiply $(A.40)$ and $(A.41)$

$$\begin{aligned}
P_{ij}X_{ij} =& Y_i^{\frac{-1}{\gamma+\sigma}} Y_j^{\frac{1}{\gamma+\sigma}} C_{ij}^{-\frac{\sigma}{\gamma+\sigma}} T_{ij}^{-\frac{\sigma}{\gamma+\sigma}} Z_{ij}^{-\frac{\sigma}{\gamma+\sigma}} \omega_i^{\frac{\sigma}{\gamma+\sigma}} \\
& \cdot \left[\left(\sum_{k=1,k\neq i}^{N} P_{ik}^{1+\gamma} \right)^{\frac{1}{1+\gamma}} \right]^{\frac{\gamma-\eta}{\gamma+\sigma}} \left[\left(\sum_{k=1,k\neq j}^{N} P_{kj}^{1-\sigma} \omega_k^{\sigma} \right)^{\frac{1}{1-\sigma}} \right]^{\frac{\sigma-\mu}{\gamma+\sigma}} \\
& \cdot \left\{ \left[\left(\sum_{k=1,k\neq i}^{N} P_{ik}^{1+\gamma} \right)^{\frac{1}{1+\gamma}} \right]^{1+\eta} + P_{ii}^{1+\eta} \right\}^{\frac{1}{\gamma+\sigma}} \left\{ \left[\left(\sum_{k=1,k\neq j}^{N} P_{kj}^{1-\sigma} \omega_k^{\sigma} \right)^{\frac{1}{1-\sigma}} \right]^{1-\mu} + P_{jj}^{1-\mu} \omega_j^{\mu} \right\}^{\frac{-1}{\gamma+\sigma}} \\
& \cdot Y_i Y_i^{\frac{-\gamma}{\gamma+\sigma}} Y_j^{\frac{\gamma}{\gamma+\sigma}} C_{ij}^{-\frac{\gamma\sigma}{\gamma+\sigma}} T_{ij}^{-\frac{\gamma\sigma}{\gamma+\sigma}} Z_{ij}^{-\frac{\gamma\sigma}{\gamma+\sigma}} \omega_i^{\frac{\gamma\sigma}{\gamma+\sigma}} \\
& \cdot \left[\left(\sum_{k=1,k\neq i}^{N} P_{ik}^{1+\gamma} \right)^{\frac{1}{1+\gamma}} \right]^{\frac{-\sigma(\gamma-\eta)}{\gamma+\sigma}} \left[\left(\sum_{k=1,k\neq j}^{N} P_{kj}^{1-\sigma} \omega_k^{\sigma} \right)^{\frac{1}{1-\sigma}} \right]^{\frac{\gamma(\sigma-\mu)}{\gamma+\sigma}} \\
& \cdot \left\{ \left[\left(\sum_{k=1,k\neq i}^{N} P_{ik}^{1+\gamma} \right)^{\frac{1}{1+\gamma}} \right]^{1+\eta} + P_{ii}^{1+\eta} \right\}^{\frac{-\sigma}{\gamma+\sigma}} \left\{ \left[\left(\sum_{k=1,k\neq j}^{N} P_{kj}^{1-\sigma} \omega_k^{\sigma} \right)^{\frac{1}{1-\sigma}} \right]^{1-\mu} + P_{jj}^{1-\mu} \omega_j^{\mu} \right\}^{\frac{-\gamma}{\gamma+\sigma}}
\end{aligned}$$

Summarize exponents and add indexes

$$P_{ij}X_{ij} = Y_i^{\frac{\sigma_j-1}{\gamma_i+\sigma_j}} Y_j^{\frac{\gamma_i+1}{\gamma_i+\sigma_j}} C_{ij}^{-\frac{(\gamma_i+1)\sigma_j}{\gamma_i+\sigma_j}} T_{ij}^{-\frac{(\gamma_i+1)\sigma_j}{\gamma_i+\sigma_j}} Z_{ij}^{-\frac{(\gamma_i+1)\sigma_j}{\gamma_i+\sigma_j}} \omega_i^{\frac{(\gamma_i+1)\sigma_j}{\gamma_i+\sigma_j}}$$
$$\cdot \left[\left(\sum_{k=1,k\neq i}^{N} P_{ik}^{1+\gamma_i}\right)^{\frac{1}{1+\gamma_i}}\right]^{-\frac{(\sigma_j-1)(\gamma_i-\eta_i)}{(1+\gamma_i)(\gamma_i+\sigma_j)}} \left[\left(\sum_{k=1,k\neq j}^{N} P_{kj}^{1-\sigma_j}\omega_k^{\sigma_j}\right)^{\frac{1}{1-\sigma_j}}\right]^{\frac{(\gamma_i+1)(\sigma_j-\mu_j)}{(1-\sigma_j)(\gamma_i+\sigma_j)}}$$
$$\cdot \left\{\left[\left(\sum_{k=1,k\neq i}^{N} P_{ik}^{1+\gamma_i}\right)^{\frac{1}{1+\gamma_i}}\right]^{1+\eta_i} + P_{ii}^{1+\eta_i}\right\}^{-\frac{(\sigma_j-1)}{\gamma_i+\sigma_j}}$$
$$\cdot \left\{\left[\left(\sum_{k=1,k\neq j}^{N} P_{kj}^{1-\sigma_j}\omega_k^{\sigma_j}\right)^{\frac{1}{1-\sigma_j}}\right]^{1-\mu_j} + P_{jj}^{1-\mu_j}\omega_j^{\mu_j}\right\}^{-\frac{(\gamma_i+1)}{\gamma_i+\sigma_j}}$$

The same as Bergstrand's 1985 representation if ω converges to 1

$$P_{ij}X_{ij} = Y_i^{\frac{\sigma_j-1}{\gamma_i+\sigma_j}} Y_j^{\frac{\gamma_i+1}{\gamma_i+\sigma_j}} C_{ij}^{-\frac{(\gamma_i+1)\sigma_j}{\gamma_i+\sigma_j}} T_{ij}^{-\frac{(\gamma_i+1)\sigma_j}{\gamma_i+\sigma_j}} Z_{ij}^{-\frac{(\gamma_i+1)\sigma_j}{\gamma_i+\sigma_j}} \left(\frac{1-M_{ij}}{M_j}\alpha_{ij} + \frac{M_{ij}}{M_j}\zeta_{ij}\right)^{\frac{(\gamma_i+1)\sigma_j}{\gamma_i+\sigma_j}}$$
$$\cdot \left[\left(\sum_{k=1,k\neq i}^{N} P_{ik}^{1+\gamma_i}\right)^{\frac{1}{1+\gamma_i}}\right]^{-\frac{(\sigma_j-1)(\gamma_i-\eta_i)}{(1+\gamma_i)(\gamma_i+\sigma_j)}}$$
$$\cdot \left\{\left[\sum_{k=1,k\neq j}^{N} P_{kj}^{1-\sigma_j}\left(\frac{1-M_{kj}}{M_j}\alpha_{kj} + \frac{M_{kj}}{M_j}\zeta_{kj}\right)^{\sigma_j}\right]^{\frac{1}{1-\sigma_j}}\right\}^{\frac{(\gamma_i+1)(\sigma_j-\mu_j)}{(1-\sigma_j)(\gamma_i+\sigma_j)}}$$
$$\cdot \left\{\left[\left(\sum_{k=1,k\neq i}^{N} P_{ik}^{1+\gamma_i}\right)^{\frac{1}{1+\gamma_i}}\right]^{1+\eta_i} + P_{ii}^{1+\eta_i}\right\}^{-\frac{(\sigma_j-1)}{\gamma_i+\sigma_j}}$$
$$\cdot \left\langle\left\{\left[\sum_{k=1,k\neq j}^{N} P_{kj}^{1-\sigma_j}\left(\frac{1-M_{kj}}{M_j}\alpha_{kj} + \frac{M_{kj}}{M_j}\zeta_{kj}\right)^{\sigma_j}\right]^{\frac{1}{1-\sigma_j}}\right\}^{1-\mu_j}\right.$$
$$\left. + P_{jj}^{1-\mu_j}\left(\frac{1-M_{jj}}{M_j}\alpha_{jj} + \frac{M_{jj}}{M_j}\zeta_{jj}\right)^{\mu_j}\right\rangle^{-\frac{(\gamma_i+1)}{\gamma_i+\sigma_j}} \tag{A.42}$$

A.4 Derivative with respect to migrants

$$\text{Since } \delta > \phi > 1 \Rightarrow \gamma > \eta > 0$$
$$\gamma = \frac{1}{\phi - 1}\ ,\ -\gamma = \frac{1}{1-\phi}\ ,\ \gamma + 1 = \frac{\phi}{\phi - 1}\ ,\ \phi = \frac{1+\gamma}{\gamma}$$
$$\eta = \frac{1}{\delta - 1}\ ,\ -\eta = \frac{1}{1-\delta}\ ,\ \eta + 1 = \frac{\delta}{\delta - 1}\ ,\ \delta = \frac{1+\eta}{\eta}$$
$$\text{Since } -\infty < \psi < \theta < 1 \Rightarrow 0 < \mu < \sigma < \infty$$
$$\sigma = \frac{1}{1-\theta}, -\sigma = \frac{1}{\theta - 1}, \mu = \frac{1}{1-\psi}, -\mu = \frac{1}{\psi - 1}, \theta = \frac{\sigma - 1}{\sigma}$$
$$M_j = \sum M_{kj}, \zeta_{kj} > \alpha_{kj} \Rightarrow \zeta_{kj} - \alpha_{kj} > 0$$

First derivative with respect to the number of migrants

$$\begin{aligned}
\frac{\partial P_{ij} X_{ij}}{\partial M_{ij}} =& Y_i^{\frac{\sigma_j - 1}{\gamma_i + \sigma_j}} Y_j^{\frac{\gamma_i + 1}{\gamma_i + \sigma_j}} C_{ij}^{-\frac{(\gamma_i+1)\sigma_j}{\gamma_i + \sigma_j}} T_{ij}^{-\frac{(\gamma_i+1)\sigma_j}{\gamma_i + \sigma_j}} Z_{ij}^{-\frac{(\gamma_i+1)\sigma_j}{\gamma_i + \sigma_j}} \\
&\cdot \frac{(\gamma_i + 1)\sigma_j}{\gamma_i + \sigma_j} \left(M_j^{-1} - M_{ij} M_j^{-1} \alpha_{ij} + M_{ij} M_j^{-1} \zeta_{ij} \right)^{\frac{\gamma_i \sigma_j + \sigma_j - \gamma_i - \sigma_j}{\gamma_i + \sigma_j}} \left(-M_j^{-1} \alpha_{ij} + L_j^{-1} \zeta_{ij} \right) \\
&\cdot \left[\left(\sum_{k=1,k\neq i}^{N} P_{ik}^{1+\gamma_i} \right)^{\frac{1}{1+\gamma_i}} \right]^{-\frac{(\sigma_j - 1)(\gamma_i - \eta_i)}{(1+\gamma_i)(\gamma_i + \sigma_j)}} \\
&\cdot \left\{ \left[\sum_{k=1,k\neq j}^{N} P_{kj}^{1-\sigma_j} \left(\frac{1 - M_{kj}}{M_j} \alpha_{kj} + \frac{M_{kj}}{M_j} \zeta_{kj} \right)^{\sigma_j} \right]^{\frac{1}{1-\sigma_j}} \right\}^{\frac{(\gamma_i+1)(\sigma_j - \mu_j)}{(1-\sigma_j)(\gamma_i + \sigma_j)}} \\
&\cdot \left\{ \left[\left(\sum_{k=1,k\neq i}^{N} P_{ik}^{1+\gamma_i} \right)^{\frac{1}{1+\gamma_i}} \right]^{1+\eta_i} + P_{ii}^{1+\eta_i} \right\}^{-\frac{(\sigma_j - 1)}{\gamma_i + \sigma_j}} \\
&\cdot \left\langle \left\{ \left[\sum_{k=1,k\neq j}^{N} P_{kj}^{1-\sigma_j} \left(\frac{1 - M_{kj}}{M_j} \alpha_{kj} + \frac{M_{kj}}{M_j} \zeta_{kj} \right)^{\sigma_j} \right]^{\frac{1}{1-\sigma_j}} \right\}^{1-\mu_j} \right. \\
&\left. + P_{jj}^{1-\mu_j} \left(\frac{1 - M_{jj}}{M_j} \alpha_{jj} + \frac{M_{jj}}{M_j} \zeta_{jj} \right)^{\mu_j} \right\rangle^{-\frac{(\gamma_i+1)}{\gamma_i + \sigma_j}}
\end{aligned}$$

Rewrite

$$\begin{aligned}
\frac{\partial P_{ij}X_{ij}}{\partial M_{ij}} =& Y_i^{\frac{\sigma_j-1}{\gamma_i+\sigma_j}} Y_j^{\frac{\gamma_i+1}{\gamma_i+\sigma_j}} C_{ij}^{-\frac{(\gamma_i+1)\sigma_j}{\gamma_i+\sigma_j}} T_{ij}^{-\frac{(\gamma_i+1)\sigma_j}{\gamma_i+\sigma_j}} Z_{ij}^{-\frac{(\gamma_i+1)\sigma_j}{\gamma_i+\sigma_j}} \\
&\cdot \frac{(\gamma_i+1)\sigma_j}{\gamma_i+\sigma_j} \cdot \frac{1-M_{ij}\alpha_{ij}+M_{ij}\zeta_{ij}}{M_j}^{\frac{\gamma_i\sigma_j-\gamma_i}{\gamma_i+\sigma_j}} \frac{\zeta_{ij}-\alpha_{ij}}{M_j} \\
&\cdot \left[\left(\sum_{k=1,k\neq i}^{N} P_{ik}^{1+\gamma_i}\right)^{\frac{1}{1+\gamma_i}}\right]^{-\frac{(\sigma_j-1)(\gamma_i-\eta_i)}{(1+\gamma_i)(\gamma_i+\sigma_j)}} \\
&\cdot \left\{\left[\sum_{k=1,k\neq j}^{N} P_{kj}^{1-\sigma_j}\left(\frac{1-M_{kj}}{M_j}\alpha_{kj}+\frac{M_{kj}}{M_j}\zeta_{kj}\right)^{\sigma_j}\right]^{\frac{1}{1-\sigma_j}}\right\}^{\frac{(\gamma_i+1)(\sigma_j-\mu_j)}{(1-\sigma_j)(\gamma_i+\sigma_j)}} \\
&\cdot \left\{\left[\left(\sum_{k=1,k\neq i}^{N} P_{ik}^{1+\gamma_i}\right)^{\frac{1}{1+\gamma_i}}\right]^{1+\eta_i} + P_{ii}^{1+\eta_i}\right\}^{-\frac{(\sigma_j-1)}{\gamma_i+\sigma_j}} \\
&\cdot \left\langle \left\{\left[\sum_{k=1,k\neq j}^{N} P_{kj}^{1-\sigma_j}\left(\frac{1-M_{kj}}{M_j}\alpha_{kj}+\frac{M_{kj}}{M_j}\zeta_{kj}\right)^{\sigma_j}\right]^{\frac{1}{1-\sigma_j}}\right\}^{1-\mu_j} \right. \\
&\left. + P_{jj}^{1-\mu_j}\left(\frac{1-M_{jj}}{M_j}\alpha_{jj}+\frac{M_{jj}}{M_j}\zeta_{jj}\right)^{\mu_j}\right\rangle^{-\frac{(\gamma_i+1)}{\gamma_i+\sigma_j}}
\end{aligned}$$

Rewrite

$$\begin{aligned}
\frac{\partial P_{ij}X_{ij}}{\partial M_{ij}} =& Y_i^{\frac{\sigma_j-1}{\gamma_i+\sigma_j}} Y_j^{\frac{\gamma_i+1}{\gamma_i+\sigma_j}} C_{ij}^{-\frac{(\gamma_i+1)\sigma_j}{\gamma_i+\sigma_j}} T_{ij}^{-\frac{(\gamma_i+1)\sigma_j}{\gamma_i+\sigma_j}} Z_{ij}^{-\frac{(\gamma_i+1)\sigma_j}{\gamma_i+\sigma_j}} \\
&\cdot \frac{(\gamma_i+1)\sigma_j}{\gamma_i+\sigma_j} \cdot \frac{1+M_{ij}\left(\zeta_{ij}-\alpha_{ij}\right)}{M_j}^{\frac{\gamma_i\sigma_j-\gamma_i}{\gamma_i+\sigma_j}} \frac{\zeta_{ij}-\alpha_{ij}}{M_j} \\
&\cdot \left[\left(\sum_{k=1,k\neq i}^{N} P_{ik}^{1+\gamma_i}\right)^{\frac{1}{1+\gamma_i}}\right]^{-\frac{(\sigma_j-1)(\gamma_i-\eta_i)}{(1+\gamma_i)(\gamma_i+\sigma_j)}} \\
&\cdot \left\{\left[\sum_{k=1,k\neq j}^{N} P_{kj}^{1-\sigma_j}\left(\frac{1-M_{kj}}{M_j}\alpha_{kj}+\frac{M_{kj}}{M_j}\zeta_{kj}\right)^{\sigma_j}\right]^{\frac{1}{1-\sigma_j}}\right\}^{\frac{(\gamma_i+1)(\sigma_j-\mu_j)}{(1-\sigma_j)(\gamma_i+\sigma_j)}} \\
&\cdot \left\{\left[\left(\sum_{k=1,k\neq i}^{N} P_{ik}^{1+\gamma_i}\right)^{\frac{1}{1+\gamma_i}}\right]^{1+\eta_i} + P_{ii}^{1+\eta_i}\right\}^{-\frac{(\sigma_j-1)}{\gamma_i+\sigma_j}} \\
&\cdot \left\langle \left\{\left[\sum_{k=1,k\neq j}^{N} P_{kj}^{1-\sigma_j}\left(\frac{1-M_{kj}}{M_j}\alpha_{kj}+\frac{M_{kj}}{M_j}\zeta_{kj}\right)^{\sigma_j}\right]^{\frac{1}{1-\sigma_j}}\right\}^{1-\mu_j} \right. \\
&\left. + P_{jj}^{1-\mu_j}\left(\frac{1-M_{jj}}{M_j}\alpha_{jj}+\frac{M_{jj}}{M_j}\zeta_{jj}\right)^{\mu_j} \right\rangle^{-\frac{(\gamma_i+1)}{\gamma_i+\sigma_j}} > 0
\end{aligned}$$

Second derivative with respect to the number of migrants

$$\frac{\partial^2 P_{ij}X_{ij}}{\partial M_{ij}^2} = Y_i^{\frac{\sigma_j-1}{\gamma_i+\sigma_j}} Y_j^{\frac{\gamma_i+1}{\gamma_i+\sigma_j}} C_{ij}^{-\frac{(\gamma_i+1)\sigma_j}{\gamma_i+\sigma_j}} T_{ij}^{-\frac{(\gamma_i+1)\sigma_j}{\gamma_i+\sigma_j}} Z_{ij}^{-\frac{(\gamma_i+1)\sigma_j}{\gamma_i+\sigma_j}} \frac{(\gamma_i+1)\sigma_j}{\gamma_i+\sigma_j}$$

$$\cdot \frac{\gamma_i\sigma_j-\gamma_i}{\gamma_i+\sigma_j} \frac{1+M_{ij}\left(\zeta_{ij}-\alpha_{ij}\right)}{M_j}^{\frac{\gamma_i\sigma_j-\gamma_i-(\gamma_i+\sigma_j)}{\gamma_i+\sigma_j}} \cdot \frac{\zeta_{ij}-\alpha_{ij}}{M_j}$$

$$\cdot \frac{\zeta_{ij}-\alpha_{ij}}{M_j} \left[\left(\sum_{k=1,k\neq i}^{N} P_{ik}^{1+\gamma_i}\right)^{\frac{1}{1+\gamma_i}}\right]^{-\frac{(\sigma_j-1)(\gamma_i-\eta_i)}{(1+\gamma_i)(\gamma_i+\sigma_j)}}$$

$$\cdot \left\{\left[\sum_{k=1,k\neq j}^{N} P_{kj}^{1-\sigma_j}\left(\frac{1-M_{kj}}{M_j}\alpha_{kj}+\frac{M_{kj}}{M_j}\zeta_{kj}\right)^{\sigma_j}\right]^{\frac{1}{1-\sigma_j}}\right\}^{\frac{(\gamma_i+1)(\sigma_j-\mu_j)}{(1-\sigma_j)(\gamma_i+\sigma_j)}}$$

$$\cdot \left\{\left[\left(\sum_{k=1,k\neq i}^{N} P_{ik}^{1+\gamma_i}\right)^{\frac{1}{1+\gamma_i}}\right]^{1+\eta_i} + P_{ii}^{1+\eta_i}\right\}^{-\frac{(\sigma_j-1)}{\gamma_i+\sigma_j}}$$

$$\cdot \left\langle \left\{\left[\sum_{k=1,k\neq j}^{N} P_{kj}^{1-\sigma_j}\left(\frac{1-M_{kj}}{M_j}\alpha_{kj}+\frac{M_{kj}}{M_j}\zeta_{kj}\right)^{\sigma_j}\right]^{\frac{1}{1-\sigma_j}}\right\}^{1-\mu_j} + P_{jj}^{1-\mu_j}\left(\frac{1-M_{jj}}{M_j}\alpha_{jj}+\frac{M_{jj}}{M_j}\zeta_{jj}\right)^{\mu_j} \right\rangle^{-\frac{(\gamma_i+1)}{\gamma_i+\sigma_j}}$$

Since all of the other multipliers are bigger than zero, only

$\frac{\gamma_i\sigma_j-\gamma_i}{\gamma_i+\sigma_j}$ has to be checked

Both γ and σ are positive, therefore, if σ is > 1, the second derivative is > 0

If σ is < 1, then the second derivative is < 0

Appendix B

Migration

Table B.1: Origin countries

	Freq.	Percent	Cum.
AFG	170	7.47	7.47
BGD	166	7.29	14.76
BTN	114	5.01	19.76
CHN	202	8.87	28.63
HKG	121	5.31	33.95
IND	199	8.74	42.69
JPN	185	8.12	50.81
KOR	175	7.69	58.50
LKA	171	7.51	66.01
MAC	92	4.04	70.05
MDV	100	4.39	74.44
MNG	118	5.18	79.62
NPL	142	6.24	85.86
PAK	205	9.00	94.86
PRK	117	5.14	100.00
Total	2,277	100.00	

ISO3 alphanumeric codes of South and East Asian countries Origin countries of both migrants and trade flows

Table B.2: Destination countries

	Freq.	Percent	Cum.
AUT	195	8.56	8.56
BEL	63	2.77	11.33
CAN	47	2.06	13.39
CHE	65	2.85	16.25
DEU	39	1.71	17.96
DNK	167	7.33	25.30
ESP	161	7.07	32.37
FRA	123	5.40	37.77
GBR	70	3.07	40.84
GRC	48	2.11	42.95
IRL	25	1.10	44.05
ISL	210	9.22	53.27
ITA	78	3.43	56.70
LUX	13	0.57	57.27
NLD	240	10.54	67.81
NOR	289	12.69	80.50
PRT	9	0.40	80.90
SWE	211	9.27	90.16
TUR	15	0.66	90.82
USA	209	9.18	100.00
Total	2,277	100.00	

ISO3 alphanumeric codes of OECD founding members
Destination countries of both migrants and trade flows

Table B.3: Data set

	Obs	Mean	Std. Dev.	Min	Max
tra_{ijt}	2,170	17.466	4.248	2.558	26.371
mig_{ijt}	2,111	7.911	2.882	0	14.527
gdp_{it}	2,139	25.416	2.639	19.573	29.981
gdp_{jt}	2,277	27.031	1.680	22.821	30.487
aco_{ijt}	2,277	9.059	.251	8.214	9.843
pop_{it}	2,277	3.660	2.393	−1.384	7.218
pop_{jt}	2,277	2.549	1.731	−1.269	5.765
EUV_{it}	2,210	137.976	59.176	80.815	442.390
IUV_{jt}	2,133	158.233	37.017	94.1	228.036
DEF_{it}	2,121	6.127	6.064	−6.008	50.893
DEF_{jt}	2,277	2.360	2.523	−5.214	15.390
FTA_{ijt}	2,277	.028	.164	0	1

Lower case variables indicate the natural logarithm,
upper case variables represent the absolute values observed
Population sizes are the logarithms of 10^{-6} times the absolute value

Table B.4: Estimated effects on bilateral trade (PPML)

	(1) PPML	(2) PPML	(3) PPML	(4) PPML
mig_{ijt}	0.364***	0.277***	0.149***	0.096***
	(0.0000)	(0.0000)	(0.0000)	(0.0000)
gdp_{it}	0.860***	0.897***	0	0
	(0.0000)	(0.0000)	(.)	(.)
gdp_{jt}	0.557***	0.748***	0	0
	(0.0000)	(0.0000)	(.)	(.)
aco_{ijt}		−0.513**		−1.444***
		(0.0040)		(0.0000)
pop_{it}		0.215***		0
		(0.0000)		(.)
pop_{jt}		−0.122		0
		(0.2259)		(.)
EUV_{it}		−0.014***		0
		(0.0000)		(.)
IUV_{jt}		−0.006***		0
		(0.0000)		(.)
DEF_{it}		0.024**		0
		(0.0054)		(.)
DEF_{jt}		0.005		0
		(0.8085)		(.)
FTA_{ijt}		0.058		0.532***
		(0.4283)		(0.0000)
Exp-time FE	×	×	✓	✓
Imp-time FE	×	×	✓	✓
N	2000	1859	1984	1828

p-values in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

The dependent variable is in absolute values

Table B.5: Estimated effects on traded household consumption (PPML)

	(1) PPML	(2) PPML	(3) PPML	(4) PPML
mig_{ijt}	0.674^{***}	0.281^{***}	-0.038	-0.034
	(0.0000)	(0.0001)	(0.3757)	(0.3308)
gdp_{it}	0.691^{***}	0.657^{***}	0	0
	(0.0000)	(0.0000)	(.)	(.)
gdp_{jt}	0.195^{*}	1.045^{***}	0	0
	(0.0419)	(0.0000)	(.)	(.)
aco_{ijt}		-0.381		-0.754^{***}
		(0.2099)		(0.0000)
pop_{it}		0.648^{***}		0
		(0.0000)		(.)
pop_{jt}		-0.268		0
		(0.1418)		(.)
EUV_{it}		-0.028^{***}		0
		(0.0000)		(.)
IUV_{jt}		0.003		0
		(0.1202)		(.)
DEF_{it}		0.040^{*}		0
		(0.0242)		(.)
DEF_{jt}		0.032		0
		(0.2886)		(.)
FTA_{ijt}		-0.482		0.950^{*}
		(0.2401)		(0.0104)
Exp-time FE	×	×	✓	✓
Imp-time FE	×	×	✓	✓
N	601	548	583	524

p-values in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

The dependent variable is in absolute values

Appendix C

Distance

Table C.1: Estimated effects on bilateral trade (POLS long)

	(1)	(2)	(3)	(4)	(5)	(6)
	tra_{com_i}	tra_{com_j}	tra_{bac}	tra_{man}	tra_{imf_i}	tra_{imf_j}
gdp_i	0.484^{***}	0.515^{***}	0.466^{***}	0.447^{***}	0.450^{***}	0.468^{***}
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
gdp_j	0.400^{***}	0.400^{***}	0.602^{***}	0.597^{***}	0.394^{***}	0.411^{***}
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
dis_{ij}	-0.387^{***}	-0.318^{***}	-0.426^{***}	-0.440^{***}	-0.385^{***}	-0.372^{***}
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
pop_i	0.019^{*}	0.017	0.036^{**}	0.042^{**}	0.061^{***}	0.039^{***}
	(0.0414)	(0.1038)	(0.0078)	(0.0022)	(0.0000)	(0.0000)
pop_j	0.131^{***}	0.148^{***}	-0.032^{**}	-0.040^{**}	0.115^{***}	0.166^{***}
	(0.0000)	(0.0000)	(0.0078)	(0.0012)	(0.0000)	(0.0000)
BOR_{ij}	0.037^{***}	0.060^{***}	0.047^{***}	0.039^{***}	0.041^{***}	0.041^{***}
	(0.0002)	(0.0000)	(0.0000)	(0.0003)	(0.0001)	(0.0000)
LAN_{ij}	0.084^{***}	0.078^{***}	0.075^{***}	0.078^{***}	0.086^{***}	0.082^{***}
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
REL_{ij}	-0.009	-0.011	0.017	0.015	-0.008	0.005
	(0.3161)	(0.3292)	(0.0780)	(0.1273)	(0.3966)	(0.5895)
COL_{ij}	0.023^{*}	0.024^{*}	0.006	0.002	0.024^{*}	0.020^{*}
	(0.0130)	(0.0409)	(0.5580)	(0.8619)	(0.0159)	(0.0281)
RTA_{ij}	0.050^{***}	0.050^{***}	0.009^{***}	0.012^{***}	0.053^{***}	0.045^{***}
	(0.0000)	(0.0000)	(0.0001)	(0.0000)	(0.0000)	(0.0000)
PCI_i	-0.090^{***}	-0.084^{***}	-0.087^{***}	-0.092^{***}	-0.062^{***}	-0.070^{***}
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
PCI_j	-0.058^{***}	-0.077^{***}	-0.063^{***}	-0.062^{***}	-0.041^{***}	-0.061^{***}
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
GAT_i	0.022^{***}	0.019^{***}	0.082^{***}	0.099^{***}	0.007^{***}	0.013^{***}
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0001)	(0.0000)
GAT_j	-0.014^{***}	0.009^{***}	0.008	0.002	-0.010^{***}	-0.004^{*}
	(0.0000)	(0.0000)	(0.4410)	(0.8351)	(0.0000)	(0.0189)
WTO_i	0.034^{***}	0.027^{***}	0.010^{***}	0.011^{***}	0.030^{***}	0.046^{***}
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
WTO_j	-0.015^{*}	-0.003	-0.005^{***}	-0.003^{**}	-0.015^{*}	-0.025^{***}
	(0.0141)	(0.6493)	(0.0000)	(0.0022)	(0.0101)	(0.0000)
EUU_i	0.055^{***}	0.046^{***}	0.090^{***}	0.096^{***}	0.064^{***}	0.054^{***}
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
EUU_j	-0.048^{***}	-0.050^{***}	-0.006	-0.020	-0.040^{***}	-0.045^{***}
	(0.0000)	(0.0000)	(0.5536)	(0.0680)	(0.0000)	(0.0000)
N	62123	62559	28952	28950	61081	61384

Standardized beta coefficients; p-values in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table C.2: Estimated effects on bilateral trade (POLS short)

	(1) tra_{com_i}	(2) tra_{com_j}	(3) tra_{bac}	(4) tra_{man}	(5) tra_{imf_i}	(6) tra_{imf_j}
gdp_i	0.446^{***}	0.463^{***}	0.461^{***}	0.444^{***}	0.444^{***}	0.442^{***}
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
gdp_j	0.319^{***}	0.324^{***}	0.417^{***}	0.410^{***}	0.337^{***}	0.346^{***}
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
dis_{ij}	-0.365^{***}	-0.301^{***}	-0.424^{***}	-0.429^{***}	-0.363^{***}	-0.350^{***}
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
pop_i	0.040^{***}	0.049^{***}	-0.005	-0.008	0.018^{*}	0.026^{**}
	(0.0000)	(0.0000)	(0.6690)	(0.5252)	(0.0468)	(0.0042)
pop_j	0.191^{***}	0.254^{***}	0.107^{***}	0.096^{***}	0.150^{***}	0.235^{***}
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
BOR_{ij}	0.045^{***}	0.064^{***}	0.057^{***}	0.052^{***}	0.052^{***}	0.049^{***}
	(0.0001)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
LAN_{ij}	0.080^{***}	0.073^{***}	0.072^{***}	0.076^{***}	0.081^{***}	0.077^{***}
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
REL_{ij}	-0.013	-0.015	0.030^{**}	0.029^{**}	-0.010	0.002
	(0.2030)	(0.2441)	(0.0024)	(0.0048)	(0.3197)	(0.8445)
COL_{ij}	0.025^{*}	0.024	0.011	0.008	0.028^{*}	0.023^{*}
	(0.0174)	(0.0652)	(0.2628)	(0.4683)	(0.0107)	(0.0296)
RTA_{ij}	0.060^{***}	0.060^{***}	0.017^{***}	0.021^{***}	0.063^{***}	0.055^{***}
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
N	62123	62559	28952	28950	61081	61384

Standardized beta coefficients; p-values in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table C.3: Estimated effects on bilateral trade (PPML)

	(1)	(2)	(3)	(4)
	tra_{com_i}	tra_{com_i}	tra_{com_i}	tra_{com_i}
gdp_i	0.606614***	0.836400***	0	0
	(0.0000)	(0.0000)	(.)	(.)
gdp_j	0.597016***	0.791809***	0	0
	(0.0000)	(0.0000)	(.)	(.)
dis_{ij}	−0.735546***	−0.855616***	−0.806638***	−0.806638***
	(0.0000)	(0.0000)	(0.0000)	(0.0000)
pop_i	0.254339***	0.00756615	0	0
	(0.0000)	(0.7383)	(.)	(.)
pop_j	0.316120***	0.0989597***	0	0
	(0.0000)	(0.0000)	(.)	(.)
BOR_{ij}	0.595832***	0.478768***	0.434457***	0.434457***
	(0.0000)	(0.0000)	(0.0000)	(0.0000)
LAN_{ij}	0.403760***	0.283478***	0.116900***	0.116900***
	(0.0000)	(0.0000)	(0.0000)	(0.0000)
REL_{ij}	−0.0643493*	0.108045***	0.429197***	0.429197***
	(0.0119)	(0.0000)	(0.0000)	(0.0000)
COL_{ij}	−0.284735***	−0.328367***	−0.229219***	−0.229219***
	(0.0000)	(0.0000)	(0.0000)	(0.0000)
RTA_{ij}	0.139464***	0.136992***	0.643149***	0.643149***
	(0.0000)	(0.0000)	(0.0000)	(0.0000)
PCI_i		−0.00786358***		0
		(0.0000)		(.)
PCI_j		−0.00687157***		0
		(0.0000)		(.)
GAT_i		0.189676***		0
		(0.0000)		(.)
GAT_j		−0.117785**		0
		(0.0013)		(.)
WTO_i		0.197975*		0
		(0.0232)		(.)
WTO_j		−0.359980***		0
		(0.0000)		(.)
EUU_i		−0.200218***		0
		(0.0000)		(.)
EUU_j		−0.332058***		0
		(0.0000)		(.)
_cons	−9.685206***	−11.79556***	22.03873***	22.03873***
	(0.0000)	(0.0000)	(0.0000)	(0.0000)
N	62123	62123	62123	62123

p-values in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Appendix D

FDI

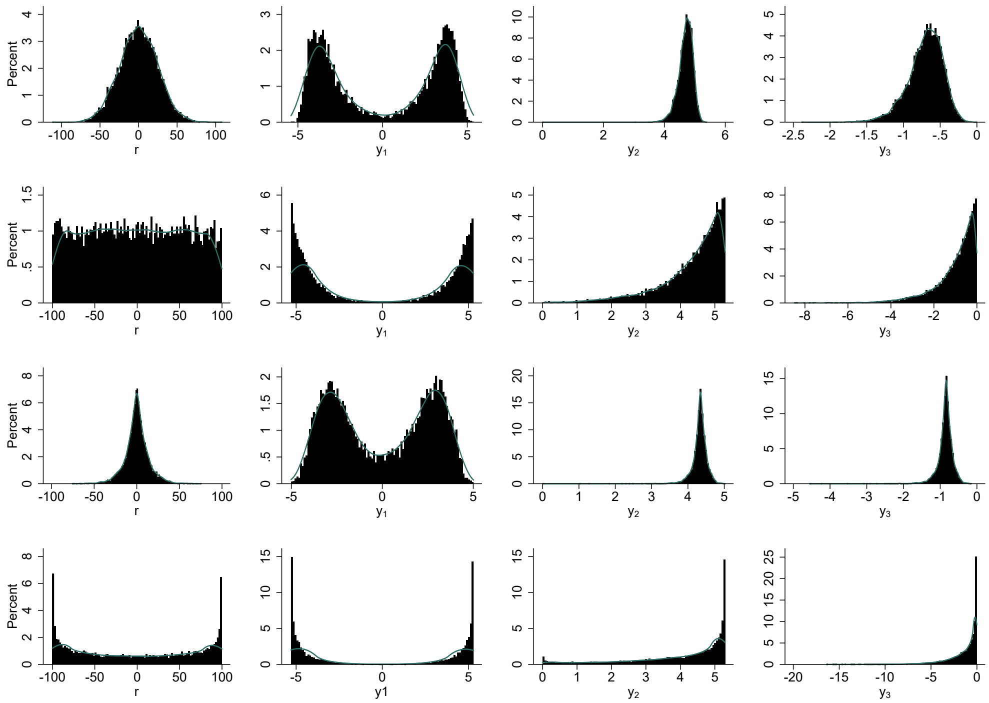

Figure D.1: Histograms of transformation functions

Table D.1: Estimated effects on bilateral FDI (GLS abs-abs)

	RE (1)	FE (2)	HT (3)	RE (4)	FE (5)	HT (6)
$INT_{L_{it}}$	−0.015***	−0.011**	−0.014**			
	(0.0004)	(0.0076)	(0.0010)			
$INT_{L_{jt}}$	−0.015**	−0.012*	−0.013**			
	(0.0039)	(0.0229)	(0.0075)			
GDP_{it}	0.530***	0.769***	0.620***	0.537***	0.778***	0.629***
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
GDP_{jt}	0.326***	0.376***	0.357***	0.333***	0.385***	0.365***
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
DIS_{ij}	−0.119***	0.000	−0.205***	−0.123***	0.000	−0.217***
	(0.0000)	(.)	(0.0005)	(0.0000)	(.)	(0.0003)
COL_{ij}	0.054*	0.000	0.055	0.053*	0.000	0.055
	(0.0445)	(.)	(0.1567)	(0.0472)	(.)	(0.1652)
BOR_{ij}	0.094***	0.000	0.060	0.096***	0.000	0.060
	(0.0006)	(.)	(0.1576)	(0.0005)	(.)	(0.1651)
LAN_{ij}	0.128***	0.000	0.148***	0.129***	0.000	0.149***
	(0.0000)	(.)	(0.0001)	(0.0000)	(.)	(0.0001)
CUR_{ijt}	−0.000	−0.005	−0.005	−0.002	−0.006	−0.007
	(0.9864)	(0.6560)	(0.6385)	(0.8821)	(0.5929)	(0.5466)
EUU_{it}	−0.025*	−0.038**	−0.033**	−0.024*	−0.037**	−0.032**
	(0.0254)	(0.0022)	(0.0038)	(0.0324)	(0.0028)	(0.0048)
EUU_{jt}	−0.019	−0.030	−0.029	−0.016	−0.029	−0.027
	(0.1845)	(0.0581)	(0.0528)	(0.2429)	(0.0685)	(0.0653)
TIM_{jt}	0.012	0.013*	0.012*	0.014*	0.014*	0.014*
	(0.0577)	(0.0408)	(0.0498)	(0.0275)	(0.0234)	(0.0257)
COS_{jt}	−0.016	−0.012	−0.015	−0.014	−0.010	−0.013
	(0.1288)	(0.3000)	(0.1592)	(0.1855)	(0.3943)	(0.2258)
PRO_{jt}	−0.042*	−0.064***	−0.050**	−0.031	−0.056**	−0.041*
	(0.0227)	(0.0010)	(0.0059)	(0.0886)	(0.0034)	(0.0239)
GOI_{jt}	0.004	−0.035	−0.007	0.000	−0.041	−0.012
	(0.8075)	(0.1020)	(0.7225)	(0.9815)	(0.0527)	(0.5361)
TAX_{jt}	−0.026*	−0.013	−0.021	−0.024	−0.009	−0.018
	(0.0388)	(0.3673)	(0.1019)	(0.0539)	(0.5096)	(0.1524)
GOS_{jt}	−0.024*	−0.021*	−0.023*	−0.020*	−0.018	−0.019*
	(0.0111)	(0.0309)	(0.0143)	(0.0291)	(0.0732)	(0.0388)
BUS_{jt}	0.039***	0.025**	0.034***	0.040***	0.025***	0.034***
	(0.0000)	(0.0010)	(0.0000)	(0.0000)	(0.0010)	(0.0000)
MON_{jt}	−0.013*	−0.009	−0.010	−0.012*	−0.008	−0.009
	(0.0337)	(0.1449)	(0.0992)	(0.0460)	(0.1642)	(0.1249)
TRF_{jt}	0.016*	0.005	0.013	0.018*	0.006	0.014*
	(0.0248)	(0.4816)	(0.0657)	(0.0127)	(0.4208)	(0.0432)
INV_{jt}	0.031***	0.014	0.023**	0.031***	0.013	0.023**
	(0.0002)	(0.1150)	(0.0040)	(0.0002)	(0.1309)	(0.0044)
FIN_{jt}	−0.030***	−0.045***	−0.037***	−0.032***	−0.047***	−0.039***
	(0.0001)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
$INT_{L_{i-jt}}$				−0.003	−0.002	−0.003
				(0.5241)	(0.6623)	(0.5488)
N	7571	7571	7571	7571	7571	7571

Standardized beta coefficients; p-values in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Declaration of originality

I confirm that this thesis is my own work and that I have not sought or used inadmissible help of third parties to produce this dissertation. I have clearly referenced all sources used. This work has not yet been submitted to another institution, neither in the same nor in a similar way and has not yet been published.

Paderborn 2022, March 7