New Developments of Parametric and Semiparametric Volatility Models with Application to Quantitative Risk Management

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Abstract

The underlying methodology of this thesis is based on the decomposition of a time series into a deterministic and stochastic component, where the latter is assumed to follow well-known time series models from the ARMA- as well as GARCH-class and the former is a non-negative slowly varying function, which can be estimated via a nonparametric smoothing method. A time series composed as described before is at best locally stationary. Hence, fitting a parametric model directly to such a series is in fact a misspecification. As a remedy, the deterministic trend or scale function has to be estimated and removed beforehand from the data, in order to fit a parametric model to the approximately stationary residuals. In this thesis local polynomial and penalised spline regression are employed for the estimation of the deterministic component. Various iterative plug-in algorithms are proposed for smoothing parameter selection and are implemented in R-packages, namely *smoots*, *esemifar* and *ufRisk*. All these packages are publicly available on *CRAN*. The wide applicability of these packages is illustrated with real data examples and the performance of the algorithms is validated within the scope of thorough simulation studies.

One of the major contributions of this thesis is the development and application of a new parametric time series model with long memory, namely the FI-Log-GARCH and various semiparametric extensions of this model as well as of other well-known time series models with short- and long memory. The FI-Log-GARCH is a fractional extension of the Log-GARCH. Theoretical properties such as necessary and sufficient conditions for stationary solutions, existence of finite fourth moments, explicit expression for the autocorrelation and central limit theorem for the sample mean are derived. Moreover, the FI-Log-GARCH is applied to forecast Value at Risk (VaR) and Expected Shortfall (ES) for various return series and is compared to conventional long memory GARCH models. All models are benchmarked via traffic light tests for both, VaR and ES, as well as via a newly developed model selection criterion. Our results show that the FI-Log-GARCH outperforms the other models in most cases. Subsequently, the semiparametric extension of the FI-Log-GARCH, namely the SEMI-FI-Log-GARCH, is proposed. Analogously, the latter and other semiparametric extensions of conventional long memory GARCH models are employed to model VaR and ES. Their performance is tested in a comprehensive comparative study where each model is applied to 20 different return series of major stock indices. Estimation of the scale function is carried out by means of a modified version of the SEMIFAR-algorithm, which is implemented in the R-package *esemifar*.

Our results indicate that Semi-LM-GARCH approaches are a meaningful substitute of parametric LM-GARCH models.

Besides local polynomial regression, another nonparametric smoothing method, namely penalised spline regression, gained more attention during the last decades due to advancing technology and especially to the upcoming complexity and scale of Big Data. So far the application of penalised splines has mainly occurred in the field of natural sciences and has rarely been applied in the context of empirical economic and financial research. An automatic iterative plug-in algorithm for uncorrelated data is proposed, which is based on an asymptotic approximation of the mean averaged squared error (MASE). Our proposal is tested in a comprehensive simulation study. Its application to real data examples shows that our proposal works very well in practice. Penalised spline regression under stationary time series errors is still a very unexplored field of research. Therefore, a newly developed iterative plug-in algorithm for correlated data is investigated in another thorough simulation study. It is found that the IPI performs very well even in scenarios where the underlying time series is composed of a complex deterministic trend and strong serial correlation in the stochastic component. In addition to that, the IPI is applied to economic time series data and benchmarked against conventional paramteric and nonparametric smoothing methods such as simple cubic regression, local cubic regression as well as the Hodrick-Prescott (HP) filter, in order to exemplify its practical relevance. Under certain conditions the HP filter is equivalent to a penalised spline. In this context, it is illustrated that the IPI for penalised splines may provide an attractive approach to obtain a data-driven estimate for the smoothing parameter of the HP filter. Furthermore, a penalised spline Log-ACD model is proposed, which is a special case of the ESEMIFAR model without long memory, and which is applied to daily average trade durations. It is observed that the IPI works very well in this context, too.

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Chapter 1

Introduction

1.1 Non- and Semiparametric regression for time series

The concept of semiparametric regression stems from the idea of decomposing a time series into a deterministic and stochastic component, where the former is estimated by a nonparametric smoothing technique and the latter via a parametric model (Dagum, 2010). This idea traces back to the astronomy of the seventeenth century. However, Persons (1919) was the first to state explicit assumptions on different components. Persons argued that a time series is composed of a secular trend, cyclical and seasonal movements as well as residual variations. In this thesis the focus lies on the decomposition of a time series into a nonparametric mean function and stationary errors. Let y_t denote a time series with $t = 1, \ldots, n$ equidistantly spaced observations. A fixed design nonparametric time series model can then be formulated as

$$y_t = m(\tau_t) + \epsilon_t, \tag{1.1}$$

where $m(\tau_t)$ denotes an at least twice differentiable smooth nonparametric mean function on $[0, 1]$ with $\tau_t = t/n \in (0, 1]$. Note that the standardization of τ_t is required for the consistent (nonparametric) estimation of m. The error process ϵ_t is assumed to be second-order stationary with zero mean, autocovariance function $\gamma_\epsilon(k)$ $(k \in \mathbb{Z})$ and variance $\sigma_\epsilon^2 = \gamma_\epsilon(0)$. We distinguish between three types of dependence structures of ϵ_t, namely (a) i.i.d. (independent and identically distributed), (b) short memory and (c) long memory such that (a) $\sum_{-\infty}^{\infty} \gamma_\epsilon(k) = \sigma_\epsilon$, (b) $0 < \sum_{-\infty}^{\infty} \gamma_\epsilon(k) < \infty$ and (c) $\sum_{-\infty}^{\infty} \gamma_\epsilon(k) = \infty$, respectively. Fitting a nonparametric regression model processes in two parts, i.e. (1.) the estimation of the deterministic and (2.) of the stochastic component. Note that the assumption that

y_t is a stationary process is a misspecification due to the nonparametric mean $m(\cdot)$. For instance, commonly used parametric models that are applied to analyse time series such as the ARIMA (autoregressive integrated moving average) and the FARIMA (fractionally integrated ARIMA, Granger and Joyeux, 1980) are likely to falsely capture deterministic changes in the mean as short- or (and) long-memory, respectively. In order to exemplify this phenomenon, we consider a simulated time series y_t with the trend function $m(\tau_t) = \tanh[6(\tau_t - 0.5)]$ and errors ϵ_t defined as White Noise with zero mean and unit variance (see Figure 1.1). Fitting a FARIMA$(0, d, 0)$ model to the non-stationary series depicted in Figure 1.1, where d stands for the long-memory or fractional differencing parameter, yields $\hat{d} = 0.228$, although $d = 0$. In contrast, if the FARIMA is fitted directly to the (stationary) error process shown in Figure 1.1 (a), we obtain $\hat{d} \approx 0$. Therefore, in order to correctly apply methods like the ARIMA or FARIMA model to analyze trend stationary processes, $m(\cdot)$ has to be estimated and removed beforehand from y_t.

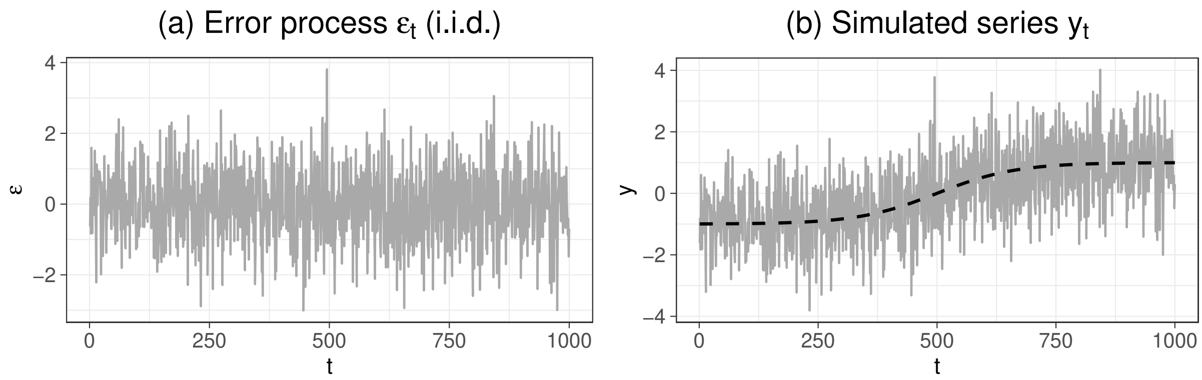

Figure 1.1: (a) - Generated error process ϵ_t modelled as White Noise with $E(\epsilon_t) = 0$ and $\text{var}(\epsilon_t) = 1$. (b) - Simulated series $y_t = m(\tau_t) + \epsilon_t$ with $n = 1000$ observations and true trend function $m(\tau_t) = \tanh[6(\tau_t - 0.5)]$ indicated by the dark-gray and black-dashed lines, respectively.

The deterministic component in model (1.1) can be estimated by a variety of nonparametric regression approaches (see e.g. Heiler, 1999; Fan, 2005, for a summarising overview). However, in this thesis only two modern smoothing techniques are discussed, namely local polynomial and penalised spline regression.

Local polynomial regression can be considered a smoothing technique that combines the local use of some parametric regression model with the moving average. Both methods have a long history, which traces back to the nineteenth century (see Loader, 2006 and Dagum, 2010, for a historical review). The moving average gained more attention not before the mid 1960s, when Nadaraya (1964) and Watson (1964) independently introduced a generalized version of the conventional moving average, namely the kernel estimator.

Similarly, local regression was primarily employed for estimating seasonality in a time series (see e.g Fisher, 1937; Jones, 1943) and Stone (1977) and Cleveland (1979) were the first ones to introduce the concept of local polynomial smoothing. From this point on, local polynomial regression rapidly became a relevant statistical tool. One of the major advantages of this method is that it provides finite sample solutions to bias correction problems which arise in kernel theory (see e.g. Hastie and Loader, 1993; Jones, 1993; Cheng et al., 1997). Moreover, local regression can be approached as an extension of kernel methods as its asymptotic properties are directly derived from results of the latter (see e.g. Wand and Jones, 1995; Fan and Gijbels, 1996). However, theoretical results from the era of the early stages of local polynomial regression, i.e. in the late 1980s and 1990s, were mostly obtained under the assumption that the errors are i.i.d. (independent and identically distributed) or at least uncorrelated random variables (see e.g Ruppert and Wand, 1994; Fan, 1992; Fan and Gijbels, 1992; Fan et al., 1996). Statistical properties of the local polynomial estimator under short range dependence were studied by Fan and Gijbels (1996), Masry and Fan (1997), Masry (1996), Härdle and Tsybakov (1997), Härdle et al. (1998), Vilar-Fernández and Vilar-Fernández (1998), Opsomer et al. (2001) and Francisco-Fernández and Vilar-Fernández (2001). The extension to long-memory errors was the next logical step, as many empirical studies revealed the presence of long-range dependence in various types of time series such as squared returns, trade duration, temperature and air pollution data (see e.g. Ding et al., 1993a; Ding and Granger, 1996; Andersen and Bollerslev, 1997; Andersen et al., 1999; Baillie and Chung, 2002; Cotter, 2005; Beran et al., 2015; Beran, 2017; Gil-Alana et al., 2020). First important theoretical results for kernel estimators with long-memory were obtained by Hall and Hart (1990) (see also Ray and Tsay, 1997) and finally extended to local polynomial regression in Beran and Feng (2002b), Beran and Feng (2002c) and Beran and Feng (2002a). The authors adapted the local polynomial estimator introduced by Stone (1977) and Cleveland (1979) to nonparametric regression with short- or long range dependent as well as antipersistent errors (see also Feng, 2007).

Approximating a function by means of basis functions such as Fourier series, B-splines or truncated polynomials has a very long history. A popular approach that stems from this branch of literature is penalised spline (P-spline) regression. The original idea of P-splines traces back to Parker and Rice (1985) and O'sullivan et al. (1986), who proposed to use a set of basis functions in combination with a penalty term. Following their approach Eilers and Marx (1996) introduced the concept of P-spline regression. This smoothing method has gained more attention during the last decades, as it offers an attractive alternative to conventional nonparametric methods. The smoothness of a P-spline is controlled via a smoothing parameter. Wand (1999) adapted the direct plug-in (DPI) method proposed by Ruppert et al. (1995) to P-spline regression and provided a closed-form asymptotic approximation for the mean average squared error (MASE), in order to determine the smoothing parameter directly from the data. Based on this approach Letmathe and Feng (2022) developed a fast, simple and reliable method to select the smoothing parameter data-driven. The authors proposed an iterative plug-in (IPI) algorithm (Gasser et al., 1991) for cross sectional data. The properties of the P-spline model with uncorrelated errors were e.g. discussed in Hall and Opsomer (2005), Kauermann (2005), Li and Ruppert (2008), Claeskens et al. (2009) and Wang et al. (2011). However P-splines under stationary time series errors are still very unexplored. Relevant studies in this context were conducted by e.g. Krivobokova and Kauermann (2007) and Feng and Härdle (2020). In the latter study the authors proposed an extension of the IPI for cross sectional data to stationary time series with short memory errors (see also Letmathe, 2022a).

The main challenge in the context of nonparametric and semiparametric regression is the selection of a smoothing parameter or bandwidth, b say, that controls the trade-off between variance and bias of $\hat{m}(\cdot)$ and affects the estimation of the dependence structure of the trend-adjusted residuals, i.e. $\hat{\epsilon}_t = y_t - \hat{m}(\tau_t)$, as well (see e.g. Beran, 2017). Two extreme cases are $b = 0$ and $b = \infty$. The former represents the most complex possibility which results in an estimated trend that is equal to the observations, i.e. $\hat{m} = y_t$ (overfitting) and the latter the most simple case with $\hat{m} \equiv \bar{y}$ (undersmoothing). To exemplify the importance of a suitably chosen smoothing parameter a local polynomial smoother with 4 different bandwidths is applied to simulated data, which is similarly generated as in the previous example (see Figure 1.1). For this example ϵ_t is generated based on a FARIMA$(1, d, 0)$ models with $\phi = 0.5$, $d = 0.3$ and unit innovation variance. Note that ϕ denotes the AR-coefficient. The resulting fits are depicted in Figure 1.2.

Apparently, the bandwidths employed in (a) and (b) are clearly too small as the estimated trends are too wiggly, whereas the bandwidth chosen in (d) leads to over-smoothing. The bandwidth used in (c) produces a reasonable fit that almost coincides with the true trend function. Fitting FARIMA$(p, d, 0)$ models to the corresponding trend adjusted residuals $\hat{\epsilon}_t$ yields: (a) short- and long-memory are clearly underestimated; (b) short-memory is overestimated whereas long-memory is underestimated; (c) short-memory and long-memory are slightly over- and underestimated, respectively; (d) short-memory is slightly underestimated and long-memory is overestimated. The FARIMA models are fitted by means of the *R* function *fracdiff* from the package under the same name. The corresponding estimates are listed in Table 1.1.

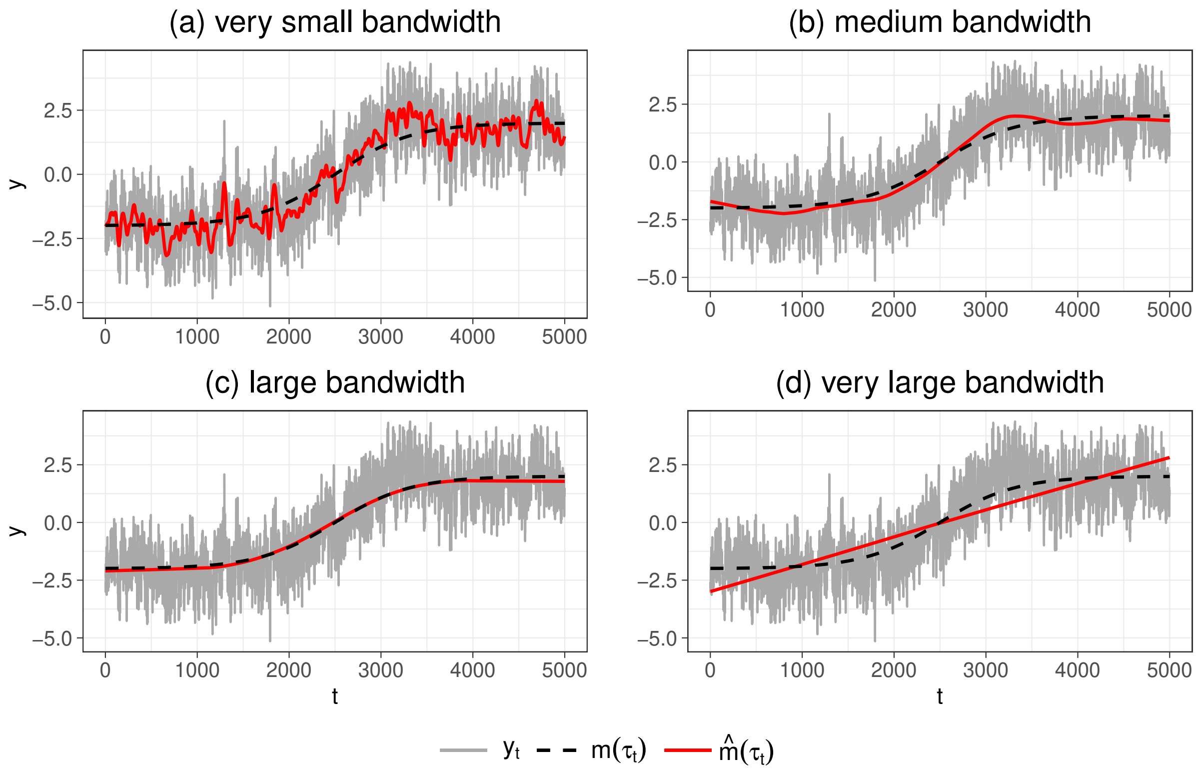

Figure 1.2: All pictures show the same simulated time series $y_t = m(\tau_t)+\epsilon_t$ with $n = 5000$ observations, trend function $m(\tau_t) = \tanh[6(\tau_t - 0.5)]$ and error process ϵ_t generated as a FARIMA$(0.5, 0.3, 0)$ with innovation variance one. In all figures local polynomial fits (red solid lines) with an epanechnikov kernel and different bandwidths are shown: (a) very small bandwidth; (b) medium bandwidth; (c) large bandwidth; (d) very large bandwidth. The true trend $m(\tau_t)$ is indicated by the black-dashed line.

Conventional methods for determining the optimal smoothing parameter are for instance Mallow's C_p (Mallows, 1973), the Akaike information criterion (Akaike, 1974), cross validation (Mosier, 1951) or generalised cross validation (Wahba, 1977; Craven and Wahba, 1978). In the presence of correlated errors, particularly under long range dependence, these criteria usually tend to select a bandwidth that is too small and, consequently, overfit the data (see e.g. Opsomer, 1997; Opsomer et al., 2001). Moreover, manually determining a bandwidth via trial and error can be misleading and prone to an erroneous estimation of the correlation structure. As a remedy for overfitting, the dependence structure of the errors has to be taken into account as illustrated by Altman (1990a), Hart (1991), Beran and Feng (2001), Ray and Tsay (1997), Opsomer (1997) and Opsomer et al. (2001) as well as Beran and Feng (2002a), Beran and Feng (2002b) and Beran and Feng (2002c) in the context of local smoothing. Within the P-spline framework relevant studies were conducted by e.g. Currie and Durban (2002), Durbán and Currie (2003), Krivobokova and Kauermann (2007) and Feng and Härdle (2020). Most of the smoothing

Table 1.1: Estimates for the long memory parameter $\hat{d}$ and AR-coefficient$\hat{\phi}$ obtained by fitting FARIMA$(1, d, 0)$ models to the trend adjusted residuals $\hat{\epsilon}_t = y_t - \hat{m}(\tau_t)$ based on (a) a very large bandwidth, (b) a medium bandwidth, (c) a large bandwidth and (d) a very large bandwidth.

	(a)	(b)	(c)	(d)
$\hat{\boldsymbol{d}}$	0.000	0.178	0.282	0.343
$\hat{\boldsymbol{\phi}}$	0.000	0.618	0.520	0.462

parameter selection criteria introduced before the 1990´s were corrected versions of the residual sum of squares (RSS) and have a very slow rate of convergence. Subsequently, innovative approaches with higher rates of convergence, namely plug-in methods (see Gasser et al., 1991; Ruppert et al., 1995 among others) and double-smoothing (DS) (see Müller, 1985; Härdle et al., 1992; Heiler and Feng, 1998), emphasized a more effective estimation of the mean integrated squared error (MISE) and mean averaged squared error (MASE). Despite the fact that plug-in bandwidth selectors have a slower rate of convergence in comparison to a DS selector, plug-in algorithms are generally more simplistic and more easily adapted to dependent errors. The main idea behind the plug-in approach is to insert suitable estimates of unknown parameters into an explicit formula for the asymptotic optimal bandwidth b_A, which is, for instance, based on an approximation of the MISE or MASE. However, in the case of correlated errors, the estimation quality of the dependence structure depends on that of $\hat{m}$ and the accuracy of $\hat{m}$ on the bandwidth b. It was shown that modified versions of the iterative plug-in approach (Gasser et al., 1991) where the estimation of the dependence structure and the bandwidth is improved in each iteration work very well for semiparametric regression with time series errors (see Beran and Feng, 2002a; Beran and Feng, 2002b; Beran and Feng, 2002c; Feng et al., 2016; Feng et al., 2020b; Feng et al., 2021; Feng et al., 2022b). Therefore, in this thesis various algorithms that are based on the iterative plug-in procedure are proposed. These algorithms are invariant point search procedures that start with an initial bandwidth b_0 and converge to a fixpoint, namely the asymptotically optimal bandwidth b_A. They are applied throughout this thesis and are implemented in recently published *R* packages called *smoots* (Feng et al., 2022a), *esemifar* (Letmathe et al., 2021b), *ufRisk* (Letmathe et al., 2022a) and *quarks* (Letmathe, 2022b). Moreover, various new semiparametric time series models are introduced. The proposed algorithms and, subsequently, the corresponding *R* packages can be easily applied to time series that are assumed to have a multiplicative form (e.g. volatility models) by log-transforming the original process, in order to establish an additive model form as given in (1.1).

1.2 Modelling volatility

The autoregressive conditional heteroscedasticity (ARCH) model and its generalization, the generalized ARCH (GARCH) model, introduced by Engle (1982) and Bollerslev (1986a), respectively, are well-known approaches for modelling non-constant conditional volatility. Both models and corresponding extensions imply exponentially decaying autocorrelations of the squared innovations and do not control for long memory in the conditional dynamics. The phenomenon of long memory in volatility was first discovered by Ding et al. (1993b) in the S&P 500 daily closing index. Subsequently, more evidence for the presence of long memory in absolute or squared observations of financial time series was found (see e.g. Ding and Granger, 1996, Andersen and Bollerslev, 1997, Andersen et al., 1999 and Cotter, 2005, among others). Consequently, various GARCH models, which are capable of modelling long memory, were proposed. The fractionally integrated GARCH model (FIGARCH) was introduced by Baillie et al. (1996) and was applied in many empirical studies to model long term dynamics in volatility of different financial time series (see e.g. Bollerslev and Mikkelsen, 1996; Tse, 1998; Beine et al., 2002; Baillie and Morana, 2009). Furthermore, Bollerslev and Mikkelsen (1996) proposed an extension of the exponential GARCH (EGARCH) (Nelson, 1991) and developed the fractionally integrated exponential GARCH (FIEGARCH). In this model the logarithm of the conditional variance is modelled as a fractionally integrated process. In addition to that, the EGARCH and FIEGARCH are capable to model so-called leverage effects, which usually have only short-term impacts on the dependence structure. Moreover, Ding et al. (1993a) proposed the so called asymmetric power GARCH (APARCH) model. It controls for the power transformation of the volatility process and the asymmetric absolute residuals, in order to avoid misspecification for non-normal data. The extension to the fractionally integrated APARCH (FIAPARCH), which was then proposed by Tse (1998), combines the FIGARCH with the APARCH. Another model that is capable of capturing persistence in volatility is the ARCH(∞) model introduced by Robinson (1991) and further investigated by Giraitis et al. (2000), Kazakevičius and Leipus (2002) as well as Douc et al. (2008). Recently, Feng et al. (2020a) introduced the FI-Log-GARCH model, which is a fractionally integrated version of the Log-GARCH (Geweke, 1986; Milhøj, 1987a; Pantula, 1986). Moreover, Royer (2022) proposes an ARCH(∞) extension of the APARCH that accounts for conditional asymmetry in the presence of severe long memory. Its specification is very general and nests the ARCH(∞) as well as the Threshold-ARCH(∞) (see Bardet and Wintenberger, 2009).

Empirical studies have revealed that long memory GARCH (LM-GARCH) models are very successful in accurately forecasting the conditional volatility of asset returns and often outperform short memory GARCH type models (see, among others, Giot and Laurent, 2003; Degiannakis*, 2004; Tang and Shieh, 2006; Grané and Veiga, 2008; Härdle and Mungo, 2008; Baillie and Morana, 2009; Demiralay and Ulusoy, 2014; Aloui and Ben Hamida, 2015; Letmathe et al., 2022b; Royer, 2022). However, another branch of literature argues that long memory in the conditional volatility might partly stem from deterministic structural shifts in the unconditional variance (see e.g. Lamoureux and Lastrapes, 1990 and Mikosch and Stărică, 2004). For instance, Beran and Ocker (2001) discovered the existence of a non-constant deterministic scale function in some volatility series by means of the semiparametric fractional autoregressive (SEMIFAR) model (Beran and Ocker, 1999). Furthermore, Feng (2004) found that financial return series exhibit conditional heteroscedasticity and slowly changing unconditional volatility simultaneously. Such a series can be transformed into a weakly stationary process by removing the deterministic component from the original series, as was illustrated by Feng (2004)and by Van Bellegem and Von Sachs (2004). The authors assume a multiplicative decomposition of volatility into a conditional and unconditional component. They proposed to estimate the latter by means of a kernel smoother of the squared residuals. Engle and Rangel (2008) as well as Brownlees and Gallo (2010) assumed another multiplicative decomposition, which is based on exponential quadratic and penalised B-splines, respectively. Moreover, Mazur and Pipień (2012) introduced the almost periodically correlated (APC-) GARCH, where the scaling function is parameterized by means of the Flexible Fourier Form (Gallant, 1981; Gallant, 1984). Recently, Amado and Teräsvirta (2014) proposed the time varying GARCH model under an equivalent assumption (see also Amado and Teräsvirta, 2008; Amado and Teräsvirta, 2013; Amado and Teräsvirta, 2017) and underline the empirical significance of taking deterministic changes in the unconditional variance of financial return series into account.

In this thesis a new parametric and various semiparametric long memory GARCH (Semi-LM-GARCH) models, which belong to a general class of non-stationary volatility models as outlined in Sucarrat (2019), are introduced and applied. It is proposed to estimate the time varying unconditional variance by means of an adapted version of the SEMIFAR algorithm (Beran and Feng, 2002a) with a local polynomial estimator. Subsequently, the deterministic component is removed from the data and a LM-GARCH model is fitted to the approximately stationary residuals.

1.3 Summary of contents

In Chapter 2 the *R* package *smoots* is introduced. One of the main features of this package is local polynomial smoothing of trend-stationary time series with automatic bandwidth selection. In *smoots* a fully data-driven iterative plug-in algorithm for bandwidth selection under stationary time series errors is implemented. This algorithm is based on minimizing the asymptotic MISE of the bandwidth. An unknown quantity in the asymptotically optimal bandwidth is the variance factor which can be estimated parametrically by assuming that the stochastic part of the time series follows for example an ARIMA model. The variance factor can also be estimated fully nonparametrically by employing another IPI-approach for a lag-window estimator of the spectral density following Bühlmann (1996). All available options for executing the algorithm, namely the order of local polynomial, the kernel weighting functions and the so-called inflation factors are explained in detail. In addition to that, functions for data-driven estimation of the first and second derivatives of the trend are implemented (see Feng, 2007). Moreover, two new semiparametric models for financial time series are proposed, in order to demonstrate further application of this package. Firstly, a Semi-Log-GARCH model is defined by adding a scale function into the Log-GARCH (logarithmic GARCH) (Pantula, 1986; Geweke, 1986; Milhøj, 1987a). Secondly, a Semi-Log-ACD model is proposed as an extension of the Type I Log-ACD (Bauwens and Giot, 2000a), which is closely related to the Semi-Log-GARCH.

An extension of the *smoots* package to data-driven trend estimation under long memory is introduced in Chapter 3. This package is called *esemifar*, and analogously to *smoots*, incorporates a data-driven iterative plug-in algorithm, which is based on the SEMIFAR (semiparametric fractional autoregressive) algorithm. The SEMIFAR (Beran and Feng, 2002c) and its exponential version the ESEMIFAR model (Beran et al., 2015), which is applicable to non-negative time series following a semiparametric multiplicative model form, are designed for simultaneous modelling of stochastic trends, deterministic trends and stationary short- and long-memory components in a time series. And, analogously to Feng et al. (2022b) *esemifar* is used within the scope of a semiparametric log-local-linear growth model for analyzing quarterly G7 GDP data. Moreover, it was first indicated by Beran et al. (2015) that the (type 1) Log-ACD model introduced by Bauwens and Giot (2000b), Bauwens et al. (2008) and Karanasos (2008) can be represented as an EFARIMA model. Subsequently, it was shown by Feng and Zhou (2015) that the EFARIMA and ESEMIFAR can be redefined as a FI-Log-ACD and a Semi-FI-Log-ACD, respectively. The use of *esemifar* is illustrated with regards to the Semi-FI-Log-ACD by modelling log-transformed trading volume of the S&P500.

In Chapter 4 a new long memory GARCH model is introduced. Due to its construction it is coined a FI-Log GARCH model. It is a fractional integrated extension of the (symmetric) Log-GARCH. Theoretical properties of the FI-Log-GARCH are derived. Moreover, the FI-Log-GARCH(1, d, 1) is applied to carry out one-day rolling forecasts of the VaR (value at risk, Morgan, 1996) and ES (expected shortfall, Acerbi and Tasche, 2002) following the requirements of the BSBC (Basel Committee on Banking Supervision, 2016; Basel Committee on Banking Supervision, 2017) for several return series. It is found that the proposed model works very well in practice. Results of the FIGARCH, FIAPARCH and FIEGARCH models, fitted by means of the `G@RCH 8.0` package implemented in the econometric software `Ox-metrics`, are used as comparisons. A reasonable model selection criterion based on different traffic light tests, including a most recently proposed traffic light test of ES by Costanzino and Curran (2018), is defined. In our empirical study the FI-Log-GARCH model often outperforms the other models, thus providing a useful alternative to existing long memory volatility models.

Semiparametric extensions of the long memory GARCH models, presented in Chapter 4, are introduced in Chapter 5. It is proposed to estimate the scale function via a modified SEMIFAR algorithm (Beran and Feng, 2002a). Practical performance is demonstrated by a comparison study, in which the parametric long memory GARCH models and their semiparametric counterparts are applied to model quantitative risk measures of daily return series of 22 major stock indices. For each model the one-step ahead out-of-sample forecasts of the value at risk (VaR) and expected shortfall (ES) at the 99%- and 97.5% confidence level with a forecast horizon of approximately one year, as required by the latest regulations proposed by the Basel committee (see Basel Committee on Banking Supervision, 2017) are calculated. The results of this comparative study reveal that the semiparametric long memory GARCH models, in particular the Semi-FI-Log-GARCH, are an attractive alternative.

The main objective of Chapter 6 is the development of an iterative plug-in algorithm for penalised spline regression under uncorrelated errors. This algorithm is based on a closed-form asymptotic approximation for the MASE obtained by Wand (1999). Moreover, the author derived a fast and simple DPI rule to determine the smoothing parameter directly from the data. Analogously to the algorithms presented in Chapter 2 and 3, we follow the idea of Gasser et al. (1991). Based on Wand's approximation three different algorithms to determine the optimal smoothing parameter are developed. In order to assess the performance of the algorithms, a comprehensive simulation study is conducted. For the estimation of the variance of the error term, a difference based variance estimator proposed by Gasser et al. (1986) is employed. The results of the simulation study and the application to real data examples illustrate the good performance of our proposal and that it works very well in practice.

In Chapter 7 the IPI for dependent data introduced by Feng and Härdle (2020) is examined within the scope of a comprehensive simulation study, in which the estimator is tested in a total of 52 different cases. The findings in this chapter confirm that the algorithm performs very well even in scenarios where the underlying time series is composed of a complex deterministic trend and strong serial correlation in the stochastic component. Moreover, the IPI is applied to economic time series data and its performance is compared to conventional paramteric and nonparametric smoothing methods such as simple cubic regression, local cubic regression as well as the Hodrick-Prescott (HP) filter (Hodrick and Prescott, 1997), in order to exemplify its practical relevance. Paige and Trindade (2010) showed that under certain conditions the HP filter is equivalent to a P-spline. In this context, it is illustrated that the IPI for P-splines may provide an attractive approach to obtain a data-driven estimate for the smoothing parameter of the HP filter. Furthermore, a P-spline Log-ACD model is proposed, which is a special case of the ESEMIFAR model (Beran et al., 2015) without long memory, and which is applied to daily average trade durations. It is observed that the IPI works very well in this context, too.

Chapter 8 summarizes the contributions of this thesis and gives prospects for further research. Throughout this thesis *R* and partly *OxMetrics* are used for the empirical studies. Mainly the *R* packages *smoots*, *esemifar* and *ufRisk* and corresponding dependencies of these packages are employed. Moreover, the *G@RCH* software implemented in *OxMetrics* is used in Chapter 5.

Chapter 8

Concluding Remarks

The contribution of this thesis can be considered an incremental process progressing from chapter to chapter. Proposed methods and models are continuously adapted, improved and extended. The development of IPI-algorithms for automatic smoothing parameter selection, their implementation in R-packages and, subsequently, the publication on CRAN enables academics and practitioners to easily work with trend stationary time series. The wide applicability of these algorithms is illustrated by their application within the scope of various semiparametric time series models throughout this work, particularly in Chapters 2, 3, 5, and 7. Moreover, a new long memory GARCH model, namely the FI-Log-GARCH, is introduced in Chapter 4. The employment of this model and its semiparametric extension, the Semi-FI-Log-GARCH (Chapter 5), to measure quantitative risk demonstrates that these models provide attractive substitutes to conventionally used approaches in this context. The main contributions of this thesis are summarized in detail as follows:

The methodological background of the R package *smoots* (version 1.0.1) is summarized in Chapter 2. The main functions in this package are explained in detail. Two new semiparametric models, namely the Semi-Log-GARCH and the Semi-Log-ACD, are introduced and applied to different kinds of non-stationary financial time series. Overall the employment of *smoots* within the scope of both models works very well. Further research based on the proposals of this chapter, such as the development of suitable forecasting procedures and stationary tests of the errors or significance of the deterministic trend, are interesting topics and it is planned to incorporate those features in following versions of the *smoots* package. Although *smoots* has been developed under the assumption of short memory errors, it is found that the errors of some real data examples used in Chapter 2 might exhibit clear long range dependence.

Subsequently, in Chapter 3 the development of a supplementing *R*-package for the *smoots* package, namely *esemifar*, is exemplified. In this regard an adapted version of the SEMIFAR-algorithm is introduced. Moreover, the implementation of this package is comprehensively described. The usage of two main functions is explained and illustrated by application to various non-stationary time series with long-memory. Analogously to Chapter 3, *esemifar* is used in the context of a Semi-FI-Log-ACD model, which is employed to model trade duration of two financial assets. Overall, the estimation results are quite satisfactory and illustrate the wide applicability of our proposal. Extensions of *esemifar* are the implementation of a forecasting procedure and the non-parametric estimation of the stochastic part of the model by means of e.g. a local Whittle-, GPH- or wavelet-estimator. A further, more comprehensive adaptation is the generalisation of the SEMIFAR-algorithm to smooth functional time series. The reader is referred to Schäfer and Feng (2021), Schäfer (2021a) and the corresponding R-package *DCSmooth* (Schäfer, 2021b). The authors propose double conditional smoothing schemes for the nonparametric estimation of functional time series under non-dependent and spatial ARMA errors. The extensions to spatial FARIMA errors is implemented in *DCSmooth*, however, only at an experimental stage. The methodological development of this extension will be subject to future studies.

In Chapter 4 a new long-memory volatility model (FI-Log-GARCH) is proposed and its theoretical properties are investigated in detail. Furthermore, the FI-Log-GARCH is employed to obtain one-day rolling forecasts of the VaR and ES for several return series. In addition to that, a new model selection criterion is proposed, in order to assess the forecasting quality for both risk measures. A small comparative study indicates that the FI-Log-GARCH provides an attractive alternative to existing long memory volatility models. Possible extensions of the FI-Log-GARCH are the implementation of long memory in the conditional mean, analogously to the FARIMA-FIGARCH, or the inclusion of asymmetric volatility effects. Moreover, the simultaneous consideration of a slowly changing unconditional mean and variance is an interesting topic for future research as well.

The semiparametric extension of the FI-Log-GARCH is discussed in Chapter 5. Different classes of semiparametric long memory GARCH models are proposed, in order to simultaneously model conditional heteroscedasticity and a slowly changing unconditional variance. The latter is estimated by means of a SEMIFARIMA model with a local polynomial smoother and, consequently, bandwidth selection is carried out by means of the corresponding algorithm, which is introduced in Chapter 3 and implemented in *esemifar*. All models are employed to obtain out-of-sample forecasts of VaR and ES. The performance of our proposals is assessed via traffic light tests and a new model selection criterion introduced in Chapter 4. A comparative study is conducted and the results show that Semi-LM-GARCH approaches are meaningful substitutes to parametric LM-GARCH models. A more comprehensive empirical study in which our proposals are benchmarked against conventional methods, e.g. nonparametric approaches such as historical simulation, is yet to be conducted. Moreover, the quality of VaR and ES forecasts might be even further improved by modifying our approach to conditional distributions that allow for modelling skewness. Another possibility is to simply bootstrap the empirical distribution function of the observed return series or estimate VaR and ES based on the empirical quantiles of the residuals.

New IPI-algorithms for selecting the smoothing parameter in the penalised spline framework are proposed in Chapter 6. A comprehensive simulation study is conducted and the corresponding results reveal the convincing performance of these algorithms. The application to real data examples illustrates the wide applicability of our proposal and that it works very well in practice. A further development of our proposal could be the estimation of the variance of the residuals, which is estimated via a difference based variance estimator. A possible improvement could be the repeated estimation of the variance based on the residuals or a combination of both approaches, where the difference based estimator is used in the first one or two iterations and an estimator obtained from the trend-adjusted residuals in the remainder. Moreover, the development of a P-spline IPI-algorithm for time series data could offer an interesting topic for future research. Possible extensions of our proposal could be the short memory and the long memory case.

The extension of the IPI-algorithm for uncorrelated data to time series with short memory errors is proposed in Chapter 7. The algorithm is tested within the scope of a comprehensive simulation study and it is revealed that the IPI performs well, even if it is applied to complex time series processes. Moreover, it is shown that our proposal may provide an attractive approach to obtain a data-driven estimate for the smoothing parameter of the Hodrick Prescott filter. A more thorough investigation on this matter is an interesting topic for future studies. Furthermore, a P-spline Log-ACD model is proposed, which is a special case of an ESEMIFAR without long memory and where the deterministic component is estimated via P-spline regression. Two application examples show that the IPI works very well in this context, too. In addition to that, our proposal can easily be applied within the scope of a P-spline GARCH and, subsequently, could be employed to model VaR and ES. Moreover, the extension of our proposal to the long memory case is another relevant topic for further research.

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I hereby declare that I prepared this thesis entirely on my own and have not used outside sources without declaration in the text. Any concepts or quotations applicable to these sources are clearly attributed to them. This thesis has not been submitted in the same or substantially similar version, not even in part, to any other authority for grading and has not been published elsewhere.

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Paderborn, October 20, 2022

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Chapter 2

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Chapter 5

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