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Abstract

Research has found that downstream bundling aggravates the problem of double marginalization in a decentralized channel, but reduces the intensity of downstream price competition when trading homogeneous goods. We study the validity of those results in a set-up where the traded goods have heterogeneous product qualities. We find that the quality relation between the goods determines whether the competition reduction effect of bundling outweighs the aggravation of double marginalization in a decentralized channel. Thus, the quality relation between the goods determines the profitability of downstream bundling. The underlying market consists of a distribution channel with two downstream firms and two price-setting monopolistic upstream producers. One upstream firm sells *good 1* exclusively to one downstream firm and the other upstream firm sells *good 2* to both downstream firms. The downstream firms compete in prices and the two-product downstream firm has the option to bundle both goods. In particular, we find bundling to be profitable for the two-product downstream firm only when the quality of *good 2* exceeds the quality of *good 1*. However, we find bundling *always* to be profitable when the production process is controlled by the downstream industry. The impact on total welfare is ambiguous and depends on the distribution of market power in the channel and the quality levels of the goods.

JEL classification: D21; D61; L11; L15

Keywords: double marginalization; downstream bundling; leverage theory; quality differentiation

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1 Introduction

Selling individual goods as a bundle for one price is a selling strategy often considered by firms. Whether this strategy is attractive for a company and might lead to higher profits than selling the goods on a standalone basis might depend on whether the traded goods are homogeneous or heterogeneous. This paper focuses on heterogeneous products that differ in their quality levels. It studies the impact of various qualities between goods on the retail bundling incentives in a decentralized channel and its impact on welfare. Following Häckner (2000), we interpret the customer's valuation for a good as product quality and use this interpretation to implement the goods' quality levels into our theoretical model.

Our market follows the market proposed in Heinzel (2019) and consists of a decentralized market with two goods, two upstream firms, and two downstream firms. Each of the two upstream firms is a monopolistic producer of one good. One upstream firms sells its *good 1* exclusively to one downstream firm, whereas the other one sells its *good 2* to both downstream firms. Thus, one downstream firm supplies one product as a monopolist and competes with the other downstream firm in the second product market. This two-product downstream firm has the option to purely bundle the goods or to sell them separately to the final customers. The downstream firms compete in prices. We argue that this exclusivity relationship between upstream and downstream industry can be found in the retail industry, for instance: Retail stores often purchase goods specifically for them produced from manufacturers and sell them as so-called store brands to the final customers. Additionally, streaming service providers such as Netflix often supply exclusive content on their platforms for which they acquired the exclusive streaming rights.

Our analysis enables to gain insides into the motives and market consequences of downstream bundling. It is already shown that downstream bundling aggravates the problem of double marginalization (e.g., Bhargava, 2012), and that it might reduce the intensity of downstream price competition (e.g., Carbajo et al., 1990). Heinzel (2019) studies the interplay of both effects in a vertically related market with competition in one downstream market when the traded goods are of the same quality. Contrary to that, the present study assumes the goods to have *heterogeneous* and exogenous qualities. Therefore, we study whether the effects announced by Bhargava (2012) and Carbajo et al. (1990) also hold for heterogeneous goods with different quality levels and how both effects affect each other, and consequently the profitability of downstream bundling, in a market set-up as described by Heinzel (2019).

We investigate the following research questions: *How do various degrees of product qualities impact the selling decision of the two-product downstream firm? How do profitable downstream bundling and product qualities affect market results, especially welfare outcomes? How does the distribution of market power in the channel affect the findings qualitatively?*

The bundling literature that deals with a market set-ups as our downstream market is two-fold. One part of the literature focuses on bundling as a strategic tool to affect the competition in the second market (see e.g., Carbajo et al., 1990; Martin, 1999; Egli, 2007; Vamosiu, 2018). The other part of this literature deals with bundling as a strategy to deter entry or to foreclose the second market (e.g., Whinston, 1990; Choi and Stefanadis, 2001; Carlton and Waldman, 2002; Nalebuff, 2004; Peitz, 2008; Hurkens et al., 2019). The very same part of the literature stream is called leverage theory of bundling, in which the multi-product downstream firm might leverage its market power from the monopoly into the duopoly by bundling. Since the distinction between these two parts of the literature is not always clear, we refer to both parts of the literature as *leverage theory* but focus in our work on the competitive aspect of bundling.

Close to our paper are the articles by Carbajo et al. (1990) and Martin (1999). Carbajo et al. (1990) consider a set-up with a two-product firm that competes with a one-product firm in one product market but is a monopolist in the other product market. They observe that bundling lowers the degree of competition between the firms, given that they engage in price competition. This effect leads to bundling always being more profitable than separate selling. Given quantity competition, separate selling may be more profitable than bundling in their set-up. They additionally find that bundling always reduces consumer surplus but has ambiguous effects on social welfare. This implies that a firm's bundling strategy may affect the intensity of oligopolistic competition and this, in turn, may have an impact on the bundling decision itself. Martin (1999) considers the same market structure as Carbajo et al. (1990) but concentrates on quantity competition and considers complementarity as well as substitutability between the goods. He finds that bundling may change or create substitutability relationships between the traded goods. Furthermore, bundling always reduces consumer surplus and social welfare in the equilibrium in Martin's model.

Besides that, our study also connects to the literature of vertical product differentiation even though it does not directly add to it since we mainly focus on different quality levels between independent goods. One study that examines the interplay of product quality and bundling is by Banciu et al. (2010). They study a monopoly market and introduce goods with different quality ratings as well as fixed available resources and zero variable costs. They find that pure bundling is the optimal strategy compared to mixed bundling and separate selling when the products are vertically differentiated and the capacities are unconstrained. Honhon and Pan (2017) study a monopolist who either offers vertically differentiated components or bundles those components but, contrary to Banciu et al. (2010), they consider positive variable cost. They contradict the results of Banciu et al. (2010) by showing that pure bundling, mixed bundling, or separate selling can be the optimal strategy under unlimited capacity. Honhon and Pan also demonstrate that pure bundling may be optimal over mixed bundling or separate selling for a superadditive quality relationship and that separate selling may be optimal for all product categories.

In summary, Banciu et al. (2010) and Honhon and Pan (2017) show that the quality relationship between the components affects the optimality of the bundling strategies. They illustrate that the introduction of vertical product differentiation results in situations where pure bundling dominates mixed bundling and pure components selling. This finding is in contrast to previous literature on bundling (Schmalensee, 1984; McAfee et al., 1989) that shows that mixed bundling (weakly) dominates the other two strategies. These literature results demonstrate that product qualities should be considered when analyzing bundling.

Our paper also contributes to the literature strand that evaluates downstream bundling in a decentralized channel. Other articles within this research field are, for instance, Bhargava (2012), Chakravarty et al. (2013), Girju et al. (2013), Cao et al. (2015), Chen et al. (2016), Ma and Mallik (2017), Cao et al. (2019), and Giri et al. (2020). The article by Bhargava (2012) is especially connected to our paper. He illustrates that in a channel with a monopolistic retailer and two monopolistic manufacturers, retail bundling induces both manufacturers to overprice their goods. Thus, bundling aggravates the double marginalization problem and this makes bundling the inferior strategy compared to separate selling for the retailer. Also related to our work is the paper by Ma and Mallik (2017). They evaluate bundling in a channel that consists of one retailer, one manufacturer, and two vertically differentiated goods (a premium and a basic good). They show that the results of Banciu et al. (2010) and Honhon and Pan (2017) regarding the (possible) dominance of pure bundling under vertical differentiation over mixed bundling and separate selling hold under vertical differentiation and double marginalization.

Recapitulating, the existing leverage theory research has mainly focused on non-vertical markets. The existing literature on downstream bundling has mainly investigated distribution channels without downstream competition and widely disregarded the impact of qualities. We add to the bundling literature by being, best to our knowledge, the first paper to evaluate downstream bundling in a distribution channel, where the goods differ in qualities and the downstream market is of a leverage theory set-up. Hitherto only Heinzel (2019) evaluates downstream bundling in a distribution channel with such a leverage structure in the retail market. He finds that under price competition, the positive effect of bundling in the form of a reduction in the intensity of competition can outweigh the negative effect of bundling in the form of an aggravated double marginalization problem. The final effects depend on the upstream firms' marginal production costs. Contrary to this, under quantity competition, retailer bundling is never profitable. In Heinzel's model, both traded goods have symmetric quality levels and are not in the focus of his analysis, which is in contrast to our model.

The major findings of our study can be summarized as follows. We find that the quality of good 2 needs to exceed the quality of good 1 for downstream bundling to be profitable for the two-product firm. However, bundling also aggravates the problem of double marginalization for the bundling firm and thus may not be a profitable strategy. Put differently, for a sufficiently low quality of good 2, the two-product downstream firm prefers to price its products independently. This is the case even though bundling reduces the intensity of competition in the downstream duopoly and leads to an extension of the two-product firm's monopoly power regarding good 1 into the downstream market for good 2. In short, the product qualities determine the profitability of bundling by influencing the magnitudes of both the competition reduction effect of bundling and the aggravation of double marginalization. Thereby we confirm Heinzel's (2019) results that the positive effect of bundling in the form of a reduction in the intensity of competition can outweigh the negative effect of bundling in the form of an aggravated double marginalization problem under price competition. Contrary to him, we identify the product qualities as the determining factor for the final effect.

To illustrate the impact of double marginalization, we analyze a centralized channel where the full market power lies with the downstream firms and therefore double marginalization is eliminated. We observe that bundling is always the two-product firm's best strategy in the centralized channel. Hence, we identify the double marginalization problem and its aggravation by bundling as a factor to lower the bundling incentive in the channel. Yet, when we consider that both goods are produced by a single upstream firm with upstream market power – and therefore also have double marginalization – bundling is again always the two-product firm's best strategy. Consequently, Bhargava's (2012) result that it is a combination of vertical externalities and horizontal externalities upstream that weakens the downstream firm's bundling incentives in the decentralized channel holds when the traded goods differ in quality and there is downstream competition in the channel in one product market.

Our observation that bundling is not always the two-product firm's best strategy is especially interesting considering that parts of the previous leverage theory literature find bundling under price competition to be always profitable (compare Carbajo et al., 1990; Peitz, 2008). Chung et al. (2013) already identify the degree of inter-brand differentiation between the competing products as a pivotal factor to drive the bundling decision. Our paper additionally identifies on the one hand the product qualities and differences in these levels and, on the other hand, the channel effects as decisive factors that drive the profitability of bundling under price competition.

Furthermore, we identify downstream bundling as a welfare deteriorating strategy in the decentralized channel since it reduces both consumer surplus and producer surplus in equilibrium. In the centralized channel, profitable bundling reduces consumer surplus but increases producer surplus, which can lead to an increase in overall welfare. The ultimate effects in the centralized channel are determined by the quality levels: Total welfare is increased by profitable bundling for a *low* quality level of good 2, and decreased for a sufficiently *high* quality level of good 2.

In a discussion, we relax the strict additivity assumption of the products in the bundle and find that when the goods are subadditively or superadditively valued, then bundling may be also profitable when good 2 is of higher quality than good 1. Furthermore, if the bundled goods exhibit superadditivity, bundling can be profitable independent of the quality relation between the bundled goods.

The rest of the paper is structured as follows. We analyze the decentralized channel in Section 2 and investigate the centralized channel in Section 3. In Section 4 we discuss our results for relaxing the non-negativity constraint and relaxing the strict additivity assumption of the bundled goods. Section 5 concludes.

2 Decentralized Channel: Framework and Analysis

2.1 Basics of the model

The distribution channel consists of two downstream firms (D_A and D_B), two upstream firms (U_1 and U_2), and two products (*good 1* and *good 2*). There is a continuum of final customers. Good 1 is manufactured by upstream firm U_1 and good 2 by upstream firm U_2. Both upstream firms are monopolists in their respective markets and both goods are produced at symmetric constant marginal cost $k \geq 0$.[1] We assume that upstream firm U_1 and downstream firm D_A have an exclusive relationship. In particular, we assume that both firms behave according to an exclusivity contract, which allows U_1 to sell its good 1 only to D_A, making D_A the downstream monopolist for good 1. Such an exclusive relationship can, for instance, be found in the streaming service industry and might reflect a producing company that sells certain productions exclusively to one streaming service. Another example for exclusive agreements is the 'Amazon Exclusives' program. Manufacturers involved in this program must sell their goods only via Amazon.com and not via any other online marketplace.[2] Good 2 is sold to both downstream firms by U_2, leading to a downstream duopoly. We assume that the downstream firms engage in price competition in the market for good 2.

The goods manufactured by the upstream firms are the input goods of the downstream firms and are resold without any changes in their characteristics as final goods by the downstream firms. This implies that 1) the downstream firms transform the inputs into output on a one-to-one basis at zero cost and that 2) the downstream firms supply the products to the final consumers with the quality provided by the upstream firms. Moreover, neither D_A nor D_B have any production costs (e.g., for repackaging or bundling) when selling the goods to the final customers.

In the subsequent sections, we solve the following game for the subgame perfect Nash equilibrium in pure strategies by applying backward induction. Thereby, we consider the following timing (Figure 1): At first, the two-product downstream firm D_A decides whether to bundle the products or not, whereas it only bundles if it leads to a higher profit than selling the products separately. Afterwards, both upstream firms set their equilibrium prices. In particular, upstream firm U_1 (U_2) sets the input price c_1 (c_2), which depends on the two-product downstream firm's selling strategy. In the last step, both downstream firms choose their profit-maximizing prices.

Our timing implies that the upstream firms already know whether D_A bundles when making their pricing decision. This can be observed in reality in the streaming service industry, for example. As Netflix's pure bundling strategy is common knowledge the producers set their prices being already aware of Netflix bundling strategy. Furthermore, you can also interpret this timing in the sense that the upstream firms predict the bundling strategy due to the historical actions of the downstream firm. Note that we could also simply assume that D_A makes a

[1] The following analysis reveals that our qualitative results hold for $k = 0$ without loss of generality.

[2] Regarding the reasoning for such exclusive relationships see Heinzel (2019).

precommitment to either bundling or separate selling in the beginning and that in the first stage the upstream firms set their prices and in the second stage the downstream firms make their pricing decisions, which is how the game in Bhargava (2012) is played.

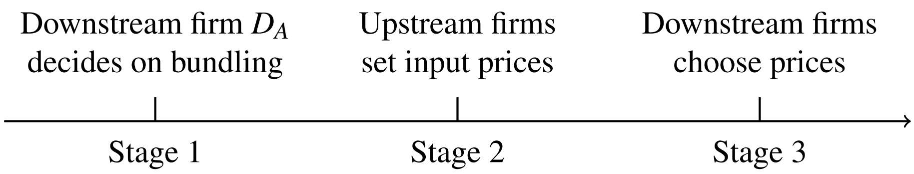


Figure 1: Timing of the game

We first solve the model for the case in which D_A is assumed to sell its products separately, and then for the case in which D_A is assumed to bundle its products. We only consider pure bundling as a bundling strategy for D_A. In a last step, we compare the market results under separate selling and bundling in order to determine D_A's incentives to bundle. All proofs, first- and second-order conditions as well as comparisons can be found in the Appendix.

2.2 Separate selling: Nash equilibrium outcomes

Assume that downstream firm D_A plays a separate selling strategy (the superscript S denotes, mostly, the equilibrium solutions for this case) such that firm D_A supplies good 1 and good 2 separately and downstream firm D_B offers good 2 to the final customers. Good 2 is perfectly substitutable between the downstream firms. Hence the final customers are indifferent between buying good 2 from either one of the two downstream firms. Figure 2 provides the market structure in the separate selling case.

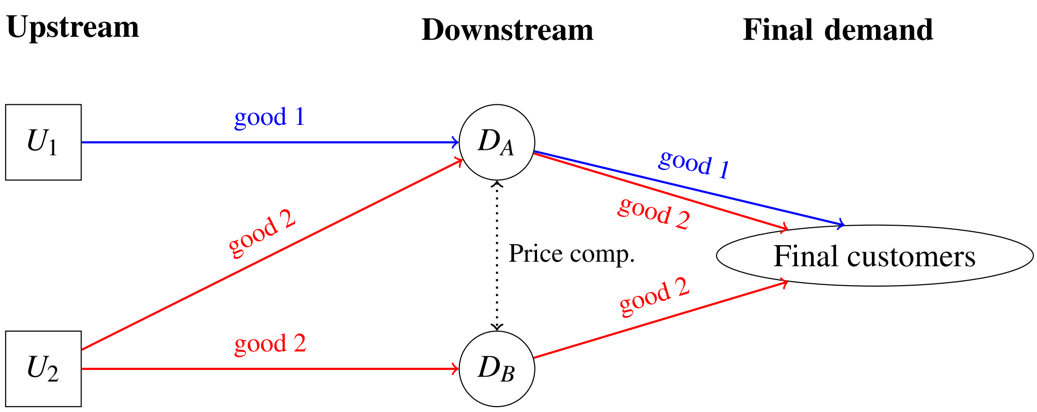


Figure 2: Market Structure under separate selling

The aggregate final customers' preferences regarding good 1 and good 2 are given by the representative customer's utility and follow Dixit (1979):[3]

$$V = m + a_1 Q_1 + a_2 Q_2 - \frac{1}{2}\left(Q_1^2 + Q_2^2\right), \tag{1}$$

where Q_1 (Q_2) is the quantity of good 1 (good 2) purchased by the representative customer and m is the quantity of other goods he consumes. The parameter $a_1 > 0$ $(a_2 > 0)$ denotes the customer's valuation for good 1 (good 2), which represents the customer's reservation price for the respective good. We assume $a_1, a_2 > k$ to ensure market transactions. As already pointed out, the customer valuation for a good can be interpreted as the product quality of the respective good such as in Häckner (2000), for instance. We adopt this interpretation in our model and thus denote a_1 (a_2) as the quality of good 1 (good 2). We allow for $a_1 = a_2$ but focus on the cases where we have $a_1 \neq a_2$. We assume the quality of each good to be *exogenously* given and the two standalone goods to be *independent* in demand, where the latter is incorporated in the customers' preferences. The parameter m denotes the composite good whose price is normalized to one.

The price of good 1 (good 2) is given by p_1 (p_2). Solving the representative customer's optimization problem gives us the following inverse demand for the two standalone goods:

$$p_1 = a_1 - Q_1,$$

$$p_2 = a_2 - Q_2.$$

It holds that $Q_2 = q_{A2} + q_{B2}$, where q_{A2} is firm D_A's supplied quantity of good 2 and q_{B2} is firm D_B's supplied quantity of good 2. The downstream quantity of good 1 supplied by D_A is $Q_1 = q_{A1}$. By the inverse demand, we derive the demand of the two goods as

$$Q_1 = a_1 - p_1, \tag{2}$$

$$Q_2 = a_2 - p_2. \tag{3}$$

The profit that downstream firm D_A maximizes is compounded by the profit it gains in the monopoly regarding good 1 and the profit it gains in the duopoly regarding good 2. Firm D_B's profit depends solely on the profits it gains in the market for good 2. Finally, the equilibrium downstream profits are $\pi_{D_A}^S = (p_1^S - c_1^S)q_{A1}^S + (p_2^S - c_2^S)q_{A2}^S$ and $\pi_{D_B}^S = (p_2^S - c_2^S)q_{B2}^S$ with equilibrium prices

$$p_1^S = \frac{a_1 + c_1}{2}, \tag{4}$$

$$p_2^S = c_2. \tag{5}$$

The downstream price for good 2 is driven down to marginal cost in equilibrium due to the

[3] Strictly spoken, we are depicting the utility function $V(m, Q_1, Q_2)$. For better readability, we refrain from depicting the functions' arguments in our following calculations.

price competition for a homogeneous good between the downstream firms. The downstream firms' marginal costs are given by the respective input prices set by the upstream firms.

We now turn to the upstream side of the supply chain. In order to receive firm D_A's input demand regarding good 1, we substitute Equation (4) into Equation (2). The input demand regarding good 2 is obtained by inserting Equation (5) into Equation (3). The input resulting demands are

$$Q_1 = \frac{a_1 - c_1}{2},$$

$$Q_2 = a_2 - c_2.$$

Finally, the profits of the upstream firms are given as

$$\pi_{U_1} = (c_1 - k)\,Q_1,$$

$$\pi_{U_2} = (c_2 - k)\,Q_2.$$

Maximizing the profits with respect to the input prices leads to the equilibrium input prices:

$$c_1^S = \frac{a_1 + k}{2}, \tag{6}$$

$$c_2^S = \frac{a_2 + k}{2}. \tag{7}$$

We receive the final market results by inserting (6) and (7) into the other market entities. Further below, Lemma 1 lists the equilibrium input prices and the residual equilibrium values under separate selling.

2.3 Bundling: Nash equilibrium outcomes

Now suppose that downstream firm D_A purely bundles its products (the superscript BL denotes, mostly, the equilibrium solutions for this case). Bundling in our set-up means that firm D_A ties good 1 with good 2 and sells solely the resulting product combination (called *bundle A*) at a single price. We assume that bundle A contains one unit of good 1 and one unit of good 2, which can be denoted as bundle $A : (1,1)$. For notational purposes, we denote firm D_B's product as *bundle* $B : (0,1)$, which consists only of one unit of good 2. Figure 3 depicts the market structure in the bundling case.

The relationships between the quantities of the two component goods and the bundles are

$$Q_1 = b_A, \tag{8}$$

$$Q_2 = b_A + b_B, \tag{9}$$

where b_A is the quantity of the bundles sold by firm D_A and b_B the quantity of bundles sold

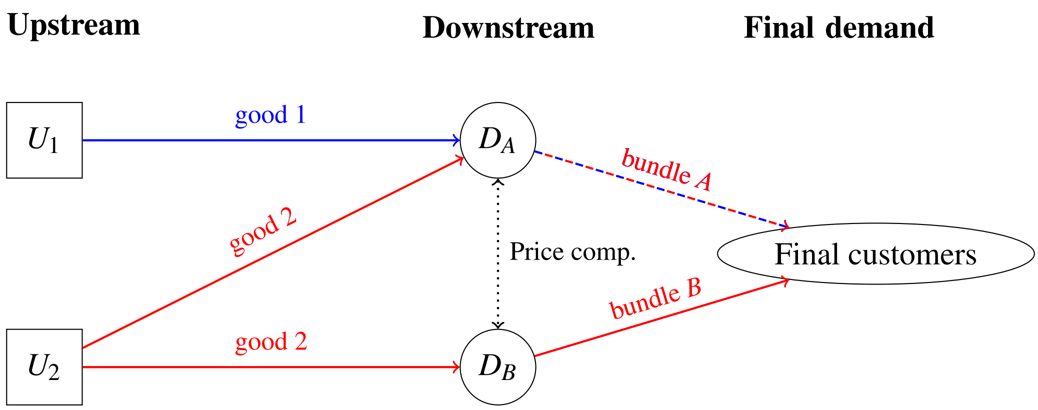


Figure 3: Market structure under bundling

by firm D_B. Following a method used by Martin (1999), we substitute (8) and (9) into (1). This method yields V, which describes the utility the representative customer derives from consuming the bundles and other goods m:

$$V = m + (a_1 + a_2)b_A + a_2 b_B - \frac{1}{2}\left(2b_A^2 + 2b_A b_B + b_B^2\right).$$

The same assumptions regarding the quality parameters a_1 and a_2 are imposed as in the separate selling case. We suppose *strict additivity* concerning the qualities of the two standalone goods.[4] This means that a bundle provides the same total quality as the sum of the qualities of its component goods. Consequently, bundling does not add (*superadditivity*) or reduce any value (*subadditivity*) in product quality.[5]

The price of bundle A is denoted by p_A and the price of bundle B by p_B. Solving the representative customer's optimization problem regarding bundle A (respectively bundle B) gives us the inverse demand for the bundles:

$$p_A = a_1 + a_2 - 2b_A - b_B, \tag{10}$$

$$p_B = a_2 - b_A - b_B. \tag{11}$$

Thus, the demand for the bundles are

$$b_A = p_B - p_A + a_1,$$

$$b_B = p_A - 2p_B + a_2 - a_1.$$

We observe that $\frac{\partial b_A}{\partial p_B} > 0$ and $\frac{\partial b_B}{\partial p_A} > 0$. This means that the two bundles pose *imperfect*

[4] Venkatesh and Kamakura (2003) and Honhon and Pan (2017) refer to products with a strict additivity relationship as independently valued, which is consistent with our assumptions.

[5] See Subsection 4.2 for the effects of sub- or superadditivity regarding the qualities on the incentives for downstream bundling.

substitutes, whereas under separate selling both downstream firms' products in the market for good 2 are *perfect* substitutes. Consequently, bundling differentiates the products sold by both downstream firms in the duopoly. Additionally, the standalone goods are independent in demand under separate selling, but the bundles are substitutes. Hence, bundling creates substitutability between the goods, which is in line with Martin (1999). The product differentiation in the duopoly enables both downstream firms to charge under bundling a downstream price above input price.

Under bundling, the downstream firms' profits are

$$\pi_{D_A} = (p_A - c_1 - c_2)\, b_A, \tag{12}$$

$$\pi_{D_B} = (p_B - c_2)\, b_B. \tag{13}$$

Solving the downstream firms' optimization problems leads to the equilibrium price for bundle A, respectively bundle B:

$$p_A^{BL} = \frac{3a_1 + 4c_1 + a_2 + 6c_2}{7},$$

$$p_B^{BL} = \frac{2a_2 - a_1 + c_1 + 5c_2}{7}.$$

We insert Equations (2.3) and (2.3) into Equations (2.3) and (2.3) to receive the input demand. We directly obtain the input demand for good 1 since $b_A = Q_1$. We get the input demand for good 2 by calculating $b_A + b_B = Q_2$. Note that we obtain $b_B = \frac{2(2a_2 - a_1 - 2c_2 + c_1)}{7}$. We ultimately receive

$$Q_1 = \frac{3a_1 + a_2 - 3c_1 - c_2}{7},$$

$$Q_2 = \frac{a_1 + 5a_2 - c_1 - 5c_2}{7}.$$

We observe $\frac{\partial Q_1}{\partial c_2} < 0$ and $\frac{\partial Q_2}{\partial c_1} < 0$. This means that the two standalone goods become complementary input goods because of being tied together in bundle A.

The profits of both upstream firms under bundling are analogous to the ones under separate selling. The upstream firms' equilibrium input prices under bundling are

$$c_1^{BL} = \frac{29a_1 + 5a_2 + 25k}{59},$$

$$c_2^{BL} = \frac{3a_1 + 29a_2 + 27k}{59}.$$

Since the two separate goods are complements under bundling, a raise in the quality of either good and therefore in the customer valuation induces higher input prices. This means that $\frac{\partial c_{1,2}^{BL}}{\partial a_{1,2}} > 0$. Inserting the equilibrium input prices under separate selling and bundling into the residual entities generates Lemma 1.

Lemma 1. *The equilibrium values under separate selling and bundling are as follows:*

	Separate selling	**Bundling**
Profit of downstream firms	$\pi^S_{D_A} = \frac{(a_1-k)^2}{16}$	$\pi^{BL}_{D_A} = \frac{9(29a_1+5a_2-34k)^2}{170569}$
	$\pi^S_{D_B} = 0$	$\pi^{BL}_{D_B} = \frac{2(-36a_1+65a_2-29k)^2}{170569}$
Profit of upstream firms	$\pi^S_{U_1} = \frac{(a_1-k)^2}{8}$	$\pi^{BL}_{U_1} = \frac{3(29a_1+5a_2-34k)^2}{24367}$
	$\pi^S_{U_2} = \frac{(a_2-k)^2}{4}$	$\pi^{BL}_{U_2} = \frac{5(3a_1+29a_2-32k)^2}{24367}$
Input prices	$c^S_1 = \frac{a_1+k}{2}$	$c^{BL}_1 = \frac{29a_1+5a_2+25k}{59}$
	$c^S_2 = \frac{a_2+k}{2}$	$c^{BL}_2 = \frac{3a_1+29a_2+27k}{59}$
Final prices	$p^S_1 = \frac{3a_1+k}{4}$	$p^{BL}_A = \frac{311a_1+253a_2+262k}{413}$
	$p^S_2 = \frac{a_2+k}{2}$	$p^{BL}_B = \frac{-15a_1+268a_2+160k}{413}$
Quantities	$Q^S_1 = \frac{a_1-k}{4}$	$Q^{BL}_1 = \frac{3(29a_1+5a_2-34k)}{413}$
	$Q^S_2 = \frac{a_2-k}{2}$	$Q^{BL}_2 = \frac{5(3a_1+29a_2-32k)}{413}$
Downstream quantities	$q^S_{A1} = \frac{a_1-k}{4}$	$b^{BL}_A = \frac{3(29a_1+5a_2-34k)}{413}$
	$q^S_{A2} = \frac{a_2-k}{4}$	
	$q^S_{B2} = \frac{a_2-k}{4}$	$b^{BL}_B = \frac{2(-36a_1+65a_2-29k)}{413}$

Proof. See Appendix A.1 and A.2. □

Note that $b^{BL}_B > 0$ only holds for $a_2 > \frac{36a_1+29k}{65}$ and $p^{BL}_B > 0$ only for $a_2 > \frac{15a_1-160k}{268}$, where $\frac{36a_1+29k}{65} > \frac{15a_1-160k}{268}$. Therefore, we impose the restriction that $a_2 > \frac{36a_1+29k}{65} =: \underline{a}^S_2$. The assumptions $a_2 > \underline{a}^S_2 > k$ and $a_1 > k$ ensure non-negativity for all equilibrium market magnitudes.[6]

The differentiation of the goods in the duopoly reduces the intensity of the hard price competition between the downstream firms and therefore allows D_B to charge a price for bundle B above the input price of good 2. This in turn enables firm D_A to set a very high price for bundle A, which is clearly larger than the sum of input prices of both components. This observed reduction in the intensity of competition under price competition has a positive effect on both downstream firms' bundling profits and is in line with previous papers on leverage theory, such as Carbajo et al. (1990); Egli (2007); Chung et al. (2013); Heinzel (2019). It is the effect that may make bundling profitable for D_A as illustrated in the following.

[6]See Subsection 4.1 for the equilibrium values without the non-negativity constraint and the respective analysis of equilibrium results.

2.4 Bundling Decision and the Consequences of Bundling

In this section, we first illustrate under which conditions bundling represents an equilibrium strategy for downstream firm D_A. Firm D_A bundles only when its profit in bundling is higher than its profit in separate selling, which we refer to as *profitable bundling*. More specifically, we first derive the constellations and degrees of product quality levels that ensure the existence of a bundling equilibrium. In the next step, we investigate the role of input prices regarding firm D_A's motivation for bundling. Then, we analyze how profitable bundling affects the market magnitudes, such as other firms' prices and profits, in comparison to separate selling. Finally, we examine the effects of profitable bundling on social welfare.

2.4.1 Downstream bundling incentives

We assume $p_1^S > p_2^S$ in our framework and hereby follow the reasoning of Carbajo et al. (1990).[7] They argue that it is the goal of the firm's bundling strategy to raise the downstream price of good 2 in order to extract more consumer surplus from consumers who buy good 1 under separate selling. Given that firm D_A bundles, all consumers that want to consume good 1 can only receive it by purchasing the bundle. Finally, in order to obtain good 1, they would also be willing to pay a higher price for good 2. Considering $p_1^S > p_2^S$, we compare D_A's separate selling profit with its bundling profit and identify the quality levels of good 1 and good 2 under which bundling is D_A's preferred strategy. Proposition 1 summarizes our findings.

Proposition 1. *In the decentralized channel, downstream firm D_A prefers bundling over separate selling if the quality of good 2 is sufficiently large, i.e., if $a_2 \in \left(\underline{a}_2^{BL}, \overline{a}_2\right)$, where $\underline{a}_2^{BL} := \frac{13a_1-k}{12}$ and $\overline{a}_2 := \frac{3a_1-k}{2}$.*

Proof. See Appendix B.1. □

The condition $p_1^S > p_2^S$ gives us $\overline{a}_2$ as the upper quality bound of good 2 for profitable bundling (depicted by the blue dashed line in Figure 4). Considering our boundaries $\underline{a}_2^S < a_2 < \overline{a}_2$, we find that bundling is more beneficial for firm D_A than separate selling when the quality of good 2 is sufficiently high, i.e., larger than $\underline{a}_2^{BL}$. This lower bound of the *profitable bundling interval* is illustrated by the red dashed line in Figure 4. The profitable bundling interval is marked by the blue shaded area in Figure 4. However, when $\underline{a}_2^S < a_2 < \underline{a}_2^{BL}$ (green shaded area in Figure 4), downstream firm D_A prefers to offer its products separately.

Moreover, Proposition 1 implies that within the profitable bundling interval it always holds that the quality of good 2 exceeds the quality of good 1 ($a_2 > a_1$).[8] This leads to the following insight:

[7]Note that this assumption is not crucial for the existence of a bundling equilibrium in our framework.

[8]In Figure 4, the area below the black solid line marks where good 1 is of higher quality compared to good 2 ($a_1 > a_2$); the area above the black solid line marks where good 2 is of higher quality compared to good 1 ($a_1 < a_2$). The regions above $\overline{a}_2$ and below $\underline{a}_2^S$ are excluded due to our already mentioned assumptions.

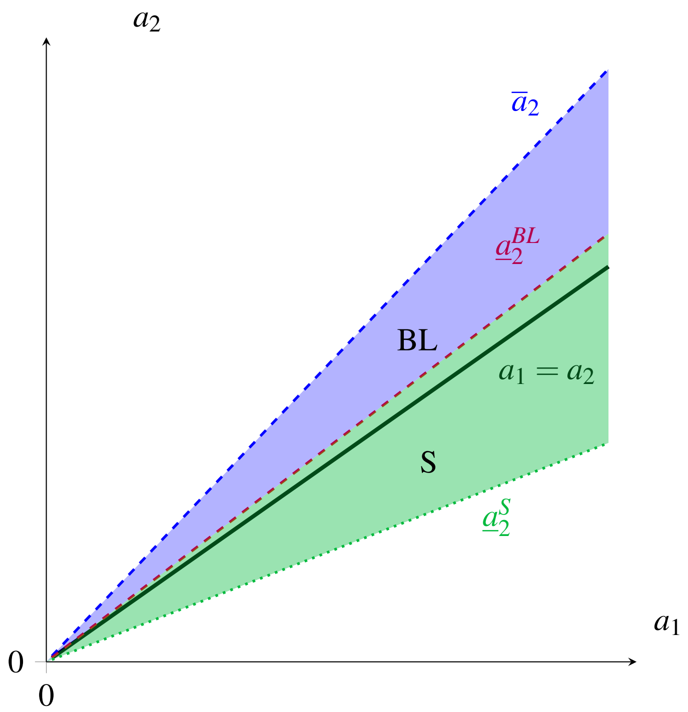


Figure 4: Separate selling vs. bundling (with $k = 0$)

Corollary 1. *The quality of good 2 must be larger than the quality of good 1 for downstream firm D_A to prefer bundling over separate selling.*

Notice that $a_2 > a_1$ is a necessary but not sufficient condition for D_A to prefer bundling, since D_A's separate selling profit exceeds its bundling profit in the region $\underline{a}_2^S < a_2 < \underline{a}_2^{BL}$, in which $a_2 > a_1$ can hold too (compare Figure 4). However, the reverse holds true as $a_2 < a_1$ implies that D_A's separate selling profit *certainly* exceeds its bundling profit so that $\pi_{D_A}^S > \pi_{D_A}^{BL}$.

The intuition behind Proposition 1 and Corollary 1 is as follows. In separate selling, changes in the quality of good 2 do not affect D_A's profit $\left(\frac{\partial \pi_{D_A}^S}{\partial a_2} = 0\right)$. That is because of two reasons: for one thing, the downstream price of good 2 is equal to its input price due to the intense price competition between the downstream firms in this market. For another thing, the standalone goods are independent in demand. Contrary to that, a higher quality of good 1 positively affects D_A's separate selling profit $\left(\frac{\partial \pi_{D_A}^S}{\partial a_1} > 0\right)$. That is because firm D_A charges the monopoly price of good 1 under separate selling and a higher customer's valuation for good 1 – thus a higher a_1 – allows firm D_A to set an even higher monopoly price $\left(\frac{\partial p_1^S}{\partial a_1} > 0\right)$.

Now consider the bundling setting. The differentiation of goods in the duopoly and the subsequent reduction in the degree of downstream competition clearly raises firm D_A's incentives to bundle, as already indicated above. In addition, downstream firm D_A is able to extend the monopoly power it has in the market for good 1 to the second product market by bundling: It charges a higher price for bundle A than the sum of input prices and the sum of the prices it charges for the standalone goods under separate selling. Aside from those rather general implications of bundling on the firms' pricing behavior, we find the following when focusing on the impact of the goods' quality levels: An increase in the quality of good 2 makes firm D_A charge an even higher price for bundle A $\left(\frac{\partial p_A^{BL}}{\partial a_2} > 0\right)$. It follows from the softened competition under bundling that this quality increase of good 2 and the consequential price increase raise D_A's bundling profit $\left(\frac{\partial \pi_{D_A}^{BL}}{\partial a_2} > 0\right)$.

To sum up, a lower quality of good 1 makes the separate selling strategy less attractive for downstream firm D_A, while a higher quality of good 2 makes the bundling strategy more attractive for D_A. As a consequence, when the quality of good 2 is sufficiently large such that it exceeds the quality of good 1, D_A's bundling profit exceeds its separate selling profit and therefore D_A prefers to bundle.

We study next how the input prices c_1 and c_2 are affected by downstream firm D_A's selling strategy to identify the influence of the upstream firms' price setting behavior on D_A's bundling incentives. We find that the upstream firms' price setting reactions to firm D_A's bundling strategy weaken the attractiveness of bundling:

Proposition 2. *When firm D_A bundles, then both upstream firms raise their input prices.*

Proof. See Appendix B.2. □

The intuition behind the increase of input prices is as follows. Both upstream firms want to benefit from D_A's bundling strategy. The two standalone goods become complementary inputs due to bundling, which increases the need for both goods. Furthermore, since bundle A consists of the goods of both upstream firms, an increase in the input price of good 1 only partially impacts U_1's sales. By contrast, a raise in the input price of good 2 lowers the quantities of both bundles. However, in separate selling, a raise of the input price of good 2 has a rather strong negative effect on the downstream demand (and consequently the input demand) for good 2 due to good 2 being priced at its input price in the downstream market. Under bundling, this negative effect of a raised input price of good 2 on good 2's sales is weakened since the bundle prices are set above input prices. Ultimately, these effects induce both upstream firms to always raise their prices under bundling.

We can conclude that bundling aggravates the double marginalization problem for downstream firm D_A and creates double marginalization for downstream firm D_B. First, in the separate selling market, there is only double marginalization in the supply chain of the bilateral monopoly regarding good 1 (as $p_2^S = c_2^S$ in the second product market), whereas in the bundling market double marginalization emerges with respect to both bundles. In addition, the sum of the two input prices with bundling is greater than without bundling, which directly affects D_A as it sets a bundling price above the sum of the input prices. Finally, the problem of double marginalization is worsened for D_A and for the whole channel. This effect might lead to a bundle price higher than optimal for D_A and consequently to too little bundle sales. Thus, the upstream firms' price setting reaction to bundling weakens the incentives for downstream bundling.

Nevertheless, we observe that despite an increase in both input prices and consequently a relatively heavy double marginalization problem, it might be more profitable for firm D_A to bundle than to sell the goods separately, depending on the product qualities. This means that the positive impacts of bundling on D_A's profit, such as a lower degree of competition and the extension of market power, outweigh the negative influences – as an exacerbated double marginalization problem – given the right constellation of qualities. We discuss the impact of double marginalization on bundling in more detail in Section 3 where we abstract from vertical externalities.

2.4.2 Consequences of Profitable Bundling

In this section, we investigate the effects of bundling in equilibrium, which implies that bundling is profitable for D_A. We refer to D_A's equilibrium bundling strategy in this section as *profitable bundling* and *bundling* synonymously.

Overall, the downstream industry benefits from the bundling decision of the two-product downstream firm D_A. Not only the profit of D_A but also the profit of downstream firm D_B increases by bundling. Whereas D_B gains a profit of zero in separate selling, in the bundling equilibrium it gains a positive profit due to the bundles being differentiated. The differentiation, the softened competition, and the increased input price of good 2 result in higher downstream prices set by D_B and D_A, which yields a decrease in the downstream quantity of each downstream firm regarding good 2. Therefore, the total quantity of good 2 falls. In contrast to that, firm D_A's quantity and thus the total quantity of good 1 rises due to bundling. This can be explained by the fact that good 1 is in the bundle tied with a product of higher quality and more intense competition. Consequently, not only consumers with a high reservation price for the standalone good 1 but also consumers with a relatively high valuation for good 2 are willing to buy the bundle despite the increase in downstream prices.

As one unit of input represents one unit of output, we can directly derive the impact of bundling on the upstream quantities from the impact of bundling on the downstream quantities. The divergence in the influence of bundling on the upstream quantities leads to a consequential divergence in the effect of bundling on the upstream firms' profits as stated by

Proposition 3. *Profitable bundling leads to an increase in upstream firm U_1's profit and a decrease in upstream firm U_2's profit.*

Proof. See Appendix B.3. □

Upstream firm U_1 sells a higher quantity of good 1 at a higher price and hence its profit rises due to bundling. By contrast, U_2's profit is reduced by bundling even though it raises its price for good 2 as well. The softening in downstream competition and subsequent aggravation of the double marginalization problem caused by the increase in input prices results in too low sales for U_2 and, consequentially, bundling lowers U_2's profit. This illustrates that raising its price is rather detrimental for U_2. However, note that despite the increase in U_1's quantity and profit, U_1's sales and profit with profitable bundling are lower than the quantity and profit of upstream firm U_2, i.e., $\pi_{U_1}^{BL} < \pi_{U_2}^{BL}$ and $Q_1^{BL} < Q_2^{BL}$.

We now turn to the welfare analysis of bundling. The producer surplus PS is defined as the sum of profits of two upstream and two downstream firms. Total welfare W is described by the sum of the consumer surplus CS and the producer surplus PS. Lemma 2 summarizes the welfare results for the decentralized channel.

Lemma 2. *The welfare results for bundling and separate selling are as follows:*

Producer Surplus	$PS^S = \frac{7k^2-8a_2k-6a_1k+4a_2^2+3a_1^2}{16}$ $PS^{BL} = \frac{72202k^2-82700a_2k-61704a_1k+38635a_2^2+5430a_1a_2+28137a_1^2}{170569}$
Consumer Surplus	$CS^S = \frac{5k^2-8a_2k-2a_1k+4a_2^2+a_1^2}{32}$ $CS^{BL} = \frac{18002k^2-24730a_2k-11274a_1k+10625a_2^2+3480a_1a_2+3897a_1^2}{170569}$
Social welfare	$W^S = \frac{19k^2-24a_2k-14a_1k+12a_2^2+7a_1^2}{32}$ $W^{BL} = \frac{3\left(30068k^2-35810a_2k-24326a_1k+16420a_2^2+2970a_1a_2+10678a_1^2\right)}{170569}$

Proof. See Appendix A.1 and A.2. □

By comparing the welfare results in Lemma 2, we find downstream bundling to be welfare harming in the decentralized channel as stated by

Proposition 4. *Profitable bundling results in a decrease in consumer surplus, a decrease in producer surplus, and a decrease in total welfare.*

Proof. See Appendix B.4. □

This overall reduction of social welfare induced by bundling has been observed in other parts of the existing bundling literature with a comparable set-up as well (see e.g., Martin, 1999). The intuition for the decrease in consumer surplus is straightforward. Both downstream firms raise their prices, and this causes the consumer surplus to fall. The reduction in producer surplus is, however, somewhat surprising. The respective profits of both downstream firms as well as the profit of upstream firm U_1 are raised by bundling. Yet, the overall industry profit falls. Consequently, the decrease in upstream firm U_2's profit outweighs the total increase in profits of the three residual firms. As a consequence, total welfare always decreases when the two-product downstream firm D_A bundles in equilibrium.

In the next section, we investigate a centralized channel which we compare with the decentralized channel. The centralized channel case provides additional insights about the interplay of downstream bundling and the distribution of market power in the channel.

3 Centralized channel

3.1 Basics of the model

In the centralized channel (this case is denoted by a *Tilde*), the regarded market has the same structure as the decentralized market, but the full market power lies with the downstream industry, resulting in the upstream firms being *price-takers*.

The centralized channel allows us to investigate the bundling incentives, the impact of the products' quality levels on the market outcomes, and the welfare effects of bundling without double marginalization. Thus, it allows us to exclude double marginalization as a factor influencing the bundling incentives.

Note that except for the distribution of market power in the channel, all assumptions remain the same as in the decentralized case. The downstream firms' optimization problems are analogous to the respective ones in the decentralized channel. The timing is now as follows: A t first, firm D_A decides on bundling, then both downstream firms decide on the input prices and finally set the downstream prices. Our approach for studying this centralized channel is the same as in the decentralized channel.

As the two downstream firms have full market power, they set the input prices equal to the upstream firms' marginal costs of production for both goods to keep their input costs as low as possible. Therefore, we have in equilibrium $\tilde{c}_1^i = \tilde{c}_2^i = k$, where $i \in \{S, BL\}$. Consequently, the equilibrium price of good 1 under separate selling is given by $\tilde{p}_1^S = \frac{\tilde{a}_1+k}{2}$ and the equilibrium price of good 2 by $\tilde{p}_2^S = k$. Again, as a precondition for a bundling equilibrium must hold that $\tilde{p}_1^S > \tilde{p}_2^S$, which is always satisfied due to the assumption $\tilde{a}_1 > k$. Note that we have $\tilde{b}_B^{BL} > 0$ for $\tilde{a}_2 > \frac{\tilde{a}_1+k}{2} =: \underline{\tilde{a}}_2$ and $\tilde{p}_B^{BL} > 0$ for $\tilde{a}_2 > \frac{\tilde{a}_1-6k}{2}$, where $\underline{\tilde{a}}_2 > \frac{\tilde{a}_1-6k}{2}$. Therefore, we assume $\tilde{a}_1 > k$ and $\tilde{a}_2 > \underline{\tilde{a}}_2$, where $\underline{\tilde{a}}_2 > k$, for the centralized channel which ensures non-negativity for all market entities. An overview of all market results can be found in Appendix A.3.

3.2 Analysis

3.2.1 Downstream bundling incentives

Comparing firm D_A's profit under bundling and separate selling generates Proposition 5, which is graphically illustrated by Figure 5.[9]

Proposition 5. *In the centralized channel, downstream firm D_A always prefers bundling over separate selling.*

Proof. See Appendix B.5. □

Consequently, if the downstream firms have the full market power in the channel, bundling is always profitable for D_A. Proposition 5 also implies that in the centralized channel case it

[9] Note that there is only a blue shaded area in Figure 5 as bundling is always profitable.

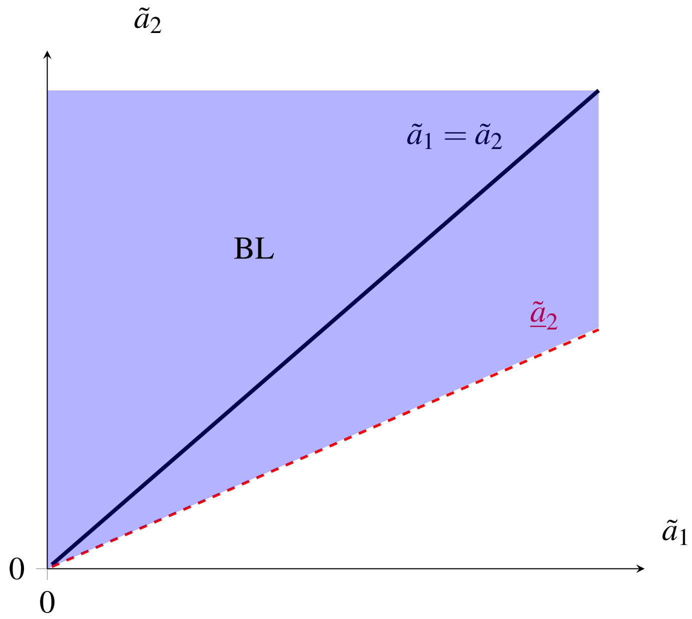


Figure 5: Centralized channel: separate selling vs. bundling (with $k = 0$)

is not a necessary condition that good 2 is of higher quality than good 1 for bundling to be profitable (compare Figure 5), which is in contrast to the decentralized channel. Additionally, bundling is profitable at a lower quality level of good 2 than in the decentralized channel because of $\underline{\tilde{a}}_2 < \underline{a}_2^{BL}$, where $\underline{a}_2^{BL}$ is the lower bound of the profitable bundling interval in the decentralized channel. This lower bound is greater than the lower bound for profitable bundling in the centralized channel. Analogous to the decentralized channel, a higher quality of good 2 means a higher bundling profit for D_A since it allows for a higher bundle price but has no effect on D_A's separate selling profit, whereas a higher quality of good 1 raises D_A's separate selling profit. Nevertheless, even when $\tilde{a}_1 > \tilde{a}_2$, bundling is more profitable for D_A than pricing the goods independently. We conclude from our findings that D_A's bundling incentives are stronger in the centralized than in the decentralized channel.

The intuition behind this result is as follows. In the centralized channel, there is no double marginalization for either one of the downstream firms and thus no aggravation of the double marginalization problem by bundling. Therefore, the effects of a softened competition and the extension of D_A's monopoly power, which we observe in either channel, have such a strong positive impact on D_A's bundling profit that bundling is always profitable here.

In the decentralized channel, we identified the perspective of gaining a share of D_A's additional bundling profit as one main factor driving the upstream firms' desire to increase their prices. However, the horizontal externalities between the upstream firms also play a pivotal role, which is in line with Bhargava (2012) and Heinzel (2019). To illustrate this insight, consider the following change in the decentralized model: In order to rule out horizontal externalities upstream, we investigate the case where the powerful upstream firms merge to a *multi-product upstream monopoly*, in which we again assume $p_1^S > p_2^S$.[10] Consequently, we have one powerful upstream firm U producing both goods and selling good 1 only to firm D_A and good 2 to both downstream firms. Upstream firm U sets its input prices in the bundling case exactly as in the separate selling case, i.e., $c_1^i = \frac{a_1+k}{2}$ and $c_2^i = \frac{a_2+k}{2}$. Hence, the input prices in the multi-product upstream monopoly are independent of the two-product downstream firm's selling strategy choice. This is because the multi-product upstream monopolist internalizes the demand externalities between the two goods, which represent complementary inputs under bundling. Considering the bundling incentives in the multi-product upstream monopoly, we find that downstream firm D_A always prefers bundling over separate selling regardless of the quality relations of the goods, such as in the centralized channel. Even though there is a double marginalization problem regarding good 1 for D_A, it is not aggravated by bundling. Double marginalization in the market for good 2 is again created by bundling, but only to D_A's benefit as it can set higher prices under bundling but has to bear the same costs as with separate selling.

In conclusion, aside from the structure of the decentralized channel with powerful upstream firms, we identify also given retail competition the horizontal externalities between the independently operating upstream firms as pivotal factor for the worsened double marginalization problem.

3.2.2 Consequences of profitable bundling

We now analyze the equilibrium effects of profitable bundling. As it turns out, bundling is always profitable so we use the terms *bundling* and *profitable bundling* again as synonyms in this section. As in the decentralized channel, the softened competition in the downstream duopoly due to bundling leads to an increase in downstream prices. In particular, the price of bundle A is higher than the added prices of the standalone goods and the price of bundle B is higher than the input price of good 2, which is also its downstream price in separate selling. Therefore, even without double marginalization, and hence without an aggravation of the double marginalization problem, bundling results in higher final market prices. Consequently, bundling again results in a positive profit for firm D_B, in comparison to the zero profit under separate selling.

Moreover, bundling may either reduce both downstream firms' output levels in the market for good 2 or raise the output of one firm but lower the competitor's output. Hence, D_A might help its competitor to strengthen its relative market position by bundling. Nevertheless, firm D_A would prefer to bundle due to the consequential raise in its own profit. Furthermore, the total quantity of good 2 is always lowered due to profitable bundling, whereas the total quantity of good 1 is raised. The former clearly follows from the softening in competition in the duopoly and the latter from good 1 being sold together in a bundle with good 2, as in the decentralized channel. Notably, the quantity of good 1 increases even when good 2 has a lower quality level

[10] Exact values for the multi-product upstream monopoly case can be found in Appendix A.4.

than good 1.

Turning to the welfare analysis of bundling, we find the following:

Proposition 6. *In the centralized channel, profitable bundling results in a decrease in consumer surplus, an increase in producer surplus, an increase in total welfare when* $\tilde{a}_2 \in \left(\underline{\tilde{a}}_2, \frac{19\tilde{a}_1-9k}{10}\right)$*, and a decrease in total welfare when* $\tilde{a}_2 > \frac{19\tilde{a}_1-9k}{10}$.

Proof. See Appendix B.6. □

As in the decentralized channel, bundling reduces the consumer surplus because it increases the prices of the final goods. Nevertheless, the consumers partly benefit from the centralized market structure since they pay lower prices for the bundles than in the decentralized channel, due to the lack of double marginalization.[11] In contrast to the decentralized channel, the producer surplus increases since both downstream firms' profits rise due to bundling, whereas both upstream firms earn zero profits independent of D_A's selling strategy. This raise in producer surplus is larger than the loss in consumer surplus if $\tilde{a}_2 \in \left(\underline{\tilde{a}}_2, \frac{19\tilde{a}_1-9k}{10}\right)$. Consequently, bundling raises social welfare for a sufficiently low quality of good 2. By contrast, when the quality of good 2 is sufficiently large, i.e., $\tilde{a}_2 > \frac{19\tilde{a}_1-9k}{10}$, then the loss in consumer surplus is greater than the raise in producer surplus and bundling reduces social welfare like in the decentralized channel. Ambiguous results for the effects of bundling on social welfare are also observed by in other models, e.g., in Carbajo et al. (1990), the final effect of social welfare depends on the average costs of production.

The intuition behind our observations is as follows. An increase in $\tilde{a}_2$ raises consumer surplus under bundling and under separate selling $\left(\frac{\partial \tilde{CS}^S}{\partial \tilde{a}_2}, \frac{\partial \tilde{CS}^{BL}}{\partial \tilde{a}_2} > 0\right)$ because of the increased customers' reservation prices and therefore higher equilibrium quantities. This increase is greater under separate selling than under bundling $\left(\frac{\partial \tilde{CS}^S}{\partial \tilde{a}_2} > \frac{\partial \tilde{CS}^{BL}}{\partial \tilde{a}_2}\right)$. Thus, an increase in the quality level of good 2 leads to even further diverging consumer surpluses under bundling and separate selling. The reason for this is that changes in the quality level of good 2 have no effect on the downstream prices under separate selling but a higher $\tilde{a}_2$ induces even higher bundles prices $\left(\frac{\partial \tilde{p}_A^{BL}}{\partial \tilde{a}_2}, \frac{\partial \tilde{p}_B^{BL}}{\partial \tilde{a}_2} > 0\right)$. This has an additional negative impact on consumer surplus under bundling. As a consequence, for a sufficiently high quality level of good 2, the loss in consumer surplus is so high, that it cannot be outweighed by the gain in producer surplus and welfare consequently falls. Notice that the gain in producer surplus is even larger with a high $\tilde{a}_2$ under bundling since a raise in $\tilde{a}_2$ fosters producer surplus under bundling $\left(\frac{\partial \tilde{PS}^{BL}}{\partial \tilde{a}_2} > 0\right)$, but does not affect the producer surplus under separate selling $\left(\frac{\partial \tilde{PS}^S}{\partial \tilde{a}_2} = 0\right)$.

[11] The price of bundle A (bundle B) is smaller in the centralized channel than its counterpart in the decentralized model for $a_2 > \frac{-67a_1+164k}{97}$ ($a_2 > \frac{-22a_1+97k}{75}$) which is always satisfied because $k > \frac{-67a_1+164k}{97}, \frac{-22a_1+97k}{75}$.

4 Discussion

4.1 Non-negativity constraints

In the decentralized channel, we restrict our analysis to values of $a_2 > \frac{36a_1+29k}{65}$ to guarantee that $b_B^{BL} > 0$ and $p_B^{BL} > 0$.[12] We now relax this restriction and analyze what happens when $a_2 < \frac{36a_1+29k}{65}$. When $a_2 < \frac{36a_1+29k}{65}$ and firm D_A bundles, then it would be optimal for D_B to set $p_B^{BL} = 0$ and $b_B^{BL} = 0$ in order to guarantee zero profits and prevent losses. Anticipating D_B's behavior, D_A could strategically threaten D_B to bundle to practically drive D_B out of the market. Nevertheless, this is only a credible threat by D_A if bundling leads to a higher profit for D_A. However, when D_B leaves the market due to D_A's decision to bundle, then bundling does not necessarily generate a greater profit for D_A than selling the products separately while D_B is staying in the market. Only if $a_1 < 1.32k$ and $a_2 \in \left(0, \frac{8.243k-6.243a_1}{2}\right)$, is D_A better off with bundling. The main reason for this rather surprising result is the upstream firms' pricing behavior. In the region where bundling is profitable, the sum of upstream prices is decreased by bundling due to the rather low customer's valuation of the goods. However, the sum of upstream prices may be increased by bundling which can then make bundling unprofitable despite the elimination of a competitor.

4.2 Strict additivity relaxed

In our main analysis we assume that the bundle is of the same total quality as the sum of qualities of the standalone goods. We now relax this assumption and investigate whether changing the additivity relationship of the goods in the bundle impacts the bundling incentives of the two-product downstream firm.

In order to do that, we follow an approach by Venkatesh and Kamakura (2003) and consider a factor for the goods' degree of contingency. Thereby, we model whether the customers value the goods differently when they are sold in a bundle compared to how they value the sum of the two standalone goods. The costumer's valuation for bundle A is assumed to be constant and is denoted by a_{12}.[13] We introduce for the contingency between good 1 and good 2 in bundle A the parameter $\varepsilon := \frac{a_{12}-(a_1+a_2)}{a_1+a_2}$. When $\varepsilon > 0$, bundling adds value to the bundle compared to the sum of the standalone goods' valuations (*superadditivity*), implying complementarity between the product pair. When $\varepsilon < 0$, the customer's valuation for the bundle is smaller than the valuation for the sum of the standalone goods (*subadditivity*), which implies that the goods in the bundle have a substitute relationship. When ε is zero, the goods' valuations are strictly additive (*strict additivity*), thus being independently valued, as in the main scenario. Solving the contingency parameter ε for the costumer's valuation for bundle A leaves us with

[12] Exact values for the non-negativity case can be found in Appendix A.5.

[13] Note that bundle B consists only of good 2, which is why bundle B's quality level is denoted by a_2 independent of any value additivity assumption.

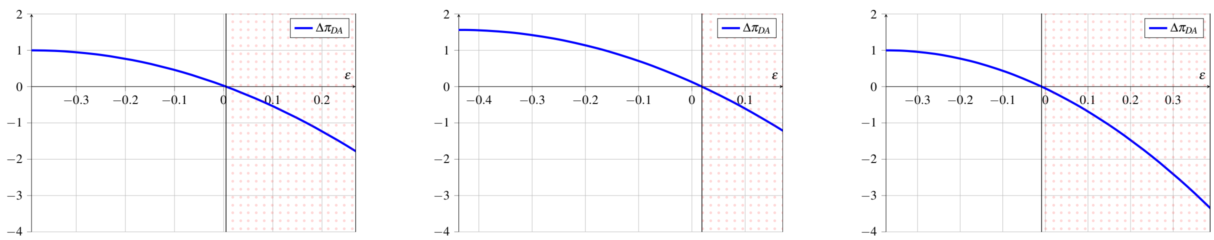
$a_{12} = (1+\varepsilon)(a_1 + a_2)$. Inserting a_{12} for the quality of bundle A in the utility W from bundling of the main decentralized case, thus Equation (2.3), yields a utility O that describes the utility of the representative customer under bundling as follows:

$$O(m, b_A, b_B) = m + (1+\varepsilon)(a_1 + a_2)b_A + a_2 b_B - \frac{1}{2}\left(2b_A^2 + 2b_A b_B + b_B^2\right).$$

We solve this case analogously to the main decentralized channel case for bundling.[14] Notice that we consider a change in the valuation for the *bundled* goods, not for the standalone goods. This is why the separate selling equilibrium results for this case are the same as in the main decentralized channel.

We use a numeric example to simplify our analysis. We calculate our results for three cases, varying the quality relationship between the standalone goods. In particular, we analyze Case a) with $a_1 = a_2 = 6$, so where both goods are of the same quality, Case b) with $7 = a_1 > a_2 = 6$, where good 1 is of higher quality then good 2, and Case c) with $6 = a_1 < a_2 = 7$, in which good 2 is of higher quality then good 1. Suppose that the marginal cost of production is constant at $k = 2$ for all three cases. Note that those values fulfill our separate selling condition $p_1^S > p_2^S$ for all three cases. In order to ensure non-negativity for the equilibrium entities (for $b_A, b_B > 0$) we impose for Case a) $\varepsilon \in (-0.3908, 0.2685)$, for Case b) $\varepsilon \in (-0.4377, 0.1709)$, and for Case c) $\varepsilon \in (-0.3740, 0.3868)$. The assumption $a_1, a_2 > k$ ensures non-negativity for the residual equilibrium values.

We determine the profitable bundling intervals (which implies $\Delta\pi_{D_A} = \pi_{D_A}^S - \pi_{D_A}^{BL} < 0$) for the respective cases as in the previous chapters. Figure 6 depicts the bundling incentive of the two-product downstream firm D_A within the non-negativity range for each case. The red dotted area marks the profitable bundling interval for the respective case.



(a) $a_1 = a_2 = 6$ **(b)** $7 = a_1 > a_2 = 6$ **(c)** $6 = a_1 < a_2 = 7$

Figure 6: Bundling incentive of D_A with contingency parameter ε ($k = 2$)

We observe that in Case a) bundling is profitable for $\varepsilon \in (0.0048, 0.2685)$ and in Case b) bundling is profitable for $\varepsilon \in (0.0188, 0.1709)$. Consequently, for both cases, bundling is only profitable for a positive contingency parameter so that $\varepsilon > 0$. This implies that when the goods have equal qualities (as in Case a) and when good 1 is of higher quality than good 2 (as in Case b), bundling is only profitable when bundle A is valued in a *superadditive* way by the

[14] Exact values for this case can be found in Appendix A.6.

customer.

Case c) reflects the quality relationship $a_1 < a_2$ between the standalone goods, which we identify in the previous chapter to be necessary for bundling to be profitable in equilibrium. Considering our numeric example, bundling is in Case c) profitable for $\varepsilon \in (-0.0088, 0.3868)$. Therefore, we observe that bundling is profitable for a negative and a positive contingency parameter as well as a contingency parameter equal zero. This implies that when good 2 is of higher quality than good 1, bundling can be profitable irrespective of whether the customer's valuation of the bundle is subadditive, strictly additive or superadditive.

We can draw several insights from this analysis: For one thing, bundling is for $a_1 < a_2$ not only profitable when the customer's valuations of the goods are strictly additive – as assumed in the main decentralized model – but also when the goods are valued as subadditive or superadditive. For another, when comparing the bundling incentives between the three cases, we can conclude that the bundling incentives in Case c), when $a_1 < a_2$, are higher than in Case a) or b). This is reflected in the fact that the profitability of bundling in the latter two cases only holds for a superadditive relationship between the goods, whereas in Case c), bundling can also be profitable for a subadditive and a strictly additive relationship between the goods. Last, this numeric analysis shows that when the goods in the bundle indeed have a superadditive relationship, thus $\varepsilon > 0$, then their quality relation is not crucial for the existence of a profitable bundling interval in general.

5 Conclusion

In this paper, we study the influence of heterogeneous product qualities on a downstream firm's bundling decision in a supply chain. We consider the downstream market to be of a common leverage theory market structure and the upstream producers to be powerful monopolists. In the downstream market, there is a two-product firm that is a monopolist in one product market but competes in prices with another firm in the second market. We analyze the incentives of the two-product downstream firm to choose pure bundling as a selling strategy. Additionally, we investigate the impact of profitable bundling on the market outcomes, especially on welfare outcomes. Furthermore, we analyze the role of product qualities as well as the distribution of market power in the channel regarding the effects of bundling. We consider a centralized channel where the downstream firms have the full market power as reference case.

We find that bundling is profitable for the two-product downstream firm only when the quality of the product sold in the downstream duopoly (*good 2*) is sufficiently high such that it also exceeds the quality of the product sold exclusively by the two-product downstream firm (*good 1*). This is because the two-product downstream firm especially benefits from the positive effects of bundling on its profit in form of a reduction in the intensity of downstream competition and extension of its monopoly power if good 2 is of high quality. The reason for this is that a high quality implies that customers have a high valuation and thus a high willingness to pay for good 2, which allows for high bundle prices. Then, bundling is profitable for the two-product downstream firm despite an aggravation of the problem of double marginalization as a consequence thereof. Whereas we find these results under the assumption of strict value additivity of the bundled goods, we confirm by means of a numeric example that for the identified quality relation bundling is also profitable when the bundled products have a superadditive value (complementary) or subadditive value (substitutable) relationship. Interestingly, we additionally demonstrate that given superadditivity among the traded goods, bundling may also be profitable for the downstream firm when the quality of good 2 is equal or lower than the quality of good 1.

In the centralized case, bundling is always profitable for the two-product downstream firm independent of the quality levels of the goods. The stronger bundling incentives in the centralized compared to the decentralized case result from the lack of double marginalization in the centralized channel. However, when we assume that the powerful upstream firms in the decentralized channel merge, but the downstream market still is of a common leverage market set-up, bundling is again always profitable for the two-product firm. This result illustrates that it is the combination of vertical externalities and horizontal externalities upstream that lowers the incentives for downstream bundling in the decentralized channel, which is in line with Bhargava (2012) and Heinzel (2019). Consequently, it shows that Bhargava's result that bundling aggravates the double marginalization problem in a channel also holds when the downstream market of the channel is of oligopolistic nature in one product market. It further shows that Heinzel's result that under price competition the positive effect of bundling in the form of a reduction in the intensity of competition can outweigh the negative effect of bundling in the form of an aggravated double marginalization problem holds when assuming that the traded goods are of different qualities.

Regarding social welfare, we find that in the decentralized channel bundling reduces consumer and producer surplus in the equilibrium. The consumer surplus is decreased because bundling induces both downstream firms to raise their prices. Interestingly, only the upstream firm selling to both downstream firms suffers from bundling due to the softening in downstream competition. This loss, however, is greater than the total gain of the other firms due to bundling, which results in an overall decrease in producer surplus. By contrast, bundling increases the producer surplus in the centralized channel because both downstream firms' profits increase and the upstream firms as price-takers gain zero profits. The consumer surplus decreases in the centralized channel due to the increased prices in bundling, where a high quality of good 2 exacerbates this negative impact due to even higher downstream prices. Finally, bundling raises (reduces) social welfare in the centralized channel when the quality level of good 2 is sufficiently low (high).

To sum up, we find that separate selling may be the superior selling strategy in comparison to bundling for a downstream firm. More specifically, in our model a two-product downstream firm in a leverage theory set-up may prefer separate selling over bundling. This result is in contrast to some parts of the previous literature on the leverage theory and can be explained by the channel structure and the consideration of powerful upstream firms. In addition, we identify the quality levels of the traded goods as a deciding factor regarding the profitability of bundling in a decentralized channel with downstream competition. The welfare effects of downstream bundling are ambiguous and are affected by the product qualities and the distribution of market power in the channel.

We derive the following managerial and economic implications from our results. Our findings suggest that downstream firms should always take the qualities of the traded products into account when deciding on bundling. We illustrate that in some cases unbundling could raise a downstream firm's profits when it procures goods from powerful producers. Additionally, we highlight that downstream bundling should not be free of antitrust concerns as it may have a negative impact on the market efficiency. Still, depending on the qualities of the goods, downstream bundling can also increase welfare when the full market power in a distribution channel is with the downstream industry.

Our work provides a solid basis on which future research can be connected. One possibility would be to allow for mixed bundling, meaning the two-product downstream firm sells the goods bundled as well as separately. While the focus of the work would shift rather to finding the optimal selling strategy, shedding light on this issue considering our market set-up could provide further important implications. Additional research might be done regarding issues related to competition policy, such as potential regulation methods for downstream bundling. Such extensions would allow for further interesting research at the interface of management and economics.

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Appendix

A Equilibrium Calculations

A.1 Separate Selling

A.1.1 Equilibrium Conditions

As we have standard Bertrand competition assumptions in the market for good 2, the equilibrium price of good 2 simply equals the input price of good 2. Moreover, π_{D_A} is strictly concave in p_1 since $\frac{\partial^2 \pi_{D_A}}{\partial p_1^2} = -2 < 0$. The equilibrium monopoly downstream price of good 1 that D_A charges is determined by the first-order condition (FOC)

$$\frac{\partial \pi_{D_A}}{\partial p_1} = a_1 - 2p_1 + c_1 \overset{!}{=} 0. \tag{A.1}$$

Solving for p_1 gives us the monopoly price.

In the upstream market, the profit of U_1 and U_2 is strictly concave in c_1 and c_2, respectively, since $\frac{d^2 \pi_{U_1}}{dc_1^2} = -1 < 0$ and $\frac{d^2 \pi_{U_2}}{dc_2^2} = -2 < 0$. The FOCs regarding the optimal input prices for both upstream firms are given as

$$\frac{d\pi_{U_1}}{dc_1} = \frac{a_1 - 2c_1 + k}{2} \overset{!}{=} 0, \tag{A.2}$$

$$\frac{d\pi_{U_2}}{dc_2} = a_2 - 2c_2 + k \overset{!}{=} 0. \tag{A.3}$$

Solving for c_1 and c_2, respectively, gives us the equilibrium input prices c_1^S and c_2^S under separate selling.

A.1.2 Welfare

The producer surplus is the sum of all firms' profits such that

$$\begin{aligned} PS^S &= \pi_{D_A}^S + \pi_{D_B}^S + \pi_{U_1}^S + \pi_{U_2}^S \\ &= \frac{7k^2 - 8a_2k - 6a_1k + 4a_2^2 + 3a_1^2}{16}. \end{aligned} \tag{A.4}$$

The consumer surplus for the respective good is given by

$$CS_1^S = \frac{a_1 - p_1}{2} Q_1^S = \frac{(a_1 - k)^2}{32}, \tag{A.5}$$

$$CS_2^S = \frac{a_2 - p_2}{2} Q_2^S = \frac{(a_2 - k)^2}{8}. \tag{A.6}$$

Thus, total consumer surplus is

$$\begin{aligned} CS^S &= CS_1^S + CS_2^S \\ &= \frac{5k^2 - 8a_2k - 2a_1k + 4a_2^2 + a_1^2}{32}. \end{aligned} \tag{A.7}$$

Finally, total welfare is

$$\begin{aligned} W^S &= PS^S + CS^S \\ &= \frac{19k^2 - 24a_2k - 14a_1k + 12a_2^2 + 7a_1^2}{32}. \end{aligned} \tag{A.8}$$

A.2 Bundling

A.2.1 Equilibrium Conditions

Under bundling, we have $\frac{\partial^2 \pi_{D_A}}{\partial p_A^2} = -2 < 0$ and $\frac{\partial^2 \pi_{D_B}}{\partial p_B^2} = -4 < 0$ in the downstream market. The downstream firms' FOCs with respect to the optimal downstream prices for bundle A, respectively, bundle B are

$$\frac{\partial \pi_{D_A}}{\partial p_A} = a_1 + c_1 + c_2 + p_B - 2p_A \stackrel{!}{=} 0, \tag{A.9}$$

$$\frac{\partial \pi_{D_B}}{\partial p_B} = a_2 - a_1 + 2c_2 - 4p_B + p_A + \stackrel{!}{=} 0. \tag{A.10}$$

From the FOCs we can derive the downstream firms' reaction functions with respect to their equilibrium prices as

$$p_A(p_B) = \frac{a_1 + c_1 + c_2 + p_B}{2}, \tag{A.11}$$

$$p_B(p_A) = \frac{a_2 - a_1 + 2c_2 + p_A}{4}. \tag{A.12}$$

The intersection of the two reaction functions generates the equilibrium prices of the two bundles.

In the upstream market, we have $\frac{\partial^2 \pi_{U_1}}{\partial c_1^2} = -\frac{6}{7} < 0$ and $\frac{\partial^2 \pi_{U_2}}{\partial c_2^2} = -\frac{10}{7} < 0$. The FOCs that determine the profit-maximizing input prices for the upstream firms are

$$\frac{\partial \pi_{U_1}}{\partial c_1} = \frac{3a_1 + a_2 - 6c_1 - c_2 + 3k}{7} \stackrel{!}{=} 0, \tag{A.13}$$

$$\frac{\partial \pi_{U_2}}{\partial c_2} = \frac{a_1 + 5a_2 - c_1 - 10c_2 + 5k}{7} \stackrel{!}{=} 0. \tag{A.14}$$

Solving the FOCs for c_1 and c_2, respectively, generates the upstream firms' reaction functions regarding their optimal prices. We get

$$c_1(c_2) = \frac{3a_1 + a_2 - c_2 + 3k}{6}, \tag{A.15}$$

$$c_2(c_1) = \frac{5a_2 + a_1 + 5k - c_1}{10}. \tag{A.16}$$

The intersection of the two reaction functions generates the equilibrium input prices c_1^{BL} and c_2^{BL}.

A.2.2 Welfare

The producer surplus under bundling is described by

$$\begin{aligned} PS^{BL} &= \pi_{D_A}^{BL} + \pi_{D_B}^{BL} + \pi_{U_1}^{BL} + \pi_{U_2}^{BL} \\ &= \frac{72202k^2 - 82700a_2k - 61704a_1k + 38635a_2^2 + 5430a_1a_2 + 28137a_1^2}{170569}. \end{aligned} \tag{A.17}$$

The consumer surplus for bundle A and bundle B, respectively, is given by

$$CS_A^{BL} = \frac{a_1 + a_2 - p_A}{2} b_A = \frac{3(-5a_2 - 29a_1 + 34k)(-51a_1 - 80a_2 + 131k)}{170569}, \tag{A.18}$$

$$CS_B^{BL} = \frac{a_2 - p_B}{2} b_B = \frac{5(36a_1 - 65a_2 + 29k)(-3a_1 - 29a_2 + 32k)}{170569}. \tag{A.19}$$

Hence, final consumer surplus is

$$\begin{aligned} CS^{BL} &= CS_A^{BL} + CS_B^{BL} \\ &= \frac{18002k^2 - 24730a_2k - 11274a_1k + 10625a_2^2 + 3480a_1a_2 + 3897a_1^2}{170569}. \end{aligned} \tag{A.20}$$

Consequently, total welfare in the bundling market amounts to

$$\begin{aligned} W^{BL} &= PS^{BL} + CS^{BL} \\ &= \frac{3\left(30068k^2 - 35810a_2k - 24326a_1k + 16420a_2^2 + 2970a_1a_2 + 10678a_1^2\right)}{170569}. \end{aligned} \tag{A.21}$$

A.2.3 Comparisons for the Decentralized Channel

- We have $\Delta p_A = p_1^S + p_2^S - p_A^{BL} = \frac{191k - 186a_2 - 5a_1}{1652} < 0$ for $a_2 > \frac{191k - 5a_1}{186}$, which is clearly fulfilled because of $a_2 > k > \frac{191k - 5a_1}{186}$. Consequently, the price of bundle A is larger than the sum of the prices of the standalone goods.
- We get $\Delta q_{A2} = q_{A2}^S - b_A^{BL} = \frac{353a_2 - 348a_1 + 5k}{1652} > 0$ if $a_2 > \frac{348a_1 + 5k}{353}$. This condition is always satisfied under profitable bundling since $\underline{a}_2^{BL} > \frac{348a_1 + 5k}{353}$. That is, D_A's quantity of good 2

decreases due to bundling in the equilibrium.

- We obtain $\Delta q_{B2} = q_{B2}^S - b_B^{BL} = \frac{288a_1 - 107a_2 - 181k}{1652} > 0$ if $a_2 < \frac{288a_1 - 181k}{107}$, which is always met in the profitable bundling interval since $\overline{a}_2 < \frac{288a_1 - 181k}{107}$. Consequently, profitable bundling reduces firm D_B's supplied quantity.

- We have $\Delta Q_1 = Q_1^S - b_A^{BL} = \frac{5(13a_1 - 12a_2 - k)}{1652} < 0$ when $a_2 > \underline{a}_2^{BL}$, which is obviously satisfied under profitable bundling. In conclusion, downstream firm D_A's quantity of good 1 is raised by profitable bundling.

- It holds that $\Delta Q^{BL} = Q_1^{BL} - Q_2^{BL} = \frac{2(36a_1 - 65a_2 + 29k)}{413} < 0$ for $a_2 > \underline{a}_2^S$, which is fulfilled by assumption. Therefore, the quantity of good 2 is always larger than the quantity of good 1 under bundling.

- We obtain $\Delta \pi_U^{BL} = \pi_{U_1}^{BL} - \pi_{U_2}^{BL} = \frac{2(-6a_1k + 10a_2k + 3a_1^2 - 5a_2^2 - 2k^2)}{59}$. Note that $\Delta \pi_U^{BL}$ is quadratic and strictly concave in a_2 as $\frac{\partial^2 \Delta \pi_U^{BL}}{\partial a_2^2} < 0$. Thus, we have $\Delta \pi_U^{BL} < 0$ for $a_2 < \frac{-\sqrt{15}a_1 + (\sqrt{15}+5)k}{5}$ and for $a_2 > \frac{\sqrt{15}a_1 - (\sqrt{15}-5)k}{5}$. However, we have $\underline{a}_2^{BL} > \frac{\sqrt{15}a_1 - (\sqrt{15}-5)k}{5}$. Consequently, given profitable bundling, the profit of U_2 is larger than the profit of U_1 in any case.

A.3 Centralized Channel

The full market power is with the downstream firms in the centralized case. The optimization problems of the downstream firms are analogous to the ones in the decentralized channel (Appendix A.1). The same holds for the welfare calculations. Hence, we simply set $\tilde{c}_1^S = \tilde{c}_1^{BL} = k$ and $\tilde{c}_2^S = \tilde{c}_2^{BL} = k$ and substitute the input prices into the market entities of the decentralized channel. Tables 1 and 2 provide an overview of the respective results in the centralized case.

	Separate Selling	**Bundling**
Profit Downstream Firms	$\tilde{\pi}^S_{D_A} = \frac{(\tilde{a}_1-k)^2}{4}$	$\tilde{\pi}^{BL}_{D_A} = \frac{(3\tilde{a}_1+\tilde{a}_2-4k)^2}{49}$
	$\tilde{\pi}^S_{D_B} = 0$	$\tilde{\pi}^{BL}_{D_B} = \frac{2(\tilde{a}_1-2\tilde{a}_2+k)^2}{49}$
Profit Upstream Firms	$\tilde{\pi}^S_{U_1} = 0$	$\tilde{\pi}^{BL}_{U_1} = 0$
	$\tilde{\pi}^S_{U_2} = 0$	$\tilde{\pi}^{BL}_{U_2} = 0$
Input Prices	$\tilde{c}^S_1 = k$	$\tilde{c}^{BL}_1 = k$
	$\tilde{c}^S_2 = k$	$\tilde{c}^{BL}_2 = k$
Prices	$\tilde{p}^S_1 = \frac{\tilde{a}_1+k}{2}$	$\tilde{p}^{BL}_A = \frac{3\tilde{a}_1+\tilde{a}_2+10k}{7}$
	$\tilde{p}^S_2 = k$	$\tilde{p}^{BL}_B = \frac{-\tilde{a}_1+2\tilde{a}_2+6k}{7}$
Quantities	$\tilde{Q}^S_1 = \frac{\tilde{a}_1-k}{2}$	$\tilde{Q}^{BL}_1 = \frac{3\tilde{a}_1+\tilde{a}_2-4k}{7}$
	$\tilde{Q}^S_2 = \tilde{a}_2 - k$	$\tilde{Q}^{BL}_2 = \frac{\tilde{a}_1+5\tilde{a}_2-6k}{7}$
Downstream Quantities	$\tilde{q}^S_{A1} = \frac{\tilde{a}_1-k}{2}$	$\tilde{b}^{BL}_A = \frac{3\tilde{a}_1+\tilde{a}_2-4k}{7}$
	$\tilde{q}^S_{A2} = \frac{\tilde{a}_2-k}{2}$	
	$\tilde{q}^S_{B2} = \frac{\tilde{a}_2-k}{2}$	$\tilde{b}^{BL}_B = \frac{2(-\tilde{a}_1+2\tilde{a}_2-k)}{7}$

Table 1: Centralized Channel: Equilibrium Values

Producer Surplus	$\tilde{PS}^S = \frac{(\tilde{a}_1-k)^2}{4}$
	$\tilde{PS}^{BL} = \frac{18k^2-16\tilde{a}_2k-20\tilde{a}_1k+9\tilde{a}_2^2-2\tilde{a}_1\tilde{a}_2+11\tilde{a}_1^2}{49}$
Consumer Surplus	$\tilde{CS}^S = \frac{5k^2-8\tilde{a}_2k-2\tilde{a}_1k+4\tilde{a}_2^2+\tilde{a}_1^2}{8}$
	$\tilde{CS}^{BL} = \frac{26k^2-34\tilde{a}_2k-18\tilde{a}_1k+13\tilde{a}_2^2+8\tilde{a}_1\tilde{a}_2+5\tilde{a}_1^2}{49}$
Welfare	$\tilde{W}^S = \frac{7k^2-8\tilde{a}_2k-6\tilde{a}_1k+4\tilde{a}_2^2+3\tilde{a}_1^2}{8}$
	$\tilde{W}^{BL} = \frac{44k^2-50\tilde{a}_2k-38\tilde{a}_1k+22\tilde{a}_2^2+6\tilde{a}_1\tilde{a}_2+16\tilde{a}_1^2}{49}$

Table 2: Centralized Channel: Welfare

A.3.1 Comparisons for the Centralized Channel

- We get $\Delta\tilde{q}_{A2} = \tilde{q}^S_{A2} - \tilde{b}^{BL}_A = \frac{6\tilde{a}_1+5\tilde{a}_2+k}{14} < 0$ for $\tilde{a}_2 < \frac{6\tilde{a}_1-k}{5}$. We observe that $\underline{\tilde{a}}_2 < \frac{6\tilde{a}_1-k}{5}$. That is, firm D_A's quantity of good 2 may rise due to bundling. Moreover, $\Delta\tilde{q}_{B2} = \tilde{q}^S_{B2} - \tilde{b}^{BL}_B = \frac{4\tilde{a}_1-\tilde{a}_2-3k}{14} < 0$ holds when $\tilde{a}_2 > 4\tilde{a}_1 - 3k$. We have $\underline{\tilde{a}}_2 < 4\tilde{a}_1 - 3k$. Thus, firm D_B's quantity of good 2 may increase as well due to bundling. However, as $4\tilde{a}_1 - 3k > \frac{6\tilde{a}_1-k}{5}$, we can rule out a situation where both firms' quantities of good 2 increase in the bundling equilibrium. In contrast, when $4\tilde{a}_1 - 3k > \tilde{a}_2 > \frac{6\tilde{a}_1 - k}{5}$, both firms' quantities regarding good 2 would fall.

- We have $\Delta\tilde{Q}_2 = \tilde{Q}_2^S - \tilde{Q}_2^{BL} = \frac{-\tilde{a}_1+2\tilde{a}_2-k}{7} > 0$ for $\tilde{a}_2 > \underline{\tilde{a}}_2$. Clearly, the total quantity of good 2 is reduced as a consequence of bundling.

- It holds that $\Delta\tilde{Q}_1 = \tilde{Q}_1^S - \tilde{Q}_1^{BL} = \frac{\tilde{a}_1-2\tilde{a}_2+k}{14} < 0$ for $\tilde{a}_2 > \underline{\tilde{a}}_2$. Obviously, the supplied quantity of good 1 increases due to bundling.

- We have $\Delta\tilde{p}_A = \tilde{p}_1^S + \tilde{p}_2^S - \tilde{p}_A^{BL} = \frac{\tilde{a}_1-2\tilde{a}_2+k}{14} < 0$ for $\tilde{a}_2 > \underline{\tilde{a}}_2$. Hence, the price of bundle A is larger than the sum of the prices of the standalone goods.

- We obtain $\Delta\frac{\partial \tilde{CS}}{\partial \tilde{a}_2} = \frac{\partial \tilde{CS}^S}{\partial \tilde{a}_2} - \frac{\partial \tilde{CS}^{BL}}{\partial \tilde{a}_2} > 0$ when $\tilde{a}_2 > \frac{8\tilde{a}_1+15k}{23}$, which is always fulfilled because $\tilde{a}_2 > \underline{\tilde{a}}_2 > \frac{8\tilde{a}_1+15k}{23}$.

A.3.2 Decentralized versus Centralized Channel

- We have $\Delta p_A^{VS} = \tilde{p}_A^{BL} - p_A^{BL} = \frac{2(-67a_1-97a_2+164k)}{413} < 0$ when $a_2 > \frac{-67a_1+164k}{97}$, which is always satisfied because $a_1 > k > \frac{-67a_1+164k}{97}$. We can conclude that the price of bundle A is lower in the centralized channel than in the decentralized channel.

- We have $\Delta p_B^{VS} = \tilde{p}_B^{BL} - p_B^{BL} = \frac{2(-22a_1-75a_2+97k)}{413} < 0$ when $a_2 > \frac{-22a_1+97k}{75}$, which is satisfied because $a_1 > k > \frac{-22a_1+97k}{75}$. Hence, the price of bundle B is lower in the centralized channel than in the decentralized channel.

- Downstream firm D_A's profit in the decentralized case exceeds its profit in the centralized case if and only if

$$\begin{aligned}\Delta\pi_{D_A}^{BL} &= \tilde{\pi}_{D_A}^{BL} - \pi_{D_A}^{BL} \\ &= \frac{45292k^2 + (-24788a_2 - 65796a_1)k + 3256a_2^2}{170569} \\ &\quad + \frac{18276a_1a_2 + 23760a_1^2}{170569} < 0. \end{aligned} \tag{A.22}$$

Note that $\Delta\pi_{D_A}^{BL}$ is quadratic and strictly convex in a_2 $\left(\frac{\partial^2 \Delta\pi_{D_A}}{\partial a_2^2} = \frac{6512}{170569} > 0\right)$. We obtain $\Delta\pi_{D_A}^{BL} < 0$ for $a_2 \in \left(\frac{-132a_1+169k}{37}, \frac{-45a_1+67k}{22}\right)$. However, we have $a_2 > k > \frac{-45a_1+67k}{22}$ and hence we always get $\Delta\pi_{D_A}^{BL} > 0$. Thus, firm D_A's bundling profit is always higher in the centralized case than in the decentralized case.

A.4 Multi-Product Upstream Monopoly

Consider the case that both products, good 1 and good 2, are produced solely by one independent upstream firm, which is called firm U.

A.4.1 Separate Selling

In separate selling, the multi-product upstream monopolist has the following profit function:

$$\pi_U(c_1, c_2) = (c_1 - k) Q_1(c_1) + (c_2 - k) Q_2(c_2). \tag{A.23}$$

Both produced goods, good 1 and good 2, are independent in demand. Thereby, the profit is strictly concave in input prices and we derive the same FOCs as in the case with two independent upstream producers. Consequently, solving the optimization problem of the multi-product upstream monopoly firm leads to the same input prices as with separate producers.

A.4.2 Bundling

In case downstream firm D_A bundles its products, multi-product upstream monopoly firm U's profit function is analogous to under separate selling. We have $\frac{\partial^2 \pi_U}{\partial c_1^2} = -\frac{6}{7}$ and $\frac{\partial^2 \pi_U}{\partial c_2^2} = -\frac{10}{7}$ here. The FOCs for the multi-product upstream monopoly firm are given as

$$\frac{\partial \pi_U}{\partial c_1} = \frac{3a_1 + a_2 + 4k - 6c_1 - 2c_2}{7} \stackrel{!}{=} 0, \tag{A.24}$$

$$\frac{\partial \pi_U}{\partial c_2} = \frac{a_1 + 5a_2 + 6k - 2c_1 - 10c_2}{7} \stackrel{!}{=} 0. \tag{A.25}$$

Solving the equation system of FOCs above for c_1 and c_2 leads to the optimal input prices, which are identical to the respective ones of the multi-product upstream monopoly in the separate selling case.

The following Tables 3 and 4 provide an overview of the market results after inserting the optimal input prices for the separate selling case and the bundling case. We have $b_B > 0$ for $a_2 > \frac{a_1+k}{2}$ and $p_B > 0$ for $a_2 > \frac{a_1-6k}{9}$, where $\frac{a_1+k}{2} > \frac{a_1-6k}{9}$. We thus assume $a_1 > k$ and $a_2 > \frac{a_1+k}{2}$, where $\frac{a_1+k}{2} > k$, which ensures non-negativity for all parameters in the multi-product upstream monopoly case.

	Separate Selling	Bundling
Profit Downstream Firms	$\pi^S_{D_A} = \frac{(a_1-k)^2}{16}$	$\pi^{BL}_{D_A} = \frac{(3a_1+a_2-4k)^2}{196}$
	$\pi^S_{D_B} = 0$	$\pi^{BL}_{D_B} = \frac{(-a_1+2a_2-k)^2}{98}$
Profit Upstream Firm U	$\pi^S_U = \frac{3k^2-4a_2k-2a_1k+2a_2^2+a_1^2}{8}$	$\pi^{BL}_U = \frac{10k^2-12a_2k-8a_1k+5a_2^2+2a_1a_2+3a_1^2}{28}$
Input Prices	$c^S_1 = \frac{a_1+k}{2}$	$c^{BL}_1 = \frac{a_1+k}{2}$
	$c^S_2 = \frac{a_2+k}{2}$	$c^{BL}_2 = \frac{a_2+k}{2}$
Prices	$p^S_1 = \frac{3a_1+k}{4}$	$p^{BL}_A = \frac{5a_1+4a_2+5k}{7}$
	$p^S_2 = \frac{a_2+k}{2}$	$p^{BL}_B = \frac{-a_1+9a_2+6k}{14}$
Quantities	$Q^S_1 = \frac{a_1-k}{4}$	$Q^{BL}_1 = \frac{3a_1+a_2-4k}{14}$
	$Q^S_2 = \frac{a_2-k}{2}$	$Q^{BL}_2 = \frac{a_1+5a_2-6k}{14}$
Downstream Quantities	$q^S_{A1} = \frac{a_1-k}{4}$	$b^{BL}_A = \frac{3a_1+a_2-4k}{14}$
	$q^S_{A2} = \frac{a_2-k}{4}$	
	$q^S_{B2} = \frac{a_2-k}{4}$	$b^{BL}_B = \frac{-a_1+2a_2-k}{7}$

Table 3: Multi-Product Upstream Monopoly: Equilibrium Values

Producer Surplus	$PS^S = \frac{7k^2-8a_2k-6a_1k+4a_2^2+3a_1^2}{16}$
	$PS^{BL} = \frac{22k^2-25a_2k-19a_1k+11a_2^2+3a_1a_2+8a_1^2}{49}$
Consumer Surplus	$CS^S = \frac{5k^2-8a_2k-2a_1k+4a_2^2+a_1^2}{32}$
	$CS^{BL} = \frac{26k^2-34a_2k-18a_1k+13a_2^2+8a_1a_2+5a_1^2}{196}$
Welfare	$W^S = \frac{19k^2-24a_2k-14a_1k+12a_2^2+7a_1^2}{32}$
	$W^{BL} = \frac{114k^2-134a_2k-94a_1k+57a_2^2+20a_1a_2+37a_1^2}{196}$

Table 4: Multi-Product Upstream Monopoly: Welfare

A.4.3 Bundling Decision

In the multi-product upstream monopoly setting, $p^S_1 > p^S_2$ holds when $a_2 < \frac{3a_1-k}{2}$. Notice that $\frac{3a_1-k}{2} > \frac{a_1+k}{2}$. For that reason, we restrict the quality of good 2 from above by $\frac{3a_1-k}{2}$, which differs from the centralized case. In the multi-product upstream monopoly, downstream firm

D_A's profit under bundling exceeds its separate selling profit if and only if

$$\Delta\pi_{D_A} = \pi^S_{D_A} - \pi^{BL}_{D_A} = -\frac{(-2a_2 + a_1 + k)(-2a_2 - 13a_1 + 15k)}{784} < 0. \tag{A.26}$$

Notice that $\Delta\pi_{D_A}$ is quadratic and strictly concave in a_2 $\left(\frac{\partial^2 \Delta\pi_{D_A}}{\partial a_2^2} = -\frac{1}{98} < 0\right)$. We get $\Delta\pi_{D_A} < 0$ for $a_2 < \frac{15k-13a_1}{2}$ and for $a_2 > \frac{a_1+k}{2}$. The latter is given by assumption and thus we always have $\Delta\pi_{D_A} < 0$. Therefore, bundling is always profitable for D_A when both goods are produced by one monopolistic producer, as in the centralized case.

A.5 Non-negativity Constraints

In this case, we relax the non-negativity constraints of the decentralized channel that $b^{BL}_B > 0$ and $p^{BL}_B > 0$, which was secured by $a_2 > \frac{36a_1+29k}{65}$. Thus, we analyze the equilibrium values for $a_2 < \frac{36a_1+29k}{65}$.

A.5.1 Equilibrium Values

Table 5 provides an overview about the market results for the relaxed non-negativity case under bundling and separate selling. Note that the equilibrium results under separate selling are the same as in the decentralized case with constraints. The values under bundling arise from a market setting in which the rival of the two-product downstream firm D_A, meaning downstream firm D_B, may leave the market provided that D_A bundles, which makes D_A the sole downstream firm on the market.

A.5.2 Bundling Decision

In the case, where we relax the non-negativity restriction, downstream firm D_A's profit under bundling exceeds its separate selling profit if and only if

$$\Delta\pi^{neg}_{D_A} = \pi^S_{D_A} - \pi^{BL}_{D_A} = \frac{-4a_1a_2 - 10a_1k + 8a_2k + 7a_1^2 - 2a_2^2 + k^2}{144} < 0. \tag{A.27}$$

Note that $\Delta\pi^{neg}_{D_A}$ is quadratic and strictly concave in a_2 $\left(\frac{\partial^2 \Delta\pi^{neg}_{D_A}}{\partial a_2^2} = -\frac{1}{36} < 0\right)$. We get $\Delta\pi^{neg}_{D_A} < 0$ for $a_2 < \frac{-6.243a_1+8.243k}{2}$ and for $a_2 > \frac{2.243a_1-0.243k}{2}$. Since $\frac{2.243a_1-0.243k}{2} > \frac{36a_1+29k}{65}$ and $a_1, a_2, k > 0$, profitable bundling is only feasible when $a_2 \in \left(0, \frac{-6.243a_1+8.243k}{2}\right)$. Furthermore, $0 < a_2 < \frac{-6.243a_1+8.243k}{2}$ can only hold for $a_1 < 1.32k$. To summarize, if $a_1 < 1.32k$ and $a_2 \in \left(0, \frac{-6.243a_1+8.243k}{2}\right)$, then D_A is better off with bundling.

	Separate Selling	**Bundling**
Profit Downstream Firms	$\pi^S_{D_A} = \frac{(a_1-k)^2}{16}$	$\pi^{BL}_{D_A} = \frac{(a_1+a_2-2k)^2}{72}$
	$\pi^S_{D_B} = 0$	$\pi^{BL}_{D_B} = 0$
Profit Upstream Firms	$\pi^S_{U_1} = \frac{(a_1-k)^2}{8}$	$\pi^{BL}_{U_1} = \frac{(a_1+a_2-2k)^2}{36}$
	$\pi^S_{U_2} = \frac{(a_2-k)^2}{4}$	$\pi^{BL}_{U_2} = \frac{(a_1+a_2-2k)^2}{36}$
Input Prices	$c^S_1 = \frac{a_1+k}{2}$	$c^{BL}_1 = \frac{a_1+a_2+k}{3}$
	$c^S_2 = \frac{a_2+k}{2}$	$c^{BL}_2 = \frac{a_1+a_2+k}{3}$
Final Prices	$p^S_1 = \frac{3a_1+k}{4}$	$p^{BL}_A = \frac{5a_1+5a_2+2k}{6}$
	$p^S_2 = \frac{a_2+k}{2}$	$p^{BL}_B = 0$
Quantities	$Q^S_1 = \frac{a_1-k}{4}$	$Q^{BL}_1 = \frac{a_1+a_2-2k}{12}$
	$Q^S_2 = \frac{a_2-k}{2}$	$Q^{BL}_2 = \frac{a_1+a_2-2k}{12}$
Downstream Quantities	$q^S_{A1} = \frac{a_1-k}{4}$	$b^{BL}_A = \frac{a_1+a_2-2k}{12}$
	$q^S_{A2} = \frac{a_2-k}{4}$	
	$q^S_{B2} = \frac{a_2-k}{4}$	$b^{BL}_B = 0$

Table 5: Relaxed Non-Negativity Constraints: Equilibrium Values

A.6 Strict Additivity Relaxed

In this case we relax the assumption about the goods having a strictly additive value relationship. In the following analysis, we allow for subadditivity, strict additivity and superadditivity and analyze which impact those additivity relations have on the bundling incentives of the two-product downstream firm.

A.6.1 Equilibrium Values

The calculations for the bundling case are conducted analogously to the main decentralized case. The calculations for separate selling are identical to the ones in the decentralized case since the additivity assumption only concerns the customer's valuation of bundle A. Table 6 provides an overview about the equilibrium results for the additivity case.

	Separate Selling	**Bundling**
Profit Downstream Firms	$\pi^S_{D_A} = \frac{(a_1-k)^2}{16}$	$\pi^{BL}_{D_A} = \frac{9(29a_1+5a_2-34k+29a_1\varepsilon+29a_2\varepsilon)^2}{170569}$
	$\pi^S_{D_B} = 0$	$\pi^{BL}_{D_B} = \frac{2(36a_1-65a_2+29k+36a_1\varepsilon+36a_2\varepsilon)^2}{170569}$
Profit Upstream Firms	$\pi^S_{U_1} = \frac{(a_1-k)^2}{8}$	$\pi^{BL}_{U_1} = \frac{3(29a_1+5a_2-34k+29a_1\varepsilon+29a_2\varepsilon)^2}{24367}$
	$\pi^S_{U_2} = \frac{(a_2-k)^2}{4}$	$\pi^{BL}_{U_2} = \frac{5(3a_1+29a_2-32k+3a_1\varepsilon+3a_2\varepsilon)^2}{24367}$
Input Prices	$c^S_1 = \frac{a_1+k}{2}$	$c^{BL}_1 = \frac{29a_1+5a_2+25k+29a_1\varepsilon+29a_2\varepsilon}{59}$
	$c^S_2 = \frac{a_2+k}{2}$	$c^{BL}_2 = \frac{3a_1+29a_2+27k+3a_1\varepsilon+3a_2\varepsilon}{59}$
Final Prices	$p^S_1 = \frac{3a_1+k}{4}$	$p^{BL}_A = \frac{311a_1+253a_2+262k+311a_1\varepsilon+311a_2\varepsilon}{413}$
	$p^S_2 = \frac{a_2+k}{2}$	$p^{BL}_B = \frac{-15a_1+268a_2+160k-15a_1\varepsilon-15a_2\varepsilon}{413}$
Quantities	$Q^S_1 = \frac{a_1-k}{4}$	$Q^{BL}_1 = \frac{3(29a_1+5a_2-34k+29a_1\varepsilon+29a_2\varepsilon)}{413}$
	$Q^S_2 = \frac{a_2-k}{2}$	$Q^{BL}_2 = \frac{5(3a_1+29a_2-32k+3a_1\varepsilon+3a_2\varepsilon)}{413}$
Downstream Quantities	$q^S_{A1} = \frac{a_1-k}{4}$	$b^{BL}_A = \frac{3(29a_1+5a_2-34k+29a_1\varepsilon+29a_2\varepsilon)}{413}$
	$q^S_{A2} = \frac{a_2-k}{4}$	
	$q^S_{B2} = \frac{a_2-k}{4}$	$b^{BL}_B = \frac{2(-36a_1+65a_2-29k-36a_1\varepsilon-36a_2\varepsilon)}{413}$

Table 6: Equilibrium Results Additivity: Equilibrium Values

A.6.2 Bundling Decision

We conduct our analysis using a numeric example. In Case a) we suppose the relation $a_1 = a_2 = 6$, in Case b) we suppose $7 = a_1 > a_2 = 6$, and in Case c) we suppose $6 = a_1 < a_2 = 7$. We further assume for all three cases $k = 2$. Note that those values fulfill the condition $p^S_1 > p^S_2$.

- In Case a) downstream firm D_A's profit under bundling exceeds its separate selling profit if and only if

$$\begin{aligned}\Delta\pi^{a)}_{D_A} &= \pi^S_{D_A} - \pi^{BL}_{D_A}\\ &= \frac{(892-4176(\varepsilon+1))\,(4176(\varepsilon+1)-4196)}{2729104} < 0. \qquad \text{(A.28)}\end{aligned}$$

  Note that $\Delta\pi^{a)}_{D_A}$ is quadratic and strictly concave in ε $\left(\frac{\partial^2\Delta\pi^{a)}_{D_A}}{\partial\varepsilon^2} = -\frac{2179872}{170569} < 0\right)$. We get $\Delta\pi^{a)}_{D_A} < 0$ for $\varepsilon < -0.7864$ and for $\varepsilon > 0.0048$. To guarantee $b_A, b_B > 0$, we impose $\varepsilon \in (-0.3908, 0.2685)$. Since $-0.3908 > -0.7864$ and $0.2685 > 0.0048$, we find that D_A is better of with bundling for $\varepsilon \in (0.0047, 0.2685)$.

- In Case b) downstream firm D_A's profit under bundling exceeds its separate selling profit if and only if

$$\begin{aligned}\Delta\pi_{D_A}^{b)} &= \pi_{D_A}^{S} - \pi_{D_A}^{BL}\\ &= \frac{(479 - 4524(\varepsilon+1))(4524(\varepsilon+1) - 4609)}{2729104} < 0.\end{aligned} \tag{A.29}$$

Note that $\Delta\pi_{D_A}^{b)}$ is quadratic and strictly concave in ε $\left(\frac{\partial^2 \Delta\pi_{D_A}^{b)}}{\partial\varepsilon^2} = -\frac{2558322}{170569} < 0\right)$. We get $\Delta\pi_{D_A}^{b)} < 0$ for $\varepsilon < -0.8941$ and for $\varepsilon > 0.0188$. To guarantee $b_A, b_B > 0$, we impose $\varepsilon \in (-0.4377, 0.1709)$. Since $0.4377 > -0.8941$ and $0.1709 > 0.0188$, we find that D_A is better of with bundling for $\varepsilon \in (0.0188, 0.1709)$.

- In Case c) downstream firm D_A's profit under bundling exceeds its separate selling profit if and only if

$$\begin{aligned}\Delta\pi_{D_A}^{c)} &= \pi_{D_A}^{S} - \pi_{D_A}^{BL}\\ &= \frac{(1180 - 4524(\varepsilon+1))(4524(\varepsilon+1) - 4484)}{2729104} < 0.\end{aligned} \tag{A.30}$$

Note that $\Delta\pi_{D_A}^{c)}$ is quadratic and strictly concave in ε $\left(\frac{\partial^2 \Delta\pi_{D_A}^{c)}}{\partial\varepsilon^2} = -\frac{2558322}{170569} < 0\right)$. We get $\Delta\pi_{D_A}^{c)} < 0$ for $\varepsilon < -0.7392$ and for $\varepsilon > -0.0088$. To guarantee $b_A, b_B > 0$, we impose $\varepsilon \in (-0.3740, 0.3868)$. Since $-0.3740 > -0.7392$ and $0.3868 > -0.0088 > -0.3740$, we find that D_A is better of with bundling for $\varepsilon \in (-0.0088, 0.3868)$.

B Proofs of Propositions

B.1 Proof of Proposition 1

Downstream firm D_A's bundling profit exceeds its separate selling profit if and only if

$$\begin{aligned}\Delta\pi_{D_A} &= \pi_{D_A}^{S} - \pi_{D_A}^{BL}\\ &= \frac{5(13a_1 - 12a_2 - k)(761a_1 + 60a_2 - 821k)}{2729104} < 0.\end{aligned} \tag{B.1}$$

Notice that $\Delta\pi_{D_A}$ is quadratic and strictly concave in a_2 $\left(\frac{\partial^2 \Delta\pi_{D_A}}{\partial a_2^2} = -\frac{450}{170569} < 0\right)$. Thus, we obtain $\Delta\pi_{D_A} < 0$ for $a_2 < a_2^1 := \frac{-761a_1+821k}{60}$ or $a_2 > a_2^2 := \frac{13a_1-k}{12}$. As we assume $a_1, a_2 > k$, we clearly have $a_2^2 > k \geq 0$, $k > a_2^1$ and $\overline{a}_2 > a_2^2$. Consequently, firm D_A prefers bundling over separate selling when $a_2 \in \left(\underline{a}_2^{BL}, \overline{a}_2\right)$, where $\underline{a}_2^{BL} := a_2^2$ stands for the lower bound and $\overline{a}_2$ for the upper bound of the profitable bundling interval. Notice that the upper bound is derived from the assumption $p_1^S > p_2^S$. In case $a_2 \in \left(\underline{a}_2^S, \underline{a}_2^{BL}\right)$, where $\underline{a}_2^S := \frac{36a_1+29k}{65}$, firm D_A prefers separate selling over bundling since then bundling leads to a lower profit than separate selling.

B.2 Proof of Proposition 2

We get $c_1^S < c_1^{BL}$ when $a_2 > \frac{a_1+9k}{10}$, where $\underline{a}_2^S > \frac{a_1+9k}{10}$. Consequently, for any $a_2 \in \left(\underline{a}_2^S, \overline{a}_2\right)$, bundling increases the input price of good 1.

We have $c_2^S < c_2^{BL}$ if $a_2 < 6a_1 - 5k$. Note that $\overline{a}_2 < 6a_1 - 5k$ since we assume $a_1 > k$. Consequently, for any $a_2 \in \left(\underline{a}_2^S, \overline{a}_2\right)$, bundling increases the input price of good 2.

B.3 Proof of Proposition 3

The proof of the proposition is as follows:

- Upstream firm U_1's profit increases due to bundling if and only if

$$\Delta\pi_{U_1} = \pi_{U_1}^S - \pi_{U_1}^{BL} = \frac{4183a_1^2 - 600a_2^2 - 6960a_1a_2 + 8160a_2k - 1406a_1k - 3377k^2}{194936} < 0. \qquad \text{(B.2)}$$

  Notice that $\Delta\pi_{U_1}$ is quadratic and strictly concave in a_2 $\left(\frac{\partial^2 \Delta\pi_{U_1}}{\partial a_2^2} = -\frac{150}{24367} < 0\right)$. We find that $\Delta\pi_{U_1} < 0$ for $a_2 < \frac{(59\sqrt{42}+408)k+(-59\sqrt{42}-348)a_1}{60}$ or $a_2 > -\frac{(59\sqrt{42}-408)k+(348-59\sqrt{42})a_1}{60}$. The lower bound of the profitable bundling interval is greater than the larger root of $\Delta\pi_{U_1}$, i.e. $\underline{a}_2^{BL} > -\frac{\left(59\sqrt{42}-408\right)k+\left(348-59\sqrt{42}\right)a_1}{60}$. Consequently, in the profitable bundling interval, $a_2 > -\frac{(59\sqrt{42}-408)k+(348-59\sqrt{42})a_1}{60}$ is satisfied in any case and thus we always have $\Delta\pi_{U_1} < 0$. Therefore, the profit of upstream firm U_1 increases due to profitable bundling.

- Upstream firm U_2's profit is increased by bundling if and only if

$$\Delta\pi_{U_2} = \pi_{U_2}^S - \pi_{U_2}^{BL} = \frac{3887k^2 - 11614a_2k + 3840a_1k + 7547a_2^2 - 3480a_1a_2 - 180a_1^2}{97468} < 0. \qquad \text{(B.3)}$$

  The function $\Delta\pi_{U_2}$ is strictly convex and quadratic in a_2 $\left(\frac{\partial^2 \Delta\pi_{U_2}}{\partial a_2^2} = \frac{7547}{48734} > 0\right)$. We have $\Delta\pi_{U_2} < 0$ for $a_2 \in \left(\frac{(354\sqrt{35}+5807)k+(1740-354\sqrt{35})a_1}{7547}, -\frac{(354\sqrt{35}-5807)k+(-354\sqrt{35}-1740)a_1}{7547}\right)$. However, it holds that $\underline{a}_2^{BL} > -\frac{\left(354\sqrt{35}-5807\right)k+\left(-354\sqrt{35}-1740\right)a_1}{7547}$. That is, the lower bound of the profitable bundling interval is greater than the upper bound of the interval of a_2, in which $\Delta\pi_{U_2} < 0$. Thus, for any a_2 in the profitable bundling interval, $\Delta\pi_{U_2} > 0$ is given. Therefore, profitable bundling reduces U_2's profit.

B.4 Proof of Proposition 4

We prove the cases according to the cases in the proposition:

- The consumer surplus increases as a consequence of bundling if and only if

$$\begin{aligned}\Delta CS &= CS^S - CS^{BL}\\ &= \frac{276781k^2 - 573192a_2k + 19630a_1k + 342276a_2^2}{5458208}\\ &+ \frac{-111360a_1a_2 + 45865a_1^2}{5458208} < 0.\end{aligned} \tag{B.4}$$

  Notice that ΔCS is strictly convex and quadratic in a_2 $\left(\frac{\partial^2 \Delta CS}{\partial a_2^2} = \frac{85569}{682276} > 0\right)$ with its vertex regarding a_2 at $V\left(\frac{4640a_1+23883k}{28523} \middle| \frac{6155(a_1-k)^2}{912736}\right)$. It holds that $\frac{6155(a_1-k)^2}{912736} > 0$ and therefore ΔCS is always greater zero. This means that bundling reduces the consumer surplus in any case.

- The producer surplus rises due to bundling if and only if

$$\begin{aligned}\Delta PS &= PS^S - PS^{BL}\\ &= \frac{3(12917k^2 - 13784a_2k - 12050a_1k)}{2729104}\\ &+ \frac{3(21372a_2^2 - 28960a_1a_2 + 20505a_1^2)}{2729104} < 0.\end{aligned} \tag{B.5}$$

  The function ΔPS is strictly convex and quadratic in a_2 $\left(\frac{\partial^2 \Delta PS}{\partial a_2^2} = \frac{16209}{341138} > 0\right)$. It has its vertex with respect to a_2 at $V\left(\frac{3620a_1+1723k}{5343} \middle| \frac{335(a_1-k)^2}{28496}\right)$. Note that $\frac{335(a_1-k)^2}{28496} > 0$ holds and thus $\Delta PS > 0$. Consequently, the producer surplus decreases as a consequence of bundling.

- The previous two cases show that bundling reduces consumer as well as producer surplus and consequently total welfare.

B.5 Proof of Proposition 5

In the centralized channel, firm D_A's bundling profit exceeds its separate selling profit if and only if

$$\begin{aligned}\Delta\tilde{\pi}_{D_A} &= \tilde{\pi}^S_{D_A} - \tilde{\pi}^{BL}_{D_A} \\ &= \frac{(\tilde{a}_1 - 2\tilde{a}_2 + k)(13\tilde{a}_1 + 2\tilde{a}_2 - 15k)}{196} < 0. \end{aligned} \tag{B.6}$$

Note that $\Delta\tilde{\pi}_{D_A}$ is quadratic and strictly concave in $\tilde{a}_2$ $\left(\frac{\partial^2 \Delta\tilde{\pi}_{D_A}}{\partial \tilde{a}_2^2} = -\frac{2}{49} < 0\right)$. Solving for $\tilde{a}_2$ yields that we have $\Delta\tilde{\pi}_{D_A} < 0$ for $\tilde{a}_2 < \frac{-13\tilde{a}_1+15k}{2}$ or for $\tilde{a}_2 > \frac{\tilde{a}_1+k}{2} =: \underline{\tilde{a}}_2$. Since we assume $\tilde{a}_2 > \underline{\tilde{a}}_2$, we always have $\Delta\tilde{\pi}_{D_A} < 0$, which means that firm D_A always prefers bundling over separate selling in the centralized channel.

B.6 Proof of Proposition 6

We prove the cases in the order proposed in the proposition:

- The consumer surplus is raised by bundling if and only if

$$\begin{aligned}\Delta\tilde{CS} &= \tilde{CS}^S - \tilde{CS}^{BL} \\ &= \frac{(\tilde{a}_1 - 2\tilde{a}_2 + k)(9\tilde{a}_1 - 46\tilde{a}_2 + 37k)}{392} < 0. \end{aligned} \tag{B.7}$$

  We observe that $\Delta\tilde{CS}$ is quadratic and strictly convex in $\tilde{a}_2$ $\left(\frac{\partial^2 \Delta\tilde{CS}}{\partial \tilde{a}_2^2} = \frac{23}{49} > 0\right)$. Furthermore, we observe that $\Delta\tilde{CS} < 0$ for $\tilde{a}_2 \in \left(\frac{9\tilde{a}_1+37k}{46}, \underline{\tilde{a}}_2\right)$. However, when $\tilde{a}_2 > \underline{\tilde{a}}_2$, we get $\Delta\tilde{CS} > 0$, where $\tilde{a}_2 > \underline{\tilde{a}}_2$ is given by assumption. Consequently, the consumer surplus is reduced by bundling with certainty.

- When D_A bundles, it earns a higher profit than under separate selling. Additionally, D_B gains a positive profit under bundling in contrast to a zero profit under separate selling. Clearly, the producer surplus consisting of the two downstream firms' profits and the zero profits of the two upstream firms is raised by bundling.

- Total welfare rises as a consequence of bundling if and only if

$$\begin{aligned}\Delta\tilde{W} &= \tilde{W}^S - \tilde{W}^{BL} \\ &= \frac{19\tilde{a}_1^2 + 20\tilde{a}_2^2 - 48\tilde{a}_1\tilde{a}_2 - 9k^2 - k(-8\tilde{a}_2 - 10\tilde{a}_1)}{392} < 0. \end{aligned} \tag{B.8}$$

  Note that $\Delta\tilde{W}$ is quadratic and strictly convex in $\tilde{a}_2$ $\left(\frac{\partial^2 \Delta\tilde{W}}{\partial \tilde{a}_2^2} = \frac{5}{49} > 0\right)$. Further note that when $\tilde{a}_2 \in \left(\underline{\tilde{a}}_2, \frac{19\tilde{a}_1-9k}{10}\right)$, we obtain $\Delta\tilde{W} < 0$. Consequently, for $\tilde{a}_2 \in \left(\underline{\tilde{a}}_2, \frac{19\tilde{a}_1-9k}{10}\right)$, bundling increases total welfare. If $\tilde{a}_2 > \frac{19\tilde{a}_1-9k}{10}$, we have $\Delta\tilde{W} > 0$ and thus bundling decreases total welfare in the centralized channel. Remember that $\tilde{a}_2 < \underline{\tilde{a}}_2$ is ruled out by assumption.