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The Impact of Product Differentiation on Retail Bundling in a Vertical Market

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Abstract

We study the effects of product differentiation on the bundling incentives of a two-product retailer. Two monopolistic manufacturers each produce a differentiated good. One sells it to both retailers, while the other only supplies a single retailer. Retailers compete in prices. Retail bundling is profitable when the goods are close substitutes. Only then is competition so intense that the retailer uses bundling to relax competition both within and across product markets, despite an aggravation of the double marginalization problem. Our asymmetric market structure arises endogenously for the case of close substitutes. In this case, bundling reduces social welfare.

JEL classification: D43; L13; L42

Keywords: retail bundling; upstream market power; double marginalization; product differentiation

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1 Introduction

Retailers often have to ponder whether to sell their products separately or as bundles, combining several products into one. Presumably, the products' characteristics have a major impact on this decision. In this paper, we therefore study the impact of product differentiation between two products on the incentive to bundle them in a vertically related market. More specifically, we examine how a change in the degree of complementarity or substitutability affects the equilibrium in this market, in particular the prices, quantities, and profits, and we identify a two-product retailer's incentive to sell the goods as a (pure) bundle. Further, we analyze the welfare implications of this decision.

Our framework follows Heinzel (2019) and consists of two manufacturers, two retailers, and two differentiated products. The upstream side of the market is characterized by two monopolistic manufacturers, each producing one of the products. One retailer buys one good exclusively from one manufacturer, while the other retailer purchases from both manufacturers. In this asymmetric set-up, the two-product retailer has the option to bundle the two goods. Consequently, both retailers compete in one of the product markets, while the other good is sold exclusively by the two-product retailer. We assume the retailers compete in prices. In order to model product differentiation, we adopt the approach pioneered by Dixit (1979) and Singh and Vives (1984).

Our results apply to the digital sector, for example. The upstream firms may be content providers of some digital content, such as movies, and the downstream firms may be streaming service providers or TV stations, which purchase the content from the upstream firms and sell them, separately or bundled, to the final customers. For example, a customer could watch two series from the same genre, which might have a substitute character to each other or a long movie complementary to a short comedy series. Rennhoff and Serfes (2009) already mentioned the increasing importance of bundling decisions in the modern digital age, where digital contents such as movies, news, or games are traded as goods by content providers who could bundle the different services.

We investigate the following questions: *How does product differentiation, that is, the degree of substitution or complementarity, affect the equilibrium prices, quantities, and profits under the two selling regimes (separate selling and pure bundling)? How does the degree of product differentiation change the two-product retailer's incentive to bundle the goods? What are the welfare consequences of retail bundling in our framework?*

Our main result is that the two-product retailer will only bundle the products when the goods constitute close substitutes. While this might seem counterintuitive, one can often observe bundles of close substitutes in reality. For example, in the grocery store, packs of pepper are sold in bundles, either as (almost) perfect substitutes of identical color or as close substitutes, just differing in color. Furthermore, the clothing industry frequently offers bundles of either the same clothes or in packs where the items just differ in pattern or color from each other.

The intuition underlying our main result runs as follows. In our market set-up, bundling reduces competition in two ways, both within and across product markets. The lower the degree of product differentiation, the stronger the competition across the differentiated products. Eventually, when the products become close substitutes and competition gets very intense, the anticompetitive effect of bundling is strongest. Only then, the competition-reducing effect of bundling outweighs the aggravation of the double marginalization problem that occurs due to bundling in our vertical market structure. Therefore, retail bundling is profitable only for close substitutes.

Considering the impact of product differentiation on the market outcome, we observe that with increasing complementarity, both under bundling and separate selling, prices and demand increase for both goods (for the exclusive good, this only applies up to a point where the goods become close to perfect complements). This holds because, for complementary products, an increase in one product's demand fosters the demand of the respective other product. Moreover, an increase in complementarity raises the consumers' willingness to pay, which is why the demand increases despite the increase in prices.

Regarding social welfare, retail bundling reduces the consumer surplus due to higher downstream prices, but it enhances the producer surplus, since all firms, both in the downstream and in the upstream market, gain from bundling. Social welfare, however, only increases when the original products are close to perfect substitutes and decreases otherwise.

We complete our analysis, exploring how our asymmetric market set-up can arise once we endogenize the manufacturers' choice of retailers. It turns out that an asymmetric distribution system occurs in equilibrium when the original goods constitute close, but not too close substitutes. In this case, bundling represents an equilibrium outcome and reduces social welfare.

Our paper is organized as follows. In Section 2, we relate our paper to the existing bundling literature. In Section 3, we present the general model and solve it for the market equilibrium under separate selling and bundling, respectively. In Section 4, we discuss the impact of product differentiation on the market equilibrium under both separate selling and bundling, we investigate the bundling decision, and we examine the consequences of bundling for the market equilibrium and for social welfare. In Section 5, we provide a foundation for our market set-up, endogenizing the manufacturers' distribution choice. Section 6 concludes.

2 Literature

Our paper relates to two strands of literature. The first strand explores how different market structures impact on the incentives for (retail) bundling. The second strand investigates how product differentiation affects the bundling incentives. Given that our focus lies on the latter effect, we keep the treatment of the former strand rather short.[1]

[1] An extensive overview on how the market structure affects the incentives for (retail) bundling can be found in Heinzel (2019).

Some contributions to the first strand study retail bundling in a (decentralized) distribution channel. This literature considers a market with a monopolistic retailer and either one manufacturer (see e.g. Girju et al., 2013; Ma and Mallik, 2017; Cao et al., 2019) or several manufacturers (see e.g. Bhargava, 2012; Chakravarty et al., 2013; Cao et al., 2015; Chen et al., 2016). Other contributions examine the role of competition in an integrated market. One part of this literature focuses on the competitive bundling aspect in the sense that a multi-product firm that is a monopolist in one market, might strategically use bundling to affect, especially soften, the intensity of competition in a secondary competitive market (see e.g. Carbajo et al., 1990; Martin, 1999; Egli, 2007; Chen et al., 2016; Vamosiu, 2018b). The other part of this literature concentrates on the analysis of bundling as a strategy that deters entry or forecloses the second market and thereby monopolizes it (see e.g. Whinston, 1990; Choi and Stefanadis, 2001; Carlton and Waldman, 2002; Nalebuff, 2004; Peitz, 2008; Hurkens et al., 2019). The latter strand of literature is also known as *leverage theory*.[2]

Heinzel (2019) models a leverage theory set-up in the downstream market of his decentralized distribution channel while studying the bundling incentives of a retailer. He identifies that under retail price competition, retail bundling can be profitable depending on the manufacturers' marginal costs. He also finds that retail bundling lowers consumer and producer welfare. His research differs from ours in that he does not consider product differentiation between the goods. In an extension of his model, Endres and Heinzel (2019) examine the impact of product qualities on retail bundling. They find that the quality relations of the products are pivotal for the profitability of retail bundling. By contrast, the present study concentrates on the impact of substitutability and complementarity between the products that exhibit the same quality.

Our paper most closely relates to the literature on bundling with product differentiation. A seminal paper is by Lewbel (1985), who finds that pure bundling might be an integrated two-product firm's best selling strategy when the goods have a substitutable relationship. Dansby and Conrad (1984) demonstrate that mixed bundling can be favored over pure bundling by a two-product monopolist when the products have a subadditive customer valuation and hence represent substitutes. Venkatesh and Kamakura (2003) show that a decrease in the level of substitution, or an increase in the level of complementarity, between the goods has a positive effect on the incentive of an integrated two-product monopolist to purely bundle the goods. For non-linear demand relations, Telser (1979) points out that the integrated monopolist seller can only extract the full consumer surplus when he sells complementary goods as a bundle.[3] There are two major differences between our set-up and the ones in the above papers on product differentiation. First, they all consider integrated markets while we consider a vertically

[2] As a clear distinction between the two parts of the strand is not always possible, we refer to papers of either type as leverage theory papers. However, we do not focus on the entry-exit aspect of this literature, but on the competitive bundling aspect.

[3] There are various articles examining the impact of different ranges of substitutability and complementarity, e.g. the seminal ones by Spence (1976a,b), Dixit (1979), Singh and Vives (1984), Häckner (2000). However, these authors do not consider bundling as a selling strategy, but rather focus on different aspects of oligopolistic or monopolistic competition.

separated market. Second, they all focus on pure monopolists while we assume the bundling firm to engage in duopolistic competition in one retail market.

The literature on bundling of differentiated goods that indeed considers competition between (integrated) firms within a leverage theory context takes on several directions. Some studies examine the impact of complementarity on entry deterrence or on the exclusion of a rival in the competitive market; see e.g. Whinston (1990), Choi and Stefanadis (2001), Carlton and Waldman (2002), Nalebuff (2004). These authors generally demonstrate that bundling complementary goods has the potential to foreclose the market or to defend it against potential entry. Other papers focus on the aspect of *(in)compatibility* of bundling differentiated goods in a leverage market set-up such as, for instance, Denicolo (2000) or Vamosiu (2018a).[4]

Closer to our model is the literature considering how bundling of differentiated goods can be used to strategically affect the downstream competition in a leverage theory or related structure. For instance, Egli (2007) examines a Hotelling framework where one firm sells two products in a bundle, while another firm competes against one of the two products. It turns out that the firms choose minimum differentiation, but charge different prices in equilibrium. Mantovani (2013) analyzes a market with a producer of two complementary goods, who, for each of the goods, faces a rival competing with a differentiated product. He finds that the multi-product firm chooses bundling under price competition when the degree of substitutability between two variants of one good is sufficiently high and/or when the two goods offered by the two-product firm represent weak complements. Exploring a similar market set-up, Gwon (2015) illustrates that the two-product firm chooses bundling when the goods are strong complements. Chung et al. (2013) analyze a set-up with a two-product firm and a one-product rival. They find that a decrease in the degree of differentiation between the *bundled* goods strengthens the bundling incentives. Furthermore, they observe that the welfare effects of bundling are ambiguous and differ depending on the mode of (price vs. quantity) competition. The main difference between our work and this strand of literature is that we investigate the impact of product differentiation *and* the vertical market structure on a firm's bundling incentives and on the welfare effects of (retail) bundling.

In the literature on retail bundling in a supply chain, some studies examine the bundling of *complementary* products. For example, Shao (2016) finds that cooperation between suppliers increases the profitability of bundling when the degree of complementarity is high. Pan and Zhou (2017) obtain that retail bundling is profitable when the goods are moderately weak complements. Liu et al. (2020) find that the retailer prefers separate selling when the products represent weak complements. Contrary to this literature, we derive a retailer's bundling incentive for both complementary and substitutable products, taking retail competition into account, and analyze the welfare consequences of bundling.

Our paper further relates to the literature on bundling in oligopolies. Also considering com-

[4]Notice that Matutes and Regibeau (1992) also deal with compatibility, but that they examine a duopoly in which both firms produce two goods, each one representing a differentiated version of the other firm's good.

petition in the downstream market, Rennhoff and Serfes (2009) combine retail bundling and product differentiation. They investigate the bundling incentives in a market with two upstream and two downstream firms, where the original products are horizontally differentiated. It turns out that both upstream and downstream firms benefit from downstream bundling. In contrast to us, Rennhoff and Serfes do not analyze the strategic motive of a downstream retailer to use bundling as a tool to leverage its monopoly power on one market to impact competition in the other market. Apart from this difference, they do not allow for complementarity between the goods. Zhou (2017) observes that with more than two firms in the market, bundling may relax competition and thereby reduce consumer welfare but raise the firms' profits.

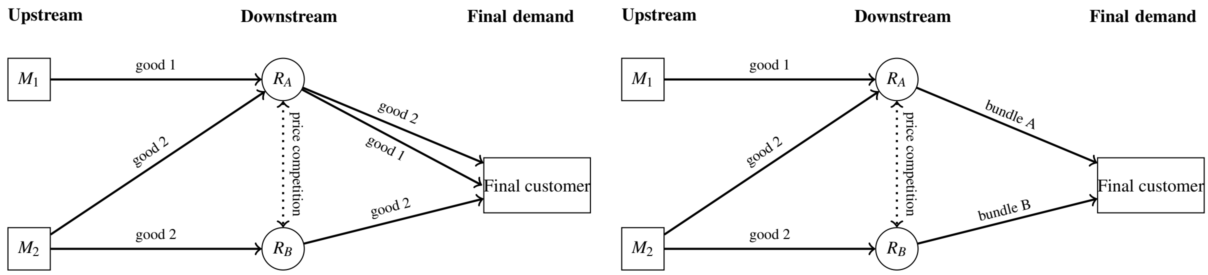
In summary, our paper contributes to the literature on bundling as follows: First, adding a *vertical market structure,* we enrich the efforts already made to study the consequences of *product differentiation* on a firm's bundling decision and on social welfare. Furthermore, we consider a decentralized channel with *downstream competition* in one market as well as *monopoly power* in the other and in the upstream tier. Our set-up allows us to analyze how the interplay of monopoly power, retail competition, double marginalization, and product differentiation affects the incentives for retail bundling and its consequences. Incorporating these aspects in our model allows us to derive implications from both an antitrust perspective and a managerial point of view.

3 The model and its market equilibria

Consider a market with two monopolistic manufacturers, M_1 and M_2, and two retailers, R_A and R_B (see Figure 1). Each manufacturer M_i produces a single good i ($i = 1, 2$). While manufacturer M_2 sells his good to *both* retailers, manufacturer M_1 maintains an exclusive relationship with R_A, selling his good *only* to R_A. We suppose that R_A has sufficient market power to establish an exclusive relationship with M_1 even though it may be disadvantageous for the producer. In Section 5, we relax this assumption and endogenize the manufacturers' distribution choice. Our set-up mirrors electronic retail markets, for example, where retailers make the producer sell certain products exclusively to them which are often labeled as store brands. Such an exclusivity agreement may serve to dampen the degree of interbrand competition in the retail market (Moner-Colonques et al., 2004).

Retailer R_A offers both goods to the final customer, while R_B only offers good 2. Retailer R_A has the option to sell his goods separately or as a bundle (see Figure 1). The final customers can only purchase the goods from either of the two retailers. Both retailers compete in prices. Even though the manufacturers may be inferior to the downstream firms in terms of market power and cannot set their distribution relations themselves, they still have market power in the upstream market. Hence, M_1 and M_2 act as *price-setters*.

We assume the goods 1 and 2 are differentiated, allowing them to represent (imperfect) complements, (imperfect) substitutes, or to be independent in demand of each other. In contrast,



(a) Separate selling

(b) Bundling

Figure 1: Market structures

the versions of good 2 sold by retailer R_A and R_B constitute perfect substitutes. Goods 1 and 2 are further characterized by their customers' marginal willingness to pay a_i, where $a_i > 0$ and $i = 1, 2$, which can be interpreted as the product quality (see e.g. Häckner, 2000). In our model, both goods have the same quality level, that is, we assume $a_1 = a_2 = a$.[5] For simplicity, we assume the manufacturers' marginal cost of production to be equal for both goods and denote it by $k \in [0, a)$. We assume $k < a$ to ensure market transactions occur in equilibrium.

The timing of the game involves three stages (see Figure 2). At stage 1, the two-product retailer R_A decides whether to bundle or to sell his products separately. At stage 2, the manufacturers each set their profit-maximizing wholesale price. At stage 3, the retailers choose their prices. At each stage, the decisions are taken independently and simultaneously. We solve this sequential game by backward induction to derive its subgame perfect equilibrium (SPE).

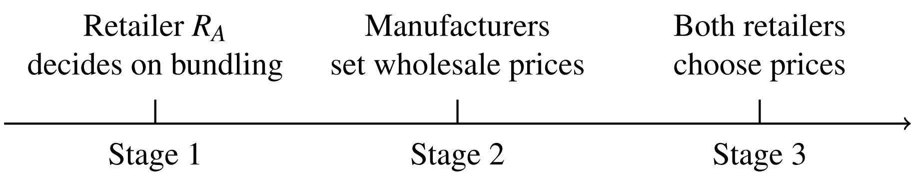


Figure 2: Timing of the game

This timing is reflected in several industries, for instance, in the streaming service industry. Consider Netflix as a retailer that offers its media content only with pure bundling. The content producers are aware of the streaming firm's selling strategy and set their prices accordingly. Afterwards, Netflix adjusts its downstream price when necessary.

In the following, we examine the market first for the case of separate selling and then for the case of bundling (Sections 3.1 and 3.2). For each of the cases, we derive the subgame perfect equilibrium of the pricing subgame starting at stage 2. The corresponding equilibrium results are denoted by superscripts S and BL , respectively. The calculations can be found in Appendix A.

[5]For a study of bundling incentives that considers asymmetric product qualities, see e.g. Endres and Heinzel (2019).

3.1 Market equilibrium under separate selling

Suppose the two-product retailer R_A sells his goods separately to the final customer. Figure 1a depicts the market structure of this case.

The (representative) customer's utility from consuming goods 1 and 2 and a bundle of other goods, m, follows Dixit (1979):[6]

$$V = m + a(Q_1 + Q_2) - \frac{1}{2}\left(Q_1^2 + 2\theta Q_1 Q_2 + Q_2^2\right), \tag{1}$$

where Q_1 and Q_2 denote the consumption of goods 1 and 2, respectively. The parameter $\theta \in (-1,1)$ captures the degree of product differentiation between the goods.[7] In case of $\theta = 0$, the two goods are independent in demand.[8] For $\theta > 0$ ($\theta < 0$), the goods constitute substitutes (complements). Finally, in the limit of $\theta \to 1$ ($\theta \to -1$), the goods represent perfect substitutes (perfect complements). In the following, however, we restrict our analysis to $\theta \in (-1,1)$. Moreover, let p_1 and p_2 denote the prices for good 1 and good 2, respectively. The composite good is denoted by m and its price is normalized to one.

Solving the customer's utility maximization problem, we obtain the inverse demand for both products:

$$p_1 = a - Q_1 - \theta Q_2, \tag{2}$$

$$p_2 = a - Q_2 - \theta Q_1, \tag{3}$$

where $Q_1 = q_{A1}$ and $Q_2 = q_{A2} + q_{B2}$. Here, q_{A1} and q_{A2} denote the quantities of goods 1 and 2 supplied by retailer R_A and q_{B2} denotes the quantity of good 2 supplied by retailer R_B.

To derive the demand for the two products, we invert the system of inverse demands, (2) and (3):

$$Q_1 = \frac{a - p_1 - (a - p_2)\theta}{1 - \theta^2}, \tag{4}$$

$$Q_2 = \frac{a - p_2 - (a - p_1)\theta}{1 - \theta^2}. \tag{5}$$

Both retailers set their prices independently and simultaneously, aiming to maximize profit. Selling both good 1 and good 2, retailer R_A sets the corresponding prices p_1 and p_{A2} to maximize

$$\pi_A = (p_1 - w_1)q_{A1} + (p_{A2} - w_2)q_{A2}, \tag{6}$$

where w_1 and w_2 denote the wholesale prices for goods 1 and 2, set by the upstream firms. The

[6] Strictly speaking, V represents a utility *function* $V(m, Q_1, Q_2)$. However, for better readability we omit function arguments in the following. Moreover, as usual, we think of the customer as representing the whole mass of customers (or consumers). Therefore, we also make use of the plural form in our discussions.

[7] Notice that we refer to the goods as *becoming* or *being* complements or substitutes even though they are only *perceived* as such by the customers. Still, within the course of our analysis, we use both expressions interchangeably.

[8] For an analysis of the bundling incentives for independent goods, see Heinzel (2019).

profit of retailer R_A consists of two parts. The first part represents the profit from selling good 1 in the monopoly market. The second part comprises the profit earned in the duopoly market of good 2. In contrast, retailer R_B is only active in the market of good 2 and sets his price p_{B2} in order to maximize

$$\pi_B = (p_{B2} - w_2) q_{B2}. \tag{7}$$

The price equilibrium in the retail market is then given by

$$p_1^S = \frac{a(1-\theta) + w_1 + \theta w_2}{2}, \tag{8}$$

$$p_2^S = p_{A2}^S = p_{B2}^S = w_2. \tag{9}$$

In the market of the homogeneous good 2, price competition drives the price down to marginal cost, which is given by the wholesale price w_2. In the market of good 1, retailer R_A exploits his monopoly power and charges the monopoly price given that $p_2^S = w_2$. Observe that, unlike in a two-product monopoly, product differentiation θ affects the equilibrium price in the monopoly market of good 1, which is caused by the competition externality in the market of good 2.

Inserting the equilibrium prices (8) and (9) into demand (4) and (5), we obtain the equilibrium demand:

$$Q_1 = \frac{a(1-\theta) - w_1 + w_2\theta}{2(1-\theta^2)}, \tag{10}$$

$$Q_2 = \frac{a(1-\theta)(2+\theta) + w_1\theta - w_2\left(2-\theta^2\right)}{2(1-\theta^2)}. \tag{11}$$

Notice that the signs of the cross-price derivatives coincide with the sign of θ. Thus, the goods 1 and 2 represent substitutes (complements) in the upstream market if, and only if, they do so in the downstream market.[9]

In the upstream market, the manufacturers M_1 and M_2 each set their wholesale price w_1 and w_2 in order to maximize profit:

$$\pi_1 = (w_1 - k) Q_1, \tag{12}$$

$$\pi_2 = (w_2 - k) Q_2, \tag{13}$$

where Q_1 and Q_2 are given by (10) and (11), respectively.

Solving for the price equilibrium in the upstream market, we obtain the wholesale prices for goods 1 and 2:

$$w_1^S = \frac{4(a+k) - 2(a-k)\theta - (3a+2k)\theta^2 + (a-k)\theta^3}{8-5\theta^2}, \tag{14}$$

$$w_2^S = \frac{4(a+k) - (a-k)\theta - (3a+2k)\theta^2}{8-5\theta^2}. \tag{15}$$

[9] We have $\frac{\partial Q_2}{\partial w_1} = \frac{\partial Q_1}{\partial w_2} < 0$ for $\theta \in (-1,0)$ and $\frac{\partial Q_2}{\partial w_1} = \frac{\partial Q_1}{\partial w_2} > 0$ for $\theta \in (0,1)$.

The remaining equilibrium results under separate selling can be derived by inserting the equilibrium wholesale prices (14) and (15) into the other expressions. Table 1 in the Appendix A.5 contains an extensive list of equilibrium outcomes, such as prices, quantities, profits in both the upstream and the downstream market, producer surplus, consumer surplus, and social welfare.

In Section 4, we explore the impact of product differentiation θ on the market equilibrium in greater detail. Before, however, we continue with deriving the market equilibrium under bundling.

3.2 Market equilibrium under bundling

Now assume retailer R_A combines one unit of good 1 and one unit of good 2 to one unit of the final product, which we call bundle A. That is, R_A uses *pure bundling* as his selling strategy. As before, retailer R_B offers good 2 to the final customers, which we call bundle B for notation purposes. Bundle B consists of one unit of good 2 only. The upstream side of the market remains the same. Figure 1b depicts the market structure in the case of bundling.

Accordingly, the total demand of good 1 equals the demand of bundle A, denoted by b_A, while the demand of good 2 is given by the sum of the demands for bundles A and B, the demand for the latter being denoted by b_B. The total demand for good 1 and good 2 can hence be expressed by

$$Q_1 = b_A, \tag{16}$$

$$Q_2 = b_A + b_B, \tag{17}$$

respectively.

As proposed in Martin (1999), we insert (16) and (17) into (1) to derive the customer's utility under bundling:

$$V = m + a(2b_A + b_B) - \frac{1}{2}\left(2(1+\theta)b_A^2 + 2(1+\theta)b_A b_B + b_B^2\right).$$

Solving the customer's utility maximization problem with respect to b_A and b_B, we obtain the inverse demand for bundle A and bundle B:

$$p_A = 2a - 2(1+\theta)b_A - (1+\theta)b_B, \tag{18}$$

$$p_B = a - (1+\theta)b_A - b_B, \tag{19}$$

where p_A and p_B denote the prices of bundle A and bundle B, respectively.

To derive the demand for the two bundles, we invert the system of inverse demands, (18) and (19). This yields

$$b_A = \frac{a(1-\theta) - p_A + p_B(1+\theta)}{1-\theta^2}, \tag{20}$$

$$b_B = \frac{p_A - 2p_B}{1-\theta}. \tag{21}$$

Notice that the cross-price effects of the two bundles are always positive.[10] Thus, bundling creates a substitute relationship between the bundles no matter whether the standalone goods represent substitutes or complements (see the result of Martin, 1999). Furthermore, observe that the retail firms do not compete for the customers of good 2 on a product-to-product basis anymore, since customers now choose between bundle A and bundle B, which are imperfect substitutes. Therefore, bundling softens price competition for good 2.

Under bundling, the profits of retailers R_A and R_B are given by

$$\pi_A = (p_A - w_1 - w_2)\, b_A, \tag{22}$$

$$\pi_B = (p_B - w_2)\, b_B. \tag{23}$$

The price equilibrium in the downstream market can then be derived as

$$p_A^{BL} = \frac{4a(1-\theta) + 4w_1 + 2w_2(3+\theta)}{7-\theta}, \tag{24}$$

$$p_B^{BL} = \frac{a(1-\theta) + w_1 + 5w_2}{7-\theta}. \tag{25}$$

To derive the input demand for goods 1 and 2 in the upstream market, we insert the equilibrium retail prices (24) and (25) into the demands (20) and (21). We hence obtain

$$Q_1 = \frac{4a(1-\theta) - w_1(3-\theta) - w_2(1-3\theta)}{(7-\theta)(1-\theta)(1+\theta)}, \tag{26}$$

$$Q_2 = \frac{2a(3-2\theta-\theta^2) - w_1(1-3\theta) - w_2(5-\theta-2\theta^2)}{(7-\theta)(1-\theta)(1+\theta)}. \tag{27}$$

Unlike separate selling, bundling alters the substitute relationship in the upstream market. When the customer perceives goods 1 and 2 as weak substitutes in the downstream market, bundling induces the input goods 1 and 2 in the upstream market to become complements. This occurs for $\theta \in (0, 1/3)$. Overall, the input goods represent complements for $\theta \in (-1, 1/3)$, while for $\theta \in (1/3, 1)$ they constitute substitutes.[11]

In order to obtain the upstream firms' profit functions, π_1 and π_2, we insert the demands (26) and (27) into (12) and (13), respectively. Solving for the price equilibrium in the upstream market yields the equilibrium wholesale prices under bundling:

$$w_1^{BL} = \frac{2a(17-13\theta-9\theta^2+5\theta^3) + k(25-11\theta^2-2\theta^3)}{59-26\theta-29\theta^2+8\theta^3}, \tag{28}$$

[10] We have $\frac{\partial b_A}{\partial p_B} = \frac{1+\theta}{1-\theta^2} > 0$ and $\frac{\partial b_B}{\partial p_A} = \frac{1}{1-\theta} > 0$ for $\theta \in (-1, 1)$.

[11] We have $\frac{\partial Q_2}{\partial w_1} = \frac{\partial Q_1}{\partial w_2} < 0$ for $\theta \in (-1, 1/3)$ and $\frac{\partial Q_2}{\partial w_1} = \frac{\partial Q_1}{\partial w_2} > 0$ for $\theta \in (1/3, 1)$.

$$w_2^{BL} = \frac{4a(8-5\theta-4\theta^2+\theta^3)+k(27-6\theta-13\theta^2+4\theta^3)}{59-26\theta-29\theta^2+8\theta^3}. \tag{29}$$

Like under separate selling, we obtain the remaining equilibrium results by inserting the equilibrium wholesale prices (28) and (29) into the other expressions. Table 1 also collects the equilibrium results for the case of bundling (see Appendix A.5).

Compared to separate selling, bundling modifies the nature of competition. First, it affects product differentiation in the downstream market in two ways. On the one hand, it eliminates the perfect substitutability between the two versions of good 2 in the retail market. While under separate selling, customers could buy identical copies from either retailer, under bundling, a customer has to choose between bundle b_A, which also contains one unit of good 1, or bundle b_B, which only contains a unit of good 2. On the other hand, it also creates a complementarity between goods 1 and 2 for the case of weak substitutes. An increase in product differentiation reduces the incentive to compete aggressively with each other and hence mitigates the price competition between the retailers in the duopoly (which is in line with the literature, see e.g. Carbajo et al., 1990; Egli, 2007; Mantovani, 2013; Chung et al., 2013; Endres and Heinzel, 2019). This enables retailer R_B to set a price for bundle B above the wholesale price of good 2. It also enables retailer R_A to charge a price for bundle A above the sum of wholesale prices of both standalone goods. Second, bundling changes competition in the upstream market as well. Under separate selling, the manufacturer's products in the upstream market constitute substitutes (complements) if, and only if, the final customers in the downstream market perceive goods 1 and 2 as substitutes (complements). In contrast, under bundling, the manufacturers' outputs represent complements even if the final consumers perceive them to be weak substitutes.

In Section 4, we now explore the market outcomes and the implications of product differentiation in greater detail.

4 Equilibrium analysis

We first investigate the effect of product differentiation on prices and quantities in the market equilibrium under separate selling and bundling, respectively. Subsequently, we examine the incentive of retailer R_A to bundle his goods rather than to sell them separately. Finally, we explore how optimal bundling affects the market outcome and social welfare.

4.1 Price and demand behavior under separate selling and bundling

We begin with discussing the effect of product differentiation on the level of prices (see Figure 3). Starting from the case of independent goods ($\theta = 0$), the equilibrium prices in both the downstream and the upstream market monotonically increase when goods 1 and 2 become more complementary. Only the wholesale price of good 1 eventually decreases when goods 1 and 2 become close to perfect complements. This occurs both under separate selling and bundling. Similarly, the retail price of good 1 decreases under separate selling, while the retail price of bundle A continues to increase when goods 1 and 2 become strong complements.[12]

The reason for the price increase is that with *increasing complementarity* the goods become more essential to each other and thus more valuable for the customers. Both the upstream and the downstream firms skim off this rent by asking higher prices. Furthermore, independent of where the customer buys good 2, he can only buy the complementary good 1 at R_A, which strengthens retailer R_A's monopoly position for good 1 and allows him to increase his price for good 1 and bundle A, respectively. For very close complements he is even able to set a price for bundle A very close to the sum of reservation prices for the standalone goods and thereby comes close to extracting the full consumer surplus. This effect is similar to the one described in Telser (1979), who, for the case of non-linear demand, finds that a two-product seller can extract the full consumer surplus by bundling when the goods represent complements.

When the complementarity of the products approaches perfect complementarity, *under separate selling,* both the wholesale price and the retail price of good 1 eventually decrease while the respective prices of good 2 continue to increase. This difference is caused by the different modes of competition. While good 1 is sold in a monopoly market, there is competition in the market for good 2. The two-product retailer R_A (partly) internalizes the negative cross-price effects between products 1 and 2 by offering product 1 at a lower price in order to promote the demand for good 2. This induces manufacturer M_2 to raise his wholesale price, the increase of which retailer R_A passes on to his customers by increasing the retail price of good 2. Anticipating R_A's behavior, M_1 reduces his price in order to stimulate the demand.

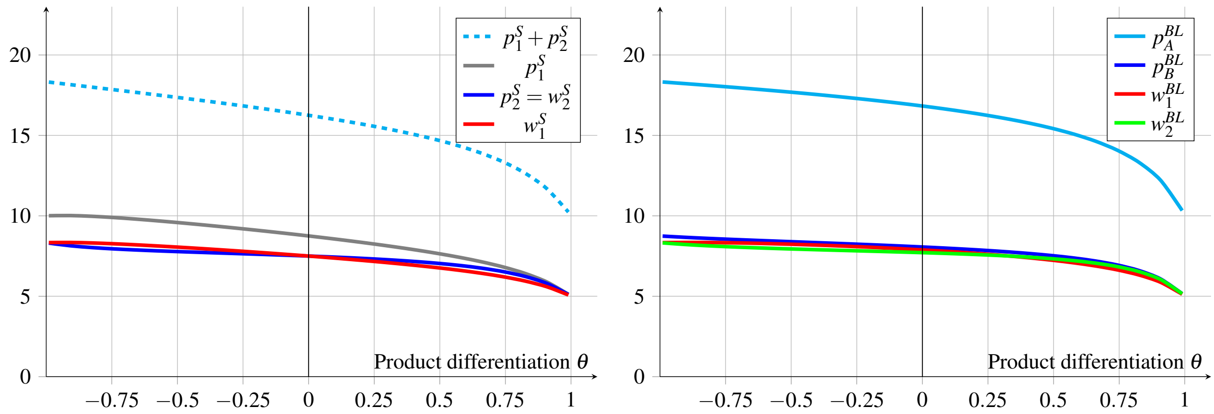
Under bundling, M_1 too reduces his price once the complementarity becomes sufficiently strong, since this induces retailer R_A to buy more of good 1. Due to the complementarity between goods 1 and 2, retailer R_A also buys more of good 2. In contrast to separate selling, however, retailer R_A does not respond by reducing his price, but rather raises it further. This points to a crucial difference between separate selling and bundling. As complementarities become sufficiently strong, bundling allows retailer R_A to leverage his market power from the market of good 1 to the market of good 2. As a consequence of this, he need not reduce the price of good 1 to stimulate the demand for good 2, but can rather afford to raise the price of the bundle.

When the degree of *substitution* between goods 1 and 2 increases, competition between the goods and bundles intensifies under both separate selling and bundling. Under *separate selling,* this primarily affects the competition between the two goods offered by retailer R_A, since competition between retailers R_A and R_B is already extreme, driving the price of good 2

[12] More specifically, we find $\frac{\partial p_1^S}{\partial \theta} = \frac{\partial w_1^S}{\partial \theta} < 0$ for $\theta \in (-0.9407, 1)$ and $\frac{\partial p_1^S}{\partial \theta} = \frac{\partial w_1^S}{\partial \theta} > 0$ for $\theta \in (-1, -0.9408)$, $\frac{\partial w_1^{BL}}{\partial \theta} < 0$ for $\theta \in (-0.8933, 1)$ and $\frac{\partial w_1^{BL}}{\partial \theta} > 0$ for $\theta \in (-1, -0.8934)$, $\frac{\partial p_2^S}{\partial \theta} = \frac{\partial w_2^S}{\partial \theta} < 0$ for $\theta \in (-1, 1)$, and $\frac{\partial p_B^{BL}}{\partial \theta}$, $\frac{\partial p_A^{BL}}{\partial \theta}$, and $\frac{\partial w_2^{BL}}{\partial \theta} < 0$ for $\theta \in (-1, 1)$.

down to marginal cost w_2. There is, however, an indirect effect on the price of good 2 as manufacturer M_2 responds to the increased competition between goods 1 and 2 by reducing the price w_2. Similarly, manufacturer M_1 reduces his price w_1. These indirect effects in the upstream market occur under both separate selling and bundling.

Bundling softens the competition between retailers R_A and R_B. This effect is stronger the lower the degree of substitution between goods 1 and 2. Moreover, bundling reduces the singularity of good 1 and bundle A, since consumers can increasingly substitute them with good 2 in bundle B (compare Venkatesh and Kamakura, 2003). Thus, with an increasing degree of substitution θ bundle A becomes less attractive, which explains the pronounced drop in his price.



(a) Separate selling **(b)** Bundling

Figure 3: Equilibrium prices (for $a = 10, k = 5$)

We continue with discussing the effect on the *equilibrium quantities* (see Figure 4). Starting from the case of independent goods, the total demand of goods 1 and 2, Q_1 and Q_2, increases with increasing complementarity and it (eventually) increases with increasing substitutability. These observations apply to both separate selling and bundling.[13] In the retail market under bundling, however, the demand for bundle B, b_B, changes in the opposite direction. With increasing complementarity, it goes down and similarly so when the goods become close to perfect substitutes.[14]

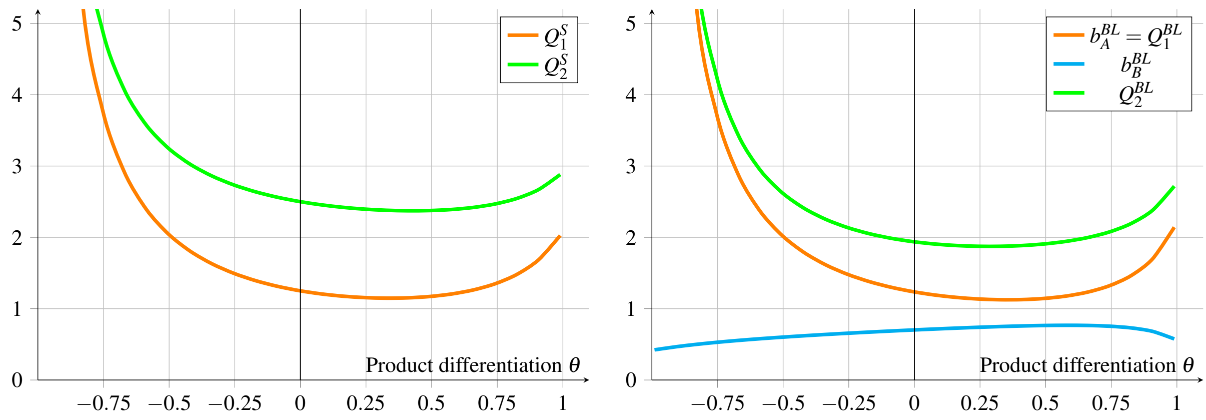
As to the former observation, notice that a change in the degree of product differentiation θ has two effects. First, the higher θ is, the more intense is the competition, the lower is the equilibrium price, and the larger is the total demand for goods 1 and 2. Second, θ also affects the customers' willingness to pay. The more complementary the goods 1 and 2, the higher is the customers' willingness to pay and the higher is their demand. The U-shaped curve then results because the marginal effect of product differentiation is stronger for limit values of θ

[13]More specifically, we find that $\frac{\partial Q_1^S}{\partial \theta} < 0$ for $\theta \in (-1, 0.3393)$, $\frac{\partial Q_2^S}{\partial \theta} < 0$ for $\theta \in (-1, 0.4272)$, and $\frac{\partial b_A^{BL}}{\partial \theta} < 0$ for $\theta \in (-1, 0.3567)$, while $\frac{\partial Q_1^S}{\partial \theta} > 0$ for $\theta \in (0.3394, 1)$, $\frac{\partial Q_2^S}{\partial \theta} > 0$ for $\theta \in (0.4273, 1)$, and $\frac{\partial b_A^{BL}}{\partial \theta} > 0$ for $\theta \in (0.3568, 1)$.

[14]Notice that $\frac{\partial b_B^{BL}}{\partial \theta} > 0$ for $\theta \in (-1, 0.5872)$, while $\frac{\partial b_B^{BL}}{\partial \theta} < 0$ for $\theta \in (0.5873, 1)$.

(viz. for $|\theta|$ close to 1).

As to the demand for bundle B, two effects contribute to its reduction. First, the stronger the complementarity between goods 1 and 2, the more attractive bundle A becomes as opposed to bundle B, because customers need to buy both goods 1 and 2, once their complementarity becomes stronger. As a consequence, consumers increasingly buy bundle A, which contains both goods, rather than bundle B, which only contains good 2. Second, at the other extreme, when goods 1 and 2 become close substitutes, the price of bundle A drops sharply, while the price drop for bundle B is weaker. Therefore, the demand for bundle A increases at the expense of the demand for bundle B.



(a) Separate selling **(b)** Bundling

Figure 4: Equilibrium quantities (for $a = 10, k = 5$)

4.2 Bundling incentives

In this subsection we first examine the decision of retailer R_A whether to bundle goods 1 and 2 or not. Afterwards, we explore how various assumptions of our modeling framework affect this bundling incentive. If not stated otherwise, the comparisons henceforth refer to differences of the equilibrium values under separate selling and under bundling.

In a similar integrated market structure, Carbajo et al. (1990) consider a higher price of the exclusive good, i.e. $p_1^S > p_2^S$, as a precondition for bundling to be considered by a multi-product firm. In this case, customers who exhibit a high valuation for good 1 under separate selling, might also accept a higher price of good 2 in bundle A, only to receive good 1 in the bundle. This allows the firm to skim off a larger share of the consumer surplus. Notice that this condition is satisfied for all $\theta \in (-1, 1)$ in our model because of $a > k$. In order to investigate the profitability of bundling, we compare the profit of the two-product retailer R_A under the two selling regimes, bundling and separate selling. Interestingly, bundling is only profitable when the two goods represent close substitutes:

Proposition 1. *Retailer R_A prefers bundling over separate selling for high degrees of substitution, viz. for $\theta \in (0.8827, 1)$.*

Proof. See Appendix F.1. □

The intuition runs as follows. When goods 1 and 2 represent close substitutes, bundling by retailer R_A reduces competition in two ways. First, retailer R_A introduces product differentiation in the market of good 2, differentiating his copy of good 2 from the copy of good 2 sold by R_B. This weakens the competition with retailer R_B. Second, bundling also softens the competition between good 1 sold by R_A and the copy of good 2 sold by R_B, and it entirely eliminates the internal competition between good 1 and the copy of good 2, both sold by R_A. Therefore, bundling becomes profitable once goods 1 and 2 become sufficiently close substitutes.

When goods 1 and 2 constitute complements, however, the competition-reducing effect of bundling in fact harms retailer R_A. Competition leads to low prices and a low price of, say good 2, also promotes the demand for R_A's good 1 in the case of complementary goods. This effect is the stronger the more complementary the goods 1 and 2 become, and it positively influences R_A's profit (compare Whinston, 1990). Thus, for complements, competition benefits the retailers and bundling is never profitable.

Lewbel (1985) has identified bundling to be profitable for substitute goods as well. He shows that pure bundling can be a two-product firm's optimal selling strategy when the goods constitute imperfect substitutes, while it may not be optimal when the goods represent complements. Notice, however, that Lewbel (1985) studies an integrated set-up with a multi-product monopolist, and neither considers competition in the downstream market nor a vertical structure. Our result also contrasts with Venkatesh and Kamakura (2003), who find the bundling incentive to increase with decreasing substitutability. They argue that customers only demand one of the substitutes in a bundle and discard the other, which makes selling the goods separately more profitable with a high degree of substitution. Unlike us, Venkatesh and Kamakura consider a monopolistic supplier while we consider duopolistic competition. As mentioned above, in our model bundling reduces competition in multiple ways, which is not the case in Venkatesh and Kamakura's model.

Thus, it seems that the ambiguity in results is caused by the different assumptions on the underlying market structure. We therefore continue with exploring how core assumptions of our market framework impact on the incentive to bundle differentiated goods. We explore to what extent retailer R_A is capable of *leveraging* his market power from the market for good 1 to the market for good 2. We then proceed with examining the problem of double marginalization, which is caused by the *upstream pricing* decisions of the manufacturers. Finally, we investigate the interplay of these vertical externalities with the horizontal externalities in the upstream market, considering two variations of our model.

Leveraging An incentive to bundle goods 1 and 2 arises from retailer R_A's monopoly position in the downstream market of good 1. Bundling differentiates the goods in the downstream duopoly, which allows retailer R_A to leverage his market power from the monopoly to the duopoly. This is reflected in a higher downstream price of the bundle compared to the sum of the single goods' prices under separate selling.

For low and medium degrees of substitutability, the higher prices under bundling entail a reduction in quantities and a lower profit of retailer R_A.[15] For high degrees of substitutability, we observe an increase in both the price and the demand for bundle A (compared to the total quantities of goods 1 and 2 under separate selling). This positively affects R_A's profit under bundling and explains why bundling becomes more profitable than separate selling for close substitutes. For all degrees of complementarity, bundling leads to lower demand, which causes separate selling to be more profitable than bundling for the two-product retailer R_A.

Upstream pricing The manufacturers' pricing decisions in the upstream market represent another factor impacting on the bundling incentive of retailer R_A. This has been illustrated, for instance, by Rennhoff and Serfes (2009), Bhargava (2012) or Cao et al. (2015). Proposition 2 summarizes the manufacturers' pricing response to bundling, irrespectively of whether bundling is profitable for retailer R_A or not.

Proposition 2. *The wholesale price for good 1 is higher under bundling than under separate selling for* $\theta \in (-1, -0.9174)$, *and it is lower for* $\theta \in (-0.9175, 1)$. *The wholesale price for good 2 is higher under bundling for all* $\theta \in (-1, 1)$. *The sum of wholesale prices is always greater under bundling than under separate selling.*

Proof. See Appendix F.2. □

The intuition behind Proposition 2 is that the manufacturers anticipate that retailer R_A has decided in favor of bundling in order to extract more surplus from consumers by charging a higher price. Accordingly, the manufacturers set higher prices as well in order to skim part of retailer R_A's higher profit. Only when goods 1 and 2 constitute close to perfect complements, does manufacturer M_1 charge a lower wholesale price for good 1 under bundling. This raises the demand for good 1, which, by complementarity of goods 1 and 2, stimulates the demand for good 2, which, in turn, promotes the demand for good 1 even more. Manufacturer M_2, however, charges a higher price since his demand is less elastic under bundling than under separate selling.

Even when the wholesale price of good 1 is lower under bundling, the sum of both input prices is always higher. Bundling thus aggravates the problem of double marginalization between retailer R_A and manufacturer M_1 in the decentralized channel of good 1. Bundling also creates a double marginalization problem in the market for good 2, which is not present under separate selling.

In order to shed further light on the impact of double marginalization, we proceed with inspecting two variations of the model. The first one eliminates the double marginalization

[15] More specifically, we have $Q_1^S - Q_1^{BL} > 0$ for $\theta \in (-1, 0.8826)$ and $q_{A2}^S - b_A^{BL} > 0$ for $\theta \in (-0.0433, 0.6199)$, while $Q_1^S - Q_1^{BL} < 0$ for $\theta \in (0.8827, 1)$ and $q_{A2}^S - b_A^{BL} < 0$ for $\theta \in (-1, -0.0434) \cup (0.6200, 1)$.

problem by considering a *centralized channel*, where the entire market power lies with the retailers. The second variation replaces the upstream market by a monopolistic *multi-product manufacturer* who produces and sells both good 1 and 2. In this way, we dispose of the horizontal externalities in the upstream market. Notice that in both settings all other assumptions remain the same as in the decentralized channel.

Centralized channel In our first variation of the model, we assume that the retailers determine the wholesale prices, which they set as low as possible such that, for both goods, the price equals the marginal cost. We hence obtain $w_1^j = w_2^j = k$, for $j \in \{S, BL\}$, i.e., under both bundling and separate selling.[16] It immediately follows that the price-taking manufacturers neither make any profit under separate selling nor under bundling.

We find that, in the centralized channel, retailer R_A always prefers bundling over separate selling independent of the degree of product differentiation between goods 1 and 2. Comparing this result with that of the decentralized channel, we see that the aggravation of the double marginalization problem reduces R_A's bundling incentive in the decentralized channel. It is the distribution of market power between the upstream and downstream market that causes the double marginalization problem. These vertical externalities negatively impact on the bundling incentive of the two-product retailer in the decentralized channel. Thus, our analysis shows that the latter result is robust to considering goods 1 and 2 as imperfect complements or substitutes.

Multi-product manufacturer Our second variation serves to explore the role of horizontal externalities in the upstream market. We replace the upstream manufacturers by a single monopolistic multi-product manufacturer who produces both goods and supplies them to the retailers in the same way as in our main model with the decentralized channel. This removes the horizontal externalities of the upstream market.

The multi-product manufacturer optimally sets both wholesale prices equal to $w_1^j = w_2^j = \frac{a+k}{2}$, that is, under both bundling and separate selling.[17] Interestingly, the profit-maximizing prices neither depend on the selling strategy of retailer R_A nor on the degree of product differentiation θ.

As with the centralized channel, it turns out that bundling is always profitable for retailer R_A, independent of the degree of product differentiation between the goods. While the monopolistic multi-product manufacturer causes a double marginalization problem, bundling does not exacerbate this problem. Therefore, the bundling incentive of the two-product retailer is stronger in this variation than in our main model with two manufacturers.

Combining our observations from the centralized channel and the multi-product manufacturer, we hence find that the presence of upstream market power *alone* does not weaken the

[16] All calculations and proofs can be found in Appendix B.

[17] All calculations and proofs can be found in Appendix C.

bundling incentives. Rather, it is the alliance of horizontal externalities (due to the upstream market competition) and vertical externalities (resulting from upstream market power) that causes the dilution of the bundling incentive in our main model. Observe that this insight tallies with similar observations by Bhargava (2012) and Endres and Heinzel (2019), but extends them to horizontally differentiated products for the entire range of complements and substitutes.

Scrutinizing an integrated set-up, Chung et al. (2013) find that the bundling incentive decreases in the degree of substitutability for a two-product firm that competes in a duopoly for one good, holding a monopoly position in the market of the other good. In contrast to Chung et al. (2013) and Venkatesh and Kamakura (2003), in our model the degree of substitutability of the bundled products enhances the bundling incentive. On the one hand, this shows that the combination of vertical and horizontal externalities can have such a strong effect on the bundling incentive that it prevents the retailer from bundling even though customers exhibit a high willingness to pay for bundles with differentiated products. On the other hand, it demonstrates that, in addition to product differentiation, the competition-reducing and the market extension effect have a major impact on the bundling decision, which is consistent with Chung et al. (2013). Ultimately, it depends on the degree of product differentiation between the bundled goods which effect prevails.

4.3 Consequences of bundling on the market equilibrium and social welfare

In the remainder of this section, we explore the consequences of bundling on the market outcome, assuming that bundling occurs. Henceforth, we refer to the corresponding range of θ as the *profitable bundling interval* and to bundling within this interval as *profitable bundling* or simply *bundling*. For this purpose, we return to considering our main model.

In the downstream market, retailer R_B benefits from R_A's decision to bundle. While under separate selling, retailer R_B prices at marginal cost and earns zero profit, under bundling, he sets a price above the wholesale price of manufacturer M_2, which yields a strictly positive profit. Moreover, the wholesale price of M_2 is always higher under bundling than under separate selling. Therefore, retailer R_B charges a higher price under bundling.

Proposition 3 summarizes the impact of bundling in the downstream market:

Proposition 3. *The demand for good 1 (good 2) is higher (lower) under profitable bundling than under separate selling. The decrease in the total demand for good 2 consists of an increase in retailer R_A's quantity of good 2 and a decrease of retailer R_B's quantity of good 2. Retailer R_B earns higher profit when retailer R_A decides to bundle.*

Proof. See Appendix F.3. □

As pointed out in Subsection 4.2, the demand of good 1 is higher under bundling for close substitutes. Since good 2 is part of both bundles, the overall effect on good 2 also depends on retailer R_B's quantity of good 2, which decreases due to the raise in the retail price of good 2 or bundle B, respectively. In total, this decrease in the demand for R_B's version of good 2 outweighs the increase in demand for R_A's version of good 2. Therefore, bundling entails an overall decrease in the demand for good 2.

In the upstream market, both manufacturers earn higher profit under bundling than under separate selling. For manufacturer M_1 this is immediate, since both the wholesale price and the demand for good 1 are higher under bundling. Manufacturer M_2 gains from bundling, since the increase in the wholesale prices of good 2 overcompensates the loss in sales.

Proposition 4. *The profits of manufacturers M_1 and M_2 increase under profitable bundling.*

Proof. See Appendix F.4. □

Unlike here, bundling always reduces the profit of the manufacturer selling to both downstream retailers in Endres and Heinzel (2019), who consider the case of independent goods and allow for heterogeneous quality (in the sense of Häckner, 2000). In our model, manufacturer M_2 always benefits from bundling, since the increase in retailer R_A's quantity of good 2 mitigates the decrease in retailer R_B's quantity of good 2. In combination with the increased wholesale price of good 2, this leads to an increase in manufacturer M_2's profit. By contrast, in Endres and Heinzel (2019), the individual quantities of good 2 decrease for both firms, which entails such a strong reduction in quantities that the increase in the wholesale price of good 2 does not compensate for the loss in demand. Since the result in Endres and Heinzel (2019) also obtains for close to homogenous qualities, this suggests that it is the presence of product differentiation that affects the relative strength of the effects, entailing an increase in the profit of manufacturer M_2 under profitable bundling.

We end this section examining the welfare consequences of bundling. Notice that the producer surplus is defined to include the profits of all retailers and manufacturers. By Propositions 3 and 4 producer surplus is higher under bundling:

Proposition 5. *When bundling is profitable to retailer R_A, consumer surplus is always lower and producer surplus always higher than under separate selling. Social welfare is lower under bundling for $\theta \in (0.8827, 0.9986)$, but higher for $\theta \in (0.9987, 1)$.*

Proof. See Appendix F.5. □

The decrease in consumer surplus is caused by the increase in retail prices of both bundles. Interestingly, bundling raises social welfare when both goods are close to perfect substitutes, viz. for $\theta \in (0.9987, 1)$. In this case, the increase in producer surplus compensates for the loss in consumer surplus. By contrast, for $\theta \in (0.8827, 0.9986)$, the usual case obtains that the increase in prices results in a higher producer surplus and a lower consumer surplus. Most importantly, however, it involves a deadweight loss, which causes social welfare to decrease.

The intuition is as follows. Further above, we have seen that the retail prices under bundling are always greater than under separate selling, while the former converge to the (sum of the) latter as the degree of substitutability increases (recall Figure 3). Accordingly, for low degrees of substitutability $\theta \in (0.8827, 0.9986)$, the difference in retail prices between bundling and separate selling is large, and bundling has a relatively strong impact on consumer surplus compared to its impact on producer surplus. By contrast, as the degree of substitutability approaches perfect substitutability, the impact of bundling on the consumer surplus vanishes as the difference in retail prices between bundling and separate selling converges to zero. Therefore, for close substitutes $\theta \in (0.9987, 1)$, the positive impact of bundling on the producer surplus ultimately overcompensates its negative impact on the consumer surplus, and thus bundling leads to higher social welfare.

Observe that bundling also results in lower social welfare when the products constitute complements or when they exhibit a small or medium degree of substitutatability, viz. for $\theta \in (-1, 0.8826)$. This reduction in social welfare is mainly driven by the higher prices under bundling. Thus, from a welfare perspective, bundling should raise *antitrust concerns* in our market set-up for almost all degrees of substitutability *except for close to perfect substitutes.*

Other authors, for instance Carbajo et al. (1990) or Chung et al. (2013), also find a negative effect of bundling on consumer surplus and ambiguous results on social welfare. Similar to us, Chung et al. (2013) identify that bundling raises welfare under price competition when the goods are close substitutes. We add to this literature by showing that, under price competition, the degree of product differentiation represents a major factor when assessing the welfare consequences of bundling.

5 Endogenous distribution channels

In this section, we endogenize the manufacturers' distribution choice. This lends support for the distribution structure of our model.

We use a framework similar to the one in Moner-Colonques et al. (2004).[18] At the first stage of our game, the manufacturers independently and simultaneously decide on their distribution choice. Each manufacturer i picks a strategy s_i from the strategy set $S = \{A, B, AB\}$, where $s_i = A$ and $s_i = B$ indicate that manufacturer i sells to retailer R_A and R_B, respectively, while $s_i = AB$ means that he sells to both retailers.[19] Consequently, there are nine strategy combinations (s_1, s_2) to which we refer as *distribution systems*. At the second stage, the manufacturers independently choose their price, and at the last stage, the retailers engage in price competition as in our main set-up. We solve the game by backward induction to derive its subgame perfect

[18] Just like us, Moner-Colonques et al. (2004) utilize a quadratic utility function such as proposed by Dixit (1979) and Singh and Vives (1984). However, they differ in that they examine quantity competition in the downstream market, focus on asymmetric qualities, and consider substitutes, but no complements.

[19] Notice that *not selling to anyone* is a strictly dominated strategy.

equilibrium (SPE).

Proposition 6. *For $\theta \in (0.9122, 0.9716)$, any SPE of the three-stage game involves an asymmetric distribution system s^* at stage one, that is, we have*

$$s^* \in \{(A,AB),(B,AB),(AB,A),(AB,B)\}.$$

Proof. See Appendix E.7. □

The intuition is as follows. On the one hand, when one manufacturer distributes to both retailers, then it is optimal to the other manufacturer to pick a single retailer when the products are close substitutes, viz. for $\theta > 0.9122$. In this way, he avoids the strong interbrand competition in the downstream market that would result when selling to both retailers. On the other hand, when one manufacturer picks a single retailer, the other manufacturer only introduces interbrand competition at one retail store when distributing to both retailers. Selling to both retailers, he benefits from the output expansion effect provided the products are not too close substitutes, viz. for $\theta < 0.9716$.

Thus, our finding provides support for the asymmetric distribution system that lies at the core of our modeling framework. Observe that the interval $(0.9122, 0.9716)$ lies entirely within the interval of profitable bundling, $(0.8827, 1)$.

6 Conclusion

In this paper we have studied a distribution channel with two monopolistic manufacturers, each producing a differentiated good, and two retailers. One manufacturer exclusively sells to one of the retailers, while the other manufacturer sells to both of them. The two-product retailer has the option to bundle the goods or to sell them separately. We have investigated how product differentiation of the goods impacts on the equilibrium outcome and on the incentives of the two-product retailer to bundle these goods. Furthermore, we have analyzed the consequences of retail bundling for social welfare.

We find that, with increasing complementarity, both under separate selling and under bundling prices and quantities increase. For complementary products, the demand of each product stimulates the demand of the respective other product. This mutual demand-enhancing effect is so strong that it outweighs the negative effect caused by the increase in prices.

Regarding the incentives for bundling, we obtain that the two-product retailer only sells his products in a bundle when the products constitute close substitutes. Only then does the competition-reducing effect of bundling compensate for the negative effects of bundling, caused by the aggravation of the double marginalization problem. For substitutes, the two-product retailer not only competes with the other retailer for the homogeneous good, he also faces competition between the two goods offered by himself. Accordingly, bundling reduces competition in two ways, both within and across the two product markets. When the two goods become close substitutes, eventually the competition-reducing effect of bundling becomes so strong that bundling becomes profitable.

Our analysis complements the findings of Lewbel (1985) or Whinston (1990), who illustrate that bundling may *not* be profitable in an integrated set-up when the original goods represent complements. We show that the negative impact of an aggravated double marginalization problem can never be outweighed by the positive effects of bundling for any degree of complementarity. This holds even though bundling reduces the intensity of competition in the downstream duopoly and though customers derive a high utility from consuming complementary goods.

To gain a better understanding of the driving factors behind our results, we have examined two variations of our set-up. First, we have analyzed a centralized channel, in which the downstream firms have all market power. This eliminates the double marginalization problem. Second, we have investigated a decentralized channel with a multi-product manufacturer who is the sole producer of the two products. In both market variations, bundling is always profitable, independently of the degree of product differentiation. We thus conclude that it is a combination of vertical externalities and horizontal externalities upstream that negatively impacts on the retailer's bundling incentives. In this vein, we extend findings identified by Bhargava (2012), Heinzel (2019), and Endres and Heinzel (2019) to the case of horizontal product differentiation.

Regarding social welfare, we find that bundling always reduces consumer surplus and always increases producer surplus in our main set-up. The overall effect of bundling on social welfare depends on the degree of product differentiation. Bundling raises social welfare for close to perfect substitutes. Even though the prices under bundling are always higher than under separate selling, we observe that the difference becomes smaller with increasing substitutability. Only when the goods represent very close substitutes, the positive effect of higher prices on the producer surplus outweighs their negative effect on the consumer surplus.

Finally, we have provided support for our asymmetric market set-up. Endogenizing the manufacturers' distribution decision, we have demonstrated that our market structure indeed arises in equilibrium, provided the products are close, but not too close substitutes. Notice, however, that bundling has a negative impact on social welfare in the corresponding range of product differentiation.

Our paper stresses the importance of product differentiation for (retail) bundling, considering retail competition in a vertically related market that is characterized by double marginalization. While the existing literature has remained inconclusive about the effect of product differentiation on retail bundling and welfare, this paper sheds further light on the issue. More importantly, our analysis shows that, within our market framework, product bundling should raise serious antitrust concerns.

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Appendix

A Decentralized channel (main model)

A.1 Separate selling: market equilibrium

Under separate selling, the two-product retailer R_A maximizes (6), where we substitute $q_{A1} = Q_1$ and insert (4) to obtain

$$\pi_A = (p_1 - w_1)\frac{a - p_1 - (a - p_2)\theta}{1 - \theta^2} + (p_{A2} - w_2)q_{A2}. \tag{A.1}$$

Notice that π_A is strictly concave in p_1 because of $\frac{\partial^2 \pi_A}{\partial p_1^2} = -\frac{2}{1-\theta^2} < 0$ for $\theta \in (-1, 1)$.

Maximizing (A.1) yields the corresponding first order condition (FOC):

$$\frac{\partial \pi_A}{\partial p_1} = \frac{a(1-\theta) + w_1 - 2p_1 + p_2\theta}{1 - \theta^2} \overset{!}{=} 0,$$

which characterizes the monopoly price of good 1.

Turning to the upstream market, we observe that the profits π_1 and π_2 are strictly concave in w_1 (respectively w_2) because of $\frac{\partial^2 \pi_1}{\partial w_1^2} = \frac{-1}{1-\theta^2} < 0$ and $\frac{\partial^2 \pi_2}{\partial w_2^2} = \frac{-2+\theta^2}{1-\theta^2} < 0$. Maximizing π_1 and π_2 results in the following FOCs:

$$\frac{\partial \pi_1}{\partial w_1} = \frac{a(1-\theta) + k - 2w_1 + w_2\theta}{2(1 - \theta^2)} \overset{!}{=} 0,$$

$$\frac{\partial \pi_2}{\partial w_2} = \frac{a(2-\theta-\theta^2) + k(2-\theta^2) + w_1\theta - 2w_2(2-\theta^2)}{2(1 - \theta^2)} \overset{!}{=} 0.$$

Solving the FOCs, we obtain the equilibrium wholesale prices (14) and (15), which we insert into (6)-(13) to obtain the equilibrium prices, quantities, and profits under separate selling, such as summarized in Table 1 (see Appendix A.5).

A.2 Separate selling: welfare

The producer surplus includes the profits of all firms in the market:

$$\begin{aligned} PS^S &\equiv \pi_A^S + \pi_B^S + \pi_1^S + \pi_2^S \\ &= \frac{(a-k)^2(1-\theta)(112 + 144\theta - 8\theta^2 - 60\theta^3 - 15\theta^4)}{4(1+\theta)(8 - 5\theta^2)^2}. \end{aligned}$$

Consumer surplus is given by $CS^S \equiv CS_1^S + CS_2^S$. Following Carbajo et al. (1990), we set $CS_1^S = \frac{a-p_1^S}{2} Q_1^S$ and $CS_2^S = \frac{a-p_2^S}{2} Q_2^S$. Social welfare amounts to $W^S \equiv PS^S + CS^S$.

$$\begin{aligned} CS^S &= \frac{(a-k)^2(4+2\theta-\theta^2)(4-2\theta-3\theta^2)}{8(8-5\theta^2)^2} \\ &\quad + \frac{(a-k)^2(4+3\theta)(2-\theta^2)(4\theta-2\theta^2)}{4(1+\theta)(8-5\theta^2)^2} \\ &= \frac{(a-k)^2(80+96\theta-48\theta^2-76\theta^3+5\theta^4+15\theta^5)}{8(1+\theta)(8-5\theta^2)^2}, \end{aligned}$$

$$W^S = \frac{(a-k)^2(304+160\theta-352\theta^2-180\theta^3+95\theta^4+45\theta^5)}{8(1+\theta)(8-5\theta^2)^2}.$$

A.3 Bundling: market equilibrium

Under bundling, retailers R_A and R_B maximize (22) and (23) subject to (20) and (21), respectively. Solving the corresponding FOCs,

$$\frac{\partial \pi_A}{\partial p_A} = \frac{a-2p_A+p_B+w_1+w_2-a\theta+p_B\theta}{(1-\theta)(1+\theta)} \stackrel{!}{=} 0,$$

$$\frac{\partial \pi_B}{\partial p_B} = \frac{p_A-4p_B+2w_2}{1-\theta} \stackrel{!}{=} 0,$$

yields the equilibrium retail prices (24) and (25). Notice that π_A and π_B are strictly concave because of $\frac{\partial^2 \pi_A}{\partial p_A^2} = \frac{-2}{1-\theta^2} < 0$ and $\frac{\partial^2 \pi_B}{\partial p_B^2} = \frac{-4}{1-\theta} < 0$ for $\theta \in (-1,1)$.

In the upstream market, maximizing π_1 and π_2 results in the following FOCs:

$$\frac{\partial \pi_1}{\partial w_1} = \frac{4a+3k-6w_1-w_2-4a\theta-k\theta+2w_1\theta+3w_2\theta}{(7-\theta)(1-\theta)(1+\theta)} \stackrel{!}{=} 0, \tag{A.2}$$

$$\frac{\partial \pi_2}{\partial w_2} = \frac{6a+5k-w_1-10w_2-4a\theta-k\theta+3w_1\theta+2w_2\theta-2a\theta^2-2k\theta^2+4w_2\theta^2}{(7-\theta)(1-\theta)(1+\theta)} \stackrel{!}{=} 0. \tag{A.3}$$

Notice that the profit functions are strictly concave because of $\frac{\partial^2 \pi_1}{\partial w_1^2} = \frac{2(-3+\theta)}{(7-\theta)(1-\theta)(1+\theta)} < 0$ and $\frac{\partial^2 \pi_2}{\partial w_2^2} = \frac{2(-5+\theta+2\theta^2)}{(7-\theta)(1-\theta)(1+\theta)} < 0$ for $\theta \in (-1,1)$. Solving (A.2) and (A.3), we get the equilibrium wholesale prices (28) and (29).

Inserting these into (20)-(27), we obtain the equilibrium prices, quantities, and profits under bundling, such as summarized in Table 1 (see Appendix A.5).

The cross-derivatives of the upstream quantities with respect to the wholesale prices show the following signs: $\frac{\partial Q_2}{\partial w_1} = \frac{\partial Q_1}{\partial w_2} < 0$ for $\theta \in (-1,1/3)$ and $\frac{\partial Q_2}{\partial w_1} = \frac{\partial Q_1}{\partial w_2} > 0$ for $\theta \in (1/3,1)$.

A.4 Bundling: welfare

Producer surplus is denoted by $PS^{BL} \equiv \pi_A^{BL} + \pi_B^{BL} + \pi_1^{BL} + \pi_2^{BL}$ and consumer surplus by $CS^{BL} \equiv CS_A^{BL} + CS_B^{BL}$. Following Carbajo et al. (1990), we set $CS_A^{BL} = \frac{2a - p_A^{BL}}{2} b_A^{BL}$ and $CS_B^{BL} = \frac{a - p_B^{BL}}{2} b_B^{BL}$. Social welfare amounts to $W^{BL} \equiv PS^{BL} + CS^{BL}$.

$$
\begin{aligned}
PS^{BL} &= (36101 + 64669\theta - 272242\theta^2 - 2294\theta^3 + 5409\theta^4 - 415\theta^5 - 236\theta^6 + 32\theta^7) \\
&\quad \times \frac{2(a-k)^2(1-\theta)}{(7-\theta)^2(1+\theta)(59-26\theta-29\theta^2+8\theta^3)^2},
\end{aligned}
$$

$$
\begin{aligned}
CS^{BL} &= \frac{2(a-k)^2(3-\theta)(17+4\theta-5\theta^2)(131+9\theta-67\theta^2-5\theta^3+4\theta^4)}{(7-\theta)^2(1+\theta)(59-26\theta-29\theta^2+8\theta^3)^2} \\
&\quad + \frac{2(a-k)^2(29-10\theta-19\theta^2+4\theta^3)(80-15\theta-38\theta^2+9\theta^3)}{(7-\theta)^2(59-26\theta-29\theta^2+8\theta^3)^2} \\
&= \frac{2(a-k)^2(9001+889\theta-9658\theta^2-662\theta^3+3365\theta^4-11\theta^5-388\theta^6+56\theta^7)}{(7-\theta)^2(1+\theta)(59-26\theta-29\theta^2+8\theta^3)^2},
\end{aligned}
$$

$$
\begin{aligned}
W^{BL} &= (45102 - 28743\theta - 433369\theta^2 + 242868\theta^3 + 110668\theta^4 - 58335\theta^5 - 209\theta^6 \\
&\quad + 324\theta^7 - 32\theta^8) \times \frac{2(a-k)^2}{(7-\theta)^2(1+\theta)(59-26\theta-29\theta^2+8\theta^3)^2}.
\end{aligned}
$$

A.5 Summary of the market outcomes

Table 1 collects the equilibrium values under separate selling and bundling:

	Separate selling	**Bundling**
Downstream prices	$p_1^S = \frac{12a+4k-(6a-6k)\theta-(9a+k)\theta^2+(3a-3k)\theta^3}{16-10\theta^2}$ $p_2^S = \frac{4a+4k-(a-k)\theta-(3a+2k)\theta^2}{8-5\theta^2}$	$p_A^{BL} = \frac{2(282a+131k-250a\theta+9k\theta-110a\theta^2-67k\theta^2+90a\theta^3-5k\theta^3+4k\theta^4-12a\theta^4)}{(7-\theta)(59-26\theta-29\theta^2+8\theta^3)}$ $p_B^{BL} = \frac{253a+160k-211a\theta-30k\theta-101a\theta^2-76k\theta^2-8a\theta^4+18k\theta^3+67a\theta^3}{(7-\theta)(59-26\theta-29\theta^2+8\theta^3)}$
Downstream quantities	$q_{A1}^S = \frac{(a-k)(4+2\theta-\theta^2)}{2(1+\theta)(8-5\theta^2)}$ $q_{A2}^S = \frac{(a-k)(4+3\theta)(2-\theta^2)}{4(1+\theta)(8-5\theta^2)}$ $q_{B2}^S = \frac{(a-k)(4+3\theta)(2-\theta^2)}{4(1+\theta)(8-5\theta^2)}$	$b_A^{BL} = \frac{2(a-k)(3-\theta)(17+4\theta-5\theta^2)}{(7-\theta)(1+\theta)(59-26\theta-29\theta^2+8\theta^3)}$ $b_B^{BL} = \frac{2(a-k)(29-10\theta-19\theta^2+4\theta^3)}{(7-\theta)(59-26\theta-29\theta^2+8\theta^3)}$
Quantities	$Q_1^S = \frac{(a-k)(4+2\theta-\theta^2)}{2(1+\theta)(8-5\theta^2)}$ $Q_2^S = \frac{(a-k)(4+3\theta)(2-\theta^2)}{2(1+\theta)(8-5\theta^2)}$	$Q_1^{BL} = \frac{2(a-k)(3-\theta)(17+4\theta-5\theta^2)}{(7-\theta)(1+\theta)(59-26\theta-29\theta^2+8\theta^3)}$ $Q_2^{BL} = \frac{4(a-k)(8+3\theta-\theta^2)(5-\theta-2\theta^2)}{(7-\theta)(1+\theta)(59-26\theta-29\theta^2+8\theta^3)}$
Manufacturer profits	$\pi_1^S = \frac{(a-k)^2(1-\theta)(4+2\theta-\theta^2)^2}{2(1+\theta)(8-5\theta^2)^2}$ $\pi_2^S = \frac{(a-k)^2(1-\theta)(2-\theta^2)(4+3\theta)^2}{2(1+\theta)(8-5\theta^2)^2}$	$\pi_1^{BL} = \frac{4(a-k)^2(3-\theta)(1-\theta)(17+4\theta-5\theta^2)^2}{(7-\theta)(1+\theta)(59-26\theta-29\theta^2+8\theta^3)^2}$ $\pi_2^{BL} = \frac{16(a-k)^2(1-\theta)(8+3\theta-\theta^2)^2(5-\theta-2\theta^2)}{(7-\theta)(1+\theta)(59-26\theta-29\theta^2+8\theta^3)^2}$
Retailer profits	$\pi_A^S = \frac{(a-k)^2(1-\theta)(4+2\theta-\theta^2)^2}{4(1+\theta)(8-5\theta^2)^2}$ $\pi_B^S = 0$	$\pi_A^{BL} = \frac{4(a-k)^2(3-\theta)^2(1-\theta)(17+4\theta-5\theta^2)^2}{(7-\theta)^2(1+\theta)(59-26\theta-29\theta^2+8\theta^3)^2}$ $\pi_B^{BL} = \frac{2(a-k)^2(1-\theta)(29-10\theta-19\theta^2+4\theta^3)^2}{(7-\theta)^2(59-26\theta-29\theta^2+8\theta^3)^2}$
Producer surplus Consumer surplus Social welfare	$PS^S = \frac{(a-k)^2(1-\theta)(112+144\theta-8\theta^2-60\theta^3-15\theta^4)}{4(1+\theta)(8-5\theta^2)^2}$ $CS^S = \frac{(a-k)^2(80+96\theta-48\theta^2-76\theta^3+5\theta^4+15\theta^5)}{8(1+\theta)(8-5\theta^2)^2}$ $W^S = \frac{(a-k)^2(304+160\theta-352\theta^2-180\theta^3+95\theta^4+45\theta^5)}{8(1+\theta)(8-5\theta^2)^2}$	$PS^{BL} = \frac{2(a-k)^2(1-\theta)(36101+64669\theta-272242\theta^2-2294\theta^3+5409\theta^4-415\theta^5-236\theta^6+32\theta^7)}{(7-\theta)^2(1+\theta)(59-26\theta-29\theta^2+8\theta^3)^2}$ $CS^{BL} = \frac{2(a-k)^2(9001+889\theta-9658\theta^2-662\theta^3+3365\theta^4-11\theta^5-388\theta^6+56\theta^7)}{(7-\theta)^2(1+\theta)(59-26\theta-29\theta^2+8\theta^3)^2}$ $W^{BL} = \frac{2(a-k)^2(45102-28743\theta-433369\theta^2+242868\theta^3+110668\theta^4-58335\theta^5-209\theta^6+324\theta^7-32\theta^8)}{(7-\theta)^2(1+\theta)(59-26\theta-29\theta^2+8\theta^3)^2}$

Table 1: Market outcomes in a decentralized channel

B Centralized channel

B.1 Summary of the market outcomes

Inserting the equilibrium wholesale prices of the centralized channel, $w_1^S = w_2^S = k$, into (6)-(13) and (20)-(27), we obtain the equilibrium outcomes under separate selling and bundling, respectively, such as collected in Table 2. Note that the condition in Carbajo et al. (1990), $p_1^S > p_2^S$, is satisfied for all $\theta \in (-1,1)$.

	Separate selling	**Bundling**
Downstream prices	$p_1^S = \frac{a+k-(a-k)\theta}{2}$ $p_2^S = k$	$p_A{}^{BL} = \frac{4a+10k-(4a-2k)\theta}{7-\theta}$ $p_B{}^{BL} = \frac{a+6k-a\theta}{7-\theta}$
Downstream quantities	$q_{A1}^S = \frac{a-k}{2(1+\theta)}$ $q_{A2}^S = \frac{(a-k)(2+\theta)}{4(1+\theta)}$ $q_{B2}^S = \frac{(a-k)(2+\theta)}{4(1+\theta)}$	$b_A^{BL} = \frac{4(a-k)}{(7-\theta)(1+\theta)}$ $b_B^{BL} = \frac{2(a-k)}{7-\theta}$
Quantities	$Q_1^S = \frac{a-k}{2(1+\theta)}$ $Q_2^S = \frac{(a-k)(2+\theta)}{2(1+\theta)}$	$Q_1^{BL} = \frac{4(a-k)}{(7-\theta)(1+\theta)}$ $Q_2^{BL} = \frac{2(a-k)(3+\theta)}{(7-\theta)(1+\theta)}$
Retailer profits	$\pi_A^S = \frac{(a-k)^2(1-\theta)}{4(1+\theta)}$ $\pi_B^S = 0$	$\pi_A^{BL} = \frac{16(a-k)^2(1-\theta)}{(7-\theta)^2(1+\theta)}$ $\pi_B^{BL} = \frac{2(a-k)^2(1-\theta)}{(7-\theta)^2}$
Manufacturer profits	$\pi_1^S = 0$ $\pi_2^S = 0$	$\pi_1^{BL} = 0$ $\pi_2^{BL} = 0$
Producer surplus Consumer surplus Social welfare	$PS^S = \frac{(a-k)^2(1-\theta)}{4(1+\theta)}$ $CS^S = \frac{(a-k)^2(5+3\theta)}{8(1+\theta)}$ $W^S = \frac{(a-k)^2(7+\theta)}{8(1+\theta)}$	$PS^{BL} = \frac{2(a-k)^2(1-\theta)(9+\theta)}{(7-\theta)^2(1+\theta)}$ $CS^{BL} = \frac{2(a-k)^2(13+5\theta)}{(7-\theta)^2(1+\theta)}$ $W^{BL} = \frac{2(a-k)^2(22-3\theta-\theta^2)}{(7-\theta)^2(1+\theta)}$

Table 2: Market outcomes in a centralized channel

B.2 Bundling decision

Bundling is optimal to retailer R_A for all $\theta \in (-1,1)$, since $a > k$ implies

$$\Delta\pi_A \equiv \pi_A^S - \pi_A^{BL} = \frac{(a-k)^2(15-\theta)(-1+\theta)}{4(7-\theta)^2} < 0.$$

C Multi-product monopoly

C.1 Market outcome under separate selling

Under separate selling, a multi-product monopolist maximizes

$$\pi_U = (w_1 - k)Q_1 + (w_2 - k)Q_2 \tag{C.1}$$

subject to (10) and (11). Notice that π_U is strictly concave in both w_1 and w_2 because of $\frac{\partial^2 \pi_U}{\partial w_1^2} = \frac{-1}{(1-\theta)(1+\theta)} < 0$ and $\frac{\partial^2 \pi_U}{\partial w_2^2} = \frac{-2+\theta^2}{(1-\theta)(1+\theta)} < 0$.

Maximizing (C.1) yields the FOCs

$$\frac{\partial \pi_U}{\partial w_1} = \frac{a + k - 2w_1 - a\theta - k\theta + 2w_2\theta}{2(1-\theta)(1+\theta)} \stackrel{!}{=} 0,$$

$$\frac{\partial \pi_U}{\partial w_2} = \frac{2a + 2k - 4w_2 - a\theta - k\theta + 2w_1\theta - a\theta^2 - k\theta^2 + 2w_2\theta^2}{2(1-\theta)(1+\theta)} \stackrel{!}{=} 0,$$

which characterize the unique solution $w_1^S = w_2^S = \frac{a+k}{2}$.

Inserting these values into (6)-(13), we obtain the market outcome, such as summarized in Table 3 (see Appendix C.3).

C.2 Market outcome under bundling

Under bundling, the multi-product monopolist maximizes $\pi_U = \pi_A + \pi_B$ subject to (20), (21), (24), and (25). Notice that $\frac{\partial^2 \pi_U}{\partial w_1^2} = \frac{2(\theta-3)}{(\theta-7)(\theta-1)(\theta+1)} < 0$ and $\frac{\partial^2 \pi_U}{\partial w_2^2} = \frac{2\left(2\theta^2+\theta-5\right)}{(\theta-7)(\theta-1)(\theta+1)} < 0$.

Solving the maximization problem, we obtain the following FOCs:

$$\frac{\partial \pi_U}{\partial w_1} = \frac{2(2a + 2k - 3w_1 - w_2 - 2a\theta - 2k\theta + w_1\theta + 3w_2\theta)}{(7-\theta)(1-\theta)(1+\theta)} \stackrel{!}{=} 0,$$

$$\frac{\partial \pi_U}{\partial w_2} = \frac{2(3a + 3k - w_1 - 5w_2 - 2a\theta - 2k\theta + 3w_1\theta + w_2\theta - a\theta^2 - k\theta^2 + 2w_2\theta^2)}{(7-\theta)(1-\theta)(1+\theta)} \stackrel{!}{=} 0,$$

which again yield the unique solution $w_1^{BL} = w_2^{BL} = \frac{a+k}{2}$.

Table 3 also displays the market outcome under bundling. Notice that $p_1^S > p_2^S$ for all $\theta \in (-1, 1)$.

C.3 Summary of the market outcomes

	Separate selling	**Bundling**
Downstream prices	$p_1^S = \frac{3a+k-a\theta+k\theta}{4}$ $p_2^S = \frac{a+k}{2}$	$p_A^{BL} = \frac{4a+5(a+k)-(3a-k)\theta}{7-\theta}$ $p_B^{BL} = \frac{a+3(a+k)-a\theta}{7-\theta}$
Quantities	$q_{A1}^S = \frac{a-k}{4(1+\theta)}$ $q_{A2}^S = \frac{(a-k)(2+\theta)}{8(1+\theta)}$ $q_{B2}^S = \frac{(a-k)(2+\theta)}{8(1+\theta)}$	$b_A^{BL} = \frac{2(a-k)}{(7-\theta)(1+\theta)}$ $b_B^{BL} = \frac{a-k}{7-\theta}$
Downstream quantities	$Q_1^S = \frac{a-k}{4(1+\theta)}$ $Q_2^S = \frac{(a-k)(2+\theta)}{4(1+\theta)}$	$Q_1^{BL} = \frac{2(a-k)}{(7-\theta)(1+\theta)}$ $Q_2^{BL} = \frac{(a-k)(3+\theta)}{(7-\theta)(1+\theta)}$
Manufacturer profits	$\pi_U^S = \frac{(a-k)^2(3+\theta)}{8(1+\theta)}$	$\pi_U^{BL} = \frac{(a-k)^2(5+\theta)}{2(7-\theta)(1+\theta)}$
Retailer profits	$\pi_A^S = \frac{(a-k)^2(1-\theta)}{16(1+\theta)}$ $\pi_B^S = 0$	$\pi_A^{BL} = \frac{4(a-k)^2(1-\theta)}{(7-\theta)^2(1+\theta)}$ $\pi_B^{BL} = \frac{(a-k)^2(1-\theta)}{2(7-\theta)^2}$
Producer surplus Consumer surplus Social welfare	$PS^S = \frac{(a-k)^2(7+\theta)}{16(1+\theta)}$ $CS^S = \frac{(a-k)^2(5+3\theta)}{32(1+\theta)}$ $W^S = \frac{(a-k)^2(19+5\theta)}{32(1+\theta)}$	$PS^{BL} = \frac{(a-k)^2(22-3\theta-\theta^2)}{(7-\theta)^2(1+\theta)}$ $CS^{BL} = \frac{(a-k)^2(13+5\theta)}{2(7-\theta)^2(1+\theta)}$ $W^{BL} = \frac{(a-k)^2(57-\theta-2\theta^2)}{2(7-\theta)^2(1+\theta)}$

Table 3: Market outcomes in a multi-product monopoly

C.4 Bundling decision

Bundling is always optimal to retailer R_A:

$$\Delta\pi_A \equiv \pi_A^S - \pi_A^{BL} = \frac{(a-k)^2(15-\theta)(-1+\theta)}{16(7-\theta)^2} < 0,$$

which holds true because of $a > k$ and $\theta \in (-1, 1)$.

D Further analysis

D.1 Comparisons: downstream prices

We have

$$\begin{aligned} p_1^S - p_2^S &= \frac{(a-k)(1-\theta)(4-3\theta^2)}{2(8-5\theta^2)} > 0, \\ \Delta p_A &\equiv p_1^S + p_2^S - p_A^{BL} \\ &= -\frac{(a-k)(1-\theta)(1+\theta)(764+124\theta-589\theta^2+34\theta^3+135\theta^4-24\theta^5)}{2(7-\theta)(8-5\theta^2)(59-26\theta-29\theta^2+8\theta^3)} < 0, \\ \Delta p_B &\equiv p_2^S - p_B^{BL} \\ &= -\frac{(a-k)(1-\theta)(372+61\theta-306\theta^2+45\theta^3+72\theta^4-16\theta^5)}{(7-\theta)(8-5\theta^2)(59-26\theta-29\theta^2+8\theta^3)} < 0, \end{aligned}$$

since the terms in parentheses are strictly positive because of $a > k$ and $\theta \in (-1,1)$.

D.2 Partial derivatives: downstream prices

Consider $\frac{\partial p_1^S}{\partial \theta} = -\frac{3(a-k)(16+8\theta-14\theta^2+5\theta^4)}{2(8+5\theta^2)^2}$. Its roots in $(-1,1)$ correspond to those of $\varphi_1(\theta) \equiv 16+8\theta-14\theta^2+5\theta^4 = 0$, since the remaining terms in parentheses are strictly positive. Notice that $\varphi_1(\theta)$ has a unique root $\hat{\theta}_1$ in $(-1,1)$, which is approximately given by $\hat{\theta}_1 \approx -0.9407$. Moreover, we have $\frac{\partial p_1^S}{\partial \theta} > 0$ for $\theta \in (-1,\hat{\theta}_1)$ and $\frac{\partial p_1^S}{\partial \theta} < 0$ for $\theta \in (\hat{\theta}_1, 1)$.

We have $\frac{\partial p_2^S}{\partial \theta} = -\frac{(a-k)(8+8\theta+5\theta^2)}{(8-5\theta^2)^2} < 0$, for all $\theta \in (-1,1)$.

We have $\frac{\partial p_A^{BL}}{\partial \theta} = -\frac{8(a-k)(8822-2242\theta-5465\theta^2+2920\theta^3+976\theta^4-622\theta^5+75\theta^6)}{(7-\theta)^2(59-26\theta-29\theta^2+8\theta^3)^2} < 0$, for all $\theta \in (-1,1)$.

We have $\frac{\partial p_B^{BL}}{\partial \theta} = -\frac{2(a-k)(13085-30668\theta-2746\theta^2+772\theta^3+1277\theta^4-608\theta^5+72\theta^6)}{(7-\theta)^2(59-26\theta-29\theta^2+8\theta^3)^2} < 0$, for all $\theta \in (-1,1)$.

Consider $\frac{\partial \Delta p_A}{\partial \theta} = -(a-k)\Big(\frac{1882688-3621656\theta-2172272\theta^2+69389680\theta^3-9639993\theta^4-50203660\theta^5+2150921\theta^6}{2(7-\theta)^2(8-5\theta^2)^2(59-26\theta-29\theta^2+8\theta^3)^2} + \frac{1523428\theta^7-910263\theta^8-986300\theta^9+118807\theta^{10}-20400\theta^{11}+960\theta^{12}}{2(7-\theta)^2(8-5\theta^2)^2(59-26\theta-29\theta^2+8\theta^3)^2}\Big)$, which has unique root $\hat{\theta}_2$ in $(-1,1)$, given by $\hat{\theta}_2 \approx 0.6265$. Moreover, we have $\frac{\partial \Delta p_A}{\partial \theta} < 0$ for $\theta \in (-1,\hat{\theta}_2)$ and $\frac{\partial \Delta p_A}{\partial \theta} > 0$ for $\theta \in (\hat{\theta}_2, 1)$. Since $\Delta p_A < 0$ for all $\theta \in (-1,1)$, the absolute difference $|\Delta p_A|$ is decreasing in θ for $\theta > \hat{\theta}_2$.

Consider $\frac{\partial \Delta p_B}{\partial \theta} = (a-k)\Big(\frac{310328-1647280\theta-1000437\theta^2+10458020\theta^3+583471\theta^4-792500\theta^5-121743\theta^6}{(7-\theta)^2(8-5\theta^2)^2(59-26\theta-29\theta^2+8\theta^3)^2} + \frac{197474\theta^7+124130\theta^8-24112\theta^9+3280\theta^{10}}{(7-\theta)^2(8-5\theta^2)^2(59-26\theta-29\theta^2+8\theta^3)^2}\Big)$. This has unique root $\hat{\theta}_3$ in $(-1,1)$, which is given by $\hat{\theta}_3 \approx -0.5594$. Moreover, we have $\frac{\partial \Delta p_B}{\partial \theta} < 0$ for $\theta \in (-1,\hat{\theta}_3)$ and $\frac{\partial \Delta p_B}{\partial \theta} > 0$ for $\theta \in (\hat{\theta}_3, 1)$.

D.3 Partial derivatives: quantities

Consider $\frac{\partial Q_1^S}{\partial \theta} = -\frac{(a-k)(16-24\theta-62\theta^2-20\theta^3+5\theta^4)}{2(1+\theta)^2(8-5\theta^2)^2}$. This has a unique root $\hat{\theta}_4$ in $(-1,1)$, which approximately equals $\hat{\theta}_4 \approx 0.3394$. Moreover, we have $\frac{\partial Q_1^S}{\partial \theta} < 0$ for $\theta \in (-1,\hat{\theta}_4)$ and $\frac{\partial Q_1^S}{\partial \theta} > 0$

for $\theta \in (\hat{\theta}_4, 1)$.

Consider $\frac{\partial Q_2^S}{\partial \theta} = -\frac{(a-k)(16-16\theta-46\theta^2-12\theta^3+5\theta^4)}{2(1+\theta)^2(8-5\theta^2)^2}$. Notice that this has a unique root $\hat{\theta}_5$ in $(-1,1)$, which is given by $\hat{\theta}_5 \approx 0.4272$. We have $\frac{\partial Q_2^S}{\partial \theta} < 0$ for $\theta \in (-1, \hat{\theta}_5)$ and $\frac{\partial Q_2^S}{\partial \theta} > 0$ for $\theta \in (\hat{\theta}_5, 1)$.

Consider $\frac{\partial b_A^{BL}}{\partial \theta} = -\frac{2(a-k)(10837-26942\theta-14913\theta^2+14908\theta^3+643\theta^4-2766\theta^5+841\theta^6-80\theta^7)}{(7-\theta)^2(1+\theta)^2(59-26\theta-29\theta^2+8\theta^3)^2}$, which has a unique root $\hat{\theta}_6$ in $(-1,1)$. We have $\hat{\theta}_6 \approx 0.3568$. Observe that $\frac{\partial b_A^{BL}}{\partial \theta} < 0$ for $\theta \in (-1, \hat{\theta}_6)$ and $\frac{\partial b_A^{BL}}{\partial \theta} > 0$ for $\theta \in (\hat{\theta}_6, 1)$.

Consider $\frac{\partial b_B^{BL}}{\partial \theta} = \frac{2(a-k)(2859-5428\theta+370\theta^2+700\theta^3+667\theta^4-304\theta^5+32\theta^6)}{(7-\theta)^2(59-26\theta-29\theta^2+8\theta^3)^2}$. This has a unique root $\hat{\theta}_7$ in $(-1,1)$, which is given by $\hat{\theta}_7 \approx 0.5873$. We have $\frac{\partial b_B^{BL}}{\partial \theta} > 0$ for $\theta \in (-1, \hat{\theta}_7)$ and $\frac{\partial b_B^{BL}}{\partial \theta} < 0$ for $\theta \in (\hat{\theta}_7, 1)$.

E Endogenous distribution channels

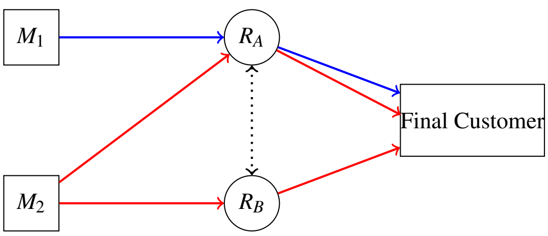
In this appendix, we analyze the endogenous choice of distribution channels.

The consumers' demand functions are given by (4) and (5), since they do not depend on the distribution choice of the manufacturers. Similarly, the profit functions of manufacturers M_1 and M_2 are given by (12) and (13), respectively.

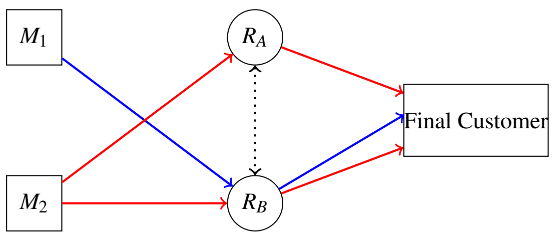
We compare several market set-ups, some of which represent mirror cases of another set-up. While we depict all market set-ups in the following, we present the equilibrium solutions only for one representative distribution system. The equilibria of the remaining distribution systems can be determined analogously.

E.1 Partial exclusivity – manufacturer 1 sells exclusively

Manufacturer M_1 either sells to retailer R_A or to retailer R_B. The distribution system is (A, AB) or (B, AB).



Distribution system (A, AB)

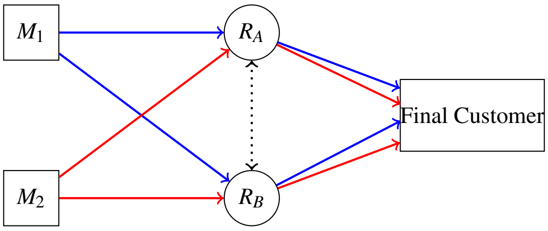


Distribution system (B, AB)

The following calculations refer to distribution system (A, AB), where M_1 sells exclusively to R_A. This is exactly the distribution system of our main set-up. Thus we can adopt our results from Appendix A. Table 4 collects the equilibrium results of all distribution systems.

E.2 No exclusivity – both manufacturers sell non-exclusively

Both manufacturers sell to both retailers. The unique distribution system is (AB, AB).



Distribution system (AB, AB)

The profits of retailers R_A and R_B are given by

$$\pi_A = (p_{A1} - w_1)q_{A1} + (p_{A2} - w_2)q_{A2},$$

$$\pi_B = (p_{B1} - w_1)q_{B1} + (p_{B2} - w_2)q_{B2}.$$

In this case, the equilibrium retail prices correspond to the input prices:

$$p_1 = p_{A1} = p_{B1} = w_1 \quad \text{and} \quad p_2 = p_{A2} = p_{B2} = w_2.$$

We insert the equilibrium prices into (4) and (5) to obtain the equilibrium upstream demand:

$$Q_1 = \frac{a - w_1 - (a - w_2)\theta}{1 - \theta^2} \quad \text{and} \quad Q_2 = \frac{a - w_2 - (a - w_1)\theta}{1 - \theta^2}.$$

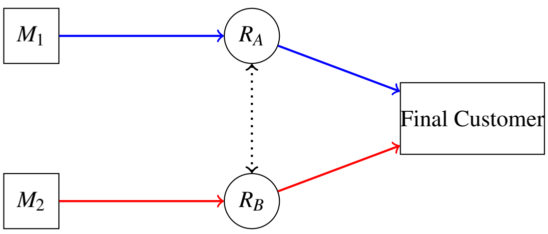
Inserting these in turn into (12) and (13) yields

$$\pi_1 = \frac{(w_1 - k)(a - w_1 - (a - w_2)\theta)}{1 - \theta^2},$$

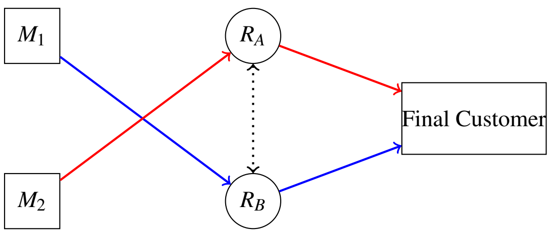
$$\pi_2 = \frac{(w_2 - k)(a - w_2 - (a - w_1)\theta)}{1 - \theta^2}.$$

E.3 Full exclusivity – dual exclusive relationships

Both manufacturers sell exclusively, each one to a different retailer. The distribution system is (A, B) or (B, A). The following calculations refer to distribution system (A, B).



Distribution system (A, B)



Distribution system (B, A)

The profits of retailers R_A and R_B are given by

$$\pi_A = (p_1 - w_1)Q_1,$$

$$\pi_B = (p_2 - w_2)Q_2,$$

and the equilibrium retail prices by

$$p_1 = \frac{2a + 2w_1 - (a - w_2)\theta - a\theta^2}{4 - \theta^2},$$

$$p_2 = \frac{2a + 2w_2 - (a - w_1)\theta - a\theta^2}{4 - \theta^2}.$$

We insert the equilibrium prices into (4) and (5) to obtain

$$Q_1 = \frac{2a - 2w_1 - a\theta + w_2\theta - a\theta^2 + w_1\theta^2}{(2-\theta)(1-\theta)(1+\theta)(2+\theta)},$$

$$Q_2 = \frac{2a - 2w_2 - a\theta + w_1\theta - a\theta^2 + w_2\theta^2}{(2-\theta)(1-\theta)(1+\theta)(2+\theta)}.$$

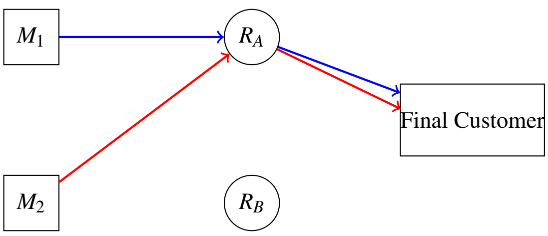
Inserting these in turn into (12) and (13), we get

$$\pi_1 = \frac{(w_1 - k)(2a - 2w_1 - a\theta + w_2\theta - a\theta^2 + w_1\theta^2)}{(2-\theta)(1-\theta)(1+\theta)(2+\theta)},$$

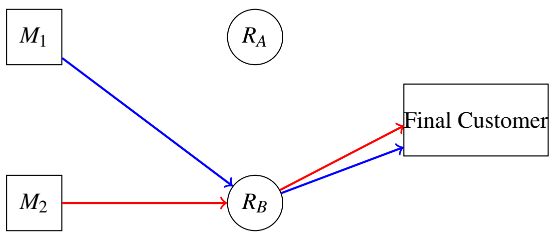
$$\pi_2 = \frac{(w_2 - k)(2a - 2w_2 - a\theta + w_1\theta - a\theta^2 + w_2\theta^2)}{(2-\theta)(1-\theta)(1+\theta)(2+\theta)}.$$

E.4 Full exclusivity – exclusive selling to same retailer

Both manufacturers exclusively supply the same retailer. The distribution system is (A,A) or (B,B). Our calculations refer to distribution system (A,A).



Distribution system (A,A)



Distribution system (B,B)

The profit of retailer R_A is given by

$$\pi_A = (p_1 - w_1)Q_1 + (p_2 - w_2)Q_2.$$

Since no product is distributed to retailer R_B, retailer R_A sets monopoly retail prices:

$$p_1 = \frac{a+w_1}{2} \quad \text{and} \quad p_2 = \frac{a+w_2}{2}.$$

We insert the equilibrium prices to obtain the demands

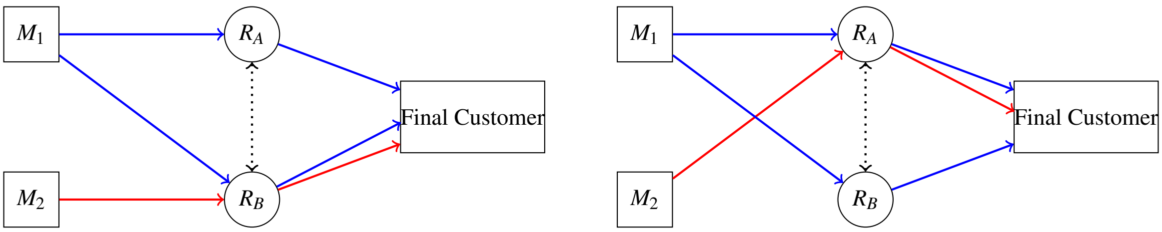
$$Q_1 = \frac{a-w_1-a\theta+w_2\theta}{2(1-\theta)(1+\theta)} \quad \text{and} \quad Q_2 = \frac{a-w_2-a\theta+w_1\theta}{2(1-\theta)(1+\theta)}.$$

Inserting these in turn into (12) and (13), we get

$$\pi_1 = \frac{(w_1-k)(a-w_1-a\theta+w_2\theta)}{2(1-\theta)(1+\theta)} \quad \text{and} \quad \pi_2 = \frac{(w_2-k)(a-w_2-a\theta+w_1\theta)}{2(1-\theta)(1+\theta)}.$$

E.5 Partial exclusivity – manufacturer 2 sells exclusively

Manufacturer M_2 either sells to retailer R_A or to retailer R_B. The distribution system is (AB,B) or (AB,A). This case is analogous to the distribution system (A,AB), with the two manufacturers switching roles. The equilibrium market outcome is listed in Table 4.



Distribution system (AB,B) **Distribution system** (AB,A)

E.6 Market outcomes across distribution systems

	(A,AB)	(AB,AB)	(A,B)	(A,A)	(AB,B)
w_1	$\frac{4a+4k-(2a-2k)\theta-(3a+2k)\theta^2+(a-k)\theta^3}{8-5\theta^2}$	$\frac{a+k-a\theta}{2-\theta}$	$\frac{2a+2k-a\theta-(a+k)\theta^2}{4-\theta-2\theta^2}$	$\frac{a+k-a\theta}{2-\theta}$	$\frac{4a+4k-(a-k)\theta-(3a+2k)\theta^2}{8-5\theta^2}$
w_2	$\frac{4a+4k-(a-k)\theta-(2k+3a)\theta^2}{8-5\theta^2}$	$\frac{a+k-a\theta}{2-\theta}$	$\frac{2a+2k-a\theta-(a+k)\theta^2}{4-\theta-2\theta^2}$	$\frac{a+k-a\theta}{2-\theta}$	$\frac{4a+4k-(2a-2k)\theta-(3a+2k)\theta^2+(a-k)\theta^3}{8-5\theta^2}$
p_1	$\frac{12a+4k-(6a-6k)\theta-(9a+k)\theta^2-(3k-3a)\theta^3}{2(8-5\theta^2)}$	w_1	$\frac{6a+2k-6a\theta-2a\theta^2+2a\theta^3-k\theta^2}{(2-\theta)(4-\theta-2\theta^2)}$	$\frac{3a+k-2a\theta}{2(2-\theta)}$	w_1
p_2	w_2	w_2	$\frac{6a+2k-6a\theta-2a\theta^2+2a\theta^3-k\theta^2}{(2-\theta)(4-\theta-2\theta^2)}$	$\frac{3a+k-2a\theta}{2(2-\theta)}$	$\frac{12a+4k-(6a-6k)\theta-(9a+k)\theta^2+(3a-3k)\theta^3}{2(8-5\theta^2)}$
Q_1	$\frac{(a-k)(4+2\theta-\theta^2)}{2(1+\theta)(8-5\theta^2)}$	$\frac{a-k}{(2-\theta)(1+\theta)}$	$\frac{(a-k)(2-\theta^2)}{(2-\theta)(1+\theta)(4-\theta-2\theta^2)}$	$\frac{a-k}{2(2-\theta)(1+\theta)}$	$\frac{(a-k)(4+3\theta)(2-\theta^2)}{2(1+\theta)(8-5\theta^2)}$
Q_2	$\frac{(a-k)(4+3\theta)(2-\theta^2)}{2(1+\theta)(8-5\theta^2)}$	$\frac{a-k}{(2-\theta)(1+\theta)}$	$\frac{(a-k)(2-\theta^2)}{(2-\theta)(1+\theta)(4-\theta-2\theta^2)}$	$\frac{a-k}{2(2-\theta)(1+\theta)}$	$\frac{(a-k)(4+2\theta-\theta^2)}{2(1+\theta)(8-5\theta^2)}$
π_1	$\frac{(a-k)^2(1-\theta)(4+2\theta-\theta^2)^2}{2(1+\theta)(8-5\theta^2)^2}$	$\frac{(a-k)^2(1-\theta)}{(2-\theta)^2(1+\theta)}$	$\frac{(a-k)^2(1-\theta)(2+\theta)(2-\theta^2)}{(2-\theta)(1+\theta)(4-\theta-2\theta^2)^2}$	$\frac{(a-k)^2(1-\theta)}{(2-\theta)^2(1+\theta)}$	$\frac{(a-k)^2(1-\theta)(4+3\theta)^2(2-\theta^2)}{2(1+\theta)(8-5\theta^2)^2}$
π_2	$\frac{(a-k)^2(1-\theta)(4+3\theta)^2(2-\theta^2)}{2(1+\theta)(8-5\theta^2)^2}$	$\frac{(a-k)^2(1-\theta)}{(2-\theta)^2(1+\theta)}$	$\frac{(a-k)^2(1-\theta)(2+\theta)(2-\theta^2)}{(2-\theta)(1+\theta)(4-\theta-2\theta^2)^2}$	$\frac{(a-k)^2(1-\theta)}{(2-\theta)^2(1+\theta)}$	$\frac{(a-k)^2(1-\theta)(4+2\theta-\theta^2)^2}{2(1+\theta)(8-5\theta^2)^2}$
π_A	$\frac{(a-k)^2(1-\theta)(4+2\theta-\theta^2)^2}{4(1+\theta)(8-5\theta^2)^2}$	0	$\frac{(a-k)^2(1-\theta)(2-\theta^2)^2}{(2-\theta)^2(1+\theta)(4-\theta-2\theta^2)^2}$	$\frac{(a-k)^2}{2(2-\theta)^2(1+\theta)}$	0
π_B	0	0	$\frac{(a-k)^2(1-\theta)(2-\theta^2)^2}{(2-\theta)^2(1+\theta)(4-\theta-2\theta^2)^2}$	-	$\frac{(a-k)^2(1-\theta)(4+2\theta-\theta^2)^2}{4(1+\theta)(8-5\theta^2)^2}$

Table 4: Market outcomes across distribution systems

E.7 Determination of Nash equilibria

We first examine the best response correspondence of the manufacturers. Subsequently, we derive the Nash equilibrium distribution systems of Proposition 6.

Lemma. *Let i and j, $j \neq i$, denote manufacturers i and j, respectively. Then there exist $\underline{\theta}, \overline{\theta} \in (-1,1)$ with $\underline{\theta} < \overline{\theta}$ such that:*

(i) if $\theta \in (-1, \underline{\theta})$, then, for any strategy $s_j \in S$, the set of best responses by manufacturer i is given by $s_i^* = \{AB\}$;

(ii) if $\theta \in (\underline{\theta}, \overline{\theta})$, then the set of best responses by manufacturer i is $s_i^* = \{AB\}$ for $s_j \in \{A,B\}$, and it is $s_i^* = \{A,B\}$ for $s_j = AB$;

(iii) if $\theta \in (\overline{\theta}, 1)$, then the set of best responses by manufacturer i is given by $s_i^* = \{B\}$ for $s_j = A$, it is $s_i^* = \{A\}$ for $s_j = B$, and it is $s_i^* = \{A,B\}$ for $s_j = AB$.

Moreover, we have that $\underline{\theta} \approx 0.9121$ and $\overline{\theta} \approx 0.9716$.

Proof of the Lemma: Without loss of generality, let $i = 1$ and $j = 2$. First, consider $s_2 = A$. For M_1, strategy A does strictly worse than AB, since $a > k$ and $\theta \in (-1,1)$ imply

$$\pi_1(AB,A) - \pi_1(A,A) = \frac{(a-k)^2(1-\theta)(64-72\theta^2+16\theta^3+21\theta^4-9\theta^5)}{2(2-\theta)^2(8-5\theta^2)^2} > 0.$$

It is hence sufficient to compare $\pi_1(AB,A)$ and $\pi_1(B,A)$. We have

$$\pi_1(AB,A) - \pi_1(B,A) = \frac{(a-k)^2(1-\theta)(2-\theta^2)\psi_1(\theta)}{2(2-\theta)(1+\theta)(4-\theta-2\theta^2)^2(8-5\theta^2)^2}, \tag{E.1}$$

where $\psi_1(\theta) \equiv 256+128\theta-512\theta^2-288\theta^3+318\theta^4+189\theta^5-60\theta^6-36\theta^7$. Because of $a > k$ and $\theta \in (-1,1)$, the roots of (E.1) and $\psi_1(\theta)$ coincide. There exists a unique root $\overline{\theta} \in$

$(-1,1)$, which is approximately given by $\overline{\theta} \approx 0.9716$. Moreover, we have $\pi_1(AB,A) > \pi_1(B,A)$ for $\theta \in (-1,\overline{\theta})$ and $\pi_1(AB,A) < \pi_1(B,A)$ for $\theta \in (\overline{\theta},1)$. The third column of Table 5 further below collects our findings for this case.

Second, for $s_2 = R_B$, the analysis is analogous to the previous case, by symmetry between strategies A and B. The fourth column of Table 5 corresponds to this case.

Finally, suppose $s_2 = AB$. By symmetry between strategies $s_1 = A$ and $s_1 = B$, it suffices to compare AB and A. Consider the corresponding profit difference:

$$\pi_1(AB,AB) - \pi_1(A,AB) = \frac{(a-k)^2(1-\theta)(2-\theta^2)\psi_2(\theta)}{2(2-\theta)^2(1+\theta)(8-5\theta^2)^2}, \tag{E.2}$$

where $\psi_2(\theta) \equiv 32 - 32\theta^2 - 8\theta^3 + \theta^4$. Because of $a > k$ and $\theta \in (-1,1)$, the roots of (E.2) and $\psi_2(\theta)$ coincide. There exists a unique root $\underline{\theta} \in (-1,1)$, which is given by $\underline{\theta} \approx 0.9121$. Moreover, we have $\pi_1(AB,AB) > \pi_1(A,AB) = \pi_1(B,AB)$ for $\theta \in (-1,\underline{\theta})$ and $\pi_1(AB,AB) < \pi_1(A,AB) = \pi_1(B,AB)$ for $\theta \in (\underline{\theta},1)$. The last column of Table 5 summarizes this case. □

Table 5 displays the best response sets characterized in the above lemma:

		M_j $(j \neq i)$		
		A	B	AB
M_i	$\theta \in (-1,\underline{\theta})$	$\{AB\}$	$\{AB\}$	$\{AB\}$
	$\theta \in (\underline{\theta},\overline{\theta})$	$\{AB\}$	$\{AB\}$	$\{A,B\}$
	$\theta \in (\overline{\theta},1)$	$\{B\}$	$\{A\}$	$\{A,B\}$

Table 5: Best response correspondence of manufacturer M_i (for $i = 1,2$)

Proof of Proposition 6: Consider $\theta \in (0.9122, 0.9716)$ and notice that $(0.9122, 0.9716) \subset (\underline{\theta},\overline{\theta})$. Hence, part (ii) of the above lemma applies. It follows that $s_i^* = A$ is a best response to $s_j^* = AB$ and $s_j^* = AB$ is a best response to $s_i^* = A$. Since this holds for any $i \in \{1,2\}$ and $j = 3-i$, the distribution systems (A,AB) and (AB,A) constitute a Nash equilibrium. A similar argument applies when we replace $s_i^* = A$ by $s_i^* = B$. Therefore, the distribution systems (B,AB) and (AB,B) constitute a Nash equilibrium as well. □

F Proofs of Propositions 1 – 5

F.1 Proposition 1

Bundling is profitable for R_A if, and only if, $\Delta\pi_A \equiv \pi_A^S - \pi_A^{BL} < 0$. Notice that

$$\begin{aligned}\Delta\pi_A &= \frac{(a-k)^2(1-\theta)(4+2\theta-\theta^2)^2}{4(1+\theta)(8-5\theta^2)^2} - \frac{4(a-k)^2(3-\theta)^2(1-\theta)(17+4\theta-5\theta^2)^2}{(7-\theta)^2(1+\theta)(59-26\theta-29\theta^2+8\theta^3)^2} \\ &= \frac{(a-k)^2(1-\theta)\varphi_2(\theta)}{4(7-\theta)^2(8-5\theta^2)^2(59-26\theta-29\theta^2+8\theta^3)^2} \\ &\quad \times(3284-298\theta-3231\theta^2+487\theta^3+695\theta^4-201\theta^5+8\theta^6), \qquad \text{(F.1)}\end{aligned}$$

where the roots of $\varphi_2(\theta) \equiv 20+2\theta+23\theta^2-56\theta^3-9\theta^4+8\theta^5$ coincide with those of $\Delta\pi_A$, since the last term of (F.1) is strictly positive. Observe that there exists a unique root $\hat{\theta}_8$ in $(-1,1)$, which approximately equals $\hat{\theta}_8 \approx 0.8826$. Moreover, it holds that $\Delta\pi_A > 0$ for $\theta \in (-1,\hat{\theta}_8)$ and $\Delta\pi_A < 0$ for $\theta \in (\hat{\theta}_8, 1)$.

F.2 Proposition 2

Bundling involves a higher wholesale price for good 1 if, and only if,

$$\begin{aligned}\Delta w_1 &\equiv w_1^S - w_1^{BL} \\ &= -\frac{(a-k)(1-\theta)(1+\theta)(36+14\theta-37\theta^2-3\theta^3+8\theta^4)}{(8-5\theta^2)(59-26\theta-29\theta^2+8\theta^3)} < 0.\end{aligned}$$

Observe that Δw_1 has a unique root $\hat{\theta}_9$ in $(-1,1)$, which is given by $\hat{\theta}_9 \approx -0.9174$. Moreover, it holds that $\Delta w_1 > 0$ for $\theta \in (-1,\hat{\theta}_9)$ and $\Delta w_1 < 0$ for $\theta \in (\hat{\theta}_9, 1)$.

Bundling entails a higher wholesale price of good 2 because $\theta \in (-1,1)$ and $a > k$ imply

$$\begin{aligned}\Delta w_2 &\equiv w_2^S - w_2^{BL} \\ &= -\frac{(a-k)(1-\theta)(1+\theta)(20+3\theta-4\theta^3-\theta^2)}{(8-5\theta^2)(59-26\theta-29\theta^2+8\theta^3)} < 0.\end{aligned}$$

Finally, the sum of wholesale prices is higher under bundling because of

$$\begin{aligned}\Delta w &\equiv (w_1^S + w_2^S) - (w_1^{BL} + w_2^{BL}) \\ &= \frac{8a+8k-3a\theta+3k\theta-6a\theta^2-4k\theta^2+a\theta^3-k\theta^3}{8-5\theta^2} \\ &\quad + \frac{2\left(33a+26k-23a\theta-3k\theta-17a\theta^2-12k\theta^2+7a\theta^3+k\theta^3\right)}{59-26\theta-29\theta^2+8\theta^3} \\ &= -\frac{(a-k)(1-\theta)(1+\theta)\left(56+17\theta-38\theta^2-7\theta^3+8\theta^4\right)}{(8-5\theta^2)(59-26\theta-29\theta^2+8\theta^3)} < 0,\end{aligned}$$

where the strict inequality follows from $\theta \in (-1,1)$ and $a > k$.

F.3 Proposition 3

Bundling implies a higher Q_1 if, and only if,

$$\begin{aligned}\Delta Q_1 &\equiv Q_1^S - Q_1^{BL} \\ &= \frac{(a-k)\varphi_2(\theta)}{2(7-\theta)(8-5\theta^2)(59-26\theta-29\theta^2+8\theta^3)} < 0,\end{aligned}$$

where $\varphi_2(\theta)$ has been defined in section F.1. Because of $a > k$, its unique root $\hat{\theta}_8$ in $(-1,1)$ also represents the unique root of ΔQ_1. Moreover, we have $\Delta Q_1 > 0$ for $\theta \in (-1, \hat{\theta}_8)$ and $\Delta Q_1 < 0$ for $\theta \in (\hat{\theta}_8, 1)$.

Bundling entails a lower Q_2 for all $\theta \in (-1,1)$, because $a > k$ implies

$$\begin{aligned}\Delta Q_2 &\equiv Q_2^S - Q_2^{BL} \\ &= \frac{(a-k)(744-642\theta-736\theta^2+679\theta^3+110\theta^4-167\theta^5+24\theta^6)}{2(7-\theta)(8-5\theta^2)(59-26\theta-29\theta^2+8\theta^3)} > 0.\end{aligned}$$

Bundling entails a higher quantity of good 2 for retailer R_A if, and only if,

$$\begin{aligned}\Delta q_{A2} &\equiv q_{A2}^S - b_A \\ &= \frac{(a-k)(40+870\theta-1258\theta^2-1177\theta^3+1117\theta^4+343\theta^5-223\theta^6+24\theta^7)}{4(7-\theta)(1+\theta)(8-5\theta^2)(59-26\theta-29\theta^2+8\theta^3)} < 0.\end{aligned}$$

Observe that Δq_{A2} has two roots, which approximately equal $\hat{\theta}_{10} \approx -0.0434$ and $\hat{\theta}_{11} \approx 0.6200$. Moreover, we have $\Delta q_{A2} < 0$ for $\theta \in (-1, \hat{\theta}_{10})$ and for $\theta \in (\hat{\theta}_{11}, 1)$, while $\Delta q_{A2} > 0$ holds for $\theta \in (\hat{\theta}_{10}, \hat{\theta}_{11})$. Thus, under profitable bundling, the quantity of good 2 sold by firm A increases.

Bundling entails a lower quantity of good 2 for retailer R_B, since $\theta \in (-1,1)$ and $a > k$ imply

$$\begin{aligned}\Delta q_{B2} &\equiv q_{B2}^S - b_B \\ &= \frac{(a-k)(1448-666\theta-1498\theta^2+1063\theta^3+461\theta^4-457\theta^5-63\theta^6+24\theta^7)}{4(7-\theta)(1+\theta)(8-5\theta^2)(59-26\theta-29\theta^2+8\theta^3)} > 0.\end{aligned}$$

Bundling is beneficial for R_B, since $\pi_B^S = 0$ implies

$$\begin{aligned}\Delta \pi_B &\equiv \pi_B^S - \pi_B^{BL} \\ &= -\frac{2(a-k)^2(1-\theta)\left(29-10\theta-19\theta^2+4\theta^3\right)^2}{(7-\theta)^2(59-26\theta-29\theta^2+8\theta^3)^2},\end{aligned}$$

which is strictly negative because of $\theta \in (-1,1)$ and $a > k$.

F.4 Proposition 4

Bundling is beneficial for manufacturer M_1 if, and only if, $\Delta\pi_1 \equiv \pi_1^S - \pi_1^{BL} < 0$, where

$$\begin{aligned}\Delta\pi_1 &= \frac{(a-k)^2(1-\theta)(4+2\theta-\theta^2)^2}{2(1+\theta)(8-5\theta^2)^2} - \frac{4(a-k)^2(3-\theta)(1-\theta)(17+4\theta-5\theta^2)^2}{(7-\theta)(1+\theta)(59-26\theta-29\theta^2+8\theta^3)^2} \\ &= \frac{(a-k)^2(1-\theta)\varphi_3(\theta)}{2(7-\theta)(8-5\theta^2)^2(59-26\theta-29\theta^2+8\theta^3)^2}\end{aligned}$$

and where $\varphi_3(\theta)$ is defined by

$$\begin{aligned}\varphi_3(\theta) &\equiv -54032-163336\theta+122148\theta^2-940\theta^3-98321\theta^4+18700\theta^5 \\ &\quad +32438\theta^6-9160\theta^7-3297\theta^8+1232\theta^9-64\theta^{10}.\end{aligned}$$

Notice that $\Delta\pi_1 = 0$ if, and only if, $\varphi_3(\theta) = 0$ because of $\theta \in (-1,1)$ and $a > k$. There exists a unique root $\hat{\theta}_{12}$ in $(-1,1)$, which is given by $\hat{\theta}_{12} \approx -0.7799$. Moreover, we have $\Delta\pi_1 > 0$ for $\theta \in (-1,\hat{\theta}_{12})$ and $\Delta\pi_1 < 0$ for $\theta \in (\hat{\theta}_{12},1)$.

Bundling entails a higher profit of M_2 if, and only if,

$$\begin{aligned}\Delta\pi_2 &\equiv \pi_2^S - \pi_2^{BL} \\ &= \frac{(a-k)^2(1-\theta)(2-\theta^2)(4+3\theta)^2}{2(1+\theta)(8-5\theta^2)^2} \\ &\quad - \frac{16(a-k)^2(1-\theta)(8+3\theta-\theta^2)^2(5-\theta-2\theta^2)}{(7-\theta)(1+\theta)(59-26\theta-29\theta^2+8\theta^3)^2} \\ &= \frac{(a-k)^2(1-\theta)\varphi_4(\theta)}{2(7-\theta)(8-5\theta^2)^2(59-26\theta-29\theta^2+8\theta^3)^2} < 0,\end{aligned}$$

where $\varphi_4(\theta)$ is defined by

$$\begin{aligned}\varphi_4(\theta) &\equiv 124384-113840\theta-300962\theta^2+222240\theta^3+250173\theta^4-158444\theta^5 \\ &\quad -83188\theta^6+49188\theta^7+7889\theta^8-5648\theta^9+576\theta^{10}.\end{aligned}$$

Again, we have $\Delta\pi_2 = 0$ if, and only if, $\varphi_4(\theta) = 0$ because of $a > k$ and $\theta \in (-1,1)$. There exists a unique root $\hat{\theta}_{13}$ in $(-1,1)$, which approximately equals $\hat{\theta}_{13} \approx 0.7151$. We obtain $\Delta\pi_2 > 0$ for $\theta \in (-1,\hat{\theta}_{13})$ and $\Delta\pi_2 < 0$ for $\theta \in (\hat{\theta}_{13},1)$.

F.5 Proposition 5

Consumer surplus is lower under bundling because of

$$\begin{aligned}\Delta CS &\equiv CS^S - CS^{BL}\\ &= \frac{(a-k)^2(80+96\theta-48\theta^2-76\theta^3+5\theta^4+15\theta^5)}{8(1+\theta)(8-5\theta^2)^2}\\ &\quad -\frac{2(a-k)^2(9001+889\theta-9658\theta^2-662\theta^3+3365\theta^4-11\theta^5-388\theta^6+56\theta^7)}{(7-\theta)^2(1+\theta)(59-26\theta-29\theta^2+8\theta^3)^2}\\ &= -\frac{(a-k)^2(1-\theta)\varphi_5(\theta)}{8(7-\theta)^2(8-5\theta^2)^2(59-26\theta-29\theta^2+8\theta^3)^2}>0,\end{aligned}$$

where $\varphi_5(\theta)$ is given by

$$\begin{aligned}\varphi_5(\theta) &\equiv -4428496+4609920\theta+85077760\theta^2-19290440\theta^3-59258930\theta^4+21260270\theta^5\\ &\quad +16867660\theta^6-8739820\theta^7-1078170\theta^8+1177510\theta^9-200800\theta^{10}+9600\theta^{11}.\end{aligned}$$

and where the strict inequality follows from $\varphi_5(\theta)<0$, $a>k$, and $\theta\in(-1,1)$.

As to *producer surplus,* notice that bundling raises the profits of retailer R_B and of both manufacturers by Propositions 3 and 4, respectively. Thus, under profitable bundling, the profits of all firms and hence the producer surplus are higher than under separate selling.

Social welfare is higher under bundling if, and only if, $\Delta W \equiv W^S - W^{BL} < 0$, where

$$\begin{aligned}\Delta W &= \frac{(a-k)^2(304+160\theta-352\theta^2-180\theta^3+95\theta^4+45\theta^5)}{8(1+\theta)(8-5\theta^2)^2}\\ &\quad -2(a-k)^2\left(\frac{(45102-287430\theta-433690\theta^2+242860\theta^3+110680\theta^4)}{(7-\theta)^2(1+\theta)(59-26\theta-29\theta^2+8\theta^3)^2}\right.\\ &\quad \left.+\frac{(-58350\theta^5-2090\theta^6+3240\theta^7-320\theta^8)}{(7-\theta)^2(1+\theta)(59-26\theta-29\theta^2+8\theta^3)^2}\right)\\ &= -\frac{(a-k)^2(1-\theta)\varphi_6(\theta)}{8(7-\theta)^2(8-5\theta^2)^2(59-26\theta-29\theta^2+8\theta^3)^2}\end{aligned}$$

and where $\varphi_6(\theta)$ is defined by

$$\begin{aligned}\varphi_6(\theta) &\equiv -5668528+37921920\theta+108706880\theta^2-70966520\theta^3-72412310\theta^4+50383370\theta^5\\ &\quad +17743620\theta^6-16108900\theta^7-25230\theta^8+1851250\theta^9-423200\theta^{10}+28800\theta^{11}.\end{aligned}$$

Because of $a>k$ and $\theta\in(-1,1)$, the roots of ΔW and $\varphi_6(\theta)$ coincide. There exists a unique root in $(-1,1)$, which is given by $\hat{\theta}_{14}\approx 0.9987$. Moreover, we have $\Delta W>0$ for $\theta\in(-1,\hat{\theta}_{14})$ and $\Delta W<0$ for $\theta\in(\hat{\theta}_{14},1)$. Thus, profitable bundling leads to higher welfare for $\theta\in(\hat{\theta}_8,\hat{\theta}_{14})$ and to lower welfare for $\theta\in(\hat{\theta}_{14},1)$, where $\hat{\theta}_8$ represents the lower boundary of profitable bundling characterized in section F.1.