

**FACULTY OF BUSINESS ADMINISTRATION  
AND ECONOMICS**

**Working Paper Series**

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Working Paper No. 2022-10

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Channel Structure in a Manufacturer-Driven Supply  
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Angelika Endres-Fröhlich

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April 2022

# The Impact of Product Differentiation on the Channel Structure in a Manufacturer-Driven Supply Chain

Angelika Endres-Fröhlich<sup>1</sup>

## Abstract

We study the impact of product differentiation on different distribution systems in a supply chain. Our market is characterized by an asymmetric supply chain with two retailers and three manufacturers that each produce one differentiated good. We determine that a non-exclusive distribution system is a Nash equilibrium for all degrees of product differentiation between the three goods. Furthermore, we assess the welfare implications of various distribution systems. We identify that the non-exclusive equilibrium channel structure provides the highest social welfare and highest consumer surplus for all degrees of product differentiation. Aside from that, we find a strong incentive for the manufacturers to form an exclusive selling cartel for close substitutes, which would cause a Pareto improvement for all firms but harm overall welfare.

*JEL classification:* D21; D47; L22

*Keywords:* product differentiation; endogenous markets; supply chains

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<sup>1</sup>Paderborn University, angelika.endres@upb.de

# 1. Introduction

Goods in a supply chain can be sold exclusively or non-exclusively via any firm. The firms' decisions which distribution relations to set depends on various factors, such as the degree of product differentiation between the goods in the market. The distribution links the firms maintain finally determine the resulting distribution system. This paper examines the impact of product differentiation on the upstream firms' distribution choice and assesses the welfare implications of the resulting distribution systems. Our main contribution is that a non-exclusive channel structure yields a Nash equilibrium for all degrees of product differentiation. We further identify the equilibrium distribution to provide the highest social welfare for all degrees of product differentiation.

Our model is as follows: Our market is asymmetric and consists of three manufacturers, two retailers, and three goods. We suppose that the retailers compete in prices. Supply chains with several upstream firms are common in retailing or service industries where retailers outsource the production of the components as in the automotive industry or the computer industry (Edirisinghe et al., 2011). Especially the number of physical retail stores is indeed limited and thus might be smaller than the number of manufacturers selling to the retailers. Traditionally, the literature around (non-)exclusive dealing structures assumes that the upstream industry has more market (or channel) power (see Wang et al., 2019). We follow that assumption.<sup>1</sup> Examples of powerful manufacturers are upstream suppliers such as Intel or Microsoft that might be more dominant in a supply chain than the downstream firms they deliver to (Zhao et al., 2014).

Furthermore, we assume that the traded goods are (imperfect) complements, (imperfect) substitutes, or independent in demand. An application for our model might be powerful electronic producers such as Apple, Samsung, and Xiaomi that establish the market's distribution relations by selling their goods solely via authorized retailers. One could imagine their smartphones, display covers, and power banks as traded goods. Their respective smartphones with similar features might be perceived as (almost) perfect substitutes and a smartphone and a display cover as (almost) perfect complements. For the intermediate ranges of product differentiation one can think about a power bank by Samsung as complementary to most smartphones but only delivering 'Samsung adaptive fast charging' combined with a Samsung device (Samsung, 2021).

We investigate the following research questions: *Which channel structure arises in a manufacturer-driven supply chain? What impact has the degree of product differentiation on the equilibrium channel structure? and Is the equilibrium channel structure efficient in terms of providing the highest social welfare?*

Previous literature has mostly studied the endogeneity of channel structures considering

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<sup>1</sup>In the relevant game theoretic literature in marketing, channel power is indicated by the relative profitability of any channel side (Ailawadi, 2001). Since the manufacturers earn traditionally more than the retailers in a supply chain, especially in markets with strong retail competition (Karray and Sigué, 2018), we allocate the market power to the supply side of the market.

narrower supply chains with one or two manufacturer(s) (e.g., Dobson and Waterson, 1996; Moner-Colonques et al., 2004; Endres-Fröhlich et al., 2022). Besanko and Perry (1993) investigate a supply chain with three manufacturers and impose the restriction that being in one exclusive upstream relationship prevents the retailer from entering additional upstream selling relationships. We study an asymmetric setting with two retailers and *three* manufacturers, in which the retailers might sell *several* (non-)exclusively purchased goods. Thereby, our model mirrors that in reality a retailer often buys his products from more than two manufacturers.

As main result that a non-exclusive channel structure yields a Nash equilibrium for all degrees of product differentiation implies that all three manufacturers sell their differentiated (or independent) goods non-exclusively to both retailers. Consequently, the retailers are multiproduct dealers. When the goods are complements or weak substitutes, the equilibrium occurs due to the manufacturers profiting greatly from the output expansion of selling non-exclusively. For very close substitutes, all manufacturers would be better off jointly selling exclusively since it would mitigate the strong degree of competition in the market. However, they behave as in a prisoner's dilemma and therefore do not change to a more profitable exclusive distribution system. This is why the non-exclusive selling structure yields a Nash equilibrium also for very close substitutes.

Previous literature with smaller market set-ups already identified exclusive selling systems for certain degrees of product differentiation. Bako (2016) observes an exclusive selling system equilibrium to solve the problem of contracting externalities for slight substitutes. Wu and Mallik (2010) detect an exclusive channel arrangement in equilibrium to dampen the strong degree of competition for weak product differentiation. Contrary to them, we establish the non-exclusive channel structure to exhibit an equilibrium for *all degrees of product differentiation*. To the best of our knowledge, this result still is unaccounted for.

Our result indicates that the number of competing goods in the downstream market and the retailers' ability to sell several exclusively purchased goods crucially impacts the manufacturers' equilibrium distribution choice. *Unilaterally* selling exclusively would barely soften the degree of competition in the market and the associated output reduction would cause the exclusively selling manufacturer considerable losses. However, when the manufacturers *jointly* agree on an exclusive distribution system, we identify a strong incentive to form a manufacturer cartel since all firms would experience a Pareto improvement. Notice that the retailers also profit from such a manufacturer cartel and thereby have no incentive to reveal any observed collusive behavior. Furthermore, we ascertain that the non-exclusive channel structure in equilibrium is robust to an increase in the number of manufacturers by one, thus for two retailers and *four* manufacturers.

Regarding the welfare implications of the distribution systems we establish the following: For all degrees of product differentiation, the non-exclusive equilibrium channel structure is efficient in terms of providing the highest social welfare among all possible distribution systems. Furthermore, consumer surplus is highest under the non-exclusive channel structure due to the

low wholesale and retail prices in equilibrium. Hence, even though the one-sided distribution of market power might rise anti-trust concerns, our model shows that from a social welfare and consumer surplus perspective, this concern is uncalled for. However, if the manufacturers pursue any collusive agreement to increase their joint profits, this would lessen social welfare and would again raise concerns for a social planner.

This paper contributes most closely to the literature about the impact of product differentiation on the channel structure in a decentralized channel. A seminal paper is by [Chang \(1992\)](#), who studies an optimally endogenous channel structure without product differentiation and concludes that each manufacturer would supply his homogeneous good exclusively to one retailer. His paper was often extended with differentiated products. The literature that previously investigated the impact of product differentiation on the channel structure considers aspects as retail investments ([Besanko and Perry, 1993](#)), differentiated retail services ([Dobson and Waterson, 1996](#)), quality asymmetries ([Moner-Colonques et al., 2004](#)), partial integration ([Wu and Mallik, 2010](#)), bilateral link formation ([Mauleon et al., 2011](#)), private contracts ([Bako, 2016](#)), or wide ranges of product differentiation ([Endres-Fröhlich et al., 2022](#)). Despite the various impact factors, most studies identify a non-exclusive distribution structure for high product differentiation and an exclusive distribution for low product differentiation ([Besanko and Perry, 1993; Dobson and Waterson, 1996; Moner-Colonques et al., 2004; Wu and Mallik, 2010; Endres-Fröhlich et al., 2022](#)). Additionally, [Besanko and Perry \(1993\)](#) find an asymmetric distribution system for intermediate levels of interbrand competition, [Moner-Colonques et al. \(2004\)](#) for sufficiently large brand asymmetry and product differentiation, and [Endres-Fröhlich et al. \(2022\)](#) for relatively (but not very) low product differentiation. The difference between those studies and ours seems to be caused by the exclusive dealing assumptions in [Besanko and Perry \(1993\)](#), the additional differentiation between the brands assumed in [Moner-Colonques et al. \(2004\)](#), and the number of goods in [Endres-Fröhlich et al. \(2022\)](#).

The identified effects of the channel structure on welfare are ambiguous. [Besanko and Perry \(1993\)](#) find that social welfare is higher in the exclusive dealing channel structure compared to the mixed and the non-exclusive channel structure. [O'Brien and Shaffer \(1993\)](#) agree with that and state that selling exclusively to two retail firms may enhance social welfare as compared to selling via a common retailer. Contrary to that, [Dobson and Waterson \(1996\)](#) find that exclusive contracts harm social welfare independent of the degree of product differentiation, while [Bako \(2016\)](#) agrees with this only when the goods are slight substitutes.

Equilibrium distribution structures have also been studied in other contexts. Some studies focus on the strategic effects of vertical separation ([Bonanno and Vickers, 1988](#)) and vertical restraints on competition ([Rey and Stiglitz, 1995](#)), or the anti-competitive effects of exclusive contracts ([Bernheim and Whinston, 1998; Fumagalli and Motta, 2006; Lee, 2013](#)). Various factors were previously identified to impact the kind of distribution system that arises, e.g., a (non-)linear demand function ([Choi, 1991](#)), waiting costs ([Gabrielsen, 1997](#)), two-sidedness of a market ([Armstrong and Wright, 2007](#)), or risk aversion ([Hansen and Motta, 2019](#)).

Differing from most of these studies, we investigate an extended market set-up by considering an asymmetric supply chain set-up with three manufacturers and two retailers and allow the retailers to sell several (non-)exclusively purchased goods. Both assumptions induce market situations in which at least one retailer has several product relations with the upstream industry, even when all three manufacturers choose an exclusive selling strategy. This already results in a certain competitive pressure in the downstream market independent of the degree of product differentiation. Our assumptions further allow for a wider application to reality.

The remainder of the paper is structured as follows: Section 2 introduces the basic model, considers the possible distribution structures that might arise, and investigates their respective equilibrium values. Section 3 derives the equilibrium distribution system for the manufacturers and the welfare implications of all distribution systems. It further analyzes a possibility for a Pareto improvement for all firms. Section 4 varies the number of manufacturers and analyzes the robustness of the main result. Section 5 concludes the paper.

## 2. Model and equilibrium analysis

Next, we introduce the market set-up and the behavior of the market players to determine the equilibrium entities for all possible channel structures.

### 2.1. Market set-up

The market consists of three monopolistic manufacturers  $M_i$  producing good  $i$ , with  $i = 1, 2, 3$ , two competing retailers  $R_j$ , with  $j = A, B$ , and a representative customer. The customer can purchase the goods only from the retailers, not directly from the manufacturers. We assume the retailers compete in prices. We furthermore interpret *selling exclusively* purely from a manufacturer's perspective as selling exclusively to one retailer. This differs from the understanding of 'exclusive dealing' as an agreement in which the retailers sell the good of one manufacturer exclusively (see [Besanko and Perry, 1993](#)). By considering a larger number of manufacturers than retailers, we ensure that at least one of the retailers sells several goods. This implies that even when all manufacturers sell exclusively, our set-up guarantees competition in one retail store for two products. We assume that the supply chain is dominated by the manufacturers, implying that they move first.

The timing of the game involves two stages (see Figure 1). At stage 1, manufacturers  $M_i$  make their distribution choice and choose their wholesale prices accordingly. This means that they decide whether to sell their goods exclusively to any retailer or non-exclusively to both retailers. At stage 2, both retailers  $R_j$  choose independently and simultaneously their prices. We solve the sequential game by backward induction to derive its subgame perfect equilibrium (SPE).

In the first stage, each manufacturer chooses a strategy from the strategy set  $S_i = \{A, B, AB\}$ .

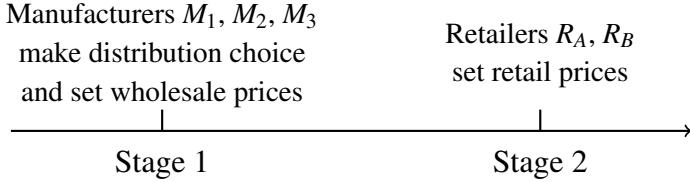


Figure 1: Timing

Strategy  $s_1 = A$  implies that manufacturer  $M_1$  chooses to distribute exclusively to retailer  $R_A$ . Reasons for the agents to pursue an exclusive contract might be to enlarge horizontal market power by vertical market foreclosure, to gain superior efficiency (Chang, 1992), or to dampen the degree of interbrand competition in the retail market (Moner-Colonques et al., 2004). Strategy  $s_1 = AB$  implies that manufacturer  $M_1$  sells his good non-exclusively to both retailers. This strategy may be strategically used by a manufacturer to introduce intrabrand competition between the goods that then might coexist with interbrand competition in the downstream market (Moner-Colonques et al., 2004). In this model, we concentrate on the manufacturer's motive to impact the degree of competition for pursuing (non-)exclusive contracts.<sup>2</sup>

Taken together, we denote our distribution systems by the strategy choices of all manufacturers. Distribution system  $(A, A, AB)$ , for example, implies that manufacturers  $M_1$  and  $M_2$  sell exclusively to retailer  $R_A$  and manufacturer  $M_3$  sells non-exclusively to both retailers. The retailers also may supply several goods if several manufacturers sell them their goods (non-)exclusively. Consequently, 27 possible strategy combinations exist, which we refer to as *distribution systems*.<sup>3</sup> Some of the channels are symmetric. Thus, we reduce our analysis to six distribution systems which are depicted in Figure 2.

Depending on the channel structure, various dimensions of competition take place, which are impacted by the degree of product differentiation (e.g., Moner-Colonques et al., 2004; Mauleon et al., 2011). *Interbrand competition* happens between the goods of different manufacturers and intensifies with decreasing product differentiation. *Intrabrand competition* takes place among retailers that sell homogeneous goods and occurs independently of the degree of product differentiation.

These modes of competition give rise to two opposing effects that greatly affect the manufacturers' distribution choice: the *competition effect* and the *output expansion effect*. The output expansion effect covers the additional upstream profits gained from installing a new distribution relationship. Selling a product non-exclusively intensifies interbrand competition and

<sup>2</sup>Notice that the strategy of *not selling to anybody* is always a dominated one.

<sup>3</sup>In particular, all manufacturers might distribute exclusively  $(A, A, A)$ ,  $(A, A, B)$ ,  $(A, B, A)$ ,  $(B, A, A)$ ,  $(B, B, A)$ ,  $(B, A, B)$ ,  $(A, B, B)$ ,  $(B, B, B)$ , one manufacturer might sell to two retailers  $(A, A, AB)$ ,  $(A, AB, A)$ ,  $(AB, A, A)$ ,  $(AB, B, B)$ ,  $(B, AB, B)$ ,  $(B, B, AB)$ ,  $(A, B, AB)$ ,  $(B, A, AB)$ ,  $(A, AB, B)$ ,  $(B, AB, A)$ ,  $(AB, A, B)$ ,  $(AB, B, A)$ , two manufacturer might sell to two retailers  $(AB, AB, A)$ ,  $(AB, AB, B)$ ,  $(AB, A, AB)$ ,  $(AB, B, AB)$ ,  $(B, AB, AB)$ ,  $(A, AB, AB)$ , or all three manufacturers might sell to two retailers  $(AB, AB, AB)$ . Notice that we use the terms *distributions* and *channel structures* interchangeably.

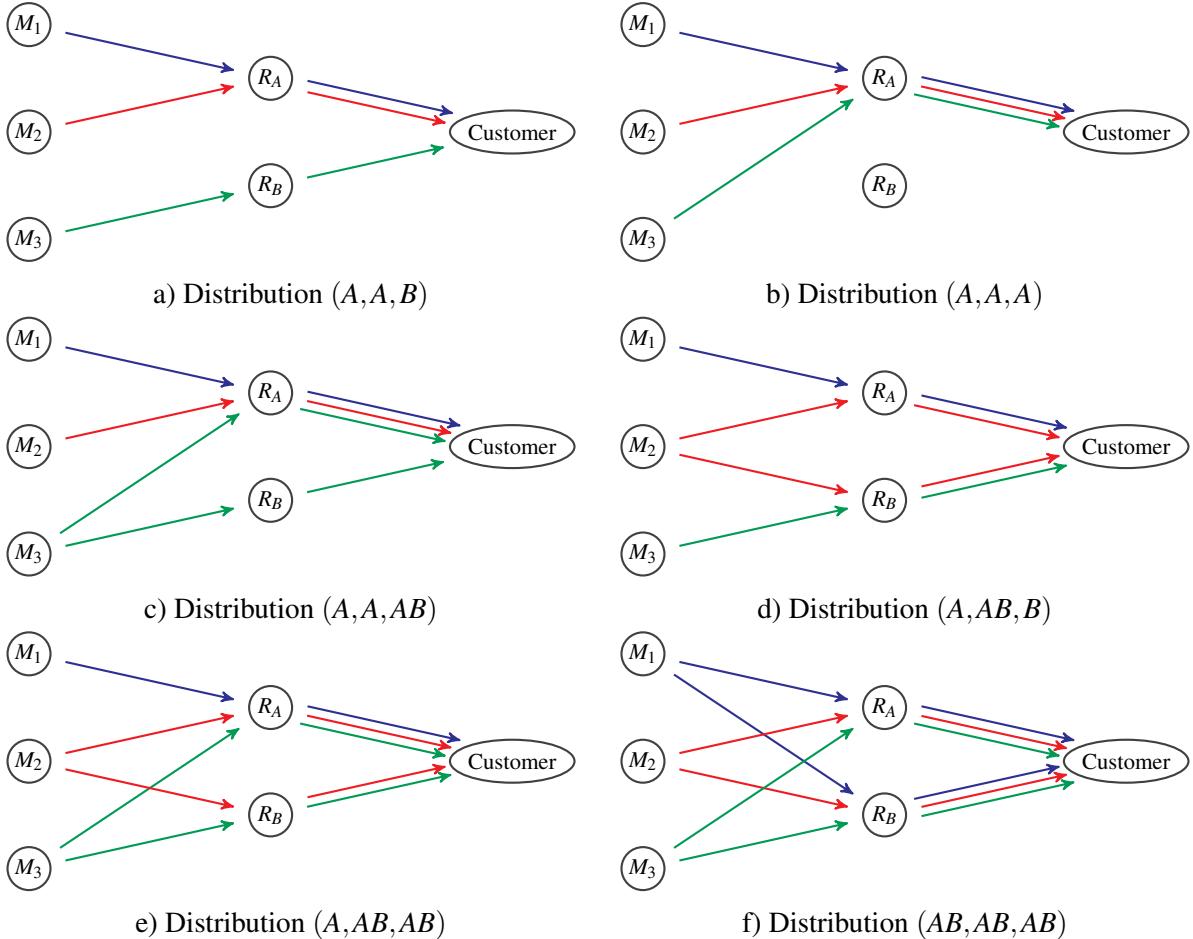


Figure 2: Distribution systems

additionally installs intrabrand competition; both cause lower prices and a higher output for this good. The competition effect covers the negative effect of an additional selling relationship on the manufacturers' profits via the lower prices charged for the good. The magnitudes of the effects decide about the profitability of any selling relationship for the manufacturers and thus the distribution strategy they choose in equilibrium.

In the following, we will discuss the demand side of the market.

## 2.2. Demand functions

The representative consumer's utility for goods  $i$  and a bundle of other goods  $m$  follows Dixit (1979); Singh and Vives (1984):<sup>4</sup>

$$V = m + a(Q_1 + Q_2 + Q_3) - \frac{1}{2} (Q_1^2 + Q_2^2 + Q_3^2 + 2\theta Q_1 Q_2 + 2\theta Q_2 Q_3 + 2\theta Q_1 Q_3).$$

The parameter  $Q_i$  denotes the quantity of good  $i$  that the consumer buys and  $\theta$  captures

<sup>4</sup>For better readability we refrain from depicting the functions' arguments in the following analysis. Further, we presume the customer represents the whole mass of consumers, as common with this type of utility function. We therefore also use the plural form in the following discussions.

the degree of product differentiation between the goods. Following Häckner (2000) and Amir et al. (2017), we set  $\theta \in (-\frac{1}{i-1}, 1)$ , i.e.  $\theta \in (\underline{\theta}, \bar{\theta})$  with  $\underline{\theta} = -0.5$  and  $\bar{\theta} = 1$ , to ensure strict concavity for our quadratic utility function.<sup>5</sup> For  $\theta < 0$  ( $\theta > 0$ ) the goods have an imperfect complementary (substitute) relationship, and in case of  $\theta = 0$  the goods are independent in demand. We refrain from analyzing the boundary cases of  $\underline{\theta} = -0.5$  ( $\bar{\theta} = 1$ ), where the goods pose perfect complements (substitutes).

All goods  $i$  have the same quality level  $a$ , which represents the customer's willingness to pay for this good, and are produced for the same marginal costs of production  $k \in [0, a]$ .<sup>6</sup> We denote  $p_i$  as the retail price for good  $i$ . The composite good is denoted by  $m$  and its price is normalized to one. Solving the customer's utility maximization problem, we obtain the inverse demand for the respective good:

$$p_1 = a - Q_1 - \theta(Q_2 + Q_3), \quad (1)$$

$$p_2 = a - Q_2 - \theta(Q_1 + Q_3), \quad (2)$$

$$p_3 = a - Q_3 - \theta(Q_1 + Q_2), \quad (3)$$

where  $Q_i = q_{Ai} + q_{Bi}$ . Here,  $q_{Ai}$  denotes the quantity of good  $i$  supplied by retailer  $R_A$  and  $q_{Bi}$  the quantity of good  $i$  supplied by retailer  $R_B$ . We invert the systems of inverse demand, (1)-(3), to derive the demand for the three goods as

$$Q_1 = \frac{a - p_1 - (a + p_1 - p_2 - p_3)\theta}{1 + \theta - 2\theta^2}, \quad (4)$$

$$Q_2 = \frac{a - p_2 - (a - p_1 + p_2 - p_3)\theta}{1 + \theta - 2\theta^2}, \quad (5)$$

$$Q_3 = \frac{a - p_3 - (a - p_1 - p_2 + p_3)\theta}{1 + \theta - 2\theta^2}, \quad (6)$$

respectively.

As common with this type of utility, we set the constraint that quantities are non-negative, which is fulfilled by  $a > k$  for all possible distribution systems.<sup>7</sup>

### 2.3. Retailers' price setting behavior

We begin with deriving the equilibrium behavior of both retailers for each of the possible channel structures. For the sake of exposition, we only present the calculations for one of the possible distribution systems, which is the non-exclusive distribution system  $(AB, AB, AB)$ , where

<sup>5</sup>Notice that  $\theta \in (-\frac{1}{i-1}, 1)$  is a restriction for the commonly used a priori possible range for  $i = 2$  goods of  $\theta \in (-1, 1)$ , for our case of  $i > 2$  firms.

<sup>6</sup>Due to all three goods having the same quality level  $a$ , we do not have any inferior good that might incentivize a manufacturer to foreclose it from the market even if the goods are substitutes (see Yehezkel, 2008).

<sup>7</sup>Further information can be found in Spence (1976).

all three manufacturers distribute their goods to both retailers.<sup>8</sup>

The retailers set their prices simultaneously and independently, while aiming at maximizing profit. Retailers  $R_A$  and  $R_B$  sell all three goods  $i$  and set their prices  $p_{Ai}$ , respectively  $p_{Bi}$ , to maximize

$$\pi_A = (p_{A1} - w_1)q_{A1} + (p_{A2} - w_2)q_{A2} + (p_{A3} - w_3)q_{A3},$$

$$\pi_B = (p_{B1} - w_1)q_{B1} + (p_{B2} - w_2)q_{B2} + (p_{B3} - w_3)q_{B3},$$

where  $w_i$  denotes the wholesale prices set by the manufacturers for good  $i$ . Both retailers' total profits sum up the profits gained from selling the three goods.

The price equilibrium in the retail market is thus given by

$$p_1^* = p_{A1}^* = p_{B1}^* = w_1, \quad (7)$$

$$p_2^* = p_{A2}^* = p_{B2}^* = w_2, \quad (8)$$

$$p_3^* = p_{A3}^* = p_{B3}^* = w_3, \quad (9)$$

where  $*$  denotes all equilibrium entities. The price competition, which takes place for each of the three goods in the retail market, drives the retail prices down to marginal costs, which is given by the respective wholesale price  $w_i$ .

Inserting the equilibrium prices (7)-(9) into demand (4)-(6), we obtain the equilibrium demand:

$$Q_1 = \frac{a - w_1 - (a + w_1 - w_2 - w_3)\theta}{1 + \theta - 2\theta^2}, \quad (10)$$

$$Q_2 = \frac{a - w_2 - (a - w_1 + w_2 - w_3)\theta}{1 + \theta - 2\theta^2}, \quad (11)$$

$$Q_3 = \frac{a - w_3 - (a - w_1 - w_2 + w_3)\theta}{1 + \theta - 2\theta^2}. \quad (12)$$

Notice that the cross-price derivatives coincide with the sign of  $\theta$ . Thus, good  $i$  represents a substitute, respectively a complement, in the upstream market, if and only if it does so in the downstream market.<sup>9</sup> The retailers' equilibrium retail prices and profits for the other distribution systems are provided in Table 1.

## 2.4. Manufacturers' price setting behavior

Depending on the second-stage decision of the retailers we can calculate the equilibrium wholesale prices of the manufacturers for all six cases. The results can be found in Table 2.

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<sup>8</sup> All calculations can be found in Appendix A.

<sup>9</sup> It holds  $\frac{\partial Q_i}{\partial w_j} = -\frac{\theta}{2\theta^2 - \theta - 1}$  with  $j \neq i$ . We have  $\frac{\partial Q_i}{\partial w_j} < 0$  for  $\theta \in (\underline{\theta}, 0)$  and  $\frac{\partial Q_i}{\partial w_j} > 0$  for  $\theta \in (0, \bar{\theta})$ .

Distribution	Equilibrium retail prices $p_i^*$ and equilibrium retail profits $\pi_j^*$
$(A, A, A)$	$p_1^* = \frac{a+w_1}{2}; p_2^* = \frac{a+w_2}{2}; p_3^* = \frac{a+w_3}{2}$ $\pi_A^* = \frac{-(2w_1+2w_2+2w_3)a+(3a^2-(2w_1+2w_2+2w_3)a+(2w_2+2w_1)w_3+2w_1w_2-w_1^2-w_2^2-w_3^2)\theta+3a^2+w_1^2+w_2^2+w_3^2}{4+4\theta-8\theta^2}; \pi_B^* = 0$
$(A, A, B)$	$p_1^* = \frac{4a+4w_1+(2a+4w_1+2w_3)\theta-(6a+w_1-w_2-2w_3)\theta^2}{8+8\theta-4\theta^2}; p_2^* = \frac{4a+4w_2+(2a+4w_2+2w_3)\theta-(6a-w_1+w_2-2w_3)\theta^2}{8+8\theta-4\theta^2}; p_3^* = \frac{2a+2w_3+(w_1+w_2+2w_3)\theta-2a\theta^2}{4+4\theta-2\theta^2}$ $\pi_A^* = \frac{(6a\theta^2-2w_3\theta^2-3w_2\theta^2-w_1\theta^2-2a\theta-2w_3\theta+4w_2\theta-4a+4w_2)(2w_2\theta^3-2w_1\theta^3-6a\theta^2+2w_3\theta^2-w_2\theta^2+5w_1\theta^2+2a\theta+2w_3\theta-8w_2\theta+4w_1\theta+4a-4w_2)}{16(\theta-1)(2\theta+1)(\theta^2-2\theta-2)^2}$ $\pi_B^* = \frac{(6a\theta^2-2w_3\theta^2-w_2\theta^2-3w_1\theta^2-2a\theta-2w_3\theta+4w_1\theta-4a+4w_1)(2w_2\theta^3-2w_1\theta^3+6a\theta^2-2w_3\theta^2-5w_2\theta^2+w_1\theta^2-2a\theta-2w_3\theta-4w_2\theta+8w_1\theta-4a+4w_1)}{16(\theta-1)(2\theta+1)(\theta^2-2\theta-2)^2}$ $\pi_B^* = \frac{2a-2w_3-(1+\theta)(-w_1\theta-w_2\theta+2w_3\theta+2a\theta^2-2w_3\theta^2)}{4(\theta-1)(1+2\theta)(-2-2\theta+\theta^2)^2}$
$(A, A, AB)$	$p_1^* = \frac{a+w_1-(a-w_3)\theta}{2}; p_2^* = \frac{a+w_2-(a-w_3)\theta}{2}; p_3^* = w_3$ $\pi_A^* = \frac{a-(2w_1+2w_2)+2a^2+w_1^2+w_2^2+(-4a^2+(2w_1+2w_2+4w_3)a-(2w_1+2w_2)w_3-2w_1w_2+w_1^2+w_2^2)\theta+(2a^2-4w_3a+2w_3^2)\theta^2}{4+4\theta-8\theta^2}; \pi_B^* = 0$
$(A, AB, B)$	$p_1^* = \frac{2a+2w_1+(a+4w_1+2w_2+w_3)\theta-(3a-2w_1-3w_2-w_3)\theta^2}{4+8\theta+3\theta^2}; p_2^* = w_2; p_3^* = \frac{2a+2w_3+(a+w_1+2w_2+4w_3)\theta-(3a-w_1-3w_2-2w_3)\theta^2}{4+8\theta+3\theta^2}$ $\pi_A^* = -\frac{(1+\theta)(-2a+2w_1-a\theta+4w_1\theta-2w_2\theta-w_3\theta+3a\theta^2+w_1\theta^2-3w_2\theta^2-w_3\theta^2)^2}{(-1+\theta)(2+\theta)^2(1+2\theta)(2+3\theta)^2}; \pi_B^* = -\frac{(1+\theta)(-2a+2w_3-a\theta-w_1\theta-2w_2\theta+4w_3\theta+3a\theta^2-w_1\theta^2-3w_2\theta^2+w_3\theta^2)^2}{(-1+\theta)(2+\theta)^2(1+2\theta)(2+3\theta)^2}$
$(A, AB, AB)$	$p_1^* = \frac{a+w_1-(a-w_1-w_2-w_3)\theta}{2+2\theta}; p_2^* = w_2; p_3^* = w_3$ $\pi_A^* = \frac{(a-w_1-a\theta-w_1\theta+w_2\theta+w_3\theta)^2}{4(-1+\theta)(1+\theta)(1+2\theta)}; \pi_B^* = 0$
$(AB, AB, AB)$	$p_1^* = p_2^* = p_3^* = \frac{3a+k-a\theta+k\theta}{4}$ $\pi_A^* = \pi_B^* = 0$

Table 1: First-stage Nash equilibrium prices and profits of retailers

The manufacturers each set their wholesale prices in order to maximize profit:

$$\pi_1 = (w_1 - k) Q_1, \quad (13)$$

$$\pi_2 = (w_2 - k) Q_2, \quad (14)$$

$$\pi_3 = (w_3 - k) Q_3, \quad (15)$$

where  $Q_i$  is provided by (10)-(12), respectively. Solving for the upstream price equilibrium, we gain the wholesale prices for the goods as indicated in Table 2.

Distribution	Wholesale prices
$(A, A, A)$	$w_1^* = w_2^* = w_3^* = \frac{a+k-(a-k)\theta}{2}$
$(A, A, B)$	$w_1^* = w_2^* = \frac{4a+4k+(7a+13k)\theta-(7a-7k)\theta^2-(9a+9k)\theta^3+(5a-4k)\theta^4+2k\theta^5}{8+20\theta-18\theta^3+\theta^4+2\theta^5}$ $w_3^* = \frac{8a+8k+(16a+24k)\theta-(9a-9k)\theta^2-(20a+16k)\theta^3+(3a-k)\theta^4+(2a+2k)\theta^5}{16+40\theta-36\theta^3+2\theta^4+4\theta^5}$
$(A, A, AB)$	$w_1^* = w_2^* = \frac{2a+2k+(a+5k)\theta-(4a-k)\theta^2+(a-3k)\theta^3}{4+6\theta-3\theta^2-2\theta^3}$ $w_3^* = \frac{2a+2k+(2a+4k)\theta-3a\theta^2-(a+k)\theta^3}{4+6\theta-3\theta^2-2\theta^3}$
$(A, AB, B)$	$w_1^* = w_3^* = \frac{8a+8k+(20a+32k)\theta-(2a-36k)\theta^2-(23a-5k)\theta^3-(3a+5k)\theta^4}{16+52\theta+34\theta^2-18\theta^3-8\theta^4}$ $w_2^* = \frac{8a+8k+(22a+30k)\theta+(3a+31k)\theta^2-(24a-6k)\theta^3-(9a-k)\theta^4}{16+52\theta+34\theta^2-18\theta^3-8\theta^4}$
$(A, AB, AB)$	$w_1^* = \frac{4a+4k+(6a+14k)\theta-(5a-13k)\theta^2-(5a-k)\theta^3}{8+20\theta+8\theta^2-4\theta^3}$ $w_2^* = w_3^* = \frac{4a+4k+(3a+9k)\theta-(7a-3k)\theta^2}{8+12\theta-4\theta^2}$
$(AB, AB, AB)$	$w_1^* = w_2^* = w_3^* = \frac{a+k-(a-k)\theta}{2}$

Table 2: Second-stage Nash equilibrium wholesale prices

Substituting the equilibrium wholesale prices into the other expressions yields the remaining equilibrium results. Notice that  $a > k$  ensures non-negativity for all equilibrium market entities.

## 2.5. Welfare results

We now turn to the welfare analysis of the non-exclusive distribution system  $(AB, AB, AB)$ . The producer surplus  $PS$  includes the profits of all firms in the market:

$$PS^* = \pi_1^* + \pi_2^* + \pi_3^* + \pi_A^* + \pi_B^* = \frac{3(a-k)^2(1-\theta)(1+\theta)}{4(1+2\theta)}.$$

Consumer surplus  $CS$  is provided by  $CS_1 = \frac{a-p_1}{2}Q_1$ ,  $CS_2 = \frac{a-p_2}{2}Q_2$ , and  $CS_3 = \frac{a-p_3}{2}Q_3$ , respectively, and amounts to

$$CS^* = CS_1^* + CS_2^* + CS_3^* = 3 \times \frac{(a-k)^2(1+\theta)^2}{8(1+2\theta)}.$$

The sum of consumer and producer surplus provides social welfare  $W$  as

$$W^* = PS^* + CS^* = \frac{3(a-k)^2 (3-\theta) (1+\theta)}{8(1+2\theta)}.$$

Similarly, we calculate the welfare equilibria for the other distribution systems. More details can be found in Appendix A.

### 3. Analysis

We first investigate the *equilibrium* distribution system. Subsequently, we explore how the equilibrium distribution system affects social welfare.

#### 3.1. Mutually best responses

When we examine the best responses of manufacturer  $M_i$  to the remaining two manufacturers' choices, we establish that it is for  $M_i$  always a best response to distribute his goods *non-exclusively* to both retailers. This is captured by the following result:

**Proposition 1.** *For all  $\theta \in (\underline{\theta}, \bar{\theta})$ , the Nash equilibrium distribution system is the non-exclusive market distribution  $(AB, AB, AB)$ .*

*Proof.* See Appendix B. □

In other words, the non-exclusive distribution  $(AB, AB, AB)$  exhibits a Nash equilibrium *independent* of the degree of product differentiation. To the best of our knowledge, this result yet is unaccounted for. For complements, this result has been obtained in past literature and is quite intuitive. [Cai et al. \(2012\)](#) identify that intuitively a manufacturer is reluctant to be in an exclusive upstream-downstream relationship for complementary products since this lowers demand and thereby his profit. For close substitutes, however, an exclusive distribution system might occur to mitigate the strong degree of intra- and interbrand competition in the market (see [Endres-Fröhlich et al. 2022](#); [Besanko and Perry 1993](#)). Opposed to that, the manufacturers choose to sell non-exclusively when the goods are close substitutes.

The underlying effects that induce selling non-exclusively in equilibrium are as follows: When the goods are complements, i.e.,  $\theta \in (\underline{\theta}, 0)$ , the customers perceive them as being more essential for each other. The necessity to buy the respective other goods increases, same as the customers' willingness to pay for the goods. In turn, the demand for each good is fostered by the increasing demand for the other complementary goods. This allows manufacturers to raise wholesale prices and finally subtract higher consumer surplus. Even though this higher wholesale price – that is transferred to the retail prices via the downstream price competition – dampens the demand for the goods, we still observe a strong output expansion effect. Overall,

all manufacturers benefit more from the additional profits gained by the output expansion of selling non-exclusively than they lose from the intensification of the competitive situation on the market associated with it.

When the goods are substitutes, i.e.,  $\theta \in (0, \bar{\theta})$ , the manufacturers' market power decreases with low product differentiation. This is reflected in a lower upstream price mark-up and consequently lower wholesale prices. Selling non-exclusively intensifies the already very competitive situation on the market and pushes wholesale and retail prices down even more. These lower prices negatively affect manufacturers' profits but also cause an output expansion, which in turn mitigates the negative impact on profits. Put together, the *higher* demand by the decreasing prices dampens the *lower* demand induced by the goods becoming less differentiated and the reduced necessity to buy substitute goods. Despite the strong aggravation of the competitive situation, the manufacturers still choose the non-exclusive channel structure in equilibrium for substitutes.

This behavior can be ascribed to the fact that *unilaterally* deviating from selling exclusively causes a strong profitable output expansion effect for the deviating manufacturer for  $\theta \in (0, \bar{\theta})$ . The remaining non-deviating manufacturers lose from any deviation of another manufacturer to a non-exclusive selling strategy. In turn, they would choose to unilaterally deviate in order to profit from the deviation.<sup>10</sup> Following that logic, the manufacturers end up in the non-exclusive selling distribution.

However, for  $\theta \in (\check{\theta}_1, \bar{\theta})$ , with  $\check{\theta}_1 \approx 0.9018$ , the competition effect might dominate the output expansion effect for all manufacturers. This takes place when *several* manufacturers jointly deviate from selling exclusively.<sup>11</sup> This implies that the degree of competition is so high that mitigating competition by *jointly* selling exclusively provides higher manufacturer profits than selling non-exclusively.<sup>12</sup> Still, the non-exclusive channel structure exhibits an equilibrium in this range. The reason is that the manufacturers find themselves captured to sell in the non-exclusive equilibrium structure, even though they would be better off selling in the exclusive distribution. Thus, they behave as if they were in a prisoner's dilemma.

Our equilibrium result is partially in line with [Bako \(2016\)](#) who discovers a non-exclusive channel structure in equilibrium for close substitutes. Opposed to us, she assumes private contract information between the agents, which causes reduced joint profits compared to a setting with open contract information. Selling exclusively would solve the contracting externality and allow joint profit maximizing outcome of the firms. However, she observes a prisoner's dilemma that prevents the manufacturers from selling exclusively for low degrees of product differentiation.

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<sup>10</sup>See Lemma 3 and part a) of Lemma 4. Further, for  $\theta \in (\theta, \bar{\theta})$ , the incentive to unilaterally deviate from selling exclusively exists for each distribution system with at least one exclusive selling strategy, which makes our game an acyclic one.

<sup>11</sup>See part b) of Lemma 4

<sup>12</sup>Notice that this is only valid for the exclusive distribution system  $(A, A, B)$ , not the exclusive system  $(A, A, A)$  as described later.

Contrary to us, [Besanko and Perry \(1993\)](#), [Endres-Fröhlich et al. \(2022\)](#), and [Bako \(2016\)](#) determine an *exclusive* structure for weak substitutes. In [Besanko and Perry \(1993\)](#) this is caused by a prisoner's dilemma. Different from us, they consider investment competition that introduces an additional dimension of competition affecting the results.

To summarize, we confirm the existence of several upstream firms ([Bako, 2016](#)) and price competition in the downstream market ([Choi, 1991](#)) as highly influential for a non-exclusive channel structure to arise. Nevertheless, our result cannot only be ascribed to that since we identify the non-exclusive structure to yield as equilibrium for *all* degrees of product differentiation.

We furthermore identify the competitive consequences of two other model assumptions as pivotal for the manufacturers' continual non-exclusive distribution choice, the asymmetric set-up with three manufacturers and two retailers and the assumption of the retailer being able to pursue several (non-)exclusive selling relationships with the upstream market. This causes a strong competitive pressure in the downstream market by inducing competition in at least one retail store with at least two competing products independent of the manufacturers' selling choice. Combined with the strong competitive situation by the downstream price competition prevails an intense degree of competition in the market. Any firm deviating from selling non-exclusively cannot sufficiently reduce this strong degree of competition. Thus, no other distribution system than the non-exclusive one might arise in equilibrium. Consequently, the identified Nash equilibrium is unique.

Aside from the prisoner's dilemma for very close substitutes, we detect an incentive for the manufacturers to jointly increase their individual profits by forming a cartel:

**Proposition 2.** *There exists a  $\tilde{\theta}$  such that for  $\theta > \tilde{\theta}$  pursuing the exclusive selling structure  $(A, A, B)$  causes a Pareto improvement for all firms.*

*Proof.* See Appendix [H.1](#). □

In particular, the distribution system, say,  $(A, A, B)$  causes higher profits than the  $(AB, AB, AB)$  distribution system for manufacturers  $M_1$  and  $M_2$  when  $\theta \in (\check{\theta}_2, \bar{\theta})$ , with  $\check{\theta}_2 \approx 0.6954$  and for  $M_3$  when  $\theta \in (\check{\theta}_3, \bar{\theta})$ , with  $\check{\theta}_3 \approx 0.5279$ . We identify  $\tilde{\theta} := \max(\check{\theta}_3, \check{\theta}_2) = \check{\theta}_2$ . Thus, the manufacturers have a joint incentive to insist on exclusive contracts for  $\theta \in (\tilde{\theta}, \bar{\theta})$ . Besides, the exclusive selling structure benefits both retailers; they earn positive profits in the exclusive channel structure as opposed to zero profits in the non-exclusive equilibrium one. To sum up, the manufacturers can enforce a Pareto improvement for all firms by cooperatively selling their goods in the exclusive distribution system  $(A, A, B)$ . This represents a strong incentive to collectively decide about the distribution system a priori and therefore should raise competition policy concerns.

Notice that this effect is not observable in the exclusive  $(A, A, A)$  distribution system since the negative effects of the interbrand competition on prices can be internalized by the monop-

olistically acting sole retailer.<sup>13</sup> The retail prices for all goods would be higher which reduces their demand compared to the  $(AB, AB, AB)$  distribution system. This negatively affects manufacturers' profits such that they have no incentive to choose the  $(A, A, A)$  distribution system.

### 3.2. Welfare effects

Subsequently, we study the welfare implications of the distribution systems. We compare producer surplus, consumer surplus, and social welfare between all considered distribution systems (see Figures 3 and 4 for a depiction of welfare in each distribution system).

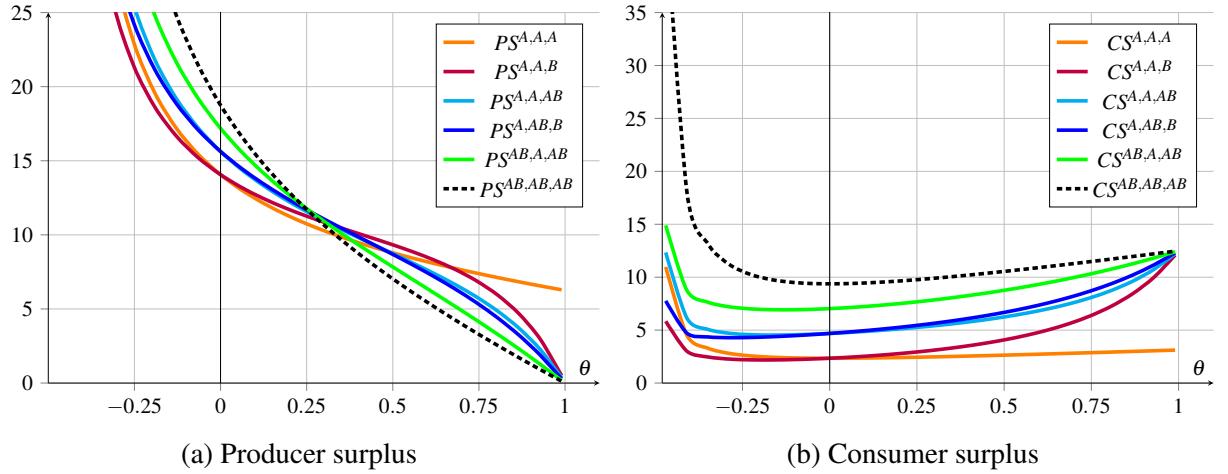


Figure 3: Consumer and producer surplus (with  $a = 10, k = 5$ )

Note: Since we exclude the boundaries  $\underline{\theta} = -0.5$  and  $\bar{\theta} = 1$  in our analysis, we do not depict them in the graphs.

Our analysis regarding producer surplus yields the following insight:

**Lemma 1.** *Producer surplus for  $\theta \in (\underline{\theta}, \check{\theta}_4)$  is highest under the non-exclusive channel structure, for  $\theta \in (\check{\theta}_4, \check{\theta}_5)$  it is highest under partially exclusive distribution systems, and for  $\theta \in (\check{\theta}_5, \bar{\theta})$  it is highest under the exclusive channel structure, with  $\underline{\theta} < \check{\theta}_4 < \check{\theta}_5 < \bar{\theta}$ .*

*Proof.* See Appendix E. □

Notice that  $\check{\theta}_4 \approx 0.2377$  and  $\check{\theta}_5 \approx 0.6694$ . Generally, exclusive distribution systems gain producer surplus with decreasing product differentiation. To be specific, for  $\theta \in (\underline{\theta}, \check{\theta}_4)$ , the non-exclusive market distribution  $(AB, AB, AB)$  reveals the highest producer surplus. This is caused by the large output expansion fortified by the goods being complements, which strongly benefits the manufacturers and thus producer surplus. With decreasing product differentiation, the positive demand effect of complements subsides, and the higher degree of competition causes the wholesale prices and, consequently, the manufacturers' profit and producer surplus to decrease.

<sup>13</sup>See Appendix C.1 for detailed calculations.

For  $\theta \in (\check{\theta}_4, \check{\theta}_5)$ , the partially exclusive distribution systems surpass the non-exclusive distribution system in producer surplus. The gain in producer surplus is attributed to the retailers that can set prices above marginal costs for exclusively sold goods. This lowers demand and reduces manufacturers' profits. Still, the retailers' increased profits override the manufacturers' lost profits and cause the partially exclusive distribution systems to provide the highest producer surplus for weak substitutes.

When the goods are close substitutes, this causes an intense degree of competition and induces retail and wholesale prices to approach marginal costs. Hence, for  $\theta \in (\check{\theta}_5, \bar{\theta})$ , the only distribution system in which the retailers' profits do not approach zero for (almost) perfect substitutes is distribution system  $(A, A, A)$ . In this distribution, all manufacturers sell to one retailer, who acts as monopolist and can internalize the negative effects of the intrabrand and interbrand competition. He can still set a price mark-up even though the goods are (almost) perfect substitutes, which drives wholesale prices to approach marginal cost. This is the only distribution system in which producer surplus stays positive for (almost) perfect substitutes.

When comparing consumer surplus between the different channel structures, we find the following:

**Lemma 2.** *For all  $\theta \in (\underline{\theta}, \bar{\theta})$ , consumer surplus is highest under the non-exclusive equilibrium channel structure  $(AB, AB, AB)$ .*

*Proof.* See Appendix F. □

We identify the low retail prices as decisive for the non-exclusive channel structure to exhibit the highest consumer surplus. Since both retailers sell homogeneous goods, the intraproduct competition drives retail prices down to marginal costs irrespective of the degree of product differentiation. Additionally, the interbrand competition induced by the manufacturers selling non-exclusively further exerts competitive pressure on the wholesale (and thus retail) prices. This proves advantageous for the consumers and causes the non-exclusive market distribution to provide the highest consumer surplus of all distribution systems for  $\theta \in (\underline{\theta}, \bar{\theta})$ . The effect of an increasing consumer surplus with a decreasing degree of product differentiation due to decreasing input prices is in line with the literature (e.g., [Heinzel and Hoof, 2020](#)).

Considering social welfare as the sum of producer surplus and consumer surplus, our next proposition is as follows:

**Proposition 3.** *For all  $\theta \in (\underline{\theta}, \bar{\theta})$  social welfare is highest under the non-exclusive equilibrium channel structure  $(AB, AB, AB)$ .*

*Proof.* See Appendix H.2. □

In sum, social welfare reveals in the non-exclusive market distribution the highest social welfare for  $\theta \in (\underline{\theta}, \bar{\theta})$ . Even for decreasing product differentiation, when the (more) exclusive distribution systems provide the highest producer welfare, consumer surplus in the non-exclusive

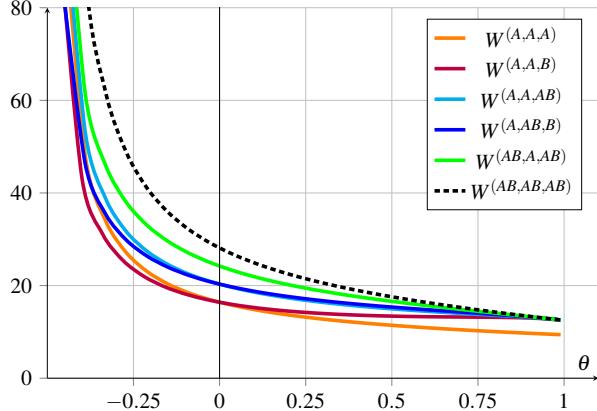


Figure 4: Behavior of social welfare (with  $a = 10, k = 5$ )

Note: Since we exclude the boundaries  $\theta = -0.5$  and  $\theta = 1$ , we do not depict that in the graph.

channel is high enough to cause social welfare to be highest in this distribution structure as well. This result implies that in case the manufacturers decide on any collusive agreement other than the non-exclusive distribution system, social welfare would be lessened for all degrees of product differentiation. This emphasizes the negative implications of the manufacturers' incentive to form a cartel also from a social welfare perspective.

This result is contrary to [Bako \(2016\)](#), who observes social welfare in the non-exclusive market set-up to exceed social welfare in the exclusive contracts case only for highly differentiated products. The difference to their results is caused by our very competitive situation in the downstream market, which benefits consumer surplus and thereby social welfare strongly.

## 4. Robustness: Changing numbers of manufacturers

We are now validating the robustness of our results concerning changes in the number of manufacturers, *ceteris paribus*. We first investigate the case of  $i = 4$  manufacturers while assuming  $j = 2$  retailers and establish again the non-exclusive distribution system as a Nash equilibrium independent of the degree of product differentiation.<sup>14</sup> The reason for this is the same as in the main case. Selling exclusively by any manufacturer would not sufficiently dampen the degree of competition in the market. The high competitive pressure induced by the high number of traded goods in the market and by the retailers being able to uphold several (non-)exclusive selling relations explains this. This finally induces the non-exclusive channel structure to arise in equilibrium.<sup>15</sup> Hence, our equilibrium channel structure result is robust to an increase in the number of manufacturers to  $i = 4$ .

When we consider a decrease in the number of manufacturers, i.e.,  $i = j = 2$ , we can refer to the analysis of [Endres-Fröhlich et al. \(2022\)](#). They identify that the non-exclusive struc-

<sup>14</sup>See Appendix G for detailed calculations.

<sup>15</sup>Notice that we did not conduct a thorough analysis of equilibrium channel structures. We rather analyzed the robustness of the non-exclusive channel structure as a Nash equilibrium for increasing the number of manufacturers to  $i = 4$ . Consequently, we cannot rule out that there might be other equilibria.

ture arises in equilibrium only for complements up to close substitutes. They ascertain an asymmetric distribution system (in which one manufacturer sells exclusively and the other one non-exclusively to the retail industry) for relatively close substitutes. For very low degrees of product differentiation, they obtain an exclusive channel structure in equilibrium. The difference is caused by the lower degree of intrabrand and interbrand competition in the market due to the lower number of traded goods. Then, one manufacturer selling exclusively indeed causes a significant reduction of competition that provokes – depending on the degree of product differentiation – different equilibrium outcomes.

This indicates that the non-exclusive channel structure as equilibrium is robust to increasing the number of manufacturers in the market by one as long as the number of retailers stays unchanged. It might even seem as if a number of  $i \geq 3$  manufacturers for  $j = 2$  retailers might be the critical threshold for which the non-exclusive equilibrium occurs in equilibrium since the competitive situation on the downstream market further intensifies with each additional manufacturer that produces one good. Additional research would be necessary to formally confirm this.

## 5. Conclusion

This paper investigates the impact of product differentiation on the equilibrium channel structure and its welfare implications. We consider an asymmetric manufacturer-driven supply chain that consists of three manufacturers, each selling a differentiated (or independent) good to two price competing retailers. The manufacturers can either sell exclusively or non-exclusively to the retailers.

Our main finding is that the non-exclusive channel structure exhibits a Nash equilibrium for all degrees of product differentiation. Previously, literature mostly identified an exclusive channel structure in equilibrium for low product differentiation. The reason is that selling exclusively dampens the strong degree of competition in the market associated with low degrees of product differentiation. Contrary to that, our model ascertains the non-exclusive channel structure in equilibrium even for low product differentiation, which might seem unintuitive. However, in reality, we indeed see that powerful electronic producers such as Samsung, Apple, and Xiaomi sell not just complementary goods non-exclusively to several electronic retailers (such as smartphones, power banks, and display protectors). All of them also sell their own smartphone versions that account for close substitutes via the same electronic retailers.

The intuition runs as follows. When the goods are complements or weak substitutes, there is a strong output expansion that comes along with selling non-exclusively. The additional profit caused by the output expansion outweighs the loss in profits caused by the increase in the degree of competition associated with decreasing degrees of product differentiation. When the goods are complements, this relation is valid for all manufacturers, which is straightforward. Each product's demand fosters the other products' demand and thereby raises overall demand

and manufacturers' profits.

When the goods are sufficiently substitutable, only the manufacturer that unilaterally builds another non-exclusive selling relation experiences a dominating output expansion effect. For very close substitutes, the reverse is true for all firms. The level of competition in the market is then so strong that mitigating it by selling exclusively indeed would be more lucrative for the manufacturers. However, for close substitutes the manufacturers behave as in a prisoner's dilemma and therefore do not sell exclusively. Thus, only the non-exclusive selling structure arises in equilibrium. This bears a high incentive for the manufacturers to form a collusive agreement and thus gain higher individual profits.

In light of our motivating example of the electronic producers Apple, Samsung, and Xiaomi, this would indicate that their decision to sell their complementary and substitute goods non-exclusively is in accordance with our theoretical predictions. Still, it might not be the most profitable distribution choice considering selling their close substitute smartphones versions.

By means of a robustness check, we furthermore conclude that our equilibrium distribution system is robust to increasing the number of manufacturers by one.

Moreover, the non-exclusive channel structure yields the highest social welfare and consumer surplus among all possible distribution systems. This is mostly attributed to the low retail prices in the equilibrium channel structure. Therefore, the welfare implications of our study are the following: From a consumer and social welfare perspective, the identified equilibrium channel structure does not have to be of any concern to a social planner, even though the market power lies fully with one industry side. Suspicion of the authorities with regard to welfare and cartel behavior should only be raised in case the manufacturers sell close substitutes exclusively.

The managerial implications of our results are that not only the manufacturers' individual considerations but also their rival manufacturers' selling decisions crucially matter for the arising distribution system. It is particularly important to consider the terms of the selling contracts with the retail industry, specifically whether an exclusive selling structure on the upstream market induces a one-good-per-store selling policy on the downstream market. Especially when the goods are close substitutes the rivals' behavior should be monitored closely.

This study provides several points to connect future research to. One aspect would be varying the sequentiality of the players' decisions to investigate the impact of the succession of the moves. When interpreting the first mover as the one with the largest market power, this would shed some light on the impact of the distribution of market power on the equilibrium distribution system. Furthermore, one could model brand differentiation between the goods, which would be particularly interesting if the goods are sold non-exclusively. This would allow connecting to reality even more closely. Another interesting point would be to study our setting when the retailers compete in quantities to determine the impact of the mode of competition on the final results.

## Acknowledgments

This work was partially supported by the German Research Foundation (DFG) within the Collaborative Research Centre “On-The-Fly Computing” (SFB 901) under the project number 160364472-SFB901/3. We further thank Burkhard Hehenkamp, Joachim Heinzel, Britta Hoyer, and the team members of Claus-Jochen Haake for their valuable comments.

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# Appendix

## A. Equilibrium calculations of distribution systems

### A.1. Case a) – (A,A,A)

Suppose that all manufacturers sell their goods exclusively to one retailer. This implies the distribution systems (A,A,A) or (B,B,B). Following, we describe distribution system (A,A,A) as depicted as Case a) in Figure 2.

The profit of retailer  $R_A$  is provided by

$$\pi_A = (p_1 - w_1)Q_1 + (p_2 - w_2)Q_2 + (p_3 - w_3)Q_3. \quad (\text{A.1})$$

Inserting (4)-(6) into (A.1), provides

$$\begin{aligned} \pi_A = & \frac{(p_1 - w_1)(a - p_1 - (a + p_1 - p_2 - p_3)\theta)}{1 + \theta - 2\theta^2} \\ & + \frac{(p_2 - w_2)(a - p_2 - (a - p_1 + p_2 - p_3)\theta)}{1 + \theta - 2\theta^2} \\ & + \frac{(p_3 - w_3)(a - p_3 - (a - p_1 - p_2 + p_3)\theta)}{1 + \theta - 2\theta^2}. \end{aligned} \quad (\text{A.2})$$

Notice that (A.2) is strictly concave in  $p_i$  because of  $\frac{\partial^2 \pi_A}{\partial p_i^2} = -\frac{2(1+\theta)}{1+\theta-2\theta^2} < 0$ .

Maximizing (A.2) yields the corresponding first order conditions (FOCs):

$$\begin{aligned} \frac{\partial \pi_A}{\partial p_1} &= \frac{a - 2p_1 + w_1 - a\theta - 2p_1\theta + 2p_2\theta + 2p_3\theta + w_1\theta - w_2\theta - w_3\theta}{(1 - \theta)(1 + 2\theta)} \stackrel{!}{=} 0, \\ \frac{\partial \pi_A}{\partial p_2} &= \frac{a - 2p_2 + w_2 - a\theta + 2p_1\theta - 2p_2\theta + 2p_3\theta - w_1\theta + w_2\theta - w_3\theta}{(1 - \theta)(1 + 2\theta)} \stackrel{!}{=} 0, \\ \frac{\partial \pi_A}{\partial p_3} &= \frac{a - 2p_3 + w_3 - a\theta + 2p_1\theta + 2p_2\theta - 2p_3\theta - w_1\theta - w_2\theta + w_3\theta}{(1 - \theta)(1 + 2\theta)} \stackrel{!}{=} 0, \end{aligned}$$

which respectively characterize the monopoly prices of good 1, good 2, and good 3 as depicted in Table 1.

Inserting the equilibrium prices into (10)-(12) yields the goods' demand in equilibrium:

$$Q_1 = \frac{a - w_1 - a\theta - w_1\theta + w_2\theta + w_3\theta}{2(1 - \theta)(1 + 2\theta)}, \quad (\text{A.3})$$

$$Q_2 = \frac{a - w_2 - a\theta + w_1\theta - w_2\theta + w_3\theta}{2(1 - \theta)(1 + 2\theta)}, \quad (\text{A.4})$$

$$Q_3 = \frac{a - w_3 - a\theta + w_1\theta + w_2\theta - w_3\theta}{2(1 - \theta)(1 + 2\theta)}. \quad (\text{A.5})$$

Inserting (A.3)-(A.5) into (13)-(15) yields:

$$\pi_1 = \frac{(w_1 - k) (a - w_1 - a\theta - w_1\theta + w_2\theta + w_3\theta)}{2(1-\theta)(1+2\theta)}, \quad (\text{A.6})$$

$$\pi_2 = \frac{(w_2 - k) (a - w_2 - a\theta + w_1\theta - w_2\theta + w_3\theta)}{2(1-\theta)(1+2\theta)}, \quad (\text{A.7})$$

$$\pi_3 = \frac{(w_3 - k) (a - w_3 - a\theta + w_1\theta + w_2\theta - w_3\theta)}{2(1-\theta)(1+2\theta)}. \quad (\text{A.8})$$

Notice that (A.6)-(A.8) are strictly concave in  $w_i$  because of  $\frac{\partial^2 \pi_i}{\partial w_i^2} = -\frac{1+\theta}{(1-\theta)(1+2\theta)} < 0$ . Maximizing (A.6)-(A.8) results in the following FOCs, respectively:

$$\frac{\partial \pi_1}{\partial w_1} = \frac{a+k-2w_1-a\theta+k\theta-2w_1\theta+w_2\theta+w_3\theta}{2(1-\theta)(1+2\theta)} \stackrel{!}{=} 0, \quad (\text{A.9})$$

$$\frac{\partial \pi_2}{\partial w_2} = \frac{a+k-2w_2-a\theta+k\theta+w_1\theta-2w_2\theta+w_3\theta}{2(1-\theta)(1+2\theta)} \stackrel{!}{=} 0, \quad (\text{A.10})$$

$$\frac{\partial \pi_3}{\partial w_3} = \frac{a+k-2w_3-a\theta+k\theta+w_1\theta+w_2\theta-2w_3\theta}{2(1-\theta)(1+2\theta)} \stackrel{!}{=} 0. \quad (\text{A.11})$$

Solving (A.9)-(A.11) leads to the equilibrium wholesale prices as depicted in Table 2, which we insert into the residual equations to obtain the resulting equilibrium prices, quantities, and profits, such as summarized in Table 3.

The producer surplus includes the profits of all firms. The consumer surplus is provided by  $CS_1^* = \frac{a-p_1^*}{2}Q_1^*$ ,  $CS_2^* = \frac{a-p_2^*}{2}Q_2^*$ , and  $CS_3^* = \frac{a-p_3^*}{2}Q_3^*$ , respectively, and social welfare amount to

$$\begin{aligned} PS^* &= \pi_1^* + \pi_2^* + \pi_3^* + \pi_A^* + \pi_B^* = \frac{3(a-k)^2(3-\theta)(1+\theta)}{16(1+2\theta)}, \\ CS^* &= CS_1^* + CS_2^* + CS_3^* = 3 \times \frac{(a-k)^2(1+\theta)^2}{32(1+2\theta)}, \\ W^* &= PS^* + CS^* = \frac{3(a-k)^2(7-\theta)(1+\theta)}{32(1+2\theta)}. \end{aligned}$$

Quantities	$Q_1^* = Q_2^* = Q_3^* = \frac{(a-k)(1+\theta)}{4(1+2\theta)}$
Retail prices	$p_1^* = p_2^* = p_3^* = \frac{3a+k+k\theta-a\theta}{4}$
Manufacturer profits	$\pi_1^* = \pi_2^* = \pi_3^* = \frac{(a-k)^2(1-\theta)(1+\theta)}{8(1+2\theta)}$
Retail profits	$\pi_A^* = \frac{3(a-k)^2(1+\theta)^2}{16(1+2\theta)}; \pi_B^* = 0$

Table 3: Equilibrium values distribution system  $(A, A, A)$

The assumption  $a > k$  ensures non-negativity for all equilibrium market entities.

## A.2. Case b) – (A,A,B)

Suppose that two manufacturers sell exclusively to one retailer and that one manufacturer sells exclusively to the other retailer. This implies the distribution systems (A,A,B), (B,A,A), (A,B,A), (B,B,A), (A,B,B), and (B,A,B). Following, we describe the calculations for distribution system (A,A,B) as depicted as Case b) in Figure 2.

The profits of retailers  $R_A$  and  $R_B$  are provided by

$$\pi_A = (p_1 - w_1)Q_1 + (p_2 - w_2)Q_2, \quad (\text{A.12})$$

$$\pi_B = (p_3 - w_3)Q_3. \quad (\text{A.13})$$

Inserting (4)-(6) into (A.12) and (A.13), respectively, provides

$$\begin{aligned} \pi_A &= \frac{(p_1 - w_1)(a - p_1 - (a + p_1 - p_2 - p_3)\theta)}{1 + \theta - 2\theta^2} \\ &+ \frac{(p_2 - w_2)(a - p_2 - (a - p_1 + p_2 - p_3)\theta)}{1 + \theta - 2\theta^2}, \end{aligned} \quad (\text{A.14})$$

$$\pi_B = \frac{(p_3 - w_3)(a - p_3 - (a - p_1 - p_2 + p_3)\theta)}{1 + \theta - 2\theta^2}. \quad (\text{A.15})$$

Notice that (A.14) and (A.15) are strictly concave in  $p_1$ ,  $p_2$ , and  $p_3$  because of  $\frac{\partial^2 \pi_A}{\partial p_1^2} = \frac{\partial^2 \pi_A}{\partial p_2^2} = -\frac{\partial^2 \pi_B}{\partial p_3^2} = \frac{2(1+\theta)}{1+\theta-2\theta^2} < 0$ .

Maximizing (A.14) and (A.15) yields the corresponding first order conditions (FOCs):

$$\begin{aligned} \frac{\partial \pi_A}{\partial p_1} &= \frac{a + w_1 - 2p_1 - a\theta - 2p_1\theta + 2p_2\theta + p_3\theta + w_1\theta - w_2\theta}{(1 - \theta)(1 + 2\theta)} \stackrel{!}{=} 0, \\ \frac{\partial \pi_A}{\partial p_2} &= \frac{a + w_2 - 2p_2 - a\theta + 2p_1\theta - 2p_2\theta + p_3\theta - w_1\theta + w_2\theta}{(1 - \theta)(1 + 2\theta)} \stackrel{!}{=} 0, \\ \frac{\partial \pi_B}{\partial p_3} &= \frac{a + w_3 - 2p_3 - a\theta + p_1\theta + p_2\theta - 2p_3\theta + w_3\theta}{(1 - \theta)(1 + 2\theta)} \stackrel{!}{=} 0, \end{aligned}$$

which respectively characterize the monopoly price of goods 1, good 2, and good 3 as depicted in Table 1.

Inserting the equilibrium prices into the demand for the respective good from (10)-(12) yields the goods' demand in equilibrium:

$$Q_1 = \frac{-4a + 4w_1 + (2a + 2w_3 - 4w_2 + 8w_1)\theta - (6a - 2w_3 - 5w_2 + w_1)\theta^2 + (2w_1 - 2w_2)\theta^3}{8 + 16\theta - 12\theta^2 - 20\theta^3 + 8\theta^4}, \quad (\text{A.16})$$

$$Q_2 = \frac{4a - 4w_2 + (2a + 4w_1 - 8w_2 + 2w_3)\theta - (6a - 5w_1 + w_2 - 2w_3)\theta^2 - (2w_1 - 2w_2)\theta^3}{8 + 16\theta - 12\theta^2 - 20\theta^3 + 8\theta^4}, \quad (\text{A.17})$$

$$Q_3 = \frac{2a - 2w_3 + (2a + w_1 + w_2 - 4w_3)\theta - (2a - w_1 - w_2)\theta^2 - (2a - 2w_3)\theta^3}{4 + 8\theta - 6\theta^2 - 10\theta^3 + 4\theta^4}. \quad (\text{A.18})$$

Inserting (A.16)-(A.18) into (13)-(15) yields:

$$\pi_1 = \frac{(k - w_1)\lambda_1}{4(1 - \theta)(1 + 2\theta)(2 + 2\theta - \theta^2)}, \quad (\text{A.19})$$

where  $\lambda_1$  is denoted by

$$\begin{aligned} \lambda_1 = & -4a + 4w_1 - 2a\theta + 8w_1\theta - 4w_2\theta - 2w_3\theta + 2w_2\theta^3 - 2w_1\theta^3 \\ & + 6a\theta^2 + w_1\theta^2 - 5w_2\theta^2 - 2w_3\theta^2, \end{aligned}$$

$$\pi_2 = \frac{(k - w_2)\lambda_2}{4(1 - \theta)(1 + 2\theta)(2 + 2\theta - \theta^2)}, \quad (\text{A.20})$$

where  $\lambda_2$  is denoted by

$$\begin{aligned} \lambda_2 = & -4a + 4w_2 - 2a\theta - 4w_1\theta + 8w_2\theta - 2w_3\theta + 6a\theta^2 - 5w_1\theta^2 \\ & + w_2\theta^2 - 2w_3\theta^2 + 2w_1\theta^3 - 2w_2\theta^3, \\ \pi_3 = & \frac{(k - w_3)}{2(1 - \theta)(1 + 2\theta)(2 + 2\theta - \theta^2)}. \end{aligned} \quad (\text{A.21})$$

Notice (A.19)-(A.21) are strictly concave in  $w_1$ ,  $w_2$  and  $w_3$  because of  $\frac{\partial^2 \pi_1}{\partial w_1^2} = \frac{\partial^2 \pi_2}{\partial w_2^2} = -\frac{4 + 8\theta + \theta^2 - 2\theta^3}{2(1 - \theta)(1 + 2\theta)(2 + 2\theta - \theta^2)} < 0$  and  $\frac{\partial^2 \pi_3}{\partial w_3^2} = -\frac{2(1 + \theta)(1 + \theta - \theta^2)}{(1 - \theta)(1 + 2\theta)(2 + 2\theta - \theta^2)} < 0$ . Maximizing (A.19)-(A.21) results in the following FOCs, respectively:

$$\begin{aligned} \frac{\partial \pi_1}{\partial w_1} = & -\frac{1}{8 + 16\theta - 12\theta^2 - 20\theta^3 + 8\theta^4} \times \\ & (-4a - 4k + 8w_1 + (-2a - 8k + 16w_1 - 4w_2 - 2w_3)\theta \\ & + (6a - k + 2w_1 - 5w_2 - 2w_3)\theta^2 + (2k - 4w_1 + 2w_2)\theta^3) \\ \stackrel{!}{=} & 0, \end{aligned} \quad (\text{A.22})$$

$$\begin{aligned}
\frac{\partial \pi_2}{\partial w_2} &= -\frac{1}{8+16\theta-12\theta^2-20\theta^3+8\theta^4} \times \\
&\quad (-4a-4k+8w_2+(-2a-8k-4w_1+16w_2-2w_3)\theta \\
&\quad +(6a-k-5w_1+2w_2-2w_3)\theta^2+(2k+2w_1-4w_2)\theta^3) \\
&\stackrel{!}{=} 0,
\end{aligned} \tag{A.23}$$

$$\begin{aligned}
\frac{\partial \pi_3}{\partial w_3} &= -\frac{1}{4+8\theta-6\theta^2-10\theta^3+4\theta^4} \times \\
&\quad (-2a-2k+4w_3+(-2a-4k-w_1-w_2+8w_3)\theta \\
&\quad +(2a-w_1-w_2)\theta^2+(2a+2k-4w_3)\theta^3) \\
&\stackrel{!}{=} 0.
\end{aligned} \tag{A.24}$$

Solving (A.22)-(A.24) leads to the equilibrium wholesale prices as depicted in Table 2, which we insert into the residual equations to obtain the resulting equilibrium prices, quantities, and profits, such as summarized in Table 4.

Quantities	$Q_1^* = Q_2^* = \frac{(k-a)(4+8\theta+\theta^2-2\theta^3)(4+11\theta+4\theta^2-5\theta^3)}{4(1+2\theta)(-2-2\theta+\theta^2)(8+20\theta-18\theta^3+\theta^4+2\theta^5)}$ $Q_3^* = \frac{(a-k)(1+\theta)(1+\theta-\theta^2)(8+24\theta+15\theta^2-5\theta^3-2\theta^4)}{2(1+2\theta)(2+2\theta-\theta^2)(8+20\theta-18\theta^3+\theta^4+2\theta^5)}$
Retail prices	$p_1^* = p_2^* = \frac{48a+16k+148a\theta+76k\theta+16a\theta^2+112k\theta^2-249a\theta^3+25k\theta^3-77a\theta^4-59k\theta^4+121a\theta^5-25k\theta^5+3a\theta^6+9k\theta^6-10a\theta^7+2k\theta^7}{4(2+2\theta-\theta^2)(8+20\theta-18\theta^3+\theta^4+2\theta^5)}$ $p_3^* = \frac{24a+8k+72a\theta+40k\theta+59k\theta^2+5a\theta^2-119a\theta^3+7k\theta^3-33a\theta^4-35k\theta^4+55a\theta^5+6k\theta^6-4a\theta^7-7k\theta^5}{2(2+2\theta-\theta^2)(8+20\theta-18\theta^3+\theta^4+2\theta^5)}$
Manufacturer	$\pi_1^* = \pi_2^* = \frac{(a-k)^2(1-\theta)(4+8\theta+\theta^2-2\theta^3)(4+11\theta+4\theta^2-5\theta^3)^2}{4(1+2\theta)(2+2\theta-\theta^2)(8+20\theta-18\theta^3+\theta^4+2\theta^5)^2}$
Profits	$\pi_3^* = \frac{(a-k)^2(1-\theta)(1+\theta)(1+\theta-\theta^2)(8+24\theta+15\theta^2-5\theta^3-2\theta^4)^2}{4(1+2\theta)(2+2\theta-\theta^2)(8+20\theta-18\theta^3+\theta^4+2\theta^5)^2}$
Retail profits	$\pi_A^* = \frac{(a-k)^2(1-\theta)(4+8\theta+\theta^2-2\theta^3)^2(4+11\theta+4\theta^2-5\theta^3)^2}{8(1+2\theta)(2+2\theta-\theta^2)^2(8+20\theta-18\theta^3+\theta^4+2\theta^5)^2}$ $\pi_B^* = \frac{(a-k)^2(1-\theta)(1+\theta)(1+\theta-\theta^2)^2(8+24\theta+15\theta^2-5\theta^3-2\theta^4)^2}{4(1+2\theta)(2+2\theta-\theta^2)^2(8+20\theta-18\theta^3+\theta^4+2\theta^5)^2}$

Table 4: Equilibrium values distribution system ( $A, A, B$ )

The assumption  $a > k$  ensures non-negativity for all equilibrium market entities.

The producer surplus includes the profits of all firms in the market:

$$\begin{aligned}
PS^* &= \pi_1^* + \pi_2^* + \pi_3^* + \pi_A^* + \pi_B^* \\
&= \frac{(a-k)^2(1-\theta)\lambda_3}{8(1+2\theta)(2+2\theta-\theta^2)^2(8+20\theta-18\theta^3+\theta^4+2\theta^5)^2},
\end{aligned}$$

where  $\lambda_3$  is defined as

$$\begin{aligned}
\lambda_3 &= 1152 + 10240\theta + 35792\theta^2 + 58112\theta^3 + 30422\theta^4 - 32034\theta^5 - 43873\theta^6 \\
&\quad - 1240\theta^7 + 16788\theta^8 + 2866\theta^9 - 2833\theta^{10} - 376\theta^{11} + 156\theta^{12} + 16\theta^{13}.
\end{aligned}$$

The consumer surplus is provided by  $CS_1^* = \frac{a-p_1^*}{2}Q_1^*$ ,  $CS_2^* = \frac{a-p_2^*}{2}Q_2^*$ , and  $CS_3^* = \frac{a-p_3^*}{2}Q_3^*$ , respectively as

$$\begin{aligned}
CS^* &= CS_1^* + CS_2^* + CS_3^* = \\
&= 2 \times \frac{(a-k)^2 (1+\theta) (4+8\theta+\theta^2-2\theta^3) (4+11\theta+4\theta^2-5\theta^3)}{32(1+2\theta)(2+2\theta-\theta^2)^2} \\
&\quad \times \frac{(16+60\theta+52\theta^2-27\theta^3-32\theta^4+7\theta^5+2\theta^6)}{(8+20\theta-18\theta^3+\theta^4+2\theta^5)^2} \\
&\quad + \frac{(a-k)^2 (1+\theta) (1+\theta-\theta^2) (8+24\theta+15\theta^2-5\theta^3-2\theta^4)}{8(2+2\theta-\theta^2)^2} \\
&\quad \times \frac{(8+24\theta+11\theta^2-15\theta^3-5\theta^4+3\theta^5)}{(8+20\theta-18\theta^3+\theta^4+2\theta^5)^2} \\
&= \frac{(a-k)^2 (1+\theta) \lambda_4}{16(1+2\theta)(2+2\theta-\theta^2)^2 (8+20\theta-18\theta^3+\theta^4+2\theta^5)^2},
\end{aligned}$$

where  $\lambda_4$  is denoted as

$$\begin{aligned}
\lambda_4 &= 384 + 3328\theta + 11120\theta^2 + 16384\theta^3 + 5098\theta^4 - 13426\theta^5 - 11975\theta^6 \\
&\quad + 3054\theta^7 + 5434\theta^8 - 272\theta^9 - 973\theta^{10} + 52\theta^{11} + 44\theta^{12}.
\end{aligned}$$

Social welfare amount to

$$W^* = PS^* + CS^* = \frac{(a-k)^2 \lambda_5}{16(1+2\theta)(2+2\theta-\theta^2)^2 (8+20\theta-18\theta^3+\theta^4+2\theta^5)^2},$$

where  $\lambda_5$  is denoted as

$$\begin{aligned}
\lambda_5 &= 2688 + 21888\theta + 65552\theta^2 + 72144\theta^3 - 33898\theta^4 - 133240\theta^5 - 49079\theta^6 + 76345\theta^7 \\
&\quad + 44544\theta^8 - 22682\theta^9 - 12643\theta^{10} + 3993\theta^{11} + 1160\theta^{12} - 236\theta^{13} - 32\theta^{14}.
\end{aligned}$$

### A.3. Case c) – (A,A,AB)

Suppose that two manufacturers sell their goods exclusively to one retailer and that one manufacturer sells to both retailers. This implies the distribution systems (A,A,AB), (A,AB,A), (AB,A,A), (B,B,AB), (B,AB,B), and (AB,B,B). Following, we describe distribution system (A,A,AB) as depicted as Case c) in Figure 2.

The profits of retailers  $R_A$  and  $R_B$  are provided by

$$\pi_A = (p_1 - w_1)Q_1 + (p_2 - w_2)Q_2 + (p_{A3} - w_3)q_{A3}, \quad (\text{A.25})$$

$$\pi_B = (p_{B3} - w_3)q_{B3}. \quad (\text{A.26})$$

Inserting (4)-(6) into (A.25), provides

$$\begin{aligned} \pi_A = & \frac{(p_1 - w_1)(a - p_1 - (a + p_1 - p_2 - p_3)\theta)}{1 + \theta - 2\theta^2} \\ & + \frac{(p_2 - w_2)(a - p_2 - (a - p_1 + p_2 - p_3)\theta)}{1 + \theta - 2\theta^2} \\ & + (p_{A3} - w_3)q_{A3}. \end{aligned} \quad (\text{A.27})$$

Notice that (A.14) is strictly concave in  $p_1$  because of  $\frac{\partial^2 \pi_A}{\partial p_1^2} = \frac{\partial^2 \pi_A}{\partial p_2^2} = -\frac{2(1+\theta)}{1+\theta-2\theta^2} < 0$ . Maximizing (A.14) yields the corresponding first order conditions (FOCs):

$$\frac{\partial \pi_A}{\partial p_1} = \frac{a - 2p_1 + w_1 - a\theta - 2p_1\theta + 2p_2\theta + p_3\theta + w_1\theta - w_2\theta}{(1 - \theta)(1 + 2\theta)} \stackrel{!}{=} 0,$$

$$\frac{\partial \pi_A}{\partial p_2} = \frac{a - 2p_2 + w_2 - a\theta + 2p_1\theta - 2p_2\theta + p_3\theta - w_1\theta + w_2\theta}{(1 - \theta)(1 + 2\theta)} \stackrel{!}{=} 0,$$

which respectively characterize the monopoly price of good 1 and good 2 as depicted in Table 1. Notice that the equilibrium retail price for good 3 is driven down to marginal costs due to the price competition in the market for good 3.

Inserting the equilibrium prices into the demand for the respective good from (10)-(12) yields their demand in equilibrium:

$$Q_1 = \frac{a - w_1 - a\theta - w_1\theta + w_2\theta + w_3\theta}{2(1 - \theta)(1 + 2\theta)}, \quad (\text{A.28})$$

$$Q_2 = \frac{a - w_2 - a\theta + w_1\theta - w_2\theta + w_3\theta}{2(1 - \theta)(1 + 2\theta)}, \quad (\text{A.29})$$

$$Q_3 = \frac{2a - 2w_3 + w_1\theta + w_2\theta - 2w_3\theta - 2a\theta^2 + 2w_3\theta^2}{2(1 - \theta)(1 + 2\theta)}. \quad (\text{A.30})$$

Inserting (A.28)-(A.30) into (13)-(15) yields

$$\pi_1 = \frac{(k - w_1) (-a + w_1 + a\theta + w_1\theta - w_2\theta - w_3\theta)}{2(1 - \theta)(1 + 2\theta)} \quad (\text{A.31})$$

$$\pi_2 = \frac{(k - w_2) (-a + w_2 + a\theta - w_1\theta + w_2\theta - w_3\theta)}{2(1 - \theta)(1 + 2\theta)} \quad (\text{A.32})$$

$$\pi_3 = \frac{(k - w_3) (-2a + 2w_3 - w_1\theta - w_2\theta + 2w_3\theta + 2a\theta^2 - 2w_3\theta^2)}{2(1 - \theta)(1 + 2\theta)} \quad (\text{A.33})$$

Notice that (A.31)-(A.33) are strictly concave in  $w_1$ ,  $w_2$ , and  $w_3$  because of  $\frac{\partial^2 \pi_1}{\partial w_1^2} = -\frac{\partial^2 \pi_2}{\partial w_2^2} = \frac{1+\theta}{(1-\theta)(1+2\theta)} < 0$  and  $\frac{\partial^2 \pi_3}{\partial w_3^2} = -\frac{2(1+(1-\theta)\theta)}{1+\theta-2\theta^2} < 0$ .

Maximizing (A.31)-(A.33) results in the following FOCs, respectively:

$$\frac{\partial \pi_1}{\partial w_1} = \frac{a + k - 2w_1 - a\theta + k\theta - 2w_1\theta + w_2\theta + w_3\theta}{2(1 - \theta)(1 + 2\theta)} \stackrel{!}{=} 0, \quad (\text{A.34})$$

$$\frac{\partial \pi_2}{\partial w_2} = \frac{a + k - 2w_2 - a\theta + k\theta + w_1\theta - 2w_2\theta + w_3\theta}{2(1 - \theta)(1 + 2\theta)} \stackrel{!}{=} 0, \quad (\text{A.35})$$

$$\frac{\partial \pi_3}{\partial w_3} = \frac{2a + 2k - 4w_3 + 2k\theta + w_1\theta + w_2\theta - 4w_3\theta - 2a\theta^2 - 2k\theta^2 + 4w_3\theta^2}{2(1 - \theta)(1 + 2\theta)} \stackrel{!}{=} 0. \quad (\text{A.36})$$

Solving (A.34)-(A.36) leads to the equilibrium wholesale prices as depicted in Table 2, which we insert into the residual equations to obtain the resulting equilibrium prices, quantities, and profits, such as summarized in Table 5.

The producer surplus includes the profits of all firms in the market:

$$\begin{aligned} PS^* &= \pi_1^* + \pi_2^* + \pi_3^* + \pi_A^* + \pi_B^* \\ &= \frac{(a - k)^2 (1 - \theta) (20 + 92\theta + 131\theta^2 + 38\theta^3 - 38\theta^4 - 16\theta^5 - \theta^6)}{2(1 + 2\theta) (4 + 6\theta - 3\theta^2 - 2\theta^3)^2}. \end{aligned}$$

The consumer surplus is provided by  $CS_1^* = \frac{a - p_1^*}{2} Q_1^*$ ,  $CS_2^* = \frac{a - p_2^*}{2} Q_2^*$ , and  $CS_3^* = \frac{a - p_3^*}{2} Q_3^*$ , respectively as

$$\begin{aligned} CS^* &= CS_1^* + CS_2^* + CS_3^* \\ &= 2 \times \frac{(a - k)^2 (1 + \theta) (2 + 3\theta - \theta^2) (2 + 7\theta + 5\theta^2 - 3\theta^3 - \theta^4)}{8(1 + 2\theta) (4 + 6\theta - 3\theta^2 - 2\theta^3)^2} \\ &\quad + \frac{(a - k)^2 (1 + \theta - \theta^2) (2 + 4\theta + \theta^2) (2 + 4\theta - \theta^3)}{2(1 + 2\theta) (4 + 6\theta - 3\theta^2 - 2\theta^3)^2} \\ &= \frac{(a - k)^2 (12 + 64\theta + 109\theta^2 + 39\theta^3 - 54\theta^4 - 30\theta^5 + 7\theta^6 + 3\theta^7)}{4(1 + 2\theta) (4 + 6\theta - 3\theta^2 - 2\theta^3)^2}, \end{aligned}$$

Social welfare amount to

$$W^* = PS^* + CS^* = \frac{(a-k)^2 (52 + 208\theta + 187\theta^2 - 147\theta^3 - 206\theta^4 + 14\theta^5 + 37\theta^6 + 5\theta^7)}{4(1+2\theta)(4+6\theta-3\theta^2-2\theta^3)^2}.$$

Quantities	$Q_1^* = Q_2^* = \frac{(a-k)(1+\theta)(2+3\theta-\theta^2)}{2(1+2\theta)(4+6\theta-3\theta^2-2\theta^3)}$ $Q_3^* = \frac{(a-k)(1+\theta-\theta^2)(2+4\theta+\theta^2)}{(1+2\theta)(4+6\theta-3\theta^2-2\theta^3)}$
Retail prices	$p_1^* = p_2^* = \frac{6a+2k+5a\theta+7k\theta-11a\theta^2+5k\theta^2-a\theta^3-3k\theta^3+a\theta^4-k\theta^4}{2(4+6\theta-3\theta^2-2\theta^3)}$ $p_3^* = \frac{2a+2k+2a\theta+4k\theta-3a\theta^2-a\theta^3-k\theta^3}{(4+6\theta-3\theta^2-2\theta^3)}$
Manufacturer profits	$\pi_1^* = \pi_2^* = \frac{(a-k)^2(1-\theta)(1+\theta)(2+3\theta-\theta^2)^2}{2(1+2\theta)(4+6\theta-3\theta^2-2\theta^3)^2}$ $\pi_3^* = \frac{(a-k)^2(1-\theta)(1+\theta-\theta^2)(2+4\theta+\theta^2)^2}{(1+2\theta)(4+6\theta-3\theta^2-2\theta^3)^2}$
Retail profits	$\pi_A^* = \frac{(a-k)^2(1-\theta)(1+\theta)^2(2+3\theta-\theta^2)^2}{2(2\theta+1)(4+6\theta-3\theta^2-2\theta^3)^2}$ $\pi_B^* = 0$

Table 5: Equilibrium values distribution system  $(A, A, AB)$

The assumption  $a > k$  ensures non-negativity for all equilibrium market entities.

#### A.4. Case d) – (A,AB,B)

Suppose that two manufacturers sell their goods exclusively to one retailer and one manufacturer sells to both retailers. This implies the distribution systems (A,B,AB), (A,AB,B), (AB,A,B), (B,A,AB), (B,AB,A), and (AB,A,B). Following, we describe the distribution system (A,AB,B) as depicted as Case d) in Figure 2.

The profits of retailers  $R_A$  and  $R_B$  are provided by

$$\pi_A = (p_1 - w_1)Q_1 + (p_{A2} - w_2)q_{A2}, \quad (\text{A.37})$$

$$\pi_B = (p_{B2} - w_2)q_{B2} + (p_3 - w_3)Q_3. \quad (\text{A.38})$$

Inserting (4)-(6) into (A.37) and (A.38), respectively, provides

$$\pi_A = \frac{(p_1 - w_1)(a - p_1 - (a + p_1 - p_2 - p_3)\theta)}{1 + \theta - 2\theta^2} + (p_{A2} - w_2)q_{A2}, \quad (\text{A.39})$$

$$\pi_B = (p_{B2} - w_2)q_{B2} + \frac{(p_3 - w_3)(a - p_3 - (a - p_1 - p_2 + p_3)\theta)}{1 + \theta - 2\theta^2}. \quad (\text{A.40})$$

Notice that (A.37) and (A.38) are strictly concave in  $p_1$  and  $p_3$  because of  $\frac{\partial^2 \pi_A}{\partial p_1^2} = \frac{\partial^2 \pi_B}{\partial p_3^2} = -\frac{2(1+\theta)}{1+\theta-2\theta^2} < 0$ . Maximizing (A.39) and (A.40) yields the corresponding first order conditions (FOCs):

$$\frac{\partial \pi_A}{\partial p_1} = \frac{a - 2p_1 + w_1 - a\theta - 2p_1\theta + p_2\theta + p_3\theta + w_1\theta}{(1 - \theta)(1 + 2\theta)} \stackrel{!}{=} 0,$$

$$\frac{\partial \pi_A}{\partial p_3} = \frac{a + w_3 - 2p_3 - a\theta + p_1\theta + p_2\theta - 2p_3\theta + w_3\theta}{(1 - \theta)(1 + 2\theta)} \stackrel{!}{=} 0,$$

which respectively characterize the monopoly price of good 1 and good 3 as depicted in Table 1. Notice that the equilibrium retail price for good 2 is driven down to marginal costs due to the price competition in the market for good 2.

Inserting the equilibrium prices into (10)-(12) yields the goods' demand in equilibrium:

$$Q_1 = \frac{(1 + \theta)(2a - 2w_1 + a\theta - 4w_1\theta + 2w_2\theta + w_3\theta - 3a\theta^2 - w_1\theta^2 + 3w_2\theta^2 + w_3\theta^2)}{(1 - \theta)(2 + \theta)(1 + 2\theta)(2 + 3\theta)}, \quad (\text{A.41})$$

$$Q_2 = \frac{2a - 2w_2 + a\theta + w_1\theta - 3w_2\theta + w_3\theta - 3a\theta^2 + w_1\theta^2 + w_2\theta^2 + w_3\theta^2}{(1 - \theta)(2 + \theta)(1 + 2\theta)}, \quad (\text{A.42})$$

$$Q_3 = \frac{(1 + \theta)(2a - 2w_3 + a\theta + w_1\theta + 2w_2\theta - 4w_3\theta - 3a\theta^2 + w_1\theta^2 + 3w_2\theta^2 - w_3\theta^2)}{(1 - \theta)(2 + \theta)(1 + 2\theta)(2 + 3\theta)}. \quad (\text{A.43})$$

Inserting (A.41)-(A.43) into (13)-(15) yields

$$\pi_1 = \frac{(w_1 - k)(1 + \theta)(2a - 2w_1 + a\theta - 4w_1\theta + 2w_2\theta + w_3\theta - 3a\theta^2 - w_1\theta^2 + 3w_2\theta^2 + w_3\theta^2)}{(1 - \theta)(2 + \theta)(1 + 2\theta)(2 + 3\theta)}, \quad (\text{A.44})$$

$$\pi_2 = \frac{(w_2 - k)(2a - 2w_2 + a\theta + w_1\theta - 3w_2\theta + w_3\theta - 3a\theta^2 + w_1\theta^2 + w_2\theta^2 + w_3\theta^2)}{(1 - \theta)(2 + \theta)(1 + 2\theta)}, \quad (\text{A.45})$$

$$\pi_3 = \frac{(w_3 - k)(1 + \theta)(2a - 2w_3 + a\theta + w_1\theta + 2w_2\theta - 4w_3\theta - 3a\theta^2 + w_1\theta^2 + 3w_2\theta^2 - w_3\theta^2)}{(1 - \theta)(2 + \theta)(1 + 2\theta)(2 + 3\theta)}. \quad (\text{A.46})$$

Notice that (A.44)-(A.46) are strictly concave in  $w_1$ ,  $w_2$ , and  $w_3$  because of  $\frac{\partial^2 \pi_1}{\partial w_1^2} = \frac{\partial^2 \pi_3}{\partial w_3^2} = -\frac{2(1+\theta)(\theta^2+4\theta+2)}{(1-\theta)(2+\theta)(1+2\theta)(2+3\theta)} < 0$  and  $\frac{\partial^2 \pi_2}{\partial w_2^2} = -\frac{2(2+3\theta-\theta^2)}{(1-\theta)(2+\theta)(1+2\theta)} < 0$ .

Maximizing (A.44)-(A.46) results in the following FOCs, respectively:

$$\begin{aligned} \frac{\partial \pi_1}{\partial w_1} &= \frac{1}{4 + 12\theta + 3\theta^2 - 13\theta^3 - 6\theta^4} \times (2a + 2k - 4w_1 + (3a + 6k - 12w_1 + 2w_2 + w_3)\theta \\ &\quad + (-2a + 5k - 10w_1 + 5w_2 + 2w_3)\theta^2 + (-3a + k - 2w_1 + 3w_2 + w_3)\theta^3) \\ &\stackrel{!}{=} 0, \end{aligned} \quad (\text{A.47})$$

$$\frac{\partial \pi_2}{\partial w_2} = \frac{2a + 2k - 4w_2 - (-a - 3k - w_1 + 6w_2 - w_3)\theta + (3a + k - w_1 - 2w_2 - w_3)\theta^2}{2 + 3\theta - 3\theta^2 - 2\theta^3} \stackrel{!}{=} 0, \quad (\text{A.48})$$

$$\begin{aligned} \frac{\partial \pi_3}{\partial w_3} &= \frac{1}{4 + 12\theta + 3\theta^2 - 13\theta^3 - 6\theta^4} \times (2a + 2k - 4w_3 + (3a + 6k + w_1 + 2w_2 - 12w_3)\theta \\ &\quad + (-2a + 5k + 2w_1 + 5w_2 - 10w_3)\theta^2 + (-3a + k + w_1 + 3w_2 - 2w_3)\theta^3) \\ &\stackrel{!}{=} 0. \end{aligned} \quad (\text{A.49})$$

Solving (A.47)-(A.49) leads to the equilibrium wholesale prices as depicted in Table 2, which we insert into the residual equations to obtain the resulting equilibrium prices, quantities, and profits, such as summarized in Table 6.

Quantities	$Q_1^* = Q_3^* = \frac{(a-k)(1+\theta)(2+4\theta+\theta^2)(4+8\theta+\theta^2)}{2(2+\theta)(1+2\theta)(8+26\theta+17\theta^2-9\theta^3-4\theta^4)}$ $Q_2^* = \frac{(a-k)(2+3\theta)(2+3\theta-\theta^2)(4+9\theta+3\theta^2)}{2(2+\theta)(1+2\theta)(8+26\theta+17\theta^2+9\theta^3+4\theta^4)}$
Retail prices	$p_1^* = p_3^* = \frac{12a+4k+36a\theta+24k\theta+11a\theta^2+49k\theta^2-37a\theta^3+36k\theta^3-20a\theta^4+3k\theta^4-2a\theta^5-2k\theta^5}{(2+\theta)(8+26\theta+17\theta^2-9\theta^3-4\theta^4)}$ $p_2^* = \frac{8a+8k+22a\theta+30k\theta+3a\theta^2+31k\theta^2-24a\theta^3+6k\theta^3-9a\theta^4+k\theta^4}{2(8+26\theta+17\theta^2-9\theta^3-4\theta^4)}$
Manufacturer profits	$\pi_1^* = \pi_3^* = \frac{(a-k)^2(1-\theta)(1+\theta)(2+3\theta)(2+4\theta+\theta^2)(4+8\theta+\theta^2)^2}{4(2+\theta)(1+2\theta)(8+26\theta+17\theta^2-9\theta^3-4\theta^4)^2}$ $\pi_2^* = \frac{(a-k)^2(1-\theta)(2+3\theta)^2(2+3\theta-\theta^2)(4+9\theta+3\theta^2)^2}{4(2+\theta)(1+2\theta)(8+26\theta+17\theta^2-9\theta^3)-4\theta^4)^2}$
Retail profits	$\pi_A^* = \frac{(a-k)^2(1-\theta)(1+\theta)(2+4\theta+\theta^2)^2(4+8\theta+\theta^2)^2}{4(2+\theta)^2(1+2\theta)(8+26\theta+17\theta^2-9\theta^3-4\theta^4)^2}$ $\pi_B^* = \frac{(a-k)^2(1-\theta)(1+\theta)(2+4\theta+\theta^2)^2(4+8\theta+\theta^2)^2}{4(2+\theta)^2(1+2\theta)(8+26\theta+17\theta^2-9\theta^3-4\theta^4)^2}$

Table 6: Equilibrium values distribution system ( $A, AB, B$ )

The assumption  $a > k$  ensures non-negativity for all equilibrium market entities.

The producer surplus includes the profits of all firms in the market:

$$PS^* = \pi_1^* + \pi_2^* + \pi_3^* + \pi_A^* + \pi_B^* = \frac{(a-k)^2(1-\theta)\lambda_6}{4(2+\theta)^2(1+2\theta)(8+26\theta+17\theta^2-9\theta^3-4\theta^4)^2},$$

where  $\lambda_6$  is denoted as

$$\lambda_6 = 640 + 5888\theta + 22544\theta^2 + 46320\theta^3 + 54784\theta^4 + 36928\theta^5 + 12801\theta^6 + 1393\theta^7 - 321\theta^8 - 73\theta^9,$$

The consumer surplus is provided by  $CS_1^* = \frac{a-p_1^*}{2}Q_1^*$ ,  $CS_2^* = \frac{a-p_2^*}{2}Q_2^*$ , and  $CS_3^* = \frac{a-p_3^*}{2}Q_3^*$ , respectively, and social welfare amount to:

$$\begin{aligned} CS^* &= CS_1^* + CS_2^* + CS_3^* \\ &= 2 \times \frac{(a-k)^2(1+\theta)(2+4\theta+\theta^2)(4+8\theta+\theta^2)(4+16\theta+17\theta^2+2\theta^3-\theta^4)}{4(2+\theta)^2(8+26\theta+17\theta^2-9\theta^3-4\theta^4)^2} \\ &\quad + \frac{(a-k)^2(2+3\theta)(2+3\theta-\theta^2)(4+9\theta+3\theta^2)(8+30\theta+31\theta^2+6\theta^3+\theta^4)}{8(2+\theta)(1+2\theta)(8+26\theta+17\theta^2-9\theta^3-4\theta^4)^2} \\ &= \frac{(a-k)^2\lambda_7}{8(2+\theta)^2(1+2\theta)(8+26\theta+17\theta^2-9\theta^3-4\theta^4)^2}, \end{aligned}$$

where  $\lambda_7$  is denoted as

$$\lambda_7 = 384 + 3840\theta + 16080\theta^2 + 36384\theta^3 + 47760\theta^4 + 36064\theta^5 + 14209\theta^6 + 1866\theta^7 + 448\theta^8 - 170\theta^9 - 17\theta^{10},$$

$$W^* = PS^* + CS^* = \frac{(a-k)^2 \lambda_8}{8(2+\theta)^2 (1+2\theta) (8+26\theta+17\theta^2-9\theta^3-4\theta^4)^2},$$

where  $\lambda_8$  is denoted by

$$\begin{aligned} \lambda_8 = & 1664 + 14336\theta + 49392\theta^2 + 83936\theta^3 + 64688\theta^4 + 352\theta^5 - 34045\theta^6 \\ & - 20950\theta^7 - 3876\theta^8 + 326\theta^9 + 129\theta^{10}. \end{aligned}$$

### A.5. Case e) – (A,AB,AB)

Suppose that two manufacturers sell their goods non-exclusively and one manufacturer sells exclusively to a retailer. This implies the distribution systems (A,AB,AB), (AB,A,AB), and (AB,AB,A), (B,AB,AB), (AB,B,AB), (AB,AB,B). Following, we describe the distribution system (A,AB,AB) as depicted as Case e) in Figure 2.

The profits of retailers  $R_A$  and  $R_B$  are provided by

$$\pi_A = (p_1 - w_1)Q_1 + (p_{A2} - w_2)q_{A2} + (p_{A3} - w_3)q_{A3}, \quad (\text{A.50})$$

$$\pi_B = (p_{B2} - w_2)q_{B2} + (p_{B3} - w_3)q_{B3}. \quad (\text{A.51})$$

Inserting (4)-(6) into (A.50) provides

$$\begin{aligned} \pi_A = & \frac{(p_1 - w_1)(a - p_1 - (a + p_1 - p_2 - p_3)\theta)}{1 + \theta - 2\theta^2} \\ & + (p_{A2} - w_2)q_{A2} + (p_{A3} - w_3)q_{A3}. \end{aligned} \quad (\text{A.52})$$

Notice that (A.52) is strictly concave in  $p_1$  because of  $\frac{\partial^2 \pi_A}{\partial p_1^2} = -\frac{2(1+\theta)}{1+\theta-2\theta^2} < 0$ . Maximizing (A.52) yields the corresponding first order condition (FOC):

$$\frac{\partial \pi_A}{\partial p_1} = \frac{a + w_1 - 2p_1 - a\theta - 2p_1\theta + p_2\theta + p_3\theta + w_1\theta}{(1 - \theta)(1 + 2\theta)} \stackrel{!}{=} 0, \quad (\text{A.53})$$

which characterizes the monopoly price of good 1 as depicted in Table 1. Notice that the equilibrium retail price for good 2 and good 3 is driven down to marginal costs, respectively, due to the price competition in the market for good 2 and good 3.

Inserting the equilibrium prices into (10)-(12) yields the goods' demand in equilibrium:

$$Q_1 = \frac{a - w_1 - a\theta - w_1\theta + w_2\theta + w_3\theta}{2(1 - \theta)(1 + 2\theta)}, \quad (\text{A.54})$$

$$Q_2 = \frac{2a - 2w_2 + a\theta + w_1\theta - 4w_2\theta + 2w_3\theta - 3a\theta^2 + w_1\theta^2 - w_2\theta^2 + 3w_3\theta^2}{2(1 - \theta)(1 + \theta)(1 + 2\theta)}, \quad (\text{A.55})$$

$$Q_3 = \frac{2a - 2w_3 + a\theta + w_1\theta + 2w_2\theta - 4w_3\theta - 3a\theta^2 + w_1\theta^2 + 3w_2\theta^2 - w_3\theta^2}{2(1 - \theta)(\theta + 1)(1 + 2\theta)}. \quad (\text{A.56})$$

Inserting (A.54)-(A.56) into (13)-(15) yields

$$\pi_1 = \frac{(k - w_1)(-a + w_1 + a\theta + w_1\theta - w_2\theta - w_3\theta)}{2(1 - \theta)(1 + 2\theta)}, \quad (\text{A.57})$$

$$\pi_2 = \frac{(k-w_2) (-2a+2w_2-a\theta-w_1\theta+4w_2\theta-2w_3\theta+3a\theta^2-w_1\theta^2+w_2\theta^2-3w_3\theta^2)}{2(1-\theta)(1+\theta)(1+2\theta)}, \quad (\text{A.58})$$

$$\pi_3 = \frac{(k-w_3) (-2a+2w_3-a\theta-w_1\theta-2w_2\theta+4w_3\theta+3a\theta^2-w_1\theta^2-3w_2\theta^2+w_3\theta^2)}{2(1-\theta)(1+\theta)(1+2\theta)}. \quad (\text{A.59})$$

Notice that (A.57)-(A.59) are strictly concave in  $w_1$ ,  $w_2$ , and  $w_3$  because of  $\frac{\partial^2 \pi_1}{\partial w_1^2} = -\frac{1+\theta}{(1-\theta)(1+2\theta)} < 0$  and  $\frac{\partial^2 \pi_2}{\partial w_2^2} = \frac{\partial^2 \pi_3}{\partial w_3^2} = -\frac{2+4\theta+\theta^2}{(1-\theta)(1+\theta)(1+2\theta)} < 0$ .

Maximizing (A.57)-(A.59) results in the following FOCs, respectively:

$$\frac{\partial \pi_1}{\partial w_1} = -\frac{a+k-2w_1-a\theta+k\theta-2w_1\theta+w_2\theta+w_3\theta}{2(1-\theta)(1+2\theta)} \stackrel{!}{=} 0, \quad (\text{A.60})$$

$$\frac{\partial \pi_2}{\partial w_2} = \frac{2a+2k-4w_2+(a+4k+w_1-8w_2+2w_3)\theta+(-3a+k+w_1-2w_2+3w_3)\theta^2}{2+4\theta-2\theta^2-4\theta^3} \stackrel{!}{=} 0, \quad (\text{A.61})$$

$$\frac{\partial \pi_3}{\partial w_3} = \frac{2a+2k-4w_3+(a+4k+w_1+2w_2-8w_3)\theta+(-3a+k+w_1+3w_2-2w_3)\theta^2}{2+4\theta-2\theta^2-4\theta^3} \stackrel{!}{=} 0. \quad (\text{A.62})$$

Solving (A.60)-(A.62) leads to the equilibrium wholesale prices as depicted in Table 2, which we insert into the residual equations to obtain the resulting equilibrium prices, quantities, and profits, such as summarized in Table 7.

Quantities	$Q_1^* = \frac{(a-k)(4+10\theta+5\theta^2)}{8(1+2\theta)(2+3\theta-\theta^2)}$ $Q_2^* = Q_3^* = \frac{(a-k)(4+7\theta)(2+4\theta+\theta^2)}{8(\theta+1)(1+2\theta)(2+3\theta-\theta^2)}$
Retail prices	$p_1^* = \frac{12a+4k+18a\theta-15a\theta^2+22k\theta+31k\theta^2-15a\theta^3+7k\theta^3}{8(1+\theta)(2+3\theta-\theta^2)}$ $p_2^* = p_3^* = \frac{4a+4k+3a\theta+9k\theta-7a\theta^2+3k\theta^2}{4(2+3\theta-\theta^2)}$
Manufacturer profits	$\pi_1^* = \frac{(a-k)^2(1-\theta)(4+10\theta+5\theta^2)^2}{32(1+\theta)(1+2\theta)(2+3\theta-\theta^2)^2}$ $\pi_2^* = \pi_3^* = \frac{(a-k)^2(1-\theta)(4+7\theta)^2(2+4+\theta^2\theta)}{32(1+\theta)(1+2\theta)(2+3\theta-\theta^2)^2}$
Retail profits	$\pi_A^* = \frac{(a-k)^2(1-\theta)(4+10\theta+5\theta^2)^2}{64(1+\theta)(1+2\theta)(2+3\theta-\theta^2)^2}$ $\pi_B^* = 0$

Table 7: Equilibrium values distribution system ( $A, AB, AB$ )

The assumption  $a > k$  ensures non-negativity for all equilibrium market entities.

The producer surplus includes the profits of all firms in the market:

$$\begin{aligned} PS^* &= \pi_1^* + \pi_2^* + \pi_3^* + \pi_A^* + \pi_B^* \\ &= \frac{(a-k)^2 (1-\theta) (176 + 944\theta + 1772\theta^2 + 1308\theta^3 + 271\theta^4)}{64 (1+\theta) (1+2\theta) (2+3\theta-\theta^2)^2}. \end{aligned}$$

The consumer surplus is provided by  $CS_1^* = \frac{a-p_1^*}{2} Q_1^*$ ,  $CS_2^* = \frac{a-p_2^*}{2} Q_2^*$ , and  $CS_3^* = \frac{a-p_3^*}{2} Q_3^*$ , respectively, and social welfare amount to

$$\begin{aligned} CS^* &= CS_1^* + CS_2^* + CS_3^* \\ &= \frac{(a-k)^2 (4 + 10\theta + 5\theta^2) (4 + 22\theta + 31\theta^2 + 7\theta^3)}{128 (1+\theta) (1+2\theta) (2+3\theta-\theta^2)^2} \\ &\quad + 2 \times \frac{(a-k)^2 (4 + 7\theta) (2 + 4\theta + \theta^2) (4 + 9\theta + 3\theta^2)}{64 (1+\theta) (1+2\theta) (2+3\theta-\theta^2)^2} \\ &= \frac{(a-k)^2 (144 + 896\theta + 2052\theta^2 + 2072\theta^3 + 861\theta^4 + 119\theta^5)}{128 (1+\theta) (1+2\theta) (2+3\theta-\theta^2)^2}, \end{aligned}$$

$$W^* = PS^* + CS^* = \frac{(a-k)^2 (496 + 2432\theta + 3708\theta^2 + 1144\theta^3 - 1213\theta^4 - 423\theta^5)}{128 (1+\theta) (1+2\theta) (2+3\theta-\theta^2)^2}.$$

## A.6. Case f) – (AB,AB,AB)

Suppose a market in which each manufacturer sells to both retailers in the distribution systems (AB,AB,AB) as described in Subsection 2.3 and Subsection 2.4. This distribution system is depicted as Case f) in Figure 2. An overview about the equilibrium values in the non-exclusive market distribution (AB,AB,AB) is provided in Table 8.

Turning to the upstream market, inserting (10)-(12) into (13)-(15) yields

$$\pi_1 = \frac{(w_1 - k)(a - k)(1 + \theta)}{2(1 + 2\theta)} \quad (\text{A.63})$$

$$\pi_2 = \frac{(w_2 - k)(a - k)(1 + \theta)}{2(1 + 2\theta)} \quad (\text{A.64})$$

$$\pi_3 = \frac{(w_3 - k)(a - k)(1 + \theta)}{2(1 + 2\theta)} \quad (\text{A.65})$$

Notice that (A.63)-(A.65) are strictly concave in  $w_i$  because of  $\frac{\partial^2 \pi_i}{\partial w_i^2} = \frac{2(\theta+1)}{(\theta-1)(2\theta+1)} < 0$ . Maximizing (A.63)-(A.65) results in the following FOCs, respectively:

$$\frac{\partial \pi_1}{\partial w_1} = \frac{a + k - 2w_1 - a\theta + k\theta - 2w_1\theta + w_2\theta + w_3\theta}{(1 - \theta)(1 + 2\theta)} \stackrel{!}{=} 0, \quad (\text{A.66})$$

$$\frac{\partial \pi_2}{\partial w_2} = \frac{a + k - 2w_2 - a\theta + k\theta + w_1\theta - 2w_2\theta + w_3\theta}{(1 - \theta)(1 + 2\theta)} \stackrel{!}{=} 0, \quad (\text{A.67})$$

$$\frac{\partial \pi_3}{\partial w_3} = \frac{a + k - 2w_3 - a\theta + k\theta + w_1\theta + w_2\theta - 2w_3\theta}{(1 - \theta)(1 + 2\theta)} \stackrel{!}{=} 0. \quad (\text{A.68})$$

Solving (A.66)-(A.68) leads to the equilibrium wholesale prices as depicted in Table 2, which we insert into the residual equations to obtain the resulting equilibrium prices, quantities, and profits, such as summarized in Table 8.

Quantities	$Q_1^* = Q_2^* = Q_3^* = \frac{(a-k)(1+\theta)}{2(1+2\theta)}$
Retail prices	$p_1^* = p_2^* = p_3^* = \frac{a+k-(a-k)\theta}{2}$
Manufacturer profits	$\pi_1^* = \pi_2^* = \pi_3^* = \frac{(a-k)^2(1-\theta)(1+\theta)}{4(1+2\theta)}$
Retail profits	$\pi_A^* = \pi_B^* = 0$

Table 8: Equilibrium values distribution system (AB,AB,AB)

The assumption  $a > k$  ensures non-negativity for all equilibrium market entities.

## B. Mutually best responses

In Lemma 3, we first examine the best responses of manufacturer  $i$  to manufacturer  $j$ 's strategies depending on  $\theta$ . Subsequently, we derive the Nash equilibrium distribution system of Proposition 1.

Since the game is symmetric across players, it suffices to determine the set of best responses of manufacturer  $M_i$  to any strategy  $s_j$  of manufacturer  $M_j$ .

**Lemma 3.** *Let  $i$ ,  $j$ , and  $l$  ( $i \neq j \neq l$ ) denote manufacturers  $i$ ,  $j$ , and  $l$ , respectively. Independent of manufacturer  $l$ 's strategy, there exist  $\theta \in (\underline{\theta}, \bar{\theta})$ , with  $\underline{\theta} < \bar{\theta}$ , such that the best responses of  $i$  and  $j$  are described as*

		$M_j$		
		$A$	$B$	$AB$
$M_i$	$A$	NBR ; NBR	NBR ; NBR	NBR ; $\theta \in (\underline{\theta}, \bar{\theta})$
	$B$	NBR ; NBR	NBR ; NBR	NBR ; $\theta \in (\underline{\theta}, \bar{\theta})$
	$AB$	$\theta \in (\underline{\theta}, \bar{\theta})$ ; NBR	$\theta \in (\underline{\theta}, \bar{\theta})$ ; NBR	$\theta \in (\underline{\theta}, \bar{\theta})$ ; $\theta \in (\underline{\theta}, \bar{\theta})$

Note: The given  $\theta$  interval states when the respective strategy is a best response; NBR = Never best response

### Proof of Lemma 3:

Without loss of generality, let  $i = 1$ ,  $j = 2$ , and  $l = 3$ . First, consider manufacturer  $M_3$  plays  $s_3 = A$ . Notice that the comparisons for  $s_3 = B$  are analogous.

Provided that  $M_2$  plays  $s_2 = A$ , strategy  $s_1 = A$  is strictly dominated by  $s_1 = AB$  since  $a > k$  and  $\theta \in (-0.5, 1)$  as indicated by

$$\pi_1^{(A,A,A)} - \pi_1^{(AB,A,A)} = -\frac{(a-k)^2(1-\theta)(1+2\theta)(16+32\theta+4\theta^2-8\theta^3-5\theta^4-\theta^5)}{8(4+6\theta-3\theta^2-2\theta^3)^2} < 0.$$

Thus, it is sufficient to compare  $\pi_1^{(B,A,A)}$  and  $\pi_1^{(AB,A,A)}$ . We have for  $\theta \in (-0.5, 1)$  due to  $a > k$  and  $\theta \in (-0.5, 1)$

$$\begin{aligned} \Delta\pi_1^a &\equiv \pi_1^{(B,A,A)} - \pi_1^{(AB,A,A)} \\ &= -\frac{(a-k)^2(1-\theta)(1+\theta-\theta^2)\lambda_{10}}{4(1+2\theta)(2+2\theta-\theta^2)(4+6\theta-3\theta^2-2\theta^3)^2(8+20\theta-18\theta^3+\theta^4+2\theta^5)^2} < 0, \end{aligned}$$

where  $\lambda_{10}$  is denoted by

$$\begin{aligned} \lambda_{10} &\equiv 1024 + 10240\theta + 39936\theta^2 + 69120\theta^3 + 21360\theta^4 - 101664\theta^5 - 124732\theta^6 \\ &\quad + 15584\theta^7 + 99415\theta^8 + 28105\theta^9 - 30779\theta^{10} - 13809\theta^{11} + 3416\theta^{12} \\ &\quad + 2288\theta^{13} + 12\theta^{14} - 128\theta^{15} - 16\theta^{16}. \end{aligned}$$

Second, consider  $M_2$  plays  $s_2 = B$ . Manufacturer  $M_1$ 's profit in distribution system  $(A, B, A)$  is the same as in distribution system  $(B, B, A)$ . Thus, it suffices to compare the following:

$$\begin{aligned}\Delta\pi_1^b &\equiv \pi_1^{(A,B,A)} - \pi_1^{(AB,B,A)} \\ &= -\frac{(a-k)^2(1-\theta)\lambda_{11}}{4(2+\theta)(2+2\theta-\theta^2)(8+26\theta+17\theta^2-9\theta^3-4\theta^4)^2(8+20\theta-18\theta^3+\theta^4+2\theta^5)^2},\end{aligned}$$

where  $\lambda_{11}$  is denoted by

$$\begin{aligned}\lambda_{11} &\equiv 8192 + 110592\theta + 632832\theta^2 + 1940992\theta^3 + 3181024\theta^4 + 1730064\theta^5 - 3000800\theta^6 \\ &\quad - 5808736\theta^7 - 2206678\theta^8 + 2963651\theta^9 + 2887296\theta^{10} - 190611\theta^{11} - 1060310\theta^{12} \\ &\quad - 183373\theta^{13} + 178890\theta^{14} + 48644\theta^{15} - 14469\theta^{16} - 4685\theta^{17} + 459\theta^{18} + 162\theta^{19}.\end{aligned}$$

We observe  $\Delta\pi_1^b < 0$  for  $\theta \in (-0.5, 1)$  because of  $a > k$  and  $\theta \in (-0.5, 1)$ .

Finally, suppose  $M_2$  plays  $s_2 = AB$ . Then, strategy  $s_1 = A$  is again strictly dominated by  $s_1 = AB$  since  $a > k$  and  $\theta \in (-0.5, 1)$  as depicted by:

$$\begin{aligned}\Delta\pi_1^c &\equiv \pi_1^{(A,AB,A)} - \pi_1^{(AB,AB,A)} \\ &= -\frac{(a-k)^2(1-\theta)\lambda_{12}}{32(1+\theta)(2+3\theta-\theta^2)^2(4+6\theta-3\theta^2-2\theta^3)^2} < 0,\end{aligned}$$

where  $\lambda_{12}$  is denoted by

$$\begin{aligned}\lambda_{12} &\equiv 256 + 1792\theta + 4384\theta^2 + 3360\theta^3 - 2888\theta^4 - 5704\theta^5 - 1746\theta^6 \\ &\quad + 1220\theta^7 + 833\theta^8 + 90\theta^9.\end{aligned}$$

As before, it thus suffices to compare  $\pi_1^{(B,AB,A)}$  and  $\pi_1^{(AB,AB,A)}$ . We have for  $\theta \in (-0.5, 1)$

$$\begin{aligned}\Delta\pi_1^d &\equiv \pi_1^{(B,AB,A)} - \pi_1^{(AB,AB,A)} \\ &= -\frac{(a-k)^2(1-\theta)(2+4\theta+\theta^2)\lambda_{13}}{32(1+\theta)(2+\theta)(1+2\theta)(2+3\theta-\theta^2)^2(8+26\theta+17\theta^2-9\theta^3-4\theta^4)^2} < 0,\end{aligned}$$

where  $\lambda_{13}$  is denoted by

$$\begin{aligned}\lambda_{13} &\equiv 1024 + 10752\theta + 46080\theta^2 + 99968\theta^3 + 102344\theta^4 + 5756\theta^5 - 92634\theta^6 \\ &\quad - 81743\theta^7 - 16664\theta^8 + 10201\theta^9 + 5688\theta^{10} + 760\theta^{11},\end{aligned}$$

since the terms in parentheses and  $\lambda_{13}$  are strictly positive because of  $a > k$  and  $\theta \in (-0.5, 1)$ .

The best responses of manufacturer  $M_2$  to manufacturer  $M_1$ 's strategies are analogous due to the symmetry between firms. The analogous comparisons apply for the case that  $M_3$  plays  $s_3 = AB$ .

Thus, distributing *non-exclusively* is a mutual best response. Depending on  $M_3$ 's distribution choice, the arising distribution system might be then distribution system  $(AB,AB,A)$ ,  $(AB,AB,B)$ , or  $(AB,AB,AB)$ .

**Proof of Proposition 1:**

Manufacturer  $M_3$  has the same profit in distribution system  $(AB,AB,A)$  as in distribution system  $(AB,AB,B)$ . Thus, it is sufficient to compare  $\pi_3^{(AB,AB,A)}$  and  $\pi_3^{(AB,AB,AB)}$ . We find

$$\begin{aligned}\Delta\pi_3^e &\equiv \pi_3^{(AB,AB,A)} - \pi_3^{(AB,AB,AB)} \\ &= -\frac{(a-k)^2(1-\theta)(16+80\theta+124\theta^2+28\theta^3-73\theta^4-32\theta^5+8\theta^6)}{32(1+\theta)(1+2\theta)(2+3\theta-\theta^2)^2} < 0,\end{aligned}$$

for  $\theta \in (-0.5, 1)$  since the terms in parentheses are strictly positive because of  $a > k$  and  $\theta \in (-0.5, 1)$ .

It follows directly from Lemma 3 and the above comparison that  $s_3^* = AB$  is a best response to  $s_1 = A, B, AB$  and  $s_2 = A, B, AB$  for  $\theta \in (-0.5, 1)$ . Thus, the distribution system  $(AB,AB,AB)$  constitutes a Nash equilibrium.

## C. Other comparisons

### C.1. Deviation from $(AB,AB,AB)$ to $(A,A,A)$

We have  $\pi_i^{(AB,AB,AB)} - \pi_i^{(A,A,A)} = \frac{(a-k)^2(1-\theta)(1+\theta)}{8(1+2\theta)} > 0$  for  $\theta \in (-0.5, 1)$  since the terms in parentheses are strictly positive because of  $a > k$  and  $\theta \in (-0.5, 1)$  with  $i = \{1, 2, 3\}$ .

### C.2. Equilibrium quantities and equilibrium prices

Consider  $\Delta w_1 \equiv w_1^{(A,A,B)} - w_1^{(AB,A,B)} = \frac{(k-a)(1-\theta)\theta(1+2\theta)(16+60\theta+38\theta^2-74\theta^3-70\theta^4+13\theta^5+9\theta^6)}{2(8+26\theta+17\theta^2-9\theta^3-4\theta^4)(8+20\theta-18\theta^3+\theta^4+2\theta^5)}$ . This has a unique root  $\hat{\theta}_1$  in  $(-0.5, 1)$ , which is given by  $\hat{\theta}_1 = 0$ . We have  $\Delta w_1 > 0$  for  $\theta \in (-0.5, \hat{\theta}_1)$  and  $\Delta w_1 < 0$  for  $\theta \in (\hat{\theta}_1, 1)$ .

We have

$$p_1^{(AB,AB,AB)} - p_1^{(A,A,A)} = -\frac{(a-k)(1+\theta)}{4} < 0,$$

$$Q_i^{(AB,AB,AB)} - Q_i^{(A,A,A)} = \frac{(a-k)(1+\theta)}{4(1+2\theta)} > 0,$$

$$\begin{aligned} \Delta Q_1 &\equiv Q_1^{(A,A,B)} - Q_1^{(AB,A,B)} \\ &= -\frac{(a-k)\lambda_9}{4(2+\theta)(2+2\theta-\theta^2)(8+26\theta+17\theta^2-9\theta^3-4\theta^4)(8+20\theta-18\theta^3+\theta^4+2\theta^5)} < 0, \end{aligned}$$

where  $\lambda_9$  is defined by

$$\begin{aligned} \lambda_9 &\equiv 256 + 1792\theta + 4336\theta^2 + 3008\theta^3 - 3592\theta^4 - 5476\theta^5 + 135\theta^6 \\ &\quad + 2263\theta^7 + 79\theta^8 - 347\theta^9 - 4\theta^{10} + 18\theta^{11}, \end{aligned}$$

for  $\theta \in (-0.5, 1)$  since the terms in parentheses, same as  $\lambda_9$ , are strictly positive because of  $a > k$  and  $\theta \in (-0.5, 1)$  for  $i = \{1, 2, 3\}$ .

## D. Consequences of a manufacturer's strategy choice on profits

Lemma 4 describes the consequences of a manufacturer's strategy choice on his and the remaining manufacturers' profits for certain ranges of  $\theta$ .

**Lemma 4.** *Let  $i$ ,  $j$ , and  $l$  ( $i \neq j \neq l$ ) denote manufacturers  $i$ ,  $j$ , and  $l$ , respectively. Then,*

- a) *any manufacturer  $j$  and  $l$  is worse off for strategy  $s_i^* = \{AB\}$  than for any strategy  $s_i = \{A, B\}$  for  $\theta \in (0, \bar{\theta})$ ,*
- b) *each manufacturer  $i$  that is multilaterally choosing any strategy  $s_i = \{A, B\}$  is better off than choosing strategy  $s_i^* = \{AB\}$  for  $\theta \in (\check{\theta}_1, \bar{\theta})$ .*

Moreover, we have  $0 < \check{\theta}_1 < \bar{\theta}$ , where  $\check{\theta}_1 \approx 0.9018$  and  $\bar{\theta} = 1$ .

**Proof of Lemma 4 Part a):**

For strategy  $s_i^* = \{AB\}$ , manufacturers  $j$  and  $l$  are worse off than if manufacturer  $i$  played any strategy  $s_i = \{A, B\}$  for  $\theta \in (0, \bar{\theta})$ , which is indicated by the following profit comparisons. Without loss of generality, let be  $i = 1$ ,  $j = 2$ , and  $l = 3$ . We have

$$\begin{aligned} \Delta\pi_2^f &\equiv \pi_2^{(A,A,B)} - \pi_2^{(AB,A,B)} \\ &= \frac{(a-k)^2(1-\theta)\theta^2\lambda_{14}}{4(2+\theta)(2+2\theta-\theta^2)(8+26\theta+17\theta^2-9\theta^3-4\theta^4)^2(8+20\theta-18\theta^3+\theta^4+2\theta^5)^2}, \end{aligned}$$

while  $\lambda_{14}$  is defined as

$$\begin{aligned} \lambda_{14} &\equiv 1024 + 14336\theta + 86048\theta^2 + 285936\theta^3 + 559264\theta^4 + 602816\theta^5 + 198902\theta^6 \\ &\quad - 303667\theta^7 - 373440\theta^8 - 92205\theta^9 + 85290\theta^{10} + 57691\theta^{11} + 1684\theta^{12} \\ &\quad - 8259\theta^{13} - 1864\theta^{14} + 311\theta^{15} + 121\theta^{16} + 6\theta^{17}. \end{aligned}$$

This has a root at  $\hat{\theta}_{2a} \approx -0.3701$  and a root at  $\hat{\theta}_{2b} = 0$  in  $(-0.5, 1)$ . We have  $\Delta\pi_2^f < 0$  for  $\theta \in (-0.5, \hat{\theta}_{2a})$  and  $\Delta\pi_2^f > 0$  for  $\theta \in (\hat{\theta}_{2a}, \hat{\theta}_{2b}]$  and  $\theta \in [\hat{\theta}_{2b}, 1)$ .

$$\begin{aligned} \Delta\pi_3^f &\equiv \pi_3^{(A,A,B)} - \pi_3^{(AB,A,B)} \\ &= \frac{(a-k)^2(1-\theta)\theta(1+\theta)\lambda_{15}}{4(2+\theta)(2+2\theta-\theta^2)(8+26\theta+17\theta^2-9\theta^3-4\theta^4)^2(8+20\theta-18\theta^3+\theta^4+2\theta^5)^2}, \end{aligned}$$

while  $\lambda_{15}$  is defined as

$$\begin{aligned} \lambda_{15} &\equiv 4096 + 49664\theta + 253184\theta^2 + 686208\theta^3 + 990592\theta^4 + 510504\theta^5 - 537708\theta^6 \\ &\quad - 880758\theta^7 - 188113\theta^8 + 344357\theta^9 + 171563\theta^{10} - 58186\theta^{11} - 42687\theta^{12} \\ &\quad + 3321\theta^{13} + 4741\theta^{14} + 218\theta^{15} - 205\theta^{16} - 26\theta^{17}. \end{aligned}$$

This has a unique root  $\hat{\theta}_3$  in  $(-0.5, 1)$ , which is given by  $\hat{\theta}_3 = 0$ . We have  $\Delta\pi_3^f < 0$  for  $\theta \in (-0.5, \hat{\theta}_3)$  and  $\Delta\pi_3^f > 0$  for  $\theta \in (\hat{\theta}_3, 1)$ .

$$\begin{aligned}\Delta\pi_2^g &\equiv \pi_2^{(B,A,A)} - \pi_2^{(AB,A,A)} \\ &= \frac{(a-k)^2(1-\theta)\theta\lambda_{16}}{4(1+2\theta)(2+2\theta-\theta^2)(4+6\theta-3\theta^2-2\theta^3)^2(8+20\theta-18\theta^3+\theta^4+2\theta^5)^2},\end{aligned}$$

while  $\lambda_{16}$  is defined as

$$\begin{aligned}\lambda_{16} &\equiv 512 + 5184\theta + 20800\theta^2 + 38656\theta^3 + 20064\theta^4 - 42336\theta^5 - 65148\theta^6 \\ &\quad - 687\theta^7 + 49298\theta^8 + 16106\theta^9 - 18272\theta^{10} - 7197\theta^{11} + 3824\theta^{12} \\ &\quad + 1030\theta^{13} - 342\theta^{14} - 48\theta^{15} + 8\theta^{16}.\end{aligned}$$

This has a unique root  $\hat{\theta}_4$  in  $(-0.5, 1)$ , which is given by  $\hat{\theta}_4 = 0$ . We have  $\Delta\pi_2^g < 0$  for  $\theta \in (-0.5, \hat{\theta}_4)$  and  $\Delta\pi_2^g > 0$  for  $\theta \in (\hat{\theta}_4, 1)$ . Manufacturer  $M_3$ 's profit comparison is analogous.

$$\begin{aligned}\Delta\pi_2^h &\equiv \pi_2^{(A,AB,B)} - \pi_2^{(AB,AB,B)} \\ &= \frac{(a-k)^2(1-\theta)\theta\lambda_{17}}{32(1+\theta)(2+\theta)(1+2\theta)(2+3\theta-\theta^2)^2(8+26\theta+17\theta^2-9\theta^3-4\theta^4)^2},\end{aligned}$$

while  $\lambda_{17}$  is defined as

$$\begin{aligned}\lambda_{17} &\equiv 2048 + 25344\theta + 135296\theta^2 + 405360\theta^3 + 741224\theta^4 + 839468\theta^5 + 566178\theta^6 \\ &\quad + 199814\theta^7 + 17581\theta^8 - 19020\theta^9 - 18769\theta^{10} - 8696\theta^{11} - 1432\theta^{12}.\end{aligned}$$

This has a unique root in  $\hat{\theta}_5$  in  $(-0.5, 1)$ , which is given by  $\hat{\theta}_5 = 0$ . We have  $\Delta\pi_2^h < 0$  for  $\theta \in (-0.5, \hat{\theta}_5)$  and  $\Delta\pi_2^h > 0$  for  $\theta \in (\hat{\theta}_5, 1)$ .

$$\begin{aligned}\Delta\pi_3^h &\equiv \pi_3^{(A,AB,B)} - \pi_3^{(AB,AB,B)} \\ &= \frac{(a-k)^2(1-\theta)\theta\lambda_{18}}{32(1+\theta)(2+\theta)(1+2\theta)(2+3\theta-\theta^2)^2(8+26\theta+17\theta^2-9\theta^3-4\theta^4)^2},\end{aligned}$$

while  $\lambda_{18}$  is defined as

$$\begin{aligned}\lambda_{18} &\equiv 1024 + 12160\theta + 62656\theta^2 + 183552\theta^3 + 336864\theta^4 + 402736\theta^5 + 315008\theta^6 \\ &\quad + 152494\theta^7 + 30507\theta^8 - 14836\theta^9 - 13361\theta^{10} - 3800\theta^{11} - 376\theta^{12}.\end{aligned}$$

This has a unique root  $\hat{\theta}_6$  in  $(-0.5, 1)$ , which is given by  $\hat{\theta}_6 = 0$ . We have  $\Delta\pi_3^h < 0$  for  $\theta \in$

$(-0.5, \hat{\theta}_6)$  and  $\Delta\pi_3^h > 0$  for  $\theta \in (\hat{\theta}_6, 1)$ .

$$\begin{aligned}\Delta\pi_2^i &\equiv \pi_2^{(A,A,AB)} - \pi_2^{(AB,A,AB)} \\ &= \frac{(a-k)^2(1-\theta)\theta^3(4+7\theta)(32+128\theta+136\theta^2-12\theta^3-55\theta^4-6\theta^5)}{32(1+\theta)(2+3\theta-\theta^2)^2(4+6\theta-3\theta^2-2\theta^3)^2},\end{aligned}$$

which has a unique root  $\hat{\theta}_7$  in  $(-0.5, 1)$ , which is given by  $\hat{\theta}_7 = 0$ . We have  $\Delta\pi_2^i < 0$  for  $\theta \in (-0.5, \hat{\theta}_7)$  and  $\Delta\pi_2^i > 0$  for  $\theta \in (\hat{\theta}_7, 1)$ .

$$\pi_3^{(A,A,AB)} - \pi_3^{(AB,A,AB)} = \frac{(a-k)^2(1-\theta)\theta(2+4\theta+\theta^2)\lambda_{19}}{32(1+\theta)(2+3\theta-\theta^2)^2(4+6\theta-3\theta^2-2\theta^3)^2},$$

while  $\lambda_{19}$  is defined as

$$\lambda_{19} \equiv 128 + 624\theta + 912\theta^2 + 196\theta^3 - 332\theta^4 - 73\theta^5 - 58\theta^6 - 16\theta^7,$$

which has a unique root  $\hat{\theta}_8$  in  $(-0.5, 1)$ , which is given by  $\hat{\theta}_8 = 0$ . We have  $\pi_3^{(A,A,AB)} < \pi_3^{(AB,A,AB)}$  for  $\theta \in (-0.5, \hat{\theta}_8)$  and  $\pi_3^{(A,A,AB)} > \pi_3^{(AB,A,AB)}$  for  $\theta \in (\hat{\theta}_8, 1)$ .

$$\pi_2^{(A,AB,AB)} - \pi_2^{(AB,AB,AB)} = \frac{(a-k)^2(1-\theta)\theta(16+74\theta+124\theta^2+97\theta^3+32\theta^4-8\theta^5)}{32(1+\theta)(1+2\theta)(2+3\theta-\theta^2)^2},$$

which has a unique root  $\hat{\theta}_9$  in  $(-0.5, 1)$ , which is given by  $\hat{\theta}_9 = 0$ . We find  $\pi_2^{(A,AB,AB)} < \pi_2^{(AB,AB,AB)}$  for  $\theta \in (-0.5, \hat{\theta}_9)$  and  $\pi_2^{(A,AB,AB)} > \pi_2^{(AB,AB,AB)}$  for  $\theta \in (\hat{\theta}_9, 1)$ . Manufacturer  $M_3$ 's profit comparison is analogous.

We further observe that for joint strategies  $s_i^* = \{AB\}$  and  $s_j^* = \{AB\}$ , manufacturer  $l$  is worse off than if manufacturers  $i$  and  $j$  played any strategy  $s_i^* = \{A, B\}$  and  $s_j^* = \{A, B\}$  for  $\theta \in (0, \bar{\theta})$ , which is indicated by the following profit comparisons. Let again be  $i = 1$ ,  $j = 2$ , and  $l = 3$  without loss of generality. We have

$$\pi_3^{(A,A,AB)} - \pi_3^{(AB,AB,AB)} = \frac{(a-k)^2(1-\theta)\theta(16+68\theta+88\theta^2+23\theta^3-25\theta^4-20\theta^5-4\theta^6)}{4(1+2\theta)(4+6\theta-3\theta^2-2\theta^3)^2},$$

which has a unique root  $\hat{\theta}_{10}$  in  $(-0.5, 1)$ , which is given by  $\hat{\theta}_{10} = 0$ . We have  $\pi_3^{(A,A,AB)} < \pi_3^{(AB,AB,AB)}$  for  $\theta \in (-0.5, \hat{\theta}_{10})$  and  $\pi_3^{(A,A,AB)} > \pi_3^{(AB,AB,AB)}$  for  $\theta \in (\hat{\theta}_{10}, 1)$ .

$$\pi_3^{(A,B,AB)} - \pi_3^{(AB,AB,AB)} = \frac{(a-k)^2(1-\theta)\theta\lambda_{20}}{2(2+\theta)(1+2\theta)(8+26\theta+17\theta^2-9\theta^3-4\theta^4)^2},$$

while  $\lambda_{20}$  is defined as

$$\lambda_{20} \equiv 64 + 512\theta + 1656\theta^2 + 2760\theta^3 + 2484\theta^4 + 1105\theta^5 + 92\theta^6 - 137\theta^7 - 60\theta^8 - 8\theta^9,$$

which has a unique root  $\hat{\theta}_{11}$  in  $(-0.5, 1)$ , which is given by  $\hat{\theta}_{11} = 0$ . We have  $\pi_3^{(A,B,AB)} < \pi_3^{(AB,AB,AB)}$  for  $\theta \in (-0.5, \hat{\theta}_{11})$  and  $\pi_3^{(A,B,AB)} > \pi_3^{(AB,AB,AB)}$  for  $\theta \in (\hat{\theta}_{11}, 1)$ .

$$\begin{aligned} \Delta\pi_3^j &\equiv \pi_3^{(A,B,A)} - \pi_3^{(AB,AB,A)} \\ &= \frac{(a-k)^2(1-\theta)\theta\lambda_{21}}{32(1+\theta)(1+2\theta)(2+3\theta-\theta^2)^2(2+2\theta-\theta^2)(8+20\theta-18\theta^3+\theta^4+2\theta^5)^2}, \end{aligned}$$

while  $\lambda_{21}$  is defined as

$$\begin{aligned} \lambda_{21} \equiv & 1024 + 11392\theta + 53504\theta^2 + 135872\theta^3 + 194464\theta^4 + 134408\theta^5 - 14232\theta^6 \\ & - 103992\theta^7 - 74024\theta^8 - 2696\theta^9 + 27056\theta^{10} + 11798\theta^{11} - 3650\theta^{12} \\ & - 2415\theta^{13} + 300\theta^{14} + 100\theta^{15}, \end{aligned}$$

which has a unique root  $\hat{\theta}_{12}$  in  $(-0.5, 1)$ , which is given by  $\hat{\theta}_{12} = 0$ . We have  $\Delta\pi_3^j < 0$  for  $\theta \in (-0.5, \hat{\theta}_{12})$  and  $\Delta\pi_3^j > 0$  for  $\theta \in (\hat{\theta}_{12}, 1)$ .

$$\begin{aligned} \Delta\pi_3^k &\equiv \pi_3^{(A,A,B)} - \pi_3^{(AB,AB,B)} \\ &= \frac{(a-k)^2(1-\theta)\theta\lambda_{22}}{32(1+\theta)(1+2\theta)(2+3\theta-\theta^2)^2(2+2\theta-\theta^2)(8+20\theta-18\theta^3+\theta^4+2\theta^5)^2}, \end{aligned}$$

while  $\lambda_{22}$  is defined as

$$\begin{aligned} \lambda_{22} \equiv & 2048 + 23552\theta + 113664\theta^2 + 291744\theta^3 + 402304\theta^4 + 213768\theta^5 - 154792\theta^6 \\ & - 274736\theta^7 - 82696\theta^8 + 66928\theta^9 + 43520\theta^{10} - 2386\theta^{11} - 6274\theta^{12} \\ & - 839\theta^{13} + 300\theta^{14} + 68\theta^{15}, \end{aligned}$$

which has a unique root  $\hat{\theta}_{13}$  in  $(-0.5, 1)$ , which is given by  $\hat{\theta}_{13} = 0$ . We have  $\Delta\pi_3^k < 0$  for  $\theta \in (-0.5, \hat{\theta}_{13})$  and  $\Delta\pi_3^k > 0$  for  $\theta \in (\hat{\theta}_{13}, 1)$ .

### Proof of Lemma 4 Part b):

For  $\theta \in (\check{\theta}_1, \bar{\theta})$ , with  $\check{\theta}_1 \approx 0.9018$ , we observe that *jointly* selling exclusively provides higher profits than selling non-exclusively for those manufacturers, as shown by the following calculations. Let again be  $i = 1$ ,  $j = 2$ , and  $l = 3$ , without loss of generality. We have

$$\pi_1^{(A,A,AB)} - \pi_1^{(AB,AB,AB)} = \frac{(a-k)^2(\theta-1)(1+\theta)(2+4\theta+\theta^2)(4+4\theta-9\theta^2-4\theta^3+4\theta^4)}{4(1+2\theta)(4+6\theta-3\theta^2-2\theta^3)^2},$$

which has a unique root  $\hat{\theta}_{14}$  in  $(-0.5, 1)$ , which is given by  $\hat{\theta}_{14} \approx 0.9018$ . We have  $\pi_1^{(A,A,AB)} < \pi_1^{(AB,AB,AB)}$  for  $\theta \in (-0.5, \hat{\theta}_{14})$  and  $\pi_1^{(A,A,AB)} > \pi_1^{(AB,AB,AB)} 0$  for  $\theta \in (\hat{\theta}_{14}, 1)$ . Notice that we refer to  $\hat{\theta}_{14}$  as  $\check{\theta}_1$  in the text for notational reasons. Manufacturer  $M_2$ 's profit comparison is analogous.

$$\pi_1^{(A,B,AB)} - \pi_1^{(AB,AB,AB)} = \frac{(a-k)^2 (\theta-1) (1+\theta) \lambda_{23}}{4 (2+\theta) (1+2\theta) (8+26\theta+17\theta^2-9\theta^3-4\theta^4)^2},$$

while  $\lambda_{23}$  is defined as

$$\lambda_{23} \equiv 64 + 416\theta + 904\theta^2 + 412\theta^3 - 1174\theta^4 - 1725\theta^5 - 686\theta^6 + 86\theta^7 + 104\theta^8 + 16\theta^9,$$

which has a unique root  $\hat{\theta}_{15}$  in  $(-0.5, 1)$ , which is given by  $\hat{\theta}_{15} \approx 0.7995$ . We have  $\pi_1^{(A,B,AB)} < \pi_1^{(AB,AB,AB)}$  for  $\theta \in (-0.5, \hat{\theta}_{15})$  and  $\pi_1^{(A,B,AB)} > \pi_1^{(AB,AB,AB)}$  for  $\theta \in (\hat{\theta}_{15}, 1)$ . Manufacturer  $M_2$ 's profit comparison is analogous.

$$\begin{aligned} \Delta\pi_1^l &\equiv \pi_1^{(A,A,B)} - \pi_1^{(AB,AB,B)} \\ &= \frac{(a-k)^2 (1-\theta) \lambda_{24}}{32 (1+\theta) (1+2\theta) (2+3\theta-\theta^2)^2 (2+2\theta-\theta^2) (8+20\theta-18\theta^3+\theta^4+2\theta^5)^2}, \end{aligned}$$

while  $\lambda_{24}$  is defined as

$$\begin{aligned} \lambda_{24} \equiv & -2048 - 23552\theta - 109696\theta^2 - 248576\theta^3 - 209024\theta^4 + 230368\theta^5 + 655208\theta^6 \\ & + 374088\theta^7 - 280168\theta^8 - 372888\theta^9 - 12988\theta^{10} + 116044\theta^{11} + 23980\theta^{12} \\ & - 14930\theta^{13} - 4319\theta^{14} + 812\theta^{15} + 196\theta^{16}, \end{aligned}$$

which has a unique root  $\hat{\theta}_{16}$  in  $(-0.5, 1)$ , which is given by  $\hat{\theta}_{16} \approx 0.8982$ . We have  $\Delta\pi_1^l < 0$  for  $\theta \in (-0.5, \hat{\theta}_{16})$  and  $\Delta\pi_1^l > 0$  for  $\theta \in (\hat{\theta}_{16}, 1)$ . Manufacturer  $M_2$ 's profit comparison is analogous.

$$\begin{aligned} \Delta\pi_1^m &\equiv \pi_1^{(A,B,A)} - \pi_1^{(AB,AB,A)} \\ &= \frac{(a-k)^2 (1-\theta) \lambda_{25}}{32 (1+\theta) (1+2\theta) (2+3\theta-\theta^2)^2 (2+2\theta-\theta^2) (8+20\theta-18\theta^3+\theta^4+2\theta^5)^2}, \end{aligned}$$

while  $\lambda_{25}$  is defined as

$$\begin{aligned} \lambda_{25} = & -2048 - 23552\theta - 109696\theta^2 - 248576\theta^3 - 209024\theta^4 + 230368\theta^5 + 655208\theta^6 \\ & + 374088\theta^7 - 280168\theta^8 - 372888\theta^9 - 12988\theta^{10} + 116044\theta^{11} + 23980\theta^{12} \\ & - 14930\theta^{13} - 4319\theta^{14} + 812\theta^{15} + 196\theta^{16}, \end{aligned}$$

which has a unique root  $\hat{\theta}_{17}$  in  $(-0.5, 1)$ , which is given by  $\hat{\theta}_{17} \approx 0.8982$ . We have  $\Delta\pi_1^m < 0$

for  $\theta \in (-0.5, \hat{\theta}_{17})$  and  $\Delta\pi_1^m > 0$  for  $\theta \in (\hat{\theta}_{17}, 1)$ .

$$\begin{aligned}\Delta\pi_2^m &\equiv \pi_2^{(A,B,A)} - \pi_2^{(AB,AB,A)} \\ &= \frac{(a-k)^2(1-\theta)\lambda_{26}}{32(1+\theta)(1+2\theta)(2+3\theta-\theta^2)^2(2+2\theta-\theta^2)(8+20\theta-18\theta^3+\theta^4+2\theta^5)^2},\end{aligned}$$

while  $\lambda_{26}$  is defined as

$$\begin{aligned}\lambda_{26} &\equiv -2048 - 22528\theta - 97536\theta^2 - 188416\theta^3 - 53152\theta^4 + 438208\theta^5 + 734568\theta^6 \\ &\quad + 233528\theta^7 - 450912\theta^8 - 381560\theta^9 + 56636\theta^{10} + 132508\theta^{11} + 9796\theta^{12} \\ &\quad - 17554\theta^{13} - 2743\theta^{14} + 812\theta^{15} + 164\theta^{16},\end{aligned}$$

which has a unique root  $\hat{\theta}_{18}$  in  $(-0.5, 1)$ , which is given by  $\hat{\theta}_{18} \approx 0.7018$ . We have  $\Delta\pi_2^m < 0$  for  $\theta \in (-0.5, \hat{\theta}_{18})$  and  $\Delta\pi_2^m > 0$  for  $\theta \in (\hat{\theta}_{18}, 1)$ .

$$\pi_1^{(A,A,B)} - \pi_1^{(AB,AB,AB)} = \frac{(a-k)^2(1-\theta)\lambda_{27}}{4(1+2\theta)(2+2\theta-\theta^2)(8+20\theta-18\theta^3+\theta^4+2\theta^5)^2},$$

while  $\lambda_{27}$  is defined as

$$\begin{aligned}\lambda_{27} &\equiv -64 - 416\theta - 812\theta^2 + 192\theta^3 + 2465\theta^4 + 2190\theta^5 - 1090\theta^6 \\ &\quad - 1972\theta^7 + 59\theta^8 + 666\theta^9 + 11\theta^{10} - 91\theta^{11} + 4\theta^{13},\end{aligned}$$

which has a unique root  $\hat{\theta}_{19}$  in  $(-0.5, 1)$ , which is given by  $\hat{\theta}_{19} \approx 0.6954$ . We have  $\pi_1^{(A,A,B)} < \pi_1^{(AB,AB,AB)}$  for  $\theta \in (-0.5, \hat{\theta}_{19})$  and  $\pi_1^{(A,A,B)} > \pi_1^{(AB,AB,AB)}$  for  $\theta \in (\hat{\theta}_{19}, 1)$ . Notice that we refer to  $\hat{\theta}_{19}$  as  $\check{\theta}_2$  in the text for notational reasons. Manufacturer  $M_2$ 's profit comparison is analogous.

$$\pi_3^{(A,A,B)} - \pi_3^{(AB,AB,AB)} = \frac{(a-k)^2(1-\theta)(1+\theta)\lambda_{28}}{4(1+2\theta)(2+2\theta-\theta^2)(8+20\theta-18\theta^3+\theta^4+2\theta^5)^2},$$

while  $\lambda_{28}$  is defined as

$$\begin{aligned}\lambda_{28} &\equiv -64 - 320\theta - 240\theta^2 + 1168\theta^3 + 2161\theta^4 + 43\theta^5 - 1890\theta^6 \\ &\quad - 433\theta^7 + 677\theta^8 + 82\theta^9 - 91\theta^{10} - 4\theta^{11} + 4\theta^{12},\end{aligned}$$

which has a unique root  $\hat{\theta}_{20}$  in  $(-0.5, 1)$ , which is given by  $\hat{\theta}_{20} \approx 0.5279$ . We have  $\pi_3^{(A,A,B)} < \pi_3^{(AB,AB,AB)}$  for  $\theta \in (-0.5, \hat{\theta}_{20})$  and  $\pi_3^{(A,A,B)} > \pi_3^{(AB,AB,AB)}$  for  $\theta \in (\hat{\theta}_{20}, 1)$ . Notice that we refer to  $\hat{\theta}_{20}$  as  $\check{\theta}_3$  in the text for notational reasons.

## E. Comparison producer surplus

### Proof of Lemma 1:

Distribution system  $(AB, AB, AB)$  has the highest producer surplus of all distribution systems for  $\theta \in (-0.5, 0.2377)$  due to  $a > k$  and  $\theta \in (\underline{\theta}, \bar{\theta})$  as shown by following comparisons:

$$PS^{(A,A,A)} - PS^{(AB,AB,AB)} = \frac{3(a-k)^2(1+\theta)(3\theta-1)}{16(1+2\theta)}.$$

This has a unique root  $\hat{\theta}_{21}$  in  $(-0.5, 1)$ , which is given by  $\hat{\theta}_{21} \approx 0.3333$ . We have  $PS^{(A,A,A)} < PS^{(AB,AB,AB)}$  for  $\theta \in (-0.5, \hat{\theta}_{21})$  and  $PS^{(A,A,A)} > PS^{(AB,AB,AB)}$  for  $\theta \in (\hat{\theta}_{21}, 1)$ .

$$PS^{(A,A,B)} - PS^{(AB,AB,AB)} = \frac{(a-k)^2(\theta-1)\lambda_{29}}{8(1+2\theta)(2+2\theta-\theta^2)^2(8+20\theta-18\theta^3+\theta^4+2\theta^5)^2},$$

where  $\lambda_{29}$  is defined as

$$\begin{aligned} \lambda_{29} \equiv & 384 + 2048\theta - 80\theta^2 - 22400\theta^3 - 57686\theta^4 - 45726\theta^5 + 26593\theta^6 + 57400\theta^7 + 7524\theta^8 \\ & - 24826\theta^9 - 6167\theta^{10} + 5632\theta^{11} + 1002\theta^{12} - 610\theta^{13} - 48\theta^{14} + 24\theta^{15}. \end{aligned}$$

This has a unique root  $\hat{\theta}_{22}$  in  $(-0.5, 1)$ , which is given by  $\hat{\theta}_{22} \approx 0.2849$ . We have  $PS^{(A,A,B)} < PS^{(AB,AB,AB)}$  for  $\theta \in (-0.5, \hat{\theta}_{22})$  and  $PS^{(A,A,B)} > PS^{(AB,AB,AB)}$  for  $\theta \in (\hat{\theta}_{22}, 1)$ .

$$\begin{aligned} \Delta PS_1 \equiv & PS^{(A,A,AB)} - PS^{(AB,AB,AB)} \\ = & \frac{(a-k)^2(\theta-1)(8+8\theta-82\theta^2-196\theta^3-125\theta^4+23\theta^5+50\theta^6+12\theta^7)}{4(1+2\theta)(4+6\theta-3\theta^2-2\theta^3)^2}. \end{aligned}$$

This has a unique root  $\hat{\theta}_{23}$  in  $(-0.5, 1)$ , which is given by  $\hat{\theta}_{23} \approx 0.2667$ . We have  $\Delta PS_1 < 0$  for  $\theta \in (-0.5, \hat{\theta}_{23})$  and  $\Delta PS_1 > 0$  for  $\theta \in (\hat{\theta}_{23}, 1)$ .

$$PS^{(A,AB,B)} - PS^{(AB,AB,AB)} = \frac{(a-k)^2(\theta-1)\lambda_{30}}{2(2+\theta)^2(1+2\theta)(8+26\theta+17\theta^2-9\theta^3-4\theta^4)^2},$$

where  $\lambda_{30}$  is defined as

$$\begin{aligned} \lambda_{30} \equiv & 64 + 320\theta - 112\theta^2 - 4128\theta^3 - 12236\theta^4 - 17492\theta^5 - 13611\theta^6 \\ & - 5144\theta^7 - 63\theta^8 + 686\theta^9 + 228\theta^{10} + 24\theta^{11}. \end{aligned}$$

This has a unique root  $\hat{\theta}_{24}$  in  $(-0.5, 1)$ , which is given by  $\hat{\theta}_{24} \approx 0.2523$ . We have  $PS^{(A,AB,B)} <$

$PS^{(AB,AB,AB)}$  for  $\theta \in (-0.5, \hat{\theta}_{24})$  and  $PS^{(A,AB,B)} > PS^{(AB,AB,AB)}$  for  $\theta \in (\hat{\theta}_{24}, 1)$ .

$$\begin{aligned}\Delta PS_2 &\equiv PS^{(AB,A,AB)} - PS^{(AB,AB,AB)} \\ &= \frac{(a-k)^2(\theta-1)(16+16\theta-188\theta^2-540\theta^3-559\theta^4-192\theta^5+48\theta^6)}{64(1+\theta)(1+2\theta)(2+3\theta-\theta^2)^2}.\end{aligned}$$

This has a unique root  $\hat{\theta}_{25}$  in  $(-0.5, 1)$ , which is given by  $\hat{\theta}_{25} \approx 0.2377$ . We have  $\Delta PS_2 < 0$  for  $\theta \in (-0.5, \hat{\theta}_{25})$  and  $\Delta PS_2 > 0$  for  $\theta \in (\hat{\theta}_{25}, 1)$ . Notice that we refer to  $\hat{\theta}_{25}$  as  $\check{\theta}_4$  in the text for notational reasons.

Distribution system  $(A,A,A)$  has the highest producer surplus of all distribution systems for  $\theta \in (0.6694, 1)$  due to  $a > k$  and  $\theta \in (\underline{\theta}, \bar{\theta})$  as shown by the previous comparisons and the following ones:

$$PS^{(A,A,A)} - PS^{(A,A,B)} = -\frac{(a-k)^2\theta\lambda_{31}}{16(1+2\theta)(2+2\theta-\theta^2)^2(8+20\theta-18\theta^3+\theta^4+2\theta^5)^2},$$

where  $\lambda_{31}$  is defined as

$$\begin{aligned}\lambda_{31} &\equiv 512 + 3680\theta + 8928\theta^2 + 3372\theta^3 - 21904\theta^4 - 36638\theta^5 - 7614\theta^6 \\ &\quad + 27668\theta^7 + 17252\theta^8 - 8878\theta^9 - 7470\theta^{10} + 1955\theta^{11} + 1190\theta^{12} \\ &\quad - 257\theta^{13} - 60\theta^{14} + 12\theta^{15}.\end{aligned}$$

$$PS^{(A,A,A)} - PS^{(A,A,AB)} = \frac{(a-k)^2(-16-16\theta+68\theta^2+68\theta^3-11\theta^4+20\theta^5-7\theta^6-6\theta^7)}{16(4+6\theta-3\theta^2-2\theta^3)^2}.$$

This has a unique root  $\hat{\theta}_{27}$  in  $(-0.5, 1)$ , which is given by  $\hat{\theta}_{27} \approx 0.4871$ . We have  $PS^{(A,A,A)} < PS^{(A,A,AB)}$  for  $\theta \in (-0.5, \hat{\theta}_{27})$  and  $PS^{(A,A,A)} > PS^{(A,A,AB)}$  for  $\theta \in (\hat{\theta}_{27}, 1)$ .

$$PS^{(A,A,A)} - PS^{(A,AB,B)} = \frac{(a-k)^2\lambda_{32}}{16(2+\theta)^2(1+2\theta)(8+26\theta+17\theta^2-9\theta^3-4\theta^4)^2},$$

where  $\lambda_{32}$  is defined as

$$\begin{aligned}\lambda_{32} &\equiv -256 - 2176\theta - 6192\theta^2 - 3232\theta^3 + 19016\theta^4 + 46944\theta^5 + 51301\theta^6 \\ &\quad + 33368\theta^7 + 14410\theta^8 + 3352\theta^9 - 223\theta^{10} - 312\theta^{11} - 48\theta^{12}.\end{aligned}$$

This has a root  $\hat{\theta}_{28a}$ , which is given by  $\hat{\theta}_{28a} \approx -0.3402$ , and a root  $\hat{\theta}_{28b}$ , which is given by  $\hat{\theta}_{28b} \approx 0.4805$  in  $(-0.5, 1)$ . We find  $PS^{(A,A,A)} > PS^{(A,AB,B)}$  for  $\theta \in (-0.5, \hat{\theta}_{28a})$  and for  $\theta \in$

$(\hat{\theta}_{28b}, 1)$  and  $PS^{(A,A,A)} < PS^{(A,AB,B)}$  for  $\theta \in (\hat{\theta}_{28a}, \hat{\theta}_{28b})$ .

$$PS^{(A,A,A)} - PS^{(AB,A,AB)} = \frac{(a-k)^2 (-32 - 32\theta + 184\theta^2 + 276\theta^3 + 77\theta^4 + 45\theta^5 - 6\theta^6)}{64(1+\theta)(2+3\theta-\theta^2)^2}.$$

This has a unique root  $\hat{\theta}_{29}$  in  $(-0.5, 1)$ , which is given by  $\hat{\theta}_{29} \approx 0.3820$ . We have  $PS^{(A,A,A)} < PS^{(AB,A,AB)}$  for  $\theta \in (-0.5, \hat{\theta}_{29})$  and  $PS^{(A,A,A)} > PS^{(AB,A,AB)}$  for  $\theta \in (\hat{\theta}_{29}, 1)$ .

Distribution system  $(A, A, B)$  has the highest producer surplus of all distribution systems for  $\theta \in (0.3494, 0.6694)$  due to  $a > k$  and  $\theta \in (\underline{\theta}, \bar{\theta})$  as shown by the previous comparisons and the following ones:

$$\begin{aligned} \Delta PS_3 &\equiv PS^{(A,A,B)} - PS^{(A,A,AB)} \\ &= \frac{(a-k)^2 (1-\theta) \lambda_{33}}{8(1+2\theta)(2+2\theta-\theta^2)^2 (4+6\theta-3\theta^2-2\theta^3)^2 (8+20\theta-18\theta^3+\theta^4+2\theta^5)^2}, \end{aligned}$$

where  $\lambda_{33}$  is defined as

$$\begin{aligned} \lambda_{33} &\equiv -2048 - 18432\theta - 48384\theta^2 + 58624\theta^3 + 589984\theta^4 + 1223104\theta^5 + 628072\theta^6 \\ &\quad - 1340544\theta^7 - 1995758\theta^8 + 22394\theta^9 + 1562911\theta^{10} + 544460\theta^{11} - 593892\theta^{12} \\ &\quad - 296506\theta^{13} + 123547\theta^{14} + 65548\theta^{15} - 12208\theta^{16} - 6752\theta^{17} + 308\theta^{18} + 272\theta^{19} \\ &\quad + 16\theta^{20}. \end{aligned}$$

This has a unique root  $\hat{\theta}_{30}$  in  $(-0.5, 1)$ , which is given by  $\hat{\theta}_{30} \approx 0.3260$ . We have  $\Delta PS_3 < 0$  for  $\theta \in (-0.5, \hat{\theta}_{30})$  and  $\Delta PS_3 > 0$  for  $\theta \in (\hat{\theta}_{30}, 1)$ .

$$\begin{aligned} \Delta PS_4 &\equiv PS^{(A,A,B)} - PS^{(A,AB,B)} \\ &= \frac{(a-k)^2 (1-\theta) \lambda_{34}}{8(2+\theta)^2 (2+2\theta-\theta^2)^2 (8+26\theta+17\theta^2-9\theta^3-4\theta^4)^2 (8+20\theta-18\theta^3+\theta^4+2\theta^5)^2}, \end{aligned}$$

where  $\lambda_{34}$  is defined as

$$\begin{aligned} \lambda_{34} &\equiv -32768 - 409600\theta - 1968128\theta^2 - 3479552\theta^3 + 6057728\theta^4 + 44516608\theta^5 \\ &\quad + 99632224\theta^6 + 104008864\theta^7 + 15320528\theta^8 - 83316112\theta^9 - 77511202\theta^{10} \\ &\quad + 963854\theta^{11} + 36235437\theta^{12} + 12675738\theta^{13} - 7214793\theta^{14} - 4512764\theta^{15} \\ &\quad + 612615\theta^{16} + 756746\theta^{17} + 13525\theta^{18} - 67076\theta^{19} - 7066\theta^{20} + 2534\theta^{21} + 420\theta^{22}. \end{aligned}$$

This has a unique root  $\hat{\theta}_{31}$  in  $(-0.5, 1)$ , which is given by  $\hat{\theta}_{31} \approx 0.3494$ . We find  $\Delta PS_4 < 0$  for

$\theta \in (-0.5, \hat{\theta}_{31})$  and  $\Delta PS_4 > 0$  for  $\theta \in (\hat{\theta}_{31}, 1)$ .

$$\begin{aligned}\Delta PS_5 &\equiv PS^{(A,A,B)} - PS^{(AB,A,AB)} \\ &= \frac{(a-k)^2(\theta-1)\lambda_{35}}{64(1+\theta)(1+2\theta)(2+3\theta-\theta^2)^2(2+2\theta-\theta^2)^2(8+20\theta-18\theta^3+\theta^4+2\theta^5)^2},\end{aligned}$$

where  $\lambda_{35}$  is defined as

$$\begin{aligned}\lambda_{35} &\equiv 8192 + 81920\theta + 264704\theta^2 - 71680\theta^3 - 2790208\theta^4 - 8130112\theta^5 - 10168656\theta^6 \\ &\quad - 2723264\theta^7 + 7363304\theta^8 + 7260632\theta^9 - 352712\theta^{10} - 3205416\theta^{11} - 727500\theta^{12} \\ &\quad + 611800\theta^{13} + 200448\theta^{14} - 48708\theta^{15} - 22809\theta^{16} + 1372\theta^{17} + 956\theta^{18}.\end{aligned}$$

This has a unique root  $\hat{\theta}_{32}$  in  $(-0.5, 1)$ , which is given by  $\hat{\theta}_{32} \approx 0.3083$ . We have  $\Delta PS_5 < 0$  for  $\theta \in (-0.5, \hat{\theta}_{32})$  and  $\Delta PS_5 > 0$  for  $\theta \in (\hat{\theta}_{32}, 1)$ .

Distribution system  $(A, AB, B)$  has the highest producer surplus of all distribution systems for  $\theta \in (0.2667, 0.3494)$  due to  $k < a$  and  $\theta \in (\underline{\theta}, \bar{\theta})$  as shown by the previous comparisons and the following ones.

$$\begin{aligned}\Delta PS_6 &\equiv PS^{(A,A,AB)} - PS^{(A,AB,B)} \\ &= \frac{(a-k)^2(\theta-1)\theta\lambda_{36}}{4(2+\theta)^2(1+2\theta)(4+6\theta-3\theta^2-2\theta^3)^2(8+26\theta+17\theta^2-9\theta^3-4\theta^4)^2},\end{aligned}$$

where  $\lambda_{36}$  is defined as

$$\begin{aligned}\lambda_{36} &\equiv 1024 + 9856\theta + 35712\theta^2 + 45600\theta^3 - 68736\theta^4 - 349592\theta^5 - 586712\theta^6 \\ &\quad - 513058\theta^7 - 194136\theta^8 + 62095\theta^9 + 116017\theta^{10} + 65265\theta^{11} + 20547\theta^{12} \\ &\quad + 4002\theta^{13} + 492\theta^{14} + 32\theta^{15}.\end{aligned}$$

This has a root  $\hat{\theta}_{33a}$ , which is given at  $\hat{\theta}_{33a} = 0$ , and a root  $\hat{\theta}_{33b}$ , which is given at  $\hat{\theta}_{33b} \approx 0.4543$  in  $(-0.5, 1)$ . We have  $\Delta PS_6 < 0$  for  $\theta \in (\hat{\theta}_{33a}, \hat{\theta}_{33b})$  and  $\Delta PS_6 > 0$  for  $\theta \in (-0.5, \hat{\theta}_{33a})$  and  $\theta \in (\hat{\theta}_{33b}, 1)$ .

$$\begin{aligned}\Delta PS_7 &\equiv PS^{(A,AB,B)} - PS^{(AB,A,AB)} \\ &= \frac{(a-k)^2(\theta-1)\lambda_{37}}{64(1+\theta)(2+\theta)^2(1+2\theta)(2+3\theta-\theta^2)^2(8+26\theta+17\theta^2-9\theta^3-4\theta^4)^2},\end{aligned}$$

where  $\lambda_{37}$  is defined as

$$\begin{aligned}\lambda_{37} \equiv & 4096 + 38912\theta + 113408\theta^2 - 117504\theta^3 - 1705152\theta^4 - 5354560\theta^5 - 9252608\theta^6 \\ & - 9917136\theta^7 - 6614956\theta^8 - 2410444\theta^9 - 47013\theta^{10} + 445050\theta^{11} + 237591\theta^{12} \\ & + 57080\theta^{13} + 5504\theta^{14}.\end{aligned}$$

This has a unique root  $\hat{\theta}_{34}$  in  $(-0.5, 1)$ , which is given by  $\hat{\theta}_{34} \approx 0.2667$ . We find  $\Delta PS_7 < 0$  for  $\theta \in (-0.5, \hat{\theta}_{34})$  and  $\Delta PS_7 > 0$  for  $\theta \in (\hat{\theta}_{34}, 1)$ .

Distribution system  $(AB, A, AB)$  has the highest producer surplus of all distribution systems in  $\theta \in (0.2377, 0.2667)$  due to  $a > k$  and  $\theta \in (\underline{\theta}, \bar{\theta})$  as shown by the previous comparisons and the following ones.

$$PS^{(A,A,AB)} - PS^{(AB,A,AB)} = \frac{(a-k)^2(\theta-1)\lambda_{38}}{64(1+\theta)(2+3\theta-\theta^2)^2(4+6\theta-3\theta^2-2\theta^3)^2},$$

where  $\lambda_{38}$  is defined as

$$\begin{aligned}\lambda_{38} \equiv & 256 + 1024\theta - 1024\theta^2 - 11136\theta^3 - 20784\theta^4 - 14816\theta^5 - 1456\theta^6 \\ & + 4128\theta^7 + 3199\theta^8 + 710\theta^9 + 16\theta^{10}.\end{aligned}$$

This has a unique root  $\hat{\theta}_{35}$  in  $(-0.5, 1)$ , which is given by  $\hat{\theta}_{35} \approx 0.2936$ . We find  $PS^{(A,A,AB)} < PS^{(AB,A,AB)}$  for  $\theta \in (-0.5, \hat{\theta}_{35})$  and  $PS^{(A,A,AB)} > PS^{(AB,A,AB)}$  for  $\theta \in (\hat{\theta}_{35}, 1)$ .

## F. Comparison consumer surplus

**Proof of Lemma 3.2:**

We have

$$CS^{(A,A,A)} - CS^{(AB,AB,AB)} = -\frac{9(a-k)^2(1+\theta)^2}{32(1+2\theta)} < 0,$$

$$\begin{aligned} \Delta CS_1 &\equiv CS^{(A,A,B)} - CS^{(AB,AB,AB)} \\ &= -\frac{(a-k)^2(1-\theta)(1+\theta)\lambda_{39}}{16(1+2\theta)(2+2\theta-\theta^2)^2(8+20\theta-18\theta^3+\theta^4+2\theta^5)^2} < 0, \end{aligned}$$

where  $\lambda_{39}$  is defined by

$$\begin{aligned} \lambda_{39} &\equiv 1152 + 10112\theta + 34704\theta^2 + 54032\theta^3 + 21670\theta^4 - 42664\theta^5 - 47969\theta^6 + 5137\theta^7 \\ &\quad + 24015\theta^8 + 2327\theta^9 - 5700\theta^{10} - 496\theta^{11} + 618\theta^{12} + 24\theta^{13} - 24\theta^{14}, \end{aligned}$$

$$\begin{aligned} \Delta CS_2 &\equiv CS^{(A,A,AB)} - CS^{(AB,AB,AB)} \\ &= -\frac{(a-k)^2(1-\theta)(24+136\theta+290\theta^2+272\theta^3+59\theta^4-91\theta^5-66\theta^6-12\theta^7)}{8(1+2\theta)(4+6\theta-3\theta^2-2\theta^3)^2} < 0, \end{aligned}$$

$$CS^{(A,AB,B)} - CS^{(AB,AB,AB)} = -\frac{(a-k)^2(1-\theta)\lambda_{40}}{4(2+\theta)^2(1+2\theta)(8+26\theta+17\theta^2-9\theta^3-4\theta^4)^2} < 0,$$

where  $\lambda_{40}$  is defined by

$$\begin{aligned} \lambda_{40} &\equiv 192 + 1920\theta + 8304\theta^2 + 20304\theta^3 + 30612\theta^4 + 28708\theta^5 + 15365\theta^6 \\ &\quad + 2774\theta^7 - 1673\theta^8 - 1162\theta^9 - 276\theta^{10} - 24\theta^{11}, \end{aligned}$$

$$\begin{aligned} \Delta CS_3 &\equiv CS^{(AB,A,AB)} - CS^{(AB,AB,AB)} \\ &= -\frac{(a-k)^2(1-\theta)(48+304\theta+796\theta^2+1076\theta^3+695\theta^4+96\theta^5-48\theta^6)}{128(1+\theta)(1+2\theta)(2+3\theta-\theta^2)^2} < 0, \end{aligned}$$

since the terms in parentheses, same as  $\lambda_{39}$  and  $\lambda_{40}$  are strictly positive because of  $a > k$  and  $\theta \in (-0.5, 1)$ .

## G. Robustness: Varying number of manufacturers

### G.1. Model and equilibrium calculations

Suppose we have  $i = 4$  manufacturers. All other assumptions remain as in the main case. In channel structure  $(AB, AB, AB, AB)$ , all four manufacturers choose the strategy  $s_i = AB$ . The equilibrium calculations are analogous to distribution system  $(AB, AB, AB)$ . In channel structure  $(A, AB, AB, AB)$ , respectively  $(B, AB, AB, AB)$ , manufacturer  $M_1$  sells exclusively and manufacturers  $M_2, M_3$ , and  $M_4$  sell non-exclusively. The equilibrium calculations are analogous to distribution system  $(A, AB, AB)$ .

Figure 5 depicts both distributions systems and Table 9 presents their respective equilibrium values.

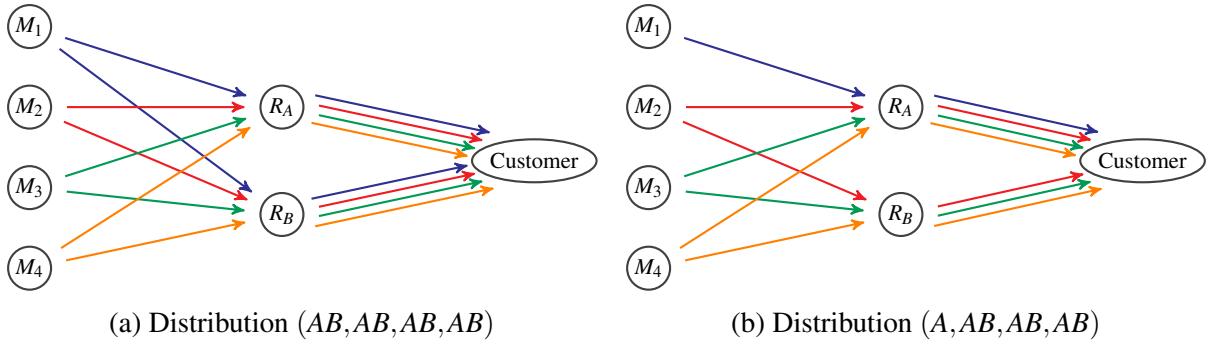


Figure 5: Distribution systems (with  $i = 4$ )

Distribution system	$(AB, AB, AB, AB)$	$(A, AB, AB, AB)$
Quantities	$Q_1^* = Q_2^* = Q_3^* = Q_4^* = \frac{(a-k)(1+2\theta)}{(2+\theta)(1+3\theta)}$	$Q_1^* = \frac{(a-k)(1+2\theta)}{2(2+\theta)(1+3\theta)}$ $Q_2^* = Q_3^* = Q_4^* = \frac{(a-k)(2+5\theta)}{2(2+\theta)(1+3\theta)}$
Wholesale prices	$w_1^* = w_2^* = w_3^* = w_4^* = \frac{a+k-(a-2k)\theta}{2+\theta}$	$w_1^* = w_2^* = w_3^* = w_4^* = \frac{a+k-(a-2k)\theta}{2+\theta}$
Retail prices	$p_1^* = p_2^* = p_3^* = p_4^* = \frac{a+k+(2k-a)\theta}{2+\theta}$	$p_1^* = \frac{3a+k-3a\theta+5k\theta}{2(2+\theta)}$ $p_2^* = p_3^* = p_4^* = \frac{a+k-(a-2k)\theta}{2+\theta}$
Manufacturer profits	$\pi_1^* = \pi_2^* = \pi_3^* = \pi_4^* = \frac{(a-k)^2(1-\theta)(1+2\theta)}{(2+\theta)^2(1+3\theta)}$	$\pi_1^* = \frac{(a-k)^2(1-\theta)(1+2\theta)}{2(2+\theta)^2(1+3\theta)}$ $\pi_2^* = \pi_3^* = \pi_4^* = \frac{(a-k)^2(1-\theta)(2+5\theta)}{2(2+\theta)^2(1+3\theta)}$
Retail profits	$\pi_A^* = 0$ $\pi_B^* = 0$	$\pi_A^* = \frac{(a-k)^2(1-\theta)(1+2\theta)}{4(2+\theta)^2(1+3\theta)}$ $\pi_B^* = 0$

Table 9: Equilibrium values (with  $i = 4$ )

### G.2. No profitable deviation

We have  $\pi_1^{(AB, AB, AB, AB)} - \pi_1^{(A, AB, AB, AB)} = \frac{(a-k)^2(1-\theta)(1+2\theta)}{2(2+\theta)^2(1+3\theta)} > 0$  for  $\theta \in (-0.5, 1)$  since  $a > k$  and  $\theta \in (-0.5, 1)$ .

## H. Proof of Propositions

### H.1. Proof of Proposition 2

For all manufacturer  $i$ , we already know from Lemma 4 that distribution system  $(A, A, B)$  is more profitable than distribution system  $(AB, AB, AB)$  for certain ranges of product differentiation.

We further have

$$\begin{aligned}\Delta\pi_A &\equiv \pi_A^{(AB,AB,AB)} - \pi_A^{(A,A,B)} \\ &= -\frac{(a-k)^2(1-\theta)(4+8\theta+\theta^2-2\theta^3)^2(4+11\theta+4\theta^2-5\theta^3)^2}{8(1+2\theta)(2+2\theta-\theta^2)^2(8+20\theta-18\theta^3+\theta^4+2\theta^5)^2} < 0,\end{aligned}$$

$$\begin{aligned}\Delta\pi_B &\equiv \pi_B^{(AB,AB,AB)} - \pi_B^{(A,A,B)} \\ &= -\frac{(a-k)^2(1-\theta)(1+\theta)(1+\theta-\theta^2)^2(8+24\theta+15\theta^2-5\theta^3-2\theta^4)^2}{4(1+2\theta)(2+2\theta-\theta^2)^2(8+20\theta-18\theta^3+\theta^4+2\theta^5)^2} < 0,\end{aligned}$$

for  $\theta \in (-0.5, 1)$  since the terms in parentheses are strictly positive because of  $a > k$  and  $\theta \in (-0.5, 1)$ .

### H.2. Proof of Proposition 3

We have

$$W^{(A,A,A)} - W^{(AB,AB,AB)} = -\frac{3(a-k)^2(1+\theta)(5-3\theta)}{32(1+2\theta)} < 0,$$

$$W^{(A,A,B)} - W^{(AB,AB,AB)} = -\frac{(a-k)^2(1-\theta)\lambda_{41}}{16(1+2\theta)(2+2\theta-\theta^2)^2(8+20\theta-18\theta^3+\theta^4+2\theta^5)^2} < 0,$$

where  $\lambda_{41}$  is defined by

$$\begin{aligned}\lambda_{41} &\equiv 1920 + 15360\theta + 44656\theta^2 + 43936\theta^3 - 39670\theta^4 - 112446\theta^5 - 37447\theta^6 + 71968\theta^7 \\ &\quad + 44200\theta^8 - 23310\theta^9 - 15707\theta^{10} + 5068\theta^{11} + 2126\theta^{12} - 578\theta^{13} - 96\theta^{14} + 24\theta^{15},\end{aligned}$$

$$\begin{aligned}\Delta W_1 &\equiv W^{(A,A,AB)} - W^{(AB,AB,AB)} \\ &= -\frac{(a-k)^2(1-\theta)(40+152\theta+126\theta^2-120\theta^3-191\theta^4-45\theta^5+34\theta^6+12\theta^7)}{8(1+2\theta)(4+6\theta-3\theta^2-2\theta^3)^2} < 0,\end{aligned}$$

$$W^{(A,AB,B)} - W^{(AB,AB,AB)} = -\frac{(a-k)^2 (1-\theta) \lambda_{42}}{4(2+\theta)^2 (1+2\theta) (8+26\theta+17\theta^2-9\theta^3-4\theta^4)^2} < 0,$$

where  $\lambda_{42}$  is defined by

$$\begin{aligned} \lambda_{42} \equiv & 320 + 2560\theta + 8080\theta^2 + 12048\theta^3 + 6140\theta^4 - 6276\theta^5 - 11857\theta^6 \\ & - 7514\theta^7 - 1799\theta^8 + 210\theta^9 + 180\theta^{10} + 24\theta^{11}, \end{aligned}$$

$$\begin{aligned} \Delta W_2 \equiv & W^{(AB,A,AB)} - W^{(AB,AB,AB)} \\ = & -\frac{(a-k)^2 (1-\theta) (80 + 336\theta + 420\theta^2 - 4\theta^3 - 423\theta^4 - 288\theta^5 + 48\theta^6)}{128 (1+\theta) (1+2\theta) (2+3\theta-\theta^2)^2} < 0, \end{aligned}$$

for  $\theta \in (-0.5, 1)$ , since the terms in parentheses, same as  $\lambda_{41}$  and  $\lambda_{42}$ , are strictly positive because of  $a > k$  and  $\theta \in (-0.5, 1)$ .