Capacity investments in a competitive energy market

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Abstract

We study the abilities of competitive markets to produce sufficient energy capacities to meet a fixed energy demand. Renewable energy producers with stochastic outputs and no variable costs compete against conventional energy producers with deterministic, pollutant outputs and increasing marginal costs. We find that either market forces are strong enough to serve the entire demand, or they are too weak such that the market fails and nothing is produced. This crucially depends on the relative cost of renewable energy investments, such that relatively cheap renewable energy causes the market to fail. Welfare analyses show that with increasing levels of conventional energy pollution the ability of the market to produce an efficient outcome further declines. As a policy implication, our findings refute the use of a strategic reserve as a blackout backstop solution. Instead, a capacity mechanism consisting of a tax-and-subsidy scheme can align the market outcome with the efficient solution for all pollution levels and relative costs of renewable energy capacities.

Keywords: Renewable versus conventional energy, capacity mechanisms, strategic reserves, capacity payments

JEL Classification Numbers: D41, L11, Q48

1 Introduction

In recent years, the energy market has experienced various changes. Starting from a market dominated by conventional energies (coal, nuclear power, oil, and gas) and state monopolies, we experienced both a liberalization and the entry of renewable energy producers in great numbers, mainly thanks to technological progress and generous state subsidies. The idea of these subsidies is ecologically motivated, but at the same time the challenge of ensuring energy security is increasingly at stake. Both in politics and in science there is an ongoing debate about how to tackle the problem of how to ensure energy security when more and more energy production is volatile (presented in the literature review below).

At the center of attention are capacity mechanisms where compared to energy-only markets, producers are not (only) paid for energy supply but for capacity provision. These mechanisms aim at creating an environment in which potential power blackouts are compensated by market participants whenever renewable energies fail to produce. Such backup energy can, however, only be accessed if sufficient capacity has been built up and is available at the moment of the energy shortage.

Behind the discussion about the functioning of capacity markets is a concrete question for market designers and institutions: Can there be a market solution to the energy security problem, that is, are there sufficient incentives for market participants to provide enough energy? And, if this is not the case, how could appropriate measures to solve this problem look like? While the existing literature gives some convincing arguments why the first question should be negated , the second question is much harder to answer. With this paper we contribute to deepen the understanding of the market solution to the energy security problem.

There are various reasons why market incentives are too weak to induce sufficient investments in energy capacities. Joskow and Tirole (2007) and Cramton and Ockenfels (2016) point at the deficit elements of energy-only markets. Their arguments center around three major issues. First, consumers hardly respond to price changes and they do not reduce their consumption quickly enough so that demand cannot be met at times where energy is scarce, even if all producers produce at their capacity limit. Second, there is the "missing-money" problem (see also Cramton and Stoft, 2006). This problem is again due to the low price elasticity of energy demand. In times of scarcity, this implies great market power of energy producers, which is currently restricted by market regulators in many countries by means of price caps. Consequently, the price for energy during demand peaks does not induce adequate investment incentives and so the price caps contribute to the supply shortage. Third, and finally, it is argued that investments in energy capacities have the nature of a public good. In case of a power blackout, no producer is able to sell its energy. However, the costs of building up the capacity to avoid such a blackout are born by single investors. As these investments rescue all energy producers, the ratio of costs and benefits is out of balance and causes underinvestments.

As a response to these institutional obstacles to the energy security problem, the subsequent literature has discussed and tested various forms of capacity mechanisms. Bublitz et al. (2019) provide a detailed classification and cluster the different streams of this literature.[1] Notable are especially the contributions by Cramton and Stoft (2005) and Cramton et al. (2013) who address the questions of how the pricing of capacity units and the procedure should be designed. Common to all these designs is the idea that energy producers earn an additional revenue stream on the part of their capacity that they withhold from the regular market and only provide when energy is scarce.

An open question in this literature is, however, under which conditions it is beneficial or even essential to replace conventional energy-only markets by capacity mechanisms? Put differently,

[1] The latest literature review can be found in Duggan (2020).

when are market incentives no longer strong enough to provide sufficient energy supply? And, how does this depend on the available energy technology? In this paper, we set up a theoretical model to shed light on these questions. In particular, our model starts from two types of available production technologies: a renewable energy technology which allows production without any variable costs, but which results in a stochastic output; and a conventional technology, which results in a reliable output but has an increasing marginal cost of production. One might think of wind and solar energy, on the one hand, and a mix of coal, oil, and gas plants, on the other hand, whereby oil and gas plants are fired up when coal plants are at their limit. Further, the market is split into three sequential parts in our model: a capacity market (if it exists), a future market, and a spot market. In particular, we assume that the demand on the future market is fixed and that supply is perfectly competitive with free entry into the future market (or capacity market in case it exists). Moreover, the actual renewable energy output becomes known after the closing of the future market but before the opening of the spot market. Thus, in our model, producers of conventional energy may not offer all their capacity on the future market but instead speculate on a low renewable energy output and thus a high price on the spot market.

In our first set of results, we identify different market scenarios depending on which types of energy technologies are active in the future market, ranging from the traditional markets with only conventional energy to the markets of the future where only renewable energy is traded. Next, we show that only two of these scenarios survive when producers build up investments without a capacity market: in the unique stable free-entry equilibrium, either only conventional energy suppliers enter the market or a mixture of conventional and renewable energy producers such that the total conventional energy capacity is large enough so that conventional energy producers alone can ensure energy security. Which of the two equilibria arises crucially depends on the relative costs of renewable energy investments, such that the mixed equilibrium requires renewable energy costs to be on an intermediate level. Most importantly here, we find that it is not possible that demand is met by a technology mix dominated by renewable energies. Technically, our model admits for no free-entry equilibrium when the investment costs of the renewable technology are relatively low.

In other words, our model suggests that a fundamental shift away from energy-only markets will become unavoidable when the costs of renewable energy technologies tend to fall as fast as they currently do. Moreover, it suggests that a strategic reserve provided by a system operator is of little use because either market participants can ensure system stability on their own or the system operator has to serve the entire demand on its own, which is similar to returning to a state-owned energy system.[2]

[2]In some countries, such as Belgium or Germany, the strategic reserve is an intermediate form of a capacity mechanism, as the system operator reserves a fixed amount of capacity from every producer that is no longer available for the wholesale market. Either producers can actively apply to put (parts) of their capacity into the reserve, or the operator prohibits the deactivation of older power plants. In other countries like France the system operator runs its own power plants. To make the distinction with the capacity mechanism clear, we assume the latter form of strategic reserve in our model.

In a welfare analysis, we evaluate if and when a regime switch is also socially desirable. Welfare consists of the probability to serve the energy demand, investment costs, variable costs, and a pollution cost on conventional energy. Our analysis of the optimal technology mix shows that without pollution and a relative expensive renewable technology, the two types of stable equilibria mentioned above are also aligned with the efficient solution. However, with positive pollution costs or low renewable energy costs, a technology mix dominated by renewable energy is preferred even in the parameter range where a stable market equilibrium exists.

In order to close the gap between stability and efficiency, we finally investigate the potential of capacity payments. In particular, we study a subsidy on renewable energy investments and a tax on conventional energy investments and compute for all parameter constellations of our model, the optimal tax-and-subsidy policy that induces a perfect alignment between investment incentives and efficient investment levels. Our results here are the following: There is only one parameter constellation where conventional energy producers should be taxed: that is to keep them out of the market when only renewable energy technologies are desired and the society accepts power blackouts. Otherwise, in all other parameter constellations, conventional energy is needed to ensure system stability when renewable energy output is low. Under the optimal policy mix, renewable energy, by contrast, is always subsidized to increase their share in the technology mix. When only renewable energy is desired, the optimal subsidy level is even as high as the total cost of a renewable energy investment.

Our model and findings are related to the following studies that also extend on the fundamental insights by Joskow and Tirole (2007) and Cramton and Ockenfels (2016). The private investment incentives in energy capacity and the role of capacity mechanisms have been studied in a number of papers. In an environment with perfect competition, Stoft (2002) investigates the combination of a capacity mechanism with a price cap on units of energy. Fabra (2018) extends on his analysis by adding imperfect competition and market power to the picture. In her approach, price caps are combined with capacity payments of the type also studied in our paper. Holmberg and Ritz (2020) follow a similar approach but also incorporate a strategic reserve into their model. Neuhoff and De Vries (2004) add risk-averse consumers and producers and study the role of long-term contracts. Finally, Grimm and Zoettl (2013) analyze investment incentives under different degrees of competition in the energy market.

Strategic reserves have been previously compared to capacity mechanisms as well. Hary et al. (2016) run a simulation study and find that both approaches can reduce cyclical investments but the strategic reserve is less efficient in terms of social welfare. In contrast, Neuhoff et al. (2016) argue that capacity mechanisms should be used as one component of an integrated approach. Then the strategic reserve has the advantage of being a rather mild market impact and is therefore quicker installed and uninstalled again. Finally, Traber (2017) computes the optimal strategic reserve size alongside with an optimal reserve payment by a system operator. His welfare analysis provides ambiguous results in terms of which mechanism should be implemented.

The main contribution of our paper is that we set up a basic framework by introducing two different types of available technologies: a renewable energy technology with a stochastic output and no variable production costs and a conventional energy technology with a deterministic, pollutant output and increasing marginal costs of production. Our main finding is that the market can either provide the entire supply or none at all. In neither case a strategic reserve is helpful. Instead, a tax-and-subsidy system can close the gap between stable market outcomes and welfare efficient solutions. In Section 2, we set up our model and characterize the market scenarios. The welfare analysis is done in Section 3, and Section 4 introduces the optimal tax-and-subsidy scheme. Section 5 concludes.

2 Model

We consider a three-stage model consisting of an entry and investment stage (supported by a tax-and-subsidy scheme in Section 4), a future market, and a spot market, which take place in that order. There are two types of potential entrants, both taking the prices, investments, and energy supplies of all other producers as fixed and given when making their decisions (perfect competition): first, renewable energy producers, which we denote windmill operator w from now on, and second, conventional energy producers, which we denote the thermal power operators t.

In the investment stage investors choose to enter the market. We assume for simplicity that each potential entrant i has the choice between either entering the market with one unit of output with either wind energy, thermal energy, or not entering at all. There are fixed investment costs for each unit built, which are specified below. With perfect competition n^w and n^t producers enter the market until the marginal entrant and the marginal unit of capacity has an expected profit of zero.[3] The capacities fix the outputs the producers can supply on the day of delivery after the closure of the spot market. Denote with $q^w \leq 1$ and $q^t \leq 1$ units of wind and thermal energy outputs and total capacities with Q^w and Q^t respectively. For users of the wind technology, these are the only costs of production as there are no further variable costs of wind generation. Users of the thermal technology, by contrast, have an additional variable cost of $c(q^t)$, which satisfies $c(6) = 0$ and which is increasing and convex in q^t, that is, $c'(q^t) > 0$ and $c''(q^t) > 0$. In addition to increasing the capacity limit, any unit of investment in thermal energy has an indirect effect on the variable production costs of a thermal producer because this producer can spread out its total output on a larger capacity. Concretely, we assume a linearly increasing marginal cost function

$$c'(q^t) = \beta q^t \qquad \text{and} \quad c(q^t) = \int_0^{q^t} c'(x)dx = \frac{\beta(q^t)^2}{2} \tag{1}$$

so that β measures the marginal cost at the capacity limit and the marginal cost at $q^t < 1$ is simply

[3]Throughout, we ignore the integer problem and allow n^w and n^t to be real numbers.

given by the ratio between q^t and 1.

On the future market, the incumbent producers commit to energy outputs q_f^w and q_f^t respectively, which they plan to deliver on the day of delivery. Consumer demand is price inelastic and given by D up until a reservation price $p = 1$. For $p > 1$, demand is 0. We assume that every producer can promise to supply at most its capacity limit, that is, $q_f \leq 1$. The n^t thermal energy producers and the n^w wind energy producers then offer total outputs of Q_f^t and Q_f^w units respectively, against the market price p_f. This price is the same for all market participants and market clearing thus requires a $p_f \leq p = 1$ such that $Q_f^t + Q_f^w = D$ or a $p_f > 1$ such that $Q_f^t + Q_f^w = 0$. Due to perfect competition, the market price p_f is equal to the highest unit cost of delivery. That unit cost is the outcome of a random variable because the wind output on the day of delivery is uncertain and the actual wind output will only become common knowledge after the future market closes.

After the closing of the future market, the actual wind output v per unit of investment in windmills will be revealed. The wind output v is a random variable that can be interpreted as the velocity of the wind. For simplicity, let us assume that wind outputs are perfectly correlated among the windmills and that there are only two states of the wind: with probability θ the wind blows at full velocity ($v = 1$) and with probability $1 - \theta$ the wind does not blow at all ($v = 0$). Whether or not the output $Q_f^t + Q_f^w = D$ promised on the future market can be supplied on the day of delivery is thus dependent on the actual state of the wind. If $v = 1$, then windmill operators can supply their promised output. As the total supply is then larger or equal to D, every producer can deliver what he has promised against the already paid market price p_f. By contrast, if the actual wind output is zero and $Q_f^w > 0$, then there is a supply shortage. In this case, thermal energy producers have the possibility to provide the missing energy on a spot market.

On the spot market, thermal producers receive a price p_v for every unit of energy they provide in addition. We assume that this price is paid by the windmill operators and can be understood as a penalty on every promised unit they fail to deliver. Denote by q_f^t the quantity that a thermal energy producer schedules on the future market and by q_v^t the actual quantity the producer supplies on the delivery day. Then, $q_v^t - q_f^t$ measures the producer's supply on the spot market. For reasons that become clear below, a thermal producer will supply as much as he can on the spot market, but it might occur that there is insufficient total thermal capacity to compensate for the missing wind output. Thus, three scenarios are possible on the spot market depending on the velocity of the wind: first, actual wind output meets or even exceeds the wind output scheduled on the future market. This scenario we call a *long* market. Second, the wind does not blow, but there is enough thermal capacity to cover this shortage. This is what we call a *short* market. Third, there is no wind output and thermal producers do *not* have the capacities to close the gap, which is what we call a *failed* market. Formally, the spot market is

- *long* if $v = 1$,

- *short* if $v = 0$ and $Q^t \geq D$,
- *failed* if $v = 0$ and $Q^t < D$.

Hence, there might be an energy shortage on the delivery day because windmill operators cannot supply whatever they have promised on the future market. In that case, thermal energy producers who still have available capacity can sell additional units of energy in the spot market.

In case of a failed market, the system operator needs to step in to prevent a power blackout. For that matter, we assume that the system operator has built up an amount $R \geq 0$ of energy capacity as a strategic reserve. This reserve is large enough to cover whatever is needed. Moreover, it has to consist of conventional energy to guarantee its availability on the delivery day so that the fixed and variable costs of the strategic reserve are the same as the costs of a thermal energy plant. This reserve cannot be used in the future market or in the spot market but only for whatever shortage is left after the spot market closes. Thus, implementing such a state's safety measurement influences the private investors' decisions by guaranteeing a functional market, while prices on future and spot market are not affected. This allows us to focus the analysis on the market's ability to provide sufficient capacities.

2.1 Profit functions

From an ex-ante perspective, the choice for a certain technology is associated with the following expected profit function. Let there be normalized investment costs for thermal and wind energy respectively with $k^t \frac{\beta}{2}$ and $k^w \theta\beta$. Thus, investment costs in thermal energy are expressed in terms of the average marginal production cost, $\beta/2$, and investment costs in wind energy are expressed in terms of the expected maximal avoided penalty, $\theta\beta$. Then thermal energy producers earn:

$$\mathbb{E}[\pi^t | q_f^t, q_v^t] \;=\; p_f q_f^t \;-\; \theta\, c(q_f^t) + (1-\theta)\Big(p_{v=0}(q_{v=0}^t - q_f^t) - c(q_{v=0}^t)\Big) \;-\; k^t \frac{\beta}{2} \tag{2}$$

subject to the constraint that $0 \leq q_f^t \leq 1$, and $q_f^t \leq q_{v=0}^t$. Windmill operators earn:

$$\mathbb{E}[\pi^w | q_f^w] \;=\; p_f q_f^w \;-\; (1-\theta) p_{v=0}\, q_f^w \;-\; k^w \theta\beta\,. \tag{3}$$

subject to the constraints that $0 \leq q_f^w \leq 1$.

Thus, the expected profit of the thermal technology consists of the revenue on the future market $p_f q_f^t$ and the revenue on the spot market for selling additional energy $p_{v=0}(q_{v=0}^t - q_f^t)$ when $v = 0$. From this, the fixed costs $k^t \frac{\beta}{2}$ and the variable costs are deducted, which might either be $c(q_f^t)$ when $v = 1$ or $c(q_{v=0}^t)$ when $v = 0$. The expected profit of the wind technology, by contrast, consists of the spot market revenue $p_f q_f^w$ minus the fixed investment cost $k^w \theta\beta$ and the penalty $p_{v=0}\, q_f^w$ if $v = 0$.

3 Free entry equilibrium

3.1 Spot market

We solve the model via backward induction. Since we assumed that wind output is perfectly correlated among producers, only thermal producers can become active on the spot market whenever the wind does not blow ($v = 0$). With perfect competition, the following conditions hold for an optimal quantity q_v^t and the equilibrium spot market price p_v:

$$(q_{v=0}^t,\ p_{v=0}) = \begin{cases} \left(\frac{D}{n^t},\ c'(\frac{D}{n^t})\right) & \text{for} \quad Q^t \geq D\,, \\ \left(1,\ \beta\right) & \text{for} \quad Q^t < D\,. \end{cases}$$

The equilibrium price p_v is equal to the unit production cost of the last unit of thermal energy that is needed to restock the energy shortage, both in a *short* and in a *failed* market. This price is also what windmill operators have to pay for every unit of energy they promised on the future market but failed to deliver. In case of a *failed* market, the highest marginal cost is the marginal cost at the capacity limit, β and the system operator needs to step in to stabilize the market with an additional supply of $R_{v=0} = D - Q^t$.[4]

3.2 Future market

Thermal producers maximize (2), anticipating on the p_v and q_v^t in the spot market. Windmill operators maximize (3). For any $p_f \geq 0$, the optimal quantities of both types of producers must thus satisfy the Karush-Kuhn-Tucker conditions (Chiang, 1984, pp.722):

$$\begin{aligned} \text{thermal plants:} \quad & \Big[p_f - (1-\theta)p_{v=0} \ - \ \theta c'(q_f^t)\Big] q_f^t \left(q_f^t - 1\right) = 0 \\ \text{windmills:} \quad & \Big[p_f - (1-\theta)p_{v=0}\Big] q_f^w \left(q_f^w - 1\right) = 0\,. \end{aligned}$$

The square brackets show the partial derivatives $\partial\mathbb{E}[\pi^t]/(\partial q_f^t)$ and $\partial\mathbb{E}[\pi^w]/(\partial q_f^w)$, the other factors cover the possible corner solutions of either scheduling nothing on the future market or scheduling the capacity limit.

[4]Note, that even though there is not enough privately provided energy in a failed market, thermal energy producers cannot exert any market power and increase the price above marginal costs of β. This goes back to the assumption that the central operator has sufficient reserve to keep at the level of Q^t.

Thus, the optimal outputs are dependent on the price p_f as follows:

$$q_f^w = \begin{cases} 0 & \text{if } p_f < (1-\theta)p_{v=0} \\ (0,1) & \text{if } p_f = (1-\theta)p_{v=0} \\ 1 & \text{otherwise}\,, \end{cases} \tag{4}$$

$$q_f^t = \begin{cases} 0 & \text{if } p_f < (1-\theta)p_{v=0} \ + \ \theta c'(q_f^t) \\ (0,1) & \text{if } p_f = (1-\theta)p_{v=0} \ + \ \theta c'(q_f^t) \\ 1 & \text{otherwise}\,. \end{cases} \tag{5}$$

To grasp the intuition, recall that windmill operators anticipate the price on the spot market to be equal to the expected penalization costs. Windmill operators compare these costs to the price on the future market. When the expected costs are higher, no wind energy is sold on the future market. When the price is higher, the operators schedule at their capacity limit. Thermal energy producer make a similar comparison. This means that an interior equilibrium, the price p_f must be such that producers are indifferent between scheduling and not.

Turning to the equilibrium prices and quantities, the first thing to note is that $p_f < (1-\theta)p_{v=0}$ cannot occur in equilibrium because supply would be zero at that price, but demand would be strictly positive. Second, our model predicts the merit order of "clean energy first" to endogenously arise in equilibrium. In fact, (4) and (5) together imply that for a price p^f where thermal producers become active on the future market, windmill operators already schedule at their capacity limit. This means that only the following equilibrium configurations may occur: The demand is covered by thermal energy alone (scenario *i*), or demand is covered by both types of energies, such that only windmills operate at their capacity limit (scenario *ii*) or both types of producers (scenario *iii*), or demand is covered by wind energy alone (scenario *iv*).

i) *Just Thermal:* It is possible that only thermal producers are active when no windmill operators enter the market ($n^w = 0$). In this scenario, it must be that $Q^t \geq D$. Moreover, the future market price is equal to $p_f = c'(\frac{D}{n^t})$. The expected profit of thermal producers thus becomes

$$\mathbb{E}[\pi^t] \;=\; c'(\frac{D}{n^t})\,\frac{D}{n^t} - c(\frac{D}{n^t}) \;-\; k^t\frac{\beta}{2}\,,$$

with $\mathbb{E}[\pi^t]+k^t\frac{\beta}{2} = c'(\frac{D}{n^t})\,\frac{D}{n^t} - c(\frac{D}{n^t}) > 0$ since $c(6) = 0$ and since marginal costs are increasing in q_v^t.

ii) *Wind and Thermal:* $Q^t \geq Q_f^t = D - Q_f^w$ and because thermal producers jump in at a higher price than windmill operators, it must be that $Q_f^w = Q^w$. The price p_f satisfies the no arbitrage condition

$$p_f = (1-\theta)p_{v=0} \ + \ \theta c'(q_f^t)\,.$$

where $p_{v=0} = \beta$ when $D > Q^t$ and $p_{v=0} = c'\big(\frac{D}{n^t}\big)$ when $D \leq Q^t$. Expected profits become

$$\begin{aligned}
\mathbb{E}[\pi^t] \ &= \ \theta\Big(c'(\frac{D-Q^w}{n^t})\,\frac{D-Q^w}{n^t} \ - \ c(\frac{D-Q^w}{n^t})\Big) \ - \ k^t\frac{\beta}{2} \\
&+ \ \begin{cases} (1-\theta)\Big(c'(\frac{D}{n^t})\frac{D}{n^t} - c(\frac{D}{n^t})\Big) & \text{if} \quad D \leq Q^t \\ (1-\theta)\Big(\beta - \frac{\beta}{2}\Big) & \text{if} \quad D > Q^t \end{cases} \\
\mathbb{E}[\pi^w] \ &= \ \theta c'(\frac{D-Q^w}{n^t}) \ - \ k^w\theta\beta
\end{aligned}$$

with $\mathbb{E}[\pi^t] + k^t\frac{\beta}{2} > 0$ and $\mathbb{E}[\pi^w] + k^w\theta\beta > 0$.

Even though we have not specified the regulator's role yet, it may have a special interest in the scenario $D \leq Q^t$. Here we observe that thermal energy producers have entered with such large capacities that they can always prevent a system failure on their own. This can be understood as a *natural* reserve, which emerges purely via the market mechanism.

iii) *Capacity limit:* A scenario with $Q^t + Q^w = D$. Note that no less capacity would suffice because this would mean that demand cannot be met even in a long market ($v = 1$). In this scenario, the future market price is equal to the marginal cost of thermal producers at their capacity limit, $p_f = \beta$. The system will *not* always fail because in a long market both types of suppliers can deliver their scheduled amounts. Expected profits thus become

$$\begin{aligned}
\mathbb{E}[\pi^t] \ &= \ \beta - \frac{\beta}{2} \ - \ k^t\frac{\beta}{2} \\
\mathbb{E}[\pi^w] \ &= \ \theta\beta \ - \ k^w\theta\beta\,.
\end{aligned}$$

iv) *Just Wind:* $Q_f^t = 0$, $Q_f^w = D \leq Q^w$, and the price satisfies the no-arbitrage condition

$$p_f = (1-\theta)p_{v=0},$$

where $p_{v=0} = \beta$ when $D > Q^t$ and $p_{v=0} = c'\big(\frac{D}{n^t}\big)$ when $D \leq Q^t$.

Note that p_f cannot be higher than the expected marginal penalty of the windmill operators. If it it would be higher, then p_f would exceed their expected marginal cost of supply of $(1-\theta)p_{v=0}$ according to (5), because thermal producers have a marginal cost of zero at $q_f^t = 0$. This, however, leads to a contradiction to $Q_f^t = 0$. Since thermal producers are only active on the spot market, their expected profit consists of the gross profit on the spot market minus their investment costs. The expected profit of windmill operators consists, in contrast, of the certain profit on the future market minus the penalty when they cannot deliver it and their investment costs. With the future market price exactly compensating for the penalty, we get $\mathbb{E}[\pi^w] = -k^w\theta\beta$. Without further calculation, we can immediately formulate our first proposition.

Proposition 1. *A free entry equilibrium in the Just Wind scenario does not exist.*

The proof is straightforward because wind energy producers have negative expected profits in this scenario. This is a direct consequence of perfect competition between producers without variable production costs but positive fixed costs. Hence, windmill operators will not enter the market, unless there is at least one thermal energy producer active on the future market that drives up the price p_f. This is notable on a more general dimension: If a designer prefers an energy market with only renewable (zero variable costs) energy, for instance for ecological reasons, an energy-only market cannot produce the necessary incentives to guarantee a stable supply.

Comparing the equilibrium prices p_f between the different scenarios, we can observe an additional effect: The higher the share of wind energy in the future market and the larger the total capacity, the lower is the price on the future market.

3.3 Entry and investments

We now turn to the investment phase, in which potential producers choose one of the two technologies and so many of them enter the market until the expected profit of the entrants reaches zero.

In the following, we describe the conditions under which each of the four future market scenarios described above can be supported in a free entry equilibrium. We also study the stability of equilibria by means of tâtonnement stability. We thus say that an equilibrium is stable when any perturbation to the number of entrants n^t or n^w causes a bouncing back to the original point via a dynamic adjustment process.[5]

Scenario *iv*) can be ignored since it cannot arise in a free entry equilibrium by Proposition 1. The following results characterize the conditions under which Scenario *i*) to *iii*) arise.

Proposition 2. *There exists a unique and stable 'Just Thermal' equilibrium with* $n^t = D/\sqrt{k^t}$ *and* $n^w = 0$ *when* $k^t \leq \min\{1, (k^w)^2\}$.

Proof. Given the functional form of the cost function (1), the expected profits as a function of the number of entrants can be written as

$$\begin{aligned} \mathbb{E}[\pi^t|n^t] &= \frac{\beta}{2}(\frac{D}{n^t})^2 - k^t\frac{\beta}{2} \\ \mathbb{E}[\pi^w|n^w = 0] &= \theta\beta\frac{D}{n^t} - \theta\beta k^w \end{aligned} \quad (6)$$

[5]We refer to Mas-Colell et al. (1995) for a profound background of tâtonnement stability.

Entrants will choose the thermal technology until expected profits are zero, giving us $n^t = D/\sqrt{k^t}$. The expected profit of the wind technology needs to be smaller zero at $n^w = 0$. This gives $k^t \leq (k^w)^2$. Finally, $Q^t \geq D$ needs to hold, giving $k^t \leq 1$. Together, these conditions ensure existence of a free entry equilibrium. The equilibrium is moreover stable because $\mathbb{E}[\pi_t|n^t] < 0$ if and only if $n^t > D/\sqrt{k^t}$ so that thermal suppliers exit (enter) the market after a small, random perturbation to n^t. □

A *Just Thermal* equilibrium arises when the wind technology is relatively expensive compared to the thermal technology, and $k^t \leq 1$ such that at least one thermal producer would enter the market. Obviously, there is no need for a spot market or a strategic reserve in this scenario.

Proposition 3. *There exists a unique and stable 'Wind and Thermal' equilibrium with $n^w < D$, $n^t \geq D$, and $R_{v=0} = 0$ when $1 > k^t > (k^w)^2 \geq \frac{k^t - 1 + \theta}{\theta}$. In particular, the equilibrium numbers of firms are given by*

$$
\begin{aligned}
n^w &= D\Big(1 - k^w\sqrt{\frac{1-\theta}{k^t - \theta(k^w)^2}}\Big) \\
n^t &= D\sqrt{\frac{1-\theta}{k^t - \theta(k^w)^2}} \\
R &= 0
\end{aligned}
$$

Proof. Profits as functions of the number of firms become:

$$
\begin{aligned}
\mathbb{E}[\pi^t|n^t] &= \theta\Big(\frac{\beta(D-n^w)}{n^t}\frac{D-n^w}{n^t} - \frac{\beta(D-n^w)^2}{2(n^t)^2}\Big) - k^t\frac{\beta}{2} + (1-\theta)\frac{\beta}{2}(\frac{D}{n^t})^2 \\
&= \frac{\beta}{2}\Big(\theta(\frac{D-n^w}{n^t})^2 + (1-\theta)(\frac{D}{n^t})^2 - k^t\Big) \\
\mathbb{E}[\pi^w|n^w] &= \theta c'(\frac{D-n^w}{n^t}) - k^w \\
&= \theta\beta\Big(\frac{D-n^w}{n^t} - k^w\Big).
\end{aligned}
\tag{7}
$$

Applying the free entry condition, we get the following best reaction function to compute the number of entrants:

$$
\begin{aligned}
n^w := r(n^t) &= D - n^t k^w \\
n^t := r(n^w) &= \sqrt{\frac{D^2 - 2\theta D n^w + \theta(n^w)^2}{k^t}}
\end{aligned}
\tag{8}
$$

Plugging in the n^w expression into the n^t expression in (8), the equilibrium number of firms are given by

$$
\begin{aligned}
n^w &= D\Big(1 - k^w\sqrt{\frac{1-\theta}{k^t - \theta(k^w)^2}}\Big) \\
n^t &= D\sqrt{\frac{1-\theta}{k^t - \theta(k^w)^2}}\,.
\end{aligned} \tag{9}
$$

As we require that $Q^t = n^t \geq D$, we need to have $\frac{k^t}{\theta} > (k^w)^2 \geq \frac{k^t-1+\theta}{\theta}$. Moreover, because we want $n^w > 0$, $(k^w)^2 < k^t$ must be satisfied. Together, this gives the condition $k^t > (k^w)^2 \geq \frac{k^t-1+\theta}{\theta}$. In order for both inequalities to be fulfilled, $k^t < 1$ also needs to hold.

To prove the stability of the equilibrium, it is

$$
\frac{\partial r^t}{\partial n^w}\frac{\partial r^w}{\partial n^t} \;=\; \big(-\frac{\theta(D-n^w)}{n^t}\big)\big(-k^w\big) \;=\; (k^w)^2\theta \;<\; 1\,.
$$

Thus, any perturbation $\epsilon > 0$ to an equilibrium value of n^t (or similarly n^w) initiates a monotonically converging process that leads back to the initial point after an infinite number of steps, that is, a sequence with $(n^t)_1 = n^t \pm \epsilon$, $|(n^t)_{t+1} - n^t| < |(n^t)_t - n^t|$ for $t \geq 1$, and $\lim_{t\to\infty}(n^t)_t = n^t$.

□

In this equilibrium type, there is no need for a strategic reserve because the low investment costs attract enough thermal producers to cover any potential shortages. It emerges when the thermal technology is still comparatively cheap, but not so cheap to keep wind energy producers out of the market. The equilibrium has the *Just Thermal* equilibrium as the borderline case.

Proposition 4. *Multiple 'Wind and Thermal' equilibria with $n^t \in \big(0\,,\,D\big)$, $n^w = D - k^w n^t$, and $n^w + n^t > D$ exist when $(k^w)^2 = \frac{k^t-1+\theta}{\theta} < 1$. None of these equilibria is stable.*

Proof. In this scenario, expected profits are given by

$$
\begin{aligned}
\mathbb{E}[\pi^t|n^t] &= \theta\Big(\frac{\beta(D-n^w)}{n^t}\frac{D-n^w}{n^t} - \frac{\beta(D-n^w)^2}{2(n^t)^2}\Big) - k^t\frac{\beta}{2} + (1-\theta)\frac{\beta}{2} \\
&= \frac{\beta}{2}\Big(\theta(\frac{D-n^w}{n^t})^2 + (1-\theta) - k^t\Big) \\
\mathbb{E}[\pi^w|n^w] &= \theta\beta\Big(\frac{D-n^w}{n^t} - k^w\Big)
\end{aligned} \tag{10}
$$

Free entry implies that the numbers of firms must satisfy the best response functions

$$
\begin{aligned}
n^w := r^{t'}(n^t) &= D - k^w n^t \\
n^t := r^{w'}(n^w) &= \sqrt{\frac{\theta}{k^t-1+\theta}}\,(D-n^w)\,.
\end{aligned}
$$

These two equations have a unique intersection point at $n^w = D$ and $n^t = 0$, unless

$$(k^w)^2 = \frac{k^t - 1 + \theta}{\theta},$$

in which case any $n^t \in (0\,,\,D)$ and $n^w = D - k^w n^t$ is an equilibrium when the following additional condition is satisfied: As we need to have $n^t + n^w > D$, we additionally require $1 - \theta < k^t < 1$. This also implies that $k^w < 1$.

To prove the instability, adding (removing) a small amount $\epsilon > 0$ to (from) an equilibrium value of n^t, $0 < n^t < D$ leads to a new equilibrium point $(n^t)' = n^t + \epsilon$ and $(n^w)' = D - k^w(n^t)'$, from which there is no return to (n^t, n^w). □

We observe that a *Wind and Thermal* equilibrium with $n^t \in \big(0\,,\,D\big)$ only emerges under a particular parameter constellation. Moreover, if such an equilibrium emerges, it is unstable.

Proposition 5. *Multiple 'capacity limit' equilibria with $n^w, n^t > 0$ and $n^t + n^w = D$ exist when $k^t = k^w = 1$. None of these equilibria is stable.*

Proof. Filling in the the cost function (1) into the expected profit functions gives

$$\begin{aligned} \mathbb{E}[\pi^t|n^t] &= \frac{\beta}{2} - k^t\frac{\beta}{2} \\ \mathbb{E}[\pi^w|n^w] &= \theta\beta - \theta\beta k^w\,. \end{aligned} \tag{11}$$

Hence, in order to satisfy the break even condition, k^t and k^w must be equal to 1. Any numbers of n^w and n^t is an equilibrium in this case as long as $n^t + n^w = D$. Yet, all equilibria are unstable. To see this, consider an equilibrium point (n^w, n^t). Removing a small amount ϵ from n^t has no impact on the expected profits of wind and thermal producers, respectively. Yet, the market clearing condition is violated, which yields several ways to return to an equilibrium: (i) add ϵ to n^t, (ii) add ϵ to n^w, or (iii) any combination of the two until we end up at a new equilibrium point with $(n^t)' + (n^w)' = D$. Since (ii) and (iii) are options, the initial equilibrium is unstable. □

Similar to the previous scenario, this equilibrium type also only emerges under a particular parameter constellation. Total energy capacity is just enough to serve the demand, therefore a strategic reserve as high as the total wind capacity is needed to close the gap between demand and what thermal producers can supply under a failed market.

So far, we have examined all possible equilibrium configurations and calculated the necessary parameter conditions on the investment costs. If we combine these conditions, we make two observations:

Corollary 1. *There is a unique stable free entry equilibrium when $k^t \leq 1$ and $(k^w)^2 \geq \frac{k^t - 1 + \theta}{\theta}$. In contrast, no free entry equilibrium exist when $k^t > 1$ or $(k^w)^2 < \frac{k^t - 1 + \theta}{\theta}$.*

To develop some intuition, consider first the case where $k^t > 1$. Note that, by comparison of the profit terms in (6), (11), (7), and (10), the highest equilibrium profit that a thermal energy producer can collect is $\frac{\beta}{2} - \frac{\beta}{2}k^t$. This is what a producer earns when no wind is offered on the future market and the total thermal capacity is just enough to cover the demand, i.e, $n^t = D$ (see in particular (6)). When $k^t > 1$, this however means that thermal profits are negative in any equilibrium constellation. In other words, a free entry equilibrium does not exist when $k^t > 1$ because the thermal technology is too expensive.

Consider next the case where $(k^w)^2 < \frac{k^t-1+\theta}{\theta}$ and thus where the wind technology is relatively cheap. In fact, it is so cheap in this case that, by (9), it must be that $0 < n^t < D$ and, thus, that relatively few thermal producers are active. This, however, means by (10) that the expected profit of the thermal technology becomes $(\beta/2)(\theta(k^w)^2+1-\theta-k^t)$, which is negative for any $0 < n^t < D$ when $(k^w)^2 < \frac{k^t-1+\theta}{\theta}$. Hence, the wind technology crowds out the thermal technology in this case. Eventually, this leads us to the *Just Wind* scenario described in Proposition 1, where we know that also no wind producer can make a positive profit. In other words, there does not exist a free entry equilibrium when $(k^w)^2 < \frac{k^t-1+\theta}{\theta}$ because of the excessive entry of wind producers.

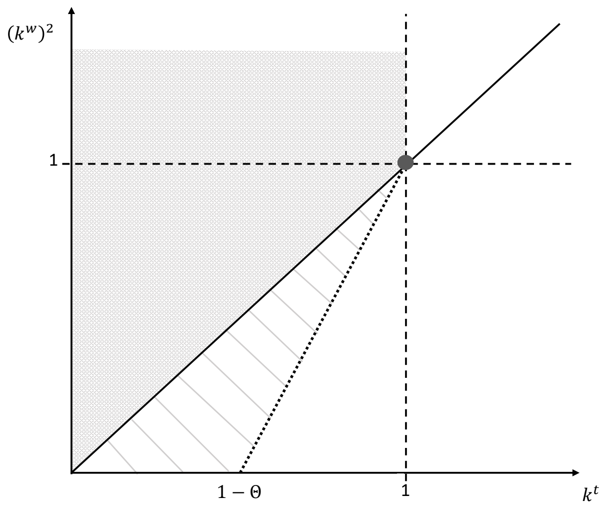

Figure 1: Free entry equilibrium

We can combine the previous statements in following figure. It shows the free entry equilibria for any combination of investment costs. The gray solid area marks the stable *Just Thermal* equilibria of Proposition 2 and the dashed area the stable *Wind and Thermal* equilibria of Proposition 3. The dotted line marks the locations of the unstable *Wind and Thermal* equilibria of Proposition 4 and the dot at $(1, 1)$ the locus of the unstable *capacity limit* equilibrium. In the remaining parameter regimes, we do not have any free entry equilibrium.

So far, we have investigated the conditions under which a competitive market is able to serve the energy market on its own. One of the findings is that in none of the *stable* equilibria sketched above, a strategic reserve is a meaningful supplement to the market solution. Either the market is able to provide a sufficient amount of thermal capacity on its own to avoid *system failure* on the delivery day or the investment incentives are so weak that a system operator would need to build up a strategic reserve as large as the entire demand to ensure system stability under all states of the wind.

This raises the question for an alternative solution, such as a capacity payment scheme.[6] Moreover, it raises the question whether a benevolent system operator does actually want to secure the system under costs. We turn to the second question first.

4 Welfare analysis

A benevolent system operator is on the one hand obviously interested in maintaining a properly running energy system. This is why thermal energy capacities may be justified. However, on the other hand the operator may also want to avoid the associated environmental damages. Therefore, we determine here the socially optimal levels of thermal, n^t, and wind capacities, n^w, assuming that thermal energy production pollutes the environment.[7] Hence, in this section we do not consider free market entry of private firms. Instead, we analyze the market constellation a central planner would choose. To perform the analysis, we need to add one more assumption: we said that consumers' reservation price is given by $p = 1$ up until amount D and that the unit cost per thermal output is $\beta \leq 1$ at the capacity limit. Suppose, now, that ρ measures the social cost of pollution per unit of thermal energy burned and we assume that $0 \leq \rho < 1$. This upper bound ensures that at least some thermal energy can make sense from a societal viewpoint.

Social welfare is a function of n^t and n^w and consists of the likelihood that the built-up capacity can meet the demand D, minus variable costs of thermal energy production, minus the amount of pollution, and minus the total investment costs in both thermal and wind capacities. We can distinguish between six regimes of how a socially optimal capacity mix may look like. Note that these regimes only partially overlap with the scenarios of Section 3 because first, we do not differentiate between privately and publicly thermal energy here and second, we consider regimes where partial blackouts may be acceptable for the system operator.

(1) First, the system operator just builds up thermal capacities. The welfare expression becomes

$$W(n^t) = D - n^t \frac{\beta}{2} \big(\frac{D}{n^t}\big)^2 - \rho D - n^t k^t \frac{\beta}{2} \,,$$

[6] Another alternative solution, which we do not look at, is to allow a single or a few suppliers enter the market who can then set their prices above their production cost on the future market. Also, we ignore the possibility that a system operator might offer its strategic reserve on the future market to lower the residual demand $D - R$ below the capacity of the thermal producers, n^t. Instead, we assume that the strategic reserve is only meant as a backup on the delivery day.

[7] For comparison with the free entry model of the previous section, the socially optimal level of thermal capacity, n^t, computed here corresponds to the sum of privately provided and strategic reserve capacities computed there, which are both of thermal energy origin.

with $n^t \geq D$. The first-order condition is $-k^t + D^2/(n^t)^2 = 0$ and thus, we get $n^t = D/\sqrt{k^t}$. Moreover, for $n^t \geq D$ to be satisfied, we require that $k^t \leq 1$. This regime is equivalent to the *Just Thermal* scenario of Section 3.

(2) Here, the system operator chooses a thermal capacity stock that is sufficient to serve the entire demand alone, and additionally a stock of wind capacities, which are, however, insufficient to serve the demand. In both states of the wind, $v = 1$ and $v = 0$, energy supply is thus secured by thermal energy, but with more pollution when $v = 0$. The welfare expression becomes

$$\begin{aligned} W(n^t, n^w) &= \theta\Big[D - n^t\frac{\beta}{2}(\frac{D-n^w}{n^t})^2 - \rho(D - n^w)\Big] \\ &+ (1-\theta)\Big[D - n^t\frac{\beta}{2}(\frac{D}{n^t})^2 - \rho D\Big] - n^w k^w \theta\beta - n^t k^t \frac{\beta}{2} \end{aligned}$$

under the constraints that $n^t \geq D$ and $0 < n^w < D$. The Karush-Kuhn-Tucker conditions are[8]

$$\begin{aligned} -k^t + \theta\frac{(D-n^w)^2}{(n^t)^2} + (1-\theta)\frac{D^2}{(n^t)^2} &= 0 \quad \text{and} \\ -k^w + \frac{\rho}{\beta} + \frac{D-n^w}{n^t} &= 0 \quad \text{and} \\ n^t &\geq D \quad \text{and} \\ 0 < n^w &< D\,. \end{aligned}$$

Combining these conditions result in the unique interior solution

$$\begin{aligned} n^w &= D\Big(1 - (k^w - \frac{\rho}{\beta})\sqrt{\frac{1-\theta}{k^t - \theta(k^w - \frac{\rho}{\beta})^2}}\Big) \\ n^t &= D\sqrt{\frac{1-\theta}{k^t - \theta(k^w - \frac{\rho}{\beta})^2}}\,. \end{aligned}$$

We thus call this solution the *Wind and Thermal* solution. Note that it satisfies the second-order condition for a local maximum, i.e., the Hessian matrix at this point is negative definite. Moreover, the solution requires that $1 > k^t > (k^w - \frac{\rho}{\beta})^2 \geq (k^t - 1 + \theta)/\theta$ for $n^t \geq D$ and $0 < n^w < D$ to hold true.

(3) The system operator may alternatively build up a capacity stock so that the entire demand is satisfied by wind energy when $v = 1$ and by thermal energy when $v = 0$. This regime is thus equivalent to the *Just Wind* equilibrium of Section 3. The welfare expression is given by

$$W(n^t, n^w) = \theta D + (1-\theta)\Big[D - n^t\frac{\beta}{2}(\frac{D}{n^t})^2 - \rho D\Big] - n^w k^w \theta\beta - n^t k^t \frac{\beta}{2}\,,$$

[8] We omit the alternative optimality condition $n^t - D = 0$ because the existing condition in line must be satisfied even at $n^t = D$ for otherwise the system operator would prefer a capacity regime with $n^t < D$.

under the constraints that $n^t, n^w \geq D$. Obviously, $n^w = D$ is optimal here. Moreover, the first-order condition for an interior n^t, i.e., $-k^t + (1-\theta)(D/n^t)^2 = 0$, gives

$$n^t \quad = \quad D\sqrt{\frac{1-\theta}{k^t}}\,.$$

This solution satisfies the second-order condition $\partial^2 W/\partial(n^t)^2 < 0$ for a local maximum. Moreover, it requires that $k^t \leq 1-\theta$ for $n^t \geq D$ to hold true.

(4) In this regime, the system operator may build up a capacity mix where thermal capacities are insufficient to serve the total demand alone, but jointly with wind energy demand can be met. Since $n^t < D$ in this case, the system crashes if $v = 0$ (which happens with probability $1-\theta$) so that neither positive welfare nor pollution is generated. Welfare can thus be written as

$$W(n^t, n^w) = \theta\Big[D - n^t\frac{\beta}{2}\big(\frac{D-n^w}{n^t}\big)^2 - \rho(D-n^w)\Big] - n^w k^w \theta\beta - n^t k^t \frac{\beta}{2}\,,$$

with the inequality and non-negativity constraints $n^t + n^w \geq D$, $0 \leq n^t < D$, and $n^w \geq 0$. Obviously, there is no reason to build up wind capacities in excess of D because when the system is positive, the total demand can be met with wind alone even when $n^w = D$. Hence, the Karush-Kuhn-Tucker conditions become[9]

$$\begin{aligned}
n^t\Big[\theta\frac{(D-n^w)^2}{(n^t)^2} - k^t + y\Big] &= 0 \quad \text{and}\\
\frac{\rho}{\beta} + \frac{D-n^w}{n^t} - k^w + y &= 0 \quad \text{and}\\
\Big[n^t + n^w - D\Big]y &= 0 \quad \text{and}\\
0 \leq n^t &< D \quad \text{and}\\
0 \leq n^w &\leq D \quad \text{and}\\
y &\geq 0\,,
\end{aligned}$$

where y denotes the Lagrangian multiplier that appears because of the inequality constraint $n^t + n^w \geq D$ (Chiang, 1984, pp.722).

These conditions have four potential solutions, all of which lead to the same solution $n^t = 0$ and $n^w = D$. Hence, intuitively, the planner prefers just "clean" energy in this capacity regime. This is intuitive because thermal capacities are insufficient to serve the demand alone ($n^t < D$) so that the system crashes anyway when $v = 0$.

Note that this solution is not identical to the *Just Wind* equilibrium described in Section 3

[9] We omit the alternative optimality condition $n^w - D = 0$ from this set of conditions because the existing condition in line two must be satisfied even at $n^w = D$ for otherwise the system operator would prefer a capacity $n^w \neq D$.

since there we assumed that a crash of the market will be avoided by the system operator by means of a strategic reserve. We therefore call this regime here a *Pure Wind* regime because the system operator only builds up wind capacities.

(5) Finally, the system operator may build up capacities so small that the demand can never be met in no state of the wind, that is, $n^t + n^w < D$. This, however, means that it is optimal to set $n^t = n^w = 0$.

Now that we have allocated welfare to the different regimes, we can add efficiency to the previous discussion of the market outcomes. We can distinguish between two cases, one with low pollution, i.e. $\rho \leq 1 - \beta$, and one with high pollution $\rho > 1 - \beta$

Proposition 6. *Let $\rho \leq 1-\beta$. Then, for $k^w > \rho/\beta$ the welfare maximizing regime is*

- *regime (1) when $(k^w - \frac{\rho}{\beta})^2 \geq k^t$ and $k^t \leq 1$,*
- *regime (2) when $k^t > (k^w - \frac{\rho}{\beta})^2 \geq (k^t - 1 + \theta)/\theta$,*
- *regime (4) when $(k^w - \frac{\rho}{\beta})^2 < (k^t - 1 + \theta)/\theta$ and $1 + \frac{\rho}{\beta} \geq k^w$, and*
- *regime (5) when $k^w > 1 + \frac{\rho}{\beta}$ and $k^t > 1$.*

Whereas for $k^w \leq \rho/\beta$ the welfare maximizing regime is

- *regime (3) when $k^t \leq 1 - \theta$*
- *regime (4) when $k^t > 1 - \theta$*

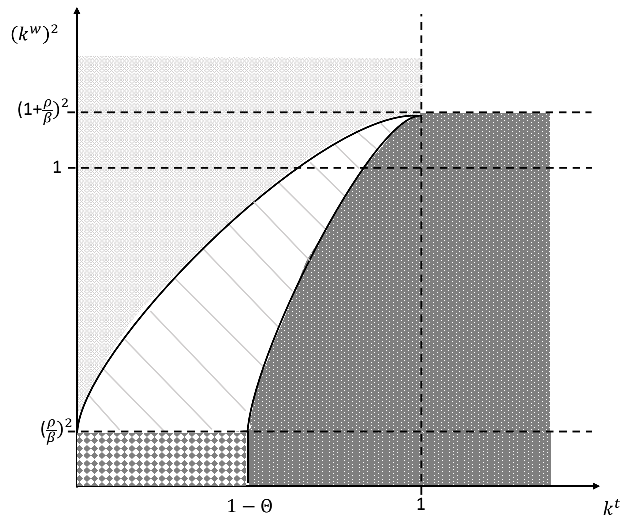

Figure 2: Welfare efficiency for $\rho \leq 1 - \beta$

The proof is in the appendix. Broadly speaking, the socially optimal capacity mix depends on the relative investment costs for thermal and wind capacities, respectively. Interestingly, it can be socially desirable to just invest in wind energy and to afford blackouts as in regime (4). This is the case when the social costs of pollution and/or the thermal investment costs exceed the benefits of a steady energy supply. Another interesting regime is regime (3), where the system operator maintains an overcapacity that is sufficient to cover twice the demand when $v = 1$. This regime is preferred when investment costs for both technologies are low.

Figure 2 sketches the welfare efficient regimes when pollution is low ($\rho \leq 1 - \beta$). In the light gray area and in the shaded area, regime (1) and regime (2) respectively is welfare maximizing. These regimes are already known from Figure 1 as the *Just Thermal* and the *Wind and Thermal* equilibrium. New is the dotted area where regime (3) is welfare efficient. In the dark gray area regime (4) and in the remaining white areas regime (5) are welfare-maximizing.

We get similar results for efficient regimes with high pollution, i.e. $\rho > 1 - \beta$.

Proposition 7. *Let $\rho > 1 - \beta$ and $k^w > \rho/\beta$. Then, the welfare maximizing regime is*

- *regime (1) when $(k^w - \frac{\rho}{\beta})^2 \geq k^t$ and $k^t \leq (\frac{1-\rho}{\beta})^2$,*
- *regime (2) when $k^t > (k^w - \frac{\rho}{\beta})^2 \geq (k^t - (1-\theta)(\frac{1-\rho}{\beta})^2)/\theta$,*
- *regime (4) when $(k^w - \frac{\rho}{\beta})^2 < (k^t - (1-\theta)(\frac{1-\rho}{\beta})^2)/\theta$ and $1/\beta \geq k^w$, and*
- *regime (5) when $k^w > 1/\beta$ and $k^t > (\frac{1-\rho}{\beta})^2$,*

When $k^w \leq \rho/\beta$ instead, the welfare maximizing regime is

- *regime (3) when $k^t \leq (1-\theta)\left(\frac{1-\rho}{\beta}\right)^2$*
- *regime (4) when $k^t > (1-\theta)\left(\frac{1-\rho}{\beta}\right)^2$.*

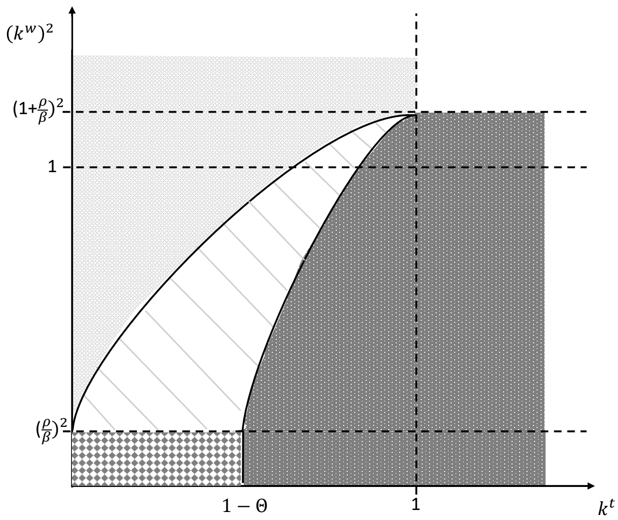

Figure 3: Welfare efficiency for $\rho > 1 - \beta$

The proof is also in the appendix. Figure 3 illustrates the welfare-efficient regimes. The same legend of Figure 2 applies to this figure, i.e. light gray is regime (1), shaded is regime (2), dotted is regime (3), dark gray is regime (4) and the remaining white area regime (5). In addition, we take $\bar{k}^t = (1-\theta)\left(\frac{1-\rho}{\beta}\right)^2$. We observe the same pattern in the high pollution case as in the low pollution, just with a stronger impact. The intuition behind it is that with high pollution the range where the social planner prefers the pure wind regime naturally increases.

So far we have identified all scenarios where the market is able to provide energy as well as the regimes that a social planner would choose. Now we are in the position to compare the market outcomes with the efficient solutions.

Corollary 2. *When $\rho = 0$, then every stable market equilibrium is efficient.*

The result is a simple combination of Propositions 2, 3, and 6, and can be seen as an application of the first welfare theorem. However, do note that even when $\rho = 0$ there can be an important source of inefficiency in our model. In particular, when $(k^w)^2 < (k^t - 1 + \theta)/\theta$ and $k^t \leq 1$ then a benevolent system operator would want to establish a *Pure Wind* regime (4), but as we know, such a system cannot be maintained in an unregulated market.

If we consider cases with $\rho > 0$ instead, we observe the following:

Corollary 3. *When $\rho > 0$, only the 'Just Thermal' equilibrium is also efficient if in addition (a) $(k^w - \frac{\rho}{\beta})^2 \leq k^t < 1$ when $\rho < 1 - \beta$ or (b) $(k^w - \frac{\rho}{\beta})^2 \leq k^t < (\frac{1-\rho}{\beta})^2$ when $\rho \geq 1 - \beta$.*

Here we combine Propositions 2, 3, 6, and 7, such that we are within the borders of regime (1). Thus, the only case that is both welfare efficient and yields a stable market outcome is when a social planner neglects the pollution factor. Yet, with increasing pollution the range of this result becomes successively smaller. Further, we observe that no unique and stable *Wind and Thermal* equilibria of Proposition 3 can also be efficient, because pollution always induces the social planner to choose less thermal energy. Hence, it becomes even less likely that the system operator can rely on market logic to provide the needed energy.

5 Capacity payments

We now consider a policy to attain a perfect match between the market outcome of Section 3 and the welfare efficient solution. A simple and effective way to stimulate wind investments (and to stifle thermal investments) is to subsidize entry with the wind technology by $t^w \leq 0$ and to tax entry with the thermal technology by $t^t \geq 0$ in the investment stage of the game. To obtain tax neutrality, the system operator could then reclaim (reimburse) the subsidy (tax income) from (to) consumers. Given the inelastic demand, one way to do this is in the form of a use-independent lump sum transfer, another equivalent way is a consumption-dependent tax (subsidy).

In the following, we first write down the profits of thermal and wind producers in the different solutions developed in Section 4 as functions of t^t and t^w. We then determine the levels of t^t and t^w that support these solutions in a free entry equilibrium.

i) For a *Just Thermal* equilibrium with subsidies/taxes corresponding to welfare regime (1), i.e, with $n^w = 0$ and $n^t \geq D$, we need to set t^w and t^t such that

$$\begin{aligned}\mathbb{E}[\pi_t | n^t, n^w = 0, t^t] &= \frac{\beta}{2}\Big((\frac{D}{n^t})^2 \; - \; (k^t + t^t) \Big) = 0 \\ \mathbb{E}[\pi_w | n^t, n^w = 0, t^w] &= \theta\beta\Big(\frac{D}{n^t} - (k^w + t^w) \Big) \leq 0\,.\end{aligned}$$

The number of thermal producers as a function of t^t then becomes $n^t = D/\sqrt{k^t + t^t}$.

ii) In order to get a *Wind and Thermal* equilibrium (type I) corresponding to welfare regime (2), i.e, with $n^t \geq D$ and $n^w < D$, subsidies and taxes need to ensure that

$$\begin{aligned}\mathbb{E}[\pi_t | n^t, n^w, t^t] &= \frac{\beta}{2}\Big(\theta(\frac{D - n^w}{n^t})^2 \; + \; (1-\theta)(\frac{D}{n^t})^2 \; - \; (k^t + t^t) \Big) \; = \; 0 \\ \mathbb{E}[\pi_w | n^t, n^w, t^w] &= \theta\beta\Big(\frac{D - n^w}{n^t} \; - \; (k^w + t^w) \Big) \; = \; 0\,.\end{aligned}$$

The equilibrium number of firms as functions of t^t and t^w thus become

$$\begin{aligned}n^w &= D\Big(1 - (k^w + t^w)\sqrt{\frac{1-\theta}{k^t + t^t - \theta(k^w + t^w)^2}} \Big) \\ n^t &= D\sqrt{\frac{1-\theta}{k^t + t^t - \theta(k^w + t^w)^2}}\,.\end{aligned}$$

iii) For a *Wind and Thermal* equilibrium (type II) corresponding to welfare regime (3), i.e. $n^t \geq D$ and $n^w = D$, profits need to satisfy the same conditions as under *iii*). The equilibrium number of thermal producers as a function of t^t then becomes

$$n^t \;=\; D\sqrt{\frac{1-\theta}{k^t + t^t}}\,.$$

iv) To maintain a *Pure Wind* equilibrium with subsidies/taxes corresponding to welfare regime (4), i.e. $n^w = D$ and $n^t = 0$, we need to set t^w and t^t such that

$$\begin{aligned}\mathbb{E}[\pi_t | n^t = 0, n^w = D, t^t] &= \frac{\beta}{2}\Big(1 - \theta - (k^t + t^t) \Big) \leq 0 \\ \mathbb{E}[\pi_w | n^t = 0, n^w = D, t^w] &= 0 \; - \; \theta\beta(k^w + t^w) \; = \; 0\,.\end{aligned}$$

v) Finally, let a *breakdown equilibrium* correspond to welfare regime (5), i.e, with $n^w = n^t = 0$, we require that

$$\begin{aligned}\mathbb{E}[\pi_t | n^t = n^w = 0, t^t] &= \frac{\beta}{2}\Big(1 \; - \; (k^t + t^t) \Big) \leq 0 \\ \mathbb{E}[\pi_w | n^t = n^w = 0, t^w] &= \theta\beta\Big(1 - (k^w + t^w) \Big) \leq 0\,.\end{aligned}$$

These conditions enable us to compute the optimal subsidy/tax system which induces an efficient result. The following result summarizes.

Proposition 8. *The optimal level of taxes and subsidies are given by*

i) $t^w \geq -\rho/\beta$ *and* $t^t = 0$ *when the parameter conditions of welfare regime (1) are met,*

ii) $t^w = -\rho/\beta$ *and* $t^t = 0$ *when the parameter conditions of welfare regime (2) are met,*

iii) $t^w = -k^w$ *and* $t^t = 0$ *when the parameter conditions of welfare regime (3) are met.*

iv) $t^w = -k^w$ *and* $t^t \geq 1 - \theta - k^t$ *when the parameter conditions of welfare regime (4) are met,*

v) $t^w \geq 1 - k^w$ *and* $t^t \geq 1 - k^t$ *when the parameter conditions of welfare regime (5) are met.*

Proof. Simply note that for the taxes and subsidies specified above, the equilibrium numbers of suppliers, n^w and n^t, are identical to the efficient number of plants derived in Section 4. □

Let us interpret this result step by step. Both in regime (3) and (4), wind energy is supposed to cover full demand as long as it is available. However, since the market cannot generate this outcome, investment costs have to be fully covered by subsidies in *iii*) and *iv*). Thus, windmill operators are just paid for availability when the price on the future market is zero.

Different among these two settings is that in the *Pure Wind* regime thermal investors should be kept out of the market. Hence, a tax of $t^t > 1 - \theta - k^t$ prevents that thermal investors can make profit. In contrast, no taxes are collected from thermal energy producers in all other cases, except for the *breakdown* case *v*). The reason for $t^t = 0$ is that society needs thermal producers to cover the demand if there is no wind available.

The *Just Thermal* regime (1) implies that windmill entry is socially not desirable. Whether wind energy investors are not subsidized too much with $t^w \geq -\rho/\beta$ or not at all, lead to the same result: Only thermal investors are active and do so without the need of further incentives. Thus, since no windmills are actually built in this equilibrium constellation, no state intervention is needed and the policy is revenue neutral.

In case *ii*), the optimal policy is similar to the described equilibrium constellation *iii*) above: Thermal investments are not penalized unlike what is expected to be optimal for a standard market with a global externality. The reason is that society needs thermal production in order to ensure system balance. Windmills are, in contrast, subsidized to increase their share in the production mix. The optimal policy is thus not demand neutral. It is also interesting that the higher β, the smaller the optimal subsidy t^w needs to be. This is reminiscent of earlier results that less money is missing the higher the markup over (average) costs.

6 Conclusion

In this paper we have discussed the challenge of the energy market to deal with an increasing share of renewable energy sources and volatile supply. In order to understand the advantages and disadvantages of different policy measures, we set up a model to identify the possibilities of the market to come up with a solution by itself. The model reflects a competitive setting of renewable energy producers with volatile output but no variable costs vs conventional energy producers with reliable output and variable costs. In three stages producers first decide to build up capacities, and then sell energy in a perfectly competitive future market. If renewable energy falls short to deliver the promised energy, conventional energy producers can compensate the shortage on the spot market.

First, we consider a rather mild impact of a capacity mechanism with the strategic reserve. Thus, the state is able to prevent a market crash, but the reserved energy has no further effect on future or spot market. We show that in this setup the market can only establish two scenarios. On the one hand, there is just conventional energy in the market, if investments in this energy is much cheaper than in renewable energy. On the other hand, there is a scenario with some renewable energy and enough conventional to serve the entire market. In both cases a strategic reserve is not needed. In contrast, a scenario with just renewable energy cannot occur because investment costs can never be compensated when the competition pushes the price to zero. Other scenarios where a strategic reserve would be needed does not produce a stable equilibrium if we apply tâtonnement stability. Hence, market incentives are either strong enough to create the entire supply or none at all.

Next, we run a welfare analysis with a central planner taking investment costs, variable costs and additionally a pollution factor for conventional energy into consideration. We identify the market constellation of renewable and conventional energy that is socially desirable and thus welfare efficient. A pollution factor of zero causes each stable equilibrium to be also efficient. However, with increasing pollution the capacity of the market to come up with a solution is constantly shrinking.

Last, we formulate an alternative approach to nationalizing total energy production if the market cannot provide sufficient energy. We introduce a system of subsidizing investments in renewable energy and taxing investments in conventional energy. Taxes only make sense to keep conventional energy out of the market whenever the central planner prefers a blackout if renewable energy is not available. Contrarily, renewable energy should be subsidized either partly in order to boost investments, or even with total investment costs. The latter case occurs when renewable energy is supposed to cover total demand if it is available.

Further research could tackle the question how other types of capacity mechanisms would affect the outcome in our competitive setup. The role of market power, which could be an answer to the challenges mentioned in this paper, could also be investigated in more detail. And finally, empirical research could check whether or not the theoretical results of this paper pass an application test.

A Appendix

The following proof serves for both Proposition 6 and 7.

Proof. If we combine regime (1) and (2), welfare can be written as

$$W = \begin{cases} 0 & \text{if } (6) \\ \theta D(1-k^w\beta) & \text{if } (2) \\ D(1-\sqrt{k^t}\beta-\rho) & \text{if } (3) \\ D(1-\theta k^w\beta) - D\beta\sqrt{(1-\theta)(k^t-\theta(k^w-\frac{\rho}{\beta})^2)} - (1-\theta)\rho D & \text{if } (4) \\ D(1-\theta k^w\beta) - \beta D\sqrt{(1-\theta)k^t} - (1-\theta)\rho D & \text{if } (5) \end{cases}$$

In particular, welfare in regime (4) follows from the intermediate expressions

$$\begin{aligned} & D - \theta k^w\beta D\big(1-(k^w-\frac{\rho}{\beta})Z\big) - k^t\frac{\beta}{2}ZD - \theta\Big(\frac{\beta}{2}(k^w-\frac{\rho}{\beta})^2 + \rho(k^w-\frac{\rho}{\beta})\Big)ZD \\ - & (1-\theta)\Big(\rho D + \frac{\beta}{2}D/Z\Big) \\ = & D(1-\theta k^w\beta) + \theta k^w\beta(k^w-\frac{\rho}{\beta})ZD - k^t\beta ZD + \frac{\beta}{2}\Big(k^t-\theta(k^w-\frac{\rho}{\beta})^2\Big)ZD - \theta\rho(k^w-\frac{\rho}{\beta})ZD \\ - & (1-\theta)\Big(\rho D + \frac{\beta}{2}D/Z\Big) \\ = & D(1-\theta k^w\beta) + \theta\beta(k^w-\frac{\rho}{\beta})^2ZD - k^t\beta ZD - (1-\theta)\rho D \\ = & D(1-\theta k^w\beta) - \beta(k^t-\theta(k^w-\frac{\rho}{\beta})^2)ZD - (1-\theta)\rho D \\ = & D(1-\theta k^w\beta) - \beta\sqrt{(1-\theta)(k^t-\theta\beta(k^w-\frac{\rho}{\beta})^2)}D - (1-\theta)\rho D\,, \end{aligned}$$

where $Z \equiv \sqrt{\frac{1-\theta}{k^t-\theta(k^w-\frac{\rho}{\beta})^2)}} \geq 1$. The first and the second line is nothing but optimal n^w and n^t filled in. The third and the fourth line follows from expansion (and reduction) by the same summand $-k^t\frac{\beta}{2}ZD$. Going to line five, note that $(k^t-\theta(k^w-\frac{\rho}{\beta})^2)Z = (1-\theta)/Z$ so that two summands cancel against each other. Moreover, $\theta k^w\beta(k^w-\frac{\rho}{\beta})ZD - \theta\rho(k^w-\frac{\rho}{\beta})ZD = \theta\beta(k^w-\frac{\rho}{\beta})^2ZD$. Line six summarizes the terms and line seven uses that $(k^t-\theta(k^w-\frac{\rho}{\beta})^2)Z = \sqrt{(1-\theta)(k^t-\theta\beta(k^w-\frac{\rho}{\beta})^2)}$.

Case $k^w \leq \rho/\beta$: By inspection, welfare under (5) dominates welfare under (4) and also (3) whenever $k^w \leq \rho/\beta$. Comparing (5) to (2) we see that (5) is dominant for $k^t \leq (1-\theta)\big(\frac{1-\rho}{\beta}\big)^2$. However, regime (5) is by definition only feasible for $k^t \leq 1-\theta$. Hence, the lower bound for this regime is $(1-\theta)\big(\frac{1-\rho}{\beta}\big)^2$ if $\rho > 1-\beta$ and $k^t \leq 1-\theta$ otherwise. Moreover, (6) is no option when $k^w \leq \rho/\beta$ because it is dominated by (2).

Case $k^w > \rho/\beta$ and $\rho \leq 1-\beta$ and $k^t \leq 1$: Next, we consider $k^w > \rho/\beta$, such that (4) is strictly better than (5). Moreover, it is $W(3) \geq W(5)$ if and only if $k^t \leq (\frac{\theta}{1-\sqrt{1-\theta}})^2(k^w-\frac{\rho}{\beta})^2$, whereby

$(\frac{\theta}{1-\sqrt{1-\theta}})^2 > 1$ for all feasible $0 < \theta < 1$. Finally, it is $W(3) > W(4)$ if and only if $k^t < (k^w - \frac{\rho}{\beta})^2$. This means that out of the three, (5) is preferred when $(k^w - \frac{\rho}{\beta})^2 < (k^t - 1 + \theta)/\theta$, (4) is preferred when $k^t > (k^w - \frac{\rho}{\beta})^2 \geq (k^t - 1 + \theta)/\theta$ and (3) is preferred when $(k^w - \frac{\rho}{\beta})^2 \geq k^t$. However, since (5) is only valid if $k^t \leq 1 - \theta$, the condition for (5) can never be met. Hence, (5) is dominated by (3) and (4), which makes intuitive sense since the social planner would want to avoid overinvestments.

Adding regime (2) to the picture, it is $W(4) \geq W(2)$ if and only if $(k^w - \frac{\rho}{\beta})^2 \geq (k^t - (1-\theta)(\frac{1-\rho}{\beta})^2)/\theta$. When $\rho \leq 1 - \beta$, then the lower bound $(k^t - (1-\theta)(\frac{1-\rho}{\beta})^2)/\theta$ is weakly smaller than $(k^t - 1 + \theta)/\theta$. Thus, out of (2), (3), and (4), (2) is preferred when $(k^w - \frac{\rho}{\beta})^2 < (k^t - 1 + \theta)/\theta$, (4) is preferred when $k^t > (k^w - \frac{\rho}{\beta})^2 \geq (k^t - 1 + \theta)/\theta$, and (3) is preferred when $(k^w - \frac{\rho}{\beta})^2 \geq k^t$.

Adding regime (6) to the picture, it is $W(4) \geq W(6)$ if and only if $(k^w - \frac{\rho}{\beta})^2 \geq (k^t - (1-\theta)(\frac{1-\beta\theta k^w}{\beta(1-\theta)} - \frac{\rho}{\beta})^2)/\theta$, $W(2) \geq W(6)$ if and only if $k^w \leq 1/\beta$, and $W(3) \geq W(6)$ if and only if $k^t \leq (\frac{1}{\beta} - \frac{\rho}{\beta})^2$. This last condition is only binding when $\rho > 1 - \beta$, since (3) also requires that $k^t \leq 1$. Hence, we have $W(3) \geq W(6)$ if and only if $k^t \leq 1$ and $\rho \leq 1 - \beta$. This means that (6) is no option whenever $k^t \leq 1$ and $\rho \leq 1 - \beta$ because it is dominated by (3).

Case $k^w > \rho/\beta$ and $\rho > 1 - \beta$ and $k^t \leq (\frac{1-\rho}{\beta})^2$: When $\rho > 1 - \beta$, then the lower bound $(k^t - (1-\theta)(\frac{1-\rho}{\beta})^2)/\theta$ is larger than $(k^t - 1 + \theta)/\theta$. Yet, since $\rho < 1$ it is never larger than k^t. Hence, out of (2), (3), and (4), (2) is preferred when $(k^w - \frac{\rho}{\beta})^2 < (k^t - (1-\theta)(\frac{1-\rho}{\beta})^2)/\theta$, (4) is preferred when $k^t > (k^w - \frac{\rho}{\beta})^2 \geq (k^t - (1-\theta)(\frac{1-\rho}{\beta})^2)/\theta$, and (3) is preferred when $(k^w - \frac{\rho}{\beta})^2 \geq k^t$.

Adding regime (6) to the picture, it is $W(3) \geq W(6)$ if and only if $k^t \leq (\frac{1-\rho}{\beta})^2$ and $\rho > 1 - \beta$. This means that (6) is no option when $k^t \leq (\frac{1-\rho}{\beta})^2$ and $\rho > 1 - \beta$.

Case $k^w > \rho/\beta$ and $\rho \leq 1 - \beta$ and $k^t > 1$: When $k^t > 1$, then the race is only between (6) and (2), which is won by (2) if and only if $\rho/\beta < k^w \leq 1/\beta$. With $\rho \leq 1 - \beta$ and $\rho < 1$ we also get $\beta < 1$, such that we end up with $\rho/\beta < k^w \leq 1 + \frac{\rho}{\beta}$.

Case $k^w > \rho/\beta$ and $\rho > 1 - \beta$ and $k^t > (\frac{1-\rho}{\beta})^2$: When $\rho > 1 - \beta$ and $k^t > (\frac{1-\rho}{\beta})^2$, then the race is between (6), (2), and (4). (4) is better than when $k^t > (k^w - \frac{\rho}{\beta})^2 \geq (k^t - (1-\theta)(\frac{1-\rho}{\beta})^2)/\theta$, which is never satisfied within the given range, which leaves (2) and (6). Then (2) dominates (6) as long as $k^w < \frac{1}{\beta}$. In contrast (6) is the dominating regime for $k^w \geq \frac{1}{\beta}$, which is only valid for $k^t > (\frac{1-\rho}{\beta}$ □

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