UNIVERSITÄT PADERBORN
Die Universität der Informationsgesellschaft

Asymmetry, Contestedness and Distortion in Two-Person Bargaining

Der Fakultät für Wirtschaftswissenschaften der Universität Paderborn zur Erlangung des akademischen Grades

Doktor der Wirtschaftswissenschaften
– Doctor rerum politicarum –

vorgelegte Dissertation von

Thomas Streck
geboren am 20.08.1991 in Münster

Schloß Holte- Stukenbrock, Juli 2025

Erstgutachter
Prof. Dr. Claus-Jochen Haake
Fakultät für
Wirtschaftswissenschaften
Universität Paderborn

Zweitgutachter
Prof. Dr. Burkhard Hehenkamp
Fakultät für
Wirtschaftswissenschaften
Universität Paderborn

Diese Arbeit entstand in Teilen im Rahmen meiner Tätigkeit als wissenschaftlicher Mitarbeiter im Teilprojekt A3 “The market for services: Incentives, algorithms, implementation” des Sonderforschungsbereichs SFB 901 “On-The-Fly Computing” an der Universität Paderborn.

Acknowledgements

August 28th, 2015, was a pretty hot day in a small town called Huércal-Overa, located in the desolate border region between Murcia and Almería. While staying at the home of my friend Abraham's parents, I opened my laptop to check my emails (yes, smartphones already existed at the time, but I hadn't quite arrived in that world yet). That day, a message from Claus-Jochen Haake popped into my inbox. In his view, I had apparently gathered some "micro-experience" and he could imagine me joining the team starting in the upcoming winter semester.

Ten years later - two of them as a student assistant, six as a research associate, and nearly two more in the private sector - we now communicate via WhatsApp and have long since dropped the formal tone. During those years, I had the opportunity to immerse myself in a world I previously knew very little about: academia. It demanded a lot from me, but gave back just as much. In hindsight, I am deeply grateful for this period. Not only did it offer privileges such as access to exclusive knowledge and international conference travel, but, most importantly, it gave me the time to reflect on the societal role I want to consciously embrace in the future.

This journey, culminating in the submission of this dissertation, is not one that is walked alone. I have had many companions along the way to whom I would like to express my heartfelt thanks.

First and foremost, I thank my doctoral advisor, Claus-Jochen Haake. Thank you for seeing something in me and opening a door I never would have passed through on my own. At first, the university felt like an anonymous place to me. Of course, I made lasting friendships early on, but the distance to the professors felt immense, especially in a program like economics with massive enrollment numbers. It was only in your Game Theory class that I began to build a personal connection with professors, especially with you. Through the way you taught the material, so interactive and vivid for those who were interested, I discovered a deep delight in the subject, and as you can see, I have remained loyal to cooperative game theory ever since. You gave me a lot of freedom, often letting me try out procedural approaches and algorithms on my own before offering guidance. I truly appreciated that. Later, I had the chance to witness and learn from your game-theoretic expertise firsthand, especially while co-authoring research papers together, sometimes deep into the night during the pandemic. Your enthusiasm for developing game-theoretic ideas was infectious. I value our friendship and am deeply thankful for everything you have taught me.

I am equally grateful to my colleagues at the chair and to our associated researchers. From every professional connection, without exception, true friendships have emerged. Be it the informal discussions about B-movies, the overly competitive pub quizzes, or the countless dissertation-relevant conversations: I will never forget them. While I would love to name each and every colleague here, I have promised myself to keep these acknowledgments within three pages. Therefore, I want to particularly thank Papatya Duman and Nadja Stroh-Maraun. You patiently dealt with my many questions and battled through countless pages of my written thought experiments. You always took time for me and answered my questions in a way I could truly understand. I have looked up to you both many times. I would also like to thank Britta Hoyer and Simon Hoof, with whom I shared wonderful experiences at conferences, summer schools, in shared offices, or during our time at SFB901.

Which brings me to the next point: I am grateful to SFB901. It taught me the practical side of collaborative research, something that takes real effort when trying to get computer scientists and economists to speak the same language. This research program enabled me to attend the aforementioned conferences and summer schools and even supported a training course in management. I may have been quickly lost whenever the Wednesday afternoon meetings at the Heinz Nixdorf Institute shifted too far into technical IT territory, but I appreciated the process, and I absolutely approve of the octopus logo that ended up representing the Proof of Concept.

Outside the academic world, I am deeply thankful to the companions who helped shape me into the person I am today. I thank my Mama and Papa, who raised me, helped me through school, and prepared me for the world. They gave me a compass of values I gladly follow to this day, and I could not wish for better parents. Likewise, my older brother and sister have always been role models. I thank them for raising me alongside my parents, letting me follow them, and for letting me look at their homework back when I was still in kindergarten. Even now, though they have moved away from Ostwestfalen-Lippe, I still enjoy every moment I get to spend with them. Pet owners will understand this (the rest may feel free to skip the next sentences), but I am also endlessly grateful to my dogs. Columbo, Andy, and Toni were always by my side: comforting me during difficult times, keeping me grounded during the good ones, and, you have guessed it, never once eating my homework.

I also want to thank my long-time school friends and those I met during my university years. Whether it was through university sports, leading to all sorts of other activities and even club memberships, or the late-night study sessions and well-earned celebrations, or through shared semesters abroad: you all contributed to making my memories of student life overwhelmingly positive.

And last but not least, I want to thank my partner Laura, who, in many ways, co-authored this dissertation with me. You stood by my side through it all, motivated me, supported me mentally, and always had an ear for me, even while managing your own demanding studies and job. Economic math class may have never convinced you, but without you, this dissertation would never have been completed. I am deeply impressed by everything you do.

Finally, a few honorable mentions that simply cannot be left out: a big thank you to Pizza Toni on Heiersstraße, who often saved me from having to cook. Thanks to UFC Paderborn and university sports for helping me stay balanced. Thanks to Hops Bierbar, Globetrotter, and Josefs Brauerei, for reasons that should be fairly obvious. Thanks to the NFL Game Pass and the large white wall in office Q4.307, which allowed Britta Hoyer and me to watch the Winter Olympics via projector. And thanks to the many university events, mostly organized by Stefan Jungblut, which kept us in great contact with the international students in our master's program. While this last list may sound a bit trivial and has expanded these acknowledgments onto a fourth page, each of these mentions holds incredible value to me, and they have more than earned their place on these pages.

Schloß Holte- Stukenbrock, July 2025 *Thomas Streck*

Contents

List of Figures

List of Tables

CHAPTER 1
Introduction

Since the dawn of mankind, individuals have been confronted with bargaining situations on a daily basis. Whether it concerned the allocation of tasks within a family or the trade of goods and services on early marketplaces, the fundamental questions have remained the same to this day. What is the best outcome – for me, for my family, or for society? How strong is my bargaining position, and what if I cannot defend it? Is the result fair in my eyes? The bargaining situation itself may take different forms, but Muthoo (1999) managed to concisely describe what these situations broadly have in common:

> "[A] bargaining situation is a situation in which [...] players have common interest to co-operate, but have conflicting interests over exactly how to co-operate."[1]

The common interest in exchanging goods or services arises from the recognition of a potential surplus by all parties involved. While they prefer agreement over disagreement, the question of how to divide this surplus remains contested. Usually, products may perish or lose their value over time[2], such that outside options or disagreements become a viable threat.

While many would characterize a result in bargaining as a consequence of the participants' negotiation skills, the theory typically models and analyzes different bargaining situations with the aim of proposing processes and solutions with desirable features. A key component of such a solution may include, for example, a form of efficiency that emerges from how a surplus is distributed. It

[1] Muthoo (1999), Preliminaries, p. 1.

[2] Rubinstein (1982) carried out pioneering work in this area by the implementation of discount factors in case of no immediate agreement on how to share a pie.

would be inefficient if a model assigned a good to someone who cannot benefit from it, while another person in the same context could derive substantial utility. Another crucial aspect of a bargaining solution is the relation between the participants. Equal agents should expect equal utility. Any deviation from equality may be justified by differences in their status, entitlements, or *fallback options*. Samuels (1973) observed that Adam Smith (see, e.g., Smith (2016a)) already discussed inequalities in the employer-employee relationship when it comes to wages, which he interpreted as a structural asymmetry ultimately resulting in power inequalities. These considerations relate to the concept of asymmetry of bargaining power in labor economics, which is essential to explain unequal bargaining outcomes (see Vercherand (2014)).

Asymmetry of bargaining power is not limited to labor economics. In fact, unequal bargaining positions are ubiquitous across economic and social contexts. The concept of *bargaining power* captures the ability of a party to influence the outcome of a bargaining process in its favor. In practice, bargaining power may stem from a variety of sources, including but not limited to outside options, legal entitlements, information advantages, or structural positions in the bargaining environment. Importantly, these sources are often unequally distributed among the participants.

Real-life settings frequently reveal such asymmetries. One illustrative example arises in digital platforms, where buyers and sellers interact through an intermediary. In the CRC 901 (Sonderforschungsbereich SFB 901)[3], we studied allocation mechanisms for platform-based services. These markets are characterized by a multi-sided structure, where different types of participants interact via a central platform. In our setting, the platform coordinates between two distinct roles: *service providers*, who offer modular service components (e.g., data processing routines or software modules), and *OTF providers*, who combine and assemble such services into customized workflows tailored to the needs of an end-user. A transaction only occurs if both sides agree to cooperate on a given composition. Thus, negotiation takes place over the inclu-

[3]For further information to subproject A3 of the CRC 901, see Sonderforschungsbereich 901 - On-The-Fly Computing (2023): https://sfb901.uni-paderborn.de/projects/project-area-a/subproject-a3 (last access 2025/05/19).

sion of a specific service module into a larger, dynamically composed system, typically involving questions of compatibility, price, and quality of service. A central insight from subproject A3 is that platforms do not merely facilitate exchanges. They shape the structure and outcome of negotiations. The rules that govern matching, timing, or information access can systematically favor one side over the other. In such settings, both service providers and OTF providers must simultaneously agree to a transaction. If this coordination fails, the system shifts to an alternative pairing. These fallback options, embedded in the platform architecture, act as endogenous outside options and directly influence the bargaining position of each side. Consequently, the negotiation outcome is shaped not only by preferences or costs, but by how the platform distributes power through its design. For instance, fallback options that are predefined in the system architecture, such as automated reassignment to alternative providers, can strengthen one party's position without any change in their preferences. However, from a modeling perspective, such mechanisms are reflected in the shape of the feasible set or in alternative bargaining opportunities, which in turn affect the resulting utility levels. These observations illustrate that bargaining power is not an exogenous feature, but can be shaped by institutional or technical design choices.

This perspective directly motivates the theoretical arguments developed in this thesis. Chapter 2 analyzes how different formal representations of bargaining power, such as exogenous weights or shifted disagreement points, capture such asymmetries, and how these modeling choices affect the negotiated outcome. In the platform context, this corresponds to whether the fallback structure is embedded in individual entitlements or in the configuration of alternatives. Chapter 3 then explores how different platform architectures, reflected in the shape of the feasible bargaining set, systematically influence the extent to which one party benefits from asymmetry. Even when bargaining power is held constant, the design of the set itself can tilt the result. Finally, Chapter 4 introduces a structural measure of how contested a platform-based negotiation is, depending on how aligned or divergent the participants' interests are. This measure helps identify when the negotiation process is likely to be cooperative, and when it is characterized by deeper structural conflict.

Another example concerns inheritance disputes. According to German statutory inheritance law, heirs typically receive fixed shares depending on their degree of kinsmanship.[4] For example, if one child of the deceased has already passed away, that share is usually inherited by their children in equal parts. While this rule-based approach simplifies allocation, it may ignore individual preferences and result in inefficiencies. Heirs often assign different emotional or practical value to specific items, such as a house, a piece of land, or a family heirloom. Applying rigid monetary division rules may thus yield outcomes that are legally correct but perceived as unfair.

Cooperative bargaining theory, which this thesis primarily focuses on, offers an alternative perspective: it enables heirs to negotiate personalized agreements that better reflect their individual valuations, particularly when no testament exists and the default legal solution would divide the estate in strictly monetary terms. If the parties reach a mutual agreement, they are free to assign specific items, such as a house or heirloom, to individual heirs, even if their monetary value differs. This flexibility opens the door for more efficient outcomes. However, even in such cooperative settings, bargaining power plays a crucial role. Some parties may possess greater legal knowledge, stronger emotional claims, or superior fallback positions, for instance, the financial means to buy out co-heirs or the ability to delay negotiations strategically. In particular, the statutory inheritance shares may serve as an implicit threat point: if no agreement is reached, the estate will be divided according to the legal default. As a result, those entitled to a larger legal share can exert greater influence on the negotiated outcome. These asymmetries not only shape the final agreement, but also affect the feasibility and perceived fairness of any cooperative solution.

Moreover, certain entitlements are not negotiable at all. Under German law, for instance, even if a testator excludes their spouse or children from the will, they are still entitled to claim a compulsory share of the inheritance in

[4] See §§ 1924–1931 BGB (Bundesministerium der Justiz, 2025). Closer relatives exclude more distant ones; children inherit before grandchildren, and spouses receive a fixed share in addition to relatives. In the absence of a testament, the estate is usually divided in monetary terms among the heirs.

monetary terms.[5] These legal fallback rights constitute a legally guaranteed form of bargaining power and illustrate how formal legal structures can create or reinforce asymmetries within bargaining environments.

The examples above illustrate that bargaining power can arise from diverse and often asymmetric conditions. These considerations raise a broader question: how do different bargaining procedures account for such asymmetries, and are their results neutral with respect to power imbalances? This thesis addresses this theme by formulating three central research questions, each corresponding to one of the main chapters. These questions investigate how asymmetry enters a bargaining problem, how its effects depend on the structure of the feasible set, and how we can formally characterize the intensity of the resulting conflict. The analysis focuses on three axiomatic bargaining solutions, the *Nash solution*, the *Kalai–Smorodinsky solution*, and, in a later chapter, the *Perles–Maschler solution*, each offering a different theoretical perspective on these questions. This leads to the first of three central research questions:

> *What are the consequences of introducing asymmetry into a bargaining problem, and how do different solution concepts respond to it?*

In Chapter 2, we investigate two principal modeling approaches for asymmetry: one based on weighted bargaining solutions and the other based on shifting the *disagreement point*. While both methods are widely used in the literature, they produce markedly different outcomes. Our analysis compares these approaches systematically, revealing how the choice of modeling technique can distort the negotiated solution and how this distortion affects the disadvantaged player in particular.

Building on this analysis, the next natural question concerns the underlying structure of the bargaining problem itself. While most axiomatic bargaining solutions are defined on convex and compact sets, their behavior may differ significantly depending on the specific shape and curvature of the feasible set.

[5] See § 1931 BGB (Bundesministerium der Justiz, 2025) on the statutory inheritance share of the spouse, and §§ 2303 ff. BGB (Bundesministerium der Justiz, 2025) on the compulsory portion law.

Even within the class of convex domains, the form of the *Pareto frontier* can vary substantially, and such differences may affect the negotiated outcome, particularly in highly asymmetric settings.

> *Does the distortion caused by asymmetry become more severe depending on the specific shape of the bargaining set?*

Chapter 3 addresses this second research question by systematically varying the shape of the feasible set while keeping the degree of asymmetry constant. We analyze how different solutions respond to changes in the set's shape and investigate under which conditions these solutions exhibit stronger or weaker bias in favor of the more powerful player. The results show that the shape of the set matters: the way asymmetry is modeled interacts with it in subtle but important ways, and the same environment can yield very different outcomes.

While Chapter 3 explores how the shape of the bargaining set affects outcomes under asymmetry, it also raises a deeper interpretative question: can the Pareto frontier itself tell us something about the nature of the bargaining problem? In particular, are there characteristics of the frontier that help us distinguish between scenarios where bargaining is genuinely contested and those where outcomes are essentially uncontentious?

> *Can the Pareto frontier be used to identify whether a bargaining scenario is contested or characterized by non-overlapping interests?*

Chapter 4 addresses this third research question by developing a formal measure that captures the extent to which a bargaining scenario is *contested*, based solely on the structure of the feasible set and the form of the Pareto frontier. Rather than focusing on bargaining power asymmetries or disagreement points, the analysis investigates how the distribution of interests across the set determines the intensity of the bargaining conflict. The results reveal that some problems involve areas where interests directly conflict, while others exhibit more separable preferences, reducing the scope for disagreement. This approach provides a new perspective on the internal structure of bargaining problems, beyond what standard axiomatic solutions can capture.

While this thesis focuses on bargaining power in settings where the rules and structure of negotiation are clearly defined, such as cooperative bargaining problems, it is important to distinguish this concept from market power. Although both relate to the distribution of surplus, they originate from different sources and follow different logics. Bargaining power emerges from the dynamics of the negotiation process itself, such as disagreement options, timing, or commitment ability, whereas market power typically arises from a firm's position in a competitive environment.

Several studies have emphasized this distinction. Dukes et al. (2006) show that stronger market presence does not automatically translate into stronger bargaining power. A retailer with broad customer access may still end up in a weak bargaining position if the manufacturer has better outside options or more leverage in the negotiation. This observation resonates with the analysis in Chapter 3 of this thesis, where alternative bargaining sets can be interpreted as fallback configurations, and switching between them can shift the outcome in favor of one side. Similarly, the service allocation problems studied in CRC 901 follow this logic: if a match between a service provider and an OTF provider is unfavorable, the system transitions to a more efficient pairing, effectively implementing an adjustment based on fallback options. In a related line of work, Iyer & Villas-Boas (2003) show that bargaining structures and protocols within distribution channels crucially affect surplus division, even when market positions remain unchanged. Their analysis highlights the importance of the negotiation process itself, particularly the sequence of offers and the bilateral nature of the agreements. This insight parallels the arguments developed in Chapter 2, where we demonstrate how different formalizations of asymmetry, such as weights or disagreement shifts, can substantially alter the outcome, despite identical feasible sets.

From an empirical perspective, Draganska et al. (2010) provide a framework to estimate bargaining power in manufacturer–retailer relationships based on observed pricing behavior, independent of market share. Such empirical models support the argument that bargaining power is a separate dimension of strategic interaction. Additional empirical and structural studies, such as Grennan (2013) in the context of medical devices and Chiang et al. (2003)

on multichannel coordination, reinforce the view that bargaining power is shaped by both observable and unobservable factors and cannot be inferred from market characteristics alone.

Taken together, these contributions make clear that bargaining power and market power are conceptually distinct. Bargaining power depends on the structure of the negotiation itself: on who has alternatives, timing advantages, or influence over the process, and cannot be inferred from market shares or concentration alone. Understanding these dynamics requires dedicated modeling tools. The remainder of this thesis develops and applies such tools within the framework of cooperative bargaining theory.

The next section introduces the main concepts of cooperative bargaining theory. It outlines the key axioms behind well-known solution concepts and discusses how the literature has approached questions of fairness, efficiency, and asymmetry in bargaining problems.

1.1 Game theoretic background

Game theory provides two main perspectives for analyzing bargaining situations: the cooperative and the non-cooperative approach. While both aim to model strategic interaction between agents, they differ fundamentally in their modeling assumptions and conceptual focus. The non-cooperative approach describes the bargaining process explicitly as a strategic game in extensive or normal form. It focuses on how rational players behave under specific rules, such as who moves when, what information they have, or what actions are available. The seminal work of Rubinstein (1982) falls into this category: he models bargaining as an alternating-offers game with discounting, where the equilibrium outcome depends on the players' patience and strategic positioning. The outcome emerges as a fixed point of recursive strategic anticipation, reflecting a dynamic and procedural notion of bargaining power akin to backwards induction.

In contrast, the cooperative approach abstracts from the detailed negotiation process. It assumes that players can agree on any feasible outcome and focuses on the set of possible agreements, typically represented as a convex, compact subset of utility pairs. This approach uses axiomatic reasoning to define what a “reasonable” solution to a bargaining problem should look like, based on normative principles such as Pareto efficiency, symmetry, or invariance to affine transformations. Instead of modeling the sequential dynamics of negotiation, it selects outcomes that satisfy fairness or consistency principles, under the assumption that rational players could enforce such an agreement if they chose to. In this sense, cooperative solutions are best understood as benchmarks, idealized but insightful representations of how surplus might be divided under certain normative assumptions.

This thesis adopts the cooperative perspective for two main reasons. First, it makes it possible to examine how different bargaining solutions respond to asymmetries, based on clearly defined normative criteria such as fairness, efficiency, or invariance. Second, it offers a structured framework to compare solution concepts by their axioms and functional properties. This makes the cooperative approach a suitable foundation for the research questions addressed in this thesis. In addressing these questions, this thesis focuses on three central axiomatic bargaining solutions: the Nash bargaining solution, the Kalai–Smorodinsky solution, and the Perles–Maschler solution. These concepts are chosen because they offer distinct yet structurally comparable perspectives on fairness and asymmetry, and because they can be meaningfully contrasted both analytically and geometrically. Other classical solutions, such as the egalitarian or utilitarian solution, are excluded from the core analysis due to their lack of scale invariance or insufficient reflection of fallback positions and bargaining power. The reasons for this selection are discussed in more detail in Chapter 5.

A variety of solution concepts have been proposed within cooperative bargaining theory to formalize what constitutes a reasonable agreement between players. Among these, three solutions stand out and form the theoretical foundation of this thesis: the Nash bargaining solution (Nash, 1950), the Kalai-Smorodinsky solution (Kalai & Smorodinsky, 1975), and the Perles-Maschler solution (Perles & Maschler, 1981). Each solution rests on a distinct set of axioms that express normative principles such as fairness, efficiency, and symmetry. They differ in how they balance these principles when they come into conflict, for instance, between equal treatment and the strategic relevance of fallback positions, and thus offer complementary perspectives on what makes a bargaining outcome acceptable. Their development reflects broader theoretical concerns of the time. Nash's approach aimed to capture rational compromise through functional axioms. Kalai and Smorodinsky responded with a stronger emphasis on *proportional fairness*[6], replacing the independence of irrelevant alternatives with a monotonicity condition. The Perles-Maschler solution, in turn, emerged from a more geometric and demand-oriented reasoning, with roots in linear optimization and *duality theory*[7]. It was formally introduced in Perles & Maschler (1981) as a variant of the Nash bargaining problem. While the Perles–Maschler solution was developed independently in the context of two-person bargaining, it resonates with broader fairness principles discussed in earlier cooperative game theory, including claims-based and dominance-oriented concepts (Aumann & Maschler, 1961; Maschler, 1992). These earlier contributions laid the groundwork for solution concepts that give particular weight to what each player demands and how strong their position is in the negotiation. This perspective anticipated the formal structure later used in the *superadditive* solution developed by Perles and Maschler.

The Nash bargaining solution is widely regarded as a foundational concept in cooperative bargaining theory. It selects the unique outcome that maximizes the product of the players' utility gains over the disagreement point. Formally, it is characterized by four axioms: Pareto efficiency, symmetry, invariance to affine transformations, and the independence of irrelevant alternatives (IIA). Nash's (1950) formulation showed that a combination of relatively simple axioms suffices to single out a specific solution in cooperative bargaining settings. The IIA axiom has been the subject of considerable criticism. It requires that the solution remains unchanged when unchosen but feasible alternatives

[6]Proportional fairness refers to the idea that each player should receive a share of the surplus that reflects the proportion of their maximum feasible gain, relative to the disagreement point. In the Kalai-Smorodinsky solution, this implies that the players' utility gains maintain the same ratio as their ideal points.

[7]Duality in linear programming refers to the correspondence between a "primal" optimization problem and its associated "dual" problem. In the context of bargaining, duality-based methods are used to characterize optimal responses to constraints and to formalize notions of balance or equilibrium between competing claims.

are added or removed. Although logically consistent, this condition has been viewed as problematic in applied contexts, particularly in situations where the availability of alternatives influences the decision context. Early doubts were raised by Luce & Raiffa (2012) in the decision-theoretic literature, who questioned whether rational behavior in actual choice settings could plausibly satisfy IIA. Further theoretical analysis by Roth et al. (1977) demonstrated that dispensing with IIA does not necessarily lead to inconsistency, but instead allows for the construction of alternative solution concepts with meaningful structure. Empirical studies have reinforced these concerns. Tversky (1972) showed that in individual choice problems, preferences can shift depending on the presence of additional options that should, by the IIA criterion, have no impact on the outcome. In bargaining experiments, Nydegger & Owen (1974) observed that participants frequently opted for equal splits, even when the formal conditions of the Nash solution pointed elsewhere. These findings suggest that the IIA axiom may misrepresent actual negotiation behavior, and they contributed to the emergence of alternative formulations.

The Kalai-Smorodinsky solution (Kalai & Smorodinsky, 1975) retains all of Nash's axioms except for IIA, which it replaces with a *monotonicity condition*[8]. This axiom requires that if one player's feasible utility improves, without any loss to the other, then the improving player should not end up worse off in the final agreement. The solution selects the point on the Pareto frontier where each player's utility gain corresponds proportionally to their maximal attainable utility. In other words, the relative share of each player's ideal gain is preserved. This reflects a concept of *proportional fairness* which places greater emphasis on fairness in relative terms than the Nash solution. Kalai and Smorodinsky presented their solution explicitly as a response to Nash, arguing that monotonicity avoids some of the counterintuitive implications of IIA. In doing so, they shifted the focus from consistency across irrelevant alternatives

[8]Monotonicity in this context refers to the idea that if one player's feasible utility improves while the other's remains unchanged, the solution should not shift to the disadvantage of the improving player. This property ensures that expansions of the feasible set benefit the relevant player, unlike the IIA axiom, which disregards such "context-sensitive" adjustments. See Kalai & Smorodinsky (1975) for the original formulation and Moulin (2004) for a more recent discussion of this principle.

to fairness in how the solution reacts to feasible improvements. This emphasis on proportional fairness has been supported by later contributions, including Thomson (1987) and Moulin (1991), who highlight its intuitive appeal in settings with unequal maximum gains.

The Perles-Maschler solution (Perles & Maschler, 1981) presents a conceptually distinct approach within cooperative bargaining theory. While less widely known than the Nash and Kalai-Smorodinsky solutions, it addresses similar fairness concerns from a different angle. Rather than focusing on abstract symmetry or proportionality, this solution emphasizes the structure of the feasible set and the players' relative bargaining claims. It is particularly sensitive to how each party's demand interacts with the geometry of the Pareto frontier, reflecting the idea that the players' claims depend not only on abstract fairness norms but also on their relative position within the feasible set. The solution is particularly suited to settings where the feasible set is curved or asymmetric, as it reveals how fairness considerations interact with the underlying geometry of the problem. It lies between egalitarian and utilitarian ideals: it neither enforces strict equality of utility gains nor seeks to maximize total surplus. Instead, it balances fairness and structural considerations by accounting for both players' positions and the shape of the feasible set (Perles & Maschler, 1981).

From a technical point of view, the Perles-Maschler solution can be illustrated through a geometric analogy that is especially intuitive in the case of a *normalized bargaining problem*, where the disagreement point is at the origin and the ideal points are at $(1, 0)$ and $(0, 1)$. Imagine two donkey carts starting simultaneously at these endpoints of the Pareto frontier. Each cart moves along the curve toward the other, but the local curvature of the frontier affects how quickly each side progresses: flatter sections are traversed more rapidly, while more curved regions slow the movement. The point at which the two carts meet determines the solution. This construction reflects how the geometry of the feasible set, its curvature and asymmetry, shapes the negotiated outcome, without relying solely on criteria such as symmetry or proportionality. From a formal perspective, the Perles-Maschler solution can be described as the outcome of a *linear optimization problem* (Perles & Maschler, 1981). Each player formulates a demand that reflects what they believe to be a fair share, subject to the constraint that both demands must still lie within the feasible set. The solution identifies the point where these opposing demands are just balanced: neither player can insist on more without violating feasibility. This idea has been illustrated in applied work such as Haake (2009), where players' evaluations of feasible allocations can be interpreted as structured demands, constrained by the geometry of the problem. The idea of balancing opposing claims under feasibility constraints builds on earlier notions of claims-based fairness. It was Perles and Maschler (1981) who first formalized this principle in a superadditive variant of the Nash problem, using tools from linear programming and duality theory. The solution has also been recognized for its normative plausibility and implementation potential: Rosenmüller (2004) highlights its conceptual robustness, while Gul & Pesendorfer (2020) show that it can arise as an equilibrium outcome in a collective choice market, thereby linking fairness and market-based allocation mechanisms.

Many classical bargaining solutions are built on a symmetry assumption: both players are treated as equals in terms of entitlement, strategic position, and fallback options. Yet real-world bargaining situations often involve asymmetries, whether due to differences in legal standing, negotiation leverage, or differences in market power. To address such contexts, asymmetric extensions of standard solutions have been developed. A well-known example is the weighted Nash bargaining solution, which incorporates exogenous weights into the Nash product to reflect differences in bargaining power, priority, or entitlement (Harsanyi, 1986). This approach was first motivated in the context of uncertainty about utility functions (Harsanyi, 1962), but later gained broader relevance as a tool for reflecting distributive asymmetries. In contrast to the symmetric case, the weighted solution no longer treats both players equally, but adjusts the outcome in line with the specified weights, and thus reflects predefined distributional priorities in the bargaining result. Another modeling approach shifts the disagreement point itself to reflect disparities in fallback options or outside threats. Such shifts can represent legal advantages, institutional guarantees, or market-based outside options available to only one party. Both approaches maintain the essential structure of the original problem while allowing for richer distributional analysis.

Dubra (2001) provides a unified framework for analyzing these asymmetric extensions. He shows how various solution concepts, especially the Kalai-Smorodinsky solution, can be generalized via weighting schemes or transformed disagreement points, while preserving desirable axiomatic properties such as Pareto efficiency or monotonicity. These modifications offer tools for examining distributive questions in settings where symmetry is neither empirically realistic nor justified. At the same time, some bargaining solutions are less straightforward to adapt. The Perles-Maschler solution, for example, is based on a geometric construction that closely follows the shape of the Pareto frontier. While its intuition, such as the donkey cart analogy, suggests ways of incorporating asymmetry (e.g., by allowing the carts to move at different speeds), a generally accepted asymmetric variant has not yet been formulated. A formal asymmetric extension has been proposed by Ervig & Haake (2005), who adapt the original construction by introducing asymmetric bargaining weights. However, this approach has not yet received broader axiomatic justification. The possibility of such an extension is revisited in the outlook of this thesis (see Chapter 5).

Not all classical bargaining solutions are suitable for the type of analysis pursued in this thesis. The *egalitarian solution*, originally motivated by Harsanyi's (1953) fairness criterion of equal utility gains, equalizes the players' improvements relative to the disagreement point. It has an intuitive appeal due to its focus on equal treatment and distributive fairness. However, as noted by Kalai (1977), it is not scale-covariant: outcomes may change under affine transformations of utility, making comparisons across preference systems problematic. It also ignores differences in fallback positions or entitlements (Kalai, 1977), treating all players as if their claims were interchangeable. The *utilitarian solution*, in contrast, maximizes the sum of utilities. It is invariant under affine transformations and clearly efficient, but it neglects distributional fairness. Small utility gains for one player can be outweighed by large gains for the other, regardless of initial positions or constraints. As noted by d'Aspremont & Gevers (1977), this can lead to highly asymmetric outcomes even in fairly symmetric problems. Both solutions fail to reflect bargaining power or strategic structure, and are thus excluded from the core analysis in this thesis.

An alternative approach to solving bargaining problems focuses not just on identifying fair outcomes through axioms, but on designing procedures that actively lead participants toward fair and efficient agreements. Instead of defining fairness solely through formal properties of utility allocations, these procedural methods define concrete steps by which agreements can be reached. They place importance not only on the properties of the final allocation, but also on the perceived fairness of the process itself. These procedural perspectives differ in orientation from purely axiomatic solutions like the Nash or Perles-Maschler solutions, which define fairness through mathematical principles in the utility space. However, they can align closely with outcome-oriented concepts such as the Kalai-Smorodinsky solution, especially in structured environments with divisible goods. In such settings, algorithms like the *Adjusted Winner* procedure can approximate or even reproduce axiomatic solutions, thus offering a practical bridge between theory and application. A prominent example of this procedural approach is the aforementioned *Adjusted Winner* (AW) procedure developed by Brams & Taylor (1996). Designed for two-player disputes involving multiple items or issues, AW guarantees *envy-freeness*, *equitability*, and Pareto efficiency under certain assumptions. In environments with linear utilities and perfectly divisible items, its outcome coincides with the Kalai–Smorodinsky solution. This connection highlights that AW can be interpreted as a procedural implementation of the Kalai-Smorodinsky solution, translating axiomatic fairness principles into a transparent negotiation process. Each party assigns points to the issues, and an algorithm adjusts the allocation until these fairness criteria are met. The method has gained practical relevance in contexts ranging from divorce settlements to international conflict resolution.[9] Its transparency and procedural clarity are often cited as factors that enhance the legitimacy and acceptance of the outcome, especially when trust in the theoretical fairness axioms is limited. Building on the same logic,

[9] For instance, the AW method has been used to resolve divorce settlements involving the division of property and personal items (see illustrations in Brams & Taylor (1996)). In the context of international conflict, the method has been proposed for resolving disputes such as the Israeli–Palestinian territorial conflict (see Massoud (2000)).

John & Raith (1999) and Raith (2000) introduced the issue-option framework, which represents bargaining problems as collections of issues with discrete options. This structure allows for the algorithmic implementation of fairness principles, particularly in complex multi-issue negotiations. Rather than relying solely on properties of utility functions, it emphasizes the decomposability of negotiation contents. Issues can be weighted, compared, and allocated in structured rounds, facilitating both fairness and transparency.

Importantly, these procedural frameworks can be linked back to axiomatic bargaining theory. For example, John & Raith (1999) demonstrate how specific axioms, such as efficiency and equal treatment, can be translated into concrete negotiation rules. In this sense, the procedures do not reject the principles of classical bargaining theory. Instead, they provide a method to implement or validate such principles in practice. Schneider & Krämer (2004) further highlight that fair outcomes are often perceived not just through mathematical elegance, but through the visible and comprehensible structure of the process itself. This notion of procedural legitimacy, how a solution is reached, can be just as important for acceptance as the outcome's normative properties. Haake & Su (2005) advance this perspective by showing how procedural and axiomatic reasoning can reinforce each other. Their work combines the Adjusted Winner algorithm with game-theoretic incentive considerations, identifying conditions under which fair outcomes are both attainable and strategically stable. By linking mechanism design to classic fairness principles, they build a bridge between strategic implementation and normative justification. In their conceptual orientation, these procedural approaches differ from bargaining solutions as the Nash, Kalai-Smorodinsky, or Perles-Maschler solution by emphasizing procedural implementation and clearly defined fairness criteria. However, they are not in conflict with classical models. On the contrary, they can complement them usefully: procedural methods, such as AW or the issue-option framework, can help implement the outcomes of axiomatic solutions, test their plausibility in applied settings, or improve their acceptance among negotiating parties. In this way, they help bridge the gap between theoretical fairness and practical negotiation and offer additional tools for analyzing and implementing bargaining solutions in applied contexts.

Taken together, these classical and more recent bargaining solutions provide a rich conceptual toolbox for analyzing fairness, asymmetry, and structure in cooperative bargaining. This theoretical foundation enables a systematic investigation of how different modeling choices affect bargaining outcomes under asymmetric conditions, and how the resulting variation can be traced back to the formal properties of each solution concept.

1.2 Illustrative numerical examples

To complement the theoretical discussion in Section 1.1, this section presents a set of numerical examples that make the differences between bargaining solutions more transparent. These examples serve two purposes: they illustrate how even minimal changes in fallback positions or bargaining power can influence outcomes, and they offer a first comparative perspective on solution concepts such as the Nash, Kalai–Smorodinsky, and Perles–Maschler solutions. For reference and contrast, the egalitarian and utilitarian solutions are also included. The settings are deliberately stylized to highlight specific effects, such as curvature, asymmetry, or shifts in the disagreement point, that affect the resulting allocations. While not intended to reflect empirical bargaining problems, they help isolate structural mechanisms that play a central role in later chapters.

Figure 1.1 provides an overview of four such examples. The top row shows symmetric cases: on the left, a linear Pareto frontier with a kink at $(0.75, 0.75)$; on the right, a smooth curved frontier defined by the function $y = 1 - x^3$. The bottom row uses the same piecewise linear setting to explore the effects of introducing weighted bargaining power (left, with player 1 having a relative weight of $\alpha = 2/3$) and a shifted disagreement point (right, shifting the disagreement point in favor of player 1). These comparisons help to clarify how the various solutions respond to changes in fallback positions or asymmetries, even in highly stylized environments.

To complement these visual comparisons and to provide a systematic overview of the underlying computation steps, Table 1.1 summarizes the general procedures for determining each bargaining solution in symmetric and

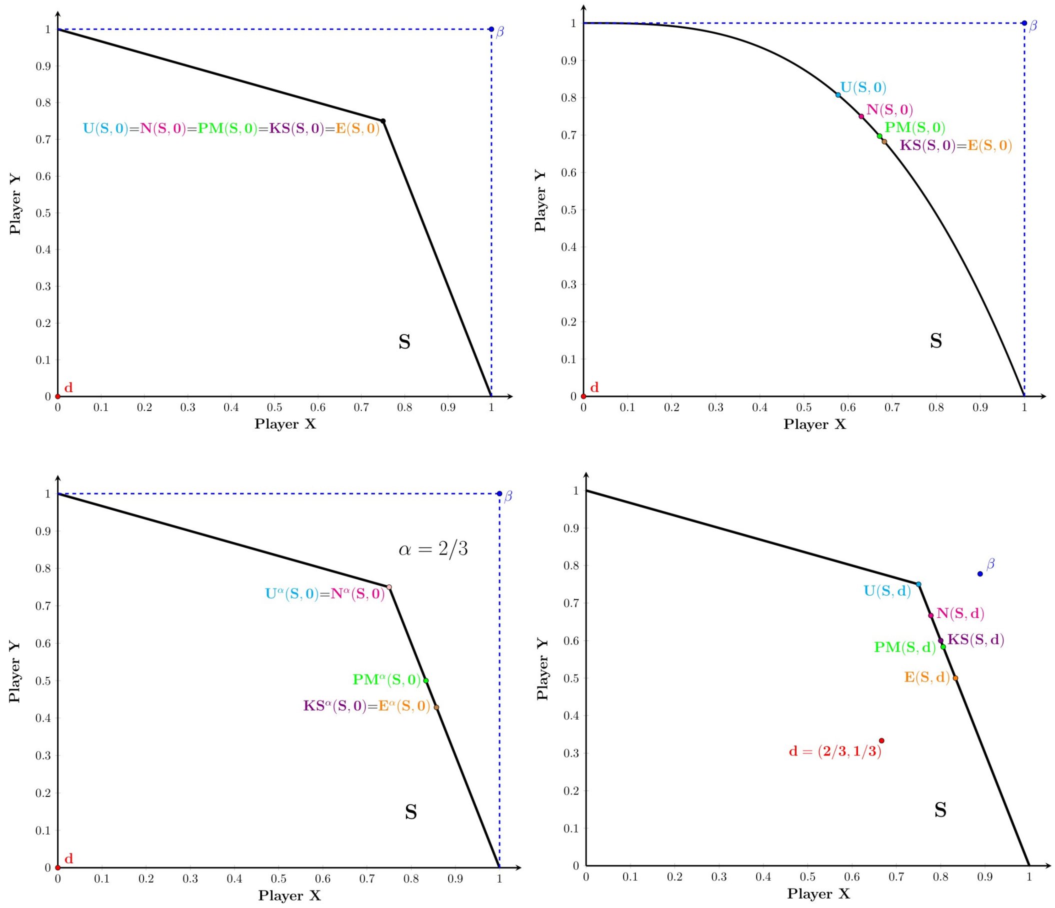

Figure 1.1: Visual comparison of five bargaining solutions across four settings. Top row: symmetric cases with linear (left) and curved (right) Pareto frontiers. Bottom row: weighted bargaining power (left) and shifted disagreement point (right) in the linear setting.

asymmetric settings. In all but the bottom right example, the disagreement point is fixed at the origin, simplifying the feasible set and ensuring direct comparability of fallback positions. Only in the bottom right example is the disagreement point shifted, which fundamentally alters the feasible set and requires recalculating all solutions relative to this new fallback structure.

Across all examples, a clear pattern emerges: in the first (top left) case, where the kink and the overall symmetry of the Pareto frontier were chosen to ensure full alignment, all solutions coincide at $(0.75, 0.75)$ and no deviation can be observed. In the curved environment of the second (top right) scenario,

Solution	**Symmetric setting (equal weights)**	**Asymmetric setting (weights α and $1-\alpha$)**
Nash	Maximize the product $(x-d_x)(y-d_y)$ along the Pareto frontier.	Maximize the weighted product $(x-d_x)^\alpha(y-d_y)^{1-\alpha}$.
Kalai-Smorodinsky	Find the point on the Pareto frontier that equalizes the relative shares: $\frac{x-d_x}{x^{\max}-d_x} = \frac{y-d_y}{y^{\max}-d_y}$.	Use weighted relative shares: $\frac{x-d_x}{x^{\max}-d_x} = \frac{1-\alpha}{\alpha}\frac{y-d_y}{y^{\max}-d_y}$.
Egalitarian	Find the point on the Pareto frontier where $x-d_x = y-d_y$.	Use weighted gains: $\alpha(x-d_x) = (1-\alpha)(y-d_y)$.
Utilitarian	Maximize the sum $(x-d_x)+(y-d_y)$ along the Pareto frontier.	Use the weighted sum: $\alpha(x-d_x)+(1-\alpha)(y-d_y)$.
Perles-Maschler	Find the point where the “donkey carts” meet (calculate the arc lengths along the frontier from each endpoint and find the point where the arc length is cut in half): $L(0,x^*) = L(x^*,1)$.	The “donkey cart” of player A travels α of the total arc length, the “donkey cart” of player B $(1-\alpha)$ of the total arc length.

Table 1.1: General computation for the five solutions in symmetric and asymmetric two-player bargaining problems

differences between the solutions emerge purely due to this curvature. While the egalitarian and Kalai-Smorodinsky solutions remain close and relatively proportional, the utilitarian solution shifts further upwards to maximize the total sum of payoffs. The Perles-Maschler solution, in contrast, is not primarily driven by the overall payoff sum or strict proportionality. Instead, it can be interpreted as the geometric midpoint between the endpoints of the curved Pareto frontier, which in this particular example results in a location slightly to the left of the Kalai-Smorodinsky and egalitarian solutions. Finally, the

Nash solution balances these considerations in a different way: it maximizes the product of gains relative to the disagreement point, leading it to settle at a compromise location that is typically closer to the center of the feasible region and that reflects both the fallback positions and the potential gains of the players.

The bottom left example highlights how explicit weighting of the players' bargaining power affects the various solution concepts. In this setting, the Kalai-Smorodinsky and egalitarian solutions move rapidly away from the symmetric point $(0.75, 0.75)$, directly reflecting the new weighting scheme in the proportional fairness conditions they satisfy. In contrast, the Nash and utilitarian solutions remain at $(0.75, 0.75)$ in this particular example. For the Nash solution, this apparent rigidity arises because of its IIA property: as long as the feasible set includes the same fallback structure and the kink point $(0.75, 0.75)$ still satisfies product maximization under the new weighting, the Nash solution does not move. For the utilitarian solution, the explanation is different: it depends on finding the point on the Pareto frontier that is tangent to an isoquant of the form $\alpha x + (1 - \alpha)y = c$. Because the kink at $(0.75, 0.75)$ is particularly attractive in this linear setting (it touches an entire family of such isoquants simultaneously, essentially stabilizing the utlitarian solution at this point), the utilitarian solution remains there until the weighting becomes strong enough to move the solution to another part of the Pareto frontier.[10] These observations highlight that the unchanged location of the Nash and utilitarian solutions is not a universal feature of these solution concepts. Rather, it depends on the specific shape of the Pareto frontier and the interplay of fallback positions and the structure of possible payoff allocations in this example. The Perles-Maschler solution in this scenario also moves away from $(0.75, 0.75)$, but in a distinctive way. It can be interpreted as one possible generalization to incorporate asymmetry: assigning donkey carts to travel α and $(1 - \alpha)$ of the total arc length of the Pareto frontier, respectively. This approach maintains the intuitive midpoint character of the

[10]This observation also matches with the general pattern that the *relative utilitarian bargaining solution* typically lies between the Kalai-Smorodinsky and the Nash solution, as discussed in Chapter 2. In this particular example, the relative utilitarian solution coincides with the Nash solution, further emphasizing this intermediate character.

Perles-Maschler solution along the frontier, while adjusting to the explicit weighting scheme. The term *arc length*[11] emphasizes that the Perles–Maschler solution reacts directly to changes in the shape of the Pareto frontier, unlike the Nash and utilitarian solutions, which remain focused on product or sum maximization and do not respond to small geometric changes in the frontier as long as the relevant fallback structure remains intact.

The bottom right example introduces a shifted disagreement point, which changes the feasible set itself and thus influences all solution concepts. The Kalai-Smorodinsky solution continues to equalize relative shares, now recalculated from the new disagreement point to the (new) utopia point. This ensures that it remains directly tied to proportional fairness. In contrast, the egalitarian solution in this setting no longer coincides with the Kalai-Smorodinsky solution: it requires equal absolute gains measured from the new disagreement point, which typically leads to a different location when the fallback positions are not symmetric. The Nash solution again centers on maximizing the product of gains relative to the new fallback positions, resulting in a compromise that reflects both the disagreement point and the shape of the frontier. The utilitarian solution, interestingly, remains at the kink point $(0.75, 0.75)$. This stability arises because the kink continues to maximize the weighted sum of payoffs. Under the new fallback positions, the isoquant $\alpha x + (1-\alpha)y = c$ remains tangent at this point for the symmetric weighting ($\alpha = 1/2$) used here. Finally, the Perles-Maschler solution adjusts in a geometric way to the new fallback position, maintaining its interpretation as a midpoint along the total arc-length of the Pareto frontier, as before. This highlights that the Perles-Maschler solution responds directly to changes in the fallback structure and the shape of the feasible frontier, offering a distinct compromise that is neither purely focused on relative shares nor driven by overall payoff maximization.

To summarize these numerical insights and to provide a compact overview of the key results, Table 1.2 lists the exact numerical outcomes for the five

[11] The term is not used in Perles & Maschler (1981), but they refer to the lengths of boundary segments of polygonal feasible sets, which constitute the efficient frontier in their setting. This supports a geometric interpretation based on arc length. A modern surface-based variant with similar sensitivity properties is presented in Rosenmüller (2021).

solutions in the four scenarios.

Solution	**Top Left**	**Top Right**	**Bottom Left**	**Bottom Right**
Nash	(0.75, 0.75)	(0.63, 0.75)	(0.75, 0.75)	(0.7778, 0.6667)
Kalai-Smo.	(0.75, 0.75)	(0.6823, 0.6823)	(0.8571, 0.4286)	(0.8, 0.6)
Egalitarian	(0.75, 0.75)	(0.6823, 0.6823)	(0.8571, 0.4286)	(0.8333, 0.5)
Utilitarian	(0.75, 0.75)	(0.5774, 0.8075)	(0.75, 0.75)	(0.75, 0.75)
Perles-Ma.	(0.75, 0.75)	(0.6714, 0.6973)	(0.8333, 0.5)	(0.8056, 0.5832)

Table 1.2: Solution points in the four example settings

Each solution concept reacts in its own way to variations in fallback positions and to the introduction of asymmetry. While the Kalai-Smorodinsky and egalitarian solutions directly translate changes in fallback positions into proportional or absolute fairness notions, the Nash and utilitarian solutions often remain focused on product or sum maximization and respond only to more substantial changes in the shape of the Pareto frontier. The Perles-Maschler solution, with its geometric midpoint character, adjusts in a way that combines fairness considerations with the overall structure of the feasible frontier. These examples highlight an important insight: no single bargaining solution can simultaneously satisfy all fairness or efficiency goals in two-player bargaining. Rather, the choice of solution determines how differences in disagreement points and variations in fallback positions are translated into final bargaining outcomes. As discussed by Kalai (1977) and Moulin (1991), proportional fairness and compromise principles such as the independence of irrelevant alternatives (as embodied in the Nash solution) are generally incompatible, reflecting a fundamental trade-off between fairness and disagreement-based compromise. The examples here also show how even small changes in fallback positions or in the curvature of the feasible frontier can have significant effects on the final bargaining outcome, depending on the solution concept that is applied. These numerical examples already demonstrate how distortion can emerge and vary systematically across solution concepts, laying the foundation for the more formal distortion analyses that follow in Chapters 2 and 3. While they do not address contestedness directly, they help illustrate how asymmetry shapes the bargaining outcome, a topic that connects to the broader structural analysis in Chapter 4.

1.3 Contribution

Building on the conceptual and illustrative discussion in Sections 1.1 and 1.2, this thesis addresses open questions in the study of cooperative bargaining solutions under asymmetry. While classical solutions, such as the Nash and Kalai-Smorodinsky solutions, are well-understood in symmetric settings, their properties in asymmetric situations, especially when disagreement points or bargaining powers differ, have not been fully resolved. In addition, the influence of the shape and structure of the bargaining set on these solutions has received limited attention. The thesis examines these aspects in a structured way and contributes to a better understanding of how asymmetry and the form of the bargaining set affect cooperative bargaining solutions. It consists of three papers, each focusing on one particular aspect.

The first contribution of this thesis is to examine how different formalizations of bargaining power influence cooperative bargaining solutions. In Chapter 2, two established approaches are compared: modeling bargaining power by assigning explicit weights to the players or by shifting the disagreement point within the feasible set. Both methods capture the idea that one party may have a stronger position, but they lead to different outcomes. The analysis focuses on the Nash and Kalai-Smorodinsky solutions and investigates how these two ways of representing asymmetry affect the allocation of payoffs. In particular, for the Kalai-Smorodinsky solution, a sufficiently weak player prefers introducing bargaining power by shifting the disagreement point, while for the Nash solution, this player instead prefers the weighted version of the solution, where bargaining power is directly expressed through weights. Furthermore, a direct comparison of the two weighted solutions shows that the weaker player always favors the weighted Nash solution over the weighted Kalai-Smorodinsky solution. These results highlight that there is no neutral way to account for asymmetric bargaining power in cooperative bargaining models. Depending on how bargaining power is incorporated, different players are systematically favored. This finding is relevant for practical applications, such as platform-mediated bargaining or automated negotiation procedures, where the designer must choose a solution concept and a way to incorporate bargaining power. The results provide clear guidance on which players benefit from each modeling approach and thus contribute to a deeper understanding of the role of bargaining power in cooperative bargaining theory.

The second contribution builds on the analysis of bargaining power in Chapter 2 by focusing on how the form of the bargaining set itself influences the outcomes. Chapter 3 studies how the curvature of the Pareto frontier interacts with asymmetry in bargaining power. The analysis compares outcomes across different bargaining sets that differ only in their shape, while the degree of bargaining power asymmetry remains constant. The main finding is that the shape of the bargaining set can significantly amplify the distortion between different solution approaches, especially in highly asymmetric situations. In particular, for the asymmetric Nash solution and when comparing two differently shaped bargaining sets, a more curved Pareto frontier systematically increases the distortion relative to the asymmetric Kalai-Smorodinsky solution. This means that the curvature of the bargaining set itself has substantial effects on the extent to which the solutions differ in their outcomes. However, the extent of this effect is not consistently observed for all solution concepts or for all ways of introducing bargaining power. Depending on the exact bargaining problem and the type of asymmetry, the influence of the bargaining set's curvature can differ. The chapter also establishes that when two bargaining sets are nested, the number of indifference points, where players are indifferent between the sets, is always odd. These insights highlight that not only the method of incorporating bargaining power, but also the structure of the bargaining environment itself can systematically favor one side. This is particularly relevant for social planners or platform designers who have to choose or shape the environment in which negotiations take place. The results thus extend the understanding of how structural features of bargaining problems shape outcomes under asymmetric conditions.

The third contribution of this thesis is to introduce and characterize a formal measure for the *contestedness* of a two-person bargaining problem. In Chapter 4, contestedness is defined as a measure that quantifies how severe the conflict is between the players, based solely on the structure of the feasible set and the shape of the Pareto frontier. This measure addresses the research question raised in the introduction of how one can formally capture the competitiveness or conflict intensity in a bargaining situation. Unlike traditional bargaining solutions that focus on the allocation of surplus, contestedness captures how overlapping or divergent the players' preferences are, offering a complementary perspective on the bargaining environment. The measure is derived axiomatically: a set of natural axioms is identified, and it is shown that there exists a unique mapping satisfying them. This mapping corresponds to a normalized version of the *standard traveling time* from the Perles-Maschler solution and describes structural aspects of the bargaining problem itself, independent of the particular bargaining solution used. The chapter also explores the practical relevance of contestedness, for instance in political decision-making processes where it can quantify how much voters' and parties' preferences align. By providing a formal tool to assess the intensity of conflict in bargaining problems, this contribution extends the conceptual framework of cooperative bargaining theory beyond calculating payoff allocations. It highlights that the nature of the bargaining problem itself, apart from the chosen solution, can be systematically analyzed and compared, offering valuable insights for social planners and other stakeholders (economists or politicians, e.g.) involved in cooperative negotiations.

Altogether, these three contributions provide a deeper understanding of how asymmetry and structural properties of bargaining problems shape the behavior of classical solution concepts. They show that even small changes in how bargaining power is incorporated and how the feasible set is structured can lead to systematic differences in the resulting allocations. This offers important theoretical insights and helps to identify critical aspects that may be relevant in practical applications. To conclude, Chapter 5 offers a short outlook, outlining key open questions, limitations, and future research directions that arise from the contributions of Chapters 2 to 4.

CHAPTER 2

Distortion through modeling asymmetric bargaining power

Claus-Jochen Haake and Thomas Streck

Reference:
Haake, C.-J. & Streck, T. (2024). *Distortion through modeling asymmetric bargaining power.* Working Papers Dissertations 143, Paderborn University, Faculty of Business Administration and Economics

Acknowledgments:
We thank Papatya Duman and two anonymous referees for valuable comments on an earlier version. The paper was presented at the 13th Conference on Economic Design. An earlier version was further presented at SING 15 and the 15th International Conference on Game Theory and Management. This work was partially supported by the German Research Foundation (DFG) within the Collaborative Research Center "On-The-Fly Computing" (SFB 901) under the project number 160364472-SFB901.

CHAPTER 3

Sensitivity of bargaining solutions to set curvature

Thomas Streck

Reference:
Streck, T. (2025). *Sensitivity of bargaining solutions to set curvature.* Working Papers Dissertations 145, Paderborn University, Faculty of Business Administration and Economics

Acknowledgments:
I thank Papatya Duman and Claus-Jochen Haake for valuable comments. This work was partially supported by the German Research Foundation (DFG) within the Collaborative Research Center "On-The-Fly Computing" (SFB 901) under the project number 160364472-SFB901.

CHAPTER 4

A measure for contestedness of a two-person bargaining problem

Claus-Jochen Haake and Thomas Streck

Reference:

Haake, C.-J. & Streck, T. (2025). *A measure for contestedness of a two-person bargaining problem.* Working Papers Dissertations 144, Paderborn University, Faculty of Business Administration and Economics

Acknowledgments:

We thank Papatya Duman, Stefan Napel, William Thomson and Walter Trockel for valuable comments. An earlier version was presented at the 2024 Conference on Mechanism and Institution Design. This work was partially supported by the German Research Foundation (DFG) within the Collaborative Research Center "On-The-Fly Computing" (SFB 901) under the project number 160364472-SFB901.

CHAPTER 5
Outlook

This final chapter outlines several open questions and possible directions for further research that naturally arise from the analysis in Chapters 2 to 4. While the preceding chapters focused on two-player bargaining problems and on a systematic investigation of how asymmetry interacts with different solution concepts, the insights gained raise broader questions about how these findings might generalize to more complex bargaining environments and what practical implications they might have.

A natural extension is to consider bargaining problems with three or more players. Such generalizations are not just theoretical exercises but directly relevant in many real-world settings, from inheritance disputes involving several heirs to platform-based marketplaces that must balance the interests of multiple stakeholders. Extending the well-established (symmetric) Nash and Kalai-Smorodinsky solutions to more than two players is theoretically straightforward: for the Nash solution, the product of the players' utility gains relative to the disagreement point is maximized, while the Kalai-Smorodinsky solution equalizes the relative shares of each player's maximum attainable payoff. These multiplayer extensions preserve the key features of the original two-player formulations, such as Pareto efficiency, symmetry (when players are treated equally), and invariance under positive affine transformations of utility. However, the IIA axiom is still not fulfilled by the Kalai-Smorodinsky solution in multidimensional settings (see Kalai (1977) and Moulin (1991)). Moreover, both solutions can be generalized to incorporate asymmetric bargaining power by introducing exogenous weights that directly reflect differences in the players' bargaining strength (see Thomson (1987) and Dubra (2001)). This allows these concepts to remain applicable even in settings where players have unequal fallback positions or structural advantages. The Perles-Maschler solution cannot be directly generalized to three or more players while preserving its original axiomatic foundation (Perles, 1982). This is because some of the axioms, such as *superadditivity*, become incompatible when moving to higher dimensions. Later work by Calvo & Gutiérrez (1994) and more recently by Rosenmüller (2021) have proposed modified procedures that approximate the spirit of the Perles-Maschler solution for multiplayer bargaining, but these methods necessarily relax some of the original axiomatic requirements.

Beyond the formal obstacles of extending two-player solutions, moving to three or more players raises broader conceptual questions. Many axioms that are natural and compatible in the two-player case, such as symmetry or IIA, become increasingly difficult to maintain simultaneously in higher dimensions. This insight is central to the work of Kalai (1977), who introduced the idea of "partial compromise solutions" to balance fairness principles when full axiom compatibility cannot be achieved. In such environments, no single solution can simultaneously satisfy all fairness principles that might be desirable in simpler two-player settings. In light of these complexities, this thesis deliberately focuses on the two-player case to provide a clear and consistent framework for analyzing distortion under asymmetry. While the conceptual logic of distortions, arising from different ways of modeling bargaining power or from variations in the shape of the bargaining set, remains relevant in multiplayer settings, the specific patterns and magnitude of these distortions cannot be directly inferred from the two-player results. This is because the introduction of additional players fundamentally alters the structure of the feasible set and the interactions between fallback options. In particular, the interplay of multiple fallback options can create new sources of distortion that do not exist in "simple" two-player cases. Moreover, the intuitive notion of a distortion curve, which is clear and well-defined in the two-dimensional setting, no longer applies directly in higher dimensions. Instead, the distortion itself becomes a multidimensional region, making comparisons and the measurement of systematic biases considerably more complex. Extending these analyses to higher dimensions would thus require a reassessment of which axiomatic principles are essential and how distortion patterns can be meaningfully compared. Consequently, the generalization to three or more players remains an open topic for future work.

Beyond the numerical examples introduced in Section 1.2, several broader conceptual questions remain open that go beyond the systematic distortion analyses in Chapters 2 and 3. In particular, they concern the treatment of non-covariant solutions, the theoretical extension of the Perles-Maschler solution to explicitly asymmetric contexts, and the influence of substitution elasticities in the players' value functions, especially in regard to the Nash solution. While these aspects are not systematically investigated in this thesis, they highlight conceptual challenges that could guide future research and deepen our understanding of how distortion arises and interacts with preferences and fairness ideals in bargaining scenarios.

The exclusion of non-covariant solutions in the systematic distortion analysis reflects a key methodological choice. The analysis focuses exclusively on covariant bargaining solutions, namely, the Nash, Kalai-Smorodinsky, relative utilitarian, and Perles-Maschler solutions, because these solutions share the crucial property of *scale invariance* or *covariance* under positive linear transformations of utility functions (see e.g. Moulin (1991); Thomson (1987)). This property ensures that observed distortions truly reflect differences in bargaining power or fallback positions, rather than artifacts of measurement conventions or unit rescaling. In contrast, non-covariant solutions like the utilitarian or egalitarian bargaining solutions lack this invariance and are therefore excluded to preserve comparability and generalizability of the distortion results.

Another conceptual consideration concerns possible ways to extend the Perles-Maschler solution to incorporate explicit asymmetries. Conceptually, there are two possible pathways: first, adjusting the traveling distances along the Pareto frontier, where each player's share of the total arc length reflects their bargaining weight, and second, adjusting the traveling speeds at which the players traverse the Pareto frontier according to their bargaining weights. In this thesis, the first approach has been used in Figure 1.1, but its broader theoretical implications have not been systematically explored. The second possibility, involving different traveling speeds, remains even less investigated in the cooperative bargaining literature. For both generalizations, it remains largely unclear how they would collectively affect the broader set of desirable properties typically associated with the Perles-Maschler solution. While individual axioms such as scale invariance, Pareto efficiency, individual rationality, and superadditivity are important reference points, no systematic analysis exists to determine which of these properties might still hold, be modified, or require new interpretations in an asymmetric setting. As far as I am aware, no fully developed and widely accepted extension of the Perles-Maschler solution along either of these lines currently exists. While Ervig & Haake (2005) propose a promising approach to incorporating bargaining weights into the Perles-Maschler framework, a comprehensive axiomatic analysis of its implications for properties such as Pareto efficiency, scale invariance, or superadditivity remains open. These questions suggest a possible direction for future research, especially in contexts such as digital platforms, where players may face different search costs or other forms of asymmetric frictions that influence their relative bargaining power.

Finally, a further important aspect that remains to be systematically explored is how *substitution elasticities* within players' value functions influence the relative positions of different bargaining solutions. In cooperative bargaining theory, substitution elasticity captures how willing a player is to trade off gains in their own payoff for gains in the other player's payoff. When the players' preferences are more substitutive (higher elasticity), they are more open to moving away from strict proportional fairness; when they are more complementary (lower elasticity), they tend to remain closer to fairness notions like the Kalai-Smorodinsky solution. In such contexts, the placement of the Nash solution can be seen as adjusting to these substitution preferences.[1] It often lies between the relative utilitarian solution, which itself reflects a weighted average of individual payoffs, and the Kalai-Smorodinsky solution (cf. Thomson (1981) and Cao (1982)). As players become more willing to substitute (higher elasticity), the Nash solution tends to move closer to the relative utilitarian outcome. Conversely, when players treat payoffs as closer complements (lower elasticity), the Nash solution shifts towards the Kalai-

[1] This interpretation builds on standard microeconomic reasoning about marginal rates of substitution and their role in determining the location of bargaining outcomes (see e.g., Binmore et al. (1986)).

Smorodinsky solution, which more rigidly enforces proportional fairness. This behavior highlights that what appears as a "distortion" in the Nash outcome is not solely a geometric or structural feature of the Pareto frontier, but also depends on the players' preferences. In the Nash bargaining formulation, this emerges from the way the Nash product $(x - d_x)(y - d_y)$ balances players' marginal rates of substitution along the frontier, which are shaped by these elasticities. In practical contexts such as the aforementioned digital platforms or labor markets, where players may have heterogeneous preferences for substitutability, some prioritizing absolute payoffs, others focusing on fairness, such differences could systematically shape how bargaining outcomes are perceived and accepted.

These open questions and conceptual considerations complement the preceding analyses and outline directions for future research on distortion, contestedness, and structural features of bargaining problems.

Bibliography

Anbarci, N. & Feltovich, N. (2013). How sensitive are bargaining outcomes to changes in disagreement payoffs? *Experimental economics*, 16, 560–596.

Anbarci, N. & Feltovich, N. (2018). How fully do people exploit their bargaining position? The effects of bargaining institution and the 50–50 norm. *Journal of Economic Behavior & Organization*, 145, 320–334.

Aumann, R. J. & Maschler, M. (1961). *The bargaining set for cooperative games*, volume 10. Princeton University Princeton, NJ.

Banerjee, S. (2020). Effect of reduced opportunities on bargaining outcomes: an experiment with status asymmetries. *Theory and Decision*, 89(3), 313–346.

Binmore, K., Rubinstein, A., & Wolinsky, A. (1986). The Nash bargaining solution in economic modelling. *The RAND Journal of Economics*, 17(2), 176–188.

Brams, S. J. & Taylor, A. D. (1996). *Fair Division: From cake-cutting to dispute resolution*. Cambridge University Press.

Brooks, J. M., Dor, A., & Wong, H. S. (1997). Hospital-insurer bargaining: An empirical investigation of appendectomy pricing. *Journal of health economics*, 16(4), 417–434.

Brooks, J. M., Doucette, W., & Sorofman, B. (1999). Factors affecting bargaining outcomes between pharmacies and insurers. *Health Services Research*, 34(1 Pt 2), 439–451.

Bundesministerium der Justiz (2025). Bürgerliches Gesetzbuch. `https://www.gesetze-im-internet.de/bgb/` (in German, last access 2025/05/20).

Bundeszentrale für politische Bildung (2025). Wahl-o-mat. `https://www.bpb.de/themen/wahl-o-mat/` (in German, last access 2025/05/06).

Calvo, E. & Gutiérrez, E. (1994). Extension of the Perles-Maschler solution to n-person bargaining games. *International Journal of Game Theory*, 23, 325–346.

Calvo, E. & Peters, H. (2000). Dynamics and axiomatics of the equal area bargaining solution. *International Journal of Game Theory*, 29, 81–92.

Camerer, C. F., Nave, G., & Smith, A. (2019). Dynamic unstructured bargaining with private information: theory, experiment, and outcome prediction via machine learning. *Management Science*, 65(4), 1867–1890.

Cao, X. (1982). Preference functions and bargaining solutions. In *1982 21st IEEE Conference on Decision and Control*, number 21 (pp. 164–171).: IEEE.

Chambers, C. P. & Echenique, F. (2014). On the consistency of data with bargaining theories. *Theoretical Economics*, 9(1), 137–162.

Cherchye, L., Demuynck, T., & Rock, B. D. (2011). Revealed preference analysis of non-cooperative household consumption. *The Economic Journal*, 121(555), 1073–1096.

Chiang, W.-y. K., Chhajed, D., & Hess, J. D. (2003). Direct marketing, indirect profits: A strategic analysis of dual-channel supply-chain design. *Management science*, 49(1), 1–20.

Chiappori, P.-A., Donni, O., & Komunjer, I. (2012). Learning from a piece of pie. *The Review of Economic Studies*, 79(1), 162–195.

d'Aspremont, C. & Gevers, L. (1977). Equity and the informational basis of collective choice. *The Review of Economic Studies*, 44(2), 199–209.

Draganska, M., Klapper, D., & Villas-Boas, S. B. (2010). A larger slice or a larger pie? An empirical investigation of bargaining power in the distribution channel. *Marketing Science*, 29(1), 57–74.

Dubra, J. (2001). An asymmetric Kalai-Smorodinsky solution. *Economics Letters*, 73(2), 131–136.

Duffy, J., Lebeau, L., & Puzzello, D. (2021). Bargaining under liquidity constraints: Nash vs. Kalai in the laboratory.

Dukes, A. J., Gal-Or, E., & Srinivasan, K. (2006). Channel bargaining with retailer asymmetry. *Journal of Marketing Research*, 43(1), 84–97.

Ervig, U. & Haake, C.-J. (2005). Trading bargaining weights. *Journal of Mathematical Economics*, 41(8), 983–993.

Feltovich, N. (2019). Is earned bargaining power more fully exploited? *Journal of Economic Behavior & Organization*, 167, 152–180.

Grennan, M. (2013). Price discrimination and bargaining: Empirical evidence from medical devices. *American Economic Review*, 103(1), 145–177.

Gul, F. & Pesendorfer, W. (2020). Lindahl equilibrium as a collective choice rule. *arXiv preprint arXiv:2008.09932*.

Haake, C.-J. (2009). Dividing by demanding: Object division through market procedures. *International Game Theory Review*, 11(01), 15–32.

Haake, C.-J. & Streck, T. (2024). *Distortion through modeling asymmetric bargaining power*. Working Papers Dissertations 143, Paderborn University, Faculty of Business Administration and Economics.

Haake, C.-J. & Streck, T. (2025). *A measure for contestedness of a two-person bargaining problem*. Working Papers Dissertations 144, Paderborn University, Faculty of Business Administration and Economics.

Haake, C.-J. & Su, F. E. (2005). Fair division procedures: Why use mathematics?

Harsanyi, J. C. (1953). Cardinal utility in welfare economics and in the theory of risk-taking. *Journal of Political Economy*, 61(5), 434–435.

Harsanyi, J. C. (1962). Bargaining in ignorance of the opponent's utility function. *Journal of Conflict Resolution*, 6(1), 29–38.

Harsanyi, J. C. (1986). *Rational behavior and bargaining equilibrium in games and social situations*. CUP Archive.

Horn, H. & Wolinsky, A. (1988). Bilateral monopolies and incentives for merger. *The RAND Journal of Economics*, (pp. 408–419).

Iyer, G. & Villas-Boas, J. M. (2003). A bargaining theory of distribution channels. *Journal of marketing research*, 40(1), 80–100.

John, R. & Raith, M. G. (1999). Strategic step-by-step negotiation. *Journal of Economics*, 70, 127–154.

Kalai, E. (1977). Proportional solutions to bargaining situations: interpersonal utility comparisons. *Econometrica: Journal of the Econometric Society*, 45(7), 1623–1630.

Kalai, E. & Smorodinsky, M. (1975). Other solutions to Nash's bargaining problem. *Econometrica: Journal of the Econometric Society*, 43(3), 513–518.

Kampas, A. & White, B. (2003). Selecting permit allocation rules for agricultural pollution control: a bargaining solution. *Ecological Economics*, 47(2-3), 135–147.

Karagözoğlu, E. & Urhan, Ü. B. (2017). The effect of stake size in experimental bargaining and distribution games: A survey. *Group Decision and Negotiation*, 26, 285–325.

Luce, R. D. & Raiffa, H. (2012). *Games and decisions: Introduction and critical survey*. Courier Corporation.

Maschler, M. (1992). The bargaining set, kernel, and nucleolus. *Handbook of game theory with economic applications*, 1, 591–667.

Maschler, M. & Perles, M. A. (1981). The present status of the super-additive solution. *Essays in Game Theory and Mathematical Economics in Honor of Oskar Morgenstern, RJ Aumann, ed., Mannheim: Bibliographisches Institut*.

Massoud, T. G. (2000). Fair division, adjusted winner procedure (AW), and the Israeli-Palestinian conflict. *Journal of Conflict Resolution*, 44(3), 333–358.

Moulin, H. (1991). *Axioms of cooperative decision making*. Number 15. Cambridge university press.

Moulin, H. (2004). *Fair division and collective welfare*. MIT press.

Muthoo, A. (1999). *Bargaining theory with applications*. Cambridge University Press.

Nash, J. F. (1950). The bargaining problem. *Econometrica: Journal of the Econometric Society*, 18(2), 155–162.

Navarro, N. & Veszteg, R. F. (2020). On the empirical validity of axioms in unstructured bargaining. *Games and Economic Behavior*, 121, 117–145.

Nydegger, R. V. & Owen, G. (1974). Two-person bargaining: An experimental test of the Nash axioms. *International Journal of game theory*, 3, 239–249.

Pallaschke, D. & Rosenmüller, J. (2006). Computing the Minkowski sum of prisms. *Journal of Global Optimization*, 35(2), 321–341.

Pallaschke, D. & Rosenmüller, J. (2007). A superadditive solution for cephoidal bargaining problems. *International Journal of Game Theory*, 35(4), 569–590.

Perles, M. A. (1982). Non-existence of super-additive solutions for 3-person games. *International Journal of Game Theory*, 11, 151–161.

Perles, M. A. & Maschler, M. (1981). The super-additive solution for the Nash bargaining game. *International Journal of Game Theory*, 10, 163–193.

Peters, H. & Van Damme, E. (1991). Characterizing the Nash and Raiffa bargaining solutions by disagreement point axioms. *Mathematics of Operations Research*, 16(3), 447–461.

Raith, M. G. (2000). Fair-negotiation procedures. *Mathematical Social Sciences*, 39(3), 303–322.

Rosenmüller, J. (2004). The Maschler- Perles solution: 2 simple proofs for superadditivity. *Economics Bulletin*, 3(35), 1–8.

Rosenmüller, J. (2013). *Game theory: Stochastics, information, strategies and cooperation*, volume 25. Springer Science & Business Media.

Rosenmüller, J. (2021). *Bargaining solutions via surface measures*. Technical report, Center for Mathematical Economics Working Papers.

Roth, A. E. et al. (1977). Independence of irrelevant alternatives, and solutions to Nash's bargaining problem. *Journal of Economic Theory*, 16(2), 247–251.

Rubinstein, A. (1982). Perfect equilibrium in a bargaining model. *Econometrica: Journal of the Econometric Society*, (pp. 97–109).

Samuels, W. J. (1973). Adam Smith and the economy as a system of power. *Review of Social Economy*, 31(2), 123–137.

Schneider, G. & Krämer, U. S. (2004). The limitations of fair division: An experimental evaluation of three procedures. *Journal of Conflict Resolution*, 48(4), 506–524.

Smith, A. (2016a). *The wealth of nations*. Aegitas.

Smith, H. (2016b). The economics of retailer–supplier pricing relationships: theory and evidence. *Handbook on the Economics of Retailing and Distribution*, (pp. 97–136).

Sonderforschungsbereich 901 - On-The-Fly Computing (2023). Subproject A3 - the market for services: Incentives, algorithms, implementation. `https://sfb901.uni-paderborn.de/projects/project-area-a/subproject-a3` (last access 2025/05/19).

Streck, T. (2025). *Sensitivity of bargaining solutions to set curvature*. Working Papers Dissertations 145, Paderborn University, Faculty of Business Administration and Economics.

Svejnar, J. (1980). On the empirical testing of the Nash-Zeuthen bargaining solution. *ILR Review*, 33(4), 536–542.

Svejnar, J. (1986). Bargaining power, fear of disagreement, and wage settlements: Theory and evidence from US industry. *Econometrica: Journal of the Econometric Society*, 54(5), 1055–1078.

Takeuchi, A., Veszteg, R. F., Kamijo, Y., & Funaki, Y. (2022). Bargaining over a jointly produced pie: The effect of the production function on bargaining outcomes. *Games and Economic Behavior*, 134, 169–198.

Thomson, W. (1981). Nash's bargaining solution and utilitarian choice rules. *Econometrica: Journal of the Econometric Society*, 49(2), 535–538.

Thomson, W. (1987). Monotonicity of bargaining solutions with respect to the disagreement point. *Journal of Economic Theory*, 42(1), 50–58.

Thomson, W. (1994). Cooperative models of bargaining. *Handbook of game theory with economic applications*, 2, 1237–1284.

Thomson, W. (2010). *Bargaining and the theory of cooperative games: John Nash and beyond*. Edward Elgar Publishing.

Thomson, W. (2022). On the axiomatic theory of bargaining: a survey of recent results. *Review of Economic Design*, 26(4), 491–542.

Thomson, W. (2023). The axiomatics of economic design: An introduction to theory and methods. volume 1. *Studies in Choice and Welfare. Springer, Cham*.

Tversky, A. (1972). Elimination by aspects: A theory of choice. *Psychological review*, 79(4), 281.

Vercherand, J. (2014). *Labour: A heterodox approach*. Springer.