Abstract

Let Q be a quiver. M. Reineke and A. Hubery investigated the connection between the composition monoid $\mathcal{CM}(Q)$, as introduced by M. Reineke, and the generic composition algebra $\mathcal{C}_q(Q)$, as introduced by C. M. Ringel, specialised at $q = 0$. In this thesis we continue their work. We show that if Q is a Dynkin quiver or an oriented cycle, then $\mathcal{C}_0(Q)$ is isomorphic to the monoid algebra $\mathbb{Q}\mathcal{CM}(Q)$. Moreover, if Q is an acyclic, extended Dynkin quiver, we show that there exists a surjective homomorphism $\Phi\colon \mathcal{C}_0(Q) \to \mathbb{Q}\mathcal{CM}(Q)$, and we describe its non-trivial kernel.

Our main tool is a geometric version of BGP reflection functors on quiver Grassmannians and quiver flags, that is varieties consisting of filtrations of a fixed representation by subrepresentations of fixed dimension vectors. These functors enable us to calculate various structure constants of the composition algebra.

Moreover, we investigate geometric properties of quiver flags and quiver Grassmannians, and show that under certain conditions, quiver flags are irreducible and smooth. If, in addition, we have a counting polynomial, these properties imply the positivity of the Euler characteristic of the quiver flag.