Abstract

The central objects of investigation in this thesis are the thick subcategories as well as the exact abelian extension closed subcategories of the category of quiver representations. A full additive subcategory $\mathcal{C}$ of an abelian category $\mathcal{A}$ is called thick, provided that $\mathcal{C}$ is closed under taking direct summands, kernels of epimorphisms, cokernels of monomorphisms and extensions. The category $\mathcal{C}$ is called exact abelian if it is abelian, the embedding functor preserves exact sequences, hence closed under arbitrary kernels and cokernels.

First we consider the category of locally nilpotent representations over the path algebra of the cyclic quiver. We show that any thick subcategory is exact abelian. Then we give a combinatorial description of thick subcategories via non-crossing arcs on the circle and using generating functions, we calculate their number. Furthermore, we establish a bijection between thick subcategories with a projective generator, thick subcategories without a projective generator, support-tilting and cotilting modules. Then we study exact abelian extension closed subcategories for Nakayama algebras, and we find a recursive formula for their number.

For a finite and acyclic quiver, we consider the category of its quiver representations. We show that any thick subcategory generated by preprojective or preinjective representations is exact abelian. Then we specialise to Euclidian quiver case and we verify that any thick subcategory is exact abelian. Furthermore, we extend a result of Ingalls and Thomas and we give a complete combinatorial classification of thick subcategories in that case.

For a hereditary algebra A, we consider the tilted algebra $B = \operatorname{End}_A(T_A)$, where T_A is a tilting module. We establish a bijection between the exact abelian extension and torsion closed subcategories of $\operatorname{mod} A$ and the exact abelian extension closed subcategories of $\operatorname{mod} B$.