

Abstract of the thesis:

Geometry and Quantization of Howe Pairs of Symplectic Actions

Motivated by representation-theoretic Howe duality, we seek an analogous symplectic construction in the sense that its geometric quantization decomposes with Howe duality. We find that a symplectically correct setting is given by two Lie groups acting on a symplectic manifold M when these two actions commute and satisfy the *symplectic Howe condition*, i. e., these actions are Hamiltonian and their collective functions are their mutual centralizers in the Poisson algebra of smooth functions on M . Once this condition is satisfied, we obtain a natural bijection between the coadjoint orbits in one moment image and those in the other moment image – this we call *coadjoint orbit correspondence*.

We study the orbit correspondence further and show, if the acting Lie groups are compact and M is prequantizable, that it preserves integrality of the coadjoint orbits, so to both orbits in the correspondence an irreducible representation can be associated. We thus have a bijection between parts of the unitary duals of both Lie groups acting on M . Applying known results about the interchangeability of quantization and reduction, we see that for M a Kähler manifold, its quantization (as a representation of the product of both groups acting on M) decomposes into a multiplicity-free direct sum of tensor products of irreducibles of the individual groups, the pairs being given by the bijection obtained before – as one would expect according to Howe duality.

Carsten Balleier, Université de Metz and Universität Paderborn