# Increasing Stability of Aircraft and Crew Schedules

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für meine Eltern

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This work is dedicated to my parents. You have always supported me and believed in me. I owe you much more than words can ever express.

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# Contents

# 1. Introduction

Airline scheduling problems are highly complex and have for several decades now been successfully solved by Operations Research methods. Commercial airlines have to schedule huge numbers of resources like aircraft, airport gates and staff, leading to huge potential for cost savings. At the same time the huge size and the large number of dependencies of the scheduling problems at commercial airlines make it very difficult to generate feasible and cost-efficient schedules. For these reasons, these problems have received much attention from researchers and commercial software vendors leading to many sophisticated planning methods. Today most major commercial airlines use software systems with optimization methods to support their planning tasks.

With the introduction of software systems and sophisticated optimization methods for scheduling tasks airlines are now able to shorten their planning cycles and react faster to changing conditions during the planning phase as well as on the day of operations. Rescheduling in case of disruptions, however, often cannot prevent delays, because it is too expensive or simply no longer possible at that time. In such cases delays lead to additional costs of compensating customers and also possibly to additional delays due to complex dependencies between different airline resources. It is common practice today to schedule reserve resources and to place slack between flights in order to avoid delays or to have more possibilities for rescheduling. Both methods, however, are very expensive and cannot be performed extensively. Accordingly, the consideration of disruptions during the planning of schedules has recently been given more attention by the research community. The goal is to schedule slack between flights as well as reserve resources more intelligently in order efficiently to reduce the consequences of disruption.

In this work we focus on the problem of scheduling aircraft and crews under consideration of disruption. Both schedules can be easily disrupted leading to disruptions of aircraft maintenance schedules and working time regulations of crews. These disruptions are hard to recover and often lead to additional delays. Moreover, the two problems are closely interdependent due to crews changing aircraft on the day of operations. Such aircraft changes can lead to additional propagation of delays between aircraft, i.e. following the crew from one to another aircraft.

Both scheduling problems, aircraft and crew, are highly complex and difficult to solve and are therefore often solved sequentially. The literature to date shows that simultaneous consideration of these problems, however, can greatly reduce costs as well as delay propagation. The approaches to simultaneous scheduling of aircraft and crews in the literature only consider very simplified models for delay propagation. But an effective reduction of delay propagation is the most promising factor to achieve stability of airline schedules. Moreover, there is no framework within which to evaluate and compare different approaches for airline scheduling under consideration of disruptions, although there are many simulation models for airline schedules. In order to compare different scheduling approaches, however, we need a method to measure the quality of a scheduling approach which also considers effects of disruption. To summarize, the aim of this work is to research methods of evaluation of robust scheduling approaches and to develop more exact stability indicators based on delay propagation between aircraft and crews.

In this work we examine methods for simultaneously scheduling aircraft and crews under uncertainty. We present two methods to consider the delay propagation during scheduling of aircraft and crews. The first approach increases the scheduled time for aircraft changes without measuring the actual delay propagation. The second approach distributes slack over the whole schedule according to a measure of total delay propagation. The second approach involves additional complexity for the scheduling method, but promises a better measure of the delay propagation and therefore better distribution of slack. The comparison of the two methods is based on the evaluation of the operational performance of schedules, which usually involves many aspects like punctuality, recovery effort and negative effects on other resource schedules on the day of operations. We perform an extensive simulation study to evaluate the developed scheduling approaches. The study is based on a simulation model which estimates the number of additional delays and the effort of recovery and uses historical data of a major European airline.

The consideration of uncertainty during scheduling introduces additional complexity leading to even more complex problems. Thus an important aspect of this work is the development of efficient methods for solving these scheduling problems. We build on well-known optimization methods from the literature and extend them in order to cope with uncertainty. As an additional result the new techniques proposed can also be used to improve solution methods for classical scheduling problems without uncertainty. On the experience with planning under uncertainty we additionally propose a method for rescheduling of aircraft and crews due to disruptions on the day of operations.

The remainder of this work is organized as follows. The next Chapter presents background knowledge on planning processes and daily operations of commercial airlines. In Chapter 3 we discuss recent approaches to consideration of disruptions during scheduling. In Chapter 4 we describe the goal of this work in detail based on the conclusions from the previous chapter. In Chapter 5 we present a new method of efficiently scheduling aircraft and crews. In Chapter 6 we develop two approaches to consideration of disruptions during scheduling and compare them in Chapter 7 based on the evaluation of the operational performance of airline schedules. In Chapter 8 we present an additional result of this work, an approach to rescheduling aircraft and crews on the day of operations. A summary of this work and conclusions are given in Chapter 9.

# 2. Airline Scheduling Background

The topic of this work is the scheduling of crews and aircraft in the airline industry during long-term planning considering uncertainty as well as on the day of operations when irregular events lead to disruptions of schedules. The planning problems relevant to this work can be divided into three groups: *strategic planning problems*, *tactical planning problems* and *operational problems*. Solution to strategic planning problems requires general decisions about origin-destination connections, size and composition of the airline fleet as well as the location of the crew and maintenance bases (network design problem).

The tactical planning process starts several months before the day of operations with the objective of maximising the expected profit by meeting the demand of customers and minimising the costs by using the own resources efficiently. The following five planning problems are normally regarded as matters of tactical planning. The *schedule design problem* determines which connections between airports at which time are offered considering a forecast on the demand of customers. For each flight an aircraft type has to be selected considering the demand forecast as well as the availability of aircraft. This is obtained by solving the *fleet assignment problem*. Based on the resulting fleet assignment the *aircraft assignment problem* is solved to obtain maintenance schedules for aircraft as well as individual aircraft routes which cover all flights. Similarly, the *crew pairing problem* is solved to determine cost-minimal itineraries for crews covering all flights and satisfying general work-time regulations. In the next step (the *crew rostering* or *crew assignment problem*) these itineraries are assigned to individual crew members, satisfying individual work-time regulations. The different scheduling tasks are often performed by different organizational units of the airline, although the tactical planning problems are closely interdependent. Thus the scheduling tasks mainly consist of communicating with other units and repeatedly adjusting the schedules.

Operational problems are those solved on the day of operations, when the planned schedules are executed. Irregular events, like aircraft damage or airport congestion, make it necessary to change the schedules. The mathematical models for the rescheduling tasks are often similar to their planning counterparts but must be solved much faster. In contrast to long-term planning, the objective is not to maximise profit but instead to reduce the impact of the delays on the daily work.

In the remainder of this chapter we describe in Section 2.1 the tactical planning process of a commercial airline and emphasize on the interdependencies of the crew scheduling and the aircraft assignment problem. In Section 2.2 we briefly describe the daily operations of commercial airlines and focus especially on irregular operations including reasons for disruptions and possibilities to cope with those.

## 2.1. Tactical Planning Process at the Airline Industry

The tactical airline scheduling process is often decomposed into several planning problems in order to handle complexity of the different problems. Thus flight, aircraft and crew schedules are created one after another over several months prior to the day of operations. In practice, this is not a purely sequential process, compare Figure 2.1, but instead the schedules are repeatedly adjusted during the planning process in order to achieve good profitability and operational feasibility, because all five tactical planning problems are closely interdependent. For example, the actual departure and arrival times restrict the possibilities for itineraries for aircraft and crews, especially when considering maintenance restrictions and work-time regulations. The fleet assignment also restricts the possibilities for the aircraft assignment problem and crew scheduling, because members of the flight crew are normally allowed to fly one aircraft type only. The crew pairing problem depends on the aircraft assignment problem, because the robustness of the schedules can be affected when a crew changes the aircraft during a work duty. More precisely, a possible delay of a flight immediately before a scheduled aircraft change of crew affects two other flights in the schedule, the next flight of the crew itinerary due to missing crew as well as the next flight of the aircraft itinerary due to missing aircraft.

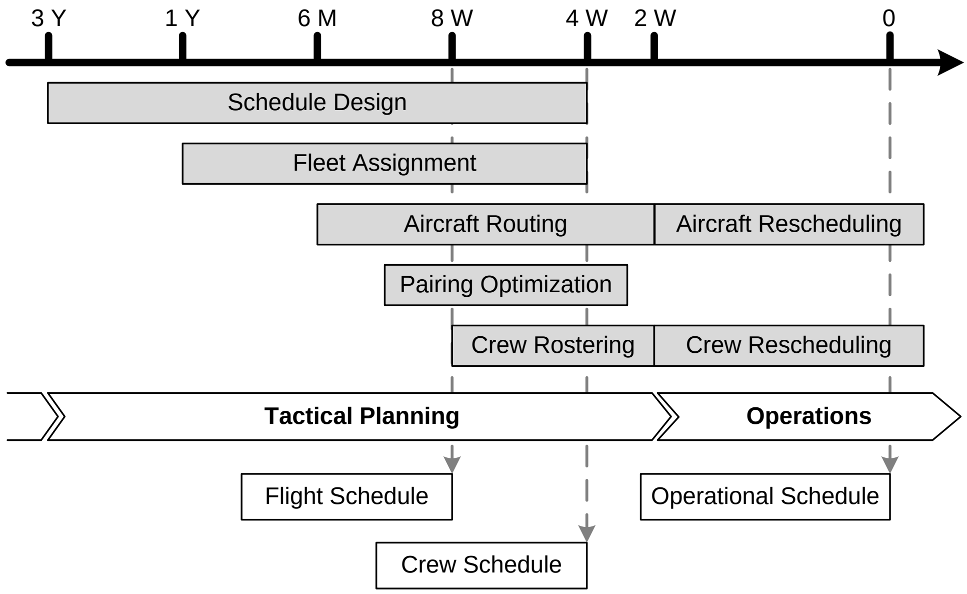


Figure 2.1.: Tactical and Operational Problems in the Airline Industry

The possibilities of manual adjustment to the schedules are limited by the complexity of the overall problem. Thus a lot of research is devoted to formulating integrated planning models and solving the scheduling problems simultaneously with the goal of creating schedules that are more profitable and of better quality. In practice, the integration of the planning process is very difficult, mainly for two reasons. Firstly, the individual planning problems are highly complex and difficult to solve, even with specially developed methods. The integration of those problems often leads to a disproportionate increase in complexity and makes it difficult to solve the problems. Secondly, today the individual planning steps are often performed by different organization units of one airline company. A partial or full integration requires substantial changes to the company processes, performance indicators and objective functions currently used by the individual units.

Nevertheless, significant decreases in costs and increases in schedule quality are possible by considering some or all problems simultaneously. For example small changes in the fleet assignment solution can lead to different crew pairings and therefore to significantly different crew costs, even if the objective of the fleet assignment solution does not change. Much research is devoted to full or partial integration of several or all planning problems with the goal of improving the total costs and schedule quality.

In the remainder of this section we follow the classical decomposition of the tactical planning process and briefly introduce each scheduling problem. Chapter 3 discusses possibilities of integrating the solution methods of the individual problems.

### 2.1.1. Schedule Design

The schedule design problem is highly interdependent with strategical decisions on market selection and route development. Decisions on flight frequency and timing are updated frequently to match experience gained with past schedules and new forecasts on customer demand. The objective of this planning step is considering the airline strategy, to reach as many customers as possible. Large airline companies today mostly operate on hub-and-spoke networks, i.e. most airports in the network have direct flights only to few airports (hubs), whereas those hub airports have direct flights to most other airports (spokes). Travel between airports that do not have a direct flight connection is possible by flying to the hub airport and from there to the destination. Flight frequency and timing have therefore also to satisfy the needs of customers who have to transfer from one flight to another at hub airports (optimisation of the hub bank structure). The schedule design problem is not focus of this work, see Etschmaier and Mathaisel [1985] and Teodorovic and Krcmar-Nozic [1989] for an overview of literature.

The decision on departure times of flights is often integrated into other tactical planning problems. In this case a flight schedule is given and only small deviations in the departure times, usually at some interval around the originally scheduled time, are allowed (Klabjan et al. [2002]). These integrated tactical problems have the appendix *with flight retiming* or *with time windows* to the name.

### 2.1.2. Fleet Assignment

The fleet assignment problem is that of assigning an aircraft type to each flight according to the passenger capacity, range and availability of the aircraft type.

The objective of this planning problem is often to maximize the number of passengers transported according to an expected demand and therefore to maximize the expected profit. A correct fleet assignment has to respect the conservation flow, i.e. each aircraft arriving at a station has to depart from the same station later. Klabjan [2005] has lately given an overview on mathematical models for the fleet assignment problem.

### 2.1.3. Aircraft Assignment

The aircraft assignment problem is that of assigning individual aircraft to flights in the schedule considering the *maintenance requirements* of the aircraft and other constraints. The aircraft assignment is performed for each fleet separately. The maintenance constraints may be classified into two main categories: long-term planned maintenance stops and minor maintenance. The long-term maintenance checks can last from several days to several weeks, where the aircraft is unavailable for flights, and must be performed at special *maintenance bases*, where certain technical equipment and staff are available. The long-term maintenance stops are planned for all aircraft in advance, since they must be regularly performed on each aircraft and the capacity of the maintenance facilities is restricted.

Minor maintenance checks must also be performed at interval, normally defined as the maximum number of flying hours. However, minor maintenance checks usually last several hours only and can be performed at night. They can be performed at more airports and are not as restricted by capacity of technical facilities as the long-term maintenance checks.

The aircraft assignment can also be restricted by *connection* and *flight restriction constraints*. The former describe which pairs of flights can be flown by one aircraft in succession. This constraint is mainly given by the time at the airport needed to unload all passengers, to clean and prepare the aircraft and to embark the passengers for the next flight. The minimal time for those activities is called *minimal turn time* and it usually differs with type of the aircraft, the departure and destination airports and even gates and terminals.

The flight restriction constraints forbid certain aircraft from flying to certain destinations, e.g. because the certain aircraft are too noisy or lack an entertainment system required for long flights. It is to be noted that this restrictions are based on properties of individual aircraft as opposed to properties of types of aircraft, which are considered in the fleet assignment problem.

The optimization criteria for the aircraft assignment problem are often related to some quality indicators rather than real monetary costs, because the operational costs are equal for all aircraft of the same fleet. Common quality indicators consider the robustness of the solution. An example is to maximize the number of *standby connections*, which are connections between successive flights longer than some limit, for example two or three hours. The motive is that during long length connections the aircraft can be used for other activities, e.g. to perform other tasks in case of schedule disruptions. Therefore the objective is to penalize medium length connections between successive flights and reward short and long connections. Another example is to minimize the number of aircraft changes performed by crews or to distribute slack in aircraft routes according to some strategy, in order to reduce the impact of delays on the day of operations.

The models and objectives of the aircraft assignment problem differ greatly between airlines. Some airlines plan aircraft assignments right after fleet assignment and before crew scheduling focusing on planning the maintenance stops. In such cases the problem is often also called *maintenance routing*. Other airlines, which are able to schedule the maintenance stops more flexibly, often plan actual aircraft assignments close before the day of operations after crew scheduling and focus on generating feasible aircraft routes or maximizing some objective function (*aircraft routing*). Lately, the name *tail assignment* was proposed for the aircraft assignment problem, where individual flight restriction constraints as well as general maintenance and connection constraints are considered.

### 2.1.4. Crew Scheduling

The crew scheduling problem is that of assigning crew members to flights. Crew scheduling at commercial airlines faces two kinds of restrictions. Firstly, general restrictions on working and flying time which apply to all crew members; secondly, restrictions applying to individual crew members or certain flights, e.g. special language requirements on certain flights or unavailability of certain crew due to vacations or training. The crew scheduling problem is then often decomposed into two planning problems based on this classification of restrictions. In the first step, that of crew pairing, anonymous crew *crew pairings* are generated. Crew crew pairings are sequences of flights a crew flies starting and ending at the same *crew base*. Crew bases are airports in the network, where crews start and end their work duties, and often correspond to the airline hubs. The basic restrictions of the problem are of course the conservation flow and the minimal connection time between two consecutive flights in a pairing. The minimal connection time between two flights is called *minimal sit time* and correspond to the minimal turn time of the aircraft if the crew stays on the aircraft. In the case of an aircraft change the minimal sit time is usually higher, because the crew eventually has to transfer from one gate to another and to perform additional checks on the aircraft.

The crew pairing problem also includes constraints concerning general work-time regulations. The actual constraints and the objective function of the problem depend on the guidelines of governing agencies, the agreements with labor organizations, the cost-structure of the airline and the scope of the optimization. In this work we use rules and objectives which are common to European airlines[1]. For rules and objectives usually used by airlines in North-America see e.g. Barnhart et al. [2003]. A crew crew pairing basically consists of several daily *duties* and night *rests* between them, see also Figure 2.2. Additionally, following rules for crew crew pairings, duties and rests apply:

- Each crew pairing starts at one crew base with a briefing of 60 minutes and ends at the same crew base with a de-briefing of 30 minutes.
- The duration of the crew pairing corresponds to the time away from home for each crew member and is restricted to 96 hours at maximum.
- The minimum duration of a night rest is 10 hours.
- The maximal number of flying hours in a duty is 8.
- The maximum duration of a duty with flying tasks is 10 hours.

[1] Appendix A is an excerpt from the regulations on flight, duty and rest requirements published by the European Parliament.

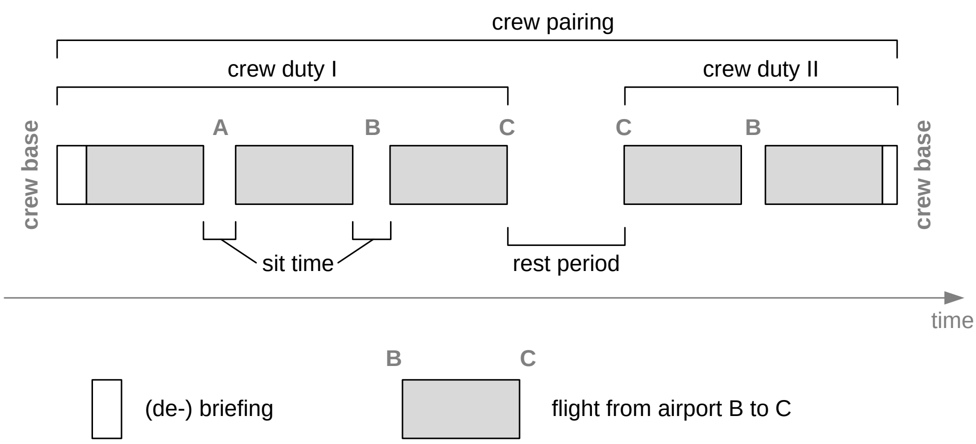


Figure 2.2.: Structure of a Crew Pairing

- The maximum duration of a duty can be increased to 12 hours, when the following rest period is longer than 12 hours.
- The maximum duration of a duty can be increased to 14 hours when the following rest period is longer than 14 hours and the intersection of the duty with night hours between 1am and 7am is less than 2.
- The maximum duration of a duty can be increased to 13 hours when the following rest period is longer than 14 hours and the intersection of the duty with night hours between 1am and 7am is less than 4.

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The objective of crew scheduling is basically to minimize the crew salaries and other crew costs. Depending on the airline, crew salaries can be based on flying time, total work time and time away from home. In the European Union crew salaries are often fixed and therefore the strategic objective is to minimize their number and the tactical objective is equally to distribute the work between all members of the staff and to generate favorable work schedules. Other possible crew costs consists e.g. of ground transport, accommodation and *deadheading*. Deadheading describes a transfer of crew as passengers on regular flights. This can be necessary to return the crew to the crew base or to transfer the crew to the next flight at another airport. In this work we use the objective of minimizing the number of daily duties, independently of the length of those duties. In particular, we do not consider costs of accommodation nor do we plan deadheading. Besides crew costs the objective function for the crew pairing problem often also considers indicators for schedule quality. Common quality measures consider regularity of daily schedules (see e.g. Klabjan et al. [2001a]) and robustness of schedules.

Many indicators for robust crew schedules are discussed in the literature. Similarly to the aircraft assignment problem the goal could be to minimize the number of aircraft changes performed by crews or to redistribute slack in crew pairings to minimize effects of delays. Other indicators for robustness consider the possibility of rescheduling some tasks of a crew to another crew on the day of operations.

In the second step, the crew assignment problem, the generated pairings are assigned to individual crew members, such that every pairing is covered by multiple crew members with the required qualification. The final crew work schedules also contain tasks other than flight duties, e.g. training or reserve duties. The objective of the crew assignment problem is to satisfy crew requests rather than to minimize costs (see Barnhart et al. [2003]). There are different approaches to finding a solution - the bidline and the personalized rostering approaches. The bidline approach is usually used in North-America and carried out in two phases. Firstly, a set of generic rosters covering all pairings is constructed. Then in the second step crew members can bid for their preferred work schedules. The personalized rostering approach, often used outside of North-America, describes the direct generation of rosters trying to maximize the individual preferences of all crew members (Kohl and Karisch [2004]).

## 2.2. Operations in the Airline Industry

On the day of operations many operational tasks are performed to execute the planned schedules, e.g. preparing, flying and maintaining the aircraft, boarding and deboarding passengers and loading and unloading baggage, and preparing and coordinate the airport facilities. Unforeseen events during the execution of those tasks as well as congestion at airports and airspace lead to delays in those tasks.

Delays may be categorized in a variety of ways. Passengers mainly distinguish between departure delay of a flight, which can cause impatience, arrival delay of the last flight in the itinerary, which can be unpleasant due to personal time restrictions at the destination, and arrival delay of any other flight in the itinerary, which can be severe if the passenger misses the next flight.

In this work three categorization systems are important. Firstly, the length of the total delay of a flight; secondly, the concept of *primary delay* and *reactionary delay*; and thirdly, the categorization according to the delay cause.

The first categorization is the *on-time performance* of a schedule in operations. An arrival or departure is on time if it is within $x$ minutes of the originally scheduled time of arrival or departure from a gate. The on-time performance is usually measured for $x$ being 0, 5, 15 and 60 minutes.

A primary delay is an immediate consequence of an unforeseen event, e.g. decrease in the landing capacity at an airport, difficulties during passenger boarding or damage to aircraft. Primary delays can lead to disrupted airline schedules, which cannot be performed as planned, due e.g. to unavailability of aircraft or crew. A common strategy to resolve those disruptions is the intentional delay of additional tasks, e.g. waiting for aircraft or crew. This intentional delay is called reactionary delay or *secondary delay*. Reactionary delays can lead to new *disruptions* of airline schedules. Delay analysis always starts with a primary delay either of a flight tasks or a task during ground operations. For example, the maintenance task of an aircraft may be delayed due to damage of the aircraft, or the passenger boarding task due to late passengers. In both cases these primary delays of non-flight tasks can lead to disruptions of the ground operations schedules and to a reactionary delay of the flight departure.

The distinction between primary and reactionary delay mainly depends on the system boundaries, e.g. scope of the planning or rescheduling problem. For example, during scheduling of crews and aircraft the ground processes between two flight are aggregated to one task for aircraft and one for crew. Thus primary delays occurring during the ground processes are also aggregated to primary delays of those tasks. In a more detailed scope, i.e. considering individual tasks at airports, the same delays can be decomposed into original delays and reactionary delays. The distinction used by European Organisation for the Safety of Air Navigation (EUROCONTROL) as well as in this work is based on flights. The first delay to a flight is called primary delay, subsequent delays caused by the unavailability of crews or aircraft are called reactionary delays. Thus a delayed departure of a flight due to difficulties during the ground process is also a primary delay.

Analyses of delay statistics carried out by airlines and air traffic control organizations are usually categorized according to the delay cause. See also EUROCONTROL [2007] for a good introduction by the EUROCONTROL. Figure 2.3 shows the relative frequency of delays in 2008 in Europe, where primary delays are grouped by accountability. Over 40% of all delays are reactionary and caused by the recovery actions performed by the airline to achieve their operational objectives. Half of all primary delays are airline-related and include initial delays caused by ground and flight operations, passengers, loading and unloading of baggage and cargo as well as damage to aircraft and other technical matters. Airport-related delays mainly occur due to congestion of the airspace and parking areas, leading to arrival and departure delay. En-route delays are caused by a lack of en-route airspace capacity, due e.g. to high demand or lack of air traffic control staff. Weather delays account only for 10% of all delays. Weather delays occur en-route as well as at departure and detination airports. At departure airports weather can lead to difficulties to move around the airport as well as to additional ground operations, like de-icing of the aircraft. At destination airports weather can lead to reduced landing rates, due to high winds or low visibility. Aircraft are actually rarely delayed en-route or on arrival at an airport: rather the departure of those aircraft which are scheduled to fly to affected airports or through affected airspace is delayed by the air traffic control. For purposes of analysis and closer comparability most airlines record detailed information on delay causes according to the International Air Transport Association (IATA) delay code system[2].

The majority of delay causes discussed here can also be categorized into three groups: delays that are related to ground operations at airports, those caused by air traffic control and reactionary delays caused by recovery actions.

[2]See Appendix B for the full list of IATA delay codes

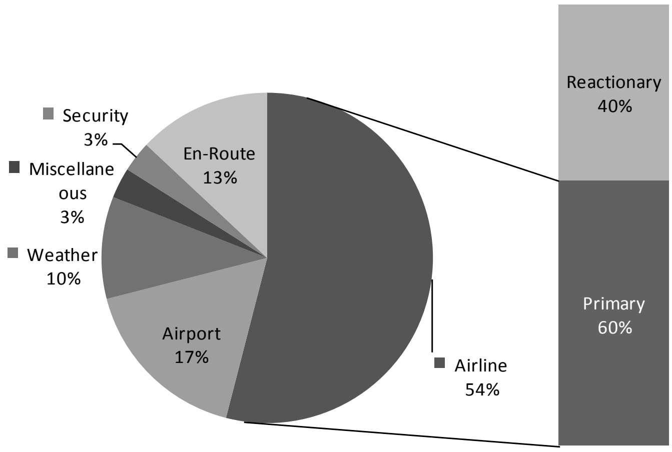



Figure 2.3.: Departure Delay Causes in Europe, 2008, EUROCONTROL [2009]

In the remainder of this section we discuss the fundamentals of airline operations related to each of those delay groups: ground operations performed at the airports, air traffic control and airline recovery processes on the day of operations.

### 2.2.1. Airport Ground Operations

The process of unloading the aircraft, preparing it for the next flight and loading it is called *airport turnaround* process. The movement of the aircraft from the runway to the gates is called *taxi-in* and the movement from the gates to the runway after push-back is called *taxi-out*. The airport turnaround process takes place between taxi-in and taxi-out. Figure 2.4 shows an overview of this process, see also Boeing [2005] for a detailed example on duration and sequence of the turnaround process. The whole process can be subdivided into two parts related to the arrival and the departure of the aircraft:

After landing and the taxi-in to the parking position, the following tasks are performed:

**Docking** is the arrival at the exact location at the gate or parking position, where the ground tasks are performed.

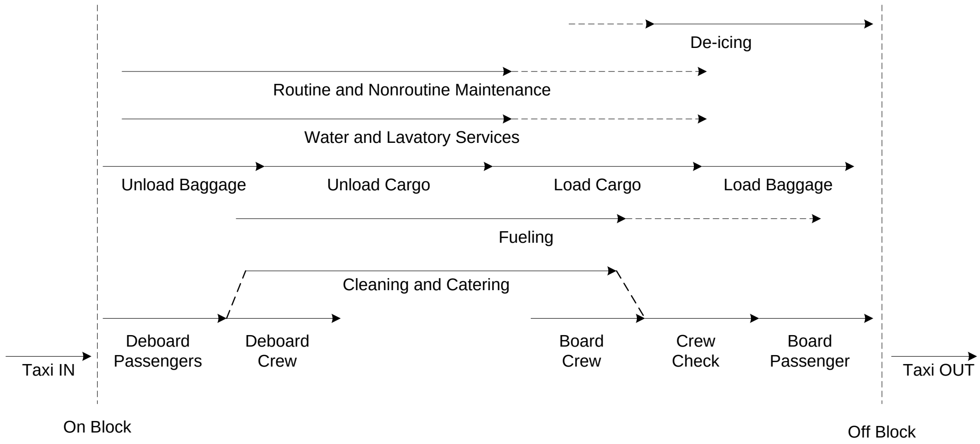



Figure 2.4.: Airport Turnaround Process of an Aircraft

**Deboarding of passengers and crew** starts with placing a boarding bridge or stairs at the aircraft. Additional personnel and buses are needed to guide the passengers.

**Baggage and cargo unloading** usually starts immediately after docking and can be concurrently performed with the deboarding of passengers and crew. In some cases cargo is unloaded at a special areas afterwards, whereas baggage is always unloaded at stand.

**Security** checks are eventually performed on arrival when passengers from certain countries are on board.

Immediately before the taxi-out to the runway and the take-off of the aircraft following tasks are performed:

**Cleaning** of the interior of the aircraft before passengers board.

**Fuelling** is performed with pump or tank vehicles during cleaning and catering.

**Catering** delivers food to the aircraft. Some airlines consider special wishes of passengers (like vegetarian meals), which requires additional planning before the turnaround process.

**Baggage and cargo loading** is performed concurrently with passenger and crew boarding. Like cargo unloading, in some cases cargo loading is performed in special cargo areas.

**Passenger boarding** right before the departure of the aircraft to prevent long waiting times for passengers inside the aircraft.

**Security** checks of all passengers and their luggage at the gate are common at some airports. In the case the security check is performed at a central area of the airport it is not included in the turnaround process.

**Aircraft checks** are performed by the crew before each flight.

**Deicing** is the process of removing frozen contaminant, snow and ice, from the aircraft. *De-icing* is performed immediately before departure of the aircraft.

**Pushback** is the process of pushing back the aircraft from the gate and starting the movement to the runway after all tasks of the turnaround process are finished.

The tasks of the airport turnaround process are preformed concurrently, as shown in Figure 2.4. The minimal time needed to complete this process is called minimal turn time and usually depends on two critical paths of tasks. Firstly, the time needed to unload and load baggage and cargo; and secondly, the time needed to deboard passengers, clean the cabin and board passengers. The time for both paths depends on the aircraft type as well as airport facilities, i.e. the size of the aircraft, the number of entrance points and the number of available boarding bridges. In the case that the crew changes the new crew usually perform additional aircraft checks, which can lead to a longer minimal turn time.

Most of airline-related delays can be allocated to certain tasks in this process. IATA delay codes 11 – 69 describe such delays, like, those caused by missing or late passengers, errors during catering, fuelling, cleaning or cargo handling as well as breakdown of technical equipment or aircraft and missing or late staff. Even the damage to aircraft during flight operations is expressed as a nonroutine maintenance task at the arrival airport. Delay codes 75 – 77 deal with weather delays, like de-icing as well as of airport facilities or other delays of the airport turnaround process due to adverse weather conditions.

### 2.2.2. Air Traffic Control

Air traffic control (ATC) can be described as the service of directing aircraft on the ground and in the air to prevent collision, providing information and support

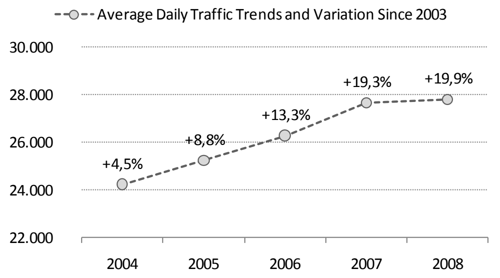



Figure 2.5.: Traffic Variation in Europe Since 2003, Source: EUROCONTROL [2008]

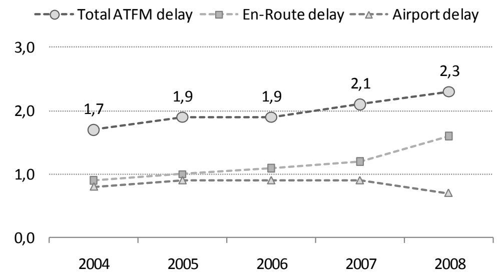



Figure 2.6.: Evolution of the Average ATFM Delay Per Flight, Source: EUROCONTROL [2008]

for pilots and controlling the overall flow of traffic in order to deal with or prevent congestion. The last task is also called air traffic flow management (ATFM). In Europe the ATFM service is centrally provided by the EUROCONTROL Central Flow Management Unit (CFMU). For a good overview of the tasks and of the organization see Leal de Matos and Ormerod [2000]. The goal of ATFM is to limit the impact of delays due to congested airports or airspace. Accordingly, on the day of operations the CFMU can delay the depart of flights to affected airports or airspace or reroute flights to avoid congested airspace. Moreover, additional planning activities together with aircraft operators (airlines), flow managers, air traffic controllers and national governments up to six months before a flight are performed to achieve a more balanced distribution of traffic in European airspace and to avoid delays on the day of operations.

One important measure is the average delay induced by ATFM, IATA codes 81 – 84. In 2008 the average delay per flight in Europe was 2.3 minutes – 1.6 minutes en-route and 0.7 minutes at an airport. The average ATFM delay has increased constantly over the last years in Europe; see Figure 2.6. The main explanation lies in the increase of the average number of daily flights in Europe. In 2008, 27.818 daily flights took place on average, that is 19,9% more than in 2003; see Figure 2.5.

Figure 2.7 shows a comparison of total traffic and flights delayed over 15 minutes due to ATFM restrictions. We can observe a slight seasonal effect on the total traffic as well as a large effect on the number of flight delays.

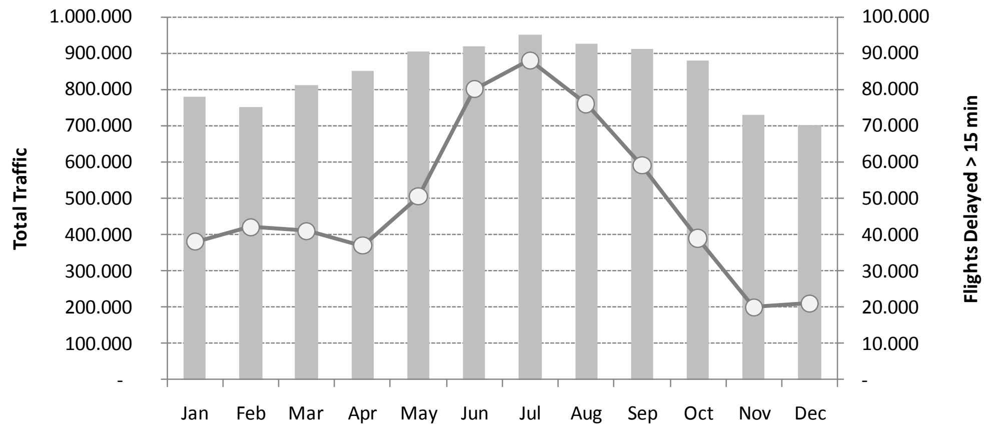



Figure 2.7.: Comparison of Total Traffic and Flights Delayed > 15 min due to ATFM Restrictions in 2008, Source: EUROCONTROL [2009]

### 2.2.3. Recovery of Disruptions

Up to two weeks before the day of operations the tactical schedules are handed over to the operation control center (OCC) of the airline, which is then responsible for the operation of the schedules. The OCC analyses all irregular events, communicates with the ATFM, identifies disruptions to the schedules and performs recovery actions to assure seamless operation of the planned schedules. The OCC has to deal with all resources, especially airport gate assignment, passenger connections as well as aircraft and crew schedules. Irregular events and delays can lead to the following disruptions of aircraft and crew schedules; see also Abdelghany et al. [2008]:

**Aircraft or crew connect rule disruption** occurs when an aircraft or crew member arrives late from the previous flight delaying the next flight. The minimal turn time for aircraft and minimal sit time for crews have to be considered. For crews this disruption is defined during duties only.

**Crew rest disruption** occurs when a crew member gets a rest period that is less than the required minimal rest duration. This happens because of the late arrival at the end of the preceding duty. The consequence is that the affected crew member cannot fly the first flight of the next duty because she/he must get her/his legal rest before starting the next duty.

**Crew duty limit disruption** occurs when the actual working time or flying time during a duty of a crew member exceeds or would exceed the corresponding limits, because of one or more delays during this duty. In this case the crew member cannot fly the remaining flights of the duty nor, start a flight when it is obvious that this disruption is going to happen during this flight.

**Aircraft or crew open position disruption** occurs when an aircraft or crew member does not show up for the flight. In the case of aircraft the reasons could be technical damage, nonroutine maintenance or cancellation of the preceding flight. In the case of crews the reasons could also be personal, e.g. illness, or operational, e.g. flight cancellation.

To resolve these aircraft and crew-related disruptions the OCC can perform a combination of following recovery actions:

**Calling standby/reserve resource.** Standby crews are located at main airports, where they can be quickly assigned to new tasks. Reserve crews are normally at home and therefore need longer preparation time before they can be assigned to tasks. If the resource disruption occurs at an airport without standby/reserve crews, the crew need to deadhead from another airport. There are rarely pure reserve or standby aircraft, due to high purchasing costs. But in some cases an unused aircraft is available, e.g. because current use of the fleet is lower than usual.

**Swapping resources** means that another crew or aircraft is assigned to the task currently disrupted and the currently unavailable resource is assigned to the tasks of the swapped resource, which are schedules for later discharge. Sometimes multiple swaps between several resources are needed to resolve these disruptions.

**Reactionary delay** means that the departure time of one or more flights is rescheduled in order to accommodate waiting for aircraft, crew members, airport facilities or passengers or load from other flights. Reactionary delays can of course lead to additional disruptions regarding own resources as well as passengers. Reactionary delays are described by IATA codes 91 – 96.

**Cancellation** of one or more flights is performed when no other recovery option is found. In this case passengers need to be rebooked to other flights and the aircraft and crew routings must be modified to prevent open position disruptions in future.

Today the recovery process at airlines is usually conducted manually on a flight-by-flight basis rather than by developing one integrated plan for all flights. Furthermore, different people are often responsible for recovery of different resources, i.e. there are aircraft routers, crew schedulers, airport gate assigner, ATFM coordinators and persons responsible for passenger connections. All recovery actions need to be coordinated by at least several of these people. The difficulty here is that the recovery actions suitable for one resource, like small delay to wait for the aircraft, can lead to disruptions at other resources, like crew duty limit disruption due to the additional delay. Therefore in practice complicated recovery actions are hardly performed: instead mainly standby/reserve resources and reactionary delays are used, leading to almost as many reactionary as primary delays.

In this work we refer to the overall process of adapting all schedules of the airline to the new situation as *recovery*. The recovery process includes using mathematical methods, such as optimization and simulation, as well as manual adjustments to the schedules especially in combination with communication between staff. Whereas *rescheduling* describes the adaption of one schedule to the new situation with the aim of performing as few changes to the schedule as possible, usually several rescheduling steps with differing schedules are performed during the recovery process.

## 2.3. Robust Scheduling

Besides disruption management on the day of operations the possibility of disruptions can be also considered during the schedule construction. This is called robust scheduling. The goal of robust scheduling is to construct schedules which remain feasible and near cost optimal under different variations of the operating environment.

The important question is how to measure robustness of a schedule. Scholl [2001] distinguishes two aspects of robustness: *stability* and *flexibility*. Whereas stable schedules are likely to remain feasible and near-cost optimal in a changing operating environment, flexible schedules can be efficiently adapted to changing operating environment. Robust schedules can also be stable and flexible at the same time. In the airline field this means that a schedule is stable as long as it remains feasible at justifiable costs without any manual changes to the schedules. A schedule is flexible if in case of disruption recovery actions are possible in order to restore the operational feasibility of the schedule without significantly increasing costs. In the case of crew and aircraft schedules reactionary delays can be considered as a manual recovery action as well as an automatic consequence if no manual recovery is performed. Thus the number and duration of reactionary delays is a main measure of the stability and flexibility of airline schedules. Additional measures of the stability of a schedule are the number of disruptions which cannot be automatically resolved by reactionary delay. Additional measures for the flexibility of a schedule are the costs of recovery as well as effects on other schedules.

Figure 2.8 summarizes this chapter showing the interrelation of disruptions for aircraft and crew schedules. If this scope is extended, additional sources of primary delays and additional recovery actions need to be considered, but the general interrelation remains.

Realistic measures of stability and flexibility of a schedule can be often computed by simulating the schedule-generating delays and using recovery actions. For the purpose of constructing robust schedules, however, these computations are far too complex for the application of optimization methods to minimize costs and maximize robustness. Thus one challenge of robust scheduling is to find more abstract indicators for robustness which are able to predict the robustness measure as well as lead to solvable mathematical representations of the scheduling tasks.

An example of an indicators for stability is the minimization of aircraft changes by crews when there is not enough slack to prevent propagation of delays to several aircraft. An example of an indicators for flexibility is the number of theoretical

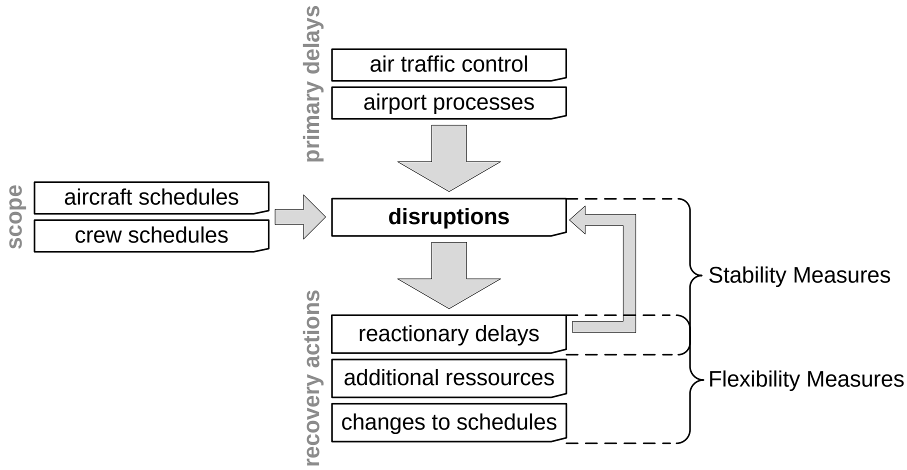



Figure 2.8.: Interrelation of Disruptions

possibilities for swapping crews in the current schedule. A high number indicates that on the day of operations more efficient crew recovery is possible. Further examples are given in Chapter 3. As in the case of those examples, most indicators primarily support one measure for stability or flexibility. A second challenge of robust scheduling is therefore to find indicators or sets of indicators to support all main measures for stability and flexibility in order to obtain robust schedules.

# 3. State Of The Art: Aircraft Assignment and Crew Pairing

In this chapter we discuss the planning problems and methods of aircraft and crew scheduling in the airline industry in detail. In addition to scheduling methods for both problems, we also discuss the integration of those planing problems, the rescheduling on the day of operation and recent approaches to robust scheduling. Other planning problems are also very important in the airline industry and often difficult to solve, but not relevant for this work. For recent surveys on airline scheduling problems we refer to Gopalan and Talluri [1998b], Klabjan [2005] and Weide [2009].

The next section deals only with the mathematical background of the relevant planning problems and methods and can therefore be omitted by readers with experience in mathematical programming.

## 3.1. Mathematical Background

The airline planning problems addressed in this work are often formulated as mathematical optimization models. In particular, the set partitioning and resource constrained shortest path problems are widely used to model the problems in this work. We therefore provide the necessary background to these two mathematical problems as well as relevant optimization techniques. For extensive surveys of combinatorial optimization we refer to Wolsey [1998].

### 3.1.1. Set Partitioning Problem

The set partitioning problem (SPP) is that of partitioning the rows of a $m \times n$ zero-one matrix by a subset of columns at the minimal cost, see Wolsey [1998]. A column $j \in N$ is a subset of rows from $M$. The element $a_{ij}$ is set to 1 if column

$j$ covers row $i$ and to 0 otherwise. $x_j$ is a binary variable that equals 1 if column $j \in N$ is part of the solution and 0 otherwise. $c_j$ is the cost associated with the corresponding column $j$. The SPP can be formulated as follows:

$$\sum_{j \in N} c_j x_j \rightarrow \min \qquad \text{SPP} \tag{3.1}$$

$$\sum_{j \in N} a_{ij} x_j = 1 \qquad \forall i \in M \tag{3.2}$$

$$x_j \in \{0, 1\} \qquad \forall j \in N. \tag{3.3}$$

The objective (3.1) minimizes the costs of the selected columns and equations (3.2) and (3.3) describe that each row must be covered by exactly one column and no fractional selection of columns is allowed.

The set covering problem (SCP) is a relaxation of the SPP that requires to cover each row by at least one column rather than exactly one column. The SCP is formulated with a greater or equals sign "≥" in equation (3.2). The optimal solution of the SCP is a lower bound of the corresponding SPP.

### 3.1.2. Resource-Constrained Shortest Path Problem and Dynamic Programming

The resource-constrained shortest path problem (RCSP) is that of finding the minimum cost path between a source node $s$ and a sink node $t$ while respecting constraints on resource consumption. Let $G = (N, A)$ be a directed graph with a set of nodes $N$ and a set of directed arcs $A$. A binary variable $x_{ij}$ is set to 1, if the directed arc $(i, j) \in A$, which is part of the solution and 0 otherwise. $c_{ij}$ is the traversal cost and $d_{ij}^r \geq 0$ is the resource consumption of resource $r \in R$ at arc $(i, j)$. A solution is resource feasible if and only if the consumption of each resource $r \in R$ is greater or equal to the lower bound $l^r$ and less or equal to the upper bound $u^r$. The RCSP is formulated as follows:

$$\sum_{(i,j)\in A} c_{ij}x_{ij} \to \min \qquad \text{RCSP} \tag{3.4}$$

$$\sum_{\{j:(i,j)\in A\}} x_{ij} - \sum_{\{j:(j,i)\in A\}} x_{ji} = \begin{cases} 1 & \text{for } i = s \\ 0 & \text{for } i \in N \setminus \{s,t\} \\ -1 & \text{for } i = t \end{cases} \tag{3.5}$$

$$l^r \leq \sum_{(i,j)\in A} d^r_{ij}x_{ij} \leq u^r \qquad \forall r \in R \tag{3.6}$$

$$x_{ij} \in \{0,1\} \qquad \forall (i,j) \in A. \tag{3.7}$$

Note that cost and resource consumption at nodes can be easily transformed to arcs. In this work the RCSP is efficiently solved using a dynamic programming method by iteratively constructing all feasible paths from source $s$ to sink $t$. Paths are encoded by *labels* at nodes, thus $L^k_{n_p}$ describes a label at node $n_p \in N$ corresponding to path $P^k_{n_p} = (s, \ldots, n_{p-1}, n_p)$, which is linked with its predecessor label $L^l_{n_{p-1}}$ at node $n_{p-1}$. The labels also store information about cost $c(P^k_n)$ and resource consumption $(d^1(P^k_n), \ldots, d^{|R|}(P^k_n))$ of the represented path. A basic pulling dynamic programming algorithm constructs (pulls) labels from all predecessor nodes to a node by updating cost and resource consumption. Thus a new label $l$ at node $j$ pulled from label $k$ at node $i$ is given by

$$L^l_j = (L^k_i, c(P^k_i) + c_{ij}, d^1(P^k_i) + d^1_{ij}, \ldots, d^{|R|}(P^k_i) + d^{|R|}_{ij}) \tag{3.8}$$

A new label is only created if the resource consumption does not exceed the upper bound for all resources, i.e. $\forall r \in R : d^r(P^l_j) \leq u^r$. In order to improve the efficiency of the algorithm, only labels that are not dominated by already existing labels are stored. A label is dominated by every label at the same node with less or equal cost and resource consumption, i.e. $L^n_i$ dominates $L^k_i$ if $c(P^n_i) \leq c(P^k_i)$ and $\forall r \in R : d^r(P^n_i) \leq d^r(P^k_i)$.

The performance of the algorithm mainly depends on the implementation of the dominance tests, but can be further improved using preprocessing techniques and problem-specific acceleration techniques. Irnich and Desaulniers [2005] give

**Algorithm 3.1**: Basic label setting algorithm for the RCSP

Set $L_s = \{(nil, 0, \ldots, 0)\}$
Set $L_t = \emptyset$
**foreach** *node* $j \in N \setminus \{s\}$ **do**
  **foreach** *predecessor node* $i$ *of* $j$ **do**
    **foreach** *label* $l$ *at node* $i$ **do**
      **if** $\exists r \in R : d^r(l) + d^r_{ij} > u^r$ **then**
        next $l$
      Set $L^m_j = (l, c(l) + c_{ij}, d^1(l) + d^1_{ij}, \ldots, d^{|R|}(l) + d^{|R|}_{ij})$
      Add $L^m_j$ to active labels at node $j$
  Remove dominated labels at node $j$

an extensive overview on the formulations of the RCSP as well as the solution methods. Hassin [1992] discuss polynomial approximation schemes for the restricted shortest path problem with one resource. Lately Steinzen [2007] presented acceleration techniques for the restricted shortest path problem solved by a labeling algorithm.

### 3.1.3. Branch-and-Bound

*Branch-and-Bound* is an exact solution method for combinatorial problems that repeatedly divides the problem into smaller subproblems and solves these subproblems recursively. The method begins with solving a relaxation of the original problem, usually a linear relaxation. If the solution of the relaxed problem is not feasible in the original problem, the relaxed problem is divided into two or more subproblems by fixing alternatives aspects of the solution. This process is called *branching*, i.e. two or more alternative branches of the solution space are created, called nodes. Through repeated branching a set of connected nodes is created, also called the branch-and-bound tree. Besides the creation of smaller problems, another aim of branching is to eliminate the current infeasible solution.

Feasible solutions are found during the process, because either the solution of a relaxation of the problem is, fortunately, feasible or the solution space only contains a feasible solution after repeated branching. The main idea of branch-and-bound is to prevent unnecessary exploration of the solution space through eliminating nodes, which certainly cannot contain an optimal solution. This step

**Algorithm 3.2**: Branch-and-Bound

Initialize the set of unprocessed nodes $N$ with the original problem
Set upper bound $Z^* = \inf$
**while** $N \neq \emptyset$ **do**
  Select next node $n \in N$ and delete it from $N$
  Solve the linear relaxation of the problem in node $n$ and store dual bound $\widehat{Z}$ and solution $\widehat{x}$
  **if** $\widehat{x}$ *is empty* **then**
    Prune node $n$ by infeasibility.
  **else if** $\widehat{Z} \geq Z^*$ **then**
    Prune node $n$ by bound.
  **else if** $\widehat{x}$ *is integer* **then**
    Set $Z^* = \widehat{Z}$ and $x^* = \widehat{x}$
    Prune node $n$ by optimality.
  **else** /* Perform Branching */
    Create two subproblems $n_1$ and $n_2$ that cut off the current fractional solution
    Add both problems to $N$
Terminate with solution $x^*$ and objective $Z^*$

is called *bounding*. Thus the objective value of infeasible solutions represents a bound to the optimal feasible objective value in this node. For minimization problems this is the lower bound. A node cannot contain a better solution, if the lower bound of this node is greater or equal to the best known objective value of a feasible solution.

Algorithm 3.2 shows the outline of the method. After solving the relaxation of the problem basically three steps are evaluated. Firstly, if no solution could be found, the node is pruned due to infeasibility. Secondly, if the solution of the relaxed problem is feasible, this node does not need to be explored further and is therefore pruned due to optimality. If the objective value is better than the best known objective value, it is stored for bounding and as a possible optimal solution. In the usual case that the solution is not feasible, however, branching is performed and two or more new nodes are added to the tree. As long as unprocessed nodes exist the algorithm continues with the next node.

The performance of the algorithm is mainly determined by the strategies for branching as well as selection of the next node. For both strategies there is extensive research and a lot of problem-specific approaches. In this work we discuss strategies for crew scheduling and aircraft routing in Chapter 5 and 8.

The algorithm can be extended in a variety of ways. A well-known generic approach is to strengthen the relaxation of the problems solved during branch-and-bound with valid inequalities (cuts), giving rise to the name *branch-and-cut*. If a column generation method is used to solve the linear relaxation of the problems during branch-and-bound, the whole algorithm is usually called *branch-and-price*. If both concepts, valid inequalities as well as column generation, are applied, the method is called *branch-and-price-and-cut*.

### 3.1.4. Column Generation

*Column generation* is a method of solving linear programs with a large number of variables. The main idea of column generation is to solve two smaller linear programs iteratively until an optimal solution to the original problem is found. Consider the following linear program

$$\sum_{j \in N} c_j x_j \to \min \qquad \text{Master Problem} \tag{3.9}$$

$$\sum_{j \in N} a_{ij} x_j = d_i \qquad \forall i \in M \tag{3.10}$$

$$x_j \in \mathbb{R} \qquad \forall j \in N. \tag{3.11}$$

We assume that $|N|$ is huge, such that it is impossible or impractical to solve the master problem directly. The idea of column generation is to solve several restricted master problems (RMPs) each with a small subset of all columns. Algorithm 3.3 shows the outline of the column generation algorithm. Firstly, the RMP is solved with an initial set of columns $N^0$ that contains a feasible solution. The dual solution of the RMP is used to select new columns, which can improve the solution of the current RMP. New columns are often obtained by solving another linear program, the *pricing problem*. The goal of the pricing problem is to find columns with negative reduced costs, i.e. columns that can improve the objective value of the current RMP. Thus the pricing problem is often formulated as that of finding the column with the smallest reduced costs:

$$\overline{c^*} \min\{c_j - \sum_{i \in M_1} a_{ij}\pi_i : j \in N\} \tag{3.12}$$

If the found column has negative reduced costs it is added to the RMP and the problem is solved again with the new set of columns. If no column with negative reduced costs can be found, the column generation algorithm terminates with the solution of the current RMP as an optimal solution to the original problem.

In order to improve the performance of the algorithm often several columns with negative reduced costs are added in each iteration of column generation (multiple pricing). Especially for degenerated, large-scale problems the value of the reduced costs is a rough indicator of the improvement of the objective value only. Thus the pricing step is often not solved to optimality in each iteration of column generation, but instead only a sufficient number of columns with negative reduced costs is selected. At the end of column generation, however, the pricing problem needs to be solved to optimality to prove that no further columns with negative reduced costs exist.

For scheduling problems such as those considered in this work, the pricing problem is often formulated as a resource-constrained shortest path problem and solved with dynamic programming methods. In order to solve integer programs with many variables such as the already-mentioned scheduling problems, a column generation algorithm is often integrated with a branch-and-bound algorithm. In this solution approach, often called branch-and-price, a column generation algorithm is used to solve the linear relaxation of the master problem in each node of the branch-and-bound tree.

We refer to Wolsey [1998] for a general discussion on column generation and to Barnhart et al. [1998] for a discussion on column generation for integer programs. Recent surveys on column generation methods are given by Desrosiers and Lübbecke [2005], Lübbecke and Desrosiers [2005] and Desaulniers et al. [2001].

**Algorithm 3.3**: Column Generation Algorithm

Initialize $N^0$
Set $t = 0$ and $N^{new} = N^0$
**while** $|N^{new}| \neq 0$ **do**
  Solve restricted master problem with $N^t$ and store dual solution $\pi^t$
  Solve pricing problem with $\pi^t$ and obtain columns $N^{new}$
  Set $N^{t+1} = N^{new} \cup N^t$ and $t = t + 1$

## 3.2. Aircraft Assignment

The aircraft assignment problem differs between airlines and research publications; compare Grönkvist [2005] and Chapter 2. The aircraft assignment step is sometimes performed before and sometimes after crew scheduling. In most publications maintenance constraints are considered; in some connection and flight restriction constraints also. There are many different objectives used for aircraft assignment, ranging from pure feasibility problems to different robustness criteria. The numerous variations of the aircraft assignment problem lead to many different solution approaches proposed in the literature.

Clarke et al. [2003] describe a mathematical formulation for the daily aircraft assignment problem with maintenance checks required every three or four days. One additional constraint is that each aircraft must fly all flights in the schedule within a certain time window. This constraint is called big-cycle constraint and is equivalent to finding a Euler-tour in the flight network. The objective of the problem is to maximize the *through revenue*. Through revenue describes the benefit to the customers if two flights are operated by the same aircraft in sequence. The benefit arises if customers booked both flights and can therefore stay on the aircraft between these. The problem is formulated as an asymmetric travelling salesman problem with additional constraints. In the first step of the solution method the sub-tour elimination constraints and maintenance constraints are relaxed in a Lagrangian way. The relaxed problem is then solved iteratively and the violated relaxed constraints are added again to the problem. The procedure is embedded in a branch-and-bound method to prove optimality. Gopalan and Talluri [1998a] propose a two-step method for solving this problem. In the first step, several daily routes of aircraft, which span several airports, are fixed by applying simple heuristics. The resulting routes end in so-called overnight-airports and lead to a reduced network with one arc for each fixed daily route. The overnight-airports can have maintenance capabilities; maintenance then is performed at night. Thus in the second step the maintenance problem is solved on the reduced network requiring the three-day maintenance constraint. Sriram and Haghani [2003] provide a multi-commodity network flow problem formulation for the weekly aircraft assignment problem with maintenance constraints. The model requires routings for each day as input and is solved by a heuristic local search.

Grönkvist [2005] consider individual aircraft for the aircraft assignment problem resulting in a set partitioning formulation, where the columns represent aircraft routes and constraints ensure that each flight is covered by exactly one aircraft. The formulation is solved by column generation with a heuristic fixing process in order to integer solutions. Constraint programming is used in a preprocessing step to reduce the size of the flight network in the pricing subproblem. Sarac et al. [2006] extend the set partitioning formulation with additional constraints to ensure sufficient capacity at maintenance bases. The solution method is also based on column generation which is embedded into a branch-and-price algorithm. The branching strategy is a combination of follow-on and aircraft-flight pair branching (see Section 5.3). The pricing subproblem is a constrained shortest path problem with one resource and is solved by dynamic programming.

Several authors propose integration of the aircraft assignment and the fleet assignment problems. Barnhart et al. [1998] consider the integrated problem with maintenance requirements and general connection restrictions. The problem is formulated as a set partitioning problem where the columns are *strings* representing a feasible sequence of flights that starts and ends at a maintenance base. The set partitioning constraint requires that each flight is covered by exactly one maintenance string. The problem is solved by branch-and-price with a resource constrained shortest path problem for generation of maintenance strings. Ioachim et al. [1999] propose a multi-commodity flow formulation for the integrated fleet and aircraft assignment with time windows. Additional constraints are added to synchronize departure times of flights on different days. The problem is also solved by branch-and-price. Desaulniers et al. [1997] propose two formulations, a multi-commodity flow and a set partitioning formulation, for the integrated fleet and aircraft assignment with time windows. Both formulations are solved with column generation and branch-and-bound.

Clausen et al. [2009] have recently provided a comprehensive review of concepts and models used for recovery of airline resources. Many solution approaches to aircraft rerouting are based on the original planning methods and incorporate flight cancellations and retiming. See Jarrah et al. [1993], Rakshit et al. [1996], Yan and Yang [1996], Cao and Kanafani [1997a] and Cao and Kanafani [1997b] for publications on methods based on network-flow algorithms. Several authors propose column generation algorithms for a set partitioning formulation of the aircraft recovery problem, see i.e. Rosenberger et al. [2003] and Andersson and Värbrand [2004].

## 3.3. Crew Pairing Optimization

The crew pairing problem is usually restricted by complicated feasibility and cost rules for the pairings. A very large number of feasible pairings exist, such that their enumeration is often impossible. Until the 1990s many heuristic local search methods were proposed to solve the problem. Today the problem is usually solved by mathematical optimization methods because these can provide better solutions as well as a bound on the quality of the solutions. In most cases column generation methods are used to avoid the enumeration of all feasible pairings. The RMP is in most cases a set partitioning or set covering problem ensuring that each flight is covered by exactly or at least one crew pairing. Feasible crew pairings are generated in the pricing subproblem. In this work we describe some of the most important formulations and solution methods based on column generation. For a comprehensive overview of state-of-the-art solution methods we refer to Barnhart et al. [2003] and Gopalakrishnan and Johnson [2005].

There are two main types of network to model the pricing subproblem. Vance et al. [1997] describe both, the *flight-based* and the *duty-based network*. In a flight-based network all flights from the schedule are connected by arcs. In the duty-based network possible duties are connected by arcs. Thus using the duty-based network either all duties have to be enumerated at the beginning or the pairing generation becomes a two-step process: in the first step promising duties are generated and in the second promising pairings are created. The two-step process limits the solution space of the pricing subproblem, because only a subset of all possible duties is used to build the network for pairing generation. The implicit consideration of duties in the flight-based network has no such limitation. Further, the duty-based network has more arcs because there are usually more duties than flights, but also the advantage that all duty rules can be built into the network. In contrast, if a flight-based network is used, duty and crew pairing rules have both to be checked by the algorithm.

Both types of network, flight-based and duty-based, can be designed as a *time-space* or as a *connection-based network*. In a connection-based network flights or duties are represented by nodes. Arcs represent possible connections between flights or duties. Each pair of flights or duties that can occur in succession in a crew pairing is connected by an arc. A time-space network uses arcs for flights and duties and nodes for start and end of each flight or duty. Additional arcs connect pairs of chronologically consecutive nodes at one airport and represent a ground waiting timespan at this airport. Thus possible connections between flights or duties are represented implicitly through one or more ground waiting arcs. Desaulniers et al. [1999] describe both network structures for the flight-based network type. The time-space network has fewer arcs in comparison to the connection-based network, but requires an explicit check of the night rest and sit-time rules by a search algorithm. In the context of branch-and-price, branching decisions on flight connections can be represented by simply removing arcs from the connection-based network. In contrast, the time-space network requires branching decisions to be enforced by the search algorithm.

In all possible networks a path represents a crew pairing. Pairings with negative reduced costs can improve the solution of the current restricted master problem. There are several approaches to finding crew pairings with negative reduced costs. A resource constrained shortest path problem can be solved by dynamic programming, where non-linear feasibility and cost rules are represented by resources as in Vance et al. [1997], Desaulniers et al. [1997]. Anbil et al. [1998] use a depth-first-search enumeration of the pairings. Klabjan et al. [2001b] use a random depth search algorithm where the probability of a connection to be selected is decreased with the duration of connection time. Galia and Hjorring [2004] solve a k-shortest path problem until a path with negative reduced cost is found. Fahle et al. [2002] propose a constraint programming approach for a column generation method used to solve the crew assignment problem.

To obtain integer feasible solutions the column generation method is often combined with a branch-and-bound method or variable fixing heuristics. If pairings are generated during the branch-and-bound or fixing heuristic, the method is called branch-and-price or fix-and-price respectively. Ryan and Foster [1981] propose a *constraint branching rule* to solve scheduling problems which are formulated as set partitioning or covering problems. Vance et al. [1997] extends this rule to the *follow-on branching rule* which is better suited for branch-and-price methods. The follow-on branching rule requires that a pair of two flights is either covered by one pairing in sequence or not covered by a same pairing. This rule is also used in Desaulniers et al. [1999]. Klabjan et al. [2001b] proposes the *time-line branching rule* which requires that in one branch the connection time in the solution between two particular flights is below a threshold and in the other branch it must be above the threshold.

Desaulniers et al. [1997] present a nonlinear multi-commodity network flow formulation and apply the Dantzig-Wolfe decomposition resulting in a set partitioning formulation for the RMP and several resource constrained shortest path subproblems for generation of feasible crew pairings. The problem is then solved by branch-and-price.

Vance et al. [1997] propose an alternative formulation for the crew pairing problem. The problem is decomposed into two stages. In the first stage a set of daily duties that covers all flights in the schedule is selected. In the second stage crew pairings are built based on the selected daily duties. A Dantzig-Wolfe decomposition is used to solve the problem with column generation resulting in two subproblems, one for pairing generation and one duty set subproblem ensuring that each flight is covered by exactly one duty. The formulation yields a better LP-bound, but is harder to solve.

The minimal sit time between two flights (crew connection) is usually higher than the minimal turn time between the same two flights (aircraft connection). In the case that the crew stays on the aircraft between two flights, however, the minimal turn time applies to both connections. Thus the feasible solution space during crew scheduling is limited by the aircraft schedule and can lead to suboptimal solutions. This limitation can lead to suboptimal solutions and is therefore the main motive for integrating aircraft assignment and crew pairing problems.

Cordeau et al. [2001] use Benders decomposition and branch-and-price to solve an integrated model for aircraft assignment and crew pairing. They model the aircraft routing problem as the master problem where optimality information from the crew pairing subproblem is transfered to the master problem. Cohn and Barnhart [2003], on the other hand, extend the crew pairing problem by representing aircraft assignment solutions as additional variables in the problem. Constraints ensure that only one variable representing an aircraft assignment solution is selected. The problems is solved by branch-and-price with two pricing problems, one for crew pairings and one for aircraft assignment solutions.

Mercier et al. [2005] consider an integrated model and use Benders decomposition and branch-and-price to solve it. The master problems is in this case the crew pairing problem and only feasibility information is transferred from the aircraft assignment problem. Mercier et al. [2005] report that their approach can solve larger problems in less computation time than the approaches proposed by Cordeau et al. [2001] and Cohn and Barnhart [2003].

Klabjan et al. [2002] partially integrate aircraft assignment and crew pairing problems. The two problems are solved sequentially, but with the crew pairing problem first. Plane count constraints in the crew pairing problem formulation ensure that a feasible aircraft assignment exists for the current crew pairing solution. The times for flight departure are varied within certain time windows in order to increase the number of feasible pairings and therefore to decrease crew costs. The crew pairing formulation is solved by branch-and-bound with column generation in the root node only.

Mercier and Soumis [2007] extend the integrated model for aircraft assignment and crew pairing of Mercier et al. [2005] with time windows for flight departure. Binary variables indicate the selected departure times and counting constraints ensure that same departure times are for the aircraft assignment as well as crew pairing. The problem is solved by Benders decomposition.

According to Clausen et al. [2009] most formulations for the crew recovery problem in the literature assume that the flight schedule is recovered before. Examples for such approaches are those of Stojković et al. [1998], Guo [2005], Nissen and Haase [2006] and Medard and Sawhney [2007]. The approaches of

Johnson et al. [1994], Lettovský et al. [2000] and Yu and Qi [2004] add the possibility of cancelling flights during crew rescheduling. Finally, Stojković and Soumis [2001], Stojković and Soumis [2005], Abdelghany et al. [2004] and Zhao et al. [2007] explicitly model departure delays in crew rescheduling.

Lettovsky [1997] presented the first approach of true integrated recovery of aircraft, crew and passengers. The integrated problem is decomposed into three subproblems (one for each resource) which are synchronized by a master problem. This master problem determines flight cancellations and delays. The problem is solved by applying Benders decomposition. In Abdelghany et al. [2008] a proactive recovery tool named DSTAR is used for integrated recovery of aircraft, cockpit crew and cabin crew based on disruption forecast of a simulation model. A rolling horizon over the schedule is used for recovery in multiple stages. At every stage the problem is solved to optimality considering only a small subset of flights.

## 3.4. Robust Scheduling of Aircraft and Crew Pairings

Regarding the robust aircraft assignment problem Ageeva [2000] propose increasing the flexibility of schedules by maximizing aircraft swap opportunities. On the flight string models for aircraft fleeting and routing by Barnhart et al. [1998], they generate aircraft schedules where flight-strings meet each other as often as possible in order to provide opportunities to swap sub-strings if disruptions occur. By contrast, Lan et al. [2006] improve the stability of schedules by considering how changes in aircraft routes may reduce potential delay propagation by aircraft connections. They use historical data about delay information and increase the stability of aircraft routes by using slack in the schedule to absorb possible disruptions. The proposed robust aircraft assignment model is solved by a Dantzig-Wolfe decomposition. Wu [2006] changes a given aircraft assignment solution by retiming flights in order to enlarge buffer times for flights with higher delay probability. Whereas most authors use indicators to measure the robustness of schedules, Fuhr [2007] proposes an analytic approach to evaluate the propagation of delays in aircraft routes. The approach considers aircraft routes only without crews and is computed approximately.

Burke et al. [2010] are the first to consider stability and flexibility of aircraft schedules simultaneously. The stability indicators (called reliability in the paper) describes the probability of propagation of delays between flights. The flexibility indicator is defined in terms of the number of aircraft swap opportunities. A multi-objective memetic algorithm is proposed to improve stability as well as flexibility of aircraft schedules simultaneously by re-routing aircraft and retiming flights. A simulation study with real world schedules is performed to explore the effects of the robustness objectives on the operational performance of schedules. For the experiments a simulation model developed at KLM Royal Dutch Airlines is used which includes a recovery module for aircraft (see Jacobs et al. [2005]). The results show that the influence of the stability objective is dominant on the operational performance of aircraft schedules. This study, however, does not consider the interdependencies of aircraft and crews.

Regarding the crew scheduling problem, Shebalov and Klabjan [2006] propose an indicator for flexibility based on task swaps as a recovery option for crew schedules. This means that a crew whose arrival is delayed next flies a flight with a later departure time than its originally assigned flight. A different crew, called the *move-up crew*, then covers the flight of the disrupted crew. The paper presents an optimization procedure to select a set of crew pairings with many opportunities for crew swapping in addition to capture the crew cost. The authors propose a new solution method based on Lagrangian relaxation and column generation for the computationally difficult problem.

Several publications consider the stability of crew schedules. Ehrgott and Ryan [2002] describe an indicator for stability of crew schedules, which considers the difference between the slack duration and the expected duration of a delay between each pair of flights. The stability indicator for a daily crew duty is then the sum of those values for each flight pair in the duty. The daily crew pairing problem is formulated as a bicriteria problem with crew costs and stability indicator as opposite optimization criteria. The problem is solved using the $\epsilon$-constraint approach. Thus the optimal crew costs are computed first and then in a second step the stability of a schedule is maximized allowing an increase of $\epsilon$% of the crew costs in comparison to the optimal costs. The approach does not consider interdepencies of pairings and aicraft nor does it consider the propagation of delays.

Schaefer et al. [2005] propose a stochastic extension to the deterministic crew scheduling problem. A classical Dantzig-Wolfe decomposition of the crew pairing optimization problem into a crew pairing generation problem and a crew pairing selection problem is used, but for each crew pairing the *expected cost is estimated through simulation*. Additionally to classical costs the crew pairings are penalized according to an estimate of the delay propagation. The simulation software used is described in Rosenberger et al. [2002]. One main assumption is that crews never wait for late aircraft. Thus no interdependency between aircraft due to crews changing aircraft is captured. Consequently, no interdependency between crew pairings using the same aircraft is captured either. The solution times are comparable with deterministic problems, only a constant effort for simulating each crew pairing once is added.

Yen and Birge [2006] are the first to consider interdependencies of crew pairings and aircraft. They model the crew pairing selection problem as a *two-stage stochastic programming problem*. The first stage corresponds to the deterministic crew pairing optimization problem. The second stage computes the propagation of delays caused by interdependencies between aircraft routes and crew pairings. The proposed branch-and-bound method starts with a given set of feasible crew pairings and branches on flight pairs with high delay propagation. During the solution process, however, an integer set partitioning subproblem must be solved frequently. These aspects lead to very high solution times, thus only small problem instances can be solved.

Mercier et al. [2005] propose an integrated aircraft assignment and crew scheduling problem incorporating an indicator for stability based on the available ground time for crews during an aircraft change. Aircraft changes with ground time below a threshold are called restricted. Schedules with less *restricted aircraft changes* are assumed to be more stable; therefore a penalty value for such aircraft changes is used. Two Benders decomposition methods combined with branch-and-price are proposed to solve the integrated model. Both methods lead to very high solution times. The proposed indicator for stability does not consider any recovery actions nor explicit delay propagation; thus only a local evaluation of the aircraft changes is carried out.

AhmedBeygi et al. [2008] redistribute slack in an existing aircraft and crew schedule in order to reduce propagation of delays by retiming flights. An optimization model is therefore formulated which considers the propagation of delays and minimizes their probability. The computational results show opportunities for improvement of robustness without increasing planned costs.

Weide et al. [2009] propose to solve the integrated aircraft assignment and crew scheduling model heuristically by solving the two problems iteratively using a shared objective function. A stability indicator is used to penalize *restricted aircraft changes* according to the duration of slack during such an aircraft change. The solution time can be dramatically reduced unlike on the integrated model of Mercier et al. [2005]. The iterative approach leads to a set of aircraft and crew schedules with different relation between planned costs and an indicator value for schedule stability. The authors highlight that it is easier for a decision maker to select a solution with the preferred relation afterwards instead of assigning monetary costs to the stability indicator. The proposed indicator for stability, however, is very difficult for a decision maker to interpret, because it does not measure delay propagation nor does it use historical data. The authors compare the iterative approach with several optimization methods and extend the model with flexible departure times. They do not, however, evaluate and compare the proposed indicator for schedule stability with other approaches to increasing robustness.

# 4. Required Work

Study of the literature on improving robustness of crew and aircraft schedules highlights several important aspects. Several authors use simulation studies to evaluate the quality of approaches for robust scheduling, but still there is no framework for evaluation and comparison of approaches. Many authors highlight the importance of interdependencies between crews and aircraft for the robustness of both schedules, but a full integrated formulation for both scheduling tasks is very difficult to solve, see Mercier and Soumis [2007]. Most approaches to incorporating historical data during scheduling only consider a simplified model of delay propagation between aircraft and crews in order to reduce the complexity of the scheduling problem. Yen and Birge [2006] propose an exact model for delay propagation, which can be solved for small problem instances only. There are no approaches in the literature which focus on stability of crew duties, although disruptions of crew duty rules can lead to major disruptions of airline schedules. There are only a few publications on flexibility of airline schedules and only one which explores the trade-off between stability and flexibility of airline schedules. To summarise, research is required into methods of evaluation of robust scheduling approaches, more exact indicators based on delay propagation between aircraft and crews, stability of crew duties, and additional approaches for increasing flexibility of airline schedules.

From this study of the state-of-the-art literature we select three objectives for this research. Firstly, we propose to develop a new more exact approach to measuring delay propagation during simultaneous scheduling of aircraft and crews. Secondly, we need a method to compare the new approach with existing approaches for robust scheduling. And thirdly, we shall show in this chapter that we need to improve the existing scheduling methods in order to cope with additional complexity induced by the consideration of uncertainty in the scheduling problems.

The main idea for the new approach to measuring delay propagation more exactly during scheduling of aircraft and crews is to solve an integrated stochastic crew and aircraft scheduling problem by decomposing this problem into separate problems of crew and aircraft scheduling. An analogous approach proposed by Weide et al. [2009] for a deterministic robust aircraft and crew scheduling leads to promising results. The additional complexity induced by the consideration of historical data cab be managed by improving well-known scheduling methods. An additional advantage of extending known scheduling methods is that the new approach can than be compared with existing approaches more easily.

Such a comparison of different approaches needs to consider two main aspects. Firstly, the evaluation of the operational quality of the planned schedules, which involves punctuality and recovery effort. A simulation model can be used to estimate the quality of airline schedules and has the advantage that possible aircraft and crew schedules created with different approaches can be evaluated using different assumptions about the reality, e.g. winter and summer conditions. The second aspect is the comparison of the approaches based on their predictability and efficiency. In this work we focus on stability of aircraft and crew schedules and therefore need a framework for evaluation of different approaches for stability, where the recovery effort is roughly estimated rather than computed exactly.

The resulting stochastic problems for crew and aircraft scheduling need to be solved fast in order to be able to solve integrated problems of practical size in an acceptable time. Thus we propose to solve the resulting stochastic problems heuristically using a state-of-the-art branch-and-price method. The concepts for such methods are well described in literature, but an efficient implementation is still highly significant for the quality of results. Moreover, main concepts of these methods need to be modified and extended to solve the stochastic problems efficiently. The extension of well-founded optimization methods for our purposes has the huge advantage that our research results can be adapted by a large research community as well as commercial software vendors.

The remainder of this work is organized as follows. In Chapter 5 we present the concept of the branch-and-price solution method with our extensions and modifications as well as results of extensive computational experiments. Based on this framework an approach to stochastic optimization of aircraft and crew schedules is developed in Chapter 6. Chapter 7 presents a concept of the evaluation of the approaches for robust scheduling as well as results of comprehensive simulation experiments. In Chapter 8 we present an approach to rescheduling aircraft and crews together with results of a simulation study. In Chapter 9 we give a summary and conclusions of this work.

# 5. A Branch-and-Price-and-Cut Framework

In this chapter we introduce a column generation based solution method for set-partitioning problems in the airline industry. We introduce several extensions to the state-of-the-art methods, which are crucial to achieving high solution quality in short solution time. Finally, extended computational experiments are performed to analyse the effects of all extensions on the results. All solution methods for integrated robust scheduling and integrated rescheduling on the day of operations presented in this work are based on this solution framework. The main motive for the method, however, is efficiently to solve crew pairing optimisation problems. Thus the extended computational experiments are performed for the crew scheduling problem only.

This chapter is organized as follows. The first section presents the set partitioning formulations of the tactical and the operational crew scheduling problem as well as an outline of the solution method based column generation. The second section presents the formulation of the subproblem for pairing generation and the corresponding solution method. The third section presents the branching strategies and the fourth presents the valid inequalities used in the proposed branch-and-bound-and-cut method. Section 5.5 describes the application of the solution method presented in this chapter to a set partitioning formulation of the aircraft assignment problem. The last section presents extensive computational results of the new method for the tactical crew scheduling problem.

## 5.1. The Formulation of the Crew Pairing Problem

The airline crew pairing problem is originally formulated as an integer, nonlinear multi-commodity network flow problem (see Desaulniers et al. [1997]). Normally a Dantzig-Wolfe decomposition is applied to avoid the nonlinear constraints resulting in a set partitioning formulation, where a cost-optimal set of legal pairings has to be selected to partition all available flights.

Equations (5.1) – (5.3) represent the set partitioning formulation of the crew pairing problem, where a set of flights must be partitioned by a set of crew pairings represented by the binary variables $x_p$ . Let $F$ be the set of all flights and $P$ be the set of possible crew pairing variables, then $P(f)$ describes the set of crew pairings covering flight $f$. Constraints (5.2) model the requirement that each flight be covered by exactly one crew pairing.

$$\sum_{p \in P} c_p x_p \rightarrow \min \qquad \text{crew pairing problem} \tag{5.1}$$

$$\sum_{p \in P(f)} x_p = 1 \qquad \forall f \in F \tag{5.2}$$

$$x_p \in \{0, 1\} \qquad \forall p \in P \tag{5.3}$$

The formulation of the crew recovery problem differs from the planning formulation due to restrictions on the availability of crews. At the time recovery begins the scheduled crews as well as standby crews are all at certain airports and have already consumed part of their resources relevant for work-time regulations, like duty working time and flying time. Moreover, we need to reschedule the existing and new crews in such a way, that all existing pairings after the recovery period are continued by same crew considering all work-time regulations. These additional restrictions can be formulated by adding availability and flow constraints to the formulation above. Let $S^P$ be the set of available crew starting positions (earliest time and airport) and $E^P$ the set of available ending positions (latest time and airport) at the end of the recovery horizon, then $P(a)$ $(P(e))$ is the set of all pairings, which start at the starting position $s$ (end at the ending position $e$). The crew availability constraints (5.4) then model that each starting position is allowed to be covered once at most and the crew flow constraints (5.5) model

| Number of night hours in the duty | Maximum duration of the duty |
|---|---|
| $\leq 2$ | 14h |
| $\leq 4$ | 13h |
| $> 4$ | 12h |

| Actual duration of the duty | Minimum duration of the night rest |
|---|---|
| $\geq$ 12h | 14h |
| $\geq$ 10h | 12h |
| $<$ 10h | 10h |

Table 5.1.: Crew Scheduling Rules

the requirement that crews must be available at certain airports at the end of the recovery horizon.

$$\sum_{r \in P(s)} x_p \leq 1 \qquad \forall s \in S^P \tag{5.4}$$

$$\sum_{r \in P(e)} x_p \geq 1 \qquad \forall e \in E^P \tag{5.5}$$

For simplicity all starting and ending positions are enumerated, but it is of course also possible to aggregate them and to increase the right-hand side of the corresponding constraints. The requirement that a certain crew must end at a certain ending position at the end of the recovery horizon can be modelled by forbidding in the pricing algorithm the generation of pairings with non-matching starting and ending positions.

Each crew pairing has to start and end at the same crew base. A crew pairing consists of several daily duties and night rests between them. During a duty a minimal connection time between each consecutive pair of flights is needed represented by the minimal sit time. The minimal sit time is needed for the crew to prepare the aircraft for the next flight and eventually to change the aircraft. The minimum duration of the night rest depends on the actual duration of the previous duty. The maximum duration of a duty depends on the intersection of the duty with the night. The actual values of the rules used in this work are shown in Table 5.1.

The objective of the crew pairing problem is to minimize the number of daily duties, independently of the duration of those duties. The formulation presented here is often solved by a column generation based method, where a linear relaxation of problem (5.1) – (5.3) is the RMP and legal crew pairings are generated using resource constrained shortest path subproblems (see Desaulniers et al. [1997]). Let $\mu \in \mathbb{R}$ be the dual variable associated with the partitioning constraint (5.2), $c_p$ the cost for crew pairing $p$, and $F(p)$ the set of flights covered by pairing $p$. Then, the reduced cost of the crew pairing $p$ are defined as follows

$$\widehat{c_p} = c_p - \sum_{f \in F(p)} \mu_f - \sigma_{s(p)} - \theta_{e(p)}, \tag{5.6}$$

where $\sigma_{s(p)}$ and $\theta_{e(p)}$ are the dual variables associated with the rescheduling constraints (5.4) and (5.5). If those constraints are not part of the problem, the corresponding duals are omitted in the computation of the reduced costs.

Each crew pairing with negative reduced cost may improve the current solution of the RMP. This means all crew pairings in the RMP have non-negative reduced cost. The goal of the pricing subproblem is to generate new crew pairings with negative reduced costs. A crew pairing generation network is used to represent all possible crew pairings implicitly, where each valid path is a valid crew pairing. By solving a resource constrained shortest path problem based on the network several crew pairings with negative reduced costs are found and added to the RMP. If no crew pairings with negative reduced costs can be found, the column generation algorithm terminates.

The RMP is usually solved using the primal or dual simplex algorithm or the interior point method (IPM). One important implication for the choice of the solution method is the presence of multiple dual optimal solutions due to high degeneracy of linear relaxations of the set partitioning problem. If the RMP is solved by a simplex method, an essentially random vertex of the face of optimal dual solutions is selected, whereas the IPM returns the relative interior point of this face. Bixby et al. [1992] and Barnhart et al. [1998] note that the IPM may lead to a better convergence of the column generation method because the interior dual solution may give a better representation of the optimal dual face. This note is supported by Jans and Degraeve [2004] who determine that a high number of columns added to the RMP actually do not change its objective value if a simplex method is used. The disadvantage of the IPM is that it cannot be restarted using the solution from the last column generation iteration. Bixby et al.

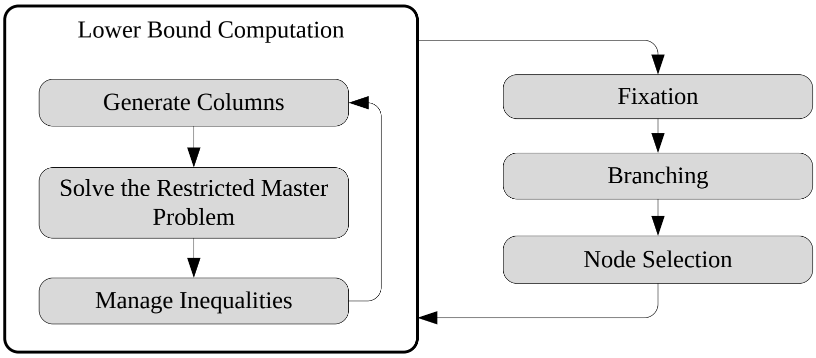



Figure 5.1.: Branch-and-Price-and-Cut Framework

[1992] present a hybrid method where an IPM is used for the first five iterations of column generation and a primal simplex method is used for the remaining iterations. Bixby et al. [1992] report substantially reduced and more predictable total computation time. Moreover, the state-of-the-art implementations of the IPM available today make use of multiple processors and are therefore often dramatically faster on modern multi-core computers. For this reason we use the IPM to solve the RMP instead of a hybrid strategy.

Our solution approach to the crew pairing problem is a combination of column and row generation for the linear relaxation of the problem (5.1) - (5.3) embedded in a branch-and-bound method with a constraint branching strategy. Figure 5.1 shows an overview of the method. The important parts of this method are presented in the following sections of this work, which is organized as follows. In Section 5.2 we describe various possibilities to model the pricing subproblem as a network. We solve the pricing subproblem by a label setting algorithm with a new backtracking scheme. The constraint branching strategy and the new search strategy used in our algorithm are presented in Section 5.3. In Section 5.4 we describe the separation of subset-row inequalities and the generation of new pairings considering such inequalities. In section 5.5 we describe the application of the solution framework developed in this chapter to the aircraft assignment problem. Finally in Section 5.6 we present results of extensive computational experiments based on the crew pairing optimization problem.

## 5.2. Pairing Generation

The quality of the overall results as well as the total solution time is highly dependent on the implementation of the pricing subproblem. In most cases the generation of new pairings consumes the most time due to complicated rules and the high number of calls to the pricing subproblem. Further, the selection of the columns may dramatically improve the convergence of the whole column generation process. Known improvement techniques are multiple pricing, where several columns are generated within each call to the subproblem, and random pricing, where a part of returned columns is selected randomly. The motive for the random selection is to generate more diverse columns. A higher diversification of columns may improve the convergence of column generation by cutting off more solutions at the same time and by indicating new search directions.

The aim of the pricing subproblem is to find the crew pairing with the lowest reduced costs. This can be modelled as a RCSP. For an extensive survey on RCSP and corresponding solution approaches we refer to Irnich and Desaulniers [2005]. Three fundamental approaches are used in the literature to solve the RCSP. The Lagrangian relaxation methods for the RCSP assume that only the resource consumption of a whole path is constrained. Constraint programming, on the other hand, may be used to model a wide range of complex constraints. The dynamic programming method is widely used to solve RCSP in a column generation context and many improvement techniques to speed up the solution process are already known (see i.e. Irnich and Desaulniers [2005]). Additionally, all crew rules described in this work can be modelled as resource restrictions thus; we choose a label setting algorithm as a solution method for the pricing subproblem.

### 5.2.1. Modelling the Pricing Subproblem

In our application possible connections between flights depend on the duration of the previous duty. An explicit representation of connections in the network would anyway require validation steps and possess many more arcs than the implicit representation. Thus we choose a time-space flight-based network to model the pricing subproblem (see Section 3.3 or Desaulniers et al. [1999]). The implicit consideration of duties allows the algorithm to increase the search space

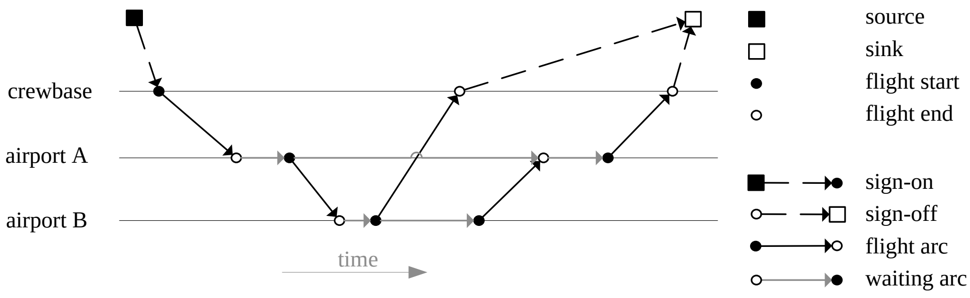



Figure 5.2.: Flight-based Time-Space Network for Pairing Generation

dynamically, dependent on the number of good pairings found. Moreover, the implicit consideration of all duties allows better methods for diversification of generated pairings. Most resource restrictions in our application are based on time. Thus if a path cannot be propagated to a node because of a violated resource restriction that is based on time, nor can this path be propagated to any other node that is later in time either. The time-space network models this relation implicitly, whereas using the connection-based network this relation must be considered by the search algorithm.

The graph is modelled as a directed acyclic network $H = (N, A)$ with source $s$ and sink $t$. Each flight is represented by an arc $(i, j) \in A$ and two nodes, one flight start node $i \in N$ and one flight end node $j \in N$. Each node is characterized by a corresponding time (arrival or departure) and an airport. Each node has at most one outgoing waiting arc to the next node in time at the same airport. We create different networks for each crew base. Thus only one airport at each network is the crew base. Each flight start node at the crew base is connected to the network source node, and each flight end node at the crew base is connected to the network sink node. Each path from the network source node s to the network sink node t is a potential crew pairing satisfying the rule that each crew pairing must start and end at the same crew base. The network structure also guarantees that there is no path connecting two flights with different arrival and departure airports. Night rests as well as duty breaks are represented by one or more waiting arcs at one airport. Figure 5.2 shows an example of the used time-space network.

The additional rescheduling constraints in the RMP require additional starting and ending arcs, which are very similar to the sign-on and sign-off arcs. The flights that may be performed by each crew as the first flight in the recovery period can be enumerated. Each flight from this enumeration is connected to the source node by a starting arc. All starting arcs from this set are associated with the same constraint in the RMP. Analogously, the ending arcs connect the possible last flights of an ending position to the sink node. The sign-on and sign-off arcs represent reserve/standby crews only. The pricing subproblem is organized in one network per crew base with the possibility of generating pairings for existing crews (following the starting arcs) as well as new reserve crews (following the sign-on arcs). Both types of pairings can finish with an ending arc as well as a sign-off arc. I.e. an existing crew can perform an unscheduled sign-off during the recovery period, and a reserve crew can be scheduled to continue a pairing after the recovery period. The validation of pairing rules after the recovery period can be performed locally in the pricing subproblem when generating new columns. Moreover, it is easy to penalize variables which schedule crews to other than original pairings after the recovery period during pairing generation, by associating each ending arc with its original crew.

### 5.2.2. New Label Setting Algorithm with Backtracking

To find a set of pairings with negative reduced costs we use a label setting algorithm that basically iterates over arcs and constructs paths from source $s$ until sink $t$ is reached. The outline of this method is described in Algorithm 5.1.

Feasible paths from source $s$ to each node are represented by labels. Each label $l_k \in L_j$ at node $j$ represents a feasible path $p = (s, \ldots, i, j)$ and is linked with its predecessor label $l_f \in L_i$ at node $i$. Besides the linking each label also stores the information about the reduced costs and the consumption of each relevant resource $r \in R$. The consumption of resources by any feasible path is bounded by maximal and minimal values. A label can only be propagated to the next node if the consumption of any resource at the label and the additional consumption by the arc are less than a given maximal value. The minimal waiting times must be reached before a label can be propagated over a flight arc. A label is propagated over an arc by creating a new label at the target node with a link

**Algorithm 5.1**: Label Setting Algorithm for Pairing Generation

**Data:** $N$ ; `/* list of nodes */`
$N_V = \{0\}$ ; `/* set of nodes to start search */`
`/* search space extension loop */`
**while** $N_V \neq \emptyset$ *and not enough labels at sink node* **do**
  $n = \max(N_V)$ ;
  $N_V = N_V\ \{n\}$ ;
  **if** $n$ *has processed labels* **then**
    propagate success status of labels to predecessor labels ;
  **if** $n$ *has unprocessed labels* **then**
    `/* network search loop */`
    **foreach** $c \in N^{\geq n}$ **do**
      **if** $s$ *has new labels* **then**
        $N_V = N_V \cup \{s\}$ ;
        DominanceTest($s$) ;
        SortLabels($s$) ;
        `/* label setting loop */`
        **foreach** *outgoing arc in* $s$ **do**
          $t$ = target node of arc ;
          **foreach** *non-dominated label in* $s$ **do**
            **if** $t$ *has memory for new label* **then**
              mark label as processed ;
              **if** *label can be propagated* **then**
                propagate label to target node ;

to the predecessor. The resource consumption of the new label at the target node is the sum of the resource consumption in the last label and at the current arc. Since duties and legal connections are implicitly represented in this network, several resources are only relevant to parts of a full crew pairing: the resource waiting time is set to zero at new labels at each flight arc, and all resources relevant for duties are set to zero after the waiting time exceeds the maximal sit time. In the case of crew rescheduling the consumption of all resources also needs to be considered at the starting and ending arcs, which represent the resource consumption of the pairings and duties before and after the recovery period. An

| **Resource** | **Relevant Rules** | **Checked at** |
|---|---|---|
| Waiting time since last flight | min / max sit time, min / max rest time | flight, waiting and ending |
| Number of duties in the pairing | max number of duties | flight and ending arcs |
| Flying hours of the current duty | max flying hours in duties | flight and ending arcs |
| Flying hours of the pairing | max flying hours in pairings | flight and ending arcs |
| Duration of the current duty | max duration of duties, max rest time | all arcs |
| Duration of the pairing | max duration of pairings | all arcs |

Table 5.2.: Relevant Resources

overview over relevant resources, the relevant rules and the arcs where these rules are checked is given in table 5.2.

Lines 8 – 19 of algorithm 5.1 describe the core of the label setting algorithm. The algorithm basically iterates through all nodes in $N$ in a topological order, starting at node $n$, and propagates selected labels to all successor nodes. The propagation of all labels is very time and memory consuming and prohibitive therefore already for small problem instances. A well-known way of avoiding the propagation of all labels and to find the shortest path anyway is the use of dominance tests. A label $l_i$ is dominated by another label $l_k$ if the reduced cost and consumption of each resource at label $l_i$ are greater than at label $l_k$ or equal. The procedure *DominanceTest(s)* tests all labels for dominance and marks dominated labels. Only non-dominated labels are propagated to successor nodes (see line 15).

We restrict the number of labels at each node to handle memory consumption of our algorithm. Thus the total number of possible labels $\left|L^T\right|$ is defined. Given the actual number of labels $|L|$ the maximal number of labels for the node $n$ is $|L^n| = \left(\left|L^T\right| - |L|\right) / \left|N^{\geq n}\right|$. This number is evaluated during the algorithm (see line 16). Since only a subset of labels is propagated we sort the labels at each node through procedure *SortLabels(in s)* to propagate only the most promising labels (see line 12). We test two sorting strategies. The first sorts the labels according to the reduced cost, and thus the columns with least reduced cost are propagated first. The second strategy first categorizes the labels according to the last two flights in the path and then propagates the least reduced cost columns from each category. The motive for the second strategy is to generate more diverse columns with negative reduced cost and thus to improve the convergence of the whole column generation method.

To improve the speed of the algorithm, it is possible to consider only a subset of all resources in the dominance tests. With such non-exact dominance tests the pricing problem is solved heuristically, because actual valid shortest paths could be marked as dominated. But we need the actual shortest paths only at the end of column generation to show that no columns with negative reduced costs exist. In the context of column generation it is more important quickly to generate many different columns with negative reduced costs. To reach this goal we propose a backtracking search algorithm. At the beginning a classical search is performed with non-exact dominance tests and relatively small total number of possible labels $\left|L^T\right|$. In case only few columns with negative reduced costs are found the algorithm continues using the labels not propagated yet (loop in line 2). Therefore during the search each label that is considered for propagation is marked as processed. After the network search (loop in line 8) terminates the nodes of the network are visited in the reverse topological order and for each label the success status is propagated back to predecessor labels.

The success status of a label indicates whether this label was extended to a crew pairing with negative reduced costs successfully. At the first node with unprocessed labels the network search starts again. But this time exact dominance tests are performed comparing the already propagated labels with the unprocessed labels at this node. Remember that only non-dominated labels were propagated in the first search using non-exact dominance tests. This time unprocessed labels are discarded if they are exactly dominated by an already propagated label with false success status. After backtracking, the maximal number of labels per node is considered again, but the labels from previous network search runs can be reused, except at the sink node of course. In this way several backtracking steps can be performed until enough columns are found or the complete search space is visited. For larger problem instances additional termination criteria should be used to avoid excessive running time.

## 5.3. Finding Integer Solutions

Column generation is suitable to solving a relaxation of the problem (5.1) – (5.3) and obtain a lower bound on the optimal objective value. Besides the fractional solution, the main problem is that the generated columns may not be sufficient to finding an optimal or even feasible solution for the original problem. Thus column generation must be embedded in other methods. One common method for this purpose is branch-and-bound, where column generation is used to solve the relaxation of the problem in each node. The whole method is then called branch and price. Barnhart et al. [1998], Desaulniers et al. [1997] and Vance et al. [1997] apply branch and price methods to the crew pairing problem.

After computing a lower bound using the column and row generation framework a branching decision is performed for the current solution of the RMP. The resulting nodes are described by the indexes of all relevant columns, the dual optimal solution and branch-and-bound path-specific termination criteria. In this framework the lower bound is not computed exactly, to avoid the tailing off effect of column generation. The computation of the lower bound terminates when the lower bound stabilizes, thus not significantly changing for six iterations of column generation. It is to be noted that this turns any search strategy into a heuristic.

The proposed framework incorporates branching strategies based on follow-on decisions. Thus after the termination of the column generation method for each follow-on $(r, s)$ the score is defined as follows

$$score(r, s) = \sum_{p \in P(r) \cap P(s)} x_p. \tag{5.7}$$

For branching we select a follow-on with a score near the target score $s_t$ and within the minimum and maximum values $s_{min}$ and $s_{max}$:

$$(r, s) = \underset{(r,s) \in F \times F}{\operatorname{argmin}} \left| s_t - score(r, s) \right|, \text{s.t. } s_{min} < score(r, s) < s_{max} \tag{5.8}$$

The values for $s_t, s_{min}$ and $s_{max}$ describe the branching strategy. The branching decisions can be easily realized. In the resulting 1-branch all columns which cover $r$ or $s$, but not both flights in succession, are deleted. In the 0-branch all columns which cover $r$ and $s$ in succession are deleted.

In the case of crew rescheduling an additional branching strategy based on the assignment of a crew to certain flights is possible. It is to be noted the problem can be solved by assigning each flight to a crew, i.e. any fractional solution contains a flight $f$ and a starting position $s$ with following score

$$0 < score(f, s) = \sum_{p \in P(f) \cap P(s)} x_p < 1 \tag{5.9}$$

A branching decision is then performed on the condition, that flight $f$ must be covered by the crew starting at position $s$ by deleting all columns with flight $f$ but without starting position $s$ in the 1-branch and deleting all columns covering flight $f$ and starting position $s$ in the 0-branch. Additionally, the generation of such columns in the pricing subproblem is forbidden. This can be easily realized by storing the starting position for each label and checking this constraint when visiting a flight-arc. Branching is performed based on the score $score(f, s)$, analogously to the follow-on branching strategy. It is possible to combine this branching strategy with follow-on branching, i.e. to compute the scores for the flight and starting position combinations as well as for the follow-ons at the same time and then to select the branching strategy as well as the actual branching decision on the score. We achieve good results with the strategy of performing several branching decisions on most fractional combinations of flights and starting positions at the beginning of the method and then to switch to the least fractional follow-on branching.

During the algorithm additional information about forbidden and fixed flight pairs is saved in the resulting node. When selecting a new node for exploration the pricing module is updated with the branching history of this node. Each label in the label setting algorithm additionally stores the last flight of the path, if this flight is part of any follow-on in the branching history. An additional feasibility check is then performed at each flight arc, indicating whether the current flight and the last flight of the path are allowed in succession. We define two default branching strategies in our framework:

- Most fractional: $s_t = 0.5$, $s_{min} = 0.4$ and $s_{max} = 0.65$
- Least fractional: $s_t = 0.8$, $s_{min} = 0.7$ and $s_{max} = 0.9$

According to these two branching strategies we present two different search strategies:

- branch-and-price: Always branch on most fractional follow-on.
- dive-and-price: At nodes with a depth smaller than five perform branching on the most fractional follow-on. At deeper nodes always fix the least fractional follow-on.

After branching is performed the resulting nodes are added to the active node list. The next node to search is selected from this list with a delayed best bound strategy. This means the current 1-branch is selected if the lower bound of this branch is not greater than $d = 5\%$ than the best bound of all other nodes in the active node list. New columns are only generated if the lower bound increases, independently of the branching strategy.

Vance et al. [1997] suggest fixing follow-ons with score 1.0. We extend this suggestion to the History Fixing strategy. Thus a follow-on is fixed if it has a score of 1.0 in ten consecutive iterations of column generation. The information about the follow-on scores is inherited to the child nodes during branching. This means in particular that this strategy considers the development of the scores for each path in the branch-and-bound tree.

We prune nodes if their lower bound is greater than a newly-found upper bound. This kind of pruning turns the branch-and-price method into a heuristic, because we do not generate all possible columns when exploring a node, but terminate when the lower bound does not change for several iterations.

When no follow-ons for branching can be found, a minimum number of columns or a maximum depth of the node is reached, the node is added to the final leaves list. A fixed number of leaves is solved using a standard mixed integer programming (MIP) solver. We solve a leaf if after a branching or fixing decision the final leaf has the best lower bound of all active nodes and other leaves.

Additionally, further leaves are solved until the desired number of explored leaves is reached or all leaves are cut off after the branch-and-bound terminates.

## 5.4. Subset Row Inequalities

Valid inequalities can be used to strengthen the formulation of the relaxation solved by column generation. Pure cutting-plane methods try to find the convex hull of the problem by generating valid inequalities iteratively. More promising is the combination of branch-and-bound and generation of valid inequalities. Valid inequalities can help to prune nodes in the branch-and-bound tree by improving bounds. They can also lead to better branching decisions by cutting off fractional solutions.

Jepsen et al. [2008] describe a branch-and-cut-and-price algorithm for the vehicle-routing problem. The authors successfully combine row and column generation for a set partitioning formulation of the problem. They introduce the Subset-Row (SR) inequalities, which are Chvátal-Gomory rank-1 cuts inspired by clique and odd hole inequalities for the set packing problem. SR inequalities are specifically linked to the rows of the set packing problem and given by

$$\sum_{j \in J} \left\lfloor \frac{1}{k} \sum_{i \in S} a_{ij} \right\rfloor \leq \left\lfloor \frac{|S|}{k} \right\rfloor, \forall S \subseteq I, 2 \leq k \leq |S| \tag{5.10}$$

where $S$ is a subset of all rows $I$. Jepsen et al. [2008] focus on inequalities defined by exactly three rows that are given by

$$\sum_{j \in J(S)} x_j \leq 1, \forall S \subseteq I \text{ and } |S| = 3 \tag{5.11}$$

where $J(S) \subseteq J$ is the subset of columns covering at least two rows in $S$. The class of inequalities defined by (5.11) is clearly a subclass of the clique inequalities. We call this class of inequalities $SR^3$.

In the pricing subproblem the dual variable $\mu_q \leq 0$ of the cut $q$ defined by three rows $S_q = \{r_1, r_2, r_3\}$ must be added to any path covering two of the rows in $S_q$. This can be easily done by computing the number of visited rows for each cut during the generation of the path. A large number of SR inequalities can slow down the pricing subproblem.

In this work we consider the class of inequalities described in (5.11) with the additional requirement that any subset of rows contain both rows of a follow-on with fractional score. Thus only a small number of possible inequalities need be enumerated by the separation algorithm and considered during the pricing subproblem. The separation of $SR^3$ inequalities is incorporated into the column generation method, rather than the branch-and-bound method. Basically, inequalities are separated for each new solution of the RMP. In the root node, however, the separation of inequalities is started after the lower bound stabilizes.

### 5.4.1. Inequality Separation

The separation algorithm is actually an enumeration of all combinations of fractional follow-ons with a third row. Thus for each follow-on $(r, s)$ with a score $0 < A_{rs}x < 1$ there exists a set of inequalities defined by

$$\left\{ \sum_{j \in J(S)} x_j \leq 1, S = \{r, s, p\} \mid \forall p > s > \right\}. \tag{5.12}$$

From this set we select at most one inequality using the following quality measures. We select only inequalities with a violation $v(C) > 0$, which is given by

$$v(C) = \sum_{j \in C} x_j - 1, \tag{5.13}$$

where C is the set of columns covered by the inequality. The violation simply is the amount by which the left-hand side of the inequality exceeds the right-hand side. If there are several inequalities with positive violation for a follow-on, the inequality with the greatest Euclidean distance between the hyper-plane defined by the inequality and the LP solution is selected. This distance is given by

$$\delta(C) = \frac{v(C)}{\sqrt{|C|}} \tag{5.14}$$

It is easy to see that the violation is normalized by considering the number of columns covered by the inequality.

Especially, we also consider existing $SR^3$ inequalities as a third row for new $SR^3$ inequalities. After the separation of new inequalities all inequalities from previous iterations with non-negative duals that are not part of any other inequality are deleted.

### 5.4.2. Generating new Pairings with $\mathrm{SR}^3$ Inequalities

For the consideration of $SR^3$ inequalities in the pricing subproblem we introduce two additional resources at the labels. The first additional resource is an array $C^P$ describing the inequalities where one of the three rows is already part of the path represented by the label. The second resource is an array $C^F$ describing the inequalities that are finally covered by the path at the current label. The resource $C^P$ is propagated to new labels and updated at flight arcs. The resource $C^F$ is not propagated to new labels. Instead this resource is checked during the column creation at the end of the pricing method and covered inequalities are added to the new column.

Assume we have a function $SR(r)$ specifying for a row $r$ (a flight or a $SR^3$ inequality) which $SR^3$ inequalities contain this row. Then at each flight-arc new potential and final $SR^3$ inequalities for the current path are computed using the procedure described in Algorithm 5.2. Basically, the algorithm checks for each inequality associated with the corresponding row of the current flight arc, whether it is already a potential inequality for this path, i.e. part of the resource $C^P$. In this case the inequality is removed from the potential list $C^P$ of the target label, the associated duals are added to the reduced costs of the target label and the list $C^F$ of found inequalities is updated. If the inequality is not yet a potential inequality, it becomes one at the target label. This procedure has to be repeated for each found inequality, considering further $SR^3$ inequalities containing the found inequality as the third row (lines 12 – 21).

**Algorithm 5.2**: Considering $SR^3$ Inequalities in Pairing Generation

**Data:** $C^P$ ; /* potential $SR^3$ inequalities at current label */
**Data:** $C^F_{new}$ ; /* covered $SR^3$ inequalities at target label */
**Data:** $C^P_{new}$ ; /* potential $SR^3$ inequalities at target label */
**Data:** $C_{tmp}$ ; /* temporary queue of $SR^3$ inequalities */
**Data:** $f$ ; /* corresponding flight of current arc */
**foreach** $c \in SR(f)$ **do**
    **if** $c \in C^P$ **then**
        Add($C^F_{new}$, c);
        Add $\mu_c$ to reduced cost of target label ;
        Push($C_{tmp}$, c) ;
    **else**
        Add($C^P_{new}$, c);
/* Consider $SR^3$ inequalities covering found inequality */
**while** $C_{tmp} \neq \emptyset$ **do**
    r = Pop($C_tmp$) ;
    **foreach** $c \in SR(r)$ **do**
        **if** $c \in C^P_{new}$ **then**
            Remove($C^P_{new}$, c);
            Add($C^F_{new}$, c);
            Add $\mu_c$ to reduced cost of target label ;
            Push($C_{tmp}$, c) ;
        **else**
            Add($C^P_{new}$, c);

## 5.5. Application of the Framework to the Aircraft Assignment Problem

The aircraft assignment problem and that of aircraft recovery can be formulated as multi commodity flow problems or, analogous to the crew recovery problem, as set partitioning problems. Usually greater success is achieved with the multi commodity flow problem formulation in combination with a commercial state-of-the-art solver. But the set partitioning formulation posses greater possibilities for consideration of path-based restrictions. We present in this section how such a set partitioning formulation for aircraft assigning and recovery can be solved with the branch-and-price-and-cut framework. We do not consider maintenance constraints, although it is straight forward to represent them in the pricing subproblem. The objective of the aircraft assignment problem in this work is always to minimize the sum of aircraft route costs. The actual costs of the aircraft routes depend on the purpose of the optimization, either recovery on the day of operations or robust scheduling during tactical planning, but can always be computed in the pricing subproblem for each variable individually.

Equations (5.15) – (5.19) represent the set partitioning formulation of the aircraft assignment problem, where a set of flights must be partitioned by a set of aircraft routes represented by the binary variables $y_r$. $R(f)$ describes the set of aircraft routoes covering flight $f$. Constraints (5.16) model the requirement that each flight must be covered by exactly one aircraft route. Let $S^R$ be the set of available aircraft starting positions (earliest time and airport) and $E^R$ the set of available ending positions (latest time and airport) at the end of the planing or recovery horizon, then $R(a)$ ($R(e)$) is the set of all aircraft, which start at the starting position $s$ (end at the ending position $e$). The aircraft availability constraints (5.17) then model that each starting position is allowed to be covered at most once and the aircraft flow constraints (5.18) model the requirement that aircraft be available at certain airports at the end of the planing or recovery horizon.

$$\sum_{r \in R} c_r y_r \rightarrow \min \qquad \text{aircraft assignment problem} \tag{5.15}$$

$$\sum_{i \in I(f)} \sum_{r \in R(f)} y_r = 1 \qquad \forall f \in F \tag{5.16}$$

$$\sum_{r \in R(s)} y_r \leq 1 \qquad \forall s \in S^R \tag{5.17}$$

$$\sum_{r \in R(e)} y_r \geq 1 \qquad \forall e \in E^R \tag{5.18}$$

$$y_r \in \{0, 1\} \qquad \forall r \in R \tag{5.19}$$

Both branching strategies, the follow-on branching as well as the assignment of flights to aircraft, are applicable to solving this problem. Moreover, all other aspects of the method, like the consideration of inequalities and fixing strategies, are based on the set-partitioning constraints (5.16) and can be reused without modification.

The pricing subproblem is also modelled as one flight-based time-space network and solved by the same dynamic programming method. For the problem of aircraft routing, however, the pricing subproblem corresponds to a shortest path problem, because there are no resource constraints. Our label setting algorithm is capable of solving the problem in polynomial time. The dominance tests are only based on reduced costs and therefore at each node only one label needs to be propagated. Due to the nature of the problem, the used network is always acyclic and the nodes can be ordered in a topological order. Thus we need to visit each node in the network only once in a topological order and to propagate the shortest path label. This dynamic programming algorithm is called reaching algorithm and is described in [Ahuja et al., 1993, pp. 107]. We extend the algorithm, however, by propagating more than one label at each node in order to generate multiple columns in each iteration.

## 5.6. Computational Experiments

The method presented in this chapter is tested for the crew scheduling problem only. We use domestic flight schedules of a European airline. The schedules available for the tests are organized in two sets. The first set consists of ten small schedules, each of 220 - 272 flights. The second test set consists of ten schedules of medium size, each of 401 - 539 flights. The planning scope of all schedules in each test set is between three and seven days[1].

The framework is implemented in .net c#. All tests are performed on a standard PC with a 64 bit dual core Intel® Core2 2.4 GHz CPU, 8 GB RAM and Windows XP x64. We use the IPM of the solver MOPS 9[2] for the master problem and the solver Cplex 11[3] for the integer problems.

[1] The test set with small problems consists of five schedules of three days, two schedules of five days and three schedules of seven days. The test set with medium problems consists of three schedules of three days, three schedules of five days and four schedules of seven days.
[2] Product of MOPS Optimierungssysteme GmbH & Co. KG [2009]
[3] Product of IBM ILOG [2009]

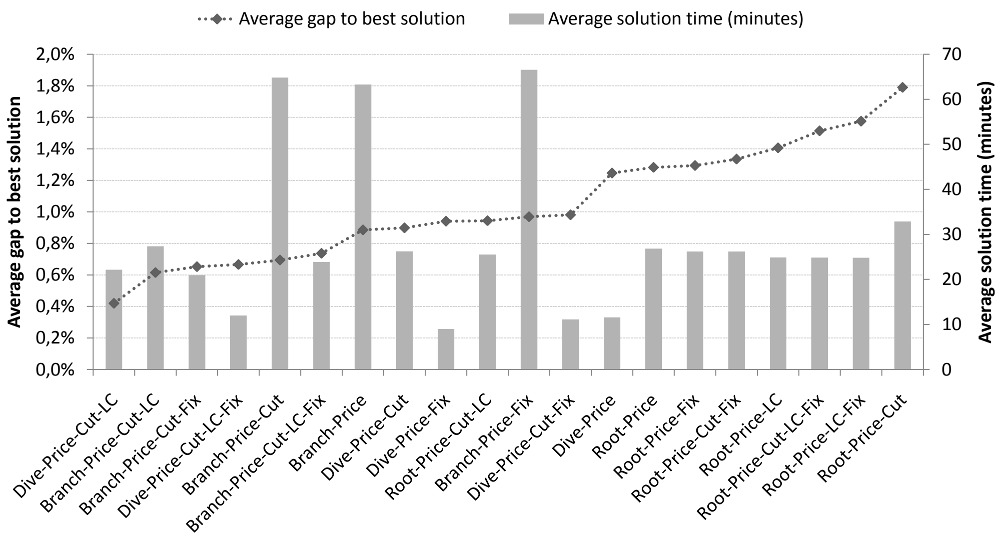



Figure 5.3.: Comparison of Different Solution Strategies for Small Problems

Figure 5.3 shows the average solution time and average gap to the best known solution value for the small problems for all tested solution strategies. In addition to strategies branch-and-price and dive-and-price, which are described in Section 5.3 in detail, we also test the strategy root-price, where columns are generated in the root node only. The generated columns are then used to formulate an integer problem, which is solved by the commercial solver IBM ILOG Cplex 11. Each strategy is combined with the improvement techniques $SR^3$ Inequalities, Label Categorizing and History Fixing. The improvement technique Label Categorizing is an alternative sorting procedure for labels in the label setting algorithm and is described in Section 5.2. With the strategy root-price the improvement techniques are only applied during the solution process of the root node because a commercial solver is used to solve the integer problem.

Same tests including all solution strategies were performed for the test set with medium problems. However, in several medium problems the branch-and-price and root-price strategies did not find any feasible solution within 24 hours. In many other cases the results were significantly worse than those of the solutions found by the dive-and-price strategies. Thus for medium problems we only show results for the dive-and-price strategies. Figure 5.4 summarizes those results.

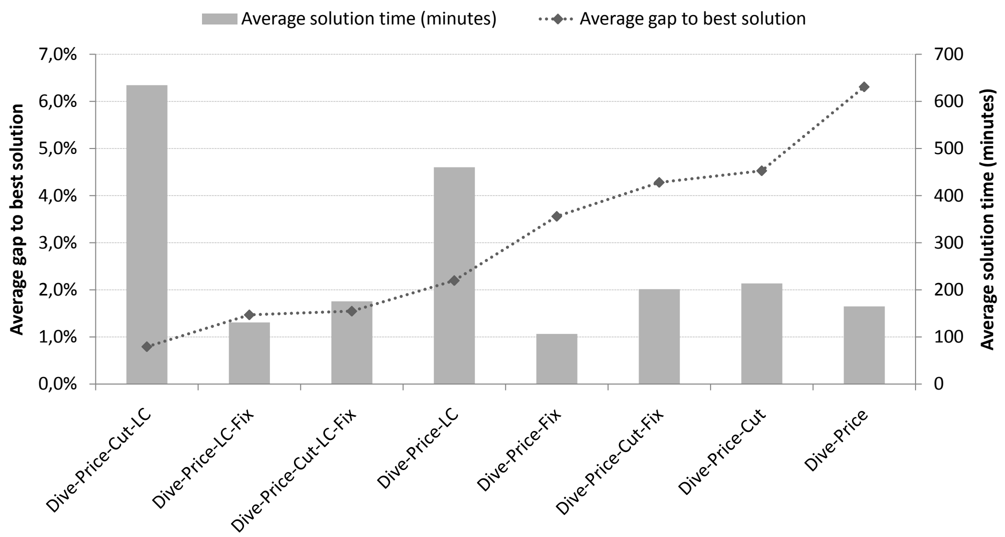



Figure 5.4.: Comparison of Different Solution Strategies for Medium Problems

For the medium problems, Figure 5.5 also shows the running times for different parts of the framework cumulated to the total solution time as well as the total number of iterations for column generation in all branch-and-bound nodes.

The solution strategies in Figures 5.3 and 5.4 are sorted according to the average gap to the best known solution value. One important result for small problems is that the dive-and-price strategy with Cut Generation and Label Categorizing often leads to better results than the corresponding branch-and-price strategy. This is possible because of the heuristic nature of the presented framework. The main termination criterion of the used column generation solver is the convergence rate of the lower bound. Thus in most cases the column generation is terminated, although more columns could be generated and therefore the optimality of the value of the lower bound is not proven. Additionally, leaves in the branch-and-bound tree with a small number of columns are declared as final leaves and solved to optimality by a commercial branch-and-bound solver without generating additional columns. The global search algorithm terminates when a maximal number of such final leaves is explored. The allowed maximal number of such final leaves for the branch-and-price strategies is three times hi-

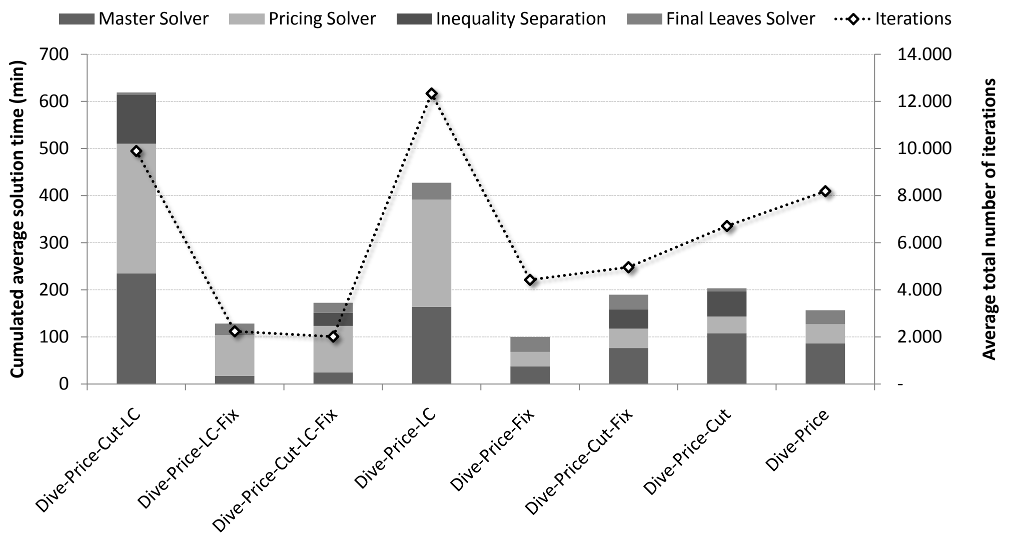



Figure 5.5.: Cumulated Average Solution Times for Different Solution Strategies for Medium Problems

gher than with the dive-and-price strategies, but nevertheless this termination criterion leads to an incomplete exploration of the branch-and-bound tree.

The results in this work show that the integration of column generation and global search for the integer solution is crucial to finding solutions of very good quality and that this integration does not unavoidably lead to higher solution times. However, an accurate comparison of the solution times of the root-price strategies with other strategies in this work is not possible, because another branch-and-bound solver is used in the root-price strategies. We implement the root-price strategy using a commercial branch-and-bound solver to show the possible improvement of solution quality as well as solution time, which can be achieved by making the effort to implement a problem-specific branch-and-bound solver.

According to the average results a positive effect of all improvement techniques on solution quality or solution time can be observed. Moreover, it seems that certain combinations of improvement techniques especially are superior in solution quality and solution time. The following detailed analysis of those effects is

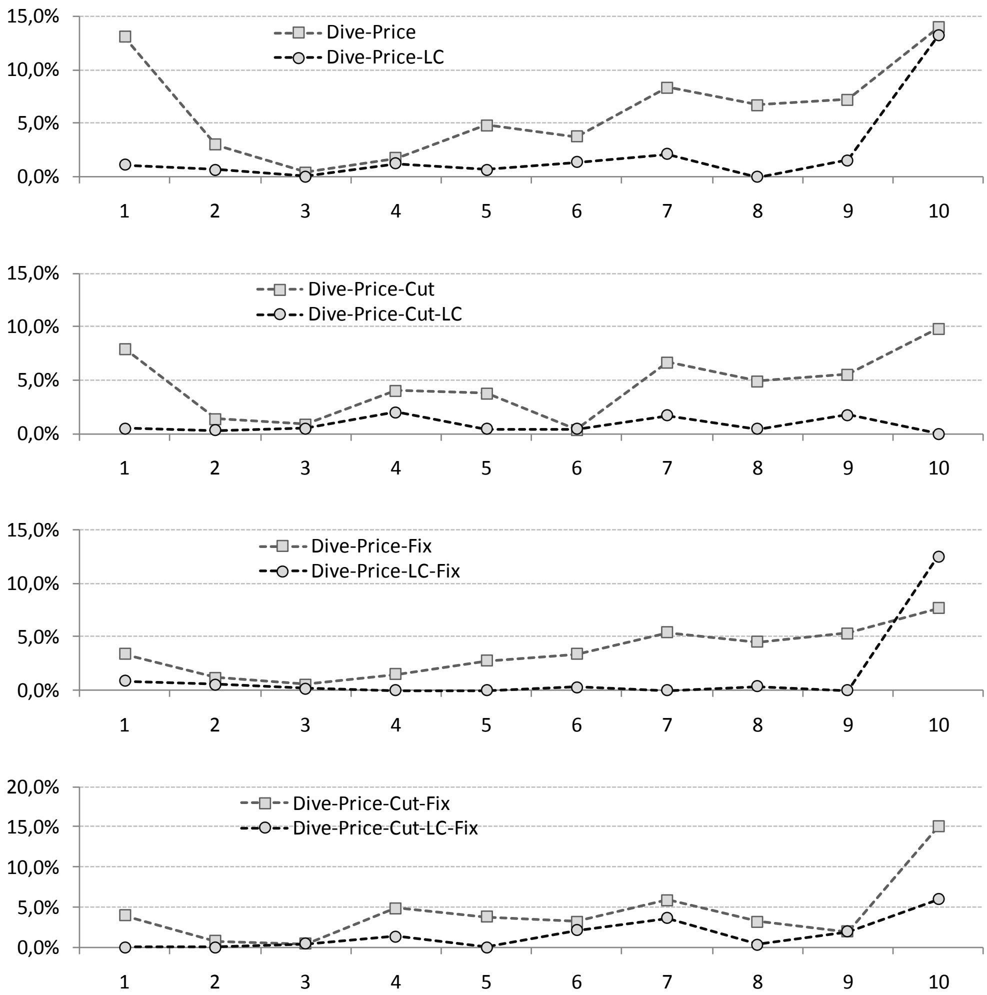



Figure 5.6.: Effect of the Improvement Technique Label Categorizing on the Solution Quality for All Medium Problems

carried out by comparing the gap to the best known solution value for all medium problems for each possible combination of two solution strategies.

### 5.6.1. Label Categorizing

One obvious result of the improvement technique Label Categorizing is the very high solution time without History Fixing; compare Figure 5.5. There are several reasons for this effect. The additional sorting and categorizing algorithms lead to higher solution times of the pricing problem. Additionally, the technique Label Categorizing leads to the effect that many less promising labels are also propagated during the label setting algorithm. Thus in comparison with the classical sorting of labels according to reduced costs, on average more labels need to be propagated to generate an equal number of columns. Moreover, due to the strong diversification of the generated columns fewer of them improve the lower bound solution, leading to a slower convergence of the column generation algorithm. In our tests on average up to 30% more iterations of the column generation algorithm are performed, although fewer branch-and-bound nodes are explored in total. Another important observation is that with this technique on average up to 35% more columns are generated in each iteration. This leads to higher solution times for the RMP. The higher number of generated columns can be explained by the restricted label storage in the pricing algorithm. After exploring the search space with a restricted number of propagated labels, the algorithm terminates if a required minimal number of columns was found or otherwise continues the search with the remaining labels. In the second case the algorithm completely explores the additional search space and returns all columns found so far or again continues the search until all labels are propagated. With the technique Label Categorizing more less promising labels are propagated in the first step and therefore it is more likely that after the first step not enough columns are found and the search has to be continued. Then, after extendeding the search more columns are often found than in the default sorting strategy where usually fewer extensions of the search are required.

The goal of the improvement technique Label Categorizing is to generate more diverse columns and therefore to improve the search for the optimal dual solution and also to use a more diversified column base during the global search. Figure 5.6 shows the effects of this improvement technique together with each of the other strategies. The result is that in only 1 of 40 cases does this technique lead to a worse solution. In all other cases an equally good and often a much better solution can be found when using Label Categorizing. Moreover, the combination with $SR^3$ Inequalities is crucial to achieving constantly good solution quality for all instances. On the other hand, this improvement technique leads to extraordinarily high solution times, as discussed above. However, together

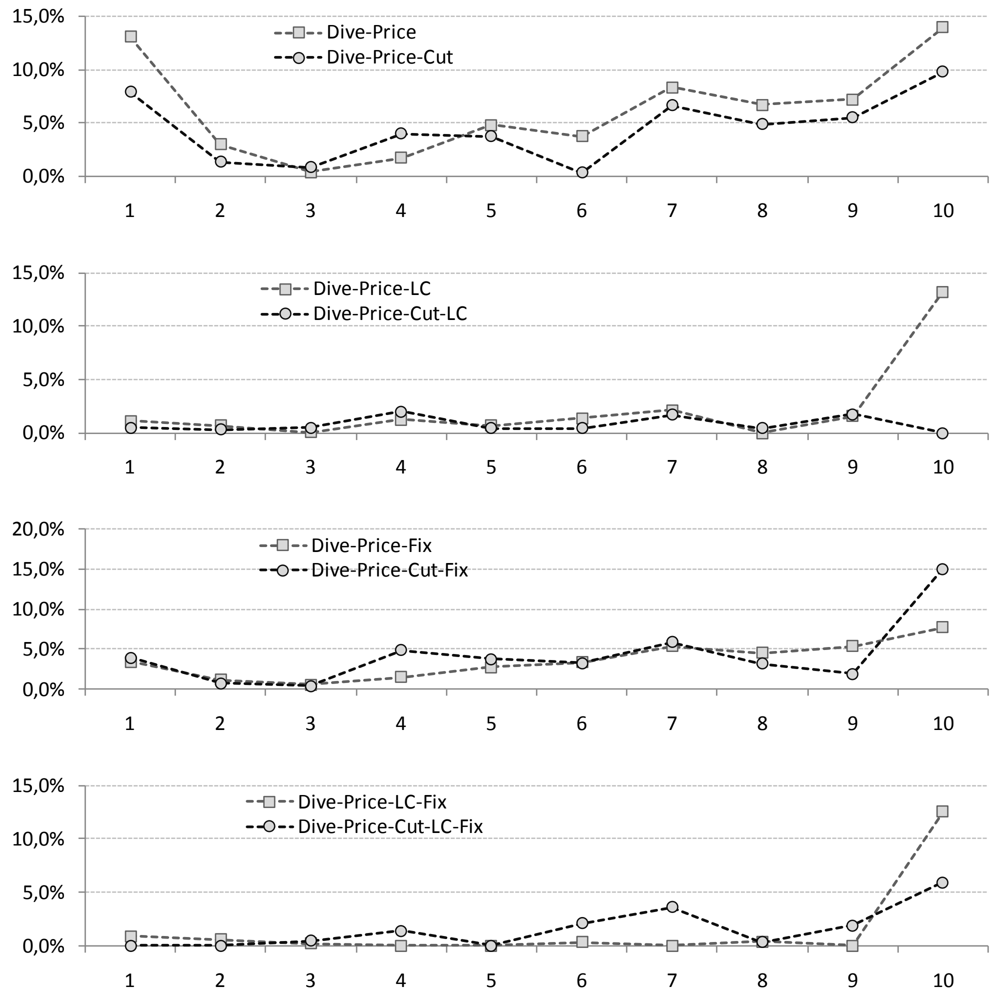



Figure 5.7.: Effect of the Improvement Technique $SR^3$ Inequalities on the Solution Quality for All Medium Problems

with the technique History Fixing a good compromise between solution time and solution quality can be achieved.

### 5.6.2. $\mathrm{SR}^3$ Inequalities

The consideration of $SR^3$ Inequalities requires additional solution time; compare Figure 5.5. In addition to the separation of inequalities and their consideration in the pricing problem, solution time increases due also to additional rows in the RMP. The potential to cut off nodes in the branch-and-bound tree is very low for the problems considered due to a normally small gap between lower and upper bound. The consequence is an increased total solution time for most problem cases. Especially in combination with the Label Categorizing technique the total solution time is noticeably increased because of the increased number of iterations in the column generation algorithm.

Figure 5.7 shows the effects of $SR^3$ Inequalities on solution quality. If used as the only improvement technique, $SR^3$ Inequalities improve the results and speed up the method. In combination with other techniques the consideration of $SR^3$ Inequalities does not generally lead to better solutions. However, the combination of $SR^3$ Inequalities with Label Categorizing is the only strategy to find near-optimal solutions for all considered schedules. In combination with $SR^3$ Inequalities and History Fixing the solution quality seems to be more stable with fewer outliers, although average results are equal. It seems that the improvement technique $SR^3$ Inequalities is more advantageous for small problems, see Figure 5.3, than for medium problems.

### 5.6.3. History Fixing

The motive for the History Fixing techniques is to reduce the search space and speed up the solution process. Especially in combination with other improvement techniques this goal can be reached without dramatic reduction in solution quality. Even strategies based on branch-and-price lead, in combination with History Fixing, to acceptable solution times for small problems. Figure 5.8 shows the effects of History Fixing in combination with each of the other strategies on solution quality of medium problems. Especially in combination with the technique Label Categorizing the solution time can be improved dramatically with only relatively small changes in solution quality. In combination with both techniques $SR^3$ Inequalities as well as Label Categorizing good stability of solution quality with little outliers can be obtained.

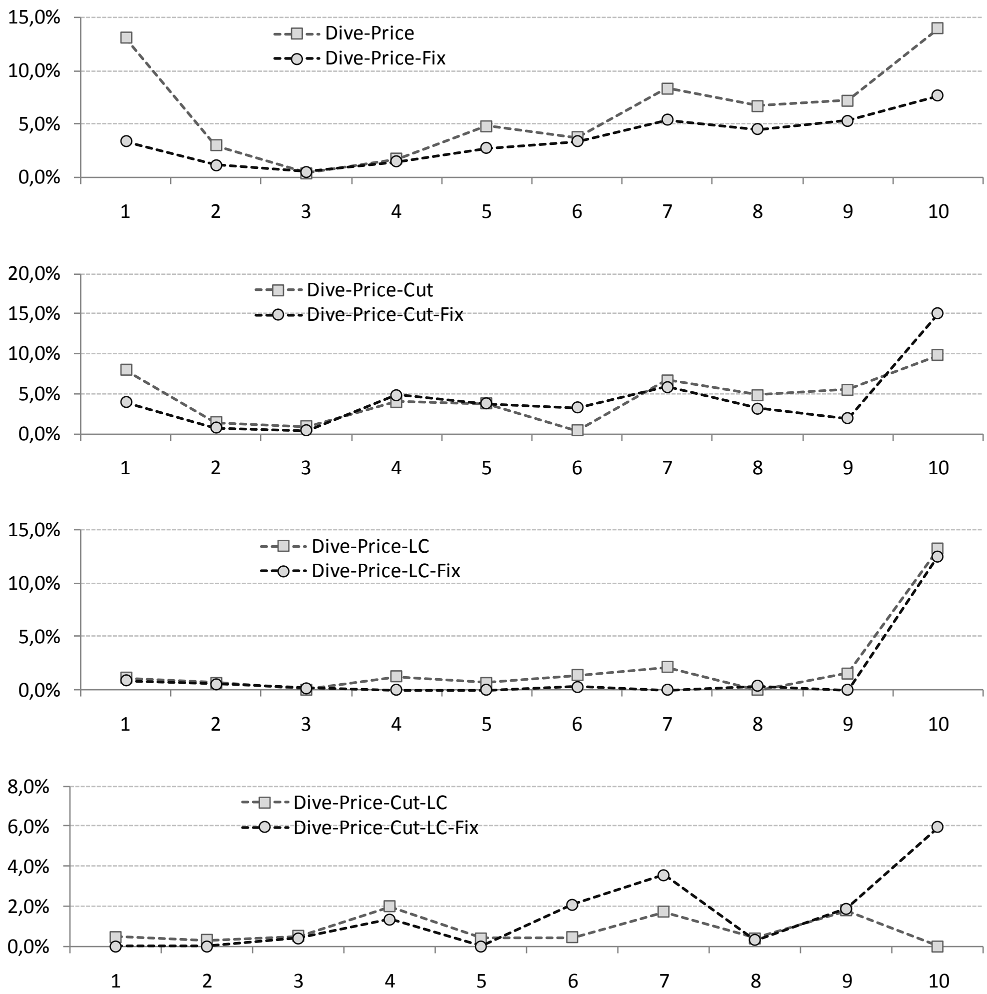



Figure 5.8.: Effect of the Improvement Technique History Fixing on the Solution Quality for All Medium Problems

# 6. A Stochastic Optimisation Approach to Crew Pairing and Aircraft Scheduling

The application of stochastic programming to crew and aircraft scheduling leads to the possibility of considering more detailed measures for stability in contrast to abstract indicators used with deterministic scheduling. Especially in combination with simultaneous scheduling of crews and aircraft new potential can be identified. The consideration of uncertainty through explicit scenarios in stochastic optimization, however, adds additional complexity to the scheduling problems.

The first step in this chapter is the formulation of an integrated stochastic crew scheduling and aircraft assignment model which considers propagation of delays due to aircraft as well as crew. The model contains several non-linear constraints and is therefore not applicable for real-world instances. The second step is a decomposition of this integrated model to achieve a more tractable stochastic model. The new model can be solved by a combination of the iterative approach, proposed by Weide et al. [2009], and classical column generation methods, thus providing a good starting point for considering robustness with real-life problem instances.

## 6.1. Formulation of the Integrated Stochastic Problem

The integrated airline crew pairing and aircraft routing problem can be formulated as a set partitioning problem, where cost-optimal sets of legal crew pairings and aircraft routes have to be selected to partition all available flights. The crew pairing and the aircraft routing problems interact through aircraft changes of crews: thus if the crew ground time during an aircraft change does not contain sufficient slack, then a possible delay of the incoming flight is propagated both to the next flight of the crew and the next flight of the aircraft. This delay propagation can be measured for given schedules by the propagation model introduced in (7.4) - (7.5). The objective is then to minimize costs induced by crew pairings and propagated delays since we assume that the operational costs of all aircraft of the same fleet are identical. Equations (6.1) - (6.7) describe the mathematical formulation of the integrated stochastic model for crew scheduling and aircraft routing.

The set $F$ represents all flights that must be covered, the sets $P$ and $R$ the possible crew pairings and aircraft routes. A solution to the problem consisting of a set of legal crew pairings and routes of flights for each aircraft is represented by $X$ and $Y$ respectively. The binary variable $x_j \in X$ ($y_i \in Y$) indicates whether the crew pairing $j$ (aircraft route $i$) is part of the solution or not. The cost $c_j$ of a crew pairing $j$ represents the costs of working time and overnight rests. The objective is to minimize these costs.

$$\sum_{p \in P} c_j x_j + \mathcal{Q}(X, Y, \Omega) \rightarrow \min \qquad \text{stochastic integrated problem} \tag{6.1}$$

$$\sum_{p \in P(f)} x_j = 1 \qquad \forall f \in F \tag{6.2}$$

$$\sum_{r \in R(f)} y_i = 1 \qquad \forall f \in F \tag{6.3}$$

$$\sum_{r \in R(s)} y_i \leq 1 \qquad \forall s \in S^R \tag{6.4}$$

$$\sum_{r \in R(e)} y_i \geq 1 \qquad \forall e \in E^R \tag{6.5}$$

$$x_j \in \{0, 1\} \qquad \forall j \in P \tag{6.6}$$

$$y_i \in \{0, 1\} \qquad \forall i \in R \tag{6.7}$$

In the following, $\mathcal{Q}(X, Y, \Omega) = c^Q Q(X, Y, \Omega)$ describes the penalty for the propagation of delays by aircraft and crews. The model for $Q(X, Y, \Omega)$ is described by (6.8) - (6.17) and computes in this case the total number of propagated delays. $\Omega$ is the set of scenarios used for evaluation of the recourse, and $p_\omega$ is the probability of the scenario $\omega \in Omega$. A scenario is a representation of the stochastic variables for the departure and arrival of flights. Deviations of these times from the scheduled times define primary delays. $r_{f\omega}$ and $d_{f\omega}$ represent actual arrival and departure time for flight $f$ under scenario $\omega$. $\widehat{r_{f\omega}}$ and $\widehat{d_{f\omega}}$ represent the arrival and departure times for flight $f$ under scenario $\omega$, if no planning decisions are considered. Thus these times include primary delays only. The scheduled arrival and departure time of flight $f$ are given by $s_f^A$ and $s_f^D$, the flight time of flight $f$ under scenario $\omega$ by $t_{f\omega}$. The scenarios for primary delays are obtained from historical data.

The primary delays are represented implicitly by the random variables $g^p_{f\omega}$, $g^c_{f\omega}$ and $t_{f\omega}$. Propagated delays result when aircraft or crews are too late for the next flight on their route. Constraints (6.9) consider the minimal ground time for the selected aircraft routes and constraints (6.10) the minimal ground time for the selected crew pairings possibly leading to delayed departure times. It is to be noted that both equations are non-linear for the integrated model (6.1) - (6.7). For a given solution $X$ and $Y$, $a_i(f)$ points to the predecessor flight of flight $f$ in aircraft route $i$, and $c_j(f)$ to the predecessor flight of flight $f$ in crew pairing $j$. $g^p_{f\omega}$ is the ground time for the final preparations of the aircraft before the flight $f$ under scenario $\omega$. $g^a_{a_i(f)f\omega} > g^p_{f\omega}$ is the total ground time between flight $f$ and the aircraft predecessor flight, and $g^c_{c_j(f)f\omega} > g^p_{f\omega}$ is the total ground time between flight $f$ and the crew predecessor flight under scenario $\omega$. Constraints (6.11) and (6.12) ensure consistency between flight arrival and departure times, and the constraints (6.13) - (6.15) represent the computation of primary delays.

The decision variables of the evaluation model $Q(X, Y, \Omega)$ are the actual arrival and departure times: thus the additional delays induced by the crew pairing and aircraft assignment decisions are minimized (equation (6.8)). $\delta_{f\omega}$ is the total arrival delay for flight $f$, whereas $\widehat{\delta_{f\omega}}$ is the number of primary delays occurring during the actual flight or the last preparations before departure under scenario $\omega$. Delay times must be nonnegative (constraints (6.16) and (6.17)).

$$Q(X,Y,\Omega) = \min \qquad \sum_{\omega\in\Omega} p_\omega \sum_{f\in F} (\delta_{f\omega} - \widehat{\delta_{f\omega}}) \tag{6.8}$$

$$d_{f\omega} - [r_{a_i(f)\omega} + g^a_{a_i(f)f\omega}]y_i \geq 0, \qquad \forall y_i \in Y, \forall f \in F, \forall \omega \in \Omega \tag{6.9}$$

$$d_{f\omega} - [r_{c_j(f)\omega} + g^c_{c_j(f)f\omega}]x_j \geq 0, \qquad \forall x_j \in X, \forall f \in F, \forall \omega \in \Omega \tag{6.10}$$

$$r_{f\omega} - d_{f\omega} \geq t_{f\omega}, \qquad \forall f \in F, \forall \omega \in \Omega \tag{6.11}$$

$$r_{f\omega} - \delta_{f\omega} \leq s^A_f, \qquad \forall f \in F, \forall \omega \in \Omega \tag{6.12}$$

$$\widehat{d_{f\omega}} - g^p_{f\omega} = s^D_f, \qquad \forall f \in F, \forall \omega \in \Omega \tag{6.13}$$

$$\widehat{r_{f\omega}} - \widehat{\delta_{f\omega}} = s^A_f, \qquad \forall f \in F, \forall \omega \in \Omega \tag{6.14}$$

$$\widehat{r_{f\omega}} - \widehat{d_{f\omega}} = t_{f\omega}, \qquad \forall f \in F, \forall \omega \in \Omega \tag{6.15}$$

$$\delta_{f\omega} \geq 0, \qquad \forall f \in F, \forall \omega \in \Omega \tag{6.16}$$

$$\widehat{\delta_{f\omega}} \geq 0, \qquad \forall f \in F, \forall \omega \in \Omega \tag{6.17}$$

## 6.2. Decomposition Strategy for the Recourse Model

In order to eliminate the non-linear constraints, the recourse function is redefined to consider the aircraft routes and crew pairings in isolation, represented by $\mathcal{Q}(X,Y,\Omega) = c^Q(\sum_j x_j Q^P(j,Y,\Omega) + \sum_i y_i Q^R(i,X,\Omega))$. The new recourse model for crew pairings is given in (6.18) - (6.27). The main difference from the integrated model defined in Section 6.1 is that $\widehat{\delta_{f\omega}}$ represents primary delays as well as those induced by aircraft routes. Thus constraints (6.24) are added and replace constraints (6.13). $\sum(\delta_{f\omega} - \widehat{\delta_{f\omega}})$ are the additional delays induced by the crew scheduling decisions only. The non-linear equation (6.10) is replaced by equation (6.20) indicating that the recourse is evaluated only for one crew pairing. Then $\forall f \in j$ is the set of all flights covered by pairing $j$.

$$
\begin{aligned}
Q^P(j,Y,\Omega) = \min \quad & \sum_{\omega\in\Omega} p_\omega \sum_{f\in F} (\delta_{f\omega} - \widehat{\delta_{f\omega}}) && (6.18)\\
d_{f\omega} - [r_{a_i(f)\omega} + g^a_{a_i(f)f\omega}]y_i \geq 0, \quad & \forall y_i \in Y, \forall f \in F, \forall \omega \in \Omega && (6.19)\\
d_{f\omega} - r_{c_j(f)\omega} - g^c_{c_j(f)f\omega} \geq 0, \quad & \forall f \in j, \forall \omega \in \Omega && (6.20)\\
r_{f\omega} - d_{f\omega} \geq t_{f\omega}, \quad & \forall f \in F, \forall \omega \in \Omega && (6.21)\\
r_{f\omega} - \delta_{f\omega} \leq s^A_f, \quad & \forall f \in F, \forall \omega \in \Omega && (6.22)\\
\delta_{f\omega} \geq 0, \quad & \forall f \in F, \forall \omega \in \Omega && (6.23)\\
\widehat{d_{f\omega}} - [\widehat{r_{a(f)\omega}} + g^a_{a_i(f)f\omega}]y_i = 0, \quad & \forall y_i \in Y, \forall f \in i, \forall \omega \in \Omega && (6.24)\\
\widehat{r_{f\omega}} - \widehat{\delta_{f\omega}} = s^A_f, \quad & \forall f \in F, \forall \omega \in \Omega && (6.25)\\
\widehat{r_{f\omega}} - \widehat{d_{f\omega}} = t_{f\omega}, \quad & \forall f \in F, \forall \omega \in \Omega && (6.26)\\
\widehat{\delta_{f\omega}} \geq 0, \quad & \forall f \in F, \forall \omega \in \Omega && (6.27)
\end{aligned}
$$

This recourse model considers the interdependencies of aircraft routes due to several aircraft changes during one crew pairing, but it does not consider interdependencies of crew pairings if those are sharing one aircraft. Moreover, through the decomposition into two separate recourse problems, the presented recourse models measure the additional delay propagation over aircraft or crew changes only. This means the presented recourse model corresponds to the principle "crew follows aircraft and vice versa", which is also found in Yen and Birge [2006] and Weide et al. [2009].

The recourse model (6.18) - (6.27) deals with the average delay for each flight having regard to all scenarios $\omega \in \Omega$. Based on this model alternative robustness indicators can easily be formulated. A straightforward way in which to consider the expected number of delayed flights and to penalize the longer delays higher is to sum up the squares of the delays in the objective. Another alternative is to count delays over a certain threshold only. Then all flights with such delays are penalized equally and shorter delays are ignored. $Q^P_t(j,Y,\Omega)$ represents the model extension for the robustness indicator based on the expected number of reactionary delays over a threshold $t_o$:

$$Q_t^P(j, Y, \Omega) = \min \sum_{\omega \in \Omega} p_\omega \sum_{f \in F} z_{f\omega} \tag{6.28}$$

subject to (6.19) - (6.27) and

$$\delta_{f\omega} - \widehat{\delta_{f\omega}} < t_o + z_{f\omega}\delta_M \qquad \forall f \in F, \forall \omega \in \Omega \tag{6.29}$$

$$z_{f\omega} \text{ is binary} \qquad \forall f \in F, \forall \omega \in \Omega \tag{6.30}$$

The new variable $z_{f\omega}$ indicates whether the additional delay of flight $f$ in scenario $\omega$ is greater than the given threshold $t_o$. $\delta_M$ belongs to the big M formulation of the constraint.

A very similar recourse model $Q_d^P(j, Y, \Omega)$ represents the robustness indicator based on the expected number of crew duty disruptions. For each flight in a crew pairing the value $D_f^{max}$ describes the duty slack, i.e. maximal possible delay of this flight $f$, which does not directly lead to a disruption of the crew duty limits (compare also Equation (6.47) on page 85). $D_f^{max}$ is only set for the last flight of each crew duty to the corresponding duty slack: for all other flights it is set to infinity. The value of the duty slack is then the individual threshold for each flight leading to the following recourse model:

$$Q_d^P(j, Y, \Omega) = \min \sum_{\omega \in \Omega} p_\omega \sum_{f \in F} z_{f\omega} \tag{6.31}$$

subject to (6.19) - (6.27)

$$\delta_{f\omega} < D_f^{max} + z_{f\omega}\delta_M \qquad \forall f \in F, \forall \omega \in \Omega \tag{6.32}$$

$$z_{f\omega} \text{ is binary} \qquad \forall f \in F, \forall \omega \in \Omega \tag{6.33}$$

The labels $Q^P$ for crew scheduling and $Q^R$ for aircraft routing refer in this chapter to any recourse model based on the computation of propagated delays for individual path variables independently of the actual solution. The models $Q_t^P$ and $Q_d^P$ are only two possible alternatives. The choice of the recourse model depends on user preference only, because the computational properties of all three models are comparable. The formulation of the analogous recourse models $Q^R$ for the aircraft routing problem is straightforward.

**Algorithm 6.1**: Iterative Method for the Stochastic Crew and Aircraft Scheduling

Set $c^Q = 0$ /* penalty value for non-robustness*/
Solve crew pairing problem without a given aircraft routing solution
**while** $c^Q \leq c^Q_{max}$ **do**
  Increase $c^Q = c^Q$ + const.
  Solve aircraft routing problem with cost $\widehat{c^R}$ based on crew schedule
  Solve crew pairing problem with cost $\widehat{c^P}$ based on aircraft schedule
  **if** $\sum_{j \in P} x_j Q^P(j, Y, \Omega) + \sum_{i \in R} y_i Q^R(i, X, \Omega)$ *does not change* **then**
    break

Note that for a given aircraft schedule the penalty costs for non-robustness can be computed during crew pairing generation for each crew pairing separately. Analogously, for a given crew schedule the additional delays and duty disruptions induced by the routing decisions for each aircraft are penalized. The decomposition of the recourse model enables the decomposition of the whole integrated problem into separate problems for crew pairing scheduling and aircraft routing, which can be solved iteratively. The interdependencies of those two problems are considered through the non-robustness penalty in the objective. For given aircraft routes the stochastic recourse for the crew scheduling problem becomes linear and vice versa. Classical column generation methods can be easily applied to each scheduling problem, because the new recourse only considers individual path variables. Algorithm 6.1 gives an overview of the complete iterative algorithm. The iterative approach starts by solving a crew pairing problem without penalty costs for non-robustness and without a given aircraft routing solution. The resulting crew costs form a lower bound on the actual optimal costs. Based on this crew schedule, the two scheduling problems are solved iteratively with increasing penalty value for non-robustness $c^Q$ until a maximum penalty value $c^Q_{max}$ is reached or the value of the recourse functions cannot be further improved.

The separate crew scheduling problem can be formulated as the following well-known set partitioning problem with recourse

$$\sum_{j \in P} x_j \left( c_j^P + c^Q Q^P(j, Y, \Omega) \right) \to \min \qquad \text{stochastic crew pairing problem} \tag{6.34}$$

$$\sum_{j \in P(f)} x_j = 1 \qquad \forall f \in F \tag{6.35}$$

$$x_j \in \{0, 1\} \quad \forall j \in P \tag{6.36}$$

An analogous set partitioning model with recourse can be used for the aircraft routing problem. By adding penalty costs for non-robustness the separate aircraft routing problem is no longer just a feasibility problem. Thus the problem formulation based on a Dantzig-Wolfe decomposition reads

$$\sum_{i \in R} y_i c^Q Q^R(i, X, \Omega) \to \min \qquad \text{stochastic aircraft assignment problem} \tag{6.37}$$

$$\sum_{r \in R(f)} y_r = 1 \qquad \forall f \in F \tag{6.38}$$

$$\sum_{r \in R(s)} y_r \leq 1 \qquad \forall s \in S^R \tag{6.39}$$

$$\sum_{r \in R(e)} y_r \geq 1 \qquad \forall e \in E^R \tag{6.40}$$

$$y_r \in \{0, 1\} \quad \forall r \in R \tag{6.41}$$

The Dantzig-Wolfe formulation of the aircraft routing problem has the advantage that it can be solved by existing column generation methods. The reduced costs are defined analogously to the reduced costs of crew pairings. The crew pairing as well as the aircraft routing problem are solved by a branch-and-price method, described in Chapter 5 and Chapter 8. The extensions to the original problems are fully represented by the recourse models. Thus these extensions need to be considered during the generation of new path variables only.

Moreover, the method for the evaluation of the recourse models $Q^P$ can be easily integrated into the solution method for the pricing subproblem. The same task network used for the pricing subproblems can be used to compute the delay propagation. In this way the costs associated with the recourse function are computed for partial paths and therefore implicitly considered in the pricing subproblem. The recourse functions, furthermore, can be evaluated recursively by dynamic programming methods analogous to the methods already used for the pricing subproblem. Equations (6.42) and (6.43) show the recursive functions for primary and reactionary delays based on the recourse models $Q^P$.

$$\widehat{\delta_{f\omega}} = -s_f^A + t_{f_\omega} + s_{a(f)}^A + g_{a_i(f)f\omega}^a + \widehat{\delta_{a(f)\omega}} \tag{6.42}$$

$$\delta_{f\omega} = -s_f^A + t_{f_\omega} + s_{c(f)}^A + g_{c_j(f)f\omega}^c + \delta_{a(f)\omega} \tag{6.43}$$

The information about primary delays as well as the aircraft schedule (in the case of crew scheduling) are available before the solution process. Thus the values for $\widehat{\delta_{f\omega}}$ can be computed in advance, if necessary for the evaluation of the recourse models. The values for $\delta_{f\omega}$ depend on the path generated and are therefore computed recursively during the solution of the pricing subproblem.

The solution method for the pricing subproblem in this work is based on dynamic programming. Dynamic programming stores information about paths at each node in the underlying network and reuses this information at all successor nodes. This storage method is also applied to the computation of delay propagation leading to additional increase in the efficiency of the method. Partial paths may be reconsidered in different pricing iterations by saving storage information for future column generation iterations. This strategy is called iteration-spanning storage and can further reduce the solution times. A greater size of iteration-spanning storage may have positive effects on the efficiency of the evaluation process, but in general it is limited by the available physical memory. For this reason, a maximum storage size is defined and elements that are less frequently considered in later iterations are removed from the storage to ensure the maximum storage size.

## 6.3. Corresponding Deterministic Indicators

A well-known indicator of punctuality of schedules is the number of aircraft changes by crews with low slack between the two flights, i.e. restricted aircraft changes, see Weide et al. [2009] and Mercier and Soumis [2007].

We represent the severity of a connection between two flights in a crew pairing in this work by the penalty factor $0 \leq \pi(f_1, f_2) \leq 1$. An aircraft change between the flights $f_1$ and $f_2$ is then penalized with the product $c_\gamma \cdot \pi(f_1, f_2)$. This indicator is based on two values, a minimal slack required during every aircraft change and a maximal slack defining the upper limit to penalize. The penalty factor is then computed using following Equation (6.44):

$$\pi(f_1, f_2) = \begin{cases} 1, & \text{if slack during connection} < \text{minimal slack} \\ 0, & \text{if slack during connection} > \text{maximal slack} \\ 1 - \left(\frac{\text{slack during connection} - \text{minimal slack}}{\text{maximal slack} - \text{minimal slack}}\right)^p, & \text{otherwise} \end{cases} \tag{6.44}$$

A connection with $\pi(f_1, f_2) > 0$ is called restricted connection. The iterative approach proposed in this chapter can also be used to minimize the number of aircraft changes at such restricted connections. Weide et al. [2009] propose to minimize the number of restricted connections in crew pairings and to maximize it in aircraft routes during the iterative approach in order to reach a minimization globally. It is to be noted that during aircraft assignment $\pi(f_1, f_2)$ describes a restricted connection in the current crew solution. Thus the goal of the aircraft assignment problem is to cover as many current restricted connections by aircraft routes in order to minimize the number of aircraft changes at such connections. The new objective functions are formulated as follows:

$$\sum_{j \in P} x_j \left( c_j^P + c^Q \sum_{(f_1, f_2) \in j} \pi(f_1, f_2) \right) \rightarrow \min \qquad \text{crew pairing} \tag{6.45}$$

$$\sum_{i \in P} y_i c^Q \sum_{(f_1, f_2) \in i} \pi(f_1, f_2) \rightarrow \max \qquad \text{aircraft route} \tag{6.46}$$

Figure 6.1 shows a visualization of the penalty factor for different values for $p$. Higher values for $p$ lead to higher penalties for shorter slack and lower penalties for longer slack than the linear penalty function with $p = 1$. The experiments in this work were all performed using the quadratic penalty function with $p = 2$. The minimal slack is set to ten minutes and the maximal to 60 minutes.

A straightforward choice for a robustness indicator facing crew duty disruptions is based on the distribution of duty slack in the crew schedule. Duty slack is the maximal duration of an arrival delay for the last flight of a duty, which does not

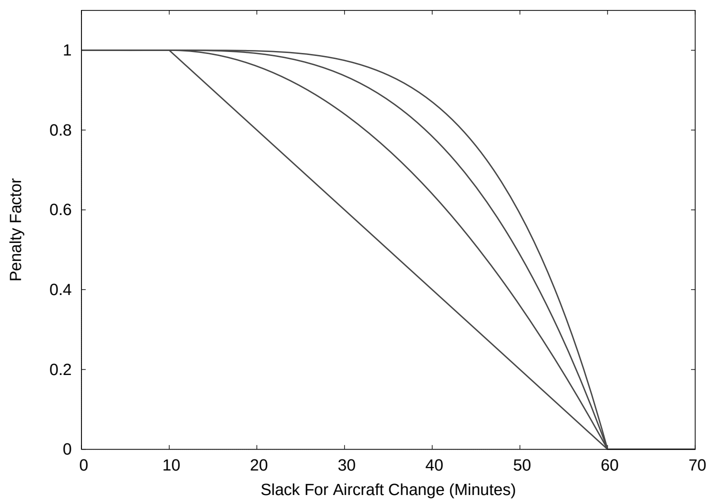



Figure 6.1.: Graph of Penalty Factor with Values 1, 2, 3 and 4 for $p$

lead to a crew duty limit or a crew rest disruption. It is to be noted that the maximal duty duration allowed can be increased depending on the following rest duration; see Section 2.1.4 on page 12 for details. Thus to identify the maximal slack it is necessary to compute the duty slack for all possible maximal duty durations using the corresponding minimal rest times. Duties with low duty slack are called restricted. The duty slack is computed as follows

$$\min \left\{ \begin{array}{l} \text{maximal allowed duty duration - scheduled duty duration} \\ \text{maximal allowed duty flight time - scheduled duty flight time} \\ \text{scheduled rest duration after duty - minimal rest duration} \end{array} \right\} \quad (6.47)$$

The penalty factor is then computed using following equation:

$$\sigma(duty) = \begin{cases} 1, & \text{if actual duty slack} < \text{minimal duty slack} \\ 0, & \text{if actual duty slack} > \text{maximal duty slack} \\ 1 - \left(\frac{\text{actual duty slack} - \text{minimal duty slack}}{\text{maximal duty slack} - \text{minimal duty slack}}\right)^2, & \text{otherwise} \end{cases} \quad (6.48)$$

The values for minimal and maximal duty slack are set to 30 and 120 minutes respectively. This robustness indicator can also be used with the iterative approach. Let $duty \in j$ and $duty \in i$ enumerate the duties ending with a flight covered by pairing $j$ or aircraft route $i$ respectively; the new objective functions are then formulated as follows:

$$\sum_{j \in P} x_j \left( c_j^P + c^Q \sum_{duty \in j} \sigma(duty) \right) \rightarrow \min \qquad \text{crew pairing} \tag{6.49}$$

$$\sum_{i \in P} y_i c^Q \sum_{duty \in i} \sigma(duty) \rightarrow \max \qquad \text{aircraft route} \tag{6.50}$$

## 6.4. Computational Experiments

The quality of the robustness indicators is discussed in detail in Chapter 7. In this chapter we discuss the solution times for four models presented in this chapter. The four models for crew and aircraft scheduling are a stochastic model for minimizing reactionary delays represented by the recourse models $Q_t^P$ and $Q_t^R$, a stochastic model for minimizing crew duty disruptions represented by the recourse models $Q_t^P$ and $Q_t^R$ and the deterministic models penalizing restricted aircraft changes and restricted duties described in Section 6.3. Figure 6.2 shows a comparison of the solution times during the iterative approach for all four models. The solution times are average values of 20 flight schedules of small size, each of 220 – 272 flights. For the stochastic models we use 1000 delay scenarios, each with an average of over 50% of flights delayed. This means, that each flight is delayed in over 500 scenarios on average.

The stochastic model for delay propagation leads to significantly higher solution times than on all other models. The average total solution time here is 3:30 hours. Three instances are very difficult to solve with total solution times over 8 hours. In these three cases over 90% of the solution time was spent in the first iteration. This means in some cases the scheduling method has the difficulty in finding the first optimal solution. Later iterations, however, are initialized with the optimal solution of the previous iteration and solved much faster. In 15 of 20 cases, however, the solution times for the stochastic model are below 2 hours. In these cases the solution time was distributed more equally over the iterations.

The average total solution times for the other three models are much lower, but the increase during the iterative method is comparable for all models. The three difficult problems lead to significantly higher solution times for the other

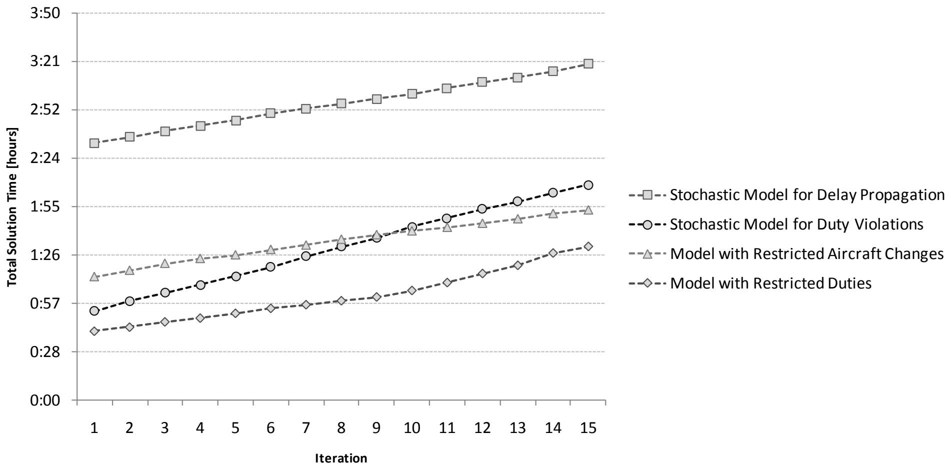



Figure 6.2.: Solution Times of the Iterative Method for all Models

three models also, although never more than 8 hours. The models evaluating restricted duties usually need less solution time because fewer evaluation steps are needed in the pricing problem.

The increase of the solution times for the stochastic models is low in comparison to the deterministic models, given that over 500 delay scenarios must be considered for each flight during the generation of new variables. This good performance is only possible because of the dynamic programming method with storage techniques used to evaluate of the recourse models. In order to show the effects of the different storage strategies we analyze the solution times of the pricing subproblem during one column generation run in detail. Figure 6.3 shows a comparison of the running times of the pricing subproblem for the first 25 iterations of the column generation algorithm. Evidently, enabling the storage has positive effects on the efficiency of the evaluation model.

While the recourse function without storage constantly needs about 50%-60% of the time in the pricing subproblem, this ratio can be reduced to around 12% in later iterations through enabling the iteration-exclusive storage, i.e. the storage is cleared in each iteration of column generation. The high relative time consumed by the evaluation method in these early iterations, however, only leads to a low increase of the total running time, because the pricing subproblem can be solved

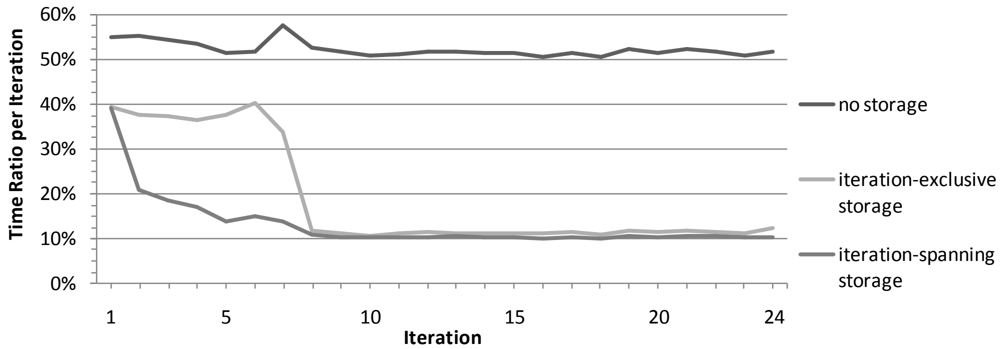



Figure 6.3.: Comparison of Solution Times for Different Storage Strategies

extremaly fast in the first iterations. Later pricing iterations need longer solution times, thus it is important that the recourse function can then be performed as efficiently as possible then. The iteration-spanning storage strategy can decrease the solution time by around 2% in later iterations compared to the iteration-exclusive storage.

# 7. Evaluation of indicators for Stability

The aim of this chapter is the evaluation of the quality of the stochastic as well as deterministic robustness indicators presented in Chapter 6. The evaluation is based on the computation of robustness measures by simulating crew and aircraft schedules. We propose to measure the quality of indicators according to the following properties:

**Predictability** describes the relation between the robustness indicator during scheduling and the robustness measures during simulation. High predictability means that there is a strong mathematical relation between the values of the indicator and those of the measure.

**Efficiency** describes the relation between the cost of the schedule and the robustness measures during simulation. High efficiency means that the increase of the robustness measure is high for already low increase of cost.

Our method of evaluation of robustness indicators using a simulation model is highlighted in Figure 7.1. In the first step a set of robust crew and aircraft schedules is generated for each flight schedule and each robustness indicator. We use the iterative approach presented in chapter 6. No specific method for generation of those schedules is, however, required, as long as each schedule in the resulting set has different costs or different value of the robustness indicator. In the second step each crew and aircraft schedule is simulated to measure the robustness, e.g. statistics on schedule disruptions and reactionary delays.

The remainder of this chapter is organized as follows. Firstly, we present assumptions underlying the simulation model used for this evaluation, e.g. the generation of primary delays. In the second step we present the simulation model. And finally we evaluate the robustness indicators presented in the previous chapter and compare the stochastic model with the deterministic indicators.

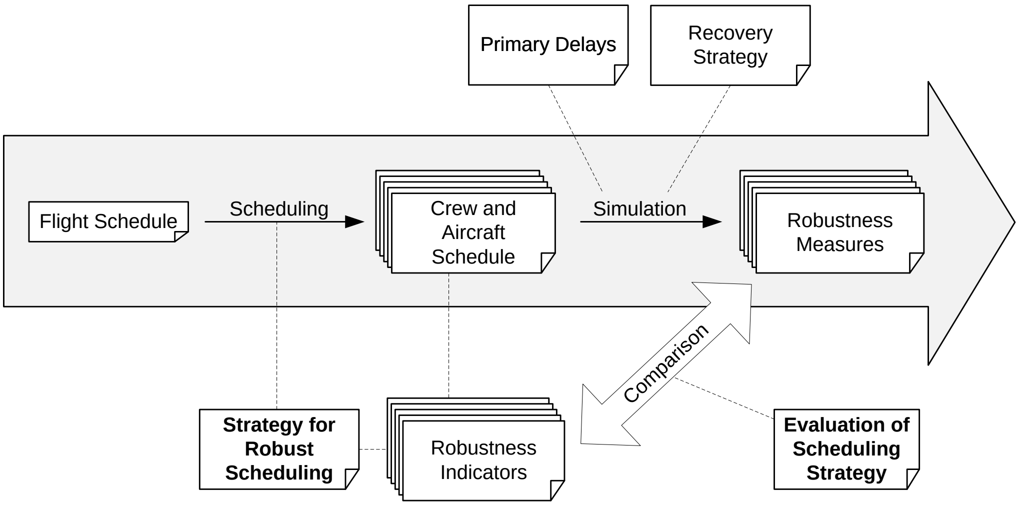



Figure 7.1.: Framework for Evaluation of Strategies for Robust Scheduling

## 7.1. Assumptions and Empirical Data

There are three important assumptions underlying the simulation model. Firstly, the ground processes for aircraft and crews at airports are represented abstractly. Secondly, no rescheduling method is incorporated into the simulation model; instead the rescheduling effort is measured by counting schedule disruptions that cannot be resolved by propagating delays. Thirdly, primary delays are generated according to probability distributions derived from historical data of a major European airline.

In this work a delay propagation model is used to evaluate the stability of aircraft and crew schedules. The model describes the interdependencies of crew pairings and aircraft routes. Thus when crews change aircraft, delays of a crew may be propagated to another aircraft and further also to other crews. The ground tasks at airports are not modelled in detail: instead one task of fixed duration is assumed to be performed immediately before departure. The duration of this task corresponds to the minimal turn time for aircraft and minimal sit time for crews respectively, i.e. the duration of all ground tasks is independent of the actual airport, daytime or season. A detailed model of ground tasks requires comprehensive knowledge of these processes, including duration and delay probabilities, which significantly differ between airports, aircraft types and even parking positions at the same airport. Whereas extensive data of processes and task durations may exist and mean considerable effort for modelling, historical data on delay probabilities for each task of these processes simply does not exist. Delays during the ground processes are often recorded as departure delays of flights only. Thus a detailed model of ground tasks in our simulation model would not lead to a better approximation of delay generation, due to lack of relevant information. For the computation of the delay propagation between two flights only the total duration of the ground processes is relevant. Thus we choose to model the ground process between two flights as one task, but distinguish between aircraft and crews.

Delay can lead to schedule disruptions, which need to be resolved to allow operation of the schedules. We distinguish between two general strategies to resolve schedule conflicts. Firstly, delays may be propagated to subsequent flights of aircraft and crew. Secondly, the schedules for aircraft and crews can be rescheduled. In order to measure stability no rescheduling, which can lead to changes of the schedule, is performed in our simulation model. Instead, the aircraft and crew connect disruptions are resolved by performing reactionary delay propagation only. Crew rest and crew duty limit disruptions are recorded for statistics, but not resolved. In particular, not even reactionary delays are propagated to resolve crew rest and crew duty limit disruptions, because this usually leads to unrealistically high delays at the beginning of the next duty. In practice such disruptions are resolved through extensive rescheduling and use of reserve resources, as shown in Chapter 8. Thus for this analysis the recovery effort is measured by counting the crew duty disruptions.

The generation of primary delays is based on historical data of a major European airline from 2003 to 2006. These empirical data on delays only contains the differences between scheduled and actual departure and arrival times respectively. In particular, primary delays during the ground process which do not lead to a delayed departure are not measured. All delays are classified according to the IATA delay codes. According to these data, we derive theoretical distributions for duration and frequency of delays for use in our simulation model. We give a brief overview of the analysis and results; for details we refer to Spengler [2009].

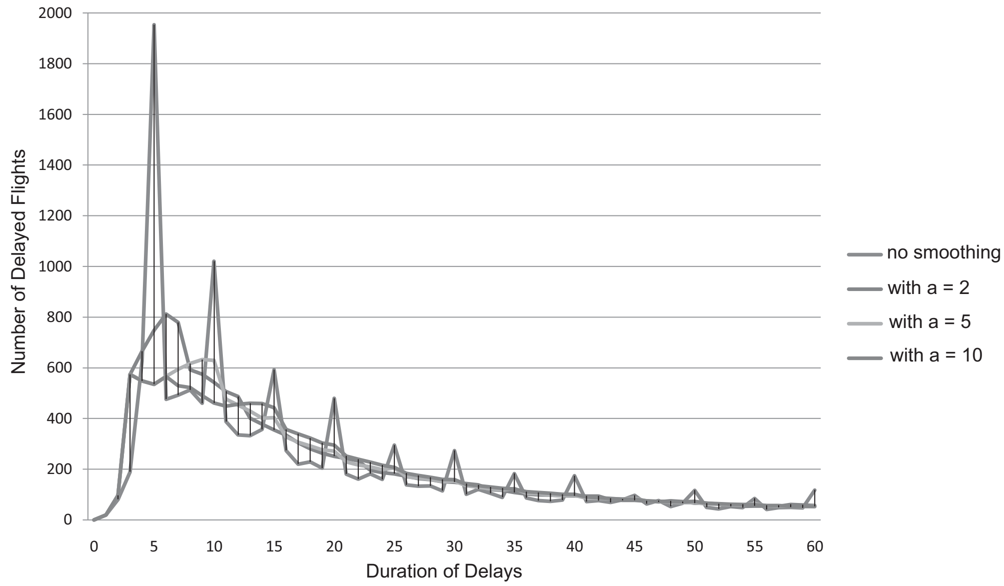



Figure 7.2.: Example for Smoothing with the Central Moving Average Method with $a$ = number data points, Source: Spengler [2009]

We group the different delay causes according to delay type as well as place and frequency of occurrence. For each delay group the empirical data of delays for the years 2003 to 2005 are then used to perform an automatic distribution fitting for duration and frequency of delays. This process consists of three steps. In the first the effects of manual input are eliminated by smoothing the empirical data by computing the central moving average of five data points. The new empirical distribution is $\widehat{F(x)}$, where $x$ is the delay duration. Figure 7.2 shows an example for the smoothing effect. In the second step, the smoothed empirical distribution $\widehat{F_{0305}}(x)$ of the years 2003 to 2005 is automatically fitted to a probability distribution using the software EasyFit[1]. The resulting probability function is $F_0(x)$. In the third step the theoretical probability distribution $F_0(x)$ is compared with the smoothed empirical probability distribution $\widehat{F_{06}}(x)$ from the year 2006 by computing the maximum distance $D$ between the two

[1]Product of MathWave Technologies [2009]

| Description | Relative Occurrence | Average Duration |
|---|---|---|
| Airline Internal | 12.6% | 2 min |
| Aircraft, passenger and baggage handling | 5.7% | 10 min |
| Aircraft maintenance | 2.7% | 34 min |
| Flight operations and Crewing | 1.28% | 13 min |
| Automated equipment failure | 1.1% | 13 min |
| No gates and wrong scheduled ground time | 0.34% | 11 min |
| Weather | 0 – 15% | 25 min |
| Airspace congestion | 0 – 15% | 13 min |
| Airport congestion | 0 – 15% | 10 min |

Table 7.1.: Distributions for Primary Delays

distributions. The empirical distributions are discrete: accordingly, the maximal distance is computed as follows. Compare also [Toutenberg, 2005, pp. 165–189]:

$$D = \max_{i \in 1 \dots n} \left\{ \left| D_i^+ \right|, \left| D_i^- \right| \right\} \tag{7.1}$$

$$\text{with} \quad D_i^+ = \widehat{F_{06}}(x_{(i-1)}) - F_0(x_{(i)}) \tag{7.2}$$

$$D_i^- = \widehat{F_{06}}(x_{(i)}) - F_0(x_{(i)}) \tag{7.3}$$

$D_{i \approx n}$ gives the distance between the two distributions at the position $n$ minutes and is a measure of the goodness of the fitted theoretical probability distribution. The theoretical distributions obtained in this work often underestimate the probability of delays shorter than five minutes and in turn overestimate the probabilities for delays longer than one hour. The values $D_{i \approx 5}$ for the distributions found during this analysis range from 2.3% to 6.7%. This deviation seems to be high, but still meets the requirements of the analysis in this work, because the average duration of delays fits and the probability of severe delays is not underestimated.

The result of this analysis is a set of nine delay groups. Table 7.1 shows a summary of the delay cause groups[2].

[2]Details on the found probability distributions are given in Appendix C.

The first six groups describe delays faced by individual flights. An independence of the events can be assumed. Thus the probability of occurrence for those delays is independent of the occurrence of other delays and can be set to a fixed value according to the relative frequency of occurrence in the empirical data. The duration of these delays is modelled by a theoretical distribution. The partitioning of the delay causes into groups follows the suggestion of IATA with slight modifications due to distinct statistical properties of different delays groups.

The last three groups represent primary delays caused by events or circumstances which lead to many delays of flights during a period; e.g. congestion at an airport leads to an increased probability of primary delays for all flights departing and arriving at this airport. In this work these groups contains delays caused by weather, airport and airspace congestion. The analysis of these delay causes shows that the occurrence probability varies either with the season or departure and arrival airport respectively. But the duration of those delays does not significantly correlate with the occurrence probability. Therefore for each delay cause group one probability distribution for the duration of the delay $d^C$ and several distributions for the occurrence probability $\pi^C$ are selected.

Given the distributions for occurrence probability, there are two ways of considering them during simulation depending on the aim of analysis. The first option is to introduce random variables in the simulation model for the occurrence probabilities of delay of each delay cause. The second option is to use a fixed occurrence probability for the whole simulation run. The first option can be realized in a two-step delay generation. In the first step the occurrence probabilities of each delay cause are determined and in the second step the actual delays of flights are generated according to the occurrence probability.

In this work we choose the second option and only consider one setting of the occurrence probability for each delay cause group during a simulation run. This option is suitable to exploring the behaviour of an airline schedule in certain circumstances, e.g. bad weather or congestion at one airport. By performing several simulation runs for different settings we can compare the performance of a schedule in different circumstances in detail. We use twelve fixed probability settings, which are presented in Table 7.2.

The stochastic models for optimization presented in Chapter 6 are similar to the simulation model used for evaluation. In order to prevent an overfitting

| Config | Weather | Airport Congestion | Airspace Congestion |
|---|---|---|---|
| medD | 20% | 10% | - |
| medC | - | 10% | 20% |
| medB | 10% | 20% | - |
| medA | 10% | 10% | 10% |
| lowD | 10% | 5% | - |
| lowC | - | 5% | 10% |
| lowB | 5% | 10% | - |
| lowA | 5% | 5% | 5% |
| highD | 30% | 15% | - |
| highC | - | 15% | 30% |
| highB | 15% | 30% | - |
| highA | 15% | 15% | 15% |

Table 7.2.: Configurations for Probabilities of Primary Delays During Simulation

effect, different primary delays for optimization and simulation are used. During optimization no primary delays due to weather, airspace congestion and airport are considered, i.e. the primary delays during optimization are underestimated.

## 7.2. Simulation of Airline Aircraft and Crew Schedules

In accordance with the assumptions presented in the previous section we here present a delay propagation model for aircraft and crews. The delay propagation model is used with a Monte Carlo method to compute schedule stability measures.

$F$ is the set of flights. The scheduled departure and arrival times are then given for each flight $f \in F$ by $s_f^D$ and $s_f^A$ and the actual departure and arrival times are given by $d_f$ and $r_f$. Equations (7.4) – (7.7) describe the delay propagation model, where the actual time of arrival $r_f$ is computed on the actual time of departure $d_f$ and the flight time $t_f$. Before a flight can depart the turn process of the aircraft abd the ground task of the crew have to be finished. $g^a_{a(f),f}$ and $g^c_{c(f),f}$ represent the ground times for aircraft and crews between two flights. The predecessor flights are given by $a(f)$ for aircraft routes and by $c(f)$ for crew pairings. $X_f$ is the primary departure delay for flight $f$ that is independent of the slack to the predecessor flight. $D_f$ is the total arrival delay for flight $f$ and $R_f$ is the reactionary arrival delay for flight $f$.

$$r_f = \max\left\{s_f^A, d_f + t_f\right\}, \qquad \forall f \in F \tag{7.4}$$

$$d_f = \max\left\{s_f^D, \max\left\{\begin{array}{l} r_{a(f)} + g^a_{a(f),f}, \\ r_{c(f)} + g^c_{c(f),f} \end{array}\right\}\right\} + X_f, \qquad \forall f \in F \tag{7.5}$$

$$D_f = r_f - s_f^A, \qquad \forall f \in F \tag{7.6}$$

$$R_f = D_f - X_f, \qquad \forall f \in F \tag{7.7}$$

where $X_f$ is the sum of several stochastic variables corresponding to different delay causes, $X_f = \sum_{C \in Causes} X_f^C$. Equation (7.8) shows a representation of the stochastic variable by two parameters: the occurrence probability $\pi^C$ and the duration $d^C$. The duration $d^C$ is always a random variable distributed by a log-normal or log-logistic distribution.

$$X_f^C = \begin{cases} 0, & \text{if } random > \pi_f^C \\ d^C \sim F^C, & \text{otherwise} \end{cases} \tag{7.8}$$

One important value of delay statistics is the fraction of flights facing reactionary delays of $n$ or more minutes:

$$0 \le P_n = \frac{|\{f \in F | R_f \ge n\}|}{|F|} \le 1.0 \tag{7.9}$$

Another measure is the number of crew duty limit and crew rest disruptions. This number corresponds to the number of flights in the following set

$$V = |\{f \in F | D_f > D_f^{max}\}|, \tag{7.10}$$

where $D_f^{max}$ is the maximal duration of an arrival delay of flight $f$, which does not lead to a crew duty limit or a crew rest disruption. In this work this value is called duty slack, $D_f^{max}$ is only set for the last flight of each crew duty to the corresponding duty slack; for all other flights it is set to infinity. The computation of the duty slack is given in Equation (6.47).

**Algorithm 7.1**: Monte Carlo Method for Simulation of Airline Schedules

Initialize $D_f^{max}$ for the last flight of each crew duty
**while** *arrival on-time performance changes significantly for any flight* **do**
  Generate primary delays $X_f$ for each flight
  Compute delay and duty rule disruption counters $D_f$, $R_f$ and $C$

Algorithm 7 shows the Monte Carlo method. In each iteration of the method two steps are performed. Firstly, a set of new primary delays is generated using the given probability distributions. Secondly, the resulting total and reactionary delays and the number of crew duty disruptions are computed using the model given by Equations 7.4 – 7.7 and 7.10. The simulation terminates when the arrival on-time performance values of each flight stabilize or a maximal number of iterations is reached. The arrival on-time performance values for a flight describe the relative number of iterations, where this flight arrived within 0, 5 and 15 minutes respectively after scheduled arrival.

We evaluate the predictability of a robustness indicator by computing the linear dependency of the robustness indicator with the corresponding robustness measures in the simulation. The efficiency of a robustness indicator is evaluated by quantifying the linear dependency of the schedule costs with the robustness measure. The absolute values of the robustness indicators, robustness measures and costs of aircraft and crew schedules differ greatly between flight schedules. Thus the evaluation of the linear dependency of the robustness indicator with the robustness measures and crew costs based on absolute values can only be made for aircraft and crew schedules of the same flight schedule. In order to evaluate the predictability and efficiency of a robustness indicator for all flight schedules, we propose to evaluate the linear dependency based on the relative difference of the robustness indicator, robustness measure and crew costs. This means for each flight schedule, a basis aircraft and crew schedule firstly is computed that is not used for evaluation. A set of subsequent aircraft and crew schedules is then computed using different robustness models. The corresponding simulation and scheduling results for each schedule in this set are compared with the basis schedule. The relative difference is then computed for the robustness indicator computed during optimization, the robustness measures computed during simulation and the crew costs.

Now, let $S = \{s_k | k = 1..n\}$ be the set of crew and aircraft schedules generated for different flight schedules. Let $ri_B(s_k)$ be the absolute value of the robustness indicator $B$ for schedule $s_k$, $rm_{P_{15}}(s_k)$ the value of the robustness measure counting propagated delays over 15 minutes, $rm_V(s_k)$ the value of the robustness measure counting crew duty disruptions and $c(s_k)$ is the value of the crew costs. Then, for each crew and aircraft schedule, $ri^{max}(s_k)$, $rm^{max}(s_k)$ and $c^{min}(s_k)$ refer to the properties of the basis schedule of the same flight schedule. $I_B$ is the set of the relative differences of the robustness indicator $B$, $M_{P_{15}}$ and $M_V$ are the relative differences of the two robustness measures from simulation and $C$ are the relative differences of the crew costs:

$$I_B = \left\{ i_k = \frac{ri_B^{max}(s_k) - ri_B(s_k)}{ri_B^{max}(s_k)} \middle| k = 1..n \right\} \tag{7.11}$$

$$M_{P_{15}} = \left\{ m_k = \frac{rm_{P_{15}}^{max}(s_k) - rm_{P_{15}}(s_k)}{rm_{P_{15}}^{max}(s_k)} \middle| k = 1..n \right\} \tag{7.12}$$

$$M_V = \left\{ m_k = \frac{rm_V^{max}(s_k) - rm_V(s_k)}{rm_V^{max}(s_k)} \middle| k = 1..n \right\} \tag{7.13}$$

$$C = \left\{ c_k = \frac{c(s_k) - c^{min}(s_k)}{c(s_k)} \middle| k = 1..n \right\} \tag{7.14}$$

The Pearson product-moment correlation factor (Equation (7.15), compare Rodgers and Nicewander [1988]) is used to evaluate a possible linear dependence between the relative decrease of a robustness indicator and the relative decrease of a robustness measure

$$Cor(I, M) = \frac{\sum_{k=1}^{n} \left(i_k - \overline{I}\right)\left(m_k - \overline{M}\right)}{\sqrt{\sum_{k=1}^{n} \left(i_k - \overline{I}\right)^2 \sum_{k=1}^{n} \left(m_k - \overline{M}\right)^2}}. \tag{7.15}$$

A correlation factor of 1 or -1 means a high linear relation, whereas a correlation factor near 0 means no linear dependence. A high linear dependence between the relative decrease of an indicator and the relative decrease of a measure indicates high predictability of the indicator. The definition of the correlation factor

$Cor(C, M)$, i.e. between the relative increase of crew costs and the relative decrease of a robustness measure is analogous.

## 7.3. Simulation Experiments

The stochastic recourse models presented in Chapter 6 are referred to as $Q_t$ for the delay propagation model and $Q_d$ for the duty slack model. We define $ri_t(s_k) = \sum_{j \in s_k} Q_t^P(j, Y_{s_k}, \omega)$ and $ri_d(s_k) = \sum_{j \in s_k} Q_d^P(j, Y_{s_k}, \omega)$. The deterministic indicators are defined as follows: $ri_\pi(s_k) = \sum_{(f_1,f_2) \in s_k} \pi(f_1, f_2)$ for the restricted aircraft changes and $ri_\sigma(s_k) = \sum_{duty \in s_k} \sigma(duty)$ for restricted duties. The sets $I_t$, $I_d$, $I_\pi$ and $I_\sigma$ then represent the relative differences of the objective values of these recourse models.

The first step of the evaluation is now to quantify the linear dependency for these indicators. The stochastic iterative method presented in this chapter is used to generate crew and aircraft schedules for 20 domestic flight schedules. Using the recourse models $Q_t$ this results in 61 aircraft and crew schedules, which are simulated using the simulation model. 20 schedules are the basis schedules leading to 41 schedules with relative differences for indicators, measures and costs. Thus the sets $M_{P_{15}}$, $M_V$, $C$ and $I_t$ contain 41 elements, which are used for the following analysis.

Figure 7.3 shows the results for the simulation configuration *highA*. There is a strong linear correlation between the relative decrease of the penalty factor for the expected reactionary delays and the relative decrease of the actual reactionary delays over 15 minutes ($Cor(I_t, M_{P_{15}}) = 0.85$) as well as a strong linear correlation between the relative increase of crew costs and the relative decrease of the actual reactionary delays over 15 minutes ($Cor(C, M_{P_{15}}) = 0.77$). The model for linear regression between the relative increase of crew costs and the relative decrease of reactionary delays is $m_k = 6.3\% + 8.6c_k$, i.e. on average a decrease in reactionary delays of 6.3% can be achieved without increasing crew costs and an additional decrease by 8.6% can be achieved by increasing crew costs by 1%.

Figure 7.4 shows the results of the correlation analysis as well as the regression analysis for all simulation configurations. The factor for linear correlation between the relative decrease of reactionary delays and the relative decrease of

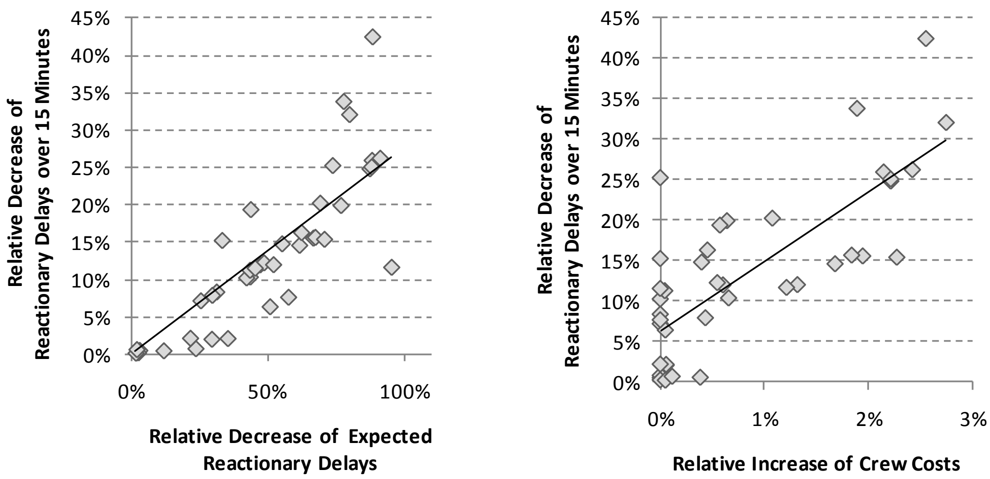



Figure 7.3.: Comparison of the Relative Decrease of Expected Reactionary Delays as well as the Relative Increase of Crew Costs with the Relative Decrease of Reactionary Delays (Configuration highA)

the corresponding penalty factor is greater than 0.82 for all configurations. The corresponding correlation factor with the relative increase of crew costs is always greater than 0.7. Both observations indicate a high predictability of the stochastic model on the punctuality of the schedules. The linear regression models are very similar for all simulation configurations, i.e. the interpretation of the model above also holds for different configurations.

The next step is to evaluate the robustness indicator for the expected number of crew duty disruptions. Figure 7.5 shows the evaluation of this robustness indicator for the configuration *highA* and Figure 7.6 for all simulation configurations. There is a high linear correlation between the relative decrease of the expected and observed crew duty disruptions ($Cor(I_d, M_V) > 0.7$) for all configurations. A detailed analysis of the relation between the relative increase of crew costs and the decrease of crew duty disruptions leads to the following regression model for the scenario highA: $m_k = 32\% + 9.8c_k$. This means an average decrease of crew duty disruptions of 32% can be achieved without increasing costs significantly. Additionally, the number of crew duty disruptions can be further decreased by 9.8% with each increase of crew costs by 1%. The correlation factor for this linear

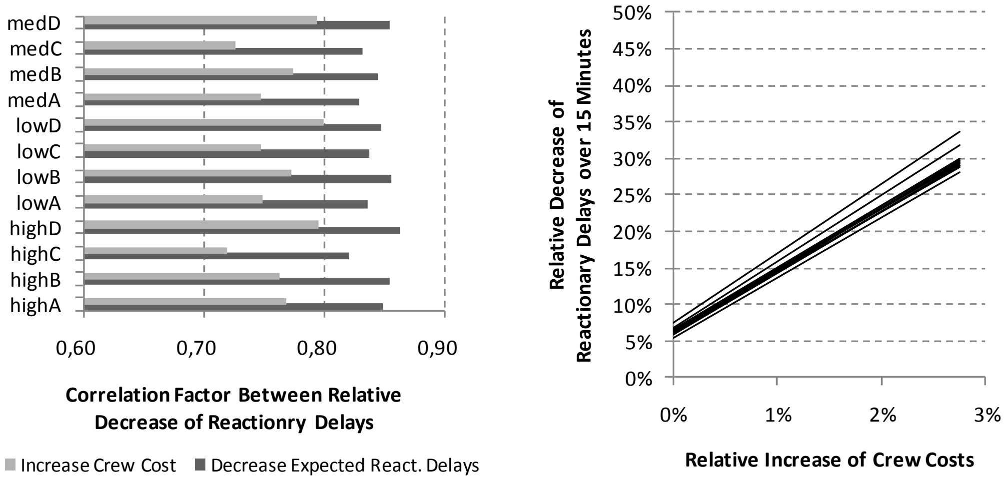



Figure 7.4.: Correlation Factor and Linear Regression Models for Penalty Factor for the Expected Number of Reactionary Delays With All Simulation Configurations

relation ($Cor(C, M_V)$), however, is only 0.5. The results for other configurations are very similar, compare Figure 7.6

Figure 7.7 shows the comparison of the delay propagation indicator with the number of crew duty disruptions as well as the comparison of the expected crew duty disruptions with the number of reactionary delays. In some cases the number of crew duty disruptions increases with a decreasing number of expected reactionary delays, but in most cases there is a low decrease of crew duty disruptions. As expected, the Pearson's correlation coefficient and the Spearman's rank correlation coefficient are both positive and low (0.27 and 0.4) for this effect. There is also no significant effect of the decrease of expected crew duty disruptions on the number of reactionary delays.

For the deterministic robustness indicators we only present the results for the simulation configuration *highA*. The results for the other configurations are very similar. The following experiments using the indicator for restricted aircraft changes are based on 56 crew and aircraft schedules generated for 20 different domestic flight schedules. 20 schedules provide bases: therefore 36 schedules with $\frac{ri_t^{max}(s_k)-ri_t(s_k)}{ri_t^{max}(s_k)} > 0$ can be used for the evaluation.

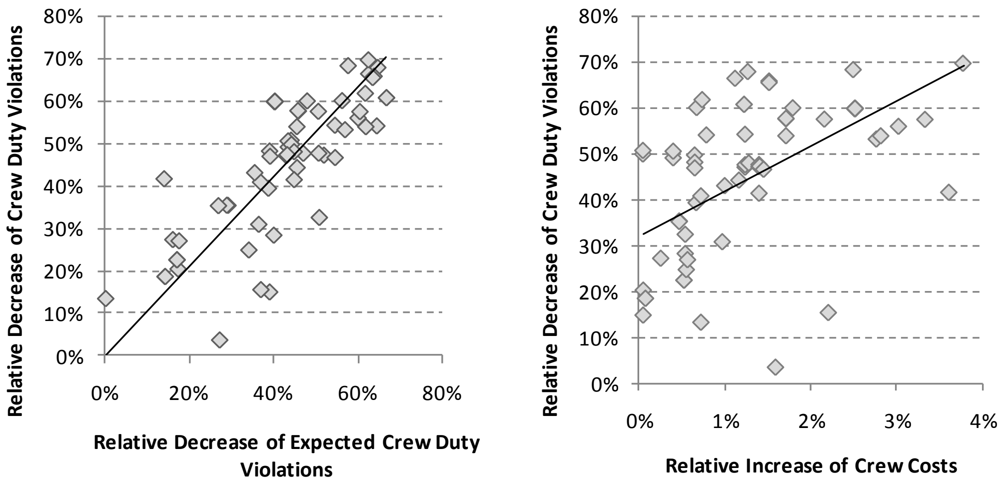



Figure 7.5.: Comparison of the Relative Decrease of Expected Crew Duty disruptions as well as the Relative Increase of Crew Costs with the Decrease of Observed Crew Duty disruptions (Configuration highA)

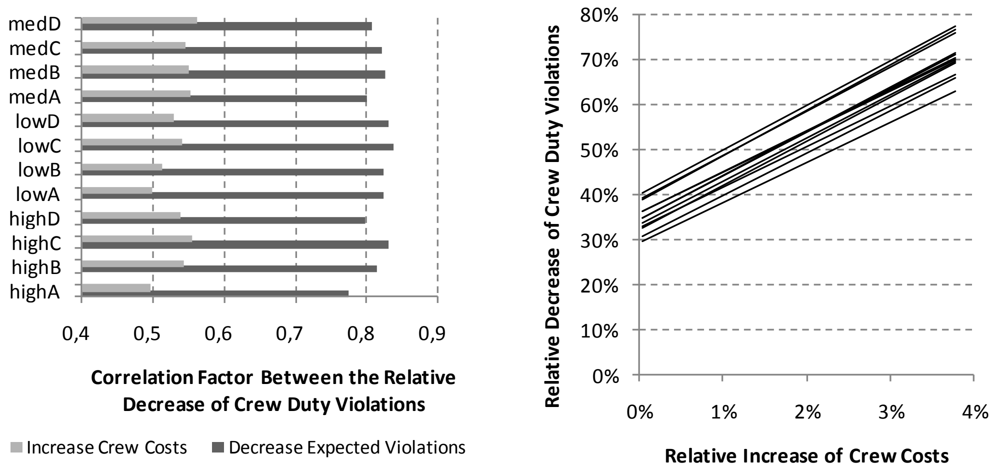



Figure 7.6.: Correlation Factor and Linear Regression Models for the Stochastic Robustness Measure Based on Expected Crew Duty disruptions With All Simulation Configurations

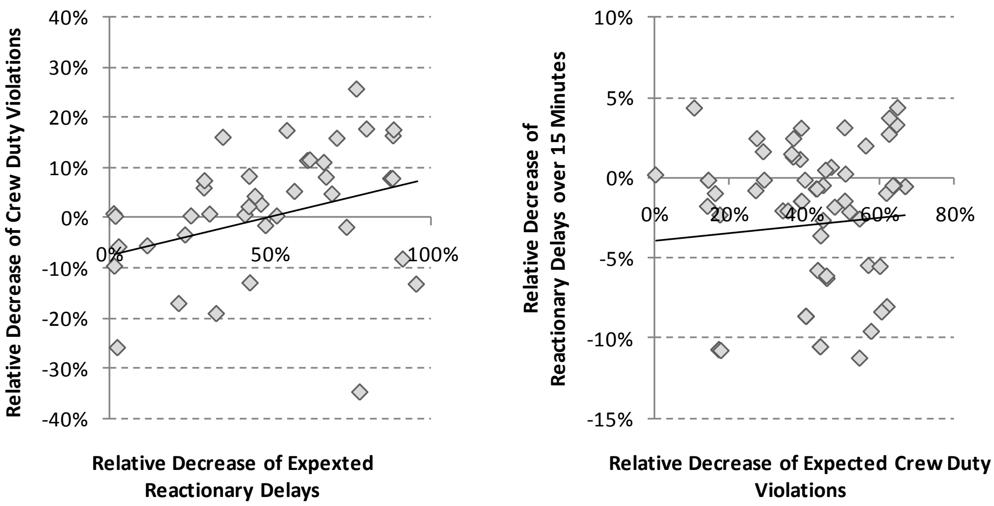



Figure 7.7.: Comparison of the Robustness Indicators with the Other Robustness Measures (Configuration highA)

Figures 7.8 show the results for the indicator based on restricted aircraft changes. The left figure shows a high linear dependency between the relative decrease of the penalty factor for restricted aircraft changes and the relative decrease of the reactionary delays over 15 minutes. The exact value of the correlation factor is $Cor(I_\pi, M_{P_{15}}) = 0.86$. The high linear dependency means a high predictability of the indicator on the number of reactionary delays.

The right figure shows a comparison of the relative increase of crew costs and the relative decrease of reactionary delays. In many cases increasing the penalty for restricted aircraft changes leads to a significant decrease in reactionary delays without increasing the crew costs at all. A regression analysis leads to the linear model $m_k = 6.4\% + 6.24c_k$. One possible interpretation of this regression model is, that the number of reactionary delays can be decreased on average by 6.4% without increasing crew costs. Additionally, a decrease of reactionary delays by 6.24% can be expexted by increasing crew costs by 1%. This, however, is only a rough estimateon, because there is only a low linear correlation.

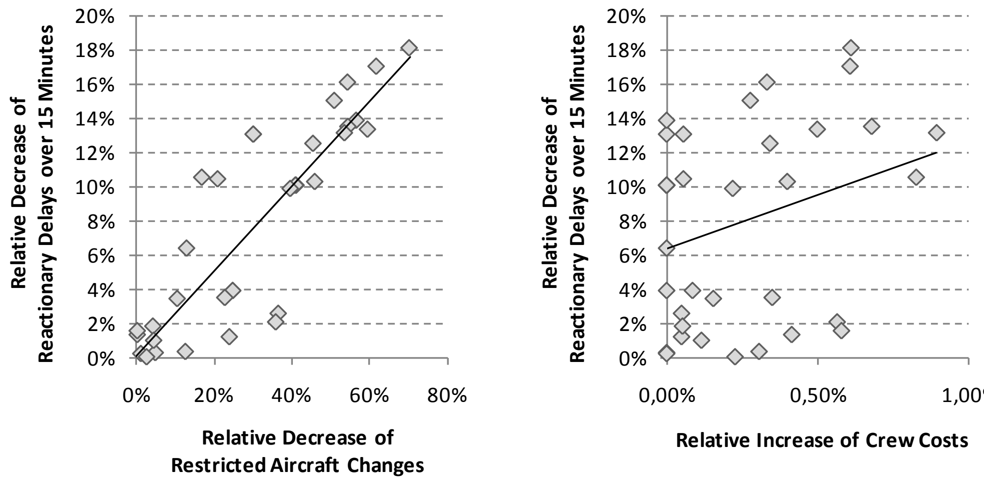
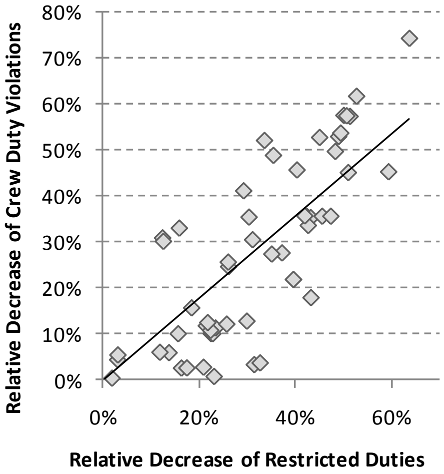



Figure 7.8.: Comparison of the Relative Decrease of Restricted Aircraft Changes as well as the Relative Increase of Crew Costs with the Relative Decrease of Reactionary Delays

For the evaluation of the deterministic indicator for restricted duties 80 crew and aircraft schedules are generated resulting in 60 schedules with positive relative decrease of the penalty factor.

The results of the evaluation of the linear correlation of the relative decrease of the penalty factor for restricted duties with the relative decrease of the crew duty disruptions in Figure 7.9 show a high predictability of the indicator ($Cor(I_\sigma, M_V) = 0.86$). In contrast to the indicator based on restricted aircraft changes, that based on restricted duties also leads to a high linear correlation between the relative increase of crew costs and the relative decrease of crew duty disruptions ($Cor(C, M_V) = 0.83$). A regression analysis of this relation leads to the linear model $m_k = 7.7\% + 5.4c_k$. The interpretation of this model is that on average the number of crew duty disruptions can be decreased by 7.7% without increasing costs. Furhtermore, an increase of crew costs by 1% leads to an average decrease in crew duty disruptions of 5.4%.

Figure 7.10 shows a comparison of the decrease of restricted aircraft changes with the decrease of crew duty disruptions observed during simulation as well as a comparison of the decrease of restricted crew duties with the decrease in reactionary delays. The result is that a low decrease of restricted aircraft changes



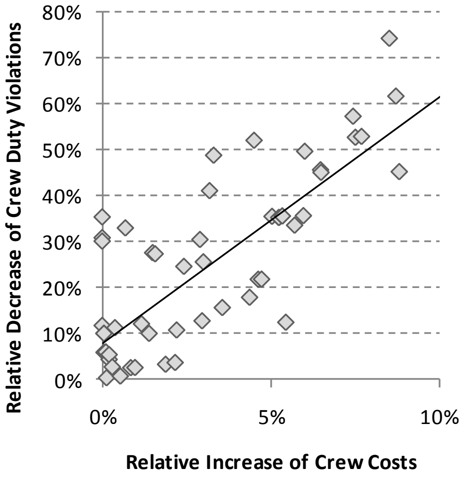
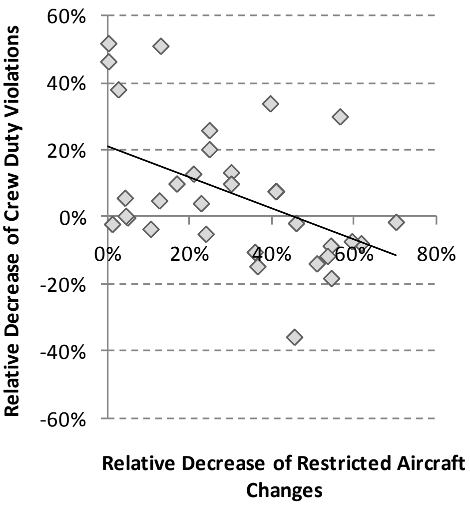




Figure 7.9.: Comparison of the Relative Decrease of Restricted Duties as well as the Relative Increase of Crew Costs with the Relative Decrease of Crew Duty Disruptions

also leads to a decrease of crew duty disruptions, but a large decrease of restricted aircraft changes can lead to an increase of crew duty disruptions. The linear regression model represents this effect and crosses the horizontal axis at 47%. The linear correlation factor, however, is only $Cor(I_\pi, M_V) = -0.47$. A decrease of restricted duties, however, has no significant effect on the number of reactionary delays.

The predictability and the efficiency of the stochastic indicator for the number of reactionary delays are similar to the properties of the corresponding deterministic indicator for punctuality. There are no samples with relative increase of crow costs by more than 1% for the deterministic indicator. The comparison of the samples with cost increase less than 1%, however, shows similar correlation and regression properties for both indicators ($I_\pi$ and $I_t$). In both cases a relative decrease in reactionary delays of up to 20% is possible. The linear correlation between the relative decrease of the indicator and the relative decrease of the measure is in both cases high, between 0.8 and 0.9. The linear correlation of the relative increase of crew costs with the relative decrease of the measure is in both cases low, between 0.3 and 0.4.



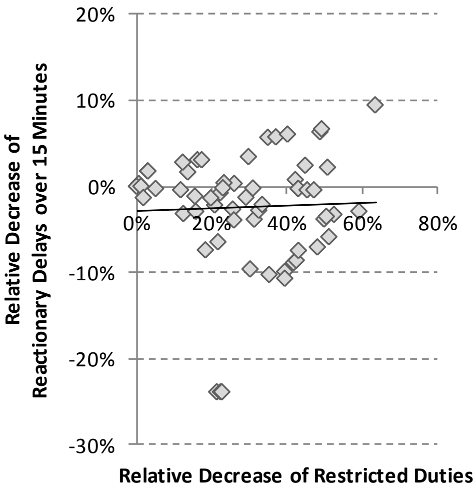




Figure 7.10.: Comparison of the Robustness Indicators with the Other Robustness Measures

Comparison of the stochastic indicator for restricted duties with the corresponding deterministic indicator shows a significantly higher efficiency of the stochastic model and comparable predictability of both indicators. Unlike with the deterministic indicator, higher decreases of crew duty disruptions can be achieved with the stochastic model in both cases; without significantly increasing costs (32% compared to 7.7%) as well as by increasing costs in steps of 1% (9.8% compared to 5.4%). A detailed analysis shows that the higher values for the relative decrease are based on higher absolute values of the decrease and not on higher reference values. The reference values $rm_d^{max}(s)$ for the stochastic model are lower than or equal to the values of the deterministic indicator $rm_\sigma^{max}(s)$ for all schedules $s$.

Both approaches for punctuality, the stochastic as well as the deterministic, lead to high predictability and efficiency of the robust scheduling. The deterministic approach, however, need an extensive calibration in order to reach these results. The first attempts with the deterministic indicator lead either to no improvement of robustness at all or to very high costs. Numerous experiments and manual calibration were needed to find a penalty function, that leads to good compromise between robustness and costs. But even this compromise has a major disadvantage. We were not able to generate schedules with more than 1% additional costs with the iterative approach. The stochastic approach on the other hand, lead to schedules with a wider range of the trade-off between costs and robustness, without any calibration. In the case of crew duty disruptions, our attempts to calibrate the deterministic indicator were not as successfull as in the case of punctuality. The stochastic model leads without any calibration to significantly better results. These results show the main advantage of the stochastic approach. Using the stochastic model less calibration and understanding of the effects of delay propagation by the decision maker is needed to achieve very good results.

# 8. Simultaneous Rescheduling of Aircraft and Crew Pairings

In this chapter a rescheduling method for crews and aircraft incorporating reactionary delays and swapping resources is presented. This method can be used to recover schedules during simulation automatically. The approach is based on a decomposition of the integrated recovery problem, analogous to the approach in Chapter 6. The resulting rescheduling problems for crews and aircraft are solved by the proposed branch-and-price-and-cut framework iteratively. A new concept for the iterative local search and additional extensions to the branch-and-price-and-cut framework are needed in order to incorporate reactionary delays. These concepts are presented in this chapter together with first simulation results.

## 8.1. Integrated Formulation for Aircraft and Crew Rescheduling

The formulation of the integrated recovery problem is based on the individual set partitioning formulations of the crew and aircraft recovery problems. The set $F$ represents all flights that must be covered, the sets $P$ and $R$ the possible crew pairings and aircraft routes. The different retiming possibilities for each flight $f \in F$ are represented by a set of arcs $I(f)$. The constraints (8.2) and (8.3) ensure that exactly one arc of each flight is covered by a crew pairing and an aircraft route. The constraints, that in the resulting crew as well as in the corresponding aircraft schedule the same arcs for retimed flights are chosen is given in (8.8). Aircraft and crew availability and flow constraints are given by (8.4) – (8.7). The objective is to minimize the use of reserve crews, additional delays of flights and changes to the schedule. This objective is represented by costs associated with crew pairing and aircraft route variables.

$$\sum_{p\in P} c_p x_p + \sum_{r\in R} c_r y_r \rightarrow \min \qquad \text{recovery problem} \tag{8.1}$$

$$\sum_{i\in I(f)} \sum_{p\in P(f)} x_p = 1 \qquad \forall f \in F \tag{8.2}$$

$$\sum_{i\in I(f)} \sum_{r\in R(f)} y_r = 1 \qquad \forall f \in F \tag{8.3}$$

$$\sum_{r\in R(s)} y_r \leq 1 \qquad \forall s \in S^R \tag{8.4}$$

$$\sum_{r\in R(e)} y_r \geq 1 \qquad \forall e \in E^R \tag{8.5}$$

$$\sum_{p\in P(s)} y_r \leq 1 \qquad \forall s \in S^P \tag{8.6}$$

$$\sum_{p\in P(e)} y_r \geq 1 \qquad \forall e \in E^P \tag{8.7}$$

$$\sum_{i\in I(f)} \left( \sum_{p\in P(i)} x_p - \sum_{r\in R(i)} y_r \right) = 0 \qquad \forall f \in F \tag{8.8}$$

$$x_p \in \{0,1\} \qquad \forall p \in P \tag{8.9}$$

$$y_r \in \{0,1\} \qquad \forall r \in R \tag{8.10}$$

The introduced formulation includes both crew pairing and aircraft route variables. The decomposition of the integrated problem into a classical set partitioning formulation for the crew pairing problem as well as the aircraft assignment problem leads to a connection of the resulting separated problems by additional objective costs, regarding the usage of retimed flights. Those additional costs are considered during the generation of the path variables. Thus the seperate crew and aircraft recovery problems can be solved iteratively.

A retiming during the solution of the crew pairing (aircraft assignment) problem which renders an existing aircraft assignment (crew pairing) solution invalid is called incompatible retiming. Incompatible retimings can easily be identified and penalized during the generation of new path variables. Given the penalty value $c^Q$, the costs of crew pairing and aircraft assignment variables become

$$\widehat{c_p} = c_p + \sum_{i \in I(p)} \sum_{r \in R(i)} (1 - y_r)\, c^Q, \tag{8.11}$$

$$\widehat{c_r} = c_r + \sum_{i \in I(r)} \sum_{p \in P(i)} (1 - x_p)\, c^Q. \tag{8.12}$$

It is to be noted that after the decomposition the arcs representing the different retiming possibilities for a flight are no longer represented in the given formulation. The compatibility of the schedules is now considered during the generation of the path variables only. Additionally, the constraint of using the earliest possible flight departure leads always to path variables with unambiguous costs. I.e. for a path variable it is always clear what departure times for the covered flights are used. In contrast, in a setting where flight retimings could be used to increase slack time, it is possible to generate two path variables which cover the same flights but have different costs due to different departure times and slack times. In this case it is no longer trivial always to ensure that the better alternative of several path possibilities is generated, without explicitly representing the different retiming possibilities in the main formulation.

## 8.2. The Iterative Solution Approach

Both resulting problems are solved iteratively with increasing penalty value for retimings that lead to incompatible schedules. Algorithm 8.1 gives an overview of the iterative method, which starts with solving a crew pairing problem without a given aircraft assignment solution. The resulting crew costs form a lower bound on the actual optimal costs. Based on this crew schedule, the two scheduling problems are solved iteratively using the new cost functions $\widehat{c^P}$ and $\widehat{c^R}$ with increasing penalty for incompatible retimings until a feasible schedule is found.

Retiming possibilities for each flight are computed in a preprocessing step before the main algorithm with the objective of enabling new connections. These connections are represented by new retiming arcs between each flight and earlier flights at the same airport. If a retiming of a flight leads to a loss of previously possible successor connections, a recursive retiming for the successor is also per-

**Algorithm 8.1**: Iterative Method for Simultaneous Recovery

```
Set c^Q = 0 ;          /* penalty cost for incompatible retimings */
Solve crew pairing problem without a given aircraft assignment solution
while c^Q ≤ c^Q_max do
    Increase c^Q
    Solve aircraft assignment problem with cost ĉ^R based on crew schedule
    if new aircraft routes do not contain any incompatible retimings then
        break
    Solve crew pairing problem with cost ĉ^P based on aircraft schedule
    if new crew pairings do not contain any incompatible retimings then
        break
```

formed. The new arcs for retiming possibilities are added to the network as additional flight arcs. Hence a flight is now represented by several arcs in the network. To ensure that the earliest arc for a flight is always selected, we only allow selected connections for the retiming arcs, which were identified during the preprocessing step. The generation of retiming arcs can be limited by limiting the set of flights enabled for retiming or limiting the depth for recursive retiming. Retiming possibilities increase the number of possible connections between flights and therefore also the complexity of the problems. The resulting network is heuristically searched for new path variables with the label setting algorithm introduced in Chapter 5.

The retiming arcs are only used in the pricing subproblems and therefore need no longer to be represented in the set partitioning formulations of the master problems. Since, we require always to use the earliest possible flight departure, the departure times as well as the objective value of each path variable are well-defined.

## 8.3. Simulation Experiments

The proposed method was tested with domestic schedules of a European airline. For our analysis we simulate the closure of a hub airport for two morning hours. All flights scheduled to arrive during this time at the closed airport are randomly delayed to arrive up to 30 minutes after the end of the disruption. Table 8.1

| Testcase | #Primary | ØPrimary [h:mm] | #Reactionary | ØReactionary [h:mm] | #Conflicts |
|---|---|---|---|---|---|
| dom094 | 4 | 1:36 | 5 | 0:29 | 6 |
| dom104 | 7 | 1:28 | 14 | 0:36 | 1 |
| dom106 | 10 | 1:38 | 27 | 1:08 | 13 |
| dom117 | 10 | 1:05 | 23 | 1:20 | 5 |
| dom121 | 7 | 1:38 | 23 | 0:43 | - |
| dom124 | 7 | 1:32 | 28 | 1:35 | 6 |
| dom128 | 3 | 1:47 | 9 | 1:10 | 1 |
| dom171 | 4 | 1:54 | 7 | 0:48 | - |

Table 8.1.: Primary and Reactionary Delays After Simple Propagation

shows the number of originally delayed flights (primary delays) and the average duration of those delays for eight problem instances with 94 - 171 flights.

As the first step of our analysis a simple propagation of all delays is performed resulting in a set of new delays (reactionary delays) and disruptions of pairing rules, like maximal daily flight or working time (conflicts). Table 8.1 also shows those results. As expected the large number of conflicts and reactionary delays shows that a simple propagation is usually not the right strategy for coping with such severe disruption. The main point of this analysis is to demonstrate the close interdependency of crews and aircraft in the used schedules, which leads to the high number of reactionary delays.

All used schedules are one day's duration. In our experiments we assume, that the occurrence of the disruption is known several hours in advance. As a result we use the whole schedule for the rescheduling and fix only the airports and times where the aircraft routes and crew pairings start at the beginning of the schedule and finish at its end.

The proposed rescheduling method does not automatically cancel flights nor uses any reserve aircraft. The used objective function is intended to minimize the number of reserve crews. Assigning a task to another pairing (task swap) is usually preferred to delaying a task; thus delays are only performed for feasibility reasons. Crew swaps, where two pairings swap their ending stations and times are not needed in this test setting, because the recovery period includes the whole schedule. In detail, the following weights were used:

| Testcase | #Reserve | #Swaps | #Reactionary | ØReactionary [h:mm] | #Conflicts |
|---|---|---|---|---|---|
| dom094 | - | 6 | 5 | 0:48 | - |
| dom104 | - | 16 | 7 | 0:40 | - |
| dom106 | 1 | 27 | 14 | 0:30 | - |
| dom117 | - | 41 | 4 | 0:38 | - |
| dom121 | 1 | 10 | 9 | 0:52 | - |
| dom124 | - | 19 | 13 | 0:45 | - |
| dom128 | - | 9 | 4 | 0:36 | - |
| dom171 | - | 2 | 2 | 0:51 | - |

Table 8.2.: Reactionary Delays and Schedule Changes After Rescheduling

- reserve crew: 200
- task swap: 5
- delayed flight: 5
- one minute delay: 1

All schedules could be rescheduled using the available aircraft resources. Table 8.2 shows a comparison of the rescheduling results performed by the proposed method and by a simple propagation. In all cases all conflicts regarding crew pairing rules could be resolved. In two cases a reserve crew was scheduled starting and ending at the same crew base. The number of retimed flights (reactionary delays) is in most cases significantly lower in comparison to the simple propagation. In one case more total reactionary delay time was needed to resolve the conflicts, but in most cases the total reactionary delay time as well as the average delay duration are significantly lower. The number of tasks assigned to a pairing other than original (swaps) varies greatly, but also closely correlates with the number of reactionary delays saved.

The results probably differ from a practical solution but match the used weights in the objective. They show that the method is capable of minimizing reactionary delays as well as constructing compatible schedules at the same time, even if large changes to both schedules are needed.

# 9. Summary and Conclusions

In this work we addressed simultaneous scheduling of aircraft and crews under consideration of disruption. These tactical scheduling tasks are usually performed several months prior to the day of operations. Both schedules are easily disrupted on the day of operations, because of restrictions on maintenance of aircraft and working time of crews. These disruptions are hard to recover and often lead to additional delays. Both problems are also closely interdependent due to crews changing aircraft on the day of operations. Such aircraft changes can lead to additional propagation of delays between aircraft, i.e. following the crew from one to another aircraft. In Chapter 2 we describe the different sources of disruptions as well as operational effects and possible responses.

The study of literature to date in Chapter 3 shows that simultaneous scheduling of aircraft and crews can greatly reduce costs as well as delay propagation. The crew and aircraft scheduling problems, however, are both computationally very difficult and therefore usually solved sequentially. Most existing approaches for robust scheduling of aircraft and crews are based on indicators for delay propagation. But most of these approaches only measure local effects of delay propagation. Moreover, there is no common framework for evaluation and comparison of different approaches for robust scheduling. On this analysis we developed in Chapter 4 the following three research objectives for this work: (1) development of a heuristic approach to simultaneous scheduling of aircraft and crews, which incorporates a more exact measure of delay propagation using historical data, (2) development of an evaluation framework to compare the new approach to robust scheduling with existing approaches and (3) development of a new optimization method for aircraft and crew schedules based on well-founded concepts from literature in order efficiently to solve the robust scheduling problems.

In Chapter 5 we proposed a common optimization method for the deterministic tactical scheduling problems for crews and aircraft as well as the corresponding rescheduling problems. The proposed branch-and-price-and-cut method was used to efficiently solve the named problems as well as the stochastic problems introduced in this work. The pricing subproblems were modelled as resource-constrained shortest-path problems and solved by a dynamic programming approach. In order to cope with the complex problems, we presented a new backtracking scheme as well as a new label categorizing technique for the dynamic programming algorithm. We showed that both techniques are crucial to solving the stochastic pricing subproblem efficiently but can also be applied to problems without stochastic scenarios. Other important developments in this work were the heuristic branching and node selection strategies as well as the consideration of subset row inequalities in the branch-and-price-and-cut method. The heuristic diving strategies in combination with problem fixing were crucial to efficiently solving large-scale scheduling problems and corresponds to the third research objective.

In accordance with the first research objective we presented in Chapter 6 an integrated stochastic model for aircraft assignment and crew pairing with the objective of minimizing planned crew costs as well as total delay propagation between aircraft and crews. The integrated model incorporates a very detailed representation of the delay propagation on the day of operations. But, it is also computationally very difficult due to non-linear constraints and binary decision variables. We proposed to solve the integrated model heuristically by decomposing it into separate stochastic problems for crews and aircraft, modelling the interdependencies of the two problems by a common objective function and applying an iterative solution approach. The heuristic solution approach enabled us to generate robust aircraft and crew schedules for weekly scheduling problems with more than 250 flights in less than four hours. Despite the decomposition of the problem and the heuristic nature of our solution approach it incorporates the most exact representation of delay propagation between aircraft and crews in current literature. The main property of the stochastic problems resulting from our decomposition is that the delay scenarios only need be considered in the pricing subproblems. We showed that the additional constraints modelling delay propagation can easily be modelled by the resource constrained shortest-path problem and thus enable us to reuse existing dynamic programming methods in the pricing subproblem. The result is an increase of the average solution time of the iterative approach by only 74% for the stochastic model, than in a determinstic model (from 2 hours to 3.5 hours). In future research the solution times for the stochastic problem can be decreased significantly by incorporating sampling methods for the consideration of delay scenarios.

Chapter 7 we devoted to the second objective and here presented a simulation model as well as a framework within which to compare different approaches for robust scheduling. With this framework we compared the stochastic scheduling approach with a similar deterministic approach to robust scheduling. The deterministic approach penalizes short slack between two flights if the crews are scheduled to change aircraft. This approach is very similar to the method proposed by Weide [2009]. The deterministic model does not consider any delay scenarios or delay propagation and thus leads to shorter solution times. We evaluated the approaches by computing the operational performance of schedules. The later is measured by simulating crew and aircraft schedules and comparing the number of reactionary delays as well as the number of crew duty disruptions, which are not resolved by delay propagation. We proposed to measure the quality of the approaches for robustness by evaluating the predictability and the efficiency of those approaches. Both approaches to increasing the punctuality, the stochastic and the deterministic, show a strong mathematical relation between the indicated values during scheduling and corresponding punctuality measures in the simulation. The approaches are also equally efficient, i.e. the relation between increase of robustness and crew costs is comparable. The deterministic indicators, however, need extensive calibration to reach comparable results.

Despite higher solution times for the stochastic model for increasing punctuality, this model offers great advantages and possibilities for future research. The two main advantages are automatic calibration and extensibility of the stochastic model. Whereas the stochastic model automatically computes propagated delays based on the delay scenarios used, the performance of the deterministic model depends on the selected minimal and maximal slack during an aircraft change. For different delay distributions or schedule properties these values need to be adjusted. Moreover, on the propagation model presented in this work it is possible to compute probabilities for severe delays and to compute delay statistics for certain flights and airports. This information may be used for positioning of reserve crews, schedule redesign and evaluation of rescheduling actions.

In Chapter 8 we proposed an approach to rescheduling aircraft and crews on the day of operations. The presented approach solves the recovery problems for crews and aircraft iteratively using flight retiming. The proposed decomposition of the integrated problem leads to a reduced complexity and therefore to lower solution times of the iterative approach than with methods for the integrated problem. All test cases in our test set could be solved in few a seconds or minutes using our prototype solver. This approach was not an original objective of this work. The main idea of this approach resulted from the experience with the optimization methods as well as the approaches to robust scheduling. The rescheduling approach could be realized very quickly by reusing the optimization methods from Chapter 5.

Future research includes the development of indicators of flexibility and the extension of the evaluation framework to measure schedule flexibility, e.g. by the approach to rescheduling proposed in this work. Examples for indicators of flexibility are move-up, stand-by and reserve crews, which may be schedules at crew bases based on expected delay. Alternatively, full recovery procedures could be considered during scheduling, e.g. as a subproblem with the objective of evaluating the flexibility of full or partial schedules. Additional research is needed to find possibilities of incorporating multiple objectives, e.g. stability and flexibility indicators, in the iterative solution approach to the integrated model.

In this work we simultaneously schedule crew pairings and aircraft. In future research additional airline scheduling problems may be integrated into simultaneous scheduling, e.g. fleet assignment and flight retiming. The fleet assignment problem may be integrated with the objective of increasing possibilities for recovery actions, because aircraft and crew swaps are easier to perform within the same fleet. Flight retiming can be used to increase slack between pairs of flights, where it cannot efficiently be obtained by crew and aircraft scheduling.

Finally, an important aspect of future research in robust scheduling shall be the estimation of the impact of delayed flights. Today most approaches treat all delayed flights equally or even totalise the duration of all delays. In practice delays often have very different side-effects and costs, e.g. stranded passengers and new bottlenecks at airport facilities. Thus additional research is needed the better to estimate the impact of delays in order to be able to deal with "right" delays at "right" cost.

# Appendix A.

# European Regulations on Crew Work-Time

Excerpt from REGULATION (EC) No 1899/2006 OF THE EUROPEAN PARLIAMENT AND OF THE COUNCIL

SUBPART Q

OPS 1.1090

**Objective and scope**

1. An operator shall establish a flight and duty time limitations and rest scheme (FTL) for crew members.
2. An operator shall ensure that for all its flights:
    a) The flight and duty time limitations and rest scheme is in accordance with both:
        i. the provisions of this subpart; and
        ii. any additional provisions that are applied by the Authority in accordance with the provisions of this subpart for the purpose of maintaining safety.
    b) Flights are planned to be completed within the allowable flight duty period taking into account the time necessary for pre-flight duties, the flight and turn-around times.
    c) Duty rosters will be prepared and published sufficiently in advance to provide the opportunity for crew members to plan adequate rest.

3. Operators' responsibilities
   a) An operator shall nominate a home base for each crew member.
   b) Operators shall be expected to appreciate the relationship between the frequencies and pattern of flight duty periods and rest periods and give due consideration to the cumulative effects of undertaking long duty hours interspersed with minimum rest.
   c) Operators shall allocate duty patterns which avoid such undesirable practices as alternating day/night duties or the positioning of crew members so that a serious disruption of established sleep/work pattern occurs.
   d) Operators shall plan local days free of duty and notify crew members in advance.
   e) Operators shall ensure that rest periods provide sufficient time to enable crew to overcome the effects of the previous duties and to be well rested by the start of the following flight duty period.
   f) Operators shall ensure flight duty periods are planned to enable crew members to remain sufficiently free from fatigue so they can operate to a satisfactory level of safety under all circumstances.
4. Crew members' responsibilities
   a) A crew member shall not operate an aeroplane if he/she knows that he/she is suffering from or is likely to suffer from fatigue or feels unfit, to the extent that the flight may be endangered.
   b) Crew members should make optimum use of the opportunities and facilities for rest provided and plan and use their rest periods properly.
5. Responsibilities of civil aviation authorities
   a) Variations
      i. Subject to the provisions of Article 8, the Authority may grant variations to the requirements in this subpart in accordance with applicable laws and procedures within the Member States concerned and in consultation with interested parties.

ii. Each operator will have to demonstrate to the Authority, using operational experience and taking into account other relevant factors such as current scientific knowledge, that its request for a variation produces an equivalent level of safety.

Such variations will be accompanied with suitable mitigation measures where appropriate.

OPS 1.1095

**Definitions**

For the purposes of this Regulation, the following definitions shall apply:

1. Augmented flight crew

   A flight crew which comprises more than the minimum number required for the operation of the aeroplane and in which each flight crew member can leave his/her post and be replaced by another appropriately qualified flight crew member.

2. Block time

   The time between an aeroplane first moving from its parking place for the purpose of taking off until it comes to rest on the designated parking position and all engines or propellers are stopped.

3. Break

   A period free of all duties, which counts as duty, being less than a rest period.

4. Duty

   Any task that a crew member is required to carry out associated with the business of an AOC holder. Unless where specific rules are provided for by this Regulation, the Authority shall define whether and to what extent standby is to be accounted for as duty.

5. Duty period

   A period which starts when a crew member is required by an operator to commence a duty and ends when the crew member is free from all duties.

6. Flight duty period

   A flight duty period (FDP) is any time during which a person operates in an aircraft as a member of its crew. The FDP starts when the crew member is required by an operator to report for a flight or a series of flights; it finishes at the end of the last flight on which he/she is an operating crew member.

7. Home base

   The location nominated by the operator to the crew member from where the crew member normally starts and ends a duty period or a series of duty periods and where, under normal conditions, the operator is not responsible for the accommodation of the crew member concerned.

8. Local day

   A 24-hour period commencing at 00:00 local time.

9. Local night

   A period of eight hours falling between 22:00 hours and 08:00 hours local time.

10. A single day free of duty

    A single day free of duty shall include two local nights. A rest period may be included as part of the day off.

11. Operating crew member

    A crew member who carries out his/her duties in an aircraft during a flight or during any part of a flight.

12. Positioning

    The transferring of a non-operating crew member from place to place, at the behest of the operator, excluding travelling time. Travelling time is defined as:

    - time from home to a designated reporting place and vice versa;
    - time for local transfer from a place of rest to the commencement of duty and vice versa.

13. Rest period

    An uninterrupted and defined period of time during which a crew member is free from all duties and airport standby.

14. Standby:

    A defined period of time during which a crew member is required by the operator to be available to receive an assignment for a flight, positioning or other duty without an intervening rest period.

15. Window of circadian low (WOCL):

    The window of circadian low (WOCL) is the period between 02:00 hours and 05:59 hours. Within a band of three time zones the WOCL refers to home base time. Beyond these three time zones the WOCL refers to home base time for the first 48 hours after departure from home base time zone, and to local time thereafter.

OPS 1.1100

**Flight and duty limitations**

1. Cumulative duty hours

   An operator shall ensure that the total duty periods to which a crew member is assigned do not exceed:

   a) 190 duty hours in any 28 consecutive days, spread as evenly as practicable throughout this period; and

   b) 60 duty hours in any seven consecutive days.

2. Limit on total block times

   An operator shall ensure that the total block times of the flights on which an individual crew member is assigned as an operating crew member does not exceed

   - 900 block hours in a calendar year; or
   - 100 block hours in any 28 consecutive days.

OPS 1.1105

**Maximum daily flight duty period (FDP)**

1. General:
   a) This OPS does not apply to single pilot operations and to emergency medical service operations.
   b) An operator shall specify reporting times that realistically reflect the time for safety related ground duties as approved by the Authority.
   c) The maximum basic daily FDP is 13 hours.
   d) These 13 hours will be reduced by 30 minutes for each sector from the third sector onwards with a maximum total reduction of two hours.
   e) When the FDP starts in the WOCL, the maximum stated in point 1c and point 1d will be reduced by 100% of its encroachment up to a maximum of two hours. When the FDP ends in or fully encompasses the WOCL, the maximum FDP stated in point 1c and point 1d will be reduced by 50% of its encroachment.
2. Extensions:
   a) The maximum daily FDP can be extended by up to one hour.
   b) Extensions are not allowed for a basic FDP of six sectors or more.
   c) Where an FDP encroaches on the WOCL by up to two hours extensions are limited to up to four sectors.
   d) Where an FDP encroaches on the WOCL by more than two hours extensions are limited to up to two sectors.
   e) The maximum number of extensions is two in any seven consecutive days.
   f) Where an FDP is planned to use an extension pre and post flight minimum rest is increased by two hours or post flight rest only is increased by four hours. Where the extensions are used for consecutive FDPs the pre and post rest between the two operations shall run consecutively.

g) When an FDP with extension starts in the period 22:00 to 04:59 hours the operator will limit the FDP to 11.45 hours.

3. Cabin Crew

   a) For cabin crew being assigned to a flight or series of flights, the FDP of the cabin crew may be extended by the difference in reporting time between cabin crew and flight crew, as long as the difference does not exceed one hour.

4. Operational robustness

   a) Planned schedules must allow for flights to be completed within the maximum permitted flight duty period. To assist in achieving this operators will take action to change a schedule or crewing arrangements at the latest where the actual operation exceeds the maximum FDP on more than 33% of the flights in that schedule during a scheduled seasonal period.

5. Positioning

   a) All the time spent on positioning is counted as duty.

   b) Positioning after reporting but prior to operating shall be included as part of the FDP but shall not count as a sector.

   c) A positioning sector immediately following operating sector will be taken into account for the calculation of minimum rest as defined in OPS 1.1110 (1a) and (1b).

6. Extended FDP (split duty)

   a) The Authority may grant approval to an operation based on an extended FDP including a break, subject to the provisions of Article 8.

   b) Each operator will have to demonstrate to the Authority, using operational experience and taking into account other relevant factors, such as current scientific knowledge, that its request for an extended FDP produces an equivalent level of safety.

OPS 1.1110

**Rest**

1. Minimum rest
   a) at least as long as the preceding duty period or 12 hours whichever is the greater;
   b) The minimum rest which must be provided before undertaking a flight duty period starting away from home base shall be at least as long as the preceding duty period or 10 hours whichever is the greater; when on minimum rest away from home base, the operator must allow for an eight-hour sleep opportunity taking due account of travelling and other physiological needs;
   c) An operator will ensure that effects on crew members of time zone differences will be compensated by additional rest, as regulated by the Authority subject to the provisions of Article 8.
   d) Notwithstanding (1a) and (1b) and subject to the provisions of Article 8, the Authority may grant reduced rest arrangements.
   e) Each operator will have to demonstrate to the Authority, using operational experience and taking into account other relevant factors, such as current scientific knowledge, that its request for reduced rest arrangements produces an equivalent level of safety.
2. Rest periods
   a) An operator shall ensure that the minimum rest provided as outlined above is increased periodically to a weekly rest period, being a 36-hour period including two local nights, such that there shall never be more than 168 hours between the end of one weekly rest period and the start of the next. As an exception to OPS 1.1095 (9), the Authority may decide that the second of those local nights may start from 20:00 hours if the weekly rest period has a duration of at least 40 hours.

OPS 1.1115

**Extension of flight duty period due to in-flight rest**

1. Subject to the provisions of Article 8 and providing each operator demonstrates to the Authority, using operational experience and taking into account other relevant factors such as current scientific knowledge, that its request produces an equivalent level of safety:

   a) Flight crew augmentation

      the Authority shall set the requirements in connection with the augmentation of a basic flight crew for the purpose of extending the flight duty period beyond the limits in OPS 1.1105 above;

   b) Cabin crew

      the Authority shall set the requirements in connection with the minimum in-flight rest by cabin crew member(s) when the FDP goes beyond the limitations in OPS 1.1105;

OPS 1.1120

**Unforeseen circumstances in actual flight operations — commander's discretion**

1. Taking into account the need for careful control of these instances implied underneath, during the actual flight operation, which starts at the reporting time, the limits on flight duty, duty and rest periods prescribed in this subpart may be modified in the event of unforeseen circumstances. Any such modifications must be acceptable to the commander after consultation with all other crew members and must, in all circumstances, comply with the following:

   a) The maximum FDP referred to in OPS 1.1105(1.3) above may not be increased by more than two hours unless the flight crew has been augmented, in which case the maximum flight duty period may be increased by not more than three hours;

      i. If on the final sector within a FDP unforeseen circumstances occur after take off that will result in the permitted increase being exceeded, the flight may continue to the planned destination or alternate;

   ii. In the event of such circumstances, the rest period following the FDP may be reduced but never below the minimum rest defined in OPS 1.1110(1b) of this subpart;

b) The Commander shall, in case of special circumstances, which could lead to severe fatigue, and after consultation with the crew members affected, reduce the actual flight duty time and/or increase the rest time in order to eliminate any detrimental effect on flight safety;

c) An operator shall ensure that:

   i. The Commander submits a report to the operator whenever a FDP is increased by his/her discretion or when a rest period is reduced in actual operation and

   ii. Where the increase of a FDP or reduction of a rest period exceeds one hour, a copy of the report, to which the operator must add his comments, is sent to the Authority no later than 28 days after the event.

OPS 1.1125

**Standby**

1. Airport standby

   a) A crew member is on airport standby from reporting at the normal report point until the end of the notified standby period.

   b) Airport standby will count in full for the purposes of cumulative duty hours.

   c) Where airport standby is immediately followed by a flight duty, the relationship between such airport standby and the assigned flight duty shall be defined by the Authority. In such a case, airport standby shall be added to the duty period referred to in OPS 1.1110 under points 1.1 and 1.2 for the purposes of calculating minimum rest.

   d) Where the airport standby does not lead to assignment on a flight duty, it shall be followed at least by a rest period as regulated by the Authority.

e) While on airport standby the operator will provide to the crew member a quiet and comfortable place not open to the public.

2. Other forms of standby (including standby at hotel)

   a) Subject to the provisions of Article 8, all other forms of standby shall be regulated by the Authority, taking into account the following:

      i. All activity shall be rostered and/or notified in advance.

      ii. The start and end time of the standby shall be defined and notified in advance.

      iii. The maximum length of any standby at a place other than a specified reporting point shall be determined.

      iv. Taking into account facilities available for the crew member to rest and other relevant factors, the relationship between the standby and any assigned flight duty resulting from the standby shall be defined.

      v. The counting of standby times for the purposes of cumulative duty hours shall be defined.

OPS 1.1130

**Nutrition**

A meal and drink opportunity must occur in order to avoid any detriment to a crew member's performance, especially when the FDP exceeds six hours.

OPS 1.1135

**Flight duty, duty and rest period records**

1. An operator shall ensure that crew member's records include:

   a) block times;

   b) start, duration and end of each duty or flight duty periods;

   c) rest periods and days free of all duties;

   and are maintained to ensure compliance with the requirements of this subpart; copies of these records will be made available to the crew member upon request.

2. If the records held by the operator under paragraph 1 do not cover all of his/her flight duty, duty and rest periods, the crew member concerned shall maintain an individual record of his/her
   a) block times;
   b) start, duration and end of each duty or flight duty periods; and
   c) rest periods and days free of all duties.
3. A crew member shall present his/her records on request to any operator who employs his/her services before he/she commences a flight duty period.
4. Records shall be preserved for at least 15 calendar months from the date of the last relevant entry or longer if required in accordance with national laws.
5. Additionally, operators shall separately retain all aircraft commander's discretion reports of extended flight duty periods, extended flight hours and reduced rest periods for at least six months after the event.

Source: European Parliament and the Council [2006]

# Appendix B.

# Standard IATA Delay Codes

Excerpt from IATA – Airport Handling Manual (Chapter AHM 011)

**Others**

| | |
|---|---|
| 00-05 | AIRLINE INTERNAL CODES |
| 06 (OA) | NO GATE/STAND AVAILABILITY DUE TO OWN AIRLINE ACTIVITY |
| 09 (SG) | SCHEDULED GROUND TIME LESS THAN DECLARED MINIMUM GROUND TIME |

**Passenger and Baggage**

| | |
|---|---|
| 11 (PD) | LATE CHECK-IN, acceptance after deadline |
| 12 (PL) | LATE CHECK-IN, congestions in check-in area |
| 13 (PE) | CHECK-IN ERROR, passenger and baggage |
| 14 (PO) | OVERSALES, booking errors |
| 15 (PH) | BOARDING, discrepancies and paging, missing checked-in passenger |
| 16 (PS) | COMMERCIAL PUBLICITY/PASSENGER CONVENIENCE, VIP, press, ground meals and missing personal items |
| 17 (PC) | CATERING ORDER, late or incorrect order given to supplier |
| 18 (PB) | BAGGAGE PROCESSING, sorting etc. |

**Cargo and Mail**

| | |
|---|---|
| 21 (CD) | DOCUMENTATION, errors etc. |
| 22 (CP) | LATE POSITIONING |
| 23 (CC) | LATE ACCEPTANCE |
| 24 (CI) | INADEQUATE PACKING |
| 25 (CO) | OVERSALES, booking errors |
| 26 (CU) | LATE PREPARATION IN WAREHOUSE |
| 27 (CE) | DOCUMENTATION, PACKING etc (Mail Only) |
| 28 (CL) | LATE POSITIONING (Mail Only) |
| 29 (CA) | LATE ACCEPTANCE (Mail Only) |

**Aircraft and Ramp Handling**

| | |
|---|---|
| 31 (GD) | AIRCRAFT DOCUMENTATION LATE/INACCURATE, weight and balance, general declaration, pax manifest, etc. |
| 32 (GL) | LOADING/UNLOADING, bulky, special load, cabin load, lack of loading staff |
| 33 (GE) | LOADING EQUIPMENT, lack of or breakdown, e.g. container pallet loader, lack of staff |
| 34 (GS) | SERVICING EQUIPMENT, lack of or breakdown, lack of staff, e.g. steps |
| 35 (GC) | AIRCRAFT CLEANING |
| 36 (GF) | FUELLING/DEFUELLING, fuel supplier |
| 37 (GB) | CATERING, late delivery or loading |
| 38 (GU) | ULD, lack of or serviceability |
| 39 (GT) | TECHNICAL EQUIPMENT, lack of or breakdown, lack of staff, e.g. pushback |

**Technical and Aircraft Equipment**

| | |
|---|---|
| 41 (TD) | AIRCRAFT DEFECTS. |
| 42 (TM) | SCHEDULED MAINTENANCE, late release. |
| 43 (TN) | NON-SCHEDULED MAINTENANCE, special checks and/or additional works beyond normal maintenance schedule. |
| 44 (TS) | SPARES AND MAINTENANCE EQUIPMENT, lack of or breakdown. |
| 45 (TA) | AOG SPARES, to be carried to another station. |
| 46 (TC) | AIRCRAFT CHANGE, for technical reasons. |
| 47 (TL) | STAND-BY AIRCRAFT, lack of planned stand-by aircraft for technical reasons. |
| 48 (TV) | SCHEDULED CABIN CONFIGURATION/VERSION ADJUSTMENTS. |

**Damage to Aircraft & EDP/Automated Equipment Failure**

| | |
|---|---|
| 51 (DF) | DAMAGE DURING FLIGHT OPERATIONS, bird or lightning strike, turbulence, heavy or overweight landing, collision during taxiing |
| 52 (DG) | DAMAGE DURING GROUND OPERATIONS, collisions (other than during taxiing), loading/off-loading damage, contamination, towing, extreme weather conditions |
| 55 (ED) | DEPARTURE CONTROL |
| 56 (EC) | CARGO PREPARATION/DOCUMENTATION |
| 57 (EF) | FLIGHT PLANS |

**Flight Operations and Crewing**

| | |
|---|---|
| 61 (FP) | FLIGHT PLAN, late completion or change of, flight documentation |
| 62 (FF) | OPERATIONAL REQUIREMENTS, fuel, load alteration |
| 63 (FT) | LATE CREW BOARDING OR DEPARTURE PROCEDURES, other than connection and standby (flight deck or entire crew) |
| 64 (FS) | FLIGHT DECK CREW SHORTAGE, sickness, awaiting standby, flight time limitations, crew meals, valid visa, health documents, etc. |
| 65 (FR) | FLIGHT DECK CREW SPECIAL REQUEST, not within operational requirements |
| 66 (FL) | LATE CABIN CREW BOARDING OR DEPARTURE PROCEDURES, other than connection and standby |
| 67 (FC) | CABIN CREW SHORTAGE, sickness, awaiting standby, flight time limitations, crew meals, valid visa, health documents, etc. |
| 68 (FA) | CABIN CREW ERROR OR SPECIAL REQUEST, not within operational requirements |
| 69 (FB) | CAPTAIN REQUEST FOR SECURITY CHECK, extraordinary |

**Weather**

| | |
|---|---|
| 71 (WO) | DEPARTURE STATION |
| 72 (WT) | DESTINATION STATION |
| 73 (WR) | EN ROUTE OR ALTERNATE |
| 75 (WI) | DE-ICING OF AIRCRAFT, removal of ice and/or snow, frost prevention excluding unserviceability of equipment |
| 76 (WS) | REMOVAL OF SNOW, ICE, WATER AND SAND FROM AIRPORT |
| 77 (WG) | GROUND HANDLING IMPAIRED BY ADVERSE WEATHER CONDITIONS |

**Air Traffic Flow Management Restrictions**

| | |
|---|---|
| 81 (AT) | ATFM due to ATC EN-ROUTE DEMAND/CAPACITY, standard demand/capacity problems |
| 82 (AX) | ATFM due to ATC STAFF/EQUIPMENT EN-ROUTE, reduced capacity caused by industrial action or staff shortage, equipment failure, military exercise or extraordinary demand due to capacity reduction in neighbouring area |
| 83 (AE) | ATFM due to RESTRICTION AT DESTINATION AIRPORT, airport and/or runway closed due to obstruction, industrial action, staff shortage, political unrest, noise abatement, night curfew, special flights |
| 84 (AW) | ATFM due to WEATHER AT DESTINATION |

**Airport and Governmental Authorities**

| | |
|---|---|
| 85 (AS) | MANDATORY SECURITY |
| 86 (AG) | IMMIGRATION, CUSTOMS, HEALTH |
| 87 (AF) | AIRPORT FACILITIES, parking stands, ramp congestion, lighting, buildings, gate limitations, etc. |
| 88 (AD) | RESTRICTIONS AT AIRPORT OF DESTINATION, airport and/or runway closed due to obstruction, industrial action, staff shortage, political unrest, noise abatement, night curfew, special flights |
| 89 (AM) | RESTRICTIONS AT AIRPORT OF DEPARTURE WITH OR WITHOUT ATFM RESTRICTIONS, including Air Traffic Services, start-up and push-back, airport and/or runway closed due to obstruction or weather, industrial action, staff shortage, political unrest, noise abatement, night curfew, special flights |

**Reactionary**

| | |
|---|---|
| 91 (RL) | LOAD CONNECTION, awaiting load from another flight |
| 92 (RT) | THROUGH CHECK-IN ERROR, passenger and baggage |
| 93 (RA) | AIRCRAFT ROTATION, late arrival of aircraft from another flight or previous sector |
| 94 (RS) | CABIN CREW ROTATION, awaiting cabin crew from another flight |
| 95 (RC) | CREW ROTATION, awaiting crew from another flight (flight deck or entire crew) |
| 96 (RO) | OPERATIONS CONTROL, re-routing, diversion, consolidation, aircraft change for reasons other than technical |

**Miscellaneous**

| | |
|---|---|
| 97 (MI) | INDUSTRIAL ACTION WITH OWN AIRLINE |
| 98 (MO) | INDUSTRIAL ACTION OUTSIDE OWN AIRLINE, excluding ATS |
| 99 (MX) | OTHER REASON, not matching any code above |

Source: IATA [2008], EUROCONTROL [2009]

# Appendix C.

# Distributions for Primary Delays

The generation of primary delays is based on historical data of a major European airline. All delays are classified according to the IATA delay codes. From these data, we derive theoretical distributions for duration and frequency of delays for use in our simulation model. We group the delay causes according to delay type as well as place and frequency of occurrence. Then, for each delay group the empirical data of delays are used to perform an automatic distribution fitting for duration and frequency of delays. In Section 7.1 we gave a brief overview of the analysis and results; here we present the detailed results. For details of the analysis method we refer to Spengler [2009].

**Airline internal** delay causes vary from airline to airline. These delay causes are mostly related to processes or equipment failure, which is specific to the airline, e.g. for identifying messages of automatic systems or problems with specific aircraft equipment. In the examined data these are the most frequent delays, but also have the shortest expected duration. The delays last on average 2 minutes and obviously follow a uniform distribution. Thus no distribution fitting and testing of goodness is performed in this case.

**IATA delay codes:** 0 – 5

**Occurrence probability:** $\pi = 12.6\%$

**Average duration:** 2 minutes

**Distribution of duration:** Uniform distribution between 1, 2 and 3 minutes.

**Aircraft, passenger and baggage handling** delay causes describe problems caused by individual passengers, airlines, suppliers as well as problems with the equipment needed for aircraft handling at the gate. Examples are late check-in of passengers, overbooking of the flight and delayed cleaning or catering by suppliers. These delay causes include most delay causes applying to the airport ground handling process described in Section 2.2.1 (pp. 16).

**IATA delay codes:** 11 – 18 and 31 – 39

**Occurrence probability:** $\pi = 5.7\%$

**Average duration:** 10 minutes

**Distribution of duration:** Log-logistic distribution with $\alpha = 2.82$, $\beta = 8.33$ and $\gamma = 0.50$; $D_{i\approx 5} = 0.06739$.

**Aircraft maintenance** delay causes describe the breakdown of the aircraft with consequential unscheduled aircraft maintenance as well as problems during the scheduled aircraft maintenance, e.g. breakdown of maintenance equipment or lack of planned stand-by aircraft for technical reasons. The delays in this group are rare but very serious, i.e. of long average duration. The two delay code ranges 41 – 48 and 51– 52 actually have different average durations, e.g. the average delay for codes 51 and 52 is 70 minutes. The common distribution, however, is extremely good, approximated by the Log-normal distribution presented.

**IATA delay codes:** 41 – 48, 51 and 52

**Occurrence probability:** $\pi = 2.7\%$

**Average duration:** 34 minutes

**Distribution of duration:** Log-normal distribution with $\sigma = 1.05$, $\mu = 2.95$ and $\gamma = 0.85$; $D_{i\approx 5} = 0.02708$.

**Flight operations and Crewing** delay causes describe operational problems, e.g. extraordinary security checks or late completion of flight plan, as well as late and missing crew for reasons other than reactionary, e.g. missing documents, sickness or disruption of individual flight time limitations.

**IATA delay codes:** 61 – 69

**Occurrence probability:** $\pi = 1.28\%$

**Average duration:** 13 minutes

**Distribution of duration:** Log-logistic distribution with $\alpha = 2.30$, $\beta = 8.94$ and $\gamma = 0.91$; $D_{i\approx 5} = 0.04765$.

**Automated equipment failure** delay causes describe problems with computer systems and automated equipment used during aircraft handling, e.g. departure control system, cargo preparation and documentation, baggage sorting and other.

**IATA delay codes:** 55 – 57

**Occurrence probability:** $\pi = 1.1\%$

**Average duration:** 13 minutes

**Distribution of duration:** Log-logistic distribution with $\alpha = 2.37$, $\beta = 8.89$ and $\gamma = 0.72$; $D_{i\approx 5} = 0.065098$.

**No gates and wrong scheduled ground time** are two, very rare delay causes. In the first case the gates are missing due to own airline activity. In the second case the scheduled ground time is less than the declared minimum.

**IATA delay codes:** 6 and 9

**Occurrence probability:** $\pi = 0.34\%$

**Average duration:** 11 minutes

**Distribution of duration:** Log-normal distribution with $\sigma = 0.75$, $\mu = 2.14$ and $\gamma = 0.32$; $D_{i\approx 5} = 0.03695$.

**Weather** delay causes describe the ATFM restrictions due to weather as well as restrictions at airports due to weather, e.g. removal of snow and ice from aircraft and airport. The average duration of such delays is long and the probability of occurrence differs according to season, less than 2% in summer and up to 12% in winter. We therefore propose three probability distributions for $\pi$: one for the period April – October, one for the period November – March and one for the whole year.

**IATA delay codes:** 71 – 79 and 84

**Occurrence probability:** $\pi$ is a random variable with the following seasonal distribution:

- Whole year: Log-normal distribution with $\sigma = 0.98$ and $\mu = -3.45$
- April – October: Log-logistic distribution with $\alpha = 2.217$ and $\beta = 0.02$
- November – March: Log-normal distribution with $\sigma = 0.97$ and $\mu = -2.99$

**Average duration:** 25 minutes

**Distribution of duration:** Log-logistic distribution with $\alpha = 1.99$, $\beta = 15.07$ and $\gamma = 0.92$; $D_{i\approx 5} = 0.02361$.

**Airspace congestion** delay causes describe the ATFM restrictions due to reduced capacity en-route. Reasons for reduced capacity include high demand by airlines, staff shortage at the control centers and industrial or military action. The analysis of the data shows a significantly higher probability of delay in the summer than winter months. This observation match the seasonal effects of total traffic and delays observed by EUROCONTROL, see Subsection 2.2.2 (pp. 18).

**IATA delay codes:** 81 and 82

**Occurrence probability:** $\pi$ is a random variable with the following seasonal distribution:

- Whole year: Log-normal distribution with $\sigma = 0.57$ and $\mu = -3.06$
- April – October: Log-normal distribution with $\sigma = 0.46$ and $\mu = -2.88$
- November – March: Log-normal distribution with $\sigma = 0.97$ and $\mu = -2.99$

**Average duration:** 13 minutes

**Distribution of duration:** Log-logistic distribution with $\alpha = 1.99$, $\beta = 15.07$ and $\gamma = 0.92$; $D_{i\approx 5} = 0.05258$.

**Airport congestion** delay causes describe general capacity reduction at the destination airport as well as the departure airport. Such capacity reductions affect all flights at those airports. Examples of causes of capacity decrease are increased demand, gate limitations, industrial actions and staff shortage at immigration or security stations. The probability of such delays varies greatly from airport to airport. This effect is demonstrated by more detailed analysis of the two hub airports in the network. Thus two possibilities of modelling the occurrence probability are presented next. The first distribution completely models the average occurrence probability for all airports in the network. Alternatively, the occurrence probability can be represented by three distributions. Two distributions model the occurrence probability of delays only at the first and second hub airport in the network respectively and another distribution models the average probability of delays for all except the two hub airports.

**IATA delay codes:** 83 and 85 – 89

**Occurrence probability:** $\pi$ is a random variable with following distribution dependent on the airport:

- All airports: Normal distribution with $\sigma = 0.03$ and $\mu = -0.13$
- Hub A: Log-normal distribution with $\sigma = 0.16$ and $\mu = -1.78$
- Hub B: Log-logistic distribution with $\alpha = 7.46$ and $\beta = 0.12$
- All airports except hubs A and B: Normal distribution with $\sigma = 0.15$ and $\mu = -0.04$

**Average duration:** 10 minutes

**Distribution of duration:** Log-normal distribution with $\sigma = 0.70$, $\mu = 2.02$ and $\gamma = 0.64$; $D_{i \approx 15} = 0.06265$

# Glossary

**Aircraft assignment problem** The aircraft assignment problem determines maintenance schedules for aircraft as well as individual aircraft routes which cover all flights. Depending on the focus, context and solution method this problem is also called maintenance routing, aircraft routing or tail assignment.

**Aircraft routing** see aircraft assignment problem.

**Airport turnaround** The process of unloading, preparing and loading the aircraft between two flights at an airport.

**Crew assignment problem** Crew rostering is the assignment of anonymous crew crew pairings to individual crew members, satisfying individual work-time regulations.

**Crew base** The location nominated by the airline to the crew member from where the crew member normally starts and ends a duty period or a series of duty periods and where, under normal conditions, the airline is not responsible for the accommodation of the crew member concerned.

**Crew pairing** A sequence of duty and rest periods of a crew, starting and ending at the same crew base.

**Crew pairing problem** The crew pairing problem determines cost minimal itineraries for crews, crew pairings, covering all flights and satisfying general work-time regulations.

**Crew rostering** see crew assignment problem.

**De-icing** is the process of removing frozen contaminant, snow, ice, from an aircraft.

**Deadheading** describes the transfer of crew members as passengers on regular flights. This can be necessary to return the crew to the crew base or to transfer the crew to the next flight at another airport. Deadheading incurs costs because the crew members need seats on the aircraft, which cannot be offered to paying customers.

**Disruption** is a conflict of invalid airline schedules, e.g. missing aircraft, missing crew or insufficient time for crew rest. Disruptions are consequences of primary or reactionary delays or of any other unforeseen events, e.g. damage to aircraft, capacity decrease at airports or missing crews.

**Duty** A daily working shift of a crew which consists of a sequence of flights. The maximal allowed duration of a duty is 10 hours, but can be extended up to 14 hours depending on the duration of the subsequent rest period.

**Fleet assignment problem** The fleet assignment problem determines for each flight an aircraft type considering the demand forecast as well as the availability of aircraft.

**Flexibility** Flexible schedules can be efficiently adapted to changing operating environment.

**Maintenance base** The location nominated by the airline to perform long-term maintenance checks at aircraft. A maintenance base needs certain technical equipment and staff to perform checks. The capacity of the maintenance facilities is restricted and therefore needs to be scheduled.

**Maintenance routing** see aircraft assignment problem.

**Minimal sit time** is the minimal time needed for a crew between two flights. It depends on the minimal turn time. In the case the crew stays on the aircraft both times are equal. When the crew changes aircraft normally the minimal sit time is longer, due to additional time needed by the crew to move at the airport and to perform additional checks on the new aircraft.

**Minimal turn time** is the minimal time needed to complete the airport turnaround process for an aircraft. It depends on the aircraft type as well as airport facilities, i.e. the size of the aircraft, the number of entrance points and the number of available boarding bridges.

**On-time performance** is the percentage of fights arriving or departing at the gate on time. An arrival or departure is on time if it is within $x$ minutes of the originally scheduled time. $x$ is usually 0, 5, 15 and 60 minutes.

**Primary delay** A primary delay is caused by any initial event. Unlike reactionary delay it is not caused by any earlier delay.

**Reactionary delay** A reactionary delay is caused by an earlier delay, a consequence of the unavailability of aircraft, crew or load due to disruptions earlier in the day. This earlier disruption can itself be a consequence of either a primary or a reactionary delay.

**Recovery** refers to the overall process of adapting the schedules to the new situation as recovery of schedules. The recovery process includes using mathematical methods such as optimization and simulation, as well as manual adjustments to the schedules especially in combination with communication between staff.

**Rescheduling** is the creation of a new schedule for a resource on the day of operation with the aim of adapting the schedule to a new situation by performing as few changes to the schedule as possible.

**Rest** A night between two duties of a crew not in the crew base, where the crew can have a rest. The required time is between 10 and 14 hours, depending on the duration of the preceding duty period.

**Schedule design problem** The schedule design problem determines which connections between airports at which time are offered considering a forecast on the demand of customers.

**Secondary delay** see reactionary delay.

**Stability** Stable airline schedules are likely to remain feasible and near-cost optimal in a changing operating environment.

**Tail assignment** see aircraft assignment problem.

**Taxi in** Movement of the aircraft from runway to the gate or parking position after landing.

**Taxi out** Movement of the aircraft from the gate or parking position to the runway before take-off.

# Acronyms

**ATC** air traffic control.

**ATFM** air traffic flow management.

**CFMU** Central Flow Management Unit.

**EUROCONTROL** European Organisation for the Safety of Air Navigation.

**IATA** International Air Transport Association.

**IPM** interior point method.

**MIP** mixed integer programming.

**OCC** operation control center.

**RCSP** resource-constrained shortest path problem.

**RMP** restricted master problem.

**SCP** set covering problem.

**SPP** set partitioning problem.

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# Bibliography


Abdelghany, A., G. Ekollu, R. Narasimhan, and K. Abdelghany (2004). A proactive crew recovery decision support tool for commercial airlines during irregular operations. *Annals of Operations Research 127*(1), 309–331.

Abdelghany, K. F., A. F. Abdelghany, and G. Ekollu (2008, March). An integrated decision support tool for airlines schedule recovery during irregular operations. *European Journal of Operational Research 185*(2), 825–848.

Ageeva, Y. (2000, August). Approaches to incorporating robustness into airline scheduling. Master's thesis, Massachusetts Institute of Technology.

AhmedBeygi, S., A. Cohn, and M. Lapp (2008). Decreasing airline delay propagation by re-allocating schedule slack. Technical report, AGIFORS. http://www.agifors.org/award/submissions2008/Ahmadbeygi_paper.pdf.

Ahuja, R., T. Magnanti, and J. Orlin (1993). *Network Flows: Theory, Algorithms, and Applications.* Englewood Cliffs, NJ, USA: Prentice Hall.

Anbil, R., J. J. Forrest, and W. R. Pulleyblank (1998). Column generation and the airline crew pairing problem. *Documenta Mathematica, Journal der Deutschen Mathematiker Vereinigung Extra Volume ICM*(III), 677–686.

Andersson, T. and P. Värbrand (2004, April). The flight perturbation problem. *Transportation Planning and Technology 27*(2), 91–117.

Barnhart, C., N. L. Boland, L. W. Clarke, E. L. Johnson, G. L. Nemhauser, and R. G. Shenoi (1998). Flight string models for aircraft fleeting and routing. *Transportation Science 32*, 208–220.

Barnhart, C., A. M. Cohn, E. L. Johnson, D. Klabjan, G. L. Nemhauser, and P. H. Vance (2003). *Handbook of Transportation Science*, Chapter Airline Crew Scheduling, pp. 517–560. Springer New York.

Barnhart, C., E. L. Johnson, G. L. Nemhauser, M. W. Savelsbergh, and P. H. Vance (1998). Branch-and-price: Column generation for solving huge integer programs. *Operations Research 46*, 316–329.

Bixby, R., J. Gregory, I. Lustig, R. Marsten, and D. Shanno (1992). Very large-scale linear programming: A case study in combining interior point and simplex methods. *Operations Research 40*(5), 885–897.

Boeing (2005). 737 - airplane characteristics for airport planning. http://www.boeing.com/commercial/airports/plan_manuals.html, last access on 19th October 2009.

Burke, E., P. De Causmaecker, G. De Maere, J. Mulder, M. Paelinck, and G. Vanden Berghe (2010). A multi-objective approach for robust airline scheduling. *Computers & Operations Research 37*(5), –.

Cao, J.-M. and A. Kanafani (1997a, March). Real-time decision support for integration of airline flight cancellations and delays part i: mathematical formulation. *Transportation Planning and Technology 20*(3), 183–199.

Cao, J.-M. and A. Kanafani (1997b, March). Real-time decision support for integration of airline flight cancellations and delays part ii: algorithm and computational experiments. *Transportation Planning and Technology 20*(3), 201–217.

Clarke, L., E. Johnson, G. Nemhauser, and Z. Zhu (2003). The aircraft rotation problem. *Anals of Operations Research 51*, 387–296.

Clausen, J., A. Larsen, J. Larsen, and N. J. Rezanova (2009, May). Disruption management in the airline industry – concepts, models and methods. *Computers & Operations Research 37*(5), 809–821.

Cohn, A. and C. Barnhart (2003). Improving crew scheduling by incorporating key maintencance routing decisions. *Operations Research 51*(3), 387–396.

Cordeau, J.-F., G. Stojkovic, F. Soumis, and J. Desrosiers (2001). Benders decomposition for simultaneous aircraft routing and crew scheduling. *Transportation Science 35*(4), 375–388.

Desaulniers, G., J. Desrosiers, Y. Dumas, S. Marc, B. Rioux, M. M. Solomon, and F. Soumis (1997). Crew pairing at Air France. *European Journal of Operational Research 97*(2), 245 – 259.

Desaulniers, G., J. Desrosiers, Y. Dumas, M. Solomon, and F. Soumis (1997). Daily aircraft routing and scheduling. *Management Science 43*(6), 841–855.

Desaulniers, G., J. Desrosiers, A. Lasry, and M. M. Solomon (1999). Crew pairing for a regional carrier. *Lecture notes in economics and mathematical systems 471*, 19–41.

Desaulniers, G., J. Desrosiers, and M. Solomon (2001). *Essays and Surveys in Metaheuristics*, Chapter Accelerating Strategies in Column Generation Methods for Vehicle Routing and Crew Scheduling Problems, pp. 309–324. Boston: Kluwer.

Desrosiers, J. and M. Lübbecke (2005). *Column Generation*, Chapter A Primer in Column Generation, pp. 1–32. New York: Springer.

Ehrgott, M. and D. M. Ryan (2002). Constructing robust crew schedules with bicriteria optimization. *Journal of Multi-Criteria Decision Analysis 11*(3), 139–150.

Etschmaier, M. and D. Mathaisel (1985). Airline scheduling: An overview. *Transportation Science 19*(2), 127–138.

EUROCONTROL (2007). Trends in air traffic volume 2 - a matter of time: Air traffic delay in europe. http://www.eurocontrol.int/statfor/, last access on 19th October 2009.

EUROCONTROL (2008). Annual report 2008. http://www.eurocontrol.int, last access on 19th October 2009.

EUROCONTROL (2009). Digest – annual 2008: Delays to air transport in europe. http://www.eurocontrol.int/coda/, last access on 19th October 2009.

European Parliament and the Council (2006). Regulation (ec) no 1899/2006.

Fahle, T., U. Junker, S. E. Karisch, N. Kohl, M. Sellmann, and B. Vaaben (2002). Constraint programming based column generation for crew assignment. *Journal of Heuristics 8*(1), 59–81.

Fuhr, B. (2007, April). Robust flight scheduling - an analytic approach to performance evaluation and optimization. Technical report, Clausthal University of Technology, Lufthansa Airlines.

Galia, R. and C. Hjorring (2004). Modelling of complex costs and rules in a crew pairing column generator. In D. Ahr, R. Fahrion, M. Oswald, and G. Reinelt (Eds.), *Operations Research Proceedings 2003 – Selected Papers of the International Conference on Operations Research (OR 2003)*, Berlin, pp. 133–140. Springer.

Gopalakrishnan, B. and E. L. Johnson (2005). Airline crew scheduling: State-of-the-art. *Annals of Operations Research 140*, 305–337.

Gopalan, R. and K. Talluri (1998a). The aircraft maintenance routing problem. *Operation Research 46*, 260–271.

Gopalan, R. and K. Talluri (1998b). Mathematical models in airline schedule planning: A survey. *Annals of Operations Research 76*, 155–185.

Grönkvist, M. (2005). *The Tail Assignment Problem.* Ph. D. thesis, Chalmers University of Technology and Göteborg University.

Guo, Y. (2005, January). *Decision support systems for airline crew recovery.* Ph. D. thesis, University of Paderborn.

Hassin, R. (1992). Approximation schemes for the restricted shortest path problem. *Mathematics of Operations Research 17*(1), 36–42.

IATA (2008). *Airport Handling Manual.* IATA.

IBM ILOG (2009). http://www.ilog.com/. last access on 19th October 2009.

Ioachim, I., J. Desrosiers, F. Soumis, and N. Bélanger (1999). Fleet assignment and routing with schedule synchronization constraints. *European Journal of Operational Research 119*(1), 75–90.

Irnich, S. and G. Desaulniers (2005). *Column Generation*, Chapter Shortest Path Problems with Resource Constraints, pp. 33–65. Springer.

Jacobs, P. H. M., A. Verbraeck, and J. B. P. Mulder (2005). Flight scheduling at klm. In *WSC '05: Proceedings of the 37th conference on Winter simulation*, pp. 299–306. Winter Simulation Conference.

Jans, R. and Z. Degraeve (2004). An industrial extension of the discrete lot-sizing and scheduling problem. *IIE Transactions 36*(1), 47–58.

Jarrah, A. I. Z., G. Yu, N. Krishnamurthy, and A. Rakshit (1993). A Decision Support Framework for Airline Flight Cancellations and Delays. *Transportation Science 27*(3), 266–280.

Jepsen, M., B. Petersen, S. Spoorendonk, and D. Pisinger (2008). Subset-row inequalities applied to the vehicle-routing problem with time windows. *Operations Research 56*(2), 497–511.

Johnson, V., L. Lettovsky, G. Nemhauser, R. Pandit, and S. Querido (1994). Final report to northwest airlines on the crew recovery problem. Technical report, The Logistic Institute, Georgia Institute of Technology, Atlanta, USA.

Klabjan, D. (2005). *Column Generation*, Chapter Large-Scale Models in the Airline Industry, pp. 163—196. Springer.

Klabjan, D., E. L. Johnson, G. L. Nemhauser, E. Gelman, and S. Ramaswamy (2001a). Airline crew scheduling with regularity. *Transportation Science 35*(4), 359–374.

Klabjan, D., E. L. Johnson, G. L. Nemhauser, E. Gelman, and S. Ramaswamy (2001b). Solving large airline crew scheduling problems: Random pairing generation and strong branching. *Computational Optimization and Applications 20*, 73–91.

Klabjan, D., E. L. Johnson, G. L. Nemhauser, E. Gelman, and S. Ramaswamy (2002). Airline crew scheduling with time windows and plane-count constraints. *Transportation Science 36*(3), 337–448.

Kohl, N. and S. Karisch (2004). Airline crew rostering: Problem types, modeling and optimization. *Annals of Operations Research 127*, 223–257.

Lan, S., J.-P. Clarke, and C. Barnhart (2006, February). Planning for robust airline operations: Optimizing aircraft routings and flight departure times to minimize passenger disruptions. *Transportation Science 40*(1), 15–28.

Leal de Matos, P. and R. Ormerod (2000). The application of operational research to European air traffic flow management – understanding the context. *European Journal of Operational Research 123*(1), 125–144.

Lettovsky, L. (1997). *Airline operations recovery: an optimization approach.* Ph. D. thesis, Georgia Institute of Technology, Atlanta, USA.

Lettovský, L., E. L. Johnson, and G. L. Nemhauser (2000). Airline crew recovery. *Transportation Science 34*(4), 337–348.

Lübbecke, M. E. and J. Desrosiers (2005). Selected topics in column generation. *Operations Research 53*(6), 1007–1023.

MathWave Technologies (2009). http://www.mathwave.com. last access on 19th October 2009.

Medard, C. P. and N. Sawhney (2007, December). Airline crew scheduling from planning to operations. *European Journal of Operational Research 183*(3), 1013–1027.

Mercier, A., J.-F. Cordeau, and F. Soumis (2005). A computational study of benders decomposition for the integrated aircraft routing and crew scheduling problem. *Computers & Operations Research 32*(6), 1451–1476.

Mercier, A. and F. Soumis (2007). An integrated aircraft routing, crew scheduling and flight retiming model. *Computers & Operations Research 34*(8), 2251–2265.

MOPS Optimierungssysteme GmbH & Co. KG (2009). http://www.mops-optimizer.com. last access on 19th October 2009.

Nissen, R. and K. Haase (2006). Duty-period-based network model for crew rescheduling in european airlines. *Journal of Scheduling 9*, 255–278.

Rakshit, A., N. Krishnamurthy, and G. Yu (1996, Mar.–Apr.). System operations advisor: A real-time decision support system for managing airline operations at united airlines. *Interfaces 26*(2), 50–58.

Rodgers, J. L. and W. A. Nicewander (1988). Thirteen ways to look at the correlation coefficient. *The American Statistician 42*(1), 59–66.

Rosenberger, J. M., E. L. Johnson, and G. L. Nemhauser (2003, November). Rerouting aircraft for airline recovery. *Transportation Science 37*(4), 408–421.

Rosenberger, J. M., A. J. Schaefer, D. Goldsman, E. L. Johnson, A. J. Kleywegt, and G. L. Nemhauser (2002, November). A stochastic model of airline operations. *Transportation Science 36*(4), 357–377.

Ryan, D. M. and B. A. Foster (1981). An integer programming approach to scheduling. *Computer Scheduling of Public Transport 1*, 269–280.

Sarac, A., R. Batta, and C. Rump (2006). A branch-and-price approach for operational aircraft maintenance routing. *European Journal of Operational Research 175*, 1850–1869.

Schaefer, A. J., E. L. Johnson, A. J. Kleywegt, and G. L. Nemhauser (2005). Airline crew scheduling under uncertainty. *Transportation Science 39*(3), 340–348.

Scholl, A. (2001). *Robuste Planung und Optimierung: Grundlagen - Konzepte und Methoden - Experimentelle Untersuchungen*, Chapter Robuste Planung, pp. 89 –172. Physica-Verlag Heidelberg.

Shebalov, S. and D. Klabjan (2006, August). Robust airline crew pairing: Move-up crews. *Transportation Science 40*(3), 300–312.

Spengler, F. (2009). *Statistische Abbildung von Verspätungen im Luftverkehr.* Decision Support & Operations Research Lab, Universität Paderborn.

Sriram, C. and A. Haghani (2003). An optimization model for aircraft maintenance scheduling and re-assignment. *Transportation Research Part A: Policy and Practice 37*(1), 29–48.

Steinzen, I. (2007). *Topics in Integrated Vehicle and Crew Scheduling in Public Transport.* Ph. D. thesis, Universität Paderborn, Paderborn.

Stojković, M. and F. Soumis (2001). An optimization model for the simultaneous operational flight and pilot scheduling problem. *Management Science 47*(9), 1290–1305.

Stojković, M. and F. Soumis (2005). The operational flight and multi-crew scheduling problem. *Yugoslav Journal of Operations Research 15*, 25–48.

Stojković, M., F. Soumis, and J. Desrosiers (1998). The operational airline crew scheduling problem. *Transportation Science 32*(3), 232–245.

Teodorovic, D. and E. Krcmar-Nozic (1989). Multicriteria model to determine flight frequencies on an airline network under competitive conditions. *Transportation Science 23*(1), 14–25.

Toutenberg, H. (2005). *Induktive Statistik* (3 ed.). Springer Berlin Heidelberg.

Vance, P. H., A. Atamtürk, C. Barnhart, E. Gelman, E. L. Johnson, A. Krishna, D. Mahidhara, G. L. Nemhauser, and R. Rebello (1997). A heuristic branch-and-price approach for the airline crew pairing problem. Technical Report LEC-97-06, Georgia Institute of Technology.

Vance, P. H., C. Barnhart, E. L. Johnson, and G. L. Nemhauser (1997, March–April). Airline crew scheduling: A new formulation and decomoposition algorithm. *Operation Research 45*(2), 188–200.

Weide, O. (2009). *Robust and Integrated Airline Scheduling.* Ph. D. thesis, The University of Auckland, Auckland.

Weide, O., D. Ryan, and M. Ehrgott (2009, May). An iterative approach to robust and integrated aircraft routing and crew scheduling. *Computers & Operations Research 37*(5), 833–844.

Wolsey, L. A. (1998). *Integer Programming*, Chapter Lagrangian Duality, pp. 167 –181. John Wiley & Sons, Inc.

Wu, C.-L. (2006, September). Improving airline network robustness and operational reliability by sequential optimisation algorithms. *Networks and Spatial Economics 6*(3), 235–251.

Yan, S. and D.-H. Yang (1996, December). A decision support framework for handling schedule perturbation. *Transportation Research Part B: Methodological 30*(6), 405–419.

Yen, J. W. and J. R. Birge (2006). A stochastic programming approach to the airline crew scheduling problem. *Transportation Science 40*(1), 3–14.

Yu, G. and X. Qi (2004). *Disruption management: framework, models and applications.* World Scientific Publishing Company.

Zhao, X., J. Zhu, and M. Guo (2007, Dec.). Application of grey programming in irregular flights scheduling. In *Industrial Engineering and Engineering Management, 2007 IEEE International Conference on*, pp. 164–168.