

Abstract

There is a curious relation between two kinds of phase space distributions associated to Laplace-eigenfunctions φ_{λ_k} on a compact hyperbolic manifold Y .

Given a pseudodifferential operator quantization $Op : C^\infty(S^*Y) \rightarrow B(L^2(Y))$, that is an assignment of bounded operators to smooth zero order symbols a on the unit (co-)tangent bundle S^*Y , the functionals $\rho_{\lambda_j, \lambda_k}(A) = \langle A\varphi_{\lambda_j}, \varphi_{\lambda_k} \rangle_{L^2(Y)}$ on the space of zero-order pseudodifferential operators give rise to Wigner distributions $W_{\lambda_j, \lambda_k}(a) = \rho_{\lambda_j, \lambda_k}(Op(a))$ on S^*Y , which are the key objects in quantum ergodicity. One studies the oscillation and concentration properties of the eigenfunctions through the so-called large energy limits of the distributions W_{λ_j, λ_k} , that is one investigates their behaviour when the eigenvalues tend to infinity.

If Y is a symmetric space of the noncompact type, the Laplace operator is replaced by the corresponding algebra of translation invariant differential operators. Given moderate eigenfunctions φ and ψ , their distributional boundary values in the sense of Helgason give rise to the Patterson-Sullivan distribution $PS_{\varphi, \psi}$ on S^*Y .

In the case of compact hyperbolic surfaces $Y = \Gamma \backslash \mathbb{H}$ it was observed by N. Anantharaman and S. Zelditch that there is an exact and an asymptotic relation between these phase space distributions.

We generalize parts of a special non-Euclidean calculus of pseudodifferential operators, which was invented by S. Zelditch for hyperbolic surfaces, to symmetric spaces $X = G/K$ of the noncompact type and their compact quotients $Y = \Gamma \backslash G/K$. We sometimes restrict our results to the case of rank one symmetric spaces. The non-Euclidean setting extends the definition of Patterson-Sullivan distributions in a natural way to arbitrary symmetric spaces of the noncompact type. Generalizing the exact formula given by Zelditch and Anantharaman, we find an explicit intertwining operator mapping Patterson-Sullivan distributions into Wigner distributions. We study the important invariance and equivariance properties of these distributions. Finally, we describe asymptotic properties of these distributions.