

# Abstract

There is a curious relation between two kinds of phase space distributions associated to Laplace-eigenfunctions  $\varphi_{\lambda_k}$  on a compact hyperbolic manifold  $Y$ .

Given a pseudodifferential operator quantization  $Op : C^\infty(S^*Y) \rightarrow B(L^2(Y))$ , that is an assignment of bounded operators to smooth zero order symbols  $a$  on the unit (co-)tangent bundle  $S^*Y$ , the functionals  $\rho_{\lambda_j, \lambda_k}(A) = \langle A\varphi_{\lambda_j}, \varphi_{\lambda_k} \rangle_{L^2(Y)}$  on the space of zero-order pseudodifferential operators give rise to Wigner distributions  $W_{\lambda_j, \lambda_k}(a) = \rho_{\lambda_j, \lambda_k}(Op(a))$  on  $S^*Y$ , which are the key objects in quantum ergodicity. One studies the oscillation and concentration properties of the eigenfunctions through the so-called large energy limits of the distributions  $W_{\lambda_j, \lambda_k}$ , that is one investigates their behaviour when the eigenvalues tend to infinity.

If  $Y$  is a symmetric space of the noncompact type, the Laplace operator is replaced by the corresponding algebra of translation invariant differential operators. Given moderate eigenfunctions  $\varphi$  and  $\psi$ , their distributional boundary values in the sense of Helgason give rise to the Patterson-Sullivan distribution  $PS_{\varphi, \psi}$  on  $S^*Y$ .

In the case of compact hyperbolic surfaces  $Y = \Gamma \backslash \mathbb{H}$  it was observed by N. Anantharaman and S. Zelditch that there is an exact and an asymptotic relation between these phase space distributions.

We generalize parts of a special non-Euclidean calculus of pseudodifferential operators, which was invented by S. Zelditch for hyperbolic surfaces, to symmetric spaces  $X = G/K$  of the noncompact type and their compact quotients  $Y = \Gamma \backslash G/K$ . We sometimes restrict our results to the case of rank one symmetric spaces. The non-Euclidean setting extends the definition of Patterson-Sullivan distributions in a natural way to arbitrary symmetric spaces of the noncompact type. Generalizing the exact formula given by Zelditch and Anantharaman, we find an explicit intertwining operator mapping Patterson-Sullivan distributions into Wigner distributions. We study the important invariance and equivariance properties of these distributions. Finally, we describe asymptotic properties of these distributions.