Abstract

In this Ph.D thesis, we study weighted spaces of holomorphic functions on the upper halfplane $\mathbf{G}$, for two kinds of weights which we call type(I) and type(II) weights. We focus on the following Banach spaces:

$\mathbf{H}_v(\mathbf{G}) := \{f \mid f : \Omega \longrightarrow \mathbb{C}$ is holomorphic and $\|f\|_v < \infty\}$ and

$\mathbf{H}_{v_0}(\mathbf{G}) := \{f \in \mathbf{H}_v(\mathbf{G}) : fv$ vanishes at infinity$\}$. Here $\|f\|_v := \sup_{\omega \in \mathbf{G}} \mid f(\omega) \mid v(\omega)$.

To obtain results about these Banach spaces we apply the Moebius transform $\alpha : \mathbb{D} \longrightarrow \mathbf{G}$ defined by $\alpha(z) = \frac{1+z}{1-z} i$ ($\mathbb{D}$ is the unit disc).

If v is of type(II) then the weight $v \circ \alpha$ is equivalent to a radial weight on $\mathbb{D}$. This enables us to transfer wellknown results about isomorphic classification of weighted spaces of holomorphic functions on unit disc $\mathbb{D}$ to $\mathbf{H}_v(\mathbf{G})$ or $\mathbf{H}_{v_0}(\mathbf{G})$. Therefore we present a complete isomorphic classification for $\mathbf{H}_v(\mathbf{G})$ and $\mathbf{H}_{v_0}(\mathbf{G})$. Unfortunately, for type(I) weights, we cannot use the same method, because in this case $\lim_{z \to 1}(v \circ \alpha)(z)$ does not exist and $v \circ \alpha$ is not equivalent to a radial weight on $\mathbb{D}$. Therefore for type(I) weights, we restrict ourselves to the following subspaces of $\mathbf{H}_v(\mathbf{G})$ and $\mathbf{H}_{v_0}(\mathbf{G})$:

$U^{\beta}_{\pm}$, $U^{\beta}_{\pm,\ 0}$, $\mathbf{H}^{2\pi}_v(\mathbf{G})$ and $\mathbf{H}^{2\pi}_{v_0}(\mathbf{G})$ and we obtain the isomorphic classifications of these spaces.

Finally, we study continuity of differentiation, composition and multiplication operators not only between weighted spaces of holomorphic functions, but also between weighted spaces of holomorphic 2π-periodic functions. We obtain sufficient (and sometimes necessary) conditions for continuity of these operators when our weights are of type(I) or type(II).