Abstract

We give a unified construction of the minimal representation of a finite cover G of the conformal group of a (non necessarily euclidean) Jordan algebra V. This representation is realized on the L^2-space of the minimal orbit $\mathcal{O}$ of the structure group L of V. We construct its corresponding $(\mathfrak{g},\mathfrak{k})$-module and show that it can be integrated to a unitary irreducible representation of G on $L^2(\mathcal{O})$.

In particular, we obtain a unified approach to the two most prominent minimal representations, namely the Segal–Shale–Weil representation of the metaplectic group $\mathrm{Mp}(n,\mathbb{R})$ and the minimal representation of $\mathrm{O}(p+1,q+1)$ which was recently studied by T. Kobayashi, G. Mano and B. Ørsted.

In the second part we investigate special functions which give rise to $\mathfrak{k}$-finite vectors in the representation. Various properties of these special functions such as differential equations, recurrence relations and integral formulas connect to the representation theory involved.

Finally, we define the conformal inversion operator $\mathcal{F}_{\mathcal{O}}$ by the action of the longest Weyl group element. $\mathcal{F}_{\mathcal{O}}$ is a unitary operator on $L^2(\mathcal{O})$ of order 2. We show that the action of $\mathcal{F}_{\mathcal{O}}$ on radial functions is given by a special case of Meijer's G-transform.