# Abstract

The emphasis of this work is put on the analysis of embeddings of hypercubes and grid-like networks in $d$-dimensional grids. Therefore, we distinguish between the examination of injective embeddings which can be proven to be optimal and the investigation of the corresponding $k$-partitioning problem. We have put particular focus on the development and exemplified application of lower bound methods for the corresponding cost-measures. These methods are based on solutions of specific extremal-set-problems hosted in the area of discrete mathematics. Moreover, these methods build the basement for almost every lower bound proof given within this work. Furthermore, in this thesis we describe results for various theoretical questions concerning the investigation of embeddings into grid networks. All the results were published before as contribution on a proceeding of an international conference or on a specific journal.

In chapter 1 we give a general introduction into the field of research considered in this work and we describe some applications that motivate our efforts.

Chapter 2 consists of a list of definitions and mathematical fundamentals needed in order to comprehend this work. The definitions include a formal description of the graph-classes and the mathematical well-defined embedding problems considered here.

In chapter 3 we describe in detail the lower bound methods developed by us for the cost-measures dilation and congestion of an embedding. Moreover, we discuss several lower bound methods for the minimal cutsize of a $k$-partitioning of a graph.

Within chapter 4 we concentrate on embeddings of binary hypercubes into higher-dimensional grids. We present an exact solution for the corresponding bijective embedding problem in such a way as to minimize the congestion. Furthermore, we solve the same problem for the cost-measure wirelength on the assumption that the side-lengths of the grid are all equal. Thus, we answer two research questions which were open for a long time. Moreover, in this chapter we also present on the assumption that the side-lengths of the grid are all equal, an asymptotically optimal solution for the bijective embedding problem in such a way as to minimize the dilation. We close this chapter with a consideration of some special questions concerning injective embeddings of hypercubes into two-dimensional grids and simulating uniaxial hypercube algorithms on a grid based network. In particular on the first mentioned topic we present lower and upper bounds which improve former published results. The material presented within this chapter build the

basement for the publications in the proceeding of the conference *Mathematical Foundations of Computer Science '98* and in the journal *Discrete Mathematics, Volume 213*.

Chapter 5 is dedicated to the problem of embedding a two-dimensional grid into another two-dimensional grid with a lower aspect-ratio. We determine lower and upper bounds for the minimal congestion of a corresponding embedding which differs by at most one. Moreover, we prove that a former published upper bound method is optimal with respect to the cost-measure dilation. This proof is done by a determination of a lower bound for the corresponding problem which match the upper bound mentioned above. Parts of the material of this chapter are published in the proceeding of the international *Workshop on Graph-Theoretic Concepts in Computer Science '98* and in the journal *Discrete Applied Mathematics, Volume 108*.

Within chapter 6 we compare the possibilities of a ring and a linear-array network for the simulation of an arbitrary tree network. We show that the linear-array is just as good as the ring with respect to the cost-measures cutwidth and wirelength. An analogous result for the bandwidth can be found in the literature. Our result can be regarded as an extension of these works and is published in the journal *Discrete Applied Mathematics, Volume 87*.

In chapter 7 we determine lower and upper bounds for the minimal cutsize of a balanced $k$-partitioning of Hamming-graphs. By applying our results on the $k$-partitioning of hypercubes, we improve a former published asymptotic for the corresponding minimal cutsize. Since the partitioning problem can be observed as a relaxation of the so called *many-to-one* embedding problem, the obtained results extend the considerations of chapter 4. We have published the results of this chapter in the proceeding of the international *Conference on Computing and Combinatorics '99* and in the journal *Discrete Applied Mathematics, Volume 95*.

Within the final chapter 8 we give at first a summary of the main results of this thesis and then we briefly describe the embedding libraries which we have implemented for the parallel runtime system PARIX (**PAR**allel extensions to Un**IX**). The works which resulted in these libraries are published in the proceedings of the conference *EURO-PAR Parallel Processing '95*, of the *Workshop on Parallel Programming and Computation '95* and of the international *Conference and Exhibition on High-Performance Computing and Networking '95*.