Thesis-Summary

Spectral Methods for Efficient Load Balancing Strategies

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The problem of balancing dynamically generated work load among the processors of a parallel machine occurs in a wide range of applications. In the past, there have been several loadbalancing methods developed and anayzed. Two subclasses of such methods are k-partitioning algorithms and diffusion schemes. In this thesis, we computed new spectral lower bounds on the k-section width of graphs and designed improved diffusion based loadbalancing algorithms.

The classical spectral lower bound on the k-section width of a graph G with n vertices has the form: $\nabla \geq \sum_{i=1}^{k} \lambda_i \cdot \frac{n}{2k}$, where $\lambda_1, \ldots, \lambda_k$ represent the k smallest eigenvalues of the Laplacian of G. We show that, depending on the topological properties of the graph, we can improve the above mentioned lower bound by specifying two constants δ and β such that the new bound has the form $\nabla \geq \delta\Big(\sum_{i=1}^{k} \lambda_i\Big)^{\beta} n(1 - o(1))$, where β is in the range $[\frac{1}{2}, 1]$. Additionally, we present several graphs for which the new bounds are tight up to a constant factor. We also obtain improved bounds on the k-section width of cartesian products of graphs by using techniques from discrete mathematics and combinatorics.

In the area of loadbalancing algorithms, we consider the so-called ADI-scheme on cartesian products of graphs and show that by combining this method with the known diffusion schemes, we obtain better convergence than the usual diffusion algorithms for these type of graphs.

Next, we use compute optimal edge-weights for a graph G, in order to maximize the condition number of the corresponding Laplacian among all Laplacians having the same communication structure. We consider edge-transitive graphs and show that for these graphs the condition number is maximized if all edges have the same weight. We also consider Cayley graphs and prove that edges generated by the same generator must be of equal weight in order to maximize the condition number. Additionally, we compute optimal values for the edge-weights of Cube-Connected-Cycles, maximizing the condition number of the corresponding Laplacian.

We conclude by considering the load balancing problem in heterogeneous processor networks. We show that known diffusion schemes can be generalized to heterogeneous processor systems and prove that these generalized load balancing algorithms compute a minimal flow w.r.t. the l_2-norm. Furthermore, we derive spectral bounds for the edge expansion of node-weighted graphs.