Property Testing and Geometry

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Abstract

Property Testing is a relaxation of a standard decision problem: Given a distance measure for a class of objects, one has to decide whether a given object O has a predetermined property Π or a distance of more than ϵ from any object having property Π. In the latter case we say O is ϵ-far from Π. If O neither has property Π nor is ϵ-far from Π a property testing algorithm may answer arbitrarily. Given the ability to perform queries about the local structure of the input object, this relaxed decision problem can often be solved without looking at the whole object.

In this thesis we apply the concept of Property Testing to *geometric* problems and we show that concepts from geometry lead to new insights into Property Testing problems:

- We develop property testing algorithms for some fundamental geometric properties including disjointness of objects, convex position, and the property that a geometric graph is a Euclidean minimum spanning tree.
- Then we present a general framework for property testing algorithms. This framework is based on a connection of Property Testing with a new class of problems which we call *abstract combinatorial programs*. Roughly speaking, we show that a property can be tested efficiently, if for any problem instance there is an equivalent (in the property testing sense) abstract combinatorial program (of small dimension and width).
- Next we introduce a new model for testing properties of point sets. We allow that our testing algorithms use certain kinds of *range queries* when accessing the input. We give property testing algorithms for convex position, map labelling, and clustering properties in this new model.
- Finally, we apply the concept of Property Testing to approximately maintain combinatorial structures of moving points under a very general model of motion.