Whitehead's Process Philosophy and Quantum Field Theory

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Contents

Introduction

Despite Quantum Field Theory's (QFT) tremendous successes in predicting empirical phenomena with unprecedented accuracy and its great importance as an instrument for planing and evaluating experiments in the area of subatomic physics, central questions concerning its proper philosophical interpretation are still not settled. In this context one of the most important questions is clearly to which ontology QFT gives rise, i.e. of what kinds of fundamental entities the world probably consists (e.g. particles, fields, events, processes, spacetime points/regions, etc.), if one assumes that QFT is telling a true story about the world. The aim of this work is the formulation of an ontological interpretation of QFT in terms of A. N. Whitehead's process philosophy. The first, and up to now, only serious attempt to use Whitehead's philosophy of process in an ontological interpretation of QFT is due to H. P. Stapp (1975, 1977, 1979). However, though there are resemblances to the overall aim of Stapp's papers, the details of the proposal for a connection of Whitehead's process philosophy and QFT presented in this book will be quite different from those developed by Stapp. In particular, Stapp only used some very general ideas of Whitehead's philosophy in his interpretation, whereas the aim of this work is an interpretation of QFT which incorporates the core concepts of Whitehead's ontology—to what extend this aim can be reached will have to be seen.

Whitehead who was familiar with the consequences of the early quantum theory of N. Bohr developed his process philosophy partly because he recognized that traditional substance ontology cannot account for the indeterminism and the quantal character of nature introduced by quantum theory. But when Whitehead developed his process philosophy in the 1920s, as an alternative to substance ontology, he probably did not know the new quantum theory due to W. Heisenberg, E. Schrödinger and P. Dirac and he clearly could not know all the later developments that finally led to QFT. Thus it is not to be expected that Whitehead's ontology can be connected to QFT without modifications. Furthermore, Whitehead's philosophy of process is explicitly designed to be an integrated ontology, that encompasses all different domains of the world, from the domain of subatomic physics to that of persons to that of cosmology, whereas this work only aims at a regional ontology for QFT—an ontology for that part of the world that can be successfully mathematically described by QFT. Therefore, it is likewise not to be expected that Whitehead's whole ontological system, with all of its complex concepts and details will be relevant for this work.

This book is structured into three parts. Part I introduces, in a self-contained way, Whitehead's process philosophy to the extend relevant for a connection with QFT. Since Whitehead's ontological writings, especially his magnum opus *Process and Reality*, are often far from being clear it is not surprising that Whitehead's ontology itself gave rise to many different, competing interpretations. The interpretation of Whitehead's philosophy given in Part I of this book in many respects follows the main lines of the interpretation proposed by J. Nobo in his book *Whitehead's Metaphysics of Extension and Solidarity*. However, it is also made use of other, competing interpretations of Whitehead's ontological writings, when these interpretations seemed to be better suited for the aim of this work. Besides already existing interpretations, some of the interpretative claims presented in Part I are grown out of the author's own study of Whitehead's writings. The ontology presented in Part I of this book is only a simplified version of Whitehead's full ontological system. For example, many of the mental aspects Whitehead ascribes to his basic entities will be ignored and instead the focus is directed towards spatiotemporal and causal aspects, in order to bring Whitehead's philosophy in contact with QFT. The neglect of those aspects of Whitehead's ontology which are not needed for the connection with QFT is justified, because it makes the work presented here much easier accessible to readers without, or with very little, previous knowledge in Whitehead's philosophy. This is an important point since this work is not only addressed to philosophers but likewise to physicists which are interested in the ontology of QFT. Furthermore, it will be no problem for readers which are familiar with Whitehead's philosophy to identify those aspects of it which have been simplified or neglected in the version presented in Part I. Besides some simplifications, the ontology presented in Part I also includes some supplementary assumptions, not (explicitly) made by Whitehead. These assumptions have been introduced, on the one hand, to close some logical gaps between Whitehead's intentions concerning his theory and his theory as explicitly formulated by him and, on the other hand, to bring his theory in closer contact to QFT. The latter is necessary because Whitehead's ontology is meant to be a metaphysical theory in the sense of describing the most general aspects of reality. As such it is, however, too far removed from a physical theory as detailed and specific as QFT to be able to ground a satisfactory ontological interpretation of the latter.

Part II of this book presents QFT in its algebraic formulation known as Algebraic QFT (AQFT). A restriction of scope had to be laid upon this presentation too, because a detailed account of all the interesting mathematical and physical consequences of QFT is not necessarily needed to enable the understanding of the main line of argument of this work and, moreover, would have made the work much more difficult from a mathematical point of view. However, it is assumed that the reader is familiar with the Special Theory of Relativity and non-relativistic Quantum Mechanics and that he has some basic knowledge of QFT. This is because it would go far beyond the scope of this book to serve the reader who is not familiar with these theories as an introductory text. Such introductions into modern physical theories are doubtlessly very important, particularly for a fruitful communication between physicists and philosophers. Part II of this book, however, is not planed as such an introductory text into QFT but merely attempts to introduce the reader who already has some basic knowledge of QFT with the most general and mathematically rigorous axiomatic formulation of this theory—AQFT.

Finally, in Part III it will be analyzed which structures of Whitehead's ontology as presented in Part I can be represented by mathematical structures available within AQFT.

Abbreviations

For convenience we will introduce the following abbreviations of books which are frequently cited in this work:

SMW: A. N. Whitehead (1967): *Science and the Modern World.* New York: The Free Press.

AI: A. N. Whitehead (1967): *Adventures of Ideas.* New York: The Free Press.

PR: A. N. Whitehead (1978): *Process and Reality.* Corrected Edition. Eds.: D. R. Griffin, D. W. Sherburne. New York: The Free Press.

RM: A. N. Whitehead (1996): *Religion in the Making.* New York: Fordham University Press.

WM: J. Nobo (1986): *Whitehead's Metaphysics of Extension and Solidarity.* Albany: State University of New York Press.

Part I

Whitehead's Process Philosophy

Chapter 1

Actuality

In Whitehead's ontological theory reality is divided into *actuality* and *potentiality*. Yet actuality and potentiality are not two disconnected modes of existence but rather are intimately related to each other. What exists by way of actuality arises out of what exists by way of potentiality and, on the other hand, what exist by way of potentiality is limited or conditioned by actuality. This bipolar structure of reality, i.e. its division into the two modes potentiality and actuality, allows Whitehead to hold that there is a real coming into being of final actualities—so-called *actual occasions* or interchangeably *actual entities*—and at the same time to deny that these actualities arise out of nothing, i.e. out of sheer non-being. That actual occasions are the final actualities means "that whatever exists in the universe by way of actuality is either itself an actual entity, a constituent aspect of an actual entity or an interrelated group—technically termed a *nexus* or *society*—of actual entities" (WM, p. 2). Thus actual occasions are the ultimate building blocks of actuality. Clearly, what has to be denied in such a conception of reality is that what exists by way of potentiality itself becomes, because this would imply that either these potentials arise out of nothing, or it would necessitate the postulation of higher grade potentials in which the becoming of lower grade ones is grounded. Thus one ends up with uncreated, i.e. eternal, potentials which ground the becoming of actualities. But Whitehead does not only hold that there is a real coming into being of final actualities which is grounded in eternal, uncreated potentials. More remarkably, he does not degrade *becom-*

ing in favour of *being*. Rather becoming and being are *two different modes of actuality—each actual occasion is both a becoming and being*. But how can each actual occasion at the same time be a becoming and a being? Are not "becoming" and "being" two disjoint ontological categories, so that no single entity can belong to both at once? This is indeed the case and Whitehead does not challenge this categorical distinction. An actual occasion is not literally at once a becoming and a being, rather it is *first* a becoming and *then* the being that is created in this act of becoming. As J. Nobo puts it,

> *becoming* and *being* are to be understood as two different modes of existence [of actual occasions]. An actual occasion ceases to exist in the former mode only to continue existing in the latter mode. Its being presupposes its becoming. (WM, p. 38)
>
> *How* an actual entity *becomes* creates *what* that actual entity *is* [...]. Its 'being' is created by its 'becoming'. (WM, p. 38)

Thus the existence of an occasion qua being is *subsequent* to and *created* by the existence of the occasion qua becoming. This conception of an actual occasion as first existing as an act of becoming and then as the being created in this act of becoming immediately raises two questions. First, what secures the *self-identity* of an occasion in the two different stages of its existence that allows one to speak of the becoming and the being of one single self-creative entity rather than of two different entities, the former creating the latter? The second question concerns the precise meaning in which the becoming of an occasion can be said to *create* its being. The answers to both question will be given in the following section.

1.1 The self-creative processes of concrescence

When Whitehead speaks of occasions as self-creating, self-causing or self-realizing[1] he does not mean that an occasion is a cause or reason for its own becoming. Such a concept of self-creativeness would be hard to defend against

[1] All of these notions, and even some more which are not mentioned here, are used interchangeably by Whitehead.

the challenge of being self-contradictory. Apart from some scholars of traditional metaphysics who try to defend the claim that god is the cause or reason for its own existence, most philosophers would agree that nothing can be meaningfully said to even partially cause its own existence (see e.g. Mellor 1995, p. 60). Whitehead belongs to the latter group, since he construes the becoming of his occasions as completely other-created/caused (WM, p. 138 ff). The aggregate consisting of all and only those occasions which are efficient causes of a new occasion's phase of becoming is termed the *actual world* of the latter occasion (see Section 1.3.1). On the other hand, that the becoming of an occasion is completely other-caused, cannot mean that this whole becoming is completely *other-determined*, since in this case there would be no ground to speak of occasions as self-creative at all. How the compatibility of the other-createdness with the self-creativeness of each occasion is achieved in Whitehead's theory will become clearer in what follows.

First of all, the becoming of an occasion is not construed by Whitehead as a single act, but rather as a process, a so-called *microscopic process of concrescence*, that consists of different phases which succeed each other. Each phase of this concrescence process is conditioned but *not completely determined* by the forgoing phases. This gives each phase of the concrescence process a certain freedom to autonomously decide its own outcome that in turn conditions the following phase and so on. Each phase of a concrescence process can be analyzed into an *active-subphase* and the *outcome* of this activity (WM, p. 72 f). In the former an autonomous decision is settled that decides the latter—the outcome or product of this active-subphase. This outcome in turn provides the "material" for the autonomous decision to be settled in the next active-subphase and so on. The material among which the autonomous decisions are to be settled are qualitative properties among which the final qualitative character of the new occasion has to be made determinate. Each becoming of an occasion starts with a range of alternative qualitative properties that has to be successively reduced to one coherent complex in the course of the concrescence process. At the point where all the phases of the concrescence are run through, the qualitative character of the new occasion is fixed, i.e. the occasion reaches its final qualitative determinateness. What is other-produced and thus completely other-determined are the initial ontological constituents of the first phase of such a concrescence process. One of these constituent elements is the initial incomplete qualitative determinateness of the new occasion that needs to be reduced to the final qualitative character by the autonomous decisions of the following concrescence process. The autonomous decisions settled in a concrescence establish in part the sense in which occasions are creative and—provided they are the self-same entities in all their phases of existence—self-creative. In connection with the doctrine of the autonomous decisions settled in a concrescence, it is important to emphasize that these decisions do not necessarily involve consciousness. Rather consciousness only plays a role in extremely high grade occasions, like those which according to Whitehead are involved in the constitution of higher animals (including humans). Whitehead uses "decision" as a technical term "in its root sense of 'cutting off' " (PR, p. 43). For Whitehead, decisions are involved in the becoming of *all* occasions, whereas conscious decisions emerge out of unconscious ones only under very special conditions which we do not have to take into account in connection with Quantum Field Theory (QFT). However, a decision is an activity, though not necessarily a conscious one, and thus *a concrescence process presupposes as a further constituent an activity that makes the decisions in regard to the determination of the final qualitative character of the occasion qua being* (AI, p. 176). The third and last element with which a new occasion is provided from the very first "moment" of its existence on, is *a finitely extended spacetime region* that serves as the unalterable spatiotemporal standpoint during its process of concrescence as well as of the completed outcome of this concrescence—the occasion qua being. This region is the stable factor in the ontological make up of an occasion that accounts for the *self-identity* of the occasion in all its phases of existence. *Thus each occasion is both other-created and self-creating in that the constituent elements from which the initial phase of its self-creative process of concrescence takes rise is completely other-created.*

A concrescence process is called microscopic, because it is *internal* to an occasion, i.e. it is part of the ontological make up of an occasion, namely its existence qua becoming. As we will see later on in Section 2.4, the mechanism by which the actual world of a new occasion in joint functioning with the eternal potentials produces the constituent elements for the very first stage of the new occasion's self-creative becoming is also a process that consists of succeeding phases—a so-called *macroscopic process of transition.* A transition process is called macroscopic, because it is not an internal constituent of an occasion, but rather mediates causal influences between an actual world and the corresponding new occasion and is thus *external* to both of them. A new occasion's process of concrescence arises from the outcome of the earlier process of transition. Contrary to the amount of freedom of the internal concrescence process of an occasion, *a transition process is completely deterministic*, i.e. its outcome—the initial qualitative character, the spacetime region, and the activity for the following concrescence—is completely determined by the corresponding actual world of already actualized occasions. Since processes of transition are the "points of intersection" between actuality and potentiality we can discuss them in detail only after we have discussed the essential structures of both actuality and potentiality.

1.1.1 Concrescence as creative process

Now we will further qualify the above remark that a concrescence process is creative in the sense of settling autonomous decisions but not in the sense of literally creating, producing or bringing about the material of its decisions. The material among which a concrescence makes decisions is its initial range of qualitative properties—its initial *definiteness.* Contrary to this the spacetime region occupied by a concrescent occasion does *not* fall under the autonomous decisions settled in a concrescence. It is the fixed and thus unalterable spatiotemporal standpoint of all phases of concrescence as well as of the completed outcome—the occasion qua being. Now the initial definiteness of an occasion is given by a range of qualitative properties. Among these the concrescent occasion has to decide its final qualitative character—its final definiteness—which is one coherent, complex property.

> [The definiteness of each occasion] starts with conditional alternatives, and by successive decisions is reduced to coherence. (PR, p. 224)
>
> Each decision represents a cutting off of some possibility for the definiteness of that occasion [...]. (WM, p. 156)

> An actual entity's process of becoming is a process of acquiring definiteness by a series of decisions to select or reject various forms of definiteness. (Sherburne 1966, p. 220 f)

The initial definiteness—the initial range of qualitative properties—of an occasion consists of *alternatives* which nevertheless can be *jointly* actualized in the same occasion, i.e. it consists of *compatible alternatives* (see Section 2.2.4). The concrescence process of an occasion is a succession of autonomous decisions by which the occasion deepens its initial definiteness. Thus a concrescence process is creative in the sense that it successively makes autonomous decisions as to the final qualitative make up of its outcome—the completed occasion. But a concrescence does *not* literally create, produce or bring about the properties among which it decides the final definiteness of its outcome. First of all, as we will see in Section 2.2, these properties are in existence as possibilities for actualizations independently of any particular occasions, since they belong to the realm of eternal potentials. Moreover, their status of existence is not even "improved" in the course of the concrescence. They do not become "more actual" with each phase of the concrescence. To the contrary, what happens to some of them during the concrescence is that they are *rejected* for entering into the next phase, so that their ontological status changes from (internal constituents of an) actuality in attainment to impossibilities for becoming (internal constituents of) an attained actuality, i.e. for integration into the occasion's final definiteness. Only at the point where the concrescence's activity is exhausted, i.e. when the concrescence process is completed, the status of the final complex of properties as well as that of its spacetime region, change from actuality in attainment to attained actuality. The attained actuality of a completed occasion is the end aimed at and actuality in attainment is part of the means to this end. But it is important to emphasize again, that this does *not* mean that the becoming of an occasion is *less actual* than its being; the becoming of an occasion is the *private* side of its actuality and its being is the *public* side of its actuality (PR, p. 151, 289; WM, p. 387 ff). In the former it functions in respect to itself and as such it is closed for anything external to it—it is alone with itself—whereas in the latter it functions as an efficient cause for new occasions in its future. Thus when it is said that a completed occasion is actualized, this does not mean that it was not an actuality in its earlier phase of becoming. Rather following the earlier quotation from Nobo, it means that an occasion ceases to exist in its private, i.e. self-creative, mode of actuality only to continue existing in its public, i.e. other-creating, mode of actuality. Thus when we speak of the actualization of the outcome of a concrescence process this is to be understood as the transformation from private actuality to public actuality, and not as a transformation from non-actuality to actuality. This transformation is what takes place at the "moment" when the final phase of a concrescence process—the phase of satisfaction—terminates. This means that the outcome of a concrescence is transformed into a public fact only at the point where all the phases of the concrescence are run through. Thus a concrescence process itself is not a gradual or even continuous unfolding from private to public actuality. Rather this transformation from privacy to publicity is one single undivided—*atomic*—act, that *at once* closes up the concrescence and throws its completed outcome into its public, other-creating mode of actuality. Since this atomic act of the transformation from private to public actuality takes place at the point where the activity of the concrescence is completely exhausted this act cannot meaningfully be attributed to the concrescence process itself. Consequently, a concrescence cannot be meaningfully said to be creative in the sense of *itself changing* the ontological status of the involved qualitative properties or the region in which they are instantiated towards attained actuality.

In sum, then, a concrescence process is creative in the sense of settling autonomous decisions as to the qualitative determinateness of its outcome but it is not creative in any stronger sense: it does not create the properties among which it makes decisions or the region in which it is located nor can the transformation of its outcome from private to public actuality, i.e. the *actualization* of the outcome, be attributed to the concrescence itself. How this actualization of the outcome of a concrescence, i.e. of the completed occasion, is to be understood will be seen in Section 2.4.1.

1.1.2 Concrescence as non-spatiotemporal process

Though a concrescence process takes place in a spacetime region, it is nevertheless not a spatiotemporal process. For a concrescence to be a spatiotemporal process its different phases would have to succeed each other in a spatiotemporal sense, i.e. they would have to be ordered in respect to some spatiotemporal order relation. Fortunately we need not investigate all kinds of possible spatiotemporal order relations to see that this is not the case. A necessary condition for a spatiotemporal order among the phases of a concrescence is that to each phase can be assigned a unique spacetime region that differs from the regions of the other phases; otherwise no spatiotemporal order relation can be applied to them. But even this necessary condition for a concrescence to count as a spatiotemporal process is not fulfilled, since the *entire* region occupied by a concrescence process belongs to *each* of its phases.

> Each phase in the genetic process [i.e. the concrescence] presupposes the entire [spatiotemporal] quantum [...]. (PR, p. 283)

Thus since the phases of which a concrescence is made up do not even belong to different subregions of the entire region, a concrescence cannot be a spatiotemporal process. Each concrescence process occupies, i.e. takes place in, a particular spacetime region, but this region is not involved in ordering the succession of the concrescence's phases.

But if the different phases of a concrescence do not succeed each other in any spatiotemporal sense, in which sense do they succeed each other at all? The same question can be asked for the manner in which the phases of a process of transition succeed each other and for the manner in which each process of transition is followed by a process of concrescence, since as we will see later on a transition is a non-spatiotemporal process, too.

1.1.3 Genetical supersession

Whitehead calls the succession of the different phases of transition- and concrescence processes as well as the succession of processes of transition by processes of concrescence *genetical supersession*, but unfortunately the nature of this genetic order is not clear from Whitehead's writings. This has led some interpreters of Whitehead to deny that a concrescence consists of different phases at all (Hartshorne 1969) or to hold that non-spatiotemporal supersession is a concept sui generis that cannot be explained by recourse to other concepts (Christian 1959, p. 80 f). In his analysis of genetical supersession W. Christian argues that this order relation cannot be understood as a temporal-, logical- or (non-spatiotemporal) whole-to-part relation. That genetical supersession cannot be understood as a temporal order is clear from our above discussion, which showed that it cannot even be a spatiotemporal order. That the phases of transition and concrescence processes are logically ordered, i.e. as premise to conclusion, is ruled out by the fact that in a concrescence process decisions are involved in passing from one phase to another. A logical order as premise to conclusion, however, would simply left no room for decisions of any kind (Christian 1959, p. 80 f). Now Christian believes that a whole-to-part understanding of genetical supersession is not appropriate because "this construction would seem to eliminate the dynamic character of the process" (Christian 1959, p. 81). Unfortunately Christian does not explain what he means by the "dynamic character" of transition and concrescence processes. Yet if it is the autonomous decisions involved in concrescence processes which are referred to—which seems to be the most likely candidate—this argument is not conclusive. For one can explicate the genetic order that obtains between the phases of transition and concrescence processes as a whole-to-part relation without doing harm to the ability for autonomous decisions in processes of concrescence. We will first discuss the case of concrescence processes and will argue later on that this explication of Whitehead's notion of genetical supersession also applies to processes of transition.

Each phase of a concrescence process has, as (part of) its outcome, a certain definiteness, i.e. a certain range of qualitative properties, and this range is more and more reduced during the concrescence. Thus the genetical order of phases in a concrescence can be explicated by reason of the outcome range of qualities of each phase: *a phase is genetically earlier than another iff (if and only if) the definiteness of the latter is strictly contained in that of the former.* The dynamic character of a concrescence process, if understood in the sense of its autonomy for decisions, is obviously not eliminated or contradicted by this understanding of the genetical order of its phases, as it would be the case with a merely logical order. To the contrary, according to Whitehead the passage from the definiteness of a given phase to that of one of its successors *requires* autonomous decisions—there is no logical way to deduce the definiteness that will be (part of) the outcome of some phase from the outcome of the preceding phase. Thus it seems very well possible to understand the Whiteheadian concept of genetic supersession or genetic order among the phases of a concrescence process in terms of a whole-to-part relation between the ranges of qualitative properties (see PR, p. 149), which result from the autonomous decisions in these phases.

The same explication of the concept of genetic order also applies to transition processes. All we need to know about this second fundamental kind of processes at this point is that a process of transition consists of two phases and that the range of qualitative properties that corresponds to the second phase—the *conformal phase*—is strictly contained in that corresponding to the first phase—the *dative phase*. Therefore, these phases too, are naturally ordered by the whole-to-part relation obtaining between the corresponding ranges of qualitative properties. Moreover, since the range of qualitative properties that is (part of) the outcome of the second (and last) phase of a transition process is the initial definiteness which is further reduced in the phases of concrescence, *this same order also accounts for the genetical supersession of processes of transition by processes of concrescence*. Thus it seems fair to conclude that, contrary to Christian's pessimistic attitude, it is very well possible to explicate Whitehead's concept of genetical supersession on the basis of the resources available within his ontological system.

Finally we have to comment on the second of the above mentioned challenges, namely the one put forward by Hartshorne, which says that concrescence processes do not consist of different phases at all, but rather only of one act of decision together with the outcome of this act. This challenge is a substantial one because Whitehead nowhere explicitly formulates a principle that blocks the possibility that a concrescence could indeed decide in one act, which of the initially given qualitative properties shall be rejected from entrance into the final definiteness of the completed occasion. However, Whitehead's ontology provides a natural resource to prohibit this. As will be seen later on, Whitehead distinguishes between simple and complex properties (see Section 2.2.2) and holds that the initial range of qualitative properties of each occasion is solely generated by simple properties (see Section 2.2.5). This opens up the possibility to answer the present challenge by assuming that *in* *each phase of a concrescence only one simple property can be eliminated, i.e. that in each phase only one simple decision can be made.* Together with the principle that a concrescence process only ends when its creative activity is completely exhausted, this assumption implies that a concrescence, in general, will consist of more than one phase. Yet this does not mean that there cannot be concrescence processes which in fact consist merely of one simple act of decision. But this is only the case if the amount of activity of this concrescence is *contingently* such that it only allows one simple decision. Although Whitehead never explicitly demanded that in each phase of a concrescence process only one simple property can be eliminated, it seems that this assumption is indeed a very natural and straight forward way of answering the challenge of the non-processual character of concrescences. We will therefore in fact add this supplementary assumption as to the structure of concrescence processes to Whitehead's ontological system, thereby making sense of the Whiteheadian demand that concrescence processes, in general, do consist of different phases and thus are in fact processes and not merely single acts.

1.2 A comment on the "birth-date" of occasions

There is some disagreement among interpreters of Whitehead as to the precise "date of birth" of an occasion. For example W. Christian (1959) as well as E. Kraus (1998) argue that a new occasion's existence already starts with the dative phase, i.e. with the first phase of a new transition process. Yet since Whitehead is clear in holding that an occasion's existence starts with the initial phase of a process of concrescence (PR, p. 210), these interpreters are forced to hold that there are in fact not two different species of processes—transition and concrescence—but merely one species—concrescence processes—which, then, have to include the dative and conformal phase, too. Nobo has shown that such a reading of Whitehead cannot be upheld. With massive textual support from PR, Nobo showed that Whitehead holds that there are indeed two different fundamental kinds of processes—concrescence *and* transition—which are involved in the creation of each occasion. Furthermore, Nobo's analysis strongly suggests, though Nobo himself is not completely clear about this point, that an occasion's existence starts with the first phase of autonomous decisions (WM, p. 90, 282, 332). This can be concluded from the two facts that occasions are, by definition, what exists qua actuality[2] (WM, p. 2) and that according to Whitehead only entities which are or have been making autonomous decisions in respect to their own determinateness, i.e. which are or have been self-creative, are actual (WM, p. 34 ff). Now in the dative and conformal phase no autonomous decisions are involved at all, since what happens in these phases is completely determined by the corresponding actual world, i.e. it is fixed by the decisions the corresponding settled occasions have felt during their self-creative modes of existence. Thus neither the dative nor the conformal phase fulfil the Whiteheadian condition for actuality and thus cannot be construed as phases in the existence of an actuality. Since, as mentioned above, Whitehead is clear in demanding that an occasion's existence begins with the first phase of concrescence, and as Nobo has shown, that Whitehead moreover holds that there are in fact two different kinds of processes involved in the creation of each occasion, both the dative and the conformal phase have to belong to the other-creating processes of transition and not to the self-creative processes of concrescence.

1.3 The world-process

The macroscopic processes of transition, which will be discussed in detail later on, and the microscopic processes of concrescence are the two fundamental kinds of processes in Whitehead's ontology.

> There are two species of process, macroscopic process, and microscopic process. The macroscopic process is the transition from attained actuality to actuality in attainment; while the microscopic process is the conversion of conditions which are merely real into determinate actuality. (PR, p. 214)

[2] Sometimes, also groups of occasions (e.g. actual worlds) or internal constituents of occasions are said to be actual. But attributing actuality to these entities is only meant in a derivative sense—what is truly actual are only the occasions themselves.

In combination processes of the two fundamental species—self-creative processes of concrescence and other-creating processes of transition—constitute the evolving *world-process* that consists of the coming into being of ever new causally related actual occasions (PR, p. 60, 210, 215, 286).

> The actualities of the Universe are processes [...], each process an individual fact. The whole Universe is the advancing assemblage of these processes. (AI, p. 197)

This world-process is moreover the *expansion* of actuality, because "actuality is cumulative and the number of its concrete components is ever increasing or shifting, with the emergence of each new creature [i.e. occasion]" (WM, p. 172; see also PR, p. 215). This means that *occasions which have been actualized do not disappear when they are superseded by new occasions but remain in being, i.e. in their static state of attained actuality*, so that each actual occasion qua being is "a stubborn fact which cannot be evaded" (PR, p. 43). Yet this does not merely mean that the world-process is an irreversible evolvement. It means that at every stage of the world-process "the past" is as actual as "the present", though in a *different* mode of actuality: the past occasions are attained actualities whereas the present ones are actualities in attainment (PR, p. 214). How this "objective immortality" of occasions can be understood, will be discussed in more detail in Section 2.3.4. One consequence of this doctrine is obviously that the notions "evolvement"and "expansion", when referring to the processual character of the world of actualities, are synonymous within Whitehead's theory: if in the evolvement of the world every attained actuality is retained, this evolvement is in fact an expansion.

According to Whitehead the aggregate of all already actualized occasions at some stage of the world-process is uniquely divided into sub-aggregates—the actual worlds at that stage. From each of these actual worlds will arise a single transition process that begets a single new but incomplete occasion that in turn completes itself in a process of concrescence. Each occasion is thus efficiently caused by the settled occasions in its corresponding actual world. All transition processes which arise from the different actual worlds at some single stage of the world-process are *causally independent* or *isolated* from one another. The same is true for the concrescence processes arising from the outcomes of these transition processes. But this mutual isolation does *not mean that jointly becoming occasions do not have common causes.* The causes of a concrescent occasion are the occasions in its actual world and two actual worlds will, in general, contain common occasions, so that two occasions of the same stage of the world-process, in general, will have common causes. Thus it is to be expected that two occasions which arise from minimally different actual worlds, i.e. which have nearly all their efficient causes in common, will be quite similar in their initial ontological make up (i.e. their spacetime regions, their creative activities and their initial definiteness). But nevertheless the transition processes from which they arose as well as their concrescence processes are completely isolated from each other in all their supersessional phases.

1.3.1 Actual worlds

The actual world of an occasion E consists of all and only those occasions which are efficient causes of *E—it is E's causal past.* In Whitehead's ontology this means that the actual world of E contains all and only those occasions which contribute to the transition process by which the initial ontological constituents of E are created (PR, p. 123 f, 320 f; see also Section 2.4). Note that no occasion belongs to its own actual world because no occasion can be a cause of its own initial constituents (see Section 1.1). Whitehead demands that each two different occasions have different actual worlds, i.e. different causal pasts (PR, p. 22 f, 28), but he makes no demands as to a connection between the spacetime regions of occasions and those belonging to their actual worlds. Therefore, one may in the first place expect that this *doctrine of actual worlds*, i.e. that different occasions necessarily arise from different causal pasts, is compatible with *any* connection between causal and spatiotemporal properties of occasions. In other words, one would expect that the occasions belonging to the actual world of some occasion can be spatiotemporally arbitrary scattered. This is, however, not the case, since the demand that two different occasions necessarily arise from different actual worlds already rules out some connections of causal and spatiotemporal properties of occasions. For example, it rules out the causal spacetime structure that follows from the assumption that causal influences can be transmitted with an arbitrary finite (but not with an infinite) velocity, because this assumption allows that the causal pasts of two simultaneous events coincide.[3]

Now by reason of the concept of actual worlds Whitehead further defines the *causal future* of an occasion and the relation of *contemporaneity* between occasions as follows: the causal future of an occasion E consists of all and only those occasions to whose actual worlds E belongs and two occasions are contemporaneous to one another if neither belongs to the actual world of the other (PR, p. 123, 319). Since solely defined in terms of actual worlds the causal futures of different occasions will also be different. Moreover, it is clear from the definitions of "causal past" and "contemporaneity" that in Whitehead's theory the latter is synonymous with "causal independence". Whitehead conceived the relation of contemporaneity as not being transitive, i.e. two occasions contemporaneous to a third occasion need not be contemporaneous to one another (PR, p. 125, 320). The fact that no occasion can be an efficient cause of itself means that the relation of contemporaneity, i.e. causal independence, is reflexive and that from its very definition it is clear that it is also symmetric (see also PR, p. 320). Thus contemporaneity is a reflexive and symmetrical but not a transitive relation.

Two well-known causal spacetime structures that fulfil the Whiteheadian assumptions on causal pasts and the relation of causal independence are those of the Special and the General Theory of Relativity (STR and GTR). That Whitehead intended to incorporate a connection between causal and spatiotemporal properties of occasions analogous to that known from STR or GTR seems to be obvious from the following quotations.

> Curiously enough, even at this early stage of metaphysical discussion, the influence of the 'relativity theory' of modern physics is important. (PR, p. 65)
>
> The differences between the actual worlds of a pair of contemporary entities, which are in a certain sense 'neighbors', are negligible

[3] The notion "event" is used instead of "occasion" when we wish to be neutral in respect to the many specifically Whiteheadian connotations with which the term "occasion" is loaded in this work. Thus especially when discussing physical or ontological theories other than Whitehead's, we will always speak of "events" instead of "occasions".

> for most human purposes. Thus the difference between the 'classical' and the 'relativity' view [...] only rarely has any important relevance. I shall always adopt the relativity view; [...] because with rare exceptions the classical doctrine can be looked on as a special case of the relativity doctrine—a case which does not seem to accord with experimental evidence. (PR, p. 66)

Although it is to be expected that because of the greater generality, domain of applicability and empirical adequacy Whitehead had GTR in mind when he designed his concepts of actual worlds and contemporaneity, we will in what follows only take into account STR when discussing the connection between causation and spacetime. This is justified by the fact that this work is concerned with an application of Whitehead's ontology in QFT, and the latter in fact incorporates the spacetime structure of STR. However, it is clear that without further assumptions Whitehead's doctrine of actual worlds, i.e. individual causal pasts of occasions, and of the non-transitivity of contemporaneity, i.e. causal independence, do not uniquely single out the general or even special relativistic connection between the spacetime regions of events and their ability to cause each other—expressed by the lightcone structures of STR respectively of GTR—*other connections are still possible.* Yet besides declaring in the above quoted passage that he "adopts the relativity view" Whitehead nowhere makes such additional assumptions explicit. That the mere declaration to "adopt the relativity view" is not sufficient for making the relativistic connection between the regions of occasions and their ability to cause each other a generally valid principle of Whitehead's theory relies on the fact that not even the spatiotemporal character of the regions of occasions is a general feature of all parts of the world-process, i.e. it is no "metaphysical necessity". As we will learn in Section 2.1, what is metaphysically necessary is that occasions embody finite regions of the so-called *extensive continuum*—but this extensive continuum is *not* to be equated with physical spacetime. In contrast to the latter, the extensive continuum is merely topologically structured and does not have dimensional or even metrical properties. According to Whitehead these further spatiotemporal structures only emerge *contingently* in some parts of the world-process, e.g. in the part that is empirically accessible to us—in our *cosmic epoch.* Now we can see why it does not suffice simply to say that one adopts the relativistic connection between causal properties of occasions and their regions within Whitehead's ontology: if these regions need not have a definite dimension and need not bear metrical-relations to each other it is far from clear how the relativistic connection shall be implemented at all—in fact, it seems that this task is impossible. Therefore, we will assume in what follows that the relativistic connection between causation and the regions of occasions is *not* a general feature of all parts of the world-process, but that it obtains merely contingently in some parts of it. However, until we will discuss the extensive continuum in detail in Section 2.1, we will for the sake of simplicity think of the regions of occasions as spatiotemporal regions.

Now in STR the constraints between spatiotemporal regions of events and their ability to cause each other arise out of the more fundamental assumption that causal influences are transmitted by reason of spatiotemporally continuous processes whose velocity is limited by a universal maximum velocity—the velocity of light in the vacuum. Therefore, relativistic causation is *local* in the sense that causes and their immediate effects are spatiotemporally contiguous and that the causes of an occasion E lie within the backward lightcone[4] of E's region. Yet in Whitehead's theory there are no spatiotemporal, let alone spatiotemporally continuous, processes which link cause and effect. Causal influences between occasions are transmitted by reason of transition processes and these are, like processes of concrescence, non-spatiotemporal processes (see Section 2.4). Consequently, there is no reason for Whitehead to demand that direct causal influences can only obtain between spatiotemporally contiguous regions. Whitehead in fact adopted the view that each occasion E' in the actual world of a given occasion E cannot only causally influence E indirectly, via a chain of contiguous occasions that reaches from E' to E, but rather that E' can also directly influence E, irrespective of the "spatiotemporal gap" between E' and E (PR, p. 307 f; WM, p. 244 f). Therefore, even if Whitehead had somehow incorporated the relativistic connection between the regions of occasions and their ability to cause each other—expressed by the lightcone structure of STR—as a general principle governing the whole world-process, his ontology would still not be local in the sense explained above, because

[4]When we speak of the backward- respectively forward lightcone of a region we not merely mean the surface of this cone but rather its surface plus the whole interior of the latter.

causation can operate across "spatiotemporal gaps".

Now apart from the missing link between causation and spacetime Whitehead's doctrine of actual worlds is problematic in its own right, i.e. as the purely causal concept that it in fact is. As we will see later on, the creation of a new incomplete occasion via a process of transition from its actual world, presupposes that this actual world is given in the following sense: before a transition process can create a new occasion's initial ontological constituents, it first of all has to be decided from which groups of occasions from all the already actualized occasions at the respective stage of the world-process new transition processes shall arise, i.e. which groups of already actualized occasions shall form the actual worlds from which new occasions will be created. Put differently, some principle is needed by reason of which the aggregate of all the occasions already actualized at some stage s of the world-process, i.e. after the actualization of all occasions of stage $s-1$, call it W_{s-1}, is arranged into subgroups $W_{s-1}(i)$—*the actual worlds at stage* s (or equivalently, *after* stage $s-1$)—such that from each of these subgroups a single transition process can arise that begets the initial components of a new occasion E_i. This is not the case in STR or any other theory according to which causes are linked to their effects by spatiotemporally continuous processes. In such a theory one can hold that a new event is created when two or more causal processes intersect spatiotemporally. Thereby, on the one hand, the region of the new event is fixed as the region of intersection and, on the other hand, the direct causes of the new event are also fixed a forteriori to be those events from which the intersecting processes took rise. The complete causal past of an event E can then simply be taken to consist of all and only those events which are direct causes of E or direct causes of the former and so on. Yet since in Whitehead's theory there are no spatiotemporally continuous processes which link causes to their effects the determination of the causes of an occasion cannot be settled in this forteriori manner (i.e. "in the moment of intersection" of the corresponding causal processes). Rather which already settled occasions shall form the actual world of a new occasion needs to be determined (genetically) *before* the very transition process that begets the initial constituents of the new occasion in question can arise from this actual world. Consequently, *the determination of the actual world of an occasion must not depend on any features* *of the new occasion which in the first place have to be created or are otherwise conditioned by reason of this actual world.* However, since *all* features—all ontological constituents—of a "newborn" occasion are in fact created in the transition process arising from its actual world and all later phases of the occasion's self-creative becoming and thus too of its being *are conditioned by its initial ontological make up* (see Section 2.4.3), no such independent features of an occasion are available by reason of which its actual world could be defined in a non-circular fashion.

For example, the determination of the actual world of an occasion by way of its spacetime region is not possible within Whitehead's ontology, because the spacetime region of an occasion itself presupposes the occasion's actual world as its efficient cause. Such a way of introducing actual worlds could only succeed if the spacetime regions of occasions would not have to be made determinate and to be realized by reason of the causal influences of other occasions, e.g. if they were eternally fixed (i.e. pre-determined), independently existing substances and if moreover this substantival spacetime would come equipped with metrical-relations between its different regions. In this case one could define the actual world of an occasion E as that group of already actualized occasions whose regions bear certain metrical-relations to E's region. However, as already mentioned above, we will learn in Section 2.1 that according to Whitehead metrical relations are *not* given on the fundamental ontological level, but rather emerge merely contingently in some parts of the world-process. Thus for this latter reason alone one cannot, at least not in all parts of the world-process, introduce the actual world of an occasion by way of its region. Yet as we have just seen, even in those parts of the world-process in which appropriate metrical-relations are available this way of singling out the actual world of an occasion (like any other way that involves E or any features of E), does not work because the region of an occasion in the first place has to be created by reason of this very actual world.

But how shall we make sense of the concept of an individual actual world of each occasion then? Is there another way of implementing this concept properly into Whitehead's theory? Later on in Section 2.5 we will see that there seems to be no other way of how this could be achieved. More importantly, we will have to modify Whitehead's ontology for the sake of eliminating an inconsistency from it, and this modification will also undermine the doctrine of actual worlds. However, until we will eventually give up this doctrine in Section 2.5.3 we will, for the sake of proceeding with the description of the philosophy of process as originally intended by Whitehead, assume that one can make sense of it within Whitehead's theory.

1.3.2 A comment on the concept of evolvement and the openness of the future

A core idea of Whitehead's philosophy of process is that the world of actualized occasions is not given once and for all as a static whole, but rather that it is an expanding pattern that grows by reason of the actualization of formerly potential occasions.

> The community of actual things [i.e. actual occasions] is an organism; but not a static organism. It is an incompletion in process of production. Thus the expansion of the universe in respect to actual things is the first meaning of 'process'; and the universe in any stage of its expansion is the first meaning of 'organism'. (PR, p. 214 f)
>
> The universe is thus a creative advance into novelty. The alternative to this doctrine is a static morphological universe [...] without unrealized potentialities; since 'potentiality' is then a meaningless term. (PR, p. 222, 46).

Thus Whitehead follows Aristotle in proposing a theory of an open future of yet unactualized—potential—entities which can be actualized at later stages of the world's evolvement. However, this openness of the future as well as the corresponding evolving or expanding character of the world are challenged by the causal spacetime structure of STR which Whitehead intended to integrate, by reason of his doctrine of actual worlds and the non-transitive relation of causal independence or contemporaneity, into his theory, too. Let us, for the sake of argument, assume that the connection between causal and spatiotemporal properties of occasions known from STR had really been implemented into Whitehead's theory as intended by him. We will see in what follows that this causal spacetime structure of STR is still not rich enough to ground the concept of an open future or the corresponding expansion of actuality. The same obviously holds for the even weaker, because purely causal, concepts of actual worlds and the non-transitive relation of causal independence.

The future (past) of an event in STR consists of those events which lie in its forward (backward) lightcone, so that two different events have different futures (pasts), even if they are simultaneous with, and thus causally independent from, one another. This, however, is not problematic as long as each event belongs to precisely one simultaneity class of events, i.e. to one and only one set of events which are mutually simultaneous to each other. In this case one can merge all the individual futures (pasts) of the events in such a simultaneity class into one unique future (past), consisting of the yet unactualized, i.e. potential, (already actualized) events, relative to this simultaneity class. Since these simultaneity classes were, by definition, disjoint each could be interpreted as representing the *frontier* of the corresponding stage of actuality, that demarcates already actualized from merely potential events. Moreover, because each event precisely belongs to one simultaneity class the family of all these classes is linearly orderable by the relation "lies in the future (past) of". Therefore, one could consistently interpret this "layer-cake structure" as the expansion of actuality by reason of the successive actualization of new layers of formerly potential events. Yet because of the non-transitivity of the relativistic simultaneity relation, that is supposed by Whitehead to be the analog of his non-transitive contemporaneity relation, each event belongs to (infinitely) many sets of mutually simultaneous events rather than to precisely one. Moreover, each two such sets L_1, L_2 to which a given event belongs are not only non-disjoint but they even *cross* each other: some events belonging to L_1 are in the future of L_2 whereas some other events belonging to L_1 are in L_2's past. Such a structure can, however, hardly be interpreted as an expansion or evolvement because it is not even linearly orderable. It neither provides us with a unique open future since the futures of L_1 and L_2 are not compatible: some events which are in the future of L_2 and thus are potential relative to L_2 are in the past of L_1 and thus are actualized relative to L_1. This, however, were still not problematic if *one* family of disjoint, linearly ordered sets of mutually simultaneous events, i.e. one family of linearly orderable simultaneity classes, were somehow privileged over all others. In this case one could still hold that this distinguished layer-cake structure represents the expansion of actuality. However, *all the incompatible layer-cake structures definable within the causal spacetime structure of STR are completely on a par with each other.* Therefore, one does not get a unique evolvement or expansion of the world but rather (infinitely) many different ones—none of which being privileged in any way over others, so that one is not licensed to speak of one unified evolving process.

Now without an ontologically distinguished layer-cake structure one can also argue that it makes no sense in the first place to introduce sets of mutually simultaneous events at all. Rather one could argue that only sets of mutually timelike separated events, i.e. sets where for each two of their members one lies within the backward lightcone of the other and which are therefore automatically linearly ordered, can be interpreted as evolving or expanding, since these sets as well as their individual linear orders are uniquely distinguished features of the relativistic causal spacetime structure (see e.g. Stein 1991). However, from our above considerations it follows that once two or more of such "timelike sets" are taken in conjunction this conjunction cannot be equipped with a unique linear order and therefore cannot be meaningfully interpreted as forming one expanding structure. Thus in this case the world could be held to consist of many individually evolving processes, *but the latter could not be combined into one unified world-process.* In Section 2.7 we will see that, at least after our modification of Whitehead's original theory, there is a rather natural way of how one can make sense of a unified expansion of actuality and of stages of this expansion and thus too, of an open future.

Chapter 2

Potentiality and its interplay with actuality

The central thesis of Whitehead's philosophy of process is that there is a real coming into being of ever new causally related, self-completing occasions. But since these occasions cannot come into being from nowhere, i.e. from non-being (PR, p. 46), Whitehead incorporated into his system *pure potentials* in which resides the unlimited potentiality for the self-creative activity, the definiteness and the spatiotemporal extensiveness which are the hallmarks of actuality. In the philosophy of process these pure potentials are necessary presuppositions of every act of becoming as well as for the definiteness and the spatiotemporal extendedness of what has become. Thus presupposed, the pure potentials cannot themselves become, since either they would have to arise out of non-being and thus nothing would be gained, or there would have to be higher grade potentials which ground the becoming of the lower grade potentials. The consequence of this latter position would be an infinite regress of potentials, which is obviously not a very attractive ontological construction. Since Whitehead's pure potentials are presupposed by everything that becomes and thus do not themselves become (or perish), they are the *eternal* entities in Whitehead's ontology.[1] To the three aspects of actual occasions—their self-

[1] Note that Whitehead's defintion of the notion "entity" differs from its usual meaning in ontology. According to the Whiteheadian definition the underlying activity and the extensive continuum are not literally entities, but rather "realities"—strictly speaking, for Whitehead, the only eternal entities are the eternal objects. But for reasons of simplicity, we will ignore

completing activity, their definiteness and their spatiotemporal extensiveness—correspond three pure potentials: the *underlying activity*, the *realm of eternal objects* and the *extensive continuum*. But these eternal or pure potentials as such cannot create any particular occasion. The eternal potentials are necessary but not sufficient for the becoming of finite actualities. The eternal potentials are the *unlimited* "sources" out of which a new occasion's very first stage of becoming is produced by the *limitations* laid upon them by the corresponding actual world. It is an ultimate principle of the philosophy of process that

> what the universe is by way of actuality always conditions and limits what the universe is by way of potentiality. This means that the pure potentiality for the becoming of an occasion is always conditioned and limited by the actual world of that occasion. The thus limited and conditioned potentiality is the *real* or *natural* potentiality—Whitehead uses both terms indifferently—from which spring the nascent occasion's successive phases of [becoming]. (WM, p. 73)

What this principle says is that from the eternal or pure potentials as such, i.e. in abstraction from their limitations due to attained actualities, can arise no single occasion. Each becoming of an actual occasion not only presupposes the eternal potentials but also the attained actualities which condition these unlimited potentials in such a way that new finite occasions can arise from the thus created *limited* or *real potentials*. This creation of real potentials, which bridge the gap between pure potentiality and actuality, out of attained actualities and the pure potentials is what takes place in processes of transition. Note that this implies that there can be no first occasion (or a first layer of causally independent, i.e. contemporaneous, occasions), since this occasion would have no antecedent occasions which could condition the pure potentials in the required way. Thus since each occasion presupposes already actualized occasions there can be no *beginning* of the coming into being of occasions. *The expanding world-process of the coming into being of actual occasions is at every*

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this subtle point and will use the term "entity" according to its usual ontological meaning, according to which it indifferently refers to every existent.

stage of its history already infinitely "old". Thus Whitehead's ontology is, in particular, not compatible with cosmological theories according to which the world has a temporal beginning, like the popular "big bang" theory. However, whether Whitehead's ontology is in accord with certain cosmological theories about the "origin" of the world is not important for us. We are only interested in its use as an ontology of QFT and the latter is based on the Minkowski spacetime of STR which is infinitely extended to the temporal past and thus fits (at least in this regard) nicely to Whitehead's ontology.

Now since in isolation from already actualized occasions the pure potentials cannot account for the becoming of new occasions and thus for the expanding world-process that is the ultimate fact to be explained by Whitehead's philosophy of process, we must conclude that the pure potentials are in fact not isolated from the already actualized occasions. They can be abstracted by reason from the limitations imposed on them. But as actually involved as internal constituents of the ever expanding world-process, they are always and inevitably found as conditioned by the already completed actualities up to the stage of the world-process in question—they are always found as limited or real potentials. The thus limited, real potentials are the germs from which new self-creative actualities immediately arise. Real potentiality is thus the created and limited mode of potentiality, that bears with it a necessary reference to a particular group of actualized occasions by the conditioning of which it is produced from the eternal unlimited potentials, as well as to a particular concrescent occasion that will necessarily arise from it (Christian 1959, p. 201). Pure potentiality, on the other hand, does "not refer to or describe any state of affairs, actual or hypothetical" it is "a 'general potentiality' unrestricted by any particular state of affairs [in the world of actualized occasions]" (Christian 1959, p. 201). Pure potentiality is merely the realm of hypothetical possibilities without any relation to particular actualities: pure potentiality is neither produced by, nor is it a potential for, particular occasions. It is only the potentiality for the becoming and being of occasions *in general*. The term "pure" is thus to be understood in the sense that these potentials are the uncreated and unlimited sources for the real, i.e. created and limited, potentials from which new concrescence processes necessarily take rise. The *whole* spectrum of qualitative, spatiotemporal and creative possibilities ever attainable in the world-process is grounded in the eternal, pure potentials. On the other hand, the *limited* spectrum of qualitative, spatiotemporal and creative possibilities attainable *by each particular occasion* is given by the corresponding created, limited—real—potentials.

Before we start our investigation of the pure potentials by examining the nature of the extensive continuum, a note on terminology is at stake. Unfortunately Whitehead's use of the terms "realization" as well as the corresponding "real", is not consistent throughout his writings. For example, he speaks of the realization of some property by an occasion when referring to the actualization of the property as constituent of the occasion's final definiteness. To avoid such ambiguities we will fix the meaning of the terms "real", "realization", "actual" and "actualization" as follows: *realization* means the transition from pure to real potentiality—from potentials for occasions in general to potentials for a particular occasion. That something is real therefore means that it has the ontological status of a real potential. The further transition from real potentiality to actuality in attainment, as well as from actuality in attainment to attained actuality is called *actualization*. This double meaning of actualization will not give rise to any confusions since it will always be clear from the context in which sense it is used.

2.1 The extensive continuum

The extensive continuum is that wherein actual occasions come to be. It can be understood as *infinite and undivided, but infinitely divisible, extension*. However, the infinite divisibility of the extensive continuum does not mean that it is divisible into *points*; rather it is merely divisible into *finitely extended regions*.[2] The reason for this is that Whitehead intends to build up his ontology on non-pointlike occasions and therefore a divisibility of the extensive continuum into points would be quite functionless—it would merely produce surplus structure. Now the notion of a region of the extensive continuum presupposes a determinate boundary by which this region is demarcated from the rest of the continuum (PR, p. 301). However, since the extensive continuum

[2] By "finitely extended" we will always mean bounded and non-pointlike.

is undivided

> in the extensive continuum, considered in itself, there are no boundaries. Therefore, the regions into which the extensive continuum is divisible are not real or proper regions—they are potential regions. Regions that might be, but are not. [...] The extensive continuum, then, is a potentiality for regions of itself. (WM, p. 208).

Thus the extensive continuum is the pure potential for the "finite extensive continua" embodied by occasions (WM, p. 212). The boundary surfaces by reason of which the, in itself undivided, extensive continuum can be divided into such finitely extended regions, belong, like the qualitative properties which make up the definiteness of an occasion, to the realm of eternal objects that will be investigated in Section 2.2. There we will see that a formerly potential finite region of the extensive continuum is realized by the incoming or instantiation of a closed boundary surface into the extensive continuum.

Besides the three ontological properties already mentioned, i.e. infiniteness, undividedness and infinite divisibility, the extensive continuum has only two further ontological properties. First, it is equipped with a primitive relation, among its potential regions, called *extensive connection* (PR, p. 288, 294). The relation of extensive connection can be understood as the disjunction of the three relations *inclusion*, *overlap* and *contact* (PR, p. 66), i.e. two potential regions of the extensive continuum which stand in the relation of extensive connection are either overlapping, in contact (i.e. contiguous with each other) or one of them is included in the other. However, Whitehead's account of extensive relations is strictly axiomatic. It presupposes only the relation of extensive connection as *primitive* and by demanding certain axioms to hold for this relation Whitehead introduces all other extensive relations, including the relations of inclusion, overlap and contact (PR, p. 294 ff). We need not discuss this axiomatic introduction of extensive relations by Whitehead here. Rather it is sufficient for our purposes to have an intuitive understanding of the mentioned relations. All the relations definable from the relation of extensive connection are purely topological ones, so that the extensive continuum as equipped with these relations is "merely" a topological structure, too. In other words, "[extensive] regions are assumed to posses just those properties which are invariant with respect to topological transformations" (Palter 1960, p. 110). Generally, a topological transformation is a bijective, continuous mapping. For example a deformation that maps a ball to a cube is a topological transformation. All regions which can be transformed into one another by reason of topological transformations are topological equivalent.[3] However, since the extensive continuum is not divisible into points the topology it is equipped with by reason of the relation of extensive connection is a *pointfree topology*. In fact Whitehead has been one of the first people who thought about the possibility of building up a topology on the concept of regions as primitives and then to construct points a forteriori from certain well behaved collections of regions, thereby reversing the conceptual order known from usual set-theoretic topology. Yet it would go far beyond the scope of this work to enter into a discussion of the differences between set-theoretic and pointfree topologies. For an introductory survey of this topic the reader is referred to (Casati and Varzi 1999). Whether equipped with a set-theoretic or pointfree topology, in a "merely" topologically structured extensive continuum the concept of the "shape" of a region as far as it goes beyond topological equivalence, is obviously absent. We will come back to the question of how regions can nevertheless have characteristics that go beyond topological ones below.

Besides the already mentioned ontological properties of the extensive continuum, i.e. its infiniteness, undividedness and its infinitely divisibility into finitely extended regions, there is only one further property termed its *separativeness*. The separativeness of the extensive continuum forbids that any two regions embodied by actual occasions do overlap, i.e. that they are not separated from each other. Thus the creation of a finite extensive region, which is part of what takes place in a process of transition, *is constraint by the separative property of the extensive continuum—the region must not overlap any other real or actual region.* By reason of this separativeness of the extensive continuum, occasions are external to each other—they are *discrete* entities (PR, p. 309; WM, p. 220, 223, 232 f). Of course, non-overlapping or separated regions can well be contiguous, i.e. in contact, with one another. But Whitehead did by no means assume that the regions embodied by occasions

[3]An example of topologically non-equivalent regions are (the surfaces of) a ball and a torus.

must always be contiguous to one another (PR, p. 307 f). Although this can contingently happen, in general, there will be "gaps" of unrealized extension between the bounded regions of occasions (PR, p. 35 f, 307 f; WM, p. 212). Thus contrary to the eternal extension that is a continuum, the aggregate of all the regions embodied by occasions, i.e. of all realized respectively actualized regions, will, in general, not even appear as a continuum. But note that even if the regions occupied by occasions were always contiguous with one another, their aggregate would still not *be* a true continuum since it would still be divided into discrete regions. Thus as in itself not divided, the extensive continuum is indifferently open to divisions by the instantiation of closed boundary surfaces, *provided* the thus created regions do not overlap, thereby being the unlimited potential for finitely extended and separated regions.

As already mentioned in Section 1.3.1, in contrast to the extensive continuum, spacetime is not a fundamental entity in Whitehead's ontology. Rather "physical time and physical space are modifications of extension brought about by the becoming of actual occasions [...]" (WM, p. 22). The extensive continuum is a more general ontological concept than physical spacetime, since "time and space are characteristics of nature which *presuppose* the scheme of extension" (PR, p. 289; italics added). For the regions of occasions to be spatiotemporal rather than merely finite extensive regions more is needed than the instantiation of closed boundary surfaces. Spatiotemporal regions have a definite dimension (in particular the dimension four) and bear metrical relations to each other. Like the boundary surfaces these further properties and relations too are eternal objects which have to be instantiated in the extensive continuum for the thus realized regions to be spatiotemporal regions. Thus "all these [spatiotemporal] properties are additional to the more basic fact of extensiveness" (PR, p. 91). However,

> the fact remains that our world is spatio-temporal; but, for Whitehead this means only that the actual entities of the contemporary [...] world inherit from antecedent actualities, and transmit to subsequent ones, the defining characteristics of our cosmic epoch, one of which characteristics is the four-dimensional structure known to us as 'space-time'. In other words, physical space and physical time are among the abstract constituents of the social order character-

> izing our cosmic epoch—characterizing, that is, what we familiarly term 'our world'. (WM, p. 216)

Our cosmic epoch is that part of the overall world-process that is empirically accessible to us and as such it is that portion of the world-process "whose detailed investigation is the topic of empirical cosmology" (WM, p. 53). In the overall world-process, however, there is room for many other cosmic epochs some of which may be spatiotemporally structured in a way similar to our epoch whereas others will not. Thus that in some cosmic epochs, and particularly in ours, spatiotemporality, i.e. four-dimensionality and the existence of metrical-relations between realized and actualized regions, are pervasive features is a contingent fact. The continued reproduction of spatiotemporal structures by all occasions belonging to our part of the world-process is an instance of what Whitehead calls a *social order*. For Whitehead there are no fixed and unchangeable laws of nature—of course, apart from the general principles governing his ontological system—that must be obeyed by occasions. According to Whitehead's *ontological principle*, it is in fact just the reverse: the laws of physics like all other contingent facts arise from the decisions of occasions (PR, p. 19, 24). Each occasion has a certain freedom in deciding its own final character and by reason of this final character it causally influences other occasions. Now the idea underlying Whitehead's concept of social orders, i.e. of laws of nature, is roughly that if (nearly all) the occasions in the causal past of a new occasion share a certain character—the defining characteristics of the social order—the freedom of the new occasion arising from this past will be strongly biased towards the reproduction of this character (see also Section 2.3.1). The spatiotemporal character of the regions of occasions is such a defining characteristic of the social order of our cosmic epoch, so that "spatio-temporality, though not a true metaphysical category, becomes, nonetheless, a 'categorial' feature of the actualities of our world [i.e. our cosmic epoch]" (WM, p. 217). However, a detailed discussion of Whitehead's account of the emergence of stable structures and ultimately of laws of nature would go far beyond the scope of this work. We will therefore simply introduce some assumptions by which we restrict the domain of application of Whitehead's extremely general theory to the domain we are interested in. Since we have, by definition, no empirical knowledge of cosmic epochs other than our own (if there are any), it is not surprising that our present physical theories, including QFT, with their laws (i.e. the theories and laws of "empirical cosmology") are precisely made for the purpose of describing merely our cosmic epoch. Thus it seems to be justified to treat in this work at least some of the contingent features—of the laws and general circumstances—of our cosmic epoch *as if* they were indeed true categorial features of Whitehead's ontological system, i.e. as if they were metaphysical necessities. In other words, it seems to be justified to *restrict the scope of Whitehead's ontology to those parts of the world-process which obey the same laws than our own cosmic epoch.* In what follows we will therefore assume that the extensive continuum has the further ontological property of four-dimensionality which is one of the most obvious lawlike features of our cosmic epoch. Since this work moreover is only concerned with an ontological interpretation of QFT, another simplifying assumption will be made. QFT does not only presuppose that spacetime is four-dimensional, it also presupposes that spacetime is "flat". More precisely, it presupposes that spacetime is equipped with the metric, i.e. the spatiotemporal distance measure, of STR. Therefore, we will deviate from Whitehead's original theory by also treating this presupposition of QFT as an *ontological property of the extensive continuum itself.*[4] Thus we assume that the distance measure—the metric—of STR is, like the relation of extensive connection and the separative property, an *inherent feature of the extensive continuum*, so that the distance between any two potential regions of the extensive continuum is now well-defined. Note that the metric of STR is understood here as a purely spatiotemporal—or more correctly extensive—relation *without any causal connotations.* That STR need not be understood as a theory linking causation and spacetime at all will be discussed later on in Section 2.8.2.

Now if the extensive continuum is equipped with the metric of STR *its potential regions are automatically determinate up to metrical properties.* Thus two potential regions of the extensive continuum are distinct iff they are distinct with respect to any topological or metrical properties. For the boundary surfaces of regions this means that they are determinate up to Poincaré transformations, which are the invariance transformations corresponding to the relativistic metric (see Chapter 5). Put differently, two boundaries are distinct

[4] This assumption will be further refined in Chapter 6.

iff they *cannot* be transformed into one another by means of some Poincaré transformation. To assume that the boundary surfaces contained in the realm of eternal objects are more determinate than up to Poincaré transformations, would merely introduce useless surplus structure. For any determinateness of boundary surfaces that goes beyond Poincaré equivalence, would be meaningless as soon as these boundaries where instantiated in the extensive continuum, because the latter, as equipped with the relativistic metric, is simply not sensitive to such a determinateness. For reasons of ontological economy one should therefore not postulate the existence of such more determinate boundaries in the realm of eternal objects.

Note, however, that *two (potential) regions* of the extensive continuum can very well differ with respect to metrical properties while being Poincaré transforms of one another. For example, two regions connected by a spatiotemporal translation bear a non-zero distance to one another and are therefore *distinct regions despite being realized by the instantiation of the same boundary surface.*

To sum up our discussion of the extensive continuum, we will assume throughout this work, that the extensive continuum is not only an uncreated, infinite, undivided, infinitely divisible, separative continuum, but that it is moreover four-dimensional and that for each two potential regions of it their distance is quantified by reason of the special-relativistic metric. Yet, with Whitehead, we do *not* assume that the regions embodied by occasions (which due to our assumptions as to the dimensionality and metricity of the extensive continuum are always spatiotemporalized regions) are also contiguous with one another. In other words, we do not assume that at any stage of the world-process the aggregate of all the already actualized regions appears like a continuum. Rather we allow for the possibility that between the spatiotemporal regions of occasions there are "gaps" of unrealized extension. In Section 3.1 we will see that this further assumption of contiguity is in fact not suggested by quantum physics.

2.2 Eternal objects

Eternal objects are the pure potentials for the spatiotemporal and qualitative character of occasions. They provide the properties and relations—understood as universals—which can be instantiated in occasions respectively in the extensive continuum.[5] It is quite usual that an entity is called a universal if it can be instantiated in more than one particular. This loose definition of universality along with the definition of particularity on which it relies shall now be specified as follows: an entity is a *particular* iff it can exist only in one connected spatiotemporal (or extensive) instance, i.e. if it can occupy, embody or be located only in a single connected region of spacetime (respectively of the extensive continuum).[6,7] Therefore, an entity is a universal iff it—as a whole—can exist in more than one separated spatiotemporal (or extensive) instances, i.e. if it—as the self-same entity—can occupy, embody or be located in two or more separated regions of spacetime (respectively of the extensive continuum). Later on in Section 8.1.7 we will strengthen this still very weak definition of universality. That this understanding of universality is indeed weak follows from the fact that according to it, substances would also count as universals. This is because, among other things, substances are usually held to have diachronic identity and thus can wholly occur in separated regions (see e.g. Seibt 1990), so that according to the above criterion they would count as universals. However, for the time being the weak criterion formulated above serves our purposes.

It is important to notice that *not* all relations in Whitehead's ontology which can obtain between occasions are eternal objects. For example, the re-

[5] For convenience we will in most cases only speak of "the instantiation of eternal objects in occasions" rather than more correctly of "the instantiation of eternal objects in occasions respectively in the extensive continum".

[6] As already mentioned, in Whitehead's original ontology spacetime regions are not necessary features of all occasions (see Section 2.1) so that in this theory one can only speak of occasions occupying extensive, rather than spatiotemporal, regions when making general statements about occasions.

[7] Usually the connectedness of a region $\mathcal{O}$ is defined as the property that each pair of points $x, y \in \mathcal{O}$ can be connected by a continuous curve that does not leave $\mathcal{O}$. However, since according to Whitehead extensive regions do not consist of points at all, one cannot introduce the property of connectedness in a way that presupposes the existence of points. Whitehead dealed with this problem by simply using the concept of a connected region as a primitive one. Although Whitehead did not made this explicit it is clear from Part IV of PR that his primitive concept "region" is in fact what one usually understands as a connected region.

latedness of two occasions as cause and effect is not given by, or reducible to, eternal objects as will become clear when we discuss processes of transition. Moreover, the primitive relation of extensive connection and all the other extensive relations defined from it are not eternal objects either. Because of our simplifying assumption that the extensive continuum comes already equipped with a fixed distance measure, the distance relations between regions too, are not given by eternal objects. Since these kinds of relatedness are the fundamental ones that can obtain between Whiteheadian occasions and since non of them is an eternal object we will in what follows only discuss Whitehead's theory of eternal objects as far as it is concerned with monadic universals, i.e. with properties understood as universals.

In itself each eternal object can be instantiated in every occasion whatsoever, i.e. in itself it is completely neutral in respect to its instantiations in particular occasions. However, not every eternal object is in fact instantiated in every occasion. The instantiation of a particular eternal object in a particular occasion is a contingent fact that is determined by the occasion's actual world via a process of transition. Thus the specific connection of a given eternal object to a given occasion is not eternally fixed and definite (SMW, p. 163), but is contingent on the actual course of the world-process.

The qualitative determinateness or definiteness of an occasion is a selection of qualitative properties, which as we will see later on belong to the so-called *subjective species* of eternal objects (see Section 2.2.1). Each occasion starts its becoming already equipped with a certain range of qualitative properties that is determined by the actual world of this occasion in a process of transition. In the subsequent phases of concrescence the occasion autonomously decides which of these initially given properties will be integrated into the final complex of properties that constitutes the determinate qualitative character—the final definiteness—of the completed occasion.

Contrary to this an occasion has no freedom to decide which region it will embody. The bounded spacetime region of an occasion is created already in the process of transition from whose outcome the self-creative becoming of the occasion takes rise. It is created by the instantiation of a boundary surface in the extensive continuum, whereby a potential finitely extended region of this continuum is transformed into a real or proper finitely extended region

(WM, p. 219, 247, 314 f). Because of our above assumption on the ontological properties of the extensive continuum, these boundary surfaces are the only *objective* eternal objects needed in the context of this work. More precisely, only surfaces which demarcate finite, four-dimensional volumes of extension are needed. In mathematical terms, our boundary surfaces will be closed, four-dimensional hypersurfaces. Due to the realization (respectively actualization) of each new region the distances between this region and the already realized (actualized) regions are also realized (actualized), whereas its distances to all potential regions are still merely potential distances.

As mentioned above the region of an occasion is that element in its ontological make up that accounts for the self-identity of the occasion in all phases of its existence. An occasion is *numerically one entity in different phases of its existence* because the spacetime region with which it is provided by its actual world via a process of transition is the fixed spatiotemporal standpoint for each phase of the occasion's concrescence as well as for the completed occasion qua attained actuality (PR, p. 283). Because the region of an occasion, once realized, will therefore necessarily also be actualized as the spatiotemporal standpoint of the occasion qua attained actuality, the distinction as to its ontological status, i.e. real potential, actuality in attainment and attained actuality, is only of minor importance.

2.2.1 Ingression and the two species of eternal objects

Instead of the instantiation of eternal objects Whitehead speaks of the *ingression* of eternal objects into occasions. As will become clear in what follows, Whitehead's notion of ingression is somewhat different from the usual notion of instantiation. What both have in common is that they are supposed to account for the entrance of universals into the constitution of particulars.

> 'Ingression' [...] is the technical term for the functioning or inclusion of an eternal object within an occasion. (WM, p. 194)

Moreover, like the notion of ingression in Whitehead's ontology, the notion of instantiation too is often construed as primitive within an ontological theory, i.e. as something that is presupposed by other aspects of the theory but that itself cannot be further analyzed within this theory.

Now Whitehead differentiates between two kinds of ingression of eternal objects in occasions—*restricted* and *unrestricted* ingression. In general, eternal objects which are ingressed in an occasion are alternatives among which the occasion can decide its final determinate character. Not all of these alternatives can be in fact selected and thus integrated into the final determinate character of the occasion, some of them necessarily have to be rejected, since the aim of each concrescence process is the intensification—the deepening—of the occasion's individuality (see Jones 1998). The aim of each concrescent occasion is a determinate character that is as specific as possible, and the less alternatives are integrated into the final determinateness of an occasion, the more heightened will be its final individuality (see Section 2.3.2). Those alternatives which are in fact integrated into a particular occasion's final determinateness, i.e. which contribute to the individuality of the completed occasion and thus "remain[ ] as an everlasting feature of the everlasting product [i.e. of the occasion qua attained actuality]" (WM, p. 197), are said to have unrestricted ingression into that occasion. The eternal objects which are ingressed into an occasion but are not integrated into the occasion's final character are said to have restricted ingression into that occasion (PR, p. 290). Therefore, restricted and unrestricted ingression are *not* two distinct ways by which eternal objects initially "enter into the constitution of", or are "made available to", occasions. Rather there is only one such way—ingression. Whether a given eternal object A has restricted or unrestricted ingression into an occasion into which it is ingressed is decided by the occasion's later, self-creative phases of concrescence. Thus "A has unrestricted (restricted) ingression into occasion E" merely means that A is ingressed into E and has (not) been integrated into E's final determinateness. Thus the usual notion of instantiation corresponds to Whitehead's unrestricted ingression, since when a property is instantiated by a particular it is understood to contribute to the particular's determinate character. Restrictedly ingressed eternal objects have been included in, and evaluated by, the concrescent occasion up to some phase of the concrescence. But then they have been rejected for entrance into all later phases and thus especially from conferring their determinate character to the completed occasion, i.e. from unrestricted ingression. Restricted ingression, therefore, is a genuine Whiteheadian conception that relies on the internal processual character of occasions and does not seem to have any parallels in other accounts of the connection between universals and particulars.

However, there are eternal objects which, if ingressed into occasions (more precisely into the extensive continuum), necessarily also have unrestricted ingression, because they cannot be eliminated in the course of the concrescence. Whitehead calls such eternal objects *objective* eternal objects. All others, i.e. which fall under the autonomous decisions of a concrescent occasion into which they are ingressed and thus can be withheld from unrestricted ingression, are called *subjective* eternal objects (PR, p. 290 f). Therefore, what has been said above, namely that the eternal objects ingressed into an occasion are alternatives among which the concrescent occasion can decide its final determinateness does not apply to objective eternal objects—*it only applies to subjective ones.* As explained above, in the context of this work the only relevant eternal objects of the objective species are the boundary surfaces of four-dimensional, finitely extended regions. That these boundary surfaces have to be objective eternal objects follows from the fact that one important function of the spacetime region created in a process of transition is that it secures the self-identity of the thereof arising occasion in all phases of its becoming and being. Therefore, this spacetime region is presupposed as the *determinate and unalterable basis*, from the very start of the concrescence process of the new occasion. Since therefore this region cannot be changed without destroying the *self-identity* of the occasion, the boundary surface by whose ingression into the extensive continuum the spacetime region is created clearly cannot fall under the autonomous decisions to be settled by the concrescent occasion. Thus boundary surfaces are eternal objects of the objective species, because their ingression necessitates their unrestricted ingression.

The only subjective eternal objects needed in this work are qualitative physical properties. Subjective eternal objects can ingress unrestrictedly as well as restrictedly. In the second and last phase of a transition process a selected range of alternative subjective eternal objects, i.e. qualitative properties, which make up the initial definiteness of the new occasion ingress into the spacetime region that has already been created in the first phase of the transition process in question. Those subjective eternal objects which are selected (rejected) by the concrescent occasion for contributing to its final definiteness—and thus to its individuality qua attained actuality—have unrestricted (restricted) ingression into that occasion.

Finally, there is another important point in connection with Whitehead's notion of the ingression of subjective eternal objects into some region $\mathcal{O}$ that needs to be mentioned here. According to Whitehead, a subjective eternal object A ingressed (restrictedly or unrestrictedly) into region $\mathcal{O}$ *indifferently belongs to the whole region* $\mathcal{O}$ *and not merely to some subregion* $\mathcal{O}' \subset \mathcal{O}$. This is important because otherwise, completed occasions were not atomic in the sense of not being divided into parts which are completed occasions in their own right (PR, p. 62, 219, 283 ff). For if different subregions $\mathcal{O}_i$ of the region $\mathcal{O}$ of an occasion E were endowed with different qualities A_i, each of them would constitute a completed occasion E_i in its own right, so that occasion E were in fact *divided* into these other occasions. Of course, since the region of an occasion is *divisible* into subregions an occasion can be *thought of* as divided into sub-occasions, each corresponding to a certain subregion. But such a division is merely a conceptual construct—a division in thought—*as long as it is not grounded in any objective features of occasions.* And when each of the subregions of an occasion is endowed with *the same (complex) quality* A as the whole region $\mathcal{O}$, there simply is no ontic feature of the whole occasion that would provide such an ontic fact of the matter, so that the undividedness of occasions is not undermined by the conceptual divisibility of their regions.

2.2.2 Simple and complex eternal objects

A complex eternal object, also called a pattern, expresses a relationship, or as Whitehead calls it, a *manner of relatedness*, among a definite set of other eternal objects (PR, p. 114 f; SMW, p. 164 ff). The eternal objects which are the relata of a given pattern are termed the *components* of the pattern (WM, p. 176). Thus the ingression of a complex eternal object is the joint ingression of all its components according to the manner of relatedness provided by the complex eternal object. However, qua pure potential, i.e. in abstraction from its ingressions into occasions, "a complex eternal object merely expresses the possibility of the joint realization [i.e. ingression], in some actual entity [...]" (WM, p. 177; see also SMW, p. 164). Eternal objects which are not complex but rather are the ultimate components of all the potential relationships established by complex eternal objects, are termed *simple eternal objects* (SMW, p. 166). Thus simple eternal objects are the *ultimate* spatiotemporal and qualitative properties in Whitehead's ontology. All complex eternal objects can ultimately be analyzed into relationships among simple eternal objects (WM, p. 177). As mentioned above, we do not need to discuss complex eternal objects which are potentials for *relations* among occasions (more precisely: for relations among eternal objects ingressed into *different* occasions). Rather we merely need to investigate complex eternal objects which can only be realized by ingression into *single* occasions, i.e. which are potentials for complex *properties* of occasions. In other words, we need only discuss those complex eternal objects which by their ingression can contribute to the determinate character of *single* occasions. Furthermore, we need not discuss all simple eternal objects inherent in Whitehead's complete theory either. For example, according to Whitehead, a definite shade of green is a simple eternal object of the subjective species. But it is clear that such qualities are not qualitative properties that can contribute to the definiteness of occasions describable by QFT. Rather the eternal objects of the subjective species relevant in the context of this work are only the physical properties as suggested by QFT, i.e. the properties connected with definite values of physical magnitudes like mass, electrical charge, spin, energy, etc.

The only eternal objects of the objective species needed in this work, namely surfaces of finitely extended four-dimensional regions, are simple. Note that the simplicity of these boundary surfaces is *not* an artefact of our simplifying assumption that the extensive continuum comes already equipped with a dimensional and a metrical structure. Of course, if this assumption had not been made the distance relations obtaining between the spacetime regions of occasions would have had to be construed as *complex* objective eternal object. Thus in this case we would have had also to take into account complex eternal objects of the objective species. But this would have had no effect on the status of boundary surfaces as *simple* eternal objects. This is so because the extensive continuum does not consist of points anyway (see Section 2.1). Therefore, the boundary surfaces by whose ingression the finitely extended regions of occasions are created, *cannot be construed as relations among points*, *but rather have to be itself the ultimate relata of distance relations*, regardless whether the latter are construed as complex eternal objects or as relations inherent in the extensive continuum itself as done in this work.

Whereas each simple eternal object is either of the objective or of the subjective species this is not the case for each complex eternal object. The reason is that there are complex eternal objects which have among their components both objective as well as subjective eternal objects and thus do not belong to either of these "pure species" of eternal objects. In other words Whitehead's distinction between subjective and objective eternal objects is an exhaustive one only among all simple eternal objects. Complex eternal objects which do not belong to either of the two pure species will henceforth be referred to as *mixed* ones. Mixed eternal objects are needed because otherwise the two pure species of eternal objects were totally isolated from each other and thus elements of both species could not jointly ingress into occasions. This, however, would have the disastrous consequence that occasions could not at the same time have spatiotemporal and qualitative properties.

Unfortunately, from Whitehead's writings it is far from clear which "manners of relatedness" can obtain between eternal objects—or in other words and more correctly (see Section 2.2.4): which (kinds of) complex eternal objects there are. Yet it is to be expected that for his theory to be applicable to the domain of physics the two logical connectives $\wedge$, i.e. conjunction/and, and $\vee$, i.e. disjunction/or, will be sufficient—in fact we will see later on that only *one* of the two connectives $\wedge$ and $\vee$ is really needed. It is clear that the negation $\neg$ *cannot* be a manner of relatedness among eternal objects, *because it simply does not relate anything at all.* In other words, there are no eternal objects of the form $\neg A$—*no negative eternal objects.* According to Whitehead all propositions, which prima facie seem to assert the existence of some negative property (or relation) are to be understood and reformulated as asserting the lack or absence of some positive properties (or relations) (PR, p. 154, 239 f, 267, 273 f; SMW, p. 162). In Section 8.1.3 we will see that, at least for physical properties, this view is well supported. Generally, the postulation of negative entities causes trouble because contrary to positive entities, negative ones have no determinate identity criteria. For example, there is obviously no non-arbitrary answer to the question "How many 'non-shortcuts' did occur yesterday?". On the other hand, it is not clear whether one can generally avoid an ontological commitment to negative entities like negative properties or relations by reformulating propositions about them, such that the problem of negative entities does not reappear in one or another form. For example, to avoid speaking of the possession of some negative property $\neg A$ one would like to reformulate all propositions in which the possession of $\neg A$ is asserted into ones asserting the non-possession of the positive property A. However, this solution could be charged of making use of the *negative relation* "non-possessing". Moreover, it is a non-trivial task to formulate criteria which in the first place unambiguously decide whether a given entity is genuinely positive or negative at all. The most promising criteria for this are either those which are based on a difference in the specificity of negative and positive entities or on a difference in their logical entailments. For example, positive but not negative properties entail both properties of the same and of different kinds as themselves: whereas non-red entails only non-crimson and other properties of the same kind, red entails both, non-green and colored, where the latter is a property of another kind than red itself. Since a more detailed discussion of the topic of negative entities would go far beyond the scope of this work we will simply assume, with Whitehead, that prima facie negative entities are ultimately reducible to positive ones. Moreover, as already mentioned above, we will see later on that this assumption is well supported, at least in regard to physical properties.

Now we want to point out that there is a tension between the existence of conjunctive eternal objects, i.e. complex eternal objects of the form $C = A \wedge B$, and Whitehead's definition of the simplicity of eternal objects by their *not having components*. The problem is that it seems that *any* property can be written as a conjunction of two (or more) other properties so that Whitehead's definition of simplicity turns out to be vacuous. For example, the property C expressible by the sentence "having a mass of 70kg" can be understood as the conjunction of the two properties A and B expressible by "having a mass between 65kg and 70kg" and "having a mass between 70kg and 79kg". On the other hand, it is clear that if one of the properties A, B and C deserves the predicate "simple" at all, it is C but not A or B. This is because we intuitively understand simplicity as having no *disjunctive* components rather than having no conjunctive components. Therefore, it seems reasonable to bring Whitehead's incomplete definition of simple eternal objects to an end by defining: *an eternal object is simple* iff it does not have disjunctive components nor do its components have disjunctive components etc. The need for the iteration in this definition derives from the fact that it is not clear whether Whitehead conceives the relation of componenthood as transitive, so that the components of a component of an eternal object A need not also be components of A (see SMW, p. 166). However, for convenience we will henceforth speak *as if* it were clear that "is a component of" is transitive. For example, instead of saying that a complex eternal object C has components whose components have the simple eternal objects A and B among their components, we will simply say that A and B are simple components of C. Of course, when arguing for a certain claim we have to be careful not to make use of transitivity in connection with the relation of componenthood.

Finally a point that will become important later on shall be mentioned. Without ultimate components—simple eternal objects—of which all complex eternal objects are "build", the latter were not fully determinate. For, by definition, the non-existence of simple eternal objects means that *each* eternal object can be written as a disjunction of other eternal objects and *thus there simply is no unique way in which a given eternal object is to be written as disjunction of others.* For example, the complex eternal object $C = A \vee B$ could also be written in the form $C = A \vee B_1 \vee B_2$ or $C = A_1 \vee A_2 \vee B_1 \vee B_2$, if $B = B_1 \vee B_2$ and $A = A_1 \vee A_2$ also hold, or as some more complicated disjunction, ad infimum. Moreover, since as argued above each eternal object can be written as a conjunction of others, without simple eternal objects *there would be the further ambiguity as to the very disjunctive or conjunctive character of a given complex eternal object.* On the other hand, if there is a stock of simple eternal objects which are the ultimate components of all other eternal objects, then there is a unique form of each complex eternal object, namely the one in which only simple eternal objects appear. Later on we will have to discuss this problem in more detail, since *QFT strongly suggests that there are no simple subjective eternal objects.*

2.2.3 Comparison with two other theories of universals

Platonic forms

Whitehead's theory of eternal objects in one respect strongly resembles Plato's theory of forms or ideas. Like Plato's forms, Whitehead's eternal objects are held to *exist independently from the fact whether they are instantiated in some occasion* (WM, p. 214). Thus like Plato and contrary to Aristotle, Whitehead promotes a strong kind of realism of universals, i.e. of universalia ante res, according to which universals exist whether they are instantiated in the concrete world or not. But in its details Whitehead's theory of eternal objects is quite different from Plato's theory of forms. First of all, unlike Plato's forms eternal objects are *not* the entities which exist in a primary sense, whereas the existence of all other entities is deficient compared to that of eternal objects. To the contrary, for Whitehead the fullest sense of existence is actuality, and this highest grade of existence is only attributed to the self-realizing and self-realized modes of existence of occasions. All other entities, including eternal objects, are only a means for the becoming and being of ever new actual occasions. Eternal objects exist as pure potentials *for* actualization in occasions, i.e. they are "meant precisely for that role of ingression" (Pols 1967, p. 7). This means that eternal objects, though existing independently from any ingressions into the concrete world, nevertheless are necessarily related to occasions in general, since "they are possibilities *for* actualization, which any actual entity can take into account, or they are indistinguishable from non-entity" (WM, p. 176; see also SMW, p. 159). Contrary to this, Plato's forms exist in a self-sufficient way with no necessary reference to things beyond themselves, i.e. their exemplifications in the concrete world are only "accidental episodes" in their self-sustained existence. Furthermore, according to Plato's account of the participation of concrete things in forms, this participation is always imperfect, i.e. a concrete thing never exemplifies the forms in which it participates in their whole perfection. In contrast to this, an eternal object as ingressed into occasions is the *self-same* eternal object as existing apart from these ingressions, i.e. eternal objects are *perfectly* reproduced in occasions (SMW, p. 159, 171; WM, p. 191) and thus eternal objects "do not have another and ideal state of being over against which the definiteness of actual entities is somehow deficient" (Pols 1967, p. 164). Thus despite the fact that Whitehead sometimes explicitly calls his eternal objects "Platonic forms" (PR, p. 43 ff) there are important differences in the details of Whitehead's and Plato's accounts of universals.

Armstrong's theory of universals

There are two aspect of D. M. Armstrong's theory of universals (1978) which are in agreement with Whitehead's theory of eternal objects. Since here we can ignore relations we will only discuss the case of properties. First of all, Armstrong shares with Whitehead the view that *besides* simple properties there have to exist complex ones: for a particular to instantiate both of the simple properties A and B there has to exist *besides* A and B, the complex property $C = A \wedge B$. The second feature Armstrong's theory shares with Whitehead's is the denial of negative properties—negative properties are assumed to be always reducible to the lack, absence or non-occurrence of positive properties and thus have no existential status of their own.

However, Armstrong and Whitehead do *not* agree about the way of existence of (positive) universals. As we have seen Whitehead follows Plato in holding that universals, including uninstantiated ones, do exist in a realm separated from the world of concrete particulars. Armstrong, however, follows Aristotle in promoting a theory of universalia in rebus. For Aristotle as well as for Armstrong *a universal only exists as far as it is a constituent of at least one particular*. This means that for example the universal roundness can be eliminated from existence by eliminating all round things in the concrete world. In other words, for Aristotle and Armstrong the existence of a universal depends on the existence of its instances. Note that the universalia in rebus view is not a variant of nominalism. A nominalistic account of universals tries to *eliminate* universals as a fundamental ontological category in that they are reduced, in one way or another, to certain classes of particulars. For example, according to one variant of nominalism, roundness is simply identified with the class of all round things in the world. Contrary to this the universalia in rebus view holds that the fact that there are many round things is explained by the existence of the universal roundness that is a common ontological constituent of all round things.

2.2.4 The compatibility of eternal objects

First of all, two eternal objects A, B are *compatible relative* to another eternal object C if both of them are components of C, i.e. if A and B stand in the manner of relatedness provided by C. Otherwise, A and B are called *incompatible relative to* C. Accordingly two eternal objects are *compatible* if there is a complex eternal object C such that A and B are components of C and *incompatible* if there is no such complex eternal object that has both A and B among its components. Now under the reasonable assumption that the logical disjunction $\vee$ is a manner of relatedness among eternal objects, it seems that *each* eternal object A *is compatible with each other* eternal object B, since $A \vee B$ is always a meaningful property if A and B are, so that the notion of (in-) compatibility would be vacuous because every two arbitrary eternal objects are compatible. However, this overlooks that *manners of relatedness are not independently existing entities* in Whitehead's theory of eternal objects—they do not exist in independence from complex eternal objects. It is just the other way round: *manners of relatedness are merely abstractions from complex eternal objects*, e.g. the manner $\vee$ does not exist as such (in the realm of eternal objects) but merely "in" complex eternal objects of the form $A \vee B$. Thus complex eternal objects are *ontologically prior* to the "relations" $\vee$ and $\wedge$—in fact, the latter are only concepts derived from certain complex eternal objects by way of abstraction and generalization. Therefore, the above argument for the vacuousness of the notion of incompatibility does not go through: whether for two given eternal objects A and B there exists the complex eternal object $A \vee B$ cannot be deduced from the mere logical meaningfulness of the latter—not all logically meaningful complexes of properties need to exist, not even qua pure potentiality. In other words, pure potentiality can very well be ontologically more restrictive than logical possibility. As the reader may expect, Whitehead did not made explicit what the precise relation between logical possibility and his notion of pure potentiality (in regard to eternal objects) is. However, we will see in a moment that there is a principle in Whitehead's ontology that can be interpreted as implying that pure potentiality is in fact the *more restrictive* *mode of possibility* compared to logical possibility—at least in regard to *mixed* eternal objects. Furthermore, in connection with QFT, it will turn out that the disjunctive subjective eternal object $A \vee B$ does not exist for each pair of subjective eternal objects A and B, so that even in regard to the realm of *subjective* eternal objects alone, the notion of incompatibility is not vacuous.

Before we go on to investigate the mentioned principle of Whitehead's theory that implies the incompatibility of some subjective eternal objects with each given objective one, we want to point out that *the only mixed eternal objects needed in Whitehead's ontology are conjunctive ones*, i.e. ones of the form $A \wedge O$. This is because each completed occasion shall have *both* a definite qualitative *and* a definite spatiotemporal character. And this obviously means that the (complex) subjective eternal object A singled out in the course of an occasion's concrescence must be synthesized with the occasion's objective eternal object O into the unity of the conjunctive mixed eternal object $A \wedge O$. Thus whenever we speak of mixed eternal objects in what follows we shall always mean conjunctive ones.

Now a subjective eternal object A is, by definition, compatible with the objective eternal object O if there is a complex eternal object—of mixed type—that has both A and O among its components, which means that at least one mixed eternal object of the form $C(\ldots, A, \ldots) \wedge O$, where $C(\ldots, A, \ldots)$ is some complex subjective eternal object that has A among its components, exists (qua pure potentiality). If the compatibility of eternal objects were only restricted by the requirement of logical consistency, as believed, for example by Kraus (1998, p. 28 ff, 34), *all conjunctions of arbitrary subjective and arbitrary objective eternal objects would exist*, since there can hardly arise a *logical* contradiction from the conjunction of qualitative properties with boundary surfaces. Yet this stands in opposition to a principle of Whitehead's ontology which says that the spatiotemporal standpoint $\mathcal{O}$ created in the first phase of a transition process by way of the ingression of an objective eternal object O into the extensive continuum, functions as a *limitation* for the following phases of transition and concrescence—in particular *for the following ingression of subjective eternal objects* and thus for the initial definiteness of the new occasion. Since if all logically possible conjunctions of subjective and objective eternal objects would really exist in the realm of eternal objects, the region $\mathcal{O}$ could not be "a limited potentiality for objectifications [i.e. for the ingression of subjective eternal objects]" (PR, p. 67; see also p. 152, 288). More explicitly, if each mixed eternal object $B \wedge O$, where B is an arbitrary subjective eternal object, would be contained in the realm of eternal objects, *each* subjective eternal object could ingress into the bounded spacetime region $\mathcal{O}$ and thus this spacetime region $\mathcal{O}$, respectively the objective eternal object O, would *not in any way constitute a limitation for such ingressions.* Thus pure potentiality *has to be more restrictive than logical possibility*—at least in regard to mixed eternal objects—since otherwise the role of the region that is realized in a transition process as the first limitation for the ingression of subjective eternal objects were undermined. Therefore, the Whiteheadian notion of (in-) compatibility is not vacuous, since at least in regard to mixed eternal objects not all logical possibilities can in fact exist in the realm of eternal objects.

2.2.5 Abstractive hierachies

The definite status of an eternal object in the realm of eternal objects is given by its being a component of particular other (more complex) eternal objects (SMW, p. 164). This status of an eternal object can be further analyzed into its status in more restricted sub-structures of the realm of eternal objects called *abstractive hierachies.* One aspect of the relevance of abstractive hierachies in the constitution of occasions lies in the fact that the range of qualitative properties, i.e. the definiteness, in each but the last stage of a concrescence process is such an abstractive hierachy (see Section 2.2.6). Whitehead defines abstractive hierachies in the following way:

> An 'abstractive hierachy based upon G', where G is a group of simple eternal objects, is a set of eternal objects which satisfy the following conditions,
>
> (i) the members of G belong to it, and are the only simple eternal objects in the hierachy,
>
> (ii) the components of any complex eternal object in the hierachy, are also members of the hierachy, and,
>
> (iii) [the elements of] any set of eternal objects belonging to the

> hierachy [...] are jointly among the components or derivative components of at least one eternal object which also belongs to the hierachy. (SMW, p. 167 f)

From the conjunction of (i) and (ii) it follows that an abstractive hierachy based upon a set G of simple eternal objects includes only complex eternal objects whose simple components are *all* among the members of G. Condition (iii) is called the *condition of connexity* because it demands that the elements of each set of members of a hierachy are again components of at least one other eternal object in that hierachy. In other words, the condition of connexity not only demands that each set of members $\{A_i\}$ of a hierachy are compatible with one another, but moreover that at least one (more) complex eternal object C relative to which the A_i are compatible is itself a member of the hierachy. This condition, for example prohibits that a group G of simple eternal objects is itself a hierachy, since in this case not all members of the hierachy would be jointly among the components of another member of the hierachy—G does not include any complex eternal object at all that could have members of G as its components, let alone all members of G. It also implies that *no element of the base G is superfluous for a hierachy based on G*, i.e. that each member of G is in fact a component of at least one complex eternal object in the hierachy and further that a set G of simple eternal objects can only be the base of a hierachy if all elements of G are compatible. But the conditions (i)-(iii) do *not* imply that an abstractive hierachy $H(G)$ contains *all* complex eternal objects whose components are in $H(G)$. In particular, they do not imply that $H(G)$ contains all complex eternal objects whose simple components are members of G. This means that, in general, many different abstractive hierachies can have the very same base. Yet in connection with QFT we will only need such hierachies $H(G)$ which in fact contain *all* complex eternal objects whose components are contained in $H(G)$ and thus especially all complex eternal objects whose simple components are contained in G. Therefore, we will from now on only take into account those hierachies which are *maximal* in this sense, i.e. which besides (i)-(iii) fulfil the further condition that[8]

> (iv) *all* complex eternal objects whose components are among the

[8] This condition is not explicitly introduced by Whitehead.

members of an abstractive hierachy $H(G)$ are themselves members of $H(G)$.

Now it is easy to see that each set G of compatible simple eternal objects (a) determines a *unique* maximal hierachy based upon G and that (b) this maximal hierachy includes all other hierachies based upon G, which justifies the qualification "maximal". For if $H(G)$ is a maximal abstractive hierachy based upon G, by condition (iv), there can be no complex eternal object C whose simple components are all in G that is not itself contained in $H(G)$. Consequently there can be no other hierachy based on G which is not completely contained in $H(G)$, which at once proofs (a) and (b). From this it moreover follows that (c) two maximal hierachies are distinct iff their bases are distinct.[9] As mentioned above this need not be the case for non-maximal hierachies—non-maximal hierachies need not be identical if their bases are.

Altogether a maximal abstractive hierachy $H(G)$ with base G is a substructure inherent in the overall relational structure of the realm of eternal objects, with the following properties:

(1) $H(G)$ is uniquely determined by G and contains all (non-maximal) hierachies based upon G.

(2) $H(G)$ contains all and only those complex eternal objects whose simple components are all among the members of the base G.

(3) All members of $H(G)$ are mutually compatible with one another.

(4) For each set of members $\{A_i\} \subseteq H(G)$ of the hierachy there is at least one (more complex) eternal object in the hierachy relative to which the elements of $\{A_i\}$ are compatible.

Thus a maximal abstractive hierachy $H(G)$ contains all (and only those) complex eternal objects which are pure potentials for joint ingressions of members of its base G. In other words, the maximal hierachy $H(G)$ exactly encompasses all the pure possibilities for the joint ingression of arbitrary subsets

[9]But note that for two maximal (or non-maximal) hierachies to be distinct their bases clearly need not be *disjoint*.

of the set G of compatible simple eternal objects. Since we have restricted our scope to conjunctive and disjunctive complex eternal objects, *the maximal abstractive hierachy $H(G)$ therefore looks as if it were generated by arbitrary combinations of disjunctions and conjunctions of elements from G.* Of course, it is not literally "generated" in this way, because this would presuppose the existence of the manners of relatedness $\wedge$ and $\vee$ independently from conjunctive and disjunctive eternal objects. However, to think of a maximal abstractive hierachy in this way makes its structure more "visible".

2.2.6 Functioning of eternal objects in the constitution of occasions

We will now summarize the functioning of eternal objects in the constitution of occasions as far as relevant for this work. The creation of a new occasion, though not the self-creation and thus not the genuine existence of a new occasion, starts with the first phase—the so-called dative-phase—of a process of transition. In this initial phase of a transition process an objective eternal object O—a boundary surface—and the position for its ingression into the extensive continuum are determined by the corresponding actual world (WM, p. 313 f, 321). By the ingression of O a finitely extended spacetime region $\mathcal{O}$ is realized as outcome of the dative phase. Furthermore, by reason of compatibility respectively incompatibility to the objective eternal object O, the region $\mathcal{O}$ constitutes a limitation for the ingression of subjective eternal objects, i.e. of qualitative properties, thereby being a limited or real potential for all following phases of transition and concrescence. Only those subjective eternal objects which are compatible with the objective eternal object O can ingress into the region $\mathcal{O}$. Recall that the compatibility of two eternal objects, in the present case of the objective eternal object O and a subjective eternal object A, means that there exists a complex eternal object, in this case a mixed one, that provides a manner for the joint ingression of O and A, i.e. that has the objective eternal object O and, perhaps besides other subjective eternal objects, also A among its components. Let $P(O)$ denote the set that contains all subjective eternal objects compatible with O as well as the mixed (and thus complex) eternal objects relative to which they are compatible with O, i.e. all eternal objects of the form $A \wedge O$ where A is an arbitrary complex subjective eternal object compatible with O. Yet, in general, not all of the subjective eternal objects in $P(O)$, i.e. which are compatible with O and thus can in principle ingress into the region $\mathcal{O}$, need also compatible with one another. In other words, the set $P(O)$, in general, contains many *distinct (maximal) abstractive hierachies*. In the conformal phase—the second and last phase of a transition process—one set of mutually compatible simple subjective eternal objects $G(O)$ is singled out from $P(O)$ and ingresses into the region $\mathcal{O}$. This set $G(O)$, together with the objective eternal object O, determines a unique maximal abstractive hierachy $H(O, G(O))$ that provides the complex properties available for the autonomous decisions in the following process of concrescence. All subjective eternal objects which are not contained in the hierachy $H(O, G(O))$ do not contribute to the initial definiteness and therefore cannot contribute to the final definiteness of the subsequently arising occasion either—they are real *im*possibilities for this occasion, but nevertheless pure possibilities for further occasions. The base $\{O, G(O)\}$ of $H(O, G(O))$ contains besides the simple objective eternal object O only simple subjective eternal objects A_i. And it is only the latter which fall under the decisions to be felt in the process of concrescence that takes rise from the outcome of the process of transition.

In this concrescence process the becoming occasion decides which of the simple qualitative alternatives A_i from $G(O)$ shall contribute to its final definiteness. More precisely, in the course of the concrescence the set $G(O)$ is successively reduced

$$G(O) \equiv G_1(O) \supset G_2(O) \supset \dots$$

until the activity of the concrescence process has reached the point where any further reduction of the set $G(O)$ would not left enough activity for the final phase of the concrescence—the phase of satisfaction. By means of this reduction the initial maximal hierachy $H(O, G(O))$ is automatically reduced

$$H(O, G(O)) = H(O, G_1(O)) \supset H(O, G_2(O)) \supset \dots$$

too, in the course of the concrescence. Let us assume that the concrescence consists of n phases, i.e. of n decisions as to the elimination of elements from

$G(O)$,[10] so that $G_n(O)$ is the subset of $G(O)$ that contains those simple subjective eternal objects which are not eliminated in the course of this concrescence, or put positively, which are selected to contribute to the final definiteness of the completed occasion.

Now for the integration of the simple subjective eternal objects $A_i \in G_n(O)$ into the unity of one complex subjective eternal object and of the latter into a unity with the objective eternal object O, a complex mixed eternal object C is needed, that provides the manner of relatedness for the elements of $\{O, G_n(O)\}$ (WM, p. 194). The complex eternal objects which provide a requisite manner are also members of the maximal abstractive hierachy $H(O, G_n(O))$ determined by the reduction of $G(O)$ to $G_n(O)$. However, in general, there will be more than one pattern in $H(O, G_n(O))$ that provides a manner for relating all elements of $G_n(O)$, i.e. that has all elements of the set $G_n(O)$ as its components. Thus the final decision to be settled in the concrescence process concerns the complex mixed eternal object $D \in H(O, G_n(O))$ according to which the simple properties $A_i \in G_n(O)$ shall be synthesized into a unity with one another and with the objective eternal object $O \in G_n(O)$ (PR, p. 154). Yet since the completed occasion shall have a determinate qualitative and a determinate spatiotemporal character, it is clear that this mixed eternal object D must be of the form $D = C(A_1, \ldots, A_m) \wedge O$, where C is its "subjective part". Thus the final decision as to the pattern $D \in H(O, G_n(O))$ *only concerns the manner of relatedness of its subjective components* $A_i \in G_n(O)$, i.e. only the subjective part C of D. This decision is felt in the last phase of the concrescence—the phase of satisfaction. The outcome of this terminal phase, i.e. the completed occasion, is the spacetime region $\mathcal{O}$ endowed with the complex quality $C(A_1, \ldots, A_m)$—it is a qualitatively endowed bounded spacetime region "that is determinate in every respect and is intolerant of any

[10] Recall that we have assumed in Section 1.1.3 that each phase of concrescence corresponds to one simple decision, i.e. to a decision as to the rejection of one simple eternal object for entrance into the next phase. Although Whitehead did not made this explicit, it seems to be the only way to support the demand that a concrescence consists of more than one phase, i.e. that a concrescence is a process of genetically sucessive phases, at all. If a concrescent occasion would have the ability for more than one simple decision at the same time, it could decide its final definiteness all at once, thereby eliminating the processual character of its self-creative becoming.

addition" (WM, p. 288; see also p. 266). We will later on in connection with QFT only need quite specific (maximal) abstractive hierachies, so that there will *always be a unique pattern* C that has all and only the simple subjective elements $A_i \in G_n(O)$ as its components. In other words, the final decision as to the pattern C will be turned into a universal principle applying to each occasion, and thus need not be construed as a decision at all (see in particular Section 11.1, equation 11.1). The final phase of the concrescence—the phase of satisfaction—then simply coincides with the last decision as to the elimination of some simple eternal object, i.e. with the reduction $G_{n-1}(O) \to G_n(O)$.

The completed occasion, that can for example be represented by the pair $(\mathcal{O}, C(A_1, \ldots, A_m))$, is a new limitation laid upon the underlying activity, and thereby causally influences all future occasions to whose actual worlds it belongs. This limiting, respectively conditioning, of the underlying activity by each completed occasion will be discussed in more detail later on.

2.3 The underlying activity

Besides a spacetime region and a range of qualitative properties the initial stage of a new occasion's becoming consists of an activity due to which this becoming is a creative process. This activity of a concrescent occasion is a *limited and individualized manifestation of the eternal underlying activity*, also called the creativity by Whitehead. This underlying activity or creativity is the pure potential

> for the becoming of determinate occasions, but is itself indeterminate or formless. In itself, then, the creativity is without a character of its own. It is [...] incapable of characterization in disconnection from its involvements in the becoming of its creatures [i.e. occasions]. (WM, p. 169)

This does not mean that the underlying activity as such has no determinate ontological properties—for sure, no entity can be indeterminate or formless in this radical sense. Rather it means that in itself the underlying activity, term it ω, though being the unbounded potential for activities, cannot create any particular occasions because *it has no determinate creative character*—it is a totally "unordered" activity—and therefore in itself cannot settle any decisions for or against particular possibilities. The underlying activity is the pure potential for the becoming of occasions *in general*: it functions in the same way for all occasions *without any valuation or decision for or against some possibility.* The characterization of the underlying activity needed for the creation of particular occasions, i.e. for the realization of particular possibilities, is due to the limitations laid upon it by the already actualized occasions at the respective stage of the world-process (WM, p. 161). The formless underlying activity needs to be manifested in limited forms equipped with determinate creative characters before it can, by way of this manifestations, contribute to the creation of particular occasions (WM, p. 308 f). In the following $\omega_{W_{s-1}}$ shall denote the manifestation of the underlying activity ω arising from the conditioning of the latter by the aggregate W_{s-1} of all already actualized occasions at stage s of the world-process.

What has been said so far about the underlying activity strongly resembles some central aspects of Aristotle's notion of *matter* as well as of Plato's *receptacle* or *chora.* The former resemblance is explicitly mentioned by Whitehead himself:

> 'Creativity' is another rendering of the Aristotelian 'matter', and of the modern 'neutral stuff'. But it is divested of the notion of passive receptivity, either of 'form', or of external relations; it is the pure notion of the activity conditioned by the objective immortality of the actual world [...]. Creativity is without a character of its own in exactly the same sense in which Aristotelian 'matter' is without a character of its own. It is that ultimate notion of the highest generality at the base of actuality. (PR, p. 31)

A detailed comparison between the Whiteheadian notion of the underlying activity or creativity and the Aristotelian notion of matter has been given by R. Fetz (1981). Like Aristotle's matter, Plato's chora is also the formless medium or substrate that is open to receive any forms. However, Plato's chora resembles the Whiteheadian underlying activity even to a higher degree, because it incorporates "tendencies for motions which, however, do not result in an ordered motion but merely in an indeterminate trembling within the chora" by reason of which the chora "supports the appearance of forms in it [...]" (Böhme 2000, p. 305 f; translation by the present author). If this interpretation is correct, the chora is—at least to a certain degree—*also an activity that supports the actualization of forms.* The resemblance of Whitehead's underlying activity and Plato's chora with regard to their role as active principles has also been recognized by Nobo. The conclusion reached by Nobo is that, although Whitehead when explicitly using the term "receptacle" mostly does so "as synonymous with 'extension' and the 'extensive continuum'" (WM, p. 256), the Platonic receptacle or chora nevertheless conveys the same principle as is conveyed by the underlying activity, namely

> that the universe is ever advancing beyond every completed multiplicity of its achieved elements, yet always retaining the achieved elements as components of the novel elements in achievement. (WM, p. 257)

How the underlying activity is involved in retaining all achieved elements—all actualized occasions—and in making them components of, in the sense of *causal factors for*, the newly arising occasions will be seen later on. In sum, one can conclude that Whitehead's conception of the underlying activity does not stand as much aside well-known ontological constructions as it may seem at first sight.

Now according to Whitehead the aggregate W_{s-1} of all already actualized occasions at some stage s of the world-process is further divided into different actual worlds $W_{s-1}(i)$. Whitehead assumes moreover that the manifestation $\omega_{W_{s-1}}$ of the underlying activity corresponding to W_{s-1} is likewise divided into partial manifestations, term them $\omega_{W_{s-1}(i)}$, each being created from the underlying activity and equipped with a determinate creative character by the conditioning of the underlying activity due to one of the actual worlds $W_{s-1}(i)$ which together make up W_{s-1}. These partial manifestations are, though equipped with a determinate creative character, not yet the activities involved in the concrescence processes of the new occasions of stage s. For the activities involved in concrescence processes are *individualized* activities (SMW, p. 177).

This individualizing of the partial activities $\omega_{W_{s-1}(i)}$ is what happens in transition processes. Yet before we discuss transition processes in more detail, we will in the following three sections elucidate the sense in which the activities involved in concrescence processes are limited and individualized (Section 2.3.1), what this means for the individuality of completed occasions (Section 2.3.2) and how the actualization of completed occasions by which they condition the underlying activity underlying activity is to be understood (Sections 2.3.3 and 2.3.4).

2.3.1 The limited and individualized activity of a concrescence process

Part of the outcome of a transition process arising from an actual world $W_{s-1}(i)$ is a limited and individualized manifestation of the underlying activity, term it $\omega^c_{W_{s-1}(i)}$, that is the activity for a new self-completing concrescent occasion. That this activity for a new concrescence process is limited means three things. First, it is only a limited "amount" of activity with which each concrescence process is provided by the genetically earlier process of transition. This secures that each concrescence process necessarily comes to an end once its amount of activity is exhausted.

The second limitation of the activity from which a new concrescence process arises is due to the restricted range of qualitative properties, i.e. a set of mutually compatible simple subjective eternal objects (see Section 2.2.6), that is available for its decisions. Like all components of the initial ontological make up of a concrescent occasion this range is determined by the occasion's actual world and is realized in the genetically earlier process of transition. Thus the activity of the new occasion is limited in that it can only make decisions among a restricted range of properties.

However, there is a third limitation of the activity of each concrescent occasion. According to Whitehead the freedom of each activity for decisions *within* the given range of properties is restricted too, i.e. its freedom to choose among the given alternatives is *no absolute freedom* (PR p. 133; WM, p. 384 ff). Each activity is the outcome of the conditioning of the underlying activity by an actual world with a determinate spatiotemporal and qualitative character.

By reason of this determinate character of this actual world the corresponding manifestation of the underlying activity thereby produced is equipped with a determinate creative character, that does not change during the course of the following concrescence. What changes during the course of a concrescence is the amount of activity but not the latter's creative character. That each activity has such an unchanging creative character means that this activity will, in general, not be neutral relative to the alternative properties among which it has to decide during the process of concrescence—its freedom for decisions is a *biased freedom*. In other words, depending on the determinate creative character the activity is provided with by its actual world, some of the possible outcomes of the corresponding concrescence process will be more likely to be decided for being integrated into the occasion's final definiteness, than others. Although Whitehead did not put it this way, the actual world of an occasion by reason of its determinate spatiotemporal and qualitative character dictates a particular chance, tendency or more technically an ontic single case probability—*a propensity*—with which each element from the range of alternative qualitative properties available at the beginning of the concrescence process will be in fact chosen for integration into its final outcome by the creative activity.[11] If this way of understanding Whitehead is correct, the creative character of an individualized manifestation of the underlying activity can be understood as a propensity-measure over the initial range of subjective eternal objects available for its decisions. That this creative character does not change during the whole concrescence process, then, means that the propensity-measure is not conditionalized after a decision has taken place—only its domain, i.e. the range of still available subjective eternal objects, is reduced by one element. Thus the third way in which the activity of each concrescence process is limited is due to the determinate creative character—the determinate propensities for decisions concerning the available subjective eternal objects—impressed on it by its actual world in the forgoing transition process. The freedom of each creative activity is not only restricted as to the range of alternatives among which it can decide, but also as to the decisions within this range. However, that Whitehead believed that there is such a freedom of decision of *each* cre-

[11] An excellent overview over the merits as well as the problems of propensity interpretations of probabilities can be found in (Rosenthal 2002).

ative activity, though only a biased one within a restricted range, is clear form the following statement.

> The doctrine of the philosophy of organism is that, however far the sphere of efficient causation be pushed in the determination of components of a concrescence [...] beyond the determination of these components there always remains the final reaction of the self-creative unity of the universe [i.e. of a concrescent occasion respectively its creative activity]. (PR, p. 47)
>
> [...] it is to be noted that the 'decided' conditions are never such as to banish freedom. They only qualify it. There is always a contingency left open for immediate decision. (PR, p. 284)

Thus according to Whitehead no occasion is *completely* other-caused, every occasion is to a greater or smaller amount self-caused, depending on how strongly the freedom of an occasion's activity is biased towards a certain outcome by its actual world—its efficient cause. The stronger this impressed bias is, the greater is the amount of efficient causation and the smaller is the amount of self-causation with respect to this occasion. In other words, the closer the propensity for a certain outcome comes to certainty (i.e. to the value one) the smaller is the freedom left for the creative activity thus biased. On the other hand, if all outcomes have the same propensity the freedom of the activity, and thus the amount of self-causation of the occasion, is as big as it can be because the actual world did not impress any preference for any possible outcome onto the creative activity. Thus when Whitehead speaks of the freedom or the autonomy of an occasion respectively of the occasion's creative activity, to decide the final definiteness of the occasion, these notions always have to be understood as limited by the corresponding actual world.

Yet this demand that *no* occasion can be *completely* other-caused, in that the creative character of its activity is maximally biased towards a certain quality—a certain element from the initial range of simple subjective eternal objects available to it—*plays no systematically fundamental role for Whitehead's theory*. Rather it seems to be completely ad hoc, since nothing else in Whitehead's ontology would have to be changed if one would allow for the more natural possibility of *completely* other-caused occasions, i.e. occasions whose final definiteness is determined by its efficient causes—by the occasions in its actual world. Since nothing speaks against it, will therefore allow for this possibility.

Before turning to the discussion of the individuality of the activities of concrescent occasions, we will introduce a simplifying assumption concerning a possibility about which Whitehead himself is silent. As mentioned above, the amount of activity can change whereas its creative character stays fixed. This is not only the case for the activity involved in a concrescence process but is likewise the case in the earlier transition process. *Thus the activities involved in a transition and the following concrescence process have the same creative character* (see also Section 2.4). Now since there is no principle that restricts the "compatibility" of certain amounts of activity and certain creative characters, each amount is compatible with each creative character. Therefore, it is very well possible that amount and creative character of an activity are *not* determined by the same occasions (from the corresponding actual world). An occasion E may, for example be an efficient cause of the creative character of the activity of some other occasion but not for this occasion's amount of activity. However, for the sake of simplicity we will henceforth assume that *amount and creative character of an activity always have the same efficient causes.* This assumption will simplify the—already quite involved—discussion of efficient causation in Whitehead's ontology in later sections (see particularly Sections 2.4.3 and 2.6.3).

Now that the limited activity of a concrescent occasion is moreover individualized means two things. The first factor of its individuality is its *particularity.* According to Section 2.2, an entity is a particular if it can exist only in one connected spatiotemporal respectively extensive instance, i.e. if it occupies, embodies or is located in a single connected spatiotemporal or extensive region. Because of our supplementary assumptions as to the ontological properties of the extensive continuum we need only speak of spatiotemporal regions in connection with the limited activities of occasions. The particularity of the activity involved in a concrescence process, therefore, means that it is located in a single connected spacetime region (WM, p. 278, 282 ff). However, one can ask what the precise meaning of the locatedness of an activity in a certain spacetime region shall mean. Since the activity in question is a decision making activity, the only reasonable meaning of its locatedness in some region seems to be that *it makes decisions for this and for no other region*, namely decisions as to the subjective eternal objects which will unrestrictedly ingress into *this* region. This understanding of the localization of activities is not in conflict with the fact that the concrescence processes in which they are involved are not spatiotemporal processes in the sense that their phases cannot be spatiotemporally ordered. This is because the entire region of an occasion is created all at once in the first phase of the forgoing process of transition and the decisions of the limited activity in each phase of the later concrescence process *refer indifferently to this entire region*. Therefore, the locatedness of a decision making activity in a certain region, by reason of its settling decisions for this (entire) region makes perfect sense.

Now the individuality of the manifestations of the underlying activity involved in concrescence processes means more than their bare particularity. The second factor that together with their particularity makes them individuals in Whitehead's sense is their already mentioned (limited) freedom or autonomy. Thus the individuality of the creative activity of an occasion is to be understood as its particularity together with the (limited) autonomy of its decisions. Since the freedom of an activity can be larger or smaller, its individuality too, varies with the degree of this freedom. In the extreme case, arbitrarily abandoned by Whitehead but allowed by us, in which there is no such freedom at all, because the creative character of the activity is maximally biased towards one possibility, its individuality is minimal—it just coincides with the activities particularity. In the other extreme case where the creative character is not biased at all, but rather is completely indifferent with respect to the different possibilities available to it, its freedom and thus too its individuality are maximal. In between these two extreme cases, the activity's creative character is neither maximally biased nor completely indifferent, corresponding to an intermediate degree of individuality.

Besides this generally merely limited freedom or autonomy there is also a sense in which each activity involved in a concrescence process is completely "autonomous"—the activity of a concrescent occasion is completely causally independent from all activities of jointly concrescent occasions. As mentioned in Section 1.3 all contemporary occasions are, by definition, completely causally isolated from one another and thus do not influence their mutual decisions in any way. Thus in regard to their contemporaries Whiteheadian occasions behave like windowless Leibnizian monads. In terms of the creative characters of occasions this therefore means that the creative character of each individualized activity is completely fixed by the occasions belonging to its own actual world—by the decisions these occasions settled during their concrescence processes—and does not depend in any way on the decisions of any contemporary occasions. However, to avoid confusions we will not speak of this causal independence as "autonomy" or "freedom". Rather the latter terms will exclusively be used to refer to the above discussed feature of the creative characters of single activities that contributes to their individuality.

In the following section we will see that the sense in which occasions are individuals during their becoming and as attained actualities is intimately related to the individuality of their creative activities.

2.3.2 The individuality of occasions

According to Whitehead occasions are individuals in each phase of their existence. Their initial constituents are an individualized, i.e. particularized and (to a certain degree) autonomous, manifestation of the underlying activity and a range of qualitative properties—the occasion's initial definiteness. The individuality of such a newborn occasion is just the individuality of its creative activity *enriched* by the initial, though incomplete, qualitative character. Thus in its initial phase of existence an occasion's individuality consists of its particularity, its autonomy and its initial definiteness. As mentioned earlier, in the following phases of the occasion's concrescence the initial definiteness is successively deepened up to the point where its creative activity is completely exhausted. Consequently, *in the course of the concrescence the occasion's individuality is successively deepened, too* (WM, p. 285 ff). The completed occasion's individuality is the result of this successive deepening. It consists of the finally attained definiteness and the unaltered particularity provided by its fixed spatiotemporal standpoint. As completed an occasion clearly no longer is an autonomous activity, since it lacks any activity at all. Thus autonomous decisions are features of an occasion's individuality in its phases of becoming, i.e. during its process of concrescence, but not of the fixed and final individuality of an actualized occasion. The distinction between the two senses of individuality of an occasion, therefore, directly corresponds to the distinction between its two modes of existence, i.e. its two modes of actuality—its dynamic self-creative mode and its static other-creating mode. An occasion starts its existence as an autonomous, minimally individualized particular and successively deepens this individuality until its autonomous activity is exhausted, thereby attaining the final and fixed individuality it will exhibit during its subsequent existence qua stubborn fact. Thus the individuality of an occasion in each phase of its existence means strictly more than bare particularity—*it is particularity enriched by autonomously deepened definiteness* (WM, p. 285). As mentioned earlier, the unique connected spacetime region, by reason of which it is a particular is also the unchanging element in an occasion's internal constitution that accounts for the self-identity of the occasion throughout all phases of its existence. Consequently, an "occasion's unique and specifiable *particularity* remains unchanged as the occasion passes through its successive phases of becoming" (WM, p. 283). An occasion is the *self-same particular*, though with a different definiteness and a different amount of creative activity, in each phase of its existence—it is a particular that autonomously deepens its individuality.

2.3.3 The envisaging property of the underlying activity

When a concrescence process is completed, its outcome—the completed occasion—is actualized. This act of actualization is the transformation of the completed occasion from its self-causing to its other-causing mode of existence and thus marks at the same time the beginning of a new transition process, by reason of which a new incomplete occasion is created (see Section 2.4). Thus by means of its actualization a completed occasion is made causally efficient for all later occasions to whose actual worlds it belongs (PR, p. 29; WM, p. 310). But this act of actualization cannot be ascribed to the individual activity of the concrescent occasion, simply because this activity is already exhausted after the last decision as to the qualitative properties of the completed occasion has been settled, i.e. at the end of the phase of satisfaction. Rather it is to be attributed to the underlying activity itself (SMW, p. 105; WM, p. 308).

The ontological property of the underlying activity by reason of which each completed occasion is actualized is called its *envisagement*. Envisagement is the Whiteheadian term for the underlying activity's *taking into account* of the outcome of each completed concrescence process, i.e. of each completed occasion. This taking into account—this envisagement—of the outcome of each concrescence process by the underlying activity *is* the act of actualization of the completed occasion. At the same time it is the act by which a new limited manifestation of the underlying activity is created from which a new transition process will take rise (WM, p. 309). Acts of envisagement are therefore the only acts which can be ascribed to the underlying activity itself (WM, p. 308), all other acts which occur during processes of transition or concrescence have to be ascribed to the limited or particularized or even individualized manifestations of the underlying activity. In sum, then, this taking into account by the underlying activity is the transformation of the completed occasion from private to public actuality, from actuality in attainment to attained actuality or equivalently from its self-creative to its other-creating mode of existence, which at the same time produces a new limited manifestation of the underlying activity and thereby marks the beginning of the dative phase of a new transition process.

This envisaging of completed occasions by the underlying activity, though acts of it, do, however, *not involve any kind of decision or valuation among the different occasions taken into account*. The underlying activity does not "privilege" any occasion over any other. It indifferently takes into account each completed occasion in the same way, i.e. it functions in the same manner in respect to every occasion whose concrescence process has terminated. *Decisions* do always presuppose an *individualized*, and therefore autonomous, manifestation of the underlying activity which settles them—the underlying activity as such as well as its merely limited and particularized manifestations, cannot settle any decisions (WM, p. 152 ff, 174; see also Section 2.4). However, though the underlying activity *takes each occasion into account in the same way*, it is nevertheless *conditioned by different occasions in different ways*. By reason of their different qualitative and spatiotemporal characters different occasions, though taken into account in exactly the same manner by the underlying activity, lead to different manifestations of it.

2.3.4 The objective immortality of occasions

Since it will not affect any argument of the present section, we will in what follows simply disregard the fact that the manifestation $\omega_{W_{s-1}}$ of the underlying activity at a some stage s of the world-process is further divided into partial manifestations $\omega_{W_{s-1}(i)}$, each being determined by one of the actual worlds $W_{s-1}(i)$ into which W_{s-1} is supposed to be divided.

According to Whitehead the actualization of an occasion makes the latter *objectively immortal*. This objective immortality is the reason that "actuality is *cumulative* and *the number of its concrete components is ever increasing* [...]" (WM, p. 172; italics added). Thus, in particular, the objective immortality of occasions is the reason for the equivalence of the evolvement and the expansion in Whitehead's ontology. How the very idea of an evolvement or expansion of the world, as discussed in Section 1.3.2, can be made sense of, will be discussed in Section 2.7. For the moment we will simply take it for granted that the world exhibits a unique layer-cake structure by reason of which it can be said to evolve or expand.

However, by objective immortality Whitehead means more than that once actualized, each occasion is a "stubborn fact which cannot be evaded". By reason of this character of a stubborn fact, each actualized occasion is also said to have "unavoidable consequences" also in the far removed future (PR, p. 43, 219). In other words, by the objective immortality of an envisaged occasion Whitehead means that (1) it is somehow retained and that by means of this, (2) it is available as an efficient cause for occasions at some arbitrary later stages of the world-process. Now according to Whitehead the reason for this objective immortality of occasions is that the

> creativity is not separable from its creatures [i.e. occasions]. Thus the creatures *remain with the creativity.* Accordingly, the creativity for a creature becomes the creativity with the creature, and thereby passes into another phase of itself. It is now the creativity for a new creature. (RM, p. 92; italics added).

> This function of creatures, that they constitute the *shifting character of creativity*, is here termed the 'objective immortality' of actual entities. (PR, p. 32; italics added)

But how is this remaining with the creativity and the latter's shifting character, that is supposed to account for the objective immortality of occasions, to be understood? We will now propose our interpretation of these matters. This will lead us to the conclusion that the most reasonable way to understand the objective immortality of occasions in sense (1), is to assume that each completed occasion genuinely remains with, i.e. is retained in, the *extensive continuum* and that the objective immortality in the causal sense (2) should better be given up in its strict form.

First of all, the only reasonable candidate for the "shifting character of creativity" is the succession $\ldots, \omega_{W_{s-1}}, \omega_{W_s}, \omega_{W_{s+1}}, \ldots$ of manifestations of the underlying activity corresponding to different stages of the world-process. For by way of the envisagement of the aggregate of occasions W_{s-1} the activity $\omega_{W_{s-1}}$ for stage s, i.e. "the creativity for the creatures" of this stage, is created from the in itself formless underlying activity ω. Now each manifestation of the underlying activity at some stage, $\omega_{W_{s-1}}$ say, is exhausted when all the occasions begotten in stage s have completed themselves, so that at the end of stage s the only activity that is left is again the underlying activity ω itself. To use some familiar physical vocabulary, the underlying activity ω is the world's *ground state of activity* and its manifestations $\omega_{W_{s-1}}$ are *excitations* of this common ground state. Yet since at the end of stage s the activity $\omega_{W_{s-1}}$ is exhausted it can hardly retain the occasions belonging to W_{s-1} plus the new occasions created in stage s—it cannot retain anything because it does no longer exist. The only activity left at this point is again the underlying activity ω. But the underlying activity cannot retain anything either, because for retaining each completed occasion the underlying activity itself would have to undergo successive changes—without any difference between the underlying activity as such and as already having retained some occasions, it is hard to see in which sense the latter can be retained at all. Thus something different from any activity seems to be needed as that wherein all completed occasions are retained so that they can again be envisaged and thus can contribute to a new manifestation of the underlying activity.

Now it is already assumed that (i) each occasion occupies a unique region of the extensive continuum, (ii) that each two occasions occupy separated regions and that (iii) the subjective eternal objects constituting an occasion's definiteness ingress into the occasion's region and thus *ultimately into the extensive continuum*. Therefore, the extensive continuum is the most natural candidate for *faithfully retaining* not only the spatiotemporal but also the qualitative character of each completed occasion—and thus each completed occasion itself. In fact, it is hard to think of any better candidate for this job. The *actualization* of a completed occasion E can then be understood as the *first* act of envisagement in which E has been taken into account by the underlying activity, because through this act E "gained" its causal efficiency, i.e. is transformed from its self-causing to its other-causing mode of existence. With the extensive continuum as retaining each completed occasion, and the further assumption that from *its first act of being envisaged—its actualization—on, an occasion will necessarily be taken into account in all following acts of envisagement as well* (see PR, p. 286), we are then able to give the following account: when the manifestation $\omega_{W_{s-1}}$ of the underlying activity is exhausted, the underlying activity ω envisages the aggregate W_s of qualitatively endowed spacetime regions (including their spatiotemporal relationships to one another) *inherent in the extensive continuum*, that consists of the occasions completed in stage s plus the aggregate W_{s-1} of all earlier actualized occasions. By the envisagement of W_s the new manifestation ω_{W_s} is created that "transmits" the causal efficiency of the occasions from all earlier stages one stage further, i.e. to stage $s+1$. After ω_{W_s} is again exhausted the underlying activity ω envisages the aggregate W_{s+1} from which its new manifestation $\omega_{W_{s+1}}$ results that, accordingly, transmits the causal efficiency of the occasions in W_{s+1} to stage $s+2$. By further iterating this mechanism, the causal efficiency of some arbitrary far removed past occasion E is made available to occasions at all later stages of the world-process and thus gives rise to E's objectively immortality as an efficient cause. Moreover, Whitehead's statement that "the creatures remain with the creativity" can then be interpreted as meaning that once envisaged, an occasion remains a conditioning factor of all later *manifestations of the underlying activity*.

However, an assumption implicit in Whitehead's idea of this causal sense of objective immortality, as well as in our above account, is that no occasion can be completely causally ineffective, in the sense that it is *not even available as an efficient cause to any other occasion*, for otherwise it would not be objectively immortal in this causal sense. But this assumption seems to be quite ad hoc since, prima facie, it seems very well possible that an occasion E makes no contribution to any later manifestation of the underlying activity, because its conditioning effect on the underlying activity is rendered irrelevant or screened off, by the conditioning effect of other earlier or contemporary occasions. In this case E would not be available as an efficient cause for *any* occasion since it does not contribute to the manifestation of the underlying activity from which this new occasion has been created. The ad hoc character of not allowing for such "cancellations" is even more strengthened in the light of the fact that the underlying activity takes into account all occasions indifferently in the same way and thus cannot account for the prohibition of cancellations. Moreover, within our above account each occasion, even if rendered completely causally ineffective, is nevertheless retained in the extensive continuum and thus is at least objectively immortal in this sense. Therefore, we will—contrary to Whitehead—allow for the possibility of completely causally ineffective occasions, since it seems to be the more natural option than to stipulate by fiat that no cancellations of conditioning effects on the underlying activity can happen. Furthermore, it could likewise be the case that an occasion E is causally effective with respect to occasions of stages $s, s+1, \ldots, s+n$ of the world-process following the stage (i.e. stage $s-1$) in which E has been envisaged for the first time but that its effectiveness for further occasions is then rendered irrelevant by the conditioning effects of occasions belonging to stage $s+(n+1)$. Therefore, it is likewise natural to allow for this possibility, too.

Thus what we end up with is essentially the above given account according to which at the end of each stage of the world-process all the occasions belonging to the latter plus the aggregate of all earlier actualized occasions, is envisaged by the underlying activity, whereby a new manifestation of it is produced from which the occasions of the next stage are begotten and so on. However, we will *not* assume that by reason of this mechanism each actualized occasion necessarily becomes an efficient cause of some other occasion. Rather we believe that this is only the case if the conditioning effect of the occasion is not rendered irrelevant by the conditioning effects of some other occasions. Note that the act of actualization of an occasion can nevertheless be understood as the transformation from the occasion's self-causing to its other-causing mode of existence—only that "other-causing mode" does no longer imply that the occasion will in fact become a cause of some other occasion.

Before turning to Whitehead's account of transition processes, it shall be pointed out that Nobo, in his interpretation of Whitehead's ontological system, also assumes that completed occasions are retained in the extensive continuum (WM, p. 205 ff). Yet for Nobo the fact that a function originally ascribed by Whitehead to "the creativity", namely the function of retaining all completed occasions, can be systematically much better understood if it is ascribed to the extensive continuum, is one of the reasons to put forward a more far reaching interpretational claim. In Chapter 6 of WM, Nobo argues that the underlying activity and the extensive continuum are not two different entities but rather merely two aspects of one and the same ultimate reality, termed the *extenso-creative matrix* that "is the ultimate ground for the becoming, the being, and the solidarity of all actual entities" (WM, p. 259). Although Nobo provides some systematic arguments as well as some textual support from Whitehead's writings for this claim, we will nevertheless be content with the orthodox view according to which extensive continuum and underlying activity are not to be fused into such an extenso-creative entity.

2.4 Other-creating processes of transition

Transition processes are the deterministic mechanisms whereby causal influences are transmitted between occasions (see Section 2.4.3). Moreover, they are non-spatiotemporal processes (see Section 2.4.4) consisting of two genetically succeeding phases—the dative phase and the conformal phase—which we will describe in this order in the following two sections.

2.4.1 The dative phase of a transition process

According to Whitehead the aggregate W_{s-1} of all actualized occasions at stage s of the world-process is uniquely divided into sub-aggregates—the actual worlds $W_{s-1}(i)$ at that stage. The manifestation $\omega_{W_{s-1}}$ of the underlying activity created by the envisagement of the occasions belonging to W_{s-1} is therefore likewise assumed to be differentiated into corresponding partial manifestations $\omega_{W_{s-1}(i)}$, each solely determined by the conditions imposed on the underlying activity ω by the occasions in *one* of the actual worlds $W_{s-1}(i)$ at stage s (SMW, p. 106, 177; WM, p. 161, 174, 168). Note that the sub-activities $\omega_{W_{s-1}(i)}$ into which the activity $\omega_{W_{s-1}}$ is supposed to be divided, are not yet located in a particular region nor do they have the ability for autonomous decisions. These features are further products of the transition process that takes rise with the creation of each activity $\omega_{W_{s-1}(i)}$ by reason of the envisagement of the corresponding actual world $W_{s-1}(i)$. The creation of the activity $\omega_{W_{s-1}(i)}$ is at the same time the initial "moment" of the dative phase of a new transition process (WM, p. 73 f). In this dative phase a new finitely extended, connected spacetime region is produced by the ingression of a boundary surface—an objective eternal object—into the extensive continuum (see Section 2.2.6). The activity for this ingression is supplied by the corresponding limited activity $\omega_{W_{s-1}(i)}$. However, it is *not a decision* of this limited activity which potential region of the extensive continuum shall be realized. This is completely determined by the qualitative and spatiotemporal characters of the occasions belonging to the corresponding actual world $W_{s-1}(i)$—it is a decision "made *by earlier* actual things [i.e. occasions] *for later* actual things [i.e. occasions]" (WM, p. 156).[12] This can be understood as meaning that the creative character of the limited activity $\omega_{W_{s-1}(i)}$, as determined by the conditions imposed on the underlying activity by the envisaged occasions of the corresponding actual world $W_{s-1}(i)$, is maximally biased towards the creation of a particular region, say $\mathcal{O}_i$. In other words, the creative character of $\omega_{W_{s-1}(i)}$ is such that it only allows the ingression of a particular boundary surface O_i at a particu-

[12] We have assumed that the ingression of eternal objects into the the extensive continuum requires an amount of activity even if no decision of this activity is involved, so that the activity at the end of the dative phase differs by this amount from the activity $\omega_{W_{s-1}(i)}$ at the beginning. However, it is not clear whether this is intended by Whitehead or not. Yet if an act of ingression that did not involve a decision of the corresponding activity should *not* reduce the latter's amount, this can easily accomodated for: in this case the activities before and after the ingression are simply identical not only as to their creative character but also as to their amount.

lar position within the yet unrealized part of the extensive continuum thereby creating the particular connected region $\mathcal{O}_i$ without involving any decisions of the activity $\omega_{W_{s-1}(i)}$ (WM, p. 159, 162). Thus besides containing propensities for the decisions as to the elimination of ingressed subjective eternal objects during the later process of concrescence, the creative character of the activity $\omega_{W_{s-1}(i)}$ also contains the "information" as to which region has to be realized in the dative phase.

Now the activity left after the creation of the region $\mathcal{O}_i$ and thus after the dative phase, term it $\omega^d_{W_{s-1}(i)}(\mathcal{O}_i)$, is supposed to be particularized, i.e. located in the connected region $\mathcal{O}_i$ (WM, p. 288). In Section 2.3.1 we have proposed that the locatedness of an activity in a region can be understood in the sense that the activity makes decisions for this and for no other region. However, this understanding does not apply to the activity in question because during the phases of transition no decisions of the activities of these phases are involved at all. Yet in Section 2.5.3 we will, for the sake of resolving two problems of Whitehead's ontology (see Sections 2.5.1 and 2.5.2), modify Whitehead's original account by assuming that all the occasions created in the same stage of the world-process arise from a *single* transition process involving a *single undivided* activity. Therefore, we need not try to find some weakened criterion for the locatedness of activities that would likewise apply to the activities of transitions. For even if such a weaker criterion could be found, it could certainly not retain Whitehead's demand of the particularity of the outcome activities of the dative phases of the transition processes at a stage of the world-process since there will not even be *distinct* activities at all which could moreover be particularized. However, until we will eventually modify Whitehead's ontology in the mentioned way, we will for the sake of proceeding with the description of transition processes as originally conceived by Whitehead, simply assume that one can find a reasonable weakening of our criterion for the locatedness of activities that allows the outcome activity $\omega^d_{W_{s-1}(i)}(\mathcal{O}_i)$ of the dative phase to be particularized.

2.4.2 The conformal phase of a transition process

In the second and last phase of a transition process—the conformal phase—the real potential from which the following phases of concrescence will take rise is completed by the ingression of a range of qualitative properties, i.e. subjective eternal objects, which will constitute the initial definiteness of the new becoming occasion (WM, p. 381; see also Section 2.2.6). The activity for the ingression of these qualitative properties is provided by the particularized activity $\omega^d_{W_{s-1}(i)}(\mathcal{O}_i)$ that is the outcome of the dative phase. As in the case of the realization of the spacetime region in the dative phase the activity is again not free to decide which abstractive hierachy of subjective eternal objects shall ingress into this region. First of all, the range of subjective eternal objects which can ingress into the already created region $\mathcal{O}_i$ is restricted by the incompatibility of some subjective eternal objects to the objective eternal object O_i (see Section 2.2.6). Moreover, which abstractive hierachy $H(O_i, G(O_i)) \subset P(O_i)$ from the set $P(O_i)$, that contains all subjective eternal objects compatible with O_i as well as the mixed complex eternal objects relative to which they are compatible with O_i, shall ingress into $\mathcal{O}_i$ is not decided by the activity $\omega^d_{W_{s-1}(i)}(\mathcal{O}_i)$ of the conformal phase either. Like the realization of the region $\mathcal{O}_i$ in the dative phase this too is completely determined by the corresponding actual world $W_{s-1}(i)$. This actual world of already settled occasions maximally biases the creative character of the corresponding manifestation $\omega_{W_{s-1}(i)}$ of the underlying activity (and thus too the creative character of $\omega^d_{W_{s-1}(i)}(\mathcal{O}_i)$) towards a particular region $\mathcal{O}_i$ as well as towards a particular set $G(O_i)$ of simple subjective eternal objects compatible with O_i, thereby also fixing the unique maximal abstractive hierachy $H(O_i, G(O_i))$ ingressing into $\mathcal{O}_i$ (see Section 2.2.6). Thus the activity involved in both phases of a transition process is only the *vehicle for bringing about what the corresponding actual world, by way of its determinate spatiotemporal and qualitative character, dictates* (WM, p. 130). The outcome of the conformal phase and thus of the transition process, is a maximal abstractive hierachy $H(O_i, G(O_i))$ that is ingressed into the finitely extended, connected spacetime region $\mathcal{O}_i$ together with the particularized amount of activity left after the two phases of transition, term it $\omega^c_{W_{s-1}(i)}(\mathcal{O}_i)$.

Now once the deterministic phases of transition are run through, the activity left has a certain freedom to make decisions within the given range of qualitative alternatives. In other words, it is assumed that the creative character of each manifestation of the underlying activity as determined by its corresponding actual world is such that this manifestation has *no* freedom in deciding the region $\mathcal{O}_i$ and the range of qualitative properties ingressed therein, but that it may have a *limited freedom or autonomy for decisions within this range* (PR, p. 47, 284; see also Section 2.3.1). The first of these decisions is the first active-subphase of the concrescence process, in the course of which the definiteness as provided by the conformal phase, i.e. by the "subjective part" of the hierachy $H(O_i, G(O_i))$, is successively reduced and thereby becomes more and more specific (see Section 2.2.6). Thus since the limited activity left at the end of the conformal phase is not only particularized but also autonomous (to the degree its fixed creative character allows) it is an individualized activity in the sense of Section 2.3.1.

The outcome of a transition—the individualized activity[13] $\omega^c_{W_{s-1}(i)}(\mathcal{O}_i)$ and the range of qualitative properties, i.e. the subjective part of $H(O_i, G(O_i))$—establishes a limited variety of possible ways the following process of concrescence can take and in this sense it is the limited or real potential for the subsequent process of concrescence. What provokes the arising of a concrescence process, out of the outcome of a transition process, is the principle that the end aimed at in each becoming is a definiteness as specific as possible (WM, p. 288). By reason of this, the range of alternative qualities needs to be further reduced to one coherent complex quality, thereby provoking the decision making process of concrescence. Thus the first phase of a new concrescence process, which consists of the first autonomous decision of the individualized activity (the active-subphase) and the deepened definiteness resulting from this decision (the outcome of this activity), is provoked by the alternatives inherent in the range of qualitative properties provided by the outcome of the deterministic transition process, and thus ultimately by the corresponding actual world.

[13]Note that the individuality of this activity inlcudes its particularity, i.e. its occupying a bounded, connected spacetime region.

2.4.3 Transition as deterministic causal process

The following remarks as to Whitehead's conception of efficient causation may seem quite vague, but this is because Whitehead did not specify it more precisely. In particular, Whitehead's ontology is far from containing a *theory of causation* as usually understood. First of all, the term "efficient causation" is used by Whitehead as a synonym for "other-causation that determines its effect (at least in part)" (WM, p.32), and thus, in particular, as the opposite of "self-causation" (PR, p. 24, 150). What is moreover clear is that neither the underlying activity itself nor its limited manifestations involved in processes of transition are the efficient causes of the new incomplete occasions thus created. Clearly, that occasions become at all can only be understood by reference to the underlying activity that is the unlimited source for all the limited activities involved and presupposed by all particular transition (as well as concrescence) processes (WM, p. 129 ff). But this underlying activity cannot by itself explain why a given occasion has the initial character that makes it that becoming occasion rather than another. However, according to Whitehead, a cause has to do precisely that—*a cause has to give a reason for a particular fact*, in this case for the particular initial character of an occasion (PR, p. 24 f, 215; WM, p. 130). The underlying activity, however, cannot function as a reason for the particular initial character of a particular occasion. This is because the underlying activity is involved, by way of one of its limited manifestation, in the transition process of each new occasion and thus the underlying activity in itself cannot explain the particular initial character of a particular new occasion. On the other hand, due to their fixed creative characters the limited manifestations of the underlying activity which are involved in transition processes *do* provide reasons for the initial character of the occasions thereby created. But these limited manifestations—in particular their creative characters—*are themselves completely determined by the corresponding actual worlds.* Therefore, in the final analysis,

> the reasons, or causes, of a particular occasion are to be found, not in the creativity, but in the completed actualities in that occasions past [...]. The creativity, on the other hand, is the active vehicle whereby these passive determinants gain their effectiveness. (WM,

p. 130)

A transition process, is thus a process whereby the settled occasions of an actual world deterministically cause the initial ontological constituents of a new self-completing occasion. That an occasion E' is an *efficient cause* of another occasion E, can therefore be understood as meaning that E' *is a relevant determining factor for the initial constituents—the initial character—of* E. This determination proceeds via the conditioning of the underlying activity by E'. In Section 2.3.4 we have argued that it would be quite ad hoc to assume with Whitehead that the conditioning effect of an occasion E' on the underlying activity (due to the envisagement of E' by the underlying activity) cannot be rendered irrelevant by the conditioning effects of other occasions. Rather the more natural position is that it can very well be the case that an occasion E' is rendered causally ineffective by the conditioning effects of other occasions on the underlying activity which screen off E''s conditioning effect. Now if these other occasions are actualized in *an earlier or in the same stage* of the world-process than E', the latter is obviously *completely* ineffective for future occasions. And if this screening off is due to some later occasion E of which E' is *not* an efficient cause, then E' is causally ineffective from that stage, say s, on in which the screening occasion E is actualized. On the other hand, if E' *is an efficient cause of the "screening occasion"* E, then E' can reasonably be called an *indirect efficient cause* of certain occasions in the future of E, namely of those which are directly caused by the latter. Accordingly one can introduce indirect causes of higher grades by their position in a chain of direct causes leading to the occasion in question. For simplicity we will, however, not make use of such a finer differentiation. Rather we will simply speak collectively of the indirect causes of an occasion without further specifying their "grade of indirectness" with respect to the latter.

The above characterization of "efficient causes" as relevant determining factors for an occasions initial constitution is therefore correctly to be understood as characterizing an occasion's *direct* efficient causes. Only these are relevant *without further qualifications*, for their effects. Contrary to this, indirect causes of an occasion are screened off from the latter by their (more) immediate effects. In the following it will be convenient to use the term "indirect efficient cause" exclusive, i.e. as not also including direct efficient causes.

For the purpose to refer indifferently to both the direct and the indirect causes of an occasion we will use the term "efficient causes".

The actual world of an occasion consists of all its direct as well as indirect causes (PR, p. 284). Note that this way of defining the actual world of an occasion E_i, suffers from the circularity already discussed in Section 1.3.1, because it makes use of the initial constituents of occasion E_i. But these are realized by means of the manifestation $\omega_{W_{s-1}(i)}$ of the underlying activity that, however, presupposes the determinate actual world $W_{s-1}(i)$ as a given fact. The actual world $W_{s-1}(i)$ needs to be singled out somehow from all of actuality W_{s-1} at stage s, for the very existence of the activity $\omega_{W_{s-1}(i)}$ and thus for the creation of (the initial constituents) of occasion E_i to take place. In Section 2.5 we will come back to this important problem of Whitehead's theory.

Direct efficient causes as probabilistic causes of the final definiteness of occasions

According to Whitehead the efficient causes of an occasion E_i remain *conditioning factors* for all the later phases of the occasion's concrescence and thus ultimately for the completed occasion's final make up—for its region and its final definiteness (WM, p. 32; see also PR, p. 47). We will see in the following that in case of *direct* efficient causes this can be understood as meaning that they are *deterministic* causes of the completed occasion's region and *probabilistic* causes of the completed occasion's definiteness. In case of the region this is clear because the region of an occasion, as determined in the dative phase of the corresponding transition process, is fixed once and for all. Therefore, the direct efficient causes of an occasion, by being deterministically relevant for this region via the creative character of the corresponding activity (see Section 2.4.1), are also deterministically relevant for the region of the completed occasion, simply because the initial and final region of an occasion are identical. We will now argue that besides this way in which the direct efficient causes of an occasion E_i are (trivially) also conditioning factors, here in the sense of deterministic causes, for E_i's region qua completed, there is also another sense in which E_i's direct efficient causes are causes of E_i as completed—namely as probabilistic causes for its final definiteness.

One of the initial ontological constituents of a new occasion E_i is the limited activity $\omega^c_{W_{s-1}(i)}(\mathcal{O}_i)$ left after the conformal phase of the corresponding transition process. Since the creative character of activities does not change during transition as well as concrescence processes, the activity $\omega^c_{W_{s-1}(i)}(\mathcal{O}_i)$ has the *same* creative character as the activity $\omega_{W_{s-1}(i)}$ at the beginning and the activity $\omega^d_{W_{s-1}(i)}(\mathcal{O}_i)$ left after the dative phase as well as all the activities of the later phases of concrescence.[14] Therefore, if occasion E' is a relevant determining factor—a direct efficient cause—for the creative character of the activity $\omega_{W_{s-1}(i)}$ (and thus of E_i's initial activity $\omega^c_{W_{s-1}(i)}(\mathcal{O}_i)$), this is also true in respect to the activities involved in all later phases of concrescence. Note that we need only be concerned with the creative characters of activities and not with their amounts here, because we have assumed that the amount and the creative character of an activity are always determined by the same occasions, i.e. have the same efficient causes (see Section 2.3.1). Now by reason of the constancy of the creative character during all phases of transition and concrescence, occasion E' can also be said to be a relevant factor for the final definiteness selected in the concrescence process. This is because the decisions settled during the phases of concrescence are not completely free but rather are biased by the creative character of the corresponding activity—and the latter being the same as the creative character of the activities of all earlier phases, and thus ultimately of $\omega_{W_{s-1}(i)}$, is in part determined by occasion E' (see Section 2.3.1). Since the creative characters of the activities, in general, merely provide propensities different from zero or one for the available subjective eternal objects, this relevance of occasion E' will, however, generally be of a probabilistic rather than a deterministic nature. Thus those occasions from the actual world of a new occasion E_i which are direct efficient causes of the latter, by reason of determining E_i's initial character, are automatically *probabilistic causes for E_i's final definiteness*, in the sense that their hypothetical non-occurrence would either raise or lower the propensities for the unrestricted ingression of the subjective eternal objects which make up the

[14]If the ingression of eternal objects not requiring a decision of the activity, does not reduce the latter's amount (see Section 2.4.1), the activities $\omega_{W_{s-1}(i)}$, $\omega^d_{W_{s-1}(i)}$ and $\omega^c_{W_{s-1}(i)}$ are even identical.

initial definiteness of occasion E_i.

Now an occasion E' from $W_{s-1}(i)$ whose occurrence *lowers* the propensity for the unrestricted ingression of a particular subjective eternal object A is usually not called a (probabilistic) cause of A. However, first of all by lowering the propensity of A (the occurrence of) occasion E' obviously influences the final definiteness of the occasion in question, and thus provides a reason for the latter's specific character. Thus at least in respect to Whitehead's understanding of causes as reasons for the particular character of occasions, it makes sense to call propensity *lowering* occasions probabilistic causes of the final definiteness of other occasions, too. More importantly, W. Salmon has convincingly argued that direct causes, *particularly in the domain of quantum physics*, can in fact lower the probability of their effects (Salmon 1984, p. 200 f; see also Dowe 2000, Chapter II.6). Therefore, it seems that any account of causation, at least if it shall be applicable to the realm of quantum physics, has to deal with the possibility of probability lowering causes.

The special case in which the direct efficient causes of E_i already constitute the *complete cause* of the occasion's final definiteness—and thus of the whole completed occasion E_i—obtains if the creative character of $\omega^c_{W_{s-1}(i)}(\mathcal{O}_i)$ is maximally biased towards one of the available alternative subjective eternal objects—the case where all but one of the propensities for these qualitative alternatives provided by the creative character of $\omega^c_{W_{s-1}(i)}(\mathcal{O}_i)$ are zero (and consequently the remaining one being one). Thus this possibility, arbitrarily abandoned by Whitehead (see Section 2.3.4), fits naturally into his theory, too.

Contrary to direct causes, indirect causes of an occasion E_i will not be probabilistically relevant for E_i's final definiteness. This is because they are not even deterministically relevant for E_i's initial constituents, including its initial activity. The latter, however, needs to be the case for a probabilistic relevance on E_i's final definiteness.

In sum, then, it seems that Whitehead's claim that the efficient causes of an occasion remain conditioning factors of all later phases seems to be justified at least in case of the direct causes, if the phrase "conditioning factors" is understood generally in a probabilistic rather than a deterministic sense. For the time being this elaboration of Whitehead's rather few hints as to his conception of causation between occasions shall suffice. We will return to this important topic in later sections, particularly in Section 2.6.3.

2.4.4 Transition as non-spatiotemporal process

Recall that a process is spatiotemporal if its phases are ordered by some spatiotemporal order relation and moreover that for this to be the case each of the phases has to belong to a unique spacetime region different from that of the others (see Section 1.1.2). Now in the first phase of a transition a single connected spacetime region is produced and thus this region belongs to this phase as (part of) its outcome. This entire region thus created in the first phase of the transition serves as the fixed spatiotemporal standpoint of the second and last phase of the transition as well as for all following phases of concrescence and for the completed occasion. Thus the entire region belongs to the transition's first phase—the dative phase—as part of its outcome and to its second phase—the conformal phase—as its given unalterable spatiotemporal standpoint. Thus in contrast to the phases of a concrescence process, this region plays *distinct* roles for the two phases of a transition: it is *presupposed* by the second phase of the transition but not by the first one which *produced* it. But is this distinct role the region plays for the two phases of a transition sufficient for a transition to be a spatiotemporal process? Obviously not, since these two different roles the same region plays for the first and the second phase of the transition is irrelevant in respect to any spatiotemporal order relation. Anything a spatiotemporal order relation is sensitive to is the region itself, with the consequence that these phases are not spatiotemporally orderable. *Thus both fundamental species of processes, transition and concrescence, are non-spatiotemporal processes.*

2.5 Two problems of Whitehead's ontology

In this section we will discuss the two main problems of Whitehead's ontology as developed so far. The first one has already been discussed in Section 1.3.1. It consists in the fact that the actual world for a new occasion cannot be determined a forteriori, i.e. it needs to be determined without making use of any features of the occasion in question and thus can , in particular, not be determined after the occasion or any of its initial features have been created. In Section 2.5.1 we will review this problem in the light of the structures of Whitehead's ontology which have been introduced since its first discussion in Section 1.3.1. After this we will show in Section 2.5.2 that even if this problem had been solved, Whitehead's ontology nevertheless contains another related problem. A proposal for the resolution of this latter problem will then be put forward in Section 2.5.3, and as we will see there, this leads to a resolution of the first problem, too.

2.5.1 The first problem

We have already argued in Section 1.3.1 that the actual world of a particular occasion E_i, begotten in stage s of the world-process say, needs to be determined before this occasion or any of its ontological constituents are created. The reason for this is that the creation of each such feature of occasion E_i is created by means of a partial manifestation of the underlying activity $\omega_{W_{s-1}(i)}$ and the existence of such a partial manifestation presupposes that there already is a division of all of actuality up to stage s, i.e. W_{s-1}, into different sub-aggregates $W_{s-1}(i)$ which will be the actual worlds for the new occasions of stage s. After our discussion in Section 1.3.1 we have introduced some further features of Whitehead's ontology: the extensive continuum, the realm of eternal objects and the underlying activity. However, none of them does provide us with a means for such a division either. First, as already argued in Section 1.3.1, the region an occasion occupies within the extensive continuum—even if it is a spatiotemporalized region—like any other of its ontological constituents, cannot be used to single out its actual world from all the occasions so far actualized. In other words, the extensive continuum, even as equipped with the supplementary properties of four-dimensionality and metricity we have provided it with, will be of no help for the present problem. The realm of eternal objects too, does not supply any means to provide us with the required division of actuality either, so that the last candidate for a solution seems to be the underlying activity. At first sight it even seems to be a good candidate since it takes into account each completed occasion, thereby actualizing it, and thus it is in "close contact" with actuality at each stage of the world-process and thus could perhaps account for its required division into different actual worlds. Yet the underlying activity itself is unable to make any decisions and thus envisages all completed occasions in precisely the same way (see Section 2.3.3). As already mentioned, it is an ultimate principle of Whitehead's ontological system that all kinds of decisions have to be traced back to the decisions of concrescent occasions (PR, p. 19, 24; WM, p. 151 ff). Consequently, the underlying activity too, cannot account for the desired differentiation of actuality at stage s into different actual worlds. To sum up, it therefore seems that the idea of a division of the world up to some stage of the world-process into different actual worlds, each constituting the causal past of a single new occasion, is not properly implementable into Whitehead's ontology.

2.5.2 The second problem

Since it will not affect the following argument, we will assume for simplicity that the world-process has a beginning (which cannot actually be the case (see Section 2)) and that in its first stage only two occasions E_1' and E_2' had been created. Now the regions (as well as the other initial constituents) of the occasions to be created in the next stage (i.e. in the second), are determined by the conditioning effects of the occasions E_1' and E_2' on the underlying activity by which the creative character of the activity ω_{W_1} for the second stage is fixed. The conditioning effects of E_1' and E_2' on the underlying activity are determined by the spatiotemporal and the qualitative determinateness of E_1' and E_2', i.e. by their regions and their final definiteness. The regions of E_1' and E_2' are not decided by them during their concrescence processes, but their final definiteness is. Now assume that the conditioning effect of E_1' on the underlying activity *due to its final definiteness* would lead to the creation of a region $\mathcal{O}_1$ and that due to the final definiteness of E_2' would lead to a region $\mathcal{O}_2$ for occasions of the second stage.

Now since occasions E_1' and E_2' are created in the same stage of the world-process, they must be causally independent. This is because the causes of an occasion must be completely determinate, and thus their concrescence processes must have terminated, genetically before the occasion in question can be created. But the obtainment of such an order of creation among occasions *created in the same stage* of the world-process, would obviously undermine the very meaning of the latter phrase. Therefore, E_1' and E_2' *cannot coordinate the decisions they settle during their concrescence processes by means of any causal influences.* On the other hand, if their decisions were *completely uncoordinated* it could happen that the regions $\mathcal{O}_1$ and $\mathcal{O}_2$, thus independently determined, will overlap, *thereby conflicting with the separative property of the extensive continuum* (see Section 2.1) that requires the regions of any two occasions to be non-overlapping.[15] This problem does occur in all cases where the initial definiteness of the occasions E_1' and E_2' is not already coordinated to such a degree that the separateness of the regions for the next stage is thereby secured. The problem in question has also been recognized by Nobo in his analysis of Whitehead's ontology (see WM, p. 307, 413 note 4). However, he does not propose any solution but merely concludes that some coordination between the decisions of concrescent occasions like E_1' and E_2' is necessary.

2.5.3 A resolution for both problems

Since direct causal influences are ruled out, the only possibility for how the concrescent occasions E_1' and E_2' could coordinate their decisions, seems to be by means of not having distinct but rather sharing *one common activity that up to some phase of concrescence decides for both.* According to this assumption, then, up to some phase of concrescence there are not two distinct concrescence processes but rather only one such process "driven" by a single undivided activity. The idea of two or more occasions arising from one undivided concrescence process has first been introduced by S. Malin (1988) as a possible way to reconcile Whitehead's ontology with the empirically confirmed violation of Bell's inequality. Our following account relies on this basic idea of Malin, but it will be far more detailed than Malin's. The most important point where we will go beyond Malin's account according to which it is not clear how more than one occasion arises from a single concrescence process,

[15]Note that the union of two such overlapping regions cannot simply be taken as the region of a single occasion, thereby avoiding this conflict. This is because by reason of the ingressed boundary surfaces of two overlapping regions their union is *not one undivided region* as required for it to be the spatiotemporal standpoint of a *single* occasion (see Section 2.1).

is by providing the following model for this: we assume that an initially undivided activity and with it the collective concrescence process, *can bifurcate into two (or more) distinct activities in some arbitrary phase of transition or concrescence, provided that the definiteness of the occasions* E_1' *and* E_2'*, are already coordinated to such a degree that the creation of overlapping regions for the next stage of the world-process is prohibited.* Since the ingression of these initial definiteness into the regions $\mathcal{O}_1'$ and $\mathcal{O}_2'$ of the occasions E_1' and E_2' takes place in the conformal phase of transition, the earliest "moment" in which a bifurcation can take place is therefore at the end of this conformal phase (see also the last section). Since each of the activities resulting from such a bifurcation, settles decisions for one and only one of the two regions $\mathcal{O}_1'$ and $\mathcal{O}_2'$, each can meaningfully said to be located in the respective region (see Section 2.3.1). Later on we will see that this account of generally undivided, bifurcating activities will eventually allow us to provide an ontological explanation for the violation of Bell's inequality and even for the different degrees of "non-separability of quantum states" (see Sections 10.2 and 11.7). Now as long as there is merely one undivided activity, settling decisions for both regions, this activity can hardly be said to be located in one of the regions and thus to be particularized. Consequently, it makes sense to speak of distinct concrescent occasions E_1' and E_2' *only after such a bifurcation* of the activity into two distinct activities, each located in one of the regions $\mathcal{O}_1'$ and $\mathcal{O}_2'$ has taken place. Moreover, if there is merely one undivided activity up to some phase of the collective concrescence process of stage s, there has consequently also been one undivided activity in all earlier phases of concrescence and transition. Therefore, Whitehead's demand of the particularity of the activities after the dative phase of transition is clearly ruled out, too—without distinct activities (and thus distinct transition processes at that stage of the world-process) there can hardly be particularized activities (see Section 2.4.1).

Another consequence of the proposed modification is that all the occasions created in the same stage of the world-process, s say, *have all their efficient causes in common and thus they have the same actual world.* This is because if there is merely one undivided initial activity $\omega_{W_{s-1}}$ for stage s, each occasion that is relevant for this activity[16] is a direct efficient cause of all the regions

[16] Note that we have assumed that amount and creative character of an activity are always

created from it. The same argument applies in case of the initial abstractive hierachies of subjective eternal objects ingressed into these regions—their ingression is directly caused by precisely the same occasions from W_{s-1}. And since the coincidence of the direct causes of these regions and initial hierachies implies the coincidence of their indirect causes (see Section 2.4.3), they have in fact all their efficient causes in common. This moreover implies, that all occasions created in the same stage of the world-process, i.e. from the same initially undivided activity, are causally independent, as it must be the case (see Section 2.5.2). This is because all these occasions have the *same* causal past, so that if one of them were a cause of another, it would belong to its own causal past, i.e. it would be an efficient cause of its own existence, which cannot be the case (see Section 1.1).

Note that the problem with the determination of actual worlds is thereby also dissolved (though not in a way intended by Whitehead). First of all, if all occasions created in the same stage arise from the same actual world, via a bifurcation activity, there simply is no need for singling out different actual worlds from W_{s-1} for the possibility of more than one occasions to be created in stage s. According to Whitehead's original, but completely ad hoc, assumption that conditioning effects of occasions cannot be rendered irrelevant, i.e. screened off, by those of other occasions, the whole aggregate W_{s-1} of all so far actualized occasions would therefore constitute the actual world of the occasions created in stage s. However, we have allowed for the possibility of causally ineffective occasions (see Section 2.4.3), so that within our account an occasion E' from W_{s-1} belongs to the actual world for stage s iff it is a relevant determining factor for the manifestation of the underlying activity for stage s, i.e. for $\omega_{W_{s-1}}$. Or more explicitly, iff E''s conditioning effect on the underlying activity is not screened off by those of other occasions from W_{s-1}. This determination of the (one) actual world for stage s does not pose any problems of the kind discussed in the last section. The activity $\omega_{W_{s-1}}$ for stage s is produced by the envisagement of the whole aggregate W_{s-1}, and those occasions from it which are irrelevant for the creative character (and the

caused by the same occasions (see Section 2.4.3), so that an occasion cannot be relevant for some activity by being relevant merely for the amount of this activity. This assumption will simplify many of the following dsicussions about efficient causation.

amount) of this activity, do not belong to the actual world for stage s. In other words, the activity and by it also the actual world for the next stage are determined in the "moment" when the underlying activity envisages the occasions belonging to W_{s-1}. And since this actual world need not be further divided for there arising more than one new occasion, this determination of the actual world of stage s is not plagued with any circularities.

Before we enter into a more detailed discussion of the consequences the assumption of initially undivided, bifurcations activities have within Whitehead's ontology, we want to point out that the possibility of there arising more than one occasion from the same actual world has also been argued for by two other authors.

Nobo and Stapp on the doctrine of actual worlds

Nobo and Stapp have come to the same conclusion as we did, in regard to Whitehead's doctrine of actual worlds that says that the causal pasts of different occasions necessarily have to differ. Yet these authors have not arrived at this conclusion by way of the above reasoning, rather they attack the doctrine of actual worlds by reason of the systematic role it plays in Whitehead's ontology. Nobo argues that the doctrine of actual worlds

> simply represents an attempt by Whitehead to apply [...] [his theory] to the cosmology of relativity physics. Nothing in Whitehead's metaphysics, as I understand it, precludes the possibility of two contemporaries having identical pasts or identical futures. Indeed, it is my conviction that the metaphysics of organism gains in coherence and adequacy when it allows for the possibility of two or more occasions having identical [...] [causal pasts and futures]. (WM, p. 279)

Stapp makes essentially the same point when he says that

> Whitehead choose to reconcile his philosophic aims with the empirical facts [of relativity physics] by imposing special ad hoc conditions on his basic ontology, rather than allowing the empirical

> facts to follow from his philosophical principles. (Stapp 1979, p. 22)
>
> [Abandoning the doctrine of actual worlds] restores unity to the world, thereby eliminating a serious structural and aesthetical defect in Whitehead's model. (Stapp 1975, p. 270 footnote 1)

Our discussion in Section 2.5.1 strengthens these conclusions as to the ad hoc and alien character of the doctrine of actual worlds within Whitehead's theory. However, as Section 2.5.2 showed, it is not enough simply to abandon the doctrine of actual worlds, because this does not solve the problem that some coordination is needed between the decisions of jointly concrescent occasions. As we have argued, following an idea of Malin, a solution of this difficulty can be achieved by the postulation of initially undivided, bifurcating activities. This postulate, then, automatically implies that the doctrine of actual worlds—apart from being not properly implementable into Whitehead's theory—has to be given up.

Now Stapp argued that, besides its ad hoc character, the idea of each two occasions necessarily arising from different causal pasts has to be given up for yet another reason:

> Whitehead imposed on his model the relativistic requirement that what happens in any given spacetime region be determined only by what has happened in its absolute past, i.e. in the backward light-cone drawn from the region. This requirement must be modified, for it is inconsistent with the implications of quantum theory expressed by a generalized version of Bell's theorem. (Stapp 1979, p. 2)

Bell's theorem and its implications for an ontological interpretation of QFT will be discussed in detail later on in Section 10.2. In that section we will see that, contra Stapp, it is *not implied* by QFT nor by Quantum Mechanics (QM) that the causal past of an event cannot be confined to its backward lightcone. However, we will see that what Bell's theorem suggests is the above assumption of generally undivided, bifurcating activities. Of course, as explained above, *by way of this assumption the doctrine of actual worlds is then* *ruled out, too.* However, the validity of this latter consequence relies on Whitehead's conception of efficient causation and will therefore not generally hold for other conceptions of causation, too. Thus Bell's theorem *in itself* does not imply that the causal pasts of events cannot be confined to their backward lightcones. Rather the ontological implications of this theorem depend on the overall interpretational framework one uses for QFT and in particular on ones conception of causation.

2.6 Bifurcating activities and their consequences for Whitehead's ontology

We have argued in the last section that two difficulties of Whitehead's ontology can be resolved by the assumption that at each stage s there is merely one undivided transition process creating the regions, say $\mathcal{O}_i$, $i = 1, \ldots, N$, with their initially ingressed hierachies of subjective eternal objects, whose outcome activity, term it $\omega^c_{W_{s-1}}$, is generally still not divided into partial activities. Generally, such a division into partial activities will only take place in some phase of the initially still undivided concrescence process, that thereby also bifurcates into partial processes. In the present section we will investigate some further consequences of the above assumption for Whitehead's theory.

2.6.1 Consequences for the individuality of occasions

The possibility of more than one completed occasion arising from a single bifurcating concrescence process, implies that one has to *restrict* the sense in which *concrescent* occasions are individuals. According to Section 2.3.2 the individuality of a concrescent occasion consists of its definiteness and *the particularity and degree of autonomy of its activity.* Consequently, the individuality of a concrescent occasion E_j with region $\mathcal{O}_j$ depends on the degree as to which the activity $\omega^c_{W_{s-1}}$ of this concrescence can be regarded as a "sum" of distinct parts

$$\omega^c_{W_{s-1}} = \omega^c_{W_{s-1}}(\mathcal{O}_j) + \omega^c_{W_{s-1}}(\mathcal{O}_1, \ldots, \mathcal{O}_{j-1}, \mathcal{O}_{j+1}, \ldots, \mathcal{O}_N), \qquad (2.1)$$

such that one of these—$\omega^c_{W_{s-1}}(\mathcal{O}_j)$ in (2.1)—can be regarded as being located in region $\mathcal{O}_j$. In other words, the individuality of a concrescent occasion depends not only on the degree of autonomy, but also on the *degree of particularity* of its activity. In Whitehead's original theory, each activity involved in a concrescence process has been assumed to be a particular in the sense of being located in a single connected region. The introduction of generally undivided, bifurcating activities therefore introduces another "parameter" on which the individuality of concrescent occasions depends, namely the degree of particularity of their activities.

That the particularity of the activities of concrescent occasions comes in degrees may at first sight look weird. However, particularity has been defined as the locatedness in a single connected region and the locatedness of a concrescent activity in a region has been understood as its settling decisions for this region (see Section 2.3.1). Therefore, it seems that one can make sense of the *degree of locatedness* of a certain activity in a certain region for example by understanding it in terms of the number of its decisions which exclusively refer to this region's final definiteness divided by the number of all its decisions. This degree of locatedness in the region in question, then, also quantifies the degree of particularity of the activity in respect to this region. Because of our assumption that in each phase of concrescence only one (simple) decision is made (see Section 1.1.3), the degree of particularity with respect to region $\mathcal{O}_j$ can also be expressed by the quotient of the number of concrescence phases in which the activity $\omega^c_{W_{s-1}}$ is in fact divided according to (2.1) and the number of all the phases of the concrescence process. This account will be further refined in Section 11.3.

Thus if a division like (2.1), according to which already the *initial* activity $\omega^c_{W_{s-1}}$ of the concrescence process is in fact divided into a particularized activity $\omega^c_{W_{s-1}}(\mathcal{O}_j)$ and another partial activity $\omega^c_{W_{s-1}}(\mathcal{O}_1, \ldots, \mathcal{O}_{j-1}, \mathcal{O}_{j+1}, \ldots, \mathcal{O}_N)$ (which need not itself be further divided into partial or even particularized activities), the concrescent occasion E_j corresponding to $\omega^c_{W_{s-1}}(\mathcal{O}_j)$ is an individual in the fullest sense, i.e. in the sense appealed to in Whitehead's original theory (see Sections 2.3.1 and 2.3.2). As already argued in Section 2.5.3, such an "early" bifurcation at the end of the conformal phase of the forgoing transition process, can very well take place without undermining the coordination of the determination of the new regions to be created in the next stage (i.e. stage $s+1$): for it is possible that already the *initial* definiteness of the regions $\mathcal{O}_1, \ldots, \mathcal{O}_N$ as determined by the corresponding causal past in the conformal phase, are already such that no further coordination, at least with respect to the initial definiteness of region $\mathcal{O}_j$, is required. The other extreme case is that such a bifurcation does not take place, not even during the following phases of concrescence, at all. In this case the only individuality attributable to the concrescent occasion in question is that due to its region and its definiteness. Thus even in this extreme case one can still say that the individuality of an occasion E_j is successively deepened during the phases concrescence, but this deepening is, then, clearly not attributable to "this occasion's autonomous decisions". Such talk only makes sense as to the degree as to which the collective activity is divisible into distinct parts one of which being located in region $\mathcal{O}_j$.

Finally we want to point out that there is no need for assuming that divisions of the kind (2.1) for different regions of a stage of the world-process, do all occur at once. Rather we can and will allow for the possibility that such "splitting offs" of partial or particularized activities corresponding to different (sets of) regions may occur in different phases.

2.6.2 The creative character of a bifurcating activity

Now we have to answer the question what happens to the creative character of an activity when the latter bifurcates into partial activities. The first possibility is that the creative characters of the resulting partial activities are always identical with one another as well as with the creative character of their "mother-activity". Of course, this is the simplest way of making sense of the dictum that only the amount of activity but not its creative character changes during the phases of transition and concrescence (see Sections 2.3.1, 2.4.1 and 2.4.2)). However, this sense of the constancy of creative character seems to be too restrictive if one allows activities to bifurcate. Rather in the light of this possibility, the more natural way of how one can understand the constancy of the creative character of activities "across" bifurcations seems to be the following: the creative characters of the partial activities of a phase (as far as there are such partial activities at all), say $\omega^{c,n}_{W_{s-1}}(\mathcal{O}_1)$, $\omega^{c,n}_{W_{s-1}}(\mathcal{O}_2, \mathcal{O}_3, \mathcal{O}_4)$ and

$\omega^{c,n}_{W_{s-1}}(\mathcal{O}_5, \ldots, \mathcal{O}_N)$ for phase n of concrescence, always combine to the creative character of the total activity of this phase, here $\omega^{c,n}_{W_{s-1}}$, without being identical to one another, the latter being the case only for the creative characters of these total activities. Thus according to this more general understanding, the constancy of the creative character during all phases of transition and concrescence holds for the *total* activities of these phases, but will generally not hold for their parts (as far as there are such parts at all). Accordingly, equation (2.1) is, then, to be understood as expressing the way in which the amounts *as well as* the creative characters of partial activities combine or "sum up" to the total activity of a phase. Later on in connection with QFT we will be able to make precise the sense of this "sum" in (2.1).

Finally there is the question of how it is *decided* in which phase a bifurcation takes place and what the resulting activities will look like (as to amount and creative character). Since according to Whitehead the activity involved in a transition process is not able to settle any autonomous decisions, bifurcations of the initial concrescent activity $\omega^{c}_{W_{s-1}}$, taking place at the end of the conformal phase of transition, *must already be determined by the efficient causes of this activity.* This need not be the case for bifurcations taking place in the phases of concrescence, because the corresponding activities have the ability of settling autonomous decisions. However, for reasons of simplicity we will assume that bifurcations taking place in phases of concrescence are also completely determined by the efficient causes which have fixed the creative character of the initial activity $\omega_{W_{s-1}}$. This determination, like the determination of the regions $\mathcal{O}_i$, $i = 1, \ldots, N$ and the initial abstractive hierachies of subjective eternal objects to be ingressed into these regions, is therefore likewise assumed to be "impressed" into the creative character of the activity $\omega_{W_{s-1}}$ by its causal past.

2.6.3 Bifurcating activities and efficient causation

Causal independence

In Section 2.5.3 we have already shown that the assumption of generally undivided, bifurcating activities implies the causal independence of the occasions created in the same stage of the world-process. There we have argued that otherwise there could be occasions which belong to its own causal past. For convenience we will begin our present discussion with a more detailed account of this argument. According to Whitehead, efficient causation proceeds via the conditioning of activities and therefore presupposes that the "effect-occasion" has its own activity different from that of the "cause-occasion". According to our modification, however, all occasions created in the same stage arise from one and the same initially undivided activity $\omega_{W_{s-1}}$, so that as long as it is undivided, causal influences between the corresponding occasions are impossible. On the other hand, as soon as two occasions of the same layer have their own activities—only then the talk of different "occasions", strictly speaking, makes sense—there is no way of how one could influence the other either. This is because the activity of one of the occasions, say E_i, would have to be conditioned by the other occasion, say E_j, qua completed outcome of its concrescence. But this presupposes that occasion E_j had conditioned the activity of occasion E_i, which, however, is only possible via conditioning the undivided activity $\omega_{W_{s-1}}$ from which occasion E_i arose (see Section 2.4.3). But this is likewise the undivided activity from which occasion E_j arose, so that one would end up with a contradictory kind of self-causation, according to which occasion E_j qua completed outcome of its concrescence is a cause of its own initial constituents. Thus, in sum, our modified account of, in general, undivided activities implies the causal independence of occasions created in the same stage of the world-process.

Causal influences up to some phase of concrescence

In Section 2.4.3 we have seen that within Whitehead's original account of efficient causation, each occasion E' that is a direct efficient cause of another occasion E_i, i.e. that is a relevant determining factor for E_i's initial ontological constituents, is automatically *a probabilistic cause of the final definiteness of* E_i. This need no longer be the case in our modified account that includes undivided, bifurcating activities. In our account it can be the case that E' is a direct efficient cause for the region and the initial definiteness of an occasion without being a cause for any features determined in phases of concrescence and thus it need, in particular, not be a probabilistic cause for the final definiteness of that occasion. Let us assume for simplicity that only two occasions, E_1 and E_2 with regions $\mathcal{O}_1$ and $\mathcal{O}_2$, will be created in stage s and let occasion E' be a direct efficient cause of both (as already shown in Section 2.5.3, each occasion E' that is an efficient cause of one occasion created in some stage is automatically an efficient cause of all other occasions created in this stage, too). Now if a bifurcation of the activity takes place already in the conformal phase of the transition process, i.e. if the activity $\omega^c_{W_{s-1}}$ is divided into particularized activities $\omega^c_{W_{s-1}}(\mathcal{O}_1)$ and $\omega^c_{W_{s-1}}(\mathcal{O}_2)$, E' need *not* be relevant for both of these resulting activities. This is because the assumed relevance of E' for the undivided "mother-activity" $\omega^c_{W_{s-1}}$ only requires E' to be relevant for *one* of the resulting activities, *but not necessarily for both.* Therefore, though E' is a direct efficient cause of both occasions with respect to their regions and initial definiteness, it need not also be a cause of the concrescent activities $\omega^c_{W_{s-1}}(\mathcal{O}_1)$ and $\omega^c_{W_{s-1}}(\mathcal{O}_2)$ of both, and thus need not be a probabilistic cause for the final definiteness of both occasions. Thus the introduction of, in general, undivided, bifurcating activities opens up the possibility that an occasion that is a direct efficient cause of at least two occasions, causes them up to different phases of their concrescence process(es)—a possibility not inherent in Whitehead's original account. However, note that the original result, namely that an occasion E' that is a direct efficient cause of another occasion is automatically also a probabilistic cause for the latter's final definiteness, *still holds for at least one of the occasions of which E' is a direct efficient cause.* This is because if E' has been relevant for an activity before its bifurcation, *E' must also be relevant for at least one of the resulting partial activities*—otherwise E' could not have been relevant for the activity before the bifurcation in the first place. By applying this result to each bifurcation of the initially undivided activity one gets the result in question.

Before we can investigate the important question whether and in what sense the account of causation presented so far is in conflict with STR, we first of all have to fix a particular connection between causal and spatiotemporal properties of occasion—otherwise obviously nothing can be said about superluminal causation and the like, at all. This will be done in the following section in which we will moreover answer another central question that has been left open up to this point—the question of how one can make sense of an evolving world-process.

2.7 The order of envisagement and the expansion of the world

For the very talk of stages and thus of an evolvement of the world or equivalently of the expansion of actuality to make sense, actuality as a whole—understood as the pattern of all occasions that have been and will ever be actualized—must exhibit a unique layer-cake structure, i.e. it has to be uniquely divisible into linearly ordered layers of occasions (see Section 1.3.2). In this section it will be seen how one can make sense of such a layer-cake structure and thus of an evolving or expanding world-process.

According to Whitehead

> the creativity [is that] whereby the actual world has its character of temporal passage to novelty. (RM, p. 90)
>
> The reason for the temporal character of the actual world can now be given by reference to the creativity and the creatures [i.e. the occasions]. For the creativity is not separable from its creatures. Thus the creatures remain with the creativity. Accordingly, the creativity for a creature becomes the creativity with the creature, and thereby passes into another phase of itself. It is now the creativity for a new creature. (RM, p. 91 f)

Thus it is fairly clear that according to Whitehead the "temporal passage to novelty" of the world is grounded in the interplay of occasions and the underlying activity and thus ultimately *in the envisagement of the former by the latter*. Unfortunately, Whitehead did not gave a more detailed account of *how* this shall be accomplished. In Section 2.3.4 we had to assume that each occasion, once envisaged will also be envisaged in all "later" acts of envisagement, to make sense of the objective immortality of occasions. In this and in the following section we will argue that by specifying the "later" relation among acts of envisagement as a *linear order*, the spatiotemporal evolvement of the world can be grounded in the envisagement of occasions by the underlying activity.

2.7.1 The order of envisagement and the causal expansion of the world

Let W denote actuality as a whole, i.e. the (hypothetical) aggregate of all the occasions which have been and will ever be actualized. Moreover, let $W_{s'}$ be the sub-aggregate of W that is taken into account by the underlying activity in the envisagement act denoted by s' and W_s the sub-aggregate envisaged in the later act denoted by s (see Section 2.3.4). Note that at this point W_s is *not* already the aggregate of *all occasions actualized after stage* s *of the world-process*. Rather this interpretation of W_s is what shall be justified in the present section. As we will argue in a moment, each envisaging act has an *immediate* predecessor. Now it would be ontologically highly uneconomical if one would allow for the possibility that in two immediately succeeding acts of envisagement $s' < s$ the very same aggregate of occasions $W_{s'} = W_s$ is envisaged, because then act s would simply fulfil no ontological function not already fulfilled by its predecessor. Therefore, it is highly reasonable to exclude this logical possibility and to assume instead that the aggregates of occasions envisaged in immediately succeeding acts of envisagement are always different. The assumption that an occasion that has been envisaged once is also to be envisaged in all succeeding acts of envisagement, that had to be made already in Section 2.3.4, then, implies that each occasion envisaged in act s' ($< s$) is also envisaged in act s, i.e. W_s *includes* $W_{s'}$. Thus there exists a unique sequence (W_s) of sub-aggregates of W, such that $W_{s'} \subset W_s$ whenever $s' < s$. The uniqueness of this sequence follows from the simple fact that there is merely a single underlying activity that envisages occasions, so that in turn there can only be one such sequence of envisaged aggregates of occasions.

Now since the world-process is infinitely old, i.e. each actualized occasion has at least one predecessor, actuality as a whole W cannot be a finite aggregate of occasions. On the other hand, occasions are spatiotemporally discrete and thus W cannot consist of uncountable many occasions, even if it would cover the whole extensive continuum without leaving any gaps of unactualized extension. Therefore, W as well as the sequence (W_s) of sub-aggregates of W have countably many elements. In other words, there can only be countably many acts of envisagement of different sub-aggregates of W. This, then, means that each act of envisagement s has a unique immediate predecessor $s-1$ and a unique immediate successor $s+1$.[17]

Because of $W_{s-1} \subset W_s$ the set $L_s \equiv W_s \backslash W_{s-1}$ of occasions belonging to W_s but *not* to W_{s-1} is non-empty and all occasions in L_s are *co-envisaged* in the following sense:

(CO-E) Two occasions E and E' are co-envisaged iff in *each* act of envisagement in which E is envisaged by the underlying activity, E' is also envisaged and vice versa.

This notion of co-envisagement is to be distinguished from the sense in which each two arbitrary occasions from some aggregate W_s are *jointly* envisaged. For, by definition, *all* occasions of W_s are taken into account together or jointly in envisaging act s by the underlying activity. Moreover, for each two arbitrary occasions in W there is at least one act of envisagement in which both are envisaged together, since W consists, by definition, of all ever actualized and thus envisaged occasions and each later envisaged aggregate of occasions includes each earlier envisaged one. But *not* all occasions in W_s are also co-envisaged. Rather the latter holds for all and only those occasions which are jointly envisaged in *all* their respective acts of being envisaged. Note that, because of the inclusion of $W_{s'}$ in W_s whenever $s' \leq s$, "being co-envisaged" is moreover equivalent to the fact that the occasions in question have been envisaged together in their respectively *first* acts of being envisaged. This is because in this case they will also be taken into account together in all their further acts of being envisaged by the underlying activity, so that they are in fact co-envisaged. On the other hand, if two occasions have not been envisaged together in their respectively first acts of being envisaged, then, there clearly has been an act of envisagement in which one of them has been envisaged but not the other so that (CO-E) is not satisfied. Therefore, "co-envisagement" is obviously a transitive relation on W: if occasions E'' and E' have been jointly

[17] Of course, if the world-process should have an end the latter is only true for all but the last stage.

envisaged in their respective first acts of being envisaged and the same is true for E' and E, then E'' and E must also be jointly envisaged in their respective first acts of being envisaged. For otherwise there would have been *more than one first* act of being envisaged for occasion E', namely the one in which it has been jointly envisaged with E'' and the one in which it has been jointly envisaged with E, which is logically impossible. Moreover, by definition, each occasion is co-envisaged with itself (reflexivity) and if E' is co-envisaged with E then E is also co-envisaged with E' (symmetry) so that *"being co-envisaged" is an equivalence relation on W*. Therefore, the latter is the disjoint union of the corresponding equivalence classes $L_s = W_s \backslash W_{s-1}$. In other words, *$W$ exhibits a unique layer-cake structure given by layers L_s of co-envisaged occasions.* Accordingly, as the notation has already suggested, the aggregate W_s that is taken into account in envisaging act s of the underlying activity, can be interpreted as the aggregate of *all actualized occasions after stage s of the world-process.* In other words, this means that the *stages of the world-process are in one-to-one correspondence with the acts of envisagement of the underlying activity.* Therefore, one can interpret the layer $L_s = W_s \backslash W_{s-1}$ as consisting of just those occasions which are begotten in stage s of the world-process. This confirms the natural expectation that the occasions arising from the same (initially) undivided activity $\omega_{W_{s-1}}$, are also co-envisaged when their (bifurcating) concrescence process(es) are run through. This could be condensed in to the slogan *"jointly begotten—co-envisaged"*. Since, as argued earlier, all occasions begotten in the same stage s of the world-process are causally independent and have the same causal past (the latter being a subset of all the so far actualized occasions W_{s-1}) the same is therefore the case for all co-envisaged occasions, i.e. for all occasions belonging to the same layer L_s. Therefore, the layer-cake structure

$$\ldots, L_{s-1}, L_s, L_{s+1}, \ldots \tag{2.2}$$

arising from the order of envisagement of the underlying activity establishes a *causal expansion* of actuality.[18] But note that unlike their common causal

[18] That the occasions in L_s are mutually causally independent and share a common causal past, can also be shown from the properties of co-envisaged occasions alone, without invoking the identity of the layer L_s with the occasions begotten in stage s. For to be co-envisaged means for two occasions E and E' that neither of them is ever envisaged before (or after)

past, not all occasions from the same layer need to have the same causal future, since they can be screened off by different occasions and thus can become causally ineffective at different stages of the world-process. Now the layer-cake structure (2.2) does not yet provide us with a spatiotemporal expansion of actuality, since the occasions in a layer L_s could be spatiotemporally arbitrarily scattered. Yet by fixing a reasonable connection between causal and spatiotemporal properties of occasions, we will be able to turn (2.2) into a spatiotemporal layer-cake structure, too. Yet before we will turn to this topic in the following section, we will answer the important question of *how it is decided which occasions shall be co-envisaged by the underlying activity* and thus shall form a layer L_s of actuality.

We know from Section 2.3 that the underlying activity itself is *not able to settle any decisions at all* and thus in particular cannot account for these decisions. Rather in Whitehead's ontology decisions are only made by concrescent occasions, more precisely by their activities, and thus the decisions in question also have to be traced back to decisions of concrescent occasions. Each occasion E has a causal past that begets its initial ontological constituents in a process of transition and thereby also fixes those occasions which will be co-envisaged with E when its concrescence process is completed, to be those occasions which are jointly begotten with E in this collective transition process (see above). Since this is the case in every stage of the world-process and the latter, moreover, does not have a beginning, i.e. no first occasion or group of co-envisaged occasions, the question of how such a "first cause" or "group of first causes" is begotten without necessitating a decision of the underlying activity *simply does not arise*. Therefore, it is fixed which occasions have to be co-envisaged and thus will belong to the same layer of actuality *without the need for any—non-available—decisions of the underlying activity*. Rather in accord with Whitehead's ontological principle, these decisions are traced back to decisions of the respective efficient causes.

Up to this point we have always used the term "causal past" in the sense of encompassing all and only those occasions which are in fact efficient causes

the other. However, the latter is necessary for one of them to be an efficient cause of the other. Therefore, they are causally independent. That E and E' moreover have the same causal past can be seen equally easily but shall not be shown here explicitly.

of the occasion in question. In the following it will be convenient to introduce another widespread use of this term, according to which it encompasses all and only those occasions which *could* have been causes of an occasion if the circumstances had been different. To avoid confusions between these two meanings of "causal past", let us introduce the term "causal past*" to refer to the latter meaning. In other words, we will understand the causal past* of an occasion E to consists of all and only those occasions which would have been efficient causes of E if their causal influences had not been screened off by other occasions (see Section 2.4.3). Thus the causal past* of an occasion in layer L_s consists of all earlier actualized occasions, i.e. of all occasions in W_{s-1}. Accordingly let us define the causal future* of E to consist of all those occasions to whose causal past* E belongs. The causal future* of an occasion from layer L_s is then given by $W \backslash W_s$, where W is again the hypothetical aggregate of all occasions that have and will ever be actualized. According this weaker notions, all occasions of the same layer do not merely have the same causal past* (as is also the case in the stronger sense) but also the same causal future*.

2.7.2 The spatiotemporal expansion of the world

Up to this point no constraints as to the region of an occasion and those of its co-envisaged occasions have been introduced, so that the occasions belonging to a layer L_s of actuality can be quite arbitrarily scattered within the extensive continuum. Therefore, one cannot yet speak of an extensive or even spatiotemporal expansion of actuality. Of course, the latter is not assumed by Whitehead to be the case in the whole world-process because according to him spatiotemporality itself is merely a contingent feature of *some* cosmic epochs (see Section 2.1) and therefore a spatiotemporal expansion can only obtain in those parts of the world-process in which the extensive regions of occasions are moreover spatiotemporally structured. Yet an extensive expansion is also held by Whitehead to be a general feature—a metaphysical necessity—of the whole world-process. As Nobo puts it,

> considered in respect to its real division, however, the extensive continuum is to be construed as forming an ever-expanding, coher-

> ent system of non-overlapping proper regions or standpoints. This coherent system of real division is termed [...] 'the region of actuality'. [...] Moreover, insofar as the actual world is always expanding through the emergence of new actualities, [...] the boundary of the region of actuality changes with each expansion of the actual world. (WM, p. 211)

It is important to notice that not the extensive continuum itself is what expands—it is the eternally fixed pure potential for actualized regions (see Section 2.1). Rather only the aggregate of actualized regions of the extensive continuum expands whenever new occasions are actualized. Fortunately we need not investigate the question of how one can make sense of an expansion of the extensive region of actuality within the merely topologically structured extensive continuum, because we have made the simplifying assumption that the extensive continuum itself is equipped with dimensional and metrical structures. More precisely, in the light of the attempted connection with QFT we have assumed in Section 2.1 that the extensive continuum is four-dimensional and that it is equipped with the metric of STR and thus we need in what follows only discuss how one can make sense of a spatiotemporal expansion. Of course, because the doctrine of actual worlds, and thus in particular the relativistic connection between causal and spatiotemporal properties of occasions, is ruled out (see Sections 2.5.3), the assumption that the extensive continuum is equipped with the relativistic metric seems to be quite arbitrary. In particular, it would presumably be more natural to choose the metric of classical Newtonian spacetime. We will return to this topic in Section 2.8. Since the relativistic metric is presupposed by QFT and assumed by Whitehead to obtain at least contingently in our cosmic epoch we should, however, try our best to incorporate it into Whitehead's theory before we may come to the conclusion that a different metric would be the better choice.

Since the world does already exhibit a causal expansion we can, by fixing an appropriate connection between causal and spatiotemporal properties of occasions turn this causal expansion into a spatiotemporal one, too. There are many possibilities of how one can connect causal and spatiotemporal properties of occasions. But the choice of a particular connection is constrained if the spatiotemporal and the causal expansion of the world do arise from the same ontological source. If there is a common ontological source for both expansions, these expansions should be compatible in the sense that the causal past* (future*) of an occasion E, i.e. those occasions which could have been causes of (could have been caused by) E, coincides with its spatiotemporal past (future). In other words, belonging to the spatiotemporal past (future) of E should be a necessary condition for being a cause of (being caused by) E. Consequently, those occasions which can neither cause nor be caused by E have to be neither spatiotemporally earlier nor later than E—they have to be in E's spatiotemporal present. In Whitehead's theory the causal and the spatiotemporal expansion of the world *are both grounded in the envisaging property* of the underlying activity and therefore should be compatible in this sense (WM, p. 213 f, 219, 223, 256).

This compatibility of the causal and spatiotemporal expansion can obviously be restated as follows:

> For any two occasions E and E', occasion E' is in the causal past* (future*) of E iff E' is in the spatiotemporal past (future) of E.

All occasions belonging to the same group of co-envisaged occasions, i.e. to the same layer, L_s say, of actuality, have the same causal past* and future* (see Section 2.7.1). From this it follows with the compatibility constraint that the spatiotemporal pasts and futures, and therefore also the spatiotemporal presents, of all co-envisaged occasions have to coincide, too. Now since we have assumed that the extensive continuum is equipped with the metric of STR, another constraint as to the spatiotemporal pasts, presents and futures of occasions is that they have to be definable in terms of this metric. Together with the fact that the spatiotemporal pasts, presents and futures of all co-envisaged occasions have to coincide, the simplest way of introducing the spatiotemporal past of an occasion seems to be the following:

(SP) The spatiotemporal past of an occasion E is the union of the backward lightcones of all the regions occupied by occasions which are co-envisaged with E, i.e. of all occasions belonging to the same layer of co-envisaged occasions than E.[19]

[19] Note that we can make use of the concept of the backward- (as well as the forward-)

From this definition of the spatiotemporal past it follows that *the regions of all co-envisaged occasions have to be mutually spacelike separated from each other*. For if the region of an occasion E' that is co-envisaged with E, would overlap E's backward lightcone, it would overlap its *own* spatiotemporal past, which cannot be the case. For the same reason the region of E cannot overlap E''s backward lightcone. Together with the fact that any two regions of occasions have to be separated, i.e. non-overlapping, it follows that any two co-envisaged occasions have to be spacelike separated.

However, one cannot define the spatiotemporal future of an occasion by simply exchanging backward- by forward lightcones in the above definition of the spatiotemporal past, because this would lead to a conflict with the compatibility constraint. To see this we need only discuss the special case where the layer of occasions co-envisaged with an occasion E merely consists of E itself, i.e. the case where at some stage of the world-process (contingently) only one new occasion, E, is created. According to (SP), the spatiotemporal past of E, then, merely consists of E's backward lightcone. Now assume that the spatiotemporal future of E were in fact given by its forward lightcone. Because of the non-pointlike character of the regions of occasions, there are, then, potential occasions within E's spatiotemporal future such that E does *not* belong to their spatiotemporal pasts, i.e. to their backward lightcones (see Figure 2.1 (a), which suppresses two spatial dimensions). Likewise there can be occasions in E's spatiotemporal past such that E does *not* lie in their spatiotemporal future, i.e. in their forward lightcones (see Figure 2.1 (b)). However, the compatibility constraint requires that for all occasions E and E', E is in the causal future* (past*) of E' iff E is in the spatiotemporal future (past) of E'. Furthermore, by definition, an occasion E' cannot be in E's causal past* without E belonging to the causal future* of E' and vice versa. Together with the compatibility constraint this implies that E has

lightcone of a region without being committed to the introduction of the surfaces of these lightcones as objective eternal obejcts which have to ingress into the extensive continuum. This is because we have equipped the extensive continuum with a metrical structure—in particular with the spatiotemporal distance measure of STR—so that statements like "region $\mathcal{O}'$ lies in the backward lightcone of region $\mathcal{O}$" are already made true by reason of the regions $\mathcal{O}'$ and $\mathcal{O}$ together with the metrical relations they bear to each other (for a refinement of this see the footnote on page 149).

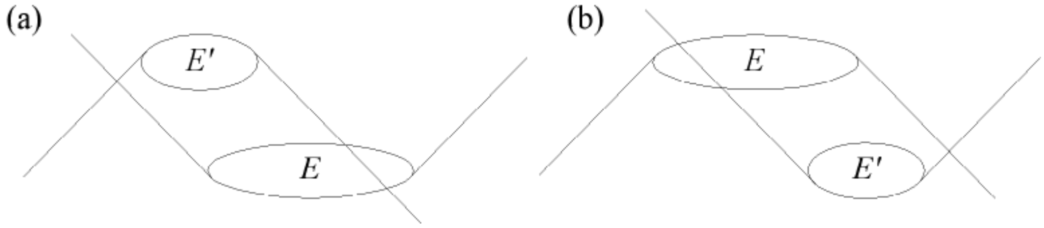

Figure 2.1: A peculiarity of extended events

to lie in the spatiotemporal future of E' iff E' is in E's spatiotemporal past and vice versa. However, as shown above this is not generally true if the spatiotemporal past *and* future of E are defined via the lightcones of E's region. Note that this is also the case in Whitehead's original ontology, at least in our cosmic epoch for which Whitehead assumed that the spatiotemporal pasts and futures of occasions are given by their lightcones. The same problem obviously obtains in STR too, if one allows events to be spatiotemporally extended instead of idealizing them as pointlike. Therefore, if we introduce the spatiotemporal past of an occasion E as above, we cannot introduce its spatiotemporal future simply by exchanging the backward with the forward lightcones of the occasions co-envisaged with E.

Because of the compatibility constraint the new layer L_s of co-envisaged occasions has to be such that all the so far actualized occasions, i.e. the ones in W_{s-1}, lie in the spatiotemporal past of L_s. Since this constraint has to be fulfilled for any particular definition of spatiotemporal pasts, presents and futures of occasions, the most economical—because least restrictive—way of introducing the spatiotemporal futures of occasions seems to be the following:

(SF) The spatiotemporal future of an occasion E is that part of the extensive continuum that is disjoint from the regions occupied by the occasions co-envisaged with E and from their (common) spatiotemporal past.

The spatiotemporal present of E is then given by the regions of the occasions which are co-envisaged, and therefore necessarily causally independent from, E. Since the present of an occasion E consists of the regions of occasions which are co-envisaged with E, and thus with one another, all the regions belonging to such a present have to be mutually spacelike separated from each other (see above).

In sum, then, the creation of a new group of regions—a new present or spatiotemporal layer of actuality—in the dative phase of the transition process arising at stage s of the world-process therefore is restricted by the following two spatiotemporal constraints: (1) the new regions have to be such that their (common) spatiotemporal past contains the regions of all the occasions so far actualized and (2) the new regions have to be spacelike separated from each other. The new group of regions thus created will be the spatiotemporal frontier of attained actuality after stage s of its expansion. As mentioned above, constraint (1) has to be obeyed independently from the particular definition of spatiotemporal pasts, presents and futures. Yet constraint (2) is a consequence of our specific definitions and it implies that, in general, the present of an occasion, unlike its past and future, will be a *disconnected* region of the extensive continuum.

Clearly, other ways of defining spatiotemporal pasts, presents and futures of occasions (by use of the relativistic metric) are possible according to which the present of each occasion is also a connected region. For example, one could demand that the regions of co-envisaged occasions lie between two parallel spacelike hyperplanes such that each region is in contact with both planes. The spatiotemporal present of an occasion could then be taken as the whole *extensive* region between these two hyperplanes. The spatiotemporal past respectively future of an occasion were then given by that part of the extensive continuum that lies on one side of the occasion's spatiotemporal present. According to this definition past, present and future were connected regions and moreover would have more "regular" shapes as in our above definition. However, this has to be paid for by a more restrictive assumption as to the creation of new layers of regions respectively occasions. According to our above definition the regions of such a layer have to be mutually spacelike separated and the union of their backward lightcones has to contain the regions of all occasions in their common causal past*. The second way of defining spatiotemporal pasts, presents and futures does not require that the regions of a new layer of occasions are spacelike separated. But it requires that these regions are in contact with the same pair of parallel spacelike hyperplanes and that each two such hyperplane-layers have to be spatiotemporally disjoint. For if they were not disjoint the present of an occasion could spatiotemporally overlap with its past and future. This, however, means that all the hyperplane-layers that have and will ever be created in the course of the world-process have to be parallel—otherwise they could not be disjoint because each is, by construction, spatially infinite. This second way of explicating the concepts in question therefore requires the determination and creation of new regions to be such that thereby a unique family of *parallel spacelike hyperplanes* is fixed. Our first definition does not impose such a strong demand on the determination and creation of new regions. It could at most be used to single out a family of non-intersecting spacelike hyper*surfaces*, and even these surfaces will not be uniquely fixed by these spatiotemporal layers of regions because there are many different possibilities to define a spacelike hypersurface from a given group of mutually spacelike separated regions.

Now since for the purposes of this work we do not need any more restrictive notions of spatiotemporal pasts, presents and futures than the ones defined with the help of the lightcones of co-envisaged occasions, we will for the rest of this work in fact adopt these definitions. Note that independently of which particular definition of spatiotemporal pasts and presents is adopted, *if* the spatiotemporal past or present of an occasion is a connected region (as is the case, according to our definition, for the past but not generally for the present) these will, in general, consist, besides the actualized respectively realized regions of past respectively present occasions, *also of unrealized extension* (the spatiotemporal future of an occasion obviously consists of unrealized extension only). This is so because we have, in accord with Whitehead, not demanded that the regions occupied by occasions have to be contiguous to each other or even that their aggregate should appear as a continuum (see Section 2.1). Therefore, it would perhaps be more appropriate to speak of the *extensive* pasts, presents and futures of occasions, even if we assume that the regions of occasions are always spatiotemporalized. However, once this point is noticed it will not cause any confusion if we simply speak of the spatiotemporal pasts, presents and futures of occasions.

2.8 On the compatibility with STR

In this section we will investigate if (the modified version of) Whitehead's ontology presented in this and the preceding chapter is in conflict with STR.

2.8.1 Superluminal causation

In Section 2.5.3 we have shown that if a layer of actuality consists of more than one occasion, these occasions are necessarily causally independent. Thus the only possibility for superluminal causation is between occasions belonging to *different* layers. The untenability of the doctrine of actual worlds that had been a consequence of our modification of Whitehead's theory clearly opens up the possibility for such superluminal causal influences. More precisely, the fact that all occasions belonging to the same layer of the world-process have all their efficient causes in common, i.e. arise from the same causal past (at least with respect to their regions and their initial definiteness) will, in general, give rise to instances of superluminal causation. However, recall that independently from our modification, the doctrine of actual worlds has been an alien element of Whitehead's original theory anyway (see Sections 2.5.1 and 2.5.2) and without a proper implementation of this doctrine, superluminal causation could likewise occur according to Whitehead's original theory.

Thus that there will, in general, be superluminal causal influences between occasions belonging to different layers of the world-process is fairly clear. And this conclusion will not merely follow within the (modified) Whiteheadian account of causation presented here. Rather it is to be expected that many other theories of causation come to the same conclusion when applied to Whitehead's ontology. Such an application, however, will be prima facie quite difficult because the notion of "occasion" as invoked by Whitehead is quite alien to those other theories. Most theories of causation take the causal relata to be "events". And though there are some differences in the precise understanding of this notion across different theories, most of them understand events either as the instantiation of properties in spacetime regions or as entities to which probabilities are ascribable. However, both of these notions of events can be made sense of within Whitehead's ontology—even by one and the same aspect of Whiteheadian occasions, namely *by the unrestricted ingression of subjective* *eternal objects into the regions of occasions at the end of their concrescence.* That this is a close analog of the instantiation of properties has already been mentioned in Section 2.2.1. And in Sections 2.3.1 and 2.4.3 we have seen that to these unrestricted ingressions of subjective eternal objects, propensities (i.e. ontic single case probabilities) are ascribed within Whitehead's ontology.

A comparison with Suppes' probabilistic theory of causation

One of the most influential probabilistic theories of causation is the one due to P. Suppes. According to Suppes, an event e' is a cause of another event e iff the following three conditions are met (Suppes 1970, p. 12 ff):

(1) e' occurs temporally earlier than e

(2) the probability of e conditional on the occurrence of e' is higher than the unconditional one

(3) there are no events which are temporally earlier then e' which screen off e''s probabilistic relevance for e

According to the last section, the Whiteheadian world-process admits a distinguished foliation of spacetime. This foliation can obviously be interpreted as establishing a temporal order among occasions, so that condition (1) makes sense in Whitehead's ontology. As argued above, the events e' and e referred to in (1)-(3) can be made sense of in Whitehead's ontology too, by identifying them with the unrestricted ingression of certain subjective eternal objects into certain regions. Recall moreover the Whiteheadian characterization of efficient causes as relevant determining factors of an occasion's initial make up, particularly of the creative character of the occasion's activity. Since this creative character provides propensities for the unrestricted ingression of the available subjective eternal objects, an occasion's efficient causes therefore condition the probabilities for the final unrestrictedly ingressed subjective eternal objects—for the occasion's final definiteness.[20] The main difference of

[20] Yet as explained in Section 2.6.3, this need not always be the case due to our postulation of bifurcating activities. However, as also argued in that section, it need to be the case for at least one occasion in each layer. And as far as there are occasions in the common causal

(1)-(3) to Whitehead's conception of causation stems from condition (2). It makes Suppes' notion of "cause" more restrictive than Whitehead's, because according to the latter also probability *lowering* events count as causes.

However, to avoid the conclusion that there is superluminal causation within Whitehead's ontology, from the point of view of Suppes' theory, it would have to be the case that *all* superluminal Whiteheadian causes are probability lowering ones. Since there is no reason why this should be the case, it is therefore to be expected that at least in some cases in which there is superluminal causation between Whiteheadian occasions E' and E, the probabilistic theory of causation due to Suppes will come to the same conclusion in regard to the corresponding events e' and e. Moreover, it is to be expected that analog conclusions can be drawn by comparison with other well-known probabilistic theories of causation, like the ones due to Lewis (1986) and Reichenbach (1956).

However, one can argue that this does not lead to a genuine conflict with STR, since the conception of causality invoked in STR is different from such "merely" probabilistic accounts. In particular, it is commonly hold that what STR prohibits is *a superluminal transfer of energy* (more precisely of energy-momentum). In the following we will argue that it is not clear whether efficient causation between Whiteheadian occasions involves something like a transfer of energy, but that it is likewise not clear whether QFT does conform to this constraint either. We will moreover point out a general argument to the effect that there is no conflict with STR, even in case of theories implying a genuine superluminal energy transfer.

Superluminal energy-momentum transfer

Energy in the sense of physics has to be compared to Whitehead's notion of activity, because Whitehead conceived his notion of activity as a metaphysical generalization of physical energy (SMW, p. 35 f; AI, p. 184 ff; PR, p. 116 f, 246, 254 f, 315; RM, p. 111 f). To see whether there is something corresponding to the *transfer* of physical energy between cause and effect in

past of this layer which are spacelike separated from the occasion in question, they are then superluminal probabilistic efficient causes of this occasion's final definiteness.

Whitehead's theory, we therefore, have to look at the way the activities of cause- and effect-occasions are related (see also Kraus 1998, p. 31). First of all, the "transfer of activity" between cause and effect, cannot be understood literally, i.e. as if the *same activity* formerly "inherent" in the cause-occasion is "given to" the effect-occasion. This is because the cause-occasion *exhausts* its activity in the determination of its final definiteness—its activity is successively reduced and thereby its definiteness is successively heightened until the activity is exhausted. By its finally attained definiteness, then, the completed occasion contributes to the determination of the new manifestation of the underlying activity for the effect-occasion. Thus the "transfer of activity" is not to be understood as a flow of a substance from one occasion to another, but rather as the "annihilation" and "recreation" of *different* activities from the underlying activity. Yet in this respect the transfer of activity between Whiteheadian occasions seems to be in agreement with the account of energy transfer provided by modern physics, particularly with what QFT is believed to tell us about this topic (see e.g. Dieks 1986, p. 88 ff; see also below). However, an important difference between the physical transfer of energy and the transfer of activity in Whitehead's ontology is that the latter does not incorporate something like a "conservation law for activity" or the like. Perhaps, Whitehead regarded the conservation of energy (or energy-momentum) as a contingent feature of our cosmic epoch and not as a metaphysical necessity governing the whole world-process. Yet without a conservation of the amount of activity it is hard to see what the rigorous meaning of a "transfer of activity from cause to effect" could mean at all. Therefore, it is not clear whether Whitehead's ontology is in conflict with STR in this respect.

However, it is not at all clear whether QFT itself does satisfy the relativistic constraint on energy–momentum transfer. Often the so-called *spectrum condition* belonging to the standard axioms of QFT, is held to prohibit a superluminal transfer of energy-momentum. However, this seems to be an overstatement because what the spectrum condition in fact rules out is merely a *specific kind* of superluminal energy-momentum transfer, namely a transfer by means of systems whose quantum states prescribe a non-vanishing probability to a superluminal velocity and to a non-zero amount of energy-momentum (see Appendix C.3). The idea underlying this kind of transfer is obviously that of a classical particle carrying some amount of energy-momentum along its continuous path. However, quantum "particles" do not have continuous world lines at all (see Section 3.1), so that they do not possess *diachronic identity*, for the latter just means that they endure to exist as the self same entities over some temporal interval. Even worse, they are not even *ontically reidentifiable*, which means that they cannot exist as the self-same entity at two different instants of time (or more generally, in two separated spacetime regions) (see e.g. Auyang 1995, Teller 1995, Clifton and Halvorson 2002, Seibt 2002). This means, however, that if the transfer of energy-momentum is supposed to involve a *spatiotemporally continuous process* or a *reidentifiable carrier of energy-momentum* (see e.g. Dowe 2000, Chapter 5), then it is ruled out *without the need to mention the spectrum condition at all.* If on the other hand, the transfer of energy-momentum does not involve continuous spatiotemporal processes or reidentifiable carriers of energy-momentum, then it is not clear what the non-existence of "superluminal velocity states" that is in fact implied by the spectrum condition means for the prohibition of superluminal energy-momentum transfer at all. For example, within the usual conceptual minimal understanding of quantum field theoretic systems as spacetime regions together with their corresponding observable algebras (see Section 10.2.5), it seems that one can argue that a superluminal energy-momentum transfer is already constituted by a sequence of spacetime regions such that the states ascribed to them have the same expectation value of energy-momentum and the first and the last region in the sequence are spacelike separated. The value of this expected energy-momentum is obviously *irrelevant* for such a transfer, *so that the spectrum-condition simply provides no means to prohibit it.* Thus it heavily depends on the particular understanding of the notion of "transfer", whether the spectrum condition does in fact prohibit the superluminal transfer of energy-momentum or not. Moreover, it seems that the kind of transfer that is in fact prohibited by this condition relies on some classical presuppositions which are already ruled out in QFT (and QM) without invoking the spectrum condition (see Section 3.1).

In sum, then, even if our modified version of Whitehead's ontology can be blamed to involve a superluminal "transfer of activities" this need not be in conflict with QFT, because the latter may very well itself admit a superluminal transfer of energy-momentum, the latter being the physical counterpart of Whitehead's notion of "activity". Thus if STR in fact constraints the transfer of energy-momentum to subluminal speeds, perhaps both, our ontology *and* QFT are in conflict with STR, depending on the precise notion of "transfer" that one invokes.

However, even if our ontology or QFT allows (something like) energy-momentum transfer and thus an undisputable kind of causal influence between spacelike separated events, this fact need still not necessarily lead to a conflict between STR. This is because such a conflict can only arise if STR incorporates claims about spatiotemporal constraints on causation at all. Although it is the by far most popular way to build up STR explicitly on assumptions as to the connection between spacetime and causation, so that STR inevitably becomes a *causal-spacetime theory*, there are of course other ways which do not rely on claims about the nature of causation at all, but rather understand STR as a pure *spacetime theory* (see e.g. Friedman 1983). In fact one can build up STR on the *special principle of relativity* alone, that demands all laws of nature to be form-invariant with respect to Poincaré transformations (Friedman 1983, p. 149 ff). According to G. Nerlich one moreover *should* do it this way, since

> special relativity is based on the principle of Lorentz invariance, not on causality. The limit principle (all causal and signal connections are slower than light) is not a basic thesis of special relativity [...]. (Nerlich 1982).

Thus if we follow Nerlich it is neither necessary nor desirable to provide STR with a causal underpinning at all (see also Nerlich 1994, Section 2.3 and Chapter 3). In this case the resulting theory would be *neutral* with respect to any claims about spatiotemporal restrictions on causation. Thus STR need not in itself be incompatible with theories that allow for superluminal causation, even in the rather strong sense of involving a transfer of energy-momentum between cause and effect.

2.8.2 Distinguished foliation of spacetime

Even if one understands STR as a pure spacetime theory, not implying any spatiotemporal constraints on causation, there is still another source for a conflict with the ontology developed up to this point. For one can object that the fact that by reason of the spatiotemporal expansion of the world-process a distinguished family of spacelike hyperplanes, i.e. a distinguished foliation of spacetime and thus a "preferred reference frame", is singled out is in conflict with STR, in particular, with the relativistic metric and its invariance group (the Poincaré group), with which the extensive continuum is equipped according to one of our supplementary assumptions of Section 2.1. Of course, as explained in Section 1.3.2, these relativistic structures are in fact not rich enough to single out a distinguished foliation, so that such a foliation is always *underdetermined by the relativistic structures.* Thus from the viewpoint of STR the introduction of a distinguished foliation is in fact *unjustified.* To avoid a conflict in this respect, one would have to argue in a first step that the distinguished foliation of spacetime is not empirically detectable—even in principle—and in a second step, that STR only describes the, in principle, empirically detectable structure of spacetime. An interpretation of STR (as well as of any other physical theory) according to which it does not unveil the (complete) spatiotemporal structure of the world, but merely describes the phenomena on the "surface" of a perhaps underlying and completely different "micro-structure", is clearly possible. However, such a "peaceful coexistence" between STR and Whitehead's ontology has to be paid for by accepting that an, according to the latter, ontologically fundamental structure of the world—the distinguished foliation—is in principle empirically inaccessible and thus is a truly "metaphysical" element of Whitehead's ontology. Whitehead himself would probably not have been worried by this consequence. Part I of PR begins with Whitehead's view of the aims of an ontological theory (he speaks of a "speculative scheme") and the methods by which these aims can be attained. According to Whitehead, the foremost aim of such a scheme is to unify *all* the empirical facts within one consistent system. Besides this "empirical completeness" and consistency Whitehead mentions some more aims, but he *nowhere claims that such a scheme must not include elements which are empirically unknowable* (PR, p. 3 ff). However, whether Whitehead was not afraid of the idea of going beyond the empirically knowable or not, most people, including the present author, will probably regard the need for the postulation of, in principle, empirically inaccessible elements within an ontological theory as a drawback for the latter.

Chapter 3

A first comparison with some results of quantum physics

In this chapter we will point out some first qualitative but nevertheless remarkable parallels between results of quantum physics and Whitehead's philosophy of process.

3.1 Discrete events

Whatever kinds of entities subatomic "particles" like electrons, protons, photons etc. may be, they definitely cannot have continuous spacetime trajectories.[1] This is a consequence of Heisenberg's uncertainty relation that implies that a quantum particle cannot posses a definite position in space and a definite momentum at the same instant of time. Consequently, quantum particles cannot posses continuous trajectories because this would obviously force them to posses a definite position *and* a definite momentum at each time of their existence. That Whitehead was aware of this consequence of quantum physics can be seen from the following passage taken from his book SMW published in 1925.[2]

[1] At least, this is the standard view on that matter, for another interpretation consult Bohm (1952).

[2] This awareness of Whitehead is even more remarkable in the light of the following statement of C. Hartshorne.

> At the present physics is troubled by the quantum theory [...]. But the point is that one of the most hopeful lines of explanation is to assume that an electron does not continuously traverse its path in space. The alternative notion as to its mode of existence is that it appears at a series of discrete positions in space which it occupies for successive durations of time. (SMW, p. 34)

But are we not able to directly observe the trajectories of subatomic particles, for example in a bubble-chamber? At first sight the result one gets from a bubble-chamber experiment, for example with electrons, might indeed look like a spatiotemporally continuous trajectory of a classical particle. But under closer inspection it turns out that this "continuous" trajectory is merely a succession of discrete, i.e. spatiotemporally non-overlapping, events. Each of these events—each bubble in the otherwise homogenous fluid with which the chamber is filled—is usually interpreted as resulting from a collision of a particle, travelling through the chamber, with one of the molecules of the fluid. But it is important to notice that in between the spatiotemporally separated bubble-events there is neither a sign of any continuous trajectory of some particle nor even some other sign for the concrete (i.e. spatiotemporal) existence of the hypothetical particle at all. All that the bubble-chamber experiment, in agreement with Heisenberg's uncertainty relation, shows is that there is *a succession of spatiotemporally discrete, finitely extended and qualitatively similar events*. But this is precisely what one would expect on the basis of Whitehead's ontology in which there are neither enduring substances (particles) which could produce continuous trajectories nor are there any spatiotemporal processes that connect or produce actual occasions. The only spatiotemporal consequence to be expected from Whitehead's ontology is indeed the observed succession of spatiotemporally discrete, finitely extended

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> Early quantum physics (as in Planck, Einstein, and Bohr) probably helped Whitehead to reach his view on this point, whereas the Uncertainty Principle came just too late to influence his doctrine of creativity. (I showed him Heisenberg's paper but as he told me, he had by that time given up trying to take the progress of physics into account because of his obligations to his Harvard students to learn all he could of the history of philosophy.) (Hartshorne 1979)

I am grateful to Michel Weber for pointing out this statement of Hartshorne to me.

regions in which certain properties are instantiated. Thus it seems that the spatiotemporal consequences of Whitehead's ontology as well as the possibility of some kind of non-spatiotemporal "transition-processes" that bridge the spatiotemporal gaps between the discrete, concrete events, seems to find experimental as well as theoretical support from quantum physics.[3] Moreover, the fact that the properties instantiated in the different regions along "the trajectory of the particle" are very similar and thus could indeed give rise to the idea that there is some object that travels through the chamber and causes the sequence of bubble-events can also be understood within Whitehead's theory. The impression that the world is made up of objects existing self-identical in time (like the hypothetical particle in the bubble-chamber) is recovered by his theory of societies of occasions, which are a kind of "Ersatz-substances" in Whitehead's system. A society is a sequence, or more generally, a pattern of occasions which by reason of the causal influences of their predecessors in the pattern instantiate (nearly) the same properties and thus give rise to the impression of objects existing self-identical in time. The mistake to take this *impression* as already representing the fundamental ontological level of the world is what according to Whitehead provided the foundation for the ontological misconception of a world that is made up of independently existing and enduring substances which move through a pre-existing "container-space".

3.2 Autonomous decisions

Another important feature of quantum physics that is in agreement with Whitehead's ontology is the fact that the actualization of quantum events can be consistently interpreted to involve an element of free choice. According

[3]Note that it is not claimed that the bubbles in the described experiment directly correspond to Whitehead's basic occasions. Rather each bubble is presumably itself a so-called society consisting of very many Whiteheadian occasions which are themselves too small to be directly observable. The bubble-chamber experiment is rather a "device" that amplifies the basic ontological make up of the world—the realm of Whiteheadian occasions—into directly observable macroscopic events—the bubbles—and it does this amplification presumably in a way that preserves some of the important structures of the underlying microrealm.

to the standard view, QM as well as QFT are truly *probabilistic* theories.[4] This means that the probabilities inherent in the formalism of QM are genuine features of this theory which cannot be eliminated—in particular it means that the probabilistic structure of QM is not a sign of its incompleteness and thus is not reducible to an underlying “more complete” deterministic theory. The latter is the case in classical statistical mechanics which is (in principle) completely reducible to Newtonian mechanics, thereby being not a truly probabilistic but merely a statistical theory. In statistical mechanics probabilities only show up because for reasons of computability many parameters are ignored as compared to Newtonian mechanics, so that a situation is, in general, no longer determined by its initial conditions. According to the standard view, QM predicts, on the basis of the complete state of the world up to some time t, probabilities for outcomes of measurements made at times later than t. But which of the possible outcomes of a measurement does in fact occur “may be regarded as a free choice of nature, limited only by the probability assignment” (Haag 1996, p. 316). This element of free choice of nature that “cannot be eliminated” (Haag 1996, p. 316), then, is the reason for the irreducibly probabilistic character of QM. But this limited freedom of nature, that according to the standard view of QM, is involved in the actualization of each quantum event, is obviously in agreement with Whitehead’s account of the actualization of the outcome of a concrescence process. According to Whitehead, which of the possible properties available to a concrescent occasion are in fact integrated into the final outcome is an autonomous decision settled solely in the concrescence, limited only by the propensities provided by the creative character of the corresponding individualized manifestation of the underlying activity.

3.3 Atomicity of actualization

The last parallel between quantum physics and Whitehead’s philosophy of process that shall be mentioned at this point is the atomic character of the act of coming into being of concrete quantum events. Concrete quantum events like the instantiation of “electronic properties”, i.e. of that complex of properties

[4] Again, this is the conviction of the great majority of physicists and philosophers working on the foundations of QM. For a different view see Bohm (1952).

by which the species "electron" is defined,[5] do not gradually or even continuously unfold in spacetime. To the contrary, either they occur fully or they do not occur at all—there is no observational or theoretical evidence for a thing like a spacetime region containing "half an electron", i.e. half of the electronic charge, mass and spin, before there is a bigger region that finally contains the "whole electron". Clearly, this fact could also be explained by the assumption that electronic events are pointlike, i.e. electronic properties are always instantiated at spacetime points rather than in finitely extended regions. Or put in the terminology of "particles" it could be explained by the assumption that electrons are "point-particles". But as we will see later on, QFT only allows non-trivial properties to be instantiated in extended regions and not at spacetime points—another important agreement between Whitehead's view and quantum physics. Since the move to assume that electronic events are located at spacetime points is thus prohibited, it seems at least possible that Whitehead's solution could be the right one. According to Whitehead's ontology the reason for the fact that we never observe half an electron is grounded in two things: first, the internal process of concrescence by which the complex of electronic properties is created is not a spatiotemporal process, so that in this internal process nothing like an unfolding of properties into a region of spacetime happens. Second, the way in which the result of this internal process is "made available" to the external world, i.e. is actualized, is an atomic act. Thus too there is no gradual unfolding of electronic properties in this act either and consequently there can be no thing like a region containing half an electron, before there is a larger region containing the whole electron.

This remarks show that Whitehead's ontology is able to capture some important features of quantum physics. Thus it seems reasonable to investigate in more detail whether Whitehead's philosophy of process is able to provide us with an adequate ontology of QFT; this will be done in Part III of this work. The following part of it will be devoted to the description of the relevant structures of the mathematical formalism of QFT, which in this work is taken to be the algebraic formalism of QFT.

[5] The properties by which the different species of "elementary particles" are defined and distinguished are the electrical charge, the rest-mass and the spin-value.

Part II

Algebraic Quantum Field Theory

Chapter 4

The role of the algebraic approach within Quantum Field Theory

Today QFT is taken to be a milestone towards the formulation of a fundamental physical theory of matter and its interactions. Being a synthesis of QM and STR, QFT is the first physical theory capable of describing three out of the four fundamental interactions of matter—gravitational-, electromagnetic-, weak- and strong interaction. From these four fundamental forces which are today believed to act in nature, only the gravitational force is excluded from a successful description within the conceptual framework of QFT. This is because on the microscopic level an influence of gravitationally interacting matter on the structure of spacetime, like the one described in the macrorealm by the GTR, is to be expected. But since QFT presupposes a given spacetime as a fixed, unchangeable background structure, its conceptual framework is simply too narrow for being able to include also the gravitational interaction. To formulate a unified theory that encompasses all four kinds of fundamental interactions and to put it to empirical tests is surely one of the great challenges of physics in the present century. However, although QFT cannot count as a fundamental physical theory, within its domain of applicability it is nevertheless the empirically best confirmed physical theory ever, so that one can expect that at least some of its structures will remain features of a future and more fundamental theory.

One major obstacle for anyone who is interested in the ontological implications of QFT is that there are many different mathematical approaches to QFT which differ significantly in the mathematical objects and methods on which they are based and in turn could suggest very different ontologies. Thus if one wants to investigate ontological questions concerning QFT, one first of all has to choose a particular formalism from which to proceed. The by far most popular formalism of QFT is the Lagrangian formalism.

4.1 The Lagrangian approach

In this approach one starts from a classical field theory (e.g. electrodynamics), specified by a concrete Lagrangian, and tries to "quantize" it following rules similar to those which have been proved to be successful in the transition from classical mechanics to QM. This scheme has been successful in the free case, i.e. in the case of classical systems consisting of non-interacting fields. Each such field can be decomposed into a positive and negative frequency part, so that after the quantization procedure has been carried through one gets annihilation and creation operators for some type of "particle", or better for some type of *field-quanta*, defined on a common Hilbert space $\mathcal{H}$ whose unit vectors $\varphi \in \mathcal{H}$ represent the physical states of systems of field-quanta. Such a model of QFT then describes an arbitrary number of non-interacting, indistinguishable field-quanta of a given type, say electrons or photons (see e.g. Teller 1995). However, there has been no easy way to extend this approach to a theory of interacting fields or field-quanta respectively. The only way to obtain results about interacting systems seemed to be by way of perturbative methods. However, perturbation theory has shown to have its own peculiar problems within QFT. The most prominent problem is the appearance of infinities within the perturbation expansions which have to be removed by procedures subsumed under the heading "renormalization". Although some of the numbers which have been calculated by use of perturbative methods based on the Lagrangian approach are the empirically best confirmed quantitative predictions a physical theory has ever yield, from a conceptual point of view the status of the Langrangian approach is merely that of a useful tool-box rather than that of a consistent theory. This conclusion as to the conceptual role of the Lagrangian approach has been further strengthened by Haag's theorem which states that a non-perturbative interacting QFT as well as the very interaction picture which is the starting point of perturbation theory do not exist in a rigorous mathematical sense within the framework used in the Lagrangian approach (see e.g. Haag 1996, p. 53 ff, Streater and Wightman 2000, p. 161 ff). Moreover, it turned out that one of the central ideas underlying the Lagrangian approach to QFT, namely the very idea of a quantum field (i.e. a "quantized" classical field) as an assignment of an operator $\Psi(x)$ to each spacetime point x is not tenable from a mathematical point of view. Frustrating results like these, together with the problems with renormalization, led a large part of the physics community in the 1950s to a rather negative attitude towards the formulation of QFT as a rigorous theory. However, instead of further investigating specific models by way of the perturbative tools of the Lagrangian approach some physicists tried to clarify the conceptual status of QFT with the hope of removing at least some of its basic problems and inconsistencies. Their strategy was to isolate those features of QFT which could be stated in mathematically rigorous terms and to extract those general postulates which looked trustworthy in the light of the lessons learned from the Lagrangian approach.

4.2 The axiomatic approach

This enterprise known as Axiomatic QFT finally led to the so-called Wightman axioms in which the assumed physical core of QFT had been summarized in a mathematically consistent way. In particular, the axiomatic approach did not start from the ill-defined notion of a quantum field as an assignment of operators to spacetime points but rather from the mathematically correct treatment of quantum fields as operator-valued distributions which become well-defined operators in the Hilbert space of state vectors $\mathcal{H}$ only when being "smeared" with an appropriate test function f with support[1] in an extended

[1]The test functions dealed with in QFT are usually taken to be infinitely often differentiable functions which together with all their derivatives decrease faster than any power of x as $|x|$ goes to infinity. The support of a test function f is that subset of spacetime for which $f(x)$ is non-zero.

region $\mathcal{O}$ of spacetime

$$\Psi(f) = \int \Psi(x) f(x)\, dx.$$

Thus within the axiomatic approach it is assumed from the outset that it is meaningless to ask for the value of a quantum field at a spacetime point x. What is meaningful is the smeared "value" of the field Ψ (i.e. the operator $\Psi(f)$) in as small a neighborhood $\mathcal{O}$ of each spacetime point x as one likes by letting it act on test functions f whose supports are contained in the chosen neighborhood $\mathcal{O}$. Thus within the axiomatic approach the fields Ψ are no longer assumed to be entirely local in the sense of specifying a well-defined operator $\Psi(x)$ for each spacetime point x. However, quantum fields understood as operator valued distributions, though less familiar mathematical objects than operators themselves serve all purposes needed from a physical point of view. In particular, the observable consequences are not affected by the loss of the localization of the fields at spacetime points. This is because each physically realizable measurement procedure requires some finitely extended region of space V and some finitely extended interval of time T to be carried out—in other words, it requires a finitely extended region $T \times V$ of spacetime. Now in the axiomatic approach the measurable physical magnitudes or observables of the theory are constructed as linear combinations of products, i.e. as polynomials, of the smeared field operators $\Psi(f)$ and since the latter are well-defined for test-functions f with supports in spacetime regions $\mathcal{O}$ as small as one likes the observables are also well-defined for each arbitrarily small *but non-pointlike* spacetime region.

Therefore, the fact forced on QFT by mathematical rigor, namely that quantum fields are at most localizable in finitely extended spacetime regions (rather than at spacetime points), does no harm to the observational content of QFT. Moreover, Axiomatic QFT has been successful in providing rigorous proofs of some important results like the spin-statistics theorem that expresses the connection between the spin of systems (i.e. integer or half-odd-integer) and the statistics obeyed by them (i.e. Bose-Einstein or Fermi-Dirac), *which has had to be put in by hand in the Lagrangian approach.*

However, another result obtained by H.-J. Borchers in 1960 indicated that the very notion of a quantum field is probably not a physically basic concept.

Borchers' result showed that *different quantum fields* Ψ, Φ *can very well lead to the same sets of observables.* Since in such a case the observable content of the theory does not depend on the choice of the underlying quantum field Ψ or Φ, this result indicates that a formulation of QFT based on quantum fields contains essential redundancies.

4.3 The algebraic approach

In the 1960s R. Haag, D. Kastler and H. Araki initiated an approach to QFT—known as Algebraic QFT (AQFT)—that can be seen as a direct advancement of the algebraic formulation of QM proposed in the late 1920s by J. von Neumann and that is *not* based on the concept of quantum fields. AQFT also being formulated in a strictly axiomatic manner gained its name from the fact that the fundamental mathematical structure on which it is based is a correspondence $\mathcal{O} \mapsto \mathcal{R}(\mathcal{O})$ between bounded spacetime regions $\mathcal{O}$ and operator algebras $\mathcal{R}(\mathcal{O})$, the self-adjoint elements of which are interpreted as the physical magnitudes or observables measurable within spacetime region $\mathcal{O}$. All other axioms which are assumed to build the basis of QFT within this approach are also of a very general nature and are expressible in the form of specifications of this fundamental correspondence. One of the ideas of the founders of AQFT was to base QFT *directly* on observables without the need of constructing them from underlying but, in general, unobservable quantum fields.

However, the algebraic approach is even more radical because it assumes that *no other characteristics of observables, besides their localization, is needed for a complete description of relativistic quantum systems.* In other words, it is assumed that the whole content of relativistic quantum physics is encoded in the correspondence $\mathcal{O} \mapsto \mathcal{R}(\mathcal{O})$ together with the axioms imposed on it. In particular, from this perspective quantum fields are to be understood merely as more or less convenient ways to *coordinatize* the algebras $\mathcal{R}(\mathcal{O})$, as has been suggested by the mentioned result of Borchers. Thus the relation between approaches based on the concept of quantum fields and the algebraic approach "may be compared with the concrete and abstract approaches to differential geometry. If one is dealing with concrete (computational) problems in geometry, it is natural to use coordinates, tensor fields, Christoffel symbols etc, whereas in the general structural analysis one relies on intrinsic concepts such as the notions of manifold, fiber bundle, connection etc." (Buchholz 2000). In the years following the first appearance of AQFT it gradually became more and more clear that the ideas underlying AQFT were in fact deep insights. That all relevant physical information is contained in the axioms of AQFT, and thus ultimately in the correspondence between spacetime regions and algebras of observables, has been confirmed by numerous results. One of the most startling of these results is of course that one can even recover the concept of quantum fields within the algebraic approach.

Because of its precise axiomatic formulation and the great generality of its axioms, AQFT seems to be best suited for the aim of this work. Especially the generality of AQFT is an advantage in the search for an ontology of QFT. Using the most general mathematical framework for QFT one need not fear that the mathematical objects eventually singled out to take the burden of representing ontological structures, are merely ephemeral consequences stemming from special assumptions made in a narrower mathematical framework. Therefore, in this work the algebraic formalism of QFT will be used. More precisely, we will use the algebraic formalism in its "concrete" version. In this concrete version the algebras $\mathcal{R}(\mathcal{O})$ are assumed to be algebras of bounded operators on a Hilbert space, whereas the "abstract" version starts with so-called C*-algebras $\mathcal{A}(\mathcal{O})$ which are *not* from the start realized as operator algebras on some Hilbert space (see Appendix B.3). One of the virtues of the abstract approach is that it provides the natural framework for the explanation of the origin of superselection rules by reason of the different unitarily inequivalent representations of the abstract algebras $\mathcal{A}(\mathcal{O})$ on concrete Hilbert spaces. However, for reasons of simplicity we will use the concrete version of AQFT in this work thereby avoiding, for example, the need to discuss the representation theory of C*-algebras.

Chapter 5

Algebraic Quantum Field Theory and its physical interpretation

We will now give a brief overview over the formalism of AQFT together with its usual *physical interpretation*. The term "physical interpretation" is meant to refer to any interpretation of a physical theory whose main aim is to extract numbers from the formalism of the theory which can be compared to the outcomes of experiments and not to make claims about the underlying ontology of the theory. The usual physical interpretation of AQFT is in fact quite neutral in respect to ontological claims since *it does not even specify the nature of relativistic quantum systems* (see also Rédei 2002). From the point of view of this work, the usual physical interpretation of AQFT therefore provides an unsuspicious framework for more specific interpretations of the formalism like the ontological interpretation that will be put forward in Part III. Since for the connection with Whitehead's philosophy of process not all aspects of the formalism of AQFT are needed, we will for the sake of simplicity discuss only those axioms of AQFT which are of direct importance for our later purposes. For a comprehensive account of all the standard axioms of AQFT as well as their physical interpretation the reader is referred to Appendix C and, in particular, to the excellent books of Haag (1996) and Horuzhy (1990). A brief summary of the basic concepts of the theory of operator algebras on Hilbert spaces is given in Appendix B. For a systematic study of this important area of mathematics the reader is referred to (Kadison and Ringrose 1983 and 1986). All mathematical notions not explicitly defined in the following can be found in one of the appendixes.

In the following M denotes Minkowski space, i.e. $\mathbb{R}^4$ equipped with the metric $\mathrm{g} : \mathbb{R}^4 \times \mathbb{R}^4 \to \mathbb{R}$, that assigns to each pair of points $x = (x^0, \mathbf{x}) \equiv (x^0, x^1, x^2, x^3)$, $y = (y^0, \mathbf{y}) \equiv (y^0, y^1, y^2, y^3) \in \mathbb{R}^4$ their Minkowski distance

$$\mathrm{g}(x, y) \equiv (x^0 - y^0)^2 - (\mathbf{x} - \mathbf{y})^2$$

where

$$(\mathbf{x} - \mathbf{y})^2 \equiv \sum_{i=1}^{3} (x^i - y^i)^2$$

is the square of the Euclidean distance in $\mathbb{R}^3$.[1] A subset of M is called an open *double cone* if it is the intersection of the open backward lightcone $V_-(z) \equiv \{x \in M : z^0 - x^0 > |\mathbf{x} - \mathbf{z}|\}$ of a point $z \in M$ with the open forward lightcone $V_+(y) \equiv \{x \in M : x^0 - y^0 > |\mathbf{x} - \mathbf{y}|\}$ of a point $y \in V_-(z)$. The set of all open double cones in M will be denoted by $D(M)$.

5.1 Local observables

The fundamental mathematical structure upon which AQFT is erected is a map

$$\mathcal{O} \mapsto \mathcal{R}(\mathcal{O}) \tag{5.1}$$

that assigns to each open, bounded, connected region $\mathcal{O}$ of Minkowski space M an algebra of linear, bounded operators $\mathcal{R}(\mathcal{O})$ on a common Hilbert space $\mathcal{H}$. Thus all the algebras $\mathcal{R}(\mathcal{O})$ are assumed to be subalgebras of the algebra $\mathcal{B}(\mathcal{H})$ of all bounded operators on a single Hilbert space $\mathcal{H}$. Moreover, the Hilbert space $\mathcal{H}$ is assumed to be separable and the algebras $\mathcal{R}(\mathcal{O})$ are assumed to be von Neumann algebras (see Appendix B). The points of Minkowski space are

[1] Throughout this book natural units will be used, so that the speed of light in the vacuum c as well as Planck's constant $\hbar$ have the numerical value 1. With a choice of units such that the value of c differs from 1, the Minkowski distance between x and y reads $\mathrm{g}(x, y) \equiv c^2(x^0 - y^0)^2 - (\mathbf{x} - \mathbf{y})^2$.

interpreted as spacetime points and the square root of the Minkowski distance $g(x,y)^{1/2}$ is accordingly interpreted as their spatiotemporal distance. Since in the fundamental correspondence (5.1) only *bounded* and *connected* spacetime regions do appear, the corresponding algebras are called *local* algebras. The underlying idea of the correspondence (5.1) is that the operators of the local algebra $\mathcal{R}(\mathcal{O})$ represent *physical operations* that can be performed within spacetime region $\mathcal{O}$ (Haag and Kastler 1964; Hellwig and Kraus 1969, 1970). The physically most important operations are those which result in a measurement of some *physical magnitude* or *observable* and as is usual practice, we will restrict the discussion to this class of operations.[2] The problems with pointlike fields in the Lagrangian approach have indicated that one should better build up the theory on non-pointlike quantities. Moreover, it is clear that no "real" measurement can be carried out at a spacetime point. Therefore, in the fundamental correspondence (5.1) only open regions are appealed to, which automatically rules out pointlike ones. Of course, it would make no difference to take closed regions with non-empty interior instead of open ones as the domain of the map (5.1), because from the physical viewpoint it is reasonable to expect that the operations performable within an open region determine the operations performable within its closure. Yet the choice of open regions has turned out to be the usual one and we will follow this trend.

Now we could proceed by stating the axioms usually required to hold for the fundamental correspondence (5.1). Yet the study of AQFT is much more simplified if one adopts some further restrictions on the set of regions appealed to in (5.1). This is because an open, bounded, connected region of M may still be of quite involved geometry and topology, which makes many investigations much more difficult. Since our primary interest are the structural properties of AQFT the optimal choice for a set of regions $\mathcal{O} \subset M$ would be one that facilitates the study of such structural properties, but is at the same time large enough to cover or approximate any open, bounded, connected region, so that nothing essential gets lost by this restriction of the domain of the map (5.1). Both conditions are perfectly met by the set of open double cones $D(M)$, which is the reason why this set is so often used especially in the study of structural

[2] For a philosophically enlightening discussion of general operations, see (Clifton & Halvorson 2001).

properties of local algebras and the relationships among them. Therefore, we will in what follows restrict the domain of the map $\mathcal{O} \mapsto \mathcal{R}(\mathcal{O})$ to the set of open double cones $\mathcal{O} \in D(M)$. For convenience we will often omit the adjective "open" and simply speak of double cones. In contexts in which it will not give rise to confusions, we will moreover often simply speak of (bounded) regions.

The observables which can be measured within $\mathcal{O} \in D(M)$—the *local observables* in region $\mathcal{O}$—are represented by the *self-adjoint* elements of $\mathcal{R}(\mathcal{O})$, i.e. by those $A \in \mathcal{R}(\mathcal{O})$ which coincide with their adjoint operator A^*. The most elementary observables are those which can take only two different values. These observables are called *simple* and are represented by projection operators, i.e. self-adjoint operators $P^* = P$ which are moreover idempotent $P^2 = P$. The most important structural feature of self-adjoint operators is that each such operator A has a unique *spectral decomposition*, which in the most simple case reads (see Appendix B.2)

$$A = \sum_i a_i P_{a_i}. \tag{5.2}$$

The real numbers a_i are the distinct eigenvalues of A and they are interpreted as the *possible values* which the corresponding observable can take in single measurements. For convenience we will in what follows simply speak of self-adjoint operators themselves as "observables" or "physical magnitudes" rather than as "operators representing observables/physical magnitudes". The set $\{P_{a_i}\}$ of so-called *eigenprojections* of A is uniquely determined by the latter and constitutes a *resolution of the identity operator* $\mathbf{1}$, which means that the eigenprojections corresponding to distinct eigenvalues are orthogonal in the sense that their product vanishes, i.e. $P_{a_i} P_{a_j} = \mathbf{0}$ whenever $a_i \neq a_j$, and the P_{a_i} sum up to the identity operator, i.e. $\sum_i P_{a_i} = \mathbf{1}$. As will be seen below this allows one to assign *probabilities to the possible values a_i of observable A*. On the other hand, given an arbitrary resolution of the identity $\{P_i\}$ one can, by combining its members with real numbers $\{a_i\}$ construct a self-adjoint operator according to (5.2). Projection operators have only two eigenvalues 1 and 0, and thus represent the conceptually simplest observables. Instead of "simple observables" projection operators are also often referred to as *properties*. Part of the reason for this is provided by the fact that because of the spectral decomposition (5.2) the occurrence of value a_i in a measurement is equivalent to the occurrence of value 1 of P_{a_i} and therefore P_{a_i} is taken to represent the property expressed by the sentence "observable A has the value a_i". More will (and has to) be said about properties in QM and QFT in Section 8.

5.2 States and probabilities

Now we turn to the representation of *states* of relativistic quantum systems. The notion of the "state" of a system suggests something like the specification of the "mode of existence" of a system and thus seems to be an ontologically much more interesting concept than that of an observable. However, from the viewpoint of physics the "mode of existence" of a quantum system is only interesting in so far as it allows the prediction of the probabilities for all the possible measurement results of each observable. The mathematical device that does precisely this is that of a *state* on $\mathcal{B}(\mathcal{H})$ as defined in Appendix B.3 (note that this time the term "state" refers to a purely mathematical object). The defining properties of a state ρ on $\mathcal{B}(\mathcal{H})$ imply that it is a *probability measure* over each resolution $\{P_i\}$ of the identity:

(i) $\rho(P_i)$ lies between 0 and 1,[3]

(ii) ρ is countably additive over each subset $\{P_{i_k}\} \subseteq \{P_i\}$, i.e. $\rho(\sum_k P_{i_k}) = \sum_k \rho(P_{i_k})$

and finally,

(iii) the sum over all $\rho(P_i)$ is 1, i.e. $\sum_i \rho(P_i) = 1$.

As explained above, the eigenprojections of a self-adjoint operator A form a resolution of the identity and each eigenprojection P_{a_i} is in one-to-one correspondence to a possible value a_i of A. Consequently, *every state defines a probability measure over the possible values of each observable.* Therefore, one assumes that the physical states of systems are represented by the mathematical states on $\mathcal{B}(\mathcal{H})$. Then the *probability for the occurrence of value a_i*

[3]Since $\rho(P_i) \geq 0$ together with (iii) automatically implies $\rho(P_i) \leq 1$ it is sufficient to require that $\rho(P_i)$ is non-negative.

of observable A *upon measurement on a system in state* ρ, abbreviated by $prob_\rho(A = a_i)$, is given by $\rho(P_{a_i})$, i.e.

$$prob_\rho(A = a_i) = \rho(P_{a_i}). \tag{5.3}$$

In Section 8.1.5 we will see how this probability ascription can be extended to the case in which the value of an observable lies within a certain range of values, rather than the observable taking a single such value.

From the possible values a_i together with their probabilities $prob_\rho(A = a_i) = \rho(P_{a_i})$ one can furthermore build the weighted sum

$$\sum_i a_i \; prob_\rho(A = a_i) = \sum_i a_i \rho(P_{a_i})$$

which is accordingly interpreted as the *expectation value* of observable A upon measurement on a system in state ρ. Making use of the linearity of state ρ and the spectral decomposition (5.2) the expectation value of A in state ρ simply turns out to be the value of A in state ρ, i.e.

$$exp_\rho(A) = \rho(A).$$

Note that this implies that for simple observables P the expectation value $exp_\rho(P) = \rho(P)$ coincides with the probability $prob_\rho^P(1) = \rho(P)$ for the occurrence of value 1 of P—or in terms of properties with the probability for the occurrence of property P.

Of course, probabilities and expectation values cannot be observed in *single* measurements. Rather the connection of these theoretical notions with empirical results has to proceed via relative frequencies and mean values in large ensembles of identically prepared systems. The probability $\rho(P_{a_i})$ with which value a_i of observable A will occur upon a measurement on a system in state ρ has to be compared with the relative frequency $\frac{N(a_i)}{N}$ with which value a_i occurs in a large series of measurements of A in an ensemble of $N \gg 1$ systems in state ρ. Accordingly, the expectation value $\rho(A)$ has to be compared with the mean value $\sum_i a_i \frac{N(a_i)}{N}$ of a_i in the ensemble.

Next we will further investigate the structure of states. As is well-known from QM each state ρ on $\mathcal{B}(\mathcal{H})$ can be represented by a *density operator* W

(i.e. by a positive operator $W \in \mathcal{B}(\mathcal{H})$ with trace $Tr(W) = 1$; see Appendix B.2) via the standard formula

$$\rho(A) = Tr(WA). \tag{5.4}$$

A *pure* state on $\mathcal{B}(\mathcal{H})$ is one that is *not* a *convex combination*

$$\rho = \sum_i c_i \rho_i \text{ with } c_i \geq 0 \text{ and } \sum_i c_i = 1 \tag{5.5}$$

of other states. It is represented via (5.4) by a one-dimensional projection operator $P \in \mathcal{B}(\mathcal{H})$. Since for a one-dimensional projection operator $Tr(PA) = \langle \psi, A\psi \rangle$, where $\psi \in \mathcal{H}$ is the unit vector that spans the one-dimensional subspace of $\mathcal{H}$ onto which P projects, each pure state ρ on $\mathcal{B}(\mathcal{H})$ can equivalently be represented by a unit vector $\psi \in \mathcal{H}$ via the special case $\rho(A) = \langle \psi, A\psi \rangle$ of formula (5.4). Pure states are often held to be the "true" physical states of quantum systems—*the ontic states*—whereas non-pure states[4] are supposed to describe our ignorance as to the true pure state of a system. However, such an *ignorance interpretation* of non-pure states cannot be upheld.

According to this ignorance interpretation, the numbers c_i in a convex combination like (5.5), where all the ρ_i are pure states, are to be interpreted as *epistemic* probabilities—probabilities due to our imperfect knowledge. More precisely, c_i is the epistemic probability for finding the system in question in the pure states ρ_i. However, it is well-known that each given non-pure state ρ can, in general, be written as a convex combination of *(infinitely) many different sets* $\{\rho_i\}$, $\{\phi_j\}$, ... of pure states

$$\rho = \sum_i c_i \rho_i = \sum_j d_j \phi_j = \ldots .$$

This, however, undermines the interpretation of such a non-pure state ρ as representing merely our ignorance as to the true pure state. For if the system under study were really in a pure state that is merely unknown to us, this pure state had better to be unique. But since the sets of pure states $\{\rho_i\}$,

[4]It is usual to speak of "mixed states" or "mixtures" instead of "non-pure states". However, such talk is quite missleading because, as we will see below, it is justified in only very special situations.

$\{\phi_j\}$, ... are different, this is not generally the case (see Ochs 1981; Rédei 1985). However, this argument against the ignorance interpretation of non-pure states is not yet conclusive. For it could be the case that there is a distinguished set of pure states $\{\rho_i\}$ into which the given non-pure state ρ can be decomposed according to (5.5). In fact it is often argued that in case the spectral decomposition

$$W = \sum_i c_i P_i \tag{5.6}$$

of the density operator W corresponding to the state ρ is non-degenerate, which means that all its eigenprojections P_i are one-dimensional and distinct from one another, a distinguished set of pure states is provided by the states $\rho_i \equiv Tr(P_i \cdot)$ corresponding to the projections P_i in (5.6). This set of pure states is distinguished from all others by the fact that the projections P_i are mutually orthogonal. Thus it seems that in case of a non-degenerate spectral decomposition of the density operator W the corresponding non-pure state $\rho = Tr(W \cdot)$ admits an ignorance interpretation.

What has been said so far already shows that if there is really a difference between certain states as to their ontological status, *the dividing line cannot simply be drawn between pure and non-pure states.* We will argue now that a peculiar feature of QFT, not inherent in QM, further undermines the ignorance interpretation of non-pure states and thus too, undermines any claim for an ontologically distinct status of pure states over non-pure ones. The feature in question is that the local algebras $\mathcal{R}(\mathcal{O})$ of QFT are von Neumann algebras of *type III* (Haag 1996, p. 118, 267 ff; Horuzhy p. 29, 35) which implies that they neither contain non-zero, finite-dimensional projections nor density operators (see e.g. Clifton 2000, p. 4). However, the latter means that a state on a local algebra $\mathcal{R}(\mathcal{O})$ is not generated by a density operator from $\mathcal{R}(\mathcal{O})$ via $\rho = Tr(W \cdot)$. Consequently, one cannot exploit any features of W—in particular, the fact that W may have a non-degenerate spectral decomposition—to single out a distinguished convex decomposition of ρ. Moreover, even if there were such a distinguished convex decomposition of the non-pure state ρ, this would not be a decomposition into pure states, because the lack of finite-dimensional projections in $\mathcal{R}(\mathcal{O})$ rules out the existence of any pure states on $\mathcal{R}(\mathcal{O})$. Thus there is simply no ground for any claims as to a distinct ontological status of pure and non-pure states on a local algebra, simply because there are no pure states at all (see also Clifton and Halvorson 2001). Now an ignorance interpreter may object that, though there are no pure states and no density operator representations of states on any local algebra, these concepts are nevertheless available for the algebra $\mathcal{B}(\mathcal{H})$ that includes all the local algebras as subalgebras. And therefore, one may reason, the ignorance interpretation can still be applied to those states on $\mathcal{B}(\mathcal{H})$ which have a distinguished convex decomposition into pure states. However, a state on $\mathcal{B}(\mathcal{H})$ is a purely *global* concept—it ascribes probabilities to the possible values of each observable $A \in \mathcal{R}(\mathcal{O})$ for all regions $\mathcal{O} \subseteq M$. Thus the system to which such a state can be reasonably ascribed is merely the universe as a whole. However, if the ignorance interpretation is only applicable to (certain states of) the universe as a whole, it is rather uninteresting. For the conclusion that the universe as a whole may be in a pure state about which we are ignorant, can already be drawn on the basis of the fact that we, as finite observers, can merely know the restriction $\rho|_{\mathcal{R}(\mathcal{O})}$ of a global state to the algebra of some finite spacetime region $\mathcal{O}$, i.e. the state induced on $\mathcal{R}(\mathcal{O})$ via $\mathcal{R}(\mathcal{O}) \ni A \mapsto \rho(A)$. And only if this restriction were a pure state (on $\mathcal{R}(\mathcal{O})$), we could be sure that the corresponding global state is also pure. However, there are no pure states on any local algebra $\mathcal{R}(\mathcal{O})$ at all, so that on the basis of the state $\rho|_{\mathcal{R}(\mathcal{O})}$ we can never be sure whether the global state ρ of the universe is pure or not. In sum, then, the ignorance interpretation of non-pure states, already severely challenged in QM, is ruled out for all interesting cases in QFT.

Note, however, that this does not mean that QFT is not able to describe systems belonging to *mixtures*. A mixture is an ensemble of systems, a fraction $0 < p_i < 1$ of which is in state ρ_i. However, if we do not know which system is in which state, the best we can do is to describe each system by the convex combination of the states ρ_i, i.e. by

$$\rho = \sum_i p_i \rho_i. \tag{5.7}$$

The numbers p_i give then the probabilities with which a system described by (5.7) is in the (non-pure) state ρ_i. Thus the probabilities p_i *are due to our ignorance with respect to the true state of the system in question*. But this is an ignorance interpretation of the non-pure state ρ. How does this relate to our above criticism? The main point of the above critic was that on the basis of a given non-pure state ρ alone, there is no fact of the matter as to the "true" set of component states of ρ about one could be ignorant. However, now we are discussing the "inverse" situation, in which a set of states $\{\rho_i\}$ together with their probabilities $\{p_i\}$ in an ensemble are given. And on the basis of this information it makes sense to use the state ρ in (5.7) as a description of a system belonging to the ensemble in question. Thus given enough information about the preparation of systems makes the ignorance interpretation of some non-pure state admissible. In the literature such non-pure states are often called *proper mixtures* whereas other non-pure states are called *improper mixtures* (see e.g. d'Espagnat 1995). However, this terminology is not very fortunate because by definition of improper mixtures they do not describe mixtures of systems, as defined above, at all. Therefore, we will not make use of this terminology. Moreover, within an ontological interpretation of QFT, epistemic deficiencies as to the true state of a system belonging to a mixture are of no interest at all. Therefore, mixtures and the corresponding ignorance interpretation of the non-pure states used to describe them will not play any role in an ontological interpretation.

Now being equipped with the fundamental correspondence (5.1) and the concepts of local observables and states we can go on to discuss those of the standard axioms of AQFT which will later on turn out to be of direct importance for the connection with Whitehead's ontology. The other axioms of AQFT not mentioned in the following, are discussed in Appendix C. We will henceforth follow the common sloppy practice of simply referring to all operators in $\mathcal{R}(\mathcal{O})$ as "observables measurable within $\mathcal{O}$"—despite the fact that only self-adjoint operators deserve this interpretation. Furthermore, when we speak of "(pure) states" without mentioning any algebra on which they are defined, we always mean "(pure) states on $\mathcal{B}(\mathcal{H})$".

5.3 Further important axioms of AQFT

The first axiom to be mentioned is called *isotony* and requires that the map (5.1) is "inclusion preserving" in the sense that the inclusion of regions implies the inclusion of the corresponding local algebras.

Isotony: For all $\mathcal{O}_1, \mathcal{O}_2 \in D(M)$, $\mathcal{O}_1 \subseteq \mathcal{O}_2$ implies $\mathcal{R}(\mathcal{O}_1) \subseteq \mathcal{R}(\mathcal{O}_2)$.

This assumption is very natural in the light of the interpretation of the operators from $\mathcal{R}(\mathcal{O})$ as observables measurable within region $\mathcal{O}$, because it simply says that in a larger region there are more (or at least: not less) observables to be measured. Mathematically this isotony-property turns the set of local algebras $\{\mathcal{R}(\mathcal{O})\}_{D(M)} \equiv \{\mathcal{R}(\mathcal{O}) : \mathcal{O} \in D(M)\}$ into a so-called *net* of von Neumann algebras, which in particular means that whenever $\mathcal{O}_1 \cup \mathcal{O}_2 \subseteq \mathcal{O}$ then $\mathcal{R}(\mathcal{O}_1) \cup \mathcal{R}(\mathcal{O}_2) \subseteq \mathcal{R}(\mathcal{O})$ holds, too.

Up to this point only topological properties of spacetime have been appealed to. The following axiom makes also use of the metrical structure of spacetime.

Spacelike Commutativity: For all $\mathcal{O}_1, \mathcal{O}_2 \in D(M)$, if $\mathcal{O}_1$ and $\mathcal{O}_2$ are spacelike separated from one another then all operators $A_1 \in \mathcal{R}(\mathcal{O}_1)$ commute with all operators $A_2 \in \mathcal{R}(\mathcal{O}_2)$.

One can show that spacelike commutativity is equivalent to the independence of the probabilities for measurement outcomes of an observable $A_1 \in \mathcal{R}(\mathcal{O}_1)$ (in any arbitrary state ρ) from the choice of the observable $A_2 \in \mathcal{R}(\mathcal{O}_2)$ measured in a spacelike separated region $\mathcal{O}_2$ (see e.g. Butterfield 1994, p. 769 f). Thus the spacelike commutativity axiom can be regarded as expressing the demand of the absence of causal influences from the choice of observable to be measured in region $\mathcal{O}_2$ on the outcome of a measurement in the spacelike separated region $\mathcal{O}_1$, since for such an influence the choice in $\mathcal{O}_2$ should at least be probabilistically relevant for the result in $\mathcal{O}_1$. However, as we will see later on, from this one *cannot* infer that there is no causal influence between spacelike separated regions *on the level of individual measurement results*, too. In other words, spacelike commutativity does not rule out causal influences between the outcomes of measurements carried out in spacelike separated regions. Thus *if one believes that STR requires that there are no superluminal causal influences simpliciter* and not merely that they are absent between the choice of the observable to be measured and the outcomes of all spacelike separated measurements, the spacelike commutativity axiom may not be the proper way of implementing this stronger restriction on causation (see e.g. Horuzhy 1990, p. 20 f). Clearly all this heavily depends on ones concept of causation. For example, if one believes that causal influences require the transfer of some amount of energy-momentum then QFT may be free of any kind of superluminal causal influences, depending, however, on the notion of "transfer" one invokes (see Section 2.8.1). However, within a weaker conception of causation, e.g. a purely probabilistic account, one can quite easily come to the conclusion that there are in fact superluminal causal influences on the level of individual measurement results (see Section 10.2.3).

Besides the restriction to at most luminal causal influences, commonly attributed to STR, the latter says that the symmetry transformations of spacetime are given by the Poincaré group $\mathcal{P}_+^\uparrow$. The Poincaré group consists of (1) spatiotemporal translations $x \to x + a$ by an arbitrary 4-vector $a = (a^0, a^1, a^2, a^3) \in M$, where a^0 corresponds to the time shift and $\mathbf{a} \equiv (a^1, a^2, a^3)$ to the spatial shift, (2) spatial rotations $\mathbf{x} \to R\mathbf{x}$ leaving the time coordinate x^0 unchanged, (3) Lorentz boosts, i.e. spatiotemporal rotations which correspond to velocity changes $\mathbf{v} \to \mathbf{v} + \mathbf{u}$ and (4) all combinations of (1)-(3). That these transformations are symmetry transformations of spacetime means that they do not change the spatiotemporal distance $\mathrm{g}(x, y)^{1/2} = ((x^0 - y^0)^2 - (\mathbf{x} - \mathbf{y})^2)^{1/2}$ between any two spacetime points, i.e. $\mathrm{g}(x, y)^{1/2} = \mathrm{g}(g(x), g(y))^{1/2}$ for all $g \in \mathcal{P}_+^\uparrow$ and all x, $y \in M$.

The sets of spatiotemporal translations (in short: translations) $\mathcal{T}$, spatial rotations $\mathcal{R}$ as well as the set of spatial rotations combined with translations and the set of spatial rotations combined with Lorentz boosts, i.e. the set of Lorentz transformations $\mathcal{L}_+^\uparrow$, are subgroups of the Poincaré group. This means that each of these subsets is again a group such that (1) it has the same neutral element e as $\mathcal{P}_+^\uparrow$, defined by $eg = ge = g$ for all $g \in \mathcal{P}_+^\uparrow$ that as a transformation maps each spacetime point into itself $x \to x$, and (2) it is closed with respect to combinations $g_1 g_2$ of the transformations it contains, i.e. each such combination is again an element of this subset. Unlike spatial rotations and Lorentz transformations the subgroup of translations $\mathcal{T}$ is moreover a commutative or Abelian group which means that for each two translations $g_1 g_2 = g_2 g_1$ holds. The Lorentz boosts alone do *not* form a subgroup of $\mathcal{P}_+^\uparrow$ because a combination of two Lorentz boosts can be a spatial rotation and thus does not again belong to the set of Lorentz boosts. Note furthermore that our restriction to double cones as admissible regions for the fundamental correspondence (5.1) is not flawed by the action of the Poincaré group because the Poincaré transform of a double cone is again a double cone.[5]

That Poincaré transformations are symmetry transformations of spacetime means for QFT that a simultaneous application of such a transformation to the source by which a system in a certain state is prepared and to the measuring device by which a certain observable is to be measured does not change the result of the measurement. However, since the result of an individual measurement of an observable A is not reproducible—QFT is a probabilistic theory—the latter cannot mean that the results of individual measurements are invariant under the simultaneous application of a Poincaré transformation $g \in \mathcal{P}_+^\uparrow$ to the system and the measurement device. Rather what needs to be invariant in such a case are the relative frequencies of measurement results. On the side of the formalism this means that under a simultaneous transformation of state $\rho \to \rho_g$ and observable $A \to A_g$ the probabilities $\rho(P_{a_i})$ of all possible values a_i of A have to be invariant. Since this obviously implies that the expectation value $\rho(A) = \sum_i a_i \rho(P_{a_i})$ of A is also invariant and, on the other hand, the probabilities $\rho(P_{a_i})$ are nothing else than the expectation values of the simple observables P_{a_i} one can equally well state the requirement of Poincaré invariance by saying that the expectation values $\rho(A)$ of all observables A in all states ρ must not be affected by the simultaneous transformations $\rho \to \rho_g$ and $A \to A_g$, i.e.

$$\rho_g(A_g) = \rho(A), \text{ for all } g \in \mathcal{P}_+^\uparrow.$$

In Appendix C it is explained in detail how this invariance condition together with the requirement that the action of Poincaré transformations on the local observables be compatible with the inherent spatiotemporal structure of the net $\{\mathcal{R}(\mathcal{O})\}_{D(M)}$, leads one to require:

Covariance: There exists a representation of the Poincaré group $\mathcal{P}_+^\uparrow$ by a group $U(\mathcal{P}_+^\uparrow) \equiv \left\{U(g) : g \in \mathcal{P}_+^\uparrow\right\}$ of unitary operators in $\mathcal{B}(\mathcal{H})$,[6] such that for all $\mathcal{O} \in D(M)$ and all $g \in \mathcal{P}_+^\uparrow$

$$U(g)\mathcal{R}(\mathcal{O})U(g)^{-1} = \mathcal{R}(g(\mathcal{O})).$$

[5]In fact, the whole set of double cones can be generated from each single double cone $\mathcal{O}_0$ by acting on it with all Poincaré transformations, i.e. $\mathcal{P}_+^\uparrow(\mathcal{O}_0) = D(M)$.

[6]Moreover, this representation is assumend to be continuous (see Appendix C).

In the following we will simply speak of the elements of $U(\mathcal{P}_+^\uparrow)$ themselves as Poincaré transformations rather than as representatives of the latter.

A problem in connection with the genuinely relativistic part of the Poincaré group, namely the subset of Lorentz boosts, is that it is parameterized by a vector $\mathbf{v}$ *that is usually interpreted as a classical velocity* (see above). The problem this causes for any ontological interpretation of QFT is that the latter does not justify the postulation of entities to which one can reasonably attribute a classical velocity at all. The attribution of a classical velocity to some entity seems to presuppose that this entity exists as the self-same entity over some (spatio-) temporal interval, i.e. possesses diachronic identity like it is believed to be the case for classical particles and macroscopic objects, or at least that it can be said to exist at two different spacetime points (or more generally, in two separated spacetime regions) and thus is ontically (but not necessarily epistemically) reidentifiable. If even this minimal requirement of ontic reidentifiability is not fulfilled, the entity in question could hardly be said to move (perhaps non-continuously) from one place to another and thus could not reasonably be attributed with a velocity. Yet as already mentioned in Section 2.8.1 it is well-known that relativistic quantum systems do not fulfil even this minimal requirement for attributing classical velocities to them. However, ontological interpretations of QFT cannot simply introduce non-quantum objects like macroscopic measuring devices, observers or reference frames, which cannot be described by QFT itself, and attribute the velocities occurring in Lorentz boosts to them. Of course, it is not claimed that it is in principle impossible to treat macroscopic objects within the conceptual framework of QFT. Rather the problem is that if this is *rigorously* done, i.e. if they are regarded as large quantum systems described by quantum states, then they too do no longer possess diachronic identity or are ontically reidentifiable in the strict sense. Thus one is faced with the dilemma that, on the one hand, it is a point in favour of an ontological theory if it does not incorporate reidentifiable or diachronically identical entities because this fits to what QFT seems to tell us. But, on the other hand, this same fact seems to make it impossible to make sense of an (at least) conceptually central piece of the formalism of QFT, namely its covariance with respect to the Poincaré group and the fact that the vacuum state of QFT is defined by its invariance with respect to the

Poincaré group (see below), because the genuinely relativistic part of the latter is constituted by the Lorentz boosts.

We will later on interpret Whiteheadian occasions as the fundamental quantum systems. Since Whiteheadian occasions are not reidentifiable or even in possession of diachronic identity (since in each phase of their existence they occupy their *whole* spatiotemporal location) Whitehead's ontology thus conforms well to what QFT is supposed to say in this respect. But for the same reason it is not to be expected that it provides us with an interpretation of Lorentz boosts and the classical velocity-parameter occurring in them.

The last axiom of AQFT to be mentioned here, is concerned with the structure of the *vacuum*. In classical, i.e. non-quantum, physics as well as in QM the vacuum is identified with "empty space". Two characteristics which are naturally to be expected from this point of view of the vacuum are that (1) the vacuum has the lowest possible amount of energy-momentum of all physical systems and that (2) it is invariant with respect to the spatiotemporal invariance group—in our case the Poincaré group. These assumptions on the vacuum are also made in QFT. However, in QFT the vacuum is regarded as a non-trivial physical system in its own right and thus it is associated with a non-zero state ω. In Appendix C it is explained that assumptions (1) and (2) (in fact even (2) alone) imply that the vacuum state is generated by a unit vector $\Omega \in \mathcal{H}$, according to $\omega = \langle \Omega, \cdot\, \Omega \rangle$, which is invariant with respect to the Poincaré group

$$U(g)\Omega = \Omega \text{ for all } g \in \mathcal{P}_+^\uparrow.$$

Moreover, as known from the treatment of free fields within the Lagrangian approach, each "material system"—understood as a system consisting of a certain number of "stable particles"—can be regarded as an excitation of the vacuum. More generally, each vector $\psi \in \mathcal{H}$ can be approximated as closely as one likes (in the norm of $\mathcal{H}$) by the application of appropriate polynomials of the local fields $\Psi(x)$, to the vacuum state.[7] In other words, the set of vectors generated by the application of polynomials of local fields to the vacuum vector is a dense subset of $\mathcal{H}$. This property of the vacuum is assumed to hold

[7] Of course, as mentioned in Chapter 4 the local fields have to be "smeared" over some non-pointlike region before they are well-defined operators that can be applied to vectors from $\mathcal{H}$.

also in the presence of interactions and therefore it is incorporated as a basic assumption into the formalism of AQFT as well. Since in AQFT there are no local fields from which the local observables are constructed, one assumes accordingly that (3) the set of vectors generated by the application of arbitrary local observables

$$A \in \mathcal{A}_{loc} \equiv \bigcup_{\mathcal{O} \in D(M)} \mathcal{R}(\mathcal{O})$$

to the vacuum vector Ω is a dense subset of $\mathcal{H}$. A vector $\varphi \in \mathcal{H}$ from which, by the application of operators from a set $\mathcal{S} \subseteq \mathcal{B}(\mathcal{H})$, a dense subset of $\mathcal{H}$ can be generated is called *cyclic* with respect to $\mathcal{S}$. Thus we can also state the assumption in question by saying that Ω is cyclic with respect to the algebra of all local observables $\mathcal{A}_{loc}$.[8] Together with the further physically reasonable assumption that the vacuum is unique, one therefore assumes the following:

Vacuum: In $\mathcal{H}$ there exists a unique unit vector Ω, that is invariant with respect to $U(\mathcal{P}_+^\uparrow)$ and cyclic with respect to $\mathcal{A}_{loc}$.

We are now equipped with the relevant principles of AQFT and their physical interpretation so that we could proceed by stating those consequences of the axioms which are important for the connection with Whitehead's process philosophy. However, the author believes that it will be more instructive and probably more convenient simply to start with developing the interpretation of AQFT in terms of Whitehead's ontology and to discuss implications of the formalism of AQFT within the relevant interpretational context.

[8] Why the set $\mathcal{A}_{loc}$ is indeed a subalgebra of $\mathcal{B}(\mathcal{H})$ is explained in Appendix C.

Part III

The Connection

Chapter 6

The representation of the extensive continuum

According to Section 2.1 the extensive continuum is an infinite, undivided but infinitely divisible continuum that is equipped with the primitive relation of extensive connection, the property of separativeness and that, according to our supplementary assumptions, is moreover four-dimensional and equipped with the spatiotemporal distance measure, i.e. the metric, of STR. The connection of this extensive continuum with the formalism of AQFT is quite straight forward because the only reasonable candidate for its representation is Minkowski space M. Since, according to Whitehead, the potential regions into which the extensive continuum can be divided—the potential regions of occasions—are non-pointlike, bounded and connected it would be most natural to represent them by the corresponding regions of Minkowski space M. However, for reasons of simplicity we have restricted the class of admissible regions for the correspondence $\mathcal{O} \mapsto \mathcal{R}(\mathcal{O})$ to a special subset of all non-pointlike, bounded, connected regions namely to the set $D(M)$ of double cones. Consequently, we will only use these double cones for the representation of the potential bounded regions into which the extensive continuum is divisible. However, once the metric of STR is assumed to be given it is very natural to assume that it is double cones into which the extensive continuum is divisible and which therefore can serve as the regions of occasions: double cones are obviously those bounded subsets of M which are most tightly connected with the relativistic metric $\mathrm{g}^{1/2}$

because they are simply given as the intersection of lightcones and the latter directly express the structure of $\mathrm{g}^{1/2}$. Thus in an extensive continuum that is structured by the metric of STR it seems natural to give double cones the privileged role as regions of the ultimate entities. For convenience we will, however, often simply speak of (bounded) regions instead of double cones.

But shall we take closed or open double cones (or such which are neither closed nor open) to represent (potential) regions of the extensive continuum? Since the realization of a potential region of the extensive continuum takes place *by the ingression of a boundary surface* into the extensive continuum, the potential regions of the latter are best represented by open subsets of M whereas the realized or actualized regions of the extensive continuum are best represented by closed subsets of M. This is because like the potential regions of the extensive continuum, open sets do not include their boundaries and like the realized or actualized regions of occasions closed sets do include their boundaries. Since, by definition, the set $D(M)$ consists of *open* double cones it is ready made for the representations of the potential regions into which the extensive continuum can be divided. The realized or actualized regions of occasions are accordingly to be represented by elements from the corresponding set of *closed* double cones. However, for convenience we will not complicate our interpretation by the explicit introduction of an own set of closed double cones. Rather we will simply represent realized or actualized regions too by elements of $D(M)$—knowing that the closure $\overline{\mathcal{O}}$ of the respective region $\mathcal{O} \in D(M)$ is the correct representative.

Now there is a potential problem with the second of our above mentioned supplementary assumptions, namely that the extensive continuum is equipped with the relativistic metric $\mathrm{g}^{1/2}$, i.e. the distance measure $(x, y) \mapsto \mathrm{g}(x, y)^{1/2}$. According to Whitehead the extensive continuum is not divisible into points but merely into finitely extended regions, so that it seems that the expression $\mathrm{g}(x, y)^{1/2}$ is not meaningful if M is interpreted as the extensive continuum because in this case one cannot make use of any expressions relying on the existence of points within M. Moreover, this divisibility of M into points is an instance of mathematical surplus structure, i.e. structure appearing in the formalism of AQFT that has no counterpart in Whitehead's ontology and therefore does not represent any ontological structure. Now Whitehead himself has shown in Part IV of PR how, by means of the primitive relation of extensive connection that is supposed to obtain between potential regions of the extensive continuum, one can recover potential points as limits of decreasing sequences $(\mathcal{O}_i)$, $\mathcal{O}_i \supset \mathcal{O}_{i+1}$ of potential regions. Therefore, one can interpret $(x,y) \mapsto \mathrm{g}(x,y)^{1/2}$ *as the limiting case of some distance measure* $\tilde{\mathrm{g}} : (\mathcal{O},\mathcal{O}') \mapsto \tilde{\mathrm{g}}(\mathcal{O},\mathcal{O}')$ *between bounded regions* $\mathcal{O},\mathcal{O}' \in D(M)$.

Of course, once a distance measure between *points* is given it is natural to expect that the distance between two bounded regions is the infimum of the distances between their points. In other words, for the two distance measures $\tilde{\mathrm{g}}$ and $\mathrm{g}^{1/2}$ to count as compatible it is not sufficient that $\mathrm{g}^{1/2}$ merely arises as a limiting case from $\tilde{\mathrm{g}}$ but that the natural extension of $\mathrm{g}^{1/2}$ to bounded regions, coincides with $\tilde{\mathrm{g}}$. Yet one cannot simply *define* $\tilde{\mathrm{g}}(\mathcal{O},\mathcal{O}')$ by the infimum of the distances between the points of $\mathcal{O}$ and $\mathcal{O}'$ because then $\tilde{\mathrm{g}}$ *would ontologically presuppose the point measure* $\mathrm{g}^{1/2}$ which would be in conflict with the assumption that $\mathrm{g}(x,y)^{1/2}$ merely arises as the limiting case $\mathcal{O} \to x$, $\mathcal{O}' \to y$ from $\tilde{\mathrm{g}}(\mathcal{O},\mathcal{O}')$, i.e. that $\mathrm{g}^{1/2}$ presupposes $\tilde{\mathrm{g}}$ and not the other way round. Fortunately, for the compatibility in the above sense it is sufficient that $\tilde{\mathrm{g}}(\mathcal{O},\mathcal{O}')$ coincides with (without being defined by) the distance between regions given by the infimum of $\left\{\mathrm{g}(x,y)^{1/2} : x \in \mathcal{O}, y \in \mathcal{O}'\right\}$. This moreover automatically secures that $\mathrm{g}(x,y)^{1/2}$ arises in the limiting case $\mathcal{O} \to x$, $\mathcal{O}' \to y$ from $\tilde{\mathrm{g}}(\mathcal{O},\mathcal{O}')$. Therefore, *we assume that the extensive continuum comes equipped with a distance measure* $\tilde{\mathrm{g}}$ *between double cones* $\mathcal{O},\mathcal{O}' \in D(M)$ *that coincides with*

$$\inf \left\{\mathrm{g}(x,y)^{1/2} : x \in \mathcal{O}, y \in \mathcal{O}'\right\},$$

but is not defined by this coincidence.[1] Since we have now sufficiently clarified

[1] In the footnote on page 101 it has been said that the truthmakers of statements of the form "region $\mathcal{O}'$ lies in the backward lightcone of region $\mathcal{O}$" are the regions $\mathcal{O}'$ and $\mathcal{O}$ together with the metrical relations obtainting between them, so that there is no need for the introduction of the surfaces of lightcones as objective eternal objects. But from the distance relation $\hat{\mathrm{g}}(\mathcal{O},\mathcal{O}')$ alone one cannot infer whether the above statement is true or not, so that it may seem that the point measure $\mathrm{g}^{1/2}$ is still needed to do this job. However, this is not the case because the above statement is true if for all bounded regions $\mathcal{O}_1$ included in $\mathcal{O}$ and all bounded regions $\mathcal{O}_2$ included in $\mathcal{O}'$, $\hat{\mathrm{g}}(\mathcal{O}_1,\mathcal{O}_2) \geq 0$ holds. Thus for our definition of spatiotemporal pasts, presents and futures we neither need to postulate lightcones as objective eternal objects nor the existence of the point measure $\mathrm{g}^{1/2}$.

the relation between $\tilde{\mathrm{g}}$ and $\mathrm{g}^{1/2}$ we will for convenience henceforth only speak of the point measure $\mathrm{g}^{1/2}$.

Now what about the primitive relation of extensive connection itself? As mentioned in Section 2.1 the obtainment of the relation of extensive connection between two bounded regions $\mathcal{O}$, $\mathcal{O}'$ means that they either overlap, one of them is included in the other or that they are in contact, i.e. contiguous to one another. These relations are given in M by reason of the *set-theoretic* relations $\subseteq$ and $\cap$ as follows: $\mathcal{O}$ is included in $\mathcal{O}'$ iff $\mathcal{O} \subseteq \mathcal{O}'$, $\mathcal{O}$ and $\mathcal{O}'$ overlap iff $\mathcal{O} \cap \mathcal{O}' \neq \varnothing$, and finally $\mathcal{O}$ and $\mathcal{O}'$ are in contact iff they overlap but there is no region $\mathcal{O}'' \in D(M)$ that is included in their overlap, i.e. $\mathcal{O}'' \subseteq \mathcal{O} \cap \mathcal{O}'$. Note that like the relation "contact" has been defined in terms of "inclusion" (i.e. $\subseteq$) and "overlap" (i.e. $\cap$) each of the two latter can solely be defined in terms of the other so that one in fact only needs to assume the existence of $\subseteq$ or $\cap$, which seems to be superior to Whitehead's way of starting with the relation of extensive connection because the latter corresponds to the disjunction of all three relations "inclusion", "overlap" and "contact"and thus seems to rely on stronger presuppositions. Yet recall that the systematic introduction of the fundamental relation of extensive connection by Whitehead did *not* proceed by defining it in terms of, or even by merely pointing to the analogy with, the disjunction of the relations "inclusion", "overlap" and "contact". Rather Whitehead proceeded in the opposite direction by introducing the relation of extensive connection in a purely axiomatic manner and then defining the relations "overlap", "inclusion" and "contact" from "extensive connection". Therefore, the fact that one can also start from the relation "inclusion" and then define "overlap", "contact" and finally "extensive connection" from it, does not show that this latter way is preferable to Whitehead's (see also PR, p. 294). Rather the problem we are faced with is that the set-theoretic relations $\subseteq$, $\cap$ and "contact" as constructed from these are defined for arbitrary subsets of M and thus in particular for pointlike subsets $\{x\} \subset M$. Therefore, one cannot make sense of the relation of extensive connection within the framework of AQFT by simply defining it in terms of the set-theoretic relations available in M because then "extensive connection" would presuppose the existence of points $x \in M$ right from the start. However, since Whitehead has shown how points can be recovered as limits when starting from bounded regions as primitives, we can again assume that the set-theoretic relations $\subseteq$, $\cap$ and "contact" arise as limiting cases from the relation of extensive connection.[2]

The last property of the extensive continuum is its separativeness, by reason of which no two overlapping regions can be realized (or even actualized). Unfortunately, this property cannot be represented by any feature of Minkowski space M. Therefore, we have to introduce it as an *independent principle*, not ontologically grounded in the properties of the extensive continuum. However, Whitehead did not made clear either how this separativeness can be understood *as a genuine feature of the extensive continuum*, so that the lack of some feature of Minkowski space that could account for it, is far from being a major drawback for our attempted Whiteheadian interpretation of QFT.

[2]Note that the coincidence of "extensive connection" with the disjunction of $\subseteq$, $\cap$ and "contact" (as defined from the former two) on bounded regions need not be assumed independently because Whitehead defined "extensive connection" in such a way that this is automatically the case.

Chapter 7

The representation of objective eternal objects

As we know there are two fundamental species of eternal objects—objective and subjective ones—as well as eternal objects which are complexes of components from both fundamental species, which we have called mixed eternal objects. In the light of what has been said so far about objective eternal objects, their representation in the formalism of AQFT should be almost obvious so that we need not say much about it. This will be different in the case of subjective and mixed eternal objects whose representation will turn out to be more difficult.

The only eternal objects of the objective species relevant in this work are the boundary surfaces by whose ingression into the extensive continuum the regions of occasions are created (see Section 2.2.1). As explained in Section 2.2.2 these boundary surfaces are moreover simple eternal objects, i.e. they have no other eternal objects as components, so that we can postpone the discussion of how to represent complex eternal objects and the related notion of compatibility until we discuss the representation of subjective and mixed eternal objects. According to the last section the potential regions of occasions are represented by the elements of the set $D(M)$. As explained in Section 2.1, it makes no sense to distinguish between boundary surfaces which arise from one another by means of Poincaré transformations—in the extensive continuum as equipped with the relativistic metric, boundary surfaces are only determined up to Poincaré transformations.[1] Consequently, objective eternal objects have to be represented by equivalence classes of regions $\mathcal{O} \in D(M)$ with respect to Poincaré transformations, i.e. by

$$\widehat{\mathcal{O}} \equiv \left\{ g(\mathcal{O}) : g \in \mathcal{P}_+^\uparrow \right\} . \tag{7.1}$$

That the set $\widehat{\mathcal{O}}$ is indeed an equivalence class follows directly from the group properties of $\mathcal{P}_+^\uparrow$. The elements $\mathcal{O}'$ of the class $\widehat{\mathcal{O}}$ that represents the boundary surface O, then, represent the *possible instances* of the universal O—or in Whiteheadian terms, the *possible or potential ingressions* of the eternal object O.

Thus objective eternal objects are represented by the very same mathematical objects than their classes of possible ingressions. A nominalist could therefore argue that what AQFT tells us about objective eternal objects is that they *are not really universals but rather classes of particulars*, namely of regions $\mathcal{O} \in D(M)$. And since there seems to be no way to represent objective eternal objects more "directly" in the formalism of AQFT, without introducing an ontologically meaningless difference between those which are connected by Poincaré transformations, this is in fact a point where QFT seems to speak against Whiteheadian ideas. In Section 8.1.7 and 8.2 we will moreover see that the situation is no better for subjective eternal objects. They too can merely be represented via their classes of possible ingressions. *Thus QFT seems not to be very willing to oblige the existence of universals.*

[1] As also mentioned in Section 2.1 this is not the case for the regions themselves because two regions which are connected by a Poincaré tranmsformation can nevertheless be distinguished by means of metrical (or even toplogical) relations when the latter inhere in the extensive continuum itself.

Chapter 8

The representation of subjective and mixed eternal objects

Subjective eternal objects are the qualitative properties, understood roughly as Platonic universals, which by their ingression into occasions constitute the qualitative characters of the latter (see Section 2.2). For this work only those physical properties are relevant, that are provided by QFT. Now in our exposition of the formalism of AQFT and its physical interpretation we have only briefly mentioned properties at all. The reason for this is that this important topic deserves a more detailed investigation that would have overloaded the exposition of the formalism of AQFT and its physical interpretation. In the following Sections 8.1.1-8.1.6 we will explain why and in what sense properties of quantum systems can be represented by projection operators and what differences to classical properties obtain. In Section 8.1.3 it will be argued that there is no need for the postulation of negative properties, both in the case of classical physics as well as in the quantum case. This result will support Whitehead's dictum against negative eternal objects (see Section 2.2.2). Section 8.1.7 will be central for the later connection with Whitehead's ontology, since it will show that the usual interpretation of single projection operators as properties has the consequence that properties, then, could not be understood as universals. However, Whiteheadian subjective eternal objects *are supposed to be properties understood as universals.* A proposal for the solution of this problem, consisting in a more indirect way of representing subjective eternal objects in the formalism of AQFT, similar to the one used to represent objective eternal objects, will then be put forward in Section 8.2.

However, until we eventually reach the result that single projections cannot be taken to represent subjective eternal objects, we will for the sake of argument tentatively speak as if subjective eternal objects could be represented by single projection operators. This is justified because the essential aspects of what will be said in the following for single projection operators, will also be true in a similar form for the classes of projections which we will finally use as representatives of subjective eternal objects. Whenever in the following a claim is valid in QM as well as in QFT we will refer to both theories collectively as Quantum Theory (QT).

8.1 Properties in quantum physics

8.1.1 From classical to quantum properties

Generally, in physics a property refers to a physical magnitude A and a set of real numbers D. It can be described by the sentence "the value of A lies in D". Thus a physical magnitude taking on a particular value or, more generally lying in some range of values, is what is meant by a physical property. Now in classical mechanics the state of a system is given by its position and momentum. The set of all states the system can be in is called its *phase space* and is denoted by Γ. Thus, for example the phase space of a point particle in one-dimension is given by $\Gamma = \mathbb{R}^2$. The physical magnitudes of a classical system are represented by the real-valued functions on its phase space Γ.[1] This means that the state γ of a system fixes the values $A(\gamma)$ of all physical magnitudes $A : \Gamma \ni \gamma \mapsto A(\gamma)$, which will no longer be true in QT (see Section 8.1.6). Physical properties are represented by the *idempotent* real-valued functions on phase space, i.e. by those functions A with $A(\gamma)^2 = A(\gamma)$ for all $\gamma \in \Gamma$ (see e.g. Griffiths 2002; Bub 1997). For convenience idempotent functions are henceforth denoted by P. This idempotence is equivalent to the fact that the function P can only take the two different values 0 and 1, so that $P(\gamma) = 1$ (0) can be taken to represent the fact that a system in

[1] In other words, all magnitudes are functions of position and momentum.

state γ does (not) possess the property represented by P. The *connection between properties and physical magnitudes* is established by the fact, well-known from elementary analysis, that each (sufficiently well behaved) real-valued function can be approximated by linear combinations of idempotent functions. In particular, a function A on Γ that can take *at most countably many different values*, i.e. whose range $R(A) \equiv \{A(\gamma) : \gamma \in \Gamma\}$ is a finite or countably infinite subset $\{a_i\}$ of $\mathbb{R}$, can uniquely be written as a linear combination of idempotent functions $\{P_{a_i}\}$

$$A = \sum_i a_i P_{a_i}, \tag{8.1}$$

such that (1) the sum $\sum_i P_{a_i}$ over all values $a_i \in R(A)$ of A equals the identity function $1_\Gamma : \Gamma \rightarrow \{1\}$ on Γ and (2) for each two different values $a_i \neq a_j$ the product of P_{a_i} and P_{a_j} yields the zero function $0_\Gamma : \Gamma \rightarrow \{0\}$ on Γ. A set of idempotent functions on Γ with these two properties is called a *resolution of the identity function* 1_Γ. Now the decomposition (8.1) of A by means of a unique resolution of the identity 1_Γ is in the first place a purely mathematical fact. Yet its physical relevance is that the idempotent functions belonging to the resolution $\{P_{a_i}\}$ are just those properties expressible by "the value of magnitude A is a_i".[2] Moreover each property of the form "the value of magnitude A lies in the set D", where $D = \{a_{i_k}\} \subseteq \{a_i\} = R(A)$ is some subset of values of A, can also be expressed by means of the P_{a_i}, namely as the sum of all P_{a_i} for which the value a_i belongs to the set $D = \{a_{i_k}\}$, i.e. by

$$P_D \equiv \sum_k P_{a_{i_k}}. \tag{8.2}$$

[2]This is because according to (8.1) the fact that the value of the magnitude A for a system in state γ is $A(\gamma) = a_i$ is equivalent to $P_{a_i}(\gamma) = 1$ and $P_{a_j}(\gamma) = 0$ for all $a_j \neq a_i$. Thus all the P_{a_i} in (8.1) are properties of the form "the value of magnitude A is a_i". Now assume there were a property P that is not contained in the above set $\{P_{a_i}\}$ but is of this form for some value a_k of magnitude A. This means that $P(\gamma) = 1$ (0) iff $A(\gamma) = a_k$ ($\neq a_k$) must hold. But according to (8.1) $A(\gamma) = a_k$ ($\neq a_k$) iff $P_{a_k}(\gamma) = 1$ (0) so that one also has $P(\gamma) = 1$ (0) iff $P_{a_k}(\gamma) = 1$ (0). However, since this must be the case for all phase space points γ one gets $P = P_{a_k}$.

The trivial property "the value of A lies in the whole range of possible values of A", is then given in terms of the P_{a_i} by

$$P_{R(A)} = \sum_i P_{a_i}, \tag{8.3}$$

which according to (1), is just the identity function 1_Γ, so that this property is always possessed by each system. The second trivial property is the one that can never be possessed by any system. It is represented by the zero function on Γ. It can also be expressed in terms of the P_{a_i} namely by a product of two of these properties corresponding to different values $a_i \neq a_j$ of A because according to (2) such a product coincides with the zero function $P_{a_i} P_{a_j} = 0_\Gamma$. Thus 0_Γ can be understood as representing the property that a system possesses at the same time two different values of one and the same magnitude which is impossible.

From the representation (8.2) of properties of the form "the value of magnitude A lies in the set D", one can easily calculate that the product $P_{D_1} P_{D_2}$ of two such properties is an idempotent function that takes value 1 iff the value of A lies in the intersection $D_1 \cap D_2$. Therefore, $P_{D_1} P_{D_2}$ should correspond to the *conjunction* $P_{D_1} \wedge P_{D_2}$ of the two properties P_{D_1} and P_{D_2}. Moreover, the expression $P_{D_1} + P_{D_2} - P_{D_1} P_{D_2}$ is the idempotent function that takes value 1 iff the value of A lies within the union $D_1 \cup D_2$ of the sets $D_1$and D_2 and should therefore correspond to the *disjunction* $P_{D_1} \vee P_{D_2}$ of the properties P_{D_1} and P_{D_2}. Finally, $1_\Gamma - P_D$ is identical with the idempotent function $P_{R(A)\setminus D}$ of the complement of the set D in the range of possible values $R(A)$ of magnitude A and thus takes value 1 respectively 0 iff P_D takes value 0 respectively 1. Therefore, it should represent the *negation* $\neg P_D$ of the property P_D.

Now operations on some set can be interpreted as the connectives "conjunction/and", "disjunction/or" and "negation/not" of standard logic between elements of this set just in case the set equipped with these operations forms a *Boolean algebra* (see e.g. Halmos and Givant 1998; Boolean algebras are defined in Appendix A). As one can easily proof, the set of "A-properties"

$$\{P_D : D \subseteq \{a_i\} = R(A)\}$$

equipped with the above operations $\wedge, \vee$ and $\neg$ forms a Boolean algebra with zero element 0_Γ and unit element 1_Γ.[3] Therefore, $P_{D_1} \wedge P_{D_2} = P_{D_1}P_{D_2}$, $P_{D_1} \vee P_{D_2} = P_{D_1} + P_{D_2} - P_{D_1}P_{D_2}$ and $\neg P_D = 1_\Gamma - P_D$ can indeed be understood as the conjunction, disjunction and negation of the involved properties. However, note that the mere fact that conjunctions, disjunctions and negations of properties can be defined *does not mean that we are automatically committed to the existence of conjunctive, disjunctive and negative properties*. Rather which of the formal expressions $P_{D_1} \wedge P_{D_2} = P_{D_1}P_{D_2}$, $P_{D_1} \vee P_{D_2} = P_{D_1} + P_{D_2} - P_{D_1}P_{D_2}$ and $\neg P_D = 1_\Gamma - P_D$ have to be taken *ontologically serious*, i.e. as representatives of genuine properties, will have to be seen. In Section 8.1.3 it will in fact turn out that negative properties are indeed not needed in classical as well as in quantum physics. Moreover, we will see that only disjunctive *or* conjunctive properties *but not both*, seem to be needed in an ontology of QFT. Yet for the moment let us simply refer to all of the above expression as "properties".

Now the set of all idempotent functions on the phase space of a classical mechanical system, denoted by $\mathcal{P}(\Gamma)$, is a Boolean algebra, too. Thus all the individual Boolean algebras consisting of the properties of single magnitudes A, are *Boolean subalgebras of* $\mathcal{P}(\Gamma)$. This means that conjunction, disjunction and negation are defined for all (pairs of) properties *irrespective* of the magnitudes and thus the Boolean algebras to which they belong. This is the reason why in classical mechanics—and more generally in classical physics—disjunctions, conjunctions, negations (and all other operations definable from these) *can be universally applied to any set of properties*. We will see below that this is no longer the case in QT—the reason for this is that not all self-adjoint operators commute with one another.

Yet our first task is to explain why in QT physical magnitudes and properties are assumed to be represented by self-adjoint operators and projection operators at all. The reason for this is that many of the structural features of classical physical magnitudes and properties also obtain for self-adjoint and projection operators. In fact, if in the above exposition the following formal

[3] If the possible values of magnitude A form a continuum (e.g. in case of position and momentum), then (8.1) goes over into an integral $A = \int_{\mathbb{R}} a \, dP_a$, where each continuous range of values $D \subseteq \mathbb{R}$ is associated with a characteristic function P_D on phase space and dP_a denotes the measure given by the map $D \mapsto P_D$. With analogous definitions as above the A-properties of a continuous magnitude A also form a Boolean algebra.

replacements are made

- phase space $\Gamma \longrightarrow$ Hilbert space $\mathcal{H}$
- point $\gamma \in \Gamma \longrightarrow$ vector $\psi \in \mathcal{H}$
- subset of $\Gamma \longrightarrow$ subspace of $\mathcal{H}$
- characteristic function $\longrightarrow$ projection operator
- real-valued function $\longrightarrow$ self-adjoint operator
- range $R(A)$ of values of function $A \longrightarrow$ spectrum of eigenvalues $\sigma(A)$ of operator A

then, apart from effects due to non-commutativity, it would also go through as a description of the quantum theoretical case. But let us look more closely at the situation in QT.

Instead of phase space one deals with a Hilbert space $\mathcal{H}$ whose unit vectors are in one-to-one correspondence with the pure states of systems.[4] As already pointed out in Section 5.1, a self-adjoint operator A—at least in the most simplest case that is the only one that needs to be discussed in this work—can be decomposed into its eigenprojections P_{a_i} according to

$$A = \sum_i a_i P_{a_i} \tag{8.4}$$

which has the same structure as (8.1) in the classical case. However, the structural similarity to the classical case goes much further. If one associates to each subset of eigenvalues $D = \{a_{i_k}\} \subseteq \{a_i\} = \sigma(A)$ of A the *spectral projection*[5]

$$P_D \equiv \sum_k P_{a_{i_k}}$$

[4]We do not discuss non-pure states here since they are of minor importance for the present argument.

[5]Note that apart from the case where the set $D = \{a_{i_k}\}$ contains only a single element the spectral projection P_D is *not* an eigenprojection of A. This is because it projects onto the smallest subspace $\uplus_k \mathcal{K}_{a_{i_k}}$ of $\mathcal{H}$ that contains the set-theoretic union of the eigenspaces $\mathcal{K}_{a_{i_k}}$ onto which the eigenprojections $P_{a_{i_k}}$ project individually (see Appendix B.2). As a subspace, $\uplus_k \mathcal{K}_{a_{i_k}}$ contains in particular all linear combinations of eigenvectors to the different eigenvalues from $\{a_{i_k}\}$ and these are not themselves eigenvectors of A.

this automatically yields $P_{\sigma(A)} = \sum_i P_{a_i} = \mathbf{1}$, $P_{\sigma(A)\setminus D} = \mathbf{1} - P_D$ and $\mathbf{0} = P_{a_i} P_{a_j}$ for $a_i \neq a_j$ because $\{P_{a_i}\}$ is a resolution of the identity (see Appendix B.2). Moreover, one can show that the resulting set

$$\{P_D : D \subseteq \sigma(A)\}$$

of spectral projections of A, equipped with the operations $P_{D_1} \wedge P_{D_2} \equiv P_{D_1} P_{D_2}$, $P_{D_1} \vee P_{D_2} \equiv P_{D_1} + P_{D_2} - P_{D_1} P_{D_2}$ and $\neg P_D \equiv \mathbf{1} - P_D$, is a Boolean algebra with zero element $\mathbf{0}$ and unit element $\mathbf{1}$.[6]

Because of these structural similarities between classical mechanical magnitudes and their corresponding properties, on the one hand, and self-adjoint operators and their spectral projections, on the other hand, it is quite reasonable to interpret self-adjoint operators as physical magnitudes, their spectral projections as the corresponding properties expressible by "the value of magnitude A lies in D" and $P_{D_1} \wedge P_{D_2} \equiv P_{D_1} P_{D_2}$, $P_{D_1} \vee P_{D_2} \equiv P_{D_1} + P_{D_2} - P_{D_1} P_{D_2}$ and $\neg P_D \equiv \mathbf{1} - P_D$ as conjunction, disjunction and negation of the involved properties.

8.1.2 Consequences of non-commutativity

Up to this point we have only considered the case of mutually commuting projection operators, namely the spectral projections of a single magnitude. In classical mechanics all Boolean algebras of properties of individual magnitudes A are embedded in the "universal" Boolean algebra $\mathcal{P}(\Gamma)$. This is no longer true in QT. The set of all projection operators in $\mathcal{B}(\mathcal{H})$, denoted by $\mathcal{P}(\mathcal{H})$, is *not* a Boolean algebra. As we will explain now, this has important consequences for conjunctions and disjunctions of quantum properties and thus too, for the

[6] The structural similarities between classical mechanical magnitudes and their corresponding properties, on the one hand, and self-adjoint operators and their spectral projections, on the other hand, also extend to self-adjoint operators with a continuous spectrum of possible values $a \in \mathbb{R}$. For them one has a unique spectral decomposition in form of an integral $A = \int_{\mathbb{R}} a \; dP_a$, where each continuous range of values $D \subseteq \mathbb{R}$ is associated with a projection operator P_D, also called a spectral projection of A, and dP_a denotes the projection-valued measure given by the map $D \mapsto P_D$ (see e.g. Reed and Simon 1980). Analogous to the discrete case one can also equip the set of spectral projections P_D of a continuous self-adjoint operator with a Boolean structure.

possibility of representing conjunctive and/or disjunctive eternal objects in QFT. The reason for the non-Boolean structure of the set $\mathcal{P}(\mathcal{H})$ of projection operators is that it contains elements which do not commute $P_1P_2 \neq P_2P_1$. Yet the product P_1P_2 (respectively P_2P_1) of two projections P_1, P_2 is again a projection iff P_1, P_2 commute (and in this case $P_1P_2 = P_2P_1$). Consequently, for non-commuting projections $P_1 \wedge P_2 = P_1P_2$ and $P_1 \vee P_2 = P_1 + P_2 - P_1P_2$ are *not* projections (in fact they are not even self-adjoint) and thus *cannot be interpreted as properties.* Moreover, this definition would turn $\wedge$ and $\vee$ into non-commutative operations, which completely undermines their interpretation as the intended connectives of standard logic.

However, one may think that there are other ways of defining $P_1 \wedge P_2$ and $P_1 \vee P_2$ that do not lead to these unwelcome consequences for non-commuting projections. For example, one can define $P_1 \wedge P_2$ as the projection operator $P_{\mathcal{K}_1 \cap \mathcal{K}_2}$ that projects onto the intersection of the subspaces $\mathcal{K}_1$ and $\mathcal{K}_2$ of $\mathcal{H}$ onto which P_1 and P_2 project individually and, accordingly, $P_1 \vee P_2$ as the projection $P_{\mathcal{K}_1 \uplus \mathcal{K}_2}$ that projects onto the smallest subspace of $\mathcal{H}$ containing $\mathcal{K}_1$ and $\mathcal{K}_2$, denoted by $\mathcal{K}_1 \uplus \mathcal{K}_2$ (see also Appendix B.2). And indeed with these definitions, $\wedge$ and $\vee$ are commutative operations and again yield projection operators. Moreover, for commuting projections one has $P_{\mathcal{K}_1 \cap \mathcal{K}_2} = P_1P_2$ and $P_{\mathcal{K}_1 \uplus \mathcal{K}_2} = P_1 + P_2 - P_1P_2$ and therefore this alternative definition of $\wedge$ and $\vee$ even retains $P_1 \wedge P_2 = P_1P_2$ and $P_1 \vee P_2 = P_1 + P_2 - P_1P_2$ whenever P_1 and P_2 commute. However, $\mathcal{P}(\mathcal{H})$ equipped with these operations and the operation $\neg P = \mathbf{1} - P$, in general called *orthocomplementation*, is still not a Boolean algebra but merely an *orthocomplemented lattice* (see Appendix A). What distinguishes a lattice, and also an orthocomplemented lattice, from a Boolean algebra is that in the latter, $\wedge$ and $\vee$ are *distributive over one another*, i.e. for all P_1, P_2, P_3 one has

$$P_1 \wedge (P_2 \vee P_3) = (P_1 \wedge P_2) \vee (P_1 \wedge P_3) \qquad (8.5a)$$

$$P_1 \vee (P_2 \wedge P_3) = (P_1 \vee P_2) \wedge (P_1 \vee P_3), \qquad (8.5b)$$

whereas in a mere (orthocomplemented) lattice this is not the case. But without distributivity, $\wedge$ and $\vee$ clearly cannot be interpreted as the connectives "conjunction/and" and "disjunction/or" of ordinary logic (see e.g. Thirring 1981, Example (2.2.37)).

Now it is important to notice that $\mathcal{P}(\mathcal{H})$ and more generally any of its subsets $\mathcal{N}$ that contains non-commuting elements cannot be turned into a Boolean algebra by *whatever* definition of $\wedge$ and $\vee$ one might try. This follows from the fact that the validity of (8.5a) and (8.5b) in an orthocomplemented lattice is equivalent to the possibility of expressing $\wedge$ and $\vee$ by $P_1 \wedge P_2 = P_1 P_2$ and $P_1 \vee P_2 = P_1 + P_2 - P_1 P_2$ for all $P_1, P_2 \in \mathcal{N}$ (see e.g. Grätzer 1998, Theorem II.9). Therefore, though $P_1 \wedge P_2 = P_{\mathcal{K}_1 \cap \mathcal{K}_2}$ and $P_1 \vee P_2 = P_{\mathcal{K}_1 \uplus \mathcal{K}_2}$ are defined for all projections $P_1, P_2 \in \mathcal{P}(\mathcal{H})$, they can only be interpreted as the conjunction and disjunction of standard logic in the framework of a Boolean algebra generated by mutually commuting projections.[7]

From what has been said so far, it is clear that there is no way to evade the fact that quantum properties cannot always be thought of in the same way as classical properties. The question is how one should deal with this fact when it comes to the interpretation of the formalism of QT. One way of dealing with the non-Boolean structure of $\mathcal{P}(\mathcal{H})$ could be to give up the requirement of distributivity for the connectives $\wedge$ and $\vee$ of standard logic and instead creating a non-standard alternative. This route has in fact been taken by G. Birkhoff and J. von Neumann when they proposed their version of such a *quantum logic* (Birkhoff and von Neumann 1936). However, despite the great deal of effort of "quantum logicians" since the pioneering work of Birkhoff and von Neumann, this approach has not turned out to be of much help for resolving the interpretational difficulties of QT, and we shall not make use of it. The other possibility—the one taken by us—*is to leave the rules of ordinary logic unchanged but restrict their applicability to sets of mutually commuting projections respectively to the Boolean algebras generated by them.* However, this poses the problem that there are many different non-commuting Boolean algebras within $\mathcal{P}(\mathcal{H})$, so that the question arises how an appropriate Boolean framework is to be singled out for a given state of affairs—which in our interpretation means at a given stage of the world-process.

Now in Section 8.1.7 we will see that subjective eternal objects cannot be represented by single projection operators, because the latter cannot be inter-

[7] Below we will see that the Boolean algebras of properties of a magnitude A are such that $\wedge$ and $\vee$ can be interpreted as ordinary conjunction and disjunction without the need for a negation $\neg$.

preted as universals. Rather similar to the representation of objective eternal objects (see Section 7), we will take certain classes of projections as representatives of subjective eternal objects. In Section 8.3.2 we will, however, be able to show that abstractive hierachies of subjective (as well as mixed) eternal objects can be represented by Boolean algebras consisting of such classes of projection operators. The question of how at each stage of the world-process a single Boolean framework of commuting Boolean algebras, each corresponding to one of the regions created at that stage, is singled out will eventually be answered in Section 10.4. The rule for this determination is then a concrete explication of Whitehead's demand that the already actualized occasions at each stage of the world-process determine the abstractive hierachies of subjective eternal objects, available for the new occasions which arise at that stage.

This way of dealing with the problem can be understood as a particular "ontological version" of the solution upheld by N. Bohr that has since then been adopted in most physical interpretations of QT. According to this approach, the choice of a single Boolean framework (in particular the Boolean algebra generated by the properties of a single magnitude A or more generally by a set of mutually commuting magnitudes), is delegated to external, i.e. non-quantum, agents—the so-called *observers*. Since, therefore, the choice of a particular magnitude by an observer is a necessary precondition for a system to possess one of the corresponding properties, magnitudes are more frequently called *observables* and the choice of the observer is understood as the initial stage of the *measurement* of the chosen observable that terminates with the possession of one of its corresponding properties by the system. Yet it is clear that the introduction of an external agent, not represented in the formalism of QT, is a rather ad hoc move that is moreover inappropriate if one is in search for an ontological interpretation of QT. However, until we have developed our own interpretation of these matters to an appropriate degree, we will simply follow the usual talk of measurements etc.

As explained in Section 2.2.2, according to Whitehead there are no negative eternal objects and more generally, no negative entities at all. We will argue in the following section that this dictum gains support at least in case of physical properties.

8.1.3 Against negative properties

In the following $\mathbf{B}(\mathcal{N})$ denotes the Boolean algebra generated from the set of mutually commuting projections $\mathcal{N} \subset \mathcal{P}(\mathcal{H})$ by means of conjunctions, disjunctions and negations of the elements of $\mathcal{N}$. As a first step it will be shown that the Boolean algebra $\mathbf{B}(\{P_D\})$ of A-properties (with zero and unit element $\mathbf{0}$ and $\mathbf{1}$), henceforth also be denoted by $\mathbf{B}(A)$, is generated by the set of eigenprojections $\{P_{a_i}\}$ of magnitude A, so that

$$\mathbf{B}(A) \equiv \mathbf{B}(\{P_D\}) = \mathbf{B}(\{P_{a_i}\}). \tag{8.6}$$

This is because each non-zero A-property P_D is the sum of the eigenprojections P_{a_i} corresponding to the eigenvalues $a_i \in D$, i.e.

$$P_D = \sum_{a_i \in D} P_{a_i}.$$

Since the eigenprojections of A form a resolution of the identity (i.e. they are mutually orthogonal and sum up to the identity) one has $P_{a_i} \wedge P_{a_j} = P_{a_i} P_{a_j} = \mathbf{0}$ for $i \neq j$ and thus too $P_{a_i} \vee P_{a_j} = P_{a_i} + P_{a_j}$. Therefore, each non-zero property P_D, including $P_{\{a_i\}} = \mathbf{1}$, is just a disjunction of eigenprojections, i.e. $P_D = \vee_{a_i \in D} P_{a_i}$. Since moreover, $\mathbf{0}$ is obtained from the set of eigenprojections by $\wedge_{a_i \in D} P_{a_i} = \mathbf{0}$, the Boolean algebra $\mathbf{B}(\{P_D\})$ is in fact generated by the set of eigenprojections $\{P_{a_i}\}$ of A so that (8.6) indeed holds.

Yet this, moreover, shows that the Boolean algebra $\mathbf{B}(\{P_{a_i}\})$ *generated from a resolution of the identity* coincides, as a set, with the *distributive lattice*, term it $\mathbf{D}(\{P_{a_i}\})$, generated from the same resolution of the identity. In general, a distributive lattice is a lattice in which the operations $\wedge$ and $\vee$ obey the distributive laws (8.5a) and (8.5b), but it is distinguished from a Boolean algebra by the fact that there exists no negation $\neg$ (see Appendix A). Therefore, the generation of the distributive lattice $\mathbf{D}(\mathcal{N})$ from a set $\mathcal{N}$ proceeds by taking all combinations of conjunctions and disjunctions of elements from the latter, whereas the Boolean algebra generated from the same set moreover includes the negations of all combinations of conjunctions and disjunctions of elements from $\mathcal{N}$. However, because of the properties of a resolution of the identity, *in the special case of a distributive lattice* $\mathbf{D}(\{P_{a_i}\})$ *generated from such a resolution, a negation can always be defined within* $\mathbf{D}(\{P_{a_i}\})$ *without adding any* *new elements, so that* $\mathbf{D}(\{P_{a_i}\})$ *and the Boolean algebra* $\mathbf{B}(\{P_{a_i}\})$ *consist of precisely the same elements.* This follows directly from what has been shown above, because in generating the Boolean algebra $\mathbf{B}(A) = \mathbf{B}(\{P_{a_i}\})$, only disjunctions and conjunctions of elements from the resolution $\{P_{a_i}\}$ have been used and thus $\mathbf{B}(\{P_{a_i}\})$ contains indeed the same elements as the distributive lattice $\mathbf{D}(\{P_{a_i}\})$.

Yet this undermines the ontological interpretation of the expression $\neg P_D$ as a genuine negative property, because *it can simply be understood as the disjunction of positive properties, namely of those* P_{a_i} *with* $a_i \in D$. Thus, at least as far as physical properties are concerned, there seems to be no need for including negative properties into the ontology of the world, at all. Since, moreover, the structure of $\mathbf{D}(\{P_{a_i}\})$ and $\mathbf{B}(\{P_{a_i}\})$ in regard to $\wedge$ and $\vee$ are identical it is clear that these operations can still be interpreted as ordinary conjunction and disjunction within $\mathbf{D}(\{P_{a_i}\})$, too.[8] Note that the above reasoning includes no specific "quantum-assumptions" but rather is likewise valid in the classical case.[9]

On the other hand, this result makes the use of Boolean algebras $\mathbf{B}(\{P_{a_i}\})$ instead of distributive lattices $\mathbf{D}(\{P_{a_i}\})$ in case the generating set $\{P_{a_i}\}$ is a resolution of the identity totally harmless from an ontological point of view, because by introducing the operation $\neg$ no new elements are added to $\mathbf{D}(\{P_{a_i}\})$, at all. Rather the difference between $\mathbf{B}(\{P_{a_i}\})$ and $\mathbf{D}(\{P_{a_i}\})$ can be seen as merely consisting in mathematical surplus structure, namely in the ontologically irrelevant operation $\neg$ (see e.g. Grätzer 1998, p. 63). Whenever possible, we will therefore make use of the more common Boolean algebras $\mathbf{B}(\{P_{a_i}\})$ instead of the corresponding distributive lattices. Moreover, if not mentioned otherwise $\{P_{a_i}\}$ or $\{P_i\}$ will always refer to resolutions of the identity.

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[8] This need, however, not be the case for more general distributive lattices, i.e. in particular for distributive lattices not generated from a resolution of the identity.

[9] For a continuous (classical or quantum) magnitude A, there is no unique set $\{P_{a_i}\}$ such that each P_D can be written as a disjunction of elements from $\{P_{a_i}\}$. However, one can show that in this case too, each A-property P_D is a disjunction of others, namely of all $P_{D'}$ with $D' \subset D$. Therefore, *in the continuous case too, each prima facie negative property can in fact be understood as a (in general non-unique) disjunction of positive ones.*

8.1.4 On conjunctive and disjunctive properties

One may think that the argument just given can also be used for arguing against the existence of non-trivial conjunctive properties. For we have shown that each non-zero element and thus, in particular, each non-zero conjunction $P_{D_1} \wedge P_{D_2} \in \mathbf{B}(\{P_{a_i}\})$ can likewise be written as a disjunction of elements from $\{P_{a_i}\}$. Thus by simply including $\mathbf{0}$ in the resolution $\{P_{a_i}\}$,[10] in fact *each element of* $\mathbf{B}(\{P_{a_i}\})$ *can be written in the form of a disjunction of elements from* $\{P_{a_i}\}$. Therefore, if complex properties are needed at all (which will be argued for in connection with QFT in Section 8.3) it seems that for reasons of ontological economy there is no need to include, besides disjunctive ones, also conjunctive ones.

However, this argument for the ontological priority of disjunctive over conjunctive properties would only be conclusive *if* the Boolean algebra $\mathbf{B}(A) = \mathbf{B}(\{P_{a_i}\})$ (or likewise the distributive lattice $\mathbf{D}(A) = \mathbf{D}(\{P_{a_i}\})$) of A-properties could not also be generated by only using conjunctions of the elements from a subset of $\mathbf{B}(A)$.[11] Yet it is easy to see that this is always possible, so that *disjunction and conjunction again stand on equal footing and therefore there is no fact of the matter for the priority of disjunctive over conjunctive properties (nor vice versa).*[12] Note that this "symmetry" between conjunction and disjunction does not only hold for Boolean algebras and distributive lattices

[10] This is possible because the zero operator is orthogonal to all other projections and adding it clearly does not change the result of the sum $\sum_i P_{a_i} = \mathbf{1}$ either, so that the defining characteristics of a resolution of the identity are not affected by the inclusion of $\mathbf{0}$.

[11] Of course, this subset will not coincide with $\{P_{a_i}\}$, i.e. with the set from which $\mathbf{B}(A)$ can be generated by disjunctions only.

[12] For example, consider the distributive lattice generated from a resolution of the identity consisting of three elements P_1, P_2 and P_3. Besides P_1, P_2 and P_3, $\mathbf{D}(P_1, P_2, P_3)$ contains the following elements: $Q_1 = P_1 \vee P_2$, $Q_2 = P_2 \vee P_3$, $Q_3 = P_1 \vee P_3$, $\mathbf{0} = P_1 \wedge P_2 \wedge P_3$ and $\mathbf{1} = P_1 \vee P_2 \vee P_3$. Thus by understanding resolutions of the identity as containing $\mathbf{0}$, all elements of $\mathbf{D}(P_1, P_2, P_3)$ are in fact generated by disjunctions of P_1, P_2, P_3 and $\mathbf{0}$. That $\mathbf{D}(P_1, P_2, P_3)$ can also be generated by conjunctions of elements from the subset $\{Q_1, Q_2, Q_3, \mathbf{1}\} \subset \mathbf{D}(P_1, P_2, P_3)$ now easily follows from the application of the distributive laws (8.5a) and (8.5b) to $Q_1 \wedge Q_2 = (P_1 \vee P_2) \wedge (P_2 \vee P_3)$, $Q_2 \wedge Q_3 = (P_2 \vee P_3) \wedge (P_1 \vee P_3)$, $Q_1 \wedge Q_3 = (P_1 \vee P_2) \wedge (P_1 \vee P_3)$ and $Q_1 \wedge Q_2 \wedge Q_3 = (P_1 \vee P_2) \wedge (P_2 \vee P_3) \wedge (P_1 \vee P_3)$. By doing so one gets $Q_1 \wedge Q_2 = P_2$, $Q_2 \wedge Q_3 = P_3$, $Q_1 \wedge Q_3 = P_1$ and $Q_1 \wedge Q_2 \wedge Q_3 = \mathbf{0}$, so that in fact $\mathbf{D}(P_1, P_2, P_3)$ is generated from $\{Q_1, Q_2, Q_3, \mathbf{1}\}$ by use of conjunctions only.

generated from a resolution of the identity **1**, but also in more general cases. For us the only more general case that will play a role in connection with Whitehead's ontology (see Section 8.4) is that the generating set $\{P_{a_i}\}$ is a set of mutually orthogonal projections whose disjunction is not identical to **1**, i.e. $\{P_{a_i}\}$ is derived from a resolution of **1** by removing some elements. In this case too, the symmetry between conjunction and disjunction holds true.[13] But note that in case of a set $\{P_{a_i}\}$ of mutually orthogonal projections that is *not* a resolution of the identity, the distributive lattice generated from $\{P_{a_i}\}$ does *not* coincide with the Boolean algebra generated from the same set. As already mentioned in the last section, this is because the latter will contain negations of elements which cannot be generated by disjunctions (and/or conjunctions) from elements of $\{P_{a_i}\}$. For example, if $\{P_{a_i}\}$ is derived from a resolution of the identity by removing one element, say P_{a_k}, then the Boolean algebra $\mathbf{B}(\{P_{a_i}\})$ does contain the latter because of $P_{a_k} = \neg \vee_{a_i \neq a_k} P_{a_i}$, whereas the distributive lattice $\mathbf{D}(\{P_{a_i}\})$ does not, since the operation $\neg$ needed to construct P_{a_k} from the elements of $\{P_{a_i}\}$ is not available.

In sum, then, the structure of the sets of properties corresponding to some (classical or quantum) magnitude A alone, does not provide any argument for whether conjunctive or disjunctive properties should be included in ones ontology—$\mathbf{B}(A)$ (and thus too $\mathbf{D}(A)$) is completely "symmetric" in respect to conjunctions and disjunctions. Moreover, such an argument can neither be derived in the slightly generalized case (needed later on) of Boolean algebras and distributive lattices, not generated by resolutions of the identity **1**, but

[13]For example, consider again the distributive lattice generated from the set of mutually orthogonal projections P_1, P_2 and P_3, but this time assume $P_1 \vee P_2 \vee P_3 < \mathbf{1}$. Besides P_1, P_2 and P_3, $\mathbf{D}(P_1, P_2, P_3)$ contains the following elements: $Q_1 = P_1 \vee P_2$, $Q_2 = P_2 \vee P_3$, $Q_3 = P_1 \vee P_3$, $\mathbf{0} = P_1 \wedge P_2 \wedge P_3$ and $Q_4 = P_1 \vee P_2 \vee P_3$. Thus by adding the trivial element $\mathbf{0}$ to the generating set $\{P_1, P_2, P_3\}$, all elements of $\mathbf{D}(P_1, P_2, P_3)$ are in fact generated by disjunctions of P_1, P_2, P_3 and $\mathbf{0}$. That $\mathbf{D}(P_1, P_2, P_3)$ can also be generated by conjunctions of elements from the subset $\{Q_1, Q_2, Q_3\, Q_4\} \subset \mathbf{D}(P_1, P_2, P_3)$ again easily follows from the application of the distributive laws (8.5a) and (8.5b) to $Q_1 \wedge Q_2 = (P_1 \vee P_2) \wedge (P_2 \vee P_3)$, $Q_2 \wedge Q_3 = (P_2 \vee P_3) \wedge (P_1 \vee P_3)$, $Q_1 \wedge Q_3 = (P_1 \vee P_2) \wedge (P_1 \vee P_3)$ and $Q_1 \wedge Q_2 \wedge Q_3 = (P_1 \vee P_2) \wedge (P_2 \vee P_3) \wedge (P_1 \vee P_3)$. By doing so one gets $Q_1 \wedge Q_2 = P_2$, $Q_2 \wedge Q_3 = P_3$, $Q_1 \wedge Q_3 = P_1$ and $Q_1 \wedge Q_2 \wedge Q_3 = \mathbf{0}$, so that in fact $\mathbf{D}(P_1, P_2, P_3)$ is generated from $\{Q_1, Q_2, Q_3, Q_4\}$ by use of conjunctions only.

merely by sets of mutually orthogonal projections.

In the following two sections, we will exploit some important consequences of the non-Boolean (or better the non-distributive) structure of $\mathcal{P}(\mathcal{H})$, deriving from its non-commutativity. In particular for the assignment of probabilities to quantum properties and for the related question as to when a system can be said to possess some property. In Section 8.3.3 we will return to the question for the ontological priority of disjunctive or conjunctive properties.

8.1.5 Probabilities for quantum properties

We have interpreted the creative characters of the activities in Whitehead's ontology as propensities, i.e. as ontic single case probabilities, for the determination of the abstractive hierachies of subjective eternal objects available to new occasions and for the decisions of concrescent occasions among the elements of these hierachies (see in particular Sections 2.3.1 and 2.4.2). Therefore, probabilities for subjective eternal objects play an important in Whitehead's ontology. We will now investigate how probabilities can be systematically assigned to quantum properties. This section extends the probability ascription introduced for the case of the eigenprojections of some magnitude (see Section 5.2) to the general case.

Whether the values $p(a)$ that a map p, defined on a set S, assigns to the elements $a \in S$ are *probabilities* of the latter, depends on the behavior of p under taking disjunctions of exclusive elements of S, i.e. of $a, b \in S$ with $a \wedge b = 0$, and on the value p assigns to the unit element 1 of S. More precisely, for $p(a)$ to be the probability of $a \in S$, the following conditions have to be satisfied by the map p:

(1) $p(a)$ lies between 0 and 1

(2) p has to assign the value 1 to the unit element $1 \in S$

(3) p is countably additive, i.e. for each countable subset $\{a_i\} \subseteq S$ with $\wedge_i a_i = 0$ one has $p(\vee_i a_i) = \sum_i p(a_i)$.

Whenever (1)-(3) hold for a map p on a set S it is called a *probability measure*. Thus for the values $p(a)$ to be probabilities, disjunctions and conjunctions need to be defined on S and S has to include a zero and a unit element. In other words, *S must be a distributive lattice with zero and unit element in which $\wedge$ and $\vee$ can be interpreted as ordinary conjunction and disjunction, i.e. a distributive lattice that coincides, as a set, with a Boolean algebra.*[14] However, the requirements (1)-(3) moreover imply that if a negation is defined on S then p automatically satisfies $p(\neg a) = 1 - p(a)$. Therefore, one can also assume from the start that S is a Boolean algebra instead of a distributive lattice, since the existence of the operation $\neg$ within S has no effect on the probability measure.

Now if a Boolean algebra $\mathbf{B}$ is even generated by a *countable* set $\{a_i\}$ of mutually exclusive elements whose disjunction equals the unit element $1 \in \mathbf{B}$, i.e. by a resolution of the unit element of $\mathbf{B}$, then $\mathbf{B}$ is itself countable and p is called a *discrete* probability measure. In this discrete case the requirements (1)-(3) are obviously equivalent to the conditions (i)-(iii) we have referred to

[14] To get a feeling why these probability assignments cannot be extended beyond Boolean algebras without leaving the realm of standard probability theory the following simple example may be instructive. Suppose P_1 and P_2 are non-commuting projections with corresponding subspaces $\mathcal{K}_1$ and $\mathcal{K}_2$. As explained above, in this case $P_1 \wedge P_2$ can be defined by the projection $P_{\mathcal{K}_1 \cap \mathcal{K}_2}$ onto the intersection of $\mathcal{K}_1$ and $\mathcal{K}_2$ and $P_1 \vee P_2$ by the projection $P_{\mathcal{K}_1 \uplus \mathcal{K}_2}$ onto the smallest subspace including $\mathcal{K}_1$ and $\mathcal{K}_2$, but $\wedge$ and $\vee$ cannot be interpreted as the conjunction and disjunction of the properties represented by P_1 and P_2.

However, one may hope that at least in case $\mathcal{K}_1 \cap \mathcal{K}_2 = \{0\}$, and thus $P_1 \wedge P_2 = P_{\mathcal{K}_1 \cap \mathcal{K}_2} = \mathbf{0}$, one can interpret P_1 and P_2 as *exclusive possibilities*, in the sense that $\rho(P_1 \vee P_2)$ is the sum of their individual probabilities $\rho(P_1)$, $\rho(P_2)$, so that $P_1 \vee P_2 = P_{\mathcal{K}_1 \uplus \mathcal{K}_2}$ would share at least this property with a true disjunction. Yet this is not the case as one can easily see in the following simple example of a two-dimensional Hilbert space $\mathcal{H}$: let P_1 be the projection onto the one-dimensional subspace $\mathcal{K}_1 = \{c\psi_1 : c \in \mathbb{C}\}$ and P_2 the projection onto the one-dimensional subspace $\mathcal{K}_2 = \{c\psi_2 : c \in \mathbb{C}\}$, where the unit vectors ψ_1 and ψ_2 are neither parallel nor orthogonal. Then one obviously has $\mathcal{K}_1 \cap \mathcal{K}_2 = \{0\}$ and thus too $P_1 \wedge P_2 = P_{\mathcal{K}_1 \cap \mathcal{K}_2} = \mathbf{0}$. Now take ρ to be the state generated by the unit vector ψ_1, so that $\rho(P_1) = \langle \psi_1, P_1\psi_1 \rangle = 1$. Since $\rho(P_1 \vee P_2) = \rho(P_1) + \rho(P_2) \leq 1$ has always to hold if $\rho(P_1 \vee P_2)$ shall be interpretable as a probability, this necessitates $\rho(P_2) = 0$. Yet since ψ_1 and ψ_2 are not orthogonal one has $\|P_2\psi_1\| > 0$ and $P_2\psi_1 / \|P_2\psi_1\| = \psi_2$ and thus $\langle \psi_1, P_2\psi_1 \rangle / \|P_2\psi_1\| = \langle \psi_1, \psi_2 \rangle$. Because of the non-orthogonality of ψ_1 and ψ_2 their scalar product $\langle \psi_1, \psi_2 \rangle$ is also non-zero so that $\rho(P_2) = \langle \psi_1, P_2\psi_1 \rangle = \|P_2\psi_1\| \langle \psi_1, \psi_2 \rangle > 0$, in contradiction to $\rho(P_1 \vee P_2) = \rho(P_1) + \rho(P_2) \leq 1$. Thus for non-commuting projections P_1 and P_2, $P_1 \vee P_2 = P_{\mathcal{K}_1 \uplus \mathcal{K}_2}$ cannot be interpreted as their disjunction even if $P_1 \wedge P_2 = P_{\mathcal{K}_1 \cap \mathcal{K}_2} = \mathbf{0}$ holds—at least not if one believes that probabilities must not exceed the value 1.

as constitutive for the concept of probability when we ascribed probabilities to properties represented by the eigenprojections of a magnitude A in Section 5.2.

Since, as just explained, probabilities can be consistently ascribed to *all* elements of a Boolean algebra we can now extend the probability ascription to *all* properties P_D corresponding to a magnitude A (and thus to arbitrary conjunctions, disjunctions and negations of them). Let A be some magnitude, ρ be some state and $P_D \in \mathbf{B}(A)$. Then *the probability for finding the value of A in the set D upon measurement of A on a system in state ρ, abbreviated by $prob_\rho(A \in D)$, is given by $\rho(P_D)$, i.e.*

$$prob_\rho(A \in D) = \rho(P_D). \tag{8.7}$$

Accordingly, *the probability for finding the value of A in D_1 and D_2, respectively in D_1 or D_2, upon a measurement of A on a system in state ρ, is by*

$$prob_\rho(A \in D_1 \wedge A \in D_2) = \rho(P_{D_1} \wedge P_{D_2}) = \rho(P_{D_1} P_{D_2}), \tag{8.8}$$

and

$$prob_\rho(A \in D_1 \vee A \in D_2) = \rho(P_{D_1} \vee P_{D_2}) = \rho(P_{D_1} + P_{D_2} - P_{D_1} P_{D_2}). \tag{8.9}$$

Moreover, since the commutativity of two magnitudes A and B is equivalent to the commutativity of their associated Boolean algebras of properties $\mathbf{B}(A)$ and $\mathbf{B}(B)$, the latter generate the Boolean algebra $\mathbf{B}(A, B) \equiv \mathbf{B}(\mathbf{B}(A) \cup \mathbf{B}(B))$ that contains besides all properties from $\mathbf{B}(A)$ and $\mathbf{B}(B)$ also all combinations of conjunctions, disjunctions and negations of the latter. Therefore, one can define *the probability for finding the value of A in the set D and the value of B in the set E, upon a joint measurement of A and B on a system in state ρ, by*

$$prob_\rho(A \in D \wedge B \in E) = \rho(P_D^A \wedge P_E^B) = \rho(P_D^A P_E^B). \tag{8.10}$$

There have been various proposals for extending the probability assignment in QT beyond Boolean algebras, in particular by introducing special "quantum probabilities" not obeying the rules of ordinary probability theory in one or another way. For example, some authors have given up the requirement that all probabilities have to be non-negative or even that they have to be real numbers, to be able to extend the assignment of such "probabilities" beyond Boolean algebras. Yet up to now none of these proposals has been proven to be helpful in clarifying the conceptual difficulties of QT. Perhaps this situation will change someday, but until then there seems to be no reason to go beyond the—both formally and intuitively—well understood realm of standard probability theory.

8.1.6 Definite, indefinite and possible quantum properties

Whiteheadian occasions can be said to possess those subjective eternal objects which are not eliminated during its process of concrescence and thus have unrestricted ingression into this occasion (see Sections 2.2.1 and 2.2.6). We will now discuss the important question when a quantum property can be said to be possessed respectively not possessed by a system in a given state. This question is rarely discussed in connection with interpretations of QT which are orientated towards the physical applicability of the formalism, i.e. physical interpretations, but it is clearly of great importance for each interpretation that attempts to unveil the ontological basis of QT. We will see that the question which properties can be said to be possessed by a system in a given state, leads to the result that in a Whiteheadian interpretation of QFT one has to adopt one part of the well-known eigenvalue-eigenstate rule whereas the other part has to be given up.

The eigenvalue-eigenstate rule

A widely held view that can be traced back at least to von Neumann (1932) is that a system in state ρ does (not) possess the property P iff the probability $\rho(P)$ assigned to P by the state ρ is 1 (0). Since in the special case where P is a one-dimensional projection $\rho(P) = 1$ ($\rho(P) = 0$) is equivalent to ρ being an eigenstate to eigenvalue 1 (0) of the projection P, this assumption has also become known as *eigenvalue-eigenstate rule.*

In the following we will argue that in our Whiteheadian interpretation of QFT,

(1) the “only-if-part”, i.e. the implication from “P is (not) possessed by a system in state ρ” to “the probability of P in state ρ is 1 (0)”

has to be adopted, whereas

(2) the “if-part”, i.e. the implication from “the probability of P in state ρ is 1 (0)” to “P is (not) possessed by a system in state ρ”

has to be denied.

Adoption of the “only-if-part” of the eigenvalue-eigenstate rule

The “only-if-part” of the eigenvalue-eigenstate rule has given rise to much dispute. As a starting point for recognizing in what respect the “only-if-part” of the eigenvalue-eigenstate rule may be regarded as dissatisfactory, note that it rules out the *property definiteness* known from classical physics. Property definiteness means that for each state ρ and each property P a system in state ρ either possesses P or not. However, if the “only-if-part” is adopted a system in state ρ with $0 < \rho(P) < 1$ neither possesses nor does it not possess P. Thus a property need not either be possessed or not possessed by a system in a given state but can have a third ontological status, unknown in classical physics, of being *indefinite* with respect to the system in this state.

For the following discussion it will be convenient to introduce a certain amount of terminology: let R be some rule which singles out possessed and not possessed properties with respect to a given state and let us denote the corresponding set of properties which are *definite*, i.e. either possessed or not possessed, for a system in state ρ by $\mathcal{D}^R_\rho$. Then the rule R can be expressed by a map

$$poss^R_\rho : \mathcal{D}^R_\rho \to \{0, 1\},$$

such that $poss^R_\rho(P) = 1$ means “a system in state ρ possesses property P (according to R)” and $poss^R_\rho(P) = 0$ means “a system in state ρ does not possess property P (according to R)”. The eigenvalue-eigenstate rule, abbreviated by R_E, can then be expressed by

$$poss^{R_E}_\rho(P) = 1 \text{ iff } \rho(P) = 1$$

and

$$poss_\rho^{R_E}(P) = 0 \text{ iff } \rho(P) = 0.$$

Now most people who challenge the "only-if-part" of these equivalences do not do so merely because it leads to the ascription of the classically unknown status of being indefinite with respect to a given state to some properties. This is because S. Kochen and E. Specker have shown that this is necessarily the case for *any* rule R that selects the properties which are definite with respect to a given state ρ in such a way that the values $poss_\rho^R(P)$ satisfy the classical logical relations for conjunctions, disjunctions and negations (Kochen and Specker 1967; see also Bub (1997) for an extensive discussion of this so-called Kochen-Specker theorem).[15] This is the case iff the map $poss_\rho^R$ satisfies

$$poss_\rho^R(\neg P_1) = 1 - poss_\rho^R(P_1) \tag{8.11a}$$

$$poss_\rho^R(P_1 \wedge P_2) = poss_\rho^R(P_1) \cdot poss_\rho^R(P_2) \tag{8.11b}$$

$$poss_\rho^R(P_1 \vee P_2) = poss_\rho^R(P_1) + poss_\rho^R(P_2) - poss_\rho^R(P_1) \cdot poss_\rho^R(P_2) \tag{8.11c}$$

for all $P_1, P_2 \in \mathcal{D}_\rho^R$, since then $\neg P_1$ expresses the non-possession of P_1, $P_1 \wedge P_2$ is possessed iff both P_1 and P_2 are possessed and $P_1 \vee P_2$ is possessed iff P_1 is possessed or P_2 is possessed or both are possessed. Note that if $\mathcal{D}_\rho^R$ were a set of *propositions* the map $poss_\rho^R$, obeying (8.11a)-(8.11c), would just be a *classical truth-functional*, that ascribes to each proposition in $\mathcal{D}_\rho^R$ the predicate "true" (= 1) or "false" (= 0). Now we already know that for pairs of non-commuting projections, the operations $\wedge$ and $\vee$[16] cannot be interpreted as conjunction and disjunction at all. Thus if (8.11b) and (8.11c) would be required to hold for non-commuting projections it would be no great surprise that a map with these properties cannot exist on the set $\mathcal{P}(\mathcal{H})$ of all projections on Hilbert space $\mathcal{H}$. However, the Kochen-Specker theorem *only presupposes that (8.11b) and (8.11c) hold for commuting projections P_1 and P_2*. The importance of the

[15] More precisely, the Kochen-Specker theorem applies only in Hilbert spaces of dimension strictly larger than 2. Yet since the descritpion of any realistic quantum system always requires a Hilbert space of infinite dimension this restriction is of minor interest and need not be discussed here.

[16] Defined for projections which do not commute by $P_1 \wedge P_2 = P_{\mathcal{K}_1 \cap \mathcal{K}_2}$ and $P_1 \vee P_2 = P_{\mathcal{K}_1 \uplus \mathcal{K}_2}$ (see Section 8.1.2).

Kochen-Specker theorem lies in the fact that even under this initially plausible conditions, no such map $poss_\rho^R$ can exist on $\mathcal{P}(\mathcal{H})$. In other words, under the assumptions (8.11a)-(8.11c) there is no rule R such that the corresponding set $\mathcal{D}_\rho^R$ of definite properties with respect to a given state ρ includes all properties $P \in \mathcal{P}(\mathcal{H})$. Thus *property definiteness cannot be retained if one is not willing to give up ordinary logical reasoning even in case of commuting properties as expressed by the map* $poss_\rho^R$. Therefore, the "only-if-part" of the eigenvalue-eigenstate rule can hardly be challenged for leading to the ascription of the classically unknown status of being indefinite to *some* properties. Rather people who challenge it do so because one can in fact formulate alternative rules which allow *more* (though not all) properties to be definite with respect to a given state (by obeying (8.11a)-(8.11c)) than allowed by the "only-if-part" of the eigenvalue-eigenstate rule. Thus this "only-if-part" is challenged because it is held to lead to the ascription "indefinite" to *too many* properties.

However, the enlargement of the set of definite properties of a state has its prize. Any rule that does not incorporate the "only-if-part" and therefore ascribes to some property P with $0 < \rho(P) < 1$ the status "definite" with respect to ρ, has the consequence that $\rho(P)$ *cannot have the status of an ontic probability*, and thus in particular of a propensity. This is obvious since, for example to hold that a P with $0 < \rho(P) < 1$ *is not possessed* by a system in state ρ is clearly not consistent with holding at the same time that $\rho(P) > 0$ measures *the likeliness with which P is possessed* by the system in question. Therefore, $\rho(P)$ can merely be some kind of *epistemic* probability that arises from our ignorance as to the true ontological status of P in respect to the system in question. Thus to deny the "only-if-part" means to give up the ontological relevance of the probabilities assigned to properties on the basis of quantum states. The situation is then as follows: one can associate a certain set $\mathcal{D}_\rho^R$ of definite properties to a system in state ρ, that will generally be larger than the set assigned by rules accepting the "only-if-part", but the probabilities $\rho(P)$ of the properties from $\mathcal{D}_\rho^R$ will, in general, not mirror the ontological status of the latter. Thus each property P from $\mathcal{D}_\rho^R$ *is determinate* as to its being possessed or not possessed by the system in question, but this ontological status of P is, in general, not correlated with the value of the probability $\rho(P)$ that merely expresses our knowledge, rational believe or something the like about the true ontological status of P. Thus any attempt to enlarge the set of definite properties for a given state ρ *beyond* those selected by the eigenvalue-eigenstate rule (by abandoning the "only-if-part") undermines the interpretation of the probabilities $\rho(P)$ as indicating the true ontological status of properties relative to a system in state ρ.

Yet for the attempted connection with Whitehead's ontology the abandonment of the "only-if-part" would be rather unfortunate because, as argued in Section 2.3, the creative character of the underlying activity can be understood as providing propensities (i.e. ontic single case probabilities), for the actualization of eternal objects by concrescent occasions. Therefore, on the side of the formalism of QFT one needs probabilities that can be reasonably interpreted as ontic rather than merely epistemic in nature. And as argued above, *this is only possible if one accepts the "only-if-part"*.

Of course, this need of the "only-if-part" of the eigenvalue-eigenstate rule is not a specific feature arising in connection with Whitehead's ontology. Rather everyone who believes that QT is *complete*, in the sense that it cannot ultimately be reduced to some more fundamental deterministic theory, has no other choice than to accept the ontic nature of (at least some) quantum probabilities. For to interpret all quantum probabilities epistemically means nothing else than to hold that they merely arise out of our ignorance with respect to some yet unknown parameters, not inherent in the formalism of QT and therefore usually called *hidden parameters* (see also Section 10.2), whose knowledge would retain determinism.

Abandoning the "if-part" of the eigenvalue-eigenstate rule

We will now argue that every interpretation of QT that (i) accepts the "only-if-part" of the eigenvalue-eigenstate rule, (ii) does not leave the realm of ordinary probability theory and (iii) that incorporates the idea of the "actualization of possibilities" with respect to properties, *has to deny the "if-part" of the eigenvalue-eigenstate rule.* As we have just argued, in a Whiteheadian interpretation the "only-if-part" has to be accepted, so that (i) is satisfied. Moreover, as mentioned in Section 8.1.5 our interpretation will stay within the realm of ordinary probability theory—therefore (ii) will likewise hold. "Actualization of possibilities with respect to properties" here means that properties have to have the ontological status of possibilities or potentialities for a system in some state ρ before they can become definite properties of that system in some *later* state ρ', where "later" need not necessarily mean (spatio-) temporally later, but rather can also refer to some other order, like e.g. the genetic order in Whitehead's ontology (see Section 1.1.3). Note that for convenience we also call the transition from "possible" to "*not* possessed", "actualization". Thus "actualization" as understood here means the transition from "possible" to "definite". Whitehead's ontology clearly contains such actualizations: transitions from real potentiality, i.e. ingressed subjective eternal objects, to possessed, i.e. unrestrictedly ingressed, ones and likewise transitions from "ingressed" to "not unrestrictedly ingressed", i.e. not possessed, subjective eternal objects. Therefore, assumption (iii) will also to be incorporated into our interpretation, so that all three assumptions (i)-(iii) will in fact be satisfied by it.

Now the states ρ and ρ', relative to which the possible respectively definite properties are defined can be *different* for two reasons. First, if the actualization of possibilities is a temporal or more generally a spatiotemporal process, the state of the system may "dynamically develop" during this process according to

$$\rho \rightarrow \rho' = \rho(U(a)^{-1} \cdot U(a)) \tag{8.12}$$

where $U(a)$ is the corresponding (spatio-) temporal translation. Yet in AQFT this form of dynamic development is incorporated by reason of the covariance condition

$$U(a)\mathcal{R}(\mathcal{O})U(a)^{-1} = \mathcal{R}(\mathcal{O} + a)$$

in the structure of the fundamental map $\mathcal{O} \mapsto \mathcal{R}(\mathcal{O})$ and thus as a transformation of observables and not as a transformation of states. Therefore, this kind of state change that could in principle occur during a spatiotemporal process of actualization in fact does not occur in AQFT. Yet it does occur in QM if the latter is formulated in the so-called *Schrödinger picture* in which the temporal development is described by unitary transformations of the states according to $\rho \rightarrow \rho_t = \rho(U(t)^{-1} \cdot U(t))$. However, QM can equivalently be formulated in the so-called *Heisenberg picture* in which the temporal evolution is described by unitary transformations of observables $A \to A(t) = U(t)AU(t)^{-1}$. Therefore, QM need not be treated separately in what follows—we merely need to understand the occurring unitary operator $U(a)$ as representing a temporal translation instead of a spatiotemporal one.

The second possibility for a change of the state of a system is known as *state-collapse. Such a collapse is any transformation of a state that is not of the form (8.12), with $U(a)$ a (spatio-) temporal translation.* Therefore, it can in particular not be interpreted as a dynamic development, but rather is an extra hypothesis not to be explained by the dynamics of QT itself. Interpretations of QT which make use, in one way or the other, of state-collapses are collectively called *collapse interpretations*. Our Whiteheadian interpretation of QFT will be a collapse interpretation which does not incorporate a further spatiotemporal process in the actualization of possibilities.

Yet the most general scheme for the actualization of possible properties in QT incorporates both, a spatiotemporal process and a non-spatiotemporal collapse. Let $\mathcal{P}_\rho \subset \mathcal{P}(\mathcal{H})$ be the set of possible properties with respect to a system in state ρ and let $\mathcal{D}^R_{\rho'}$ be the set of definite properties at the end of the actualization process. Let the actualization involve a spatiotemporal process, whose "duration" can be represented by the timelike vector a. The demand that all definite properties of the system at the end of this process have been possible properties of the system before they became definite, then, means that the set $\mathcal{D}^R_{\rho'}$ of definite properties at the end of this process, when "translated back" in (space-) time by the timelike vector $-a$ to the process' beginning, would be a subset of the set of possible properties at this beginning. Formally this can be expressed by $U(-a)\mathcal{D}^R_{\rho'}U(-a)^{-1} \subseteq \mathcal{P}_\rho$ or because of $U(-a) = U(a)^{-1}$ by

$$U(a)^{-1}\mathcal{D}^R_{\rho'}U(a) \subseteq \mathcal{P}_\rho, \tag{8.13}$$

where ρ' differs from ρ iff the actualization does moreover involve a state-collapse. If *each* of the possible properties $P \in \mathcal{P}_\rho$ becomes definite, i.e. none becomes indefinite, one has "=" instead of "⊆" in (8.13). The special case in which the actualization of possible properties does *not* incorporate a (spatio-) temporal process, as in case of Whitehead's ontology, is included in the above account by letting $a = 0$, so that $U(a) = \mathbf{1}$ and therefore (8.13) simply becomes $\mathcal{D}^R_{\rho'} \subseteq \mathcal{P}_\rho$.

Before we can proceed to show that the above assumptions (i)-(iii) together with the "if-part" of the eigenvalue-eigenstate rule lead to an inconsistency, we have to say when a property $P \in \mathcal{P}(\mathcal{H})$ shall have the ontological status "possible" with respect to a given system in state ρ. Since QT is a theory that incorporates probabilities it seems natural to assume that a property is possible with respect to a system in state ρ just in case ρ ascribes a probability to this property that, then, measures the strength of this possibility. More precisely, in interpretations of QT which do not leave the realm of standard probability theory, i.e. which satisfy (ii), it is reasonable to hold that *a property $P \in \mathcal{P}(\mathcal{H})$ has the ontological status "possible" with respect to a system in state ρ, i.e. $P \in \mathcal{P}_\rho$, iff $\rho(P)$ is a standard probability.* Note that for convenience we do allow the probability ascribed to a *possible* property to be zero. In other words, in what follows we will simply treat the impossibility with respect to ρ as if it were a special case of possibility with respect to ρ, namely the special case $\rho(P) = 0$. If one feels scruples with this wider use of the term "possible" that also includes impossibility as a special case, the term "possible" may be replaced by "possible or impossible" wherever it occurs in the rest of this section.

Now one can show that the eigenvalue-eigenstate rule R_E yields a set of definite properties $\mathcal{D}_{\rho'}^{R_E}$ for a system in state ρ' that will, in general, *not* be embeddable into a Boolean algebra because it contains projections not commuting with one another (see Bub 1997, p. 122 f; Vermaas 1999, p. 77). Since non-commutativity is invariant under unitary transformations the same is true for the translated set $U(a)^{-1}\mathcal{D}_{\rho'}^{R}U(a)$. However, according to the idea of the actualization of possibilities as expressed by (8.13), the set $U(a)^{-1}\mathcal{D}_{\rho'}^{R}U(a)$ must be included in the set $\mathcal{P}_\rho$ of possible properties at the beginning of the actualization process. Therefore, this set $\mathcal{P}_\rho$ too, contains non-commuting elements and thus cannot be embedded into any Boolean algebra, thereby undermining the assumption (ii) that the state ρ defines an ordinary probability measure on $\mathcal{P}_\rho$, for this would require $\mathcal{P}_\rho$ to be Boolean (see Section 8.1.5 and e.g. Auletta 2001, p. 194 f). Thus the eigenvalue-eigenstate rule is not compatible with the idea of the actualization of possibilities with respect to properties (assumption (iii)) and ordinary probability theory (assumption (ii)). Since the eigenvalue-eigenstate rule is just the conjunction of its "only-if-part" with its "if-part", we therefore have shown that any interpretation that satisfies (ii), (iii) and accepts the "only-if-part" (assumption (i)) must abandon the "if-part" of the eigenvalue-eigenstate rule.

In sum, then, we have shown that, in particular, a Whiteheadian interpretation of QT has to obey the following constraints: it has to accept the "only-if-part" of the eigenvalue-eigenstate rule for allowing for an ontic interpretation of quantum probabilities, and it must at the same time deny the "if-part" of the eigenvalue-eigenstate rule for being able to make sense of the Whiteheadian idea of the actualization of possibilities with respect to subjective eternal objects. We will conclude this section by taking a look at how the usual physical interpretation of QT deals with possible and definite properties relative to the state of a system.

Possible and definite properties in the usual physical interpretation

As mentioned earlier, in physical interpretations of QT the choice of a particular Boolean algebra of possible properties $\mathcal{P}_\rho$ for a system in state ρ is usually delegated to an external observer and the measurement is assumed to incorporate a state-collapse. In terms of the scheme presented above this amounts to the following: the observer sets up a measuring device to measure a particular magnitude or observable A on a system in state ρ thereby fixing the Boolean algebra of possible properties to be $\mathcal{P}_\rho = \mathbf{B}(A)$. At the end of the measurement thus set up the system possesses one of the properties $P_D \in \mathbf{B}(A)$ and its state has collapsed from ρ to $\rho' = \rho(P_D \cdot P_D)/\rho(P_D)$, so that $\rho'(P_D) = 1$. More precisely, all $P_{D'} \in \mathbf{B}(A)$ for which either $\rho'(P_{D'}) = 1$ or $\rho'(P_{D'}) = 0$ holds, are taken to be definite at the end of the measurement. This is just the eigenvalue-eigenstate rule with respect to the state ρ' but *restricted to properties from* $\mathcal{P}_\rho = \mathbf{B}(A)$, term this restricted eigenvalue-eigenstate rule R_E^A. As argued above, an unrestricted application of the eigenvalue-eigenstate rule, i.e. its application to all $P \in \mathcal{P}(\mathcal{H})$, in general, leads to collections of definite properties which are not embeddable into any Boolean algebra, and thus in particular not into $\mathbf{B}(A)$, so that either one has to leave the realm of ordinary probability theory or the measurement process cannot be understood as involving an actualization of possibilities. Since the set $\mathcal{D}_{\rho'}^{R_E^A}$ of definite properties at the end of a measurement is a subset of the set $\mathcal{P}_\rho = \mathbf{B}(A)$ of initially possible properties, *the actualization process involved in a measurement cannot be a (spatio-) temporal process* (see (8.13) above). In QM it is usual to idealize the whole measurement as instantaneous and thus to identify measurement, state-collapse and actualization, thereby making them all instantaneous and thus in particular non- (spatio-) temporal. However, in AQFT measurements cannot be idealized as instantaneous—at least not if one does not go beyond the spatiotemporal relations provided by STR. In the spacetime structure provided by STR there is no objective notion of instantaneity or equivalently simultaneity available, apart from the uninteresting one according to which each spacetime point is instantaneous with itself and with nothing else. Thus the instantaneity of measurements would imply that they are pointlike thereby undermining one of the central ideas underlying the algebraic approach, namely to build up the theory on non-pointlike quantities (see Section 4.3). More importantly, one can in fact proof that the axioms of AQFT imply that there are *no non-trivial observables at spacetime points at all*, i.e. if the region $\mathcal{O}$ "shrinks" to a point $x \in M$ the corresponding local algebras $\mathcal{R}(\mathcal{O})$ converge to the trivial algebra $\mathbb{C}\mathbf{1} \equiv \{c\mathbf{1} : c \in \mathbb{C}\}$ consisting merely of multiplies of the identity operator (see e.g. Baumgärtel 1995, Corollary 1.4.3). Since the usual physical interpretation of AQFT does not go beyond the spacetime structure provided by STR, it therefore cannot idealize measurements as instantaneous. Therefore, it incorporates the *spatiotemporal extendedness of measurements* but assumes at the same time that the *actualization involved in a measurement is not a spatiotemporal process.* Later on these somewhat strange features of the physical interpretation will be reinterpreted within the framework of non-spatiotemporal actualization processes provided by Whitehead's ontology.

8.1.7 Quantum properties as universals?

In this section we will argue that quantum properties as represented by single projection operators cannot be universals. This is an important result in connection with Whitehead's ontology since the latter does treat properties as universals. Therefore, our Whiteheadian interpretation of QFT cannot simply make use of single projection operators for the representation of Whiteheadian property-universals, i.e. subjective eternal objects. Since the presented argument will make use of specific structures of QFT *not* present in QM, the conclusion we arrive at will also be restricted to the former theory. However, since our interest in this work is QFT and not QM we will not discuss whether the arguments for the "non-universality" of properties can somehow be extended to QM or not.

Recall that according to Section 2.2 an entity is a universal iff it can occupy, embody or be located in two or more non-overlapping spacetime regions. Since AQFT is build upon the very correspondence between spacetime regions and operator algebras it is natural to hold that a property represented by the projection operator $P \in \mathcal{P}(\mathcal{H})$ can occur in region $\mathcal{O} \subset M$ *only if* P belongs to the algebra $\mathcal{R}(\mathcal{O})$ associated with $\mathcal{O}$. Note that the region $\mathcal{O}$ need not be bounded, since as explained in Appendix C, von Neumann subalgebras $\mathcal{R}(\mathcal{O})$ of $\mathcal{B}(\mathcal{H})$ can also be associated in a canonical way with unbounded regions $\mathcal{O} \subset M$, given the fundamental correspondence $\mathcal{O} \mapsto \mathcal{R}(\mathcal{O})$ for bounded regions. Thus from the above criterion for the universality of an entity we get the following necessary condition for universality of properties in AQFT: a property represented by the projection operator P is a universal *only if* it belongs to at least two algebras $\mathcal{R}(\mathcal{O}_1)$ and $\mathcal{R}(\mathcal{O}_2)$ such that $\mathcal{O}_1$ and $\mathcal{O}_2$ are (not necessarily bounded) regions which are disjoint.

Let us begin by discussing the case where *both regions are bounded*, which here always means that they are double cones.[17] L. Landau (1969) has shown that in this case the disjointness of the closures of $\mathcal{O}_1$ and $\mathcal{O}_2$, i.e. $\overline{\mathcal{O}}_1 \cap \overline{\mathcal{O}}_2 = \varnothing$, implies that the intersection of the local algebras $\mathcal{R}(\mathcal{O}_1)$ and $\mathcal{R}(\mathcal{O}_2)$ corresponding to $\mathcal{O}_1$ and $\mathcal{O}_2$ consists merely of multiplies of the identity, i.e. $\mathcal{R}(\mathcal{O}_1) \cap \mathcal{R}(\mathcal{O}_2) = \mathbb{C}\mathbf{1}$, so that the only properties contained in *both* algebras are the trivial ones $\mathbf{0}$ and $\mathbf{1}$. This means that no non-trivial property $P \in \mathcal{P}(\mathcal{H})$ can occur in any two double cones whose closures are disjoint. Now one could ask whether a property can occur in two double cones which are disjoint *but*

[17] The following discussion is an example of the earlier mentioned fact that the restriction to double cones simplifies many investigations in AQFT. For certain more general bounded regions one could reach similiar conclusions but the way to do so would be much more complicated and loaded wih many more technical assumptions.

whose closures are not, since this case is not covered by the result of Landau that only applies if the closures are disjoint. Although no general answer to this question is known (at least to the best knowledge of the author), it seems that even if this were generally true it would not be of much help to proponents of property-universals in QFT. In this case a property could occur in disjoint double cones whose closures are not disjoint, but *not* in disjoint double cones whose closures are disjoint. But it seems to be extremely unreasonable that the minor difference between regions that are not closed and, on the other hand, their closures should have so much weight as to make a difference to the answer of the crucial ontologically question whether quantum properties can be universals or not. Therefore, we conclude from what has been said so far that if a property $P \in \mathcal{P}(\mathcal{H})$ shall be a universal and thus has to have the ability to occur in at least two disjoint regions, (at least) *one of these regions has to be unbounded.* Note that in so far one is merely interested in the interpretability of projection operators as subjective eternal objects this result already provides a negative answer. This is because according to Whitehead, *subjective eternal objects are universals* which do, however, exclusively *occur in bounded regions* (because occasions only occupy bounded regions), which, as just shown, is not possible if subjective eternal objects were represented by single projection operators.

We will now argue that under a natural and often adopted strengthening of the above criterion of universality, a non-trivial $P \in \mathcal{P}(\mathcal{H})$ can only be universal if it cannot occur in any bounded region but merely in unbounded ones. But this is a rather high ontological cost for *every* interpretation of AQFT that wants to hold that quantum properties are universals. Especially in the light of the "smallness" of typical quantum systems it seems quite absurd to hold that the properties these systems can possess can only occur in unbounded regions. We therefore conclude that quantum properties cannot be understood as universals, independently from the particular interpretational framework one likes to choose. Now what is the strengthened criterion for universality? It says that if a universal can occur in some region $\mathcal{O}$ it should also have the ability to occur in a region *spacelike separated* from $\mathcal{O}$. This assumption is a strengthening of the more liberal criterion for universals formulated in Section 2.2 and restated above, because according to the latter, it would be sufficient if the entity in question could occur in two arbitrarily separated regions, whereas the strengthened criterion requires that if it can occur in some region $\mathcal{O}$ than it can also occur in a region spacelike separated from $\mathcal{O}$. However, this strengthened criterion is just the relativistic formulation of the way universals are often defined in the first place, namely *as entities which have the ability for occurrences in separated spatial regions at each instant of time (at which they can have concrete occurrences at all)* (see e.g. Loux 1998, p. 23, 54f).[18]

That this strengthened notion of universality implies that quantum properties can only be regarded as universals if one is willing to accept that they can merely occur in *unbounded* regions, is a consequence of the following result (Baumgärtel and Wollenberg 1992, Proposition 7.3.15): let $\mathcal{O}_1$ be a double cone and $\mathcal{O}_2$ be an arbitrary (possibly unbounded) region such that the closures of $\mathcal{O}_1$ and $\mathcal{O}_2$ are spacelike separated then $\mathcal{R}(\mathcal{O}_1) \cap \mathcal{R}(\mathcal{O}_2) = \mathbb{C}\mathbf{1}$. This result is an extension and at the same time a restriction of the above result of Landau. It is an extension because now one of the two regions involved can be unbounded and it is a restriction since it only applies to spacelike separated instead of merely disjoint regions. It says that if a non-trivial property $P \in \mathcal{P}(\mathcal{H})$ can occur in the bounded region $\mathcal{O} \in D(M)$ then it *cannot* occur in any other region whose closure is spacelike separated from the closure of $\mathcal{O}$, even if this other region is allowed to be unbounded.[19] Thus if one agrees to the widely accepted requirement that for being a universal, a property that can occur in some region $\mathcal{O}$ must also have the ability for occurrences in regions spacelike separated from $\mathcal{O}$, as we will do, then, a non-trivial property $P \in \mathcal{P}(\mathcal{H})$ can only be a universal *if one is willing to accept that it can exclusively occur in unbounded regions*. But since this seems to be too high a price for its universality, we conclude that non-trivial projections cannot reasonably be interpreted as property-universals. Note that in the above discussion we have not made use of any particular feature of projection operators. There-

[18] In ontological theories which incorporate substances this strengthened criterion is especially important, because our more liberal criterion, tentatively introduced earlier, does not allow to distinguish substances from universals (see Section 2.2).

[19] At this point one could again try to exploit the fact that two open regions can be spacelike separated even if their closures are not. But, as above, we do not believe that any proponent of property-universals in QFT could reasonably argue along these lines.

fore, all claims are equally valid for *arbitrary* operators $A \in \mathcal{B}(\mathcal{H})$, so that one cannot hope to make sense of property-universals in QFT by way of "simply" representing them by other operators than projections.

We are now confronted with the dilemma that, on the one hand, the only reasonable candidates for the representation of properties in QFT are projection operators but these cannot be understood as representing universals which can have local instances, and on the other hand, *subjective eternal objects are just such property-universals.* What to do? In the following we will propose the same solution as in case of objective eternal objects namely to represent each subjective eternal object *indirectly* by the representative of its class of possible ingressions. Of course, this does again open the door for nominalist attacks against universals in QFT. However, since Whitehead conceived his eternal objects as universals, we have to deal with this unfortunate state of affairs and try to make some sense of it in connection with QFT.

8.2 The "indirect" representation of subjective and mixed eternal objects

If one wants to interpret projection operators as possible instances or possible occurrences of property-universals—or in Whiteheadian terms, as possible ingressions of subjective eternal objects—one needs to group them into collections with a unity that allows for such an interpretation. In other words, and perhaps less cryptically, one clearly cannot simply take *any* set of projections to represent the possible instances of a single property-universal but rather only such sets whose members bear *sufficient similarities* to each other, i.e. *which resemble one another to a sufficient degree.* Generally "resemblance to a certain degree" need not be transitive. For example, if Peter resembles his sister Anne quite closely, and Anne resembles her mother quite closely, it is very well possible that Peter has no close resemblance to his mother. This is because Peter may resemble Anne closely only in respect of looks, while Anne resembles her mother only in respect to character, while Peter does not resemble his mother in respect to character or any other relevant feature. However, if resemblance is grounded in the fact that the resembling entities instantiate the same property-universal A, *their resemblance is always transitive.* This is because in this case the resemblance of, say a_1 and a_2 just means that "a_1 and a_2 instantiate A", and this latter relation is obviously transitive. Thus if the properties $\{P_i\}$ shall be interpretable as possible instances of the same property-universal A, then their resemblance must also be transitive. Otherwise the resemblance of the systems possessing properties from the set $\{P_i\}$ would not be transitive and thus could not be grounded in the P_i's being possible instances of the same property-universal. Besides the requirement that the resemblance relation used to construct the sets of possible instances of property-universals has to be transitive and thus to be an equivalence relation,[20] there is still another constraint that has to be obeyed by a set of projections to be interpretable as the class of possible instances of a single property-universal. According to our understanding of universality (see the last section), if the class contains a possible instance in some region $\mathcal{O}$, it must also contain a possible instance in a region spacelike separated from $\mathcal{O}$.

Moreover, apart from these general requirements there are two further specifically Whiteheadian assumptions on eternal objects which are important for our present task. And these will lead us almost directly to the resemblance relation and thus the classes we are searching for. First, since Whiteheadian occasions always occupy bounded regions, subjective eternal objects too, need only be ingressible into bounded regions. Therefore, we need only take into account classes of *local* projections. Second, subjective and objective eternal objects are eventually to be "brought together" as components of mixed eternal objects (see Sections 2.2.2 and 2.2.4). And since objective eternal objects have already been represented by equivalence classes of regions with respect to Poincaré transformations, it seems tempting to use these transformations for the construction of the classes of possible ingressions of subjective eternal objects, too. Moreover, because Poincaré transformations form a group, each set of local projections connected by Poincaré transformations were automatically an equivalence class as required for its interpretation as the class of possible ingressions of a single subjective eternal object.

Furthermore, the usual interpretation of the action of a Poincaré trans-

[20] That resemblance is always reflexive and symmetrical—not only in case it is grounded in the instantiation of the same universal—is obvious (see also Armstrong 1989, p. 40).

formation on an operator $A \in \mathcal{R}(\mathcal{O})$ also supports the view that two projections connected by a Poincaré transformation are instances of one and the same property-universal. Usually the operator $U(g)AU(g)^{-1}$ is taken to represent a measurement of the *same* magnitude as A merely carried out in region $g(\mathcal{O})$ instead of region $\mathcal{O}$. Taking this seriously means nothing else than the commitment to the assumption that magnitudes, and thus too the properties $P \in \mathbf{B}(A)$ connected with them, are universals and that the operators A and $U(g)AU(g)^{-1}$ represent the possible instances of this magnitude in the regions $\mathcal{O}$ and $g(\mathcal{O})$.

In sum, we therefore propose to interpret each equivalence class

$$\hat{P} \equiv \left\{ U(g)PU(g)^{-1} : g \in \mathcal{P}_+^\uparrow \right\}, \tag{8.14}$$

where $P \in \mathcal{P}_{loc} \equiv \mathcal{P}(\mathcal{H}) \cap \mathcal{A}_{loc}$ is some local projection, as representing the possible ingressions of a single subjective eternal object. Moreover, since there seems to be no way how the subjective eternal objects corresponding to these classes can be represented more directly in the formalism of AQFT, *we take these classes also as the representatives of subjective eternal objects themselves.*

Next we will discuss how the ability of a subjective eternal object (represented by) $\hat{P}$ to ingress into some region $\mathcal{O} \in D(M)$ can be represented in the formalism of AQFT. Recall from Section 2.2.1 that a subjective eternal object $\hat{P}$ ingressed into some region $\mathcal{O}$ *belongs indifferently to the whole region and not merely some subregion* $\mathcal{O}' \subset \mathcal{O}$. This secures the atomicity of occasions in the sense of not being divided into parts which are completed occasions in their own right.

Therefore, the ability of the subjective eternal object $\hat{P}$ to ingress into region $\mathcal{O}$ cannot merely be expressed by the existence of a projection $P' \in \hat{P}$ such that

$$P' \in \mathcal{R}(\mathcal{O}). \tag{8.15}$$

This is because the isotony property of the net of local algebras $\{\mathcal{R}(\mathcal{O})\}_{D(M)}$, i.e. the property $\mathcal{O}' \subseteq \mathcal{O} \Rightarrow \mathcal{R}(\mathcal{O}') \subseteq \mathcal{R}(\mathcal{O})$, implies that $P' \in \mathcal{R}(\mathcal{O})$ *is already satisfied if* $P' \in \mathcal{R}(\mathcal{O}')$ *for some subregion* $\mathcal{O}' \subset \mathcal{O}$. But then P' would belong to a subregion of $\mathcal{O}$, thereby undermining its role as the possible ingression of the corresponding subjective eternal object $\hat{P}$ (with $P' \in \hat{P}$) in $\mathcal{O}$. Therefore, the condition $P' \in \mathcal{R}(\mathcal{O})$ has to be supplemented by the further condition

$$P' \notin \mathcal{R}(\mathcal{O}') \text{ for all } \mathcal{O}' \subset \mathcal{O}, \tag{8.16}$$

to express the ingressibility of $\hat{P}$ into region $\mathcal{O}$. Thus if there is no projection $P' \in \hat{P}$ such that (8.15) *and* (8.16) are satisfied, $\hat{P}$ cannot be interpreted as ingressible into region $\mathcal{O}$. However, under certain technical supplementary assumptions on the structure of the correspondence $\mathcal{O} \mapsto \mathcal{R}(\mathcal{O})$ one can show that in fact *for each non-trivial local projection* P' *there are regions such that (8.15) and (8.16) are satisfied* (Kuckert 2000). Yet there may nevertheless be no *unique* region $\mathcal{O} \in D(M)$ for which these conditions hold for a given projection P'. But this does not undermine the interpretation of P' as a possible ingression of a subjective eternal object $\hat{P} \ni P'$. For since eternal objects are universals anyway there is no need for the possible ingressions of a single subjective eternal object in different regions to be different. In sum, we therefore propose that the ability for ingression of subjective eternal objects into regions of the extensive continuum is expressed by

(ING) The subjective eternal object $\hat{P}$ can ingress into the region $\mathcal{O} \in D(M)$ iff there is a $P' \in \hat{P}$ such that $P' \in \mathcal{R}(\mathcal{O})$ and $P' \notin \mathcal{R}(\mathcal{O}')$ for all $\mathcal{O}' \in D(M)$ with $\mathcal{O}' \subset \mathcal{O}$.

Thus the fundamental correspondence $\mathcal{O} \mapsto \mathcal{R}(\mathcal{O})$ between spacetime regions and operator algebras on which AQFT is erected, is likewise fundamental for expressing the ingressibility of subjective eternal objects into regions of the extensive continuum, within the formalism of AQFT. We will see in the following section that by reason of its role in grounding (ING), the correspondence $\mathcal{O} \mapsto \mathcal{R}(\mathcal{O})$ will also turn out to be fundamental for the mathematical expression of the compatibility of subjective and objective eternal objects as well as the compatibility of subjective eternal objects among each other.

Compatibility, complex subjective and mixed eternal objects

Complex subjective eternal objects The only complex eternal objects we are interested in are conjunctive and disjunctive ones (see Section 2.2.2), so that one may think that the eternal object $\hat{P}$ is complex if $P = P_1 \wedge P_2$

or $P = P_1 \vee P_2$ holds for two other local projections P_1 and P_2. We will only discuss this initially appealing mathematical criterion for the complexity of subjective eternal objects in the disjunctive case because the conjunctive case is then implicitly clear. If P is the disjunction of P_1 and P_2 one has[21]

$$\begin{aligned} \hat{P} &= \widehat{P_1 \vee P_2} \\ &= \left\{ U(g)(P_1 \vee P_2)U(g)^{-1} : g \in \mathcal{P}_+^\uparrow \right\} \end{aligned}$$

and by $P_1 \vee P_2 = P_1 + P_2 - P_1 P_2$ and the fact that $U(g)^{-1}U(g) = \mathbf{1}$ for all $g \in \mathcal{P}_+^\uparrow$ it easily follows that

$$\hat{P} = \left\{ (U(g)P_1U(g)^{-1}) \vee (U(g)P_2U(g)^{-1}) : g \in \mathcal{P}_+^\uparrow \right\}. \tag{8.17}$$

And since this seems to be the only reasonable way to define $\hat{P}_1 \vee \hat{P}_2$, one gets the desired result $\hat{P} = \hat{P}_1 \vee \hat{P}_2$.

However, there is a reason why *not each* $\hat{P}$ that can be written in the form $\hat{P}_1 \vee \hat{P}_2$ or $\hat{P}_1 \wedge \hat{P}_2$ respectively, can be regarded as a complex eternal object with components $\hat{P}_1$ and $\hat{P}_2$. Or put differently, there is a reason why the expression (8.17) (as well as the corresponding expression with $\wedge$) *cannot represent* a disjunctive (conjunctive) subjective eternal object *for two arbitrary commuting local projections* P_1 *and* P_2. The reason for this is that $\hat{P} = \hat{P}_1 \vee \hat{P}_2$ (as well as $\hat{P} = \hat{P}_1 \wedge \hat{P}_2$) according to (8.17) *does nothing to secure that there is at least one region into which both* $\hat{P}_1$ *and* $\hat{P}_2$ *can in fact ingress.* For, according to (ING) from the last section, a subjective eternal object $\hat{P}_i$ can ingress into region $\mathcal{O} \in D(M)$ iff there is a $P_i' \in \hat{P}$ such that $P_i' \in \mathcal{R}(\mathcal{O})$ and $P_i' \notin \mathcal{R}(\mathcal{O}')$ for all $\mathcal{O}' \in D(M)$ with $\mathcal{O}' \subset \mathcal{O}$. Therefore, the two subjective eternal objects $\hat{P}_1$ and $\hat{P}_2$ can *both* ingress into a common region just in case

(CO-ING) There is a region $\mathcal{O} \in D(M)$, a $P_1' \in \hat{P}_1$ and a $P_2' \in \hat{P}_2$, such that $P_1', P_2' \in \mathcal{R}(\mathcal{O})$ and $P_1', P_2' \notin \mathcal{R}(\mathcal{O}')$ for all $\mathcal{O}' \in D(M)$ with $\mathcal{O}' \subset \mathcal{O}$.

Now for two *arbitrary* local projections P_1, P_2 there is *always* a region $\mathcal{O} \in D(M)$ such that both belong to $\mathcal{R}(\mathcal{O})$: that P_1 and P_2 are local means that there are bounded regions $\mathcal{O}_1, \mathcal{O}_2 \in D(M)$ such that $P_i \in \mathcal{R}(\mathcal{O}_i)$ and because of the isotony property of the net of local algebras both $\mathcal{R}(\mathcal{O}_1)$ and

[21] Recall that this implies that P_1 and P_2 commute.

$\mathcal{R}(\mathcal{O}_2)$ are included in *each* algebra $\mathcal{R}(\mathcal{O})$ corresponding to some $\mathcal{O} \in D(M)$ with $\mathcal{O}_1 \cup \mathcal{O}_2 \subseteq \mathcal{O}$. But the second requirement, namely that for one of these regions $\mathcal{O}$ to whose algebra both P_1 and P_2 belong also "$P_1, P_2 \notin \mathcal{R}(\mathcal{O}')$ for all $\mathcal{O}' \in D(M)$ with $\mathcal{O}' \subset \mathcal{O}$" holds, is *not satisfied by two arbitrary local projections.* Thus if $P = P_1 \vee P_2$ (or $P = P_1 \vee P_2$ respectively) holds, without this supplementary requirement being fulfilled by P_1 and P_2 the corresponding "complex subjective eternal object" $\hat{P} = \hat{P}_1 \vee \hat{P}_2$ (or $\hat{P} = \hat{P}_1 \wedge \hat{P}_2$ respectively) *could not even ingress into a single region and thus not into a single occasion.* But in this case the existence of the eternal object $\hat{P}$ could hardly be justified. Note that by "existence" we here mean the existence of an eternal object qua pure potentiality and that this existence of an eternal object as such does *not* presuppose that there *is* an occasion into which this eternal object *is actually* ingressed—according to Whitehead there can very well be contingently uninstantiated universals. But if it is even *impossible* that a given eternal object ingresses *into any occasion at all*, i.e. if the eternal object is "necessarily" uninstantiated, there is no reason in the first place to postulate this eternal object—it would simply not be a potential for the determinateness of *any* occasion. Thus for the interpretation of $\hat{P} = \hat{P}_1 \vee \hat{P}_2$ (or $\hat{P} = \hat{P}_1 \wedge \hat{P}_2$ respectively) as disjunctive (conjunctive) eternal object with components $\hat{P}_1$ and $\hat{P}_2$, the projections P_1 and P_2 must not merely commute but rather the condition (CO-ING) has also to be satisfied.

Note that the satisfaction of the latter condition by P_1 and P_2 with respect to region $\mathcal{O} \in D(M)$ is equivalent to its satisfaction by their Poincaré transforms $P_1' = U(g)P_1U(g)^{-1}$ and $P_2' = U(g)P_2U(g)^{-1}$ with respect to the transformed region $g(\mathcal{O}) \in D(M)$. Thus, we always have $\hat{P} = \hat{P}_1 \vee \hat{P}_2 = \hat{P}_1' \vee \hat{P}_2'$ as well as $\hat{P} = \hat{P}_1 \wedge \hat{P}_2 = \hat{P}_1' \wedge \hat{P}_2'$ which means that the fact that $\hat{P}$ is a disjunctive or conjunctive subjective eternal object with components $\hat{P}_1$ and $\hat{P}_2$ does not depend on the chosen representatives $P_i' \in \hat{P}_i$ of the classes $\hat{P}_i$. Therefore, one can calculate with conjunctions and disjunctions (and of course also with negations) of classes $\hat{P}, \hat{Q}, \ldots$ in the same way as with these operations in case of single projections $P, Q, \ldots$. Yet we will see in Section 8.3 *that there is a fundamental ambiguity* in the way a $\hat{P}$ can be written as a disjunction or a conjunction, which stems from the fact that according to QFT there are probably no simple subjective eternal objects at all. Before we come to this important topic we will, however, first of all find mathematical expressions for the compatibility of subjective eternal objects among each other and of subjective and objective eternal objects and thus too of the existence of mixed eternal objects.

Compatibility and mixed eternal objects Recall from Section 2.2.4 that two subjective eternal objects $\hat{P}_1$ and $\hat{P}_2$ are compatible iff there is a complex subjective eternal object $\hat{P}$ of which both are components. The upshot of the above discussion has been that we assume that the subjective eternal object $\hat{P}$ is complex iff there are local projections P_1 and P_2 and a region $\mathcal{O} \in D(M)$ such that $P = P_1 \wedge P_2$ or $P = P_1 \vee P_2$ as well as $P_1, P_2 \in \mathcal{R}(\mathcal{O})$ and $P_1, P_2 \notin \mathcal{R}(\mathcal{O}')$ for all $\mathcal{O}' \in D(M)$ with $\mathcal{O}' \subset \mathcal{O}$, hold. Therefore, the following reformulation of this criterion provides us with the mathematical expression for the *compatibility of* $\hat{P}_1$ *and* $\hat{P}_2$:

(CS) The subjective eternal objects $\hat{P}_1$ and $\hat{P}_2$ are compatible iff $\hat{P}_1$ and $\hat{P}_2$ contain elements P_1' and P_2' such that

- (CS1) P_1' and P_2' commute and
- (CS2) there exists a $\mathcal{O} \in D(M)$ such that $P_1', P_2' \in \mathcal{R}(\mathcal{O})$ and $P_1', P_2' \notin \mathcal{R}(\mathcal{O}')$ for all $\mathcal{O}' \in D(M)$ with $\mathcal{O}' \subset \mathcal{O}$.

As explained in Section 8.1.2, the commutativity of the projections P_1' and P_2', i.e. condition (CS1), is necessary and sufficient for the existence of $P_1' \wedge P_2'$ and $P_1' \vee P_2'$. Condition (CS2) expresses that the condition (CO-ING) holds for the corresponding subjective eternal objects $\hat{P}_1$ and $\hat{P}_2$, i.e. that there is a region into which both can ingress.

Now the existence of a possible ingression of the subjective eternal object $\hat{P}$ in region $\mathcal{O}$, i.e. the fulfillment of (ING) by $\hat{P}$ with respect to region $\mathcal{O}$, is a reasonable necessary and sufficient condition for the existence of the *conjunctive mixed* eternal object $\hat{P} \wedge \widehat{\mathcal{O}}$. This is because the covariance property of the net of local algebras (i.e. $U(g)\mathcal{R}(\mathcal{O})U(g)^{-1} = \mathcal{R}(g(\mathcal{O}))$ for all $g \in \mathcal{P}_+^\uparrow$) implies that the fulfillment of (ING) by $\hat{P}$ with respect to region $\mathcal{O}$, i.e.

$$P' \in \mathcal{R}(\mathcal{O}) \text{ and } P' \notin \mathcal{R}(\mathcal{O}') \text{ for all } \mathcal{O}' \in D(M) \text{ with } \mathcal{O}' \subset \mathcal{O}$$

is equivalent to

$$\begin{aligned} U(g)P'U(g)^{-1} &\in \mathcal{R}(g(\mathcal{O})) \text{ and } U(g)P'U(g)^{-1} \notin \mathcal{R}(\mathcal{O}') \\ \text{for all } \mathcal{O}' &\in D(M) \text{ with } \mathcal{O}' \subset g(\mathcal{O}) \text{ and all } g \in \mathcal{P}_+^\uparrow \end{aligned}$$

which just means that each possible ingression of $\hat{P}$ is into a region from the class $\widehat{\mathcal{O}} = \{g(\mathcal{O}) : g \in \mathcal{P}_+^\uparrow\}$ representing the corresponding objective eternal object. Thus *all possible ingressions of the subjective eternal object $\hat{P}$ are in regions whose common boundary surface is (represented by) $\widehat{\mathcal{O}}$.* What else should one want for the existence of the conjunctive mixed eternal object $\hat{P} \wedge \widehat{\mathcal{O}}$?

In Section 2.2.4 we have explained why only *conjunctive mixed* eternal objects, but no disjunctive ones, are needed. Therefore, the only possibility for the compatibility of $\hat{P}$ and $\widehat{\mathcal{O}}$ is by way of the existence of the conjunctive mixed eternal object $\hat{P} \wedge \widehat{\mathcal{O}}$. Consequently, the fulfillment of (ING) by $\hat{P}$ with respect to some region $\mathcal{O}' \in \widehat{\mathcal{O}}$ which has just been seen to be a reasonable criterion for the existence of $\hat{P} \wedge \widehat{\mathcal{O}}$ is moreover already the mathematical expression for the *compatibility of the subjective eternal object $\hat{P}$ and the objective eternal object $\widehat{\mathcal{O}}$*. Explicitly, one therefore has:

(CSO) The subjective eternal object $\hat{P}$ and the objective eternal object $\widehat{\mathcal{O}}$ are compatible iff there is a $P' \in \hat{P}$ with $P' \in \mathcal{R}(\mathcal{O})$ and $P' \notin \mathcal{R}(\mathcal{O}')$ for all $\mathcal{O}' \in D(M)$ with $\mathcal{O}' \subset \mathcal{O}$.

Thus the fundamental correspondence $\mathcal{O} \mapsto \mathcal{R}(\mathcal{O})$ between spacetime regions and operator algebras on which AQFT is erected, also grounds the mathematical representation of the Whiteheadian notion of compatibility between subjective and objective eternal objects. Note, moreover, that requirement (CS2) (i.e. the condition (CO-ING)) in the expression for the compatibility of two subjective eternal objects just says that $\hat{P}_1$ and $\hat{P}_2$ are *both compatible with at least one common objective eternal object*, i.e. that both $\hat{P}_1$ and $\hat{P}_2$ fulfil (CSO) with respect to the same objective eternal object $\widehat{\mathcal{O}}$. Thus to express the compatibility of subjective eternal objects among each other in the formalism of AQFT, *one has to make use of the expression for the compatibility of objective and subjective eternal objects.* This shows that the correspondence between regions and operator algebras is not only fundamental for the compatibility of objective and subjective eternal objects, but also for the compatibility of subjective ones. In sum, then, the conceptually fundamental correspondence $\mathcal{O} \mapsto \mathcal{R}(\mathcal{O})$ of AQFT, also plays an ontologically fundamental role in connection with many central aspects of Whitehead's theory of eternal objects.

8.3 No simple properties in QFT

In this section we will argue that according to AQFT there are *no simple local projections*, i.e. local projections which are *not* disjunctions of others. In the following section we will then discuss the consequences of this state of affairs for the existence of simple subjective eternal objects. We will see that probably there are no simple subjective eternal objects at all. As already mentioned in Section 2.2.2 this leads to an indeterminateness as to the very form of subjective eternal objects. However, this is not merely a problem for a Whiteheadian interpretation of QFT, but rather for *any interpretation that uses local projections as representatives of properties or property instances.*

The ultimate reason for all this lies in the fact that the local algebras of AQFT are von Neumann algebras of type III, which implies that they do not contain *atomic* projections (see also Section 5.2). That a projection $P \neq \mathbf{0}$ from a von Neumann algebra $\mathcal{R}$ is atomic means that there is no projection $Q \neq \mathbf{0}$ in $\mathcal{R}$ with $P > Q$, i.e. with $P \geq Q$ and $P \neq Q$ ("$\geq$" is the usual partial order among projections on a Hilbert space as introduced in Appendix B.2). Thus atomic projections are the smallest non-zero elements of the lattice of projections in $\mathcal{R}$. That $\mathcal{R}$ does not contain atomic projections therefore, in particular, implies that for each $P \in \mathcal{R}$ there is an infinite sequence of smaller and smaller projections: $P > Q$ with $Q \neq \mathbf{0}$ implies that there is another non-zero projection F with $Q > F$ since otherwise Q were atomic. By iterating this argument one ends up with an infinite sequence of smaller and smaller projections. In general, there will even be many such infinite sequences for a given projection P such that projections belonging to different sequences will not commute with one another.[22]

[22] Of course, contrary to the local algebras $\mathcal{R}(\mathcal{O})$, the *global algebra* $\mathcal{R}(M) = \mathcal{B}(\mathcal{H})$, associated with the whole Minkowski space M (see Appendix C) is not of type III but

In Section 8.1.3 it had been shown that each projection contained in the Boolean algebra $\mathbf{B}(A)$ (or the corresponding distributive lattice $\mathbf{D}(A)$) corresponding to some magnitude A, can be written as a disjunction of eigenprojections of A. Now we will show that the non-existence of atomic local projections implies that every local projection $P \in \mathcal{R}(\mathcal{O})$ can be written as a disjunction of other local projections from $\mathcal{R}(\mathcal{O})$. The connection with the former result is, then, that all eigenprojections of an arbitrary local magnitude $A \in \mathcal{R}(\mathcal{O})$ can also be written as disjunctions of other projections from $\mathcal{R}(\mathcal{O})$.[23]

Let P be an arbitrary non-zero projection from the local algebra $\mathcal{R}(\mathcal{O})$. Since P is not atomic there is another non-zero projection $Q \in \mathcal{R}(\mathcal{O})$ with $P \geq Q$. Now $P \geq Q$ implies $PQ = Q$,[24] because if the subspace $\mathcal{K}_Q$ of $\mathcal{H}$ onto which Q projects is included in the subspace $\mathcal{K}_P$ of $\mathcal{H}$ onto which P projects, the further application of P to a vector $Q\psi \in \mathcal{K}_Q$ does not affect the latter any more, i.e. $PQ\psi = Q\psi$ for all $\psi \in \mathcal{H}$, and thus $PQ = Q$. By the same kind of reasoning it follows moreover that $QP = Q$ holds too,[25] so that $P \geq Q$ implies $PQ = Q = QP$ and thus, in particular, that P and Q commute. Now from $PQ = Q$ and the identity $P = Q + P - Q$ it follows $P = Q + P - PQ$ and thus $P = Q + P(\mathbf{1} - Q)$. Since P and Q as well as P and $\mathbf{1} - Q$ commute, we get $P = Q + Q'$ with $Q' \equiv P(\mathbf{1} - Q)$ where the latter projection commutes with both P and Q. Since Q and Q' are moreover orthogonal (i.e. $QQ' = \mathbf{0}$)

rather of *type I*. This means, in particular, that it contains atomic projections (Haag 1996, p. 117). However, as also explained in Appendix C, local projections are conceptually more fundamental then global ones, because the latter are merely idealizations arising as limits of sequences of the former. More importantly, global projections are by definition, not contained in any algebra $\mathcal{R}(\mathcal{O})$ corresponding to a finite region, and thus can hardly be used for the representation of properties if the latter shall be able to occur in *finitely* extended regions and not merely in infinitely extended ones. Since this latter option seems to be absurd, it seems that any interpretation of QFT that incorporates properties at all, will somehow have to use *local* projections for their representation and thus will be faced with the problems we discuss in what follows.

[23] When we speak in the following of disjunctive or conjunctive components of a projection P we always mean non-trivial ones, i.e. ones that are different from P itself as well as from $\mathbf{0}$ and $\mathbf{1}$.

[24] Both conditions are even equivalent, but the implication from the latter to the former is not needed here.

[25] This can also be seen by taking the adjoint of both sides of the equation $PQ = Q$. Because of $(PQ)^* = Q^* P^*$ and the self adjointness of P and Q one indeed gets $QP = Q$.

we have $P = Q + Q' - QQ'$ and thus in fact $P = Q \vee Q'$. Since P had been arbitrary, in fact *every* projection in $\mathcal{R}(\mathcal{O})$ can be written as the disjunction of other projections in $\mathcal{R}(\mathcal{O})$. In particular, Q and Q' can also be written as disjunctions of other projections from $\mathcal{R}(\mathcal{O})$ and since this can be iterated ad infinitum *one ends up with the result that there is no limit to the disjunctive complexity of any local projection.*

But this means that the form of local projections—and thus of the properties or property instances they are supposed to represent—is indeterminate in the sense that there is no "finest resolution" of a given projection into disjunctive components—for every such resolution $P = \vee_i Q_i$ there are always infinitely many finer ones, i.e. P can also be written in the form $P = \vee_j F_j$ such that each Q_i is itself a disjunction of F_j's and so on. But this is not the only indeterminacy as to the form of a given local projection P. For, in general, P can also be written as a disjunction of merely *two* other projections in many different ways, i.e. one has, in general, $P = Q \vee Q' = F \vee F' = G \vee G' = \cdots$, where none of the pairs of projections need commute with the others. But even if they would all commute with one another there is no way of singling out one of these representations as somehow privileged over the others, thereby eliminating this ambiguity.

In Section 8.1.4 it had been shown that each non-zero projection contained in the Boolean algebra $\mathbf{B}(A)$ (or the corresponding distributive lattice $\mathbf{D}(A)$) associated with some magnitude A, can also be written as a conjunction of elements from a subset of $\mathbf{B}(A)$ (respectively $\mathbf{D}(A)$). The non-existence of atomic local projections leads to an analog of this result on the level of the set of all projections in $\mathcal{R}(\mathcal{O})$ too, that shall be stated here without proof: each non-zero $P \in \mathcal{R}(\mathcal{O})$, different from $\mathbf{1}$, cannot only be written as a disjunction $P = Q \vee Q'$ of other projections from $\mathcal{R}(\mathcal{O})$ but also as a conjunction $P = J \wedge J'$. And again this conjunctive form of P is not unique; but what is even more important is that now *a given P is not even determinate as to whether it is conjunctive or disjunctive since one has* $P = Q \vee Q' = J \wedge J'$. Thus every interpretation of QFT that makes use of local projections in representing properties or property instances will be faced with the fact that they are infinitely complex and that QFT itself seems not to provide us with a fact of the matter as to whether a given one is disjunctive or conjunctive. We will now see which consequences this state of affairs has for our account of property-universals, i.e. of subjective eternal objects.

8.3.1 Consequences for subjective eternal objects

So far we have shown that every non-zero projection P from a local algebra $\mathcal{R}(\mathcal{O})$ can be written as the disjunction $P = Q \vee Q'$, as well as a conjunction $P = J \wedge J'$, of other projections from $\mathcal{R}(\mathcal{O})$ which commute with one another, and that both of these representations are not unique. However, this does not by itself imply that each subjective eternal object is also plagued with these ambiguities. In particular, *it does not by itself imply that there are no simple subjective eternal objects.* This is because for $\hat{P}$ with $P = Q \vee Q'$ to be interpretable as a disjunctive eternal object, $\hat{Q}$ and $\hat{Q}'$ have to be compatible. And this does not merely mean that the corresponding local projections Q and Q' need to commute (i.e. condition (CS1)), but rather that *they must also fulfil condition (CS2)* (namely that there exists a $\mathcal{O} \in D(M)$ such that $Q, Q' \in \mathcal{R}(\mathcal{O})$ and $Q, Q' \notin \mathcal{R}(\mathcal{O}')$ for all $\mathcal{O}' \in D(M)$ with $\mathcal{O}' \subset \mathcal{O}$), for

$$\hat{P} = \widehat{Q \vee Q'} = \hat{Q} \vee \hat{Q}' \tag{8.18}$$

to make sense as a subjective eternal object. Thus unless one shows that condition (CS2) is always satisfied by at least one pair of projections Q and Q' for which $P = Q \vee Q'$ holds, it could be the case that the corresponding subjective eternal object $\hat{P}$ *cannot* be written as a disjunction (8.18) at all *and thus could be held to be simple.* Unfortunately, the present author has not been able to proof or disproof this proposition. Yet, as shown in the last section, there are always infinitely many projections which are smaller than a given one P. Therefore, it seems *highly probable* that one of these infinitely many smaller projections Q, together with the corresponding projection $Q' = P(\mathbf{1} - Q)$, satisfies condition (CS2). If this were indeed the case, *each* subjective eternal object could in fact be written as a disjunction of others and thus there were no simple subjective eternal objects at all. Moreover, if more than one of the infinitely many projections which are smaller than P, say Q and F, would satisfy condition (CS2), $\hat{P}$ could be written as $\hat{P} = \hat{Q} \vee \hat{Q}'$ as well as $\hat{P} = \hat{F} \vee \hat{F}'$ (where Q and Q'need not commute with F and F'). Since this possibility too, seems to be highly probable, we should better ground our further discussions on the "worst case scenario" according to which *there are no simple subjective eternal objects and all subjective eternal object are indeterminate as to their "true" disjunctive form.* Note that this latter problem would not obtain if there were a stock of simple eternal objects which are the ultimate disjunctive components of all others. For in this case the true disjunctive form of a given $\hat{P}$ were simply that in which only these ultimate components would appear.

Now each non-zero projection $P \in \mathcal{R}(\mathcal{O})$ with $P < \mathbf{1}$ cannot only be written as a disjunction of others in infinitely many different ways, but there are also infinitely many pairs of projections J and J' in $\mathcal{R}(\mathcal{O})$ (with $P < J, J'$ $< \mathbf{1}$) such that P can be written as their conjunction $P = J \wedge J'$. By an analogous reasoning as above, this makes it at least highly probable that each subjective eternal object $\hat{P}$ can also be written in many different conjunctive forms

$$\hat{P} = \hat{J} \wedge \hat{J}' = \hat{G} \wedge \hat{G}' = \ldots . \tag{8.19}$$

If, however, each subjective eternal object $\hat{P}$ can indeed be written at the same time as a disjunction *and* as a conjunction of other subjective eternal objects, this would undermine the very ontological distinction between disjunctive and conjunctive subjective eternal objects. And, again, without simple subjective eternal objects which are the ultimate components of all other subjective eternal objects, one cannot write $\hat{P}$ in the unique form in which only these ultimate components appear and hold that this is the true form of $\hat{P}$.

On the other hand, as soon there is a fact of the matter that breaks this symmetry between disjunctions and conjunctions, one of these two possibilities can be regarded as ontologically superfluous. In Section 8.3.3 we will in fact give an argument for only including *disjunctive* subjective eternal objects into our ontology, thereby regarding the possibility of conjunctive representations (8.19) as mere mathematical surplus structure. The argument for taking only disjunctive subjective eternal objects ontologically serious derives from the fact that subjective eternal objects only ingress into occasions as members of definite (maximal) abstractive hierachies. The latter shall be reexamined in the light of the non-existence of simple subjective eternal objects in the following section.

8.3.2 Consequences for abstractive hierachies

The lack of simple subjective eternal objects also affects abstractive hierachies, since each such substructure of the realm of eternal objects is based upon a definite set of compatible *simple* eternal objects (see Section 2.2.5). However, nothing essential about the structure of an abstractive hierachies gets lost if we allow its base G to consist of a definite set of complex eternal objects instead of simple ones. What is important is that there are no eternal objects in the hierachy which are components of members of G—otherwise G could hardly be called the base of this hierachy. Therefore, we have to modify conditions (i) and (ii) in the original definition of abstractive hierachies which say that

(i) the members of the base G belong to the hierachy, and are the only simple eternal objects in the hierachy, and

(ii) the components of any complex eternal object in the hierachy, are also members of the hierachy.

Instead of (i) and (ii) we will require:

(H1) The members of the base G belong to the hierachy, and are the *only* eternal objects in the hierachy whose disjunctive components do *not* belong to it.

The appearance of *disjunctive* instead of conjunctive components in this condition derives from the fact that we naturally understand a property as *more simple* than another if the former is a disjunctive component of the latter but not vice versa (see the example in Section 2.2.2). Thus condition (H1) means that the members of G are *simple relative to the hierachy* based on G, since none of the other eternal objects contained in a hierachy based on G are disjunctive components of members of G.

The third defining condition of an abstractive hierachy—the condition of connexity—requires that

(iii) the elements of any set of eternal objects belonging to the hierachy are jointly among the components or derivative components of at least one eternal object which also belongs to the hierachy.

Since we will not exclude the case that the trivial subjective eternal object $\hat{\mathbf{1}}$ belongs to a hierachy, we have to modify (iii) too, because $\hat{\mathbf{1}}$ is not a disjunctive component of another subjective eternal object (different from itself) as necessary for the satisfaction of condition (iii). However, this problem can easily be remedied by a harmless modification of (iii) that restricts this condition to those subsets of a hierachy *not containing the trivial element* $\hat{\mathbf{1}}$:

(H2) Any set of eternal objects belonging to the hierachy *and not containing* $\hat{\mathbf{1}}$ are jointly among the components or derivative components of at least one eternal object which also belongs to the hierachy.

In the light of condition (H2) condition (H1), moreover, means that *relative to a given hierachy* the elements of its base are the *ultimate components* of all other eternal objects contained in this hierachy.

Now recall that a hierachy is maximal if it satisfies the further condition (that has earlier been denoted by (iv)):

(HM) *All* complex eternal objects whose components are among the members of an abstractive hierachy $H(G)$ are themselves members of $H(G)$.

In case of the original definition of maximal abstractive hierachies we were able to show that each set G of compatible simple eternal objects (a) determines a unique maximal hierachy based upon G, that (b) this maximal hierachy includes all other (non-maximal) hierachies based upon G and that (c) two maximal hierachies are distinct iff their bases are distinct. It is easy to see, and shall therefore not explicitly proven here, that the analogous proposition stating *the fulfillment of (a)-(c) by each set G of compatible eternal objects*, follows from our modified conditions, too. As in the case of the original definition, it therefore also holds for our modified definition that the maximal hierachy based on a set G of compatible eternal objects looks as if it were generated by arbitrary combinations of disjunctions and conjunctions from the basic elements provided by G. Although, it is not literally "generated" in this way at all, because this would require the existence of the manners of relatedness $\wedge$ and $\vee$ *independently from conjunctive and disjunctive eternal objects* (see Section 2.2.5). However, a set (on which $\wedge$ and $\vee$ are defined) that looks as if it were generated from one of its subsets by arbitrary combinations of conjunctions and disjunctions, is a distributive lattice. Thus in restriction to conjunctive and disjunctive complex eternal objects, *maximal abstractive hierachies are distributive lattices.* In particular, the maximal abstractive hierachy $H(G)$ based on G coincides with the distributive lattice $\mathbf{D}(G)$ generated from G. Altogether, it seems that replacing (i) and (ii) by (H1) and (iii) by (H3) leads to a reasonable generalization of the concept of a (maximal) abstractive hierachy as originally conceived by Whitehead, that pays tribute to the fact that probably there are no simple subjective eternal objects simpliciter.

8.3.3 Against conjunctive subjective eternal objects

The introduction of the notion of relative simplicity for subjective eternal objects, as a partial substitute for the non-available notion of simplicity simpliciter, provides a reason for breaking the symmetry between disjunction and conjunction that would obtain otherwise (i.e. in the absence of any notion of simplicity). This is because like the non-available notion of the simplicity of eternal objects simpliciter, the simplicity of an eternal object $\hat{P}$ relative to some hierachy too, is naturally tight to $\hat{P}$'s not having *disjunctive* components (see the last section). In other words, *for the notion of relative simplicity to mean what it is supposed to mean, only disjunctive eternal objects are needed.* And since we already know that only either disjunctive or conjunctive eternal objects *but not both* are needed anyway, it seems natural to discard conjunctive ones. Thereby, the ambiguity between the conjunctive and disjunctive forms in which each subjective eternal object can be written is removed. The conjunctive form in which a subjective eternal object may be representable is therefore regarded as mere mathematical surplus structure without any ontological meaning.

Of course, the ambiguity as to the (infinitely) many different disjunctive forms of each subjective eternal objects still remains. But this ambiguity too, is at least softened by the fact that, relative to a given abstractive hierachy, the form of an subjective eternal object is uniquely determined: let $\hat{P}$ be a member of the maximal hierachy $H(G)$ whose base G consists of the subjective eternal objects $\hat{P}_i$, $i = 1, \ldots, 10$. Then it makes sense to hold that, *relative* *to this hierachy*, each of its members $\hat{P}$ is of the *unique* (disjunctive) form in which only elements of the base $G = \{\hat{P}_i\}$ appear, e.g. $\hat{P} = \hat{P}_1 \vee \hat{P}_6 \vee \hat{P}_9$. Clearly the subjective eternal object $\hat{P}$ will, in general, belong to many different hierachies and moreover the $\hat{P}_i$ are themselves disjunctions of other eternal objects. Because of these two facts there is still no true disjunctive form of $\hat{P}$, *independently* from a given hierachy. But in restriction to the hierachy $H(G)$ the above disjunctive form of $\hat{P}$ is unique, since none of the disjunctive components of the $\hat{P}_i$ belongs to $H(G)$ (see condition (H1)). Thus the indeterminateness of subjective eternal objects as to their "true" form qua pure potentiality is clearly not removed—more correctly, there simply is no such "true" form. This would require the existence of simple eternal objects simpliciter, which were the ultimate components of all other eternal objects—it would require that there is a limit to the disjunctive complexity of qualities—but as we know from Section 8.3.1 this is probably not the case. However, since a subjective eternal object $\hat{P}$ ingresses into an occasion *only qua member of a definite maximal abstractive hierachy* (see Sections 2.2.6 and 2.4.2), at least in respect to $\hat{P}$'s ingression into a given occasion, its form is completely fixed. We will now argue that *this uniqueness relative to each occasion into which a subjective eternal object ingresses*, is sufficient to secure the processual character of concrescence processes.

8.3.4 Simple decisions revisited

In Section 1.1.3 we have defended the very processual character of concrescence processes by introducing the demand that in each phase of a concrescence only one *simple decision* can be felt and that a decision is simple if it eliminates *one simple eternal object* from incorporation into the following phases. Now since subjective eternal objects only ingress into occasions as members of some definite hierachy, it is, in particular, fixed whether a given subjective eternal object is simple relative to this hierachy and thus too *relative to this occasion.* This relative simplicity, however, seems to be sufficient to ground the concept of simple decisions of occasions and thus too, the processual nature of concrescence processes. For a simple decision of a given concrescent occasion could very well be understood as one in which one eternal object belonging to the base G of the ingressed hierachy is eliminated, even if the base does not consist of simple eternal objects simpliciter. The reason for this is that for *this* concrescent occasion *only the eternal objects ingressed into it are available*, so that the disjunctive substructure of the most simple ones of these—the ones belonging to the base G—simply does not matter for this occasion. Thus by demanding that the decisions of concrescent occasions are just of this type, i.e. decisions as to the elimination of one eternal object that is simple *relative to the hierachy ingressed into this occasion*, we can still make sense of simple decisions of concrescent occasions and thus too, of the processual character of concrescences without there being simple subjective eternal objects simpliciter.

8.4 The representation of abstractive hierachies

As mentioned above, *the structure of a maximal abstractive hierachy $H(G)$ of subjective eternal objects is that of a distributive lattice.* However, not every distributive lattice $\mathbf{D}(\{\hat{P}_i\})$ does represent a maximal abstractive hierachy of subjective eternal objects with the set $\{\hat{P}_i\}$ of mutually compatible elements as its base. The interpretation of the distributive lattice $\mathbf{D}(\{\hat{P}_i\})$ as the maximal abstractive hierachy based on $\{\hat{P}_i\}$ furthermore requires the set $\{\hat{P}_i\}$ to be *at most countable.* The problem is that the only reasonable way of defining a disjunction on a set of *uncountable* many projections $\{P_i\}$ (which is obviously necessary for the definition of a disjunction on the corresponding uncountable set of classes $\{\hat{P}_i\}$), namely by means of an integral of the form

$$\bigvee_{i \in J} P_i \equiv \int_J dP_i, \tag{8.20}$$

where J is a subset of the continuous set of all indices i, requires $\{P_i\}$ to lie within the range of a projection-valued measure $J \mapsto P_J$. This, however, implies that *all elements of $\{P_i\}$ are themselves disjunctions (in the sense of (8.20)) of other elements from $\{P_i\}$* (see e.g. Reed and Simon 1980, Chapter VII.3). But this means that $\{P_i\}$, and thus too, the corresponding set of classes $\{\hat{P}_i\}$, cannot be interpreted as the base of an abstractive hierachy, since requirement (H1) on abstractive hierachies is not fulfilled.

Note that the fulfillment of this requirement is moreover necessary for our answer to the challenge of the non-processuality of concrescence processes to go through. For if all disjunctive components of each element of the set $\{\hat{P}_i\}$ are themselves contained in the latter, an occasion to which $\{\hat{P}_i\}$ is available, were not able to settle simple decisions (as defined at the end of the last section), because no element of $\{\hat{P}_i\}$ were simple relative to this occasion. Thus for this reason too, *only countable sets* $\{\hat{P}_i\}$ *are reasonable candidates for representing the bases of abstractive hierachies of subjective eternal objects.*

On the other hand, it is easy to see that in fact *each countable set* $\{\hat{P}_i\}$ *whose elements are mutually compatible* generates a distributive lattice $\mathbf{D}(\{\hat{P}_i\})$, so that the latter can be interpreted as the maximal abstractive hierachy $H(G)$ based upon G, such that this base is represented by $\{\hat{P}_i\}$.

Maximal hierachies of mixed eternal objects

So far we have represented maximal abstractive hierachies solely consisting of subjective eternal objects in the formalism of AQFT. However, the bases of the complete maximal abstractive hierachies $H(O, G_n(O))$ of the succeeding phases $n = 1, 2, \ldots$ in the concrescence process of an occasion with region $\mathcal{O}$ (see Sections 2.2.6), not only contain subjective eternal objects but also the (simple) objective eternal object O (represented by the class $\widehat{\mathcal{O}}$) that has already been unrestrictedly ingressed into the extensive continuum in the dative phase of the forgoing process of transition. Since there are no disjunctive mixed eternal objects (see Section 2.2.4) and no conjunctive subjective ones (see Section 8.3.2), the structure of such a hierachy is such as if it were generated from its base $\{O, G_n(O)\}$ by arbitrary disjunctions of subjective eternal objects from $G_n(O)$ and by conjunctions of the thus "created" subjective eternal objects with the simple objective eternal object O. And since by arbitrary disjunctions of elements from $G_n(O)$ the "subjective part" $H(G_n(O))$ of the hierachy $H(O, G_n(O))$ is generated, *the latter can always be "separated" into its subjective and objective part—it is their conjunction in the sense*

$$H(O, G_n(O)) = H(G_n(O)) \wedge O \equiv \{A \wedge O : A \in H(G_n(O))\} . \tag{8.21}$$

Since moreover conjunctions of disjunctions of subjective eternal objects with the *same* objective one O can be resolved into *disjunctions of mixed conjunctive* *ones*, i.e.

$$(A \vee B) \wedge O = (A \wedge O) \vee (B \wedge O)$$

for all $A, B \in H(G_n(O))$, (8.21) is identical with the distributive lattice generated from the set $G_n(O) \wedge O$, i.e.

$$H(O, G_n(O)) = \mathbf{D}(G_n(O) \wedge O). \tag{8.22}$$

Thus the complete maximal abstractive hierachies of *mixed* eternal objects involved in the concrescence processes of occasions—and not merely their subjective parts—have the structure of distributive lattices.

Now let us denote the set of classes of projections representing the subjective part $G_n(O)$ of the base of $H(O, G_n(O))$ by $\{\hat{P}_i\}^n$. According to (8.22) the hierachy $H(O, G_n(O))$ is then represented by the distributive lattice

$$\mathbf{D}(\{\hat{P}_i\}^n \wedge \widehat{\mathcal{O}})$$

or according to (8.21) equivalently by the set

$$\mathbf{D}(\{\hat{P}_i\}^n) \wedge \widehat{\mathcal{O}} \equiv \left\{\hat{P} \wedge \widehat{\mathcal{O}} : \hat{P} \in \mathbf{D}(\{\hat{P}_i\}^n)\right\}. \tag{8.23}$$

Since each maximal hierachy $H(O, G_n(O))$ of mixed eternal objects can always be split into its subjective and objective part, and since the latter does not fall under the decisions to be settled in the concrescence at all, but is constant throughout it, we can henceforth concentrate on the subjective part $H(G_n(O))$ only. In the following paragraph we will argue that under a reasonable assumption on the subjective parts of the initial hierachies $H(O, G(O))$ of concrescent occasions, their bases $G(O)$ can be used for a reinterpretation of the notion of "local observables".

Reinterpretation of local observables

The assumption to be made on the subjective part $H(G(O))$ of the initial hierachy $H(O, G(O))$ of a concrescent occasion is that it has the structure of a distributive lattice that is *generated by a resolution of the identity* $\hat{\mathbf{1}}$, i.e. by a set of classes $\{\hat{P}_i\}$ whose elements are mutually exclusive ($\hat{P}_i \wedge \hat{P}_j = \hat{\mathbf{0}}$ for $i \neq j$)

and their disjunction $\vee_i \hat{P}_i$ equals $\hat{\mathbf{1}}$.[26] In this case we are able to reinterpret "local observables" in Whiteheadian terms. Since nothing speaks against this structural assumption on the subjective part $H(G(O))$ of the initial hierachy of each concrescent occasion, we will in fact make it. $H(G(O))$ can therefore be represented by a distributive lattice $\mathbf{D}(\{\hat{P}_i\})$ such that $\{\hat{P}_i\}$ is a resolution of $\hat{\mathbf{1}}$. However, for such distributive lattices we have shown in Section 8.1.3 that they coincide, as sets, with the corresponding Boolean algebras generated from the same resolution. Thus we can equally well use Boolean algebras $\mathbf{B}(\{\hat{P}_i\})$ generated by resolutions of $\hat{\mathbf{1}}$ as representatives of the initial hierachies of subjective eternal objects—this merely introduces the ontologically harmless operation $\neg$.

Note, however, that the subjective part $H(G_n(O))$ of the maximal hierachy $H(O, G_n(O))$ of some *later* phase $n \geq 2$ of a concrescence process *cannot be represented by a Boolean algebra*. This is because the reduced base $G_n(O) \subset G(O)$, represented by $\{\hat{P}_i\}^n \subset \{\hat{P}_i\}$, does no longer contain all the elements of the initial base (represented by $\{\hat{P}_i\}$), and thus does not have the structure of a resolution of the identity—in particular the disjunction $\vee_{\{\hat{P}_i\}^n} \hat{P}_i$ of all its elements does not equal $\hat{\mathbf{1}}$. This has the consequence that the Boolean algebra generated from the reduced set $\{\hat{P}_i\}^n$ will contain elements—in particular the negation of all elements from $\{\hat{P}_i\}^n$—not contained in the distributive lattice generated from $\{\hat{P}_i\}^n$. Therefore, $H(G_n(O))$ is to be represented by the distributive lattice $\mathbf{D}(\{\hat{P}_i\}^n)$ generated from the set $\{\hat{P}_i\}^n$ rather than by the Boolean algebra $\mathbf{B}(\{\hat{P}_i\}^n)$ since the latter is strictly larger than $\mathbf{D}(\{\hat{P}_i\}^n)$ and thus its use would not be compatible with Whitehead's dictum against negative eternal objects (see Sections 2.2.2 and 8.1.3).

Now each self-adjoint operator $A \in \mathcal{R}(\mathcal{O})$ represents some local observable and *is in one-to-one correspondence with a resolution* $\{P_i\} \subset \mathcal{R}(\mathcal{O})$ *of the identity* $\mathbf{1}$. Thus if the resolution $\{\hat{P}_i\}$ of the identity class $\hat{\mathbf{1}}$, build from $\{P_i\}$, represents the base $G(O)$ of a hierachy of subjective eternal objects, we have

[26] Note that each countable set $\{\hat{P}_i\}$ with mutually compatible elements is itself included in a distributive lattice based on such a resolution of $\hat{\mathbf{1}}$: take, for example the most simple case of two compatible elements $\hat{P}_1$ and $\hat{P}_2$, such that $\hat{P}_1 \wedge \hat{P}_2 \neq \hat{\mathbf{0}}$ and $\hat{P}_1 \vee \hat{P}_2 \neq \hat{\mathbf{1}}$. Then $\hat{Q}_1 \equiv \hat{P}_1 \wedge \hat{P}_2$, $\hat{Q}_2 \equiv \hat{P}_1 \wedge (\hat{\mathbf{1}} - \hat{P}_2)$, $\hat{Q}_3 \equiv \hat{P}_2 \wedge (\hat{\mathbf{1}} - \hat{P}_1)$ and $\hat{Q}_4 \equiv \hat{\mathbf{1}} - (\hat{P}_1 \vee \hat{P}_2)$ are easily seen to form a resolution of $\hat{\mathbf{1}}$ from which $\hat{P}_1$ and $\hat{P}_2$ can be generated (as disjunctions of the $\hat{Q}_i$).

reinterpreted the local observable in Whiteheadian terms, namely as the set of possible ingressions of the subjective eternal objects belonging to the base $G(O)$. However, this reinterpretation is *not possible for every local observable*: first of all, we have only taken into account discrete local observables, i.e. those whose corresponding Boolean algebra of spectral projections is at most countable and thus is generated by a resolution of the identity. The reason for this is that we have already argued above that only such distributive lattices or Boolean algebras respectively, can be used to represent abstractive hierachies of subjective eternal objects. Besides this general restriction of the reinterpretability of local observables to discrete ones, there is a second restriction. For it can be the case that the resolution $\{\hat{P}_i\}$ of the identity class $\hat{\mathbf{1}}$, build from $\{P_i\}$, does not represent the base of some hierachy at all. This is the case if some of the subjective eternal objects in the set $\{\hat{P}_i\}$ are *not compatible with one another*. Recall from Section 8.2 that two subjective eternal objects $\hat{P}_1$ and $\hat{P}_2$ are compatible iff there are projections $P_1' \in \hat{P}_1$, $P_2' \in \hat{P}_2$ such that P_1' and P_2' commute (condition (CS1)) and there is at least one region $\mathcal{O} \in D(M)$ such that both $\hat{P}_1$ and $\hat{P}_2$ can ingress into it, which means formally that $P_1', P_2' \in \mathcal{R}(\mathcal{O})$ and $P_1', P_2' \notin \mathcal{R}(\mathcal{O}')$ for all $\mathcal{O}' \in D(M)$ with $\mathcal{O}' \subset \mathcal{O}$ holds (condition (CS2)). As also argued in Section 8.2, *the compatibility of subjective eternal objects is necessary for the very existence of disjunctive eternal objects having the former as components.* Now that condition (CS1) is fulfilled by the eternal objects $\{\hat{P}_i\}$ is clear from the fact that the corresponding set of projections $\{P_i\}$ is a resolution of the identity and thus particularly a set of mutually commuting projections. But condition (CS2) need not be fulfilled for all pairs of subjective eternal objects in $\{\hat{P}_i\}$. And in case not all subjective eternal objects in the set $\{\hat{P}_i\}$ are mutually compatible with one another, they simply do not constitute the base $G(O)$ of a hierachy of subjective eternal objects, whereas to the resolution $\{P_i\}$ of the identity $\mathbf{1}$ corresponds a unique local observable A, which therefore is *not* reinterpretable. Since it is not to be expected that all commuting projections P_1, P_2 in $\mathcal{R}(\mathcal{O})$ fulfil the second requirement (CS2) for the compatibility of the corresponding subjective eternal objects $\hat{P}_1$ and $\hat{P}_2$, *not all local observables will therefore be ontologically relevant in our interpretation.* Rather only those local observables—or better: only those local self-adjoint operators—can be taken ontologically serious, which arise from a resolution $\{P_i\}$ that gives rise to a set $\{\hat{P}_i\}$ of *compatible* subjective eternal objects.

However, such a restriction is not in itself a drawback for an interpretation of QT. Rather the usual interpretational assumption that *each* self-adjoint operator does in fact represent some physically meaningful magnitude or observable is only made for mathematical convenience and because one simply does not have any clear-cut criterion for singling out physically meaningful operators from those which are not. Whether the restriction obtaining in our interpretation can, however, be seen as (part of) such a criterion shall not be discussed here. It should have merely been pointed out that the existence of such a restriction on the ontological relevance of the whole set of self-adjoint operators, does not in itself speak against an interpretation of QT.

Chapter 9

The representation of the underlying activity and its manifestations

9.1 The underlying activity itself

As explained in detail in Section 2.3 and Section 2.4, the underlying activity is the pure potential for all the activities involved in the other- as well as the self-creative phases in the constitution of occasions. In itself this underlying activity is formless, which means that it has no determinate creative character and therefore cannot create any particular occasions. Put differently, the underlying activity in itself cannot make any decisions for or against different spatiotemporal or qualitative possibilities—it is the potential for the becoming of occasions in general. The conditioning of the underlying activity—the "ground state of activity"—needed for the production of one of its manifestation—an "excitation" of this ground state—having a determinate creative character, is due to the limitations laid upon it by its envisagement of the already actualized occasions at the respective stage of the (infinitely old) world-process.

We have interpreted the creative character with which each manifestation of the underlying activity is equipped due to its envisagement of occasions in terms of propensities, i.e. objective single case probabilities. This means, in particular, that the manifestation of the underlying activity arising from the latter's envisaging of actuality W_{s-1} at stage s of the world-process, i.e. the activity ω_{W_s}, has to be represented in such a way that probabilities can be assigned to subjective eternal objects $\hat{P}$. Therefore, the natural way of representing ω_{W_s} is by means of some state ρ. In this case one can (under certain further constraints) interpret the value $\rho(P')$ of $P' \in \hat{P}$ as the propensity for the ingression of $\hat{P}$ into the corresponding region of the extensive continuum. Moreover, all the conditioned activities

$$\ldots, \omega_{W_{s-1}}, \omega_{W_s}, \omega_{W_{s+1}}, \ldots$$

each corresponding to some stage of the world-process, ultimately arise from the conditioning of the one underlying activity ω by the aggregates of actualized occasions

$$\cdots \subset W_{s-1} \subset W_s \subset W_{s+1} \subset \cdots .$$

Therefore, the states which represent these activities should also arise by way of a conditioning from one and the same state ρ_0. Next we will argue that the best choice for this state ρ_0 is the vacuum state of QFT, that has been denoted by ω (see Section 5.3).

First, the complete qualitative formlessness of some state ρ, i.e. its complete indifference with respect to subjective eternal objects, would mean that the value $\rho(P)$ is the same for all local projections. Yet each state is a probability measure over each Boolean algebra of projections which, in particular, means that the values $\rho(P_i)$ sum up to 1 for each resolution of the identity $\{P_i\}$. But in case $\{P_i\}$ is not finite, the latter is clearly not possible if ρ assigns the same value to each element of $\{P_i\}$. Thus there is no state that is *completely formless* in regard to all qualitative possibilities, i.e. all subjective eternal objects. Yet the vacuum state ω is among those states *which come closest to this desired qualitative formlessness.* One can show that the vacuum state assigns to each local projection a value strictly between 0 and 1, i.e. $0 < \omega(P) < 1$ for all $P \in \mathcal{P}_{loc}$ (see e.g. Redhead 1995). Thus by reason of the "only-if" part of the eigenvalue-eigenstate rule that says that P is definite with respect to ω only if $\omega(P) = 0$ or 1, and that has to be accepted if $\omega(P)$ shall be interpretable as an ontic probability (see Section 8.1.6), and thus in particular as a propensity, *every subjective eternal object is indefinite with respect to* ω. Therefore, the vacuum state is indifferent in respect to subjective eternal objects at least in the sense that all of them are alike indefinite. However, ω is *not the only state* in respect to which all $P \in \mathcal{P}_{loc}$ are indefinite—all so-called *states of bounded energy* are of this type (see Haag 1996, Theorem 5.3.2).[1] Nevertheless, the vacuum state is among the states which seem to be the best available choices for making sense of the formlessness Whitehead attributes to the underlying activity in respect to subjective eternal objects.

What distinguishes the vacuum state ω from all other states which are maximally qualitatively formless is the fact that it is the *unique* state which is maximally—though again not completely—*formless with respect to spatiotemporal regions*. Because of its Poincaré invariance the vacuum state is indifferent with respect to all regions connected by Poincaré transformations. However, since the Poincaré invariance of ω does not say anything about regions not connected by such transformations, the vacuum state cannot be said to be *completely* formless in this spatiotemporal sense. Yet it is the *only* Poincaré invariant state on $\mathcal{B}(\mathcal{H})$, so that each different state is spatiotemporally *less indifferent* and therefore the vacuum state is the best choice in this respect. Moreover, the Poincaré invariance of the vacuum state ω makes it the *unique energetic ground state* of the world (see Appendix C), which is another reason for representing the underlying activity—the "activity ground state" of the world—by the vacuum state.

A further argument for this interpretive claim is the fact that each pure state ρ on $\mathcal{B}(\mathcal{H})$ can be approximated as closely as one likes by the conditioning of some (fixed) state of bounded energy by an appropriate local operator $A \in \mathcal{R}(\mathcal{O})$, where $\mathcal{O}$ is an arbitrary bounded region (Reeh and Schlieder 1961). This result, known as the Reeh-Schlieder theorem, in particular implies that any given pure state can be approximated to any degree of accuracy by a conditioning of the *vacuum state* due to an appropriate local operator $A \in \mathcal{R}(\mathcal{O})$. Disregarding the arbitrarily

[1] A state of bounded energy is one for which the expected value of the energy-momentum operator is finite.

small error, one can thus interpret each pure state as being just an *excitation of the vacuum state* produced by the latter's conditioning

$$\omega \to \frac{\omega(A^* \cdot A)}{\omega(A^* A)}$$

by some appropriate $A \in \mathcal{A}_{loc}$ (see e.g. Redhead 1995 and Clifton and Halvorson 2001). Since each limited activity involved in the world-process can likewise be understood as an excitation of the underlying activity (see Section 2.3.4), this is another point in favour for the representation of the underlying activity by the vacuum state. In sum, then, it seems that the vacuum state is indeed the best available choice for representing Whitehead's underlying activity.

The conditioning of the underlying activity due to its envisaging a completed occasion E whose spacetime region is $\mathcal{O}$ and whose definiteness is given by the subjective eternal object $\hat{P}$ can thus be represented by the conditioning of the vacuum state ω by the projection that represents the ingression of $\hat{P}$ in region $\mathcal{O}$. For notational simplicity we assume that it is the projection P that is used to denote $\hat{P}$ that also represents $\hat{P}$'s possible ingression in region $\mathcal{O}$. Then the conditioning of the underlying activity due to its envisaging occasion E is represented by the state-collapse[2]

$$\omega \to \omega_P \equiv \frac{\langle P\Omega, \cdot P\Omega\rangle}{\langle P\Omega, P\Omega\rangle} = \frac{\omega(P \cdot P)}{\omega(P)}. \tag{9.1}$$

Recall that the act of envisagement of a completed occasion by the underlying activity is at the same time the formers actualization (see Section 2.3.3). Therefore, state-collapses (9.1) are the representatives for acts of actualization, too.

The envisaging and thus the actualization of yet another completed occasion $E' = (\mathcal{O}', \hat{P}')$ thus leads, via the collapse $\omega \to \omega_{P'P}$, to the state

$$\omega_{P'P} = \frac{\omega(PP' \cdot P'P)}{\omega(PP'P)} \tag{9.2}$$

representing the activity arising from the joint envisagement of the occasions E and E' by the underlying activity. Note that because of the self-adjointness

[2]The denominator $\langle P\Omega, P\Omega\rangle = \langle \Omega, P\Omega\rangle = \omega(P)$ merely accounts for the normalization (i.e. $\omega_P(\mathbf{1}) = 1$) of the collapsed state.

and idempotence of projections (i.e. $P = P^* = P^2$) one can (by noting that $PP' = P^*P'^* = (P'P)^*$) also write (9.2) in the form

$$\omega_{P'P} = \frac{\omega((P'P)^* \cdot P'P))}{\omega((P'P)^*(P'P))} \tag{9.3}$$

which will turn out to be convenient below, when we will generalize (9.3) to infinitely many occasion.

9.2 The manifestation of the underlying activity at some stage of the world-process

We will now show how one can represent the aggregate of all the actualized occasions and the thereby produced manifestation of the underlying activity corresponding to some stage of the world-process. Since the world-process is infinitely old, actuality at each of its stages consists of infinitely many occasions. Assume that actuality W_{s-1} at stage s of the world-process consists of the occasions $E_{i(t)} = (\mathcal{O}_{i(t)}, \hat{P}_{i(t)})$, where the argument t of the index indicates the stage to which the occasion belongs (i.e. $-\infty \leq t \leq s-1$) and the index $i(t)$ itself numbers the occasions belonging to stage t (i.e., in general, $i(t) \in \mathbb{N}$). The aggregate W_{s-1} of all the actualized occasions at stage s is then represented by the sequence

$$W_{s-1} = \left(\left\{E_{i(t)}\right\}\right)_{-\infty \leq t \leq s-1}. \tag{9.4}$$

That all occasions belonging to the same stage of the world-process are co-envisaged by the underlying activity, i.e. have been and will ever be envisaged together, so that no order of envisagement is defined among them, is displayed by representing each single stage t by a *set* $\{E_{i(t)}\}$ *and not by a sequence* of occasions.

Now by generalizing (9.3), the conditioned activity $\omega_{W_{s-1}}$ at stage s of the world-process is therefore to be represented by the state arising from the vacuum state ω due to the latter's conditioning by the infinite product[3]

$$W(s-1) \equiv \prod_{-\infty \leq t \leq s-1} \left(\prod_{i(t)} P_{i(t)} \right) = \left(\prod_{i(s-1)} P_{i(s-1)} \right) \left(\prod_{i(s-2)} P_{i(s-2)} \right) \cdots \quad (9.5)$$

of the corresponding projections, where $P_{i(t)}$ represents the ingression of the subjective eternal object $\hat{P}_{i(t)}$ into the region $\mathcal{O}_{i(t)}$.[4] However, because of the spacelike commutativity the order of two projections $P_{i(t')}$ and $P_{i(t)}$ in this product is *only* important if one region, $\mathcal{O}_{i(t')}$ say, belongs to the backward lightcone of the other $\mathcal{O}_{i(t)}$,[5] in which case $P_{i(t')}$ has to appear to the right of $P_{i(t)}$ in the product (9.5). Thus even if $t' < t$ but $\mathcal{O}_{i(t')}$ and $\mathcal{O}_{i(t)}$ are spacelike separated, $P_{i(t')}$ and $P_{i(t)}$ will commute and one can easily show that (9.5) is in fact independent of their order. Consequently, (9.5) does not incorporate the full structure inherent in (9.4). In other words, the spacelike commutativity axiom of QFT does not fit quite well with the idea of the world as consisting of layers, for it does not distinguish between spacelike separated occasions *belonging to different layers*. However, this does not introduce a new tension into our Whiteheadian interpretation of QFT. This is because the spacelike commutativity axiom is motivated by the relativistic metric, and as already argued earlier, the relativistic metric neither distinguishes between spacelike separated regions. Therefore, the root of the loss of structure in the transition from the sequence (9.4) to the operator (9.5) relies on the use of the relativistic metric in QFT. Therefore, it does not introduce a new problem into our interpretation, over and above the ones stemming from the conflict with STR (see Section 2.8).

That the operator $W(s-1)$ in (9.5) does not reveal the full structure of (9.4), implies that this is likewise the case for the corresponding manifestation ω_{W_s} of the underlying activity, which (by generalizing (9.3)) is to be represented

[3]Note that $W(s-1)$ will not itself be a self-adjoint operator (i.e. $W(s-1)^* \neq W(s-1)$) or even a projection operator, since it will, in general, contain non-commuting projections.

[4]For the sake of notational simplicity, we again assume that the projection $P_{i(t)}$ used to denote the class $\hat{P}_{i(t)}$, is already the one that represents the possible ingression of $\hat{P}_{i(t)}$ into region $\mathcal{O}_{i(t)}$.

[5]Note that the case of occasions overlapping the backward lightcone of other occasions without being included in it, is excluded by our definiton of the spatiotemporal past together with the corresponding assumptions made in Section 2.7.2.

by the state

$$\omega_{s-1} \equiv \frac{\omega(W(s-1)^* \cdot W(s-1))}{\omega(W(s-1)^* W(s-1))}. \tag{9.6}$$

Thus according to (9.6) only the occasions' final qualitative determinateness—*its final definiteness*—represented by the corresponding projections *and their relatedness as spacelike or timelike separated from one another, matters for* ω_{s-1}. Since the activity $\omega_{W_{s-1}}$ is the "vehicle" by which the causal influences from those occasions from W_{s-1} which are relevant for $\omega_{W_{s-1}}$, i.e. which are not causally ineffective, are "transmitted" to their effects, *only these aspects of actualized occasions are therefore causally relevant.*

However, this does not mean that the causally irrelevant aspects of occasions, i.e. their spatiotemporal determinateness as far as it goes *beyond* their relatedness as timelike or spacelike separated, get lost when the occasions get actualized. For we have argued in Section 2.3.4 that one can consistently hold that completed occasion are faithfully retained within the extensive continuum rather than in some activity. Thus QFT seems to support the view, also proposed by Nobo in WM, that Whitehead's doctrine that every completed occasion "remain[s] with the creativity" (RM, p. 92) has to be modified.

As already mentioned in Section 2.3.4 for Nobo this is the starting point for the more far reaching interpretational claim according to which the underlying activity and the extensive continuum are not two different entities but rather merely two aspects of one and the same ultimate reality, termed the *extenso-creative matrix.* However, we will not enter into the discussion whether this further claim can also be supported by the formalism of AQFT. Only one point shall be mentioned here, namely that the defining characteristics of the vacuum state ω (the representative of the underlying activity) *is its Poincaré invariance*, so that by letting the extensive continuum be equipped with the relativistic metric, there is indeed a *very intimate connection* between the extensive continuum (represented by Minkowski space) and the underlying activity, that can count as supporting Nobo's claim.

9.3 Reinterpreting probability statements of the physical interpretation

Together with the representation of abstractive hierachies from Section 8.3.2, we are now in a position to reinterpret the probability statements provided by the physical interpretation of AQFT (see Section 8.1.5). However, such a reinterpretation is not possible in every case. First, we have argued in Section 8.3.2 that not every local observable is ontological meaningful within our interpretation. Consequently, we can only reinterpret those probability statements involving meaningful ones, i.e. local observables $A \in \mathcal{R}(\mathcal{O})$ whose corresponding resolution of the identity $\{P_i\}$ is such that the corresponding subjective eternal objects $\{\hat{P}_i\}$ can all ingress into region $\mathcal{O}$ (i.e. fulfil condition (CS2) from Section 8.2 with respect to $\mathcal{O}$) and thus are mutually compatible as well as compatible with the objective eternal object $\widehat{\mathcal{O}}$.

Above we have moreover seen that not all states either are ontologically meaningful within our interpretation. Only those states which arise form the vacuum state by a conditioning of the from (9.6) have an ontological meaning, namely as manifestations of the underlying activity at some stage of the world-process. Thus there is another restriction on the reinterpretability of probability statements arising from the restriction to a certain subset of states. However, if the resolution $\{P_i\}$ of the identity $\mathbf{1}$ corresponding to the local observable $A \in \mathcal{R}(\mathcal{O})$ is ontologically meaningful, so that the Boolean algebra $\mathbf{B}(\{\hat{P}_i\})$ represents the subjective part $H(G(O))$ of the initial hierachy ingressed into some region of some stage s of the world-process, *and* the state ρ represents the initial activity $\omega^c_{W_{s-1}}$ for the corresponding concrescence process at the same stage, the probability for finding the value of A in the set D upon measurement of A on a system in state ρ, i.e.

$$prob_\rho(A \in D) = \rho(P_D), \tag{9.7}$$

can be reinterpreted as *the propensity provided by the creative character of the activity $\omega^c_{W_{s-1}}$ (represented by ρ) for the unrestricted ingression of the subjective eternal object represented by $\hat{P}_D \in \mathbf{B}(\{\hat{P}_i\})$ into the region $\mathcal{O}$, given that the region and the hierachy $H(G(O))$ (represented by $\mathbf{B}(\{\hat{P}_i\})$) have been realized in the forgoing transition process.* The reinterpretation of joint probabilities proceeds analogously. However, a joint probability such as

$$prob_\rho(A \in D \wedge B \in E) = \rho(P_D^A P_E^B) \tag{9.8}$$

is only ontologically meaningful if the two occasions/regions referred to belong to the *same* layer of the world-process, for otherwise the same activity, represented by the state ρ, could not provide propensities for both. If this is the case, (9.8) can be reinterpreted as *the propensity provided by the creative character of the activity $\omega^c_{W_{s-1}}$ (represented by ρ) for the unrestricted ingression of the subjective eternal object represented by $\hat{P}_D^A \in \mathbf{B}(\{\hat{P}_i^A\})$ into the region $\mathcal{O}$ and for the unrestricted ingression of the subjective eternal object represented by $\hat{P}_E^B \in \mathbf{B}(\{\hat{P}_i^B\})$ into the region $\mathcal{O}'$, given that these regions and the hierachies $H(G(O))$ and $H(G(O'))$ (represented by $\mathbf{B}(\{\hat{P}_i^A\})$ and $\mathbf{B}(\{\hat{P}_i^B\})$) have been realized in the forgoing collective transition process.*

Thus we have reinterpreted certain probability statements of the physical interpretation of QFT by means of the propensities provided by the creative characters of the activities at the *first* phase of concrescence processes. What about the later phases of concrescence? Can the creative character of the activities corresponding to these later phases also be interpreted as providing propensities for the unrestricted ingression of the subjective eternal objects not eliminated up to that phase? The particular states that will be used to represent the activities $\omega^{c,n}_{W_{s-1}}$ $(n \geq 1)$ for the different phases of a concrescence process need not be discussed here. This topic can be postponed to Chapter 11, where the representation of concrescence processes will be investigated systematically. For even without this information, it seems already clear at this point that *strictly speaking* probability statements do not make sense for concrescence phases *other than the initial one.* This is because the corresponding hierachies of subjective eternal objects $H(G_n(O))$ $(n \geq 2)$ are *not* represented by Boolean algebras or by distributive lattices coinciding as sets with Boolean algebras (see Section 8.4), so that the states representing the corresponding activities will not be probability measures on them (see Section 8.1.5). However, this would mean that the creative character of the corresponding activities cannot be interpreted as providing propensities for the unrestricted ingression of the elements of $H(G_n(O))$, at least if propensities are understood as a special kind of *probabilities*.[6]

Yet there are two reasons why we should think of this topic in a more liberal way, allowing to speak of the creative characters of these later activities as likewise providing propensities. First of all, the non-Boolean structure of the hierachies $H(G_n(O))$ for $n \geq 2$ has a quite *harmless origin*. It simply stems from the fact that the base $G_n(O))$ contains less elements than the base $G(O)$ of the initial hierachy and not from a severe structural deficit like the non-definability of disjunctions of certain elements of $G_n(O)$)—$H(G_n(O))$ has still the structure of a distributive lattice and thus is embeddable into a Boolean algebra, particularly into the initial hierachy $H(G(O))$ that is Boolean. Therefore, the only requirement on standard probabilities not satisfied by the values a state ρ ascribes to the elements of the distributive lattice $\mathbf{D}(\{P_i\}^n)$, that is assumed to represent the possible ingressions of elements of the hierachy $H(G_n(O))$ into region $\mathcal{O}$, is that $\rho(\vee_i P_i) < 1$ instead of $= 1$, whereas the other two requirements on standard probabilities are still satisfied (see Section 8.1.5). Since moreover the creative character of the activity at the beginning of the concrescence is the *same* as the creative character of all later phases, each element of $H(G_n(O))$ has the same "likeliness" for its final unrestricted ingression than it had qua member of the initial hierachy $H(G(O))$. And in case of the latter this same "likeliness" could been interpreted as a propensity. Therefore, it seems reasonable still to speak of the creative characters of the activities of later phases of concrescence as providing propensities for the unrestricted ingression of the elements of the corresponding hierachies of subjective eternal objects—despite the fact that *strictly speaking* such an interpretation is not possible if propensities are supposed to be a special kind of standard probabilities.

However, it is clear that this more liberal understanding of propensities does not enlarge the class of probability statements of the physical interpretation which can be reinterpreted in the framework of Whitehead's ontology. This is because in the physical interpretation, probability statements are bound to Boolean algebras of projections corresponding to observables, so that they can only be reinterpreted in terms of the propensities provided for the subjective eternal objects of the *initial* hierachies of concrescent occasions.

[6]For a different understanding of propensities see e.g. (Salmon 1984).

Chapter 10

The representation of transition processes

10.1 The dative phase I

Transition processes are the non-spatiotemporal, mechanisms by which the causal efficiency of occasions is transmitted to the future. A transition process takes rise from the limited manifestation $\omega_{W_{s-1}}$ of the underlying activity ω produced by the latter's envisagement of (and thus conditioning by) actuality W_{s-1} at some stage s. The activity for the transition process leading to stage s of the world-process is therefore $\omega_{W_{s-1}}$. In the first phase, the dative phase, of this transition process a new spatiotemporal layer of actuality is created, i.e. a new group of mutually spacelike separated, bounded spacetime regions $\mathcal{O}_{i(s)}$, $i(s) = 1, 2, \dots, N$.[1] These regions are determined by W_{s-1} via the creative character impressed on $\omega_{W_{s-1}}$ and each of the new regions $\mathcal{O}_{i(s)}$ has to belong to the spatiotemporal future of W_{s-1}. Therefore, we have to find a mechanism within the formalism of AQFT that can account for the determination of such a group of regions on the basis of a given state of the form ω_{s-1} (see equation (9.6)).

Now there is no a priori connection between states and (subsets of) Minkowski space M, that could be used to single out subsets of M from a given state ρ. This is because states are defined as certain maps from $\mathcal{B}(\mathcal{H})$ into the complex

[1] N may be finite or infinite.

numbers and thus the only way of singling out subsets of M on the basis of a given state ρ is *via its restriction* $\rho|_{\mathcal{R}(\mathcal{O})}$ *to the subalgebras* $\mathcal{R}(\mathcal{O}) \subseteq \mathcal{B}(\mathcal{H})$ *associated with regions* $\mathcal{O} \subseteq M$. But singling out a region $\mathcal{O}$ via the restriction of a state ρ to the algebra $\mathcal{R}(\mathcal{O})$ associated to this region, is clearly only possible if fixing the algebra $\mathcal{R}(\mathcal{O})$ also fixes a *unique* $\mathcal{O} \subset M$. In other words, it presupposes that $\mathcal{O}_1 \neq \mathcal{O}_2$ implies $\mathcal{R}(\mathcal{O}_1) \neq \mathcal{R}(\mathcal{O}_2)$, i.e. that the correspondence $\mathcal{O} \mapsto \mathcal{R}(\mathcal{O})$ is one-to-one. We are only interested in bounded regions and have moreover restricted the domain of the correspondence $\mathcal{O} \mapsto \mathcal{R}(\mathcal{O})$ to the set of double cones $D(M)$. We will now show that in this case the correspondence $\mathcal{O} \mapsto \mathcal{R}(\mathcal{O})$ is in fact one-to-one, so that fixing the local algebra $\mathcal{R}(\mathcal{O})$ also fixes a unique $\mathcal{O} \in D(M)$. We therefore have to show that from $\mathcal{O}_1 \neq \mathcal{O}_2$ $(\mathcal{O}_1, \mathcal{O}_2 \in D(M))$ it follows $\mathcal{R}(\mathcal{O}_1) \neq \mathcal{R}(\mathcal{O}_2)$. Now $\mathcal{O}_1 \neq \mathcal{O}_2$ implies that there exists a $\mathcal{O} \in D(M)$ such that either $\mathcal{O} \subset \mathcal{O}_1$ and the closures of $\mathcal{O}$ and $\mathcal{O}_2$ are disjoint, i.e. $\overline{\mathcal{O}} \cap \overline{\mathcal{O}}_2 = \varnothing$, or $\mathcal{O} \subset \mathcal{O}_2$ and $\overline{\mathcal{O}} \cap \overline{\mathcal{O}}_1 = \varnothing$. Let us assume that the first case obtains; the second one can obviously treated in complete analogy. As already mentioned in Section 8.1.7, $\overline{\mathcal{O}} \cap \overline{\mathcal{O}}_2 = \varnothing$ implies $\mathcal{R}(\mathcal{O}) \cap \mathcal{R}(\mathcal{O}_2) = \mathbb{C}\mathbf{1}$. But then $\mathcal{R}(\mathcal{O}_1) \neq \mathcal{R}(\mathcal{O}_2)$ must hold too, because otherwise one would have $\mathcal{R}(\mathcal{O}) \subset \mathcal{R}(\mathcal{O}_1) = \mathcal{R}(\mathcal{O}_2)$, so that with $\mathcal{R}(\mathcal{O}) \cap \mathcal{R}(\mathcal{O}_2) = \mathbb{C}\mathbf{1}$ it would follow that $\mathcal{R}(\mathcal{O}) = \mathbb{C}\mathbf{1}$. The latter can, however, not be the case for any $\mathcal{O} \in D(M)$ (see e.g. Horuzhy 1990, Lemma 1.3.10).

Now since we have represented subjective eternal objects by certain classes of local *projections*, only the projections contained in the local algebra of region $\mathcal{O}$ should be relevant for the question whether the latter will be realized in some stage of the world-process. That this is indeed the case stems from the fact that the set of all projections in a von Neumann algebra already generates the latter (Kadison and Ringrose 1983, p. 326), so that the set of projections in $\mathcal{R}(\mathcal{O})$, term it $\mathcal{P}(\mathcal{O})$, is already sufficient for the unique determination of the region $\mathcal{O} \in D(M)$.

Now one may think that the set $\mathcal{P}(\mathcal{O})$ of all local projections associated with region $\mathcal{O} \in D(M)$ still contains elements which will not be relevant for question whether region $\mathcal{O}$ will be realized in some stage of the world-process. Put conversely, one may think that only those projections in $\mathcal{P}(\mathcal{O})$ will be relevant for this question, which represent possible ingressions of subjective eternal objects in region $\mathcal{O}$. That, in general, not all projections in $\mathcal{P}(\mathcal{O})$ do represent such possible ingressions into region $\mathcal{O}$, stems from the Whiteheadian demand that subjective eternal objects which are ingressed into a region do indifferently belong to the *whole* region and not merely to one of its subregions (see Section 2.2.1). We have argued in Section 8.2 that therefore a local projection can only represent a possible ingression of a subjective eternal object in region $\mathcal{O}$ if, besides belonging to $\mathcal{R}(\mathcal{O})$, it does *not* belong to the algebra of any subregion $\mathcal{O}' \subset \mathcal{O}$. In other words, not all classes $\hat{P}$ where P (or another element $P' \in \hat{P}$) belongs to $\mathcal{P}(\mathcal{O})$ represent subjective eternal objects which can ingress into region $\mathcal{O}$, but only those classes which contain a projection from $\mathcal{P}(\mathcal{O})$ that does *not also belong to any subregion of* $\mathcal{O}$. This expression of the ingressibility of subjective eternal objects into regions had been called (ING) in Section 8.2. Let us denote the subset of local projections of region $\mathcal{O}$ which give rise to classes representing subjective eternal objects which can ingress into this region by $\mathcal{P}_{ING}(\mathcal{O})$.

Yet we cannot simply use the restriction of a state to the sets $\mathcal{P}_{ING}(\mathcal{O})$ for singling out the regions to be realized in the dative phase of a transition process. This is because it is to be expected that the set $\mathcal{P}_{ING}(\mathcal{O})$ will generally be too small to uniquely determine the set of all projections associated with region $\mathcal{O} \in D(M)$ and thus will not be sufficient to uniquely determine the region $\mathcal{O}$. Therefore, in singling out the regions to be realized in the dative phase of the transition process at stage s of the world-process, we have to use the restrictions of the state ω_{s-1} (representing the relevant activity $\omega_{W_{s-1}}$) to the set of *all* projections in the corresponding algebra $\mathcal{R}(\mathcal{O})$. Thus what QFT tells us is that for the determination of a region in the dative phase of a transition process it is *not* sufficient to know all the subjective eternal objects which can ingress into this region; *rather one has to know likewise which subjective eternal objects can ingress into its subregions.*

Therefore, the state ω_{s-1} and its restrictions $\omega_{s-1}|_{\mathcal{P}(\mathcal{O})}$ to the sets of all projections associated with double cones $\mathcal{O} \in D(M)$ will be the basic ingredients in singling out a unique set $\{\mathcal{O}_{i(s)}\}$ of mutually spacelike separated double cones in the future of W_{s-1}. However, contrary to the state ω_{s-1} (on $\mathcal{B}(\mathcal{H})$), its restrictions (to subsets of $\mathcal{B}(\mathcal{H})$) are not yet supplied with any ontological meaning. As shall be argued next, some such restrictions can be interpreted as *spatiotemporal perspectives* onto the undivided activity $\omega_{W_{s-1}}$ represented by the state ω_{s-1} and others as the activity arising from a *reduction of the amount* of the activity $\omega_{W_{s-1}}$.

The doctrine of perspectiveness and the reduction of the amount of an activity

Whitehead's original theory incorporates a doctrine that has not been discussed so far, but is important for our present task. For reasons that will become clear in a moment we will call this doctrine *the doctrine of perspectiveness.* It says that two (or more) occasions, even if they arise from the *same* causal past can nevertheless be causally influenced by this common past in *different* ways. This is what Whitehead tells us in the following quotation.

> [According to the doctrine of actual worlds] it is not wholly true that two contemporaries A and B enjoy a common past. [But] *even if* the occasions in the past of A be identical with the occasions in the past of B, yet A and B by reason of their difference of [extensive] status, enjoy that past under a difference of perspective [...]. Thus the objective immortality of the past in A differs form the objective immortality of the same past in B. (AI, p. 196; italics added)

Thus the reason for the difference in the objective immortality of the same past lies in the different extensive respectively spatiotemporal relations the different regions $\mathcal{O}_A$ and $\mathcal{O}_B$ of the occasions A and B bear to the occasions in their common past—*in the different extensive respectively spatiotemporal perspective on the same past* (see also PR, p. 61, 67). Now the objective immortality of past occasions is their being restored in the extensive continuum together with their thereof resulting ability to contribute to the manifestations of the underlying activity at each later stage of the world-process, that makes them available as causes for far removed future occasions (see Section 2.3.4). As a consequence of our postulation of generally undivided, bifurcating activities all occasions belonging to the same stage, s say, of the world-process have the same causal past, given by some subset of W_{s-1}. The manifestation of the underlying activity produced by the envisagement of this past is $\omega_{W_{s-1}}$. The contribution to this manifestation $\omega_{W_{s-1}}$ is the objective immortality of the past occasions *for* the new occasions to be created in stage s. Therefore, the different restrictions $\omega_{s-1}|_{\mathcal{P}(\mathcal{O}_{i(s)})}$ of the state ω_{s-1} to the sets of projections of regions $\mathcal{O}_{i(s)} \in D(M)$ in the future of W_{s-1} make sense of the statement that the new occasions of stage s "by reason of their difference of [extensive or spatiotemporal] status, enjoy that past under a difference of perspective". In other words, *the restrictions* $\omega_{s-1}|_{\mathcal{P}(\mathcal{O}_{i(s)})}$ *can naturally be interpreted as arising from the different spatiotemporal perspectives onto the same activity* $\omega_{W_{s-1}}$ *produced by this common past.* However, such perspectives obviously presuppose that the corresponding regions $\mathcal{O}_{i(s)}$ are already realized. And since this is the case only *after* the dative phase of transition, the restrictions $\omega_{s-1}|_{\mathcal{P}(\mathcal{O}_{i(s)})}$ *cannot be interpreted as perspectives onto the initial activity* $\omega_{W_{s-1}}$ *at the beginning of the dative phase, but rather have to be understood as perspectives onto the activity* $\omega^d_{W_{s-1}}$ *at its ending.*

For the latter activity itself, i.e. the outcome activity $\omega^d_{W_{s-1}}$ of the dative phase, we propose the following representation (that will be motivated below): it shall be represented by the restriction of the state ω_{s-1} (representing the initial activity $\omega_{W_{s-1}}$ at stage s) to the product, or what amounts to the same, to the conjunction

$$\prod_{i(s)} \mathcal{P}(\mathcal{O}_{i(s)}) = \wedge_{i(s)} \mathcal{P}(\mathcal{O}_{i(s)})$$

of the sets $\mathcal{P}(\mathcal{O}_{i(s)})$ corresponding to the new regions $\{\mathcal{O}_{i(s)}\}$ created in the dative phase of stage s, i.e. by

$$\omega_{s-1}|_{\wedge_{i(s)}\mathcal{P}(\mathcal{O}_{i(s)})}. \tag{10.1}$$

That products of projections from different sets $\mathcal{P}(\mathcal{O}_{i(s)})$ are well-defined and can moreover be interpreted as conjunctions, relies on the mutual spacelike separateness of the regions $\{\mathcal{O}_{i(s)}\}$ that implies the mutual commutativity of the sets $\mathcal{P}(\mathcal{O}_{i(s)})$. That only conjunction of projections associated with different regions $\mathcal{O}_j$, $\mathcal{O}_k$ of the same layer of the world-process is of interest in what follows, but not their disjunction, is because the latter does contain "less information" about the the actual definiteness of regions $\mathcal{O}_j$ and $\mathcal{O}_k$ than the conjunction, so that only the conjunction is of ontological interest. In other words, what matters for the world-process in connection with the subjective eternal objects ingressed into the regions $\mathcal{O}_j$, $\mathcal{O}_k$, is the answer to the question "what is the definiteness of region $\mathcal{O}_j$ *and* the definiteness of region $\mathcal{O}_k$ ", but not the question what the definiteness of region $\mathcal{O}_j$ *or* the definiteness of region $\mathcal{O}_k$ is, since an answer to this latter question only contains more information than the answer to the question for the definiteness of a single of these regions if the answer to the "conjunctive question" is also given. Note that a conjunction of projections belonging to different regions does not represent a possible ingression of a conjunctive subjective eternal object. We have restricted our scope to subjective eternal objects which are monadic, i.e. non-relational, universals and thus do only characterize single occasions/regions (see Section 2.2).[2] Thus contrary to the case where $P_j \wedge P_k$ represents a possible ingression of a single (conjunctive) subjective eternal object, in case P_j and P_k do represent possible ingressions of different subjective eternal objects in different regions, their conjunction does not have an existential status that goes beyond that of the possible ingressions P_j and P_k in isolation from each other (see also Section 2.2.3).

Now why should we represent the activity at the end of the dative phase by the restriction (10.1) of the state ω_{s-1} at all? First, the difference between the activities before and after some phase of transition or concrescence is just the reduced amount of the latter activity, *whereas the creative character is unchanged* (see also Section 2.3.1 as well Section 2.6.2). And since the restriction of a state to a subset of its domain *does not change the states functional form*, such a restriction will arguably not involve a change in the creative character of the corresponding activity. Moreover, what takes place in all phases of transition and concrescence, is in effect a reduction of possibilities. In particular, in the dative phase of the transition process at stage s, a *single* set $\{\mathcal{O}_{i(s)}\}$ of mutually spacelike separated regions in the future of W_{s-1}, out of the uncountable infinity of other such sets, is realized, whereby all other initially possible sets are ruled out for realization. And according to Section 2.4.1 each such act of reduction of possibilities involves a reduction of the amount of the corresponding activity, even if it does not involve a genuine decision of this activity. The operation of restricting the domain of the corresponding

[2] Apart from this we have for reasons of ontological economy abandoned *conjunctive* subjective eternal objects anyway—even monadic ones (see Section 8.3.3).

state quite nicely reflects such a reduction of possibilities, because the elements which are eliminated from its domain are no longer ascribed probabilities to by this state and thus they are no longer possibilities. Now, as explained above, there is no direct connection between states on $\mathcal{B}(\mathcal{H})$ and spacetime regions. Rather the best such connection is the one mediated by the one-to-one correspondence between double cones and their local algebras or equivalently the sets of projections included in the latter. Therefore, it seems quite reasonable to represent the realization of the set of regions $\{\mathcal{O}_{i(s)}\}$ by the restriction of the state ω_{s-1} on $\mathcal{B}(\mathcal{H})$ (representing the activity at the beginning of the dative phase) to the conjunction of the sets of projections $\mathcal{P}(\mathcal{O}_{i(s)})$ which are in one-to-one correspondence with these regions, i.e. by (10.1).

The further restriction of the state $\omega_{s-1}|_{\wedge_{i(s)}\mathcal{P}(\mathcal{O}_{i(s)})}$ to one of the set $\mathcal{P}(\mathcal{O}_{i(s)})$, say $\mathcal{P}(\mathcal{O}_j)$, results in the state

$$(\omega_{s-1}|_{\wedge_{i(s)}\mathcal{P}(\mathcal{O}_{i(s)})})|_{\mathcal{P}(\mathcal{O}_j)} = \omega_{s-1}|_{\mathcal{P}(\mathcal{O}_j)},$$

which is, however, *not* interpreted as a further reduction of the amount of activity. Rather, as proposed above, the resulting state $\omega_{s-1}|_{\mathcal{P}(\mathcal{O}_j)}$ is interpreted as a perspective onto the undivided activity $\omega^d_{W_{s-1}}$ represented by $\omega_{s-1}|_{\wedge_{i(s)}\mathcal{P}(\mathcal{O}_{i(s)})}$. Note that this different interpretation of the restriction of a state to some subset of its domain, does not introduce an inconsistency into our interpretation. Nor is it an exceptional case that one and the same mathematical object or operation is used to represent different physical structures. Rather it is quite usual that one and the same mathematical structure is interpreted in different ways within one and the same physical theory (see e.g. Schröter 1996). For example, think of the different interpretations of the real numbers within QM or QFT: they are interpreted as the possible values of (infinitely many) *different* magnitudes and moreover as time points, which is especially interesting because time is *not* a magnitude at all and thus *belongs to a completely different ontological category.* Moreover, projection operators too, have a bivalent interpretation. On the one hand, they represent properties and on the other hand states via the formula $\rho = Tr(P\cdot)$. Our bivalent use of the restriction of states, in some cases as perspectives onto an activity and in other cases as an activity reduced in amount, seems therefore likewise justified.

It is important to notice that the interpretation of the restrictions $\omega_{s-1}|_{\mathcal{P}(\mathcal{O}_{i(s)})}$ as perspectives onto one and the same activity, does *not presuppose that this activity is divided into partial activities, let alone particularized ones.* This is important, because we have assumed that such a division is at the earliest possible for the activity $\omega^c_{W_{s-1}}$ left after the transition process. Thus the perspectives (represented by) $\omega_{s-1}|_{\mathcal{P}(\mathcal{O}_{i(s)})}$ are not distinct parts of the activity $\omega^d_{W_{s-1}}$ (represented by $\omega_{s-1}|_{\wedge_{i(s)}\mathcal{P}(\mathcal{O}_{i(s)})}$) and thus are not themselves activities. Rather they are *relations* between the regions $\mathcal{O}_{i(s)}$ and the activity $\omega^d_{W_{s-1}}$, which *express the causal efficiency of the common past for each of these regions abstracted from that for all the other regions.* As we will moreover see in the following section, QFT allows for an interpretation of an activity as divided into partial activities only under very special circumstances, whereas in the generic case such an interpretation is not possible. Thus, quite surprisingly, though Whitehead did not envisage the need for undivided activities within his ontology, his original theory includes a doctrine that seems to be ready made for such a conception.

The formulation of the doctrine of perspectiveness by Whitehead is, however, far more surprising—even puzzling—in the light of the fact that Whitehead did not even believe in the possibility of two (or more) contemporary occasions arising from the same past (see e.g. the above quote). He introduced his doctrine of actual worlds precisely to abandon this possibility from his ontology. And if the doctrine of actual worlds is valid, the further doctrine of perspectiveness is rendered *completely functionless.* For if W_{s-1} were divided into different actual worlds $W_{s-1}(i)$ each giving rise to its own distinct manifestation $\omega_{W_{s-1}(i)}$ of the underlying activity, i.e. if $\omega_{W_{s-1}}$ were simply the "sum" of these distinct partial activities $\omega_{W_{s-1}(i)}$, there were no need for further spatiotemporal perspectives. This is because, according to Whitehead, after the dative phases of the corresponding transition processes, these partial activities are even particularized $\omega^d_{W_{s-1}}(\mathcal{O}_i)$, so that $\omega^d_{W_{s-1}}$ is the "sum" of spatiotemporally localized parts (see Section 2.4.1). But then each of these parts already provides much more than a mere a spatiotemporal perspective onto the past—it is a *spatiotemporal part* of the activity produced by this past. Therefore, it is quite puzzling what the genuine function of the doctrine of perspectiveness shall be if at the same time the stronger doctrine of actual worlds is upheld.

Before we can return to the main question of the present section, namely how one can single out a set of spacelike separated regions in the future of W_{s-1}, we have to discuss in some detail when QFT allows for the interpretation of states as consisting of distinct parts. This is because the mechanism we will propose for the determination of such regions in Section 10.3, will refer to the divisibility of the activity $\omega^d_{W_{s-1}}$ into partial activities. The following discussion will show that our modification of Whitehead's ontology by means of generally undivided, bifurcating activities is well supported by the formalism QFT.

10.2 Bell's theorem, non-separability, and all that

For convenience we will start the present investigation within the framework of QM and explain later on how the obtained results take over to QFT.

10.2.1 The EPR-experiment

In QM the Hilbert space of a system consisting of two distinguishable subsystems, i.e. to systems which differ in their state independent properties like mass, charge etc., is the tensor product $\mathcal{H}_1 \otimes \mathcal{H}_2$ of the Hilbert spaces associated with system 1 and 2 respectively (see Appendix B.1). The algebra of all observables of the total system is thus given by $\mathcal{B}(\mathcal{H}_1 \otimes \mathcal{H}_2)$ and the observables of system 1 and 2 alone are given by the commuting subalgebras $\mathcal{B}(\mathcal{H}_1) \otimes \mathbf{1}_2$ and $\mathbf{1}_1 \otimes \mathcal{B}(\mathcal{H}_2)$ where $\mathbf{1}_1$ and $\mathbf{1}_2$ are the identity operators in $\mathcal{B}(\mathcal{H}_1)$ and $\mathcal{B}(\mathcal{H}_2)$ respectively. Since in what follows it will always be clear which observable belongs to which of the two subsystems we will simply omit the trivial tensor product factors $\mathbf{1}_1$ and $\mathbf{1}_2$ in all operators.

Now let us consider the following experimental setup, known as *EPR-experiment* named after the physicists A. Einstein, B. Podolsky and N. Rosen who first introduced it into the discussion about the interpretation of QM (Einstein, Podolsky and Rosen 1935):[3] consider two distinguishable systems, for simplicity henceforth called particles, with spin 1/2 (e.g. an electron and

[3]In fact the experimental situation conceived by them sligthly differs from the one we will investigate here, which is the version of the EPR-experiment due to (Bohm 1951) involving

a muon) which are prepared in the so-called *singlet state* $\rho_S \equiv \langle \psi_S, \cdot \psi_S \rangle$ generated by the unit vector

$$\psi_S \equiv \frac{1}{\sqrt{2}}(\psi_1^\uparrow \otimes \psi_2^\downarrow - \psi_1^\downarrow \otimes \psi_2^\uparrow), \tag{10.2}$$

where $\psi_i^\uparrow, \psi_i^\downarrow$ are the two orthogonal eigenstates of spin 1/2-systems, corresponding to spin up $\uparrow$ and down $\downarrow$ in some given direction. The factor $1/\sqrt{2}$ is merely needed for the normalization of the resulting state $\rho_S = \langle \psi_S, \cdot \psi_S \rangle$. Two particles in this state can be prepared by a suitable decay of a spin 0-system. After this preparation the two particles move in opposite directions, so that at some time t after their emission they are spacelike separated from one another. Thus if causal influences are believed to propagate at most with the speed of light, the two particles are causally independent from each other. Now suppose each particle reaches a measurement device (i.e. a Stern-Gerlach magnet) by which the spin in some direction perpendicular to its path is measured. The probability prescribed by the singlet state ρ_S to the possible results (up and down) of a measurement of the spin of particle i in some arbitrary direction is 1/2. Assume that both measurements are made in the same direction, the z-direction with respect to a fixed spatial reference frame say. Then, if the measurement of the spin of particle 1 leads to the result up, corresponding to the projection $P_1^{\uparrow_z}$, that of particle 2 must give the result down $P_2^{\downarrow_z}$, and vice versa, since the joint probabilities $\rho_S(P_1^{\uparrow_z} P_2^{\downarrow_z})$ and $\rho_S(P_1^{\downarrow_z} P_2^{\uparrow_z})$ prescribed by the singlet state are 1. Thus the results of spin measurements of the two particles in direction z are *strictly anticorrelated*. This result may seem obvious because of the conservation of the total angular momentum, that in our case, coincides with the zero total spin of the compound system. But this overlooks two points: first, the spin of a spin 1/2-system can only take the to values up or down—there are no "intermediate" possible values. Thus unlike a classical angular momentum, the result of a spin measurement in some direction always leads to the spin being parallel or antiparallel to this direction. And second, the state ρ_S is rotational invariant, so that if instead of the z-direction the spin of the two particles had been measured in any other direction, the x-direction say, the same anticorrelation, this time between $P_1^{\downarrow_x}, P_2^{\uparrow_x}$ and $P_1^{\uparrow_x}, P_2^{\downarrow_x}$ respectively, would obtain. Therefore, the fact that the results of spin measurements

spin instead of position and momentum.

of particles 1 and 2 are always antiparallel *to one another*, independently from the chosen direction in which spin is measured is indeed strange. For if the spacelike separation between the two measurements prohibits the propagation of causal influences, and thus too of signals, between them, how can the two particles always manage to show antiparallel spins regardless of the (common) direction in which their spins are measured?

Now there is a possibility for how the particles can do the trick: it may be the case that the quantum mechanical description of the situation is not complete in that there are further parameters, not accounted for by the formalism of QM, which can explain the strict anticorrelations in the EPR-experiment. That the quantum mechanical description of the world is incomplete in this sense is the conclusion Einstein, Podolsky and Rosen have drawn from the above situation, albeit by a slightly different line of argument. Since such hypothetical supplementary parameters are not accessible within the conceptual framework of QM, they are termed *hidden parameters* or interchangeably *hidden variables*. Since QM is empirically well confirmed, hidden parameter theories (HP-theories) must, however, reproduce the empirical predictions of QM (see e.g. Auletta 2001, p. 543 f). However, it is quite easy to conceive such a HP-model for the EPR-experiment. Each of the two particles could come equipped with spin values (up or down) in *any* arbitrary direction, and to account for the strict anticorrelations, the values assigned to the two particles for a given direction must be different. To account for the probability 1/2 prescribed by QM on the basis of the state ρ_S to the results of the individual spin measurements on each particle, one can assume that the source that emits the particles produces the same fraction of particles equipped with the value up for a given direction as particles with the value down for this direction. Note that this HP-theory goes beyond QM in that it assumes that a particle can possess definite values of *non-commuting* observables, namely of the spin observables corresponding to different directions, at the same time. However, it provides an explanation for the strict anticorrelations predicted by QM without introducing any superluminal influences between the two particles or the measurements carried out in the spacelike separated regions $\mathcal{O}_1$ and $\mathcal{O}_2$. Rather it provides a *common cause* for the anticorrelated measurement results that is located in the intersection of the backward lightcones of the regions $\mathcal{O}_1$

and $\mathcal{O}_2$. This common cause is the preparation of the particles by the source that emitted them and equipped each of the them with the appropriate spin values in all directions perpendicular to its paths. Thus this HP-model of the EPR-experiment obeys two principles which are presuppositions of classical physics and were, in particular, the pillars of Einstein's world view (Howard 1985). The first is the principle of *locality* requiring that

Locality: All causes of an event lie in its backward lightcone.[4]

The fulfillment of this locality principle by the above HP-model has already been made explicit. The second principle is termed *separability* and asserts that

Separability: Each of two spatiotemporally separated systems 1 and 2 possesses its own distinct physical state, such that the physical state of the compound system 1 + 2 is wholly determined by the states of 1 and 2 (Primas 1981, p. 294; Howard 1989, p. 225 f).

That our above HP-model of the EPR-experiment is also separable is due to the fact that the physical state this model ascribes to each of the particles is just the set of spin values for all directions,[5] so that the state of the compound system 1 + 2 is simply the set-theoretic union of the states of particle 1 and particle 2 and thus is wholly determined by them. Thus the EPR-experiment as above described admits a local and separable HP-model. However, in 1964 the physicist J. S. Bell proved a theorem (Bell 1964)—today known as *Bell's theorem*—from which one can conclude that, for a slightly more general situation as the one considered in the original EPR-experiment, *no local, separable HP-model can exist.* And since QM can obviously be regarded as a HP-theory

[4]Note that locality, as formulated here, does *not* require that causes and their immediate effects are also *spatiotemporally contiguous*. The conjunction of the latter requirement with the locality principle had been termed "local causation" in Section 1.3.1. In what follows the term "local" will, however, always be used to refer to the locality principle alone and not the stronger condition of local causation.

[5]Permanent—state independent—properties like mass and charge are irrelevant for the EPR-experiment but can clearly also be incorporated into the HP-model—as "non-hidden parameters".

of itself, namely as the trivial HP-theory that supplements no further parameters to the quantum mechanical description, Bell's theorem likewise shoes that QM itself cannot be both local and separable.

10.2.2 Bell's theorem

The generalization investigated by Bell is that the directions in which the spins of the two particles are measured are not identical and that (at least) three different directions are taken into account. Let the directions in the plane normal to the paths of the particles in which their spins are measured be denoted by unit vectors $\mathbf{a}, \mathbf{b}, \ldots$. Further, let λ denote the hidden parameter state of the compound two particle system. Thus, in general, λ may include a state prescribed by QM as well as supplementary parameters not inherent in the QM formalism. The probabilities prescribed by λ to the possible results $r_{\mathbf{a}} \in \{+1$ $(=\text{up}), -1$ $(=\text{down})\}$, given a measurement on particle i in direction $\mathbf{a}$ are denoted by $p_\lambda(r_{\mathbf{a}}|\mathbf{a})$ and the joint probability for results $r_{\mathbf{a}}$ and $r_{\mathbf{b}}$ given the direction of the spin measurements on particle 1 and 2 are $\mathbf{a}$ and $\mathbf{b}$ respectively, will be denoted by $p_\lambda(r_{\mathbf{a}}, r_{\mathbf{b}}|\mathbf{a}, \mathbf{b})$. The central assumption for the proof of Bell's theorem is the following *factorizability condition*

$$p_\lambda(r_{\mathbf{a}}, r_{\mathbf{b}}|\mathbf{a}, \mathbf{b}) = p_\lambda(r_{\mathbf{a}}|\mathbf{a}) \cdot p_\lambda(r_{\mathbf{b}}|\mathbf{b}). \tag{10.3}$$

This condition says that given the state λ of the compound system, the probability for outcome $r_{\mathbf{a}}$ of the spin measurement in direction $\mathbf{a}$ on particle 1 is *probabilistically independent from the direction* $\mathbf{b}$ *as well as from the outcome* $r_{\mathbf{b}}$ *of the spacelike separated measurement on particle 2, and vice versa.* How this condition is related to the locality and separability principle will be discussed below.

Now one can show that under the assumption (10.3), the expectation values, term them $E(\mathbf{a}, \mathbf{b})$, $E(\mathbf{a}, \mathbf{c})$ and $E(\mathbf{b}, \mathbf{c})$, calculated from the joint probabilities $p_\lambda(r_{\mathbf{a}}, r_{\mathbf{b}}|\mathbf{a}, \mathbf{b})$, $p_\lambda(r_{\mathbf{a}}, r_{\mathbf{b}}|\mathbf{a}, \mathbf{c})$ and $p_\lambda(r_{\mathbf{a}}, r_{\mathbf{b}}|\mathbf{b}, \mathbf{c})$ for three directions $\mathbf{a}, \mathbf{b}, \mathbf{c}$ must satisfy the following inequality

$$|E(\mathbf{a}, \mathbf{b}) - E(\mathbf{a}, \mathbf{c})| - E(\mathbf{b}, \mathbf{c}) \leq 1. \tag{10.4}$$

This is the first of a whole family of similar inequalities known today and collectively referred to as *Bell inequalities.*

Now if λ is taken to be the singlet state of QM and $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ are chosen in such a way that the angle between $\mathbf{a}$ and $\mathbf{b}$ as well as the one between $\mathbf{b}$ and $\mathbf{c}$ is 30° (an thus the angle between $\mathbf{a}$ and $\mathbf{c}$ is 60°), the left hand side of (10.4) becomes 1.5. Thus QM violates the above Bell inequality for a certain choice of directions $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$. One can show that 1.5 is at the same time the largest value the left hand side of (10.4) can take for any quantum state λ and any choice of directions. Thus the singled state *maximally violates* Bell's inequality. However, a non-maximal violation of this inequality occurs for many other quantum states. Bell's inequality has today been tested in numerous experiments, all of which not merely show its violation but confirm with accuracy the quantitative predictions of QM (see e.g. Aspect 2002). We will now discuss the meaning of the crucial factorizability condition (10.3).

10.2.3 Analyzing the factorizability condition

In his paper from 1964 Bell himself called the factorizability condition alternately the condition of locality, of causality and of separability (Bell 1964, p. 195), which shows that at this time Bell did not regard these terms as referring to ontologically distinct features. His view was that (10.3) is the mathematical expression for the requirement "that the result of a measurement on one system be unaffected by operations on a distant system with which it has interacted in the past" (Bell 1964, p. 195). This statement, however, can be interpreted in quite different ways depending on the meaning one gives to the term "operation". However, the meaning of the factorizability condition became clearer when J. Jarrett showed that it is the conjunction of two logically independent conditions (Jarrett 1984), today usually called *parameter independence* and *outcome independence*. Parameter independence is the following condition:

$$p_\lambda(r_{\mathbf{a}}|\mathbf{a},\mathbf{b}) = p_\lambda(r_{\mathbf{a}}|\mathbf{a}) \tag{10.5}$$

$$p_\lambda(r_{\mathbf{b}}|\mathbf{a},\mathbf{b}) = p_\lambda(r_{\mathbf{b}}|\mathbf{b}), \tag{10.6}$$

where

$$p_\lambda(r_{\mathbf{a}}|\mathbf{a},\mathbf{b}) \equiv \sum_{r_{\mathbf{b}}\in\{+1,-1\}} p_\lambda(r_{\mathbf{a}}, r_{\mathbf{b}}|\mathbf{a},\mathbf{b})$$

$$p_\lambda(r_{\mathbf{b}}|\mathbf{a},\mathbf{b}) \equiv \sum_{r_{\mathbf{a}}\in\{+1,-1\}} p_\lambda(r_{\mathbf{a}}, r_{\mathbf{b}}|\mathbf{a},\mathbf{b})$$

are the marginal probabilities derived from the joint probability $p_\lambda(r_\mathbf{a}, r_\mathbf{b}|\mathbf{a}, \mathbf{b})$. Parameter independence therefore says that given the total state λ of the compound system, the result $r_\mathbf{a}$ of a measurement on particle 1 is *independent from the direction* $\mathbf{b}$ of the measurement on particle 2 and vice versa. The term *parameter* independence stems from the fact that the directions $\mathbf{a}, \mathbf{b}, \ldots$ are the parameters by which the experimenters in the EPR-experiment choose the observables to be measured on the particles—by a certain choice of these parameters the measuring devices for the two particles are set to measure the corresponding spin observables. Thus parameter independence does not refer to any *hidden* parameters possibly be included in the state λ. If parameter independence holds the choice of the observable to be measured on particle 2 is probabilistically irrelevant for the result of the measurement on particle 1, given the total state and the parameter setting for the measurement on particle 1 and vice versa. Thus parameter independence is a *screening off condition*—the total state and the observable measured on one of the particles screens off any probabilistic relevance that the choice of the observable measured on one particle might otherwise have for the result of the measurement on the other particle (Butterfield 1989). Since probabilistic relevance (in one form or another), and thus the obtainment of correlations, is commonly held to be a *necessary condition* for causal influences (see e.g. Reichenbach 1956, Suppes 1970, Lewis 1986, Mellor 1995), parameter independence is usually taken to prohibit causal influences from the choice of the observable for one particle on the result of the observable measured on the other particle (see e.g. Jarrett 1984; Shimony 1993).

Outcome independence can be formulated as follows:

$$p_\lambda(r_\mathbf{a}|\mathbf{a}, \mathbf{b}) = p_\lambda(r_\mathbf{a}|\mathbf{a}, \mathbf{b}, r_\mathbf{b}) \tag{10.7}$$

$$p_\lambda(r_\mathbf{b}|\mathbf{a}, \mathbf{b}) = p_\lambda(r_\mathbf{b}|\mathbf{a}, \mathbf{b}, r_\mathbf{a}). \tag{10.8}$$

Thus given the state and both directions $\mathbf{a}, \mathbf{b}$ for the measurements on particle 1 and 2, the result $r_\mathbf{a}$ of the measurement on particle 1 is *independent from the result* of the measurement on particle 2 and vice versa. In other words, outcome dependence too, is a screening off condition, this time the total state together with the parameter settings $\mathbf{a}, \mathbf{b}$, i.e. with the choice of the observables to be measured on *both* of the particles, renders the results of the measurements on the particles probabilistically independent from one another. Thus by denying any probabilistic relevance of these results, outcome independence in particular prohibits causal influences between these results themselves. In sum then, both parameter and outcome independence seem to be locality criteria, i.e. concrete mathematical expressions of the locality principle that denies causal influences between spacelike separated events. In case of parameter independence the events for which locality is secured are the choice of the observable to be measured on one particle and the result of the measurement on the other particle and in case of outcome independence it is the two results. However, as we will see below, outcome independence can and should be interpreted in a different way, namely as an expression for the separability principle.

The equivalence of the conjunction of parameter and outcome independence to the factorizability assumption (10.3) leading to Bell's inequality means that each theory—may it a HP-theory or not—that reproduces the empirically confirmed predictions of QM, must violate either parameter or outcome independence or both. This reduces the initial appeal of HP-theories because it shows that no such theory can reconcile the empirical data with a classical world view according to which all correlations between spacelike separated events can be explained in a locally and separable way. In the following discussion we will concentrate on QM and mention HP-theories only occasionally.

Now the real importance of the fact that factorizability can be dissected into parameter and outcome independence lies in the fact that one can locate QM's violation of Bell's inequality in its *violating outcome independence*. This is because QM *satisfies parameter independence*: let λ be an arbitrary quantum state, A and B observables of particle 1 and 2 respectively and let a and b be possible values corresponding to the eigenprojections P_a^A and P_b^B of A and B. Because A and B are observables of different systems, they commute (and so do the sets of eigenprojections $\{P_a^A\}$ and $\{P_b^B\}$). The joint probability of the results a and b—corresponding to $p_\lambda(r_{\mathbf{a}}, r_{\mathbf{b}}|\mathbf{a}, \mathbf{b})$ in the special case where A and B are the spin observables in directions $\mathbf{a}$ and $\mathbf{b}$—is then given by

$$\lambda(P_a^A P_b^B).$$

Now summing this joint probability over all possible values of observable B

and A respectively gives the marginal probabilities

$$\sum_b \lambda(P_a^A P_b^B) \text{ and } \sum_a \lambda(P_a^A P_b^B)$$

corresponding to $p_\lambda(r_\mathbf{a}|\mathbf{a},\mathbf{b})$ and $p_\lambda(r_\mathbf{b}|\mathbf{a},\mathbf{b})$ from above. Now the sets of eigen-projections $\{P_a^A\}$ and $\{P_b^B\}$ are resolutions of the identity and thus sum up to $\mathbf{1}$. Therefore, one gets

$$\sum_b \lambda(P_a^A P_b^B) = \lambda\left(P_a^A\right) \tag{10.9}$$

$$\sum_a \lambda(P_a^A P_b^B) = \lambda(P_b^B). \tag{10.10}$$

The right hand side of equation (10.9) ((10.10)) is, however, just the probability for result a (b) given a measurement of observable A (B) on particle 1 (2), corresponding to $p_\lambda(r_\mathbf{a}|\mathbf{a})$ ($p_\lambda(r_\mathbf{b}|\mathbf{b})$), so that we have in fact proven parameter independence to hold in QM for two arbitrary observables A and B of particles 1 and 2 and an arbitrary quantum state λ. As mentioned above, this means, in particular, that one cannot influence the outcome of any measurement on particle 2 by the choice of observable to be measured on particle 1 and vice versa.

Yet this satisfaction of parameter independence by QM implies that QM must violate outcome independence—otherwise it could not violate Bell's inequality. Thus in QM a violation of outcome independence is a equivalent to violation of Bell's inequality. Therefore, the result of a measurement on particle 1 will, in general, be probabilistically relevant for the result of a measurement of particle 2 (and vice versa) even given the total state and the observables to be measured. In other words, the total state and the choice of the two observables do not screen off the results from one another. Thus since according to QM there are no other causally relevant factors, it seems tempting to conclude that there is indeed a direct causal influence between the two spacelike separated results. As J. Butterfield as well as T. Maudlin have shown within the framework of Lewis' counterfactual theory of causation (Lewis 1986), outcome dependence can in fact be seen as being grounded in a direct causal link between the outcomes $r_\mathbf{b}$ and $r_\mathbf{a}$ (Butterfield 1992; Maudlin 1994, Chapter 5). However, there are good reasons *not* to think of the violation of outcome independence in terms of causal influences.

Outcome independence as a separability criterion

First of all, unlike the violation of parameter dependence, QM's violation of outcome independence cannot be utilized for any kind of (superluminal) signalling between the two spacelike separated experimenters involved in the EPR-experiment. To see why this is impossible it is instructive to first explain how one could exploit a failure of parameter independence for sending superluminal signals. The parameters **a** and **b** are under the control of the experimenters. Now if parameter independence (10.5) were violated, there were different parameters **b** and **b**′ such that

$$p_\lambda(r_\mathbf{a}|\mathbf{a},\mathbf{b}) \neq p_\lambda(r_\mathbf{a}|\mathbf{a},\mathbf{b}').$$

Thus by an appropriate choice of the parameter for the measurement on particle 2, **b**′ say, experimenter 2 could raise or lower the probabilities of the results obtained from measurements on particle 1. Of course, a single run of such an experiment may not give the other experimenter very much to go on. But with sufficient repetitions (i.e. by performing a large ensemble of such experiments (nearly) simultaneously) experimenter 1, will with great reliability, be able to detect a relative frequency different from the one he would find if experimenter 2 had chosen a different parameter $\mathbf{b} \neq \mathbf{b}'$. Contrary to this, failure of outcome independence cannot be used for such a superluminal signaling. The crucial point is that, unlike the parameters **a** and **b**, the outcomes $r_\mathbf{a}$ and $r_\mathbf{b}$ are *not* under the control of the experimenters—their occurrence is totally random. A violation of outcome independence

$$p_\lambda(r_\mathbf{a}|\mathbf{a},\mathbf{b}) \neq p_\lambda(r_\mathbf{a}|\mathbf{a},\mathbf{b},r_\mathbf{b})$$

can therefore not be exploited by experimenter 2 to systematically change the probabilities and thus the relative frequencies seen by experimenter 1. A. Shimony therefore also spoke of the failure of outcome independence as allowing some kind of "uncontrollable influences" between the two measurements (Shimony 1984). Now the fact that a failure of outcome independence cannot be used for superluminal signalling is clearly only an operational argument against seeing it as a kind of causal connection. For there is no need that all causal connections in nature must also be exploitable by us in such a way (not even in principle)—from the point of view of ontology it does not matter whether a causal influence has operational consequences or not—such as the possibility of signalling.

Yet if one interprets outcome dependence in terms of causal influences, one has to admit that *these influences go in both directions*, i.e. from outcome $r_{\mathbf{a}}$ to outcome $r_{\mathbf{b}}$ and vice versa. This is because by using the definition of the conditional probabilities

$$p_\lambda(r_{\mathbf{a}}|\mathbf{a},\mathbf{b},r_{\mathbf{b}}) \equiv \frac{p_\lambda(r_{\mathbf{a}},r_{\mathbf{b}}|\mathbf{a},\mathbf{b})}{p_\lambda(r_{\mathbf{b}}|\mathbf{a},\mathbf{b})}$$

and

$$p_\lambda(r_{\mathbf{b}}|\mathbf{a},\mathbf{b},r_{\mathbf{a}}) \equiv \frac{p_\lambda(r_{\mathbf{a}},r_{\mathbf{b}}|\mathbf{a},\mathbf{b})}{p_\lambda(r_{\mathbf{a}}|\mathbf{a},\mathbf{b})}$$

outcome independence, i.e. the conjunction of (10.7) and (10.8), turns out to be equivalent to[6]

$$p_\lambda(r_{\mathbf{a}},r_{\mathbf{b}}|\mathbf{a},\mathbf{b}) = p_\lambda(r_{\mathbf{a}}|\mathbf{a},\mathbf{b}) \cdot p_\lambda(r_{\mathbf{b}}|\mathbf{a},\mathbf{b}). \tag{10.11}$$

This, however, means that the violation of one of the two conditions (10.7) and (10.8) implies (via the violation of (10.11)) that the other condition must also be violated. This in turn means that if $r_{\mathbf{a}}$ is a cause of $r_{\mathbf{b}}$ then also vice versa, so that a failure of outcome independence implies the existence of *causal loops*. Note that this state of affairs cannot be remedied even if there were a preferred reference frame, for example in form of a distinguished foliation of spacetime as in our version of Whitehead's ontology. For whenever the two measurements were simultaneous *with respect to this preferred frame*, there were no way of distinguishing them by any further spatiotemporal means whatsoever. In this case, however, one ends up with the contradictory kind of self-causation according to which *an entity can be a cause of its own existence* (see also Section 1.1 where the difference between this inconsistent kind of self-causation and the one inherent in Whitehead's ontology is discussed). For if the occurrence of outcome $r_{\mathbf{a}}$ is a cause (not necessarily the only one) for the occurrence of $r_{\mathbf{b}}$ and the latter is at the same time a cause for the occurrence of $r_{\mathbf{a}}$, then, $r_{\mathbf{a}}$ by (partially) causing $r_{\mathbf{b}}$ also (partially) causes its own occurrence.

[6] We always assume that $p_\lambda(r_{\mathbf{a}}|\mathbf{a},\mathbf{b})$ and $p_\lambda(r_{\mathbf{b}}|\mathbf{a},\mathbf{b})$ are non-zero.

The last two arguments against interpreting outcome dependence as involving a direct causal link between the results of the two measurements—and thus as a violation of locality—derive form the fact that it can likewise be interpreted as expressing the *non-separability* of the two particles—and thus as involving a violation of separability. The possibility of understanding outcome independence as a separability criterion, i.e. a concrete instance of the separability principle, speaks against the rival interpretation as a locality criterion for the following two reasons: first, separability is ontologically more fundamental than locality since the very formulation of the latter *presupposes the satisfaction of the first part of the former, namely the possession of distinct states by two spatiotemporally separated systems.* For if spatiotemporally separated systems would not have such distinct states, but rather would all share one common state, the talk of causal influences among them would be meaningless. This is because however the notion of the "physical state of a system" may be concretized in a particular (HP-) theory, it seems that if states are ascribed to systems at all, a causal influence on a system must lead to a change in the system's state. In other words, the system's state is that on which causal influences "register". Consequently, two systems 1 and 2 sharing one and the same state, cannot causally influence one another, for if system 1 would causally influence system 2, it would inevitable also causally influence itself via the change in the common state. Thus if causation is understood as *not necessarily involving self-causation*, there must be systems with distinct states. Therefore, the first part of the separability principle has to hold for the usual talk of causation between spatiotemporally separated systems to make sense. Consequently, without the satisfaction of the first part of the separability principle, the locality principle were completely vacuous (see also Esfeld 2001). Thus if outcome independence *can* be interpreted as a concrete instance of the separability principle it *should* be understood in this way, because without such a separability criterion at hand, all talk of locality may be vacuous.[7] Secondly, as will be argued below, that two particles in the quantum mechanical singlet state violate the separability principle can be seen *indepen-*

[7] An argument as to the *epistemologically* more fundamental status of separability over locality has been given in (Howard 1989). A critizism of the latter can be found in (Belousek 1999).

dently from the route via a violation of outcome independence. Therefore, if the understanding of outcome independence as a separability criterion is possible, this understanding gains independent support at least in the case of the quantum mechanical singlet state.

Now let us start by investigating the premise for the last two arguments, namely that outcome independence can be taken as expressing the separability of the two particles involved in the EPR-experiment. The separability principle consists of two parts: first, it requires the two particles in the EPR-experiment to have distinct states and second, these distinct states must wholly determine the total state λ. Now assume that our two particles have distinct states λ_1, λ_2 as required by the principle of separability. This means that the probability $p_\lambda(r_{\mathbf{a}}|\mathbf{a},\mathbf{b})$ prescribed by the total state λ for the outcome of a measurement on particle i coincides with the probability $p_{\lambda_i}(r_{\mathbf{a}}|\mathbf{a},\mathbf{b})$ for this outcome prescribed by the state λ_i of this particle alone, i.e. it means that the following equalities hold for all parameter settings $\mathbf{a},\mathbf{b}$ and all possible results $r_{\mathbf{a}}$ and $r_{\mathbf{b}}$ of the measurements on particle 1 and 2 respectively (see e.g. Howard 1989)

$$p_{\lambda_1}(r_{\mathbf{a}}|\mathbf{a},\mathbf{b}) = p_\lambda(r_{\mathbf{a}}|\mathbf{a},\mathbf{b}) \qquad (10.12)$$

$$p_{\lambda_2}(r_{\mathbf{b}}|\mathbf{a},\mathbf{b}) = p_\lambda(r_{\mathbf{b}}|\mathbf{a},\mathbf{b}). \qquad (10.13)$$

The occurrence of the parameters $\mathbf{a}$ and $\mathbf{b}$ means that the particles can be regarded as having the distinct states λ_1 and λ_2 *conditional on* the choice of the *total* measurement context, i.e. the observables to be measured on *both* particles. Thus (10.12) and (10.13) do *not* require that each particle can be regarded as having its own state if only the observable to be measured on it (or even if no observable at all) is fixed. This makes sense for two reasons: first, outcome independence (as well as parameter independence and factorizability) are formulated as conditions on probabilities and not as conditions on states. This is because they are intended to apply not only to QM but also to a range of HP-theories as wide as possible. Now the concept of a state of a system may vary in different HP-theories, so that the conditions on these states leading to a Bell inequality will likewise vary from one theory to another. However, all these HP-theories shall reproduce the predictions of QM and since the latter are predictions of probabilities, all HP-theories must be able to produce probabilities as outputs. Therefore, it is very convenient to formulate the conditions in question as conditions on probabilities and not as conditions directly constraining the states of systems. Now in case of QM a state prescribes probabilities only within a fixed Boolean framework provided by the Boolean algebra(s) of projections of some (set of mutually commuting) observable(s) (see Section 8.1 in particular 8.1.5). This, then, necessitates the conditionalizing on the observable to measured on a particle to produce probabilities from its state. That in (10.12) as well as in (10.13) the observables to be measured on *both* particle occur, via the parameters $\mathbf{a}$ and $\mathbf{b}$, is because otherwise these conditions would already *imply that parameter independence, and thus the form of locality expressed by it, likewise holds.* For example, if one would define the state λ_1 of particle 1 by means of

$$p_{\lambda_1}(r_{\mathbf{a}}|\mathbf{a}) \equiv p_{\lambda}(r_{\mathbf{a}}|\mathbf{a}),$$

this would imply that the result of the measurement on this particle is not only independent from the *result* of the measurement on particle 2 *but also from the observable measured on particle 2, i.e. from the parameter* $\mathbf{b}$. However, parameter and outcome independence are logically independent constraints, so that in arguing for the interpretability of the latter as an expression of the separability principle, one has to conditionalize on both parameters $\mathbf{a}$ and $\mathbf{b}$ in the definition of each of the distinct states of the particles.

Now equations (10.12) and (10.13) only express the fact that each particle has its own state distinct from the state of the other particle. But the separability principle moreover requires that these distinct states do wholly determine the total state λ. This is obviously the case if the product of the two probabilities $p_{\lambda_1}(r_{\mathbf{a}}|\mathbf{a},\mathbf{b})$ and $p_{\lambda_2}(r_{\mathbf{a}}|\mathbf{a},\mathbf{b})$ equals the joint probability $p_{\lambda}(r_{\mathbf{a}}, r_{\mathbf{b}}|\mathbf{a},\mathbf{b})$ prescribed by the total state λ (see e.g. Howard 1989), i.e. if

$$p_{\lambda}(r_{\mathbf{a}}, r_{\mathbf{b}}|\mathbf{a},\mathbf{b}) = p_{\lambda_1}(r_{\mathbf{a}}|\mathbf{a},\mathbf{b}) \cdot p_{\lambda_1}(r_{\mathbf{a}}|\mathbf{a},\mathbf{b}) \qquad (10.14)$$

holds. However, according to (10.12) and (10.13), $p_{\lambda_1}(r_{\mathbf{a}}|\mathbf{a},\mathbf{b})$ equals $p_{\lambda}(r_{\mathbf{a}}|\mathbf{a},\mathbf{b})$ and $p_{\lambda_2}(r_{\mathbf{a}}|\mathbf{a},\mathbf{b})$ equals $p_{\lambda}(r_{\mathbf{b}}|\mathbf{a},\mathbf{b})$. Therefore, (10.14) *is just the statement of outcome independence* (10.11). Thus the separability principle, via its concrete manifestation in form of the requirements (10.12), (10.13) and (10.14), implies outcome independence. On the other hand, if outcome independence holds, one can read the requirements (10.12) and (10.13) as definitions of the distinct states of the two particles which determine the total state via (10.14). In sum, then, outcome independence can be interpreted as a separability criterion, i.e. as a concrete mathematical expression of the separability principle. And thus, following the above argument from the more fundamental character of the separability over the locality principle, outcome independence therefore *should* be interpreted as a separability criterion.

With this interpretation of outcome independence as a separability rather than a locality criterion, its failure means that the two particles in the EPR-experiment either do not have distinct states or that the distinct states do not determine the total state. We will now argue that this interpretation of the failure of outcome independence, besides avoiding the difficulties with the competing interpretation in terms of a direct causal link between the measurement results, moreover fits smoothly to the formalism of QM.

Generally, in QM the state of a subsystem 1 is given by the restriction of the state of the total system $1+2$ to the subsystem's observable algebra $\mathcal{B}(\mathcal{H}_1)$, i.e. by $\rho|_{\mathcal{B}(\mathcal{H}_1)}$ where ρ is the state of the compound system. On the level of the density operators by which these states are generated (see Section 5.2), this means the following: the density operator W_1 generating the state of subsystem 1 via

$$\rho|_{\mathcal{B}(\mathcal{H}_1)} = Tr_1(W_1\cdot),$$

where Tr_1 stands for the trace with respect to the Hilbert space $\mathcal{H}_1$ of subsystem 1, is obtained from the density operator W corresponding to the total state $\rho = Tr(W\cdot)$ by taking the trace of W with respect to the Hilbert space of subsystem 2, i.e. by

$$W_1 = Tr_2(W) = \sum_i \left\langle \phi_2^i, W\phi_2^i \right\rangle,$$

where $\{\phi_2^i\}$ is an orthonormal basis of the Hilbert space of subsystem 2. Applying this to our two particle system in the singlet state $\rho_S = Tr(W_S\cdot) = \langle\psi_S, \cdot\psi_S\rangle$, yields the result that the restricted states $\rho_S|_{\mathcal{B}(\mathcal{H}_1)}$ and $\rho_S|_{\mathcal{B}(\mathcal{H}_2)}$ corresponding to particles 1 and 2, are generated by the trivial density operators $W_1 = \mathbf{1}_1$ and $W_2 = \mathbf{1}_2$ respectively. Therefore, one can argue that the two particles do not have distinct states at all, since the states $\rho_S|_{\mathcal{B}(\mathcal{H}_1)}$ and $\rho_S|_{\mathcal{B}(\mathcal{H}_2)}$

have essentially the same structure and are merely distinguished by means of the indices 1 and 2.[8] This can also be seen directly from the structure of the singlet state vector

$$\psi_S = \frac{1}{\sqrt{2}}(\psi_1^\uparrow \otimes \psi_2^\downarrow - \psi_1^\downarrow \otimes \psi_2^\uparrow),$$

since it obviously doe not allow to associate one of the two state vectors $\psi^\uparrow$,$\psi^\downarrow$ to each particle. Thus in case of the singlet state, one can argue that even the first part of the separability principle, requiring the two particles to have distinct states, is violated. However, even if one could regard the states of particles 1 and 2 as distinct, the second part of the separability principle is violated anyway, because the states $\rho_S|_{\mathcal{B}(\mathcal{H}_1)} = Tr_1(W_1\cdot)$ and $\rho_S|_{\mathcal{B}(\mathcal{H}_2)} = Tr_2(W_2\cdot)$ of particles 1 and 2 do not wholly determine the total state $\rho_S = Tr(W_S\cdot)$. This is obvious since the density operators $W_1 = \mathbf{1}_1$ and $W_2 = \mathbf{1}_2$ can likewise be obtained by the above procedure of "partial tracing" from the total density operator $W = \mathbf{1}_1 \otimes \mathbf{1}_2 = \mathbf{1}$ rather than from $W_S = P_{\psi_S}$. Therefore, the states of particle 1 and 2 are also compatible with the joint state being the trivial one $\rho = Tr(\mathbf{1}\cdot)$ instead of the singlet state. Thus at least the second part of the separability principle is certainly violated by a two particle system in the singlet state, so that it is natural also to interpret the latter's violation of Bell's inequality and thus of outcome independence as a violation of separability.

10.2.4 Non-separability in QM

As just argued, one can come to the conclusion that certain quantum states violate the separability principle independently from the route through the violation of Bell's inequality. However, the great value of Bell's work has been to proof that no alternative theory whatsoever can reproduce the empirically verified predictions of QM while at the same time satisfying both locality and separability. Since QM satisfies parameter independence, the violation of outcome independence, and thus of separability, in the EPR-experiment is equivalent to the violation of Bell's inequality. And since the latter holds

[8] In case of "indistinguishable particles", i.e. particles with the same permanent properties (charge, mass and spin-value), this argument exerts its full force, for then the indices 1, 2 are arguably completely meaningless (see e.g. Dieks 1990).

iff the factorizability condition (10.3) is satisfied, in the case of QM outcome independence just tests whether a state ρ is a *product state* across the Boolean algebras of properties of the involved observables: if the observables to be measured on particle 1 and 2 are $A \in \mathcal{B}(\mathcal{H}_1)$ and $B \in \mathcal{B}(\mathcal{H}_2)$ respectively, then outcome independence just means that $\rho(P^A P^B)$ coincides with the product $\rho(P^A)\rho(P^B)$, where P^A and P^B are arbitrary spectral projections of A and B respectively. Thus in this case outcome independence tests the following equality

$$\rho(P^A P^B) = \rho(P^A)\rho(P^B) \tag{10.15}$$

for all $P^A \in \mathbf{B}(A)$ and $P^B \in \mathbf{B}(B)$ which, however, is just the defining equation for a product state across the Boolean algebras $\mathbf{B}(A)$ and $\mathbf{B}(B)$. Up to this point, we have a separability criterion that applies only *relative to a particular choice of observables.* The criterion (10.15) can, however, easily be generalized to a criterion that is free of such a restriction. Instead of the Boolean algebras of two observables of particle 1 and 2 one can define a product state across the whole observable algebras $\mathcal{B}(\mathcal{H}_1)$ and $\mathcal{B}(\mathcal{H}_2)$ of the two particles by

$$\rho(AB) = \rho(A)\rho(B) \tag{10.16}$$

for all $A \in \mathcal{B}(\mathcal{H}_1)$ and $B \in \mathcal{B}(\mathcal{H}_2)$. One can then speak of the separability of two systems in a state ρ simpliciter, *without mentioning any measurement context*, iff ρ is a product state across the algebras $\mathcal{B}(\mathcal{H}_1)$ and $\mathcal{B}(\mathcal{H}_2)$ of the two systems.[9] For convenience we will in the following simple speak of the (non-) separability of states and not of "systems in a state".

We will now argue that (10.16) is already the most general ontological separability criterion for quantum mechanical system. More precisely, we will argue that the "more general" criterion, often to be found in the literature (see e.g. Auletta 2001, p. 53, Werner 1989, Clifton and Halvorson 2000), according to which a state ρ is separable even if ρ is a convex combination of product states across $\mathcal{B}(\mathcal{H}_1)$ and $\mathcal{B}(\mathcal{H}_2)$, either does not faithfully express the separability principle at all, or is not a generalization of the above separability criterion

[9] This makes sense because if, according to this criterion, two systems are non-separable, there are (at least) two observables A and B of systems 1 and 2 such that their spectral projections do not satisfy equation (10.15) and thus the two systems are not separable *with respect to A and B.*

in an ontologically interesting sense. First, if ρ is a convex combination of product states across $\mathcal{B}(\mathcal{H}_1)$ and $\mathcal{B}(\mathcal{H}_2)$, there are states ρ_1^1, ρ_2^1 on $\mathcal{B}(\mathcal{H}_1)$ and ρ_1^2, ρ_2^2 on $\mathcal{B}(\mathcal{H}_2)$ such that

$$\rho(AB) = c\rho_1^1(A)\rho_1^2(B) + (1-c)\rho_2^1(A)\rho_2^2(B) \text{ with } 0 \leq c \leq 1 \tag{10.17}$$

for all $A \in \mathcal{B}(\mathcal{H}_1)$ and $B \in \mathcal{B}(\mathcal{H}_2)$. In the trivial case $c = 0$ (1), $\rho(AB)$ equals $\rho_2^1(A)\rho_2^2(B)$ ($\rho_1^1(A)\rho_1^2(B)$), so that the states ρ_2^1 and ρ_2^2 (ρ_1^1 and ρ_1^2) are just ρ's restrictions to the algebras of system 1 and 2 respectively. Thus in this case ρ is itself a product state, so that henceforth we need only discuss the case $0 < c < 1$. In this case the restrictions of ρ, representing the states of the subsystems 1 and 2 respectively, are given by

$$\rho|_{\mathcal{B}(\mathcal{H}_1)} = c\rho_1^1 + (1-c)\rho_2^1 \tag{10.18}$$

$$\rho|_{\mathcal{B}(\mathcal{H}_2)} = c\rho_1^2 + (1-c)\rho_2^2. \tag{10.19}$$

Thus (10.18) and (10.19) are the states ascribed to the systems 1 and 2 which together are in the total state ρ (see also the end of the last section). Now the separability principle requires that these states of 1 and 2 are distinct and together wholly determine the total state. Yet at least the last requirement is *not* fulfilled because the restrictions (10.18) and (10.19) are also compatible with the total state being the product state

$$\rho'(AB) \equiv \rho|_{\mathcal{B}(\mathcal{H}_1)}(A)\rho|_{\mathcal{B}(\mathcal{H}_2)}(B) \tag{10.20}$$

build from $\rho|_{\mathcal{B}(\mathcal{H}_1)}$ and $\rho|_{\mathcal{B}(\mathcal{H}_2)}$ instead of being the convex combination (10.17).[10] Thus the only way to safe the generalized criterion from not being expressing the separability principle at all, seems to invoke an ignorance interpretation. This is to say that the convex combination (10.17) is *not* the true state of a single (compound) system $1+2$, but merely describes a mixture of such compound systems, a fraction c of which is in the product state $\rho_1^1(A)\rho_1^2(B)$ and a fraction $1-c$ of which is in the product state $\rho_2^1(A)\rho_2^2(B)$. This is moreover the way the generalized criterion (10.17) is usually motivated: one acknowledges that systems in product states are separable and then argues that a

[10] That the product state (10.20) in fact differs form the convex combination (10.17) can easily be seen by inserting (10.18) and (10.19) into (10.20) and multiply them together.

classical mixing procedure by which an ensemble of such systems is build does not change the individual states of the systems and thus their separability by reason of being in a product state (see e.g. Auletta 2001, p. 53; Werner 1989). This is certainly true, and in case of such a mixture an ignorance interpretation is obviously appropriate (see also Section 5.2). But then this generalization of the separability criterion (10.16) is merely a conceptual one *without any further ontological consequences*, since each individual system of the mixture is judged to be separable by reason of the above criterion (10.16).

The way in which the criterion (10.16) is generalized by invoking convex combinations of product states, is therefore, in particular, of no use for our purposes because states of the form referred to in (10.17) cannot be used to represent Whiteheadian activities. For this to be possible, the probabilities ascribed on the basis of a state need to be interpretable as ontic single case probabilities—as propensities. And this is clearly in conflict with an ignorance interpretation as just mentioned.

10.2.5 Non-separability in AQFT

A general difference to QM is that in QFT the total Hilbert space $\mathcal{H}$ does, in general, not possess a tensor product structure. This particularly prohibits the common conceptual minimal understanding of a "system" in terms of a tensor product factor $\mathcal{H}_i$ of the total Hilbert space $\mathcal{H}$ together with the former's corresponding observable algebra $\mathcal{B}(\mathcal{H}_i)$ as known from QM. Because of the lack of this structure, in QFT the conceptual minimal understanding of a system is usually taken to be provided by a spacetime region $\mathcal{O}$ together with its associated algebra of observables $\mathcal{R}(\mathcal{O})$ (see e.g. Dieks 2000 and 2001). Note that the region $\mathcal{O}$ need not be bounded, since as explained in Appendix C, von Neumann subalgebras $\mathcal{R}(\mathcal{O})$ of $\mathcal{B}(\mathcal{H})$ can also be associated in a canonical way with unbounded regions $\mathcal{O} \subseteq M$, given the fundamental correspondence $\mathcal{O} \mapsto \mathcal{R}(\mathcal{O})$ for bounded regions. The analog of a quantum mechanical system consisting of two subsystems is thus, in general, provided by two separated (not necessarily bounded) regions $\mathcal{O}_1$ and $\mathcal{O}_2$ whose algebras $\mathcal{R}(\mathcal{O}_1)$ and $\mathcal{R}(\mathcal{O}_2)$ commute. The total algebra of the compound system is then given by the smallest von Neumann algebra generated by $\mathcal{R}(\mathcal{O}_1)$ and

$\mathcal{R}(\mathcal{O}_2)$, i.e. by $\mathcal{R}(\mathcal{O}_1 \cup \mathcal{O}_2) = (\mathcal{R}(\mathcal{O}_1) \cup \mathcal{R}(\mathcal{O}_2))''$ (see Appendix C). Note that the commutativity of two algebras with one another is generally secured only if the corresponding regions are *spacelike* separated to one another. Therefore, mutual commutativity has to be required independently if the regions are not spacelike separated. Without the commutativity of $\mathcal{R}(\mathcal{O}_1)$ and $\mathcal{R}(\mathcal{O}_2)$ with one another, the regions $\mathcal{O}_1$ and $\mathcal{O}_2$ together with the algebras $\mathcal{R}(\mathcal{O}_1)$ and $\mathcal{R}(\mathcal{O}_2)$, would not provide adequate substitutes for two quantum mechanical systems as defined by the tensor product structure of a quantum mechanical Hilbert space $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$, because the algebras $\mathcal{B}(\mathcal{H}_1)$ and $\mathcal{B}(\mathcal{H}_2)$ corresponding to the latter do always commute.

As already mentioned in Section 5.2, the algebras $\mathcal{R}(\mathcal{O})$ corresponding to bounded regions $\mathcal{O}$ are von Neumann algebras of type III and therefore do not contain any non-zero, finite-dimensional projections, which in turn implies that there cannot be pure states on local algebras $\mathcal{R}(\mathcal{O})$. For the existence of a pure state ρ on $\mathcal{R}(\mathcal{O})$ would imply that there is a one-dimensional projection $P \in \mathcal{R}(\mathcal{O})$ that generates ρ via $\rho = Tr(P\cdot)$ or equivalently via $\rho = \langle\psi, \cdot\psi\rangle$ where ψ is the unit vector that spans the one-dimensional subspace of $\mathcal{H}$ onto which P projects. *Thus all states on any local algebra are non-pure.* Furthermore, a non-pure state on such a local algebra $\mathcal{R}(\mathcal{O})$ is *not generated by a density operator belonging to* $\mathcal{R}(\mathcal{O})$, because $\mathcal{R}(\mathcal{O})$ does not contain such operators either. In sum, then, each state ρ on a local algebra $\mathcal{R}(\mathcal{O})$ is non-pure (i.e. there are other (non-pure) states ρ_1 and ρ_2 on $\mathcal{R}(\mathcal{O})$ such that ρ is their convex combination), but there is no density operator $W \in \mathcal{R}(\mathcal{O})$ that generates ρ via $\rho = Tr(W\cdot)$.[11] For unbounded regions $\mathcal{O}$ to which an algebra $\mathcal{R}(\mathcal{O})$ can be associated no general result concerning their type is known. However, one knows that some of these algebras are also of type III whereas others are not. Thus not only for bounded but also for some unbounded regions, the corresponding algebras do not possess any pure states nor are their states generated by projections or, more general, by density operators contained in them.

[11] Yet, as already mentioned in Section 5.2, it is nevertheless true that each extension of ρ to $\mathcal{B}(\mathcal{H})$ is generated by some density operator $W \in \mathcal{B}(\mathcal{H})$ or even by some one dimensional projection $P \in \mathcal{B}(\mathcal{H})$, the latter being the case if ρ is the restriction of a pure state on $\mathcal{B}(\mathcal{H})$ to $\mathcal{R}(\mathcal{O})$.

However, these conceptual differences to QM do not affect the conclusions about Bell's inequality and its violation, drawn in the last sections. As in QM, in QFT too, there are many states leading to a violation of Bell's inequality and such a violation can likewise by understood as a violation of the separability principle. In particular, it has been shown that QFT obeys parameter independence but violates outcome independence (see e.g. Butterfield 1994). In a sense non-separability is even more generic in QFT than in QM. One of the ontologically most interesting aspects of this is that according to QFT even the vacuum violates Bell's inequality and thus is non-separable (Summers and Werner 1985 and 1987). In QM this is trivially not the case because QM simply does not contain a non-trivial vacuum state at all. The only candidate for a quantum mechanical description of the vacuum is the trivial state $\rho = \langle 0, \cdot 0 \rangle$ (which is obviously separable)—QM treats the vacuum purely classically, as "empty nothingness". Contrary to this, QFT shows that the world is non-separable even in the absence of any "stable configurations of matter".

Now let us see how the separability criterion (10.16) can be reformulated within the framework of AQFT. Since this criterion *does not make use of any density operator or even vector representation of states* it is quite easy to accommodate it to the situation in QFT. Let us begin with the simplest case of two bounded regions. In this case we need only replace the commuting algebras $\mathcal{B}(\mathcal{H}_1)$ and $\mathcal{B}(\mathcal{H}_2)$ of the subsystems 1 and 2 referred to in (10.16) by the commuting local algebras of two separated regions $\mathcal{O}_1, \mathcal{O}_2 \in D(M)$. Moreover, since each von Neumann algebra is generated by the projections it includes, and it is only these operators we are interested in, we can and will replace the algebras $\mathcal{B}(\mathcal{H}_1)$ and $\mathcal{B}(\mathcal{H}_2)$ by the commuting sets of local projections $\mathcal{P}(\mathcal{O}_1)$ and $\mathcal{P}(\mathcal{O}_2)$ corresponding to these regions (see also Section 10.1).[12] An appropriate quantum field theoretic reformulation of the separability criterion (10.16) for this case then reads:

(SEP1) Let the sets of projections $\mathcal{P}(\mathcal{O}_1)$ and $\mathcal{P}(\mathcal{O}_2)$ corresponding to the separated regions $\mathcal{O}_1, \mathcal{O}_2 \in D(M)$ commute, then a state ρ is separable across

[12] Commutativity is required for the very notion of a product state to make sense at all, since $\rho(PQ) = \rho_1(P)\rho_2(Q)$ implies $\rho(PQ) = \rho_2(Q)\rho_1(P)$ and thus too, $\rho(PQ) = \rho(QP)$ which will generally not be satisfied for all non-commuting projections P and Q from $\mathcal{B}(\mathcal{H}_1)$ and $\mathcal{B}(\mathcal{H}_2)$ or $\mathcal{R}(\mathcal{O}_1)$ and $\mathcal{R}(\mathcal{O}_2)$ respectively.

$\mathcal{O}_1$ and $\mathcal{O}_2$ iff ρ is a product state across $\mathcal{P}(\mathcal{O}_1)$ and $\mathcal{P}(\mathcal{O}_2)$, i.e. iff there are states ρ_1 on $\mathcal{P}(\mathcal{O}_1)$ and ρ_2 on $\mathcal{P}(\mathcal{O}_2)$, such that

$$\rho(PQ) = \rho_1(P)\rho_2(Q)$$

for all $P \in \mathcal{P}(\mathcal{O}_1)$ and $Q \in \mathcal{P}(\mathcal{O}_2)$.

It is clear that the states ρ_1 and ρ_2 are just the restrictions of ρ to the sets $\mathcal{P}(\mathcal{O}_1)$ and $\mathcal{P}(\mathcal{O}_2)$, i.e. $\rho_i = \rho|_{\mathcal{P}(\mathcal{O}_i)}$.

Equipped with this criterion of separability we can now express in a mathematical rigorous way when an activity is (in-) divisible (see particularly Section 2.6.1). We assume that the activities which are divisible are just those which are represented by separable states, or more precisely, an activity represented by the state ρ is divisible into two partial activities iff there are spacelike separated regions $\mathcal{O}_1, \mathcal{O}_2 \in D(M)$ such that ρ is separable across the associated sets of projections.[13] The partial activities into which a divisible activity will be in fact divided after this division in fact took place, are then represented by the restrictions $\rho|_{\mathcal{P}(\mathcal{O}_1)}$ and $\rho|_{\mathcal{P}(\mathcal{O}_2)}$ of ρ to the projections of these regions. On the other hand, we have argued in Section 2.4.1 that the restrictions $\rho|_{\mathcal{P}(\mathcal{O}_i)}$ can generally be interpreted as perspectives onto the activity represented by ρ, and that the stronger interpretation as distinct parts of this activity is allowed by QFT only under certain specific conditions. These specific conditions are just given by the separability of ρ across the spacelike separated regions $\mathcal{O}_1$ and $\mathcal{O}_2$. In this case $\rho|_{\mathcal{P}(\mathcal{O}_1)}$ and $\rho|_{\mathcal{P}(\mathcal{O}_2)}$ can be interpreted as distinct parts of, rather than merely as perspectives onto, the activity represented by ρ. This, moreover, means that the activities $\rho|_{\mathcal{P}(\mathcal{O}_1)}$ and $\rho|_{\mathcal{P}(\mathcal{O}_2)}$ can be understood as being located in the regions $\mathcal{O}_1$ and $\mathcal{O}_2$ respectively: according to Section 2.3.1 an activity can be said to be located in a particular region if the activity makes decisions for this and no other region. And since $\rho|_{\mathcal{P}(\mathcal{O}_1)}$ and $\rho|_{\mathcal{P}(\mathcal{O}_2)}$ are distinct activities their decisions are completely independent from one another and thus each settles decisions for precisely one of the two regions $\mathcal{O}_1$ and $\mathcal{O}_2$

[13]Note that since the (in-) divisibility of activities always refers to the spacetime regions belonging to a single layer of actuality, we need only investigate spacelike separated regions (see Section 2.7.2). And in this case the spacelike commutativity axiom of AQFT automatically secures that the local algebras associated with these regions do always commute with one another, so that this commutativity need not be required independently.

only. Thus as soon as the activity ρ is divided across the regions $\mathcal{O}_1$ and $\mathcal{O}_2$, i.e. as soon as it is bifurcated, the corresponding partial activities $\rho|_{\mathcal{P}(\mathcal{O}_1)}$ and $\rho|_{\mathcal{P}(\mathcal{O}_2)}$ can be said to be located in the regions $\mathcal{O}_1$ and $\mathcal{O}_2$ respectively. And since each of these regions is in fact a single connected region, the activities $\rho|_{\mathcal{P}(\mathcal{O}_1)}$ and $\rho|_{\mathcal{P}(\mathcal{O}_2)}$ are then also particularized. Note that it makes no sense to speak of the locatedness of the activities represented by $\rho|_{\mathcal{P}(\mathcal{O}_1)}$ and $\rho|_{\mathcal{P}(\mathcal{O}_2)}$ *before* the bifurcation of ρ actually occurs. Of course, the phase in which this bifurcation occurs and the partial activities which will result from it are completely determined by the respective efficient causes of ρ. But as long as ρ is not actually divided, $\rho|_{\mathcal{P}(\mathcal{O}_1)}$ and $\rho|_{\mathcal{P}(\mathcal{O}_2)}$ are not distinct activities at all, but merely different perspectives—corresponding to the regions $\mathcal{O}_1$ and $\mathcal{O}_2$—onto the one undivided activity ρ (see Section 10.1). And it makes no sense to speak of the "locatedness of the perspective $\rho|_{\mathcal{P}(\mathcal{O}_1)}$ in the region $\mathcal{O}_1$", because a perspective is a genuinely relational entity, and as a relation between the region $\mathcal{O}_1$ and the undivided activity ρ, $\rho|_{\mathcal{P}(\mathcal{O}_1)}$ can hardly be located in $\mathcal{O}_1$.

Now (SEP1) can be generalized to countably many regions in the following way:

(SEP2) Let the sets of all projections $\mathcal{P}(\mathcal{O}_i)$ corresponding to the separated regions $\{\mathcal{O}_i\} \subset D(M)$ mutually commute with one another, then a state ρ is separable across $\{\mathcal{O}_i\}$ iff ρ is a product state across $\{\mathcal{P}(\mathcal{O}_i)\}$, i.e. iff there are states ρ_i on $\mathcal{P}(\mathcal{O}_i)$, such that

$$\rho(\prod_i P_{j_i}) = \prod_i \rho_i(P_{j_i})$$

for all $P_{j_i} \in \mathcal{P}(\mathcal{O}_i)$ and all i.

Again it is clear that ρ_i is just the restriction of ρ to the local algebra of region $\mathcal{O}_i$. Now the first activity that can be divided into partial or even particular activities is the activity $\omega^c_{W_{s-1}}$ at the beginning of the concrescence process of stage s. The earlier activities $\omega_{W_{s-1}}$ and $\omega^d_{W_{s-1}}$ are necessarily undivided for otherwise one could not account for the necessary coordination of the regions for the next stage, i.e. $s+1$, of the world-process (see Sections 2.5.3 and 2.6.1). By means of the separability criterion (SEP2) and the representations of the activities $\omega_{W_{s-1}}$ and $\omega^d_{W_{s-1}}$by the state ω_{s-1} and its restriction

$\omega_{s-1}|_{\wedge_{i(s)}\mathcal{P}(\mathcal{O}_{i(s)})}$ (see Section 10.1), this assumption of our modified version of Whitehead's ontology is thus, in principle, testable within the formalism of AQFT. Unfortunately, no general results about the (non-) separability of states of the form ω_{s-1} across spacelike separated regions is known. But there are some indications pointing into the direction of their *non*-separability (see Clifton and Halvorson 2001, p. 26 ff), so that our assumption in regard to this point seems not to be unjustified.

In Section 11.1 we will generalize the separability criterion (SEP2), so that it also covers the case of an activity that is divisible into *partial but not necessarily particularized* activities. However, for the following investigation of the question that has been left unanswered in Section 10.1, namely how a set of mutually spacelike separated regions $\{\mathcal{O}_{i(s)}\}$ in the future of W_{s-1} is determined in the dative phase of the transition process at stage s of the world-process, (SEP2) is sufficient.

10.3 The dative phase II

In Whitehead's original theory already the activity ω_{s-1} at the beginning of the dative phase at stage s of the world-process is supposed to be divided into partial activities $\omega_{W_{s-1}(i)}$ according to the division of actuality W_{s-1} into different actual worlds $W_{s-1}(i)$. Whitehead's intention, at least with respect to our cosmic epoch, was that these assumptions would reconcile his theory with the prohibition of superluminal causal influences, assumed to be implied by STR. As we have argued in Sections 2.5.1 and 2.5.2 this hope is flawed. According to our modified version of Whitehead's ontology the activity ω_{s-1} at the beginning of the dative phase is indivisible (see Sections 2.5.3 and 2.6.1). In this section we are searching for a rule that allows the determination of a collection of spacelike separated regions $\{\mathcal{O}_i\}$ in the future of W_{s-1} on the basis of this activity. Note that such a set $\{\mathcal{O}_i\}$ can consist of at most countably many regions because all of them are finitely extended.

In the following we will propose a rule that, in a sense, makes the gap between our modified ontology, on the one hand, and Whitehead's original theory and STR, on the other hand, *as small as possible.* As just mentioned Whitehead's original theory assumes that the initial activity $\omega_{W_{s-1}}$ (as well as all later activities) at stage s are divided into partial activities $\omega_{W_{s-1}(i)}$. If moreover superluminal causation shall be prohibited, the corresponding actual world $W_{s-1}(i)$ must contain only those occasions from W_{s-1} which lie within the backward lightcone $V_-(\mathcal{O}_i)$ of region $\mathcal{O}_i$. The activity $\omega_{W_{s-1}(i)}$ for the dative phase of the i-th transition process has therefore to be represented by the restriction of the state $\omega_{s-1}^{V_-(\mathcal{O}_i)}$ to set of projections $\mathcal{P}(\mathcal{O}_i)$ associated with region $\mathcal{O}_i$, where $\omega_{s-1}^{V_-(\mathcal{O}_i)}$ denotes the state that is created from the vacuum state due to the conditioning by (the projections representing the final definiteness of) those occasions from W_{s-1} which lie within the backward lightcone $V_-(\mathcal{O}_i)$ of region $\mathcal{O}_i$. Thus the partial activity $\omega_{W_{s-1}(i)}$ is to be represented by the state

$$\omega_{s-1}^{V_-(\mathcal{O}_i)}|_{\mathcal{P}(\mathcal{O}_i)}. \tag{10.21}$$

Since according to Whitehead the total activity $\omega_{W_{s-1}}$ for the dative phases of all transition processes at stage s is supposed to be *divided* into the partial activities $\omega_{W_{s-1}(i)}$, it has accordingly to be represented by the product state across the regions $\{\mathcal{O}_i\}$ build from the states (10.21) for all i, i.e. by the state[14]

$$\prod_i \omega_{s-1}^{V_-(\mathcal{O}_i)}|_{\mathcal{P}(\mathcal{O}_i)}. \tag{10.22}$$

Thus in this case the restricted states (10.21) *do not merely represent perspectives* onto the total activity $\omega_{W_{s-1}}$ (represented by (10.22)) *but rather parts of it* (see also Section 10.1)—though not yet particularized parts, because the latter require the realization of the corresponding regions $\mathcal{O}_i$.

Our proposal for the choice of regions is now as follows: from all the sets of mutually spacelike separated regions in the future of W_{s-1}, that set shall be realized in the dative phase of the collective transition process at stage s, across which the initial activity $\omega_{W_{s-1}}$ is "as close as possible" to the hypothetical divided activity represented by the product state (10.22). The closeness of two activities can be mathematically expressed by means of the norm distance between the representing states (see Appendix B.3). Since the undivided activity $\omega_{W_{s-1}}$ at the beginning of the dative phase of the one collective transition process that actually takes place instead of the hypothetical, distinct processes,

[14]The arguments $P_i \in \mathcal{P}(\mathcal{O}_i)$ of the partial states $\omega_{s-1}^{V_-(\mathcal{O}_i)}|_{\mathcal{P}(\mathcal{O}_i)}$ need not be mentioned explicitely here, because this information is already inherent in the restrictions "$\ldots|_{\mathcal{P}(\mathcal{O}_i)}$".

is represented by the state ω_{s-1}, we end up with the following mathematical expression of this proposal:

(REG) Let $\mathcal{S}_s$ be the set of all sets of mutually spacelike separated regions from $D(M)$, which lie in the future of W_{s-1}. Then that set of regions $\{\mathcal{O}_i\} \in \mathcal{S}_s$ shall be realized in stage s, for which

$$\left\| \omega_{s-1} - \prod_i \omega_{s-1}^{V_-(\mathcal{O}_i)}|_{\mathcal{P}(\mathcal{O}_i)} \right\| \tag{10.23}$$

is minimal.[15]

Thus this rule says that the new regions $\{\mathcal{O}_{i(s)}\}$ of layer L_s are to be chosen in such a way that the initial activity $\omega_{W_{s-1}}$ (represented by the state ω_{s-1}) at stage s is "as classical as possible", in the sense of being capable of approximation by a *divided* activity (represented by the product state (10.22)) whose partial activities are moreover *caused subluminally.* If (REG) is in fact the rule according to which nature determines the regions of occasions, this could explain why the superluminal causal influences obtaining according to our ontology (see Section 2.8.1) have not yet been observed. For the smaller the value of (10.23), the harder will be the empirical detection of such a superluminal causal link. And (REG) just requires this value at each stage of the world-process to be as small as possible.

Now one may object that this rule at best determines a set of local algebras of which we only know that they correspond to spacelike separated double cones in the future of W_{s-1}. However, as shown in Section 10.1 the algebra corresponding to a double cone fixes the latter completely. Therefore, this objection can be refuted, since if (REG) determines a unique set of local

[15] Since the product state (10.22) is only defined on operators of the form $\prod_i P_i$ with $P_i \in \mathcal{P}(\mathcal{O}_i)$, i.e. on $\wedge_i \mathcal{P}(\mathcal{O}_i)$, (10.23) likewise only evaluates the closeness of the two involved states on this domain. Thus what matters for (REG) is not the whole state ω_{s-1} but merely its restriction to $\wedge_i \mathcal{P}(\mathcal{O}_i)$. This restriction $\omega_{s-1}|_{\wedge_i \mathcal{P}(\mathcal{O}_i)}$ is then to be interpreted as the perspective of the union $\cup_i \mathcal{O}_i$ of the regions onto the activity $\omega_{W_{s-1}}$. It does not represent the activity $\omega^d_{W_{s-1}}$ left after the dative phase here, because the existence of this activity presupposes the realization of the regions $\{\mathcal{O}_i\}$ and thus must not be used for their determination. However, for the sake of not further complicating the present discussion, we did not mention this interpretational subtlety in the main text.

algebras corresponding to spacelike separated double cones in the future of W_{s-1}, this also uniquely fixes these double cones. Nevertheless, there is in fact a problem with (REG), stemming from the question whether (10.23) possesses a minimum at all, i.e. whether the approximation problem (10.23) possesses a unique solution. Unfortunately, the approximation problem (10.23) is not covered by known theorems, because all of them require the set of approximating objects to be convex (see e.g. Heuser 1986, p. 339, 574 ff). In our case this approximating set is given by the set of product states of the form (10.22) for all sets of mutually spacelike separated regions in the future of W_{s-1}, i.e. by the set

$$\left\{ \prod_i \omega_{s-1}^{V_-(\mathcal{O}_i)}|_{\mathcal{P}(\mathcal{O}_i)} : \{\mathcal{O}_i\} \in \mathcal{S}_s \right\}.$$

However, this set is obviously not convex because convex combinations of product states are not themselves product states (see also Section 10.2.4). Therefore, it is not secured that (REG) will in fact be able to determine a unique set $\{\mathcal{O}_{i(s)}\} \in \mathcal{S}_s$. Clearly, if it should turn out that (REG) is not able to fulfil its task, some other rule has to be found. On the other hand, as long as (REG) is not disproved we will for the sake of argument assume that it works in the required way.

10.4 The conformal phase

In the conformal phase of the collective transition process at stage s of the world-process each of the regions $\mathcal{O}_{i(s)}$ realized in the forgoing dative phase is provided with an initial definiteness, i.e. with a maximal abstractive hierachy of subjective eternal objects ingressed into it. The determination of this hierachy proceeds via the creative character of the activity $\omega_{W_{s-1}}^d$ left after the dative phase. In Section 8.4 we have seen that the initial maximal abstractive hierachy $H(G(O_{i(s)}))$ of subjective eternal objects compatible with the (objective eternal object $O_{i(s)}$ corresponding to) region $\mathcal{O}_{i(s)}$, is to be represented by a Boolean algebra $\mathbf{B}(\{\hat{P}_i\})$ where the classes of projections denoted by $\hat{P}_i$ (representing the subjective eternal objects which form the base $G(O_{i(s)})$ of the hierachy) satisfy the compatibility condition (CSO) with respect to the region $\mathcal{O}_{i(s)}$ and thus can according to condition (ING) ingress into this region (see Section 8.2). Condition (CSO) says that the subjective eternal object $\hat{P}_i$ is compatible with (and thus can ingress into) region $\mathcal{O}_{i(s)}$ iff there is a $P_i' \in \hat{P}_i$ with $P_i' \in \mathcal{P}(\mathcal{O}_{i(s)})$ and $P_i' \notin \mathcal{P}(\mathcal{O}')$ for all $\mathcal{O}' \in D(M)$ with $\mathcal{O}' \subset \mathcal{O}_{i(s)}$. The fulfillment of this condition by all the classes of projections $\{\hat{P}_i\}$, then, automatically implies their mutual compatibility with one another as expressed by the condition (CS) of Section 8.2. Therefore, our task in this section will be the formulation of a rule that determines, on the basis of the state $\omega_{s-1}|_{\wedge_{i(s)}\mathcal{P}(\mathcal{O}_{i(s)})}$ representing the relevant activity $\omega^d_{W_{s-1}}$, a Boolean algebra $\mathbf{B}(\{P_i\})$ of local projections for each of the regions $\mathcal{O}_{i(s)}$ realized in the forgoing dative phase, such that all the projections P_i satisfy $P_i \in \mathcal{P}(\mathcal{O}_{i(s)})$ and $P_i \notin \mathcal{P}(\mathcal{O}')$ for all $\mathcal{O}' \in D(M)$ with $\mathcal{O}' \subset \mathcal{O}_{i(s)}$. In this case the corresponding subjective eternal objects represented by the classes

$$\hat{P}_i = \left\{ U(g) P_i U(g)^{-1} : g \in \mathcal{P}_+^\uparrow \right\} \tag{10.24}$$

build from these projections (see Section 8.2) are compatible with region $\mathcal{O}_{i(s)}$ and thus too, mutually compatible with one another, so that the corresponding Boolean algebra of classes $\mathbf{B}(\{\hat{P}_i\})$ can be taken to represent the unique maximal hierachy of subjective eternal objects ingressed into the region $\mathcal{O}_{i(s)}$ in the conformal phase of the transition process at stage s.

First of all, there is a well-known rule, formulated by R. Clifton in the context of a proposal for a modal interpretation of AQFT, that uniquely determines a Boolean algebra of local projections from $\mathcal{P}(\mathcal{O})$ on the basis of a given state ρ (Clifton 2000). This rule makes use of the so-called *centralizer*, term it $\mathcal{C}_{\rho,\mathcal{P}(\mathcal{O})}$, of the state ρ with respect to the set of all projections in the local algebra $\mathcal{R}(\mathcal{O})$, consisting of all $P \in \mathcal{P}(\mathcal{O})$ such that

$$\rho(PQ) = \rho(QP) \text{ for all } Q \in \mathcal{P}(\mathcal{O}).$$

The *center* of the centralizer $\mathcal{C}_{\rho,\mathcal{P}(\mathcal{O})}$, term it $\mathcal{Z}(\mathcal{C}_{\rho,\mathcal{P}(\mathcal{O})})$, consists of all elements of $\mathcal{C}_{\rho,\mathcal{P}(\mathcal{O})}$ which commute with all other elements in $\mathcal{C}_{\rho,\mathcal{P}(\mathcal{O})}$, i.e.

$$\mathcal{Z}(\mathcal{C}_{\rho,\mathcal{P}(\mathcal{O})}) = \left\{ P \in \mathcal{C}_{\rho,\mathcal{P}(\mathcal{O})} : PQ = QP \text{ for all } Q \in \mathcal{C}_{\rho,\mathcal{P}(\mathcal{O})} \right\},$$

and is a *commutative* subset of $\mathcal{P}(\mathcal{O})$. Each commutative set of projections determines a unique Boolean algebra, namely the smallest Boolean algebra including this set. Therefore, the smallest Boolean algebra including $\mathcal{Z}(\mathcal{C}_{\rho,\mathcal{P}(\mathcal{O})})$, term it $\mathbf{B}_{\rho,\mathcal{P}(\mathcal{O})}$, is uniquely determined by $\mathcal{P}(\mathcal{O})$ and the state ρ. Because of the one-to-one correspondence between double cones and the sets of projections contained in their local algebras, in case of $\mathcal{O}$ being a double cone, this Boolean algebra is therefore in effect solely determined by $\mathcal{O}$ and ρ. Thus by letting ρ be the state $\omega_{s-1}|_{\wedge_{i(s)}\mathcal{P}(\mathcal{O}_{i(s)})}$ representing the activity $\omega^d_{W_{s-1}}$ after the dative phase and $\mathcal{O}$ be one of the regions $\mathcal{O}_{i(s)}$ realized in this dative phase, we get a unique Boolean algebra of local projections for each of the regions $\mathcal{O}_{i(s)}$.

However, this rule does not already satisfy all our needs. First, the Boolean algebra $\mathbf{B}_{\rho,\mathcal{P}(\mathcal{O})}$ determined by this rule, will, in general, not be generated by a resolution of the identity $\{P_i\}$ and thus will not contain at most countably many elements. But as explained in Section 8.4 uncountable Boolean algebras cannot be used for the representation of abstractive hierachies. Thus we have to supplement the "Clifton-rule" by the constraint that the Boolean algebra $\mathbf{B}_{\rho,\mathcal{P}(\mathcal{O})}$ is generated by a resolution of the identity or equivalently that it contains at most countably many elements. A second problem concerns the fact that the Clifton-rule only secures that the projections in $\mathbf{B}_{\rho,\mathcal{P}(\mathcal{O})}$ are contained in $\mathcal{P}(\mathcal{O})$ but not that they are not also contained in the local algebra of any subregion $\mathcal{O}' \subset \mathcal{O}$. The latter is, however, needed for the compatibility of the subjective eternal objects $\hat{P}$ build from the projections $P \in \mathbf{B}_{\rho,\mathcal{P}(\mathcal{O})}$ with the region $\mathcal{O}$ (see above). Thus the Clifton-rule has to be supplemented with this further requirement too, for being able to satisfy the needs of our interpretation. The two supplementary requirements needed to accommodate the Clifton-rule to our interpretational framework obviously have the effect that some regions to which a Boolean algebra $\mathbf{B}_{\rho,\mathcal{P}(\mathcal{O})}$ would be associated according to the bare Clifton-rule will not get one. Thus some of the regions created in the dative phase may not in fact get an initial and thus too not, a final definiteness, or equivalently, they are merely equipped with the *trivial* definiteness provided by the trivial subjective eternal object $\hat{\mathbf{1}}$. Consequently, no decisions have to be settled for them during the following phases of concrescence—it is already decided that they will merely be *empty spacetime regions devoid of any qualitative character*. Note that such empty spacetime regions are nevertheless distinct from mere unrealized extension—contrary to the latter the former have a definite boundary surface by reason of which they are relata of metrical relations to other regions (see Sections 2.1, 2.2.1 and 2.2.2). Although the possibility of occasions which are merely empty spacetime regions *devoid of any qualitative determinateness* has probably not been intended by Whitehead (see PR, p. 56, 92, 314), the existence of such occasions seems not to be in conflict with any other aspects of his ontology or with empirical facts.

However, there is still another problem with the Clifton-rule, that has been pointed out by Clifton himself (Clifton 2000, p. 13 ff). For there is a whole class of states ρ for which the Boolean algebra $\mathbf{B}_{\rho,\mathcal{P}(\mathcal{O})}$ consists merely of the trivial projections $\mathbf{0}$ and $\mathbf{1}$. The states in question are those which are *ergodic* with respect to $\mathcal{P}(\mathcal{O})$. Such states have the property that their centralizer with respect to $\mathcal{P}(\mathcal{O})$ is trivial, i.e. $\mathcal{C}_{\rho,\mathcal{P}(\mathcal{O})} = \{\mathbf{0}, \mathbf{1}\}$. Consequently, the center $\mathcal{Z}(\mathcal{C}_{\rho,\mathcal{R}(\mathcal{O})})$ of the latter is likewise trivial, so that the only projections contained in it, and thus too in $\mathbf{B}_{\rho,\mathcal{P}(\mathcal{O})}$, are in fact $\mathbf{0}$ and $\mathbf{1}$. The all important question is therefore whether the state we are using to represent the activity $\omega^d_{W_{s-1}}$ after the dative phase is ergodic with respect to the sets of all projections $\mathcal{P}(\mathcal{O}_{i(s)})$ or equivalently the local algebras $\mathcal{R}(\mathcal{O}_{i(s)})$ of the regions $\{\mathcal{O}_{i(s)}\}$. Now it is known that the vacuum state itself is ergodic with respect to the algebras associated with certain unbounded regions, so-called *Rindler wedges* (Longo 1979). In two special cases (namely, for free and conformal models of QFT) this ergodicity of the vacuum can even be shown to hold with respect to the algebras associated with double cones (Longo and Hislop 1982). However, whether this is generally the case is *not* known. More importantly, it is likewise unknown whether states which are created from the vacuum state by a conditioning due to local projections are ergodic with respect to double cone algebras. Since the states we are using to represent Whiteheadian activities are of the latter form (see Section 9), it is therefore an open question whether they are ergodic with respect to the local algebras of double cones (or any other regions). If these states were ergodic with respect to double cone algebras, the Clifton-rule were of no use for our interpretation, because it would merely associate the trivial Boolean algebra $\{\mathbf{0}, \mathbf{1}\}$ to *every* region created in the course of the world-process. In this case clearly some other rule had to be formulated that determines non-trivial Boolean algebras of local projections at least for some regions $\mathcal{O} \in D(M)$. However, for the sake of argument (and the lack of any better alternative) we will assume that the Clifton-rule is not rendered irrelevant for our interpretation by the structure of the states we use to represent activities.

The activity that is relevant for the determination of the abstractive hierachies of subjective eternal objects to the regions $\{\mathcal{O}_{i(s)}\}$ is the outcome activity $\omega^d_{W_{s-1}}$ of the dative phase. Therefore, what is relevant for the determination of the abstractive hierachy for a *single* of these regions, $\mathcal{O}_{i(s)}$ say, is the perspective of $\mathcal{O}_{i(s)}$ onto the undivided activity $\omega^d_{W_{s-1}}$, because it expresses the causal efficiency of the common past for region $\mathcal{O}_{i(s)}$ *abstracted from that for all the other regions* (see Section 10.1). Thus from the point of view of our interpretation as put forward so far, only the state

$$(\omega_{s-1}|_{\wedge_{i(s)}\mathcal{P}(\mathcal{O}_{i(s)})})|_{\mathcal{P}(\mathcal{O}_{i(s)})} = \omega_{s-1}|_{\mathcal{P}(\mathcal{O}_{i(s)})}$$

representing this perspective onto the activity $\omega^d_{W_{s-1}}$ (where the latter is represented by the state $\omega_{s-1}|_{\wedge_{i(s)}\mathcal{P}(\mathcal{O}_{i(s)})}$) should matter for the determination of the Boolean algebra of region $\mathcal{O}_{i(s)}$. And as is clear from the construction of the Boolean algebra $\mathbf{B}_{\rho,\mathcal{P}(\mathcal{O})}$ due to the Clifton-rule, it is in fact only the restriction of the state ρ to the set $\mathcal{P}(\mathcal{O})$ that matters for $\mathbf{B}_{\rho,\mathcal{P}(\mathcal{O})}$.

Therefore, we propose as the rule for the determination of abstractive hierachies of subjective eternal objects—of the initial definiteness—for the regions $\{\mathcal{O}_{i(s)}\}$ created in the dative phase, the Clifton-rule supplemented by the two further constraints mentioned above:

(DEF) The maximal abstractive hierachy $H(G(\mathcal{O}_{i(s)}))$ of subjective eternal objects ingressed into the region $\mathcal{O}_{i(s)}$ in the conformal phase of the transition process at stage s is represented by the Boolean algebra consisting of all classes $\hat{P}$ (of the form (10.24)) build from projections

$$P \in \mathbf{B}_{\omega_{s-1}|_{\wedge_{i(s)}\mathcal{P}(\mathcal{O}_{i(s)})},\mathcal{P}(\mathcal{O}_{i(s)})} \tag{10.25}$$

provided that (1) the latter Boolean algebra is generated by a resolution of the identity[16] included in $\mathcal{P}(\mathcal{O}_{i(s)})$, term it $\{P_i\}_{\mathcal{O}_{i(s)}}$ and (2) its elements are not contained in a local algebra $\mathcal{R}(\mathcal{O}')$ of any subregion $\mathcal{O}' \subset \mathcal{O}_{i(s)}$.

[16]As metnioned above this is equivalent to $\mathbf{B}_{\omega_{s-1}|_{\wedge_{i(s)}\mathcal{P}(\mathcal{O}_{i(s)})},\mathcal{P}(\mathcal{O}_{i(s)})}$'s consisiting of at most countably many projections.

Since according to (1) the Boolean algebra $\mathbf{B}_{\omega_{s-1}|_{\wedge_{i(s)}\mathcal{P}(\mathcal{O}_{i(s)})},\mathcal{P}(\mathcal{O}_{i(s)})}$ is generated by the resolution of the identity $\{P_i\}_{\mathcal{O}_{i(s)}}$ and thus is simply $\mathbf{B}(\{P_i\}_{\mathcal{O}_{i(s)}})$. It represents the possible ingressions into region $\mathcal{O}_{i(s)}$ of the subjective eternal objects contained in $H(G(\mathcal{O}_{i(s)}))$. The set of classes, term it $\{\hat{P}_i\}_{\mathcal{O}_{i(s)}}$, that corresponds to the resolution $\{P_i\}_{\mathcal{O}_{i(s)}}$ of the identity $\mathbf{1}$, then, represents the base $G(\mathcal{O}_{i(s)})$ of the hierachy $H(G(\mathcal{O}_{i(s)}))$ and the hierachy itself is accordingly represented by the Boolean algebra

$$\mathbf{B}(\{\hat{P}_i\}_{\mathcal{O}_{i(s)}})$$

generated from $\{\hat{P}_i\}_{\mathcal{O}_{i(s)}}$.

In the terminology of Section 8.1.6, the Boolean algebra $\mathbf{B}(\{P_i\}_{\mathcal{O}_{i(s)}})$ is just the set of possible properties $\mathcal{P}_{\omega_{s-1}|_{\mathcal{P}(\mathcal{O}_{i(s)})}}$ for a system in state $\omega_{s-1}|_{\mathcal{P}(\mathcal{O}_{i(s)})}$. In that section we have argued why, in an interpretation of QT like ours that does not leave the realm of ordinary probability theory, that interprets probabilities ontically and that incorporates the conception of an actualization of possibilities, this set of possible properties *has to be* (embeddable into) a Boolean algebra. What happens in the following phases of concrescence is, again in the terminology of that earlier section, the selection of those possible properties which will in fact be “possessed respectively not possessed by the region $\mathcal{O}_{i(s)}$” at the end of this non-spatiotemporal actualization process.

Now a possible objection that could be put forward against (DEF) is that the Boolean algebra $\mathbf{B}(\{P_i\}_{\mathcal{O}_{i(s)}})$, singled out according to the supplemented Clifton-rule, represents already a set of definite rather than genuinely possible properties. In other words, the challenge is that each element of $\mathbf{B}(\{P_i\}_{\mathcal{O}_{i(s)}})$ is either already actualized (or at least determined for actualization) or ruled out for actualization respectively, rather than being a genuine possibility as required for the Whiteheadian idea of a concrescence process as a genuine actualization of formerly ontic possibilities to make sense.

First of all, since at the end of the concrescence process to follow, each element of $\mathbf{B}(\{P_i\}_{\mathcal{O}_{i(s)}})$ will either definitely be possessed or definitely not possessed by the region $\mathcal{O}_{i(s)}$, but will not have the status “indifferent” with respect to this region, the set $\mathbf{B}(\{P_i\}_{\mathcal{O}_{i(s)}})$ in fact coincides with the set of “definite properties” at the end of the concrescence. But this does not mean that it is already determined at the concrescence’s beginning *that a particular element of*

$\mathbf{B}(\{P_i\}_{\mathcal{O}_{i(s)}})$ *will be possessed respectively not possessed at its ending.* However, for this to make sense the concrescence process has to be genuinely indeterministic. In other words, the probabilities for the elements of $\mathbf{B}(\{P_i\}_{\mathcal{O}_{i(s)}})$ impressed into the creative character of the corresponding activity have to be ontic probabilities. Yet this latter point that may be challenged, since one can show that each Boolean algebra $\mathbf{B}_{\rho,\mathcal{P}(\mathcal{O})}$ determined by the Clifton-rule on the basis of a state ρ is such that this state is a *convex combination of dispersion-free* states on $\mathbf{B}_{\rho,\mathcal{P}(\mathcal{O})}$ (Clifton 2000, p. 8). Generally, that a state σ is *dispersion-free* on a Boolean algebra $\mathbf{B}$ of projections means that the probability it ascribes to each projection in $\mathbf{B}$ is either 0 or 1. And in this case each of the properties in $\mathbf{B}$ were in fact definite, i.e. either possessed or not possessed, by a system in state σ.[17] Thus if the state ρ were in fact always dispersion free on $\mathbf{B}_{\rho,\mathcal{P}(\mathcal{O})}$ there would be no room for a further actualization process to follow. Of course, Whitehead himself believed that this can never be the case (PR, p. 47, 284). But as already remarked in Section 2.3.1 it seems that it will not cause any damage to his theory if we allow for the possibility that some occasions may be *completely* other-caused. In other words, the concrescence process of some occasions may merely consist of a single phase, the phase of satisfaction, where those of the subjective eternal objects ingressed in the conformal phase which are moreover determined for unrestricted ingression, are unrestrictedly ingressed. Such an occasion comes close to an event as usually understood, because it does not include a non-spatiotemporal actualization process in its ontological constitution; rather its ontological status changes directly from possibility to (attained) actuality without the generically Whiteheadian route through actuality in attainment. Yet if this were *always* the case (DEF) were clearly undermined, because it would have the consequence that one of the genuine features of Whitehead's ontology would not be represented in the formalism of QFT. However, the Clifton-rule does *not* imply that the state ρ is always dispersion-free on $\mathbf{B}_{\rho,\mathcal{P}(\mathcal{O})}$ but merely that it is always a *convex combination of such states.* And only if this convex combination would admit an ignorance interpretation the challenge against (DEF) would be substantial. This is because in this case one could hold that each of

[17] We assume for the sake of simplicity that $\mathbf{B}$ is at most countable as required for the application in our interpretation by (DEF).

the properties in $\mathbf{B}_{\rho,\mathcal{P}(\mathcal{O})}$ is still either possessed or not, but that this ontological status is not faithfully revealed by the probabilities $0 < \rho(P) < 1$ because they are merely epistemic, i.e. include some measure of ignorance, rather than ontic (see Section 5.2). However, such an ignorance interpretation is hardly possible because *every* state can be written as such a convex combination of dispersion-free states *on each commutative subalgebra of* $\mathcal{B}(\mathcal{H})$ and thus in particular on $\mathbf{B}_{\rho,\mathcal{P}(\mathcal{O})}$ (Clifton 2000, p. 8). Therefore, if the probabilities ρ ascribes to the elements of $\mathbf{B}_{\rho,\mathcal{P}(\mathcal{O})}$ would be interpreted as epistemic rather than ontic on the basis of the fact that ρ is a convex combination of dispersion-free states on $\mathbf{B}_{\rho,\mathcal{P}(\mathcal{O})}$, *this ignorance interpretation would apply to each state on any Boolean algebra of projections.* Consequently, *all* probabilities provided by the formalism of QFT, were epistemic rather than ontic. This, however, means nothing else than that there is some underlying (not necessarily deterministic) HP-theory, that (at least in principle) eliminates the epistemic deficiencies we are committed to due to the generically incomplete description provided by QFT. Put conversely, as far as one believes that the formalism of QFT is not generically incomplete in this sense, one cannot conclude from the fact ρ is a convex combination of dispersion-free states on $\mathbf{B}_{\rho,\mathcal{P}(\mathcal{O})}$, that the probabilities the state ρ ascribes to the elements of the Boolean algebra $\mathbf{B}_{\rho,\mathcal{P}(\mathcal{O})}$ are merely epistemic. We therefore conclude that the mentioned challenge to (DEF) does not go through.

Before we can turn to the representation of concrescence processes we have to fix *the representation of the outcome activity* $\omega^c_{W_{s-1}}$ *of the conformal phase*, and thus of the whole transition process at stage s of the world-process. In Section 10.1 we have proposed to represent the reduction in the amount of activity due to the dative phase and thus the transition from the activity $\omega_{W_{s-1}}$ at the beginning of the dative phase to the activity $\omega^d_{W_{s-1}}$ after this phase, by the restriction of the state ω_{s-1} (on $\mathcal{B}(\mathcal{H})$), representing the activity $\omega_{W_{s-1}}$, to the conjunction $\wedge_{i(s)}\mathcal{P}(\mathcal{O}_{i(s)})$ of the sets of projections corresponding to the regions created in the dative phase, i.e. by

$$\omega_{s-1} \to \omega_{s-1}|_{\wedge_{i(s)}\mathcal{P}(\mathcal{O}_{i(s)})}.$$

Accordingly we will now represent the reduction in the amount of activity due to the conformal phase and thus the transition from the activity $\omega^d_{W_{s-1}}$ at the beginning of the conformal phase to the activity $\omega^c_{W_{s-1}}$ after this phase, by the restriction of the state representing the activity $\omega^d_{W_{s-1}}$ to the conjunction $\wedge_{i(s)}\mathbf{B}(\{P_i\}_{\mathcal{O}_{i(s)}})$ of all the Boolean algebras $\mathbf{B}(\{P_i\}_{\mathcal{O}_{i(s)}})$ ingressed into the regions $\{\mathcal{O}_{i(s)}\}$ in the conformal phase:

$$\omega_{s-1}|_{\omega_{s-1}|_{\wedge_{i(s)}\mathcal{P}(\mathcal{O}_{i(s)})}} \to (\omega_{s-1}|_{\wedge_{i(s)}\mathcal{P}(\mathcal{O}_{i(s)})})|_{\wedge_{i(s)}\mathbf{B}(\{P_i\}_{\mathcal{O}_{i(s)}})} \tag{10.26}$$

Since the individual Boolean algebras $\mathbf{B}(\{P_i\}_{\mathcal{O}_{i(s)}})$ are included in the sets of all projections of regions $\mathcal{O}_{i(s)}$, so that their conjunction is likewise included in the conjunction $\wedge_{i(s)}\mathcal{P}(\mathcal{O}_{i(s)})$, the state on the right hand side of (10.26) is simply

$$\omega_{s-1}|_{\wedge_{i(s)}\mathbf{B}(\{P_i\}_{\mathcal{O}_{i(s)}})}. \tag{10.27}$$

In other words, the outcome activity $\omega^c_{W_{s-1}}$ of the conformal phase, which is at the same time the activity for the initial phase of the following concrescence process, is represented by the restriction of the state ω_{s-1} to the set $\wedge_{i(s)}\mathbf{B}(\{P_i\}_{\mathcal{O}_{i(s)}})$.

Chapter 11

The representation of concrescence processes

The final task for completing our Whiteheadian interpretation of QFT is the representation of what takes place in concrescence processes.

11.1 The hierachies and activities involved in concrescence processes

In the concrescence process—that will generally start as one undivided process bifurcating into partial or particularized processes in later phases—the maximal hierachies $H(G(\mathcal{O}_{i(s)}))$ of subjective eternal objects ingressed into the regions $\mathcal{O}_{i(s)}$ are successively reduced until the initial activity $\omega^c_{W_{s-1}}$ for the concrescence process is exhausted. Let us assume that the concrescence process of stage s of the world-process consists of M phases. In each phase of this concrescence process one of the eternal objects of the initial base $G(\mathcal{O}_{i(s)})$ is eliminated for unrestricted ingression into the corresponding region $\mathcal{O}_{i(s)}$—in other words, one of the basic elements is decided to have merely restricted ingression into that region (see Section 2.2.1). Let $G_n(\mathcal{O}_{i(s)})$ for $1 \leq n \leq M$ denote the base of the hierachy *for* phase n or equivalently *after* phase $n-1$.[1] The base of the hierachy *left at the end* of the last phase (here phase number

[1] The phrase "after phase 0 of concrescence" occuring in case $n = 1$ is to be understood as meaning "after the conformal phase of transition".

M) of the concrescence is denoted by $G_{M+1}(\mathcal{O}_{i(s)})$. It is the range of subjective eternal objects which have not been eliminated in the course of the concrescence and thus have unrestricted ingression into region $\mathcal{O}_{i(s)}$ (see below). By means of the successive reduction

$$G_1(\mathcal{O}_{i(s)}) \supset G_2(\mathcal{O}_{i(s)}) \supset \cdots \supset G_M(\mathcal{O}_{i(s)}) \supset G_{M+1}(\mathcal{O}_{i(s)})$$

of the initial base $G_1(\mathcal{O}_{i(s)}) \equiv G(\mathcal{O}_{i(s)})$, consisting of those subjective eternal objects which are simple relative to the hierachy $H(G(\mathcal{O}_{i(s)}))$ (see Section 8.3.2), the latter is successively reduced too,[2]

$$H(G_1(\mathcal{O}_{i(s)})) \supset H(G_2(\mathcal{O}_{i(s)})) \supset \cdots \supset H(G_M(\mathcal{O}_{i(s)})) \supset H(G_{M+1}(\mathcal{O}_{i(s)})).$$

As proposed in Section 8.4, the base $G_1(\mathcal{O}_{i(s)})$ of the initial hierachy is to be represented by a resolution of the identity $\hat{\mathbf{1}}$. The reduced bases $G_n(\mathcal{O}_{i(s)})$ for $2 \leq n \leq M+1$ are therefore not represented by such resolutions and as explained in Section 8.4, this implies that, contrary to the initial hierachy $H(G_1(\mathcal{O}_{i(s)}))$, the later ones $H(G_n(\mathcal{O}_{i(s)}))$ can merely be represented by distributive lattices not coinciding (as sets) with Boolean algebras. More precisely, according to the last section, the initial hierachy $H(G_1(\mathcal{O}_{i(s)}))$ is to be represented by the Boolean algebra $\mathbf{B}(\{\hat{P}_i\}_{\mathcal{O}_{i(s)}})$. If $\{\hat{P}_i\}^n_{\mathcal{O}_{i(s)}}$ denotes the set representing the reduced base $G_n(\mathcal{O}_{i(s)})$ for $2 \leq n \leq M+1$, obtained from $\{\hat{P}_i\}_{\mathcal{O}_{i(s)}}$ by removing $n-1$ elements—each eliminated in one of the phases preceding phase n—then the hierachy $H(G_n(\mathcal{O}_{i(s)}))$ is represented by the distributive lattice generated from $\{\hat{P}_i\}^n_{\mathcal{O}_{i(s)}}$ (by disjunctions of elements of the latter (see Section 8.3.3)). Let us denote this distributive lattice accordingly by $\mathbf{D}(\{\hat{P}_i\}^n_{\mathcal{O}_{i(s)}})$. Note that according to the notion just introduced, the set $\{\hat{P}_i\}_{\mathcal{O}_{i(s)}}$ representing the initial base $G_1(\mathcal{O}_{i(s)}) = G(\mathcal{O}_{i(s)})$ is also denoted by $\{\hat{P}_i\}^1_{\mathcal{O}_{i(s)}}$.

Because of the non-existence of conjunctive subjective eternal objects (see Section 8.3.3) the only available complex subjective eternal in the hierachy $H(G_{M+1}(\mathcal{O}_{i(s)})$ that has *all* the elements of the finally left set $G_{M+1}(\mathcal{O}_{i(s)})$ as its components, is the one represented by the disjunction

$$\hat{P}_{i(s)} \equiv \vee_{\{\hat{P}_i\}^{M+1}_{\mathcal{O}_{i(s)}}} \hat{P}_i \tag{11.1}$$

[2]Recall that $H(G_n(\mathcal{O}_{i(s)}))$ concides with the distributive lattice generated from the base $G_n(\mathcal{O}_{i(s)})$ (see Sections 2.2.6 and, in particular, Section 8.4).

of all these elements.[3] The subjective eternal object represented by (11.1) is therefore the one that has unrestricted ingression into the region $\mathcal{O}_{i(s)}$ (see Section 2.2.6), and thus makes up the final definiteness of the corresponding occasion. Accordingly, the subjective eternal objects which have restricted ingression into region $\mathcal{O}_{i(s)}$, i.e. which have been ingressed but are eliminated in later phases of the concrescence process are those contained in the initial hierachy $H(G_1(\mathcal{O}_{i(s)}))$ different from $\hat{P}_{i(s)}$. In other words, the set of "possessed properties" of region $\mathcal{O}_{i(s)}$ is the singleton set consisting merely of the subjective eternal object (represented by) $\hat{P}_{i(s)}$ and the set of not possessed ones is given by $H(G_1(\mathcal{O}_{i(s)}))$ without $\hat{P}_{i(s)}$. Thus the set of definite, i.e. possessed or non-possessed properties, coincides with the initial hierachy $H(G_1(\mathcal{O}_{i(s)}))$, i.e. with the set of initially possible properties.

However, this does not mean that there has been no genuine "actualization of possibilities"—this would only be the case if the probabilities of the elements of $H(G_1(\mathcal{O}_{i(s)})$ would admit an ignorance interpretation and, as argued in the last section, this is not the case. All other subjective eternal objects, i.e. which do not belong to the initial hierachy $H(G_1(\mathcal{O}_{i(s)})$ are *indefinite* with respect to region $\mathcal{O}_{i(s)}$ (see Section 8.1.6).

The unrestricted ingression of the subjective eternal object (represented by) $\hat{P}_{i(s)}$ into the region $\mathcal{O}_{i(s)}$ is at the same time the actualization of the completed occasion, represented accordingly by $E_{i(s)} = (\mathcal{O}_{i(s)}, \hat{P}_{i(s)})$, by which this occasion contributes to the new manifestation ω_{W_s} of the underlying activity (see Sections 2.3.3 and 2.3.4). According to Section 9.2 this new manifestation is therefore represented by the state

$$\omega_s = \frac{\omega(W(s)^* \cdot W(s))}{\omega(W(s)^* W(s))},$$

where the operator $W(s)$ is given by

$$W(s) = \prod_{-\infty \leq t \leq s} \left(\prod_{i(t)} P_{i(t)} \right) = \left(\prod_{i(s)} P_{i(s)} \right) \left(\prod_{i(s-1)} P_{i(s-1)} \right) \cdots .$$

In Section 9.3 we have argued that although the hierachies $H(G_n(\mathcal{O}_{i(s)}))$ for $2 \leq n \leq M$ are not Boolean algebras, we should nevertheless be licensed

[3] It is just the unit element of the distributive lattice $\mathbf{D}(\{\hat{P}_i\}_{\mathcal{O}_{i(s)}}^{M+1})$ representing the final hierachy $H(G_{M+1}(\mathcal{O}_{i(s)}))$.

to interpret the creative character of the corresponding activities $\omega^{c,n}_{W_{s-1}}$ as providing propensities (perhaps within a slightly more liberal understanding of the term) for the unrestricted ingression of the eternal objects contained in $H(G_n(\mathcal{O}_{i(s)}))$. For $n = M+1$ there is no corresponding activity, because at the end of the last phase M of concrescence, there is no activity left. Therefore, $\omega^{c,M+1}_{W_{s-1}}$ is simply to be represented by the trivial state $\rho = 0$. Now by which states shall the activities $\omega^{c,n}_{W_{s-1}}$ for $2 \leq n \leq M$ be represented?

Generalizing our claim as to the representation of the reduction of the amount of an activity due to a phase of transition, that has led us to the representation of the activity left after the dative phase and the activity left after the conformal phase respectively (see Sections 10.1 and 10.4), we propose to represent the activity $\omega^{c,n}_{W_{s-1}}$ for phase n, i.e. the one left after phase $n-1$, of concrescence *by the appropriate restriction of the state representing the outcome activity* $\omega^{c,n-1}_{W_{s-1}}$ *of the immediately preceding phase.* According to equation (10.27) of Section 10.4 the initial activity for the concrescence process $\omega^{c,1}_{W_{s-1}}$ is to be represented by the state[4]

$$\omega_{s-1}|_{\wedge_{i(s)}\mathbf{B}(\{P_i\}^1_{\mathcal{O}_{i(s)}})}.$$

Consequently, the activity left after the first phase of concrescence is to be represented by the restriction of this state to the conjunction of the distributive lattices $\mathbf{D}(\{P_i\}^2_{\mathcal{O}_{i(s)}})$ (representing the hierachies after this first phase of concrescence) obtained from the Boolean algebras $\mathbf{B}(\{P_i\}^1_{\mathcal{O}_{i(s)}})$ (which represent the initial hierachies) by removing one element from the generating set $\{P_i\}^1_{\mathcal{O}_{i(s)}}$ of each, i.e. by a state of the form[5]

$$(\omega_{s-1}|_{\wedge_{i(s)}\mathbf{B}(\{P_i\}^1_{\mathcal{O}_{i(s)}})})|_{\wedge_{i(s)}\mathbf{D}(\{P_i\}^2_{\mathcal{O}_{i(s)}})}. \tag{11.2}$$

Because of the inclusions $\mathbf{D}(\{P_i\}^2_{\mathcal{O}_{i(s)}}) \subset \mathbf{B}(\{P_i\}^1_{\mathcal{O}_{i(s)}})$ the state (11.2) simplifies to

$$\omega_{s-1}|_{\wedge_{i(s)}\mathbf{D}(\{P_i\}^2_{\mathcal{O}_{i(s)}})}.$$

By iteration, the activity $\omega^{c,n}_{W_{s-1}}$ for the n-th phase of concrescence is therefore represented by the state

$$\omega_{s-1}|_{\wedge_{i(s)}\mathbf{D}(\{P_i\}^n_{\mathcal{O}_{i(s)}})}.$$

[4]Recall that $\{P_i\}^1_{\mathcal{O}_{i(s)}}$ is just another denotation for $\{P_i\}_{\mathcal{O}_{i(s)}}$.

[5]Which particular element is removed from each generating set is not important here.

In sum, then, we have represented all the abstractive hierachies and activities involved in the succeeding phases of a concrescence process by means of objects available within the formalism of AQFT.

However, a question that has been left unanswered up to this point is how we can mathematically express the divisibility or separability of an activity $\omega^{c,n}_{W_{s-1}}$ into partial *but not necessarily particularized* activities. In Section 10.2.5 we have discussed the divisibility of an activity into two partial activities *which are also particularized.* Thus in this simplest case, *divisibility coincides with particularizability.* Yet this need not generally be the case. As discussed in Section 2.6.2, generally the activity $\omega^{c,n}_{W_{s-1}}$ may be divided into two parts $\omega^{c,n}_{W_{s-1}}(\mathcal{O}_1, \ldots, \mathcal{O}_k)$ and $\omega^{c,n}_{W_{s-1}}(\mathcal{O}_{k+1}, \ldots, \mathcal{O}_N)$ *none* of which is particularized, because each makes decisions for more than one region (namely for the regions $\mathcal{O}_1, \ldots, \mathcal{O}_k$ and $\mathcal{O}_{k+1}, \ldots, \mathcal{O}_N$ respectively) and thus cannot be said to be located in a single such region. As we will argue now, this more general possibility for the divisibility of activities can, however, also be rigorously expressed in terms of the separability of the corresponding states.

The separability criterion (SEP1) from Section 10.2.5 identifies separable states across the regions $\mathcal{O}_1$ and $\mathcal{O}_2$ with the product states across the sets of all projections $\mathcal{P}(\mathcal{O}_1)$ and $\mathcal{P}(\mathcal{O}_2)$ associated with the regions. Now the generalization of this criterion to more than two regions, termed (SEP2) in Section 10.2.5, is obviously not the right kind of generalization to be able to cover the case in question, because it simply requires that the state representing the activity $\omega^{c,n}_{W_{s-1}}$ is a product state across $\mathcal{P}(\mathcal{O}_1), \ldots, \mathcal{P}(\mathcal{O}_N)$. But a product state across $\mathcal{P}(\mathcal{O}_1), \ldots, \mathcal{P}(\mathcal{O}_N)$ just represents an activity that is divisible into N *particularized* activities, rather than an activity divisible into two *partial but non-particularized* activities. However, it is clear how the latter can be expressed mathematically: one need only require that the state in question is a product state across the sets of all projections contained in the algebras $\mathcal{R}(\cup_{i=1}^{k}\mathcal{O}_i)$ and $\mathcal{R}(\cup_{i=k+1}^{N}\mathcal{O}_i)$ associated with the union of the regions $\mathcal{O}_1, \ldots, \mathcal{O}_k$ and $\mathcal{O}_{k+1}, \ldots, \mathcal{O}_N$ respectively (see Appendix C). For if this is the case it is divisible into two parts such that one corresponds collectively to the regions $\mathcal{O}_1, \ldots, \mathcal{O}_k$ and the other corresponds collectively to the regions $\mathcal{O}_{k+1}, \ldots, \mathcal{O}_N$, without presupposing anything about a further divisibility with respect to the members of the sets $\{\mathcal{O}_1, \ldots, \mathcal{O}_k\}$ and $\{\mathcal{O}_{k+1}, \ldots, \mathcal{O}_N\}$.

Now the activities in question

$$\omega^{c,n}_{W_{s-1}}, \omega^{c,n}_{W_{s-1}}(\mathcal{O}_1, \ldots, \mathcal{O}_k) \text{ and } \omega^{c,n}_{W_{s-1}}(\mathcal{O}_{k+1}, \ldots, \mathcal{O}_N)$$

are represented by the states

$$\omega_{s-1}|_{\wedge^N_{i(s)=1}\mathbf{B}(\{P_i\}_{\mathcal{O}_{i(s)}})}, \omega_{s-1}|_{\wedge^k_{i(s)=1}\mathbf{B}(\{P_i\}_{\mathcal{O}_{i(s)}})} \text{ and } \omega_{s-1}|_{\wedge^N_{i(s)=k+1}\mathbf{B}(\{P_i\}_{\mathcal{O}_{i(s)}})} \quad (11.3)$$

in case $n = 1$, and for $2 \leq n \leq M$ by the states

$$\omega_{s-1}|_{\wedge^N_{i(s)=1}\mathbf{D}(\{P_i\}^n_{\mathcal{O}_{i(s)}})}, \omega_{s-1}|_{\wedge^k_{i(s)=1}\mathbf{D}(\{P_i\}^n_{\mathcal{O}_{i(s)}})} \text{ and } \omega_{s-1}|_{\wedge^N_{i(s)=k+1}\mathbf{D}(\{P_i\}^n_{\mathcal{O}_{i(s)}})} \quad (11.4)$$

(see Sections 10.4), and thus by restrictions to domains already strictly contained in the sets of all projections in $\mathcal{R}(\cup^k_{i(s)=1}\mathcal{O}_{i(s)})$ and $\mathcal{R}(\cup^N_{i(s)=k+1}\mathcal{O}_{i(s)})$ respectively. As explained above, these more restricted domains represent the sets of possible unrestricted ingressions of the subjective eternal objects in the hierachies $H(G_n(\mathcal{O}_{i(s)}))$ $(i(s) = 1, \ldots, N\ )$ of phase n of the concrescence process in question. Consequently, the divisibility of the activity $\omega^{c,n}_{W_{s-1}}$ need only refer to these smaller domains instead of referring to the whole sets of projections contained in the algebras $\mathcal{R}(\cup^k_{i=1}\mathcal{O}_i)$ and $\mathcal{R}(\cup^N_{i=k+1}\mathcal{O}_i)$. Therefore, the activity $\omega^{c,n}_{W_{s-1}}$ is divided into the partial activities $\omega^{c,n}_{W_{s-1}}(\mathcal{O}_1, \ldots, \mathcal{O}_k)$ and $\omega^{c,n}_{W_{s-1}}(\mathcal{O}_{k+1}, \ldots, \mathcal{O}_N)$ iff, in case $n = 1$, the corresponding state is a product state across

$$(\wedge^k_{i(s)=1}\mathbf{B}(\{P_i\}^1_{\mathcal{O}_{i(s)}})) \text{ and } (\wedge^N_{i(s)=k+1}\mathbf{B}(\{P_i\}^1_{\mathcal{O}_{i(s)}}))$$

and, in case $2 \leq n \leq M$, across

$$(\wedge^k_{i(s)=1}\mathbf{D}(\{P_i\}^n_{\mathcal{O}_{i(s)}}) \text{ and } (\wedge^N_{i(s)=k+1}\mathbf{D}(\{P_i\}^n_{\mathcal{O}_{i(s)}}),$$

i.e. iff, in case $n = 1$,

$$\omega_{s-1}|_{\wedge^N_{i(s)=1}\mathbf{B}(\{P_i\}^1_{\mathcal{O}_{i(s)}})} = \omega_{s-1}|_{\wedge^k_{i(s)=1}\mathbf{B}(\{P_i\}^1_{\mathcal{O}_{i(s)}})} \cdot \omega_{s-1}|_{\wedge^N_{i(s)=k+1}\mathbf{B}(\{P_i\}^1_{\mathcal{O}_{i(s)}})}$$

and in case $2 \leq n \leq M$

$$\omega_{s-1}|_{\wedge^N_{i(s)=1}\mathbf{D}(\{P_i\}^n_{\mathcal{O}_{i(s)}})} = \omega_{s-1}|_{\wedge^k_{i(s)=1}\mathbf{D}(\{P_i\}^n_{\mathcal{O}_{i(s)}})} \cdot \omega_{s-1}|_{\wedge^N_{i(s)=k+1}\mathbf{D}(\{P_i\}^n_{\mathcal{O}_{i(s)}})}$$

holds respectively. Since the generalization of this criterion for the divisibility of activities into partial, non-particularized activities, to more than two partial activities is straightforward, it shall not be demonstrated here explicitly.

11.2 The degree of divisibility of activities

According to Section 2.5.3 (see also Section 2.6.1), the activities $\omega^{c,n}_{W_{s-1}}$ ($1 \leq n \leq M$) involved in the phases of concrescence can bifurcate into particularized or partial activities. Section 10.2.5 and the last section have provided us with mathematical expressions for the dividedness of activities into particularized or partial activities. However, it is well-known that separability/non-separability is not an "all or nothing" alternative, rather (non-) separability of states comes in degrees. There have been a lot of different proposals for quantitative measures of the (non-) separability of states (see e.g. Auletta 2001, Section 42.8; Vedral and Plenio 1998). But none of them has become universally accepted so far. Therefore, we will not enter into the discussion which of these different proposals may quantitatively fit best to the measure of (in-) divisibility of activities we will propose below. Rather we will only attempt to give a qualitative explanation for the different degrees of (non-) separability of quantum states by means of our ontology. And for this we need not know much more than that there are such different degrees.

In Section 2.6.1 we have already argued that one can quantify the degree of *particularity* of an activity with respect to a certain region, by use of the number of concrescence phases in which the activity is divided such that one of its parts is located in this region. We will now generalize this proposal to the case where none of the partial activities need to be particularized. First of all, note that because of our assumption that the "phase of bifurcation" is impressed into the creative character of the activity in question and the constancy of this creative character throughout all phases of concrescence (in the sense explained in Section 2.6.2), one can "read off" the phase of a bifurcation already from the (creative character of the) activity $\omega^{c}_{W_{s-1}}$ *at the beginning of the concrescence process.* In fact one can even read off this "information" from the earlier activities $\omega^{d}_{W_{s-1}}$ and $\omega_{W_{s-1}}$ after and at the beginning of the dative phase of transition, because they too, have the same creative character as the outcome activity $\omega^{c}_{W_{s-1}}$ of the conformal phase of transition. But contrary to the two former activities the latter is the first activity that can in fact be divided, so that it makes sense to refer to this activity when discussing degrees of (in-) divisibility.

Now let $\mathcal{O}_1, \dots, \mathcal{O}_N$ be the regions created in the transition process of stage s. The divisibility of the activity $\omega^c_{W_{s-1}}$, across the two subsets of regions $\{\mathcal{O}_1, \dots, \mathcal{O}_k\}$ and $\{\mathcal{O}_{k+1}, \dots, \mathcal{O}_N\}$ say, is the higher the earlier the phase in which it bifurcates into the partial activities corresponding to these subsets of regions. If this bifurcation takes place in the $n-1$-th phase of concrescence[6] then the resulting partial activities are accordingly $\omega^{c,n}_{W_{s-1}}(\mathcal{O}_1, \dots, \mathcal{O}_k)$ and $\omega^{c,n}_{W_{s-1}}(\mathcal{O}_{k+1}, \dots, \mathcal{O}_N)$. Note that the explicit reference to the subset of regions across which the activity $\omega^c_{W_{s-1}}$ has a certain degree of divisibility is needed because bifurcations into partial or particularized activities with respect to different subset of the regions $\mathcal{O}_1, \dots, \mathcal{O}_N$ can take place in different phases (see Section 2.6.1). Therefore, it makes, in general, no sense to speak of the degree of divisibility of an activity simpliciter, without the mentioning of regions.

For convenience let us start by introducing a measure for the degree of *in*divisibility of an activity. A measure for the divisibility of activities will then easily be derivable from the latter. Prima facie, a good candidate for the degree of *in*divisibility of the activity $\omega^c_{W_{s-1}}$across $\{\mathcal{O}_1, \dots, \mathcal{O}_k\}$ and $\{\mathcal{O}_{k+1}, \dots, \mathcal{O}_N\}$, term it

$$N(\omega^c_{W_{s-1}}; \{\mathcal{O}_1, \dots, \mathcal{O}_k\}, \{\mathcal{O}_{k+1}, \dots, \mathcal{O}_N\}),$$

is the quotient of the number of the phase in which the activity in fact bifurcates into the partial activities $\omega^{c,n}_{W_{s-1}}(\mathcal{O}_1, \dots, \mathcal{O}_k)$ and $\omega^{c,n}_{W_{s-1}}(\mathcal{O}_{k+1}, \dots, \mathcal{O}_N)$—here phase number $n-1$—and the total number of phases of the complete concrescence process, here M, so that in this case

$$N(\omega^c_{W_{s-1}}; \{\mathcal{O}_1, \dots, \mathcal{O}_k\}, \{\mathcal{O}_{k+1}, \dots, \mathcal{O}_N\}) = \frac{n-1}{M}. \tag{11.5}$$

However, this value has to be compared with the degree one's preferred non-separability measure for states assigns to the state representing the activity $\omega^c_{W_{s-1}}$. Now there are two conditions to be fulfilled by *every* reasonable non-separability measure for states. Each such measure should assign the value 0 to product states and should be normalizable to a maximum value of 1. These

[6] As already mention in the footenote on page 263, that in case $n = 1$ the bifurcation takes place in the "0-th phase of concrescence", simply refers to a bifurcation in the conformal phase of the forgoing transition process.

extreme cases correspond to the cases where, on the one hand, already the initial activity $\omega^c_{W_{s-1}} = \omega^{c,1}_{W_{s-1}}$ for the concrescence process is in fact divided and where, on the other hand, even the activity $\omega^{c,M}_{W_{s-1}}$ of the last phase of concrescence is still undivided with respect to the the above sets of regions. Now for the first of these extreme cases, (11.5) does produce the required output

$$N(\omega^c_{W_{s-1}}; \{\mathcal{O}_1, \ldots, \mathcal{O}_k\}, \{\mathcal{O}_{k+1}, \ldots, \mathcal{O}_N\}) = \frac{1-1}{M} = 0.$$

But for the second case, where even the activity $\omega^{c,M}_{W_{s-1}}$ is still undivided across the sets of regions in question, (11.5) does not produce the required value 1, but rather the value $(M-1)/M$ that is obviously smaller than 1. However, this already suggests how to modify (11.5), so that one gets a *normalized* measure for the degree of indivisibility of activities—we simply have to take

$$N(\omega^c_{W_{s-1}}; \{\mathcal{O}_1, \ldots, \mathcal{O}_k\}, \{\mathcal{O}_{k+1}, \ldots, \mathcal{O}_N\}) \equiv \frac{n-1}{M-1}. \tag{11.6}$$

This definition covers both extreme cases because $(1-1)/(M-1) = 0$ and $(M-1)/(M-1) = 1$. Whether it is also empirically adequate in the general case, depends on the choice of one's preferred measure of non-separability for states, with which to compare (11.6). However, it is clear that there is a wide range of freedom for further modifications of (11.6) to accommodate the values prescribed by a particular non-separability measure of states, *without* undermining the underlying ontological idea that it is the quotient $(n-1)/(M-1)$ of phases that matters. For example, one can take polynomials or, more generally, real-valued functions of this quotient.[7] Thus (11.6) in fact merely represents the simplest of all the possible choices.

Now whatever further modification one may introduce into (11.6), with 1 as the maximum and 0 as the minimum value of *in*divisibility, a reasonable measure for the *degree of divisibility* of the activity $\omega^c_{W_{s-1}}$ across $\{\mathcal{O}_1, \ldots, \mathcal{O}_k\}$ and $\{\mathcal{O}_{k+1}, \ldots, \mathcal{O}_N\}$, is in any case simply given by

$$D(\omega^c_{W_{s-1}}; \{\mathcal{O}_1, \ldots, \mathcal{O}_k\}, \{\mathcal{O}_{k+1}, \ldots, \mathcal{O}_N\}) \equiv 1 - N(\omega^c_{W_{s-1}}; \{\mathcal{O}_1, \ldots, \mathcal{O}_k\}, \{\mathcal{O}_{k+1}, \ldots, \mathcal{O}_N\}). \tag{11.7}$$

[7] Provided that the range of these polynomials or functions lies in the interval $[0, 1]$.

The generalization of the degree of (in-) divisibility of activity $\omega^c_{W_{s-1}}$ across more than two subsets of the set $\{\mathcal{O}_{i(s)}\} = \{\mathcal{O}_1, \ldots, \mathcal{O}_N\}$ of all regions of stage s, say across $\{\mathcal{O}_1, \ldots, \mathcal{O}_k\}$, $\mathcal{O}_{k+1}$, $\{\mathcal{O}_{k+2}, \ldots, \mathcal{O}_l\}, \ldots, \{\mathcal{O}_{l+1}, \ldots, \mathcal{O}_N\}$, is straightforward and need therefore not be discussed here explicitly.

If our interpretation is correct, the different degrees of (non-) separability of quantum states—at least of those representing Whiteheadian activities—stem from the *internal processual structure* of Whiteheadian occasions. Put conversely, that there are such different degrees is a point in favour of our modified version of Whitehead's ontology, since one of its genuine features is an important factor in the explanation of this fact.

As already noted in Section 2.6.1 the fact that activities come in different degrees of divisibility also has important consequences for the individuality of occasions. This will be our final topic before we summarize the essential results of this work in Chapter 12.

11.3 The degree of individuality of occasions

According to Sections 2.3.1 and 2.3.2 the individuality of a concrescent occasion consists of (1) its definiteness, together with (2) the particularity and (3) the autonomy of its activity. Since the activity is exhausted in the course of the concrescence process, the individuality of a *completed* occasion accordingly consists of the particularity due to its region and the (disjunctive) subjective eternal object unrestrictedly ingressed therein—the final definiteness of this occasion. Unfortunately, the present author has no idea for how one could quantify the degree of definiteness of an occasion—either concrescent or completed. Therefore, we have to be content with the purely qualitative idea that the definiteness will be the higher the less subjective eternal objects contribute to it (as disjunctive components). Contrary to this, we will show in the following how the contributions to the individuality of concrescent occasions due to their activities (i.e. (2) and (3) above) *can be quantified.*

The last section has provided us with a means to quantify the individuality of a concrescent occasion *due to the particularity of its activity.* This contribution to the individuality of a concrescent occasion with region $\mathcal{O}_j$ belonging to stage s, depends on the degree as to which the activity $\omega^c_{W_{s-1}}$ for this concrescence is divisible into distinct parts, such that one of these, i.e. $\omega^c_{W_{s-1}}(\mathcal{O}_j)$, can be regarded as being located in region $\mathcal{O}_j$, and thus as being particularized. And since this divisibility comes in degrees, that aspect of the individuality of a concrescent occasion that comes from the particularity of its activity comes in degrees, too. More precisely, the individuality of the concrescent occasion with region $\mathcal{O}_j \in \{\mathcal{O}_{i(s)}\} = \{\mathcal{O}_1, \ldots, \mathcal{O}_N\}$ due to the particularity of its activity, coincides with the degree of divisibility of the activity $\omega^c_{W_{s-1}}$ across the region $\mathcal{O}_j$ and the subset $\{\mathcal{O}_{i(s)}\}_{i(s)\neq j}$ of the other regions of stage s, i.e. with the value of

$$D(\omega^c_{W_{s-1}}; \mathcal{O}_j, \{\mathcal{O}_{i(s)}\}_{i(s)\neq j}). \tag{11.8}$$

If already the activity $\omega^c_{W_{s-1}}$ is thus divided the individuality of the occasion in question due to the particularity of its activity is the maximum allowed by Whitehead's theory (see Sections 2.3.1 and 2.3.2), which is nicely reflected by

$$D(\omega^c_{W_{s-1}}; \mathcal{O}_j, \{\mathcal{O}_{i(s)}\}_{i(s)\neq j}) = 1.$$

And if such a bifurcation does not take place at all, i.e. if even the activity $\omega^{c,M}_{W_{s-1}}$ of the terminal phase of the concrescence process of stage s is undivided across $\mathcal{O}_j$ and $\{\mathcal{O}_{i(s)}\}_{i(s)\neq j}$, then, there simply is no particularized activity for region $\mathcal{O}_j$ and thus no individuality due to it. This too is nicely reflected by the value of (11.8) for this case, which is just

$$D(\omega^c_{W_{s-1}}; \mathcal{O}_j, \{\mathcal{O}_{i(s)}\}_{i(s)\neq j}) = 0.$$

In general, the individuality of a concrescent occasion due to the particularity of its activity will lie in between these two extreme cases.

The last contribution to the individuality of a concrescent occasion with region $\mathcal{O}_j$ comes from the *autonomy or freedom of its activity* for decisions among the range of subjective eternal objects available to it. As explained in Section 2.3.1 this freedom also comes in degrees. It is the higher the less the creative character of the activity $\omega^c_{W_{s-1}}$ is biased towards certain possibilities. In other words, it is the higher the more the propensities provided by the creative character of the activity $\omega^c_{W_{s-1}}$ are equally distributed over the qualitative possibilities given by the base $G_1(\mathcal{O}_j) = G(\mathcal{O}_j)$ of the corresponding initial hierachy $H(G_1(\mathcal{O}_j))$ of subjective eternal objects and it is accordingly minimal if the propensity for one of the possibilities from $G_1(\mathcal{O}_j)$ is 1 and all others are 0. In this latter case, the activity $\omega^c_{W_{s-1}}$ simply has no freedom or autonomy for decisions within the range $G_1(\mathcal{O}_j)$ at all. Rather the subjective eternal object that will constitute the final definiteness of the completed occasion, i.e. that will unrestrictedly ingress into the region of the occasion, is already completely determined by the occasion's efficient causes (see Section 11.1). Now according to Section 10.4 the base $G_1(\mathcal{O}_j)$ is represented by a resolution of the identity $\hat{\mathbf{1}}$, say $\{\hat{P}_k\}_{\mathcal{O}_j}$, and according to Section 11.1, the state representing the activity $\omega^c_{W_{s-1}}$ is given by $\omega_{s-1}|_{\wedge^N_{i(s)=1}\mathbf{B}(\{P_i\}^1_{\mathcal{O}_{i(s)}})}$. For simplicity of notation we will simply abbreviate the latter by ρ. The propensities for the unrestricted ingression of the subjective eternal objects from $G_1(\mathcal{O}_j)$ into region $\mathcal{O}_j$ are accordingly given by $\rho(P_k)$, where P_k is a projection representing the possible ingression of the subjective eternal object $\hat{P}_k$ in region $\mathcal{O}_j$. A quantitative measure for the freedom or autonomy of the activity $\omega^c_{W_{s-1}}$ with respect to the subjective eternal objects in $G_1(\mathcal{O}_j)$ can be constructed from the so-called *Shannon entropy* of the probability distribution $\{\rho(P_k)\}$, defined by (see e.g. Busch et al. 1995, p. 140 ff)

$$H(\{\rho(P_k)\}) \equiv -\sum_k \rho(P_k)\ln(\rho(P_k)).$$

$H(\{\rho(P_k)\})$ measures the deviation of the probability distribution $\{\rho(P_k)\}$ from the case where one of the P_k is certain (has probability one) and all others are impossible (have probability zero). A probability distribution of the latter type has Shannon entropy zero and corresponds to the case where the activity $\omega^c_{W_{s-1}}$ has no freedom at all for decisions among the range of subjective eternal objects $G_1(\mathcal{O}_j)$. In case where the set $\{P_k\}$ is finite, consisting of n elements say, $H(\{\rho(P_k)\})$ is maximal in case of the equal distribution $\rho(P_k) = 1/n$ for all P_k, where it takes the value $\ln(n)$. However, the Shannon entropy is a sensible measure of the degree of non-equality of the values $\{\rho(P_k)\}$ also in the infinite case where no equal distribution exists. This is because *any change toward the equalization of the probabilities increases the Shannon entropy.* And a higher degree of equalization of the probabilities $\{\rho(P_k)\}$ corresponds to a higher degree of freedom of decisions of the activity $\omega^c_{W_{s-1}}$ among the elements of $G_1(\mathcal{O}_j)$. Therefore, the Shannon entropy $H(\{\rho(P_k)\})$ provides a reasonable measure for the freedom or autonomy of the activity $\omega^c_{W_{s-1}}$ for decisions among the subjective eternal objects in $G_1(\mathcal{O}_j)$. However, since in the infinite case $H(\{\rho(P_k)\})$ can vary between zero and infinity, it is not yet normalized. This can, however, easily be remedied by taking for example,

$$F(\omega^c_{W_{s-1}}; G_1(\mathcal{O}_j)) \equiv 1 - e^{-H(\{\rho(P_k)\})}$$

instead of $H(\{\rho(P_k)\})$, as the measure in question.

Since the measure (11.8) for the divisibility of the activity $\omega^c_{W_{s-1}}$ is likewise normalized, we can also quantify the *total or net contribution* to the individuality of the concrescent occasion E_j due to its activity. This contribution can simply be measured by the product of the degree of particularity of the activity $\omega^c_{W_{s-1}}$ with respect to region $\mathcal{O}_j$ and the degree of freedom with respect to the set $G_1(\mathcal{O}_j)$, i.e. by

$$I(\omega^c_{W_{s-1}}; \mathcal{O}_j) \equiv D(\omega^c_{W_{s-1}}; \mathcal{O}_j, \{\mathcal{O}_{i(s)}\}_{i(s)\neq j}) \cdot F(\omega^c_{W_{s-1}}; G_1(\mathcal{O}_j)),$$

which is obviously normalized, too.

Chapter 12

Summing up

In this work we have developed an interpretation of Whitehead's writings on his "philosophy of process" and have proposed a connection of the resulting ontology with the algebraic formalism of QFT. Apart from some minor modifications and simplifications of Whiteheadian ideas, the main result of our investigation of Whitehead's philosophy has been that the ontology as originally intended by him contains an inconsistency. This inconsistency consists in the fact that the principle of the separateness of all realized (and thus too, of all actualized) regions will generally not be satisfied in the causally local and separable ontology as conceived by Whitehead. Rather to secure the separateness of all realized regions, one has to give up either the requirement of the causal independence of occasions belonging to the same layer of the world-process or the requirement of the distinctness of their concrescence processes. We have argued that the second alternative is the more appropriate one within the framework of Whitehead's ontology and have accordingly modified the latter. The most important differences of the ontology resulting from incorporating generally undivided, bifurcating activities, compared to Whitehead's original ontology have been that

(1) all occasions belonging to the same layer of the world-process have the same causes, which, in particular, rules out Whitehead's doctrine of actual worlds

(2) like the individuality of concrescent occasions due to the freedom of their

activities, the individuality due to the particularity of their activities too, becomes a matter of degree.

A consequence of (1) is that there will generally be superluminal causal influences between occasions belonging to different layers of the world-process. Whether this gives rise to a genuine conflict with STR depends on the understanding of the latter theory. In particular, if STR is understood as a pure spacetime theory, not making any claims about causation at all, then there is obviously no such conflict. On the other hand, if STR is believed to prohibit superluminal causation, but presupposes that causal influences involve a transfer of energy-momentum, it depends on the crucial concept of "transfer" and on the question whether the Whiteheadian notion of activity can be understood as an analog of physical energy, whether there is such a conflict.

However, even if STR is *not* understood as making any claims about causation, there is another source for a conflict. A central assumption of Whitehead's ontology is that the world is an expanding process. This requires that occasions are grouped into linearly ordered layers, which in turn give rise to a distinguished foliation of spacetime—a "preferred reference frame". We have shown how such a layer-cake structure can be established, by means of the underlying activities envisagement of occasions. To avoid a conflict between the thereby implemented distinguished foliation of spacetime and the Poincaré invariance implied by STR, one has to admit that STR does not deserve an ontological interpretation and that the distinguished foliation of the Whiteheadian world-process is not empirically detectable—not even in principle. Yet the fact that one can avoid a conflict with STR only if a central feature of Whitehead's ontology is empirically unknowable, is clearly a drawback for our Whiteheadian interpretation of QFT, since the latter incorporates the relativistic spacetime structure with Poincaré transformations constituting the spatiotemporal invariance group.

Our introduction of generally undivided, bifurcating activities into Whitehead's original theory has been seen to fit quite smoothly to what QFT seems to tell us by way of its violation of Bell's inequality. We have argued that the latter should be interpreted as a stemming from the non-separability of states across spacelike separated regions. The different degrees of non-separability of states can then be understood as stemming from an earlier or later bifurcation of the activities they represent, into distinct partial or particularized activities and thus from the different degrees of individuality of concrescent occasions (see (2) above).

That a concrescent occasion has a certain degree of freedom or autonomy for decisions among a range of possibilities, limited only by the creative character of its activity (understood as providing a range of propensities), nicely fits to the widely held view according to which the actualization of a possible measurement outcome involves "a free choice of nature, limited only by the probability assignment [due to the respective state]" (Haag 1996, p. 316). Measurable magnitudes or observables have been reinterpreted within our interpretation in terms of abstractive hierachies of subjective eternal objects. However, this reinterpretation, like the one of states in terms of activities, does not work in each case. Thus not all observables and states are ontologically meaningful within our Whiteheadian interpretation. This is, however, not in itself a drawback, as long as there are "enough" ontologically meaningful observables and states to secure the empirical adequacy of the interpretation. Our interpretation of the creative character of an activity as providing propensities for the ingression of subjective eternal objects, moreover allows to construct a quantitative measure for the degree of freedom of the activity by means of the Shannon entropy of the corresponding probability distribution provided by the state representing this activity.

The very Whiteheadian conception of eternal objects as universals is, however, not supported by QFT. To the contrary, we have seen that the formalism of AQFT neither provides natural candidates for a representation of objective eternal objects (i.e. boundary surfaces) nor of subjective eternal objects (i.e. qualitative properties) if these are understood as universals. Rather we had to be content with a more indirect representation of these entities by certain classes of regions and local projections respectively. This suggests that QFT is more sympathetic to a conception in which properties are not treated as universals but rather as particulars—good news for nominalists. In case of subjective eternal objects, their representation had moreover been complicated by the fact that, because of the lack of atomic local projections in QFT, there are probably no simple subjective eternal objects at all. We have partially remedied this unfortunate state of affairs by introducing the notion of "simplicity relative to an abstractive hierachy". Since subjective eternal objects only ingress into occasions as members of particular hierachies, this weaker notion is, however, sufficient to ground the idea of "simple decisions" of concrescent occasions, that had been introduced to safe Whitehead's ontology from the challenge that concrescence processes are not genuine processes at all, but merely single acts.

The Whiteheadian idea that the abstractive hierachy of subjective eternal objects, ingressing into a particular region, is determined by the corresponding causal past via the creative character of the corresponding activity, has been concretized by a particular mathematical rule we have formulated for this purpose. However, we have not been able to guaranty that this rule will in fact work appropriately. If the states which represent the relevant activities should turn out to be ergodic with respect to the local algebras of double cones, then, the rule will merely single out abstractive hierachies consisting of the two trivial subjective eternal objects represented by $\hat{\mathbf{0}}$ and $\hat{\mathbf{1}}$ respectively. In this case clearly some different rule has to be found, because otherwise the world would lack any (non-trivial) qualitative properties.

In case of the determination of the regions of occasions by their causal pasts via the creative character of the corresponding activities, the situation is similar. We have proposed a mathematical rule for how this determination shall proceed, but it is not clear whether this rule will appropriately work in all cases. This is because our rule constitutes an approximation problem in the space of states on $\mathcal{B}(\mathcal{H})$ that seems not to be covered by the known theorems of this branch of mathematics. Therefore, in this case too, the proposed rule has merely a tentative character. However, in case our proposed rules will in fact turn out to be untenable, the ideas underlying them may nevertheless be helpful to inspire some more sophisticated proposals.

The final task of fixing the degree as to which our project of establishing an adequate ontological interpretation of QFT in terms of Whitehead's philosophy of process has been successful, is left to the reader. However, we hope that those who are not satisfied with the result of this project feel encouraged to work out a more satisfactory ontology for QFT, may it be inspired by Whiteheadian ideas or not.

Appendix A

Lattices and Boolean algebras

A *partially ordered set* is a set S equipped with a relation $\leq$ such that for all $a, b, c \in S$

(P1) $a \leq a$ (reflexivity)

(P2) $a \leq b$ and $b \leq a$ implies $a = b$ (antisymmetry)

(P3) $a \leq b$ and $b \leq c$ implies $a \leq c$ (transitivity).

The relation $\leq$ is called a *reflexive partial order*. Given such a relation $\leq$ one can define another relation $<$ by $a < b :\Leftrightarrow (a \leq b$ and $a \neq b)$ with the following properties:

(P1´) $a \not< a$ (irreflexivity)

(P2´) $a < b$ implies $b \not< a$ (asymmetry)

(P3´) $a < b$ and $b < c$ implies $a < c$ (transitivity)

The relation $<$ is called *irreflexive partial order*. Given such an irreflexive partial order, one can obviously define a reflexive partial order by means of $a \leq b :\Leftrightarrow (a < b$ or $a = b)$. Thus partially ordered sets can equally well be defined by a relation satisfying (P1)-(P3) or (P1´)-(P3´) because one always has both partial orders. Since the signs $\leq$ respectively $<$ have become "rigid" denotations for reflexive and irreflexive partial orders respectively one usually omits the qualifications "reflexive" and "irreflexive" when these signs are used.

A partially ordered set S possesses a *minimal* or *zero* and a *maximal* or *unit* element, denoted by 0 and 1 respectively, if $0 \leq a$ and $a \leq 1$ respectively, holds for all $a \in S$. An element $a \neq 0$ is called *atomic* or an *atom* of S if there is no element $b \neq 0$ with $b < a$. Thus atoms are the smallest non-zero elements of a partially ordered set (with zero element).

A *lattice* L is a partially ordered set such that for each two $a, b \in L$ there exists a *supremum* $a \vee b$ and an *infimum* $a \wedge b$, where the former is the smallest element c with respect to the partial order such that $a \leq c$ and $b \leq c$ and the latter is the greatest element c with respect to $\leq$ such that $c \leq a$ and $c \leq b$. If supremum and infimum exist for every subset of L then L is called *complete*. A lattice is called *atomic* if for each $b \in L$ there is an atom a such that $a \leq b$.

A lattice can also be algebraically defined as a set L together with two binary operations $\wedge$ and $\vee$, called *meet* and *join*, such that for all $a, b, c \in L$

(L1) $a \wedge a = a$ and $a \vee a = a$ (idempotence)

(L2) $a \wedge b = b \wedge a$ and $a \vee b = b \vee a$ (commutativity)

(L3) $a \wedge (b \wedge c) = (a \wedge b) \wedge c$ and $a \vee (b \vee c) = (a \vee b) \vee c$ (associativity)

(L4) $a \wedge (a \vee b) = a$ and $a \vee (a \wedge b) = a$ (absorption laws).

Accordingly, L is complete if meet and join exist for every subset of L. If the two operations satisfy these algebraic rules then they define a partial order on L by

$$a \leq b :\Leftrightarrow b = a \vee b. \tag{A.1}$$

Note that $b = a \vee b$ is equivalent to $a = a \wedge b$ so that the latter can also be used to define the same partial order. L together with the partial order thus defined, will then be a lattice in the above order-theoretic sense. Conversely, infimum and supremum in an order-theoretically defined lattice fulfil the axioms (L1)-(L5) of an algebraically defined lattice so that it is simply a matter of taste how one introduces the concept of a lattice. Because of (A.1) the existence of a zero and unit element in L are expressed algebraically by the requirements $a = 0 \vee a$ and $1 = a \vee 1$ for all $a \in L$.

If meet and join $\wedge$, $\vee$ satisfy

(D1) $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$ (distributivity of $\wedge$ over $\vee$)

(D2) $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$ (distributivity of $\vee$ over $\wedge$),

for all $a, b, c \in L$, then the latter is called a *distributive* lattice.
If each $a \in L$ has a *complement* $\neg a \in L$, defined by

(C) $a \wedge \neg a = 0$ and $a \vee \neg a = 1$

then L is a *complemented lattice* and if the complement of each $a \in L$ satisfies

(OC) $\neg(\neg a) = a$ and $a \leq b \Leftrightarrow \neg a \leq \neg b$

L is called an *orthocomplemented lattice* and $\neg a$ is then termed the *orthocomplement* of a. The orthocomplement of each $a \in L$ is unique.

The set $\mathcal{L}(\mathcal{H})$ of all subspaces of a Hilbert space $\mathcal{H}$ is a complete, orthocomplemented, atomic lattice with zero element $\{0\}$ and unit element $\mathcal{H}$; but it is not distributive. Supremum and infimum are given for each subset $\{\mathcal{K}_i\} \subseteq \mathcal{L}(\mathcal{H})$ by $\wedge_i \mathcal{K}_i \equiv \cap_i \mathcal{K}_i$, $\vee_i \mathcal{K}_i \equiv \uplus_i \mathcal{K}_i$ and the orthocomplement of $\mathcal{K} \in \mathcal{L}(\mathcal{H})$ is $\neg \mathcal{K} \equiv \mathcal{K}^{\perp}$, where $\uplus_i \mathcal{K}_i$ is the smallest subspace of $\mathcal{H}$ including $\cup_i \mathcal{K}_i$ (see Appendixes B.1 and B.2). The corresponding partial order is given by set-theoretic inclusion $\subseteq$. The atoms of $\mathcal{L}(\mathcal{H})$ are the one-dimensional subspaces or *rays* $\{c\psi : c \in \mathbb{C}\}$ and obviously each $\mathcal{K} \in \mathcal{L}(\mathcal{H})$ includes (at least) one ray.

Finally a *Boolean algebra* **B** is a distributive, orthocomplemented lattice. A Boolean algebra is atomic respectively complete if it has these properties as a lattice. In a Boolean algebra $\wedge$, $\vee$ and $\neg$ fulfil all the requirements on the connectives conjunction/and, disjunction/or and negation/not of ordinary logic. This is not surprising because Boolean algebras were in the first place invented to capture the essence of these logical operations.

Appendix B

Operator algebras on Hilbert spaces

B.1 Hilbert spaces

A *linear space* or *vector space* $\mathcal{V}$ over the complex numbers $\mathbb{C}$ is a set on which a sum $+ : \mathcal{V} \times \mathcal{V} \to \mathcal{V}$, $(\psi, \varphi) \mapsto \psi + \varphi = \varphi + \psi$ and a product with complex numbers $\mathcal{V} \times \mathbb{C} \to \mathcal{V}$, $(\psi, c) \mapsto c\psi$ is defined, such that

(V1) $1\psi = \psi$,

(V2) $c_1(c_2\psi) = (c_1c_2)\psi$,

(V3) $c(\psi + \varphi) = c\psi + c\varphi$,

(V4) $(c_1 + c_2)\psi = c_1\psi + c_2\psi$.

A *scalar product space* is a vector space $\mathcal{V}$, on which there is a map (a *scalar product*) $\langle \cdot, \cdot \rangle : \mathcal{V} \times \mathcal{V} \to \mathbb{C}$, $(\varphi, \psi) \mapsto \langle \varphi, \psi \rangle$ such that

(S1) $\langle \psi, \psi \rangle \geq 0$ (positivity),

(S2) $\langle \varphi, \psi \rangle = \overline{\langle \psi, \varphi \rangle}$ (hermiticity),

(S3) $\langle \varphi, c_1\psi_1 + c_2\psi_2 \rangle = c_1 \langle \varphi, \psi_1 \rangle + c_2 \langle \varphi, \psi_2 \rangle$ (linearity),

(S4) $\langle \psi, \psi \rangle = 0 \Leftrightarrow \psi = 0$.

The bar denotes the complex conjugate $\overline{a+ib} = a - ib$ of the complex number $c = a + ib \in \mathbb{C}$ (a, b real numbers). From the hermiticity (S2) and the linearity in the second argument (S3) the *anti*linearity in the first argument $\langle c_1\varphi_1 + c_2\varphi_2, \psi\rangle = \bar{c}_1 \langle\varphi_1, \psi\rangle + \bar{c}_2 \langle\varphi_2, \psi\rangle$ follows. From positivity (S1) one obtains the *Cauchy-Schwartz inequality* $|\langle\varphi, \psi\rangle|^2 \leq \langle\varphi, \varphi\rangle \langle\psi, \psi\rangle$ for all $\varphi, \psi \in \mathcal{V}$. Using the basic properties of the scalar product and the Cauchy-Schwartz inequality one can define a *norm* $\|\cdot\| : \mathcal{V} \to \mathbb{R}_+ \equiv \{c \in \mathbb{R} : c \geq 0\}$ on $\mathcal{V}$ by

$$\|\psi\| \equiv \langle\psi, \psi\rangle^{1/2} \tag{B.1}$$

and proof its characteristic properties

(N1) $\|\psi\| \geq 0$ (positivity)

(N2) $\|\psi + \varphi\| \leq \|\psi\| + \|\varphi\|$ (triangle inequality),

(N3) $\|c\psi\| = |c| \, \|\psi\|$ (where $|c|$ is the absolute value of $c \in \mathbb{C}$, i.e. $|c| = \bar{c}c$),

(N4) $\|\psi\| = 0 \Leftrightarrow \psi = 0$.

Thus by reason of (B.1) each scalar product space is also a normed space. A sequence of vectors from a normed space for which $\lim_{m,n\to\infty} \|\psi_m - \psi_n\| = 0$ is called a *Cauchy sequence*. For a scalar product space $\mathcal{V}$ to be a *Hilbert space*, it moreover has to be *complete* which means that each Cauchy sequence from $\mathcal{V}$ converges to some vector $\psi \in \mathcal{V}$, i.e. $\lim_{n\to\infty} \|\psi - \psi_n\| = 0$. Thus a Hilbert space, which in the following will usually be denoted by $\mathcal{H}$, is a scalar product space over the complex numbers that is complete with respect to the norm defined from its scalar product.

A subset $\mathcal{K}$ of a Hilbert space $\mathcal{H}$ that is *closed under taking linear combinations*, i.e. for any two vectors $\psi_1, \psi_2 \in \mathcal{K}$ and $c_1, c_2 \in \mathbb{C}$ the vector $c_1\psi_1 + c_2\psi_2$ is again in $\mathcal{K}$, is called a *linear subset* of $\mathcal{H}$. If $\mathcal{K}$ is moreover complete and thus is itself a Hilbert space it is called a *subspace* of $\mathcal{H}$. Note that the completeness of a subset $\mathcal{K} \subseteq \mathcal{H}$ is equivalent to its *closedness with respect to the norm topology of* $\mathcal{H}$, i.e. to the property that the limit ψ

$$\lim_{n\to\infty} \|\psi - \psi_n\| = 0$$

of a sequence $(\psi_n) \subseteq \mathcal{K}$ again belongs to $\mathcal{K}$ (and not merely to $\mathcal{H}$). Whereas the intersection of two (or more) subspaces $\cap_i \mathcal{K}_i$ is again a subspace of $\mathcal{H}$ their union $\cup_i \mathcal{K}_i$ is obviously not even a linear subset because it does not contain linear combinations of vectors from different $\mathcal{K}_i$. Thus for $\cup_i \mathcal{K}_i$ to "become" a subspace of $\mathcal{H}$ one first of all has to add all linear combinations of vectors from different $\mathcal{K}_i$ and secondly all limits of sequences from the thus enlarged set. In other words, one has to take the closure of $\cup_i \mathcal{K}_i$ with respect to linear combinations and the closure of the resulting set with respect to the norm topology of $\mathcal{H}$. The subspace of $\mathcal{H}$ generated in this way from $\cup_i \mathcal{K}_i$ is denoted by $\uplus_i \mathcal{K}_i$, it is the *smallest subspace of of $\mathcal{H}$ including $\cup_i \mathcal{K}_i$.*

In each Hilbert space, there exists a subset of vectors $\{\phi_i\} \subset \mathcal{H}$ satisfying the following conditions

(B1) $\|\phi_i\| = 1$ (normalization),

(B2) $\langle \phi_i, \phi_j \rangle = 0$ for $i \neq j$ (orthogonality),

(B3) Each vector $\psi \in \mathcal{H}$ can be approximated as closely as one likes (in the norm $\|\cdot\|$) by finite linear combinations $\sum_{k=1}^n c_{i_k} \phi_{i_k}$ of vectors from $\{\phi_i\}$, i.e. the set of finite linear combinations of vectors from $\{\phi_i\}$ is dense in $\mathcal{H}$.

A set of vectors for which (B1)-(B3) hold is called an *orthonormal basis of $\mathcal{H}$*. One can show that each vector $\psi \in \mathcal{H}$ can be written as an (infinite) sum of the basis vectors $\{\phi_i\}$ with coefficients given by the scalar products $\langle \phi_i, \psi \rangle$, i.e. as

$$\psi = \sum_i \langle \phi_i, \psi \rangle \, \phi_i$$

. Moreover, all orthonormal bases of $\mathcal{H}$ have the same cardinality, i.e. the same number of elements, that is therefore called the *dimension* of $\mathcal{H}$. If the dimension of $\mathcal{H}$ is $n \in \mathbb{N}$, $\mathcal{H}$ is given by $\mathbb{C}^n$ equipped with the scalar product $\langle \psi, \varphi \rangle \equiv \sum_{i=1}^n a_i b_i$ for $\psi = (a_1, \ldots, a_n), \varphi = (b_1, \ldots, b_n) \in \mathbb{C}^n$. Hilbert spaces whose dimension is at most countable, i.e. finite or countably infinite, are called *separable*. In QM as well as in QFT *only* separable Hilbert spaces do appear.

For a subset $\mathcal{K}$ of $\mathcal{H}$, the set of all vectors in $\varphi \in \mathcal{H}$ which are orthogonal to any vector in $\mathcal{K}$, i.e. $\langle \varphi, \psi \rangle = 0$ for all $\psi \in \mathcal{K}$, is denoted by $\mathcal{K}^{\perp}$. Such an $\mathcal{K}^{\perp}$ is always a subspace of $\mathcal{H}$. A necessary and sufficient condition for the set $\mathcal{K}$ itself to be a subspace of $\mathcal{H}$ is

$$(\mathcal{K}^{\perp})^{\perp} = \mathcal{K}. \tag{B.2}$$

If $\mathcal{K}$ is a subspace of $\mathcal{H}$, $\mathcal{K}^{\perp}$ is called its *orthogonal complement*. Because of (B.2) the orthogonal complement is always unique. Generally, two subspaces $\mathcal{K}_1$, $\mathcal{K}_2 \subseteq \mathcal{H}$ are called orthogonal iff every vector in $\mathcal{K}_1$ is orthogonal to every vector in $\mathcal{K}_2$, i.e. iff $\mathcal{K}_1 \subseteq \mathcal{K}_2^{\perp}$ or equivalently $\mathcal{K}_2 \subseteq \mathcal{K}_1^{\perp}$. Every vector $\psi \in \mathcal{H}$ can be uniquely decomposed as $\psi = \psi_{\|} + \psi_{\perp}$ where $\psi_{\|} \in \mathcal{K}$ is the projection of ψ onto $\mathcal{K}$ and $\psi_{\perp} \in \mathcal{K}^{\perp}$ is the projection of ψ onto $\mathcal{K}^{\perp}$.

Given two Hilbert spaces $\mathcal{H}_1$ and $\mathcal{H}_2$ and orthonormal bases $\{\phi_i^1\} \subset \mathcal{H}_1$, $\{\phi_j^2\} \subset \mathcal{H}_2$ we can define the Hilbert space $\mathcal{H}_1 \otimes \mathcal{H}_2$, called the *tensor product of $\mathcal{H}_1$ and $\mathcal{H}_2$*, as follows: a basis of $\mathcal{H}_1 \otimes \mathcal{H}_2$ shall be given by $\{\phi_i^1 \otimes \phi_j^2\}$ (where any combinations of i, j are considered) and the expression

$$\psi = \sum_{i,j} c_{ij} \phi_i^1 \otimes \phi_j^2$$

be a general vector in $\mathcal{H}_1 \otimes \mathcal{H}_2$ whenever

$$\sum_{i,j} |c_{ij}|^2 < \infty$$

for the set of complex numbers $\{c_{ij}\}$. Linear combinations and scalar product are defined by

$$a \sum_{i,j} c_{ij} \phi_i^1 \otimes \phi_j^2 + b \sum_{i,j} d_{ij} \phi_i^1 \otimes \phi_j^2 \equiv \sum_{i,j} (a c_{ij} + b d_{ij}) \phi_i^1 \otimes \phi_j^2$$

respectively

$$\left\langle \sum_{i,j} c_{ij} \phi_i^1 \otimes \phi_j^2, \sum_{i,j} d_{ji} \phi_i^1 \otimes \phi_j^2 \right\rangle \equiv \sum_{i,j} \bar{c}_{ij} d_{ij}. \tag{B.3}$$

Finally, for the general vectors $\varphi = \sum_i c_i \phi_i^1 \in \mathcal{H}_1$ and $\chi = \sum_j d_j \phi_j^2 \in \mathcal{H}_2$ we define their tensor product $\varphi \otimes \chi \in \mathcal{H}_1 \otimes \mathcal{H}_2$ by

$$\varphi \otimes \chi \equiv \sum_{i,j} c_i d_j \phi_i^1 \otimes \phi_j^2. \tag{B.4}$$

Then this tensor product $\varphi \otimes \chi$ is linear in both arguments, i.e.

$$\begin{aligned}(a\varphi_1 + b\varphi_2) \otimes \chi &= a(\varphi_1 \otimes \chi) + b(\varphi_2 \otimes \chi),\\ \varphi \otimes (a\chi_1 + b\chi_2) &= a(\varphi \otimes \chi_1) + b(\varphi \otimes \chi_2),\end{aligned}$$

and is associative with respect to multiplication with complex numbers

$$(a\varphi) \otimes \chi = \varphi \otimes (a\chi) = a(\varphi \otimes \chi).$$

Furthermore, the scalar product (B.3) of two tensor products $\varphi \otimes \chi$ and $\eta \otimes \zeta$ turns out to be the product of the scalar product $\langle \varphi, \eta \rangle_1$ in $\mathcal{H}_1$ with the scalar product $\langle \chi, \zeta \rangle_2$ in $\mathcal{H}_2$, i.e.

$$\langle \varphi \otimes \chi, \eta \otimes \zeta \rangle = \langle \varphi, \eta \rangle_1 \langle \chi, \zeta \rangle_2 . \tag{B.5}$$

It follows that for any orthonormal bases $\{\xi_i^1\}$ and $\{\xi_j^2\}$ of $\mathcal{H}_1$ and $\mathcal{H}_2$, $\{\xi_i^1 \otimes \xi_j^2\}$ constructed according to (B.4) becomes an orthonormal basis of $\mathcal{H}_1 \otimes \mathcal{H}_2$ and thus the latter does not depend on the choice of the orthonormal bases used in the above definition.

B.2 Bounded operators

A map $A : \mathcal{H} \to \mathcal{H}$, $\psi \mapsto A(\psi)$ that satisfies

(O1) $A(c_1\psi_1 + c_2\psi_2) = c_1 A(\psi_1) + c_2 A(\psi_2)$ for all $c_1, c_2 \in \mathbb{C}$ and $\psi_1, \psi_2 \in \mathcal{H}$ (linearity),

(O2) $\|A(\psi)\| \leq a \|\psi\|$ for some $a \in \mathbb{R}_+$ (boundedness)

is called a *linear*, *bounded operator* on $\mathcal{H}$. The set of all linear, bounded operators on $\mathcal{H}$ will be denoted by $\mathcal{B}(\mathcal{H})$. The restriction to linear maps $A : \mathcal{H} \to \mathcal{H}$ is natural because of the linear structure of the domain $\mathcal{H}$. At first sight the requirement that only linear, bounded maps which are defined on the *whole* space $\mathcal{H}$ are taken into account may seem to be a severe restriction. However, one can show that each linear, bounded map with domain $D \subset \mathcal{H}$ can be uniquely extended to all of $\mathcal{H}$ so that one can without loss of generality require $D = \mathcal{H}$. This is no longer true for linear, unbounded maps for which $\|A(\psi)\| = \infty$ for some $\psi \in \mathcal{H}$, which makes their study much more difficult (see e.g. Reed and Simon 1980, Chapter VIII). Since in what follows we will only deal with linear, bounded operators the adjectives "linear" and "bounded" will often be omitted. Moreover, it is usual to simply write $A\psi$ instead of $A(\psi)$. The most trivial operators are the zero operator $\mathbf{0}$ that maps each vector ψ to $0 \in \mathcal{H}$, i.e. $\mathbf{0}\psi = 0$, and the identity operator $\mathbf{1}$ that maps each vector onto itself, i.e. $\mathbf{1}\psi = \psi$.

For $A, B \in \mathcal{B}(\mathcal{H})$ there is a natural notion of a *product* be given by their combination as maps, i.e. AB is defined as the map $\psi \mapsto A(B\psi)$ and BA as the map $\psi \mapsto B(A\psi)$. Both AB and BA are again linear, bounded operators but in general AB will be different from BA. Therefore, it makes sense to define the *commutator* of A and B by

$$[A, B] \equiv AB - BA.$$

Two operators whose commutator vanishes are called *commuting*.

For each operator $A \in \mathcal{B}(\mathcal{H})$ there exists a unique operator A^* satisfying

$$\langle A^*\varphi, \psi\rangle = \langle \varphi, A\psi\rangle \text{ for all } \varphi, \psi \in \mathcal{H}$$

which is called the *adjoint* of A. The adjoint of A is unique and one has $(A^*)^* = A$. Furthermore, one can show that $(AB)^* = B^*A^*$ and $(aA + bB)^* = \bar{a}A^* + \bar{b}B^*$ $(a, b \in \mathbb{C})$ hold. The latter property of the map $A \mapsto A^*$ is called *antilinearity*.

Operators which are identical with their adjoints, i.e. $A^* = A$, are called *self-adjoint*. The product of two self-adjoint operators A and B is obviously again self-adjoint, i.e. $(AB)^* = (BA)^*$, iff A and B commute.

If for $A \in \mathcal{B}(\mathcal{H})$ there exists a $B \in \mathcal{B}(\mathcal{H})$ such that

$$AB = BA = \mathbf{1}$$

then B is called the *inverse* of A. *If* A has an inverse B it is unique and A is the inverse of B so that, denoting the inverse of A by A^{-1}, one has $(A^{-1})^{-1} = A$. Furthermore, one can show that $(AB)^{-1} = B^{-1}A^{-1}$ and $(A^*)^{-1} = (A^{-1})^*$ hold.

An operator $U \in \mathcal{B}(\mathcal{H})$ whose inverse is identical to its adjoint

$$U^{-1} = U^*$$

is called *unitary*. This condition is equivalent to

$$\langle U\varphi, U\psi\rangle = \langle U^*U\varphi, \psi\rangle = \left\langle U^{-1}U\varphi, \psi\right\rangle = \langle \varphi, \psi\rangle \text{ for all } \varphi, \psi \in \mathcal{H}, \tag{B.6}$$

so that unitary operators preserve all the structures of a Hilbert space. Interestingly, (B.6) is also equivalent to the prima facie weaker condition

$$\|U\psi\| = \langle U\psi, U\psi\rangle^{1/2} = \langle \psi, \psi\rangle^{1/2} = \|\psi\| \text{ for all } \psi \in \mathcal{H},$$

so that the unitary operators in $\mathcal{B}(\mathcal{H})$ can also be characterized as the *norm-preserving or isometric, bijections of* $\mathcal{H}$. For each two orthonormal bases $\{\phi_i\}$ and $\{\varphi_j\}$ of $\mathcal{H}$ there is a unique unitary operator such that $\{U\phi_i\} = \{\varphi_j\}$ and thus too, $\{U^{-1}\varphi_j\} = \{\phi_i\}$.

The map $P_{\mathcal{K}} : \mathcal{H} \to \mathcal{K}$ that assigns to each vector $\psi = \psi_{\|} + \psi_{\perp} \in \mathcal{H}$ its unique projection $\psi_{\|} \in \mathcal{K}$ onto the subspace $\mathcal{K} \subseteq \mathcal{H}$ is a linear, bounded operator. It is obvious geometrically that it is moreover *idempotent* $P_{\mathcal{K}}^2 = P_{\mathcal{K}}$ in the sense $P_{\mathcal{K}}(P_{\mathcal{K}}\psi) = P_{\mathcal{K}}\psi$. Moreover, one can easily see that it is also self-adjoint, i.e. $P_{\mathcal{K}}^* = P_{\mathcal{K}}$. Conversely, for each self-adjoint, idempotent operator, i.e. $P^2 = P = P^*$, the range of P

$$\mathcal{K} = P\mathcal{H} = \{P\psi : \psi \in \mathcal{H}\}$$

is a subspace of $\mathcal{H}$ and $P = P_{\mathcal{K}}$. Thus the subspaces of $\mathcal{H}$ are in a one-to-one correspondence with the self-adjoint, idempotent operators on $\mathcal{H}$. Because of their geometrical interpretation the latter are called *projection operators* or in short *projections*. The zero and the identity operator are also projection operators which project onto the trivial subspaces $\{0\}$ respectively $\mathcal{H}$. A projection is called *n-dimensional* just in case the dimension of the subspace onto which it projects has dimension n, where $n \in \{0, 1, 2, \ldots, \infty\}$ and the zero operator is the unique projection of dimension 0.

The product of two projection operators P_1 and P_2 is again a projection operator iff P_1 and P_2 commute. The resulting projection $P_1P_2 = P_2P_1$ projects onto the subspace $\mathcal{K}_1 \cap \mathcal{K}_2$, i.e. onto the intersection of the subspaces onto which P_1 and P_2 project individually. Thus, if two subspaces $\mathcal{K}_1, \mathcal{K}_2 \subseteq \mathcal{H}$ are orthogonal and thus $\mathcal{K}_1 \cap \mathcal{K}_2 = \{0\}$ the product of the corresponding projections vanishes, i.e. $P_1P_2 = \mathbf{0}$. Therefore, one also calls two projections whose product vanishes *orthogonal*. Note that $P_1P_2 = \mathbf{0}$ implies via

$(P_1P_2)^* = P_2^*P_1^* = P_2P_1$ that $P_2P_1 = \mathbf{0}$, so that two orthogonal projections are always commuting. The sum of two projection operators P_1 and P_2 is again a projection operator iff P_1 and P_2 are orthogonal. The resulting projection $P_1 + P_2$ projects onto $\mathcal{K}_1 \uplus \mathcal{K}_2$, i.e. onto the smallest subspace including the union $\mathcal{K}_1 \cup \mathcal{K}_2$ of the subspaces onto which P_1 and P_2 project individually.

The *spectrum* of an operator A is defined as the set of complex numbers a for which the operator $a\mathbf{1} - A$ does *not* have an inverse, i.e.

$$\sigma(A) \equiv \left\{a \in \mathbb{C} : (a\mathbf{1} - A)^{-1} \notin \mathcal{B}(\mathcal{H})\right\}.$$

Thus, in particular, all values a for which $a\mathbf{1} - A$ is not injective (i.e. if there is a $\psi \in \mathcal{H}$ such that $(a\mathbf{1} - A)\psi = 0$) belong to $\sigma(A)$. Such an a with $A\psi = a\psi$ is called an *eigenvalue* of A and ψ is called *eigenvector* of A to eigenvalue a. Another reason why $(a\mathbf{1} - A)^{-1}$ may not be a linear, bounded operator on $\mathcal{H}$ is that although $a\mathbf{1} - A$ is injective it is not surjective and thus $(a\mathbf{1} - A)^{-1}$ is simply not defined for some $\psi \in \mathcal{H}$. Therefore, the spectrum of an operator A generally consists of two parts, a part $\sigma_d(A)$ consisting of the eigenvalues of A and a part $\sigma_c(A)$ that consists of those values $a \in \mathbb{C}$ for which $a\mathbf{1}-A$ is injective but not surjective. One can show that the set $\sigma_d(A)$ is always countable, i.e. it is a discrete set, whereas the set $\sigma_c(A)$ can also be uncountable. Note that in case $\mathcal{H}$ is finite-dimensional, i.e. $\mathcal{H} = \mathbb{C}^n$ for some $n \in \mathbb{N}$, $\sigma_c(A)$ is always empty and $\sigma_d(A)$ is finite, so that the spectrum of each operator (which in this case is simply an $n \times n$-matrix) coincides with its finite set of eigenvalues.

The spectrum of a self-adjoint operator is always a subset of the real numbers and conversely each operator with a real spectrum is self-adjoint. Moreover, for self-adjoint operators $\sigma_c(A)$ is always purely continuous, i.e. a union of intervals from $\mathbb{R}$. A self-adjoint operator whose spectrum is non-negative, i.e. $\sigma(A) \subseteq \mathbb{R}_+$, is called *positive*. Because of the spectrum of the trivial projections $\mathbf{0}$ and $\mathbf{1}$ is given by $\sigma(\mathbf{0}) = \{0\}$ and $\sigma(\mathbf{1}) = \{1\}$ and that of each non-trival one P by $\sigma(P) = \sigma_d(P) = \{0, 1\}$, projection operators are examples of positive operators. One can show that the positivity of an operator is equivalent to the condition $\langle\psi, A\psi\rangle \geq 0$ for all $\psi \in \mathcal{H}$. By reason of

$$B \geq A :\Leftrightarrow \langle\psi, B\psi\rangle \geq \langle\psi, A\psi\rangle \;\text{ for all } \psi \in \mathcal{H}$$

one can therefore introduce a *partial order* (see Appendix A) among positive operators. For projection operators this partial order has a geometrical interpretation in terms of the corresponding subspaces because $P_2 \geq P_1$ is equivalent to $\mathcal{K}_2 \supseteq \mathcal{K}_1$, where $\mathcal{K}_i$ is the subspace of $\mathcal{H}$ onto which P_i projects.

Now there is an intimate connection between general self-adjoint and projection operators, called the *spectral decomposition of self-adjoint operators.* Since the set of eigenvalues $\sigma_d(A)$ of an operator is always countable, self-adjoint operators whose spectrum coincides with their set of eigenvalues, i.e. $\sigma(A) = \sigma_d(A)$, are called *discrete.* For reasons internal to Whitehead's ontology we will only need discrete self-adjoint operators. For this reason we will not discuss the spectral decomposition of self-adjoint operators with continuous spectra (apart from some scattered remarks). In this simplest discrete case the spectral decomposition of a self-adjoint operator A reads

$$A = \sum_i a_i P_{a_i}. \tag{B.7}$$

The a_i are the distinct eigenvalues and the P_{a_i} are the corresponding *eigenprojections* of A, i.e. P_{a_i} is the projection operator that projects onto the subspace $\mathcal{K}_{a_i}$ of $\mathcal{H}$ that consists of all the eigenvectors of A to eigenvalue a_i (i.e. the vectors $\psi_i \in \mathcal{H}$ with $A\psi_i = a_i\psi_i$). The eigenprojections corresponding to two different eigenvalues are always orthogonal, i.e. $P_{a_i}P_{a_j} = \mathbf{0}$ if $i \neq j$, and since they are projection operators, so that ${P_{a_i}}^2 = P_{a_i}$, one has $P_{a_i}P_{a_j} = \delta_{ij}P_{a_i}$ (where $\delta_{ij} = 1$ if $i = j$ and 0 otherwise). Moreover, the sum over all eigenprojections of a self-adjoint operator is identical with the identity operator, i.e. $\sum_i P_{a_i} = \mathbf{1}$. Thus the set $\{P_{a_i}\}$ of eigenprojections of a self-adjoint operator constitutes a *resolution of the identity operator* $\mathbf{1}$. On the other hand, given an arbitrary resolution of the identity $\{P_i\}$ one can, by combining its members with real numbers $\{a_i\}$ construct a self-adjoint operator according to (B.7). Each projection operator is itself self-adjoint and thus has also a unique spectral decomposition. However, the spectral decomposition of a projection P is merely of the trivial form $P = 1 \cdot P + 0 \cdot (\mathbf{1}-P)$.

Given a positive operator A and an orthonormal basis $\{\phi_i\}$, the *trace* of A is defined by

$$Tr(A) \equiv \sum_i \langle \phi_i, A\phi_i \rangle$$

and is independent from the particular orthonormal basis $\{\phi_i\}$. The trace of a positive operator can take any value in $[0, \infty]$, in particular, one has $Tr(\mathbf{1}) =$

∞. However, *if* for some positive operator $Tr(A)$ is finite one has

(T1) $Tr(AB)$ is finite for every $B \in \mathcal{B}(\mathcal{H})$ (finiteness)

(T2) $Tr(A(b_1 B_1 + b_2 B_2)) = b_1 Tr(AB_1) + b_2 Tr(AB_2)$ for all $b_1, b_2 \in \mathbb{C}$

and $B_1, B_2 \in \mathcal{B}(\mathcal{H})$ (linearity)

(T3) $Tr(AB) = Tr(BA)$ for all $B \in \mathcal{B}(\mathcal{H})$ (cyclic invariance)

Form the cyclic invariance (T3) it follows

$$Tr(U^{-1}ABU) = Tr(ABUU^{-1}) = Tr(AB)$$

for all unitary operators $U \in \mathcal{B}(\mathcal{H})$, so that the trace is unitarily invariant.

If the Hilbert space $\mathcal{H}$ is the tensor product of the Hilbert spaces $\mathcal{H}_1$ and $\mathcal{H}_2$, i.e. $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$, then each bounded operator $C \in \mathcal{B}(\mathcal{H})$ can be written in the form

$$C = \sum_{i,j} c_{ij} A_i^1 \otimes A_j^2$$

where the A_i^1 and A_j^2 are bounded operators on the Hilbert spaces $\mathcal{H}_1$ and $\mathcal{H}_2$ respectively, and $A^1 \otimes A^2$ is the tensor product of $A^1 \in \mathcal{B}(\mathcal{H}_1)$ and $A^2 \in \mathcal{B}(\mathcal{H}_2)$ defined on each vector $\psi \in \mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$ by

$$\begin{aligned}
(A^1 \otimes A^2)\psi &= (A^1 \otimes A^2)\psi \\
&= (A^1 \otimes A^2) \sum_{i,j} d_{ij} \phi_i^1 \otimes \phi_j^2 \\
&= \sum_{i,j} d_{ij} (A^1 \phi_i^1 \otimes A^2 \phi_j^2).
\end{aligned}$$

Thus like each vector $\psi \in \mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$ is a (countable) linear combination of tensor product vectors $\phi_i^1 \otimes \phi_j^2$, each bounded operator $C \in \mathcal{B}(\mathcal{H}_1 \otimes \mathcal{H}_2)$ is a (countable) linear combination of tensor product operators $A_i^1 \otimes A_j^2$. One can show that the thus defined tensor product $A \otimes B$ of bounded operators respects the factoring of scalar products (see (B.5)) in the following sense

$$\langle \varphi \otimes \chi, (A \otimes B)\varphi' \otimes \chi' \rangle = \langle \varphi, A\varphi' \rangle \langle \chi, B\chi' \rangle .$$

B.3 Algebras of bounded operators

An algebra $\mathcal{A}$ is a vector space (over $\mathbb{C}$) on which there is moreover a map—a *product*—$\mathcal{A} \times \mathcal{A} \to \mathcal{A}$, $(A, B) \mapsto AB$ such that for all $A, B, C \in \mathcal{A}$, $c \in \mathbb{C}$

(A1) $A(BC) = (AB)C$ (associativity)

(A2) $(A + B)C = AC + BC$ and $A(B + C) = AB + AC$ (distributivity)

(A3) $(cA)B = A(cB) = cAB$.

If there is furthermore a *unit element* $\mathbf{1} \in \mathcal{A}$, such that for all $A \in \mathcal{A}$

(A4) $A\mathbf{1} = \mathbf{1}A$

then $\mathcal{A}$ is called a *unitial algebra*. In QM and QFT only unitial algebras are considered. An algebra in which $AB = BA$ holds for each two of its elements is called *commutative* or *Abelian*. $\mathcal{B}(\mathcal{H})$ with the usual operator product and the identity operator as unit element is an unitial algebra that is not commutative. Furthermore, each subset $\mathcal{S} \subseteq \mathcal{B}(\mathcal{H})$ that contains with $A, B \in \mathcal{S}$ also $aA + bB \in \mathcal{S}$ and $AB \in \mathcal{S}$ (and the identity $\mathbf{1} \in \mathcal{S}$) is a (unitial) algebra. Each subset of $\mathcal{B}(\mathcal{H})$ that is itself a (unitial) algebra is called a (unitial) *subalgebra* of $\mathcal{B}(\mathcal{H})$.

A **-algebra* is an algebra $\mathcal{A}$ on which there is a map $^* : \mathcal{A} \to \mathcal{A}$, $A \mapsto A^*$ termed *involution*, such that for all $A, B \in \mathcal{A}$, $c \in \mathbb{C}$

(*1) $(AB)^* = B^* A^*$

(*2) $(A + B)^* = A^* + B^*$

(*3) $(cA)^* = \bar{c}A^*$

(*4) $(A^*)^* = A$

Because of (*3) the involution is not linear but *antilinear* $(aA + bB)^* = \bar{a}A^* + \bar{b}B^*$. On $\mathcal{B}(\mathcal{H})$ an involution is given by the map that assigns to each operator A its adjoint A^* and each subalgebra of $\mathcal{B}(\mathcal{H})$ that contains with A also its adjoint A^* is itself a *-algebra and thus a **-subalgebra* of $\mathcal{B}(\mathcal{H})$.

A *-algebra $\mathcal{A}$ that is at the same time a *Banach space* (i.e. a normed vector space that is complete with respect to its norm $\|\cdot\|$), and for all $A, B \in \mathcal{A}$ one has moreover

(B) $\|AB\| \leq \|A\| \, \|B\|$,

is called a *Banach algebra*. If $\mathcal{A}$ is at the same time a *-algebra and a Banach algebra, such that for all $A \in \mathcal{A}$

(C*) $\|A^*A\| = \|A\|^2$

then it is a *C*-algebra*. $\mathcal{B}(\mathcal{H})$ equipped with the *operator norm*

$$\|A\| \equiv \sup \left\{ \frac{\|A\psi\|}{\|\psi\|} : \psi \in \mathcal{H}, \psi \neq 0 \right\},$$

where the norm inside the supremum is that of $\mathcal{H}$, is a C*-algebra. Moreover, each *-subalgebra $\mathcal{A}$ of $\mathcal{B}(\mathcal{H})$ that is closed with respect to the operator norm topology, i.e. for which the limit A

$$\lim_{n \to \infty} \|A - A_n\| = 0$$

of a sequence $(A_n) \subseteq \mathcal{A}$ is again in $\mathcal{A}$ (and not merely in $\mathcal{B}(\mathcal{H})$), is itself a C*-algebra and thus a *C*-subalgebra* of $\mathcal{B}(\mathcal{H})$.

Another important topology on $\mathcal{B}(\mathcal{H})$—the *weak operator topology*—is defined by means of seminorms. A seminorm $p(A)$ obeys the properties (N1)-(N3) of a norm but not (N4), so that $p(A) = 0$ does not imply $A = \mathbf{0}$. Nevertheless, a topology can be generated by a (not necessarily countable) system of seminorms $\{p_i\}$ for which $(\forall i : p_i(A) = 0) \Rightarrow A = \mathbf{0}$. However, for topologies generated by a system of seminorms it is not enough to consider only the convergence of sequences. Rather the closure of a set is obtained by adding the limit points of all "generalized sequences"—of all so-called *nets*. Yet for the understanding of this work this subtlety is not important, so that we will speak only of ordinary sequences. The weak operator topology is obtained if one uses the absolute values of "matrix elements" $|\langle \psi, A\varphi \rangle|$ between arbitrary unit vectors $\psi, \varphi \in \mathcal{H}$, $\|\psi\| = \|\varphi\| = 1$, as a set of seminorms. Thus a sequence $(A_n) \subset \mathcal{B}(\mathcal{H})$ *converges weakly* to some $A \in \mathcal{B}(\mathcal{H})$ iff

$$\lim_{n \to \infty} |\langle \psi, A\varphi \rangle - \langle \psi, A_n\varphi \rangle| = 0, \text{ for all } \psi, \varphi \in \mathcal{H}.$$

The weak operator topology is weaker than the operator norm topology which means that each norm convergent sequence of operators also converges weakly but the converse is not generally true.

Now a *von Neumann algebra* is defined to be a unitial *-subalgebra $\mathcal{R}$ of $\mathcal{B}(\mathcal{H})$ that is closed with respect to the weak operator topology, i.e. that contains all weak limits of sequences $(A_n) \subset \mathcal{R}$. Interestingly, one can also characterize von Neumann algebras solely by algebraic means without the use of any topological considerations. The concept needed for this is that of the *commutant* $\mathcal{S}'$ of a set $\mathcal{S} \subseteq \mathcal{B}(\mathcal{H})$ of operators. $\mathcal{S}'$ consists of all operators in $\mathcal{B}(\mathcal{H})$ which commute with each member of $\mathcal{S}$, i.e.

$$\mathcal{S}' \equiv \{A \in \mathcal{B}(\mathcal{H}) : [A, B] = \mathbf{0}, \forall B \in \mathcal{S}\}\,.$$

Accordingly, the *bicommutant* of $\mathcal{S}$ is the set of all operators in $\mathcal{B}(\mathcal{H})$ which commute with all operators in $\mathcal{S}'$, i.e. $\mathcal{S}'' \equiv (\mathcal{S}')'$. Now von Neumann's famous bicommutant theorem implies that a unitial *-subalgebra $\mathcal{A}$ of $\mathcal{B}(\mathcal{H})$ is a von Neumann algebra iff it coincides with its bicommutant, i.e. $\mathcal{A} = \mathcal{A}''$ (see e.g. Haag 1996, Theorem 2.1.4).

Moreover, like C*-algebras, von Neumann algebras can also be defined "abstractly", i.e. by not defining them as special subalgebras of $\mathcal{B}(\mathcal{H})$ (see e.g. Kadison and Ringrose 1986, p. 498). However, this will not be done here because it would require the introduction of further technical concepts and is of no importance for this work.

Since the operator norm topology is stronger than the weak operator topology, each weakly closed subset is also strongly closed because with decreasing strength of the topology one gets more and more limit points. Thus each von Neumann algebra is also a C*-algebra but in general the converse is not the case. Besides the weak operator topology one can define some other important topologies on $\mathcal{B}(\mathcal{H})$ by means of systems of seminorms: the *ultra weak* and the *strong operator topology.* All the topologies mentioned so far are different and thus one would expect that one gets different kinds of *-algebras by taking closures of *-subalgebras of $\mathcal{B}(\mathcal{H})$ with respect to each of these topologies. However, von Neumann has shown that on *-subalgebras of $\mathcal{B}(\mathcal{H})$ the closures with respect to the ultra weak, the strong operator and the weak operator topology coincide. Therefore, from the mentioned topologies one gets only *two* different kinds of *-algebras: C*-algebras and von Neumann algebras and it is only the latter which are considered in this work.

Finally we will consider the notion of a state on a von Neumann algebra $\mathcal{R}$. A map

$$\rho : \mathcal{R} \mapsto \mathbb{C}$$

such that

(S1) $\rho(aA + bB) = a\rho(A) + b\rho(B)$ for all $a, b \in \mathbb{C}$ and $A, B \in \mathcal{R}$ (linearity)

(S2) $\rho(A) \geq 0$ for all positive operators $A \in \mathcal{R}$ (see Appendix B.2) (positivity)

(S3) $\rho(\mathbf{1}) = 1$ (normalization)

(S4) $\rho(\sum_i P_i) = \sum_i \rho(P_i)$ for each countable set of mutually orthogonal projections $\{P_i\}$ (countable additivity)

is called a *state* on the von Neumann algebra $\mathcal{R}$. Usually a state is characterized by requirements (S1)-(S3) only and if (S4) is moreover satisfied one speaks of a *normal state*. However, since *only* normal states are regarded as physically meaningful (see e.g. Haag 1996, Chapter III.2.2), we will simply include the countable additivity (S4) into the characterization of a state.

Now the set of all states on $\mathcal{R}$ is *convex*, which means that with ρ and σ also their *convex combination*

$$c\rho + c'\sigma \text{ with } c, c' \geq 0 \text{ and } c + c' = 1$$

is a state on $\mathcal{R}$. If $\mathcal{R}_1$ is a von Neumann subalgebra of $\mathcal{R}$, the *restriction* of a state ρ on $\mathcal{R}$ to $\mathcal{R}_1$ defined by

$$\rho|_{\mathcal{R}_1} : \mathcal{R}_1 \ni A \mapsto \rho(A)$$

is a state on $\mathcal{R}_1$. In general, there is more than one *extension* of a state on $\mathcal{R}_1$ to $\mathcal{R}$, or in other words, a state on $\mathcal{R}_1$ is the restriction of more than one state on a larger von Neumann algebra to $\mathcal{R}_1$. More generally, a state on $\mathcal{R}$ can be restricted to an arbitrary *subset* $\mathcal{S} \subset \mathcal{R}$ not being an algebra or even a von Neumann algebra. In this more general case too, the same symbol $\rho|_{\mathcal{S}}$ is used to denote the restriction of the domain of the state in question.

The set of states on a von Neumann algebra $\mathcal{R}$ can be equipped with several different topologies. We will, however, only need the topology induced by the norm distance

$$\|\rho - \sigma\| \equiv \sup \left\{ \frac{|\rho(A) - \sigma(A)|}{\|A\|} : A \in \mathcal{R} \right\},$$

where $\|A\|$ is the operator norm of $A \in \mathcal{R}$ as introduced earlier. Thus $\|\rho - \sigma\|$ is the lowest bound for the number c, such that $|\rho(A) - \sigma(A)| \leq c \|A\|$ holds for all operators in $\mathcal{R}$.

Appendix C

The standard axioms of AQFT and their physical interpretation

All mathematical notions not explicitly defined in the following can be found in one of the earlier appendixes. In the following M denotes Minkowski space, i.e. $\mathbb{R}^4$ equipped with the metric $\mathrm{g} : \mathbb{R}^4 \times \mathbb{R}^4 \to \mathbb{R}$, that assigns to each pair of points $x = (x^0, \mathbf{x}) \equiv (x^0, x^1, x^2, x^3)$, $y = (y^0, \mathbf{y}) \equiv (y^0, y^1, y^2, y^3) \in \mathbb{R}^4$ their Minkowski distance $\mathrm{g}(x, y) \equiv (x^0 - y^0)^2 - (\mathbf{x} - \mathbf{y})^2$ where $(\mathbf{x} - \mathbf{y})^2 \equiv \sum_{i=1}^{3}(x^i - y^i)^2$ is the square of the Euclidean distance in $\mathbb{R}^3$.[1] A subset of M is called an open *double cone* if it is the intersection of the open backward lightcone $V_-(z) \equiv \{x \in M : z^0 - x^0 > |\mathbf{x} - \mathbf{z}|\}$ of a point $z \in M$ with the open forward lightcone $V_+(y) \equiv \{x \in M : x^0 - y^0 > |\mathbf{x} - \mathbf{y}|\}$ of a point $y \in V_-(z)$. The set of all open double cones in M will be denoted by $D(M)$.

C.1 Local observables

The fundamental mathematical structure upon which AQFT is erected is a map

$$\mathcal{O} \mapsto \mathcal{R}(\mathcal{O}) \tag{C.1}$$

[1]Throughout this book natural units will be used, so that the speed of light in the vacuum c as well as Planck's constant $\hbar$ have the numerical value 1. With a choice of units such that the value of c is differs from 1, the Minkowski distance between x and y reads $\mathrm{g}(x, y) \equiv c^2(x^0 - y^0)^2 - (\mathbf{x} - \mathbf{y})^2$.

that assigns to each open, bounded, connected region $\mathcal{O}$ of Minkowski space M a von Neumann algebra $\mathcal{R}(\mathcal{O})$ on a common Hilbert space $\mathcal{H}$. Thus all the algebras $\mathcal{R}(\mathcal{O})$ are assumed to be subalgebras of the algebra $\mathcal{B}(\mathcal{H})$ of all bounded operators on a single Hilbert space $\mathcal{H}$. Moreover, the Hilbert space $\mathcal{H}$ is assumed to be separable. The points of Minkowski space are interpreted as spacetime points and the square root of the Minkowski distance $\mathrm{g}(x,y)^{1/2}$ is accordingly interpreted as their spatiotemporal distance. Since in the fundamental correspondence (C.1) only *bounded* and *connected* spacetime regions do appear, the corresponding algebras are called *local* algebras. The underlying idea of the correspondence (C.1) is that the operators of the local algebra $\mathcal{R}(\mathcal{O})$ represent *physical operations* that can be performed within spacetime region $\mathcal{O}$ (Haag and Kastler 1964; Hellwig and Kraus 1969 and 1970). The physically most important operations are those which result in a measurement of some *physical magnitude* or *observable* and as is usual practice, we will restrict the discussion to this class of operations. The problems with pointlike fields in the Lagrangian approach have indicated that one should better build up the theory on non-pointlike quantities. Moreover, it is clear that no "real" measurement can be carried out at a spacetime point. Therefore, in the fundamental correspondence (C.1) only open regions are appealed to, which automatically rules out pointlike ones.. Of course, it would make no difference to take closed regions with non-empty interior instead of open ones as the domain of the map (C.1), because from the physical viewpoint it is reasonable to expect that the operations performable within an open region determine the operations performable within its closure. Yet the choice of open regions has turned out to be the usual one and we will follow this trend.

Now we could proceed by stating the axioms usually required to hold for the fundamental correspondence (C.1). Yet the study of AQFT is much more simplified if one adopts some further restrictions on the set of regions appealed to in (C.1). This is because an open, bounded, connected region of M may still be of quite involved geometry and topology, which makes many investigations much more difficult. Since our primary interest are the structural properties of AQFT the optimal choice for a set of regions $\mathcal{O} \subset M$ would be one that facilitates the study of such structural properties, but is at the same time large enough to cover or approximate any open, bounded, connected region, so that nothing essential gets lost by this restriction of the domain of the map (C.1). Both conditions are perfectly met by the set of open double cones $D(M)$, which is the reason why this set is so often used especially in the study of structural properties of local algebras and the relationships among them. Therefore, we will in what follows restrict the domain of the map $\mathcal{O} \mapsto \mathcal{R}(\mathcal{O})$ to the set of open double cones $\mathcal{O} \in D(M)$. For convenience we will often omit the adjective "open" and simply speak of double cones. In contexts in which it will not give rise to confusions, we will moreover often simply speak of (bounded) regions.

The observables which can be measured within $\mathcal{O} \in D(M)$—the *local observables* in region $\mathcal{O}$—are represented by the self-adjoint elements of $\mathcal{R}(\mathcal{O})$. The important properties of self-adjoint operators in respect to their interpretation are that they are those operators whose spectrum solely consists of real numbers and that each self-adjoint operator A has a unique spectral decomposition. For reasons internal to Whitehead's ontology we will only need self-adjoint operators with a *completely discrete spectrum*, i.e. a spectrum $\sigma(A)$ solely consisting of eigenvalues (see Appendix B.2). In this simplest case the spectral decomposition reads

$$A = \sum_i a_i P_{a_i}.$$

Since all the eigenvalues a_i of a self-adjoint operator A are real they are interpreted as the *possible values* which the corresponding observable can take in single measurements. For convenience we will in what follows simply speak of self-adjoint operators themselves as "observables" or "physical magnitudes" rather than as "operators representing observables/physical magnitudes". From their idempotence $P = P^2$ together with their self-adjointness, it follows that projection operators have only two eigenvalues 1 and 0,[2] and thus represent the conceptually simplest observables. Instead of "simple observables" projection operators are also often referred to as *properties*. Part of the reason for this is provided by the fact that because of the unique spectral decomposition of each self-adjoint operator A, the occurrence of the possible value a_i in a measurement of the observable A is equivalent to the occurrence

[2] More precisely, apart form the zero and the identity operator which project onto the trivial subspaces $\{0\}$ and $\mathcal{H}$ of $\mathcal{H}$, every projection operator has precisely the two eigenvalues 1 and 0.

of value 1 of the corresponding eigenprojection P_{a_i} and therefore P_{a_i} is taken to represent the property expressed by the sentence "observable A has the value a_i".

Since by reason of the unique spectral decomposition of self-adjoint operators, the discussion of observables can be reduced to the discussion of simple observables or properties represented by projection operators, this suggests to specify the local algebras $\mathcal{R}(\mathcal{O})$ to be von Neumann algebras. This is because a von Neumann algebra is generated by the projections it contain in the sense that it is the weak closure of the set of all polynomials of the projections it contains (see e.g. Emch 1972, p.116). This then guaranties that the properties $\{P_{a_i}\}$ corresponding via the spectral decomposition to the local observable $A \in \mathcal{R}(\mathcal{O})$, measurable in region $\mathcal{O}$, do also belong to $\mathcal{R}(\mathcal{O})$ (and not merely to $\mathcal{B}(\mathcal{H})$), and thus can in fact occur in region $\mathcal{O}$ as it should be the case.[3]

C.2 States and probabilities

Now we turn to the representation of *states* of relativistic quantum systems. The notion of the "state" of a system suggests something like the specification of the "mode of existence" of a system and thus seems to be an ontologically

[3]Moreover, that the algebras $\mathcal{R}(\mathcal{O})$ contain sufficiently many projections makes the limitation to bounded operators less restrictive. Prima facie this is a restriction because one knows from QM that some important observables like position, momentum and in many cases also the energy of systems have to be represented by unbounded self-adjoint operators. In contrast to bounded operators, for unbounded operators $\|A\psi\|$ is not bounded for all $\psi \in \mathcal{H}$. Therefore, an unbounded operator is not defined on the whole Hilbert space $\mathcal{H}$ but merely on the subset of those vectors for which $\|A\psi\| < \infty$. This fact makes the theory of unbounded operators much more complicated than that of bounded operators. However, if the subset of $\mathcal{H}$ on which $\|A\psi\| < \infty$ is dense in $\mathcal{H}$ one can generalize all physically relevant concepts known from bounded operators to unbounded ones. In particular, one can generalize the concepts of self-adjointness, of the spectrum and of the spectral decomposition to unbounded densely definend operators. Since all physically relevant statements about unbounded self-adjoint operators can also be expressed by means of their spectral projections which, as projections are always bounded operators, it suffices for an unbounded observable A to count as measurable within region $\mathcal{O}$ that all its spectral projections belong to $\mathcal{R}(\mathcal{O})$. Unbounded self-adjoint operators whose spectral projections belong to a von Neumann algebra $\mathcal{R}$ are said to be affiliated with $\mathcal{R}$.

much more interesting concept than that of an observable. However, from the viewpoint of physics the "mode of existence" of a quantum system is only interesting in so far as it allows the prediction of the probabilities for all the possible measurement results of each observable. The mathematical device that does precisely this is that of a *state* on $\mathcal{B}(\mathcal{H})$ as defined in Appendix B.3 (note that this time the term "state" refers to a purely mathematical object). The defining properties of a state ρ on $\mathcal{B}(\mathcal{H})$ imply that it is a *probability measure over each resolution* $\{P_i\}$ *of the identity*:

(i) $\rho(P_i)$ lies between 0 and 1,[4]

(ii) ρ is countably additive over each subset $\{P_{i_k}\} \subseteq \{P_i\}$, i.e. $\rho(\sum_k P_{i_k}) = \sum_k \rho(P_{i_k})$

and finally, because of (ii), $\sum_i P_i = \mathbf{1}$ and the normalization $\rho(\mathbf{1}) = 1$,

(iii) the sum over all $\rho(P_i)$ is 1, i.e. $\sum_i \rho(P_i) = 1$.

Since the eigenprojections $\{P_{a_i}\}$ of a self-adjoint operator A form a resolution of the identity and each eigenprojection P_{a_i} is in one-to-one correspondence to a possible value a_i of A, *every state* ρ *defines a probability measure over the possible values of each observable*. Therefore, one assumes that the (physical) states of systems are represented by the mathematical states on $\mathcal{B}(\mathcal{H})$. Then the *probability for the occurrence of value* a_i *of observable* A *upon measurement on a system in state* ρ, abbreviated by $prob_\rho(A = a_i)$, is given by $\rho(P_{a_i})$, i.e.

$$prob_\rho(A = a_i) = \rho(P_{a_i}).$$

From the possible values $a_i \in \sigma(A)$ together with their probabilities $prob_\rho(A = a_i) = \rho(P_{a_i})$ one can furthermore build the weighted sum

$$\sum_i a_i \; prob_\rho(A = a_i) = \sum_i a_i \rho(P_{a_i})$$

which is accordingly interpreted as the *expectation value* of observable A upon measurement on a system in state ρ. Making use of the linearity of state ρ and

[4] Since $\rho(P_i) \geq 0$ together with (iii) automatically implies $\rho(P_i) \leq 1$ it is sufficient to require that $\rho(P_i)$ is non-negative.

the spectral decomposition of A, the expectation value of A in state ρ simply turns out to be the value of A in state ρ, i.e.

$$\begin{aligned} exp_\rho(A) &= \sum_i a_i \; prob_\rho(A = a_i) = \sum_i a_i \rho(P_{a_i}) \\ &= \rho\left(\sum_i a_i P_{a_i}\right) = \rho(A). \end{aligned}$$

Note that this implies that for simple observables P the expectation value $exp_\rho(P) = \rho(P)$ coincides with the probability $prob_\rho^P(1) = \rho(P)$ for the occurrence of value 1 of P—or in terms of properties with the probability for the occurrence of property P. It shall be mentioned that the countable additivity of a state is equivalent to its continuity with respect to the weak operator topology: if the sequence (A_n) converges weakly to an operator A then the sequence of numbers $\rho(A_n)$ converges to the number $\rho(A)$. In other words, the expectation values of two operators which are close to one another with respect to the weak operator topology are also close to each other. Therefore, one can approximate the expectation value of the limit operator of a weakly converging sequence (A_n) by the expectation values of elements of this sequence such that the approximation is the better the closer the chosen element A_n is to the limit operator A.

Of course, probabilities and expectation values cannot be observed in *single* measurements. Rather the connection of these theoretical notions with empirical results has to proceed via relative frequencies and mean values in large ensembles of identically prepared systems. The probability $\rho(P_{a_i})$ with which value a_i of observable A will occur upon a measurement on a system in state ρ has to be compared with the relative frequency $\frac{N(a_i)}{N}$ with which value a_i occurs in a large series of measurements of A in an ensemble of $N \gg 1$ systems in state ρ. Accordingly, the expectation value $\rho(A)$ has to be compared with the mean value $\sum_i a_i \frac{N(a_i)}{N}$ of a_i in the ensemble.

As is well-known from QM each state ρ on $\mathcal{B}(\mathcal{H})$ can be represented by a *density operator* W (i.e. by a positive operator $W \in \mathcal{B}(\mathcal{H})$ with trace $Tr(W) = 1$; see Appendix B.2) via the standard formula

$$\rho(A) = Tr(WA). \tag{C.2}$$

A *pure* state on $\mathcal{B}(\mathcal{H})$ is one that is not a *convex combination*

$$\rho = \sum_i c_i \rho_i \text{ with } c_i \geq 0 \text{ and } \sum_i c_i = 1 \tag{C.3}$$

of other states. It is represented via (C.2) by a one-dimensional projection operator $P \in \mathcal{B}(\mathcal{H})$. Since for a one-dimensional projection operator $Tr(PA) = \langle \psi, A\psi \rangle$, where $\psi \in \mathcal{H}$ is the unit vector that spans the one-dimensional subspace of $\mathcal{H}$ onto which P projects, each pure state ρ on $\mathcal{B}(\mathcal{H})$ can equivalently be represented by a unit vector $\psi \in \mathcal{H}$ via the special case $\rho(A) = \langle \psi, A\psi \rangle$ of formula (C.2).

Now being equipped with the fundamental correspondence (C.1) and the concepts of local observables and states we can go on to introduce the further standard axioms of AQFT. We will henceforth follow the common sloppy practice of simply referring to all operators in $\mathcal{R}(\mathcal{O})$ as "observables measurable within $\mathcal{O}$"—despite the fact that only self-adjoint operators deserve this interpretation. Furthermore, when we speak of "(pure) states" without mentioning any algebra on which they are defined, we always mean "(pure) states on $\mathcal{B}(\mathcal{H})$".

C.3 The further axioms of AQFT

The first axiom to be mentioned is called *isotony* and requires that the map (C.1) is "inclusion preserving" in the sense that the inclusion of regions implies the inclusion of the corresponding local algebras.

Isotony: For all $\mathcal{O}_1$, $\mathcal{O}_2 \in D(M)$, $\mathcal{O}_1 \subseteq \mathcal{O}_2$ implies $\mathcal{R}(\mathcal{O}_1) \subseteq \mathcal{R}(\mathcal{O}_2)$.

This assumption is very natural in the light of the interpretation of the operators from $\mathcal{R}(\mathcal{O})$ as observables measurable within region $\mathcal{O}$, because it simply says that in a larger region there are more (or at least: not less) observables to be measured. Mathematically this isotony-property turns the set of local algebras $\{\mathcal{R}(\mathcal{O})\}_{D(M)} \equiv \{\mathcal{R}(\mathcal{O}) : \mathcal{O} \in D(M)\}$ into a so-called net of von Neumann algebras, which in particular means that whenever $\mathcal{O}_1 \cup \mathcal{O}_2 \subseteq \mathcal{O}$ then $\mathcal{R}(\mathcal{O}_1) \cup \mathcal{R}(\mathcal{O}_2) \subseteq \mathcal{R}(\mathcal{O})$ holds, too. As a consequence of this the set-theoretic union

$$\mathcal{A}_{loc} \equiv \bigcup_{\mathcal{O}\in D(M)} \mathcal{R}(\mathcal{O})$$

of all local algebras itself becomes a *-subalgebra of $\mathcal{B}(\mathcal{H})$, termed the *algebra of all local observables.* Without the isotony-property this would not be the case, because for $A_1 \in \mathcal{R}(\mathcal{O}_1)$, $A_2 \in \mathcal{R}(\mathcal{O}_2)$ with $\mathcal{O}_1 \neq \mathcal{O}_2$ nothing would secure that their products A_1A_2 respectively A_1A_2 and their sum $A_1 + A_2$ belong to $\mathcal{R}(\mathcal{O}_1) \cup \mathcal{R}(\mathcal{O}_2)$ or $\mathcal{R}(\mathcal{O}_1 \cup \mathcal{O}_2)$ or any other local algebra $\mathcal{R}(\mathcal{O})$ with $\mathcal{O} \supset \mathcal{O}_1 \cup \mathcal{O}_2$. However, in general $\mathcal{A}_{loc}$ will not already be a von Neumann algebra, since it will not be closed with respect to the weak operator topology or equivalently it will not coincide with its bicommutant. The weak closure $\overline{\mathcal{A}_{loc}}^w$ or equivalently the bicommutant $\mathcal{A}''_{loc}$ of $\mathcal{A}_{loc}$ is a von Neumann algebra, denoted by $\mathcal{R}(M)$, that is assumed to be the algebra of all observables associated to the whole Minkowski space. The algebra $\mathcal{R}(M)$ contains, by definition, besides all local observables also all the weak limits of sequences of local observables and as far as these "limit operators" arise from sequences (A_i) of local observables such that there is no region $\mathcal{O} \in D(M)$ with $(A_i) \subset \mathcal{R}(\mathcal{O})$, they do not themselves belong to any local algebra. For this reason the operators in $\mathcal{R}(M)\backslash\mathcal{A}_{loc}$ are called *global observables* and the algebra $\mathcal{R}(M)$ in which they are contained is termed the *global algebra.* Since global observables are not measurable in any bounded spacetime region and are moreover merely a forteriori "constructs" from the basic local observables they are only regarded as physical idealizations.

That the same construction principle by which the global algebra $\mathcal{R}(M)$ has been defined from the local algebras $\{\mathcal{R}(\mathcal{O})\}_{D(M)}$, namely to take a covering of M by double cones $\{\mathcal{O}_i\}$ and to define the algebra corresponding to M as the uniquely determined weak closure or bicommutant of the set-theoretic union of the corresponding local algebras $\{\mathcal{R}(\mathcal{O}_i)\}$, can also be used to associate a von Neumann algebra $\mathcal{R}(\mathcal{O}) \subset \mathcal{B}(\mathcal{H})$ to unbounded regions $\mathcal{O} \subset M$ other than M itself, is implied by the next axiom termed *additivity.*

Additivity: To $\mathcal{O} \subseteq M$ with $\mathcal{O} = \cup_i \mathcal{O}_i$, $\{\mathcal{O}_i\} \subset D(M)$ corresponds the von Neumann algebra

$$\mathcal{R}(\mathcal{O}) = \Big(\bigcup_i \mathcal{R}(\mathcal{O}_i)\Big)''.$$

Thus additivity allows the extension of the correspondence $\mathcal{O} \mapsto \mathcal{R}(\mathcal{O})$ to all spacetime regions which are arbitrary unions of double cones. This includes also *disconnected* regions, like $\mathcal{O} = \mathcal{O}_1 \cup \mathcal{O}_2$ with $\mathcal{O}_1$ and $\mathcal{O}_2$ disjoint.

Up to this point only topological properties of spacetime have been appealed to in the axioms. The following axiom makes also use of the metrical structure of spacetime, because it says that all operators $A_1 \in \mathcal{R}(\mathcal{O}_1)$ commute with all operators $A_2 \in \mathcal{R}(\mathcal{O}_2)$, i.e. $[A_1, A_2] = \mathbf{0}$, if the regions $\mathcal{O}_1$ and $\mathcal{O}_2$ are spacelike separated from each other.

Spacelike Commutativity: For all $\mathcal{O}_1$, $\mathcal{O}_2 \in D(M)$, if $\mathcal{O}_1$ and $\mathcal{O}_2$ are spacelike separated from one another then $\mathcal{R}(\mathcal{O}_1) \subseteq \mathcal{R}(\mathcal{O}_2)'$.

One can show that spacelike commutativity is equivalent to the independence of the probabilities for measurement outcomes of an observable $A_1 \in \mathcal{R}(\mathcal{O}_1)$ (in any arbitrary state ρ) from the choice of the observable $A_2 \in \mathcal{R}(\mathcal{O}_2)$ measured in a spacelike separated region $\mathcal{O}_2$ (see e.g. Butterfield 1994, p. 769 f; and also Section 10.2.3).

Now according to STR the symmetry transformations of spacetime are given by the Poincaré group $\mathcal{P}_+^\uparrow$. The Poincaré group consists of (1) spatiotemporal translations $x \to x + a$ by an arbitrary 4-vector $a = (a^0, a^1, a^2, a^3) \in M$, where a^0 corresponds to the time shift and $\mathbf{a} \equiv (a^1, a^2, a^3)$ to the spatial shift, (2) spatial rotations $\mathbf{x} \to R\mathbf{x}$ leaving the time coordinate x^0 unchanged, (3) Lorentz boosts, i.e. spatiotemporal rotations which correspond to velocity changes $\mathbf{v} \to \mathbf{v} + \mathbf{u}$ and (4) all combinations of (1)-(3). That these transformations are symmetry transformations of spacetime means that they do not change the spatiotemporal distance $\mathrm{g}(x, y)^{1/2} = ((x^0 - y^0)^2 - (\mathbf{x} - \mathbf{y})^2)^{1/2}$ between any two spacetime points, i.e. $\mathrm{g}(x, y)^{1/2} = \mathrm{g}(g(x), g(y))^{1/2}$ for all $g \in \mathcal{P}_+^\uparrow$ and all x, $y \in M$.

The sets of spatiotemporal translations (in short: translations) $\mathcal{T}$, spatial rotations $\mathcal{R}$ as well as the set of spatial rotations combined with translations and the set of spatial rotations combined with Lorentz boosts, i.e. the set of Lorentz transformations $\mathcal{L}_+^\uparrow$, are subgroups of the Poincaré group. This means that each of these subsets is again a group such that (1) it has the same neutral element e as $\mathcal{P}_+^\uparrow$, defined by $eg = ge = g$ for all $g \in \mathcal{P}_+^\uparrow$ that as a transformation maps each spacetime point onto itself $x \to x$, and (2) it is closed with respect to combinations $g_1 g_2$ of the transformations it contains, i.e. each such combination is again an element of this subset. Unlike spatial rotations and Lorentz transformations the subgroup of translations $\mathcal{T}$ is moreover a commutative or Abelian group which means that for each two translations $g_1 g_2 = g_2 g_1$ holds. The Lorentz boosts alone do *not* form a subgroup of $\mathcal{P}_+^\uparrow$ because a combination of two Lorentz boosts can be a spatial rotation and thus does not again belong to the set of Lorentz boosts. Note furthermore that our restriction to double cones as admissible regions for the fundamental correspondence (C.1) is not flawed by the action of the Poincaré group because the Poincaré transform of a double cone is again a double cone.[5]

That Poincaré transformations are symmetry transformations of spacetime means for QFT that a simultaneous application of such a transformation to the source by which a system in a certain state is prepared and to the measuring device by which a certain observable is to be measured does not change the result of the measurement. However, since the result of an individual measurement of an observable A is not reproducible—QFT is a probabilistic theory—the latter cannot mean that the results of individual measurements are invariant under the simultaneous application of a Poincaré transformation $g \in \mathcal{P}_+^\uparrow$ to the system and the measurement device. Rather what needs to be invariant in such a case are the relative frequencies of measurement results. On the side of the formalism this means that under a simultaneous transformation of state $\rho \to \rho_g$ and observable $A \to A_g$ the probabilities $\rho(P_{a_i})$ of all possible values $a_i \in \sigma(A)$ of A have to be invariant. Since this obviously implies that the expectation value $\rho(A) = \sum_i a_i \rho(P_{a_i})$ of A is also invariant and, on the other hand, the probabilities $\rho(P_{a_i})$ are nothing else than the expectation values of the simple observables P_{a_i} one can equally well state the requirement of Poincaré invariance by saying that the expectation values $\rho(A)$ of all observables A in all states ρ must not be affected by the simultaneous transformations $\rho \to \rho_g$ and $A \to A_g$, i.e.

$$\rho_g(A_g) = \rho(A), \text{ for all } g \in \mathcal{P}_+^\uparrow. \tag{C.4}$$

[5]In fact, the whole set of double cones can be generated from each single double cone $\mathcal{O}_0$ by acting on it with all Poincaré transformations, i.e. $\mathcal{P}_+^\uparrow\ (\mathcal{O}_0) = D(M)$.

Now for the terms ρ_g and A_g to make mathematical sense, one needs a representation[6] of the group $\mathcal{P}_+^\uparrow$ that implements the transformations $g \in \mathcal{P}_+^\uparrow$ on the level of observables and states. Moreover, this implementation must be such that the invariance condition (C.4) is fulfilled. Under the assumption that the global algebra $\mathcal{R}(M)$ coincides with the algebra $\mathcal{B}(\mathcal{H})$ of all bounded operators on $\mathcal{H}$, which will be discussed at end of this appendix, one can (without loss of generality) implement $\mathcal{P}_+^\uparrow$ by means of a *weakly continuous group of unitary operators*

$$U(\mathcal{P}_+^\uparrow) \equiv \left\{ U(g) : g \in \mathcal{P}_+^\uparrow \right\}$$

from $\mathcal{B}(\mathcal{H})$, such that the observables transform like

$$A_g \equiv U(g) A U(g)^{-1} \tag{C.5}$$

(Horuzhy 1990, p. 17). That $U(P_+^\uparrow)$ is a representation of the Poincaré group means that it is the image of a map $U : \mathcal{P}_+^\uparrow \to \mathcal{B}(\mathcal{H})$ that "models" the group $\mathcal{P}_+^\uparrow$ within $\mathcal{B}(\mathcal{H})$, i.e. $U(\mathcal{P}_+^\uparrow)$ is the image of $\mathcal{P}_+^\uparrow$ under a map that preserves all group structures—a so-called group-homomorphism. In particular, this means that to the identity transformation $e \in \mathcal{P}_+^\uparrow$ corresponds the identity operator (i.e. $U(e) = \mathbf{1} \in U(P_+^\uparrow)$), to the inverse transformation g^{-1} of g corresponds the inverse of the operator $U(g)$ (i.e. $U(g^{-1}) = U(g)^{-1}$) and to the product of two transformations $g_2 g_1$ corresponds the product of the corresponding operators (i.e. $U(g_2 g_1) = U(g_2)U(g_1)$). The "weak continuity of the group $U(\mathcal{P}_+^\uparrow)$" is to be understood as the weak continuity of the corresponding group-homomorphism $U : g \mapsto U(g)$. This means that if the sequence $(g_n) \subset \mathcal{P}_+^\uparrow$ converges to $\tilde{g} \in \mathcal{P}_+^\uparrow$ then $(U(g_n)) \subset U(\mathcal{P}_+^\uparrow)$ converges to $U(\tilde{g})$ where the latter convergence is with respect to the weak operator topology. One can show that each such limiting operator $U(\tilde{g})$ is again unitary and belongs to $U(\mathcal{P}_+^\uparrow)$, so that the weak continuity of the group-homomorphism $U : g \mapsto U(g)$ secures that $U(\mathcal{P}_+^\uparrow)$ is closed with respect to limiting processes within $\mathcal{P}_+^\uparrow$.[7]

[6] Note that the term "representation" is meant this time in the mathematical sense of there being a group-homomorphism $h : G \to N$ (i.e. a map that preserves all group structures and thus makes the image $h(G) \subseteq N$ a "model" of the group G within the set N), and not in the sense of an interpretation of mathematical objects by non-mathematical ones as in case of the representation of observables by self-adjoint operators etc.

[7] In the overwhelming part of the literature it is said that $U(\mathcal{P}_+^\uparrow)$ respectively $U : g \mapsto U(g)$ has to be strongly continuous, i.e. continuous with respect to the so-called strong operator

Now one can show that the transformation law for observables under the representation $U(\mathcal{P}_+^\uparrow)$, i.e. (C.5) together with the invariance condition (C.4) already fixes the transformation law for states to be of the form

$$\rho_g = \rho(U(g)^{-1} \cdot U(g)), \tag{C.6}$$

so that the latter need not be assumed independently.[8] That (C.5) together with (C.6) in fact fulfil the invariance condition (C.4) immediately follows by inserting A_g according to (C.5) into ρ_g according to (C.6)

$$\rho_g(A_g) = \rho(U(g)^{-1}U(g)AU(g)^{-1}U(g)) = \rho(A).$$

Thus the existence of the unitary representation $U(\mathcal{P}_+^\uparrow)$ of the Poincaré group in fact secures the Poincaré invariance of the theory expressed by (C.4).

We will now see that for the action of the representation $U(\mathcal{P}_+^\uparrow)$ of the Poincaré group on the local observables to be compatible with the inherent spatiotemporal structure of the net $\{\mathcal{R}(\mathcal{O})\}_{D(M)}$, the former has to act *covariantly* on the latter, which means that the Poincaré transform $A_g = U(g)AU(g)^{-1}$ of a local observable $A \in \mathcal{R}(\mathcal{O})$ has to belong to the local algebra $\mathcal{R}(g(\mathcal{O}))$ of the Poincaré transformed region $g(\mathcal{O})$. To see that this has to be the case, let us consider for example a translation g_a by a 4-vector $a \in M$. It transforms a spacetime region $\mathcal{O}$ into the region $g_a(\mathcal{O}) = \mathcal{O} + a$. Now if for a local observable $A \in \mathcal{R}(\mathcal{O})$ the transformed observable $U(g_a)AU(g_a)^{-1}$ would *not* belong to the local algebra $\mathcal{R}(\mathcal{O}+a)$ of the transformed region it would not be measurable within this region $\mathcal{O}+a$. However, in this case the transformation $A \to U(g_a)AU(g_a)^{-1}$ could hardly count as a proper implementation of the translation by 4-vector a on the level of local observables. Since an analog line of thought applies in regard to all Poincaré transformations, one altogether requires that the following holds:

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topology. Of course, this is true because the weak limit of a sequence of unitary operators will not be unitary *if the sequence is not also strongly converging* (see e.g. Thirring 1994, p. 16). However, since von Neumann has shown that for von Neumann algebras the weak- and strong operator toplogies are equivalent, i.e. each weakly converging sequence is also strongly converging and vice versa, there is no need to complicate the present exposition by further introducing the strong operator topology.

[8]Equivalently, (C.6) together with the invariance condition (C.4) fixes (C.5) (Boguliubov et al. 1990, p. 249 ff).

Covariance: There exists a weakly continuous unitary representation $U(\mathcal{P}_+^\uparrow) \subset \mathcal{B}(\mathcal{H})$ of the Poincaré group $\mathcal{P}_+^\uparrow$ such that for all $\mathcal{O} \in D(M)$ and all $g \in \mathcal{P}_+^\uparrow$

$$U(g)\mathcal{R}(\mathcal{O})U(g)^{-1} = \mathcal{R}(g(\mathcal{O})).$$

We will now investigate more closely the representation of the translation group $\mathcal{T} \subset \mathcal{P}_+^\uparrow$; this will lead us to the formulation of the last two axioms of AQFT. Since the translation group is a commutative subgroup of the Poincaré group its representation $U(\mathcal{T}) \equiv \{U(g) : g \in \mathcal{T}\}$ is a commutative subgroup of the representation $U(\mathcal{P}_+^\uparrow)$ of the Poincaré group. Moreover, each translation $g \in \mathcal{T}$ is in one-to-one correspondence to a 4-vector $a \in M$—the vector by which g translates. Therefore, the translation group $\mathcal{T}$ as well as its representation $U(\mathcal{T}) \equiv \{U(g) : g \in \mathcal{T}\}$ can be parameterized by four real numbers a^μ—the components of a 4-vector $a = (a^0, a^1, a^2, a^3) \in M$—so that we can simply write a or $U(a)$ respectively when referring to an element of $\mathcal{T}$ or $U(\mathcal{T})$ respectively. Thus altogether the representation $U(\mathcal{T})$ of the translation group is a commutative, weakly continuous, four-parameter group of unitary operators. According to the so-called SNAG theorem[9] each element of $U(\mathcal{T})$ can therefore be written in the form[10]

$$U(a) = e^{ia \bullet P} \tag{C.7}$$

where P is a 4-vector whose components P^μ, $\mu = 0, \ldots, 3$, are commuting self-adjoint operators—the so-called *generators* of $U(\mathcal{T})$—and "$\bullet$" stands for the Minkowski scalar product, which in terms of $a \in M$ and $P = (P^0, P^1, P^2, P^3)$ reads $a \bullet P \equiv a^0P^0 - \mathbf{a} \cdot \mathbf{P}$, where $\mathbf{a} \cdot \mathbf{P} \equiv a^1P^1 + a^2P^2 + a^3P^3$ is the Euclidean scalar product in $\mathbb{R}^3$.[11] Because of the operators P^μ are pairwise commuting, the unitary operator

$$U(a) = e^{ia \bullet P} = e^{i(a^0P^0 - a^1P^1 - a^2P^2 - a^3P^3)}$$

[9] "SNAG" stands for Stone, Naimark, Ambrose and Godement which were primarily involved in the elaboration of this theorem.

[10] With a choice of units such that the value of $\hbar$ differs from 1, (C.7) would read $U(a) = \exp(\frac{i}{\hbar}\, a \bullet P)$.

[11] One can show that the P^μ are necesarily unbounded self-adjoint operators. However, as mentioned in the footnote on page 304, unbounded self-adjoint operators also have a unique spectral decomposition, so that one can make sense of functions of unbounded self-adjoint operators, like the exponential $\exp(ia \bullet P)$, too (see e.g. Reed and Simon1980).

can also be written as a product

$$U(a) = e^{ia^0 P^0} e^{-ia^1 P^1} e^{-ia^2 P^2} e^{-ia^3 P^3}$$

of the pairwise commuting unitary operators

$$U(a^0) = e^{ia^0 P^0},\ U(a^j) = e^{-ia^j P^j}\ \ (j = 1, 2, 3).$$

Thus with respect to some fixed inertial reference frame the zeroth component P^0 is the generator of temporal translations

$$U(a^0) = e^{ia^0 P^0}$$

and the remaining three components P^j $(j = 1, 2, 3)$, collected up to the 3-vector $\mathbf{P} \equiv (P^1, P^2, P^3)$, generate spatial translations

$$U(\mathbf{a}) = e^{-i\mathbf{a}\cdot\mathbf{P}}.$$

The zeroth component P^0 of P is therefore interpreted as the energy- and $\mathbf{P} \equiv (P^1, P^2, P^3)$ as the momentum observable with respect to the chosen inertial reference frame.[12] Consequently, $P = (P^0, \mathbf{P})$ represents the *energy-momentum* observable of the kind of systems in question. Since the P^μ are pairwise commuting each of them also commutes with each translation $U(a)$ (and not merely with the translation $U(a^\mu)$ generated by P^μ itself) so that energy P^0 and momentum $\mathbf{P}$ are *translation invariant*, i.e.

$$U(a)P^0U(a)^{-1} = P^0 \text{ and } U(a)\mathbf{P}U(a)^{-1} = \mathbf{P}, \text{ for all } a \in M.$$

Thus in particular both energy and momentum are *conserved quantities* because they are invariant under all timelike translations.

Physically it is to be expected that, in general, the spectrum of the energy-momentum observable P will have a discrete as well as a continuous part. For example, for systems which consist of subsystems attracting each other by electromagnetic forces, the discrete part of the spectrum will correspond to the regime where the subsystems are bound together by reason of their attracting interactions, whereas the continuous part of the spectrum corresponds

[12] For a more detailed discussion and justification of this interpretation, see (Araki 1999, p. 75 ff) and (Thaller 1992, Chapter 2).

to the regime where the kinetic energy of the subsystems is large enough to separate and eventually behave like non-interacting, i.e. free, systems. So far we have only discussed observables with a purely discrete spectrum. However, all that we need to now at present about non-discrete observables is that the values from the continuous part of their spectrum are also regarded as possible values which the observable can take upon measurement. This leads to some important physical restrictions on the spectrum of the energy-momentum observable. First of all, the spectrum of the energy has to be bounded from below, i.e. there has to be a number $E \in \mathbb{R}$ such that $\sigma(P^0) \geq E$,[13] for otherwise one could (in principle) arbitrarily lower the energy of a system and transfer this energy to its environment, thereby using the system as an infinite reservoir of energy. A particular choice of E merely fixes the scale of possible energy values and thus from the standpoint of physics each choice is as good as any other—however, for convenience one takes $E = 0$. Now from the relativistic kinematical connection between energy p^0, momentum $\mathbf{p}$ and rest mass m given by $(p^0)^2 = \mathbf{p}^2 + m^2$, together with the relativistic identity $p^2 = p \bullet p = (p^0)^2 - \mathbf{p}^2$, it follows that the square of the energy-momentum of a system equals its rest mast, i.e. $p^2 = m^2$. Therefore, in QFT the operator $\sqrt{P^2} = \sqrt{(P^0)^2 - \mathbf{P}^2}$ is interpreted as the *rest mass* observable. For the physical interpretation of $\sqrt{P^2}$ as the observable of rest mass (as well as for it to be a well-defined self-adjoint operator), the spectrum of P^2 has to to be positive, i.e. $\sigma(P^2) \geq 0$, because otherwise one could get negative values for the square of the rest mass and thus imaginary values for the rest mass itself. The condition $\sigma(P^2) \geq 0$ implies that the spectrum of P does not simply coincide with the full Cartesian product $\times_\mu \sigma\left(P^\mu\right)$ of the spectra of its components P^μ as it were to be expected from a purely mathematical point of view, but rather that it is a proper subset of the Cartesian product. More precisely, the two requirements $\sigma(P^0) \geq 0$ and $\sigma(P^2) \geq 0$ taken together say that only those energy-momentum values $p = (p^0, \mathbf{p}) \in \mathbb{R}^4$ are possible for which $p^0 \geq 0$ and $p^2 = (p^0)^2 - \mathbf{p}^2 \geq 0$ or equivalently for which $p^0 \geq 0$ and $p^0 \geq |\mathbf{p}|$, where $|\mathbf{p}| = \sqrt{(p^1)^2 + (p^2)^2 + (p^3)^2}$ is the Euclidian length of the vector $\mathbf{p} \in \mathbb{R}^3$. Thus the spectrum of the energy-momentum observable is con-

[13]Since the P^μ are unbounded operators, this condition is not automatically fulfilled, as in case of bounded operators (see also the footnote on page 304).

fined to the closed forward cone $\overline{V}_+ \equiv \{p = (p^0, \mathbf{p}) \in \mathbb{R}^4 : p^0 \geq 0,\, p^0 \geq |\mathbf{p}|\}$. This requirement is known as the

Spectrum condition: $\sigma(P) \subseteq \overline{V}_+$.

The spectrum condition is often held to prohibit a superluminal transfer of energy-momentum. However, this seems to be an overstatement because what the spectrum condition in fact rules out is the existence of states ρ with a non-zero expectation value of energy-momentum and a non-zero expectation value for a superluminal velocity. Now for the latter to make sense one first of all needs a self-adjoint operator that can reasonably be taken to represent a velocity observable of relativistic quantum systems. Since the task to introduce such a relativistic quantum velocity operator is mathematically quite involved (see e.g. Thaller 1992), we will—as it is quite usual—only give an heuristic argument for the claim in question. According to relativistic kinematics, energy p^0, momentum $\mathbf{p}$ and rest mass m are connected with the velocity $\mathbf{v}$ via $p^0 = m\gamma$ and $\mathbf{p} = m\gamma\mathbf{v}$, where $\gamma \equiv (1 - |\mathbf{v}|^2)^{-1/2}$,[14] so that the velocity is given by $\mathbf{v} = \mathbf{p}/p^0$. But this means that $p^0 \geq |\mathbf{p}|$ and $|\mathbf{v}| > 1$ are inconsistent, so that if there were some state ρ to which a superluminal velocity $|\mathbf{v}| > 1$ could be ascribed with some non-zero probability this state would violate the spectrum condition because it would prescribe a non-zero probability to energy-momentum values with $p^0 < |\mathbf{p}|$. Yet this merely rules out a certain kind of superluminal energy-momentum transfer, namely a transfer by means of systems which are in the described superluminal velocity states (see also Section 2.8.1).

The last axiom of AQFT is concerned with the structure of the vacuum. One characteristic of the vacuum is that it the system with the lowest possible amount of energy-momentum. This means that the state that represents the vacuum is a pure state on $\mathcal{B}(\mathcal{H})$ whose generating unit vector $\Omega \in \mathcal{H}$—*the vacuum vector*—is an eigenvector of P to eigenvalue 0, i.e. $P\Omega = 0$ or equivalently $P^\mu\Omega = 0$ for $\mu = 0, \ldots, 3$. Because of

$$P\Omega = 0$$

$$\Leftrightarrow e^{ia\bullet P}\Omega = \Omega, \text{ for all } a \in M$$

[14] Remember that we are using natural units, so that $c = 1$.

$$\Leftrightarrow U(a)\Omega = \Omega, \text{ for all } a \in M$$

this is equivalent to the fact that Ω is an eigenvector of all translations $U(a) \in U(\mathcal{T})$ to eigenvalue 1 and thus is translation invariant. Yet the vacuum is not only expected to be translation invariant but also to be invariant under Lorentz transformations $U(g) \in U(\mathcal{L}_+^\uparrow)$ and thus under the whole Poincaré group $U(\mathcal{P}_+^\uparrow)$.[15] A further physically reasonable assumption is that the vacuum is unique, which formally means that the vacuum vector Ω is the *only* vector (up to an irrelevant phase factor $c \in \mathbb{C}$, $|c| = 1$) that is invariant under the Poincaré group. The assumptions as to the properties of the vacuum so far mentioned, namely that it is Poincaré invariant (or equivalently that it has zero energy-momentum and is Lorentz invariant) and that it is unique, would have been assumed in any classical, i.e. non-quantum, theory, too. However, classically the vacuum simply coincides with "empty space" so that there is no need to regard and study it as a physical system in its own right. This attitude towards the vacuum has dramatically changes since the advent of QFT. As is known from the treatment of free fields within the Lagrangian approach, each "material system"—understood as a system consisting of a certain number of "stable particles"—can be regarded as an excitation of the vacuum. More generally, each vector $\psi \in \mathcal{H}$ can be approximated as closely as one likes (in the norm of $\mathcal{H}$) by the application of appropriate polynomials of the local fields $\Psi(x)$, to the vacuum state.[16] In other words, the set of vectors generated by the application of polynomials of local fields to the vacuum vector is a dense subset of $\mathcal{H}$. This property of the vacuum is assumed to hold also in the presence of interactions and therefore it is incorporated as a basic assumption into the formalism of AQFT. Since in AQFT there are no local fields from which the local observables are constructed, one assumes accordingly that the set of vectors generated by the application of arbitrary local observables

$$A \in \mathcal{A}_{loc} = \bigcup_{\mathcal{O} \in D(M)} \mathcal{R}(\mathcal{O})$$

[15] We will simply call $U(\mathcal{T})$, $U(\mathcal{L}_+^\uparrow)$ and $U(\mathcal{P}_+^\uparrow)$ translation-, Lorentz- and Poincaré group because this will not cause any confusions but enhanches the readability of the text.

[16] Of course, as mentioned in Chapter 4, the local fields have to be "smeared" over some non-pointlike region before they are well-defined operators that can be applied to vectors from $\mathcal{H}$.

to the vacuum vector Ω is a dense subset of $\mathcal{H}$. A vector $\varphi \in \mathcal{H}$ from which, by the application of operators from a set $\mathcal{S} \subseteq \mathcal{B}(\mathcal{H})$, a dense subset of $\mathcal{H}$ can be generated is called *cyclic* with respect to $\mathcal{S}$. Thus we can also state the assumption in question by saying that Ω is cyclic with respect to $\mathcal{A}_{loc}$. The assumptions in regard to the vacuum can therefore be summarized as follows:

Vacuum: In $\mathcal{H}$ there exists a unique unit vector Ω (up to a phase factor), that is invariant with respect to $U(\mathcal{P}_+^\uparrow)$ and cyclic with respect to $\mathcal{A}_{loc}$.

In what follows we will denote the vacuum state, i.e. the pure state $\langle\Omega, \cdot\, \Omega\rangle$ on $\mathcal{B}(\mathcal{H})$ generated by the vacuum vector Ω, by ω. In connection with the implementation of the Poincaré group we have made the assumption that the global algebra $\mathcal{R}(M)$ coincides with the algebra $\mathcal{B}(\mathcal{H})$ of all bounded operators on $\mathcal{H}$. However, one can show that $\mathcal{R}(M) = \mathcal{B}(\mathcal{H})$ is equivalent to the cyclicity and uniqueness of the vacuum vector (Horuzhy 1990, p. 107), so that one need not assume the coincidence of $\mathcal{R}(M)$ and $\mathcal{B}(\mathcal{H})$ independently.

As explained above, because of spatiotemporal translations $U(a) = \exp(ia\bullet P)$ are generated by the energy-momentum operator P, the latter is automatically translation invariant. Now one can show that no non-trivial operator A (i.e. $A \neq \lambda\mathbf{1}$, $\lambda \in \mathbb{C}$) that is translation invariant (i.e. $U(a)AU(a)^{-1} = A$, for all $a \in M$) can belong to any local algebra $\mathcal{R}(\mathcal{O})$ (Landau 1969). That the energy-momentum observable cannot be of the trivial form $P = \lambda\mathbf{1}$ follows for example, from the requirement that the vacuum vector Ω is an eigenvector of P to eigenvalue 0. This could only be the case if $\lambda = 0$ and thus only if P would be the zero-operator, which means that such a theory only describes the vacuum and thus is quite uninteresting. Therefore, *the energy-momentum P has to be a global observable, not measurable in any bounded region of spacetime.*

However, each global observable is the weak limit of a sequence (A_i) of local observables, and thus should at least be “approximately measurable” in bounded spacetime regions by means of the measurement of appropriate local observables from (A_i). Moreover, because of the continuity of normal states with respect to the weak operator topology it is secured that the expectation value of a global observable A is also approximated the better the closer A is approximated by some local observable. As the great accuracy between theoretical predictions and experimental results concerning the energy-momenta and rest masses of elementary particles show, sufficiently good local approximations to measurements of P (respectively $\sqrt{P^2}$) seem to be experimentally available. Yet the fact remains that if one takes the very idea underlying AQFT serious, that says that only local observables should be regarded as physically meaningful, energy-momentum and with it all global observables are only idealizations.[17]

[17] There is a further respect in which the energy-momentum observable is merely an idealization. As mentioned in the footnote on page 313, P is necessarily an unbounded self-adjoint operator. One consequence of this is that P's spectrum can only be bounded from above or from below but not both ways. Since for physical reasons the spectrum of P has to be bounded from below it therefore cannot also be bounded from above. But this means that the energy-momentum of a system can (in principle) become infinite, which is another idealization concerning P.

Note furthermore that since P is an unbounded operator it cannot belong to the algebra $\mathcal{R}(M) = \mathbf{B}(\mathcal{H})$ because the latter, by defintion, only contains bounded operators. Yet all spectral projections of P belong to $\mathcal{R}(M) = \mathbf{B}(\mathcal{H})$, so that P is affiliated with $\mathcal{R}(M) = \mathbf{B}(\mathcal{H})$ (see the footnote on page 304).

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