Spaces of continuous and holomorphic functions with growth conditions

Abstract: In the first part of this thesis we investigate the weighted (PLB)-spaces $\mathcal{A}C(X)$ and $\mathcal{A}_0C(X)$ of continuous functions, i.e. for a double sequence $\mathcal{A} := ((a_{n,k})_{k\in\mathbb{N}})_{n\in\mathbb{N}}$ of strictly positive continuous functions (weights) on a locally compact space X with

$$a_{n,k+1}(x) \leq a_{n,k}(x) \leq a_{n+1,k}(x) \ \forall\ n,k \in \mathbb{N}, x \in X,$$

we form the projective limit (with respect to n) of the inductive limits (with respect to k) of the weighted Banach spaces of continuous functions $Ca_{n,k}(X)$ and $C(a_{n,k})_0(X)$, respectively. We analyse their topological structures. For o-growth conditions we can characterise when the (PLB)-space $\mathcal{A}_0C(X)$ and the (LF)-space $\mathcal{V}_0C(X)$ (where the inductive and the projective limit are interchanged) are equal algebraically and topologically.

The second part of the thesis deals with weighted Banach spaces of holomorphic functions on the upper half-plane G. Let $v : G \to \mathbb{R}_+$ be a strictly positive, continuous function. The space $Hv_0(G)$ is defined as follows:

$$Hv_0(G) := \{f \in H(G); v|f| \text{ vanishes at infinity on } G\}.$$

This chapter is motivated by a question of Bierstedt. In a survey about weighted inductive limits of spaces of holomorphic functions he asked if the space $Hv_0(G)$ has the approximation property under some conditions of Holtmanns. The problem remains open in general, but we give a positive answer for weights with two additional conditions. Actually, using a theorem of Lusky, we can even show the existence of a basis.