

# Numerical Analysis of Transport in Dynamical Systems

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Transport processes play an important role in many natural phenomena. Prominent examples are the chaotic advection of fluid particles in geophysical flows or the transport of asteroids and comets in the solar system. Similar transport mechanisms are also at work in chemical physics explaining for example the transition between different conformations of molecules or the kinematics of chemical reactions. Therefore, in the numerical analysis of such dynamical systems one is interested in the identification of those regions in phase space that are involved in the transport process. In this context, invariant manifolds of hyperbolic objects play a crucial role as these structures are known to form natural barriers to transport.

This thesis contributes to the current research on the detection of invariant manifolds in dynamical systems with general time dependencies. To be more precise, we extend well-established methods related to finite-time Lyapunov exponents and relative dispersion and embed them in the set oriented approach first proposed by Dellnitz&Hohmann (1996). This results in new multilevel techniques which allow for the efficient detection, approximation, and continuation of stable and unstable manifolds of hyperbolic objects.

Moreover, we present a set oriented technique for the computation of transport rates in dynamical systems with general time dependencies. The method relies on a discretization of the Perron-Frobenius operator of the underlying non-autonomous dynamical system.

The methods are demonstrated by several examples, for example, from astrodynamics and physical chemistry.

The main application is the analysis of geophysical fluid data. In particular, we are able to identify Lagrangian coherent structures, finite-time analogues to invariant manifolds, in two- as well as three-dimensional ocean flows.