Mean behaviour of uniformly summable $\mathcal{Q}$-multiplicative functions

Abstract: In this thesis, we prove, both for the q-adic case and general $\mathcal{Q}$-adic representations, new theorems about the average of multiplicative functions without the assumption $|f| \leq 1$; it turns out that the class of *uniformly summable functions* is the appropriate generalization. In this context, we also investigate α-almost-periodic q-multiplicative functions.

(I) For uniformly summable q-multiplicative functions:
We give a complete characterization of the means $\frac{1}{N}\sum_{n<N} f(n)$ and $\frac{1}{N}\sum_{n<N} |f(n)|^\alpha$ as $N \to \infty$, $\alpha > 0$, where f is uniformly summable and q-multiplicative.
To our surprise, we find that for q-multiplicative functions the space $\mathcal{L}^\alpha$ for every $\alpha > 0$ coincides with the space $\mathcal{L}^*$. Furthermore, applying our main results, we investigate finitely distributed q-additive functions and find characterizations for q-multiplicative functions belonging to the space $\mathcal{D}^1$ of limit-periodic functions and the space $\mathcal{A}^1$ of almost-periodic functions by their respective spectrum $\sigma(f)$.

(II) For uniformly summable $\mathcal{Q}$-multiplicative functions:
In the case of a bounded sequence $\{q_r\}_{r\geq 1}$ we have similar theorems as in the q-adic case. In the case of an unbounded sequence $\{q_r\}_{r\geq 1}$ the situation is quite different. Unavoidable for unbounded sequences $\{q_r\}_{r\geq 1}$ is the existence of a so-called first digit phenomenon.
We investigate the mean behaviour of uniformly summable $\mathcal{Q}$-multiplicative functions that belong to $\mathcal{L}^2$ and for which the first digit condition

$$\max_{1\leq j\leq q_r-1} \frac{1}{j+1}\sum_{a=0}^{j} |f(a\mathcal{Q}_{r-1}) - 1|^2 \to 0 \qquad \text{as } r \to \infty$$

holds.