Analysis of Non-Newtonian and Two-Phase Flows

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Non-Newtonian fluids are characterized by different features, such as viscosity, elasticity, or memory effects. An important feature of many polymeric liquids is the fact that their viscosity changes with the shear rate, so-called *generalized Newtonian fluids* on which this thesis is focused on. The mathematical model describing the flow of these fluids in the whole space is given by the system

$$\begin{aligned} \frac{\partial}{\partial t}\mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u} + \nabla p &= \operatorname{div}\mathbf{S} + \mathbf{f}, && \text{on } [0,T]\times\mathbb{R}^n \\ \operatorname{div}\mathbf{u} &= 0, && \text{on } [0,T]\times\mathbb{R}^n \\ \mathbf{u}_{|t=0} &= \mathbf{u}_0, && \text{on } \mathbb{R}^n. \end{aligned} \tag{1}$$

Here, the stress tensor $\mathbf{S}$ is given by $\mathbf{S} = 2\ \mu(\|\mathbf{D}\|^2)\mathbf{D}$ with the rate-of-deformation tensor $\mathbf{D} = \frac{1}{2}\left[\nabla\mathbf{u} + (\nabla\mathbf{u})^T\right]$, and the viscosity function μ depending on $\|\mathbf{D}\|^2$ respectively on the shear rate $\dot{\gamma} = \sqrt{2}\,\|\mathbf{D}\|$, where $\|\mathbf{D}\|$ denotes the Hilbert-Schmidt norm.

The main result of this thesis is the proof of existence of problem (1) in the maximal L_p-regularity class. By means of maximal L_p-regularity, local (in time) strong well-posedness of this model is obtained under certain restrictions concerning the viscosity function. For example, for the viscosity function

$$\mu(\|\mathbf{D}\|^2) = \mu_0(1 + \|\mathbf{D}\|^2)^{\frac{m-2}{2}}$$

with $\mu_0 > 0$ which is often used in the mathematical literature, we obtain local existence in $\mathbb{R}^n, (n > 1)$ for $m > \frac{3n-4}{2(n-1)}$, i.e., in the 3D case $m > 5/4$ is sufficient. We emphasize that according to the engineering literature the range of interest concerning m is $m > 1$.

In the numerical part, the emphasis is on two-phase flows, since many interesting problems appear in this context. Moreover, we compare experiment and simulation of a binary droplet collision using non-Newtonian fluids, the behavior of which is assumed to be that of generalized Newtonian fluids. Furthermore, assuming the situation of system (1), we show the admissibility of the viscosity function used in the simulation.