Average and Smoothed Complexity of Geometric Structures

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In many areas one encounters applications from computational geometry, e.g. in computer-aided design, operations research, geographic information systems, computer graphics, and combinatorial optimization. In these applications the input data often comes from experimental and physical measurements and is thus afflicted with some error since measuring devices have only limited accuracy. One standard assumption in physics is that this error is distributed according to the Gaussian normal distribution.

In smoothed analysis, one perturbs now input instances by adding a small random noise vector to each input point. If the random noise comes from a Gaussian normal distribution, this provides a model for imprecise or noisy data. Furthermore, one would like to analyze the effect of rounding errors when computations are done on a computer using fixed precision arithmetic. By assuming that the added random noise is uniformly distributed in a hypercube centered at the exact real position of the input point, one is able to model such rounding errors. Smoothed analysis provides then the expected worst case complexity for these classes of perturbed input instances.

In this thesis, smoothed analysis is applied to some fundamental problems in the area of computational geometry. A comprehensive analysis of the number of vertices of the convex hull of a point set in d-dimensional space is provided. For moving objects, the notion of worst and smoothed case motion complexity is introduced. E.g. in the area of kinetic data structures it is the goal to maintain a combinatorial attribute of a moving point set efficiently. The efficiency of such a data structure is measured with respect to the motion complexity of that attribute.