Decision Support Expert System

for Process Selection

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Dissertation

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To my beloved parents and sisters

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Abstract

This thesis presents a new methodology to automate decision making in engineering. Decision making for the selection problems in the field of engineering has become more complex due to larger number of alternatives and multiple goals that sometimes conflict with each other. As a decision aid for engineers, it is necessary to design a decision support expert system for the engineering selection problems. For the case study, we apply our framework to the domain of chemical engineering, specializing in the domain of microencapsulation process selection.

The proposed system incorporates Expert System (ES) module and Multiple Attribute Decision Method (MADM) module that consists of three submodules, i.e. Analytical Hierarchy Process (AHP), Base Reference Analytical Hierarchy Process (BR-AHP) and fuzzy Base Reference Analytical Hierarchy Process (fuzzy BR-AHP) modules. The ES module provides a list of feasible alternatives and then the MADM module is used to rank the alternatives.

The Analytical Hierarchy Process (AHP) is a MADM approach that utilizes structured pairwise comparisons. Although pairwise comparisons have been seen as an effective way for eliciting qualitative data, a major drawback is that the exhaustive pairwise comparison is tiresome and time consuming when there are many alternatives to be considered. We propose a new approach to improve this limitation, the so-called Base Reference Analytical Hierarchy Process (BR-AHP).

Since many real-world engineering systems are too complex to be defined precisely, there exist imprecisions or approximations. The available information for making a decision may also be vague and uncertain. Thus, a more realistic approach is to incorporate fuzzy theory. Therefore, we propose a new approach to cope with imprecision, uncertainties and vagueness in the judgments of the decision makers, the so-called fuzzy Base Reference Analytical Hierarchy Process (fuzzy BR-AHP). In many cases, data in the MADM problems are imprecise and easy to change. Therefore, the framework proposed in this thesis also incorporates sensitivity analysis for handling changeable data.

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Chapter 1

Introduction

Decision making in engineering is often regarded as an 'art' rather than a 'science'. Experience and intuition are considered to be important in decision making because frequently there lacks quantitative data. Uncertainty in data has also significantly prevented reliable decision making. Having reliable qualitative or quantitative data analysis to evaluate risk and its impact on the engineering life cycle is both challenging as well as significant. Decision makers need to be concerned not only with the intangible and qualitative factors such as flexibility and quality but also with the tangible and quantitative factors such as cost in engineering selection problems.

Design problems are very complex and require numerous factors to be considered. Design decision making often requires not only to evaluate the subjective criteria but also the objective criteria. Qualitative and quantitative data always exist simultaneously in real world decision making situations. For example, in the case of a robot selection, according to Braglia & Gabbrielli (2000), there are two types of robot attributes that need to be considered: objective attributes and subjective attributes. Objective attributes are measured and defined in numerical terms, engineering attributes (load capacity, accuracy, repeatability, speed, etc.) or cost attributes (purchase and installation cost, maintenance cost, training cost, etc.). On the other hand, the subjective attributes (such as the vendor's service quality, the programming flexibility, the man-machine interface, etc.) are qualitative thus cannot be precisely and numerically measured by the decision maker.

The decision making process may be qualitative, quantitative or a combination of both. The problem structuring and analysis process in an engineering decision making process is conceptualized in Figure 1.1. The qualitative analysis is based primarily on the judgment, knowledge and experience of an expert (or a team of experts). When there is substantial experience and expertise within an analytical team, then the qualitative analysis can be emphasized. However,

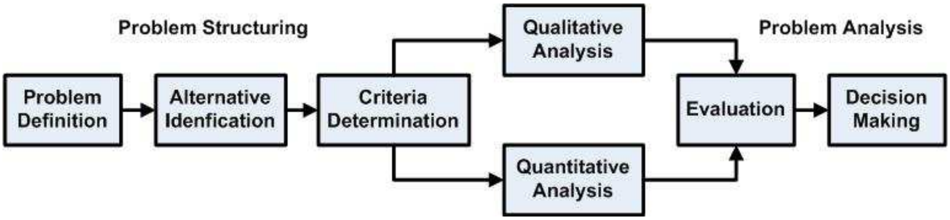


Figure 1.1: *Problem Structuring and Analysis in Engineering Decision Making Process*

in cases where there is either limited experience or plenty data available, then a quantitative analysis may be more appropriate. In a quantitative analysis, the main focus is on facts and data associated with a problem then focus on a mathematical formulation that encompasses the objectives, variables and constraints of the particular problem.

A distinction can be made between "discrete" and "continuous" decision problems. Discrete decision problems involve a finite set of alternatives and are often referred to as "selection problems." Continuous decision problems are characterized by an infinite number of feasible alternatives and are referred to as "synthesis problems." This thesis focuses on the development of decision support expert system framework to support engineering decision making which utilize qualitative and quantitative data, specializing in the selection problems. Decision making for the selection problems in the field of engineering has become more complex due to a large number of alternatives and multiple goals that sometimes conflict with each other. For the case study, we will apply our framework to the domain of chemical engineering, especially in the domain of microencapsulation process selection.

Every decision making problem involves multiple criteria by nature. Engineering problems are no exception; they often require addressing multiple criteria which often contradict one another. For instance, an automobile manufacturer may want to build a new vehicle that incurs low R&D cost yet possesses good performance characteristics. However, lower R&D cost compromises performance, necessitating a design trade-off.

Multiple Criteria Decision Making (MCDM) was developed in the recent couple of decades as a response to the problems faced by decision makers when confronting complex issues. MCDM is divided into two main groups (Hwang & Yoon, 1981): Multiple Attribute Decision Making (MADM) and Multiple Objective Decision Making (MODM). MADM is also known as Multiple Criteria Decision Analysis (MCDA), whereas MODM is also known as Multi Objective Optimization (MOO), Multi Criteria Optimization (MCO) or vector optimization. MADM corresponds to the discrete decision problems or the selection problems which involve choosing one of several possible alternatives. MODM corresponds to the continuous decision problems or the synthesis problems which involve creating solutions that aim to attain a set of goals.

In our case study of the microencapsulation selection problem, feasible alternatives (i.e. the candidates to be selected) are explicitly given rather than being implicitly defined by the model. The decision making problem is, thus, a discrete one: it involves selecting the best alternatives from a finite set of feasible ones based on the evaluation of each alternative against a given set of criteria. Therefore, we will focus ourselves on discrete MCDM methods or Multiple Attribute Decision Making (MADM), rather than the continuous MCDM methods. The problem with multiple criteria considered in this thesis belongs to the class of MADM problems.

As the basis of the MADM technique, we adopted and extended the Analytical Hierarchy Process (AHP) method. The AHP is a multiple criteria decision analysis method that was developed by Thomas L. Saaty (Saaty, 1980, 2001). The principle behind AHP is that, in decision making, the use of factual data, knowledge, and experience, each play an equally important role. The AHP approach, which enables qualitative analysis using a combination of subjective and objective information/data, is a MADM approach that uses hierarchical structured pairwise comparisons. One of the drawbacks of AHP is that the exhaustive pairwise comparison is tiresome and time consuming when there are many alternatives to be considered. This thesis presents a new approach to improve this limitation of AHP, the so-called Base Reference Analytical Hierarchy Process (BR-AHP). It also describes how the new methodology BR-AHP can be used to create the basis of a Decision Support Expert System (DSES) for the engineering selection problems. Besides extending the AHP method, we also incorporate fuzzy technique and sensitivity analysis for handling imprecise, vague or uncertain data.

In the following section, we present the background and motivation of our research followed by the research objective and area. Finally, the organization of the thesis is given in the last section.

1.1 Background and Motivation

Providing intelligent decision support system for engineering design tasks is the key to enhancing productivity in the process industries. However, design tasks are difficult and not that easy to automate, mostly because they are based on heuristics and their solution spaces are open-ended. In designing, the elements of the designed artifact are not constrained by a predefined set, but they are subject only to the constraints of the manufacturing methods and the characteristics of the raw materials. Hence, the knowledge for general design tasks is open-ended. Configuration and selection are special cases of design as shown in Figure 1.2. The knowledge necessary for configuration tasks is more bounded, and deals with defining and characterizing the set of possible parts, which are designed so that they can be combined systematically and would cover the desired range of possible functions (Stefik, 1995). On the other hand, selection involves making a choice

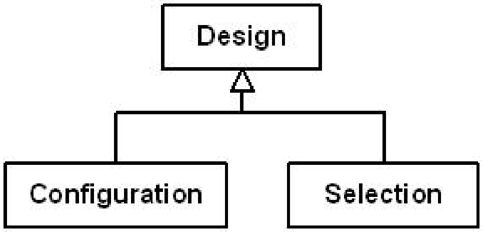


Figure 1.2: *The Hierarchy of Design Classes*

within a predefined or existing enumerated set of alternatives which can be expanded. Hence, the solution space for selection is similar to that for configuration, and is more bounded than that for design. In contrast to configuration however, selection does not involve the systematic combination of components, but simply choosing one or a few alternatives from a set of available alternatives to ensure that a desired function can be accomplished.

This thesis investigates the problem of choosing from a list of design alternatives based upon multiple, uncertain and possibly conflicting attributes. It is known that in engineering, decisions are rarely made using a single attribute. Moreover, these decision problems also have built-in uncertainties whose effects need to be incorporated into the decision making process. Several sources of uncertainties in multiple attribute decision problems are the imprecise attribute values, the inabilities of the decision maker to precisely state his/her preferences of the attributes or the lack of a complete set of alternatives amongst which a choice has to be made. Additionally, it is essential that the decision making method used for multiple attribute selection problems provides a winning alternative that is insensitive to changes in relative attribute importance, given the same decision maker preferences.

The approach presented in this thesis may be appropriate to tackle various kinds of selection problems in different domains. However, we will concentrate on a particular part of chemical engineering selection problem: the selection of microencapsulation process.

Microencapsulation is a technique for encapsulating substances into tiny packaged materials called microcapsules. This technique has been used in a wide range of fields from pharmaceutical and chemicals to agriculture, textile and printing. The number of microencapsulation technologies to choose from is very extensive, making it difficult to select a proper process for a specific product. Many different techniques have been proposed for the production of microcapsules by academics and industrial researchers. Up till now a wide range of microencapsulation techniques have been developed (Thies, 1996). The selection of microencapsulation technique depends on many factors, such as the nature of the polymer, the application, the intended use and many others. Some mathematical models like Partial Differential Equations (PDE) and Computational Fluid Dynamics (CFD) have been proposed to provide representation of the process of microencapsulation. But these techniques are only useful as aids to illustrate microencapsulation process behavior and its formation, they do not help the process engineers for selecting the appropriate microencapsulation methods. Moreover, most of them are too complex and are not so easily understood by managers, experts and engineers in industry.

To create a decision aid for process engineers, it is necessary to design a decision support expert system (DSES) that provides help for the selection of microencapsulation techniques. The proposed system will incorporate two main modules, i.e. the modules of Decision Support System (DSS) (i.e. Multiple Attribute Decision Making (MADM) module) and Expert System (ES). The ES module will provide a list of feasible microencapsulation alternatives and then the MADM module is used to rank the microencapsulation alternatives.

Analytical Hierarchy Process (AHP) is one of the most widely used MADM methods which deals with the problem of choosing one alternative from a set of alternatives characterized in terms of their attributes by using pairwise comparison technique. However, this exhaustive pairwise comparison is tiresome and time consuming when there are many alternatives to be considered. Nowadays there are more than 1000 methods that can be identified in the patent literature for the microencapsulation selection problem (Gouin, 2004). In this case, we need the approach that will be able to cope with many alternatives. Therefore, we propose a new approach to amend this limitation of pairwise comparison, the so-called base pairwise comparison.

In the engineering domain in general and particularly in the microencapsulation domain, the available information in order to make a decision is possibly vague or uncertain. In this case, a crisp pairwise comparison in the BR-AHP would seem to be insufficient and imprecise to capture the degree of importance of the engineering requirements. A more natural way to cope with uncertain and imprecise judgments is to express the comparison ratios as fuzzy numbers, which incorporate the vagueness of the human thinking. Therefore, we propose a new approach to improve this limitation, the so-called fuzzy base pairwise comparison.

Fuzzy decision making is useful in describing imprecise problems more accurately than the conventional methods. Imprecision may arise due to the following reasons (Yeo et al., 2004; Chen & Hwang, 1992):

- Unquantifiable information such as level of satisfaction. Some properties can not easily be described using numbers, then linguistic terms are usually used. For example, level of satisfaction can be evaluated with terms as good, fair, poor, etc. These are qualitative data that cannot be physically measured.

- Incomplete information where the data is not exact, such as due to poor resolution in the measuring instruments. Obtaining a precise numerical value for some measurements is sometimes a difficult task, because the measurement equipment is not precise enough, such as the velocity of a car.

- Non-obtainable information, such as when the cost of obtaining the data is too high or when the data is not available. When the methodology involved in a measurement is complex and time consuming approximations of the value are used.
- Partial ignorance, when the situation is not fully understood. The experts that provide the data do not always know all the details of all criteria for all alternatives. This natural ignorance about some criteria or alternatives introduces imprecision in the global process.

Therefore, this study will dedicate itself to develop a decision support expert system using the fuzzy sets theory combined with the BR-AHP for solving the problem of the selection of the right microencapsulation techniques.

1.2 Research Objective

The objective of this thesis is to develop a decision support expert system framework for the engineering selection problems, specializing in the field of chemical engineering in the case of microencapsulation selection problems. The proposed system is called Decision Support Expert System (DSES) because it incorporates the modules of Decision Support System (DSS), i.e. Multiple Attribute Decision Making (MADM) techniques, and Expert System (ES). The ES module provides a list of feasible alternatives and then the MADM module is used to rank the alternatives.

The Analytical Hierarchy Process (AHP), one of the most popular Multiple Attribute Decision Method (MADM) techniques, has been used intensively by many researchers in the academics and industry to aid the selection problems. AHP has been applied to a wide area (Golden et al., 1989), including a variety of selection problems (Akash et al., 1999; Braglia & Petroni, 1999; Karacal et al., 1996; Kontio, 1996), e.g. technology selection, supplier selection, plants selection, reusable software selection, and many others.

There are several benefits in using AHP for selection problems. First, a selection problem is reconstructed in hierarchical manner showing the overall goal of the decision at the highest level, the decision criteria at the next lower level, and the sub-criteria (if any) and all decision alternatives replicated under each criterion at the lowest levels of the hierarchy. Second, pairwise preferences are elicited from the decision maker and captured in matrix form. The unique essence of the AHP is displayed as the complexity of multicriteria and multi decision alternative problem is reduced to a series of simple pairwise comparison. Decision makers can much more readily express a preference of one alternative versus another alternative if there are only two alternatives at a time being compared to each other in the context of only one decision criterion. However, pairwise comparisons can be tiresome and time consuming, especially in the engineering selection problems where there are many alternatives to be considered. Additionally, the user cannot be completely consistent in every case if the size of pairwise comparisons is large. This thesis presents a new approach to amend this limitation of AHP, the so-called Base Reference Analytical Hierarchy Process (BR-AHP).

However in some cases, the information available for making a decision is vague and uncertain. In this case, a crisp pairwise comparison in the BR-AHP would seem to be insufficient and imprecise to capture the degree of importance of microencapsulation requirements. A more realistic approach is to incorporate fuzzy theory. Fuzzy decision making is useful in describing imprecise problems more accurately than the conventional methods. By applying fuzzy sets theory in the field of multiple attribute decision making, this study aims to develop a more appropriate approach for the engineering selection problems, particularly in dealing with multiple goals of development and inherent imprecision. It is to be expected that the fuzzy multiple attribute decision making approach developed in this work could have provided a broader and more comprehensive perspective to produce more realistic meanings than the existing methods so far employed. Therefore, we propose a new approach to amend this limitation, the so-called fuzzy base pairwise comparison. Therefore, the proposed decision support expert system should also provide the approach using the fuzzy sets theory being integrated with BR-AHP for solving the engineering selection problem, i.e. the problem of the selection of the right microencapsulation techniques.

In many cases, data in the MCDM problems are imprecise and changeable. Therefore, it is important to perform a sensitivity analysis to the input data. The decision support expert system should also be able to perform a sensitivity analysis to assess the robustness of the preference ranking to the changes in the criteria scores and/or the assigned weights. Thus, we need to implement the decision support expert system with a methodology for performing sensitivity analysis on the weights of the decision criteria and the preference values of the alternatives with respect to the decision criteria. Most of the commercial decision support systems which implement AHP methodology, can only perform one type of elementary sensitivity analysis. There is just one option being to alter the weights of the decision criteria. In the proposed decision support expert system, besides altering the weights of the decision criteria, we can also analyze the result by altering the performance values of the alternatives with respect to the criteria and obtain a graphical result.

1.3 Research Area

As illustrated in Figure 1.3, the research area of this work is a fusion of Computer Science, Operations Research, Computational Intelligence bonded by Engineering. This fusion offers a unified approach to the design of a system to solve engineer-

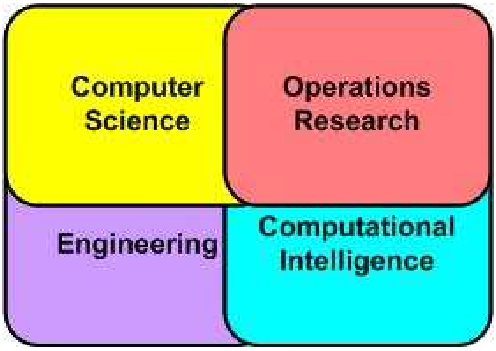


Figure 1.3: *Research Area*

ing problems. The result of this is the proposed decision support expert system. The technique of Computational Intelligence used to this end is the fuzzy logic approach. The specific areas of Computer Science where this work belongs to are as follows: Knowledge-based System, Expert System and Artificial Intelligence. The research in this work belongs to the following areas of Operations Research: Decision Support System, Decision Analysis and Multiple Criteria Decision Making. Engineering is the application or the problem domain. Therefore, the result of this research is acceptable in the respectively broad areas mentioned above.

The work in this thesis focuses on a specific design problem, i.e. the selection problem in the chemical engineering discipline in the field of microencapsulation. This thesis deals with the problem of engineering selection, although the decision model presented here is not limited to this specific problem and could very well be applied to other more general selection problems.

Although this work is to be applied to engineering area, it can also be applied to decision making in any discipline. It can be applied to management area, marketing area or even to the decisions to be made in day-to-day life. The research illustrated in this thesis combines the concepts from decision theory and artificial intelligence fields to be applied into the field of engineering, specializing in the case of microencapsulation process selection.

1.4 Organization of the Thesis

Chapter one outlines a brief summary of the background and motivation of the study, mainly focusing on the engineering selection problems, the introduction to the problem domain, the objectives to be achieved and the organization of the study.

Chapter two presents the decision support expert system framework for the engineering selection problem. We begin this chapter with the background and motivation why we need to develop such a system. Then, we describe the decision support expert system architecture. The proposed system consists of the modules of Expert System (ES) and Multiple Attribute Decision Making (MADM). In the selection process, the ES module provides a list of technically feasible alternatives and the MADM module is used to rank the alternatives based on selected criteria. Afterward we present each module of the decision support expert system.

Chapter three presents the problem domain, i.e. the microencapsulation domain. We will present the concept, morphology, release mechanisms, and the reasoning behind microencapsulation in this chapter. We will also provide the criteria and the methods of the microencapsulation that we are interested in.

Chapter four presents the application of Analytical Hierarchy Process (AHP) for microencapsulation process selection problem. In this chapter, we start with the engineering problem solving method, followed by an overview of the AHP. Then we present a methodology for performing a sensitivity analysis on the weights of decision criteria and the preference values of the alternatives with respect to each decision criterion. Afterward we apply the application of the AHP method to the engineering process selection problem, i.e. to the domain of chemical engineering for the selection of the appropriate microencapsulation method.

Chapter five presents the methodology of the Base Reference Analytical Hierarchy Process (BR-AHP) approach. In this chapter, we will present a case study to show the application of BR-AHP in an engineering process selection, i.e. in microencapsulation process selection problem. We will also demonstrate the sensitivity analysis method used for BR-AHP approach.

Chapter six presents the extension of the aforementioned approach in fuzzy environment, the so-called fuzzy Base Reference Analytical Hierarchy Process (fuzzy BR-AHP) approach. In this chapter, we will also present a case study to show the application of fuzzy BR-AHP as the basis of the Decision Support System (DSS) for solving the engineering selection problem, i.e. the problem of the selection of the appropriate microencapsulation techniques.

Chapter seven presents the conclusions of the study. Some recommendations for further research are also given in this chapter.

Bibliographical Note: A subset of the results presented in this thesis has been published in the field of Operations Research and Industrial Engineering (Hotman & Alke, 2005), the field of Computer Science/Artificial Intelligence (Hotman, 2005a), and the field of Computational Intelligence and Engineering Systems (Hotman, 2005b).

Chapter 2

Decision Support Expert System Framework

The selection of appropriate methods or tools is one of the most critical decisions in the field of engineering. The selection problems in this field have become more and more complex since there are a large number of alternatives to be selected from and multiple goals that sometimes conflict with each other. Therefore, as a decision aid for engineers, it is necessary to design a tool that can help the engineers to solve those problems. This chapter presents a Decision Support Expert System (DSES) framework for engineering selection problems, especially for microencapsulation process selection. The proposed system is called Decision Support Expert System because it incorporates Decision Support System (DSS) module, i.e. Multiple Attribute Decision Making (MADM) module, and Expert System (ES) module. The ES module provides a list of technically feasible alternatives and the MADM module is used to rank the alternatives based on selected criteria. In addition, the robustness of the selection procedure may be evaluated using sensitivity analysis.

We will start this chapter with the background and motivation why we need to develop decision support expert system in the next section. Then in Section 2.2 we will discuss our decision support expert system architecture. We provide an overview to expert system module and introduce fundamental concepts of expert system in Section 2.3. A decision support system framework will be presented in Section 2.4. Finally, the conclusions of this chapter are given in Section 2.5.

2.1 Background and Motivation

Due to the rapid growth of microencapsulation technology, the selection of the most appropriate microencapsulation process has become increasingly important.

Microencapsulation is the name given to a novel technique for the preparation of small substances which was developed about 50 years ago. The number of microencapsulation technologies to choose from is very extensive, making it difficult to select a proper process for a specific product. Many different techniques have been proposed for the production of microcapsules by academics and industrial researchers. Nowadays more than 1000 methods (Gouin, 2004) can be identified in the patent literature. The conventional approach for the microencapsulation process selection has been entrusted to the process/chemical engineers.

Mathematical models like Partial Differential Equations (PDE) and Computational Fluid Dynamics (CFD) have been proposed to represent the real effects of the widely known microencapsulation techniques as near as possible. But most of them are too complex and are not so easy for managers, experts and engineers in industries to comprehend. However, all these techniques are only useful as aids for describing the microencapsulation behavior and the microencapsulation formation but they do not help the process/chemical engineers to choose the right techniques. So the engineers are left with only a few tangible options: 1) consult microencapsulation experts; 2) rely on the advice of industrial partners or equipment vendors; 3) employ the use of analytical models; or 4) rely on textbooks, handbooks, and their own experience.

The service from consultants are expensive, most research institutes and companies are not be able to afford them. The advice given by industrial partners is free, but however, they usually give recommendation of the techniques that they are most familiar with. The advice given by vendors is also free, however, they have an inherent interest in selling their own products, and hence, their advice cannot be always trusted. Finally, analytical models are very rarely used for microencapsulation selection due to over simplification. Therefore, engineers usually end up selecting the microencapsulation technique that they are most familiar with, and thus these may not represent the most cost-effective solution.

Therefore, as a decision aid for process/chemical engineers, it is necessary to design a decision support expert system that provides help for selecting the right microencapsulation technique. Expert Systems have been used to model and to automate problem solving and decision making in engineering. They are used to perform a variety of complicated tasks that in the past could be performed by only a limited number of highly trained human experts (Rolston, 1988). Similar to an expert possessing knowledge and experience in a specialized domain uses reasoning rules and expertise to solve a problem, an expert system can embody the logic of such expertise. Logical decision steps can be programmed into the computer to solve problems or provide information in a specialist's domain. The engineering problems are usually translated into a set of rules. However, not all the problems can be converted this way. Experts often have difficulties to express their decisions in understandable terms which are easy to convert into a set of rules. Two experts may not completely agree on the answer to a given problem. Additionally, these decisions are rarely made based upon a single attribute. There are usually many attributes upon which these selections have to be made and there also exist conflicts amongst these attributes. Therefore, there is a need to trade-off amongst these attributes in order to determine the best alternative. Moreover, these decision problems also have built-in uncertainties whose effects need to be incorporated into the decision making process.

Multiple Criteria Decision Making (MCDM) was developed in the recent couples of decades as a response to the problems faced by decision makers when confronting complex issues in the presence of multiple and possibly conflicting criteria. MCDM is divided into two main groups (Hwang & Yoon, 1981): Multiple Attribute Decision Making (MADM) and Multiple Objective Decision Making (MODM). MADM corresponds to the selection/evaluation problems where there are a finite set of alternatives in terms of a finite number of decision criteria, often such criteria may be in conflict with each other. Whereas MODM corresponds to the design/synthesis problems where associated with designing the best alternative solution, i.e. the one that is most attractive over all criteria. In MODM, it is not associated with the problem where the alternatives are predetermined. The approach we adopted and proposed belong to MADM approach because in our study the alternatives of microencapsulation techniques are predetermined and also the focus of this thesis is to solve the selection problems.

MADM techniques are necessary in this decision support expert system framework because the expert system framework typically generates multiple alternatives and a method is needed to evaluate these alternatives. The expert system framework only makes recommendations based on the technical aspects of microencapsulation selected by the user. MADM techniques offer powerful ways of dealing with the decision maker's preferences and for ranking alternatives. For this reason MADM module has been included in the system's framework. The Analytical Hierarchy Process (AHP) is a MADM method that utilizes structured pairwise comparisons. AHP is widely used in industry to aid selection process (Golden et al., 1989; Akash et al., 1999; Braglia & Petroni, 1999; Karacal et al., 1996; Kontio, 1996), e.g. technology selection, supplier selection, plants selection, reusable software selection, and others. This approach is well-suited for a variety of different problems in the engineering domain and it is easily understood by the experts. The methodology is applicable to a wide range of engineering problems and it is easily implemented to an expert system.

In the AHP, the decision maker models a problem as a hierarchy of criteria, sub-criteria, and alternatives. After the hierarchy is constructed, the decision maker assesses the importance of each element at each level of the hierarchy. This is accomplished by generating entries in a pairwise comparison matrix where elements are compared to each other. Although pairwise comparisons have been seen by many people as an effective way for eliciting qualitative data for multiple criteria decision making problems, a major drawback is that the number of the required comparisons increases quadratically with the number of the alternatives to be compared. Thus, often even data for medium size decision problems may be impractical to be elicited via pairwise comparisons. The more the comparisons are, the higher is the likelihood that the decision maker will make erroneous data.

We propose a new approach to improve this limitation of AHP, the so-called Base Reference Analytical Hierarchy Process (BR-AHP) in Chapter 5.

Since many real-world engineering systems are too complex to be defined in precise terms, imprecision or approximation is often involved in the selection of either design parameters or empirical formulation. Imprecision may originate from indirect measurements, estimation routines, subjective interpretation, and expert judgment of available information. Imprecise information such as may arise from incomplete data, vague description or subjective interpretations of expert judgments often play a major role in data acquisition for the study of complex systems. In this case, a crisp pairwise comparison in the BR-AHP would seem to be insufficient and imprecise to capture the degree of importance of engineering requirements. A more realistic approach is to incorporate fuzzy theory. A methodology based on fuzzy set theory is presented to express imprecision of input data. Fuzzy decision making is useful in describing imprecise problems more accurately than the conventional methods. By applying fuzzy sets theory in the field of multiple attribute decision making, this study aims at developing a more appropriate approach for the engineering selection problems, particularly in dealing with multiple goals of development and inherent imprecision. It is expected that the fuzzy multiple attribute decision making approach developed in this work could give a broader, more comprehensive perspective and produce more realistic meaning than the existing methods so far employed. Therefore, we propose a new approach to improve this limitation, the so-called fuzzy base pairwise comparison. The new approach which utilizes fuzzy base pairwise comparison is called fuzzy Base Reference Analytical Hierarchy Process (fuzzy BR-AHP) approach.

Since the proposed system incorporates Decision Support System (DSS) module, i.e. Multiple Attribute Decision Making (MADM) module which consists of AHP, BR-AHP and fuzzy BR-AHP modules, and Expert System (ES) module. Thus, we called our proposed system as Decision Support Expert System (DSES). The ES module gives suggestion to the user by listing the technically feasible alternatives. The MADM module is used to rank the alternatives based on selected criteria. Additionally, the robustness of the selection procedure may be evaluated using sensitivity analysis. It is very often that the values used for the parameters in a MADM model are just estimates. Therefore, sensitivity analysis needs to be performed to investigate what happens if these estimates are changed.

2.2 Decision Support Expert System Architecture

The overall decision support expert system architecture is shown in Figure 2.1. The system architecture is using a three-tier architecture, which consists of front-end (presentation layer), middleware (application layer) and back-end (data layer). At the bottom layer is the knowledge base which consists of rules and facts. The middle consists of application server and inference engine. The top layer is the graphical user interface, i.e. the web browser. Figure 2.2 shows the interaction of the components in the presentation layer.

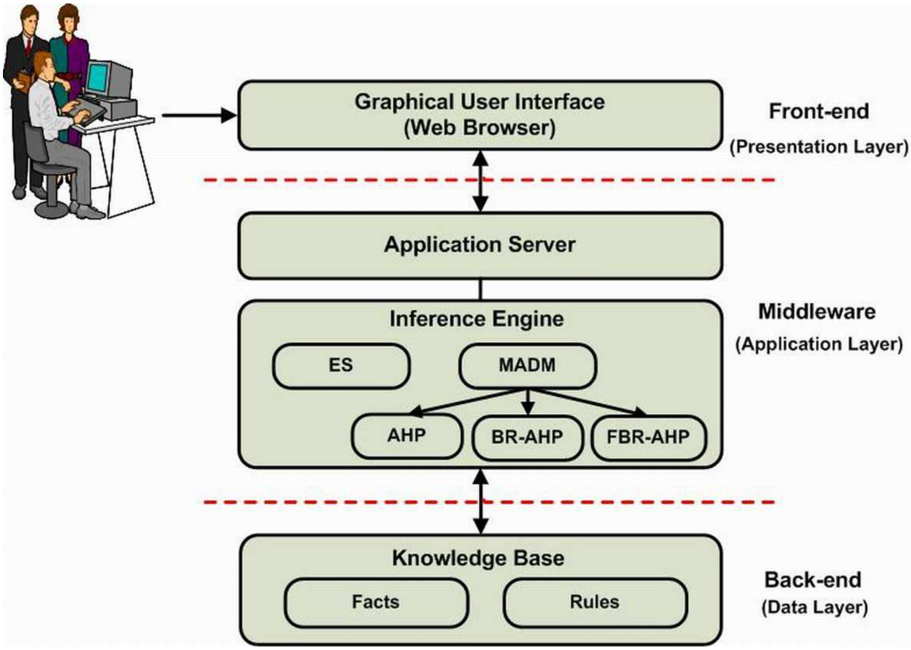


Figure 2.1: *The Decision Support Expert System Architecture*

The Decision Support Expert System (DSES) framework consists of four major components and they are:

1. the graphical user interface,
2. the application server,
3. the inference engine, and
4. the knowledge base.

We will briefly discuss each of these components and then how the whole system works in the following subsections.

2.2.1 The Graphical User Interface

Decision support systems (DSS) and Expert Systems (ES) were traditionally developed for desktop use by individual users. All components of those systems usually resided on the same machine. As with other software, those system softwares needed to be customized for the operating platform it was to be run on.

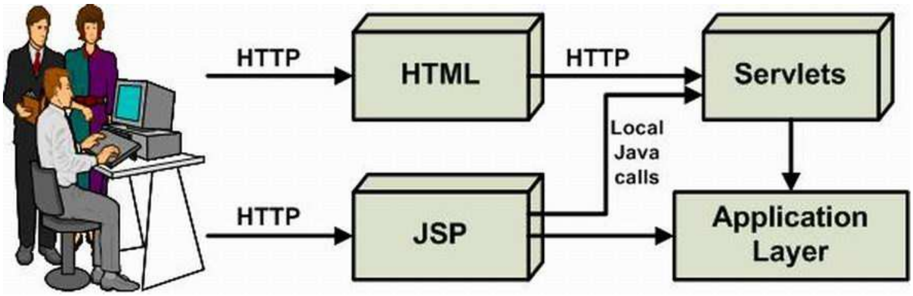


Figure 2.2: *The Presentation Layer Components Interaction*

Recently, with the emergence and widespread use of technologies that enable distributed computing in a heterogeneous computing environment – precisely, the technologies associated with the World Wide Web and the Internet – it has become possible to make DSS and ES accessible to a large number of users, irrespective of the computing platform used by them. Here, a standard Web browser like Netscape Navigator, Mozilla and Internet Explorer serves as the user interface, accommodating both textual and multimedia information. The data and model management features are provided by DSS and ES programs on a remote Web server and/or via a platform-independent programming language such as Java.

Therefore, an important and distinctive feature of the DSES is that it should be usable over the World Wide Web via a standard Web browser. Thus, it is widely and easily accessible. In addition, by implementing these features they can be made available to a large number of users/decision makers, requiring only that they have access to a Web browser. All execution and computational processes occur on the application server, and data is exchanged between the users and application server via HTML pages and HTML forms.

2.2.2 The Application Server

The Application Server is used as the Web Server. The Web Server is a computer responsible for serving web pages, mostly HTML documents, via the HTTP protocol to clients, mostly web browsers. The Web Server program used here is Apache HTTP Server from the Apache Software Foundation.

2.2.3 The Inference Engine

The inference engine is used to derive answers from a knowledge base. It is the brain of the Decision Support Expert System that provides a methodology for the information reasoning in the knowledge base, and to formulate conclusions. It executes facts and rules in a given order to solve a problem. Doing this, it can deduce new facts. This inference engine has two main elements for reasoning processes. There are two modules to be included in the inference engine which are:

1. Expert System (ES), and
2. Multiple Attribute Decision Making (MADM).

The Expert System (ES) module and Multiple Attribute Decision Making (MADM) module are working together for the selection process. The flowchart of how these two modules are working together is shown in Figure 2.3.

The selection process involves a two-step process:

1. Generation of concept alternatives.
2. Comparing alternatives and selecting the best one.

A two-layered architectural framework for representing the selection task, which includes the two sub-processes of listing and recommendation, is proposed. The two layer of selection framework consists of expert rules to provide the list of technically feasible alternatives in the top layer, and in the bottom layer, it uses Multiple Attribute Decision Making (MADM) methods to recommend the result of the selection process.

If there is only one alternative solution as a result of the output evaluation of the ES module, then the DSES will give that result as the only recommendation to the decision maker. But normally, there will be several alternatives solution as a result of the output evaluation of the ES module. In this case, then the decision maker needs to proceed the second module, i.e. the MADM module. In MADM module, the results from the ES module will be evaluated and ranked. In the end the recommendation will be given to the alternative solution with the highest priority.

Expert System Module

The Expert System (ES) module provides a list of technically feasible alternatives using rules. Expert System always possesses certain heuristics that form the static knowledge base, and the inference and search processes. The rules allow the system to deduce new results from an initial set of data (premises). A rule is basically constituted by the following structure:

IF *conditions* THEN *actions*

Further explanation concerning this module will be given in Section 2.3.

Multiple Attribute Decision Making Module

The Multiple Attribute Decision Making (MADM) module is used to rank the alternatives based on selected criteria. This part contains the three sub-modules

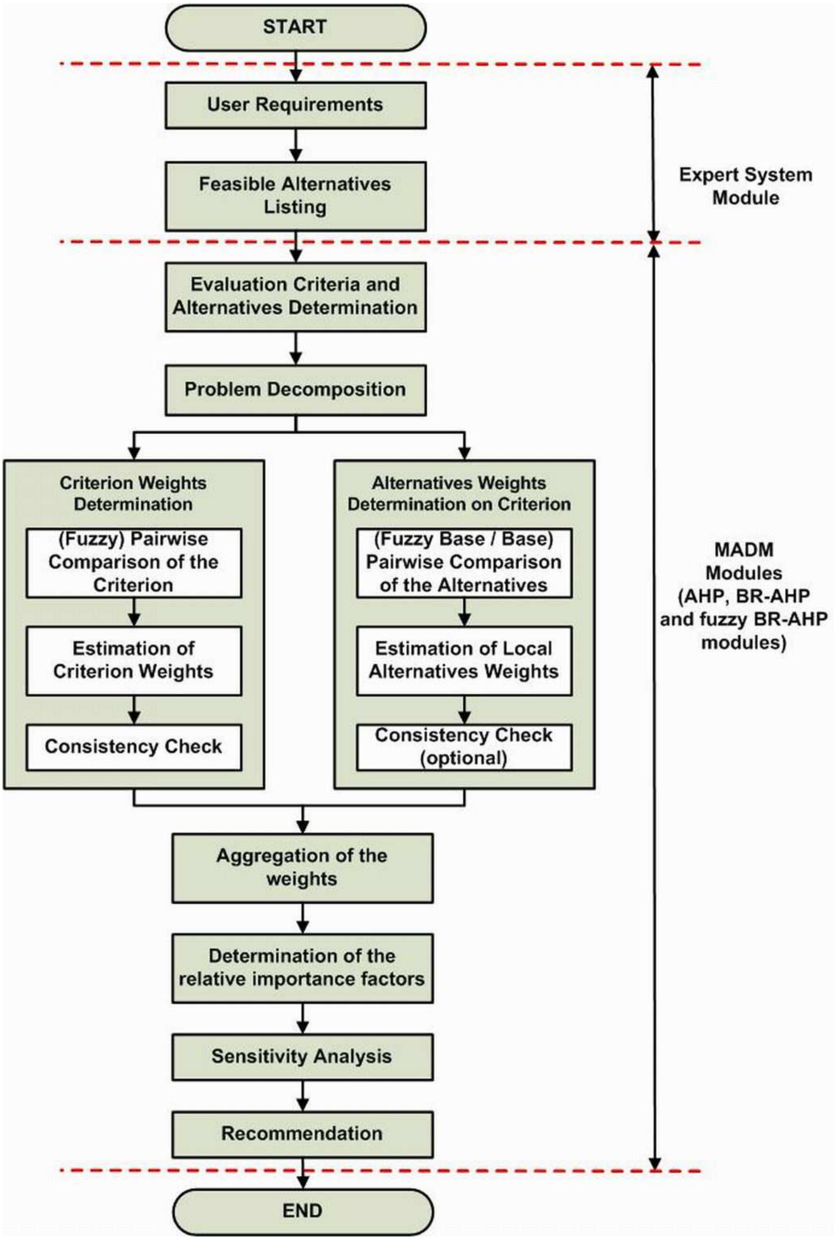


Figure 2.3: *Decision Support Expert System Flowchart*

to perform multiple-attribute decision making. The first module uses AHP (Analytical Hierarchy Process). The second module uses BR-AHP (Base Reference Analytical Hierarchy Process). The third module uses fuzzy BR-AHP (fuzzy Base Reference Analytical Hierarchy Process). More explanation of these modules will be presented in Section 2.4.

2.2.4 The Knowledge Base

The knowledge base represents a storage of the knowledge (i.e., basic facts, procedural rules and heuristics) available to the system. In general, knowledge is stored in the form of facts and rules, but the specific schemes used for storing the information vary greatly. The knowledge base in the back-end layer is stored as databases in the database management system (DBMS). A DBMS is a computer program used to manage a database and runs operations on the data requested by the clients. The DBMS used in this thesis is an open source database MySQL. A JDBC driver is used as a standard interface that enables communication between Java-based applications and database management systems.

2.3 Expert System

In order to tackle a design/selection problem in the field of engineering, expertise from the domain experts needs to be elicited. Thus, most of the system models and automated decision making in engineering developed in the past are called knowledge-based systems or expert systems. This subsection contains a brief introduction to experts systems and discusses briefly their elements and the reason that has motivated the industry to adopt knowledge-based techniques.

2.3.1 Expert System Elements

Expert systems, which are often called knowledge-based systems, are comprised of a software technology that can replicate certain aspects of expertise and can manipulate both qualitative and quantitative knowledge. This technology offers users new ways of organizing, formalizing, and manipulating context-specific knowledge and problems.

The classical view on conventional computer software is as follows:

$$\textit{Software} = \textit{Data} + \textit{Algorithm}$$

Here, the algorithm processes data in a top-down sequential manner until one arrives at the result. In contrast, computer software used in Expert Systems can be described as follows:

$$\textit{Expert System} = \textit{Knowledge} + \textit{Inference}$$

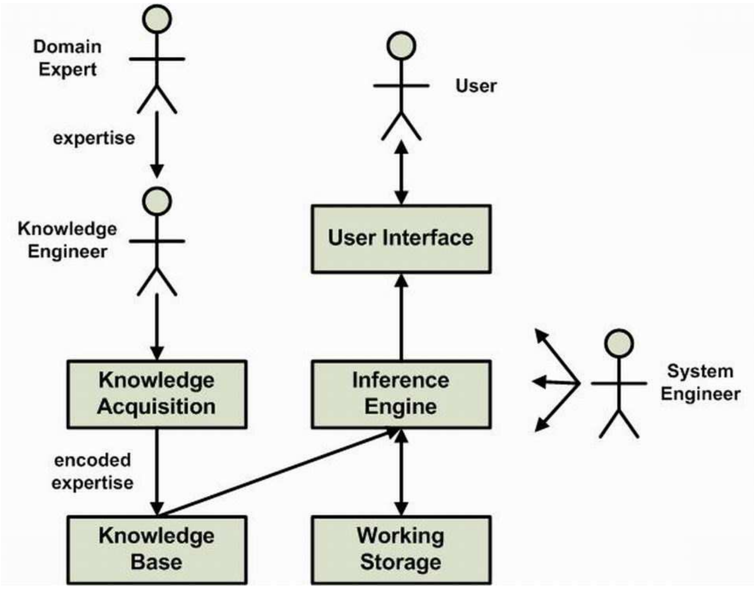


Figure 2.4: *Elements and Interfaces of Expert System*

In this case, the system structure differs radically and the principal elements are the *knowledge base*, which is a depository of all the available domain specific knowledge and the *inference engine*, the software whose function is to infer decisions. The process of codifying an expert's knowledge in a form that is accessible to a non-expert through an expert system is called *knowledge engineering*.

An Expert System can be characterized as an *intelligent knowledge-based system* provided it reproduces knowledge in the form of rules. The most significant characteristic of this class of systems is that it draws on human knowledge and emulates human experts in the manner with which they arrive at decisions. One definition of an Expert System which we adopted in this thesis is (Stefik, 1995):

> *"An Expert System is a computer program whose performance is guided by specific, expert knowledge in solving problems."*

Expert systems have a number of major system components and many interfaces with individuals in various roles. These are shown in Figure 2.4. An Expert System includes the following elements:

- Knowledge base – which comprises of facts and rules, is a declarative representation of the expertise, often in production rules (IF–THEN rules);

- Working storage – through which the data is stored during the process, is the data which is specific to the problem being solved;

- Inference engine – which processes the data in the knowledge base in order to arrive at logical conclusions, is the code at the core of the system which derives recommendations from the knowledge base and problem-specific data in working storage;
- User interface – through which the human user interacts with the system, is the code that controls the dialog between the user and the system.

In general, knowledge that is useful in solving real industrial problems has two components:

- facts, which constitute transient information subject to changes with time; and
- procedural knowledge, which refers to the manner in which experts in the specific field of application arrive at their decisions.

The major roles of individuals who interact with the expert system are:

- Domain expert – the individual(s) who are experts in solving the problems the expert system is intended to solve or in our case study the engineers who have years of experience with working with the problem domain;
- Knowledge engineer – the individual who acquires the expert's knowledge and passes it on the knowledge base in a form that can be used by the expert system;
- User – the individual who will be using or consulting the expert system to get advice which would have been provided by an expert.
- System engineer – the individual who builds the user interface, designs the declarative format of the knowledge base, and implements the inference engine. Depending on the size of the expert system, the knowledge engineer and the system engineer might be the same person.

One of the major bottlenecks in building expert systems is the knowledge engineering process. Coding the expertise into a declarative rule format may also be a difficult and tedious task.

2.3.2 Inference methods

There are two inference methods which are commonly used in expert systems.

- Goal driven reasoning or backward chaining – an inference technique which uses production rules (IF–THEN rules) to break a goal into smaller subgoals which are easier to prove;
- Data driven reasoning or forward chaining – an inference technique which uses production rules (IF–THEN rules) to deduce a problem solution from initial input data;

Goal Driven Reasoning

Goal driven reasoning, or backward chaining, is an efficient way to solve problems that can be modeled as "structured selection" problems. That is, the aim of the system is to pick the best choice from many enumerated possibilities. For example, an identification problem falls in this category. Diagnostic systems also fit this model, since the aim of the system is to pick the correct diagnosis.

The knowledge is structured in rules which describe how each of the possibilities might be selected. The rule breaks the problem into sub-problems. For example, the following hypothetical top level rules are in a system which identifies microencapsulation methods.

IF

> class is chemical AND core material is liquid

THEN

> method is coacervation.

IF

> class is chemical AND core material is solid

THEN

> method is in-situ polymerization.

The system would work out all of the rules which gave information satisfying the goal of identifying the microencapsulation method. Each would trigger sub-goals. In the case of these two rules, the sub-goals of determining the class and the core material would be pursued. The following rule is one example that satisfies the class sub-goal:

IF

> equipment is batch reactor AND particle size medium AND
> cost limited

THEN

> class is chemical.

The sub-goals of determining equipment, particle size, and cost would be satisfied by asking the user. By having the lowest level sub-goal satisfied or denied by the user, the system effectively carries on a dialog with the user. The user sees the system asking questions and responding to answers as it attempts to find the rule which correctly identifies the microencapsulation method.

Data Driven Reasoning

For some problems, e.g. in configuration problems, it is not possible to list all of the possible answers before hand and have the system select the correct one. Since the inputs vary and can be combined in an almost infinite number of ways, the goal driven approach will not work. To cope with these problems we can use the data driven or forward chaining approach. It uses rules similar to those used for backward chaining, however, the inference process is different. The system keeps track of the current state of problem solution and looks for rules which will move that state closer to a final solution.

For example, the following hypothetical top level rules are in a system which identifies microencapsulation methods. Here is a rule from such a system which determined the preferred method to be complex coacervation.

IF

> shell material coalescence with core material at interface AND microcapsules are forming

THEN

> preferred method is complex coacervation

IF

> core material is oil

THEN

> core material is in liquid state

IF

> core material is dispersed in the solution in liquid state at a temperature lower than 90 °C AND core material is in liquid state

THEN

> microcapsules are forming

And we have the following facts:

Core material is oil.
Oil is dispersed in the solution at 40 °C.
Shell material is gelatin.
Gelatin coalescence with oil at interface.

The first rule would check whether the shell material coalescence with core material at interface and whether microcapsules are forming. Since none of these conditions satisfied, then this rule will not fire. Then the second rule would check whether the core material is oil. Since we have this fact in knowledge base, then this rule will fire. This rule will result in additional fact, i.e. core material is in liquid state, which will be stored in working storage. After that the third rule would check whether the core material is dispersed in the solution in liquid state at a temperature lower than 90 °C and whether the core material is in liquid state. Since we have a fact that oil is dispersed in the solution at 40 °C, then the first condition is satisfied. The second condition will also be satisfied because of our new fact from the fired second rule. The third rule will fire because both conditions are satisfied. It will result in another fact, i.e. microcapsules are forming. Next the system will check the first rule again. This time the first rule will fire since both of the conditions are satisfied. This will result in a fact, i.e. preferred method is complex coacervation. When all of the facts and the rules were checked, the system would be finished, and the output would be the final state.

Figure 2.5 illustrates the difference between forward and backward chaining systems for two simplified rules. For a data driven (forward chaining) system, the system must be initially populated with data, in contrast to the goal driven system which gathers data as it needs it. The forward chaining system starts with the data of state = solid and size = medium and uses the rules to derive microencapsulation is achieved by emulsification. The backward chaining system starts with the goal of finding the approach for microencapsulation and uses the two rules to reduce the previous goal to the problem of finding values for state and size.

2.3.3 Data Representation

In all rule based systems, the rules refer to the data. The data representation can be simple or complex, depending on the problem. There are four levels of common used data representation for expert systems which are illustrated in Figure 2.6.

The most fundamental scheme uses attribute-value pairs as seen in the rules for identifying particles. Examples are state–solid, and size–small. When a system is reasoning about multiple objects, it is necessary to include the object as well as the attribute-value. For example the microcapsules identification system might be dealing with multiple particles with different attributes, such as diameter. The data representation in this case must include the object. Once there are objects in the system, they each might have multiple attributes. This leads to a record-based structure where a single data item in working storage contains an object name and all of its associated attribute-value pairs. Frames are a more complex way of storing objects and their attribute-values. Frames add intelligence to the data representation, and allow objects to inherit values from other objects. In a microcapsule identification system each piece of particle can inherit default values

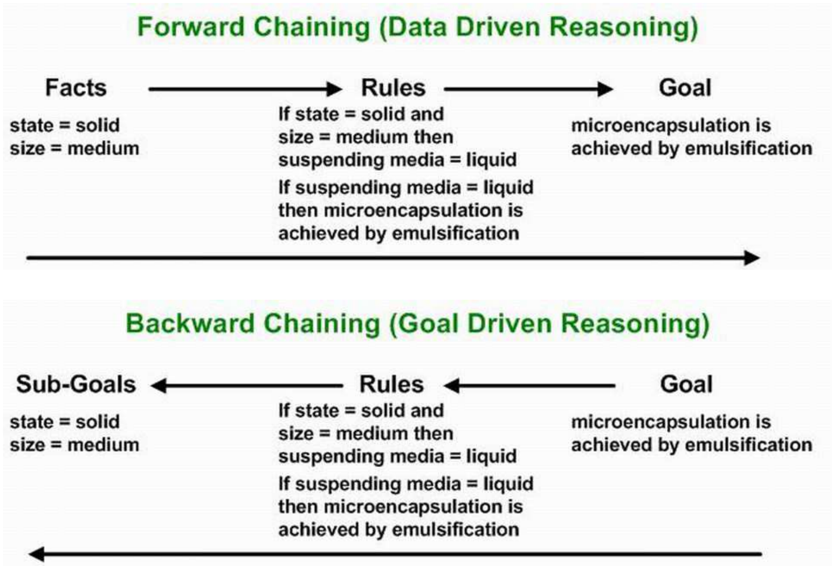


Figure 2.5: *Difference between Forward and Backward Chaining*

for the total number of core and shell.

2.3.4 Prolog

In general, expert systems programming focuses on issues of inference and heuristic search and depends heavily on the symbols manipulation. The programming languages LISP and Prolog are the most common languages in the expert system development, although conventional languages such as C are becoming more common to use. LISP is a functional language because every statement in the language is a description of a function. Prolog was developed as a method of programming computers using logic rather than conventional programming languages. It is also called a logic language because every statement in the language is an expression in a formal logic syntax. The details of building expert systems are illustrated in this thesis through the use of Prolog language. In this section, the basic concept of Prolog will be introduced. They will be needed in the this work as the programming language for expert system module. For a more thorough discussion of Prolog for artificial intelligence application, please refer to Bratko (1986).

The expressiveness of Prolog is due to three major features of the language: rule-based programming, built-in pattern matching, and backtracking execution. The rule-based programming allows the program code to be written in a form which is more declarative than procedural. This is made possible by the built-in pattern matching and backtracking which automatically provide for the flow

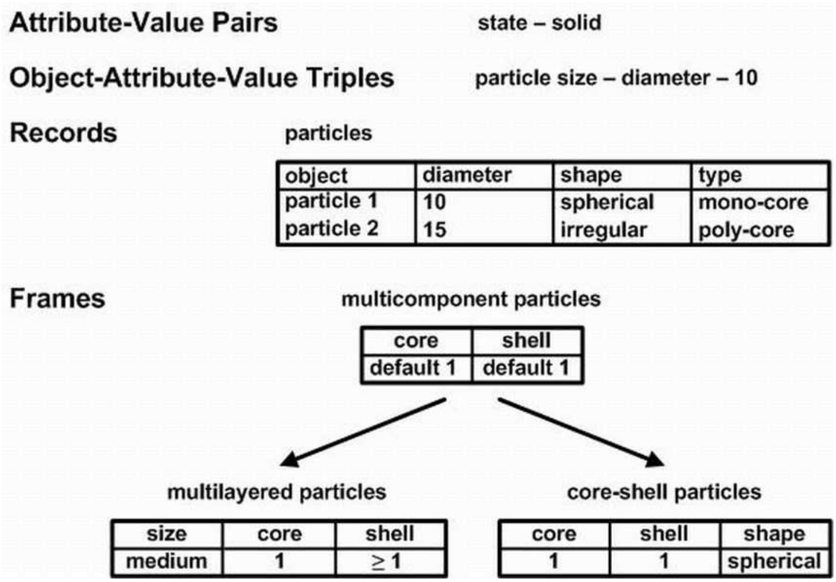


Figure 2.6: *Four Levels of Data Representation*

of control in the program. Together these features make it possible to elegantly implement many types of expert systems.

Prolog has a built-in backward chaining inference engine which can be used to partially implement some expert systems. Prolog rules are used for the knowledge representation, and the Prolog inference engine is used to derive conclusions. The Prolog inference engine does simple backward chaining. Each rule has a goal and a number of sub-goals. The Prolog inference engine either proves or disproves each goal. There is no uncertainty associated with the results.

Prolog System Example

A system which identifies microencapsulation methods will be used to illustrate a native Prolog expert system. The expertise in the system is a small subset of microencapsulation methods derived from the literature and the chemical engineers. The rules of the system were designed to illustrate how to represent various types of knowledge, rather than to provide accurate identification.

Prolog formats Prolog provides facilities for representing various types of data as shown in Figure 2.7. At the bottom level, atoms are used to represent objects. Atoms are represented by alpha-numeric strings, starting with a lower case letter. Values are used to represent the attributes of entities and may be integers, real numbers or quoted string characters. Atoms and values make up the set of Prolog

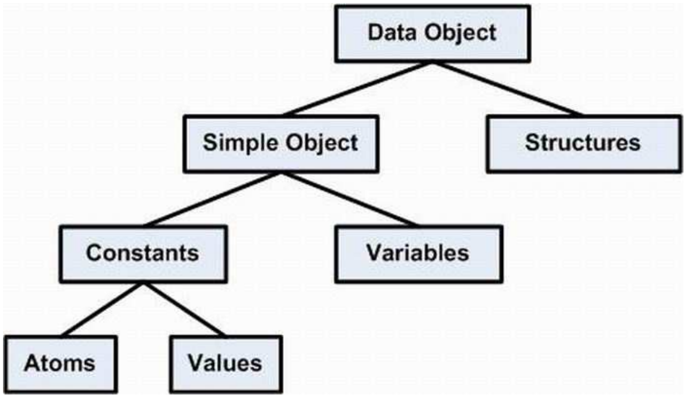


Figure 2.7: *Prolog Data Structures*

constants.

Variables in Prolog correspond to universally quantified variables in predicate logic. They are represented by alpha-numeric strings, starting with an upper case letter.

The most important type of data object in Prolog is the structure. Prolog data structures are used to represent logical predicates. The name of predicate is represented by a functor and its parameters, thus:

functor(parameter_1, parameter_2, ...)

A parameter is either a simple object or another structure.

The rules for expert systems are usually written in the form:

IF first premise, and second premise, and ...

THEN conclusion

The IF side of the rule is referred to as the left hand side (LHS), and the THEN side is referred to as the right hand side (RHS). This is semantically the same as a Prolog rule:

conclusion :- first_premise, second_premise, ...

Rules about microencapsulation methods The most fundamental rules in the system identify the various production methods of microencapsulation. Using the normal IF–THEN format, a rule for identifying a particular coacervation method is:

IF

class is chemical AND core material is liquid

THEN

method is coacervation.

In Prolog the same rule is:

method(coacervation) :- class(chemical), core_material(liquid).

The following rules distinguish between methods of in-situ polymerization and spray drying. They are clauses of the predicate method/1:

method(in-situ_polymerization) :- class(chemical), core_material(solid).

method(spray_drying) :- class(physical), core_material(solid).

In order for these rules to succeed in distinguishing the two microencapsulation methods, we would have to store facts about a particular microcapsule that needed identification in the program. For example if we added the following facts to the program:

class(chemical).

core_material(solid).

The following query could be used to identify the microencapsulation method:

?- method(X).

X = in-situ_polymerization

Figure 2.8 shows an example of the results of Prolog query about the following questions:

1. Which microencapsulation methods should we try if we want to produce microcapsules with a diameter 1 to 500 μm?
2. Which microencapsulation methods should we try if we want to produce microcapsules with a diameter 1 to 500 μm and the methods should belong to the physical method?
3. Which microencapsulation methods should we try if we want to produce microcapsules with a diameter 1 to 500 μm and the methods should belong to the chemical method?

The example shows that Prolog can be used as a declarative language for the knowledge representation of an expert system. The rules lend themselves to solving identification and other types of selection problems that do not require dealing with uncertainty.

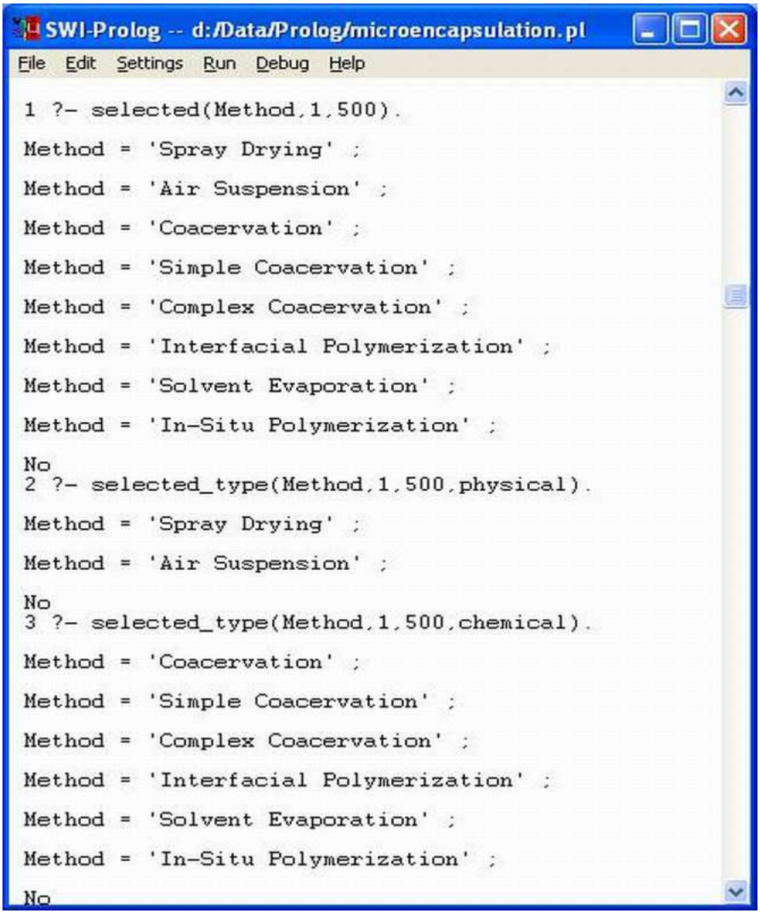


Figure 2.8: *Example of Prolog Query about Microencapsulation Methods*

2.3.5 Reasons for Expert Systems

The developments in the field of Expert Systems rapidly found proponents in industry despite the inherent reluctance to adopt new technology. The reasons that has motivated industry to adopt knowledge-based techniques are the following:

- the lack of an explicit quantitative description of the physical process,
- the existence of the knowledge and experience to control the process, and
- the ability of a class of knowledge-based systems to deal with vagueness and uncertainty that is characteristic of many industrial processes.

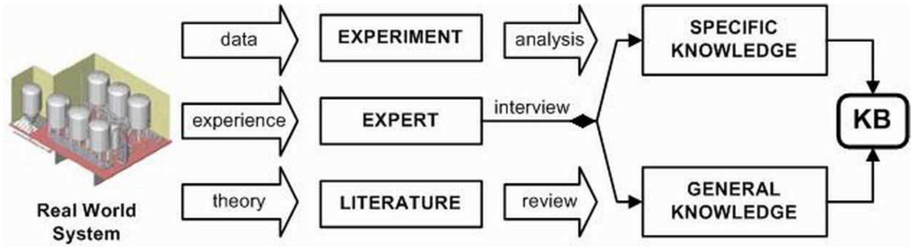


Figure 2.9: *Knowledge Acquisition Methods*

A common feature in industrial and manufacturing systems is that their quantitative models that are supposed to predict their dynamic behavior are either unknown or do not possess sufficient fidelity. This is particularly true in the case of large-scale industrial processes whose quantitative description is a difficult, tedious and occasionally impossible task for lack of sufficient deep knowledge. **Deep knowledge** is the result of microscopic knowledge of the physical laws that govern the behavior of a process. In contrast, **shallow knowledge** is the result of holistic or macroscopic knowledge and is readily available from human domain experts. This knowledge is acquired after years of experience in operating the process and observing its behavior.

2.3.6 Knowledge Acquisition

A variety of methods were used to develop the knowledge base for this study. The source of knowledge used to develop the expert system for this study includes the following: (1) literature review, (2) experts interviews, and (3) experimental data analysis. Figure 2.9 illustrated the main sources and methods that will be used to acquire both general and specific knowledge on the microencapsulation domain.

The acquisition, organization and representation of the knowledge required to model the microencapsulation selection process was conducted in two parts: (i) general knowledge acquisition from literature review in order to obtain an understanding of the problem domain and (ii) specific knowledge acquisition from the experts to obtain detailed knowledge about some microencapsulation methods.

In addition to the human expert, a secondary knowledge source was found in the paper and literature given by the domain expert. The literature includes the descriptions of the microencapsulation methods and the condition in which they are suitable. A third source of knowledge we used in this thesis is experimental data analysis. Based on the information we got from the domain experts and also the literature given by the domain expert, we performed some experiments on the data and then we analyzed the resulting data.

2.4 Decision Support System

Decision support systems (DSS) are computer technology solutions that can be used to support complex decision-making and problem solving (Turban & Aronson, 1998; Mittra, 1986). These systems assist decision-makers in choosing between beliefs or actions by applying knowledge about the decision domain to arrive at recommendation for the various option. Research in the decision sciences has resulted in the development of a variety of scientific problem-solving and model-based methods for many decision problems.

We will start this section with the introduction of decision making process. Then we introduce the definition, the characteristics and the formulation of the MCDM problem. In our case study, the microencapsulation selection problem, the feasible alternatives (i.e. the candidates to be selected) are explicitly given rather than being implicitly defined by the model. The decision making problem is, thus, a discrete one: it involves selecting the best alternatives from a finite set of feasible ones based on the evaluation of each alternative against a given set of criteria. Therefore, we will focus ourselves on discrete MCDM methods or Multiple Attribute Decision Making (MADM), rather than the continuous MCDM methods. The problem with multiple criteria considered in this thesis belongs to the class of MADM problems.

2.4.1 Decision Making Process

Decision making is a process of choosing/selecting among alternative courses of action for the purpose of achieving a goal or goals.

Decision making process is executed in four major phases, intelligence, design, choice or decision and implementation as shown in Figure 2.10.

- **Intelligence Phase:** also called problem formulation phase, where the situation is analysed for the problem and prospects.

- **Design Phase:** involves problem understanding, generating alternatives, selecting criteria and establishing relationships among them.

- **Choice/Decision Phase:** involves the evaluation of the alternatives using the set criteria to achieve the objective.

- **Implementation Phase:** involves the realization of the process into practice.

Decision making is classified based on what the decision maker knows about the results. We can classify this knowledge into three categories ranging from complete knowledge to incomplete knowledge as shown in Figure 2.11. These categories are

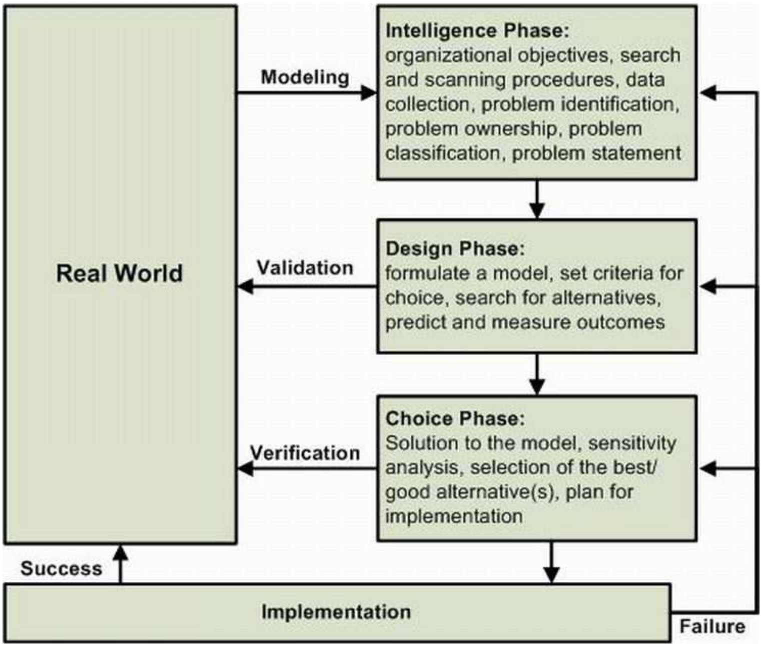


Figure 2.10: *The Decision Making Process according to Turban & Aronson (1998)*

Certainty A decision making under certainty is assumed that complete knowledge is available so that the decision maker knows exactly what the outcome of each course of action will be (as in a deterministic environment).

Risk A decision making under risk is when the decision maker must consider several possible outcomes for each alternative, each with a given probability of occurrence. This is known as a probabilistic or stochastic decision situation.

Uncertainty A decision making under uncertainty is when the decision makers considers situations in which several outcomes are possible for each course of action. In contrast to the risk situation, the decision maker does not know or cannot estimate the probability of occurrence of the possible outcomes.

In order to improve the quality of decisions, some knowledge of decision theory is necessary. A decision problem consists of the following elements: decision maker, candidate alternatives, states of nature, outcome, and decision criteria. Different decision problems have their own contexts and thus their own characteristics. We can also classify them into different groups according to these characteristics.

1) Based on decision maker, decision problems can be divided into individual decision making and group decision making problems. In the former,

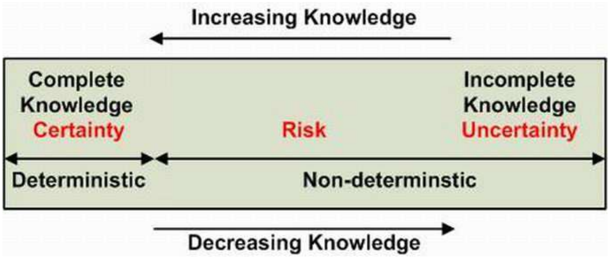


Figure 2.11: *The Classification of Decision Making Process*

there is only one decision maker to make the decision. In the latter, there are multiple decision makers with different backgrounds and interests and reconciliation needs to be made to reach a final decision.

2) Based on the candidate alternatives, decision problems can be treated as descriptive models, in which a limited fixed number of alternatives are evaluated to choose a best choice to the decision maker, or prescriptive models, in which there are infinite alternatives and the task of the models is to indicate good choices for decision makers.

3) Based on states of nature, the classification like we already defined above can be made, i.e. a decision is under certainty if the decision maker knows which state of nature will obtain and what the outcome of each alternative will be, under risk if the decision maker can identify each state of nature and the probability of occurrence of each state of nature and knows the outcome associated with the each alternative and state of nature, and under uncertainty if the decision maker knows the specific outcomes associated within each alternative under each state of nature, but he/she does not know, or is unwilling to express quantitatively the probabilities associated with the states.

4) Based on decision criteria, decision problems can be divided two groups: single criterion if there is only one criterion, and multi criteria if several conflicted criteria are involved.

2.4.2 Multiple Criteria Decision Making Problem

Multiple Criteria Decision Making (MCDM) refers to making decisions in the presence of multiple and possibly conflicting criteria. MCDM is an area of work relating to the use of sets of criteria to assist in making decisions.

The multiple criteria problems arise naturally in many real life situations. The common factors (Sen & Yang, 1998):

(i) there is a range of possible actions.

(ii) each action is characterized by a set of consequences some of which are beneficial and others less so.

(iii) the decision maker is required to weigh up the pros and cons before arriving at a preferred action, and to do this he/she wight use a range of decision rules.

In many real life problems, choosing between possible courses of action may be difficult because it requires balancing several factors. MCDM deals with situations where the decision maker has several conflicting objectives (see, e.g., Ehrgott & Gandibleaux (2002); Figueira et al. (2005) for a recent survey). There is generally no perfect alternative, and a good compromise must be identified.

Classification of MCDM Problems

According to Sen & Yang (1995, 1998), the MCDM problems can be broken down into two distinct types of problems, i.e. selection problems and synthesis problems. Selection problems involve choosing one of several possible alternatives. Synthesis involves creating solutions whose aim is to attain a set of goals. Synthesis can also be looked upon as an investigation into what is achievable within the constraints of a problem.

For multiple criteria problems there is rarely, if ever, one clear "best" alternative. It is often clear, however, what is the best alternative with respect to particular criteria (e.g. minimum production cost, and maximum safety). This leads to the concept of Pareto optimality:

Definition 2.4.1. *The alternative A dominates the alternative B if A is at least as good as B on all attributes, and strictly better on at least one. A feasible point which is not dominated by any other point is called Pareto optimal, and may be referred to as an non-dominated point or a Pareto point. The set of all Pareto points for a multiple criteria problem is called the Pareto set or Pareto frontier.*

Hwang & Yoon (1981), Triantaphyllou (2000), and Zimmermann (1991) classified that the problems of MCDM can be split into two problem-solving categories:

1. **Multiple Attribute Decision Making (MADM)** - associated with the selection/evaluation of a problem where there are a finite set of alternatives in terms of a finite number of decision criteria, often such criteria may be in conflict with each other. MADM is also known as multiple criteria decision analysis (MCDA). MADM problem has usually a limited number of predetermined alternatives, which have an associated a level of achievement of the attributes, and the final decision is made based on these attributes. Associated attributes/criteria are likely to be both quantitative and qualitative (fuzzy), and be of different units of measure. For the recent reviews on MCDA methods see Figueira et al. (2005).

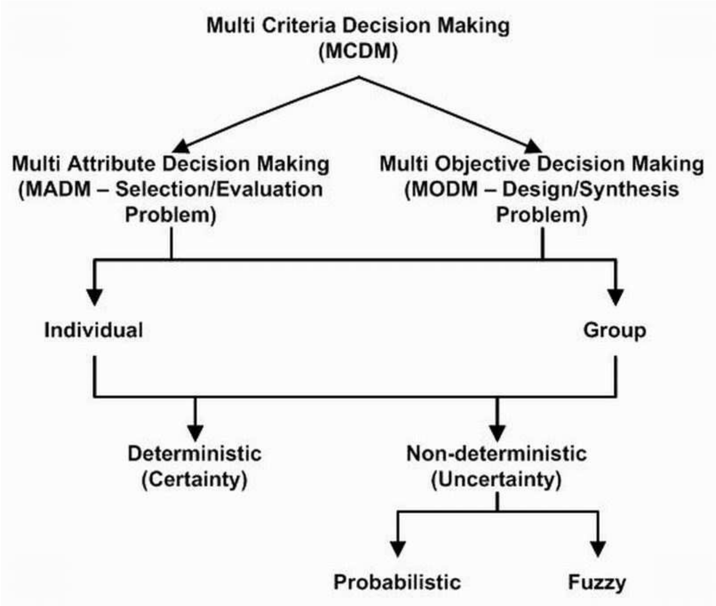


Figure 2.12: *Multiple Criteria Decision Making Classification*

2. **Multiple Objective Decision Making (MODM)** - designing the best alternative solution, i.e. the one that is most attractive over all criteria. MODM is also known as multi objective optimization (MOO), multi criteria optimization (MCO) or vector optimization. MODM is associated with the problems where alternatives are not predetermined. For the recent reviews on MODM methods see Ehrgott & Gandibleaux (2002).

For each categories, two dichotomies can be distinguished:

(1) Individual versus group decision maker problems

(2) Deterministic (decision under certainty) versus non-deterministic (decision under uncertainty) problems

Figure 2.12 shows the classification of MCDM methods. There are many MCDM methods available in the literature. As they have each its characteristics, there are many ways to classify them. One classification method is according to the type of data they use, which can be deterministic, stochastic, or fuzzy. Another way to classify MCDM method corresponds to the number of decision maker involved in the decision making process. There can only be one decision maker, or a group of decision makers.

In the MODM approach, contrary to the MADM approach, the decision alternatives are not given or defined implicitly. Instead, MODM provides a mathemat-

Table 2.1: *The Main Features of MADM and MODM Problems*

Characteristic	MADM Problem	MODM Problem
Criteria (defined by)	Attributes	Objectives
Objective	Implicit (ill-defined)	Explicit
Attribute	Explicit	Implicit
Constraint	Inactive (incorporate into attributes)	Active
Alternative	Finite number, discrete, and predetermined	Infinite number, continuous
Application	Selection/evaluation	Design/synthesis

ical framework for designing a set of decision alternatives. For MODM problems the decision maker must come up with the most preferred alternative or design it by assigning values to decision variables. Each alternative, once identified, is judged by how close it satisfies an objective or multiple objectives. In this approach, the number of potential decision alternatives may be very large (infinite).

For MADM problems, the decision maker must choose from a set of alternatives that are usually explicitly defined. Of course not all the alternatives may be known a priori. In other words, the decision maker selects a most preferred alternative. Table 2.1 summarizes the common characteristics of MADM and MODM problems.

Having a defined set A of actions and a set C of criteria on A, a multiple criteria decision problem is the one that, with respect to a goal G, aims to find one of the below:

a) a subset of A that contains the best actions,

b) an assignment of the actions into predefined categories, or

c) a rank of the actions in A from best to worst.

Each of these objectives defines a different multiple criteria decision problem, called: (a) choice problem, (b) classification or sorting problem (depending on if the categories are preferentially ordered or not) and (c) ranking problem. The main difficulty lies in the fact that it is an ill-defined mathematical problem because there is no objective or optimal solution for all the criteria. Thus, some trade-off must be done among the different points of view to determine an acceptable solution for the decision problem.

In general, the engineering design involves only two tasks: (1) determine all possible design alternatives and (2) choose the best one. While optimization theory deals with search techniques (given objective functions and constraints), Decision theory deals with proper formulation of the objective function and selection. Optimization and selection problems are part of decision making and are, thus, needed to design.

In the scope of this thesis we are applying the former as the aim is to assist with selection problem, not design problem. In our case study, microencapsulation selection problem, the feasible alternatives (i.e. the candidates to be selected) are explicitly given rather than being implicitly defined by the model. The decision making problem is, thus, a discrete one: it involves selecting the best alternatives from a finite set of feasible ones based on the evaluation of each alternative against a given set of criteria. The methods employed in this thesis will, therefore, be based on MADM approach, rather than the general MCDM approach. The problem with multiple criteria considered in this thesis belongs to the class of MADM problems.

The terms criterion and attribute are often used synonymously in the MCDM literature. Attribute is often used to refer to a measurable criterion. Although the method we use here is based on MADM, in this thesis we will use the term criterion instead of attribute.

In the literature, the terms MCDM or MCDA often refer only to the second class of the problems, MADM, which is the one we are working on. For the first class of the problems, MODM, the terms used are usually MCO (Multi Criteria Optimization) or MOO (Multi Objective Optimization).

2.4.3 Multiple Attribute Decision Making Problem

An MADM problem can be easily expressed in matrix format. A decision matrix A is an $(n \times m)$ matrix in which element a_{ij} indicates the performance of alternative A_i when it is evaluated in terms of decision criterion C_j, (for $i = 1, 2, 3, \ldots, n$, and $j = 1, 2, 3, \ldots, m$). It is also assumed that the decision maker has determined the weights of relative performance of the decision criteria (denoted as W_j, for $j = 1, 2, 3, \ldots, m$). This information is best summarized in Table 2.2. Given the previous definitions, then the general MADM problem can be defined as follows (Zimmermann, 1991):

Definition 2.4.2. *Let $A = \{A_i,\ \text{for } i = 1, 2, 3, \ldots, n\}$ be a (finite) set of decision alternatives and $G = \{g_j,\ \text{for } j = 1, 2, 3, \ldots, m\}$ a (finite) set of goals, attributes or criteria according to which the desirability of an alternative is judged. The aim of MADM is to determine the optimal alternative A^* with the highest degree of desirability with respect to all relevant goals g_j.*

Basically MADM consists of three phases:

1. Modeling phase
   In this phase, we look for appropriate models for constructing the partial scores of the alternatives with respect to each criterion and also for determining the importance of each criterion (i.e., the weights).

2. Aggregation phase
   In this phase, we try to find a global (total) score for each alternative, on the basis of the partial scores and the weights.

Table 2.2: *A Typical Decision Matrix.*

	Criteria				
	C_1	C_2	C_3	$\ldots$	C_m
Alternatives	W_1	W_2	W_3	$\ldots$	W_m
A_1	a_{11}	a_{12}	a_{13}	$\ldots$	a_{1m}
A_2	a_{21}	a_{22}	a_{23}	$\ldots$	a_{2m}
A_3	a_{31}	a_{32}	a_{33}	$\ldots$	a_{3m}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
A_n	a_{n1}	a_{n2}	a_{n3}	$\ldots$	a_{nm}

3. Ranking/Exploitation phase
   In this phase, we transform the global information about the alternatives either into a complete ranking of the elements in A, or into a global choice of the best alternatives in A.

The MADM approach requires that the choice (selection) be made among the decision alternatives described by their attributes (criteria as attributes). MADM problems are assumed to have a predetermined, finite number of decision alternatives. Solving a MADM problem involves sorting and ranking the decision alternatives.

If the decision matrix is known, the dominance relation on the alternatives can be identified. One alternative is said to dominate another if the first one is at least as good as the second in every criterion and strictly better in at least one criterion. Alternatives which are not dominated by any other alternative are called non-dominated alternatives.

A generic framework of the principal steps in the application of MADM models, and the concepts and procedure involved, has been discussed by Dodgson et al. (2001). They identify the following sequence of steps in a typical application (see Figure 2.13):

1. Establish the decision objectives (goals), and identify the decision maker(s).

2. Identify the alternatives.

3. Identify the criteria (attributes) that are relevant to the decision problem.

4. Assign scores to measure the performance of the alternatives for each of the criteria and construct an evaluation matrix.

5. Standardize the raw scores to generate a priority scores matrix or decision table.

6. Determine a weight for each criterion to reflect how important it is to the overall decision.

7. Use aggregation functions to compute an overall assessment measure for each decision alternatives by combining the weights and priority scores and forms the basis of a preference ranking.

8. Perform a sensitivity analysis to assess the robustness of the preference ranking to the changes in the criteria scores and/or the assigned weights.

9. Examine the preference ranking to make a final recommendation.

In our decision support expert system framework, the MADM module contains three sub-modules for performing multiple-attribute decision making. The first module uses AHP (Analytical Hierarchy Process). The second module uses BR-AHP (Base Reference Analytical Hierarchy Process). The third module uses fuzzy BR-AHP (fuzzy Base Reference Analytical Hierarchy Process).

The following subsections present briefly about those three modules. For the details about each of these modules please refer to Chapter 4, Chapter 5, and Chapter 6, respectively.

2.4.4 Analytical Hierarchy Process

Applying Analytical Hierarchy Process (AHP) to a decision-making problem involves four fundamental steps:

Problem Decomposition/Hierarchy Construction. We construct the structure of the problem according to its main components: goal/objective, set of criteria for evaluation, and the decision alternatives.

Pairwise Comparison of Decision Criteria. The relative importance of criteria within each category and of each category within the group of categories is established through pairwise comparisons using a square matrix structure. For n number of criteria an $n \times n$ square matrix is formed. Hence $n(n-1)/2$ judgments are to be made on the importance of criteria which is done with the aid of Saaty's nine-point scale. In this step, we use prioritization method to attain the criteria weight.

Ratings of Alternatives. Pairwise comparison is applied to obtain ratings for qualitative data. If quantitative data is available then the rating is done by existing or estimated performance data.

Rankings Finally ratings of the alternative are combined with the ratings of the criteria to form an overall rating for each alternative. The alternative with the highest rating is ranked the best choice, taking into account the relative importance of each criterion and the relative desirability of the alternatives with respect to each criterion.

AHP requires a specific consistency check of the pairwise comparisons in order to ensure that the decision maker is being neither inconsistent nor random in

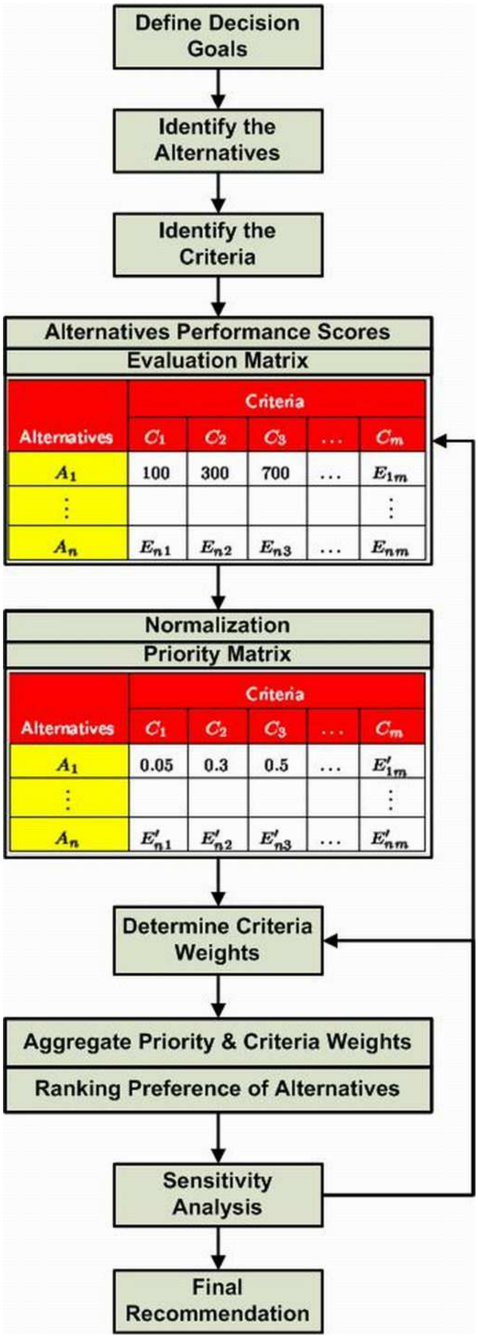


Figure 2.13: *The Steps in Applying MADM Models*

his/her pairwise comparisons. Saaty (1980, 2001) proposed utilizing consistency index (CI) and consistency ratio (CR) to verify the consistency of the comparison matrix. CI and CR are defined as follows:

$$CI = (\lambda_{\max} - n)/(n-1), \qquad CR = CI/RI,$$

where RI represents the average consistency index over numerous random entries of same order reciprocal matrices. If $CR \leq 0.1$, the estimate is accepted; otherwise, a new comparison matrix is solicited until $CR \leq 0.1$. Figure 2.14 shows the AHP module.

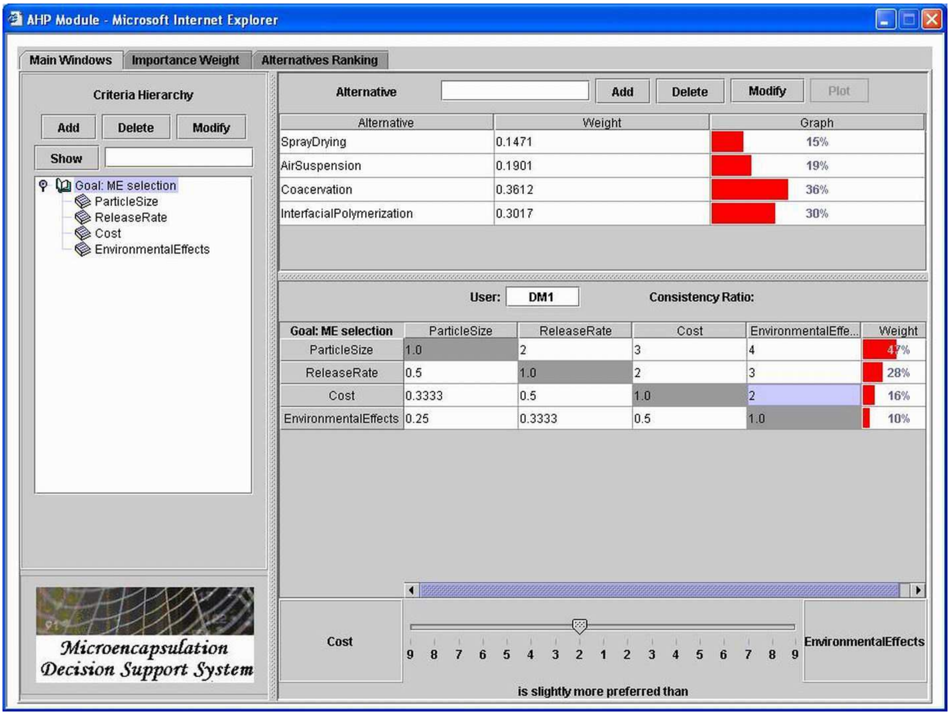


Figure 2.14: *Analytical Hierarchy Process Module*

2.4.5 Base Reference Analytical Hierarchy Process

The Base Reference Analytical Hierarchy Process (BR-AHP) is a process that consists of four steps:

1. Construct the hierarchy structure of the selection problem.

2. Calculate the relative importance of the criteria using pairwise comparison method.

3. Choose the base alternative. Compare the base alternative relative to each other alternatives and calculate the relative weights of each alternative on the basis of each selection criterion this is achieved by performing base pairwise comparison method of the alternatives.

4. Combine the ratings derived in steps 2 and 3 to obtain an overall relative ranking for each potential alternative.

In BR-AHP most procedures follow the original architecture of AHP. The difference is only in step 3 where it uses a base pairwise comparison method. By using the pairwise comparison method it needs to estimate $n(n-1)/2$ judgments for n alternatives, but with the base pairwise comparison method it only needs $n-1$ judgments. Figure 2.15 shows the BR-AHP module.

Figure 2.15: *Base Reference Analytical Hierarchy Process Module*

To demonstrate how the BR-AHP works, a hypothetical example for selection of the best technology for advanced manufacturing is provided (Hotman, 2005a). Due partly to the rapid growth of manufacturing technology, the method of selecting of the most appropriate manufacturing process to meet users' requirements from among a number of manufacturing systems has become increasingly important. The first step in the BR-AHP method is to construct the hierarchical structure of manufacturing technology selection problem. The goal is to determine the best technology for manufacturing system. There are four criteria for the selection, i.e. the flexibility, the capability, the learning ability and the cost of the system. There are ten alternative technologies to be considered (M1 - M10). The alternative manufacturing technologies are analyzed with respect to the criteria in the second level of the hierarchy, as shown in Figure 2.16.

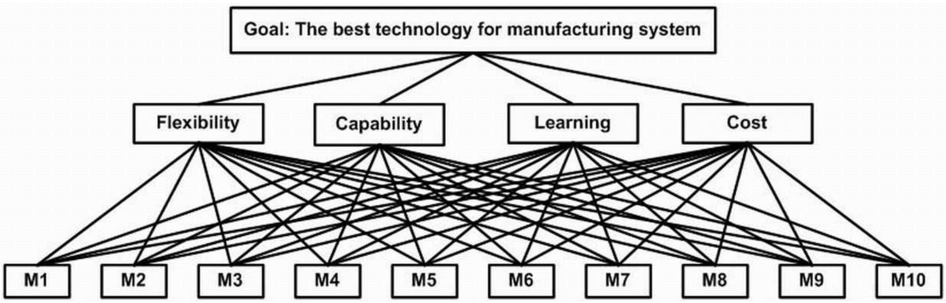
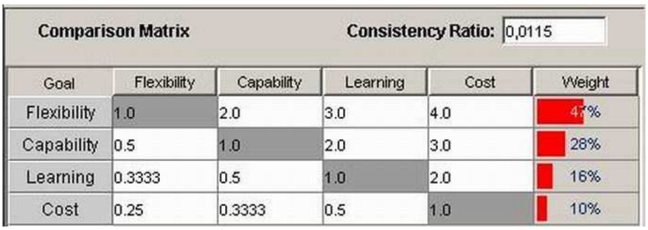


Figure 2.16: *Technology Selection Hierarchy*

The next step is the pairwise comparison of the importance of the technology selection criteria. This is done by assigning a rating on a scale from 1 (equally good/indifferent) to 9 (absolutely better) to the more important criteria, whilst the less important criteria in the pairing is awarded a rating reciprocal to this value. Figure 2.17 shows the result of pairwise comparison of the technology selection criteria. Here we have a comparison of 4 criteria, so the decision maker needs to determine 6 judgments.



Comparison Matrix Consistency Ratio: 0,0115

Goal	Flexibility	Capability	Learning	Cost	Weight
Flexibility	1.0	2.0	3.0	4.0	47%
Capability	0.5	1.0	2.0	3.0	28%
Learning	0.3333	0.5	1.0	2.0	16%
Cost	0.25	0.3333	0.5	1.0	10%

Figure 2.17: *Pairwise Comparison of Technology Selection Criteria*

The next step is the base pairwise comparison of the technology alternatives (M1 - M10). First, the decision maker needs to select the base alternative. Then, he/she compares his/her base alternative with other alternatives. Figure 2.18 shows the base pairwise comparison of technology alternatives based on capability criterion. In this case, the decision maker selected the alternative M1 as the base alternative and then compared M1 with (M2 - M10). In this step, the decision maker only needed to give 9 judgments.

Comparison Matrix **Consistency Ratio:** 0,0029 **Base:** 1

Capability	M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	Weight
M1	1.0	3.0	0.5	1.0	2.0	0.3333	1.0	3.0	0.5	4.0	9%
M2	0.3333	1.0	0.1667	0.3333	0.5	0.1111	0.3333	1.0	0.1667	1.0	3%
M3	2.0	6.0	1.0	2.0	4.0	0.5	2.0	6.0	1.0	8.0	17%
M4	1.0	3.0	0.5	1.0	2.0	0.3333	1.0	3.0	0.5	4.0	9%
M5	0.5	2.0	0.25	0.5	1.0	0.1667	0.5	2.0	0.25	2.0	5%
M6	3.0	9.0	2.0	3.0	6.0	1.0	3.0	9.0	2.0	9.0	27%
M7	1.0	3.0	0.5	1.0	2.0	0.3333	1.0	3.0	0.5	4.0	9%
M8	0.3333	1.0	0.1667	0.3333	0.5	0.1111	0.3333	1.0	0.1667	1.0	3%
M9	2.0	6.0	1.0	2.0	4.0	0.5	2.0	6.0	1.0	8.0	17%
M10	0.25	1.0	0.125	0.25	0.5	0.1111	0.25	1.0	0.125	1.0	2%

Figure 2.18: *Base Pairwise Comparison of Technology Alternatives based on 'Capability' Criterion*

We compared this step with the usual AHP method. Using AHP method, the decision maker needed to give 45 judgments. Figure 2.19 shows the pairwise comparison of technology alternatives based on capability criterion.

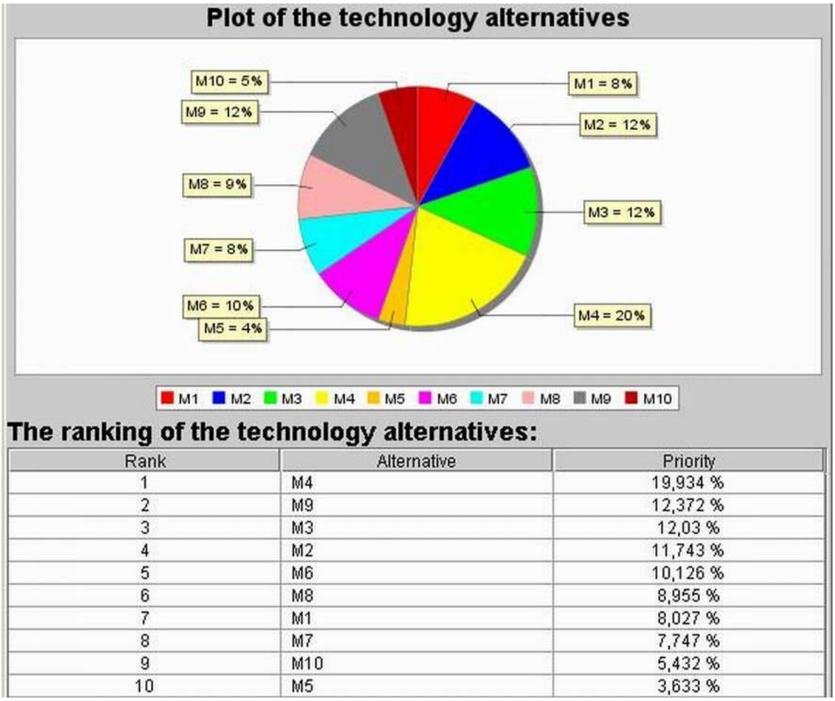
Comparison Matrix **Consistency Ratio:** 0,0415

Capability	M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	Weight
M1	1.0	3.0	0.5	1.0	2.0	0.3333	1.0	3.0	0.5	4.0	10%
M2	0.3333	1.0	0.2	0.5	1.0	0.1667	0.5	1.0	0.2	1.0	4%
M3	2.0	5.0	1.0	2.0	2.0	0.3333	2.0	3.0	1.0	2.0	14%
M4	1.0	2.0	0.5	1.0	1.0	0.3333	1.0	3.0	0.3333	0.5	7%
M5	0.5	1.0	0.5	1.0	1.0	0.1667	1.0	1.0	0.5	0.5	5%
M6	3.0	6.0	3.0	3.0	6.0	1.0	4.0	4.0	1.0	3.0	24%
M7	1.0	2.0	0.5	1.0	1.0	0.25	1.0	1.0	0.3333	2.0	7%
M8	0.3333	1.0	0.3333	0.3333	1.0	0.25	1.0	1.0	0.3333	0.5	4%
M9	2.0	5.0	1.0	3.0	2.0	1.0	3.0	3.0	1.0	3.0	17%
M10	0.25	1.0	0.5	2.0	2.0	0.3333	0.5	2.0	0.3333	1.0	7%

Figure 2.19: *Pairwise Comparison of Technology Alternatives based on 'Capability' Criterion*

In Figures 2.18 and 2.19, we see that the consistency ratio by using base pairwise comparison method ($CR = 0.0029$) is smaller than by using pairwise comparison method ($CR = 0.0415$). This means that the base pairwise comparison method is more consistent. Additionally, the time needed by the decision maker to fill in base pairwise comparison matrix is much less than the time to fill in pairwise comparison matrix.

The final step in the BR-AHP method is to combine all the weights derived in the previous steps to obtain the overall ranking for the alternative. The result of the ranking of the technology alternatives is shown in Figure 2.20.



Rank	Alternative	Priority
1	M4	19,934 %
2	M9	12,372 %
3	M3	12,03 %
4	M2	11,743 %
5	M6	10,126 %
6	M8	8,955 %
7	M1	8,027 %
8	M7	7,747 %
9	M10	5,432 %
10	M5	3,633 %

Figure 2.20: *Technology Selection Result*

2.4.6 Fuzzy Base Reference Analytical Hierarchy Process

The fuzzy Base Reference Analytical Hierarchy Process (fuzzy BR-AHP) methodology is consisted of four main steps:

1. Construction of the hierarchical structure of the decision problem.

2. Evaluation of the criteria using fuzzy pairwise comparison method.

3. Evaluation of the alternatives using fuzzy base pairwise comparison method.

4. Aggregation of the results in steps 2 and 3 to obtain an overall relative ranking for each potential alternative.

In step 3 we use fuzzy base pairwise comparison method for eliciting the judgments from decision makers. By using the fuzzy pairwise comparison method each decision maker needs to estimate $n(n-1)/2$ judgments for n alternatives, but with the fuzzy base pairwise comparison method it only needs $n-1$ judgments. Figure 2.14 shows the fuzzy BR-AHP module.

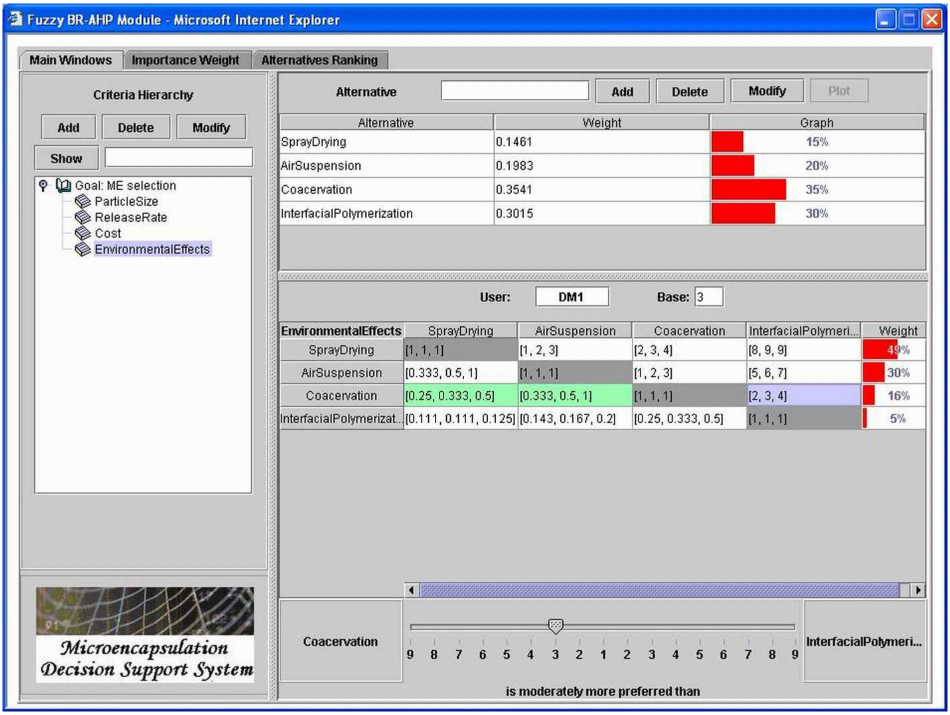


Figure 2.21: *Fuzzy Base Reference Analytical Hierarchy Process Module*

2.4.7 Group Decision Making

The application of three MADM modules can be used for group decision making by aggregating individual judgments or individual priorities. In a real case problem, the weights or the judgments derived for each of experts are somewhat different. To determine a compromised value for the weights attribute, two different methods are available.

Aggregation of individual judgment: Take the pairwise comparison of experts and determine a new pairwise comparison matrix based on a combination of all the experts values utilizing the relation below:

$$\hat{a}_{ij} = \left(\prod_{k=1}^{n} a_{ij}^{k} \right)^{1/n}$$

where n is the total number of experts.

Aggregation of individual priorities: Aggregate the values of the weights determined for each of the experts. The most common methods for aggregating are the geometric mean method and the arithmetic mean method.

2.5 Summary

This chapter presents a decision support expert system framework for engineering selection problems, specializing in the microencapsulation process selection problem. A wide range of microencapsulation techniques have been developed to date. The process of selecting microencapsulation method is complicated by the large number of microencapsulation alternatives with overlapping capabilities. The selection of a microencapsulation technique depends on many factors. As a decision aid for process/chemical engineers, it is necessary to design a decision support system that provides help to select an appropriate microencapsulation technique. In this chapter, we proposed a decision support expert system for the selection of microencapsulation process, which incorporated the expert system and the multiple attribute decision making modules. The Decision Support Expert System (DSES) combines expert system tools and multiple-attribute decision making techniques. The expert system module provides a list of technically feasible alternatives. The multiple-attribute decision making modules are used to rank the alternatives based on selected criteria.

Chapter 3

Problem Domain: Microencapsulation

In this chapter, we present the problem domain that we worked with in our case study, i.e. microencapsulation domain. We will begin with the introduction of microencapsulation in the next section. The morphology, release mechanisms, and the reasons to use the microencapsulation will be presented afterward. Next we present the criteria and the methods of the microencapsulation that we are interested in. Finally, we will summarize this chapter in the last section.

3.1 Introduction

Microencapsulation is the process of enveloping gases, liquid droplets, or fine solid particles to produce capsules with a diameter range between 1 and 1000 μm, known as microcapsules. The capsules that are in the size range of 1 μm to 1000 μm are referred to as microcapsules. Capsules below the size of 1 μm are frequently referred to as nanocapsules and they are made using very specialized methods. The term capsule refers to macro objects in the order of 1 mm or larger. This capsule term is frequently used in the delivery of pharmaceuticals.

Microcapsules are in general small particles containing an active agent or a core material surrounded by a coating layer or a shell. These microcapsules release their contents at a later time by means appropriate to the application. In other words, microencapsulation provides a means of wrapping, separating, and storing materials on a microscopic scale for later release under controlled conditions. This technology has been used in several fields for various engineering applications including pharmaceuticals, agriculture, food, printing and cosmetics (Benita, 1996). Microencapsulation may be performed by a large number of techniques with many purposes.

The microencapsulation technology remains something of an art, although firmly grounded in science (Kondo, 1978). Combining the right shell materials with the most efficient production process for any given core material and its intended use requires extensive scientific knowledge of all the materials and processes involved and a good feel for how materials behave under various conditions.

The structure of the material for a microcapsule as shown in Figure 3.1 consists of:

- Core Material: The substance that is encapsulated could be called the core material, the active ingredient or agent, fill, payload, nucleus, or internal phase. These materials may vary depending on the intended use of the capsules. Many different active materials have been successfully encapsulated using a variety of coatings including gelatin, cellulose, polyethylene glycol and waxes.

- Wall Material: The material encapsulating the core is often called coating, membrane, shell, envelop or wall material. This material, which may consist of natural, semisynthetic, or synthetic polymer, can be made permeable, semipermeable or impermeable. The purpose of wall material is to seal off the core material from the external surrounding. Microcapsules may have one wall or multiple shells arranged in strata of varying thicknesses around the core.

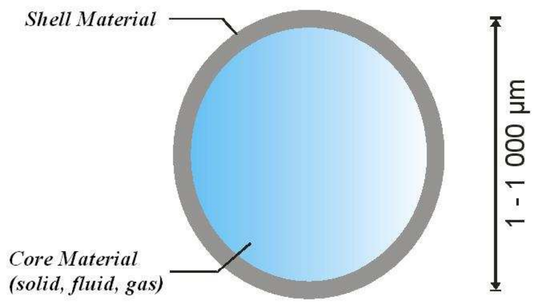


Figure 3.1: *The Structure of the Microcapsule*

Core material of almost any material can be encompassed in a impervious wall material and thus isolated from reactive, corrosive, or hostile atmospheres or surroundings. Microencapsulation technology is very broad so that many diverse characteristics, properties, and physical forms can be built into capsule structures. Microcapsules can have many different types and structures ranging from simple droplets of liquid core material surrounded by a spherical shell, to irregularly-shaped particles containing small droplets of core material dispersed in a continuous polymer shell matrix. There are numerous basic microencapsulation techniques employed to produce this wide range of structures and many of these techniques have several variations.

3.2 Microcapsule Morphology

The architecture of microcapsules is generally divided into several arbitrary and overlapping classifications. The most common or well known type of a microcapsule is that with a spherical structure. In its simplest form, a microcapsule is a small sphere with a uniform wall around it. Many microcapsules however bear little resemblance to these simple spheres. The core may be a crystal, a jagged adsorbent particle, an emulsion, a suspension of solids, or a suspension of smaller microcapsules. Another structure is known as the matrix structure. In this structure, the matrix particle resembles that of a peanut cluster. The core material is buried in varying depths inside of the wall material.

If the core material is an irregular material, which occurs with a ground particle, then the wall will slightly follow the contour of the irregular particle and one achieves an irregular microcapsule. The last well known design for a microcapsule is that of a multiple wall. In this case, the multiple walls are placed around a core to achieve multiple purposes related to the manufacture of the capsules, their subsequent storage and controlled release. It is also possible to design other microcapsules that have multiple cores where the multiple cores may actually be an agglomerate of several different types of microcapsules.

Microcapsules can be classified into three basic categories according to their morphology as mono-cored, poly-cored, and matrix types, as shown in Figure 3.2. Morphological control is important and much effort has been given in to controlling internal structures, which largely depend on the protocol and the microencapsulation methods employed.

All three states of matter (solids, liquids, and gases) are possible to be microencapsulated. This allows liquid and gas phase materials to be handled more easily as solids, and can afford some measure of protection to those handling hazardous materials.

The most significant feature of microcapsules is their microscopic size that allows for a huge surface or interface area. Through selection of the composition materials (core material and membrane), we can endow microcapsules with a variety of functions. Generally, membrane materials are chosen to assert the effects of microencapsulation. Therefore, not only synthetic and natural polymers but also lipids and inorganic materials are used for the preparation of microcapsules.

One should also keep in mind that the whole process of microencapsulation actually covers three separate processes on a time scale. The first process consists of forming a wall around the core material. The second process involves keeping

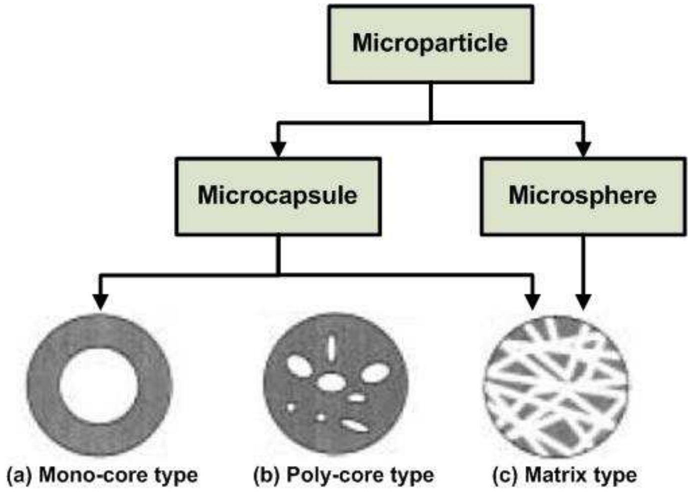


Figure 3.2: *Classification of Microparticles from their Morphology*

the core inside the wall material so that it is not released. Also, the wall material must prevent undesirable materials that may harm the core from entering. And finally, it is necessary to let the core material out starting at the right time and at the right rate.

3.3 Release Mechanisms

One important feature of a microcapsule is that it preserves the core materials, i.e., it protects the core material from being contaminated, impaired, or altered until the content is to be taken out and used. It isolates the core such that the content cannot react with other materials. Another important feature is that the core material can be subsequently released, usually either by breakage of the shell material under pressure or heat, or by slow diffusion of the core material through the shell wall. A shell material breakage releases all of the material at once; this method is used for pressure-sensitive copy paper, adhesives, and perfume printing. Release by diffusion through the shell wall makes it possible to control the speed at which the core material is released; this method is used for agricultural chemicals, medicines, and aromatics.

The four methods of releasing the inner core material are as follows:

1. physical/mechanical rupture of the capsule wall (outer layer is broken) – e.g. scratch-and-sniff stickers, carbonless paper

2. dissolve outer layer – e.g. detergents

3. melt outer layer – e.g. some baking mixes

4. diffusion through outer layer (tiny amounts come out through the layer over time) – e.g. aspirin

3.3.1 Release Rates

The release rates that are attainable from a single microcapsule are generally zero, half or first order. Zero order occurs when the core is a pure material and is released through the wall of a reservoir microcapsule as a pure material. Half order release generally occurs with matrix particles. First order release occurs when the core material is actually a solution. As the solute material is released from the capsule the concentration of solute material in the solvent decreases and a first order release is achieved. A mixture of microcapsules will include a distribution of capsules varying in size and wall thickness. The effect, therefore, is to produce a release rate different from zero, half or first order because of the ensemble of microcapsules. It is, therefore, very desirable to carefully examine on an experimental basis the release rate from an ensemble of microcapsules and to recognize that the deviation from theory is due to the distribution in size and wall thickness.

3.4 Reasons for Microencapsulation

The reasons for microencapsulation are countless. In some cases, the core must be isolated from its surroundings, as in isolating vitamins from the deteriorating effects of oxygen, retarding evaporation of a volatile core, improving the handling properties of a sticky material, or isolating a reactive core from chemical attack. In other cases, the objective is not to isolate the core completely but to control the rate at which it leaves the microcapsule, as in the controlled release of drugs or pesticides. The problem may be as simple as masking the taste or odor of the core, or as complex as increasing the selectivity of an adsorption or extraction process.

Microcapsules have a number of interesting advantages and the main reasons for microencapsulation can be exemplified as

1. controlled release of encapsulating drugs,

2. protection of the encapsulated materials against oxidation or deactivation due to reaction in the environment,

3. masking of odor and/or taste of encapsulating materials,

4. isolation of encapsulating materials from undesirable phenomena, and

5. easy handling as powder-like materials.

3.5 Applications of Microencapsulation

The applications of microencapsulation are numerous. Microencapsulated materials are utilized in agriculture, pharmaceuticals, foods, cosmetics and fragrances, textiles, paper, paints, coatings and adhesives, printing applications, and many other industries.

The concept of encapsulation has been applied by many industries through the years. Mechanical encapsulation techniques dates back to the late 1800s. The pharmaceutical industry, in particular, has used this technology to develop large gelatin capsules which constitute a distinctive dosage form for drugs. The coating of pills or solid drug granulates have been used by the pharmaceutical industry.

In the following years, there has been an increasing need for smaller and smaller size of capsules, and to improve the protection and containment of liquids. The first industrial product employing microencapsulation was carbonless copy paper developed by Green and Schleicher in the 1950s. The microcapsules used in it were prepared by complex coacervation of gelatin and gum arabic (Green & Schleicher, 1957). A coating of microencapsulated colorless ink is applied to the top sheet of paper, and a developer is applied to the subsequent sheet. When pressure is applied by writing, the capsules break and the ink reacts with the developer to produce the dark color of the copy.

To this day, carbonless copy paper is one of the most significant products utilizing microencapsulation technology, and is still produced commercially. The technology developed for carbonless copy paper have led to the development of various microcapsule products in recent years.

Recently, the microencapsulation process has been adopted to a number of fields of advanced technology like an electronic paper. When an electric field is applied between microcapsules, the microparticles move in the low dielectric constant solution toward the oppositely charged electrode in the phenomenon of electrophoretic migration. If the bottom electrode is positively charged, black microparticles with negative charges should move toward the bottom electrode. At the same time, an opposite electric field pulls the white particles to the top of the microcapsules, making the surface appear white at that spot. By reversing this process, the white microparticles move toward the bottom of the microcapsules and the black particles appear at the top of the microcapsules, which makes the surface become black at that spot. This is how microencapsulated ink (E-ink) forms letters and pictures on the display. Figure 3.3 shows this process.

Food Industry The application of microencapsulation in food industry are:

- Liquid delivery by coatings with pre-designed release mechanisms;
- The retention of volatile compounds for release under desired conditions;
- Protection against the effects of evaporation and moisture, oxygen and ultraviolet light;

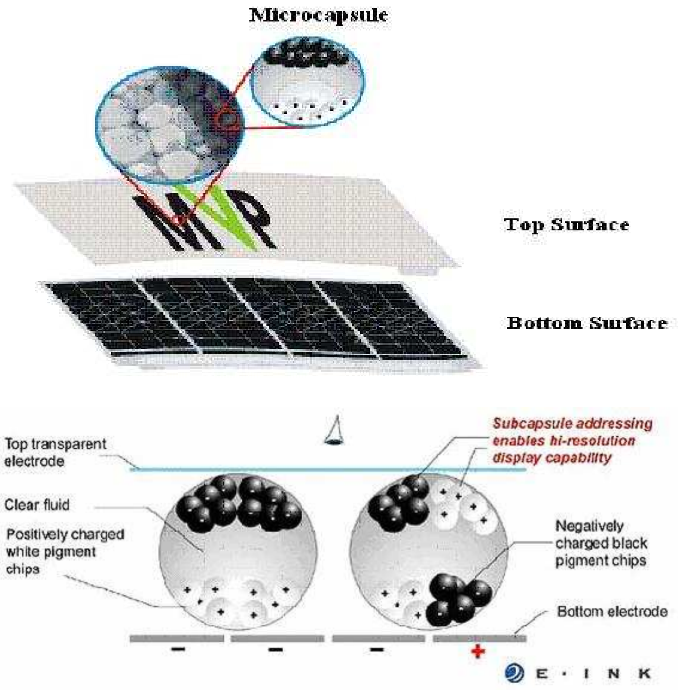


Figure 3.3: *Microencapsulation Method for Display Technology*

- Mixing of normally incompatible ingredients;
- Taste and odor masking - usually by encapsulation in coatings that resist release in the mouth, but allow release in the digestive system;
- Use of coatings to change the texture or density of solid materials;
- Special effects with unusual release systems.

Textile Industry Today's textile industry makes use of microencapsulated materials to enhance the properties of finished goods. One application that has been increasingly utilized is the incorporation of microencapsulated phase change materials (PCMs). Phase change materials absorb and release heat in response to changes in environmental temperatures. When temperatures rise, the phase change material melts, absorbing excess heat, and feels cool. Conversely, as temperatures fall, the PCM releases heat as it solidifies, and feels warm. This property of microencapsulated phase change materials can be harnessed to increase the comfort level for users of sports equipment, military gear, bedding, clothing, building materials, and many other consumer products. Microencapsulated PCMs have even been used in NASA-patented thermal protection systems for spacecraft.

Agriculture Industry Agricultural chemicals have been used to fertilise land and to protect plants from insects. The controlled release of the substance which can limit the replication is achievable by implementing microencapsulation.

Pesticides are encapsulated to be released over time, allowing farmers to apply the pesticides less often resulting in the decrease usage of very highly concentrated and perhaps toxic initial applications followed by repeated applications to combat the loss of efficacy due to leaching, evaporation, and degradation. Protecting the pesticides from full exposure to the elements lessens the risk to the environment and those that might be exposed to the chemicals and provides a more efficient strategy to pest control.

Pharmaceutical Industry The application of controlled release in pharmaceutical industry, for example aspirin, can be achieved by the microencapsulation method (i.e. phase separation and coacervation). It is an example where drugs can be encapsulated to improve their product performance by taste or color masking to prevent oxidation, and enhancing other product characteristics.

3.6 Microencapsulation Criteria

The design of a microencapsulation system must take into account the total system (Finch & Bodmeier, 2002): the active and carrier materials, the mechanism of release, and the ultimate fate of all the ingredients. Each of these parameters must be optimized if a satisfactory product is to be obtained. Many methods exist for the production of microparticles which allow many variations, depending on core and wall-polymer solubility, capsule size, wall thickness and permeability, type and rate of release of core contents, and physical properties.

In choosing processes for the production of microcapsules for particular applications, several physical properties must be considered (Finch & Bodmeier, 2002), including:

1. *Core Wettability.* The key property is the ability of the core to be wetted by the wall material. Theoretically it is possible to forecast this property, but in practice, it is usually determined during the microencapsulation process.

2. *Core Solubility.* The contents of the microcapsule core should not be soluble in the solvent for the wall polymer, and the polymer should not dissolve significantly in the liquid core. A water-soluble solid can be coated with a water-soluble polymer solution, for example with spray coating, since the water should evaporate quickly during the microcapsule formation.

3. *Wall elasticity* is determined by the nature of the wall polymer, the thickness of the wall, and the size of the microcapsules formed.

4. *Wall permeability* determines whether the microcapsule core contents can be retained indefinitely until ruptured (impermeable wall), or may be released at a predetermined rate as in controlled-release applications (permeable wall).

5. *Wall Polymer Adhesive characteristics* are considerably affected by solution temperature and concentration. They also depend on the physical properties of the wall polymer, especially the melting point and glass transition temperatures, the degree of crystallinity, and the degradation rate (under microcapsule formation conditions). Stickiness during capsule formation, and "stringiness" during Spray Drying can be major problems affecting microcapsule manufacture and storage behavior.

Many factors affect the size and quality of microcapsules. Some of these factors influence the performance of controlled release of active components. Factor affecting the quality of microcapsules during production are (Finch & Bodmeier, 2002):

- Choice of solvent
- Mixtures of solvents used
- Aqueous solubility of active agent
- Rate of solvent removal
- Drying conditions
- Type and molecular mass of carrier
- Crystallinity of polymer

Factors affecting the size of microcapsules during production are (Finch & Bodmeier, 2002):

- Stirring rate
- Solids content of organic phase
- Viscosity of organic phase
- Viscosity of aqueous phase
- Concentration and type of surfactant (if any)
- Configuration of vessel and stirrer

- Quantity of organic and aqueous phases
- Temperature profile during production
- Nozzle design

3.7 Selection of Microencapsulation Methods

Many different methods have been proposed to produce microcapsules (several hundred methods/modifications have been identified in the patent literature) with many detailed variations. They depend on core and wall-polymer solubility, particle size, wall thickness and wall permeability, type and rate of release of core contents required, physical properties, and also the cost of manufacture. The choice of a particular preparation method and a suitable polymer will depend on the physicochemical properties of the active substance, the desired release characteristics, the therapeutic goal for drug substances, the route of administration, the biodegradability/biocompatibility of the carrier material and the regulatory considerations. From a technological point of view, the successful selection of a preparation method will be determined by the ability to achieve high loadings with the active substance, high encapsulation efficiencies, high product yields, and the potential for easy scale-up. For example, methods with high encapsulation efficiencies but with only low active loading capacity are limited to very potent active substances.

The various microencapsulation processes allow product formulators to create capsules ranging from less than a micrometer to several thousand micrometers in size. Each process offers specific attributes, such as high production rates, large production volume, high product yield, and different capital and operating costs. Other process variables include greater flexibility in shell material selection and differences in microcapsule morphology, particle size, and distribution.

Microencapsulation processes are divided into physical, physicochemical and mechanical systems (Kondo, 1978), or are classified as physical/mechanical and chemical processes (Thies, 1996). Physical methods use commercially available equipment to create and stabilize the capsules. Chemical techniques apply ionic chemistry to create the microspheres in batch reactors.

A) Physical Methods

Spray-Drying Spray drying serves as a microencapsulation technique when an active material is dissolved or suspended in a melt or polymer solution and becomes trapped in the dried particle. The main advantages is the ability to handle labile materials because of the short contact time in the dryer, and additionally the operation is economical.

Pan Coating The Pan Coating process, widely used in the pharmaceutical industry, is among the oldest industrial procedures for forming small, coated particles or tablets. The particles are tumbled in a pan or other device while the coating material is applied slowly.

Air-Suspension Coating Air-suspension coating of particles by solutions or melts gives better control and flexibility. The particles are coated while suspended in an upward-moving air stream. They are supported by a perforated plate having different patterns of holes inside and outside a cylindrical insert. Just sufficient air is permitted to rise through the outer annular space to fluidize the settling particles. Most of the rising air (usually heated) flows inside the cylinder, causing the particles to rise rapidly. At the top, as the air stream diverges and slows, they settle back onto the outer bed and move downward to repeat the cycle. The particles pass through the inner cylinder many times in a few minutes.

B) Chemical Methods

Coacervation Coacervation is the process of separating a solution of colloid into a new phase. Coacervation is also known as phase separation. The original one phase system becomes a two phase system. One is rich in its colloid concentration and the other is poor in its colloid concentration. The colloid rich phase in a disperse state appears as coacervate droplets. These coacervate droplets will coalesce with the other coacervate droplets into a bigger colloid-rich liquid layer, known as the coacervate layer which can be deposited to produce the wall material of the microcapsule.

In Figure 3.4 the first system is two phase system, i.e. oil phase and polymer solution phase (a gelatine-gum arabic solution). After adding acid or precipitant three phase system is formed, i.e. the oil phase, the polymer rich phase which consists of, e.g. 15% polymer solution and 85% water and the polymer phase which consists of almost all element of it is water, e.g. $\sim$ 100% water.

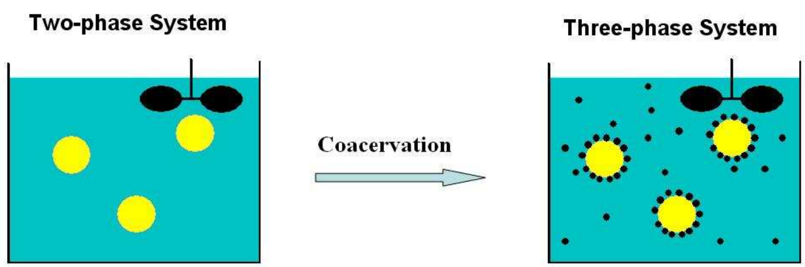


Figure 3.4: *Coacervation Process*

Coacervation is triggered by the addition of precipitant or by the addition of acid which cause a change in pH. As the coacervation continues the fine coacervate droplets coalesce into larger one, until a coherent coacervate phase is formed containing practically all of the polymer.

When only one material is present in the polymer solution, this process is referred to simple coacervation process and when two or more materials of opposite charge are present it is referred to complex coacervation process.

Interfacial Polymerization In Interfacial Polymerization, the two reactants in a polycondensation meet at an interface and react rapidly. Under the right conditions, thin flexible walls form rapidly at the interface. Condensed polymer walls form instantaneously at the interface of the emulsion droplets.

In-Situ Polymerization In a some microencapsulation processes, the direct polymerization of a single monomer is carried out on the particle surface, e.g. Cellulose fibers are encapsulated in polyethylene while immersed in dry toluene.

3.8 Summary

Since the objective of the study in this thesis is to develop a decision support expert system for the engineering selection problems, specializing in the domain of microencapsulation process selection, in this chapter we have presented the domain we have worked in, i.e. the microencapsulation domain. First we discussed the definition of microencapsulation. Then, we introduced the morphology, the release mechanisms, and the reason of microencapsulation. Lastly, we presented the criteria and the methods of the microencapsulation that we will further used later on the case study.

Chapter 4

Microencapsulation Process Selection using the AHP

Decision making problems in the field of engineering are usually associated with multiple, and potentially conflicting requirements. The rapid growth of development in material and manufacturing technology has brought many exciting changes in this field, thus enabling the engineers to benefit from wider selection of materials and processing techniques. The Analytical Hierarchy Process (AHP), one of the most popular Multiple Attribute Decision Making (MADM) methods, has been used intensively by many researchers in the academics and industry to aid the selection problems. This chapter examines the use of AHP to solve the engineering selection problem. We present here an AHP-based decision support system to select the most suitable engineering process. In this chapter, first we discuss about engineering problem solving method, followed by an overview of the AHP. Uncertainties are usually present in any realistic decision situation, especially in engineering decision making. Sensitivity analysis is a commonly used method for checking the robustness of the ranking to small changes in the input values. Here we present a methodology for performing a sensitivity analysis on the weights of decision criteria and the preference values of the alternatives with respect to each decision criterion. Afterward we apply the application of the AHP method to the engineering process selection problem, i.e. to the domain of chemical engineering for the selection of the appropriate microencapsulation method. Then, the ranking results of the different alternatives are given along with sensitivity analysis studies of the effects of different assumptions on the obtained rankings. Finally, we will summarize this chapter.

4.1 Engineering Problem Solving

Engineering problems are complex and include many and different kinds of concerns. It may include various needs and require different kinds of solution domain knowledge, various goals, different abstractions, etc. The major difficulty in solving complex problems is caused by the ill-posed, large-scale, and fuzzy structure of real life problems, whereas a simple problem is a well-defined problem that can be handled by some known engineering theory.

For large and complex problems it is just practically impossible to cope with all these concerns at once and by only one engineer. This means that the problem cannot be solved in one step. One technique for coping with this complexity is the decomposition of a problem into sub-problems. Engineering disciplines apply this technique and decompose the overall engineering process into some manageable engineering processes.

The strategy of solving complex problems by decomposition into partial problems is called the "Divide and Conquer" (DAC) approach. The principle of DAC suggests that: (1) complex decision problems should be decomposed into smaller, more manageable parts and (2) these smaller parts should be logically aggregated to derive an overall value for each alternative. The Analytical Hierarchy Process (Saaty, 1980) is a multiple attribute decision making (MADM) method, which uses the DAC principle.

The AHP is a powerful and flexible MADM method for complex problems, and has been used in many management and industrial applications (Hwang & Yoon, 1981; Zahedi, 1986; Golden et al., 1989; Vargas, 1990; Saaty, 2001; Vaidya & Kumar, 2006). The AHP combines qualitative and quantitative aspects of complex problems by means of a hierarchical structure. The principle of AHP suggests as follows: (1) break down a complex and unstructured problem into its component parts, (2) use facts and judgments of decision makers to relate and prioritize the components, and (3) synthesize the results. The AHP approach will be described in more detail in the following section.

4.2 The Analytical Hierarchy Process

The Analytical Hierarchy Process (AHP) (Saaty, 1980) which was first developed by Thomas L. Saaty in the 1970s is a widely used Multiple Attribute Decision Method (MADM) for complex problems. MADM problems are dealing with the priority of alternatives with respect to many attributes. The AHP is widely used in various industrial applications (Zahedi, 1986; Vaidya & Kumar, 2006) in general, in engineering selection problems in particular, e.g., selection of diagnostic techniques and instrumentation in a predictive maintenance program (Carnero, 2005), selection of welding process (Ravisankar et al., 2006), selection of computer-integrated manufacturing technologies (Luong, 1998), selection of laboratory reactor (Hanratty & Joseph, 1992) selection of casting process (Tiwari & Banerjee, 2001), selection of architectural consultants (Cheung & Kuen, 2002), selection of machine tools (Yurdakul, 2004), etc.

AHP is based on three basic principles: decomposition, comparative judgments, and hierarchic composition or synthesis of priorities. The decomposition principle is applied by structuring a complex problem into a hierarchy of clusters, sub-clusters, sub-sub clusters and so on. The principle of comparative judgments is applied by constructing pairwise comparisons of all combinations of elements in a cluster with respect to the parent of the cluster. These pairwise comparisons are then used to derive 'local' priorities of the elements in a cluster with respect to their parent. The principle of hierarchic composition or synthesis is applied by multiplying the local priorities of elements in a cluster by the priority of the parent element, producing 'global' priorities throughout the hierarchy and then adding up the global priorities for the lowest level elements (the alternatives).

In practice after the global priority is achieved, sensitivity analysis should be carried out to determine the robustness of such decisions with respect to variations in the pairwise rankings and changes of the judgments.

In the following subsections, we discuss the principle of AHP in more detail.

4.2.1 Decomposition

The AHP transforms a complex, multiple criteria problem into a hierarchical structure. The number of levels in the structure depends upon the complexity of the problem and the degree of detail in the problem. The main objective or the goal of the problem is represented at the top level of the hierarchy. Then, each level of the hierarchy contains criteria or sub-criteria that influence the decision. The bottom level of the structure contains the alternatives.

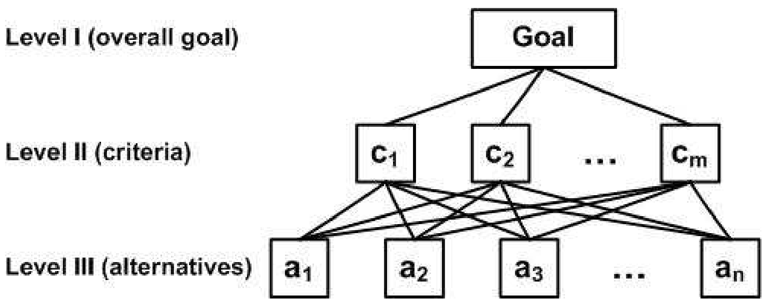


Figure 4.1: *A Hierarchy with Single Criteria Layer Structure*

Figure 4.1 shows a three layer hierarchical structure with n alternatives (a_i, $i = 1, \ldots, n$) and m criteria (c_j, $j = 1, \ldots, m$). In the first (or top) layer is the overall goal of the decision problem. In the second (or middle) layer is the criteria layer. In the last (or bottom) layer is the alternative layer.

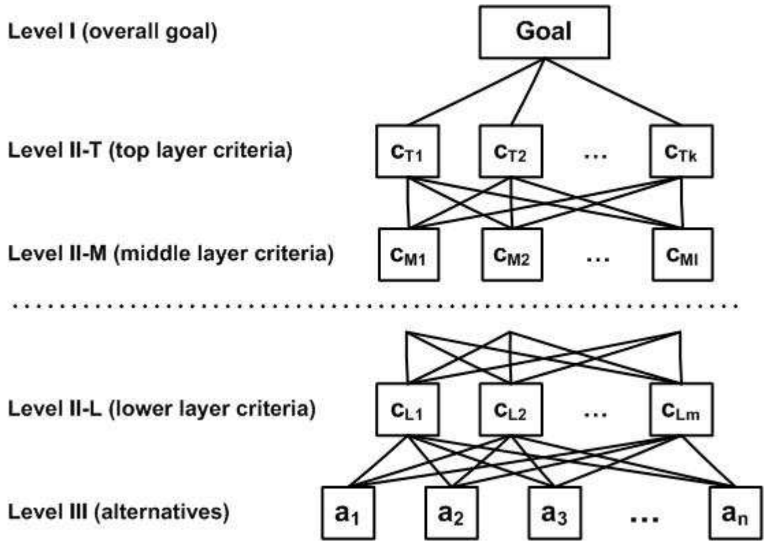


Figure 4.2: *A Hierarchy with Multi Criteria Layer Structure*

Figure 4.2 shows a more general multi level hierarchical structure. In this case, we can have multi criteria layer depend on the complexity of the decision problem.

4.2.2 Comparative Judgments

Once a hierarchical representation of the problem has been achieved, we would like to establish priorities of the criteria and evaluate each of the alternatives with respect to the corresponding criterion. Pairwise comparisons are used to determine the relative importance of each criterion and each alternative in terms of each criterion.

Without loss of generality, we can formalize the problem by considering the pairwise comparison of n elements $A_1, A_2, \ldots, A_n$ at a given level of hierarchy. The decision maker semantically compares any two elements A_i and A_j and indirectly (verbally) or directly (numerically) assigns the value a_{ij} that represents his/her judgment of the relative importance of decision element A_i over A_j by using a scale. If the elements A_i and A_j are of the same importance for the decision maker, then $a_{ij} = 1$, and if A_i is preferred to A_j, then $a_{ij} > 1$. The reciprocal property $a_{ji} = 1/a_{ij}$ by assumption always holds, and $a_{ii} = 1$ for all $i = 1, 2, \ldots, n$. This way a positive reciprocal matrix of pairwise comparisons $A = \{a_{ij}\}$ is constructed having a dimension $n \times n$. The elements of the main diagonal are all equal to 1, and symmetrical elements are mutually reciprocal. This means that only $n(n-1)/2$ judgments are required to construct the matrix.

Table 4.1: *Fundamental Scale used in Pairwise Comparison*

Intensity of Importance	Qualitative Definition	Explanation
1	Equal importance	Two activities contribute equally to the objective
2	Weak	
3	Moderate importance	Experience and judgments slightly favor one activity over another
4	Moderate plus	
5	Strong importance	Experience and judgment strongly favor one activity over another
6	Strong plus	
7	Very strong or demonstrated importance	An activity is favored very strongly over another and dominance is demonstrated in practice
8	Very, very strong	
9	Extreme importance	The evidence favoring one activity over another is of the highest possible order of affirmation

Let $A_1, A_2, \ldots, A_n$ be any set of elements and $w_1, w_2, \ldots, w_n$ their corresponding weights. We want to compare the corresponding weights of each elements with the weights of all the other elements in the set with respect to a goal or property that they have in common. The comparison weights can be formulated as matrix A in equation (4.1).

$$A = \{a_{ij}\}_{n\times n} = \begin{bmatrix} 1 & a_{12} & \cdots & a_{1n} \\ a_{21} & 1 & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & 1 \end{bmatrix} = \begin{bmatrix} 1 & \frac{w_1}{w_2} & \cdots & \frac{w_1}{w_n} \\ \frac{w_2}{w_1} & 1 & \cdots & \frac{w_2}{w_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{w_n}{w_1} & \frac{w_n}{w_2} & \cdots & 1 \end{bmatrix} \tag{4.1}$$

where a_{ij} represents the relative importance of A_i over A_j with respect to a goal or property that they have in common, $a_{ji} = 1/a_{ij}$ for all $i, j = 1, \ldots, n$; due to symmetry comparisons.

Pairwise comparisons are quantified by using a scale. The scale is an one-to-one-mapping between the set of discrete linguistic choices available to the decision maker and a discrete set of numbers which represent the importance or weight of the linguistic choices. There were several approaches in developing such scales (Triantaphyllou, 2000), e.g. the linear, logarithmic, and exponential scales. These approaches are based on some psychological theories and the researchers developed these numbers to be used based on these psychological theories.

In this thesis, we use nine-point Saaty's scale, and the scale automatically transforms the decision maker's judgments into numbers. To fill the matrix A, Saaty (1980) proposed the use of a nine-point scale to express the decision maker's preference and intensity of that preference for one element over the other. Table 4.1 contains the recommended scale from $1 - 9$, which is used to assign a judgment in comparing pairs of elements on each level of hierarchy against their parent in the next higher level.

4.2.3 Synthesis of Priorities

After we have the positive reciprocal matrices from each level of the hierarchy, the next step is to estimate the relative weights of the decision elements by using prioritization techniques. The estimation of priorities from pairwise comparison matrices is the central part of the AHP. By deriving priority vectors for all matrices in the hierarchy created for given decision problem, it is possible to perform aggregation and obtain the final composite vector of priorities for alternatives at the bottom level of the hierarchy. There are different techniques to extract priorities vectors from the comparison matrices and significant effort of researchers has been directed to find the best estimation method. The methods for deriving priorities from comparison matrices are as follows: additive normalization, eigenvector, geometric mean, least-squares, weighted least-squares, and logarithmic least-squares method.

The eigenvector method (EM) was first proposed by Saaty (1980) who proved that the principal eigenvector of the comparison matrix can be used as a required priority vector, both for consistent and inconsistent judgments of the decision maker. Saaty also suggested several approximate methods to obtain required vector. The simplest one may be referred to additive normalization method (ANM). It derives priorities by taking the sums of columns in a comparison matrix and then averaging obtained values in rows. The better way to approximate the eigenvector is the geometric mean method (GMM). The other methods for deriving priorities from comparison matrices such as least-squares, weighted least-squares, and logarithmic least-squares method are based on some optimization approach. The problem of priority derivation is stated as: minimize the given objective function that measures the deviations between an "ideal" and the actual solution, subject to some additional constraints.

When the decision maker makes judgments, either he/she will be consistent or inconsistent. If he/she is perfectly consistent, then all elements a_{ij} have exact values $a_{ij} = w_i/w_j$ and transitive condition $a_{ij} = a_{ik}a_{kj}$ is satisfied for all $i, j, k = 1, 2, \ldots, n$, thus the comparison matrix is said to be consistent and can be represented as $A_c = \{w_i/w_j\}$. The relative priorities of compared elements are unique and can be calculated by taking the average of the elements in any column of the matrix, and then dividing each of them by the sum of all elements of the column. However, the decision maker's evaluations a_{ij} are rarely perfect and the transitive rule is rather frequently violated. In this case, comparison matrix is said to be inconsistent which can be represented as $A_{ic} \approx \{w_i/w_j\}$. The elements of this matrix are only estimates ($a_{ij} \approx w_i/w_j$). Furthermore, the inconsistent priorities are not unique, a prioritization method should be used for their estimation.

Additive Normalization Method (ANM)

To obtain the priority vector w by this method it is enough to divide the elements of each column of matrix A by the sum of that column (i.e. normalize the column), then add the elements in each resulting row and finally divide this sum by the number of elements in the row. This procedure is described by relations Eq. (4.2) and Eq. (4.3).

$$a'_{ij} = \frac{a_{ij}}{\sum\limits_{i=1}^{n} a_{ij}}, \quad i,j = 1,2,\ldots,n, \tag{4.2}$$

$$w_i = \left(\frac{1}{n}\right)\sum_{j=1}^{n} a'_{ij}, \quad i = 1,2,\ldots,n. \tag{4.3}$$

Geometric Mean Method (GMM)

According to Saaty the eigenvector can be generated in different ways, but the geometric mean method (GMM) is the best way which is calculated as follows:

1. Multiply out each row in the matrix.
2. Take the n-th root of the multiplication, since there are n entries in each row.
3. Normalize those roots by deriving the total and dividing them by the total.

This procedure is described as the following equations:

$$w'_i = \left(\prod_{j=1}^{n} a_{ij}\right)^{1/n}, \quad i = 1,2,\ldots,n, \tag{4.4}$$

$$w_i = \frac{w'_i}{\sum\limits_{i=1}^{n} w'_i}, \quad i = 1,2,\ldots,n. \tag{4.5}$$

Least Squares Method (LSM)

Least Squares Method (LSM) is an optimization problem, i.e. minimization problem, of the $\mathcal{L}^2$ norm of $(A - w\frac{1}{w})^T$, where $\frac{1}{w}^T$ denotes the row vector $(\frac{1}{w_1}, \frac{1}{w_2}, \ldots, \frac{1}{w_n})$.

$$\begin{aligned} &\min \sum_{i=1}^{n}\sum_{j=1}^{n}\left(a_{ij} - \frac{w_i}{w_j}\right)^2 \\ &\text{subject to} \quad \sum_{i=1}^{n} w_i = 1, \quad w_i > 0, \quad i = 1,2,\ldots,n. \end{aligned} \tag{4.6}$$

Weighted Least Squares Method (WLSM)

Chu et al. (1979) proposed this method as a modification of the least-square method (LSM). WLSM minimizes $\mathcal{L}^2$ distance function defined for elements of the unknown priority vector w and known judgment ratios $a_{ij} = w_i/w_j$ by solving the following constrained non-linear optimization problem:

$$\begin{aligned} &\min \sum_{i=1}^{n} \sum_{j=1}^{n} (w_i - a_{ij} w_j)^2 \\ &\text{subject to} \quad \sum_{i=1}^{n} w_i = 1, \quad w_i > 0, \quad i = 1, 2, \ldots, n. \end{aligned} \tag{4.7}$$

Logarithmic Least Squares Method (LLSM)

The LLSM also uses $\mathcal{L}^2$ metric in defining objective function of the following optimization problem:

$$\begin{aligned} &\min \sum_{i=1}^{n} \sum_{j>i}^{n} [\ln a_{ij} - (\ln w_i - \ln w_j)]^2 \\ &\text{subject to} \quad \prod_{i=1}^{n} w_i = 1, \quad w_i > 0, \quad i = 1, 2, \ldots, n. \end{aligned} \tag{4.8}$$

Crawford & Williams (1985) have shown that the solution for problem above is unique and can be found simply as the geometric means of the rows of matrix A:

$$w_i = \prod_{j=1}^{n} a_{ij}^{1/n}, \quad i = 1, 2, \ldots, n. \tag{4.9}$$

Eigenvector Method (EM)

The objective is to find eigenvalues w, for each w_i:

$$w = (w_1, w_2, \ldots, w_n)$$

where w is eigenvector and a column matrix.

This is the original method suggested by Saaty (1980). He proposed the principal eigenvector of A as the desired priority vector w. To find this vector the linear system:

$$Aw = \lambda w, \quad e^T w = 1$$

should be solved where λ is the principal eigenvalue of matrix A. If the decision maker is consistent, then $\lambda = n$; otherwise $\lambda > n$.

It was shown by various researchers that for small deviations around the consistent ratios w_i/w_j, EM method gives reasonably good approximation of the priorities vector. However, when the inconsistencies are large, it is generally accepted that solutions are not so satisfactory.

Let $R_n^+ = \left\{ w = (w_1, w_2, \ldots, w_n)^T \,|w_i > 0, i = 1, 2, \ldots, n \right\}$.

Lemma 4.2.1 (Perron). *Let $A = (a_{ij})$ be an $n \times n$ positive matrix and $\lambda_{\max}$ be the maximal eigenvalue of A. Then, we have*

$$\lambda_{\max} = \min_{x \in R_n^+} \max_i \sum_{j=1}^{n} a_{ij} \frac{w_j}{w_i}.$$

Let A and $\lambda_{\max}$ be as in Lemma 4.2.1. The positive right eigenvector corresponding to $\lambda_{\max}$ is called the principal right eigenvector of A.

Lemma 4.2.2. *(Saaty, 1980) Let A be an $n \times n$ positive reciprocal matrix and $\lambda_{\max}$ be the maximal eigenvalue of A. Then, $\lambda_{\max} \geq n$ and equality holds if and only if A is consistent.*

Definition 4.2.3. *The judgment matrix $A = (a_{ij})_{n \times n}$ is called a consistent matrix if*

$$a_{ij} a_{ik} = a_{kj}, \quad i, j, k \in \Omega$$

where $\Omega = \{1, 2, \ldots, n\}$. By the property of consistent matrices, we have $a_{ij} = w_i/w_j$, $i, j \in \Omega$.

However, we seldom have consistent matrices because the decision maker's judgments a_{ij} are rarely perfect. The consistency index which was suggested by Saaty to verify the consistency of the comparison matrix is defined as follows:

$$\text{Consistency Index } (CI) = \frac{\lambda_{\max} - n}{n - 1} \tag{4.10}$$

This is a measure to assess the difference of the pairwise comparisons consistency with respect to the perfect consistency. The numerator defines the deviation of the maximum eigenvalue ($\lambda_{\max}$) from perfect consistency, which is n. The denominator is needed to compute an average deviation of each pairwise comparison from perfectly consistent judgment. A value of one is subtracted from the order of matrix n, because one of the pairwise comparisons is a self-comparison, and there should be no inconsistency involved in self-comparison.

The consistency check of pairwise comparison is done by comparing the computed consistency index with the average consistency index of randomly generated reciprocal matrices using the nine-point scale. Such a consistency index is called the random index (RI). Table 4.2 shows the random indices for matrices of order 1 through 10.

Table 4.2: *Average Random Consistency Index (RI)*

Size of matrix	1	2	3	4	5	6	7	8	9	10
Random consistency	0	0	0.58	0.9	1.12	1.24	1.32	1.41	1.45	1.49

AHP measures the overall consistency of judgments by means of a consistency ratio (CR). The consistency ratio is obtained by dividing the computed consistency index by the random index as follows:

$$CR = \frac{CI}{RI} \tag{4.11}$$

If the matrix is consistent (as it defined in Lemma 4.2.2), then $\lambda_{\max} = n$, so $CI = 0$ and $CR = 0$ as well. On the other hand, if the comparison are carried out randomly, the expected value of CR is 1. Saaty stated that a consistency ratio of 0.10 or less can be considered acceptable, otherwise the judgment should be improved. This improvement can be done by double-checking the data entry and by omitting bad judgments that have high inconsistency ratios.

Suppose that we have a decision problem with n alternatives (a_i, $i = 1, \dots, n$) and m criteria (c_j, $j = 1, \dots, m$) with the structure shown in Figure 4.1. Suppose that we have all the weights of criteria and all the performances of the alternatives with respect to each criterion. Let $w_{C_1}, w_{C_2}, \dots, w_{C_m}$ denotes the weight of the criteria and w_{ij} ($i = 1, 2, \dots, n, j = 1, 2, \dots, m$) is the performance of the i-th alternative with respect to j-th criterion. The weight of the i-th alternative can be obtained as a weighted sum of performances:

$$w_i = \sum_{j=1}^{m} w_{C_j} w_{ij}\ , \quad i = 1, 2, \dots, n. \tag{4.12}$$

4.3 Sensitivity Analysis

The relative weights of criteria and alternatives in the AHP process are determined by the decision maker, so some degree of uncertainty or subjectivity exists. Also, often the data in MCDM problems are imprecise and changeable. Therefore, to explore the response of model solutions (i.e. the solution robustness) to potential shifts in the overall priority of the alternatives, sensitivity analysis need to be performed. The functions of the sensitivity analysis are as follows: (1) to provide insight into how the overall scores were generated, and (2) to identify how greater emphasis on different criteria/alternatives would influence the results.

Many scientist agree on the importance of a sensitivity analysis of a trade-off study in order to show how the results respond to the changes in the importance of the criteria and the preferences of the alternatives. Triantaphyllou & Sánchez (1997) proposed a methodology for performing sensitivity analysis on the weight of the decision criteria and the performance values of the alternatives in terms of the criteria for deterministic multiple criteria decision making methods. Their methodology can be used to determine the criteria in the model that requires only a small change in weight to cause a switch in rank of one or more alternatives. In the Triantaphyllou & Sánchez (1997) methodology, in order to perform a sensitivity analysis for any multi criteria decision making problem, the n alternatives must be arranged such that the following relation is always satisfied:

$$P_1 \geq P_2 \geq P_3 \geq \ldots \geq P_n \tag{4.13}$$

where P_i is the preference value for the i-th alternative. In the case of an AHP-based selection problem, the first alternative (A_1) is ranked first, the second alternative (A_2) is ranked second and so on.

In this section, we propose a methodology for the more general problems, so that the constraint in Eq. (4.13) need not to be satisfied. That is, the best alternative can be any alternative not necessary always the first alternative. Beside extending the methodology presented by Triantaphyllou & Sánchez (1997), we also implemented our methodology to the decision support system framework so the user can have the visualization of the sensitivity analysis of the changes in the preference of the criteria and also in the preference of the alternatives. Commercial software package which performs AHP such as Expert Choice only performs a type of elementary sensitivity analysis. The user only has the option to graphically alter the weights of the decision criteria and the software does not offer any means to study the effects of changes on the preference of the alternatives.

Let us consider a decision problem with m criteria and n alternatives. Criteria are denoted as $C_y(y = 1, 2, \ldots, m)$ and alternatives as $A_x(x = 1, 2, \ldots, n)$. Assume that for each criterion C_y the decision maker has determined its preference, or weight, w_y. It is also assumed that the decision maker has determined the preference of alternative A_x with respect to criterion C_y, $a_{xy}(x = 1, 2, \ldots, n;\ y = 1, 2, \ldots, m)$. The weight vector $(w_1, w_2, \ldots, w_m)$ is derived from $m \times m$ reciprocal matrix which determined by pairwise comparisons of the criteria. In a similar way, the vector $(a_{1y}, a_{2y}, \ldots, a_{ny})$, $y = 1, 2, \ldots, m$ is derived from $n \times n$ positive reciprocal matrix which is determined by pairwise comparisons of the preference of the n alternatives on the y-th criterion. Without loss of generality, assume that these weight vector and preference vectors are normalized:

$$\sum_{j=1}^{m} w_j = 1 \tag{4.14}$$

$$\sum_{i=1}^{n} a_{ij} = 1 \quad j = 1, 2, \ldots, m \tag{4.15}$$

Let us consider $P_x(x = 1, 2, \ldots, n)$ represent the final preference of alternatives A_x, which is calculated according to the weighted sum principle:

$$P_x = \sum_{y=1}^{m} w_y a_{xy} \quad x = 1, 2, \ldots, n. \tag{4.16}$$

4.3.1 Sensitivity Analysis on the Weights of the Criteria

Definition 4.3.1. *Let $\theta_{xy}^{k}(1 \leq x < y \leq n$ and $1 \leq k \leq m)$ denote the minimum change in the current weight w_k for criterion C_k such that the ranking of* *alternatives A_x and A_y will be reversed. Also, define:*

$$\theta_{xy}^{k}{}' = \theta_{xy}^{k} \times \frac{100}{w_k} \quad (1 \leq x < y \leq n \text{ and } 1 \leq k \leq m), \tag{4.17}$$

which expresses its minimum changes in relative terms (in %).

We will define the most critical criterion in the next four definitions. The first two definitions apply when one is interested only in changes in the best/top alternative (i.e. alternative A_t), while the last two definitions apply when one is interested in changes in the ranking of any alternative.

Definition 4.3.2. *The Absolute Top Critical (ATC) criterion is the criterion that corresponds to the smallest $\left|\theta_{xy}^{k}\right|$ value $(1 \leq x < y \leq n$ and $1 \leq k \leq m)$, where $x = t$ or $y = t$, and alternative A_t is the best/top alternative.*

Definition 4.3.3. *The Relative Top Critical (RTC) criterion is the criterion that corresponds to the smallest $\left|\theta_{xy}^{k}{}'\right|$ value $(1 \leq x < y \leq n$ and $1 \leq k \leq m)$, where $x = t$ or $y = t$, and alternative A_t is the best/top alternative.*

Definition 4.3.4. *The Absolute Overall Critical (AOC) criterion is the criterion that corresponds to the smallest $\left|\theta_{xy}^{k}\right|$ value $(1 \leq x < y \leq n$ and $1 \leq k \leq m)$.*

Definition 4.3.5. *The Relative Overall Critical (ROC) criterion is the criterion that corresponds to the smallest $\left|\theta_{xy}^{k}{}'\right|$ value $(1 \leq x < y \leq n$ and $1 \leq k \leq m)$.*

We will define the critical degree and sensitivity coefficient of a given decision criterion in the next two definitions.

Definition 4.3.6. *The critical degree of criterion C_k, denoted as $\delta_k{}'$, is the smallest quantity (in %) by which the current value of w_k must change, such that the current ranking of the alternatives will change. We define:*

$$\delta_k{}' = \min_{1 \leq x < y \leq n} \left\{ \left|\theta_{xy}^{k}{}'\right| \right\}, \quad \text{for all } 1 \leq k \leq m. \tag{4.18}$$

Definition 4.3.7. *The sensitivity coefficient of criterion C_k, denoted as $S'(C_k)$, is the reciprocal of its critical degree. We define:*

$$S'(C_k) = \frac{1}{\delta_k{}'}, \quad \text{for all } 1 \leq k \leq m. \tag{4.19}$$

If the critical degree is non-feasible (i.e., impossible to change any alternative rank with any weight change), then the sensitivity coefficient is set equal to 0.

Theorem 4.3.8. *The threshold value $\theta_{xy}^{k}(1 \leq x < y \leq n$ and $1 \leq k \leq m)$, by which the current weight w_k of the criterion C_k needs to be modified (before* *normalization process) so that the ranking of the alternatives A_x and A_y will be reserved, is given as follows:*

$$\theta_{xy}^k = \frac{P_y - P_x}{a_{yk} - a_{xk}} \tag{4.20}$$

If $P_x > P_y$, then

$$\begin{cases} \theta_{xy}^k < \dfrac{P_y - P_x}{a_{yk} - a_{xk}} & \text{if } a_{yk} > a_{xk}, \\ \theta_{xy}^k > \dfrac{P_y - P_x}{a_{yk} - a_{xk}} & \text{if } a_{yk} < a_{xk}. \end{cases} \tag{4.21}$$

If $P_x < P_y$, then

$$\begin{cases} \theta_{xy}^k < \dfrac{P_y - P_x}{a_{yk} - a_{xk}} & \text{if } a_{yk} < a_{xk}, \\ \theta_{xy}^k > \dfrac{P_y - P_x}{a_{yk} - a_{xk}} & \text{if } a_{yk} > a_{xk}. \end{cases} \tag{4.22}$$

Furthermore, the following condition should also be satisfied for the value of θ_{xy}^k to be feasible:

$$\theta_{xy}^k \leq w_k \tag{4.23}$$

Proof. Let the new modified weight of criterion C_k, denoted as w_k^*, is:

$$w_k^* = w_k - \theta_{xy}^k \tag{4.24}$$

To preserve property (4.14), it is necessary that all weights be normalized. Therefore, the new normalized weights will be defined as follows:

$$\begin{aligned} w_i{}' &= \frac{w_i}{\sum\limits_{j=1}^{m} w_j - \theta_{xy}^k} = \frac{w_i}{w_k^* + \sum\limits_{j \neq k} w_j} \qquad i = 1, 2, \ldots, k-1, k+1, \ldots, m. \\ w_k{}' &= \frac{w_k^*}{\sum\limits_{j=1}^{m} w_j - \theta_{xy}^k} = \frac{w_k^*}{w_k^* + \sum\limits_{j \neq k} w_j} \end{aligned} \tag{4.25}$$

Given the new weights w_i' (for $i = 1, 2, \ldots, m$), let $P_x{}'$ and $P_y{}'$ denote the new final preference values for the two alternatives A_x and A_y, respectively. The threshold value is achieved when $P_x{}' = P_y{}'$. Using the Eq. (4.16) and Eq. (4.25), the following relation is derived

$$
\begin{aligned}
P_x{}' &= P_y{}' \\
\sum_{j=1}^{m} w_j{}' a_{xj} &= \sum_{j=1}^{m} w_j{}' a_{yj} \\
\frac{w_k^* a_{xk}}{w_k^* + \sum_{j \neq k} w_j} + \frac{\sum_{j \neq k} w_j a_{xj}}{w_k^* + \sum_{j \neq k} w_j} &= \frac{w_k^* a_{yk}}{w_k^* + \sum_{j \neq k} w_j} + \frac{\sum_{j \neq k} w_j a_{yj}}{w_k^* + \sum_{j \neq k} w_j} \\
w_k^* a_{xk} + \sum_{j \neq k} w_j a_{xj} &= w_k^* a_{yk} + \sum_{j \neq k} w_j a_{yj} \qquad (4.26)
\end{aligned}
$$

Substitute Eq. (4.24) into Eq. (4.26), the following relation is derived:

$$
\begin{aligned}
\left(w_k - \theta_{xy}^k\right) a_{xk} + \sum_{j \neq k} w_j a_{xj} &= \left(w_k - \theta_{xy}^k\right) a_{yk} + \sum_{j \neq k} w_j a_{yj} \\
-\theta_{xy}^k a_{xk} + \sum_{j=1}^{m} w_j a_{xj} &= -\theta_{xy}^k a_{yk} + \sum_{j=1}^{m} w_j a_{yj} \\
-\theta_{xy}^k a_{xk} + P_x &= -\theta_{xy}^k a_{yk} + P_y \\
\theta_{xy}^k \left(a_{yk} - a_{xk}\right) &= P_y - P_x \\
\theta_{xy}^k &= \frac{P_y - P_x}{a_{yk} - a_{xk}} \qquad (4.27)
\end{aligned}
$$

If $P_x > P_y$, then due to the fact that it is necessary to have the new ranking of these two alternatives reversed, the following relation should be satisfied:

$$P_x{}' < P_y{}'$$

So we get

$$
\begin{aligned}
\theta_{xy}^k \left(a_{yk} - a_{xk}\right) &< P_y - P_x \\
\theta_{xy}^k &< \frac{P_y - P_x}{a_{yk} - a_{xk}} \qquad \text{if } a_{yk} > a_{xk}, \text{ or} \\
\theta_{xy}^k &> \frac{P_y - P_x}{a_{yk} - a_{xk}} \qquad \text{if } a_{yk} < a_{xk}. \qquad (4.28)
\end{aligned}
$$

If $P_x < P_y$, then due to the fact that it is necessary to have the new ranking of these two alternatives reversed, the following relation should be satisfied:

$$P_x{}' > P_y{}'$$

So we get

$$\begin{aligned}\theta^k_{xy}\,(a_{yk} - a_{xk}) &> P_y - P_x \\ \theta^k_{xy} &> \frac{P_y - P_x}{a_{yk} - a_{xk}} \qquad \text{if } a_{yk} > a_{xk}, \text{ or} \\ \theta^k_{xy} &< \frac{P_y - P_x}{a_{yk} - a_{xk}} \qquad \text{if } a_{yk} < a_{xk}.\end{aligned} \tag{4.29}$$

Furthermore, the following condition should also be satisfied for the new weight $w^*_k = w_k - \theta^k_{xy}$ to be feasible:

$$w^*_k \geq 0$$

which implies

$$w_k - \theta^k_{xy} \geq 0 \quad \Rightarrow \quad \theta^k_{xy} \leq w_k \tag{4.30}$$

Therefore, in order to have a feasible value, Eq. (4.23) should be satisfied. ■

If alternative A_x dominates alternative A_y (i.e. $a_{xk} \geq a_{yk}$, for all $k = 1, 2, \ldots, m$), then it is impossible to make alternative A_x more preferred than alternative A_y by changing the weights of the criteria.

Definition 4.3.9. *A criterion C_k is a **robust criterion** if all θ^k_{xy} quantities (for $1 \leq x < y \leq n$ and $1 \leq k \leq m$) associated with the criterion are non-feasible. In other words, if Eq. (4.23) is violated for all $x, y = 1, 2, \ldots, n$, for some criterion C_k, then any change on the weight of that criterion does not affect the existing ranking of any of the alternatives and thus, this criterion is a robust criterion.*

In a sensitivity analysis, we can drop a robust criterion from further consideration. If one is interested in determining the *most critical criterion*, then all possible ${\theta^k_{xy}}'$ $(1 \leq x < y \leq n$ and $1 \leq k \leq m)$ values need to be calculated. In total there are $m \times n(n-1)/2$ such possible ${\theta^k_{xy}}'$ quantities.

4.3.2 Sensitivity Analysis on the Preferences of the Alternatives

Definition 4.3.10. *Let $\Omega^k_{xy}(1 \leq x, y \leq n, x \neq y$ and $1 \leq k \leq m)$ denote the threshold value of a_{xk}, which is the minimum change which has to occur on the current value of a_{xk} such that the ranking of alternatives A_x and A_y will be reversed. Also, define:*

$${\Omega^k_{xy}}' = \Omega^k_{xy} \times \frac{100}{a_{xk}} \quad (1 \leq x, y \leq n, x \neq y \text{ and } 1 \leq k \leq m), \tag{4.31}$$

which expresses the changes in relative terms (in %).

There are n alternatives, so we have a total of $(n-1)$ such threshold values for each a_{xk} performance measure.

In the next definition, we will define the alternative that is associated with the smallest threshold value as the "most sensitive alternative". Also, as before, one may be interested in changes of the ranking of only the best/top alternative, or in changes in the ranking of any alternative.

Definition 4.3.11. *The critical degree of alternative A_x with respect to the criterion C_k, denoted as $\Delta_{xk}{}'$, is the smallest quantity (in %) by which the current value of a_{xk} must change, such that the existing ranking of the alternatives will change. Define:*

$$\Delta_{xk}{}' = \min_{\substack{1 \le x,y \le n \\ x \ne y}} \left\{ \left| \Omega_{xy}^{k}{}' \right| \right\}, \quad \textit{for all } 1 \le k \le m. \tag{4.32}$$

Definition 4.3.12. *Alternative A_c is the most critical alternative if it is associated with the smallest critical degree. That is, if and only if the following relation is true:*

$$\Delta_{cy}{}' = \min_{1 \le x \le n} \left\{ \min_{1 \le k \le m} \left\{ \Delta_{xk}{}' \right\} \right\}, \quad \textit{for some } 1 \le y \le n \tag{4.33}$$

Definition 4.3.13. *The sensitivity coefficient of alternative A_x with respect to criterion C_k, denoted as $S'(a_{xk})$, is the reciprocal of its critical degree. We define:*

$$S'(a_{xk}) = \frac{1}{\Delta_{xk}{}'}, \quad \textit{for all } 1 \le x \le n, \textit{ and } 1 \le k \le m. \tag{4.34}$$

If the critical degree is non-feasible, then we set $S'(a_{xk}) = 0$.

Definition 4.3.11 shows that the smaller the critical degree $\Delta_{xk}{}'$ is, the easier the ranking of alternative A_x can change. Definition 4.3.13 indicates that as the sensitivity coefficient $S'(a_{xk})$ are higher, then the ranking are easier to change. The most sensitive alternative is the one with the highest sensitivity coefficient.

Theorem 4.3.14. *The threshold value $\Omega_{xy}^{k}(1 \le x, y \le n$ and $1 \le k \le m)$, by which the measure of performance of the alternative A_x in terms of the criterion C_k needs to be modified so that the ranking of the alternatives A_x and A_y will be reserved, is given as follows:*

$$\Omega_{xy}^{k} = \frac{P_x - P_y}{P_x - P_y + w_k \left(a_{yk} - a_{xk} + 1\right)} \tag{4.35}$$

If $P_x < P_y$, then

$$\Omega_{xy}^{k} \quad < \quad \frac{P_x - P_y}{P_x - P_y + w_k \left(a_{yk} - a_{xk} + 1\right)} \tag{4.36}$$

If $P_x > P_y$, then

$$\Omega_{xy}^{k} \quad > \quad \frac{P_x - P_y}{P_x - P_y + w_k \left(a_{yk} - a_{xk} + 1\right)} \tag{4.37}$$

Furthermore, the following condition should also be satisfied for the threshold value of Ω_{xy}^{k} to be feasible:

$$\Omega_{xy}^{k} \le a_{xk} \tag{4.38}$$

Proof. Let the new modified measure of performance of alternative A_x in term of the criterion C_k, denoted as a^*_{xk}, is:

$$a^*_{xk} = a_{xk} - \Omega^k_{xy} \tag{4.39}$$

To preserve property (4.15), it is necessary that all the column preference values be normalized after changing the value of a_{xk} to a^*_{xk}. Therefore, the new normalized preference values will be defined as follows:

$$\begin{aligned} a_{ik}' &= \frac{a_{ik}}{\sum\limits_{j=1}^{n} a_{jk} - \Omega^k_{xy}} = \frac{a_{ik}}{1-\Omega^k_{xy}} \quad i = 1, 2, \ldots, x-1, x+1, \ldots, n. \\ a_{xk}' &= \frac{a^*_{xk}}{\sum\limits_{j=1}^{n} a_{jk} - \Omega^k_{xy}} = \frac{a^*_{xk}}{1-\Omega^k_{xy}} = \frac{a_{xk}-\Omega^k_{xy}}{1-\Omega^k_{xy}} \end{aligned} \tag{4.40}$$

The threshold value is achieved when $P_x' = P_y'$, then:

$$\begin{aligned} w_k a_{xk}' + \sum_{j\neq k} w_j a_{xj} &= w_k a_{yk}' + \sum_{j\neq k} w_j a_{yj} \\ w_k a_{xk}' + w_k\left(a_{xk} - a_{xk}\right) + \sum_{j\neq k} w_j a_{xj} &= w_k a_{yk}' + w_k\left(a_{yk} - a_{yk}\right) + \sum_{j\neq k} w_j a_{yj} \\ w_k a_{xk}' - w_k a_{xk} + \sum_{j=1}^{m} w_j a_{xj} &= w_k a_{yk}' - w_k a_{yk} + \sum_{j=1}^{m} w_j a_{yj} \\ w_k a_{xk}' - w_k a_{xk} + P_x &= w_k a_{yk}' - w_k a_{yk} + P_y \end{aligned} \tag{4.41}$$

Substitute (4.40) on (4.41), we get:

$$\frac{w_k\left(a_{xk} - \Omega^k_{xy}\right)}{1-\Omega^k_{xy}} - w_k a_{xk} + P_x = \frac{w_k a_{yk}}{1-\Omega^k_{xy}} - w_k a_{yk} + P_y \tag{4.42}$$

which can be further reduced to:

$$\Omega^k_{xy} = \frac{P_x - P_y}{P_x - P_y + w_k\left(a_{yk} - a_{xk} + 1\right)} \tag{4.43}$$

If before the modification we have $P_x < P_y$, then after changing the value of a_{xk} to a^*_{xk} we have

$$P_x' > P_y'. \tag{4.44}$$

Then, we have

$$\Omega^k_{xy} < \frac{P_x - P_y}{P_x - P_y + w_k\left(a_{yk} - a_{xk} + 1\right)} \tag{4.45}$$

Note that the denominator on the right-hand side is always a positive number. If before the modification we have $P_x > P_y$, then after changing the value of a_{xk} to a^*_{xk} we have

$$P_x{}' < P_y{}'. \tag{4.46}$$

Then, we have

$$\Omega^k_{xy} \quad > \quad \frac{P_x - P_y}{P_x - P_y + w_k\,(a_{yk} - a_{xk} + 1)} \tag{4.47}$$

Furthermore, the following condition should also be satisfied for the new Ω^k_{xy} value to have a feasible value:

$$\begin{array}{rcccl} 0 & \leq & a_{xk}{}' & \leq & 1 \\ 0 & \leq & \dfrac{a_{xk} - \Omega^k_{xy}}{1 - \Omega^k_{xy}} & \leq & 1 \\ 0 & \leq & a_{xk} - \Omega^k_{xy} & \leq & 1 - \Omega^k_{xy} \\ \Omega^k_{xy} & \leq & a_{xk} & \leq & 1 \end{array} \tag{4.48}$$

Therefore, in order to have a feasible value, Eq. (4.38) should be satisfied. ■

4.4 AHP for Microencapsulation Process Selection

The first stage in the Analytical Hierarchy Process (AHP) approach is the construction of a hierarchical structure to present the problem, with the top level representing the overall objective, the middle level representing the criteria, and the bottom level representing the alternatives. The first step in this stage is to determine the objective of the problem. In our case study, the objective is to evaluate which microencapsulation method would be best for a specific application. Therefore, the main objective ”selection of the best microencapsulation method” is placed at the top level of the AHP hierarchy. The second step is to identify key evaluation criteria for assessing the objective. Here, five selection criteria were considered to be relevant to this particular application: Core Wettability (CW), Core Solubility (CS), Wall Elasticity (WE), Wall Permeability (WP), and Wall Polymer Adhesive (WPA). These five criteria are placed at the second level of the AHP model. Finally, six microencapsulation methods were considered – Spray Drying (SD), Pan Coating (PC), Air Suspension (AS), Coacervation (C), Interfacial Polymerization (IP), and In-Situ Polymerization (ISP). The first three methods are the physical methods and the last three methods are the chemical methods. These six microencapsulation alternatives are placed at the bottom of the AHP model. The constructed hierarchy structure is shown in Figure 4.3.

Once the decision hierarchy is developed, the next stage is to compare and evaluate the elements at the same level in the hierarchy in pairs and then measure their comparative contribution to the main objective. A comparison matrix is set up by comparing pairs of criteria or alternatives. A predetermined scale of relative

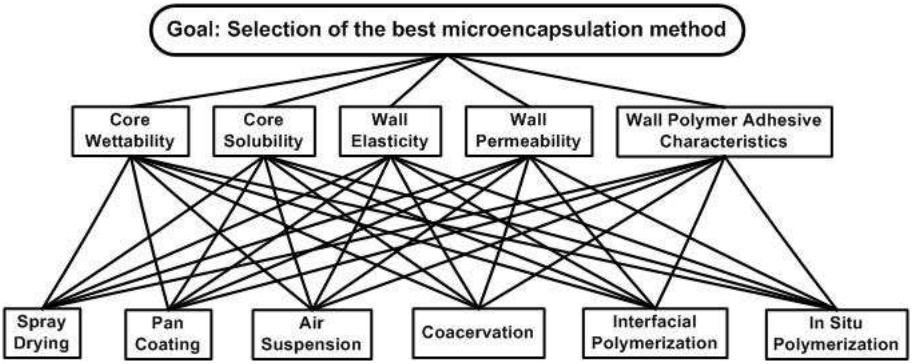


Figure 4.3: *The AHP Hierarchy for Microencapsulation Selection*

importance assigns values to the pairwise comparisons. For acquiring experts' preferences, we assigned a value between 1 (equal preference/indifference) and 9 (extremely more preferred), and its reciprocal. The scale that we used in this case study is given in Table 4.3.

Table 4.3: *Ratio Scale for Pairwise Comparisons*

Value	Definition
1	Equal preference/Indifference of elements
3	Moderate preference of one element over the other
5	Strong preference of one element over the other
7	Very strong preference of one element over the other
9	Extreme preference of one element over the other
2, 4, 6, 8	Intermediate values between two adjacent judgments

This pairwise comparison enabled the decision maker to evaluate the contribution of each element to the objective independently, thereby simplifying the decision making process. Here, the five microencapsulation criteria were compared in pairs to measure their impacts on the objective. Also, the six microencapsulation alternatives were compared in pairs to measure their importance based on each criterion. In this case study, we have a pairwise comparison of 5 criteria, so a total of 10 pairwise comparisons had to be made at the criteria level. The results of this operation are presented in Table 4.4. The interpretation of the results shown in Table 4.4 is as follows: (1) Core Wettability (CW) is moderately more preferred than Core Solubility (CS); (2) Core Wettability (CW) is strongly more preferred than Wall Elasticity (WE); (3) Core Wettability (CW) is very strongly more preferred than Wall Permeability (WP); (4) Core Wettability (CW) is extremely more preferred than Wall Polymer Adhesive (WPA); (5) Core Solubility (CS) is slightly more preferred than Wall Elasticity (WE); (6) Core Solubility (CS) is moderately to strongly more preferred than Wall Permeability (WP); (7)

Table 4.4: *Normalized Pairwise Rating of Selection Criteria*

Criterion	CW	CS	WE	WP	WPA
Core Wettability (CW)	1	3 (1)	5 (2)	7 (3)	9 (4)
Core Solubility (CS)	1/3	1	2 (5)	4 (6)	6 (7)
Wall Elasticity (WE)	1/5	1/2	1	2 (8)	4 (9)
Wall Permeability (WP)	1/7	1/4	1/2	1	2 (10)
Wall Polymer Adhesive (WPA)	1/9	1/6	1/4	1/2	1
Column Sum	1.7873	4.9167	8.75	14.5	22

$CR = 0.0175$

Core Solubility (CS) is strongly to very strongly more preferred than Wall Polymer Adhesive (WPA); (8) Wall Elasticity (WE) is slightly more preferred than Wall Permeability (WP); (9) Wall Elasticity (WE) is moderately to strongly more preferred than Wall Polymer adhesive (WPA); and (10) Wall Permeability (WP) is slightly more preferred than Wall Polymer Adhesive (WPA). Reciprocal values for these comparisons are automatically entered into the appropriate corresponding cells of the table. For this process, the decision maker also had to check the consistency ratio (CR). If the $CR > 0.1$, then the decision maker needs to readjust his/her judgments.

In the final stage, synthesis of priorities was conducted to calculate a composite weight for each alternative, based on preferences derived from the comparison matrix. After calculating the composite weight, we obtained the relative priority of the microencapsulation alternatives with respect to each microencapsulation criterion. Using the values entered for the pairwise comparisons, mathematical techniques were applied to establish the weights assigned to each criterion. The weights are actually a measure of the relative importance of the criteria. We could use some prioritization techniques to estimate the relative weights of the decision elements. In Section 4.2.3, we discussed some prioritization methods for deriving priorities from comparison matrices.

The original AHP method establishes weights for the criteria by solving the matrix using the eigenvector method (Saaty, 1980). The eigenvector solution involves a rigorous mathematical approach to matrix calculation. An approximate method of the eigenvector solution, i.e. Additive Normalization Method (ANM), can be calculated by: (1) converting the fraction pairwise comparisons to decimal equivalents; (2) creating a normalized matrix by dividing each element in the matrix by its respective column total; (3) summing the rows of the normalized matrix; and (4) dividing the row sums by the order of the matrix (n), i.e., the number of elements compared. The resulting column of values is an approximation of the eigenvector, which is actually the weight assigned to each of the elements. This solution to the matrix is also referred to as the weight vector.

The entries in Table 4.4 are then normalized, by dividing each entry in a column by the sum of all the entries in that column, so that they add up to one. Following normalization, the weights are then averaged across the rows to give an average weight for each criterion as shown in Table 4.5. The resulting weight

Table 4.5: *Pairwise Rating of Selection Criteria*

Criterion	CW	CS	WE	WP	WEP	
CW	0.5595	0.6102	0.5714	0.4828	0.4091	0.5266
CS	0.1865	0.2034	0.2286	0.2759	0.2727	0.2334
WE	0.1119	0.1017	0.1143	0.1379	0.1818	0.1295
WP	0.0799	0.0508	0.0571	0.0690	0.0909	0.0696
WEP	0.0622	0.0339	0.0286	0.0345	0.0455	0.0409

Table 4.6: *Pairwise Rating of Alternative Microencapsulation Methods with respect to Core Wettability*

	SD	PC	AS	C	IP	ISP
SD	1	1/2	1/4	1/3	2	3
PC	2	1	1/2	1/2	4	6
AS	4	2	1	1	8	9
C	3	2	1	1	6	9
IP	1/2	1/4	1/8	1/6	1	2
ISP	1/3	1/6	1/9	1/9	1/2	1
	10.8333	5.9167	2.9861	3.1111	21.5	30

$CR = 0.0052$

vector from the pairwise comparisons of Table 4.4 is given as the vector in the last column of Table 4.5.

The next step is the pairwise comparison of the microencapsulation methods evaluating how well they satisfy each of the criteria. For each pairing within each criterion, the better microencapsulation method is awarded a rating on a scale from 1 (equally good/indifferent) to 9 (absolutely better), whilst the other method in the pairing is awarded a rating reciprocal to this value. Here we evaluate 6 microencapsulation alternatives for each microencapsulation criterion. For each criterion, the decision maker needs to determine $6(6-1)/2 = 15$ pairwise comparisons. For each process, the decision maker also has to check the consistency ratio (CR) of his/her judgments. If the $CR > 0.1$, the decision maker needs to readjust his/her judgments. It becomes a tiresome process if the number of alternatives increases. And it also becomes a burden to the decision maker to check the consistency if he/she always has to modify his/her judgment. If the matrix is large, the decision maker sometimes doesn't even know where the problem is and which value to modify.

The results for the 'Core Wettability' criterion are given in Table 4.6. Each entry in this matrix records how well the microencapsulation method in the corresponding row is when it is compared to the method in the corresponding column with respect to the 'Core Wettability' criterion. The ratings in these comparison matrices are normalized as before and averaged across the rows to give an average normalized rating by criterion for each microencapsulation method, as illustrated

Table 4.7: *Normalized Pairwise Rating of Alternatives with respect to Core Wettability*

	SD	PC	AS	C	IP	ISP	
SD	0.0923	0.0845	0.0837	0.1071	0.0930	0.1000	0.0935
PC	0.1846	0.1690	0.1674	0.1607	0.1860	0.2000	0.1780
AS	0.3692	0.3380	0.3349	0.3214	0.3721	0.3000	0.3393
C	0.2769	0.3380	0.3349	0.3214	0.2791	0.3000	0.3084
IP	0.0462	0.0423	0.0419	0.0536	0.0465	0.0667	0.0495
ISP	0.0308	0.0282	0.0372	0.0357	0.0233	0.0333	0.0314

Table 4.8: *Pairwise Rating of Alternative Microencapsulation Methods with respect to Core Solubility*

	SD	PC	AS	C	IP	ISP
SD	1	1/3	1/2	1/5	2	3
PC	3	1	2	1/3	4	5
AS	2	1/2	1	1/4	3	4
C	5	3	4	1	6	7
IP	1/2	1/4	1/3	1/6	1	1
ISP	1/3	1/5	1/4	1/7	1	1
	11.8333	5.2833	8.0833	2.0929	17	21

$CR = 0.0243$

in Table 4.7 for 'Core Wettability' criterion.

Tables 4.8, 4.10, 4.12, and 4.14 show the pairwise comparison of the six microencapsulation methods with respect to the criterion 'Core Solubility', 'Wall Elasticity', 'Wall Permeability', and 'Wall Polymer Adhesive', respectively. Tables 4.9, 4.11, 4.13, and 4.15 show normalized pairwise rating for each microencapsulation method with respect to the criterion 'Core Solubility', 'Wall Elasticity', 'Wall Permeability', and 'Wall Polymer Adhesive', respectively. Table 4.16 summarizes the average normalized ratings of microencapsulation alternatives with respect to each microencapsulation selection criterion.

The Consistency Ratio (CR) of each decision maker's pairwise comparison matrix should be less than the threshold value of 0.1. throughout the evaluation

Table 4.9: *Normalized Pairwise Rating of Alternatives with respect to Core Solubility*

	SD	PC	AS	C	IP	ISP	
SD	0.0845	0.0631	0.0619	0.0956	0.1176	0.1429	0.0943
PC	0.2535	0.1893	0.2474	0.1593	0.2353	0.2381	0.2205
AS	0.1690	0.0946	0.1237	0.1195	0.1765	0.1905	0.1456
C	0.4225	0.5678	0.4948	0.4778	0.3529	0.3333	0.4415
IP	0.0423	0.0473	0.0412	0.0796	0.0588	0.0476	0.0528
ISP	0.0282	0.0379	0.0309	0.0683	0.0588	0.0476	0.0453

Table 4.10: *Pairwise Rating of Alternative Microencapsulation Methods with respect to Wall Elasticity*

	SD	PC	AS	C	IP	ISP
SD	1	1/3	1/2	1/6	2	3
PC	3	1	2	1/2	4	5
AS	2	1/2	1	1/4	3	4
C	6	2	4	1	8	9
IP	1/2	1/4	1/3	1/8	1	1
ISP	1/3	1/5	1/4	1/9	1	1
	12.8333	4.2833	8.0833	2.1528	19	23

$CR = 0.0126$

Table 4.11: *Normalized Pairwise Rating of Alternatives with respect to Wall Elasticity*

	SD	PC	AS	C	IP	ISP	
SD	0.0779	0.0778	0.0619	0.0774	0.1053	0.1304	0.0885
PC	0.2338	0.2335	0.2474	0.2323	0.2105	0.2174	0.2291
AS	0.1558	0.1167	0.1237	0.1161	0.1579	0.1739	0.1407
C	0.4675	0.4669	0.4948	0.4645	0.4211	0.3913	0.4510
IP	0.0390	0.0584	0.0412	0.0581	0.0526	0.0435	0.0488
ISP	0.0260	0.0467	0.0309	0.0516	0.0526	0.0435	0.0419

Table 4.12: *Pairwise Rating of Alternative Microencapsulation Methods with respect to Wall Permeability*

	SD	PC	AS	C	IP	ISP
SD	1	3	2	4	2	3
PC	1/3	1	1/2	1	1/2	1
AS	1/2	2	1	2	1	2
C	1/4	1	1/2	1	1/2	2
IP	1/2	2	1	2	1	3
ISP	1/3	1	1/2	1/2	1/3	1
	2.9167	10	5.5	10.5	5.3333	12

$CR = 0.0144$

Table 4.13: *Normalized Pairwise Rating of Alternatives with respect to Wall Permeability*

	SD	PC	AS	C	IP	ISP	
SD	0.3429	0.3000	0.3636	0.3810	0.3750	0.2500	0.3354
PC	0.1143	0.1000	0.0909	0.0952	0.0938	0.0833	0.0963
AS	0.1714	0.2000	0.1818	0.1905	0.1875	0.1667	0.1830
C	0.0857	0.1000	0.0909	0.0952	0.0938	0.1667	0.1054
IP	0.1714	0.2000	0.1818	0.1905	0.1875	0.2500	0.1969
ISP	0.1143	0.1000	0.0909	0.0476	0.0625	0.0833	0.0831

Table 4.14: *Pairwise Rating of Alternative Microencapsulation Methods with respect to Wall Polymer Adhesive*

	SD	PC	AS	C	IP	ISP
SD	1	1	1/2	1	1/2	1
PC	1	1	1/3	1/2	1/3	1
AS	2	3	1	2	1	2
C	1	2	1/2	1	1/2	1
IP	2	3	1	2	1	2
ISP	1	1	1/2	1	1/2	1
	8	11	3.8333	7.5	3.8333	8

$CR = 0.0081$

Table 4.15: *Normalized Pairwise Rating of Alternatives with respect to Wall Polymer Adhesive*

	SD	PC	AS	C	IP	ISP	
SD	0.1250	0.0909	0.1304	0.1333	0.1304	0.1250	0.1225
PC	0.1250	0.0909	0.0870	0.0667	0.0870	0.1250	0.0969
AS	0.2500	0.2727	0.2609	0.2667	0.2609	0.2500	0.2602
C	0.1250	0.1818	0.1304	0.1333	0.1304	0.1250	0.1377
IP	0.2500	0.2727	0.2609	0.2667	0.2609	0.2500	0.2602
ISP	0.1250	0.0909	0.1304	0.1333	0.1304	0.1250	0.1225

Table 4.16: *Average Normalized Ratings of Microencapsulation Methods with respect to Each Criterion*

	Criterion				
Alternative	CW	CS	WE	WP	WPA
SD	0.0935	0.0943	0.0885	0.3354	0.1225
PC	0.1780	0.2205	0.2291	0.0963	0.0969
AS	0.3393	0.1456	0.1407	0.1830	0.2602
C	0.3084	0.4415	0.4510	0.1054	0.1377
IP	0.0495	0.0528	0.0488	0.1969	0.2602
ISP	0.0314	0.0453	0.0419	0.0831	0.1225

process. We see that the results of the consistency test and the CR of the comparison matrices from the Tables 4.4, 4.6, 4.8, 4.10, 4.12, and 4.14 are all ≤ 0.1, indicating the judgments of the decision maker are consistent enough.

The final stage in the AHP approach is to combine the average normalized microencapsulation method ratings (Table 4.16) with the average normalized criterion weights (Table 4.5), producing an overall rating for each microencapsulation method, i.e. the extent to which the methods satisfy the criteria is weighted according to the relative importance of the criteria. This is done as follows:

$$P_j = \sum_i (w_i a_{ij})$$

where

P_j	=	overall relative rating for the j-th microencapsulation alternative
w_i	=	average normalized weight for the i-th criterion
a_{ij}	=	average normalized rating for the j-th microencapsulation method with respect to the i-th criterion

Table 4.17 delivers the results of this final step. These results show clearly that 'Coacervation' method is the most preferred microencapsulation method, followed by 'Air Suspension' and 'Pan Coating' method in the second and third place, respectively. Conversely, 'In-Situ Polymerization' is the least preferred microencapsulation method.

Figure 4.4 shows the weights of every microencapsulation methods in this case study with respect to all the considered microencapsulation criteria and the goal. Figure 4.5 shows the weights of all considered microencapsulation criteria.

Table 4.17: *Overall Microencapsulation Method Ratings*

Microencapsulation Method	P_j	Final Rank
Spray Drying	0.1110	4
Pan Coating	0.1855	3
Air Suspension	0.2542	2
Coacervation	0.3368	1
Interfacial Polymerization	0.0691	5
In-Situ Polymerization	0.0433	6

In practice, sensitivity analysis should be carried out to determine the robustness of such decisions with respect to variations in the pairwise ratings and the changes of the weights of the decision criteria and the preference values of the alternatives. A sensitivity analysis was carried out to determine the critical factors. In the following section, the stability of the rank order based on the changes of the criteria weights and the preference values of the alternatives within each criterion was tested in a sensitivity analysis.

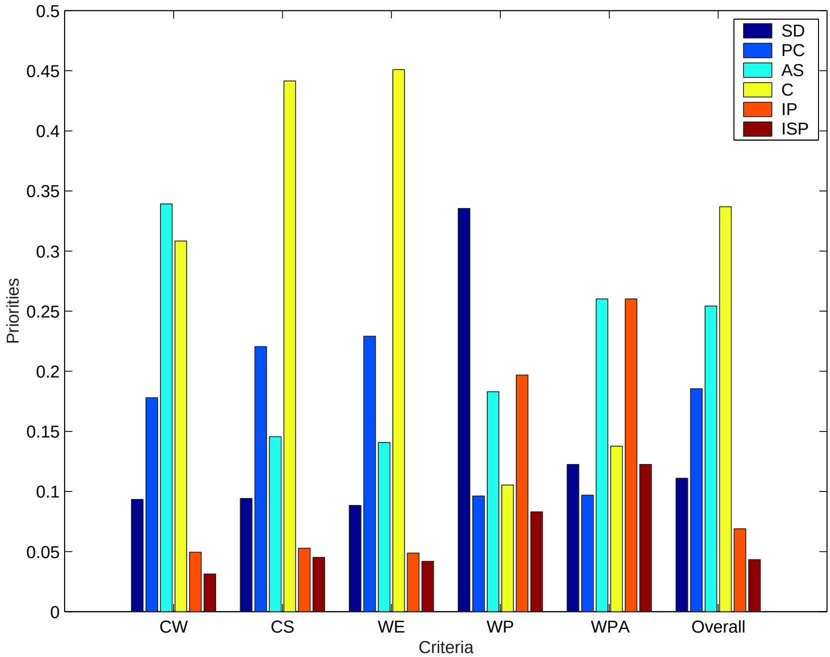


Figure 4.4: *Weights of the Microencapsulation Alternatives with respect to the Criteria and the Goal*

4.5 Sensitivity Analysis Results

Consider a decision making problem for microencapsulation process selection with six microencapsulation alternatives and five microencapsulation criteria. Table 4.16 represents the corresponding decision matrix and Table 4.17 represents the final preferences and ranking of the six microencapsulation alternatives. The criteria weights for the five microencapsulation criteria are presented in Figure 4.5. The results indicate that the criterion 'Core Wettability' has the highest weight (53%), followed by the criteria 'Core Solubility' and 'Wall Elasticity' which have relative weights of 23% and 13%, respectively. 'Wall Permeability' and 'Wall Polymer Adhesive' received only 7% and 4%, respectively.

The final priorities of the six microencapsulation methods, depicted in Figure 4.4, are the result of an assessment of the weighted criteria. This result shows that 'Coacervation' is the most preferred microencapsulation alternative. From Table 4.17, we see that it captured 33.68% of the total weight. The next-ranked microencapsulation alternatives with the weights of 25.42% and 18.55% respectively are 'Air Suspension' and 'Pan Coating'. In our assessment, we observe that 'In-Situ Polymerization' is the lowest-ranked microencapsulation alternative and can hardly be seen as a viable option for chemical engineers.

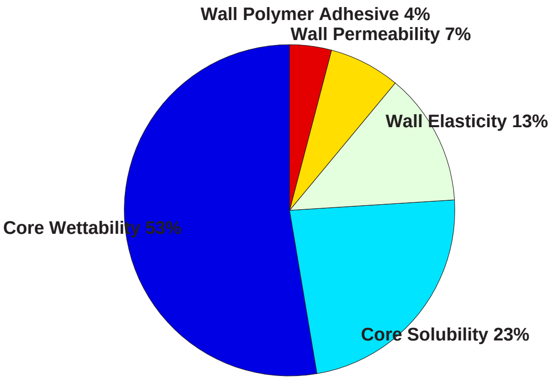


Figure 4.5: *Weights of the Microencapsulation Criteria*

A look at the performance of the microencapsulation alternatives vis-á-vis the individual criteria allows a more detailed analysis of the final ranking (see Figure 4.4). The leading position of 'Coacervation' is based on its high performance on the criteria 'Core Solubility' and 'Wall Elasticity'. For both sets of criteria, it has by far the strongest performance of all the microencapsulation alternatives. Since the weights of these criteria are relatively high, 'Coacervation' turns out to be the most preferred microencapsulation method. Comparing the second and third-ranked microencapsulation alternatives, 'Air Suspension' and 'Pan Coating', the former reveals a high performance on the 'Core Wettability' criterion (by far the highest of all the alternatives). On the other hand, the latter shows significant performance on three criteria, i.e. 'Core Wettability', 'Core Solubility' and 'Wall Elasticity'.

Since the weights of the criteria and the performance of the alternatives with respect to each criterion are critical to determine the final ranking of microencapsulation alternatives, the stability of the rank order based on different weighting schemes needs to be tested in a sensitivity analysis. The objective of a sensitivity analysis is to explore how the ranking of the alternatives change when the input data (preference judgments and degrees of fuzziness) are changed. For this purpose, the weights of the important criteria are separately altered, simulating weights between 0 and 1 (note that the weights of the other criteria change accordingly, reflecting the relative nature of the weights, i.e. total weight has to add up to 1). Figures 4.6a - 4.6e show the sensitivity analysis diagram of microencapsulation alternatives for changing weights between 0 and 1 of the criteria 'Core Wettability', 'Core Solubility', 'Wall Elasticity', 'Wall Permeability' and 'Wall Polymer Adhesive', respectively. The solid patterned lines in those figures represent the development of the final priorities (y-axis) of the six microencapsulation alternatives for changing criteria weights (x-axis). The original weight of the respective criterion is marked by a solid vertical line labeled 'Baseline' on the top of it. The dashed vertical lines are the rank-reversal lines that indicate at which corresponding criterion weight the rank reversal occurs. The robust criterion exists if there is no rank-reversal line. In our case study, there is no robust criterion since all the figures in Figure 4.6a - 4.6e have at least one rank-reversal line.

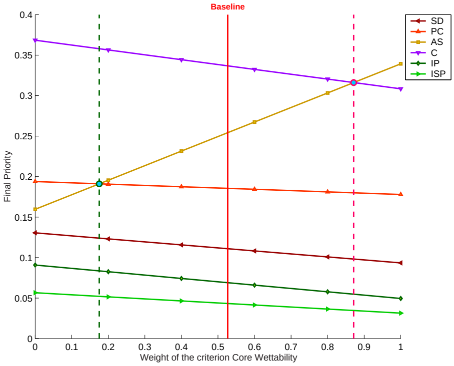
The results of the sensitivity analysis are summarized in Tables 4.18 and 4.19, where the symbols SD, PC, AS, C, IP and ISP denote the six microencapsulation alternatives, i.e. Spray Drying (SD), Pan Coating (PC), Air Suspension (AS), Coacervation (C), Interfacial Polymerization (IP), and In-Situ Polymerization (ISP); and the symbols CW, CS, WE, WP, and WPA denote the five microencapsulation criteria, i.e. Core Wettability (CW), Core Solubility (CS), Wall Elasticity (WE), Wall Permeability (WP), and Wall Polymer Adhesive (WPA). Table 4.18 shows the absolute value of changes for each criterion necessary to cause the reversal of ranks for all 15 pairs of microencapsulation alternatives, while Table 4.19 presents the relative changes of current weights necessary to cause the reversal of ranks (in %).

Table 4.18: *Absolute Value Changes in Criteria Weights for the Reversal of Ranks*

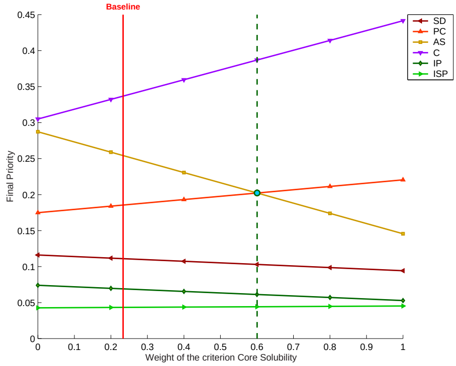
	Criterion				
Pair of Alternatives	CW	CS	WE	WP	WPA
SD - PC	–	–	–	-0.3116	-2.9101
SD - AS	–	–	–	-0.9397	–
SD - C	–	–	–	-0.9817	–
SD - IP	–	–	–	–	**-0.3048**
SD - ISP	–	–	–	–	–
PC - AS	0.4261	-0.9182	-0.7772	–	–
PC - C	–	–	–	–	–
PC - IP	–	–	–	-1.1575	-0.7133
PC - ISP	–	–	–	–	-5.5536
AS - C	-2.6738	–	–	-1.0643	**-0.6741**
AS - IP	–	–	–	-13.3340	–
AS - ISP	–	–	–	–	–
C - IP	–	–	–	-2.9269	-2.1857
C - ISP	–	–	–	–	–
IP - ISP	–	–	–	–	–

In the next paragraph, we will discuss in detail the results of the sensitivity analysis we obtained from Tables 4.18 and 4.19 and from Figures 4.6a - 4.6e.

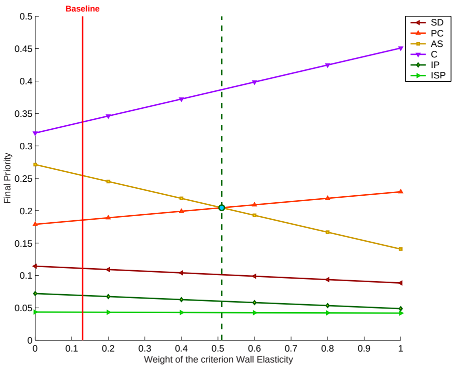
In Table 4.17 we observe that the first alternative is not the best alternative as required in the Triantaphyllou & Sánchez (1997) methodology. Triantaphyllou & Sánchez (1997) proposed a methodology for performing sensitivity analysis on the weights of the decision criteria and the performance values of the alternatives in terms of the criteria for deterministic multiple criteria decision making methods. Their methodology can be used to determine the criteria in a model requiring only



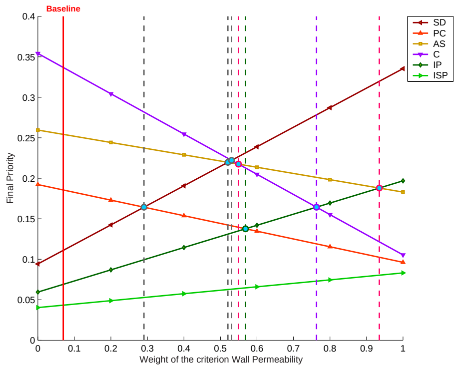
(a) Core Wettability



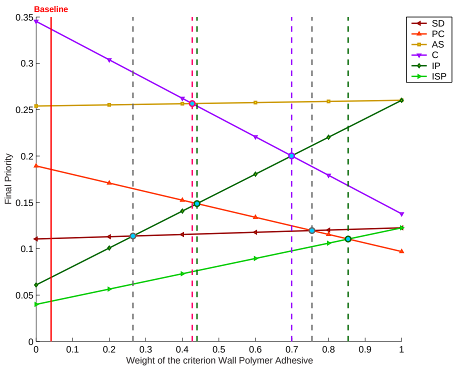
(b) Core Solubility



(c) Wall Elasticity



(d) Wall Permeability



(e) Wall Polymer Adhesive

Figure 4.6: *Sensitivity Analysis Diagram for Microencapsulation Criteria*

Table 4.19: *Percentage Changes in Criteria Weights for the Reversal of Ranks*

Pair of Alternatives	Criterion				
	CW	CS	WE	WP	WPA
SD - PC	–	–	–	-447.9	-7112.6
SD - AS	–	–	–	-1351	–
SD - C	–	–	–	-1411.4	–
SD - IP	–	–	–	–	-744.85
SD - ISP	–	–	–	–	–
PC - AS	**80.913**	-393.38	-600.02	–	–
PC - C	–	–	–	–	–
PC - IP	–	–	–	-1664.1	-1743.4
PC - ISP	–	–	–	–	-13574
AS - C	**-507.76**	–	–	-1530	-1647.6
AS - IP	–	–	–	-19169	–
AS - ISP	–	–	–	–	–
C - IP	–	–	–	-4207.8	-5342
C - ISP	–	–	–	–	–
IP - ISP	–	–	–	–	–

a small change in weight in order to cause one or more alternatives to switch ranks. However, their methodology requires the n alternatives to be arranged in a specific way in order to perform a sensitivity analysis. The arrangement is made such that the following relation is always satisfied: $P_1 \geq P_2 \geq P_3 \geq \ldots \geq P_n$ where P_i is the preference value for the i-th alternative. The methodology proposed in this thesis is to be applied more generally thus the relation above may be not satisfied.

Observe in Table 4.5 that according to the weights of the five criteria, the criterion 'Core Wettability' (criterion 1) appears to be the most important having the weight $w_1 = 0.5266$. As shown in Table 4.17, the most preferred alternative is 'Coacervation' (alternative 4) with $P_4 = 0.3368$ and the second preferred alternative is 'Air Suspension' (alternative 3) with $P_3 = 0.2542$. From Table 4.16, we have the performance value of alternatives 'Coacervation' and 'Air Suspension' with respect to criterion 'Core Wettability' as follows: $a_{41} = 0.3084$ and $a_{31} = 0.3393$. Thus the minimum change θ_{34}^1 needed to alter the current weight w_1 such that the current ranking of the two alternatives A_3 and A_4 will be reversed, can be calculated by using Eq. (4.22) because $P_3 < P_4$ of Theorem 4.3.8 as follows:

$$\begin{aligned}
\theta_{34}^1 &< \frac{(P_4 - P_3)}{(a_{41} - a_{31})} \\
\theta_{34}^1 &< \frac{(0.3368 - 0.2542)}{(0.3084 - 0.3393)} \\
\theta_{34}^1 &< -2.6738
\end{aligned}$$

This means that the alternatives A_3 and A_4 will reverse their ranks if we increase the weight of w_1 greater than 2.6738.

The quantity $\theta_{34}^1 = -2.6738$ satisfies the feasiblity condition (Eq. (4.23)), because it is less than w_1. Thus, the modified weight w_1^* of the first criterion (before normalization) can be calculated as follows:

$$w_1^* = 0.5266 - (-2.6738) = 3.2004.$$

After the normalization process, it becomes:

$$w_1{}' = 3.2004/(1 - (-2.6738)) = 0.8711.$$

This result corresponds to the sensitivity analysis diagram of the criterion 'Core Wettability' which is shown in Figure 4.6a. This diagram shows for the weights greater than 0.8711 the rank order of the alternatives 'Coacervation' and 'Air Suspension' will reverse.

Doing the steps as explained above for all possible combinations of criteria and pairs of alternatives, we derive Table 4.18. In total there are $5 \times 6(6-1)/2 = 75$ such possible θ_{xy}^k quantities. In Table 4.18, the symbol '–' indicates non-feasible values (i.e. the corresponding θ_{xy}^k value does not satisfy relation in Eq. (4.23)). The gray shaded rows represent all values which are corresponding to the best alternative 'Coacervation', and the bold-faced numbers indicate minimum critical changes. The bold-faced number within the gray shaded row (−0.6741) indicates minimum critical change corresponding to the best alternative 'Coacervation'. The bold-faced number not within the gray shaded row (−0.3048) indicates minimum critical change corresponds to any alternative.

Table 4.19 shows the changes in relative terms (i.e., the ${\theta_{xy}^k}'$ values which are derived using Definition 4.3.1). The symbol '–', gray shaded rows and bold-faced numbers have the same meaning as explained above for Table 4.18. The negative values in Tables 4.18 and 4.19 indicate increases, while the positive values indicate decreases. The values of changes (either in absolute terms or in percentages) in Tables 4.18 and 4.19 are not normalized.

From Table 4.18, we observe that the three criteria, Core Wettability (CW), Wall Permeability (WP) and Wall Polymer Adhesive (WPA), may cause the best alternative 'Coacervation' to switch rank with other alternatives. The minimal absolute value changes for these three criteria are CW (−2.6738), WP (−0.9817) and WPA (−0.6741). We see that in Table 4.18 the minimal absolute value for criteria 'Core Wettability' and 'Wall Polymer Adhesive' is reached by the pair of alternatives 'Air Suspension' and 'Coacervation' and the criterion 'Wall Permeability' has a minimum for the pair of alternatives 'Spray Drying' and 'Coacervation'. This means that the best alternative switches from 'Coacervation' to 'Air Suspension' after we increase the criteria 'Core Wettability' and 'Wall Polymer Adhesive' by the corresponding minimal absolute value changes. This is shown in Figures 4.6a and 4.6e. The best alternative switches from 'Coacervation' to 'Spray Drying' if we increase the weight of criterion 'Wall Permeability' by 0.9817 (before normalization process). This phenomenon is shown in Figure 4.6d. As we see in Table 4.18, for the criteria 'Core Solubility' and 'Wall Elasticity', there are only non-feasible values inside the gray shaded row. This means that there is no way to switch the alternative 'Coacervation' from the best position. In other words, these two criteria 'Core Solubility' and 'Wall Elasticity' are insensitive with respect to the best alternative 'Coacervation'. Figures 4.6b and 4.6c show that 'Coacervation' will always be the best alternative even though we change the weights of the criteria 'Core Solubility' and 'Wall Elasticity'.

In Table 4.19, we find that the criterion 'Core Wettability' requires a much smaller change in relative weights than that required by the others. Hence, we conclude that 'Core Wettability' appears to be the most critical criterion.

Figure 4.6a presents how the weights for the six microencapsulation alternative vary when the weight for 'Core Wettability' is varied between 0 and 1. We observe that the overall weight for the best alternative 'Coacervation' moves in the opposite direction from that of the second best alternative 'Air Suspension'. Accordingly, the best alternative switches from 'Coacervation' to 'Air Suspension' when the weight for this criterion is greater than $(0.5266-(-2.6738))/(1-(-2.6738)) = 0.8711$.

Figure 4.6b shows the sensitivity analysis when the weight for the second most critical criterion 'Core Solubility' is varied between 0 and 1. Increasing the importance of 'Core Solubility' does not cause the rank of the best alternative 'Coacervation' to change. The best alternative is insensitive to changes in the relative importance of the weights of 'Core Solubility'. The second best alternative switches from 'Coacervation' to the next second best alternative 'Pan Coating' when the weight for this criterion is greater than $(0.2334 - (-0.9182))/(1 - (-0.9182)) = 0.6004$.

Figure 4.6c presents how the weights for the six microencapsulation alternative vary when the weight for 'Wall Elasticity' is varied between 0 and 1. For 'Wall Elasticity' criterion, the best alternative 'Coacervation' is a robust criterion because it is insensitive to the variations in the weights of 'Wall Elasticity'. The second best alternative switches from 'Coacervation' to the next second best alternative 'Pan Coating' when the weight for this criterion is greater than $(0.1295 - (-0.7772))/(1 - (-0.7772)) = 0.5102$.

Figure 4.6d presents how the weights for the six microencapsulation alternative vary when the weight for 'Wall Permeability' is varied between 0 and 1. We observe that in Table 4.18 in the column WP within the gray shaded rows, the minimum value is -0.9817 for the pair of alternatives SD – C. This means the best alternative switches from 'Coacervation' to 'Spray Drying' when the weight for this criterion is greater than $(0.0696 - (-0.9817))/(1 - (-0.9817)) = 0.5305$.

Figure 4.6e presents how the weights for the six microencapsulation alternative vary when the weight for 'Wall Polymer Adhesive' is varied between 0 and 1. We see in Table 4.18 the minimum value within column WPA and shaded gray is -0.6741 for the pair of alternatives AS – C. This means the best alternative switches from 'Coacervation' to 'Air Suspension' when the weight for this criterion is greater than $(0.0409 - (-0.6741))/(1 - (-0.6741)) = 0.4271$. We observe still within column WPA and the gray shaded rows that the other value is -2.1857 for the pair of alternatives C – IP. This means that the second-ranked alternative switches from 'Coacervation' to the third-ranked alternative 'Interfacial Polymerization' when the weight for this criterion is greater than $(0.0409-(-2.1857))/(1-(-2.1857)) = 0.6989$.

The Relative Top Critical (RTC) criterion can be found by looking for the smallest relative ${\theta_{xy}^{k}}'$ value of all gray shaded rows that are related to the best alternative (in this case 'Coacervation') in Table 4.19. The smallest of all such percentage (i.e., 507.76 %) corresponds to the criterion 'Core Wettability' when the pair of alternatives 'Air Suspension' and 'Coacervation' is considered. For criterion 'Core Wettability', an increase of its current weights by 507.76 % will make 'Air Suspension' the most preferred alternative and 'Coacervation' will not be the best alternative.

The Relative Overall Critical (ROC) criterion can be found by looking for the smallest relative ${\theta_{xy}^{k}}'$ value in Table 4.19. Such smallest value is $\theta_{23}^{1} = 80.913\%$ and it corresponds to the criterion 'Core Wettability'. Therefore, the ROC criterion is 'Core Wettability'.

At this point it should be stated that if a decision maker wishes to get the most critical criteria for absolute changes, then the previous definitions of Relative Top Critical (RTC) and Relative Overall Critical (ROC) criteria correspond to Absolute Top Critical (ATC) and Absolute Overall Critical (AOC) criterion, respectively. From Table 4.18 it can be easily verified that the ATC criterion is 'Wall Polymer Adhesive' and the AOC criterion is 'Wall Polymer Adhesive' (the corresponding minimum changes are bold-faced).

We can define the critical degree of each criterion by using Definition 4.3.6. Using the values in Table 4.19, we can determine the critical degrees of the five microencapsulation criteria are as follows: ${\delta_1}' = 80.913$, ${\delta_2}' = |-393.38| = 393.38$, ${\delta_3}' = |-600.02| = 600.02$, ${\delta_4}' = |-447.9| = 447.9$, and ${\delta_5}' = |-744.85| = 744.85$. Therefore, the sensitivity coefficients of the five microencapsulation criteria according to Definition 4.3.7 are: $S'(\text{CW}) = 0.0124$, $S'(\text{CS}) = 0.0025$, $S'(\text{WE}) = 0.0017$, $S'(\text{WP}) = 0.0022$, and $S'(\text{WPA}) = 0.0013$. That is, the most sensitive microencapsulation criterion is 'Core Wettability', followed by 'Core Solubility', 'Wall Elasticity', 'Wall Permeability' and 'Wall Polymer Adhesive'.

Figure 4.6a shows the development of the priorities for changes in the weight of 'Core Wettability'. The original weight of the respective criterion is marked by a solid vertical red line (Baseline). The other two dashed vertical lines are the Rank-reversal lines which indicate for which criteria weight the order of the ranking of the microencapsulation alternatives are reversed. In Table 4.18 column CW we see that there are two feasible values, which are the points that reverse the order of ranking. The first value is 0.4261 which shows if we decrease the weight of 'Core Wettability' more than 0.4261 we will get the reversal rank of alternative 'Pan Coating' and 'Air Suspension'. The second value is -2.6738 shows that if we increase the weight of the criterion'Core Wettability' more than 2.6738 we will get the reversal rank of alternative 'Air Suspension' and 'Coacervation'. In

Figure 4.6a, we see that if we decrease the value of 'Core Wettability' more than 0.4261 we will get 'Pan Coating' as the second preferred alternative and 'Air Suspension' as the third preferred alternative. Also, we can see that if we increase the 'Core Wettability' weight more than 2.6738 we will get 'Air Suspension' as the best preferred alternative and 'Coacervation' as the second preferred alternative.

A similar analysis can be derived from Figures 4.6b - 4.6e. In Figures 4.6b and 4.6c, we see that any increase or decrease in the weight of the 'Core Solubility' and 'Wall Elasticity' criteria can not displace the rank of 'Coacervation' from its top position. In Table 4.18, we see that in column CS and WE within the gray shaded rows all the values are '–' (non-feasible values). This means that it is impossible to reverse the rank of the top rank.

Table 4.20: *Absolute Value Changes in Alternative Weights for the Reversal of Ranks*

Alternative	Criterion					Alternative
(A_x)	CW	CS	WE	WP	WPA	(A_y)
SD	-0.1500	**-0.3956**	**-1.0174**	–	–	PC
SD	-0.2793	-1.4021	–	–	–	AS
SD	-0.5456	-2.5476	–	–	–	C
SD	**0.0769**	–	–	–	–	IP
SD	–	–	–	–	–	ISP
PC	0.1339	–	–	–	–	SD
PC	**-0.1266**	**-0.4669**	**-1.3929**	**-10.0151**	–	AS
PC	-0.3408	-1.1319	-21.7798	–	–	C
PC	–	–	–	–	–	IP
PC	–	–	–	–	–	ISP
AS	0.2651	–	–	–	–	SD
AS	**0.1347**	–	–	–	–	PC
AS	-0.1931	**-0.3756**	**-0.9479**	–	–	C
AS	0.3312	–	–	–	–	IP
AS	–	–	–	–	–	ISP
C	–	–	–	–	–	SD
C	0.2484	–	–	–	–	PC
C	**0.1321**	**0.3345**	–	–	–	AS
C	–	–	–	–	–	IP
C	–	–	–	–	–	ISP
IP	-0.0826	**-0.2086**	**-0.4526**	**-1.1266**	–	SD
IP	-0.2438	-0.7462	-3.1979	–	–	PC
IP	-0.3749	-2.6502	–	–	–	AS
IP	-0.6777	-4.7513	–	–	–	C
IP	**0.0474**	–	–	–	–	ISP
ISP	-0.1377	-0.3821	-0.9972	-3.4844	–	SD
ISP	-0.3081	-1.0763	-12.2730	–	–	PC
ISP	-0.4415	-4.5942	–	–	–	AS
ISP	-0.7746	-9.0615	–	–	–	C
ISP	**-0.0504**	**-0.1228**	**-0.2458**	**-0.4972**	**-1.2358**	IP

Notes: Bold-face indicates criticality degree Δ_{xk}(minimum values)

For the sensitivity analysis on the preference values of the alternatives with respect to the decision criteria we use Theorem 4.3.14, the absolute threshold values Θ_{xy}^{k} are shown in Table 4.20 and the relative threshold values ${\Theta_{xy}^{k}}'$ (in %)

Table 4.21: *Critical Degrees for each Alternative Performance Measure*

Alternative	Criterion				
(A_x)	CW	CS	WE	WP	WPA
SD	0.0769 (IP)	-0.3956 (PC)	-1.0174 (PC)	–	–
PC	-0.1266 (AS)	-0.4669 (AS)	-1.3929 (AS)	-10.0151 (AS)	–
AS	0.1347 (PC)	-0.3756 (C)	-0.9479 (C)	–	–
C	0.1321 (AS)	0.3345 (AS)	–	–	–
IP	**0.0474 (ISP)**	-0.2086 (SD)	-0.4526 (SD)	-1.1266 (SD)	–
ISP	-0.0504 (IP)	-0.1228 (IP)	-0.2458 (IP)	-0.4972 (IP)	-1.2358 (IP)

Notes: Bold-face indicates minimum values

Table 4.22: *Percentage Changes in Alternative Weights for the Reversal of Ranks*

Alternative	Criterion					Alternative
(A_x)	CW	CS	WE	WP	WPA	(A_y)
SD	-160.56	**-419.69**	**-1150.2**	–	–	PC
SD	-298.9	-1487.6	–	–	–	AS
SD	-583.79	-2702.9	–	–	–	C
SD	**82.317**	–	–	–	–	IP
SD	–	–	–	–	–	ISP
PC	75.219	–	–	–	–	SD
PC	**-71.145**	**-211.76**	**-607.88**	**-10405**	–	AS
PC	-191.52	-513.36	-9505.1	–	–	C
PC	–	–	–	–	–	IP
PC	–	–	–	–	–	ISP
AS	78.128	–	–	–	–	SD
AS	**39.691**	–	–	–	–	PC
AS	-56.911	**-257.92**	**-673.67**	–	–	C
AS	97.614	–	–	–	–	IP
AS	–	–	–	–	–	ISP
C	–	–	–	–	–	SD
C	80.54	–	–	–	–	PC
C	**42.819**	**75.749**	–	–	–	AS
C	–	–	–	–	–	IP
C	–	–	–	–	–	ISP
IP	-166.91	**-394.97**	**-927.57**	**-572.27**	–	SD
IP	-492.43	-1412.9	-6554.4	–	–	PC
IP	-757.33	-5018	–	–	–	AS
IP	-1369	-8996.3	–	–	–	C
IP	**95.752**	–	–	–	–	ISP
ISP	-438.38	-843.84	-2380.7	-4192.7	–	SD
ISP	-980.83	-2377.3	-29301	–	–	PC
ISP	-1405.5	-10147	–	–	–	AS
ISP	-2466.1	-20014	–	–	–	C
ISP	**-160.49**	**-271.32**	**-586.71**	**-598.28**	**-1008.6**	IP

Notes: Bold-face indicates criticality degree Δ'_{xk} (minimum values)

are shown in Table 4.22. The bold-faced entities in Table 4.20 and Table 4.22 correspond to the critical degree Δ_{xk} and $\Delta_{xk}{}'$ (i.e. the smallest entry per column in each row section), respectively. The critical degree for the absolute and rela-

Table 4.23: *Relative Critical Degrees for each Alternative Performance Measure*

Alternative (A_x)	Criterion CW	CS	WE	WP	WPA
SD	82.317 (IP)	-419.69 (PC)	-1150.2 (PC)	–	–
PC	-71.145 (AS)	-211.76 (AS)	-607.88 (AS)	-10405 (AS)	–
AS	**39.691 (PC)**	-257.92 (C)	-673.67 (C)	–	–
C	42.819 (AS)	75.749 (AS)	–	–	–
IP	95.752 (ISP)	-394.97 (SD)	-927.57 (SD)	-572.27 (SD)	–
ISP	-160.49 (IP)	-271.32 (IP)	-586.71 (IP)	-598.28 (IP)	-1008.6 (IP)

Notes: Bold-face indicates minimum values

Table 4.24: *Sensitivity Coefficient for each Alternative Performance Measure*

Alternative (A_x)	Criterion CW	CS	WE	WP	WPA
SD	0.0121 (IP)	-0.0024 (PC)	-0.0009 (PC)	0	0
PC	-0.0141 (AS)	-0.0047 (AS)	-0.0016 (AS)	-0.0001 (AS)	0
AS	0.0252 (PC)	-0.0039 (C)	-0.0015 (C)	0	0
C	0.0234 (AS)	0.0132 (AS)	0	0	0
IP	0.0104 (ISP)	-0.0025 (SD)	-0.0011 (SD)	-0.0017 (SD)	0
ISP	-0.0062 (IP)	-0.0037 (IP)	-0.0017 (IP)	-0.0017 (IP)	-0.0010 (IP)

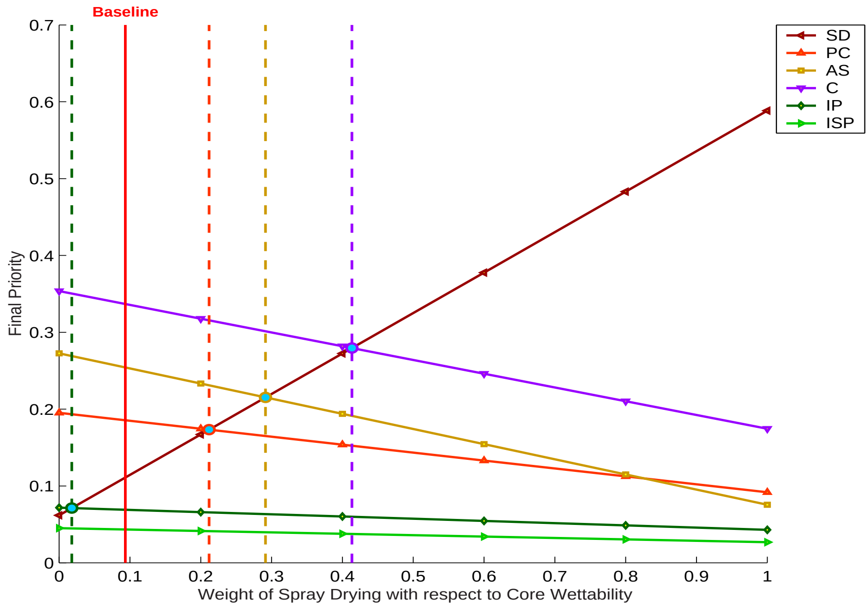


Figure 4.7: *Sensitivity Analysis Diagram for 'Spray Drying' with respect to 'Core Wettability' Criterion*

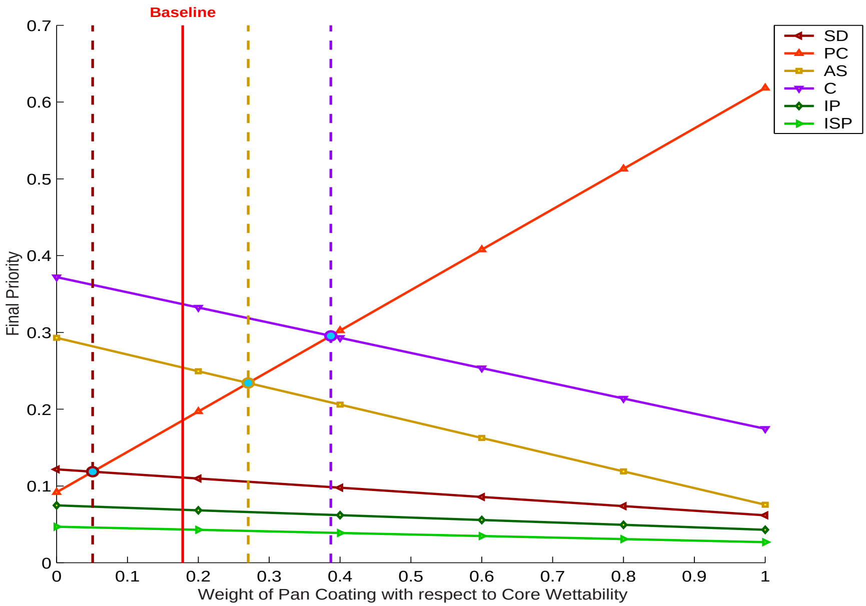


Figure 4.8: *Sensitivity Analysis Diagram for 'Pan Coating' with respect to 'Core Wettability' Criterion*

tive threshold values are summarized in Table 4.21 and Table 4.23, respectively. We notice that the corresponding alternatives from the last column in Table 4.20 and Table 4.22 are now within parentheses in the entries of Table 4.21 and Table 4.23. As before, bold-faced numbers represent corresponding minimum values. Figures 4.7 and 4.8 show the sensitivity analysis diagram for 'Spray Drying' and 'Pan Coating' with respect to 'Core Wettability', respectively. Figure 4.7 shows the performance of alternatives when the weight of the alternative 'Spray Drying' with respect to the criterion 'Core Wettability' varying between 0 and 1. Figure 4.8 shows the performance of the alternatives when weights of the alternative 'Pan Coating' with respect to the criterion 'Core Wettability' varying between 0 and 1. The rest of the sensitivity analysis diagram are shown in Appendix A.

To interpret the results in Table 4.22, let us consider one entry, for instance entry (2,1) (i.e. -298.9). This entry indicates that $\Theta_{23}^{1}{}'$ since this is the threshold value for alternative 'Spray Drying' (A_1) and alternative 'Air Suspension' (A_3) to change their position in terms of criterion 'Core Wettability' (C_1). This means that the measure of performance a_{13} must be increased by 298.9% from its current value (i.e. 0.0935 = entry (1,1) from Table 4.16) to (1+2.989)×0.0935 = 0.3730, in order for alternative 'Spray Drying' to become more preferred than alternative 'Air Suspension' (note that currently alternative 'Air Suspension' is more preferred than 'Spray Drying'). The value of the normalized: $((1 + 2.989) \times 0.0935)/((1 +$

$2.989) \times 0.0935 + 1 - 0.0935) = 0.2915$. This point is shown in Figure 4.7 where we see after 0.2915 we prefer alternative 'Spray Drying' to 'Air Suspension'. A similar interpretation holds for the rest of the entries in Table 4.22. The interpretation of the results from Table 4.20 is also the same as the interpretation from Table 4.22. The only difference is that we use the absolute value in Table 4.20 and in Table 4.22 we use the relative value.

From Table 4.21 and Table 4.23, we can conclude that the most critical alternatives based on absolute judgment is the alternative 'Interfacial Polymerization' and the one based on relative judgment is the alternative 'Air Suspension'.

Table 4.24 presents the various sensitivity analysis coefficients (as given in Definition 4.3.13). Note that if in Table 4.23 there appears a non-feasible entry (denoted by the '–' symbol), then the corresponding sensitivity coefficient in Table 4.24 is defined to be equal to 0.

4.6 Summary

In this chapter, we presented the AHP methodology as an effective decision support tool to automate decision making in the field of engineering, especially in the domain of microencapsulation process selection. AHP offers a mathematical methodology based on pairwise comparisons which is well-suited for a variety of different problems in the engineering domain and it is easily understood by the experts. To cope with the uncertainty and bias in judgments, we need to incorporate a sensitivity analysis method for handling imprecise, uncertain, and vagueness in the input data. Sensitivity analysis on the results of trade-off study is very important to reveal how the result behaves to the changes in criteria importance or performance of alternatives. This chapter also presented an adaptation and extension methodology of sensitivity analysis on the criteria weights and the preferences of alternatives.

The creation of the pairwise comparison matrices and data acquisition is a tedious and time-consuming task, especially if we want to apply this technique to a real case which has more alternatives to be compared. In this case study, the total of $10+5\times 15 = 85$ pairwise comparisons were formed. If the number of alternatives increase to 100, the decision maker needs to evaluate 24.760 pairwise comparisons. We see that although pairwise comparisons have been seen by many as an effective and intuitive way for eliciting qualitative data for multi criteria decision making problems, a major drawback is that the number of the required comparisons increases quadratically with the number of the elements to be compared. Thus, even data for medium size decision problems may be often impractical to be elicited via pairwise comparisons. The more the comparisons there are, the higher is the likelihood that the decision maker will introduce erroneous data. In the next chapter, we propose a modified AHP, called Base Reference Analytical Hierarchy Process (BR-AHP), which uses base pairwise comparison method to reduce the number of pairwise comparison of the alternatives.

Chapter 5

Base Reference Analytical Hierarchy Process

The previous chapter has shown that Analytical Hierarchy Process (AHP) can be useful for aiding the engineers to select the most appropriate engineering process, but this method had some limitations when it is applied to a larger problem when we have a lot of alternatives to select from. The drawback of AHP is that the exhaustive pairwise comparison is tiresome and time consuming when there are many alternatives to be considered. In this chapter, we propose a new approach to improve this limitation of AHP, the so-called Base Reference Analytical Hierarchy Process (BR-AHP). The BR-AHP method draws upon ideas from engineering, where most of the engineers are only familiar with one base technique and they are working by comparing the other techniques with their own base technique. This study is accompanied by a case study to show the application of BR-AHP in an engineering process selection, i.e. in microencapsulation process selection problem. This chapter proposes a new methodology which can be used to create the basis of a Decision Support System (DSS) for the engineering selection problems. This chapter extends the work in (Hotman, 2005a,b).

5.1 Introduction

Recent developments in material and manufacturing technology have brought many exciting changes in the field of engineering. Engineers can benefit from a wider selection of materials and processing techniques. They are often faced with the problem of selecting a solution from a given set of finite number of alternatives. The Multiple Attribute Decision Making (MADM) methods are used as aids for modeling engineering and management decisions in evaluating and/or selecting the desired one from a finite number of alternatives, which are characterized by multiple attributes. The Analytical Hierarchy Process (AHP) (Saaty, 1980) is a popular MADM method that utilizes structured pairwise comparisons and this method has been used widely in engineering and industry to aid the selection process. AHP is a multiple criteria decision making tool that has been used in almost all the applications related with decision making and has been applied in many different fields. The literature review and overview of the applications of AHP can be found in Zahedi (1986) and Vaidya & Kumar (2006). The AHP has been applied to a wide area (Golden et al., 1989), including a wide variety of engineering selection problems (Akash et al., 1999; Braglia & Petroni, 1999; Karacal et al., 1996; Kontio, 1996; Golden et al., 1989; Yurdakul, 2004; Hanratty & Joseph, 1992), e.g. machine tool selection, plant selection, technology selection, software selection, project selection, site selection, reactor selection and many other selection problems.

The AHP has been the subject of much controversy, particularly to do with the question of it apparently being the cause of rank reversal in some circumstances. Triantaphyllou (2001) proved that the rank reversals are not possible when a multiplicative variant of the AHP is used. The other issue which has been a concern here is the large pairwise comparisons involved in the AHP when there are many alternatives involved in the decision making process.

The AHP method involves the comparison of several candidate alternatives, which are pairwise compared using several different criteria. Pairwise comparisons can be used to determine the relative importance of each alternative with respect to each criterion. Using this method the decision maker has to express his/her opinion about the value of a single pairwise comparison at a time and this can be tiresome and time consuming, especially if there are many pairs. When pairwise comparisons are used the decision making process may become impractical when the number of the entities, i.e. alternatives or criteria, to be compared become large. If n is the number of the entities, then the number of all possible comparisons is equal to $n(n-1)/2$. For example, for $n = 100$ the decision maker would have to make a total of 4.950 pairwise comparisons for each criterion.

Many methods for reducing the number of pairwise comparisons have been proposed. One of the ideas is to use hierarchical decomposition (Scott, 2002). A large number of criteria is to be collected into smaller groups. The criteria within each group are compared to each other, and then the groups themselves are compared to each other. However, this method only works for reducing pairwise comparison among the criteria and it does not work for reducing the number of pairwise comparisons of the alternatives. Another approach is to use the duality approach, which has been proposed by Triantaphyllou (1999, 2000), to reduce the total number of pairwise comparisons when the number of alternatives is larger than the number of criteria plus one. However, with this approach the total number of pairwise comparisons is still quadratic with respect to the number of alternatives.

In engineering, most of the engineers are usually familiar only with one technique and they work by comparing their own technique with other techniques. If, for example, they want to compare two techniques they are not familiar with, they will first compare the first technique with their known technique, then compare the second technique also to the known one. Finally, they can deduce the comparison of the two techniques. So there would usually exist a base that they refer to, in this case their own technique that they know. A new method, the so-called Base Reference Analytical Hierarchy Process (BR-AHP), draws upon this idea. In this chapter, a new approach BR-AHP, which is the improvement of the AHP approach, is proposed to reduce the number of pairwise comparisons of the alternatives. This new methodology is applicable to a large range of engineering problems and it is easily implemented into an expert system.

The rest of this chapter is organized as follows. The next section describes the proposed BR-AHP algorithm. In the third section, we present analytically the reduction caused by the BR-AHP method. Then, the forth section presents the application of BR-AHP in an engineering process selection, i.e. in microencapsulation process selection problem. Next, the fifth section demonstrates the sensitivity analysis method used for BR-AHP approach. Finally, some conclusions are discussed in the final section.

5.2 Base Reference Analytical Hierarchy Process

The Base Reference Analytical Hierarchy Process (BR-AHP) is proposed to overcome the limitation of AHP when there are a large number of pairwise comparisons of the alternatives. The development of this approach was first reported in Hotman (2005a,b). In this approach, a particular pairwise comparisons technique, the so-called base pairwise comparison, was developed. The BR-AHP approach will be described in detail in the following subsections.

The BR-AHP algorithm consists of five steps:

1. The construction of the hierarchy structure of the selection problem.

2. The evaluation of the relative importance of the criteria using pairwise comparison method.

3. The selection of the base alternative and the evaluation of the base alternative relative to each other alternatives on the basis of each selection criterion using base pairwise comparison method.

4. The integration of the ratings derived in steps (2) and (3) to obtain an overall relative ranking for each potential alternative.

5. The sensitivity analysis to examine the effects of the changes of the judgments and preferences of the decision maker.

5.2.1 Decision Hierarchy Structure Construction

The first step of the BR-AHP algorithm is the construction of the hierarchical structure of the problems as in the original AHP algorithm. A decision problem is disaggregated into a hierarchy of interrelated decision elements or attributes (i.e., goal, evaluation criteria and solution alternatives). The overall objective/goal lies at the top of the hierarchy, while the lower levels of the hierarchy contain more detailed descriptions of criteria and the lowest level of the hierarchy is the alternative level. Figure 5.1 shows the structure of BR-AHP decision hierarchy which only consists of three layer.

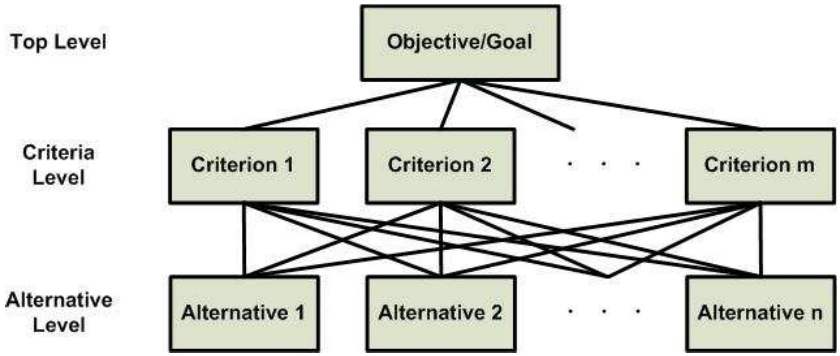


Figure 5.1: *Three Layer BR-AHP Hierarchy Structure*

5.2.2 Criteria Evaluation using Pairwise Comparison

The second step of the BR-AHP algorithm is an evaluation of criteria by pairwise comparison of the decision criteria elements. Let us consider m criteria to be evaluated, then we need to construct one $m \times m$ matrix to derive the criteria weights. We can construct the pairwise matrix between goal and criteria layer, which is shown in Eq. (5.1).

$$A = \begin{bmatrix} \frac{w_1}{w_1} & \frac{w_1}{w_2} & \cdots & \frac{w_1}{w_m} \\ \frac{w_2}{w_1} & \frac{w_2}{w_2} & \cdots & \frac{w_2}{w_m} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{w_m}{w_1} & \frac{w_m}{w_2} & \cdots & \frac{w_m}{w_m} \end{bmatrix} = \begin{bmatrix} 1 & a_{12} & \cdots & a_{1m} \\ 1/a_{12} & 1 & \cdots & a_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ 1/a_{1m} & 1/a_{2m} & \cdots & 1 \end{bmatrix} \tag{5.1}$$

where A is the pairwise comparison matrix (size $m \times m$); $\frac{w_i}{w_j}$ represents the relative importance of the i-th criterion over the j-th criterion ($i, j \in 1, 2, \ldots, m$). In general the value of $\frac{w_i}{w_j}$ is given subjectively by a decision maker. There are a total of $m(m-1)/2$ judgments which are required to develop this matrix. Reciprocals are automatically assigned in each pairwise comparison shown in Eq. (5.1). The matrices of criteria layer vs. sub-criteria layer can be considered in the same way.

5.2.3 Alternatives Evaluation using Base Pairwise Comparison

The third step of the BR-AHP algorithm is the evaluation of decision alternatives by base pairwise comparison of the decision alternative elements. In constructing the pairwise matrices we collect much more information than we need. In fact, in order to fill the right upper corner of a pairwise comparison matrix, in a problem with n elements of alternatives, the decision maker has to carry out a total of $n(n-1)/2$ judgments, whereas $n-1$ properly chosen judgments would have been sufficient. However, this major amount of information could be used in case we have to manage a base pairwise comparison matrix. The base pairwise matrix comparison between criteria layer and alternative layer is thus defined:

$$\begin{aligned} B &= \begin{bmatrix} \frac{w_1}{w_1} & \frac{w_1}{w_2} & \cdots & \frac{w_1}{w_k} & \cdots & \frac{w_1}{w_n} \\ \frac{w_2}{w_1} & \frac{w_2}{w_2} & \cdots & \frac{w_2}{w_k} & \cdots & \frac{w_2}{w_n} \\ \vdots & \vdots & \ddots & \vdots & & \vdots \\ \frac{w_k}{w_1} & \frac{w_k}{w_2} & \cdots & \frac{w_k}{w_k} & \cdots & \frac{w_k}{w_n} \\ \vdots & \vdots & & \vdots & \ddots & \vdots \\ \frac{w_n}{w_1} & \frac{w_n}{w_2} & \cdots & \frac{w_n}{w_k} & \cdots & \frac{w_n}{w_n} \end{bmatrix} \\ &= \begin{bmatrix} 1 & b_{12} & \cdots & b_{1k} & \cdots & b_{1n} \\ b_{21} & 1 & \cdots & b_{2k} & \cdots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots & & \vdots \\ b_{k1} & b_{k2} & \cdots & 1 & \cdots & b_{kn} \\ \vdots & \vdots & & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nk} & \cdots & 1 \end{bmatrix} \end{aligned} \tag{5.2}$$

where B is the base pairwise comparison matrix (size $n \times n$); k denotes the base; $\frac{w_k}{w_j}$ represents the relative importance of the k-th alternative over the j-th alternative $(k, j \in 1, 2, \ldots, n)$.

In this case, a decision maker only needs to determine the base alternative and then compare the base alternative with other alternatives (b_{kj}). Next, we need to evaluate the other values of the base pairwise comparison matrix using the information derived from the available base elements of the base pairwise comparison matrix. Here, the value of the base elements of the base pairwise comparison matrix represents the ratio between the weights of the base alternative and another alternative with respect to a defined criterion. For example b_{kj} is the ratio between the weight of the k-th alternative (w_k) and the weight of the j-th alternative $(w_j; j \in 1, \ldots, n; j \neq k)$ with respect to a common criterion. As a consequence we can write: $b_{kj} = w_k/w_j$.

Table 5.1: *A Measurement Scale for BR-AHP*

Verbal Judgment	Degree of Preference
Equally Preferred	1
Moderately Preferred	3
Strongly Preferred	5
Very Strongly Preferred	7
Extremely Preferred	9
Intermediate Values	2, 4, 6, 8

If we do not have any evaluation for b_{ij}, but we do have an evaluation for b_{ki} and b_{kj}, then we can calculate the missing value by using the relations below:

$$b_{ij} = w_i/w_j = \frac{w_k/w_j}{w_k/w_i} = \frac{b_{kj}}{b_{ki}} \tag{5.3}$$

Using this formula we can calculate the value of the missed element b_{ij} by means of information derived by the base pairwise comparison matrix without making any assumption on the value of the missed element. Using Eq. (5.3) reciprocals of base elements are also automatically assigned, which is shown in Eq. (5.4).

$$b_{ik} = \frac{b_{kk}}{b_{ki}} = \frac{1}{b_{ki}} \tag{5.4}$$

In our study, the judgment scale used here is a nine-point scale, which was proposed by Saaty. Table 5.1 shows the measurement scale used for BR-AHP. By using Saaty's nine-point scale the range of judgment is from $\frac{1}{9}$ (extremely less important) to 9 (extremely more important). To ensure consistency with the nine-point scale, we have to modify the automatic value assignment in base pairwise comparison matrix. To avoid a value higher than 9, we use the min operation to choose the lower one. And to avoid a value lower than $\frac{1}{9}$, we use the max operation to choose the higher one. Then, the modified base pairwise comparison matrix is expressed as follows:

$$\begin{aligned} B &= [b_{ij}]_{n\times n}, \ b_{ij} > 0, \ i,j = 1,2,\ldots,n \\ b_{ij} &= \begin{cases} 1 & i = j \\ \min\left\{9, \frac{b_{kj}}{b_{ki}}\right\} & \frac{b_{kj}}{b_{ki}} > 1, \ i \neq j \\ \max\left\{\frac{1}{9}, \frac{b_{kj}}{b_{ki}}\right\} & \frac{b_{kj}}{b_{ki}} < 1, \ i \neq j \end{cases} \end{aligned} \tag{5.5}$$

5.2.4 Integration of the Evaluation of Criteria and Alternatives

Finally, the weights developed at each level of the hierarchy are aggregated into an overall ranking for the alternatives, and the alternative with the highest score is considered to dominate. Let us consider that we have a decision problem with n alternatives (A_i, $i = 1, \ldots, n$) and m criteria (C_j, $j = 1, \ldots, m$). Consider that we have all the weights of criteria and all the performance values of the alternatives with respect to each criterion. Let $w_{C_1}, w_{C_2}, \ldots, w_{C_m}$ denotes the weight of the criteria and a_{ij} ($i = 1, 2, \ldots, n, j = 1, 2, \ldots, m$) is the performance value of the i-th alternative with respect to j-th criterion. The weight of the i-th alternative can be obtained as a weighted sum of performances as the follows:

$$w_{A_i} = \sum_{j=1}^{m} w_{C_j} a_{ij}, \quad i = 1, 2, \ldots, n.$$

5.2.5 Sensitivity Analysis of BR-AHP Method

The main purpose in sensitivity analysis is to examine how sensitive the choices are to the changes in the decision maker's judgments, i.e. to the changes in criteria weights or to the changes in the performances of the alternatives with respect to a defined criterion. This is useful in situations where uncertainties exist in the assessment of the importance of different factors. We proposed a methodology for performing sensitivity analysis on the weights of the decision criteria and on the weights of the alternatives with respect to each criterion in Chapter 4. In this chapter, we propose a methodology for sensitivity analysis for BR-AHP approach by only changing the preferences on the base alternatives. In BR-AHP method, a decision maker needs only to identify the base alternative and then compare the base alternative with other alternatives (b_{kj}). Thus we can perform the sensitivity analysis by changing the preference values of the comparison between base alternative and other alternatives from $\frac{1}{9}$ (extremely less preferred) to 9 (extremely more preferred).

When the base pairwise comparison is employed, the sensitivity analysis depicts how well each alternative performs on each criterion by increasing or decreasing the importance of the base alternative with respect to other alternatives based on a certain criterion.

5.3 Reduction by Base Pairwise Comparison Method

Consider a problem with m criteria and n alternatives. In the previous section, we have proposed the BR-AHP algorithm and have showed that the BR-AHP method reduces the total pairwise comparison required by the decision maker by using the base pairwise comparison method at the alternative level. In the original AHP method using the pairwise comparison method it needs to estimate $n(n-1)/2$ judgments for n alternatives, but in the BR-AHP method using the base pairwise comparison method it only needs $n - 1$ judgments.

The next question which is raised at this point is under which conditions the number of comparisons in the BR-AHP approach is smaller than in the AHP approach. We will present the answer to this question in the following theorem and corollary.

Theorem 5.3.1. *The rate of reduction of the number of comparisons (in %) by using BR-AHP approach is given by the following formula:*

$$\begin{aligned}\frac{\mathit{AHP}(m,n) - \mathit{BR\text{-}AHP}(m,n)}{\mathrm{AHP}(m,n)} \times 100 \\ = \frac{(n-1)\,(n-2)}{n\,(n-1)+m-1} \times 100 \qquad (5.6)\end{aligned}$$

where: m = *the total number of criteria*
n = *the total number of alternatives*

Proof. We consider the decision problem involving n decision alternatives and m decision criteria. In the conventional AHP approach, one $m \times m$ judgment matrix is required to derive the criteria weights and each of the m judgment matrices of size $n \times n$ are required to derive the relative weights of the n alternatives in terms of each one of the m decision criteria. Here for each criterion, $n(n-1)/2$ pairwise comparisons need to be determined. So for m criteria, $mn(n-1)/2$ pairwise comparisons need to be determined. The total pairwise comparisons needed for the criteria matrix are $m(m-1)/2$ pairwise comparisons. Thus, the total number of the required pairwise comparisons according to the conventional AHP approach is equal to

$$\begin{aligned}\frac{m(m-1)}{2} + m\frac{n(n-1)}{2} &= \frac{m}{2}\,(m-1+n(n-1)) \\ &= \frac{m}{2}\left(n^2-n+m-1\right) \qquad (5.7)\end{aligned}$$

Similarly, in the BR-AHP approach the decision maker needs to construct one matrix of size $m \times m$ for the weights of the decision criteria and m judgment matrices of size $n \times n$ by only determining $n-1$ judgments. In this approach for each criterion, only $n-1$ pairwise comparisons need to be determined. So for m criteria, $m(n-1)$ pairwise comparisons need to be determined. Therefore, the total number of pairwise comparisons using the BR-AHP approach is

$$\begin{aligned}\frac{m(m-1)}{2} + m(n-1) &= \frac{m}{2}\,(m-1+2(n-1)) \\ &= \frac{m}{2}\,(2n+m-3) \qquad (5.8)\end{aligned}$$

Then, the decrease on the number of comparisons can be written as the difference of the expressions in Eq. (5.8) and Eq. (5.7), given as Eq. (5.9):

$$\begin{aligned}\frac{m}{2}\left(n^2-n+m-1\right) - \frac{m}{2}\,(2n+m-3) &= \frac{m}{2}\left(n^2-3n+2\right) \\ &= \frac{m}{2}\,(n-1)\,(n-2) \qquad (5.9)\end{aligned}$$

Therefore, the rate of reduction (in %) of the number of comparisons between the AHP and the BR-AHP is given in Eq. (5.6). ■

Corollary 5.3.2. *The BR-AHP approach requires less pairwise comparisons than the AHP approach if the number of alternatives in the decision problem is greater than two.*

Proof. This follows directly from the fact that expression in Eq. (5.6) must be greater than zero. Thus, $n - 2 > 0$, or $n > 2$. ■

Thus, if a decision problem has more than two alternatives, then the decision making process can explicitly benefit from the smaller number of comparisons needed by the proposed BR-AHP approach. In most of the cases, especially in real life problems, the proposed method can be used to its full advantage, since the number of alternatives to be evaluated or to be compared is usually more than two.

Another way in which the BR-AHP approach is different from the AHP approach is that the use of consistency check is not essential. In AHP, the consistency check is essential to ensure the consistency of the decision maker. The consistency can be checked by calculating the consistency ratio. The index of consistency check is: $CI = (\lambda_{\max} - n)/(n - 1)$, where n is the matrix size and $\lambda_{\max}$ is the maximal eigenvalue. The ratio of the consistency check is $CR = CI/RI$, where RI is the average random consistency index as shown in Table 4.2. When $CR \leq 0.1$, it is acceptable, else, the judgment matrix must be adjusted and the decision maker needs to readjust his/her judgment. This process can be tiresome if the judgment matrix is large. This can also become a burden to the decision maker to check the consistency if he/she always has to readjust his/her judgment. If the judgment matrix is large, the decision maker sometimes doesn't even know where the problem is and which judgement to modify. In BR-AHP approach, the consistency check is not essential, since the automated generated judgments automatically improve the consistency of the decision maker.

5.4 A Case Study of BR-AHP in Engineering Process Selection

To give an illustration of the application of BR-AHP to an engineering process selection, we shall use it to select the best microencapsulation method.

Microencapsulation is the name given to a novel technique for the preparation of small substances, which began to develop about 50 years ago. Many different techniques have been proposed for the production of microcapsules by academics and industrial researchers. Nowadays more than 1000 methods can be identified in the patent literature (Gouin, 2004). Many methods exist for the production of microcapsules which vary in detail depending on core and wall-polymer solubility, capsule size, wall thickness and permeability, type and rate of release of core

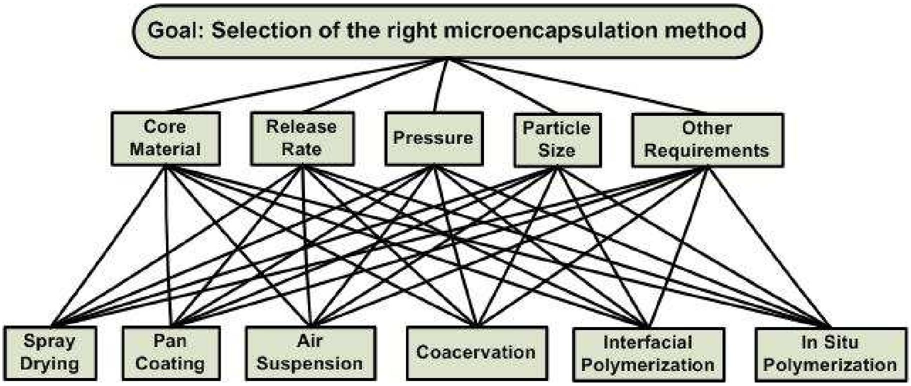


Figure 5.2: *Microencapsulation Process Selection Hierarchy*

Goal	CoreMaterial	ReleaseRate	Pressure	ParticleSize	OtherRequiements	Weight
CoreMaterial	1.0	2.0	3.0	4.0	5.0	42%
ReleaseRate	0.5	1.0	2.0	3.0	4.0	26%
Pressure	0.3333	0.5	1.0	2.0	3.0	16%
ParticleSize	0.25	0.3333	0.5	1.0	2.0	10%
OtherRequiements	0.2	0.25	0.3333	0.5	1.0	6%

Figure 5.3: *Pairwise Comparison of Microencapsulation Criteria*

contents, and physical properties. Unfortunately, there are no single microencapsulation method which is suitable for all applications. Each microencapsulation method has its own advantages and disadvantages which vary with the usage and the application (Finch & Bodmeier, 2002). In this case study, we will only consider the case of selecting the best microencapsulation technique from six alternative methods.

The first step in the BR-AHP algorithm is to construct the hierarchy structure of microencapsulation selection problem. The goal of this case study is to select the right microencapsulation method which is placed at the top level of the BR-AHP hierarchy as shown in Figure 5.2. In this case study, six microencapsulation methods were considered – Spray Drying, Pan Coating, Air Suspension, Coacervation, Interfacial Polymerization and In-Situ Polymerization method. The three former methods belong to the physical methods and the three latter methods belong to the chemical methods. After discussing with a chemical engineer, five important selection criteria for microencapsulation selection problem were considered to be relevant to this case study, i.e. core material, release rate, pressure, particle size and other requirements.

Having defined the selection criteria, the next step in the BR-AHP algorithm is the pairwise comparison of the relative importance of the microencapsulation criteria with respect to the goal. This is done by assigning a priority weight to

Table 5.2: *Number of Pairwise Comparisons by AHP and BR-AHP*

m	5			10		
n	AHP	BR-AHP	Δ	AHP	BR-AHP	Δ
2	15	15	0	55	55	0
3	25	20	5	75	65	10
4	40	25	15	105	75	30
5	60	30	30	145	85	60
6	85	35	50	195	95	100
7	115	40	75	255	105	150
8	150	45	105	325	115	210
9	190	50	140	405	125	280
10	235	55	180	495	135	360
100	24760	505	24255	49545	1035	48510
1000	2497510	5005	2492505	4995045	10035	4985010

each of the criterion by using pairwise comparison method. The results of this operation are presented in Figure 5.3. Here we have a comparison of 5 criteria, so the decision maker needs to determine 10 pairwise comparisons.

The third step in the BR-AHP algorithm is the base pairwise comparison of the microencapsulation alternatives with respect to each criterion. Figures 5.4a - 5.4e show this process. First, the decision maker had to select the base alternative. In Figures 5.4a - 5.4e, the base alternative is highlighted horizontally. In this case study, the decision maker selected a microencapsulation alternative 'Coacervation' as the base alternative because he is more familiar with this method. After choosing the base alternative, he would have to compare the selected method with other methods. In this step, the decision maker only needed to determine five judgments by comparing the base alternative 'Coacervation' with the other five microencapsulation alternatives.

The fourth step in the BR-AHP algorithm is to combine all the weights derived in the previous two steps to obtain the overall ranking for the alternatives. The result of the ranking of microencapsulation alternatives is shown in Figure 5.5.

For the case with 5 criteria and 6 alternatives, using BR-AHP method we only need a total of 35 pairwise comparisons. If we use AHP method, then we need a total of 85 pairwise comparisons. In this case, we can reduce the pairwise comparisons by 50 using BR-AHP method.

Table 5.2 shows the total pairwise comparisons needed for both methods and the reduced number of pairwise comparisons by using BR-AHP method in the case of m criteria and n alternatives. Δ is the total number of reduced pairwise comparisons by using BR-AHP method. From Table 5.2, we see that when the number of alternatives grows the BR-AHP approach reduces more (i.e. Δ becomes larger). For the case with 5 criteria and 10 alternatives, by using BR-AHP method we only need 55 pairwise comparisons. If we use AHP method, then we would have needed 235 pairwise comparisons. In this case, we can reduce pairwise

CoreMaterial	SprayDrying	PanCoating	AirSuspension	Coacervation	InterfacialPolymeri...	InSituPolymerization	Weight
SprayDrying	1.0	0.75	0.5	0.25	1.0	1.25	10%
PanCoating	1.3333	1.0	0.6667	0.3333	1.3333	1.6667	13%
AirSuspension	2.0	1.5	1.0	0.5	2.0	2.5	20%
Coacervation	4.0	3.0	2.0	1.0	4.0	5.0	39%
InterfacialPolymerizat...	1.0	0.75	0.5	0.25	1.0	1.25	10%
InSituPolymerization	0.8	0.6	0.4	0.2	0.8	1.0	8%

(a) Core Material

ReleaseRate	SprayDrying	PanCoating	AirSuspension	Coacervation	InterfacialPolymeri...	InSituPolymerization	Weight
SprayDrying	1.0	1.6667	2.5	5.0	9.0	9.0	38%
PanCoating	0.6	1.0	1.5	3.0	9.0	9.0	27%
AirSuspension	0.4	0.6667	1.0	2.0	6.0	9.0	19%
Coacervation	0.2	0.3333	0.5	1.0	3.0	5.0	10%
InterfacialPolymerizat...	0.1111	0.1111	0.1667	0.3333	1.0	1.6667	4%
InSituPolymerization	0.1111	0.1111	0.1111	0.2	0.6	1.0	3%

(b) Release Rate

Pressure	SprayDrying	PanCoating	AirSuspension	Coacervation	InterfacialPolymeri...	InSituPolymerization	Weight
SprayDrying	1.0	0.125	0.75	0.25	0.1111	1.25	4%
PanCoating	8.0	1.0	6.0	2.0	0.6667	9.0	31%
AirSuspension	1.3333	0.1667	1.0	0.3333	0.1111	1.6667	5%
Coacervation	4.0	0.5	3.0	1.0	0.3333	5.0	16%
InterfacialPolymerizat...	9.0	1.5	9.0	3.0	1.0	9.0	41%
InSituPolymerization	0.8	0.1111	0.6	0.2	0.1111	1.0	3%

(c) Pressure

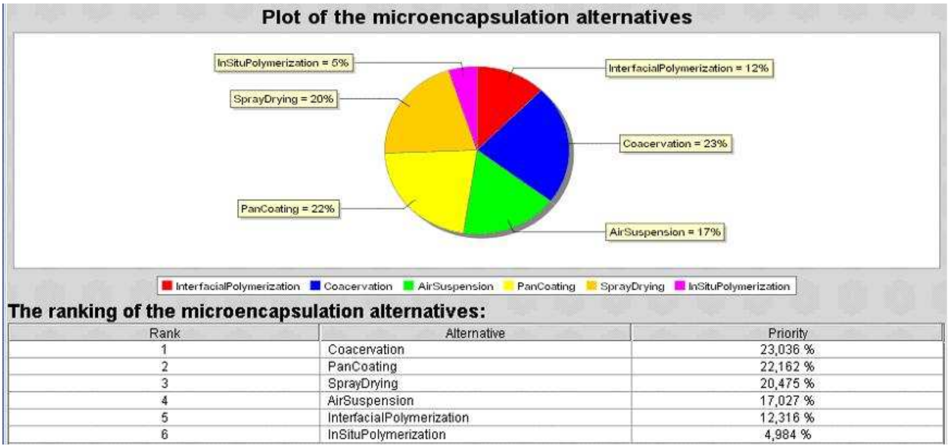
ParticleSize	SprayDrying	PanCoating	AirSuspension	Coacervation	InterfacialPolymeri...	InSituPolymerization	Weight
SprayDrying	1.0	1.25	2.5	5.0	9.0	9.0	37%
PanCoating	0.8	1.0	2.0	4.0	8.0	9.0	31%
AirSuspension	0.4	0.5	1.0	2.0	4.0	8.0	17%
Coacervation	0.2	0.25	0.5	1.0	2.0	4.0	9%
InterfacialPolymerizat...	0.1111	0.125	0.25	0.5	1.0	2.0	4%
InSituPolymerization	0.1111	0.1111	0.125	0.25	0.5	1.0	3%

(d) Particle Size

OtherRequirements	SprayDrying	PanCoating	AirSuspension	Coacervation	InterfacialPolymeri...	InSituPolymerization	Weight
SprayDrying	1.0	1.3333	2.0	4.0	9.0	9.0	34%
PanCoating	0.75	1.0	1.5	3.0	9.0	9.0	28%
AirSuspension	0.5	0.6667	1.0	2.0	6.0	9.0	20%
Coacervation	0.25	0.3333	0.5	1.0	3.0	6.0	11%
InterfacialPolymerizat...	0.1111	0.1111	0.1667	0.3333	1.0	2.0	4%
InSituPolymerization	0.1111	0.1111	0.1111	0.1667	0.5	1.0	3%

(e) Other Requirements

Figure 5.4: *Base Pairwise Comparison of Microencapsulation Alternatives with respect to Each Criterion*



Rank	Alternative	Priority
1	Coacervation	23,036 %
2	PanCoating	22,162 %
3	SprayDrying	20,475 %
4	AirSuspension	17,027 %
5	InterfacialPolymerization	12,316 %
6	InSituPolymerization	4,984 %

Figure 5.5: *Microencapsulation Process Selection Result*

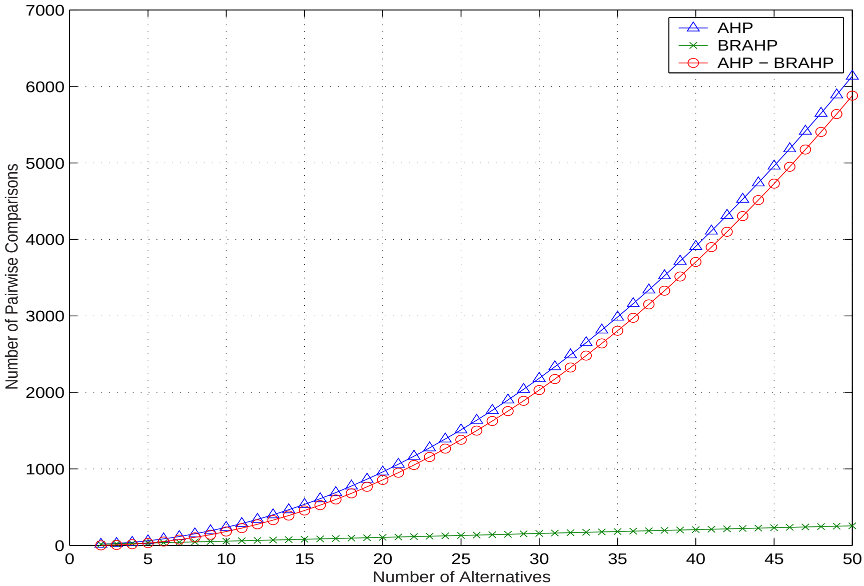


Figure 5.6: *Total Number of Comparisons and Reduction achieved when the BR-AHP is used with Number of Criteria $m = 5$.*

comparisons by 180 using BR-AHP approach. This represents a reduction of 76.6 % from the total number of pairwise comparison required using the AHP approach. In real-life cases for engineering process selections, there exist normally hundreds or even thousands of alternatives. In this case, we can reduce pairwise

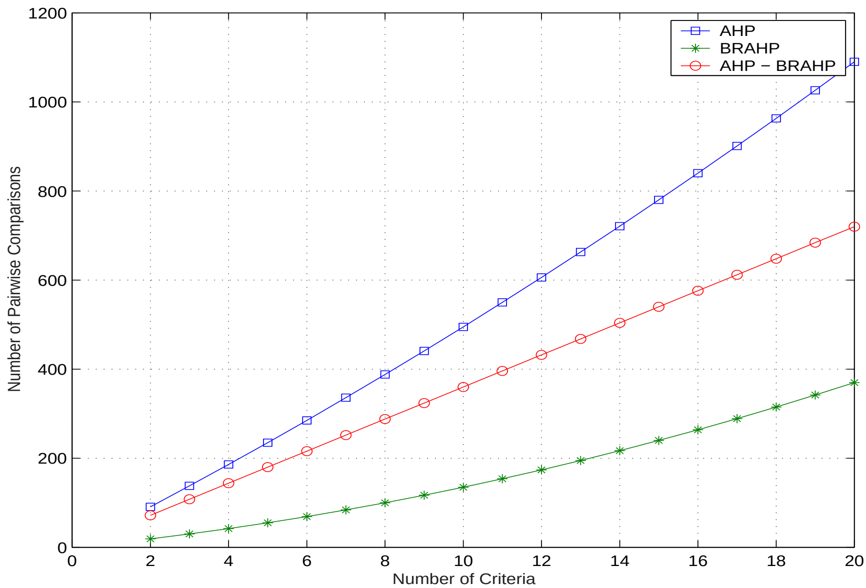


Figure 5.7: *Total Number of Comparisons and Reduction achieved when the BR-AHP is used with Number of Alternatives $n = 10$.*

comparisons even more by using BR-AHP approach.

Figure 5.6 shows the difference of pairwise comparisons of both methods (AHP and BR-AHP) and the number of comparisons reduced by using BR-AHP methods when we fixed the number of criteria $m = 5$. In Figure 5.7 we see the number of comparisons needed by both methods and the reduced number of comparisons caused by using BR-AHP methods when the number of alternatives is fixed (here we fixed the number of alternatives $n = 10$). Figure 5.8 shows the reduction of the number of comparisons (in %) when the BR-AHP approach is used.

5.5 An Illustration of the Sensitivity Analysis of BR-AHP

In practice, sensitivity analysis should be carried out to determine the robustness of such decisions with respect to variations in the pairwise rankings. An analysis can be made based on the changes in the significance of the preferences of base alternative to others. In using the base pairwise comparison method, a decision maker needs only to choose the base alternative and then compare the preference of the base alternative to other alternatives. We perform the sensitivity analysis of base pairwise comparison by changing the preferences of the comparison between base alternative and other alternatives from extremely less preferred to extremely more preferred (from $\frac{1}{9}$ to 9).

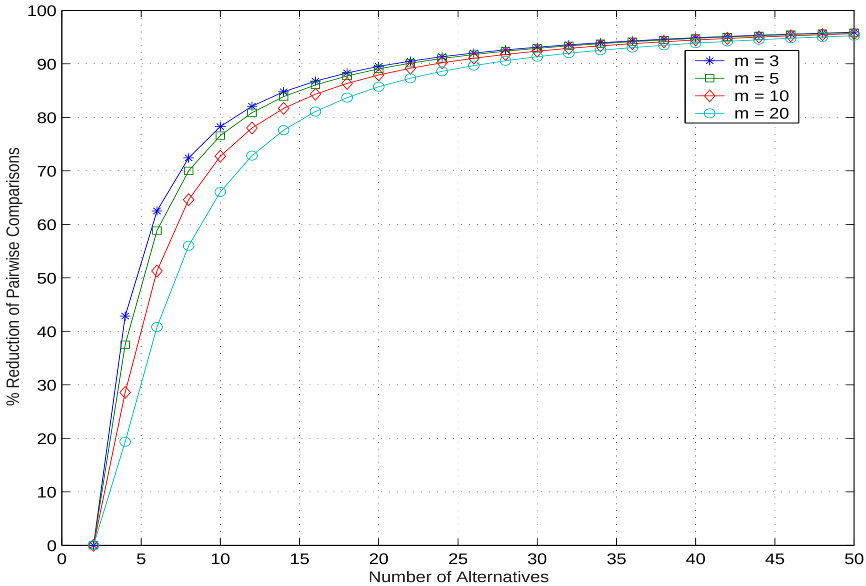
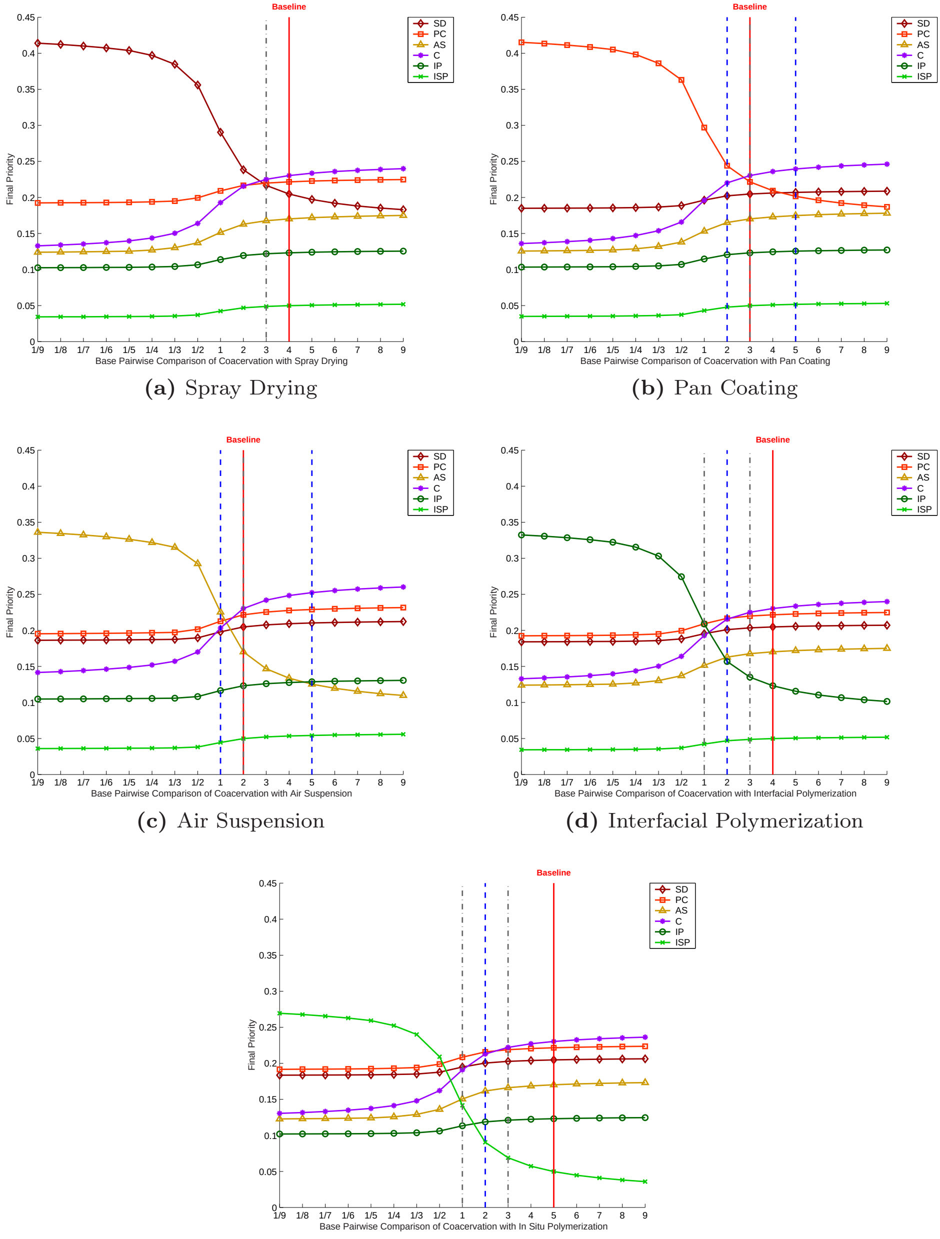


Figure 5.8: *Percent Reduction on the Number of Comparisons when the BR-AHP is used.*

Figures 5.9a - 5.9e show the sensitivity analysis results for BR-AHP method by adjusting the preferences of the base alternative 'Coacervation' to other alternatives, i.e. 'Spray Drying', 'Pan Coating', 'Air Suspension', 'Interfacial Polymerization', and 'In-Situ Polymerization', with respect to the criterion 'Core Material', respectively. The original weight of the respective comparison between base alternative 'Coacervation' and other alternatives is marked by a solid vertical line labeled 'Baseline' on the top of it. The dashed vertical lines are the rank-reversal lines that indicate at which corresponding criterion weight the rank reversal occurs. The rest of the sensitivity analysis results are shown in Appendix B.

In Figure 5.9a we see the trends of alternative preferences when we adjusted the base pairwise comparison of the base alternative 'Coacervation' to microencapsulation alternative 'Spray Drying' with respect to the criterion 'Core Material'. If we adjust the value of the comparison between base alternative 'Coacervation' and alternative 'Spray Drying' less than 3, then the alternative 'Spray Drying' will be the best alternative, otherwise the base alternative 'Coacervation' will be the best alternative.

Figure 5.9b presents the trends of alternative preferences when we adjusted the base pairwise comparison of the base alternative 'Coacervation' to microencapsulation alternative 'Pan Coating' with respect to the criterion 'Core Material'. If we adjust the value of the comparison between base alternative 'Coacervation' and alternative 'Pan Coating' less than 5, then we prefer the alternative 'Pan Coat-



(a) Spray Drying

(b) Pan Coating

(c) Air Suspension

(d) Interfacial Polymerization

(e) In Situ Polymerization

Figure 5.9: *Trends of Alternatives Preference Weights by adjusting Base Pairwise Comparison of 'Coacervation' for 'Core Material' Criterion*

ing' than alternative 'Spray Drying', otherwise we prefer the alternative 'Spray Drying' than alternative 'Pan Coating'. If we adjust the value of the comparison between base alternative 'Coacervation' and alternative 'Pan Coating' less than 3, then the alternative 'Pan Coating' will be the best alternative, otherwise the base alternative 'Coacervation' will be the best alternative. If we adjust the value of the comparison between base alternative 'Coacervation' and alternative 'Pan Coating' less than 2, then we prefer the alternative 'Spray Drying' than base alternative 'Coacervation', otherwise we prefer the base alternative 'Coacervation' than alternative 'Spray Drying'.

In Figure 5.9c we observe the trends of alternative preferences when we adjusted the base pairwise comparison of the base alternative 'Coacervation' to microencapsulation alternative 'Air Suspension' with respect to the criterion 'Core Material'. If we adjust the value of the comparison between base alternative 'Coacervation' and alternative 'Air Suspension' less than 5, then we prefer the alternative 'Air Suspension' than alternative 'Interfacial Polymerization', otherwise we prefer the alternative 'Interfacial Polymerization' than alternative 'Air Suspension'. If we adjust the value of the comparison between base alternative 'Coacervation' and alternative 'Air Suspension' less than 2, then the alternative 'Air Suspension' will be the best alternative, otherwise the base alternative 'Coacervation' will be the best alternative. If we adjust the value of the comparison between base alternative 'Coacervation' and alternative 'Air Suspension' less than 1, then we prefer the alternative 'Spray Drying' than base alternative 'Coacervation', otherwise we prefer the base alternative 'Coacervation' than alternative 'Spray Drying'.

The same way of the interpretation of the results from Figures 5.9a - 5.9c can be used also for the interpretation of the results in Figures 5.9d and 5.9e. Figure 5.9d shows the trends of alternative preferences when we adjusted the base pairwise comparison of the base alternative 'Coacervation' to microencapsulation alternative 'Interfacial Polymerization' with respect to the criterion 'Core Material'. In Figure 5.9e we see the trends of alternative preferences when we adjusted the base pairwise comparison of the base alternative 'Coacervation' to microencapsulation alternative 'In Situ Polymerization' with respect to the criterion 'Core Material'.

5.6 Summary

Decision making in the field of engineering has become more complex due to the rapid growth of this field, thus letting the engineers benefit from a broader selection of materials and technology alternatives. The selection process is a very important issue in engineering field but unfortunately an extremely complex one since many criteria and alternatives are usually involved. The Analytical Hierarchy Process (AHP) is a useful method for the selection and evaluation problems and is widely used in industry, since it makes the selection process transparent.

Although pairwise comparisons are used as an effective way to elicit qualitative data from decision makers, it can be tiresome and time consuming if there are many alternatives to consider. This chapter presents a new methodology, the so-called Base Reference Analytical Hierarchy Process (BR-AHP), which enhances the limitation of the AHP by using base pairwise comparison method. The new pairwise comparison method is called base pairwise comparison method, since it always refers to the base, comparing it with other alternatives. The BR-AHP method draws upon ideas from engineering, where most of the engineers are only familiar with one technique and they are used to comparing other techniques with their own technique. A prototype has been developed to show the effectiveness of BR-AHP algorithm. The results demonstrate that BR-AHP method enhances AHP method by reducing the number of pairwise comparison to be performed.

The analysis in this chapter has demonstrated that by using BR-AHP approach the number of comparisons required to solve a decision problem which has m criteria and n alternatives can significantly be reduced when the number of alternatives is greater than two. The achieved reductions on the required total number of pairwise comparisons are given through some analytical formulas. The reduction become more dramatic as the size of the problem increases. Thus, the proposed BR-AHP approach becomes more practical for large size decision problems. To deal with the uncertainty and bias in the decision maker's judgment, we have also incorporated a sensitivity analysis method for handling imprecise, uncertain, and vagueness in the base pairwise comparison judgments.

Chapter 6

Fuzzy Base Reference Analytical Hierarchy Process

Due to the rapid growth of microencapsulation technology, the selection of the most appropriate microencapsulation process has become increasingly important. Analytical Hierarchy Process (AHP) has been used in industry to aid selection process. However, the exhaustive pairwise comparison is tiresome and time consuming if there are many alternatives to be considered. In Chapter 5, we presented a new methodology, the so-called Base Reference Analytical Hierarchy Process (BR-AHP), to cope with the limitation of AHP. However, due to the vagueness and uncertainty existing in the judgments of real-world problems, the crisp pairwise comparison in the BR-AHP would seem to be insufficient and imprecise to capture the degree of importance of these problems. Therefore, to cope with those problems we propose a new approach using the fuzzy sets theory combined with the BR-AHP approach. The approach based on fuzzy base pairwise comparison is proposed to improve the limitation of crisp base pairwise comparison. The new approach which uses fuzzy base pairwise comparison is called fuzzy Base Reference Analytical Hierarchy Process (fuzzy BR-AHP) approach. We present the new fuzzy BR-AHP approach in this chapter. This chapter is accompanied by a case study to show the application of fuzzy BR-AHP as the basis of the Decision Support System (DSS) for solving the engineering selection problem, i.e. the problem of the selection of the appropriate microencapsulation techniques. This chapter is based on the work in (Hotman & Alke, 2005).

6.1 Background and Motivation

Analytical Hierarchy Process (AHP) is one of the most widely used Multiple Attribute Decision Making (MADM) methods which deals with the problem of choosing an alternative from a set of alternatives characterized in terms of their attributes by using pairwise comparison technique (Saaty, 1980; Golden et al., 1989; Triantaphyllou, 2000; Figueira et al., 2005). However, the exhaustive pairwise comparison is tiresome and time consuming when there are many alternatives to be considered. Therefore, in the previous chapter we proposed a new approach to improve this limitation of pairwise comparison, the so-called Base Reference Analytical Hierarchy Process (BR-AHP). The BR-AHP uses the base pairwise comparison to reduce the number of pairwise comparisons of the alternatives. In Chapter 5, we have shown that the BR-AHP approach can reduce total pairwise comparisons of the conventional AHP if the decision problem has more than two alternatives.

In the BR-AHP methodology, the pairwise comparisons for each level are conducted using a nine-point scale proposed by Saaty (1980). This scale expresses preferences between options such as equally, moderately, strongly, very strongly, or extremely preferred. These preferences are then translated into values of 1, 3, 5, 7, and 9, respectively, with 2, 4, 6, and 8 as intermediate values. The pairwise comparison ratios used in the BR-AHP approach are in crisp real numbers. However, real-world problems always contain ambiguities and multiple meanings. The descriptions of engineering design problems in the early stages are usually linguistic and vague. Furthermore, it is also known that human assessment on qualitative attributes is always subjective and thus imprecise. Therefore, a crisp number seems to be inadequate to capture the judgment from decision maker.

Fuzzy set theory has been increasingly used for tackling the uncertainty, vagueness and imprecision of information in a non-probabilistic sense lately. Fuzzy sets were first introduced by Zadeh (1965) and have been applied in different fields such as decision making and control (Dubois & Prade, 1980). In order to model uncertainty in human judgments, fuzzy sets could be incorporated in the pairwise comparisons in the BR-AHP.

There have been many fuzzy Analytical Hierarchy Process (fuzzy AHP) methods proposed by various researchers. These methods are systematic approaches to the alternative selection and evaluation problems by using the concepts of fuzzy set theory and AHP. The earliest work in fuzzy AHP appeared in van Laarhoven & Pedrycz (1983), which proposed a method of fuzzy judgment by comparison of the triangular fuzzy number. Buckley (1985) determined fuzzy priorities of comparison ratios whose membership functions are trapezoidal. Boender et al. (1989) presented a more robust approach to the normalization of the local priorities by modifying the van Laarhoven & Pedrycz method. Stam et al. (1996) explored how recently developed Artificial Intelligence (AI) techniques can be used to determine or approximate the preference ratings in AHP. They showed that the feed forward neural network formulation appeared to be a powerful tool for analyzing discrete alternative multi criteria decision problems with imprecise or fuzzy ratio-scale preference judgments. Chang (1996) introduced a new approach for handling fuzzy AHP, with the use of triangular fuzzy numbers for pairwise comparison scale of fuzzy AHP, and the use of the extent analysis method for the synthetic extent values of the pairwise comparisons. Cheng (1996) proposed a new algorithm for evaluating naval tactical missile systems by the fuzzy AHP based on grade value of membership function. Zhu et al. (1999) discussed the extent analysis method and applications of fuzzy AHP. However, all these methods were based on fuzzy pairwise comparison. Using the fuzzy pairwise comparison it needs to determine $n(n-1)/2$ judgments for n elements. The drawback of this comparison is that it is tiresome and time consuming when there are many elements to be considered. This chapter presents a new approach to improve the limitation of fuzzy pairwise comparison, the so-called fuzzy base pairwise comparison. The new approach which uses fuzzy base pairwise comparison is called fuzzy Base Reference Analytical Hierarchy Process (fuzzy BR-AHP) approach.

In this chapter, we propose a new approach which has integrated the proposed BR-AHP approach and the fuzzy set theory for tackling the vagueness, imprecision and uncertainties in the reasoning process of the decision maker's assessment. The remaining of this chapter is organized as follows. The next section presents fuzzy set theory followed by the proposed fuzzy BR-AHP methodology, the modeling of group decision and the sensitivity analysis approach for fuzzy BR-AHP. A case study, illustrating the application of the fuzzy BR-AHP approach to microencapsulation process selection problem, is given afterward. Finally, the final section will conclude the work in this chapter.

6.2 Fuzzy Set Theory

Fuzzy logic theory, which is based on the extension of the classical set theory, was introduced by Zadeh (1965) and has gained much importance in dealing with the problems involving uncertainty and vagueness of human thinking in many practical applications (Dubois & Prade, 1980; Zimmermann, 1987, 1991). Fuzzy logic is a superset of conventional logic (Aristotelian two-valued logic) that has been extended to handle the concept of "partial truth" with the possibility of expressing sets without clear boundaries or assigning partial memberships to elements of a given set.

One main contribution of the fuzzy set theory is its ability to represent uncertain and vague data. This is the first step to incorporate human knowledge into engineering systems in a systematic and efficient manner. Bellman & Zadeh (1970) first presented the decision making method in a fuzzy environment and since then fuzzy logic has often been applied to handle uncertain and subjective problems in decision making.

Some important definitions and concepts of fuzzy set theory are reviewed here in order to understand and apply fuzzy logic theory.

A tilde "$\sim$" will be placed above a symbol if it represents a fuzzy set. In this section, the basic notions of fuzzy set theory and fuzzy logic are briefly introduced. They will be needed in the remainder of this work to handle the vagueness and uncertainty. For a more thorough discussion of fuzzy sets and fuzzy logic, please refer to Zadeh (1965) and Zimmermann (1991).

6.2.1 Basic Definitions of Fuzzy Sets

In classical "crisp" set theory, a characteristic function is associated with each crisp set. This function is defined on the universe of discourse and yields the value 0 or 1 whether the argument belongs to the set or not.

Definition 6.2.1 (Set)**.** *A (crisp) set A on a universe U is characterized by its characteristic function $\mu_A : U \to \{0, 1\}$*

$$\mu_A(u) = \begin{cases} 0 & \textit{if}\, u \notin A \\ 1 & \textit{if}\, u \in A \end{cases} \tag{6.1}$$

Definition 6.2.2 (Fuzzy Set)**.** *A fuzzy set $\tilde{A}$ on a universe U is characterized by its membership function $\mu_{\tilde{A}} : U \to [0, 1]$, where $\mu_{\tilde{A}}(u)$ denotes the degree to which $u \in U$ belongs to $\tilde{A}$. $\mu_{\tilde{A}}(u)$ is called the membership degree or grade of membership of u in $\tilde{A}$.*

The closer $\mu_{\tilde{A}}(u)$ is to 1, the more u belongs to the set $\tilde{A}$; the closer it is to 0, the less it belongs to $\tilde{A}$. In this way, fuzzy sets allow flexible expression of uncertainties for set descriptions like 'the set of small sized particles'.

6.2.2 Properties of Fuzzy Sets

A horizontal representation of fuzzy sets is given by its α-level cuts. Formally it is described as follows:

Definition 6.2.3 (α-level cut, Strong α-level cut)**.** *For $\tilde{A}$ a fuzzy set on the universe U and $\alpha \in [0, 1]$, the α-level cut A_α and strong α-level cut $\bar{A}_\alpha$ are defined by*

$$A_\alpha = \{u \in U \,|\mu_{\tilde{A}}(u) \geq \alpha\} \tag{6.2}$$
$$\bar{A}_\alpha = \{u \in U \,|\mu_{\tilde{A}}(u) > \alpha\} \tag{6.3}$$

Definition 6.2.4 (Support, Kernel)**.** *For $\tilde{A}$ a fuzzy set on the universe U, the support $supp(\tilde{A})$ and kernel $ker(\tilde{A})$ are defined by*

$$supp(\tilde{A}) = \{u \in U \,|\mu_{\tilde{A}}(u) > 0\} = \bar{A}_0 \tag{6.4}$$
$$ker(\tilde{A}) = \{u \in U \,|\mu_{\tilde{A}}(u) = 1\} = A_1 \tag{6.5}$$

Definition 6.2.5 (Height)**.** *Given a universe U and a fuzzy set $\tilde{A}$ on U, the height of $\tilde{A}$ are defined by*

$$h(\tilde{A}) = \sup_{u \in U} \mu_{\tilde{A}}(u) \tag{6.6}$$

Definition 6.2.6 (Normal Fuzzy Set)**.** *A fuzzy set $\tilde{A} = \{(u, \mu_{\tilde{A}}) | u \in \mathbb{R}\}$ on U is normal if it satisfies $h(\tilde{A}) = 1$.*

When a fuzzy set $\tilde{A}$ on U is not normal, it can be normalized with the following transformation.

Definition 6.2.7 (Fuzzy Set Normalization)**.**

$$norm(\tilde{A}) = \begin{cases} \frac{\mu_{\tilde{A}}(u)}{h(\tilde{A})} & \text{if } h(\tilde{A}) \neq 0, \\ 1 & \text{if } h(\tilde{A}) = 0. \end{cases} \tag{6.7}$$

Definition 6.2.8 (Convex Fuzzy Set)**.** *Let universal set U is defined in n dimensional Euclidean Vector space $\mathbb{R}^n$. If a relation*

$$\mu_{\tilde{A}}(u) \geq \min \left[\mu_{\tilde{A}}(u_1), \mu_{\tilde{A}}(u_2)\right] \tag{6.8}$$

where $u = \alpha u_1 + (1 - \alpha) u_2$; $u_1, u_2 \in \mathbb{R}^n$; $\forall \alpha \in [0, 1]$ holds, then the fuzzy set $\tilde{A}$ is convex.

Owing to the convexity assumption of fuzzy numbers, the α-level cuts describe sets of numbers in $\mathbb{R}$ (intervals) with a given minimum likeliness (acceptance) α. When the values of α decrease, then the width of intervals of real numbers increase.

6.2.3 Fuzzy Numbers

Definition 6.2.9 (Fuzzy Number)**.** *If a fuzzy set $\tilde{A}$ is **convex** and **normal**, and its membership function $\mu_{\tilde{A}}$ is defined in $\mathbb{R}$ and piecewise continuous, it is called as "fuzzy number". Fuzzy number represents a real number interval whose boundary is fuzzy.*

Based on the definition above we can conclude that fuzzy number $\tilde{A}$ belongs to a fuzzy set, and it has a membership function $\mu_{\tilde{A}}(u) : \mathbb{R} \to [0, 1]$ and and having to fulfil the following three conditions

1. $\mu_{\tilde{A}}(u)$ is continuous mapping from real number $\mathbb{R}$ to the closed interval $[0, 1]$.
2. $\mu_{\tilde{A}}(u)$ is of a convex fuzzy subset.
3. $\mu_{\tilde{A}}(u)$ is the normality of a fuzzy subset, which means that there exists a number $u_0 \in \mathbb{R}$ that makes $\mu_{\tilde{A}}(u_0) = 1$.

The following is the interpretation for the feature of a special case of fuzzy numbers, i.e. the triangular fuzzy numbers which we use in our study.

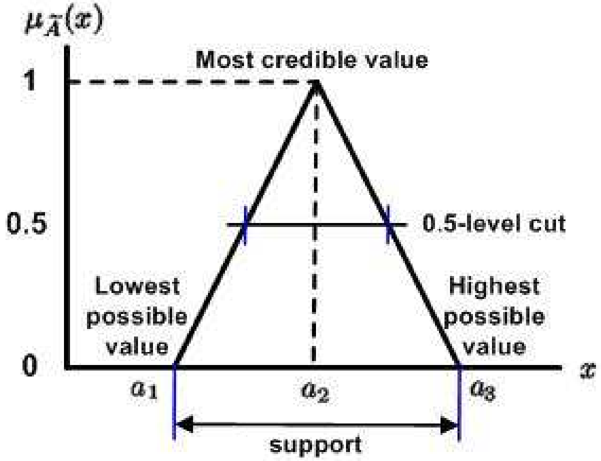


Figure 6.1: *Triangular Fuzzy Number, an α-level cut and its support.*

Definition 6.2.10 (Triangular Fuzzy Number)**.** *A triangular fuzzy number is a special class of fuzzy number whose membership can be defined by three real numbers, $\tilde{A} = (a_1, a_2, a_3)$; $a_1 \leq a_2 \leq a_3$; $a_1, a_2, a_3 \in \mathbb{R}$. Its membership function $\mu_{\tilde{A}}(u) : \mathbb{R} \to [0, 1]$ can be defined as follows:*

$$\mu_{\tilde{A}}(u) = \begin{cases} 0 & \text{if } u < a_1 \\ \frac{u-a_1}{a_2-a_1} & \text{if } a_1 \leq u \leq a_2 \\ \frac{a_3-u}{a_3-a_2} & \text{if } a_2 \leq u \leq a_3 \\ 0 & \text{if } u > a_3 \end{cases} \tag{6.9}$$

where a_2 is the most possible value of fuzzy number $\tilde{A}$, and a_1 and a_3 are the lower and upper bounds, respectively which is often used to illustrated the fuzziness of the data evaluated.

Figure 6.1 shows an example of a triangular fuzzy number with an α-level cut and its support. The most credible value is given by a membership value of 1, numbers that fall short of the lowest possible value and exceed the highest possible value get membership values of 0. Intermediate membership grades are obtained just by linear interpolation.

Mathematical operations of fuzzy numbers are extended to be defined on fuzzy sets by the use of the extension principle:

Definition 6.2.11 (Extension Principle)**.** *Let $f : \mathbb{R}\times\mathbb{R} \to \mathbb{R}$ be a binary operation over real numbers. Then it can be extended to the operation over the set $\mathbb{R}$ of fuzzy quantities. If we denote $\tilde{A}$ and $\tilde{B}$ be two fuzzy numbers and the quantity*

$\tilde{C} = f(\tilde{A}, \tilde{B})$, then the membership function $\mu_{\tilde{C}}$ is derived from the membership functions $\mu_{\tilde{A}}$ and $\mu_{\tilde{B}}$ by

$$\mu_{\tilde{C}}(z) = \sup\left[\min\left(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(y)\right) : x, y \in \mathbb{R}, z = f(x, y)\right] \qquad (6.10)$$

for any $z \in \mathbb{R}$.

Definition 6.2.12 (Interval Fuzzy Number)**.** *By defining the interval of confidence at level α, we characterize the triangular fuzzy number $\tilde{A} = (a_1, a_2, a_3)$ as interval fuzzy number $\tilde{A}_\alpha$ which can be defined as follows:*

$$\forall \alpha \in [0, 1], \quad \tilde{A}_\alpha = [a_L^{(\alpha)}, a_R^{(\alpha)}] = [a_1 + \alpha(a_2 - a_1), a_3 + \alpha(a_2 - a_3)]. \qquad (6.11)$$

6.2.4 Fuzzy Linguistic Variable

When a fuzzy number represents a linguistic concept, such as very small, small, medium, large, and so on, as interpreted in a particular content, the resulting construct is usually called linguistic variable.

Definition 6.2.13 (Linguistic Variable)**.** *(Zimmermann, 1991) A linguistic variable is characterized by a quintuple $(x, T(x), U, G, M)$*

(i) x is the name of fuzzy variable

(ii) $T(x)$ is the term set of x, that is, the set of the name of linguistic value of x, with each value being a fuzzy variable denoted by x and ranging over U

(iii) U is the universe of discourse

(iv) G is the syntactic rule which usually has the form of a grammar for generating the terms in $T(x)$

(v) M is the semantic rule which associates with each linguistic value A its meaning $M(A)$, where $M(A)$ denotes a fuzzy set in U.

An example of a linguistic variable is shown in Figure 6.2. Its name is particle size. This variable expresses the particle size (which is the base variable in the example) of a goal-oriented entity in a given context by five basic linguistic terms – very small, small, medium, large, very large – generated by a syntactic rule. Each of the basic linguistic terms are assigned one of five fuzzy numbers by a semantic rule, as shown in the figure. The fuzzy numbers, whose membership functions in this example have the triangular shapes, are defined on the interval $[0, 100]$, the range of the base variable. Each of them expresses a fuzzy restriction in this range.

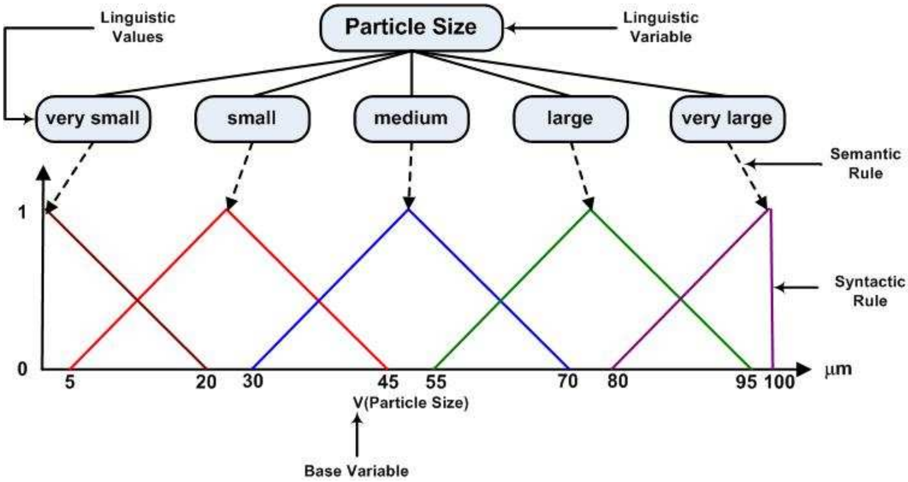


Figure 6.2: *Linguistic Variable*

6.3 The Fuzzy BR-AHP Methodology

The fuzzy Base Reference Analytical Hierarchy Process (fuzzy BR-AHP) is proposed to overcome the restriction of BR-AHP to cope with imprecision, uncertainties and vagueness in the judgments of the decision makers. The development of this approach was first reported in Hotman & Alke (2005). In this approach a particular pairwise comparisons technique, the so-called fuzzy base pairwise comparison, was developed. The fuzzy BR-AHP approach will be described in detail in the following subsections.

The proposed fuzzy BR-AHP methodology consists of four main steps:

(1) Construction of the hierarchical structure of the decision problem.

(2) Individual evaluation of the criteria using fuzzy pairwise comparison method.

(3) Individual evaluation of the alternatives using fuzzy base pairwise comparison method.

(4) Aggregation of the individual results to obtain an overall relative ranking for each potential alternative.

In step 3 we use fuzzy base pairwise comparison method for eliciting the judgments from decision makers. By using the fuzzy pairwise comparison method each decision maker needs to estimate $n(n-1)/2$ judgments for n alternatives, but with the fuzzy base pairwise comparison method it only needs $n-1$ judgments. Figure 6.3 shows the four steps of the algorithm of fuzzy BR-AHP methodology.

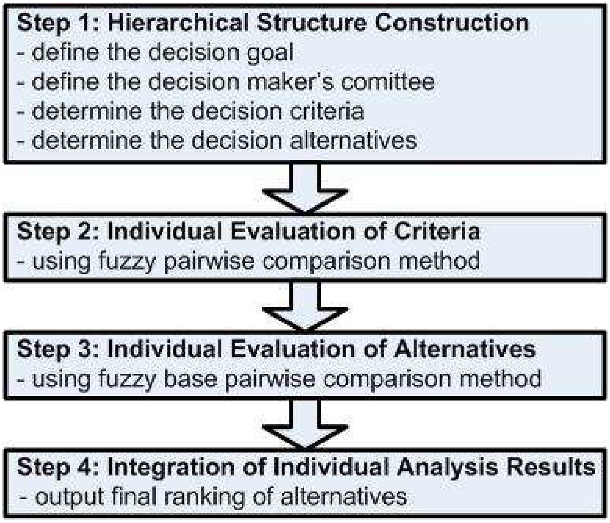


Figure 6.3: *The Procedure of Fuzzy BR-AHP Methodology*

6.3.1 Hierarchical Structure Construction

The first step in the fuzzy BR-AHP methodology is constructing the hierarchical structure of the decision problem. There are six steps to construct a hierarchy structure:

(1) Define the decision problem.

(2) Define the decision maker's committee, i.e. the people who are involved in the decision making process.

(3) Identify the overall goal of the decision problem.

(4) Identify the evaluation criteria and/or sub-criteria that must be satisfied to fulfill the overall goal.

(5) Identify the alternatives or outcomes of the decision problem.

(6) Structure the hierarchy from the highest level (the overall goal) through relevant intermediate levels to the lowest level (the alternative level).

Figure 6.4 shows an example of the constructed hierarchy structure with a single criteria layer.

6.3.2 Criteria Evaluation using Fuzzy Pairwise Comparison

After constructing the decision problem hierarchy, the decision maker is needed to compare the elements at a certain criteria level by using fuzzy pairwise com-

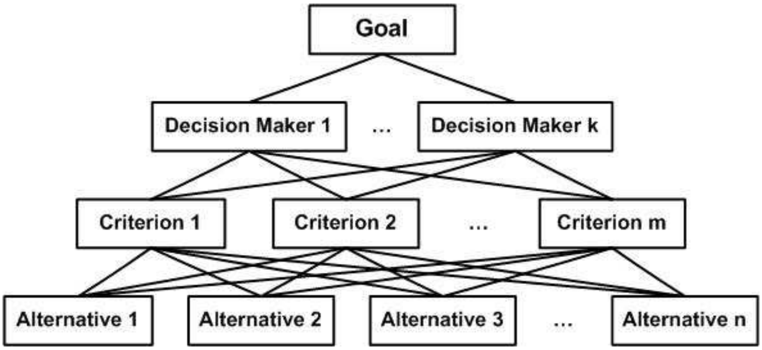


Figure 6.4: *Fuzzy BR-AHP Hierarchy with Single Criteria Layer*

parison method to estimate their relative importance with respect to the element immediate one level above. In the conventional AHP, the pairwise comparison is made using a ratio scale. The frequently used scale is the nine-point scale which shows the decision maker's preferences among the options such as equally, moderately, strongly, very strongly or extreme preferred. Even though the discrete scale of $1-9$ has the advantages of simplicity and easiness for use, it does not take into account the uncertainty associated with the mapping of one's perception to a number.

The fuzzy set theory was introduced by Zadeh (1965) to deal with problems in which vagueness or uncertainty is involved. A fuzzy number is a convex fuzzy set, characterized by a given interval of real numbers, each with a grade of membership between 0 and 1. The fuzzy number $\tilde{X}$ can be expressed as a triple (x_1, x_2, x_3). Based on Definition 6.2.10, the membership function of a fuzzy number $\tilde{X}$ is described as

$$\mu_{\tilde{X}}(x) = \begin{cases} (x-x_1)/(x_2-x_1) & x_1 \leq x \leq x_2 \\ (x_3-x)/(x_3-x_2) & x_2 \leq x \leq x_3 \\ 0 & \text{otherwise} \end{cases} \tag{6.12}$$

where x_2 is the mean value or the most possible value of the fuzzy number; and x_1 and x_3 are the lower and the upper bounds, respectively, representing the scope fuzziness.

To facilitate making pairwise comparisons, the triangular fuzzy numbers defined in Table 6.1 are used. Here, these triangular fuzzy numbers, $\tilde{1}$ to $\tilde{9}$, are used to represent subjective pairwise comparisons in order to capture the vagueness of decision maker's belief. A triangular fuzzy number $\tilde{x}$ expresses the meaning of 'about x', for $1 \leq x \leq 9$, having the membership functions as defined in Eq. (6.12).

In order to take the imprecision of human qualitative assessments into consideration, the triangular fuzzy numbers are defined with the corresponding membership functions as shown in Figure 6.5.

By using triangular fuzzy numbers $\tilde{a}_{ij} = \left(a_{ij}^L, a_{ij}^M, a_{ij}^U\right)$, via pairwise compari-

Table 6.1: *Fuzzy Numbers used for Making Qualitative Assessments*

Fuzzy number	Membership function
$\tilde{1}$	$(1, 1, 2)$
$\tilde{x}$	$(x-1, x, x+1)$ for $x = 2, \ldots, 8$
$\tilde{9}$	$(8, 9, 9)$

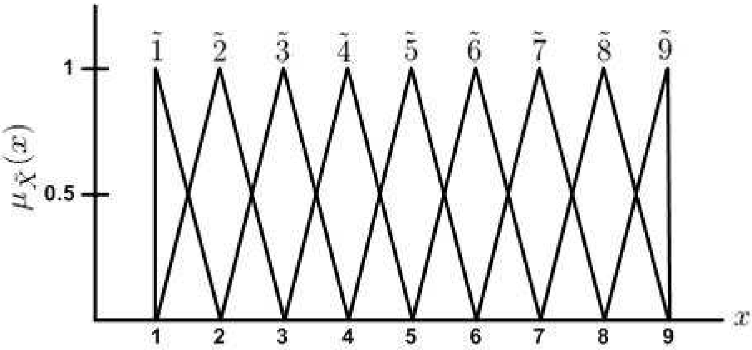


Figure 6.5: *The Triangular Fuzzy Number Membership Functions*

son, the fuzzy pairwise matrix between goal and criteria layer can be constructed, such that: $\tilde{A} = [\tilde{a}_{ij}]$ with

$$\tilde{a}_{ji} = \begin{cases} 1 & i = j \\ 1/\tilde{a}_{ij} = \left(1/a_{ij}^{U}, 1/a_{ij}^{M}, 1/a_{ij}^{L}\right) & i \neq j \end{cases} \tag{6.13}$$

In the same way, the matrices of criteria layer vs. sub-criteria layer can be considered (if any).

6.3.3 Alternatives Evaluation using Fuzzy Base Pairwise Comparison

In this section, a new approach for fuzzy comparison judgments is proposed, eliminating some drawbacks of the existing fuzzy pairwise comparison methods. This approach does not require the construction of a full set of fuzzy comparison matrices and it is able to derive priorities only based on $n-1$ set of fuzzy judgments. The fuzzy base pairwise matrix comparison between criteria layer and alternative layer is defined as follows:

$$\tilde{B} = \begin{bmatrix} 1 & \tilde{b}_{12} & \cdots & \tilde{b}_{1k} & \cdots & \tilde{b}_{1n} \\ \tilde{b}_{21} & 1 & \cdots & \tilde{b}_{2k} & \cdots & \tilde{b}_{2n} \\ \vdots & \vdots & \ddots & \vdots & & \vdots \\ \tilde{b}_{k1} & \tilde{b}_{k2} & \cdots & 1 & \cdots & \tilde{b}_{kn} \\ \vdots & \vdots & & \vdots & \ddots & \vdots \\ \tilde{b}_{n1} & \tilde{b}_{n2} & \cdots & \tilde{b}_{nk} & \cdots & 1 \end{bmatrix} \tag{6.14}$$

where $\tilde{B}$ is the fuzzy base pairwise comparison matrix (size $n \times n$); $\tilde{b}_{ij} = \left(b_{ij}^L, b_{ij}^M, b_{ij}^U\right)$; and k denotes the base reference.

The procedure of fuzzy base pairwise comparison is as follows. First, a decision maker only needs to determine the base alternative and then compare the base alternative with other alternatives ($\tilde{b}_{kj}$). Next, we have to evaluate the missing values using the information derived from the available base elements of the base pairwise matrix. Here, the value of the base elements of the base pairwise matrix represents the ratio between the weights of two different alternatives in comparison with the defined criteria. For example $\tilde{b}_{kj}$ is the approximate ratio between the weight of the k-th alternative ($\tilde{w}_k$) and the weight of the j-th alternative ($\tilde{w}_j; j \in 1, 2, \ldots, n; j \neq k$) in comparison with a common overall criteria. As a consequence we can write: $\tilde{b}_{kj} \approx \tilde{w}_k / \tilde{w}_j$.

If we do not have any evaluation for $\tilde{b}_{ij}$, but we do have an evaluation for $\tilde{b}_{ki}$ and $\tilde{b}_{kj}$, then we can calculate the missed value by considering the relations below:

$$\tilde{b}_{ij} \approx \tilde{w}_i / \tilde{w}_j \approx \frac{\tilde{w}_k / \tilde{w}_j}{\tilde{w}_k / \tilde{w}_i} \approx \frac{\tilde{b}_{kj}}{\tilde{b}_{ki}} \tag{6.15}$$

The judgment scale used here is the nine-point scale, which was proposed by Saaty (1980). To ensure the consistency with the nine-point scale, we have to modify the automatic value assignment in base pairwise matrix. The modified fuzzy base pairwise comparison matrix is expressed as follows:

$$\begin{aligned} \tilde{B} &= \left(\tilde{b}_{ij}\right), \tilde{b}_{ij} > 0, i, j = 1, 2, \ldots, n \\ \tilde{b}_{ij} &= \begin{cases} 1 & i = j \\ \min\left\{\tilde{9}, \frac{\tilde{b}_{kj}}{\tilde{b}_{ki}}\right\} & \frac{\tilde{b}_{kj}}{\tilde{b}_{ki}} > 1, i \neq j \\ \max\left\{\frac{1}{9}, \frac{\tilde{b}_{kj}}{\tilde{b}_{ki}}\right\} & \frac{\tilde{b}_{kj}}{\tilde{b}_{ki}} < 1, i \neq j \end{cases} \end{aligned} \tag{6.16}$$

$$\tilde{b}_{ij} \in \left\{\frac{1}{9}, \ldots, \frac{1}{2}, \frac{1}{1}, \tilde{1}, \tilde{2}, \ldots, \tilde{9}\right\}$$

6.3.4 Weights Determination

Several well known fuzzy prioritization methods derive fuzzy priorities $\tilde{w}_i$, $i = 1, 2, \ldots, n$, from Eq. (6.14), which approximate the fuzzy ratios $\tilde{b}_{ij}$ so that $\tilde{b}_{ij} \approx$

$\tilde{w}_i/\tilde{w}_j$. These methods are based on fuzzy versions of the logarithmic least squares method (van Laarhoven & Pedrycz, 1983; Boender et al., 1989), fuzzy modifications of the least squares method (Wagenknecht & Hartmann, 1983; Xu, 2000a), fuzzy geometric means (Buckley, 1985) and fuzzy arithmetic mean (Chang, 1996).

We use another way to calculate the fuzzy AHP weights, derived from the fuzzy extension of the multiplicative AHP method. In this case, the fuzzy weight values can be calculated using the modified geometric mean of the i-th row of the fuzzy pairwise comparison matrix:

$$\tilde{w}_i = \left[\left(\prod_{j=1}^{n} \tilde{b}_{ij}\right)^{1/n}\right] \Big/ \sum_{i=1}^{n} \left[\left(\prod_{j=1}^{n} b_{ij}^{M}\right)^{1/n}\right] \tag{6.17}$$

6.3.5 Evaluation of Weights (Defuzzification)

Now the question that arises is how to choose among several alternatives by comparing rating vectors whose components are fuzzy numbers. The simplest way of comparing two fuzzy numbers is to defuzzify the two fuzzy numbers, obtaining two crisp numbers representing in a sense typical values of the two quantities and then to compare these two crisp numbers in the usual way.

To calculate the weights, we need transform the fuzzy number in the data into crisp value; this procedure is called defuzzification method.

The Centroid Method

The defuzzification of the weight fuzzy number uses the centroid method, which preferred by most fuzzy control engineers. The function is defined as follows:

$$\tilde{X}(\text{centroid}) = \frac{\int_{supp(\tilde{A})} x\mu_{\tilde{A}}(x)dx}{\int_{supp(\tilde{A})} \mu_{\tilde{A}}(x)dx} \tag{6.18}$$

If there is a triangular fuzzy number $\tilde{w}_i = \left(w_i^L, w_i^M, w_i^U\right)$, then the centroid of the triangular fuzzy number has a formulation as in Eq. (6.19). (Proof, see Appendix D.2)

$$DF(\tilde{w}_i) = \left(w_i^L + w_i^M + w_i^U\right)/3 \quad \forall i \tag{6.19}$$

6.4 Modeling Group Decision Making

The application of fuzzy BR-AHP can be used for group decision making by aggregating individual judgments or individual priorities. The most common methods for aggregating individual pairwise comparison matrices for the purpose of group decision making can be classified under two main categories: the geometric mean (GM) method and the weighted arithmetic mean (WAM) method.

Definition 6.4.1 (Weighted Arithmetic Mean). *A Weighted Arithmetic Mean (WAM) operator is an aggregation operator defined by*

$$WAM(x_1, x_2, \ldots, x_n) = \sum_{i=1}^{n} w_i x_i \tag{6.20}$$

where $w = (w_1, w_2, \ldots, w_n) \in [0,1]^n$ is a weight vector such that $\sum_{i=1}^{n} w_i = 1$.

Definition 6.4.2 (Geometric Mean). *A Geometric Mean (GM) operator is an aggregation operator defined by*

$$GM(x_1, x_2, \ldots, x_n) = \left(\prod_{i=1}^{n} x_i\right)^{1/n} \tag{6.21}$$

Definition 6.4.3 (Weighted Geometric Mean). *A Weighted Geometric Mean (WGM) operator is an aggregation operator defined by*

$$WGM(x_1, x_2, \ldots, x_n) = \prod_{i=1}^{n} {x_i}^{w_i} \tag{6.22}$$

where $w = (w_1, w_2, \ldots, w_n) \in [0,1]^n$ is a weight vector such that $\sum_{i=1}^{n} w_i = 1$.

From the definitions above, we see that if every weight vector in WGM have the same priority then the WGM operator becomes the GM operator.

Regarding group decision making (Aczel & Saaty, 1983; Forman & Peniwati, 1998), AHP considers two different approaches: the aggregation of individual judgments (AIJ) and the aggregation of individual priorities (AIP). Forman & Peniwati (1998) proposed that the aggregation of individual judgments (AIJ) be chosen when the group is assumed to act together as an individual and the aggregation of individual priorities (AIP) be chosen when the group is assumed to act as separate individuals. They also have given details of mathematical methodologies for applying both aggregation methods. When aggregating individual priorities (AIP), both the geometric mean (GM) method and the weighted arithmetic mean (WAM) method are suitable.

In this study, we will use geometric mean method as the aggregation method because this method is more superior than weighted arithmetic mean method. One important property of the geometric mean is its ability to dampen the effect of very high or low values; whereas, such very high or very low values might bias the arithmetic mean. In other words, the geometric mean is less affected by extreme values than the arithmetic mean.

Xu (2000b) proved that the complex judgment matrix of the Weighted Geometric Mean (WGM) method is of acceptable consistency under the condition that all individual comparison matrices are of acceptable consistency. Xu (2000b) also proved that if individual comparison matrices are of acceptable consistency then the aggregated comparison matrix is also of acceptable consistency.

Let $A^k = (a_{ij}^k)$ be the judgment matrix provided by the k-th decision maker when comparing n elements $(i, j = 1, \ldots, n)$, with $w^k = (w_1^k, w_2^k, \ldots, w_n^k)$ being its priority vector $(w_i^k > 0, \sum_i w_i^k = 1)$ and r_k being the weight of the k-th decision maker $(k = 1, \ldots, m)$ in the group $(r_k > 0; \sum_k r_k = 1)$. Using the WGM method as the aggregation procedure, the group judgment matrix and the group priority vector are, respectively, given by

$$\begin{aligned} A^G &= \left(a_{ij}^G\right) \quad \textit{with } a_{ij}^G = \prod_{k=1}^{m} \left(a_{ij}^k\right)^{r_k} (i, j = 1, 2, \ldots, n), \\ w^G &= \left(w_i^G\right) \quad \textit{with } w_{ij}^G = \prod_{k=1}^{m} \left(w_{ij}^k\right)^{r_k} (i, j = 1, 2, \ldots, n). \end{aligned} \tag{6.23}$$

When the Row Geometric Mean (RGM) method prioritization procedure is employed, the final priorities of the alternatives for the two aggregation approaches (AIJ and AIP) are obtained, respectively, following the next two sequences:

AIJ: From the individual judgment matrices $A^k (k = 1, 2, \ldots, m)$, using the WGM method, we obtain the group judgment matrix A^G, and from this, using the RGM method, we derive the group priorities w^G.

AIP: From the individual judgment matrices $A^k (k = 1, 2, \ldots, m)$, we obtain the individual priorities $w^k (k = 1, 2, \ldots, m)$ using the RGM method, and from these, we derive the group priorities w^G using the WGM method.

It is simpler and more efficient to work with the AIP approach (only $O(mn)$ operations) than with the AIJ approach ($O(mn^2)$ operations). Therefore the study in this thesis will employ AIP method.

6.5 Sensitivity Analysis

Decision making is a subjective process, since the perception regarding a problem can diverge from person to person. One cannot expect a decision maker or an expert to be highly consistent while dealing with such a subjective process. The real world problems are influenced by many natural factors and processes, that are difficult to measure and model precisely. Therefore, the decision situations are surrounded by uncertainty. Sensitivity analysis is a way to address this uncertainty in estimating the parameters.

Model uncertainty arising from parameters can be analyzed using several techniques. The Monte Carlo simulation and fuzzy logic α-cuts analysis are the most common used techniques. The Monte Carlo simulation technique treats any uncertain parameter as random variable that complies with a given probabilistic distribution. This technique is widely used for analyzing probabilistic uncertainty. However, this is not our case. Here, we work with the case of uncertainty in a non-probabilistic sense. Hence, we use the fuzzy logic-based α-cut analysis in which uncertain parameters are treated as fuzzy numbers which given membership functions instead. Here in this section we will utilize interval arithmetic and α-cuts to be adopted in our methodology for the sensitivity analysis.

In fuzzy BR-AHP method, the ranking procedure starts at the determination of the criteria importance and alternative performance. By using the triangular fuzzy numbers defined in Table 6.1, a fuzzy pairwise comparison judgment matrix for criteria importance ($\tilde{C}$) and fuzzy pairwise comparison judgment matrices for alternative performances with respect to a specific criterion ($\tilde{C}_j$) can be determined by

$$\tilde{C}_j \text{ or } \tilde{C} = \begin{bmatrix} \tilde{c}_{11} & \tilde{c}_{12} & \cdots & \tilde{c}_{1k} \\ \tilde{c}_{21} & \tilde{c}_{22} & \cdots & \tilde{c}_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{c}_{k1} & \tilde{c}_{k2} & \cdots & \tilde{c}_{kk} \end{bmatrix} \tag{6.24}$$

where

$$\tilde{c}_{st} = \begin{cases} \tilde{1}, \tilde{2}, \tilde{3}, \ldots, \tilde{9}, & s < t, \\ 1, & s = t, \\ 1/\tilde{c}_{ts}, & s > t, \end{cases} \quad s, t = 1, 2, \ldots, k; \; k = m \text{ or } n, \tag{6.25}$$

Using a fuzzy prioritization method we can derive the fuzzy priorities, then we will be capable of determining the fuzzy decision matrix ($\tilde{A}$) and the weight vector ($\tilde{W}$) for the fuzzy decision problem as the follows.

$$\tilde{A} = \begin{bmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \cdots & \tilde{a}_{1m} \\ \tilde{a}_{21} & \tilde{a}_{22} & \cdots & \tilde{a}_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{a}_{n1} & \tilde{a}_{n2} & \cdots & \tilde{a}_{nm} \end{bmatrix}, \tag{6.26}$$

$$\tilde{W} = (\tilde{w}_1, \tilde{w}_2, \ldots, \tilde{w}_m), \tag{6.27}$$

where $\tilde{a}_{ij}$ represents the result of the fuzzy performance assessment of alternative $A_i (i = 1, 2, \ldots, n)$ with respect to criterion $C_j (j = 1, 2, \ldots, m)$ and w_j is the result of the fuzzy weight of the criterion C_j with respect to the overall objective of the problem.

A fuzzy performance matrix representing the overall performance of all alternatives with respect to each criterion can therefore be obtained by multiplying the weighting vector by the decision matrix. The fuzzy performance matrix is defined as follows.

$$\tilde{F} = \begin{bmatrix} \tilde{w}_1\tilde{a}_{11} & \tilde{w}_2\tilde{a}_{12} & \cdots & \tilde{w}_m\tilde{a}_{1m} \\ \tilde{w}_1\tilde{a}_{21} & \tilde{w}_2\tilde{a}_{22} & \cdots & \tilde{w}_m\tilde{a}_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{w}_1\tilde{a}_{n1} & \tilde{w}_2\tilde{a}_{n2} & \cdots & \tilde{w}_m\tilde{a}_{nm} \end{bmatrix}. \tag{6.28}$$

By using an α-cut on the fuzzy performance matrix in Eq. (6.28), an interval performance matrix can be derived as in Eq. (6.29), where $0 \leq \alpha \leq 1$. The value of α represents the decision maker's degree of confidence in his/her fuzzy assessments with respect to the alternative ratings and criteria weights. The larger α value is indicates a more confident decision maker, meaning that the decision maker's assessments are closer to the most possible value a_2 of the triangular fuzzy numbers (a_1, a_2, a_3).

$$\tilde{F}_\alpha = \begin{bmatrix} \left[f_{11L}^{(\alpha)}, f_{11R}^{(\alpha)}\right] & \left[f_{12L}^{(\alpha)}, f_{12R}^{(\alpha)}\right] & \cdots & \left[f_{1mL}^{(\alpha)}, f_{1mR}^{(\alpha)}\right] \\ \left[f_{21L}^{(\alpha)}, f_{21R}^{(\alpha)}\right] & \left[f_{22L}^{(\alpha)}, f_{22R}^{(\alpha)}\right] & \cdots & \left[f_{2mL}^{(\alpha)}, f_{2mR}^{(\alpha)}\right] \\ \vdots & \vdots & \ddots & \vdots \\ \left[f_{n1L}^{(\alpha)}, f_{n1R}^{(\alpha)}\right] & \left[f_{n2L}^{(\alpha)}, f_{n2R}^{(\alpha)}\right] & \cdots & \left[f_{nmL}^{(\alpha)}, f_{nmR}^{(\alpha)}\right] \end{bmatrix}. \tag{6.29}$$

Now we can define, for $0 < \alpha < 1$ and $\forall\ i, j$: $\quad \tilde{a}_{ij}^{(\alpha)} = \left[a_{ijL}^{(\alpha)}, a_{ijR}^{(\alpha)}\right]$.

Estimating the degree of satisfaction of the decision maker can be achieved by using an index of optimism λ. The larger an index is indicates a higher degree of optimism. The estimator for an index of optimism is defined as

$$\hat{a}_{ij}^{(\alpha)} = \lambda a_{ijR}^{(\alpha)} + (1-\lambda) a_{ijL}^{(\alpha)} \quad \forall \lambda \in [0, 1] \tag{6.30}$$

where λ denotes the degree/index of optimism of a decision maker. When $\lambda = 1$, $\hat{a}_{ij}^{(\alpha)}$ represents the viewpoint of an optimistic decision maker, while when $\lambda = 0$, $\hat{a}_{ij}^{(\alpha)}$ represents the viewpoint of a pessimistic decision maker. When $\lambda = 0.5$, $\hat{a}_{ij}^{(\alpha)}$ represents the viewpoint of a moderate decision maker

With α fixed, we use index of optimism λ to estimate the degree of satisfaction. Thus, the fuzzy performance matrix becomes

$$\hat{F} = \begin{bmatrix} 1 & \hat{f}_{12}^{(\alpha)} & \cdots & \hat{f}_{1n}^{(\alpha)} \\ \frac{1}{\hat{f}_{12}^{(\alpha)}} & 1 & \cdots & \hat{f}_{2n}^{(\alpha)} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{\hat{f}_{1n}^{(\alpha)}} & \frac{1}{\hat{f}_{2n}^{(\alpha)}} & \cdots & 1 \end{bmatrix} \tag{6.31}$$

From Eq. (6.31), let $\alpha = 0, 0.1, 0.2, \ldots, 1$, and use fixed λ values, then we obtain the wanted total priority weights for all α-cuts with fixed λ values.

6.6 Microencapsulation Decision Support System

In this section, we will present a case study to show the application of fuzzy BR-AHP in an engineering process selection, i.e. in microencapsulation process selection problem. Here we will show how the proposed methodology fuzzy BR-AHP can be used to create the basis of a Decision Support System (DSS) for the microencapsulation selection problem.

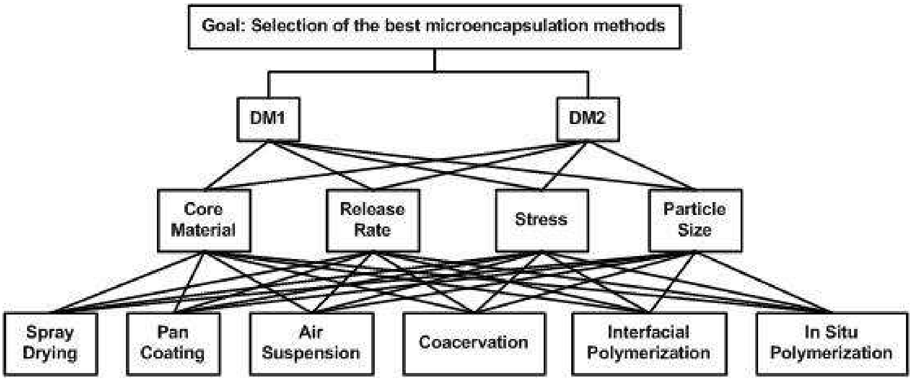
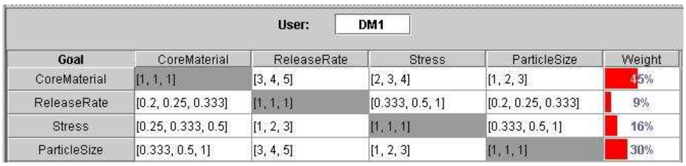


Figure 6.6: *Microencapsulation Process Selection Hierarchy*

6.6.1 Hierarchical Structure Construction

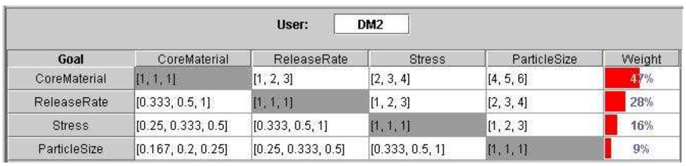
The first step of fuzzy BR-AHP algorithm is to construct a hierarchical structure of microencapsulation selection problem. The goal of our case study is to select the best microencapsulation method. There are two decision makers that are involved in the selection process, i.e. DM1 and DM2, and four criteria for the selection, i.e. Core Material, Release Rate, Stress and Particle Size. There are six microencapsulation methods to be considered, i.e. Spray Drying, Pan Coating, Air Suspension, Coacervation, Interfacial Polymerization and In-Situ Polymerization. The alternative microencapsulation methods are analysed with respect to the criteria in the third level of the hierarchy, as shown in Figure 6.6.



User: DM1

Goal	CoreMaterial	ReleaseRate	Stress	ParticleSize	Weight
CoreMaterial	[1, 1, 1]	[3, 4, 5]	[2, 3, 4]	[1, 2, 3]	45%
ReleaseRate	[0.2, 0.25, 0.333]	[1, 1, 1]	[0.333, 0.5, 1]	[0.2, 0.25, 0.333]	9%
Stress	[0.25, 0.333, 0.5]	[1, 2, 3]	[1, 1, 1]	[0.333, 0.5, 1]	16%
ParticleSize	[0.333, 0.5, 1]	[3, 4, 5]	[1, 2, 3]	[1, 1, 1]	30%

(a) by DM1



User: DM2

Goal	CoreMaterial	ReleaseRate	Stress	ParticleSize	Weight
CoreMaterial	[1, 1, 1]	[1, 2, 3]	[2, 3, 4]	[4, 5, 6]	47%
ReleaseRate	[0.333, 0.5, 1]	[1, 1, 1]	[1, 2, 3]	[2, 3, 4]	28%
Stress	[0.25, 0.333, 0.5]	[0.333, 0.5, 1]	[1, 1, 1]	[1, 2, 3]	16%
ParticleSize	[0.167, 0.2, 0.25]	[0.25, 0.333, 0.5]	[0.333, 0.5, 1]	[1, 1, 1]	9%

(b) by DM2

Figure 6.7: *Fuzzy Pairwise Comparison of Microencapsulation Criteria*

6.6.2 Individual Evaluation of Criteria

The second step is for each decision maker to evaluate the microencapsulation criteria using fuzzy pairwise comparison method. To facilitate making fuzzy pairwise comparisons, the triangular fuzzy numbers defined in Table 6.1 are used. The results of this operation by DM1 and DM2 are presented in Figure 6.7. Here we have a total of four criteria, so each decision maker needs to determine $4(4-1)/2 = 6$ pairwise comparisons.

6.6.3 Individual Evaluation of Alternatives

The third step is the individual evaluation of the microencapsulation alternatives using fuzzy base pairwise comparison. First, each decision maker should select a base alternative. Then, he/she should compare his/her base alternative with other alternatives. To facilitate the pairwise comparison, the triangular fuzzy numbers defined in Table 6.1 are employed. Figures 6.8 - 6.11 show the fuzzy base pairwise comparison of the six microencapsulation alternatives (i.e. Spray Drying, Pan Coating, Air Suspension, Coacervation, Interfacial Polymerization and In Situ Polymerization) with respect to the four microencapsulation criteria (i.e. Core Material, Release Rate, Stress and Particle Size) by two decision makers (DM1 and DM2), respectively. For the explanation of how fuzzy base pairwise comparison works, let us consider one of the previously mentioned figures. In Figure 6.8a, the base alternative is highlighted horizontally and DM1 selected an alternative "Coacervation" as the base alternative. In this step, each decision maker only needs to determine five judgments by comparing the base alternative "Coacervation" with the other five alternatives.

6.6.4 Integration of Individual Analysis Results

The final step is to combine all the weights derived from the previous steps by the two decision makers to obtain the overall ranking for the alternatives. The result of this ranking of microencapsulation alternatives is shown in Figure 6.12.

For the case with four criteria and six alternatives, by using fuzzy BR-AHP approach, each decision maker only needs a total of $4(4-1)/2 + 4 \times (6-1) = 26$ pairwise comparisons. Thus, the total required pairwise comparisons for two decision makers are equal to 52. If we use conventional fuzzy AHP, then we need $4(4-1)/2 + 4 \times 6(6-1)/2 = 66$ pairwise comparisons for each decision maker. Therefore, we need 132 pairwise comparisons for two decision makers. In this case, we have reduced the amount of comparisons by 80 in using fuzzy BR-AHP approach. In real life cases for the microencapsulation selection, we can have hundreds or even thousands alternatives. In this case, we would be able to reduce even more pairwise comparisons by using fuzzy BR-AHP approach.

Table 6.2 shows the total pairwise comparisons needed for conventional fuzzy AHP and fuzzy BR-AHP and the reduced number of pairwise comparisons by

User: DM1 Base: 4

CoreMaterial	SprayDrying	PanCoating	AirSuspension	Coacervation	InterfacialPolymeri...	InSituPolymerization	Weight
SprayDrying	[1, 1, 1]	[1, 2, 3]	[1, 2, 3]	[0.333, 0.5, 1]	[0.2, 0.25, 0.333]	[0.143, 0.167, 0.2]	8%
PanCoating	[0.333, 0.5, 1]	[1, 1, 1]	[1, 1, 2]	[0.25, 0.333, 0.5]	[0.143, 0.167, 0.2]	[0.111, 0.111, 0.125]	5%
AirSuspension	[0.333, 0.5, 1]	[0.5, 1, 1]	[1, 1, 1]	[0.2, 0.25, 0.333]	[0.111, 0.125, 0.143]	[0.111, 0.111, 0.125]	4%
Coacervation	[1, 2, 3]	[2, 3, 4]	[3, 4, 5]	[1, 1, 1]	[0.333, 0.5, 1]	[0.25, 0.333, 0.5]	15%
InterfacialPolymerizat...	[3, 4, 5]	[5, 6, 7]	[7, 8, 9]	[1, 2, 3]	[1, 1, 1]	[0.333, 0.5, 1]	28%
InSituPolymerization	[5, 6, 7]	[8, 9, 9]	[8, 9, 9]	[2, 3, 4]	[1, 2, 3]	[1, 1, 1]	42%

(a) by DM1

User: DM2 Base: 4

CoreMaterial	SprayDrying	PanCoating	AirSuspension	Coacervation	InterfacialPolymeri...	InSituPolymerization	Weight
SprayDrying	[1, 1, 1]	[1, 2, 3]	[1, 1, 2]	[0.333, 0.5, 1]	[1, 1, 2]	[1, 2, 3]	18%
PanCoating	[0.333, 0.5, 1]	[1, 1, 1]	[0.333, 0.5, 1]	[0.2, 0.25, 0.333]	[0.333, 0.5, 1]	[1, 1, 2]	9%
AirSuspension	[0.5, 1, 1]	[1, 2, 3]	[1, 1, 1]	[0.333, 0.5, 1]	[1, 1, 2]	[1, 2, 3]	17%
Coacervation	[1, 2, 3]	[3, 4, 5]	[1, 2, 3]	[1, 1, 1]	[1, 2, 3]	[3, 4, 5]	32%
InterfacialPolymerizat...	[0.5, 1, 1]	[1, 2, 3]	[0.5, 1, 1]	[0.333, 0.5, 1]	[1, 1, 1]	[1, 2, 3]	15%
InSituPolymerization	[0.333, 0.5, 1]	[0.5, 1, 1]	[0.333, 0.5, 1]	[0.2, 0.25, 0.333]	[0.333, 0.5, 1]	[1, 1, 1]	9%

(b) by DM2

Figure 6.8: *Fuzzy Base Pairwise Comparison of Microencapsulation Alternatives with respect to 'Core Material' Criterion*

User: DM1 Base: 4

ReleaseRate	SprayDrying	PanCoating	AirSuspension	Coacervation	InterfacialPolymeri...	InSituPolymerization	Weight
SprayDrying	[1, 1, 1]	[0.333, 0.5, 1]	[0.5, 1, 1]	[0.2, 0.25, 0.333]	[0.111, 0.125, 0.143]	[0.2, 0.25, 0.333]	5%
PanCoating	[1, 2, 3]	[1, 1, 1]	[1, 2, 3]	[0.333, 0.5, 1]	[0.2, 0.25, 0.333]	[0.333, 0.5, 1]	11%
AirSuspension	[1, 1, 2]	[0.333, 0.5, 1]	[1, 1, 1]	[0.25, 0.333, 0.5]	[0.143, 0.167, 0.2]	[0.25, 0.333, 0.5]	6%
Coacervation	[3, 4, 5]	[1, 2, 3]	[2, 3, 4]	[1, 1, 1]	[0.333, 0.5, 1]	[1, 1, 2]	21%
InterfacialPolymerizat...	[7, 8, 9]	[3, 4, 5]	[5, 6, 7]	[1, 2, 3]	[1, 1, 1]	[1, 2, 3]	38%
InSituPolymerization	[3, 4, 5]	[1, 2, 3]	[2, 3, 4]	[0.5, 1, 1]	[0.333, 0.5, 1]	[1, 1, 1]	19%

(a) by DM1

User: DM2 Base: 4

ReleaseRate	SprayDrying	PanCoating	AirSuspension	Coacervation	InterfacialPolymeri...	InSituPolymerization	Weight
SprayDrying	[1, 1, 1]	[0.143, 0.167, 0.2]	[0.333, 0.5, 1]	[0.25, 0.333, 0.5]	[1, 1, 2]	[0.143, 0.167, 0.2]	6%
PanCoating	[5, 6, 7]	[1, 1, 1]	[3, 4, 5]	[1, 2, 3]	[5, 6, 7]	[1, 1, 2]	33%
AirSuspension	[1, 2, 3]	[0.2, 0.25, 0.333]	[1, 1, 1]	[0.333, 0.5, 1]	[1, 2, 3]	[0.2, 0.25, 0.333]	9%
Coacervation	[2, 3, 4]	[0.333, 0.5, 1]	[1, 2, 3]	[1, 1, 1]	[2, 3, 4]	[0.333, 0.5, 1]	17%
InterfacialPolymerizat...	[0.5, 1, 1]	[0.143, 0.167, 0.2]	[0.333, 0.5, 1]	[0.25, 0.333, 0.5]	[1, 1, 1]	[0.143, 0.167, 0.2]	5%
InSituPolymerization	[5, 6, 7]	[0.5, 1, 1]	[3, 4, 5]	[1, 2, 3]	[5, 6, 7]	[1, 1, 1]	31%

(b) by DM2

Figure 6.9: *Fuzzy Base Pairwise Comparison of Microencapsulation Alternatives with respect to 'Release Rate' Criterion*

User: DM1 Base: 4

Stress	SprayDrying	PanCoating	AirSuspension	Coacervation	InterfacialPolymeri...	InSituPolymerization	Weight
SprayDrying	[1, 1, 1]	[0.143, 0.167, 0.2]	[1, 1, 2]	[0.333, 0.5, 1]	[1, 1, 2]	[1, 1, 2]	10%
PanCoating	[5, 6, 7]	[1, 1, 1]	[5, 6, 7]	[2, 3, 4]	[5, 6, 7]	[5, 6, 7]	49%
AirSuspension	[0.5, 1, 1]	[0.143, 0.167, 0.2]	[1, 1, 1]	[0.333, 0.5, 1]	[1, 1, 2]	[1, 1, 2]	9%
Coacervation	[1, 2, 3]	[0.25, 0.333, 0.5]	[1, 2, 3]	[1, 1, 1]	[1, 2, 3]	[1, 2, 3]	16%
InterfacialPolymerizat...	[0.5, 1, 1]	[0.143, 0.167, 0.2]	[0.5, 1, 1]	[0.333, 0.5, 1]	[1, 1, 1]	[1, 1, 2]	8%
InSituPolymerization	[0.5, 1, 1]	[0.143, 0.167, 0.2]	[0.5, 1, 1]	[0.333, 0.5, 1]	[0.5, 1, 1]	[1, 1, 1]	8%

(a) by DM1

User: DM2 Base: 4

Stress	SprayDrying	PanCoating	AirSuspension	Coacervation	InterfacialPolymeri...	InSituPolymerization	Weight
SprayDrying	[1, 1, 1]	[0.333, 0.5, 1]	[1, 1, 2]	[0.25, 0.333, 0.5]	[0.143, 0.167, 0.2]	[0.111, 0.111, 0.125]	5%
PanCoating	[1, 2, 3]	[1, 1, 1]	[1, 2, 3]	[0.333, 0.5, 1]	[0.2, 0.25, 0.333]	[0.143, 0.167, 0.2]	8%
AirSuspension	[0.5, 1, 1]	[0.333, 0.5, 1]	[1, 1, 1]	[0.25, 0.333, 0.5]	[0.143, 0.167, 0.2]	[0.111, 0.111, 0.125]	4%
Coacervation	[2, 3, 4]	[1, 2, 3]	[2, 3, 4]	[1, 1, 1]	[0.333, 0.5, 1]	[0.25, 0.333, 0.5]	14%
InterfacialPolymerizat...	[5, 6, 7]	[3, 4, 5]	[5, 6, 7]	[1, 2, 3]	[1, 1, 1]	[0.333, 0.5, 1]	27%
InSituPolymerization	[8, 9, 9]	[5, 6, 7]	[8, 9, 9]	[2, 3, 4]	[1, 2, 3]	[1, 1, 1]	42%

(b) by DM2

Figure 6.10: *Fuzzy Base Pairwise Comparison of Microencapsulation Alternatives with respect to 'Stress' Criterion*

User: DM1 Base: 4

ParticleSize	SprayDrying	PanCoating	AirSuspension	Coacervation	InterfacialPolymeri...	InSituPolymerization	Weight
SprayDrying	[1, 1, 1]	[0.333, 0.5, 1]	[1, 1, 2]	[0.25, 0.333, 0.5]	[0.333, 0.5, 1]	[1, 1, 2]	11%
PanCoating	[1, 2, 3]	[1, 1, 1]	[1, 2, 3]	[0.333, 0.5, 1]	[1, 1, 2]	[1, 2, 3]	19%
AirSuspension	[0.5, 1, 1]	[0.333, 0.5, 1]	[1, 1, 1]	[0.25, 0.333, 0.5]	[0.333, 0.5, 1]	[1, 1, 2]	10%
Coacervation	[2, 3, 4]	[1, 2, 3]	[2, 3, 4]	[1, 1, 1]	[1, 2, 3]	[2, 3, 4]	31%
InterfacialPolymerizat...	[1, 2, 3]	[0.5, 1, 1]	[1, 2, 3]	[0.333, 0.5, 1]	[1, 1, 1]	[1, 2, 3]	18%
InSituPolymerization	[0.5, 1, 1]	[0.333, 0.5, 1]	[0.5, 1, 1]	[0.25, 0.333, 0.5]	[0.333, 0.5, 1]	[1, 1, 1]	10%

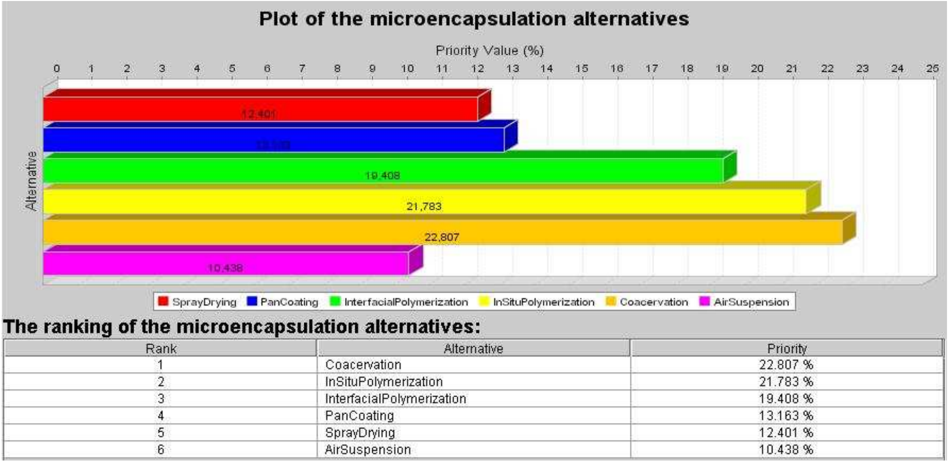
(a) by DM1

User: DM2 Base: 4

ParticleSize	SprayDrying	PanCoating	AirSuspension	Coacervation	InterfacialPolymeri...	InSituPolymerization	Weight
SprayDrying	[1, 1, 1]	[5, 6, 7]	[1, 1, 2]	[1, 2, 3]	[3, 4, 5]	[1, 1, 2]	27%
PanCoating	[0.143, 0.167, 0.2]	[1, 1, 1]	[0.143, 0.167, 0.2]	[0.25, 0.333, 0.5]	[0.333, 0.5, 1]	[0.143, 0.167, 0.2]	4%
AirSuspension	[0.5, 1, 1]	[5, 6, 7]	[1, 1, 1]	[1, 2, 3]	[3, 4, 5]	[1, 1, 2]	25%
Coacervation	[0.333, 0.5, 1]	[2, 3, 4]	[0.333, 0.5, 1]	[1, 1, 1]	[1, 2, 3]	[0.333, 0.5, 1]	14%
InterfacialPolymerizat...	[0.2, 0.25, 0.333]	[1, 2, 3]	[0.2, 0.25, 0.333]	[0.333, 0.5, 1]	[1, 1, 1]	[0.2, 0.25, 0.333]	7%
InSituPolymerization	[0.5, 1, 1]	[5, 6, 7]	[0.5, 1, 1]	[1, 2, 3]	[3, 4, 5]	[1, 1, 1]	23%

(b) by DM2

Figure 6.11: *Fuzzy Base Pairwise Comparison of Microencapsulation Alternatives with respect to 'Particle Size' Criterion*



Rank	Alternative	Priority
1	Coacervation	22.807 %
2	InSituPolymerization	21.783 %
3	InterfacialPolymerization	19.408 %
4	PanCoating	13.163 %
5	SprayDrying	12.401 %
6	AirSuspension	10.438 %

Figure 6.12: *Microencapsulation Process Selection Result*

using fuzzy BR-AHP in the case of four criteria and two decision makers.

Table 6.2: *Number of Pairwise Comparisons in the Case of Four Criteria and Two Decision Makers*

n	Conventional Fuzzy AHP	Fuzzy BR-AHP	Difference
2	20	20	0
3	36	28	8
4	60	36	24
5	92	44	48
6	132	52	80
7	180	60	120
8	236	68	168
9	300	76	224
10	372	84	288
100	39612	804	38808
1000	3996012	8004	3988008

6.6.5 Sensitivity Analysis

In this section, we will use α-cut analysis for the sensitivity analysis of microencapsulation process selection problem. By setting six α levels, i.e. $\alpha = 0$, 0.2, 0.4, 0.6, 0.8, and 1, respectively; and three different λ values, i.e. $\lambda = 0$, 0.5 and 1, we perform the sensitivity analysis of microencapsulation process selection problem. The fuzzy relative values of the six microencapsulation alternative are shown in

Figure 6.13. An α-value of 0 indicates that the decision environment is highly uncertain and α-value of 1 indicates that the problem involves no uncertainty. Intermediate values indicate uncertainty between these two extreme ranges. Here, the α-values of 0.2, 0.4, 0.6, and 0.8 are considered assuming that the decision environment is certain up to some extent.

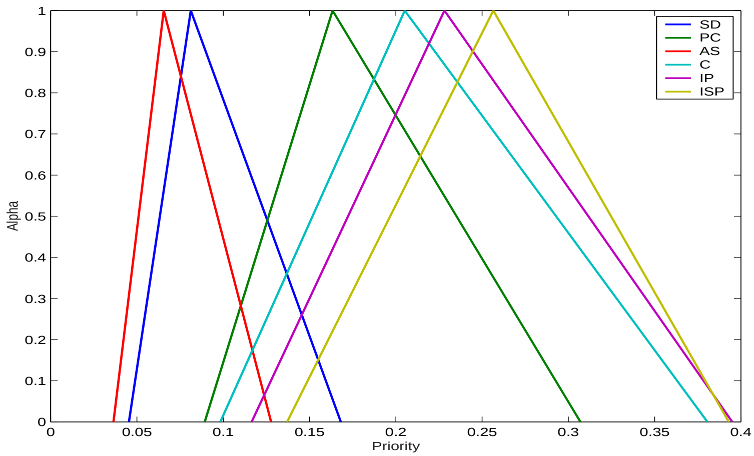
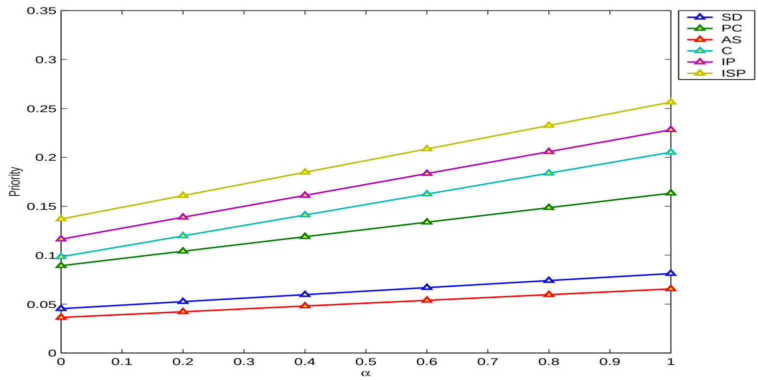


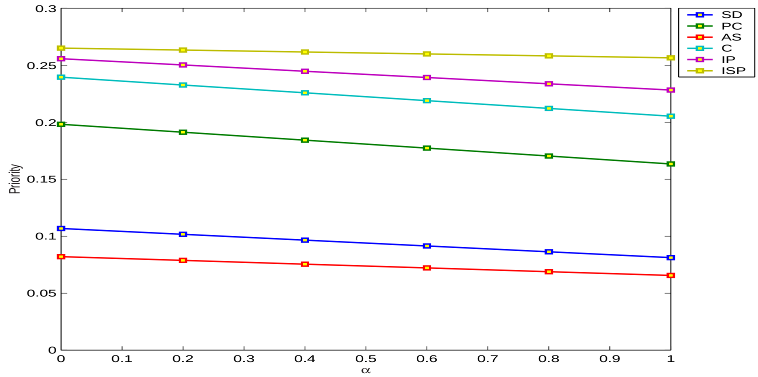
Figure 6.13: *The Fuzzy Relative Values of Microencapsulation Alternatives by DM1*

Figures 6.14a, 6.14b, and 6.14c show the sensitivity analysis results obtained by varying $\alpha-$values for the first decision maker (DM1) with the λ-values of 0, 0.5 and 1, respectively. The results of the sensitivity analysis for the second decision maker (DM2) are presented in Appendix C. The dashed vertical lines are the rank-reversal lines that indicate at which corresponding α-value the rank reversal occurs. In Figures 6.14a, 6.14b, and 6.14c, we see if $\alpha = 1$ the priority values for the six microencapsulation alternatives in these three figures are the same. That means we only have crisp values for the problem which involves no uncertainty (when $\alpha = 1$). The priority values for the six microencapsulation alternatives are 0.0812 for 'Spray Drying', 0.1634 for 'Pan Coating', 0.0655 for 'Air Suspension', 0.2052 for 'Coacervation', 0.2282 for 'Interfacial Polymerization' and 0.2565 for 'In-Situ Polymerization'. This means that the priorities of the six microencapsulation alternatives by DM1 can be written down as 'In-Situ Polymerization' $\succ$ 'Interfacial Polymerization' $\succ$ 'Coacervation' $\succ$ 'Pan Coating' $\succ$ 'Spray Drying' $\succ$ 'Air Suspension', where the symbol '$\succ$' indicates more preferred.

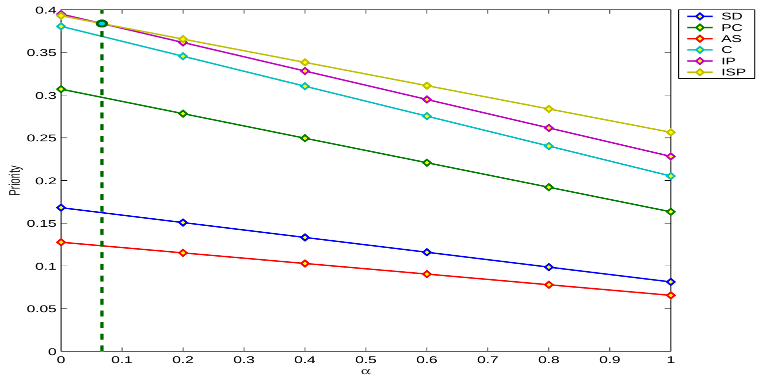
According to the sensitivity analysis performed for $\lambda = 0$ and $\lambda = 0.5$, these two cases are not sensitive. As shown in Figures 6.14a and 6.14b, while the α values increase, ranking of alternatives doesn't change. Whereas $\lambda = 1$ is slightly sensitive between $\alpha = 0$ and $\alpha = 0.2$, since the best alternative changes from 'Interfacial Polymerization' to 'In-Situ Polymerization'. If $\alpha = 0$, we have 0.3950 for 'Interfacial Polymerization' and 0.3930 for 'In-Situ Polymerization'. If



(a) $\lambda = 0$

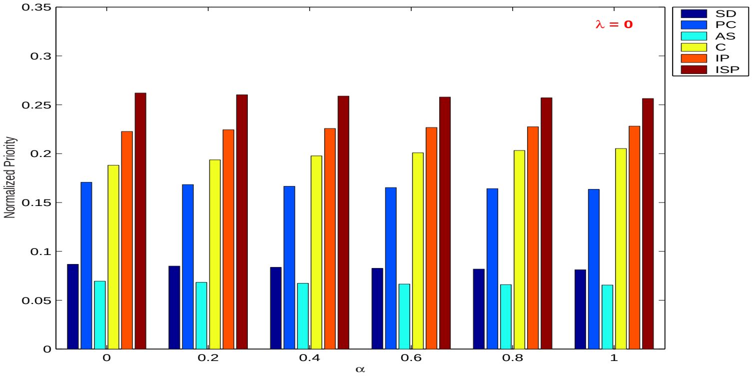


(b) $\lambda = 0.5$

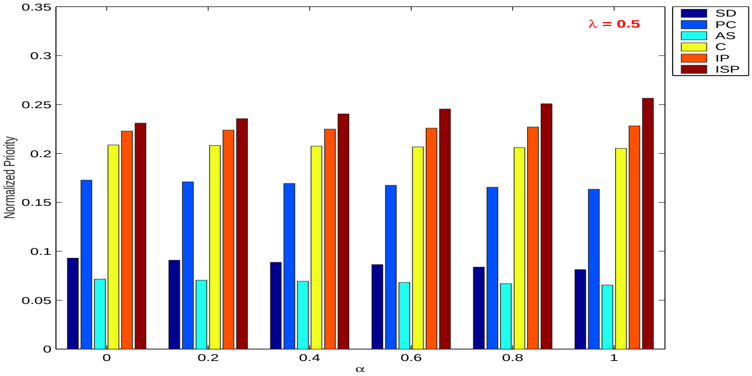


(c) $\lambda = 1$

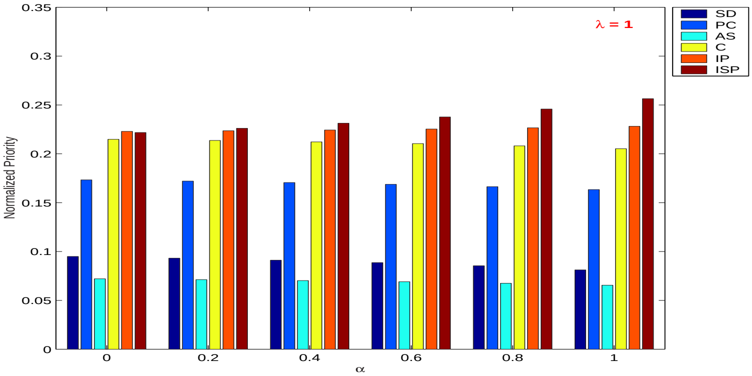
Figure 6.14: *Sensitivity Analysis of Microencapsulation Alternatives by varying* $\alpha -$ *values for the 1st Decision Maker (DM1)*



(a) $\lambda = 0$

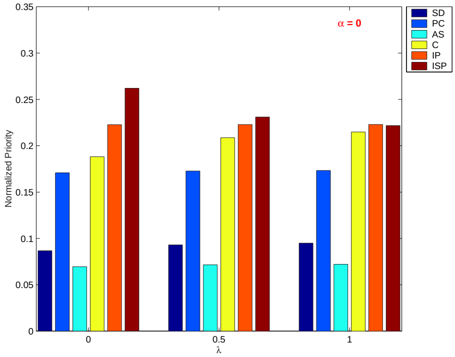


(b) $\lambda = 0.5$

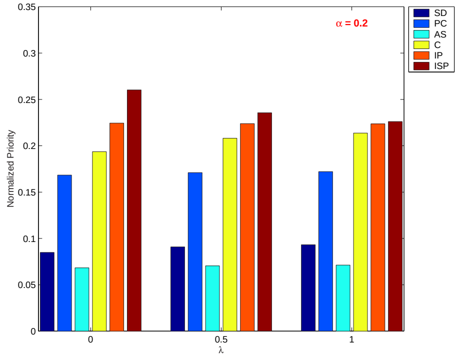


(c) $\lambda = 1$

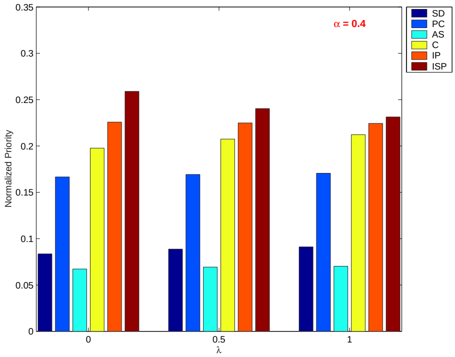
Figure 6.15: *Priority Analysis of Microencapsulation Alternatives by varying* $\alpha-$*values for the 1st Decision Maker (DM1)*



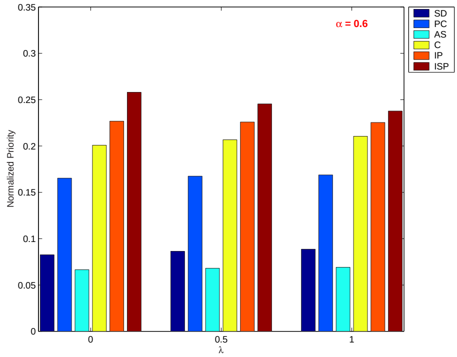
(a) $\alpha = 0$



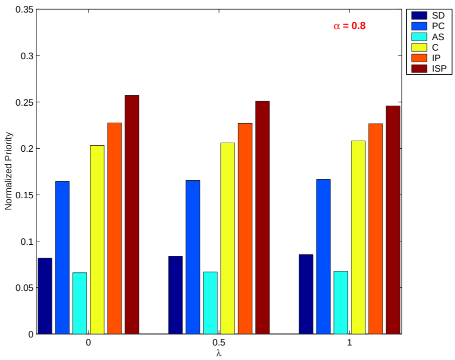
(b) $\alpha = 0.2$



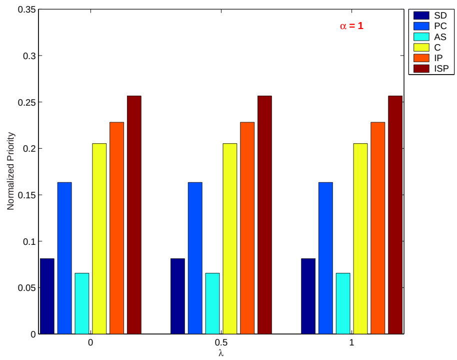
(c) $\alpha = 0.4$



(d) $\alpha = 0.6$



(e) $\alpha = 0.8$



(f) $\alpha = 1$

Figure 6.16: *Priority Analysis of Microencapsulation Alternatives by varying* $\lambda-$*values for the 1st Decision Maker (DM1)*

$\alpha = 0.2$, we have 0.3616 for 'Interfacial Polymerization' and 0.3657 for 'In-Situ Polymerization'.

Figures 6.15a, 6.15b, and 6.15c show the priority analysis results of microencapsulation alternatives obtained by varying α-values for the first decision maker (DM1) with the λ-values of 0, 0.5 and 1, respectively. The priority analysis is performed by normalizing the result of sensitivity analysis in Figure 6.14.

Besides varying the α-values, the variation for each alternative can be also depicted by fixing an α-value to show the performances for each microencapsulation alternative at different λ-values. In Figures 6.16a - 6.16f, we see these variations. Those figures show the priority analysis results of microencapsulation alternatives obtained by varying λ-values for the first decision maker (DM1) with the α-values of 0, 0.2, 0.4, 0.6, 0.8 and 1, respectively.

6.7 Summary

The main objective of this chapter is to present a new approach based on the fuzzy set theory and the Base Reference Analytical Hierarchy Process (BR-AHP) approach, which is able to cope with the imprecision and vagueness of the information data and also suitable for the development of group decision support systems. Two of the main contributions of this chapter are (1) the extension of BR-AHP method to cope with the vagueness and uncertainty in the real-world problems and (2) the possibilities that BR-AHP offers in group decision making.

In this chapter, we proposed a new fuzzy BR-AHP method for the use of microencapsulation process selection. Unlike the conventional fuzzy AHP method that needs $n(n-1)/2$ judgments for each selection criteria, the new method being the so-called fuzzy base pairwise comparison method only needs $n-1$ comparisons. The new method which uses fuzzy base pairwise comparison can significantly reduce the decision making time for each decision maker because it uses much less pairwise comparisons than the conventional approach.

The judgments given by the decision maker in decision models are often subjective or uncertain. And also the real world problems are influenced by many natural factors and processes, that are difficult to measure and model precisely. Thus, it is important to verify the final ranking of the alternatives using sensitivity analysis. Sensitivity analysis is another very important part of any decision making process, which explains the validity of the chosen approach. The level fuzzy sets (α-cut) and the index of the decision maker optimism level (λ) are employed to facilitate the sensitivity analysis in the fuzzy BR-AHP.

Chapter 7

Conclusions

7.1 Summary

The objective of this thesis is to develop a decision support expert system framework for the engineering selection problems in the field of chemical engineering in the special case of microencapsulation selection problems.

Due to the rapid growth of microencapsulation technology, the selection of the most appropriate microencapsulation process has become increasingly important. As a decision aid for process engineers, it is necessary to design a decision support expert system that provides help for selecting the appropriate microencapsulation technique for a specific application. The proposed system incorporates the modules of Expert System (ES) and Decision Support System (DSS) that uses Multiple Attribute Decision Making (MADM) techniques which consists three submodules, i.e. Analytical Hierarchy Process (AHP), Base Reference Analytical Hierarchy Process (BR-AHP) and fuzzy Base Reference Analytical Hierarchy Process (fuzzy BR-AHP) modules. The ES module provides a list of feasible microencapsulation alternatives and then the MADM modules are used to rank the microencapsulation alternatives.

Multiple Criteria Decision Making (MCDM) was developed in the recent couple of decades as a response to the problems faced by decision makers when confronting the complex problems of making decisions based upon multiple, uncertain and possibly conflicting criteria/attributes. MCDM problems can be broken down into two distinct types of problems, i.e. selection problems and synthesis problems. Selection problems involve choosing one of several possible alternatives. Synthesis involves creating solutions whose aim is to attain a set of goals. MCDM is divided into two main groups: Multiple Attribute Decision Making (MADM) and Multiple Objective Decision Making (MODM). MADM corresponds to the selection problems, while MODM corresponds to the synthesis problems. Since the focus of this thesis is to solve the selection problems in the field of engineering, thus the multiple criteria problem considered in this thesis belongs to the class of MADM problems.

Analytical Hierarchy Process (AHP) is one of the most widely used MADM methods which deals with the problem of choosing an alternative from a set of alternatives which are characterized in terms of their attributes by using pairwise comparison technique. In this thesis, we adopted and extended the Analytical Hierarchy Process (AHP) method.

Although pairwise comparisons have been seen by many as an effective way for eliciting qualitative data, a major drawback is that the exhaustive pairwise comparison is tiresome and time consuming when there are many alternatives to be considered. This thesis proposes a new approach to improve this limitation of AHP, the so-called Base Reference Analytical Hierarchy Process (BR-AHP).

Since many real-world engineering systems are too complex to be defined in precise terms, imprecision or approximation is often involved and also the information available for making a decision can be vague and uncertain. A more realistic approach is to incorporate fuzzy theory. Therefore, we propose a new approach to cope with imprecision, uncertainties and vagueness in the judgment of the decision maker, the so-called fuzzy Base Reference Analytical Hierarchy Process (fuzzy BR-AHP).

In many cases, data in the MADM problems are imprecise and easy to change. Sensitivity analysis is a commonly used method for test the ranking robustness against small changes in the input values. Therefore, it is important to perform sensitivity analysis to the input data. The framework proposed in this thesis also incorporated sensitivity analysis in each MADM module for handling imprecise, vague and uncertain data.

Finally, we have implemented the concepts described here in our prototypical decision support expert system tool to validate our approach. We have applied our approach to our test domain, i.e. microencapsulation domain.

7.2 Future Works

The developed system is only a prototype with the purpose to validate and to show the concepts proposed in this thesis thus it should definitely be upgraded and improved. As our understanding deepens and the data availability increases there is a potential to expand this model and its capabilities. Future works are needed to extend the capabilities of the proposed tool. One of the possible future development is to extend the capability of the expert system module by incorporating machine learning method in the process. Then the developed tool will also be able to store cases, learn and use them for future situations. The system can be "taught" to recognize which criteria are applicable and which are not, as well as which criteria are more important than others for particular situations.

Another possible future work of this thesis is to incorporate a CBR (Case-

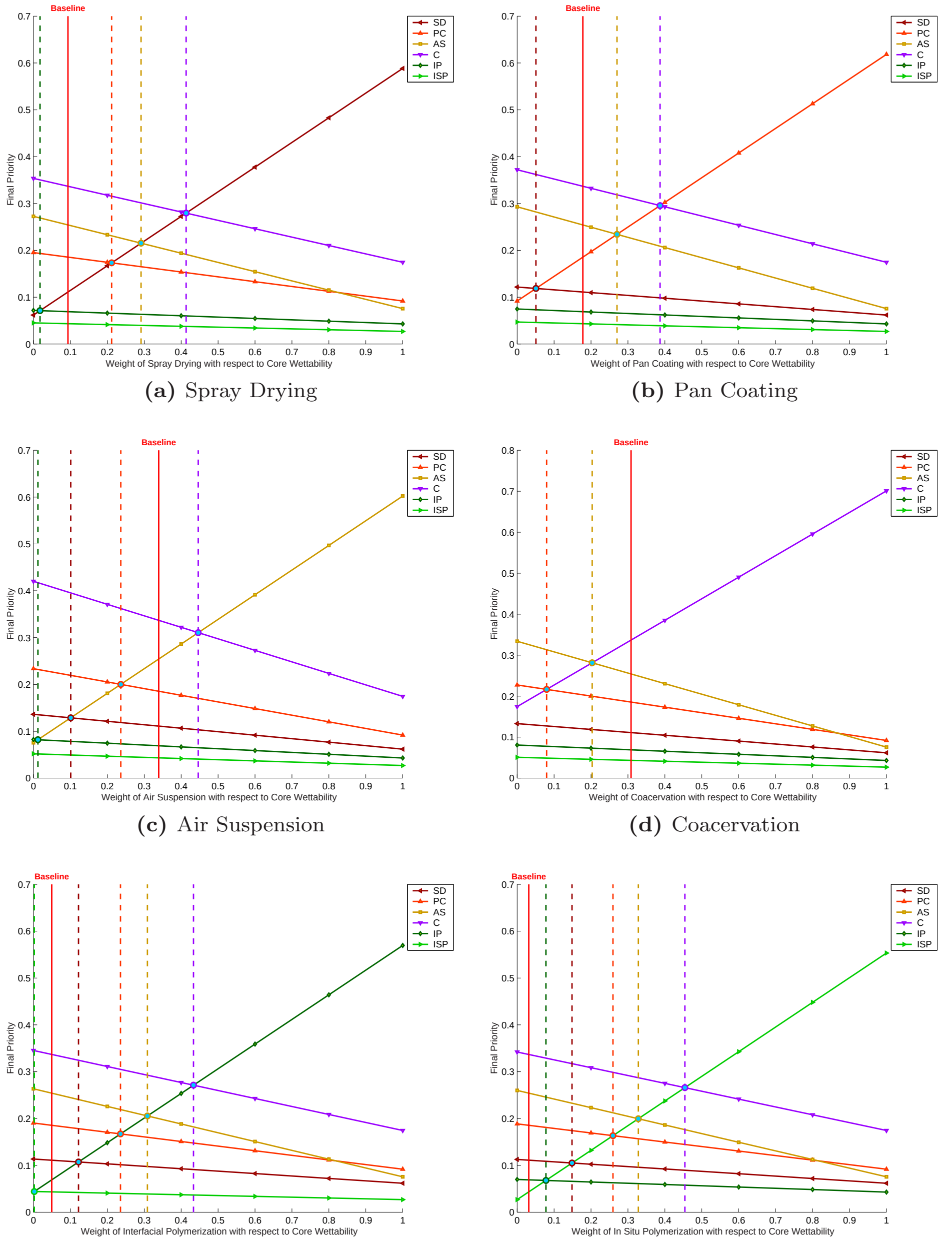
Based Reasoning) module in the inference engine as an additional tool for decision aid and as a mechanism of incremental learning. The CBR module will complement the solution, acting as a memory of past cases which can be consulted in order to identify similar cases for the new problem. This process is similar to the mechanism used by humans for the analysis of new situations. The human expert in the microencapsulation selection uses his/her previous acquired experiences as a valuable tool to explore the new scenario. The previous situations form the main source of knowledge for an expert in the microencapsulation process. An old similar episode may serve as inspiration for a new solution (with appropriate adaptations, obviously) or may represent an error in the microencapsulation decision making who must be avoided in a new analogous situation.

Besides extending the expert system part, in the future we can also extend the other part of the system, i.e. the decision support system module by integrating more advanced methods and approaches into the MADM module to extend the capability of the system in aiding the decision makers.

Appendix A

Sensitivity Analysis Diagram using AHP Methodology

This section presents the results of the sensitivity analysis using AHP method for six microencapsulation alternatives (i.e. Spray Drying, Pan Coating, Air Suspension, Coacervation, Interfacial Polymerization and In-Situ Polymerization) with respect to the criteria Core Wettability, Core Solubility, Wall Elasticity, Wall Permeability and Wall Polymer Adhesive, respectively. The discussions of these results are presented in Section 4.5.



(a) Spray Drying

(b) Pan Coating

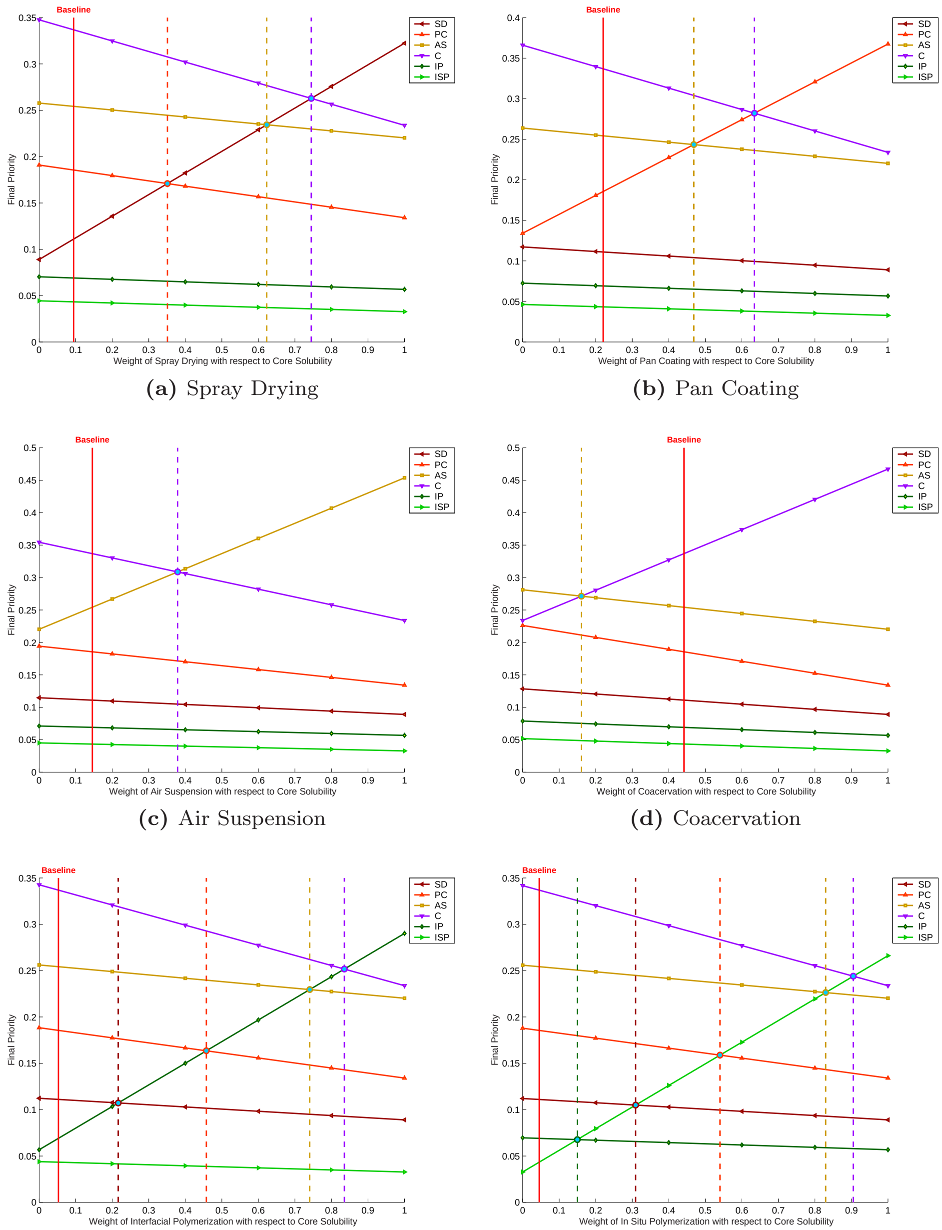
(c) Air Suspension

(d) Coacervation

(e) Interfacial Polymerization

(f) In Situ Polymerization

Figure A.1: *Sensitivity Analysis Diagram of Microencapsulation Alternatives with respect to 'Core Wettability' Criterion*



(a) Spray Drying

(b) Pan Coating

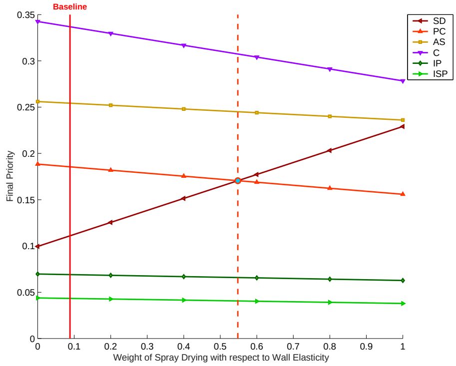
(c) Air Suspension

(d) Coacervation

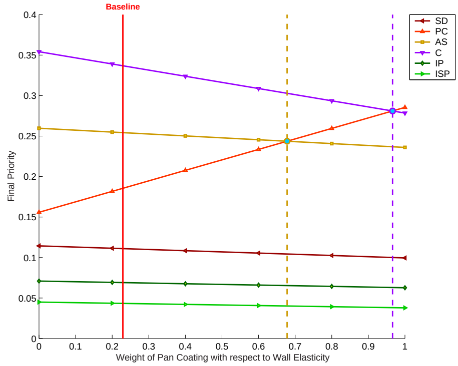
(e) Interfacial Polymerization

(f) In Situ Polymerization

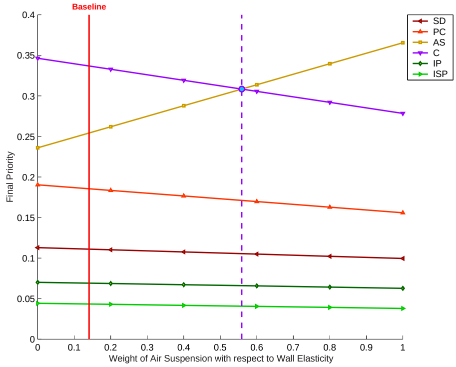
Figure A.2: *Sensitivity Analysis Diagram of Microencapsulation Alternatives with respect to 'Core Solubility' Criterion*



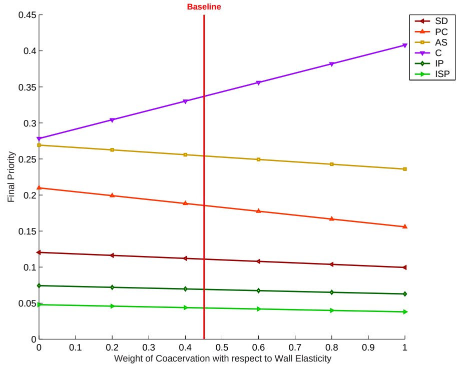
(a) Spray Drying



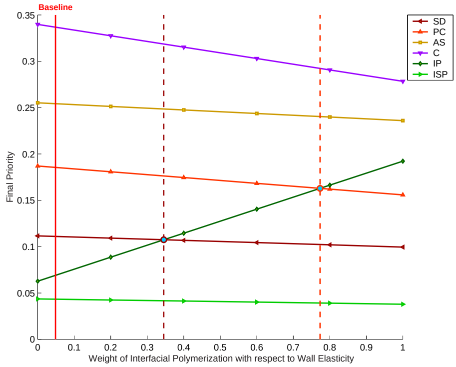
(b) Pan Coating



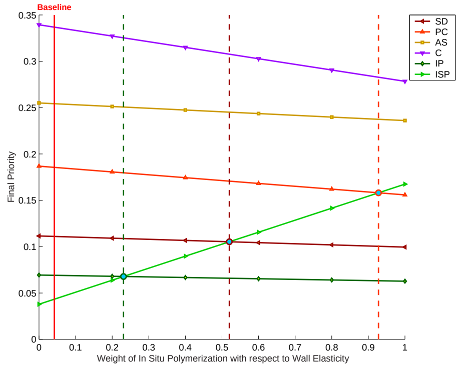
(c) Air Suspension



(d) Coacervation

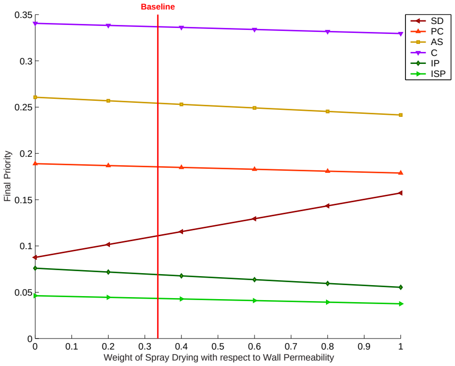


(e) Interfacial Polymerization

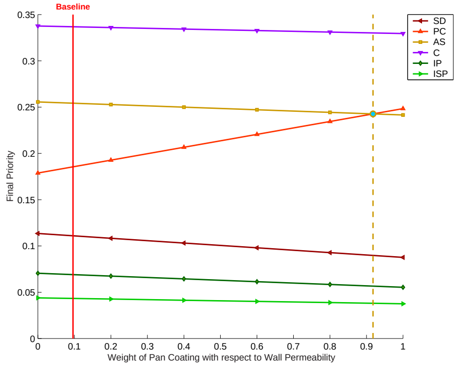


(f) In Situ Polymerization

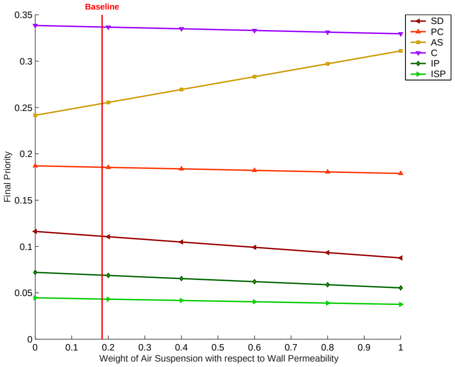
Figure A.3: *Sensitivity Analysis Diagram of Microencapsulation Alternatives with respect to 'Wall Elasticity' Criterion*



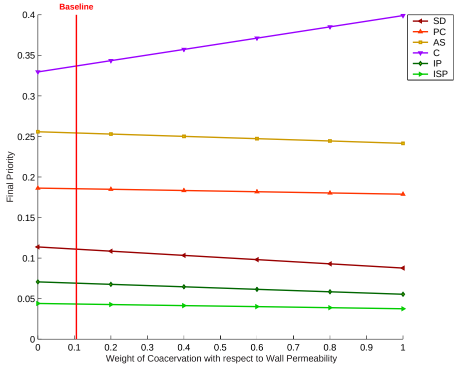
(a) Spray Drying



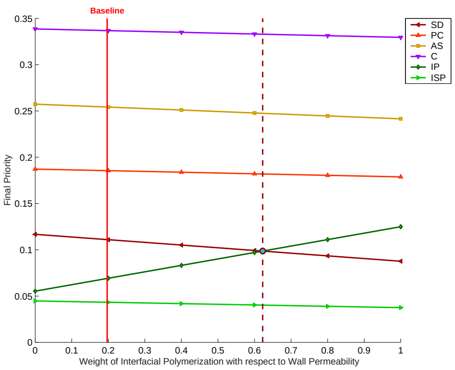
(b) Pan Coating



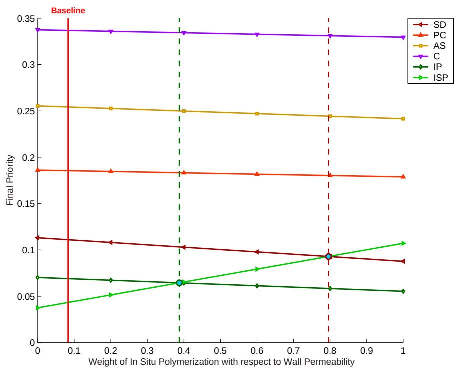
(c) Air Suspension



(d) Coacervation

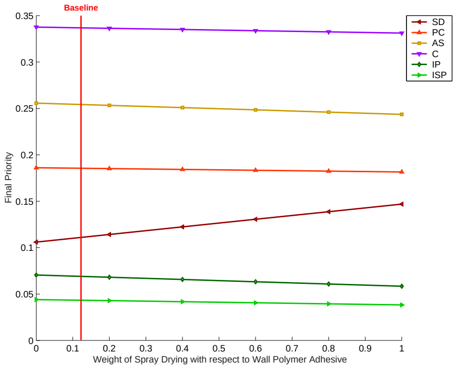


(e) Interfacial Polymerization

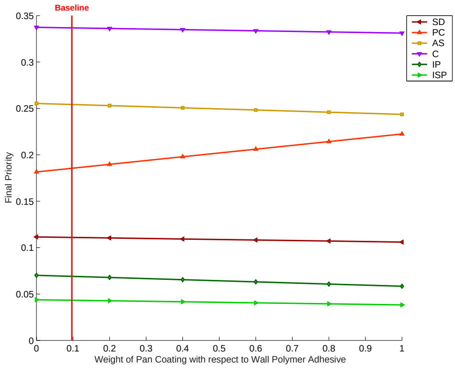


(f) In Situ Polymerization

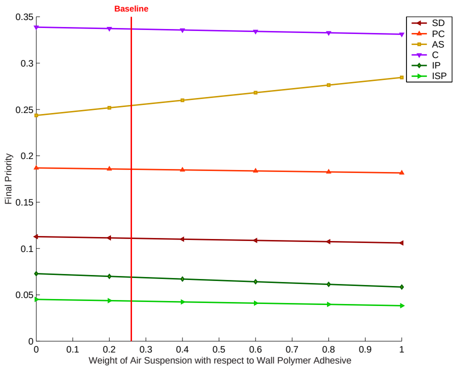
Figure A.4: *Sensitivity Analysis Diagram of Microencapsulation Alternatives with respect to 'Wall Permeability' Criterion*



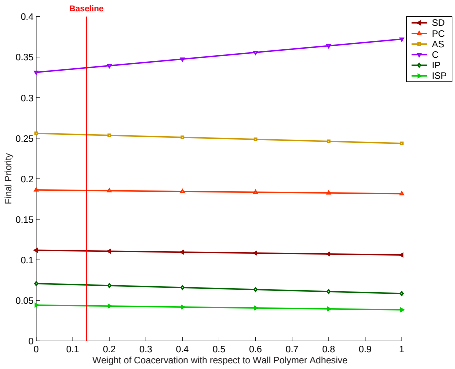
(a) Spray Drying



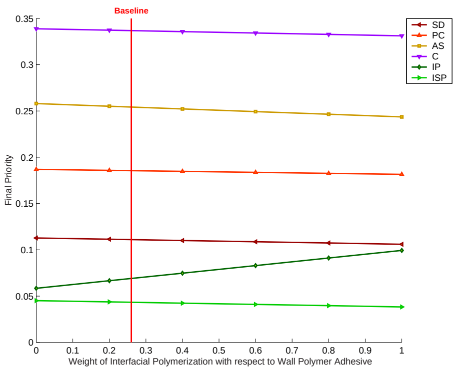
(b) Pan Coating



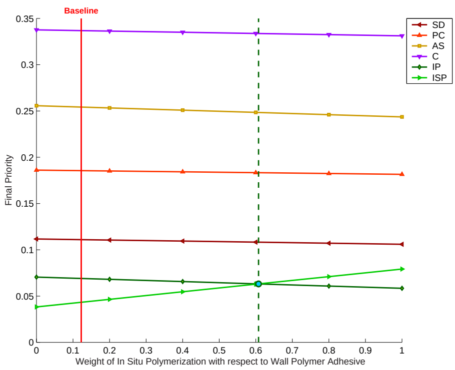
(c) Air Suspension



(d) Coacervation



(e) Interfacial Polymerization



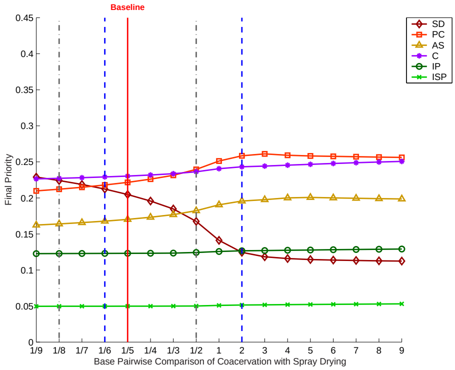
(f) In Situ Polymerization

Figure A.5: *Sensitivity Analysis Diagram of Microencapsulation Alternatives with respect to 'Wall Polymer Adhesive' Criterion*

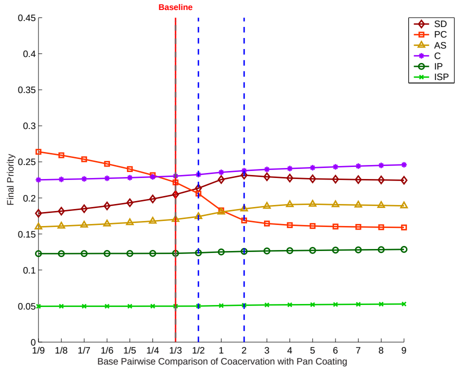
Appendix B

Sensitivity Analysis for Base Pairwise Comparison

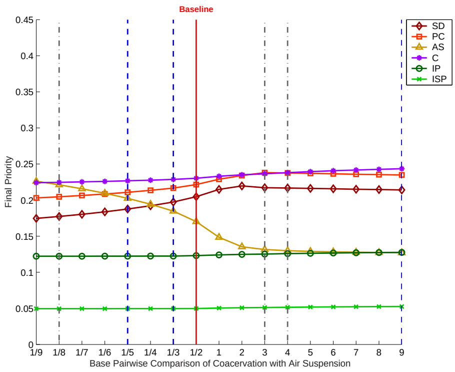
This section presents the sensitivity analysis diagram by adjusting Base Pairwise Comparison for six microencapsulation alternatives (i.e. Spray Drying, Pan Coating, Air Suspension, Coacervation, Interfacial Polymerization and In-Situ Polymerization) with respect to the criteria Release Rate, Pressure, Particle Size, and Other Requirements, respectively. The discussions of these results are presented in Section 5.5.



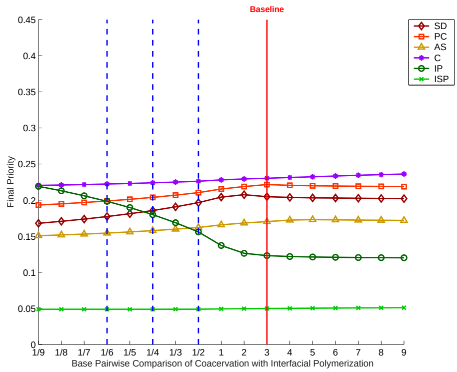
(a) Spray Drying



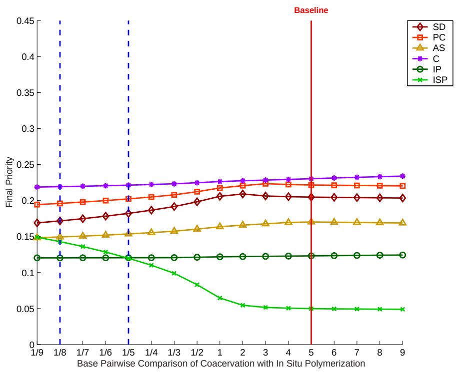
(b) Pan Coating



(c) Air Suspension

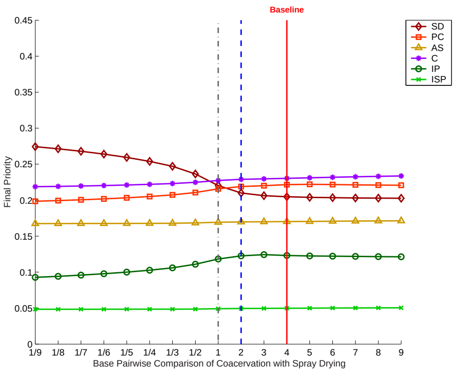


(d) Interfacial Polymerization

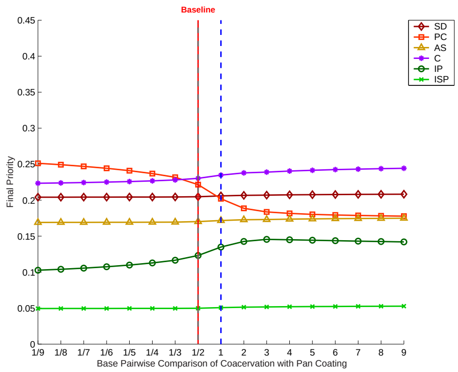


(e) In Situ Polymerization

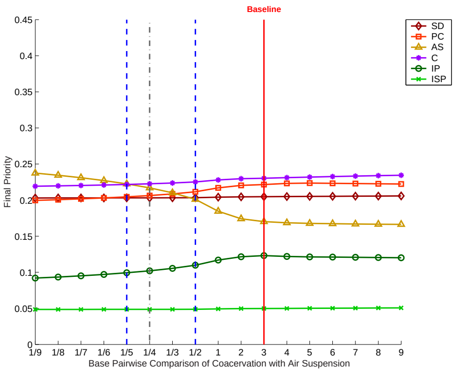
Figure B.1: *Trends of Alternatives Preference Weights by adjusting Base Pairwise Comparison of 'Coacervation' for 'Release Rate' Criterion*



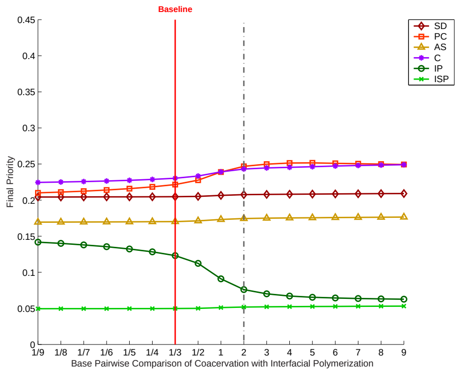
(a) Spray Drying



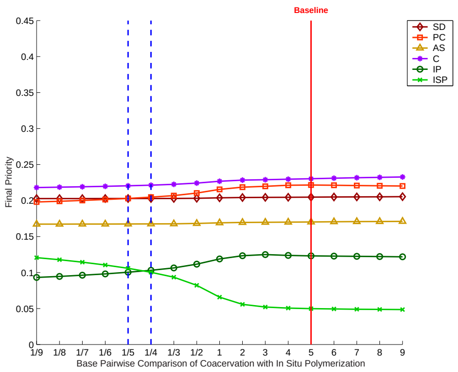
(b) Pan Coating



(c) Air Suspension

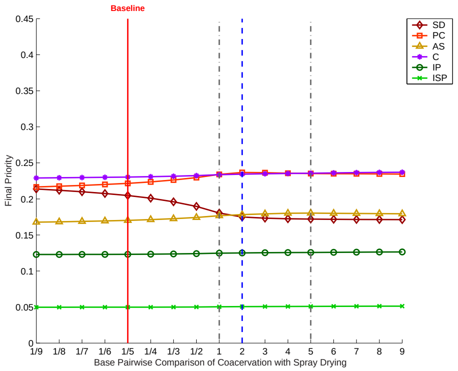


(d) Interfacial Polymerization

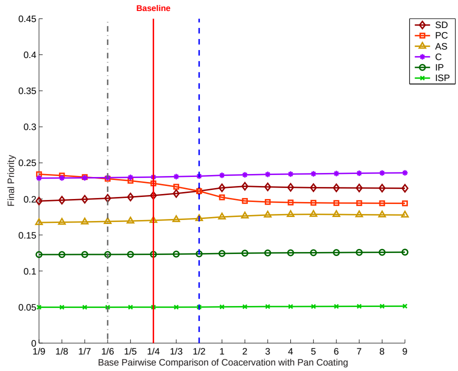


(e) In Situ Polymerization

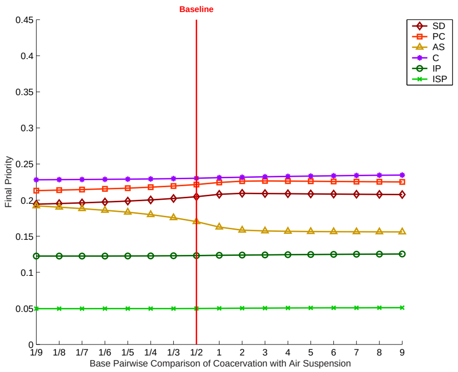
Figure B.2: *Trends of Alternatives Preference Weights by adjusting Base Pairwise Comparison of 'Coacervation' for 'Pressure' Criterion*



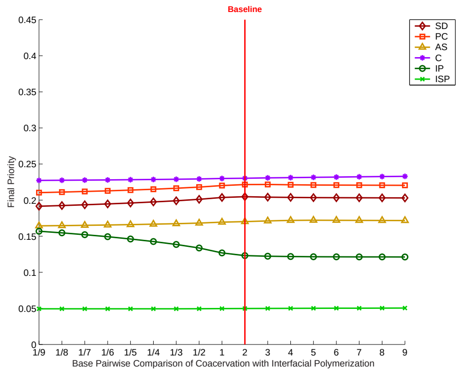
(a) Spray Drying



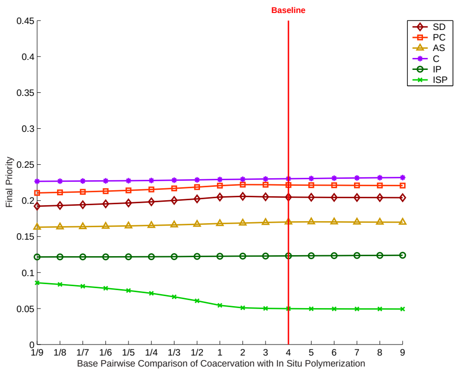
(b) Pan Coating



(c) Air Suspension

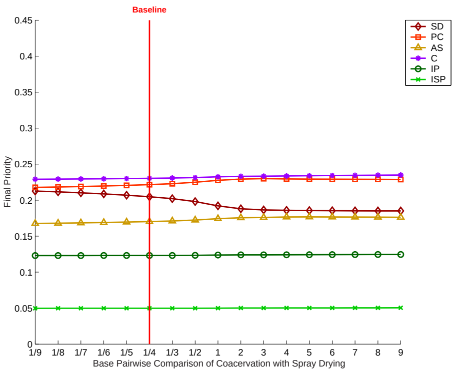


(d) Interfacial Polymerization

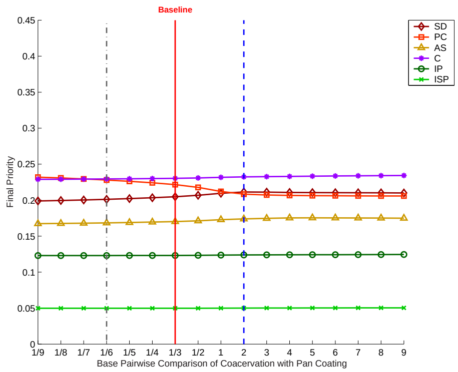


(e) In Situ Polymerization

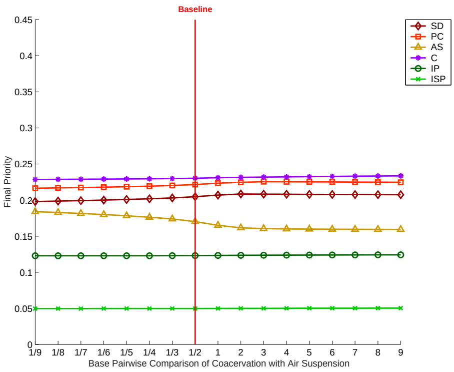
Figure B.3: *Trends of Alternatives Preference Weights by adjusting Base Pairwise Comparison of 'Coacervation' for 'Particle Size' Criterion*



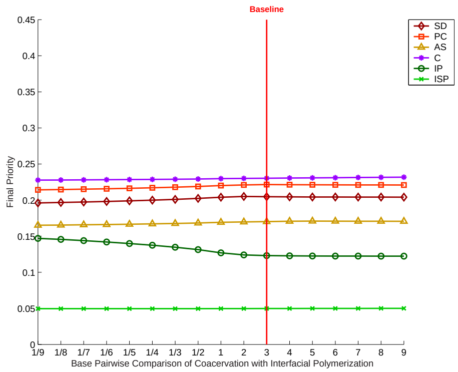
(a) Spray Drying



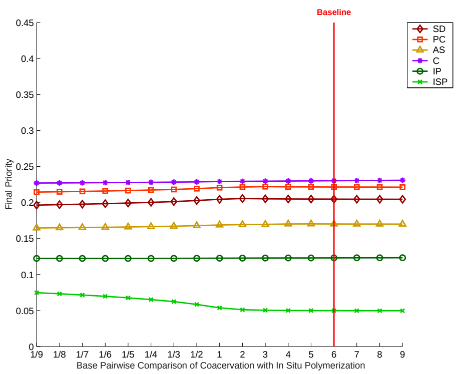
(b) Pan Coating



(c) Air Suspension



(d) Interfacial Polymerization



(e) In Situ Polymerization

Figure B.4: *Trends of Alternatives Preference Weights by adjusting Base Pairwise Comparison of 'Coacervation' for 'Other Requirements' Criterion*

Appendix C

Sensitivity Analysis of Fuzzy BR-AHP

This section presents the experimental results of the sensitivity analysis of microencapsulation alternatives of fuzzy BR-AHP method for the second decision maker (DM2) by adjusting the values of α and λ for six microencapsulation alternatives (i.e. Spray Drying, Pan Coating, Air Suspension, Coacervation, Interfacial Polymerization and In-Situ Polymerization) with respect to the criteria Release Rate, Pressure, Particle Size, and Other Requirements, respectively. The discussions of these results are presented in Section 6.6.5.

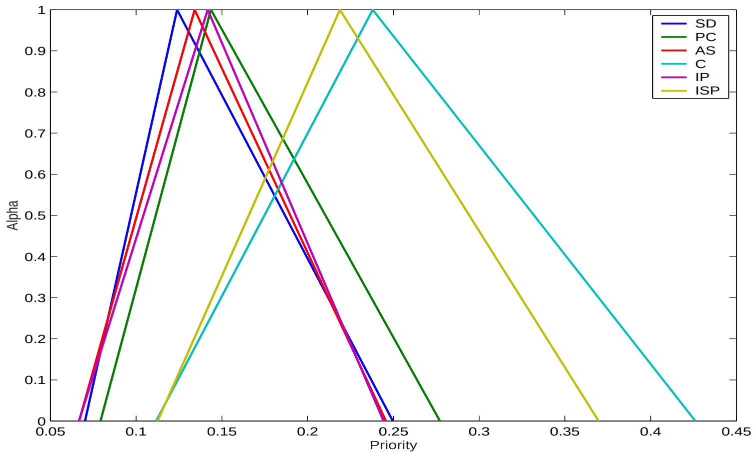
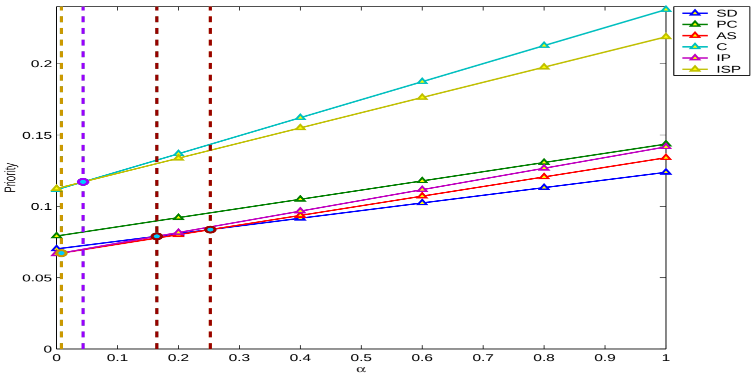
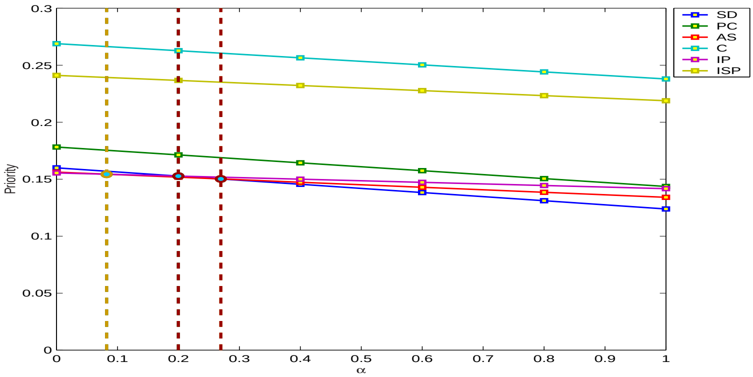


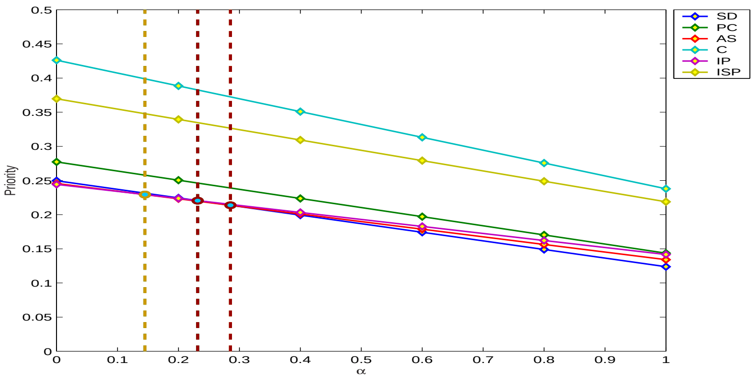
Figure C.1: *The Fuzzy Relative Values of Microencapsulation Alternatives by DM2*



(a) $\lambda = 0$

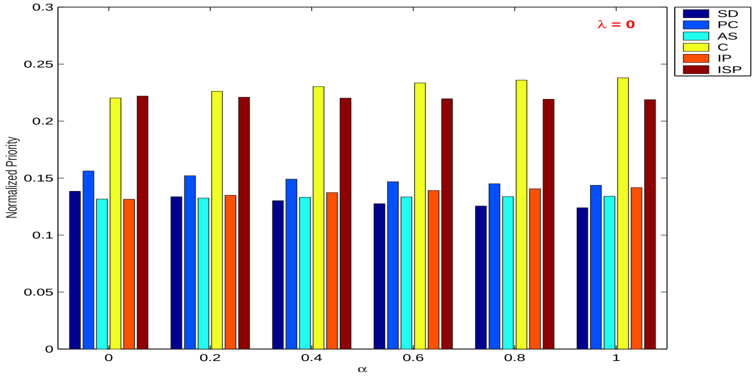


(b) $\lambda = 0.5$

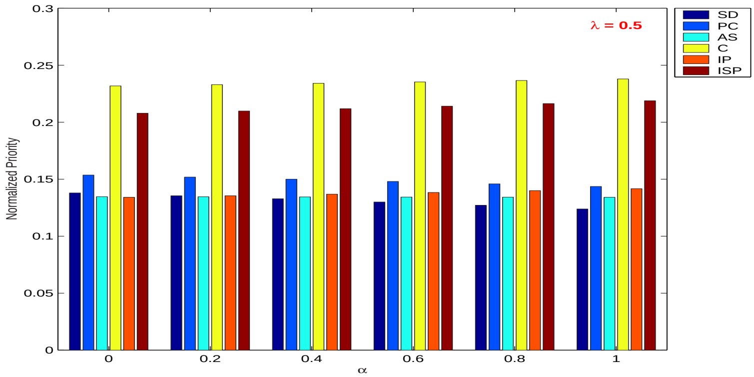


(c) $\lambda = 1$

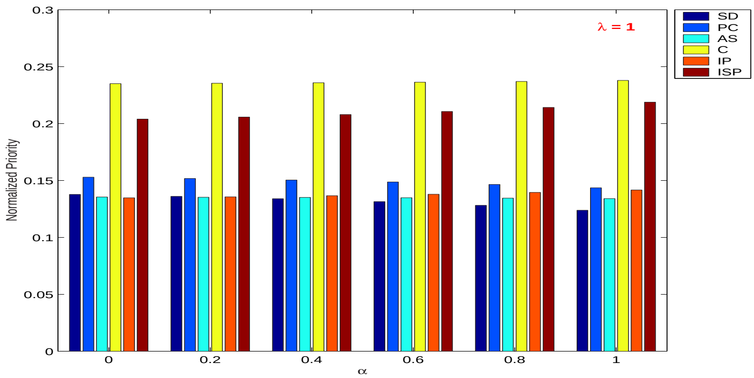
Figure C.2: *Sensitivity Analysis of Microencapsulation Alternatives by varying α – values for the 2nd Decision Maker (DM2)*



(a) $\lambda = 0$

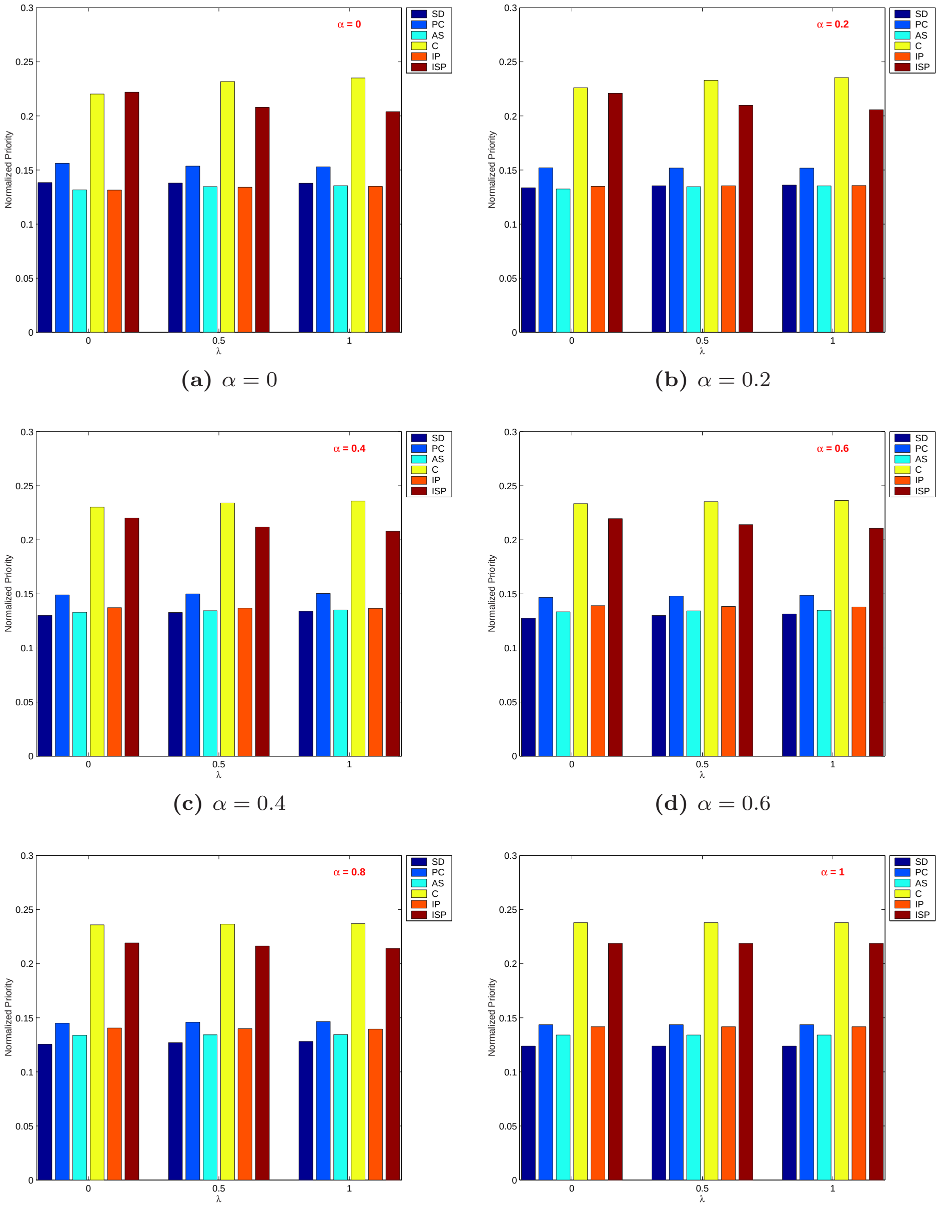


(b) $\lambda = 0.5$



(c) $\lambda = 1$

Figure C.3: *Priority Analysis of Microencapsulation Alternatives by varying* $\alpha-$ *values for the 2nd Decision Maker (DM2)*



(a) $\alpha = 0$

(b) $\alpha = 0.2$

(c) $\alpha = 0.4$

(d) $\alpha = 0.6$

(e) $\alpha = 0.8$

(f) $\alpha = 1$

Figure C.4: *Priority Analysis of Microencapsulation Alternatives by varying* $\lambda-$ *values for the 2nd Decision Maker (DM2)*

Appendix D

Fuzzy Number Operations

D.1 Operations on Fuzzy Numbers

The extension principle can be used to extend the four standard arithmetic operators: addition, subtraction, multiplication, and division to be used in operations using fuzzy numbers.

Definition D.1.1 (Operations on Triangular Fuzzy Numbers)**.** *Let $\tilde{A}$ and $\tilde{B}$ be two fuzzy numbers parameterized by the triplets (a_1, a_2, a_3) and (b_1, b_2, b_3), respectively; then the operations on triangular fuzzy numbers are expressed as*

(1) Addition operation $\tilde{A} \oplus \tilde{B}$:

$$(a_1, a_2, a_3) \oplus (b_1, b_2, b_3) = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$$

(2) Subtraction operation $\tilde{A} \ominus \tilde{B}$:

$$(a_1, a_2, a_3) \ominus (b_1, b_2, b_3) = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$$

(3) Multiplication operation $\tilde{A} \otimes \tilde{B}$:

$$(a_1, a_2, a_3) \otimes (b_1, b_2, b_3) = (a_1 \times b_1, a_2 \times b_2, a_3 \times b_3)$$

(4) Scalar Multiplication operation $\mathrm{k} \otimes \tilde{A}$:

$$\mathrm{k} \otimes (a_1, a_2, a_3) = \begin{cases} (\mathrm{k}a_1, \mathrm{k}a_2, \mathrm{k}a_3), & \forall \mathrm{k} > 0,\ \mathrm{k} \in \mathbb{R} \\ (\mathrm{k}a_3, \mathrm{k}a_2, \mathrm{k}a_1), & \forall \mathrm{k} < 0,\ \mathrm{k} \in \mathbb{R} \end{cases}$$

(5) Inverse operation $\tilde{A}^{-1}$:

$$(a_1, a_2, a_3)^{-1} = \left(\frac{1}{a_3}, \frac{1}{a_2}, \frac{1}{a_1}\right)$$

(6) Division operation $\tilde{A} \oslash \tilde{B}$:

$$(a_1, a_2, a_3) \oslash (b_1, b_2, b_3) = \left(\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1}\right)$$

where $\oplus$, $\ominus$, $\otimes$, and $\oslash$ represent fuzzy number addition, subtraction, multiplication and division, respectively.

Definition D.1.2 (Operations on Interval Fuzzy Numbers)**.** *Let $\tilde{A}_\alpha$ and $\tilde{B}_\alpha$ be two interval fuzzy numbers defined by $[a_L^{(\alpha)}, a_R^{(\alpha)}]$ and $[b_L^{(\alpha)}, b_R^{(\alpha)}]$, respectively; then the operations on interval fuzzy numbers are expressed as*

(1) Addition operation $\tilde{A}_\alpha \oplus \tilde{B}_\alpha$:

$$[a_L^{(\alpha)}, a_R^{(\alpha)}] \oplus [b_L^{(\alpha)}, b_R^{(\alpha)}] = [a_L^{(\alpha)} + b_L^{(\alpha)}, a_R^{(\alpha)} + b_R^{(\alpha)}]$$

(2) Subtraction operation $\tilde{A}_\alpha \ominus \tilde{B}_\alpha$:

$$[a_L^{(\alpha)}, a_R^{(\alpha)}] \ominus [b_L^{(\alpha)}, b_R^{(\alpha)}] = [a_L^{(\alpha)} - b_R^{(\alpha)}, a_R^{(\alpha)} - b_L^{(\alpha)}]$$

(3) Multiplication operation $\tilde{A}_\alpha \otimes \tilde{B}_\alpha$:

$$[a_L^{(\alpha)}, a_R^{(\alpha)}] \otimes [b_L^{(\alpha)}, b_R^{(\alpha)}] = [a_L^{(\alpha)} \times b_L^{(\alpha)}, a_R^{(\alpha)} \times b_R^{(\alpha)}]$$

(4) Scalar Multiplication operation $\mathrm{k} \otimes \tilde{A}_\alpha$:

$$\mathrm{k} \otimes [a_L^{(\alpha)}, a_R^{(\alpha)}] = \begin{cases} [\mathrm{k}a_L^{(\alpha)}, \mathrm{k}a_R^{(\alpha)}], & \forall \mathrm{k} > 0,\ \mathrm{k} \in \mathbb{R} \\ [\mathrm{k}a_R^{(\alpha)}, \mathrm{k}a_L^{(\alpha)}], & \forall \mathrm{k} < 0,\ \mathrm{k} \in \mathbb{R} \end{cases}$$

(5) Inverse operation $\tilde{A}_\alpha^{-1}$:

$$[a_L^{(\alpha)}, a_R^{(\alpha)}]^{-1} = \left[\frac{1}{a_R^{(\alpha)}}, \frac{1}{a_L^{(\alpha)}}\right]$$

(6) Division operation $\tilde{A}_\alpha \oslash \tilde{B}_\alpha$:

$$[a_L^{(\alpha)}, a_R^{(\alpha)}] \oslash [b_L^{(\alpha)}, b_R^{(\alpha)}] = \left[\frac{a_L^{(\alpha)}}{b_R^{(\alpha)}}, \frac{a_R^{(\alpha)}}{b_L^{(\alpha)}}\right]$$

where $\oplus$, $\ominus$, $\otimes$, and $\oslash$ represent fuzzy number addition, subtraction, multiplication and division, respectively.

D.2 Centroid Index For Triangular Fuzzy Number

Triangular fuzzy number membership function is defined as:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & \text{if } x < a_1 \\ \dfrac{x-a_1}{a_2-a_1} & \text{if } a_1 \le x \le a_2 \\ \dfrac{a_3-x}{a_3-a_2} & \text{if } a_2 \le x \le a_3 \\ 0 & \text{if } x > a_3 \end{cases} \tag{D.1}$$

where a_2 is the most possible value of fuzzy number $\tilde{A}$, and a_1 and a_3 are the lower and upper bounds, respectively.

Centroid method is defined as:

$$DF(\tilde{A}) = \frac{\int\limits_{supp(\tilde{A})} x\mu_{\tilde{A}}(x)dx}{\int\limits_{supp(\tilde{A})} \mu_{\tilde{A}}(x)dx} \tag{D.2}$$

The nominator of Eq. (D.2) has

$$\begin{aligned} \int\limits_{supp(\tilde{A})} x\mu_{\tilde{A}}(x)dx &= \int\limits_{a_1}^{a_2} x\frac{x-a_1}{a_2-a_1}dx + \int\limits_{a_2}^{a_3} x\frac{a_3-x}{a_3-a_2}dx \\ &= \frac{1}{a_2-a_1}\left[\frac{1}{3}x^3 - \frac{1}{2}a_1x^2\right]\Bigg|_{a_1}^{a_2} + \frac{1}{a_3-a_2}\left[\frac{1}{2}a_3x^2 - \frac{1}{3}x^3\right]\Bigg|_{a_2}^{a_3} \\ &= \frac{1}{a_2-a_1}\left[\left(\frac{1}{3}a_2^3 - \frac{1}{2}a_1a_2^2\right) - \left(\frac{1}{3}a_1^3 - \frac{1}{2}a_1^3\right)\right] \\ &\quad + \frac{1}{a_3-a_2}\left[\left(\frac{1}{2}a_3^3 - \frac{1}{3}a_3^3\right) - \left(\frac{1}{2}a_3a_2^2 - \frac{1}{3}a_2^3\right)\right] \\ &= \frac{1}{6}\left(a_3-a_1\right)\left(a_1+a_2+a_3\right) \end{aligned} \tag{D.3}$$

The denominator of Eq. (D.2) has

$$\begin{aligned}
\int\limits_{supp(\tilde{A})} \mu_{\tilde{A}}(x)dx &= \int\limits_{a_1}^{a_2} \frac{x-a_1}{a_2-a_1}dx + \int\limits_{a_2}^{a_3} \frac{a_3-x}{a_3-a_2}dx \\
&= \frac{1}{a_2-a_1}\left[\frac{1}{2}x^2 - a_1 x\right]\Bigg|_{a_1}^{a_2} + \frac{1}{a_3-a_2}\left[a_3 x - \frac{1}{2}x^2\right]\Bigg|_{a_2}^{a_3} \\
&= \frac{1}{a_2-a_1}\left[\left(\frac{1}{2}a_2^2 - a_1 a_2\right) - \left(\frac{1}{2}a_1^2 - a_1^2\right)\right] \\
&\quad + \frac{1}{a_3-a_2}\left[\left(a_3^2 - \frac{1}{2}a_3^2\right) - \left(a_3 a_2 - \frac{1}{2}a_2^2\right)\right] \\
&= \frac{1}{2}(a_2-a_1) + \frac{1}{2}(a_3-a_2) \\
&= \frac{1}{2}(a_3-a_1) \qquad \text{(D.4)}
\end{aligned}$$

Substituting Eqs. (D.3) and (D.4) for Eq. (D.2) , we have

$$\begin{aligned}
DF(\tilde{A}) &= \frac{\int\limits_{\sup(\tilde{A})} x\mu_{\tilde{A}}(x)dx}{\int\limits_{\sup(\tilde{A})} \mu_{\tilde{A}}(x)dx} \\
&= \frac{\frac{1}{6}(a_3-a_1)(a_1+a_2+a_3)}{\frac{1}{2}(a_3-a_1)} \\
&= \frac{1}{3}(a_1+a_2+a_3) \qquad \text{(D.5)}
\end{aligned}$$

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