Transfer operators and Zeta Functions for Spin Chains

Abstract: We study one-dimensional one-sided matrix subshifts endowed with a isotropic two-body interaction via the transfer operator method, i.e. we look for a linear operator, called the transfer operator, such that certain asymptotic properties of the partition functions can be expressed in terms of the spectrum of the operator. This approach goes back to E. Ising, H. Kramers and G. Wannier, and D. Ruelle. In works of D. Mayer, K. Viswanathan, B. Moritz, and J. Hilgert there are several examples of interactions known for which a so called dynamical trace formula holds, i.e. there exists a trace class operator, the Ruelle-Mayer transfer operator, such that the partition functions can be expressed in terms of the traces of the powers of the transfer operator. Motivated by these results we ask which is the class of interactions for which a dynamical trace formula holds. We analyse the known examples of Ruelle-Mayer transfer operators and determine what they have in common. We introduce a family of Ising type interactions which contains all the known Ising interactions with finite-range, superexponentially, exponentially, or polynomial-exponentially decaying distance function. We give some new examples, for instance Ruelle-Mayer transfer operators for M-vector models and Potts models, and new distance functions. We can formulate a general frame work for the construction of the Ruelle-Mayer transfer operator associated to interactions belonging to this class and prove a dynamical trace formula. For this class of interactions we investigate Ruelle's dynamical zeta function and show its meromorphic continuation to the entire complex plane as a quotient of regularised Fredholm determinants of the transfer operator. For polynomial-exponentially decaying interactions and also for finite range interactions we explicitly compute the conjugate of the Ruelle-Mayer transfer operator under the Bargmann transform and study its properties. Inspired by preceeding works of M. Kac, M. Gutzwiller, D. Mayer, and J. Hilgert we call this integral operator acting on the space of square-integrable functions on the n-dimensional euclidean space a Kac-Gutzwiller transfer operator.