Abstract

Given an $m \times n$-matrix A and a polyhedron $Q \subseteq \mathbb{R}^m$, we want to find a vector $b \in Q$ such that the system $Ax \leqslant b$ has no integral solution. We show that if n is fixed, there is a polynomial-time algorithm solving this problem.

Then, we consider integer programs in *standard form*,

$$\min\left\{cx : Ax = b,\, x \in \mathbb{Z}_+^n\right\},$$

and establish several bounds on the number of non-zero components in an optimum solution. It turns out that there is always an optimum solution with the number of non-zero entries bounded by a *polynomial* in the number of constraints and the maximum size of an entry in A.

For the *cutting stock prolem*, the columns of the matrix A are exactly the integral non-negative solutions of a knapsack inequality $ax \leqslant 1$; hence, their number is exponential in the input size. However, our bounds imply that an optimum solution of polynomial size exists, and therefore, the cutting stock problem belongs to NP.