Abstract

Given a (measurable) space X and a measurable map $f : X \to X$, the transfer operator for f is a linear map $P_f : \mathcal{M}(X) \to \mathcal{M}(X)$ defined by the prescription $P_f\mu(A) = \mu(f^{-1}(A))$ for all measurable sets $A \subset X$ and all $\mu \in \mathcal{M}(X)$. Here $\mathcal{M}(X)$ is a suitably chosen linear space of measures on X.

In this thesis, transfer operators are considered for a particular class of maps: those that describe a coupled cell system. A coupled cell system is a dynamical system admissible to a coupled cell network. This concept is used both in applications and in theoretical works to model dynamical systems that are built up from smaller parts (called cells), that influence each other in the temporal evolution of their internal state. Thus the state space X of a coupled cell system is the cartesian product of the state spaces X_c of the individual cells, and each component map f_c may depend on c and several other cells.

The structure of the network underlying a coupled cell system can be described in an algebraic way by means of its so-called symmetry groupoid. Admissibility of a map f on the network can then be expressed as equivariance of f with respect to a certain action of this groupoid. In the case of dynamical systems with "classical" symmetries, expressed by equivariance with respect to group actions, the results of linear representation theory have implications for a system that allow far-reaching characterisations of its dynamics. In particular, its transfer operator can be shown to possess invariant subspaces due to symmetry.

In view of these results, the aim of this thesis is to describe the structural implications that equivariance with respect to the action of the symmetry groupoid has for the transfer operator of a coupled cell system. How can the structural properties of the map f be translated into properties of the transfer operator P_f? To answer this question, this thesis shows that it is possible to decompose $\mathcal{M}(X)$ into the direct sum of subspaces $U_{\mathcal{D}}$ parametrized by the set of subsets $\mathcal{D}$ of the set of cells in such a way that the coupling structure of the network is reflected in the corresponding block decomposition of the transfer operator.

Furthermore, to analyse the structure of P_f due to symmetry properties of the network, a family $\Gamma_{\mathcal{D}}$ of symmetry groups for subsets of the set of cells is associated to the symmetry groupoid. These groups make it possible to use results from representation theory to determine further structural properties of the transfer operator.

The direct application of the theoretical results in a numerical scheme for the approximation of the transfer operator is prevented by the fact that standard methods for this task rely on the usage of bases of $\mathcal{M}(X)$ that are derived from discretizations of the state space X in a specific manner. These bases appear to be incompatible with the decomposition of $\mathcal{M}(X)$. The reasons underlying this problem are explained in detail, and an alternative method for the efficient approximation of P_f is sketched.