

Capability and complexity of computations with integer Division

In this thesis computations with an operation set including integer division DIV in the unit cost model are considered. $CC_n(S)$, the families of languages $L \subset \mathbb{Z}^n$ that can be recognized by S -computation trees S -CTs with $S \subset \{+, -, *, c, DIV, DIV_c\}$, are characterized. They are completely characterized for $S = \{+, -, DIV_c\}$ and $S = \{+, -, *, DIV_c\}$ and for $S = \{+, -, DIV\}$ and $S = \{+, -, *, DIV\}$ only in the case $n=1$ and partially in the case of $n>1$ for $S = \{+, -, DIV\}$ and $S = \{+, -, *, DIV\}$. The relations among the classes $CC_n(S)$ are completely determined and lower bounds for such S -Cts are proven.

The first nontrivial lower bound for $(\{+, -, *, DIV\}, \mathbb{Q})$ -CTs leaves a doubly logarithmic gap between the lower bounds for polynomial evaluation over $\{+, -, *\}$ and $\{+, -, *, DIV\}$. This leads to the question if a polynomial p can be evaluated in $o(\deg p)$ over $\{+, -, *, DIV\}$. This is possible with N. Bshouty's algorithm in constant time restricted to a finite input set or over all integers with bitwise conjunction as an additional operation.

These results are used for acceleration of matrix multiplication, of determinant computation over $\{+, -, *, DIV\}$ and of matrix powering over $\{+, -, *, DIV, gcd\}$.