

Capability and complexity of computations with integer Division

In this thesis computations with an operation set including integer division DIV in the unit cost model are considered. $CC_n(S)$, the families of languages $L \subset \mathbb{Z}^n$ that can be recognized by S-computation trees S-CTs with $S \subset \{+, -, *, \div, \text{DIV}, \text{DIV}_0\}$, are characterized. They are completely characterized for $S = \{+, -, \text{DIV}_0\}$ and $S = \{+, -, *, \text{DIV}_0\}$ and for $S = \{+, -, \text{DIV}\}$ and $S = \{+, -, *, \text{DIV}\}$ only in the case $n=1$ and partially in the case of $n>1$ for $S = \{+, -, \text{DIV}\}$ und $S = \{+, -, *, \text{DIV}\}$. The relations among the classes $CC_n(S)$ are completely determined and lower bounds for such S-CTs are proven.

The first nontrivial lower bound for $(\{+, -, *, \text{DIV}\}, \mathbb{Q})$ -CTs leaves a doubly logarithmic gap between the lower bounds for polynomial evaluation over $\{+, -, *\}$ and $\{+, -, *, \text{DIV}\}$. This leads to the question if a polynomial p can be evaluated in $o(\deg p)$ over $\{+, -, *, \text{DIV}\}$. This is possible with N. Bshouty's algorithm in constant time restricted to a finite input set or over all integers with bitwise conjunction as an additional operation.

These results are used for acceleration of matrix multiplication, of determinant computation over $\{+, -, *, \text{DIV}\}$ and of matrix powering over $\{+, -, *, \text{DIV}, \text{gcd}\}$.