Abstract

We present a geometric construction of cross sections for the geodesic flow on a huge class of good orbifolds with the hyperbolic plane as covering manifold. Further we realize first steps towards a generalization to other locally symmetric good orbifolds of rank one.

In the first part of this thesis we consider an arbitrary rank one Riemannian symmetric space D of noncompact type and a group Γ of isometries of D. Under weak requirements on Γ we prove the existence of isometric fundamental regions.

In the second part we specialize to orbifolds of the form $\Gamma\backslash H$ where H is the hyperbolic plane and Γ is a geometrically finite subgroup of $\mathrm{PSL}(2,\mathbb{R})$. Further we require that ∞ is a cuspidal point of Γ and that Γ satisfies an additional weak (and easy to check) condition concerning the structure of the set of isometric spheres of Γ. We construct cross sections for the geodesic flow on $\Gamma\backslash H$ for which the associated discrete dynamical systems are conjugate to discrete dynamical systems on the finite part $\mathbb{R}$ of the geodesic boundary of H. The isometric fundamental regions from the first part play a crucial rôle in this construction. The boundary discrete dynamical systems are of continued fraction type. In turn, the transfer operators produced from them have a particularly simple structure.

For each of these cross sections there is a natural labeling in terms of certain elements of Γ. The arising coding sequences of unit tangent vectors belonging to the cross section can be reconstructed from the endpoints of the associated geodesics. In some situations, the arising symbolic dynamics has a generating function for its future part. In this case, the generating function is also of continued fraction type.